

# COMPUTERS AND AUTOMATION

CYBERNETICS • ROBOTS • AUTOMATIC CONTROL

The Automation of Bank Check Processing  
. . . R. Hunt Brown

Linear Programming and Computers  
. . . Chandler Davis

Justifying the use of an Automatic Computer  
. . . Net Chapin

Charting an Automatic Data Processing System  
. . . Harry Eisenpress, James L.  
McPherson, and Julius Shiskin

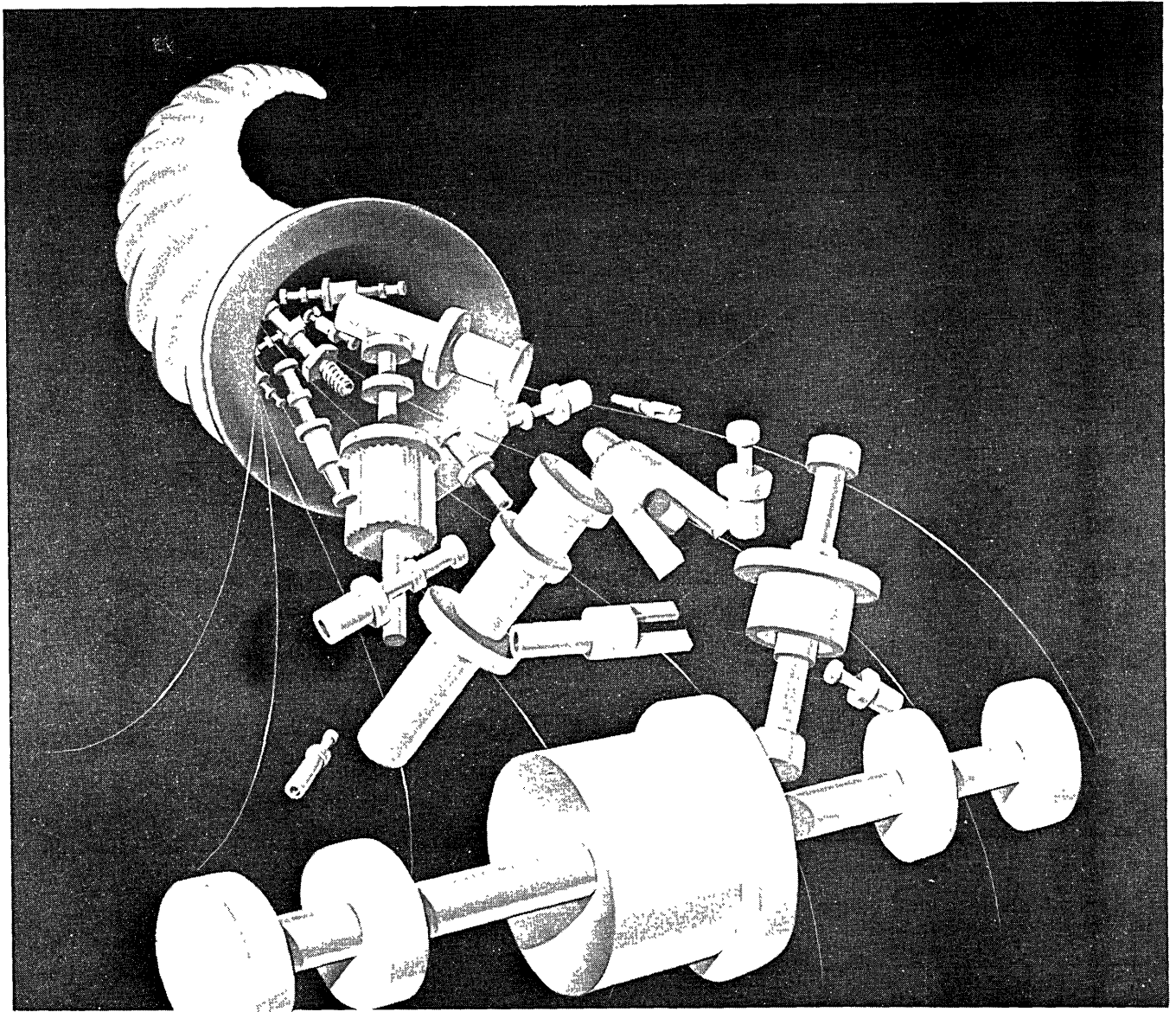
A Rotating Reading Head for Magnetic Tape and Wire  
. . . National Bureau of Standards

Some Curiosities of Binary Arithmetic Useful in  
Testing Binary Computers  
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Vol. 4  
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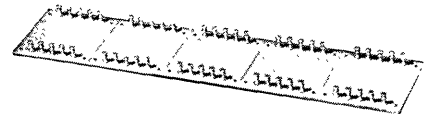
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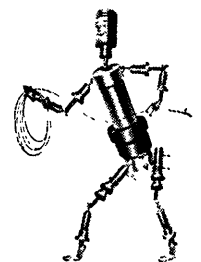
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# COMPUTERS AND AUTOMATION

## CYBERNETICS • ROBOTS • AUTOMATIC CONTROL

Vol. 4, No. 8

August, 1955

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# THE EDITOR'S NOTES

Forum

## PAYMENT FOR PAPERS

Beginning with this issue, "Computers and Automation" will make payment for accepted papers at half the rate of payment for articles -- (provided that the author wishes to be paid). This works out at \$5 to \$20 for 1000 to 4000 words. This token payment will help express our appreciation to those authors who prepare good papers for us to publish.

## PATENTS

Hans Schroeder, who provided the information we have published for many months about patents relating to computers, has entered the armed forces of the United States to fulfill his service requirements. If any reader of ours is able and willing to survey the Patent Gazette and report pertinent patents to us in the same style, we shall be glad to make an arrangement with him.

## CORRECTION

In the story "The Book Reviewer" by Rose Orente in the July issue, the name of the psychoanalyst whose theories are referred to should be corrected to read "Edmund Bergler" instead of "Edward Bergler", since he is a real person and not a fictitious character.

## EMPLOYMENT REFERENCE SERVICE

Every few days some man or some organization telephones us in regard to employment of computer people. The man says "Where are good places for me to seek employment in the computer field?" The organization says "Do you know of anybody who could fill this kind of a position....?"

"Computers and Automation" will be glad to act as a clearing house in this way, to the extent of our knowledge; and we would suggest that classified advertising, as published from time to time in these pages, would be helpful. But the more that people tell us what they are looking for, the more we may be able to help.

## INFORMATION AND REPORTERS

This brings us to the fact that a lot of the reference information we publish is dependent on what other people tell us. We are very grateful for the data that enables us to help computer people find out what they want to know.

(continued on page 32)

ELECTRONIC COMPUTER CONFERENCE,  
DARMSTADT, GERMANY,  
OCT. 25-27, 1955

A. Walther  
Darmstadt, Germany

Please let me announce that according to the enclosed German circular there will be held an international conference on

"Elektronische Rechenmaschinen und  
Informationsverarbeitung"  
(Electronic digital computers and in-  
formation processing)

at the

Institut für Praktische Mathematik (IPM)  
Technische Hochschule, Darmstadt, Germany

on October 25-27, 1955, sponsored by GAMM (Gesellschaft für Angewandte Mathematik und Mechanik) and NTG-VDE (Nachrichtentechnische Gesellschaft im Verband Deutscher Elektrotechniker).

Would you please publish this announcement in your journal "Computers and Automation"? Information on registration, which is open to all, may be obtained by writing:

Prof. Dr. A. Walther  
Institut für Praktische Mathematik (IPM)  
Technische Hochschule  
Darmstadt, Germany

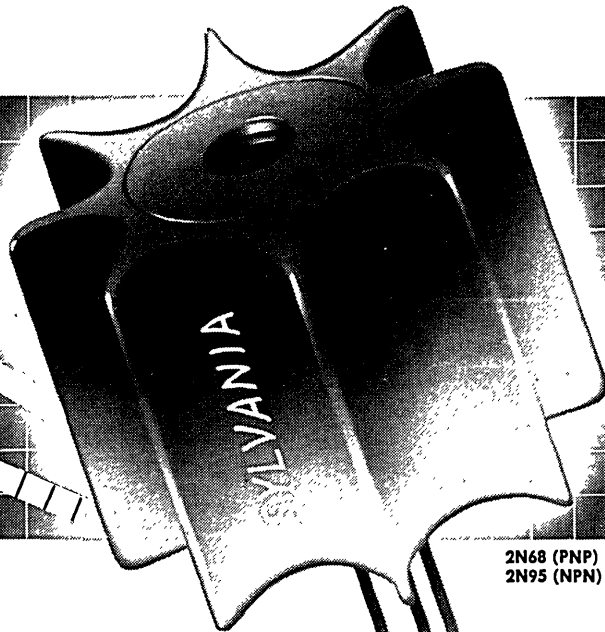
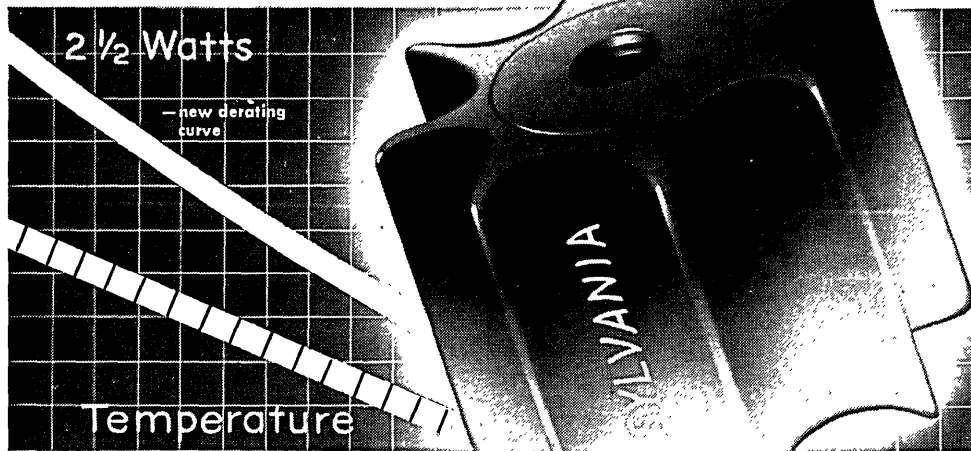
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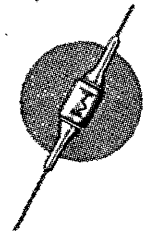
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# THE AUTOMATION OF BANK CHECK PROCESSING

R. Hunt Brown  
Automation Consultants  
New York, N. Y.

An article by the author on the savings account problem was published in the July issue of "Computers and Automation". It referred to a report by the American Bankers Association entitled "Automation of Bank Operating Procedure", in which both the checking and savings account problems are dealt with in some detail. It is clear that new equipment and systems will need to be developed to solve these problems. Some excerpts from the ABA report are given below, to show the seriousness of the situation:

"A study of the check collection system in 1954 by the Joint Committee on the Check Collection System of the American Bankers Association, the Association of Reserve City Bankers, and the Federal Reserve System revealed these interesting facts.

"The number of checking accounts in this country increased from 27 million to 47 million between 1939 and 1952. In the same period, the number of checks written annually grew from an estimated 3½ billion to nearly 8 billion. About 7 billion of the checks written in 1952 were drawn on the more than 14,000 commercial banks throughout the country; of the balance, 450 million were checks drawn on Federal Reserve Banks.

"... should the trend of the past thirteen years continue, 14 billion checks would be written in 1960; and by 1970, check volume would be 22 billion. Although these levels may not be reached, an upward trend in volume is expected to continue.

"On an average day in 1952, about 29 million checks were written, and approximately the same number were deposited in or cashed at the average bank on that day; one in five was drawn on the bank and was charged to the drawer's account. The remaining four were drawn on other banks and were collected through the check collection facilities provided by the banking system of the country.

"The average check deposited in or cashed at a bank passed through 2 1/3 banks in the process of collection from the drawee bank, and about 2 1/3 business days elapsed between the date it was deposited or cashed and the

date it was presented for payment. Checks in process of collection through the banking system on an average day in 1952 numbered about 69 million, of which 29 million were presented to drawee banks for payment on that day. Of the remaining 40 million, 15 million were payable at banks in the same cities as the collecting banks, 24 million were par items payable at out-of-town banks, and 1 million were nonpar items."

Here is a description, then, of a "phenomenal increase in the volume of activity in the handling of checking accounts", with resulting "excessive manual work in processing checks", and "increase in personnel costs — with attendant possibility of errors."

## Nonstandardization

One factor which complicates the issue is that there is no standardization of check sizes and formats. Despite efforts made in this direction for many years, little has been accomplished. Any proposed solution therefore, which contemplates changing check sizes and formats must be carefully considered. Another complication is the mutilation of checks in handling by the public. Also, a large investment in check writing machines has been made by banks and depositors, and the machines will not be jettisoned overnight. Any major sudden attempt to change the banking habits of the customers might meet disastrous defeat.

## Prequalification

The check problem may be divided into two categories, that of the travellers' checks and that of commercial and personal checks. The former category does not offer as much difficulty as the latter, as there is room for "prequalification", or the entering of certain information on checks in a form that is qualified for later automatic processing. There are three required items of information that should be in acceptable form for automatic processing, drawer bank, drawer identification, and amount:

1. The drawer bank's identification may

BANK CHECK PROCESSING

- be placed on checks when printed.
- 2. The drawer's identification and check serial number may be imprinted on the check by the bank before issuance.
- 3. The amount on the check may be imprinted at the time of writing the check in a form which can be automatically read.

Since travellers' checks are issued by only a few banks, standardization among them as to coding if desirable, should not be too difficult. The issuance of travellers' checks to contain the above three essential pieces of information, and any other fixed data such as date and place of issuance, should be easy of solution. The problem of identifying the person presenting the check and ascertaining that there is a sufficient balance to cover the check does not arise. In fact, three machines have already been developed to read travellers' checks automatically, when they are returned to the head office of the issuing bank. These machines are in the prototype or pilot plant stage.

Prototype Machines

One of these machines was made by Intelligent Machines Research Corp. of Arlington, Va., for the Bank of America. This reads serial numbers and amounts from the existing form of check in regular use, checks which have no special features to aid character sensing. The speed of reading is 100 travellers' checks per minute, and the output is the same rate of IBM punched cards.

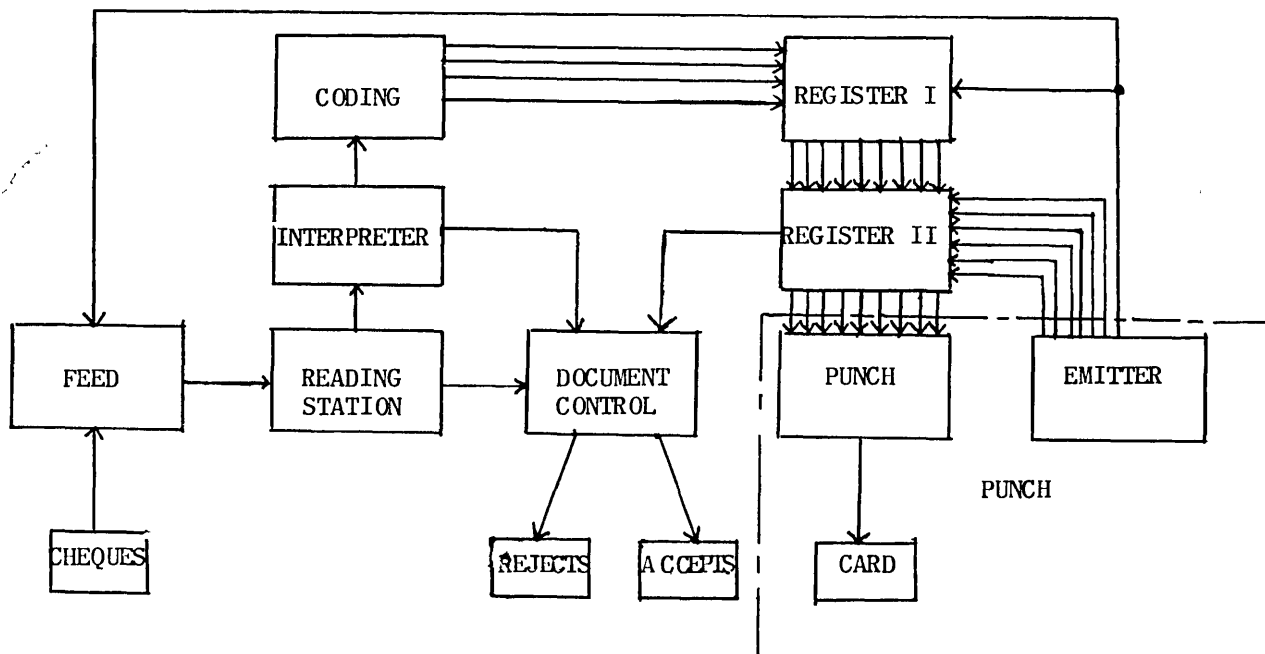
The scheme used is rather straightforward. The feed is under control of the output punch and feeds a document to the reading station on demand. As each character is identified, it is coded into a modified binary code and stored, one digit at a time, in the first register. When the punch has punched the previous card, an emitter pulse causes information to be transferred to the second register en bloc.

The punch used is of the parallel type, punching all 80 columns at once, one level at a time.

This principle could be expanded to produce a punched or magnetic tape for computer input.

The second type of travellers' check reader was made by the Control Instrument Co. of Brooklyn, N. Y., a subsidiary of Burroughs Corp. It was placed in trial operation at the First National City Bank of New York on June 15, 1955. The equipment consists of a cable-connected reader and punch, each unit approximately the size of an office desk. By means of electronics and photo cells, the machine scans the printed serial number and denomination, then punches 80 column cards at the rate of 120 per minute. The machine does the work about 8 times as fast as the previous manual method of key punching; errors are reduced by ninety percent.

The third type of machine is described in a booklet issued in 1954 by the First National Bank of Chicago, written by Mr. C.M. Weaver, Assistant Vice President. This system is one



which again produces IBM punched cards as output. Experiments have been made with travellers' checks, money orders, and punched card checks as input. A Code-Printer is used to prequalify travellers' checks by a dual printing operation; one type of ink is used for coding serial number and denomination; another type, black ink, is used for writing the name of the issuing bank. The code consists of placing one or two dots in any of four vertical positions of a column to represent each digit from zero to nine. The scanner consists of a feeding and sensing unit, which reads codes, converts, and checks itself in several ways. It makes certain that it is feeding only one check; it sorts out checks with no-coding or with illogical coding; and it counts the number of checks of each denomination. Its output is fed to a Type 519 IBM Automatic Punch, which punches at the rate of 100 cards per minute. There is a reduction of 75% in the time required for manual processing.

First National of Chicago has two other ingenious machines, one the Chex-Ray for locating staples, pins, and clips which would interfere with feeding checks to the sensor, and another the Chex-Press for restoration to proper flatness of checks mutilated by folding. (The Cummins Business Machines Division of Cummins-Chicago Corp. also has a conditioner to reclaim mutilated cards.)

#### Qualification

These notable pioneering efforts may solve the check problem in part, but only in part. They only serve where prequalification, or imprinting before issuance, can be used, producing coded dots which can be read automatically. Now let us consider "qualification", which means putting this information on the check at some later stage because it is not feasible to do so at the start.

Various ideas have been suggested for producing data in the form necessary for automatic reading. One is placing the data on the face of the check. Another is attaching to the check a punched tape or card. A third is enclosing the check in an envelope which is coded for scanning; etc. (1) (2)

A bank could issue check forms to its individual and commercial depositors who have their account number printed on each check. This problem does not arise in the case of the savings bank, where each passbook already bears an account number. Even this simple step would require a change in depositors' banking habits. However painful such a change may be, it seems it must take place before checking accounts can be automated. For example, some firms prefer to print their own checks, and it may

be difficult to persuade them to include an account number. Also, to have large firms adopt special check writers so that the amount and other data as well can be coded, will mean special inducements.

The problem of check size is less serious. The coding can be attached and anchored to one side or one end of the check. The size of the check should be within reasonable limits. Because of the close tolerance of punched card checks, standardization within such narrow limits would seem to be precluded for use by all.

Suppose that we now have the account number on a majority of checks, in a position for automatic scanning. Missing information could be coded on the check manually, and the same automatic reading process employed as for travellers' checks above. In cases where a blank is used when the account number does not appear on the check, it could be ascertained from a signature file. Such a scheme if workable might assist, but it would not solve the automation problem.

#### Large Rapid Memory

Manufacturers are beginning to announce large rapid or random access memory systems; storing up to 5 million characters per unit with an access time of one second maximum. Let us consider again the large rapid access memory proposed for savings banks in the previous article, storing billions of alphanumeric characters at a cost of less than \$0.0001 per decimal digit. This is an essential link still missing in bank automation. When this becomes available, the record of each checking account could be kept in the form of permanent magnetic impressions. At the end of each month, a statement could be printed automatically. Let us follow a check on its way through the banking channels under such a scheme.

Suppose that you present your own personal check with account number to a bank teller at any branch of your bank, for cashing it. From reference to a signature file he could identify you. If your check did not have your account number, the teller could find it in the signature book and handwrite it on your check. This signature file would be like the one used by the telephone companies for obtaining information, such as the Flexoprint of Remington Rand, updated by the Xerography process.

The teller would then key your account number into an inquiry set like the one described in the previous article. Your current balance would flash on a set of lamp indica-



tors, which could display any number from 0 to 9 in each lamp nest. If there were checks deposited which had not yet cleared, or any other restrictions, warning lights would indicate this condition. The amount of the uncleared checks could be ascertained by pushing the corresponding button. If you were making a deposit, the amount would be similarly keyed in by the teller, to bring your account into immediate balance.

The convenience to you as depositor is apparent. You do not need to go to your own branch to cash a check, and your balance is always current.

After cashing, your check would go to the file section for photostats or microfilm and later be returned with your monthly statement, as now happens.

Consider the case where you cash a check outside your bank, or mail it to a creditor in payment of a bill. This check would follow the present course through clearing, for presentation at your branch and debit to your account. When the bundle of such checks is received at the bank from the clearing house, it would go to any machine operator. If the check is prequalified, it could be put through an automatic reader, which would debit the proper account. If it is not fully marked with the necessary coding, the missing data could be entered on the check by the operator for subsequent machine reading.

Instead of punching cards, as with the traveller's check scanners above, computing circuits associated with the automatic memory would add or subtract to the balance in the account. A printer would be included in the equipment, to print out the account number and amount of check in a chronological sequence of processing. This would give a record of entries, in case data had to be reconstructed. At the end of the day, a record of entries serially by account numbers could be printed, for further record purposes. These records could be stored in a safe place, for any necessary reconstruction.

The inquiry sets at the different branches would be connected to the memory and computer at a central location, over telephone tie lines. In case of line failure, the regular telephone tie lines now used between branches could be automatically or manually cut in.

At the end of the month, statements could be prepared automatically in the same form as now used, each individual check amount being shown. The machine memory would then be erased or cleared of the previous month's transactions, and made ready for the next month. There is some question as to whether a depositor needs

a detailed monthly statement, as long as he has his cancelled checks and current balance. If this is acceptable, the amount of memory required could be substantially reduced.

This seems to be about as close to the automatic checking account as can now be reasonably expected. The teller is necessary for translation from human to machine language. The degree of prequalifying checks will gradually increase; it will never reach a full 100%. It is hardly conceivable that every depositor will eventually have a check writer which will put coding on his checks which can be read automatically. Therefore, operators or tellers will be needed at the bank to put the necessary codes on some of the checks. But a great quantity of other bank clerical work in connection with checking accounts could thus be eliminated.

One manufacturer is willing to build a prototype of such a system, which would be the first step in a bank automation development program. The system would have to be engineered by a single supplier, to avoid divided responsibility for proper functioning and servicing. Clearly, some bank would have to launch the program in order to get it started; but at present only a few of the larger banks are actively pursuing the solution of the checking account problem. Any first installation would have to be designed and built to special customer requirements, since it is doubtful that any manufacturer can yet produce a system applicable to all banks regardless of size and other variations.

The possibilities of solving the banking problems by electronic data processing machinery are tremendous. But it still is a pioneering proposition which most banks are unwilling to take on individually. Most of them are waiting for the manufacturers to produce something they can use; and that will take some time.

(1) International Telemeter Corp. of Los Angeles is now building a new type of check handling system. The first experimental equipment will be delivered to a large bank for testing in central proof-sorting operations. The initial step in processing a check is to affix a small punched tab to its edge. The data on the check is punched into the tab in very much the same way that data is punched into the cards used by conventional business machines. The check is then carried through the system with the tab affixed to it. Using the data on the tab, the sorting machine sorts checks and performs simultaneous proof operations on them. Since the machine holds the checks by the tab at every step, the shape and size of the check is not important. The checks are not damaged

(continued on page 16)

# LINEAR PROGRAMMING AND COMPUTERS

## Part II

Chandler Davis  
New York, N. Y.

(Part I was published in the July, 1955, issue of "Computers and Automation")

The job at hand is to solve the gin-sour-and-Collins problem of Part I by the simplex method.

It is embarrassing that the problem for which we haul out this medium-heavy artillery should be the simple gin-sour-and-Collins problem, which I solved handily before, you remember, by just fooling around with it a little. The point about the simplex method, of course, is that it systematizes that fooling-around into something that we know always solves all linear programming problems. So why don't I illustrate it on a somewhat bigger problem? I could do that, with not much more arithmetic. But I won't, because I want you to have a nice geometrical picture of the simplex method's operation to follow, for at least once in your life.

### Restatement of the Problem

The problem, you will remember, was this:

The first problem: Suppose we have a plant equipped to produce bottled gin sour or Tom Collins in any amounts. The only materials whose supply is limited are gin, sugar and bottles. Enough sugar is available daily to allow production of 1200 bottles of gin sour, but a bottle of Collins uses twice as much sugar as a bottle of sour. Enough gin is available daily to fill 225 bottles; the gin sour is 3/8 gin, the Collins 1/8 gin. Only 800 bottles are available daily. The profit per bottle of Collins is \$1.00; per bottle of sour, \$2.00. To maximize total daily profit, how many bottles of each should be produced daily?

$x_1$  is hundreds of bottles of Collins per day.  $x_2$  is hundreds of bottles of gin sour per day.  $x_3$ ,  $x_4$ , and  $x_5$  are artificial variables invented to throw the problem into standard form, as explained in Part I.

I won't rework the problem from the beginning, but I will express it in the standard form. Faithful readers were to have put it in this form as an exercise at the end of Part I. Here is what they should have got.

The unknowns  $x_1$ ,  $x_2$ ,  $x_3$ ,  $x_4$ , and  $x_5$  are required to satisfy the equations

$$\begin{aligned} 2x_1 + x_2 + x_3 &= 12 \\ (5.1) \quad x_1 + 3x_2 + x_4 &= 18 \\ x_1 + x_2 + x_5 &= 8. \end{aligned}$$

These variables are also required all to be non-negative:

$$(5.2) \quad x_i \geq 0 \quad \text{for} \quad i=1,2,3,4,5.$$

The problem is to find that set of values, satisfying (5.1) and (5.2), which makes the following linear function come out as big as possible:

$$(5.3) \quad f(x) = x_1 + 2x_2.$$

Phrased this way, the problem fits the standard form given in Part I. The equations (5.1) can be written as in (4.1):

$$\begin{aligned} (4.1') \quad a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 + a_{35}x_5 &= b_3 \end{aligned}$$

with  $a_{11}=2, a_{12}=1, a_{13}=1, a_{14}=a_{15}=0; b_1=12;$

$a_{21}=1, a_{22}=3, a_{23}=0, a_{24}=1, a_{25}=0; b_2=18; a_{31}=$

$a_{32}=1, a_{33}=a_{34}=0, a_{35}=1; b_3=8.$

The only inequalities required are those that keep our variables non-negative (see (5.2) above). And finally, of course, the function to be maximized is of the form (4.3):

$$(4.3') \quad f(x) = c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 + c_5x_5,$$

with  $c_1=1, c_2=2, c_3=c_4=c_5=0.$

Let me remind you also of the geometric procedure that's involved in the solution. The points in 5-space which satisfy (5.1) and (5.2) make up a convex figure X. The optimal points of X which we are looking for must include at least one extreme point. To find the optimal extreme points we will start with any old extreme point; then we will want a way -- a systematic way -- to compute from an extreme point that we know, a new adjacent extreme point. Each time we find that the new adjacent extreme point is "higher" -- that is, gives larger  $f(x)$  -- we will quickly abandon the old one for it. However if we have an extreme point such that no adjacent extreme point is "higher" we know we are at the top of the mountain, because X is a convex mountain and so can't have any companion peaks.

### The First Extreme Point

Starting with any old extreme point might not be something we can do as offhand as we can say it. How shall we find an extreme point of X? How shall we find any point of X?

Here my method is going to be a little less general than elsewhere, but still very often applicable.

In equations (5.1) you'll notice that there's a variable which comes into the first equation and none of the others, namely  $x_3$ . Similarly  $x_4$

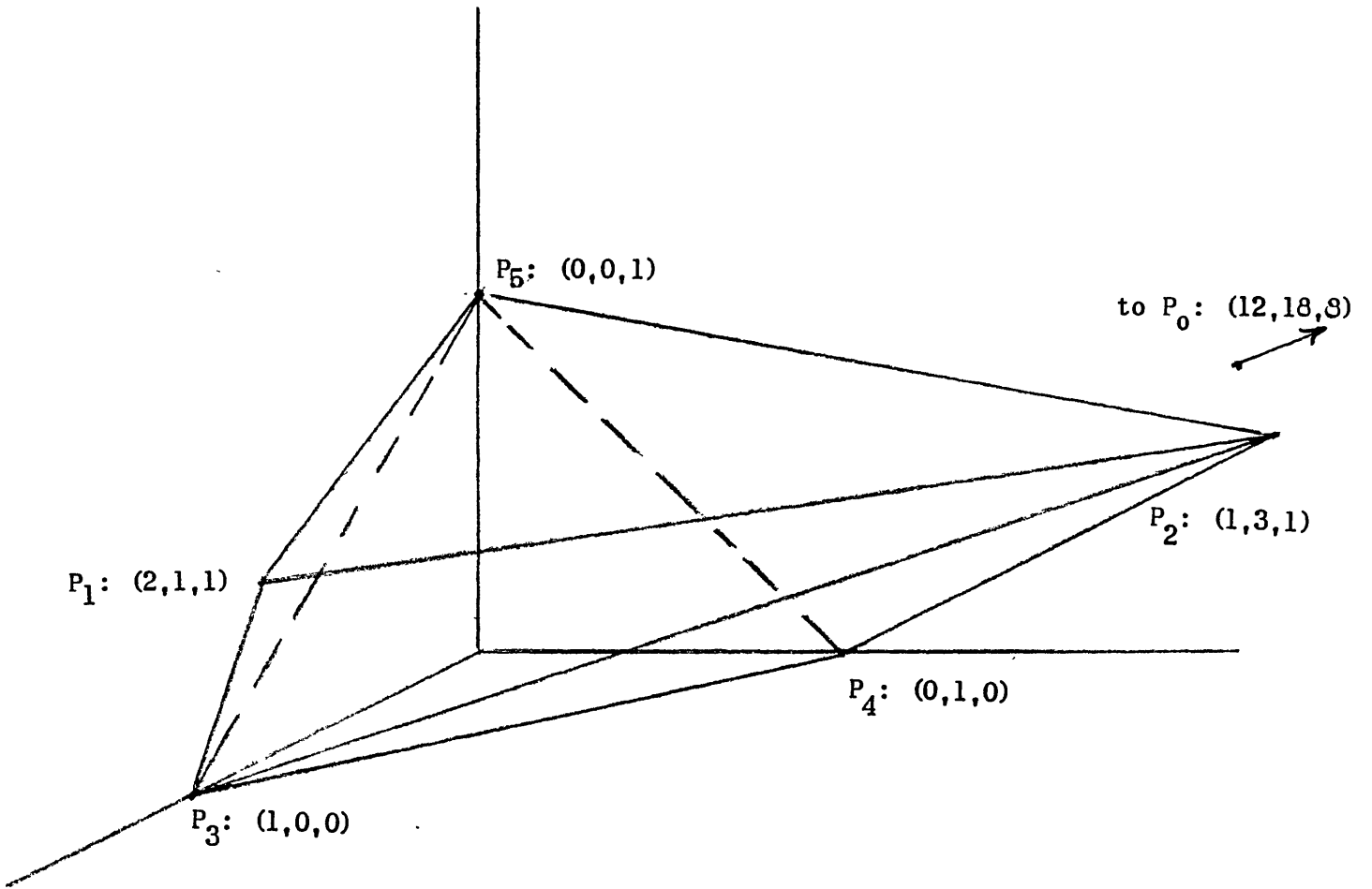


Figure 2

in the second equation and  $x_5$  in the third. This circumstance makes it a snap to write down a set of non-negative values for  $x_1, x_2, x_3, x_4, x_5$  which satisfy the equations.

Exercise: Do so.

But the circumstance which facilitated this step looks like a very fortuitous one. Why was I promising that what I was doing could be done on many harder problems?

Well, remember where those convenient variables  $x_3, x_4,$  and  $x_5$  came from. They were the ones that were made up to convert complicated inequalities (1.1), (1.2), and (1.3) into equations. I'll ask you to look back at those inequalities in Part I. Where (1.1) said that  $2x_1 + x_2$  had to be less than or equal to 12, the first equation of (5.1) says that  $2x_1 + x_2$  has to have something added to it (namely  $x_3$ ) to give 12. Since  $x_3$  can't be negative, this is equivalent. And  $x_4$  and  $x_5$  came in just the same way.

The generalization is obvious, isn't it? If all the equations required when the problem is in standard form came from inequalities, then there must be one variable per equation that was made up artificially and appears only in that equation. Then we set all the "genuine" variables equal to zero, give the artificial variables whatever values appear on the right sides of the respective equations, and we're in. Thus the answer to the exercise above was  $x_1 = x_2 = 0, x_3 = 12, x_4 = 18, x_5 = 8$ ; or briefly,  $x = (0, 0, 12, 18, 8)$ .

This procedure always gives an extreme point, too!

Exercise (fairly hard): Prove this in the numerical problem we're doing out.

Granted, there will be some linear programs which do not have this neat property. For example, the transportation problem I set up in Part I does not have this property. (We could get it to by replacing each of its equations by two inequalities, then converting each of those inequalities to an equation by an artificial new variable; but this bit of razzle-dazzle would increase the number of equations in the problem by seven, and the number of variables by the same amount, so you may justly be reluctant about it.) I will drop this particular subject here, and summarize the first step in the solution this way: Somehow, we find one extreme point of X.

#### Requirements Space

Now how do we travel to an adjacent extreme point of X?

Look again at the equations (5.1). Better yet, if you don't mind being a little less numerical, look at the more general equations (4.1'). And forget, just for the moment, all about X; because now, to answer our questions about it, we're going to need an astonishingly different geometrical interpretation of the same equations.

Remember from the discussion of convexity in Part I (or from school, if you're that well educated) that if we have several linear equations, all with the same coefficients, we can condense them into a single vector equation. That was how (3.1) became (3.2). We can do that with (4.1'). You may think when you look at (4.1') that we can't because the coefficients aren't the same in all the equations. Ah, but we will use  $x_1, x_2, x_3, x_4, x_5$ , the variables in the problem, as our coefficients! So we write

$$(6.1) \quad x_1P_1 + x_2P_2 + x_3P_3 + x_4P_4 + x_5P_5 = P_0.$$

Here  $P_1, P_2,$  etc. are the names of strings of numbers, that is, points in a new space.

The coordinates of  $P_1$  are the numbers that multiply the variable  $x_1$  in the equations:  $P_1 = (a_{11}, a_{21}, a_{31})$ . The coordinates of  $P_2$  are the numbers that multiply  $x_2$ ; and so forth. The coordinates of  $P_0$  are the numbers on the right:  $P_0 = (b_1, b_2, b_3)$

These numbers are all known, so that the points we are now considering are known points. In our numerical problem, the coordinates of the points can be read off in this way from (5.1). You can find the answers on Fig. 2, where the points are even plotted (except  $P_0$ , which would be off the page).

If we were doing a bigger problem -- any problem big enough to be difficult -- there would be more than three equations (4.1); therefore more than three coordinates for each point  $P_1, P_2, \dots$ ; therefore no nice 3-dimensional figure like Fig. 2. The geometric ideas which Fig. 2 lets you see are just what you will use on harder problems, although there, without a figure, you will be flying blind.

Excuse me for repeating myself: The space diagrammed in Fig. 2 in Part II, of which  $P_1$  and the rest are points, is a new space, not the one we discussed before. Call the previous one solution space, because the solution we are looking for is one of its points. In our example, solution space is a 5-space. The newly introduced space is called requirements space, because some of its points come from the equations stating the requirements of the problem. In our example, requirements space is a 3-space.

It could perfectly well happen that a problem would give a solution space and a requirements space of the same number of dimensions. The y should still be kept distinct in your thinking. The numbers  $x_1, x_2,$  etc. are the coordinates of a (variable) point in solution space; but in requirements space they are not a point, they are just a string of numbers appearing in a curious relation (6.1) between points.

#### Positive Combinations

What does the relation (6.1) say? It is a linear equation giving  $P_0$  in terms of the other points; and the coefficients are all non-negative.



This is almost the same as an idea we encountered in Part I in another connection. If we knew that  $x_1 + x_2 + x_3 + x_4 + x_5 = 1$ , then (6.1) would say that  $P_0$  was a "convex combination" of the rest. That idea has a very simple intuitive translation: it means  $P_0$  is "between" the others, that is, it is in the convex set which they determine.

But here we don't know that much, because the sum  $x_1 + x_2 + x_3 + x_4 + x_5$  can be any of various positive numbers. With the numbers as we have them in our particular problem, it's not even possible to choose the  $x_i$ ; so their sum comes out 1. We need a new definition. We say that the relation (6.1), with all  $x_i$  non-negative but with their sum unspecified, expresses point  $P_0$  as a positive combination of points  $P_1, P_2, P_3, P_4, P_5$ .

It's reasonable to hope that the geometric interpretation will not be much trickier, and it's not.

To say that  $P_0$  is a positive combination of the other points means that some point of the line from the origin out through  $P_0$  is a convex combination of them. That is, the line from  $(0,0,0,0,0)$  out through  $P_0$  passes between the other points. Look back at Fig. 2. The convex set determined by  $P_1, P_2, P_3, P_4, P_5$  does not contain  $P_0$ , but it does contain some points of the line  $OP_0$ .

Why is this interpretation valid? A point belongs to the line  $OP_0$  on the same side of 0 as  $P_0$  provided its coordinates are got from those of  $P_0$  by multiplying through by a positive factor. (For example, since  $P_0$  is  $(12, 18, 8)$ ,  $\frac{1}{4}P_0$  is  $(3, 4.5, 2)$ . This is the point at the base of the arrow on Fig. 2.) Now if we have an equation giving  $P_0$  as a positive combination of the other points, we can multiply it through by the appropriate positive number and get an equation giving a multiple of  $P_0$  as a convex combination of the others. And the same argument works backward.

**Exercises:** 1. Show that any point in the 3-space whose coordinates are all positive, is a positive combination of  $P_3, P_4, P_5$ .

2. Show this is consistent with the geometric interpretation of positive combinations.

3. In particular, express  $P_0$  as a positive combination of  $P_3, P_4, P_5$ .

4. Show that  $P_3$  is not a positive combination of  $P_1, P_4, P_5$ .

#### Tie-up with Solution Space

Any set of non-negative  $x_i$  which gives  $P_0$  as a positive combination of  $P_1, P_2, P_3, P_4, P_5$  satisfies (6.1), that is it satisfies (4.1); that is, it gives the coordinates of a point of  $X$  — and here we are back in solution space! Any point of  $X$ , similarly, can be reinterpreted in requirements space as a way of expressing  $P_0$  as a positive combination of  $P_1, P_2, P_3, P_4, P_5$ .

This translation back and forth between the two spaces will almost certainly be out of reach

of your intuition for a while, but I'll pursue it a little further and suggest that you try to get used to it.

There are lots of points of  $X$ , so there must be lots of different ways to express  $P_0$  as a positive combination of the others. Now you remember there were certain points of  $X$  which we were specially interested in, namely the extreme points. There is a way of spotting an extreme point ( $x_1, x_2, x_3, x_4, x_5$ ) after it has been translated into requirements space. This important device will be the justification for introducing requirements space at all.

#### Spotting Extreme Points

If  $P_0$  is given as a positive combination of  $P_1, P_2, P_3, P_4, P_5$ , with some of the coefficients zero (say,  $x_1$  and  $x_4$ ), then it would be just as correct to say that the equation gives  $P_0$  as a positive combination of the remaining  $P_i$  (say,  $P_2, P_3$ , and  $P_5$ ).

**Example:** The expression of  $P_0$  in terms of  $P_3, P_4$ , and  $P_5$  in Exercise 3 just above can just as well be called an expression of  $P_0$  as a positive combination of  $P_1, P_2, P_3, P_4, P_5$ : (7.1.1):  $P_0 = 0x_1 + 0x_2 + 12P_3 + 18P_4 + 8P_5$ . This gives a point of  $X$ , namely  $(0, 0, 12, 18, 8)$ , the one we found before.

When the list of points that actually enter into the expression can't have any points struck off it, then the coefficients give a point of  $X$  that is extreme. I'm afraid the why of this would take a little too much explaining, but I'll illustrate to make the meaning clear.

**Examples:** 1. I will use the criterion to show  $(0, 0, 12, 18, 8)$  is an extreme point. It corresponds to an expression of  $P_0$  as a positive combination of  $P_3, P_4$ , and  $P_5$ , namely (7.1.1). There is no expression of  $P_0$  as a positive combination of  $P_4$  and  $P_5$  (without  $P_3$ ); or of  $P_3$  and  $P_4$  (without  $P_5$ ). Check this. This gives the conclusion that  $(0, 0, 12, 18, 8)$  is extreme. Let's look at this in requirements space, i.e. Fig. 2. The line  $OP_0$  does have a point in common with the triangle  $P_3P_4P_5$ ; but it does not have a point in common with any of the line-segments  $P_4P_5, P_3P_5, P_3P_4$ .

2. I pull out of the hat the following point of solution space:  $(0, 4, 8, 6, 4)$ . It is a point of  $X$ , for these numbers satisfy (5.1). It is not an extreme point of  $X$ , as I show by the requirements-space criterion: It corresponds to an expression for  $P_0$  using  $P_2, P_3, P_4, P_5$  (reflecting the fact that  $OP_0$  goes through the tetrahedron  $P_2P_3P_4P_5$  in Fig. 2). But one of these points is dispensable, for we saw above that we could get along with just  $P_3, P_4, P_5$  (reflecting the fact that  $OP_0$  goes through the face  $P_3P_4P_5$  of the tetrahedron). We will see below, actually, that there is an option here as to which  $P_i$  is dispensed with. We could get along instead with  $P_2, P_3, P_5$  (reflecting the fact that  $OP_0$  also goes through the triangle  $P_2P_3P_5$ , another face of the tetrahedron).

Exercise: Show directly from the definition that  $(0,4,8,6,4)$  is not an extreme point of  $X$ .

### Finding a New Extreme Point

Having one extreme point of  $X$ , to jump to another adjacent one, this is the task in solution space.

The translation, I ask you to believe, is this: having one expression for  $P_0$  with no dispensable points, to find another such expression which brings in as few new points as possible.

All right, we start with the easily-found extreme point  $(0,0,12,18,8)$ , that is, we start with (7.1.1). Let's choose at random a new  $P_i$ ; to bring into the act:  $P_2$ , say. We want to find systematically all ways  $P_0$  can be expressed as a positive combination of  $P_2, P_3, P_4, P_5$ .

This means altering the right-hand side of (7.1.1) by adding a positive multiple of  $P_2$  while the left side stays unchanged. Something equal to this multiple of  $P_2$  must be subtracted from the right-hand side, then, if the equation is to be preserved. And it must be something involving only  $P_3, P_4$ , and  $P_5$ , because we're bringing in new points one at a time.

Is there an expression for  $P_2$  in terms of  $P_3, P_4, P_5$ ? Certainly, and because  $P_3, P_4, P_5$  have such simple coordinates it's easy to find. I'll write down the analogous expression for  $P_1$  at the same time:

$$(7.1.2) \quad P_1 = 2P_3 + P_4 + P_5$$

$$(7.1.3) \quad P_2 = P_3 + 3P_4 + P_5$$

Now we add  $yP_2$  to the right-hand side of (7.1.1), where  $y$  is any positive number; and to keep the equation true we subtract  $y$  times the right-hand side of (7.1.3). It's okay to regroup the terms of the resulting vector equation. (Because it's okay to do it for each of the numerical equations which the vector equation stands for. If this is unfamiliar to you, try to justify it to yourself before going on.) The result is:

$$(7.1.4) \quad P_0 = yP_2 + (12 - y)P_3 + (18 - 3y)P_4 + (8 - y)P_5.$$

This vector equation is valid no matter what  $y$  is; but it's of little interest to us unless all its coefficients are non-negative. If  $y$  is just a little bigger than 0, we're all right. As  $y$  increases, though, we run into trouble -- just as soon as any of the coefficients becomes 0. Which of them gets there first? The coefficient of  $P_3$  reaches zero when  $y = 12$ ; of  $P_4$  when  $y = 6$ ; of  $P_5$  when  $y = 8$ . So  $y = 6$  is as high as we can go: if we went any higher, the coefficient of  $P_4$  would be negative.

If we go just that high, and substitute  $y = 6$  in (7.1.4),  $P_4$  will be eliminated from the equation and yet no negative coefficients will appear:

$$(7.2.1) \quad P_0 = 6P_2 + 6P_3 + 2P_5$$

I think you can see that this procedure can be applied routinely and repetitively in all problems. (The only thing that can stop it is if too few  $P_i$  are used in expressing  $P_0$ . This happens if, when a new one is added, more than one drops out. This is called degeneracy; I ignore it for now.)

The procedure always gives extreme points, what's more! I will say why it does in the present instance. (7.1.4) gives, for appropriate values of  $y$ , all expressions of  $P_0$  as a positive combination of  $P_2, P_3, P_4, P_5$ ; of these, (7.2.1) is the only one that dispenses with  $P_4$ ; so there is no such expression which dispenses with  $P_4$  and one of the other three as well; so, by the criterion above,  $(0,6,6,0,2)$  is an extreme point. This reasoning generalizes.

### Is This Progress?

We don't want just any old adjacent extreme point of  $X$ ; we want a "higher" one: one giving a larger value to  $f(x)$ . Let's check that that's what we got, recalling (4.3') or (5.3). Before:

$$f((0,0,12,18,8)) = 0c_1 + 0c_2 + 12c_3 + 18c_4 + 8c_5 = 0(1) + 0(2) + 12(0) + 18(0) + 8(0) = 0$$

After:

$$f((0,6,6,0,2)) = 0(1) + 6(2) + 6(0) + 0(0) + 2(0) = 12$$

This is a distinct improvement. (The first extreme point corresponded to the unenterprising policy of producing nothing of anything; the one we have now found corresponds to the policy of producing as much gin sour as possible, with daily profit \$1200.)

Perhaps you're content with the method for climbing up  $X$  which has now been sketched: starting from one extreme point, step to any adjacent one at random, then see if  $f(x)$  has been improved.

The method is surely adequate, but it can be refined. It is not really necessary to go all the way to the new extreme point in order to find out whether it will be an improvement. Instead we could have found out, as soon as we contemplated introducing  $P_2$  on the right-hand side of (7.1.1), whether this would take us in the desired direction on  $X$ . The sophisticated reader will be able to figure out by considering (7.1.4) how to make this test, but to save space I won't give details.

### Solution of the Problem

Let us press onward from point  $(0,6,6,0,2)$ . There are two new points that could conceivably be introduced on the right-hand side of (7.2.1), namely  $P_1$  and  $P_4$ . Now introducing  $P_4$  would give us expressions for  $P_0$  using  $P_2, P_3, P_4, P_5$ . We would have an equation which, though it would look different from (7.1.4), would be equivalent to it, so it could not lead us to any point

where we haven't already been. No point in that. Better try introducing  $P_1$ .

As before, a multiple of the new point will be added to our equation's right side and an equal expression will be subtracted. Hence we need an expression for  $P_1$  in terms of  $P_2, P_3, P_5$ .

$$(7.2.2) \quad P_4 = \frac{1}{3} P_2 - \frac{1}{3} P_3 - \frac{1}{3} P_5,$$

and from this and (7.1.2),

$$(7.2.3) \quad P_1 = -\frac{1}{3} P_2 + \frac{5}{3} P_3 + \frac{2}{3} P_5.$$

Using (7.2.3) to alter (7.2.1),

$$(7.2.4) \quad P_0 = zP_1 + (6 - \frac{1}{3}z)P_2 + (6 - \frac{5}{3}z)P_3 + (2 - \frac{2}{3}z)P_5.$$

How high can  $z$  get before causing negative coefficients here? The coefficient of  $P_2$  vanishes when  $z = 18$ ; and that of  $P_3$ , when  $z = \frac{18}{5}$ ; that of  $P_5$ , when  $z = 3$ . The last is the one that stops us, substitute  $z = 3$  in (7.2.4):

$$(7.3.1) \quad P_0 = 3P_1 + 5P_2 + P_3.$$

The new extreme point does give an improved value of  $f(x)$ :

$$f(3, 5, 1, 0, 0) = 3(1) + 5(2) + 1(0) + 0(0) = 13.$$

Now to finish the solution we should verify that no new extreme point adjacent to  $(3, 5, 1, 0, 0)$  gives higher  $f(x)$ . Exercise: Do it. You should find that the only point you have to test is  $(4, 4, 0, 2, 0)$ , and that this makes  $f(x)$  equal only 12.

We have arrived at the same conclusion that the rough-and-ready rapid solution did in Part I: The first activity is to be carried on at the level 5: bottle gin sour at 500 bottles per day. The fact that  $x_3 = 1$  does not represent doing anything because  $x_5$  was one of our thrown-in variables;  $x_3 = 1$  means only that not all the sugar is being used up. Finally,  $f(x) = 13$  means the daily profit is \$1300.

#### Computation

Here is the promised summary of the arithmetic involved in a linear programming problem.

Assume the problem is in standard form. There are  $n$  variables, all required to be non-negative (4.2), and the only other requirements are in linear equations (4.1). A linear function (4.3) is to be maximized.

The following are needed repeatedly throughout the solution:

1. The coefficients of the equations. These are  $mn + m$  numbers, each associated with a row and a column index.

2. The coefficients of the function to be maximized. These are  $n$  numbers, singly indexed.

The other information need not be preserved throughout the solution. It pertains to the particular extreme point of  $X$  which the solution has

arrived at, and can be forgotten once a better extreme point has been found. (An exception might be made to this; see below.)

At the stage where an extreme point of  $X$  has just been found, we have the following information:

1. The coordinates of the extreme point itself. These can be recorded as a string of  $n$  numbers,  $n-m$  of which happen to be zero; or as  $m$  non-zero numbers associated with  $m$  chosen column indices (the  $m$  "included" indices). I will assume the second alternative.

2. The  $n-m$  expressions, in terms of the "included"  $P_i$ , of the other  $P_j$ . (In our numerical problem, (7.1.2-3), or (7.2.2-3).) These involve  $nm-m^2$  numbers, each identified by two indices.

Next one index  $i$  among those not "included" must be picked. This can be done at random. Once it is picked, the expression for the corresponding  $P_i$  is consulted. We compute —

1. Which of the included  $P_j$  is now to be dropped. This involves (see the numerical problem solution) finding  $m$  quotients and selecting the smallest positive one.

2. The new extreme point of  $X$ . The quotient from the previous step is multiplied by each of  $m$  numbers; the results are added to the coordinates of the old extreme point. It's not quite this simple, but call it  $m$  multiplications followed by  $m$  additions. Also we must record the change in what  $i$  are to be considered "included."

3. Whether the new extreme point is an improvement. This is just a question of drawing the appropriate  $m$  from among the  $n$  coefficients of  $f(x)$ , multiplying them by the old extreme point's  $m$  coordinates, adding these  $m$  numbers, and comparing the result with another number.

As hinted before, by a dodge, the last of these three steps can be as good as done without the first two. It, therefore, will be repeated as often as necessary to find a direction in which  $f(x)$  can be increased; but the other steps will be done once, with reference to this progressive direction.

Finally we compute the expressions, in terms of the new set of included  $P_i$ , for the other  $n-m$  of them. We use the  $n-m$  equations we started with. One equation is solved:  $m$  divisions and a little rearranging. In the others we substitute: essentially,  $m$  multiplications and  $m$  additions.

All these steps can be followed in our numerical problem above.

#### How Many Repetitions?

This description doesn't yet say how long the computations must be, because it doesn't say how often the procedures must be repeated.

This is something we don't know at the outset, naturally. If the first extreme point we found happened to be optimal, well — but let's not dream! Often the way the first extreme point is found guarantees it will be about as far as possible from optimal.

The total number of extreme points is always between  $n-m-1$  and  $\binom{n}{m}$ . The latter number, the number of combinations of  $n$  things  $m$  at a time, is large; but the number of extreme points is generally well below it. Of course if there are many extreme points, comparatively few of them will have to be found, as I remarked in Part I. If  $X$  is a roughly spherical diamond, our path to its optimal corner is a zigzag line across its face; translate this image to  $n$  dimensions.

A less important but easier question is this: At each extreme point, how many tries at adjacent extreme points will it take before one is reached which is an improvement? It could be as low as 1 or as high as  $n-m$ . On the average, up is as likely as down, so the average should be about 2. But toward the end this rises, as it grows harder and eventually impossible to improve the position.

If one has a record of which  $P_i$  have been dropped in earlier repetitions of the procedure, those can be tested last, since they are not such likely candidates for inclusion.

Short-cuts

It would be nice if the simplex method could be modified so that instead of inching over the surface of  $X$  we could plunge through the center of the set. But without taking up such radical changes, let's look at some available short-cuts.

1. Often a lot of the coefficients of the problem will be zero, for some reason which the statement of the problem makes evident. If there is a simple pattern to these zeros, it can be taken advantage of to simplify the plan of the computation.

2. Sometimes the problem can be replaced by a simpler one. Leaving the standard form for a moment, think of  $X$  as having many facets, each determined by an inequality. If fewer inequalities on the same variables were required, some of the slices would be repaired, and  $X$  would be replaced by a larger convex set. Suppose we keep the same expression for  $f(x)$ , and maximize it on the new set. If we have an optimal extreme point of the new set, it doesn't have to be in  $X$  at all, in which case our scheme has fallen through. But if it is in  $X$ , it is not hard to see it is an optimal extreme point of  $X$ .

Degeneracy

I have written as if, each time a new  $P_i$  was adjoined, exactly one was dropped. When this fails, that is when two or more of the  $P_i$  drop out in a single step, it is called degeneracy. On  $X$ , this means that more than  $n-m$  edges meet at a single extreme point.

It is not hard to prove, under seemingly reasonable assumptions, that the probability of having any degeneracy at all is zero. (General idea: When too many edges meet at a point, too many faces must be meeting there too. In that

case, moving one of them ever so little, in almost any direction, will increase the number of extreme points, but reduce the excess faces per extreme point.) This is deceptive; degeneracy does happen in practice. It can be handled. I won't discuss the tools that handle it, except to say that they do not involve much extra computation.

Summary

I have tried to tell what kind of problem can be set up as a linear program; what geometrical ideas motivate and "explain" the standard simplex method of solution; and what kind and volume of arithmetic is involved in working out a problem by this method.

- END -

BANK CHECK PROCESSING  
(continued from page 9)

by the process, and after a series of automatic sorting operations, may be sent through the proper channels to the banks on which they were drawn, for return to depositors.

(2) Another, different approach is represented by the recently announced automatic bank book-keeping system being developed jointly by the Burroughs, Todd, and Addressograph-Multigraph companies. In that system, an invisible code is added to checks as a byproduct of other necessary operations, and then the checks themselves will automatically govern the operation of accounting machines.

- END -

BULK SUBSCRIPTION RATES

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Table 1 -- Bulk Subscription Rates  
(United States)

Number of Simultaneous Subscriptions	Rate for Each Subscription, and Resulting Saving to Subscriber:	
	One Year	Two Year
10 or more	\$3.00, 33%	\$5.40, 40%
5 to 9	3.33, 25	6.00, 33
4	3.75, 17	7.00, 22
3	4.00, 11	7.50, 17
2	4.25, 5	8.00, 11

For Canada, add 50 cents for each year; outside of the United States and Canada, add \$1.00 for each year.



# Justifying the Use of an Automatic Computer

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Chicago, Ill.

"Should my company use an automatic computer?"

The answer to this question can properly be given only by weighing the benefits to be received against the costs to be incurred in using an automatic computer. Essentially, there are two types of these benefits and costs to be considered, those for which dollar and cents figures can be derived, and those for which such figures cannot be derived, which we shall call "irreducible".

Both the irreducible and the dollar benefits and costs are reflected in the figures found in various accounting reports and schedules, but the manner in which they appear is different. The dollar benefits and costs typically appear directly in the form of clearly identifiable dollar aggregates, as for example, the labor cost involved in inspection work. The irreducible benefits and costs are very rarely clearly identifiable in amount; they usually appear only indirectly. For example, they might appear as sales being larger (a benefit) or smaller (a cost) than they otherwise would have been, or for example, as investment in inventory being smaller (a benefit) or larger (a cost) than it otherwise would have been.

There are two major irreducible benefits from using an automatic computer. One is that the use of an automatic computer can give a company a strong competitive edge. It can make a company's service to its customers faster and more satisfactory than those of its competitors not using automatic computers. The second is that the use of an automatic computer can provide management with more accurate, more comprehensive, and more timely data on which to base decisions. This furnishing of data can apply not only to marketing operations, but also to production operations and to financial operations. Even though these benefits are usually difficult to assess in dollar terms, they can be very important, for they can mean the difference between successful growth amid competition, and failure.

There are also two major irreducible costs from using an automatic computer. One is that the use of an automatic computer makes the success of management partly dependent upon the operation of an electronic machine, and in some

lines of business, this could be a disadvantage. The second is that the use of an automatic computer typically results in a considerable rearrangement of internal company operations. If inadequately controlled, to make these rearrangements may result in a state of chaos verging on paralysis. Even though these two costs are difficult to assess in dollar figures, they can be substantial. However, since skilled and thoughtful management can make the irreducible costs small while making the irreducible benefits large, it appears that in most cases, the balance of irreducible benefits and costs is in favor of using an automatic computer. However, since all of these irreducible benefits and costs are very difficult to measure in dollar terms, to assert simply that they typically have a generally favorable balance does not clearly justify the use of an automatic computer.

To justify in dollar terms the use of an automatic computer requires a consideration of the other category of benefits and costs. These dollar benefits and dollar costs have three important characteristics. First, they can be controlled by management action. For example, the number of men on the payroll, the number of machines being operated, the number of reports being prepared and circulated, are things which management can set at any desired level. Second, these benefits and costs can be measured. In business, what can be controlled can also be measured, at least to some degree for otherwise management could not control. Third, the benefits nearly always take the form of cost savings. This means that, considering this category of dollar benefits and dollar costs, a comparison can be used, because the cost saving (the "benefit") is always a difference between two sets of costs. And an analytical comparison puts into sharp focus the differences between the alternatives of using vs. not using an automatic computer, for the advantages one alternative has over another must lie in their differences, not in their sameness.

The problem of justifying the use of an automatic computer on the basis of dollar costs therefore appears to become one of tabulating the costs under the present non-computer systems, and then tabulating the cost increases

and the cost savings expected to be produced through the use of proposed computer systems. To do this requires defining the specific applications which are to be put on the proposed computers and tracing through the cost changes that will result, for it is only the changes from the present non-computer systems that are of significance in justifying the use of an automatic computer. Thus, if the present cost of doing the applications to be converted is \$100,000 per month, and the proposed computer systems for the applications are expected to result on net balance in a \$10,000 saving per month, then this \$10,000 per month is the crucial figure in justifying the use of computers. The problem therefore appears to have two parts: how to determine or estimate the "saving", and how to draw conclusions from the "saving" to justify the use of an automatic computer.

To estimate the saving, the first step is to determine the cost of the present non-computer systems for the proposed applications for a given period of time. Usually the data available from cost accounting is not sufficiently detailed, so that it becomes necessary to gather data directly. An expedient technique usually is to identify, first, all the forms involved presently in the proposed applications, and then trace the use of these forms thru the company, taking note of all employees and supervisors involved, and all supplies and equipment involved. Written procedures, flow charts, and job descriptions if available, are very helpful in this work but they are no substitute for personal on-the-spot investigation. To be the most useful, the findings from investigation and study should be tabulated to show great detail within the three general categories of labor, supplies, and equipment. Indirect items (for example, maintenance work on adding machines) should not be omitted. However, no provision should usually be made for "burden," or "overhead", because, if it is clear that when an automatic computer is used, the total company expenditure for the components of the burden will change, then the full amount of the change usually is a known cost increase or decrease to be specifically counted in the balance appraising the automatic computer. The cost tabulation is the more useful if it clearly separates out-of-pocket costs (such as equipment rentals) from amortization charges (such as depreciation and insurance on equipment). Also, in the interests of accuracy, an accountant's amortization charges should be edited to improve their realism.<sup>1</sup>

The second step in the estimating of the saving is to find the cost of the proposed computer systems for the proposed applications. To do this requires that the written procedures and flow charts for the proposed systems be studied in comparison with the cost tabulation for the present systems. For each

item in the tabulation, the question must be asked, "How will the cost of this item change as a result of the use of an automatic computer?" If the answer is "up," then the amount of the cost increase must be estimated and recorded. If the answer is "down," then the amount of the saving must be estimated and recorded. If the answer is "no change," then the item is irrelevant to the justification procedure and may be stricken from the tabulation. Then new additional costs from the proposed procedure must be added on,--for example, the costs for programmers, for air conditioning, etc. However, the cost of amortizing the automatic computer (including the conversion loss<sup>2</sup>) should not be added on, even if part of it takes the form of a rental charge.<sup>3</sup> The net result of all these dollar savings and cost increases is the saving (if any) of the proposed computer systems over the present non-computer systems, not allowing for the capital investment involved in the use of an automatic computer.<sup>4</sup>

To interpret the saving obtained above to justify the use of an automatic computer, the saving must be compared to the acquisition cost of the automatic computer including the conversion loss. The crucial question is therefore, "Is the saving at least as large as the amortization charge against the acquisition cost and conversion loss?"

If the answer is "yes," the use of an automatic computer is ordinarily justified; if the answer is "no," or "break-even," then the justification hinges upon the estimates of the irreducible benefits and costs as noted earlier. In any event, however, to make the comparison in the case of a contemplated purchase of an automatic computer, a sound amortization period (service life) for the acquisition cost (including the conversion loss) is normally at least seven years. For, if a shorter period of time is used, it can usually be shown that because of the high obsolescence rate at present of automatic computers, an automatic computer will hardly prove to be a sound financial investment. To make the comparison in the case of a contemplated rental of an automatic computer, the amortization of the conversion loss should be added to the rentals for the same period to obtain the figure to compare with the saving.

This method of justifying the use of an automatic computer omits from specific consideration two important factors: the time value of money, and the present high obsolescence rate of automatic computers. If both of these items are to be specifically included or allowed for in the method, then the MAPI<sup>6</sup> formula should be used.<sup>5</sup> If only the time value of money is to be specifically included, then the uniform annual cost, present worth, or rate

of return formula should be used.

Two features of this justification method presented above deserve comment. First, the method is independent of the size of the company. That is, no part of the justification method directly or indirectly makes the justification a function of the size per se of the company. This means that all sizes of companies might be able to justify the use of an automatic computer. And this means that just as some small companies may not be able to justify the use of an automatic computer, so some large companies may not be able to justify the use of an automatic computer. In large companies, simply because they have larger total costs, it is usually easier to find enough savings to justify the use of an automatic computer, but there appears to be no necessary relationship between the size of a company and the fact whether or not the use of an automatic computer can be justified.

The second feature of this justification method is even more basic. The saving - the basis for justifying the use of an automatic computer in business - arises essentially because of the cost differences between the present non-computer systems on the one hand, and the proposed computer systems on the other. These cost differences arise because a different (and better costwise) way of doing things was devised making use of the proposed computer systems - a different way of combining labor, supplies, and equipment to get the company's work done. Finding better ways of combining labor, supplies, and equipment is often known as industrial engineering when applied to work in the office. Essentially the saving appears therefore only because of good systems analysis work done; this fact correctly suggests that the amount of the saving is dependent upon the quality of the systems analysis work, the nature and the merits of the proposed computer systems. This means that high-quality systems analysis may provide the justification for using an automatic computer even in a small company, but that low quality systems analysis may not be able to provide the justification even in a large company.

In short, the justification for using an automatic computer in business is primarily the amount of the cost saving expected, but since the amount of the cost saving depends on the quality of the systems analysis work, we can expect that the use of an automatic computer can be justified in almost any size of business if sufficiently good systems analysis work is done.

<sup>1</sup>If the MAPI<sup>6</sup> formula is to be used, the amortization charges should be eliminated altogether, and the changes in salvage values substi-

tuted in their place. Interest on the salvage value must also be allowed.

<sup>2</sup>This is the aggregate of all the costs and losses incurred in preparing for and in making the actual change-over from the non-computer systems to the computer systems.

<sup>3</sup>As a matter of conservatism, the unused capacity of the automatic computer should be classed as an irreducible benefit, not as a dollar saving.

<sup>4</sup>In the MAPI formula, assuming the previous note about salvage values has been observed, this net result is the defender's adverse minimum.

<sup>5</sup>In using the MAPI formula, the challenger's adverse minimum must be less than the defender's adverse minimum (see note 4) to justify the use of an automatic computer. The challenger's adverse minimum can be quickly obtained by reference to a MAPI chart given the acquisition cost (including the conversion loss) of the automatic computer, the service life, and an interest rate.

<sup>6</sup>MAPI stands for Machinery and Allied Products Institute. The MAPI formula is an aid to determining whether or not to replace equipment. The major advantage of the MAPI formula is that it makes specific allowance for the increasing costs over the course of time of keeping equipment in use. In use, the MAPI formula requires only the computation of two numbers. One of these figures is the differences in the operating and maintenance costs of the present ("defender") and proposed ("challenger") equipment. The other figure is obtained by reference to a MAPI chart (a graphic solution to the MAPI formula). These two figures ("adverse minimums") are then compared to obtain the signal of whether or not replacement should be made. A good and very easy reading reference on the MAPI formula is the MAPI Replacement Manual published in 1950 by the Machinery and Allied Products Institute.

- END -

THE TALE OF A COW

M. E. Salveson  
New York, N. Y.

Chapter I - Anonymous

Let us suppose that I am the owner of a cow. The purpose of a cow is to produce milk. If you turn the creature into a pasture and allow her to eat grass, she will produce many quarts of lacteal fluid. The process consists of eating grass, transmuting it into milk, removing the milk from the udder. However, you are infected with the system disease. You cease to be interested in milk. You hire inspectors to count the number and measure the height of the blades of grass eaten by your cow.<sup>(1)</sup> You count the number of steps she takes in the pasture. You time the number of occasions upon which she looks over the fence and test the volume of sound produced when she moos. Then you employ a bookkeeper to record these items, and various persons to make out forms for the guidance of the bookkeeper. At the end of the year you have a vast and entertaining volume of information, but you have added so greatly to the cost of your operation by systematizing in this manner that you lose money on every quart of milk you sell. This, of course, is exceedingly modern and in accord with business practice, but you find yourself compelled to sell your cow to pay your overhead.

(1) Note by M. E. Salveson: It is understood a British scientist has indeed developed a device for counting the number of blades of grass a cow eats.

Chapter II - by M. E. Salveson

Since you no longer produce and sell milk, you discharge your cow hands. This leaves you no applied direct labor, but \$10 a month overhead in space rented to store the cow's records. Your bookkeeper tells you that this rental makes your overhead rate infinity (ten divided by zero is infinity), and that you can reduce overhead tremendously - from infinity to zero - if only you get rid of the records. You give them to the farmer who purchased the cow. He does not know that your data are worthless, so he accepts the whole truckload.

He cannot read; so he employs a scientist to help him learn what is in the records. Fortunately, you kept meticulous records; on analyzing them, the farmer's scientist learns that many things simultane-

ously affect how many quarts of transmuted lacteal fluid the cow can produce. The scientist, however, is bitten with a different disease -- a strong psychological tendency toward optimization, compounded with an irrational passion for dispassionate rationality in seeking facts on productivity. In brief, he was looking for the high point on the milk production curve that corresponds to the low (sag) point on the udder.

By suitable analysis the scientist discovers from your data the number of blades of grass per day to feed the cow to get optimum milk production. Of course for the convenience of the cow and the cow hand, the scientist translates this number into the number of pitchforks full. From your data on height of the grass, he learns the height (or maturity) of the grass at which it should be mowed for optimum utilization of the cultivated lands and production of milk. Your data on the number of steps the cow had taken yield to the scientist excellent information on the optimum percent of the time she should be roaming (exercising) in the pasture or quiet in her stall. Your records on where she had looked over the fence most frequently and mooed with the greatest gusto gives the scientist clues as to what grass is most palatable, while also nutritious. This increases her protein intake and butterfat output.

In no time at all the farmer has the cow producing so much milk that even with an expensive, non-transmutive (he did not personally transmute even one drop of the lacteal fluid) scientist on the overhead payroll, the unit cost of milk was greatly reduced. He then asks the scientist to study if he should expand his herd and how rapidly. In that study he also finds the optimum amount to reduce milk prices so as to make profits soar and stay at their maximum.

As the size of the udders, the herd, and the profit grow, the farmer is so impressed with the scientific method that he has the scientist set up a regular scientific business research group to look into all phases of the business and to advise him how to manage it more profitably. This new research leads to more profit-making discoveries that are not at all obvious and quite unexpected even to this experienced and seasoned farmer. The scientist discovers that many managerial problems even of the cow business are so intricate that only by scientific methods can an optimum course of action be found. For example, he finds that mathematical computing methods are necessary to obtain the minimum cost diet (food mix) that still provides required nutrition, to determine the shortest routes for his fleet of new

(continued on page 27)



# CHARTING ON AUTOMATIC DATA PROCESSING SYSTEMS

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Bureau of the Census  
Washington 25, D. C.

An extremely rapid and mechanical method of charting time series on a high-speed printer activated by magnetic tape has recently been developed at the Bureau of the Census. This chart is a by-product of the Census Bureau's method of analyzing seasonal variations using an electronic computer, Univac.\* At present only the points are charted; connecting lines may be drawn manually. The charting method may be adapted to plot one or more time series simultaneously on a common scale, using different characters to identify the points of each series. The monthly data for Farm Income for 1940-55, in original and in seasonally-adjusted form, are shown in the upper section of the illustrative chart. With the exception of the title and legends, which have been typed in at the top prior to reproduction, this chart is shown exactly as it comes off the printer. The lower section of the chart shows the same data with the points connected manually.

Because the width of the page on the printer is fixed, the chart is run through the printer lengthwise to avoid charting the data in sections. The printer thus plots the points in chronological order — that is, the first month's data are charted on the first line; the second month's data, on the second line; and so on. As a result, the amplitude of the series must be adjusted to fit the width of the page. Furthermore, the number of discrete points which can be plotted on a line is limited to the number of printing spaces on the line, or 110. (Actually, the high-speed printer now in use at the Census Bureau provides for 130 spaces, but ten spaces are reserved on the chart for the time scale and another ten spaces may be used only to repeat the scale, or any other 10-character group, at the end of the line.) It is thus necessary to convert the data to be plotted into a series which takes on only integral values between 0 and 110. This conversion is accomplished for each month automatically on the Univac, which computes the data in plotting units according to the equation

$$(1) \quad y = (x - s) \frac{110}{r}$$

where  $y$  is the month's value in plotting units,  $x$  is the month's value in original units,  $s$  is the smallest value of all the data in original units, and  $r$  is the range of the data in original units. If more than one point is to be plotted on each line, as in the seasonal adjustment program, the same equation is applied to all the data, with  $s$  and  $r$  referring to the smallest value and the range, respectively, of all the data to be plotted.

The printing tape is then prepared on the

Univac line by line. On each line a symbol is positioned in the space corresponding to the  $y$  value for that month, and the remainder of the line is left blank. For example, for March 1955, the converted value, i.e.,  $y$ , for farm income in unadjusted form is 40; the seasonally-adjusted figure for the same month, again in  $y$  units, is 59. Therefore, for March 1955, the chart shows an "0" in space 59, and no other entries on that line. (The selection of "0" and "X" to represent the two series plotted is arbitrary; any other symbols from among the 51 available on the high-speed printer could be used.) Where the "X" and "0" coincide on the chart, only the "X" is shown since it is not possible to choose a third symbol to be printed in such cases of coincidence.

The scale used on the illustrative chart is shown on the base of the average of the original data for the period January 1947 to December 1949. The locations of the scale points are computed from equation (1), where  $x$  is taken successively as 50%, 100%, 150%, etc., of the average of the base period used. This computation is also done automatically on Univac and is printed on the chart. Similarly, the months and years are positioned on the time scale by the computer program.

As the final step in the process, the printing tape is put on the high-speed printer, and the 15-year chart is printed in less than a minute. The entire operation on the Univac and the printer takes only a few minutes.

The charting procedure, as actually developed, represents just one possible line of approach to automatic charting with electronic equipment. For example, where just one series is charted at a time, it might be desirable to construct a bar chart instead of plotting points. This could be done by positioning the same symbol in each line, not only at the space indicated by that month's  $y$  value, but also in all the spaces from the base line up to that space as well.

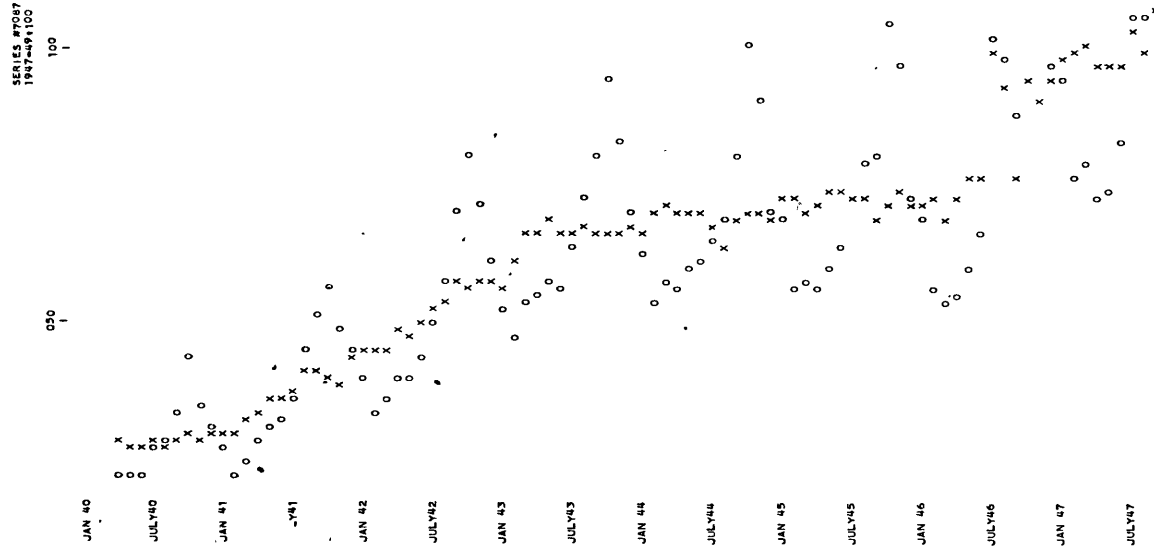
The method could also be altered to chart time series semi-logarithmically. The logarithm for each month could be computed on Univac from a suitable Taylor series expansion, using only as many terms in the expansion as are required for charting accuracy. The points could be plotted on a log scale of 1, 2, 3, ..., cycles, depending on the amplitude of the data.

While, for the present, only the points are plotted on this chart, it is conceivable that the lines can also be drawn in mechanically by the use of an interchangeable typewheel or auxiliary

U. S. FARM INCOME, 1940-50

Section 1.  
As plotted by High-Speed Printer

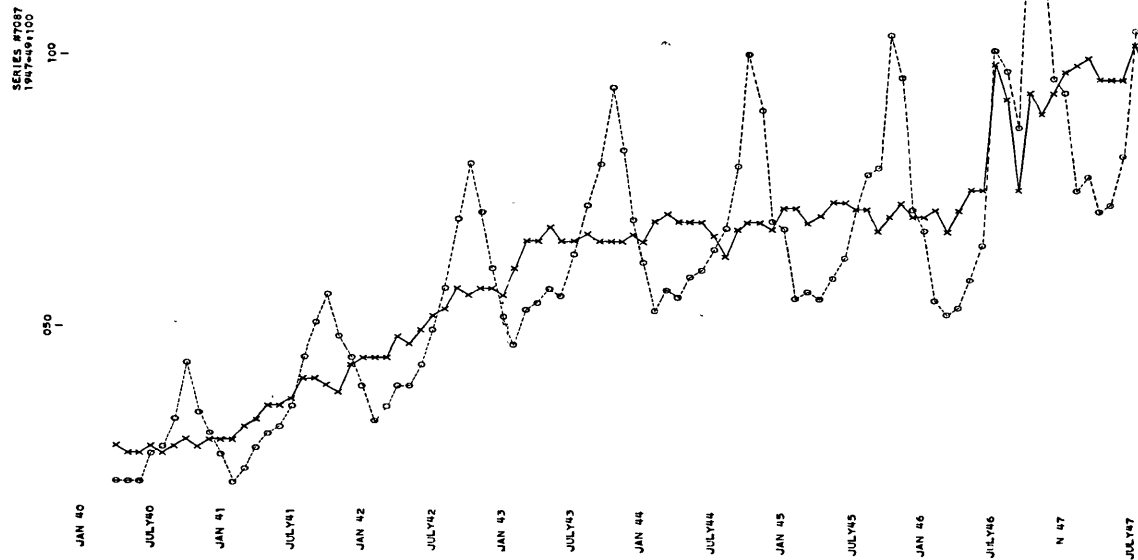
0 = original observations; X = seasonally adjusted series

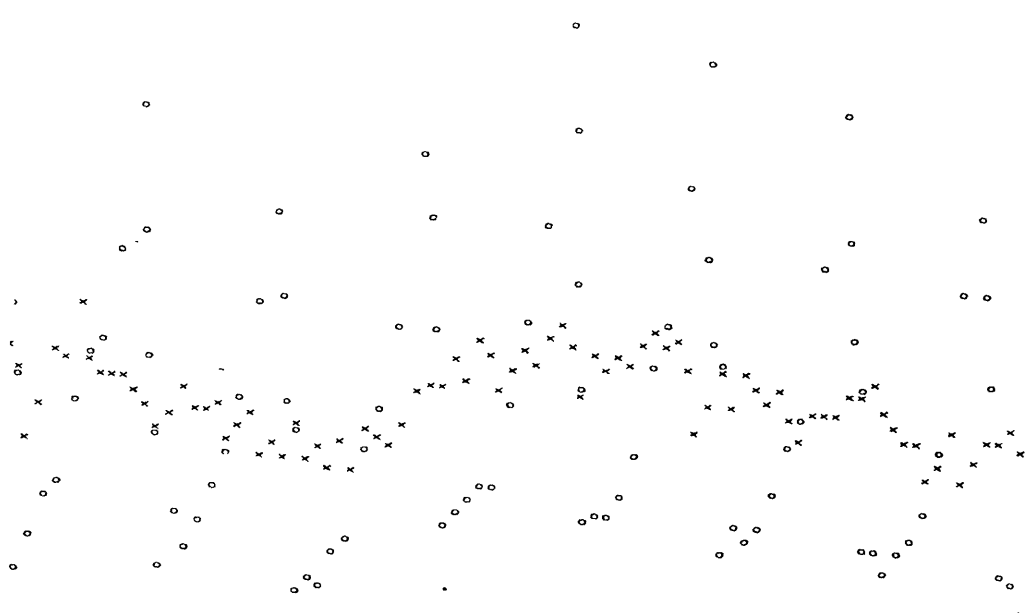
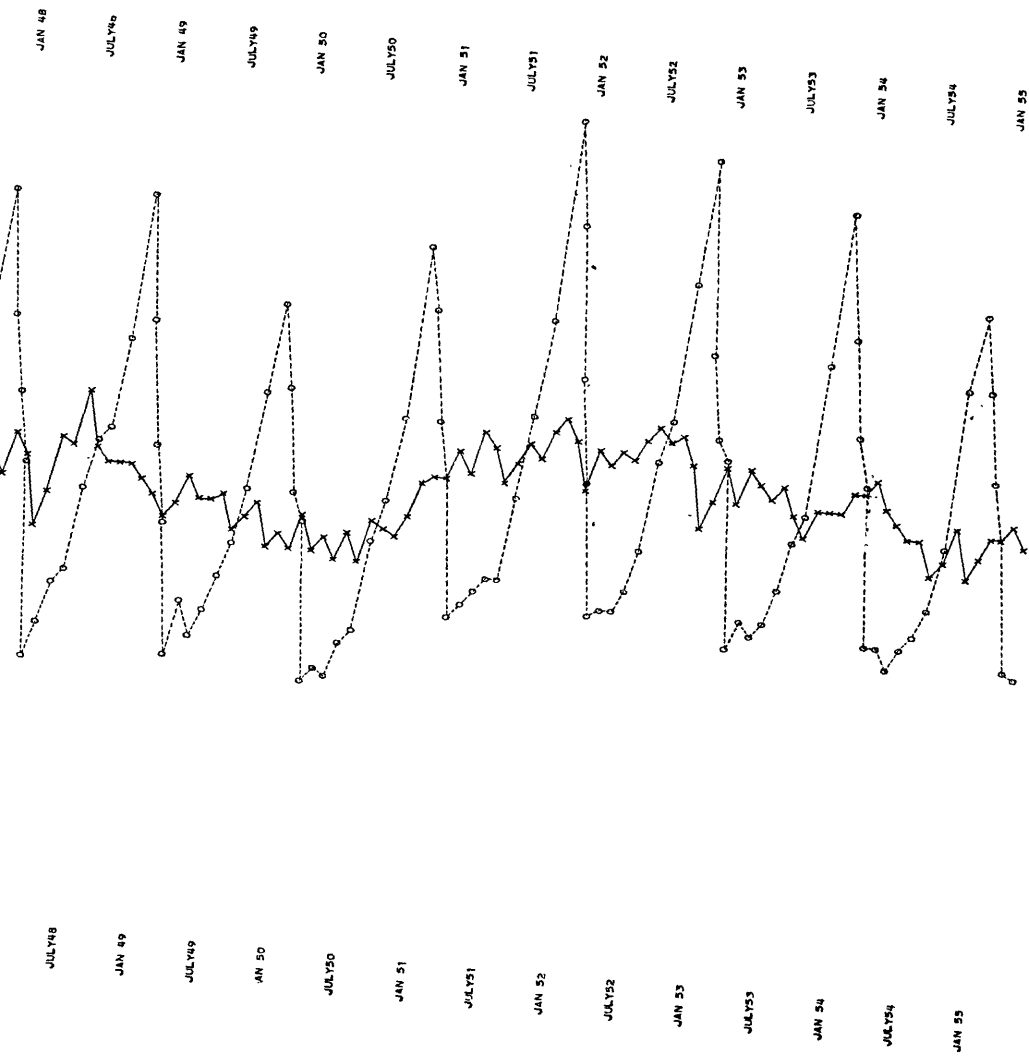


U. S. FARM INCOME, 1940-50

Section 2.  
Points plotted by High-Speed Printer and connected manually

0 - - - 0 = original observations; X — X = seasonally adjusted series





# A Rotating Reading Head for Magnetic Tape and Wire

National Bureau of Standards  
Washington 25, D. C.

A reading head that makes possible the close examination of a short section of magnetic tape or wire is now being used at the National Bureau of Standards to locate and investigate faults in magnetic recording media. Developed by J. R. Sorrells of the NBS data processing systems laboratory, this instrument makes use of a reading head mounted on a rapidly rotating drum so that the head is in contact with the tape for a part of each revolution. Since the tape is held stationary, the head reads exactly the same set of signals once each revolution, and the playback can be displayed continuously on an oscilloscope and observed as long as desired.

In addition to providing a means for closely examining the playback signals from a specific portion of the tape, the reader can be used to scan through and edit a complete tape. The observer can easily locate any defective signals along the entire length of the recorded tape. Interchangeable parts provide a means for examining several different sizes of magnetic tape or wire.

In the design and development of magnetic tape and wire equipment for external pulse storage in electronic digital computer systems, one of the primary considerations has been reliability of operation. An important factor in magnetic storage is the condition of the tape surface itself. Errors in operation can be caused by any of several types of tape faults such as "holes" and raised spots in the magnetic surface, or creases in the tape. Very often the loss of several pulses or the gain of a single pulse may be caused by a flaw that is too small to be visible to the unaided eye. Conventional means of tape reading are not suitable for locating errors, since in the usual tape transport mechanism the tape is moved continuously past a stationary head. In investigating tape for faults it is desirable to read a small specific portion of the tape over and over again at a rapid rate, and to display on an oscilloscope a steady, clear picture of the playback signals. The rotating head reading device developed at NBS provides such a repetitive method for examining tape. Once faults are located, they can be removed or else avoided in the future, thus increasing the reliability of the tape.

For convenience, the tape reader is mounted on a vertical panel. Near the two upper corners are the shafts on which the tape reels are mounted. Although tape must be reeled manually on the NBS model, a motor drive or stepping mechanism could easily be attached. The idler shafts are friction loaded to maintain the proper tension on the tape for reading as the tape reeled along.

At the lower center of the panel is the rotating drum on which the reading head is mounted. The drum is  $2\frac{7}{8}$  in. in diameter and rotates at 10 rps; thus the equivalent tape speed is 90 in/sec. The drum is made in two sections. The reading head is mounted on one section and has a groove or track machined around the periphery in which the tape or wire rides. This section is easily removed, and other similar sections may be substituted for different sizes of tape or wire.

The section of the drum nearest the panel is fastened to the drive shaft and is not ordinarily removed. This section is cup-shaped with the open side toward the panel. A photoelectric cell is mounted on the panel, inside the cup-shaped section, close to the periphery of the drum. A small aperture in the drum wall makes it possible to focus an external mounted lamp on the photocell when the drum is in the proper position.

Each time the hole in the drum wall passes the light source, the light strikes the photocell and causes a small output pulse from the cell. The pulse is applied to a cathode follower which in turn triggers the oscilloscope sweep. The aperture in the drum wall is so located that the trigger pulse occurs just before the reading head makes contact with the tape, so that the oscilloscope sweep always begins slightly before the first pulse is sensed by the head. It is this timing system that provides a steady picture of the repeated playback signals on the oscilloscope screen.

Since the reading head is mounted on a revolving drum, electrical connections must be made through slip rings and brushes. A specialized slip ring assembly using commercial brushes was designed and constructed for this purpose. The three rings are made of



ROTATING READING HEAD

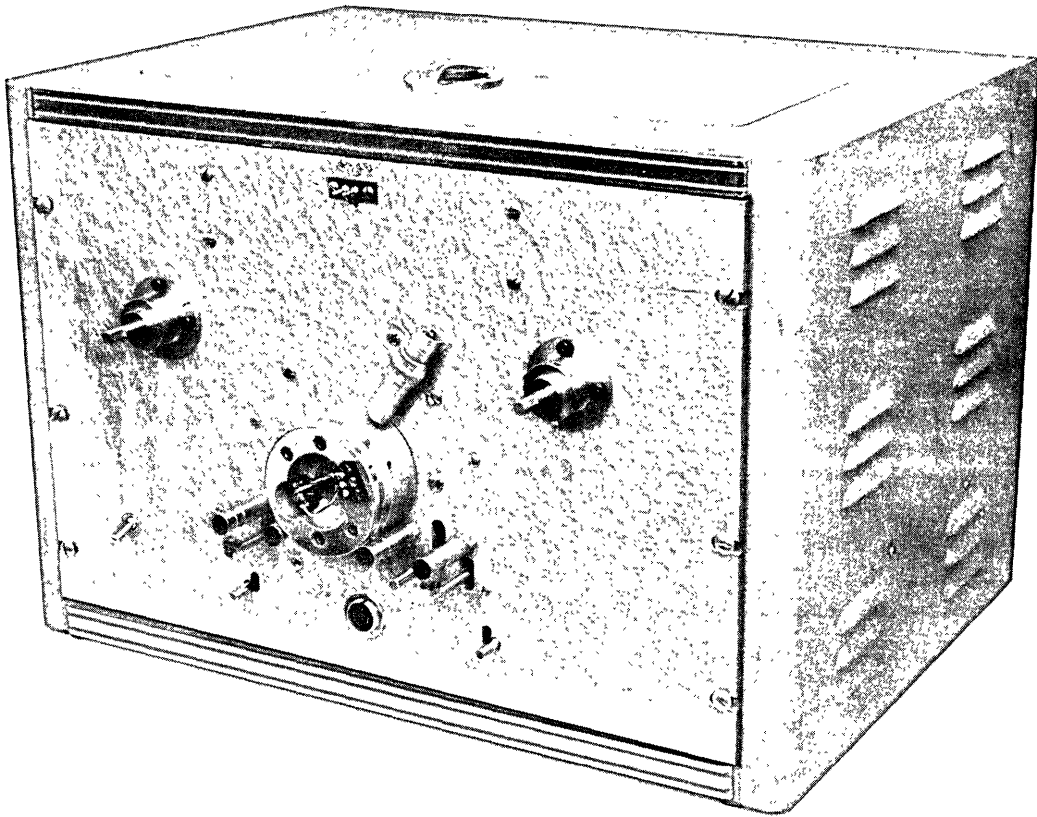


Figure 1. The rotating reading head on this machine (lower center of panel) developed at the National Bureau of Standards makes possible the close examination of a short section of magnetic tape or wire. With this instrument, the tape is held stationary; the drum revolves rapidly and for a part of each revolution the reading head is in contact with the tape. The head reads the same set of signals repetitively and the signals can be observed continuously on an oscilloscope.

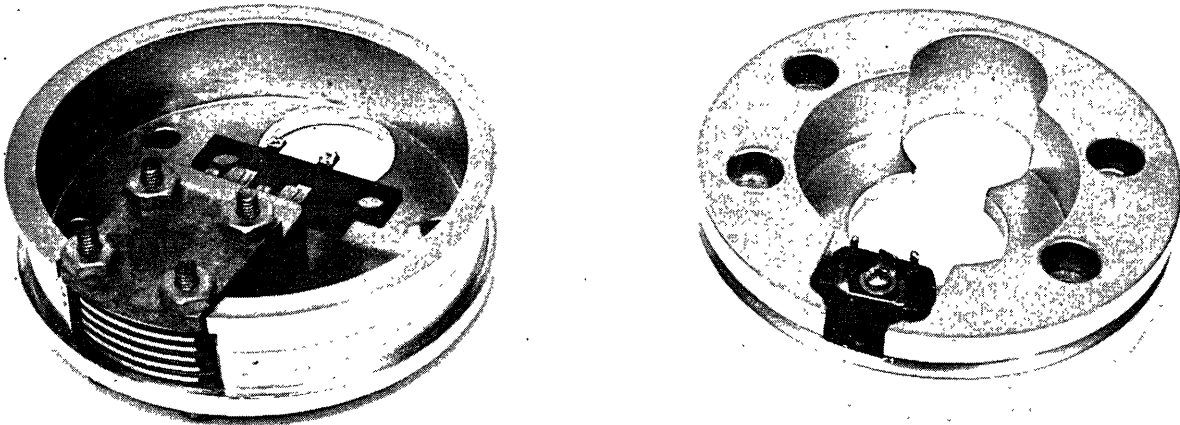


Figure 2. Two rotating reading heads: at left, the groove is wide enough to accommodate magnetic tape; at right, the groove is machined to accommodate wire only. These heads are used to locate and investigate faults in magnetic media at the National Bureau of Standards.

ROTATING READING HEAD

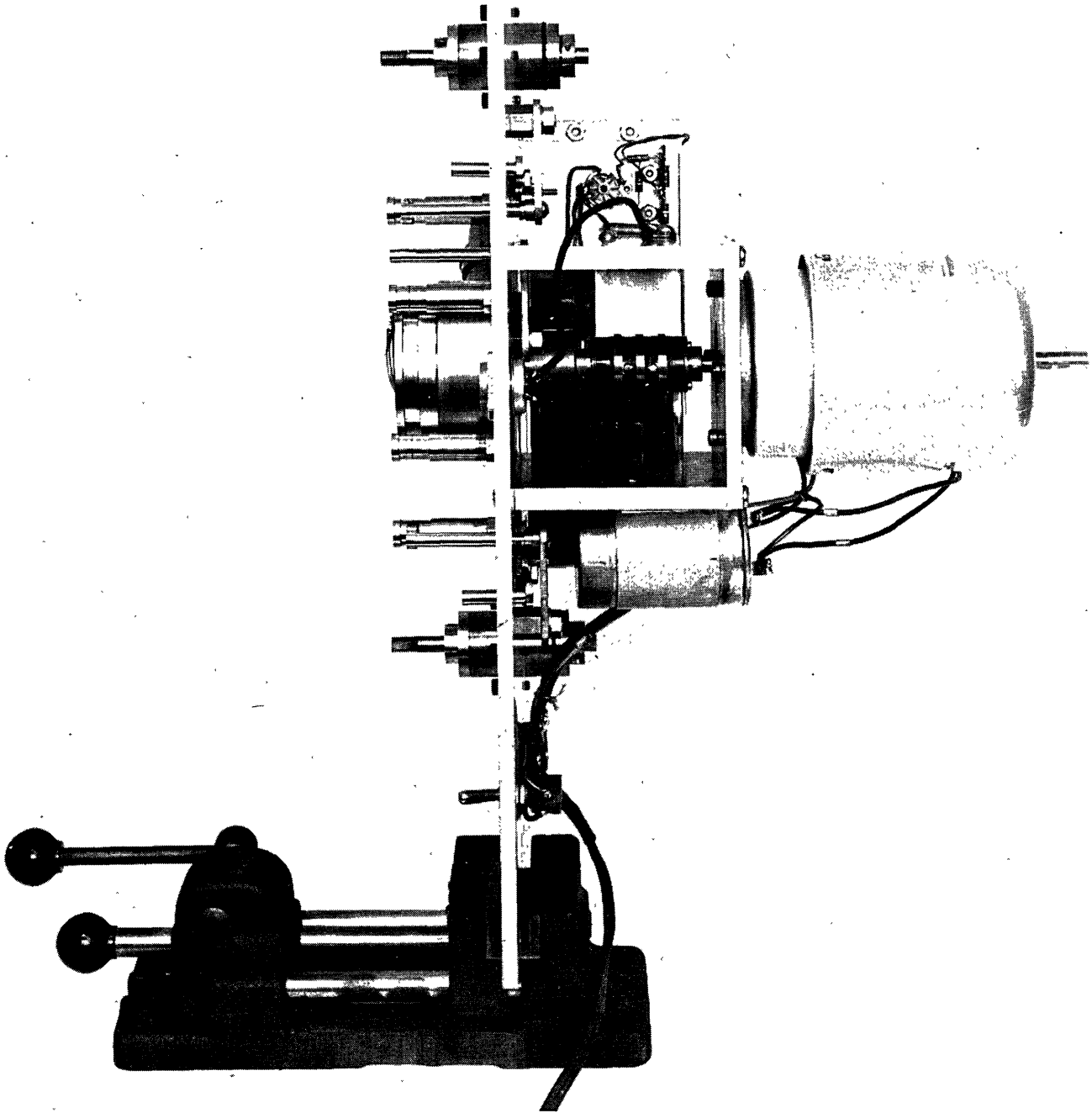


Figure 3. Edge view of rotating head reader showing interior mechanism. Electrical connections from the head are made through the slip rings and brushes in the center. At right is driving motor. On left of panel are studs for holding tape reels and for guiding the tape past the rotating drum.

ROTATING READING HEAD

electrodeposited silver on a premachined bakelite cylinder; the silver is further machined for good contact surfaces. The cylinder is mounted between the drum and the driving motor, and a two-wire shielded cable runs from the slip rings through the shaft to the reading head.

On the front of the panel are four grooved studs that guide the tape as it enters and leaves the reels and the rotating drum. Rubber shoes pressing firmly against the studs prevent the tape from creeping while being scanned. Interchangeable studs are provided for different sizes of tape and wire.

Trials of the equipment in the laboratory prove its usefulness in locating tape flaws and in reading recorded pulses. The playback signal on the oscilloscope screen shifts such a small amount that photographs taken with an exposure time of 15 sec reveal no evidence of blurring. Moreover, there is no noticeable noise from the slip ring and brush assembly.

By expanding the oscilloscope sweep, it is possible to read a computer word simply by recognizing the value of each recorded digital pulse. The rotating head has been used in this way to compare information on a magnetic wire with the paper tape from which it was recorded. This method is used to determine whether the transcribing equipment is causing trouble or the wire is at fault.

In addition to being an effective and useful means of investigating magnetic recording phenomena, the rotating-head type of the reader could also be used as a means for transcribing information directly from the keyboard to the magnetic tape. It would be most convenient to use a multichannel tape together with some provision for advancing it in short, precise steps. Each time a key on the keyboard is pressed, the corresponding character in coded form is set up in an electronic register. Then at a specific point of the rotating drum revolution, the contents are recorded on the tape in parallel form. On the next revolution of the drum, the character just recorded is compared with the character stored in the register. If the two agree, the tape advances a small distance, and the next key can be depressed to begin the next record-read-check cycle. If the two characters do not agree, the tape advance mechanism is locked out, an error indicator flashes, and the operator can either try to record again or find where the difficulty is. In this way, an operator could transcribe his problem directly from his manuscript to a magnetic tape, which then could be read directly into the computer.

-END -

TALE OF A COW  
(continued from page 20)

delivery trucks, to determine the optimum amount and mix of dairy products to carry in his optimally located warehouses, to select the most profitable mix of butter, milk, and other dairy products to make from a given batch of raw milk, to find the least cost daily work schedule and amount of equipment to purchase, to determine the most productive and profitable crop rotation plan for the cultivated lands, to compute what products should be sold now and what should be held for expected price changes, to determine the optimum balance between size of the herd, milk processing facilities, delivery fleet, and market potential. There are, naturally, other problems being solved scientifically, but they are too numerous to list here. Indeed, the farmer's customers now are too numerous to list and the problems too large to solve manually, so that he handles these with the electronic data system the scientist determined was optimal for the job.

Of course, the farmer's overhead rate, with all that scientific research, now is sky high but he does not mind. He is in business to maximize profits and not to minimize overhead costs. This, of course, is not very modern and not at all in accord with business practice, but it does pay for more vacations to California for the farmer and for his management scientist, but not for you nor your bookkeeper.

- END -

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CHARTING

(continued from page 21)

equipment.

These charts can be produced most conveniently and economically when they supplement tables computed by and printed out on an automatic data processing system. Then the input and output items in the electronic computer can be expeditiously manipulated for plotting, and the printing of the chart can follow without interruption the printing of the tables. For large-scale plotting jobs, however, just as in large-scale computing jobs, it may sometimes be preferable to use the plotting system described above rather than manual plotting, even though auxiliary operations -- card punching and card-to-tape conversion -- would be involved.

\*See Julius Shiskin, "Seasonal Computations on Univac," The American Statistician, Vol. 9, No. 1 (February 1955), pp. 19-23.

- END -

# BOOKS AND OTHER PUBLICATIONS

Jewell Bown, Gordon Spenser, and others

(List 16, COMPUTERS AND AUTOMATION, vol.4, no.8, August 1955)

This is a list of books, articles, periodicals, papers, and other publications which have a significant relation to computers or automation, and which have come to our attention. We shall be glad to report other information in future lists, if a review copy is sent to us. The plan of each entry is: author or editor / title / publisher or issuer / date, publication process, number of pages, price or its equivalent / a few comments. If you write to a publisher or issuer, we would appreciate your mentioning the listing in COMPUTERS AND AUTOMATION.

American Bankers Association, Savings and Mortgage Division, Bank Management Commission, Automation of Bank Operating Procedure / American Bankers Association, 12 East 36 St., New York 16, N. Y. / 1955, printed, 32 pp, limited distribution

A study and analysis by members of the banking industry of that field's need for automatic operations; includes suggestions for a program of automation. The commission of authors invites comments suggestions, and ideas; they include a list of specific questions addressed to machine companies. - JB

Locke, William N., and A.D. Booth, Editors, and others / Machine Translation of Languages / The Technology Press of Massachusetts Institute of Technology and John Wiley & Sons, Inc. 440 4th Ave, New York 16, N. Y. (published jointly) / May, 1955, printed, 243 pp, \$6.00

This collection of 14 essays provides an account of what has been achieved to date in the application of machines to translation. Beginning with a historical introduction, the book goes on to cover such topics as the design of an automatic dictionary, problems of the "word", speech input, storage devices, idioms and syntax, model English, and recent experiments. Interesting and readable. -JB

Haskins and Sells, Staff of / Data Processing by Electronics / Haskins and Sells, 67 Broad St, New York, N. Y. / May, 1955, printed, 113 pp, limited distribution.

The aim of this study is to convey, to the interested reader who is not a specialist a basic understanding of the operation and use of electronic data processing systems. Part I, pp 5 - 28, is a general description of electronic data processing systems and the factors involved in considering

their use; Part II, pp 29 - 64, considers basic theory, systems components, and techniques in application. The Appendix pp 67 - 113 contains summaries of the principal functional characteristics and the cost of purchase or rental of eleven electronic data-processing systems applicable to business. - JB

Hastings, Cecil, Jr, Jeanne T. Hayward, and James P. Wong, Jr. / Approximations for Digital Computers / Princeton University Press, Princeton, N. J. / June 27, 1955, printed, 201 pp, \$4

These approximations in this monograph were formerly issued in loose sheets and available only to a limited number of numerical analysts and computer operators. This publication makes them available in book form for the first time. Part I, entirely new, contains new material to introduce the collection of approximations in Part II, each approximation being presented with a carefully drawn error curve. Part II should be useful to anyone using a high-speed digital computing machine for numerical analysis. - JB

Grubbs, F. E., F. J. Murray, and J. J. Stoker, editorial committee, and others / Transactions of the Symposium on Computing Machines, Statistics and Partial Differential Equations / Interscience Publishers, Inc., 250 Fifth Avenue, New York 1, N. Y. / 1955, printed, 216 pp, \$5

This is a collection of the invited addresses presented at the second symposium on applied mathematics, held at the University of Chicago on April 29-30, 1954, and sponsored jointly by the American Mathematics Society and the Office of Ordnance Research of the U.S. Army. The papers included are "Operations Research" by P. M. Morse, "The Problem of Inductive

Inference" by J. Neyman, "Some Recent Developments in the Analysis of Variance" by H. O. Hartley, "Two Unsolved Problems in Statistical Mechanics" by J. E. Mayer, "Iterative Computational Methods" by M. R. Hestenes, "Motivations for Working in Numerical Analysis" by J. Todd, "Some Numerical Computations in Ordnance Research" by A. A. Bennett, "The Simplest Rate Theory of Pure Elasticity" by C. A. Truesdell, "On the Stability of Mechanical Systems" by J. J. Stoker, "Divergent Integrals and Partial Differential Equations" by Florent Bureau, and "On Differential Operators and Boundary Conditions" by W. Feller. The scope of Hestenes' paper is revealed by subheadings which include "Iterative Methods," "The Gradient Concept," "The Conjugate Gradient Method," "A Direct Method for Matrix Inversion," "Sources of Error," and "Eigenvalue Problems." The Section headings of the address by John Todd are "Evaluation of Polynomials," "Increasing the Speed of Convergence of Sequences," "Modified Differences," "Characteristic Roots of Finite Matrices," "Quadrature, Integral Equations," "Game Theory and Related Developments," "Recent Activity in Numerical Analysis," and "Theory of Machines or Automata." A bibliography of 75 references is included. - GS

Blundi, A. A. / What Pulses Can Do for You in "Control Engineering," July, 1955 / McGraw - Hill Publishing Co., 330 West 42 St., New York 36, N. Y. / 1955, printed, pp. 56-64, \$3 annually, 50¢ per copy. /

The author has written a popular account of how electronic pulses store information and control operations. Functions of pulse control devices described include gating, mixing, delaying, sampling, storing, counting, distributing, sequencing and the synthesis of these operations into a working system. - GS

Householder, A. S. / "Bibliography on Numerical Methods," Oak Ridge National Laboratory Report Number 1897 / Oak Ridge National Laboratory, P. O. Box P, Oak Ridge, Tenn. / 1955, multilithed, 32 pp, limited distribution

"This list of books and articles in the field of numerical analysis is intended to supplement and bring up to date one published by the . . . writer as an appen-

dix to his Principles of Numerical Analysis (McGraw-Hill, 1953). The collection of 321 items is incomplete in that it covers only that area making up the subject matter of the original book, i. e., "algebraic numerical analysis." The list is preceded by an eight-page section of comments and descriptions of the references. - GS

Householder, A. S. / "On the Convergence of Matrix Iterations," Oak Ridge National Laboratory Report Number 1883 / Oak Ridge National Laboratory, P. O. Box P, Oak Ridge, Tenn. / 1955, multilithed, 47 pp, limited distribution  
Techniques for proving convergence of sequences and series of matrices have generally been developed to cope with particular cases. The author here attempts "to gather together as many of these diverse results as possible and to develop a unifying principle that seems to include as special cases most of the techniques employed for proving convergence." Seemingly unrelated classical theorems are obtained as simple corollaries in the course of the development. - GS

Piatt, Alvin / "Computing Mechanisms" in "Automation", June 1955 / Penton Publishing Co., Penton Building, Cleveland 13, Ohio / 1955, printed, pp. 61-64, \$10 annually, \$1 per issue /  
When direct control systems become inadequate as industrial processes become more complex, the author suggests the consideration of mechanical analog devices which combine simple mechanisms with high reliability. This article describes various mechanical analog devices and their integration into complex systems. - GS

Leaver, E. W., and J. J. Brown / "A Functional Morphology of Mechanisms" in "Automation", July 1955 / Penton Publishing Co., Penton Building Cleveland 13, Ohio / 1955, printed, pp. 37-41, \$10 annually, \$1 per issue

The authors take as their thesis that the least number of concepts necessary to describe the various methods of producing work in a technological society are the parameters of energy and information. They exhibit a table which classifies various machines into (sometimes overlapping) subcategories of the two basic parameters. - GS

Goode, H. H. / "PGE Student Activities and Education in Computers", pp 49-51, in "IRE Transactions on Electronic Computers", June, 1955 / Institute of Radio Engineers, 1 East 79th St, New York 21, N. Y. / 1955, printed \$2.70

This is a valuable list of 91 colleges and universities in the United States and Canada giving the condensed status of computation education at each school. Information includes courses, facilities, the availability of assistantships, advanced degrees, seminars, and the name of a faculty member most interested in electronic computation. Analog and digital computation are distinguished. - GS

Dunn, W. H., and others / "A Digital Computer for Use in an Operational Flight Trainer", pp 55-63 in "IRE Transactions on Electronic Computers", June, 1955 / Institute of Radio Engineers, 1 East 79th St., New York 21, N. Y. / 1955, printed, \$2.70

Emphasizing the real-time aspects of the situation, the authors review the requirements for a digital computer for use in an operational flight trainer. A special purpose digital computer is described in detail to perform the necessary functions, functions for which a general purpose computer is inadequate.- GS

Murray, D. B. / "A Variable Binary Scaler" in "IRE Transactions on Electronic Computers", June, 1955 / Institute of Radio Engineers, 1 East 79th St., New York 21, N. Y. / 1955, printed, pp. 70-74, \$2.70

A class of interconnections of the binary elements of a counter are discussed in which some elements are "forward-counting" and some are "reverse-counting." Alteration of the interconnections may provide for any arbitrary integral scaling ratio up to the counter capacity. - GS.

Allen, William, and G. E. Smith / "The UNIVAC and UNIVAC SCIENTIFIC", pp 960-9, in "Instruments and Automation", June, 1955 / The Instruments Publishing Co., Inc., 845 Ridge Ave, Pittsburgh 12, Pa. / 1955, printed, \$4 annually

The first author provides a full description of the UNIVAC, its operations and auxiliary equipment. Similarly, Mr. Smith describes the UNIVAC SCIENTIFIC computer, formerly known as the ERA 1103 computer. Together, they

provide a current description of Remington Rand's large-scale computing equipment. Thirteen figures illustrate the text. - GS

Rand Corporation / A Million Random Digits with 100,000 Normal Deviates / The Free Press, Glencoe, Illinois, / 1955, printed and photooffset, 625 pp, \$10

The million random digits are presented on 400 pages each of which contains 50 lines of 50 digits arranged in ten groups of five. The normal deviates are derived from an equal number of groups of the random digits and are given to three decimal places. They are arranged in 50 lines and ten columns on each of 200 pages. A 25 page introduction describes the origin and use of the tables and describes the tests for randomness that were applied to the digits. - GS

Teichroew, D. / "Numerical Analysis Research Unpublished Statistical Tables", pp 550-556, in the Journal of the American Statistical Association", June, 1955 / American Statistical Association, 1108 16th St. N. W., Washington 6, D. C. / 1955, printed \$8 annually

The author describes a collection of statistical tables computed on the SWAC at the Numerical Analysis Research Division of the University of California at Los Angeles, formerly known as the Institute for Numerical Analysis of the National Bureau of Standards. The tables are available on punched cards. Included are 8 tables associated with the normal distribution, 4 tables with the gamma distribution, 2 with Student's t distribution, 4 with the derivation of samples by means of a computing machine, and 5 miscellaneous tables. - GS

Rubenstein, Albert H, editor, and about 28 other authors / Coordination, Control, and Financing of Industrial Research / King's Crown Press, Columbia University, New York 27, N. Y. / 1955, printed, 429 pp, cost?

This is the 4th volume in a series reporting the proceedings of the Annual Conferences on Industrial Research sponsored by the Department of Industrial Engineering at Columbia University. Included are all of the papers presented at the Fifth Conference and the discussions following them, as well as relevant and abridged excerpts from the clinic sessions. There



are four papers relating to computers:  
 "Introduction to Computer Technology"  
 by C. B. Tompkins; "Application of High  
 Speed Computers to Research Problems"  
 by R. F. Clippinger; "Automatic Data  
 Reduction" by G. Truman Hunter; "Op-  
 eration of an Industrial Computing Fac-  
 ility" by H. R. J. Grosch. Other papers  
 include: "Management of Industrial Re-  
 search"; "Economics of Industrial Re-  
 search"; "The Importance of Experi-  
 mental Design in Efficient Experiment-  
 ation"; "Research Project Evaluation";  
 clinic sessions included accounting for  
 research expenses, creativity and idea  
 sources, evaluating the returns from  
 research, market research, technical  
 communication and report writing, etc.  
 - JB

Aronson, Milton H / Electronic Circuitry for In-  
 struments and Equipment / Instruments Pub-  
 lishing Co, 921 Ridge Ave, Pittsburgh 12, Pa.  
 / 1953, printed, 318 pp, \$4

This is written in a nonmathematical  
 style primarily for technicians and en-  
 gineers, and as a textbook and home-  
 study course; it includes more than 450  
 question items for study. Covers elec-  
 trical fundamentals, resistance, cap-  
 acitance, inductance, impedance, re-  
 sonance, signal shaping, vacuum-tube  
 and gas-tube fundamentals, amplifica-  
 tion, rectification, oscillator and sig-  
 nal-shaping circuits, test equipment,  
 etc. A very good book for its purpose;  
 it would be easier to use if the print  
 were larger. - JB

Automation, staff of / "Integrated Data Processing  
 Comes to Life" in "Automation", May, 1955 /  
 Penton Publishing Co, Penton Bldg, Cleveland  
 13, Ohio / May 1955, printed, 120 pp (entire  
 magazine), yearly subscription \$10, single  
 copy \$1

A case study of the order-handling pro-  
 cedures of the Aluminum Co. of Amer-  
 ica. The order-processing system as  
 it functions at the point of origin, the  
 home office, and the factory is descri-  
 bed in detail. Packaging and shipping  
 operations and company-wide availabil-  
 ity of data are also fully explained. -JB

Cochran, William G. / Sampling Techniques /  
 John Wiley & Sons, Inc., 440 Fourth Ave, New  
 York 16, N. Y. / 1953, printed, 330 pp, \$6.50

a copy  
 A review of the principles of sampling  
 as they are applied to the field of sam-  
 ple surveys. Exposition of theory is accom-  
 panied by illustrations from survey data,  
 and since principles rather than details  
 are emphasized, the book can be used as  
 a foundation of theory for applications in  
 many fields. The book is based on the  
 finite sampling theory, yet the author  
 has indicated, wherever suitable, the  
 utility of the analysis of variance. The  
 minimum mathematical equipment nec-  
 essary for easy understanding of the  
 proofs is a knowledge of calculus as far  
 as the determination of maxima and mi-  
 nima, and an introductory statistical  
 course. Chapters include stratified  
 random sampling, and subsampling with  
 units of equal and unequal size. - JB

Fett, Gilbert Howard / Feedback Control Systems  
 / Prentice-Hall, Inc., 70 5th Ave, New York,  
 N. Y. / 1954, printed, 360 pp, \$10

A text intended to provide advanced en-  
 gineering students and practicing engi-  
 neers with a basic understanding of feed-  
 back control systems and the tools and  
 techniques required to solve the newer  
 problems in this field. Features of in-  
 terest include: use of the differential  
 equation and Laplace transform approach  
 before the complex frequency locus meth-  
 od is discussed; synthesis of the Routh-  
 Hurwitz and Nyquist analyses by the use  
 of the alpha and delta paths in the com-  
 plex plane; and the phase plane analysis  
 and the linearization method of handling  
 non-linear problems. - JB

Kraus, John D / Electromagnetics / McGraw-Hill  
 Book Company, 330 West 42 St., New York,  
 N. Y. / 1953, printed, 604 pp, \$?

The first seven chapters are written for  
 an introductory course in field and cir-  
 cuit theory in physics or electrical en-  
 gineering at 3rd or 4th year college lev-  
 el; subjects covered include static elec-  
 tric and magnetic fields, steady currents  
 and changing electric and magnetic fields.  
 The last seven chapters are written for  
 a course at senior college or graduate  
 level; they treat plane waves in dielec-  
 tric and conducting media, transmission  
 lines, wave guides, antennas, and bound-  
 ary-value problems. - JB

LaJoy, Millard H / *Industrial Automatic Controls* / Prentice-Hall, Inc. 70 5th Ave, New York 11, N. Y. / 1954, printed, 278 pp, \$6.65

This book has been written as a beginning text in the study of industrial automatic controls on the college level. Two-position, proportional, floating (integral), and rate (derivative) modes of automatic control are analyzed separately and in various combinations. The internal circuitry of pneumatic, hydraulic, and electronic controls is treated in detail. Concluding chapters consider representative industrial applications and theoretical considerations of the process. A knowledge of differential equations is required for these chapters; a knowledge of calculus for the rest of the book. - JB

puters, information theory, and automatic control. Two sections on automatic contain 5 papers dealing with trends in automation of procedures in business and industry, and 5 on other subjects. Three sections on information theory include 12 papers. Three sections on electronic computers contain a total of 10 papers dealing with magnetic memories, transistors, electrographic recording technique, and electronic generation of functions. - JB

- END -

Institute of Radio Engineers / IRE Convention Record, 1955 National Convention, "Part 4 -- Computers, Information Theory, Automatic Control" / IRE, 1 East 79 St., New York 21, N. Y. / 1955, photooffset, 208 pp, \$6.75

Contains 32 papers presented at the 1955 IRE National Convention relating to com-

THE EDITOR'S NOTES  
(continued from page 4)

The "Roster Entries" page in this issue shows some of the kind of information we always want to have. But even better would be an unofficial parttime reporter in each computer center. Any volunteers?

- END -

## ANALOG COMPUTER ENGINEERS

Bendix Research Laboratories Division, the center of advanced development activities for the Bendix Aviation Corporation is offering excellent opportunities for competent analog computer engineers. Problems in the fields of missile guidance systems, navigation studies, nuclear reactor controls, hydraulic control devices and other related projects.

The Research Laboratories is a small, separate division of a well-established reputable engineering organization, exclusively devoted to research and development of a wide variety of interesting, progressive and highly imaginative projects. Opportunity for graduate study.

### SENIOR ANALOG COMPUTER PROBLEM ANALYST:

5 - 7 years experience in dynamic analysis utilizing analog computers, must be able to direct problem from origin through computer set-up and operation, and include final analysis. Advanced degree desirable with good mathematical or physics background.

### SENIOR COMPUTER PROBLEM ENGINEER:

To assume responsibility for problem operation of an afternoon shift (4:00 PM - 12:45). 4 - 5 years of experience in computer operation. Responsible for problem set-up, checkout, operation and evaluation. Degree in math or physics.

### ANALOG COMPUTER PROBLEM ENGINEER:

3 - 4 years experience in computer operations and problems set-up. Degree in math, physics or EE necessary.

Send resume to: **Personnel Department  
Bendix Aviation Corporation  
Research Laboratories Division  
4855 Fourth Avenue  
Detroit 1, Michigan**

# Some Curiosities of Binary Arithmetic

## Useful in Testing Binary Computers

Andrew D. Booth  
Birkbeck College  
London, England

In testing an automatic computing machine that works in the binary scale, it is useful to have certain standard numbers whose square, reciprocal, square root, etc., are known. A consideration of such special numbers has led to the accumulation of a small library. It is the purpose of this note to make available to newcomers the art of some of this data.

### Fractions

Some useful binary fractions are the binary equivalents of numbers having the form  $1/n$  where  $n$  is a whole number. A regular repeating pattern is produced when  $n$  has the form  $2^p \pm 1$ . In this event:

$$1/n = \frac{1}{2^p \pm 1} = 2^{-p} (1 \mp 2^{-p} + 2^{-2p} \mp 2^{-3p} + 2^{-4p} \text{ etc.})$$

from which are readily obtained the equivalents for the fractions  $1/3, 1/5, 1/7, 1/15, 1/17, \text{ etc.}$  Also, since numbers of the form  $1/2^p$  have exact equivalents, it is easy to fill in some of the gaps which occur in the above scheme. The remaining numbers may be evaluated by expressing the desired fraction,  $1/n$ , in the form:

$$1/n = m/2^p \pm 1$$

where  $m$  is also an integer; this method was used to obtain the values of  $1/11, 1/13$  and  $1/19, \text{ etc.}$ , in Table 1 (see page 42). For example,  $1/11$  equals  $3/33$ .

### Squares

Here again, some of the most convenient "pattern producing" numbers are of the form:

$$(1-2^{-p}) \text{ for which } (1-2^{-p})^2 = 1-2^{-p+1}+2^{-2p}$$

Thus:

(.1) <sup>2</sup>	= .01
(.11) <sup>2</sup>	= .1001
(.111) <sup>2</sup>	= .110001
(.1111) <sup>2</sup>	= .11100001 etc.

Other useful numbers are:

$$(1/3)^2 = 1/9 \text{ or } (.01, 01, \text{ etc.})^2 = (.000111, 000111, \text{ etc.}) \text{ and}$$

$$(1/5)^2 = 1/25 \text{ or } (.0011, 0011, \text{ etc.})^2 = (.00001010001111010111, 00001010001111010111, \text{ etc.})$$

These numbers, together with the simple ones just given, provide a good check on the functioning of a multiplier, especially when it works upon the non-restoring principle. In the same way, by using the squares, a test can be applied both to square-rooting circuits and to programs for evaluating square roots.

(continued on page 42)

\* ————— \*

## SPECIAL ISSUES OF "COMPUTERS AND AUTOMATION"

The issue of "Computers and Automation" in June, 1955, was a special issue: "The Computer Directory, 1955", 164 pages, containing: Part 1, Who's Who in the Computer Field; Part 2, Roster of Organizations in the Computer Field; and Part 3, The Computer Field: Products and Services for Sale. It is expected that the next Computer Directory issue will be June, 1956.

The next two special issues will be December, 1955, and January, 1956. The December issue will be mainly devoted to useful information for people who have been in the computer field for some time: a "Glossary of Terms", and also cumulative editions of other pieces of reference information.

The January, 1956, issue will be mainly devoted to useful information for people who have newly entered the computer field: an introduction to computers (and to "Computers and Automation"); and reprints and revisions of some of the more introductory articles and papers that "Computers and Automation" has published.

## ADDITIONS AND REVISIONS

You are to be congratulated on the fine job being done by you and your staff in publishing the magazine "Computers and Automation." Over the many months that I have been receiving it, each edition has been read with interest.

While going through the June edition I noticed my name in the Who's Who in the Computer Field. Would you be kind enough to add title and name of my Company to your files - Associate Controller, John Hancock Mutual Life Insurance Company.

Thank you and keep up the good work.

Sincerely yours,

Harold F. Hatch  
Associate Controller  
John Hancock Mutual Life  
Insurance Company  
200 Berkeley Street  
Boston 17, Mass

In examining your excellent June 1955 computer directory issue of "Computers and Automation," I note that our organization is not listed in its present form.

The National Bureau of Standards Institute for Numerical Analysis (your page 112) was inactivated on June 30, 1954. Many of the same people, equipment and functions have been continued in the University of California, Los Angeles. Following your entry form (page 103), we have

1. and 2. University of California, Department of Mathematics, Numerical Analysis Research, Los Angeles 24, California
3. Granite 3-0971 and BRadshaw 2-6161
4. Automatic digital computers. We operate SWAC on loan from the Office of Naval Research.
5. Activity: Research and teaching in use of digital computers for scientific computation.
6. Approximately 40 employees.
7. Established 1954.
8. Remarks: Have machine, library, and other equipment owned up to 1954 by the now inactive National Bureau of Standards Institute for Numerical Analysis.

Yours very truly,

George E. Forsythe  
Research Mathematician  
Department of Mathematics  
Numerical Analysis Research  
University of California  
Los Angeles 24, California

\* \_\_\_\_\_ \*

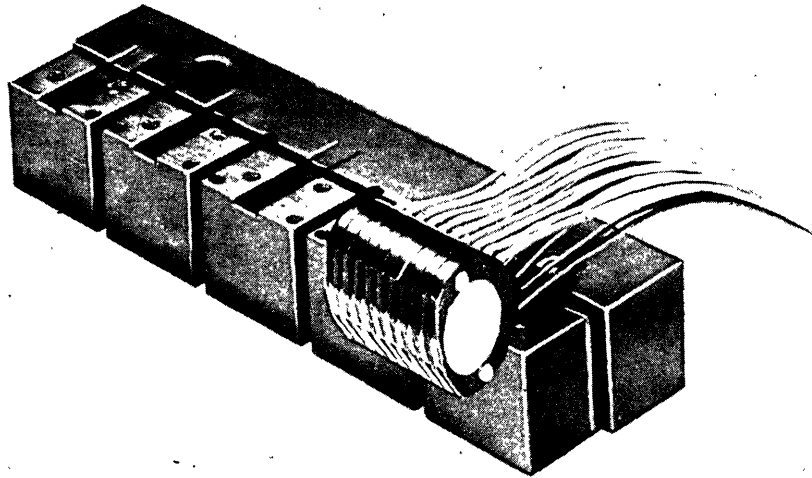
## MANUSCRIPTS

We are interested in articles, papers, and fiction relating to computers and automation. To be considered for any particular issue, the manuscript should be in our hands by the fifth of the preceding month.

Articles. We desire to publish articles that are factual, useful, understandable, and interesting to many kinds of people engaged in one part or another of the field of computers and automation. In this audience are many people who have expert knowledge of some part of the field, but who are laymen in other parts of it. Consequently a writer should seek to explain his subject, and show its context and significance. He should define unfamiliar terms, or use them in a way that makes their meaning unmistakable. He should identify unfamiliar persons with a few words. He should use examples, details, comparisons, analogies, etc., whenever they may help readers to understand a difficult point. He should give data supporting his argument and evidence for his assertions. We look particularly for articles that explore ideas in the field of computers and automation, and their applications and implications. An article may certainly be controversial if the subject is discussed reasonably. Ordinarily, the length should be 1000 to 4000 words, and payment will be, generally, \$10 to \$40 on publication. A suggestion for an article should be submitted to us before too much work is done.

Technical Papers. Many of the foregoing requirements for articles do not necessarily apply to technical papers. Undefined technical terms, unfamiliar assumptions, mathematics, circuit diagrams, etc., may be entirely appropriate. Topics interesting probably to only a few people are acceptable. Payments will be made for papers, generally \$5 to \$20 on publication, depending on length, etc.

Fiction. We desire to print or reprint fiction which explores scientific ideas and possibilities about computing machinery, robots, cybernetics, automation, etc., and their implications, and which at the same time is a good story. Ordinarily, the length should be 1000 to 4000 words, and payment will be, generally, \$10 to \$40 on publication if not previously published, and half that if previously published.



A bank of Monrobot Ring-type Heads  
on adjustable mounting

## MONROBOT COMPONENTS

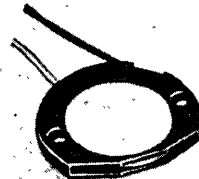
### Researched and Developed for the Electronics Field

Monroe, for many years a leader in the design and production of desk calculators, is devoting its experience and research facilities to developing not only digital electronic computers but also component parts that are unique for their originality of design and numerous advantages.

The components, illustrated here, are Monrobot Ring-type Read/Record Heads and an adjustable fixture for magnetic drum memory systems.

Monrobot Read/Record Head assemblies have a place in any magnetic storage or arithmetic drum system. Their advantages are many. They are small and compact. Their mounting design permits accurate placement with a minimum of costly machining. When assembled and mounted they permit fine, accurate adjustments both parallel and perpendicular to the drum. They are ruggedly built to last. In their pre-engineered, precision mounting they cost less than similar unmounted head stacks; also replacement of a single head is easily made.

Monrobot components of this kind offer the user of electronic equipment many worthwhile benefits. There's a lot to the Monrobot component story that's worth investigating. Inquiries are invited.



Monrobot Ring-type Head —  
small, compact



**MONROE CALCULATING MACHINE COMPANY**  
**MONROBOT LABORATORY**  
**MORRIS PLAINS NEW JERSEY**

## ROSTER ENTRY FORMS

"Computers and Automation" publishes from time to time reference information of the following three types: (1) a who's who or roster of individuals interested in the computer field; (2) a roster of organizations active in the computer field; and (3) a classified directory or roster of products and services offered in the computer field. The last cumulative roster appeared in "The Computer Directory, 1955", the June 1955 issue of "Computers and Automation." If you are interested in sending information to us for these rosters and their supplements, following is the form of entry for each of these three rosters. To avoid tearing the magazine, the form may be copied on any sheet of paper; or upon request we will send you forms for entries.

### (1) Who's Who Entry Form

1. Name (please print) \_\_\_\_\_
  2. Your Address? \_\_\_\_\_
  3. Your Organization? \_\_\_\_\_
  4. Its Address? \_\_\_\_\_
  5. Your Title? \_\_\_\_\_
  6. YOUR MAIN COMPUTER INTERESTS?
 

<input type="checkbox"/> Applications	<input type="checkbox"/> Mathematics
<input type="checkbox"/> Business	<input type="checkbox"/> Programming
<input type="checkbox"/> Construction	<input type="checkbox"/> Sales
<input type="checkbox"/> Design	<input type="checkbox"/> Other (specify): _____
<input type="checkbox"/> Electronics	_____
<input type="checkbox"/> Logic	_____
  7. Year of birth? \_\_\_\_\_
  8. College or last school? \_\_\_\_\_
  9. Year entered the computer field? \_\_\_\_\_
  10. Occupation? \_\_\_\_\_
  11. Anything else? (publications, distinctions, etc.) \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_
- \* \_\_\_\_\_ \*

### (2) Organization Entry Form

1. Your organization's name? \_\_\_\_\_
  2. Address? \_\_\_\_\_
  3. Telephone number? \_\_\_\_\_
  4. Types of computing machinery or components, or computer-field products and services that you are interested in?  
 \_\_\_\_\_  
 \_\_\_\_\_
  5. Types of activity that you engage in:  
 research       other (please explain):  
 manufacturing  
 selling      \_\_\_\_\_  
 consulting      \_\_\_\_\_
  6. Approximate number of your employees? \_\_\_\_\_
  7. Year when you were established? \_\_\_\_\_
  8. Any comments? \_\_\_\_\_  
 \_\_\_\_\_
- Filled in by \_\_\_\_\_
- Title \_\_\_\_\_ Date \_\_\_\_\_
- \* \_\_\_\_\_ \*

### (3) Product Entry Form

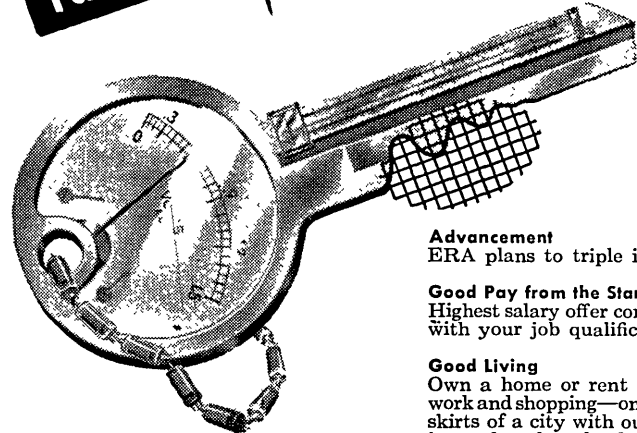
1. Name or identification of product (or service)? \_\_\_\_\_  
 \_\_\_\_\_
  2. Brief description (20 to 40 words)? \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_
  3. How is it used? \_\_\_\_\_  
 \_\_\_\_\_
  4. What is the price range? \_\_\_\_\_
  5. Under what headings should it be listed?  
 \_\_\_\_\_
  6. Your organization's name? \_\_\_\_\_  
 \_\_\_\_\_
  7. Address? \_\_\_\_\_  
 \_\_\_\_\_
- Filled in by \_\_\_\_\_
- Title \_\_\_\_\_ Date \_\_\_\_\_



**ELECTRONIC ENGINEERS MECHANICAL**

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to your  
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The Concept of Thinking -- Elliot L. Gruenberg  
General Purpose Robots -- Lawrence M. Clark

May: Ferrite Memory Devices -- Ephraim Gelbard and William Olander

Flight Simulators -- Alfred Pfanstiehl  
Autonomy and Self Repair for Computers -- Elliot L. Gruenberg

A Glossary of Computer Terminology -- Grace M. Hopper

July: Human Factors in the Design of Electronic Computers -- John Bridgewater

What is a Computer? -- Neil Macdonald

September: Computer Failures -- Automatic Internal Diagnosis (AID) -- Neil Macdonald

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The Development and Use of Automation by Ford Motor Co. -- News Dept., Ford Motor Co.

Reciprocals -- A. D. Booth

October: Flight Simulators: A New Field -- Alfred Pfanstiehl

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All-Transistor Computer -- Neil Macdonald

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Eastern Joint Computer Conference, Philadelphia, Dec. 8-10, 1954 -- Milton Stoller

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Recognizing Spoken Sounds by Means of a Computer -- Andrew D. Booth

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Computers and Weather Prediction -- Bruce Gilchrist

Random Numbers and Their Generation -- Gordon Spenser

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May: Reliability in Electronic Data Processors -- William B. Elmore

Numerical Representation in Fixed-Point Computers -- Beatrice H. Worsley

Automation -- A Report to the UAW-CIO Economic and Collective Bargaining Conference

The Skills of the American Labor Force -- James P. Mitchell

Automation Puts Industry on Eve of Fantastic Robot Era -- A. H. Raskin

The Monkey Wrench -- Gordon R. Dickson

June: THE COMPUTER DIRECTORY, 1955 (164 pages):

Part 1: Who's Who in the Computer Field

Part 2: Roster of Organizations in the Computer Field

Part 3: The Computer Field: Products and Services for Sale.

July: Mathematics, the Schools, and the Oracle -- Alston S. Householder

The Application of Automatic Computing Equipment to Savings Bank Operations -- R. Hunt Brown

The Book Reviewer -- Rose Orente

Linear Programming and Computers, Part I, -- Chandler Davis

## REFERENCE INFORMATION (in various issues):

Roster of Organizations in the Field of Computers and Automation / Roster of Automatic Computing Services / Roster of Magazines Related to Computers and Automation / Automatic Computers; List / Automatic Computers: Estimated Commercial Population / Automatic Computing Machinery: List of Types / Components of Automatic Computing Machinery: List of Types / Products and Services in the Computer Field / Who's Who in the Field of Computers and Automation / Automation: List of Outstanding Examples / Books and Other Publications / Glossary / Patents

BACK COPIES: Price, if available, \$1.25 each, except June, 1955, \$6.00. Vol. 1, no. 1, Sept, 1951, to vol. 1, no. 3, July, 1952: out of print. Vol. 1, no. 4, Oct. 1952: in print. Vol. 2, no. 1, Jan. 1953, to vol. 2, no. 9, Dec. 1953: in print except March, no. 2, and May, no. 4. Vol. 3, no. 1, Jan. 1954, to vol. 3, no. 10, Dec. 1954: in print. Vol. 4, 1955: in print.

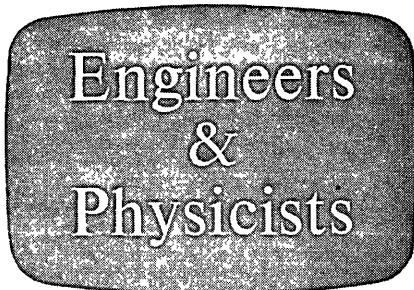
A subscription (see rates on page 4) may be specified to begin with the current month's or preceding month's issue.

WRITE TO: Berkeley Enterprises, Inc.  
Publisher of COMPUTERS AND AUTOMATION  
36 West 11 St., New York 11, N. Y.

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APPLIED TO THE DESIGN, DEVELOPMENT  
AND APPLICATION OF

*AUTOMATIC RADAR  
DATA PROCESSING,  
TRANSMISSION AND  
CORRELATION IN LARGE  
GROUND NETWORKS*



*Digital computers similar to the successful Hughes airborne fire control computers are being applied by the Ground Systems Department to the information processing and computing functions of large ground radar weapons control systems.*

The application of digital and transistor techniques to the problems of large ground radar networks has created new positions at all levels in the Ground Systems Department. Engineers and physicists with experience in fields listed, or with exceptional ability, are invited to consider joining us.

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LOGICAL DESIGN  
PROGRAMMING  
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AND TRANSMITTERS  
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*Scientific and Engineering Staff*



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Republic Aviation**

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To supervise maintenance and to design special circuitry for computers. Experience with either analogue or digital computers required. College graduate preferred.

*Please address complete resume,  
outlining details of  
your technical background, to:*

Assistant Chief Engineer  
Administration  
Mr. R. L. Bortner



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# ADVERTISING IN "COMPUTERS AND AUTOMATION"

Memorandum from Berkeley Enterprises, Inc.  
Publisher of COMPUTERS AND AUTOMATION  
36 West 11 St., New York 11, N.Y.

1. What is "COMPUTERS AND AUTOMATION"? It is a monthly magazine containing articles, papers, and reference information related to computing machinery, robots, automatic controllers, cybernetics, automation, etc. One important piece of reference information published is the "Roster of Organizations in the Field of Computers and Automation". The basic subscription rate is \$4.50 a year in the United States. Single copies are \$1.25, except June, 1955, "The Computer Directory" (164 pages, \$6.00). For the titles of articles and papers in recent issues of the magazine, see the "Back Copies" page in this issue.

2. What is the circulation? The circulation includes 1700 subscribers (as of June 20); over 300 purchasers of individual back copies; and an estimated 2000 nonsubscribing readers. The logical readers of COMPUTERS AND AUTOMATION are people concerned with the field of computers and automation. These include a great number of people who will make recommendations to their organizations about purchasing computing machinery, similar machinery, and components, and whose decisions may involve very substantial figures. The print order for the August issue was 2,300 copies. The overrun is largely held for eventual sale as back copies, and in the case of several issues the overrun has been exhausted through such sale.

3. What type of advertising does COMPUTERS AND AUTOMATION take? The purpose of the magazine is to be factual and to the point. For this purpose the kind of advertising wanted is the kind that answers questions factually. We recommend for the audience that we reach, that advertising be factual, useful, interesting, understandable, and new from issue to issue.

4. What are the specifications and cost of advertising? COMPUTERS AND AUTOMATION is published on pages 8½" x 11" (ad size, 7" x 10") and produced by photooffset, except that printed sheet advertising may be inserted and bound in with the magazine in most cases. The closing date for any issue is approximately the 10th of the month preceding. If possible, the company advertising should produce final copy. For photooffset, the copy should be exactly as desired, actual size, and assembled, and may include typing, writing, line drawing, printing, screened half tones, and any other copy that may be put under the photooffset camera without further preparation. Unscreened

photographic prints and any other copy requiring additional preparation for photooffset should be furnished separately; it will be prepared, finished, and charged to the advertiser at small additional costs. In the case of printed inserts, a sufficient quantity for the issue should be shipped to our printer, address on request.

Display advertising is sold in units of full pages (ad size 7" x 10", basic rate, \$170) and half pages (basic rate, \$90); back cover, \$330; inside front or back cover, \$210. Extra for color red (full pages only and only in certain positions), 35%. Two-page printed insert (one sheet), \$290; four-page printed insert (two sheets), \$530. Classified advertising is sold by the word (50 cents a word) with a minimum of ten words. We reserve the right not to accept advertising that does not meet our standards.

5. Who are our advertisers? Our advertisers in recent issues have included the following companies, among others:

The Austin Co.  
Automatic Electric Co.  
Cambridge Thermionic Corp.  
Federal Telephone and Radio Co.  
Ferranti Electric Co.  
Ferroxcube Corp. of America  
General Ceramics Corp.  
General Electric Co.  
Hughes Research and Development Lab.  
International Business Machines Corp.  
Laboratory for Electronics  
Lockheed Aircraft Corp.  
Logistics Research, Inc.  
Machine Statistics Co.  
Monrobot Corp.  
Norden-Ketay Corp.  
George A. Philbrick Researches, Inc.  
Potter Instrument Co.  
Raytheon Mfg. Co.  
Reeves Instrument Co.  
Remington Rand, Inc.  
Sprague Electric Co.  
Sylvania Electric Products, Inc.  
Telecomputing Corp.

Mathematical Analyst Keith Kersery loads jet transport flutter problem into one of Lockheed's two 701's. On order: two 704's to help keep Lockheed in forefront of numerical analysis and production control data processing.



The first airframe manufacturer to order and receive a 701 digital computer, Lockheed has now received a second 701 to handle a constantly increasing computing work load. It gives Lockheed the largest installation of digital computing machines in private industry.

Most of the work in process is classified. However, two significant features to the career-minded Mathematical Analyst are: 1) the wide variety of assignments caused by Lockheed's diversification and 2) the advanced nature of the work, which consists mainly of developing new approaches to aeronautical problems.

#### **Career Opportunities for Mathematical Analysts**

Lockheed's expanding development program in nuclear energy, turbo-prop and jet transports, radar search planes, supersonic aircraft and other classified projects has created a number of openings for Mathematical Analysts to work on the 701's.

Lockheed offers you attractive salaries; generous travel and moving allowances; an opportunity to enjoy Southern California life; and an extremely wide range of employee benefits which add approximately 14% to each engineer's salary in the form of insurance, retirement pension, sick leave with pay, etc.

Those interested are invited to write E. W. Des Lauriers for a brochure describing life and work at Lockheed and an application form.

## **New 701's speed Lockheed research in numerical analysis**



**LOCKHEED** AIRCRAFT CORPORATION  
BURBANK **CALIFORNIA**

# ADVERTISING INDEX

The purpose of COMPUTERS AND AUTOMATION is to be factual, useful, and understandable. For this purpose, the kind of advertising we desire to publish is the kind that answers questions, such as: What are your products? What are your services? And for each product: What is it called? What does it do? How well does it work? What are its main specifications? We reserve the right not to accept advertising that does not meet our standards.

Following is the index and a summary of advertisements. Each item contains: Name and address of the advertiser / subject of the advertisement / page number where it appears.

Bendix Aviation Corp, Research Laboratories Div., 4855 4th Ave, Detroit 1, Mich. / Analog Computer Engineers / page 32  
 Cambridge Thermionic Corp., 447 Concord Ave., Cambridge 38, Mass. / The Terminal / page 2

Ferroxcube Corp., East Bridge St., Saugerties, N. Y. / Magnetic Core Materials / page 37  
 Hughes Research and Development Laboratories, Culver City, Calif. / Engineers Wanted / page 39  
 Lockheed Aircraft Corp., Burbank, Calif. / Career Opportunities / page 41  
 Monrobot Corporation, Morris Plains, N. J. / Computer Components / page 35  
 Remington Rand, Inc., 315 4th Ave., New York 10, N.Y. / UNIVAC / page 43  
 Remington Rand, Inc., ERA Division, 1902 West Minnehaha Ave., St. Paul, Minn. / Key to your Future Engineers / page 37  
 Republic Aviation Corp., Farmingdale, L. I., N. Y. / Engineers Wanted / page 39  
 Sprague Electric Co., 377 Marshall St. North Adams, Mass. / Components / page 44  
 Sylvania Electric Products, Inc., Missile Systems Laboratory, 151 Needham St., Newton, Mass. / Help Wanted / page 5

\*-----\*

BINARY ARITHMETIC  
(continued from page 33)

TABLE 1

<u>Fraction</u>	<u>Binary Equivalent</u>
1/3	.01,01,
1/5	.0011,0011,
1/7	.001,001,
1/9	.000111,000111,
1/11	.0001011101,0001011101,
1/13	.000100111011,000100111011,
1/15	.0001,0001,
1/17	.00001111,00001111,
1/19	.000011010111100101,000011010111100101,
1/21	.000011,000011,
1/23	.00001011001,00001011001,
1/25	.00001010001111010111,00001010001111010111,
1/27	.000010010111101101,000010010111101101,
1/29	.0000100011010011110111001011, etc.
1/31	.00001,00001,
1/33	.0000011111,0000011111,
1/35	.000001110101,000001110101,
1/37	.000001101110101100111110010001010011, etc.
1/39	.000001101001,000001101001,
1/41	.00000110001111100111,00000110001111100111,
1/43	.00000101111101,00000101111101,
1/45	.000001011011,000001011011,
1/47	.00000101011100100110001,00000101011100100110001,
1/49	.000001010011100101111,000001010011100101111,

(Note: The commas divide up the repeating period of these numbers).





## Now... Univac Tells Itself What To Do!

Univac no longer asks for the detailed instruction codes required by other computers. Univac now *automatically* produces complex coded routines when given a simple instruction.

- This truly remarkable new Remington

Rand development cuts months from programming time... is easily adaptable to your individual requirements.

**If you would like more information about Univac automatic programming, write to the address below for EL264.**

ELECTRONIC COMPUTER DEPARTMENT **Remington Rand** ROOM 1818, 315 FOURTH AVE., NEW YORK 10, N. Y.  
DIVISION OF SPERRY RAND CORPORATION



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ENGINEERING DEVELOPMENT  
OF CIRCUITS USING  
PULSE TRANSFORMERS**

CHARACTERISTICS OF KIT TRANSFORMERS

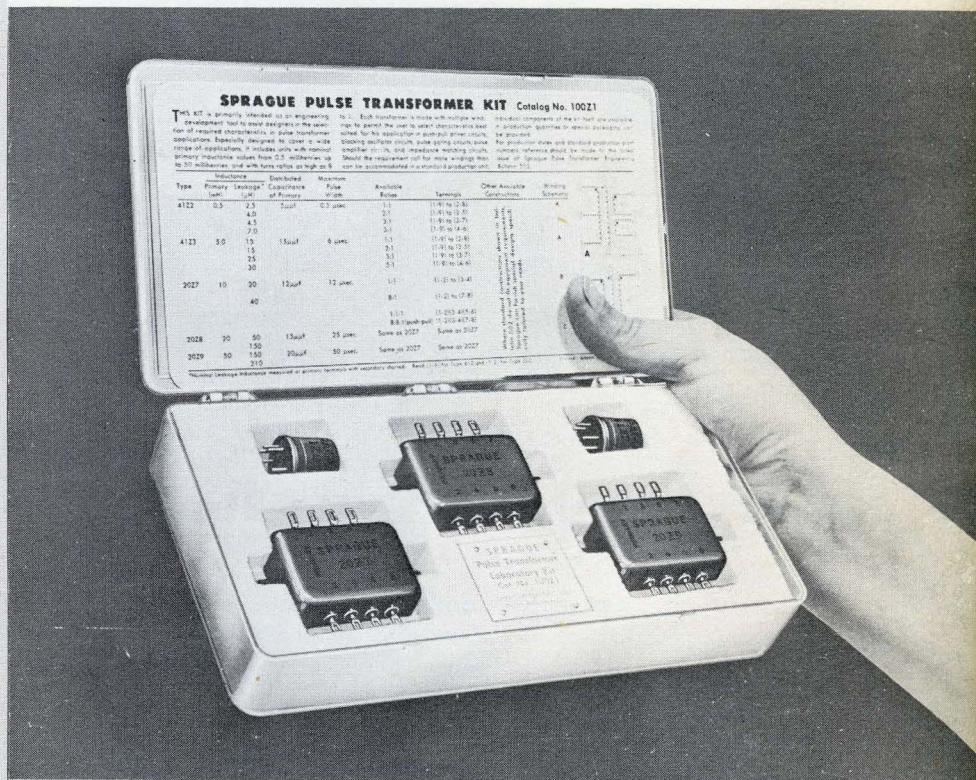
Type	Induct. Pri. ( $\mu$ H)	Leakage ( $\mu$ H)	Dist. Cap. of Pri. ( $\mu$ F)	Max. Nom. P.W. Range ( $\mu$ SEC)	Avail. Ratios.
4122	0.5	2.5	5	0.5	1:1
		4.0			2:1
		4.5			3:1
		7.0			5:1
4123	5.0	13	15	6	1:1
		15			2:1
		25			3:1
		30			5:1
2027	10	20	12	12	1:1
		40			8:1
2028	20	50	15	25	same as 2027
		150			
2029	50	150	20	50	same as 2027
		210			

**Sprague  
Pulse  
Transformer  
Kit  
Simplifies  
Circuit  
Design**

*Sprague on request will provide you with complete application engineering service for optimum results in the use of pulse transformers.*

Sprague's new Type 100Z1 Pulse Transformer Kit contains five multiple winding transformers, each chosen for its wide range of practical application. Complete technical data on each of the transformers is included in the instruction card in each kit so that the circuit designer may readily select the required windings to give transformer characteristics best suited for his applications . . . whether it be push-pull driver, blocking oscillator, pulse gating, pulse amplifier, or impedance matching. The electrical characteristics of the transformers in the kit have been designed so that they may be matched by standard Sprague subminiature hermetically-sealed pulse transformers shown in engineering bulletin 502B.

For complete information on this kit, as well as the extensive line of Sprague pulse transformers, write to the Technical Literature Section, Sprague Electric Company, 377 Marshall Street, North Adams, Massachusetts.



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*the mark of reliability*