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"SIMULTANEOUS EQUATIONS A LA KING"

Modifications or revisions to this program, as they occur, will be announced in the appropriate Catalog of Programs for the IBM Data Processing Systems. If such announcement indicates a change to the program decks or tapes, a complete new program, if needed, should be requested from the Program Distribution Center.

Mr. D. N. Leeson
IBM Corporation
425 Park Avenue
New York, New York

Eastern Regional Office
October 16, 1962

MEMORANDUM

TO: Program Information Department
DPD HQ - White Plains

SUBJECT: 1620 Program Number 5.0.008
Simultaneous Equations A La King

Please reproduce this document and include it as part of the documentation of the subject program to advise users of an error condition.

ERROR: Under certain circumstanses, the j-th column was chosen as a pivotal column when the a_{1j} element was not the maximum of the 1-st row. Generally this produced no ill effect; but occasionally a very small element (10^{-20}) was chosen which destroyed all accuracy.

CORRECTION: One card is to be added to the SPS source deck. This new card will appear after the card which bears page and line number 06180. This new card, which will bear page and line number 06190 will read -

06190 bbbbbbCFbb TEMP 3-2

The card which follows 06190 bears page and line number 07010.

Because of the dynamic storage allocation technique used in this program, this correction is given at the source level as opposed to the object level.

This card is to added at the user's location and does not affect any other portions of the program. Thus, the present directions for assembling and running may be followed.

D. N. Leeson

DNL:mk

ABSTRACT

A. Identification

1. Title

Simultaneous Equations A La King

2. Author and Date

D. N. Leeson
2/12/62

3. Installation

ERO

B. Purpose

This program inverts matrices by the elimination method and solves linear systems. All floating point arithmetic is variable in precision thus minimizing error substantially.

C. Machine Configuration

1. Core - 20, 40, 60, 80 or 100K

The maximum rank which may be attacked is a function of:

a. The mantissa size ($2 \leq L \leq 45$)
b. Core available

2. Division - Not required but allows for inversion of larger matrices if present.

3. Edit Instructions - Not required.

4. Indirect Addressing - Not required.

5. Card I/O - Required.

6. Tape I/O - Not applicable.

7. 407 Listing Device desirable but not mandatory.

D. Procedure Description

The elimination method is employed with or without pivoting. After inversion, a matrix-vector multiplication is employed for each constant vector. "Noisy" mode may be employed if desired. Re-inversion for accuracy test may also be requested.

E. Execution Time

This is a function of:

1. Matrix Rank

2. Mantissa Size

3. Presence or absence of division hardware

4. Number of constant vectors

5. Punch requests (variable)

F. Source Language

SPS

G. Accuracy

Floating point, variable precision arithmetic reduces error in most cases to:

$$1 \times 10^{-G}$$

where G is the number of digits of input data. Accuracy is the most important aspect of this program.

H. Capabilities and Limitations

If a mantissa size is chosen which is too small, round-off error produces inaccurate results. With 100K core, it is theoretically possible to invert a 99 x 99 matrix with this program.

The source program must be assembled using SPS with variable precision arithmetic each time a mantissa length or matrix rank is altered.

I, Check Out Status

The following cases were run successfully.

Matrix Rank	Word Size	Input Data Size	Error
3	8	8	$< 10^{-8}$
10*	6	8	All accuracy lost
10*	10	8	All accuracy lost
10*	15	8	$< 10^{-4}$
10*	20	8	$< 10^{-8}$
10*	40	8	$< 10^{-8}$
17	17	8	$< 10^{-8}$
25	20	8	$< 10^{-8}$
30	45	8	$< 10^{-8}$

* The matrix chosen for the 10 x 10 test was the classically ill-conditioned Hilbert matrix.

INDEX

ACKNOWLEDGEMENT

The author wishes to express his gratitude to Mr. Frank S. Beckman whose kind assistance made the preparation of this program possible. Mr. Beckman, who is a fellow-traveler* of the author of this document, gave freely of his extensive knowledge in the field of matrix theory. It was he who first brought to my attention the work of Herman Goldstein and John Von Neumann that will usually enable one to invert successfully** a matrix of order N by the elimination method. Much of the theoretical approach to this problem was done at his kitchen table on Sunday mornings over hot coffee and chopped liver sandwiches.

* We ride the same car pool.

** i.e., obtain a significantly "better", in some sense, approximation to the inverse than the identity matrix.

MACHINE REQUIREMENTS FOR UTILIZATION OF THIS PROGRAM

Any card 1620 system with 20K core can utilize this program. Indirect addressing, strip, fill, move flag and automatic division are not prerequisites. The user with direct divide may take advantage of this feature by utilizing the proper subroutine deck during the "assembly" phase of this program. The user with larger core than 20K may utilize this routine for solutions of larger systems than would be available to the 20K core owner.

INTRODUCTION

One problem which constantly plagues the numerical analyst is rounding and truncation error. This undesirable situation rears its head in all of computational mathematics and thus, although this problem is for a specific application, the ideas contained herein are more universal than the application would imply.

Technically, there are three type of errors associated with any computer program. These are:

1. Errors contained in the input data.
2. Truncation errors.
3. Round-off errors.

Absolutely nothing can be done about the first type of error since this is a problem of measurement before the computer goes into play. Truncation error arises from terminating an infinite process such as series evaluation. One can always extend the series further for better accuracy. Previously, it was not uncommon to use a variable fixed point word but floating point presented a problem in that one worked in single precision or double precision arithmetic only and even this was only possible on larger computers. This program allows for complete variability of a floating point word thus attacking the round-off error problem.

Every effort has been made to maintain accuracy and the user has a variety of choices to make when he runs data under control of this program; however, in a desire for the maintenance of accuracy timing considerations were totally ignored. This is not to say that the program was purposely built to run slowly. Abraham Lincoln was once asked, "How tall a man should be?" and his answer was "Tall enough so that his legs reach the ground". This program will run until it is finished and beyond this statement I can give no timing considerations. Two cases were timed for general information; the first was the inversion of a 10 x 10 matrix with a 45 digit mantissa and this ran 7 minutes and 20 seconds. The second was a 30 x 30 matrix with a 40 digit mantissa and this ran slightly over 2-1/2 hours. It is the contention of this author that this program is not for the most general use. There are an excellent collection of 1620 programs to invert matrices and solve systems of simultaneous equations. Because of speed factor it is preferable that these should be used. Only in those cases of extremely large matrices or ill-conditioned matrices where 8 digits of precision are not sufficient should this routine be employed.

During the run of a matrix inversion or the solution of a system of simultaneous equations the user has the option of:

1. Pivoting.*
2. Punching the source matrix.
3. Punching the inverted matrix.
4. Inverting the inverse for an accuracy test. This is discussed in the section entitled "Operating Instructions" and "Alteration Switch Settings".
5. Running with "NOISE". This is discussed in the section so headed.

* There is no rigorous statement which maintains that pivoting always yields a better inverse; thus the choice.

PARAMETER CARDS

The deck associated with this documentation is a symbolic programming system source deck and not an object deck. It must be assembled * each time the mantissa size or matrix rank is to be changed. The two cards which describe these two variable parameters will become cards 1 and 2 of the SPS source deck.

The first card has as its label the symbol "LENGTH" punched in columns 6 - 11. Columns 12 and 13 of the same card are to be punched "DS"; column 16 of this card will have a comma; and column 17 or columns 17 and 18 will have a one or two digit number specifying the length of the mantissa requested for this particular assembly. One may, if one wishes, have a page and line number in this card in columns 1 - 5. Note carefully that the mantissa length is required in this card and not the length of the floating point word which is, of course, two greater than the mantissa length.

The second card has as its label the one character symbol "N" punched in column 6. In columns 12 and 13 the declarative "DS" is punched. In column 16 a comma is punched and in column 17 or 17 and 18 a one or two digit number which specifies the rank of the matrix in question.

Thus if one were using this program to invert a 9 x 9 matrix with a mantissa size of 21 the cards would read:

```
00001LENGTHDSbb,21
00002Nbbbbb DSbb,9
```

Column 1 Column 6 Column 12 Column 16

where the small b's specified above represent blanks.

As a further example assume that one wishes to use this program to solve a system of 30 x 30 simultaneous equations and utilizes a 45 digit mantissa size. The parameter cards describing this would be as follows:

```
00001LENGTHDSbb,45
00002Nbbbbb DSbb,30
```

* Assembly, in this instance, refers to assembly utilizing SPS II for use with variable precision floating point arithmetic. SPS I, using excess 50 floating point notation, may not be employed.

OPERATING INSTRUCTIONS

1. Assemble and compress the SIMULTANEOUS EQUATIONS A LA KING source program preceding it by the two cards discussed on the previous page; the first of which has label "LENGTH" and the second of which has label "N".

2. Place behind the assembled program the data as described in the section entitled "Data Preparation". *

3. Clear memory to zeros.

4. Set console switches described in the section so entitled.

5. Place the combined assembled deck and data in the read hopper of the 1622 and depress the load key.

6. The assembled object program will load and halt at the conclusion of the loading procedure. Depress the start key on the 1620 console.

7. At the conclusion of inversion or simultaneous equation solution a re-inversion of the inverse may be obtained by manually branching to location 454. In this way, by knowing the difference between the original input matrix and the input matrix as it appears after the double inversion, one can estimate the accuracy of the results.

8. Should there be a desire to invert matrices of the same rank, or to solve systems of equations whose matrix of coefficients is of the same rank, the program is completely initializable by branching to location 402. However, the same mantissa size is to be assumed.

* Be sure to remove the 2 blank cards which appear at the end of the compressed deck. These cards are a function of the non-process run-out of the 1622.

ALTERATION SWITCH SETTINGS

SWITCH 1 OFF	-	Pivoting for size is requested.
SWITCH 1 ON	-	No pivot.
SWITCH 2 "ON"	-	Punch the source matrix after reading and floating same.
SWITCH 2 "OFF"	-	Source matrix not punched.
SWITCH 3 "ON"	-	Punch the inverted matrix.
SWITCH 3 "OFF"	-	Inverse matrix not punched. NOTE: If using this program for matrix inversion, as opposed to simultaneous equations solution, Switch 3 must be "ON" or no output will occur. This option is given should one wish to solve a system of simultaneous equations and merely receive the solution vectors, instead of the inverted matrix and solution vectors.
SWITCH 4 "ON"	-	Operate in the noisy mode, nine fill.
SWITCH 4 "OFF"	-	Operate in the standard mode, zero fill. NOTE: See description of noisy mode.

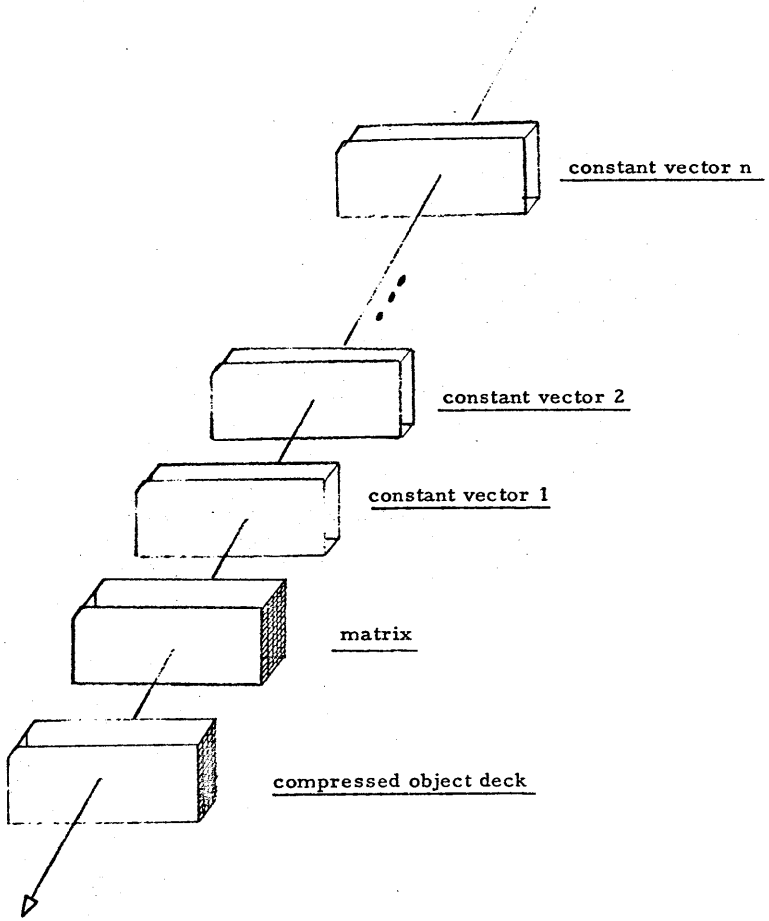
SPECIAL NOTE

This program is always conditioned to solve simultaneous equations with an infinite number of constant vectors. Thus, the program always terminates on a "READER NO FEED" condition. However, in attempting to solve a system of simultaneous equations the program will first read in and invert the matrix of coefficients and then attempt to read a constant vector. Consequently, inverting matrices with this program consists of solving a system of simultaneous equations with no constant vectors.

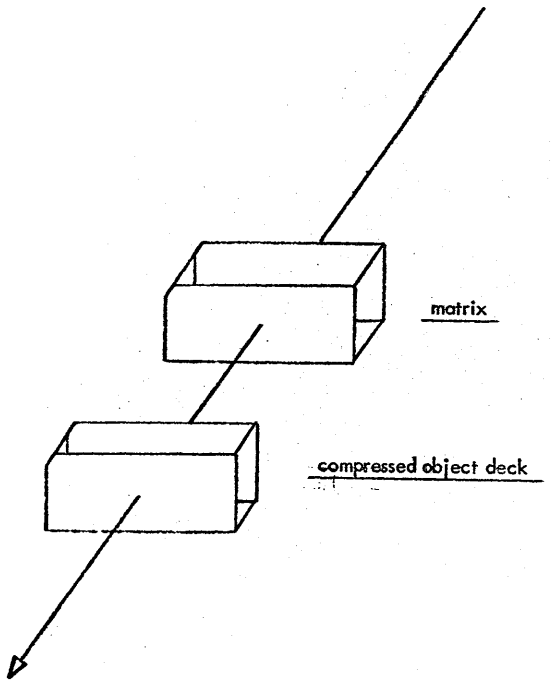
It is for this reason that blank cards placed after the last data card should not be employed, since the program is conditioned to read whatever is in the read hopper and such blank cards may damage the simultaneous equations object program. However, this will have no affect on the results since the damage will be done after the problem has been completed.

This warning is put in this program only in the event that a person wishes to invert more than one matrix of the same rank with the same mantissa size. The way this is performed is to invert number one, obtain one's results, and receive the subsequent "READER NO FEED" condition. Then branch to location 402 and place the second matrix in the read hopper, etc., ad infinitum.

INPUT DECK ARRANGEMENT FOR
SOLUTION OF SIMULTANEOUS EQUATIONS.



INPUT DECK ARRANGEMENT FOR MATRIX INVERSION



DATA PREPARATION

This author is convinced that no matter how one chooses a method of data preparation someone is bound to desire it in another fashion. I sincerely hope that the following description of data preparation will be enjoyed by a majority of the users of this program.

Each piece of datum will occupy one card. It is punched in free form, with decimal point anywhere between columns 1 and 40 inclusive. The plus sign is optional. A minus sign is, of course, mandatory. E type format is not acceptable. The matrix of coefficients is punched by rows: viz., element A12 follows element A11, etc. The following example demonstrates data preparation for a 3 x 3 system of equations with two constant vectors.

$$\begin{aligned} 3X_1+2\ X_2-\ \ 7X_3&= \\ 8X_1-9\ X_2+.0004X_3&= \\ 9527.56X_1+17X_2+\ \ 7X_3&= \end{aligned} \quad \begin{bmatrix} 15 \\ -9 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 15861 \\ .00009 \\ 38 \end{bmatrix}$$

Data input for the above problem punched anywhere between columns 1 and 40 inclusive.

```
    3.
    2.
   -7.
    8.
   -9.
     .0004
 9527.56
   17.
    7.
   15.
   -9.
    0.
15861.
     .00009
   38.
```

In the data listed above the decimal point is aligned for clarity, however, this is by no means a restriction of the system. The data may begin anywhere between columns 1 and 40.

DESCRIPTION OF CARD OUTPUT

If a 407 listing device is available to the user he may choose to wire the panel as described in this program. However, if no such listing device is available to him, or if an alternate listing device is available to him, it is necessary to know the output format of the cards produced by this program. They are as follows:

1. MATRIX CARDS. The mantissa of the floating point number has its high order position in column 3 and extends for a maximum of 45 digits. If this mantissa is negative a flagged zero appears in column 2 of the output card. The characteristic associated with this mantissa appears in columns 51 and 52. If the characteristic is negative a flagged zero appears in column 50. In columns 75 and 76 appears the row index of this particular element. In 79 and 80 appears the column index of this element.

2. CONSTANT VECTOR AND SOLUTION VECTOR CARDS. The description of these cards is identical with that of the matrix cards except that there is, of course, no row or column index in columns 75-76 and 79-80.

3. ALPHABETIC HEADER CARDS. Preceding the matrix, inverse, solution vector or constant vector will be an alphabetic card which states what is to follow. This card is always alphameric and is punched in columns 1 through 17.

The user will note for printing purposes, if it is necessary for him to wire his own panel, that an alphabetic card is determined by the presence of a digit in column 1. No numeric card ever has a digit in column 1.

NOISY MODE

During the normalization operations associated with floating point arithmetic the first significant digit appears at the high order position of the mantissa; thus, the number .01 appears as $\bar{1}00\ldots00\bar{0}\bar{1}$ and not as $\bar{0}10\ldots00\bar{0}0$.

If a number requires normalization, the general concept is to shift the number left and introduce low order zeros. This operation is called zero fill; but zero fill is generally performed because of a complete ignorance of what truly constitutes the data beyond the last known digit. Thus the number $.05\times10^{0}$ becomes $.50\times10^{-1}$ only because the digit following the number 5 is not known. Introducing digits other than zero during this fill operation is sometimes termed the "noisy" mode. This fill operation can be performed with any digit one wishes, zero fill, nine fill, and random digit fill, but the use of this becomes interesting and significant when one chooses to run a problem twice: once in the standard mode with zero fill and once in the noisy mode with nine fill. In this way, by examining the difference between results, one can form an opinion about whether the computation has been significantly affected by round-off error. The noisy mode makes no attempt at anything other than this.

If one runs the problem in both modes and no significant difference is noticeable between results of zero fill and nine fill, it is highly plausible that the results are good. Much numerical experiment of these points remains to be done.

The noisy mode may be employed at anytime the user desires it. In general, filling operations have no affect if the word chosen is larger than necessary. As an example of this one might consider the inversion of a 10 x 10 matrix with a 6 digit mantissa which is generally too small. Running this problem twice, once in the standard mode and once in the noisy mode, the user might notice that there is a significant difference between his results in case one as opposed to case two. One is able to determine, therefore, that in addition to other factors, round-off error played a significant role in the difficulties encountered.

This author presents the noisy mode for use in this program as a tool for investigational purposes.

TABLE OF MINIMUM AND SUGGESTED LENGTHS FOR SPECIFIC N

The following table gives the statistically minimum length for N ranging from 2 to 99. Use of a length greater than the suggested length is, of course, permissible should core size be available. *

* See Table of Core Requirements For Specific N and LENGTH

TABLE OF LENGTHS FOR SPECIFIC N

N	MINIMUM LENGTH	SUGGESTED LENGTH
2	5	8
3	6	9
4	6	9
5	6	9
6	7	10
7	7	10
8	7	10
9	7	10
10	7	10
11	7	10
12	7	10
13	8	11
14	8	11
15	8	11
16	8	11
17	8	11
18	8	11
19	8	11
20	8	11
21	8	11
22	8	11
23	8	11
24	8	11
25	8	11
26	8	11
27	8	11
28	9	12
29	9	12
30	9	12
31	9	12
32	9	12
33	9	12
34	9	12
35	9	12
36	9	12
37	9	12
38	9	12
39	9	12
40	9	12
41	9	12
42	9	12
43	9	12
44	9	12
45	9	12
46	9	12
47	9	12
48	9	12
49	9	12
50	9	12
51	9	12
52	9	12
53	9	12
54	9	12
55	9	12
56	9	12
57	9	12
58	9	12
59	10	13
60	10	13

61	10	13
62	10	13
63	10	13
64	10	13
65	10	13
66	10	13
67	10	13
68	10	13
69	10	13
70	10	13
71	10	13
72	10	13
73	10	13
74	10	13
75	10	13
76	10	13
77	10	13
78	10	13
79	10	13
80	10	13
81	10	13
82	10	13
83	10	13
84	10	13
85	10	13
86	10	13
87	10	13
88	10	13
89	10	13
90	10	13
91	10	13
92	10	13
93	10	13
94	10	13
95	10	13
96	10	13
97	10	13
98	10	13
99	10	13

TABLE OF CORE REQUIREMENTS FOR SPECIFIC N AND LENGTH ASSUMING AUTOMATIC DIVISION HARDWARE *

The following is a complete table showing core requirements for all possibilities of this program. For instance, should one desire to invert a 30 x 30 matrix with mantissa length 12 it is noted in the attached document that this requires 20,510 positions. Thus an individual with one module of core memory would, by necessity, choose the 11 digit mantissa which requires a total core of 19,575 positions.

For completeness, all possibilities are shown from the 2 x 2 matrix with a mantissa size of 2 to a 99 x 99 matrix with a mantissa length of 7. Much of this material is, of course, without meaning since a matrix of rank 99 probably could not be inverted with a mantissa size this small. However, previous experience has shown that whenever anything is left out of a computer program someone is bound to ask just for that. Thus peculiar cases such as a 2 x 2 matrix with a 45 digit mantissa and 99 x 99 matrix with a 2 digit mantissa are included.

Also the attached document demonstrates core requirements above 60,000 positions. This is for those cases where additional core up to 100 K is available.

It is through the use of the following pages that compromises can be made between desired mantissa size and available core; for instance, the inversion of a 52 x 52 matrix with a 18 digit mantissa requires 62,728 core positions. A compromise of a 17 digit mantissa would allow this matrix to be run successfully on a 17 digit mantissa which requires a total of 55,967 core positions.

The core requirements shown include everything necessary for the solution of the problem.

* For software divide subroutine decks add 975 core positions. For SPS subroutines with indirect addressing permitted add 109 core positions for hardware divide or 1084 core positions for software divide.

N	LENGTH	CORE REQUIREMENT
2	2	7352
2	3	7363
2	4	7374
2	5	7385
2	6	7396
2	7	7407
2	8	7418
2	9	7429
2	10	7440
2	11	7451
2	12	7462
2	13	7473
2	14	7484
2	15	7495
2	16	7506
2	17	7517
2	18	7528
2	19	7539
2	20	7550
2	21	7561
2	22	7572
2	23	7583
2	24	7594
2	25	7605
2	26	7616
2	27	7627
2	28	7638
2	29	7649
2	30	7660
2	31	7671
2	32	7682
2	33	7693
2	34	7704
2	35	7715
2	36	7726
2	37	7737
2	38	7748
2	39	7759
2	40	7770
2	41	7781
2	42	7792
2	43	7803
2	44	7814
2	45	7825
N	LENGTH	CORE REQUIREMENT
3	2	7380
3	3	7397
3	4	7414
3	5	7431
3	6	7448
3	7	7465
3	8	7482
3	9	7499
3	10	7516

3	11	7533
3	12	7550
3	13	7567
3	14	7584
3	15	7601
3	16	7618
3	17	7635
3	18	7652
3	19	7669
3	20	7686
3	21	7703
3	22	7720
3	23	7737
3	24	7754
3	25	7771
3	26	7788
3	27	7805
3	28	7822
3	29	7839
3	30	7856
3	31	7873
3	32	7890
3	33	7907
3	34	7924
3	35	7941
3	36	7958
3	37	7975
3	38	7992
3	39	8009
3	40	8026
3	41	8043
3	42	8060
3	43	8077
3	44	8094
3	45	8111
N	LENGTH	CORE REQUIREMENT
4	2	7416
4	3	7441
4	4	7466
4	5	7491
4	6	7516
4	7	7541
4	8	7566
4	9	7591
4	10	7616
4	11	7641
4	12	7666
4	13	7691
4	14	7716
4	15	7741
4	16	7766
4	17	7791
4	18	7816
4	19	7841
4	20	7866

4	21	7891
4	22	7916
4	23	7941
4	24	7966
4	25	7991
4	26	8016
4	27	8041
4	28	8066
4	29	8091
4	30	8116
4	31	8141
4	32	8166
4	33	8191
4	34	8216
4	35	8241
4	36	8266
4	37	8291
4	38	8316
4	39	8341
4	40	8366
4	41	8391
4	42	8416
4	43	8441
4	44	8466
4	45	8491

N	LENGTH	CORE REQUIREMENT
5	2	7460
5	3	7495
5	4	7530
5	5	7565
5	6	7600
5	7	7635
5	8	7670
5	9	7705
5	10	7740
5	11	7775
5	12	7810
5	13	7845
5	14	7880
5	15	7915
5	16	7950
5	17	7985
5	18	8020
5	19	8055
5	20	8090
5	21	8125
5	22	8160
5	23	8195
5	24	8230
5	25	8265
5	26	8300
5	27	8335
5	28	8370
5	29	8405
5	30	8440

5	31	8475
5	32	8510
5	33	8545
5	34	8580
5	35	8615
5	36	8650
5	37	8685
5	38	8720
5	39	8755
5	40	8790
5	41	8825
5	42	8860
5	43	8895
5	44	8930
5	45	8965

N	LENGTH	CORE REQUIREMENT
6	2	7512
6	3	7559
6	4	7606
6	5	7653
6	6	7700
6	7	7747
6	8	7794
6	9	7841
6	10	7888
6	11	7935
6	12	7982
6	13	8029
6	14	8076
6	15	8123
6	16	8170
6	17	8217
6	18	8264
6	19	8311
6	20	8358
6	21	8405
6	22	8452
6	23	8499
6	24	8546
6	25	8593
6	26	8640
6	27	8687
6	28	8734
6	29	8781
6	30	8828
6	31	8875
6	32	8922
6	33	8969
6	34	9016
6	35	9063
6	36	9110
6	37	9157
6	38	9204
6	39	9251
6	40	9298

6	41	9345
6	42	9392
6	43	9439
6	44	9486
6	45	9533

N	LENGTH	CORE REQUIREMENT
7	2	7572
7	3	7633
7	4	7694
7	5	7755
7	6	7816
7	7	7877
7	8	7938
7	9	7999
7	10	8060
7	11	8121
7	12	8182
7	13	8243
7	14	8304
7	15	8365
7	16	8426
7	17	8487
7	18	8548
7	19	8609
7	20	8670
7	21	8731
7	22	8792
7	23	8853
7	24	8914
7	25	8975
7	26	9036
7	27	9097
7	28	9158
7	29	9219
7	30	9280
7	31	9341
7	32	9402
7	33	9463
7	34	9524
7	35	9585
7	36	9646
7	37	9707
7	38	9768
7	39	9829
7	40	9890
7	41	9951
7	42	10012
7	43	10073
7	44	10134
7	45	10195

N	LENGTH	CORE REQUIREMENT
8	2	7640
8	3	7717
8	4	7794
8	5	7871

8	6	7948
8	7	8025
8	8	8102
8	9	8179
8	10	8256
8	11	8333
8	12	8410
8	13	8487
8	14	8564
8	15	8641
8	16	8718
8	17	8795
8	18	8872
8	19	8949
8	20	9026
8	21	9103
8	22	9180
8	23	9257
8	24	9334
8	25	9411
8	26	9488
8	27	9565
8	28	9642
8	29	9719
8	30	9796
8	31	9873
8	32	9950
8	33	10027
8	34	10104
8	35	10181
8	36	10258
8	37	10335
8	38	10412
8	39	10489
8	40	10566
8	41	10643
8	42	10720
8	43	10797
8	44	10874
8	45	10951

N	LENGTH	CORE REQUIREMENT
9	2	7716
9	3	7811
9	4	7906
9	5	8001
9	6	8096
9	7	8191
9	8	8286
9	9	8381
9	10	8476
9	11	8571
9	12	8666
9	13	8761
9	14	8856
9	15	8951

9	16	9046
9	17	9141
9	18	9236
9	19	9331
9	20	9426
9	21	9521
9	22	9616
9	23	9711
9	24	9806
9	25	9901
9	26	9996
9	27	10091
9	28	10186
9	29	10281
9	30	10376
9	31	10471
9	32	10566
9	33	10661
9	34	10756
9	35	10851
9	36	10946
9	37	11041
9	38	11136
9	39	11231
9	40	11326
9	41	11421
9	42	11516
9	43	11611
9	44	11706
9	45	11801

N	LENGTH	CORE REQUIREMENT
10	2	7800
10	3	7915
10	4	8030
10	5	8145
10	6	8260
10	7	8375
10	8	8490
10	9	8605
10	10	8720
10	11	8835
10	12	8950
10	13	9065
10	14	9180
10	15	9295
10	16	9410
10	17	9525
10	18	9640
10	19	9755
10	20	9870
10	21	9985
10	22	10100
10	23	10215
10	24	10330
10	25	10445

10	26	10560
10	27	10675
10	28	10790
10	29	10905
10	30	11020
10	31	11135
10	32	11250
10	33	11365
10	34	11480
10	35	11595
10	36	11710
10	37	11825
10	38	11940
10	39	12055
10	40	12170
10	41	12285
10	42	12400
10	43	12515
10	44	12630
10	45	12745

N	LENGTH	CORE REQUIREMENT
11	2	7892
11	3	8029
11	4	8166
11	5	8303
11	6	8440
11	7	8577
11	8	8714
11	9	8851
11	10	8988
11	11	9125
11	12	9262
11	13	9399
11	14	9536
11	15	9673
11	16	9810
11	17	9947
11	18	10084
11	19	10221
11	20	10358
11	21	10495
11	22	10632
11	23	10769
11	24	10906
11	25	11043
11	26	11180
11	27	11317
11	28	11454
11	29	11591
11	30	11728
11	31	11865
11	32	12002
11	33	12139
11	34	12276
11	35	12413

11	36	12550
11	37	12687
11	38	12824
11	39	12961
11	40	13098
11	41	13235
11	42	13372
11	43	13509
11	44	13646
11	45	13783

N	LENGTH	CORE REQUIREMENT
12	2	7992
12	3	8153
12	4	8314
12	5	8475
12	6	8636
12	7	8797
12	8	8958
12	9	9119
12	10	9280
12	11	9441
12	12	9602
12	13	9763
12	14	9924
12	15	10085
12	16	10246
12	17	10407
12	18	10568
12	19	10729
12	20	10890
12	21	11051
12	22	11212
12	23	11373
12	24	11534
12	25	11695
12	26	11856
12	27	12017
12	28	12178
12	29	12339
12	30	12500
12	31	12661
12	32	12822
12	33	12983
12	34	13144
12	35	13305
12	36	13466
12	37	13627
12	38	13788
12	39	13949
12	40	14110
12	41	14271
12	42	14432
12	43	14593
12	44	14754
12	45	14915

N	LENGTH	CORE REQUIREMENT
13	2	8100
13	3	8287
13	4	8474
13	5	8661
13	6	8848
13	7	9035
13	8	9222
13	9	9409
13	10	9596
13	11	9783
13	12	9970
13	13	10157
13	14	10344
13	15	10531
13	16	10718
13	17	10905
13	18	11092
13	19	11279
13	20	11466
13	21	11653
13	22	11840
13	23	12027
13	24	12214
13	25	12401
13	26	12588
13	27	12775
13	28	12962
13	29	13149
13	30	13336
13	31	13523
13	32	13710
13	33	13897
13	34	14084
13	35	14271
13	36	14458
13	37	14645
13	38	14832
13	39	15019
13	40	15206
13	41	15393
13	42	15580
13	43	15767
13	44	15954
13	45	16141

N	LENGTH	CORE REQUIREMENT
14	2	8216
14	3	8431
14	4	8646
14	5	8861
14	6	9076
14	7	9291
14	8	9506
14	9	9721
14	10	9936

14	11	10151
14	12	10366
14	13	10581
14	14	10796
14	15	11011
14	16	11226
14	17	11441
14	18	11656
14	19	11871
14	20	12086
14	21	12301
14	22	12516
14	23	12731
14	24	12946
14	25	13161
14	26	13376
14	27	13591
14	28	13806
14	29	14021
14	30	14236
14	31	14451
14	32	14666
14	33	14881
14	34	15096
14	35	15311
14	36	15526
14	37	15741
14	38	15956
14	39	16171
14	40	16386
14	41	16601
14	42	16816
14	43	17031
14	44	17246
14	45	17461

N	LENGTH	CORE REQUIREMENT
15	2	8340
15	3	8585
15	4	8830
15	5	9075
15	6	9320
15	7	9565
15	8	9810
15	9	10055
15	10	10300
15	11	10545
15	12	10790
15	13	11035
15	14	11280
15	15	11525
15	16	11770
15	17	12015
15	18	12260
15	19	12505
15	20	12750

15	21	12995
15	22	13240
15	23	13485
15	24	13730
15	25	13975
15	26	14220
15	27	14465
15	28	14710
15	29	14955
15	30	15200
15	31	15445
15	32	15690
15	33	15935
15	34	16180
15	35	16425
15	36	16670
15	37	16915
15	38	17160
15	39	17405
15	40	17650
15	41	17895
15	42	18140
15	43	18385
15	44	18630
15	45	18875

N	LENGTH	CORE REQUIREMENT
16	2	8472
16	3	8749
16	4	9026
16	5	9303
16	6	9580
16	7	9857
16	8	10134
16	9	10411
16	10	10688
16	11	10965
16	12	11242
16	13	11519
16	14	11796
16	15	12073
16	16	12350
16	17	12627
16	18	12904
16	19	13181
16	20	13458
16	21	13735
16	22	14012
16	23	14289
16	24	14566
16	25	14843
16	26	15120
16	27	15397
16	28	15674
16	29	15951
16	30	16228

16	31	16505
16	32	16782
16	33	17059
16	34	17336
16	35	17613
16	36	17890
16	37	18167
16	38	18444
16	39	18721
16	40	18998
16	41	19275
16	42	19552
16	43	19829
16	44	20106
16	45	20383

N	LENGTH	CORE REQUIREMENT
17	2	8612
17	3	8923
17	4	9234
17	5	9545
17	6	9856
17	7	10167
17	8	10478
17	9	10789
17	10	11100
17	11	11411
17	12	11722
17	13	12033
17	14	12344
17	15	12655
17	16	12966
17	17	13277
17	18	13588
17	19	13899
17	20	14210
17	21	14521
17	22	14832
17	23	15143
17	24	15454
17	25	15765
17	26	16076
17	27	16387
17	28	16698
17	29	17009
17	30	17320
17	31	17631
17	32	17942
17	33	18253
17	34	18564
17	35	18875
17	36	19186
17	37	19497
17	38	19808
17	39	20119
17	40	20430

17	41	20741
17	42	21052
17	43	21363
17	44	21674
17	45	21985

N	LENGTH	CORE REQUIREMENT
18	2	8760
18	3	9107
18	4	9454
18	5	9801
18	6	10148
18	7	10495
18	8	10842
18	9	11189
18	10	11536
18	11	11883
18	12	12230
18	13	12577
18	14	12924
18	15	13271
18	16	13618
18	17	13965
18	18	14312
18	19	14659
18	20	15006
18	21	15353
18	22	15700
18	23	16047
18	24	16394
18	25	16741
18	26	17088
18	27	17435
18	28	17782
18	29	18129
18	30	18476
18	31	18823
18	32	19170
18	33	19517
18	34	19864
18	35	20211
18	36	20558
18	37	20905
18	38	21252
18	39	21599
18	40	21946
18	41	22293
18	42	22640
18	43	22987
18	44	23334
18	45	23681

N	LENGTH	CORE REQUIREMENT
19	2	8916
19	3	9301
19	4	9686
19	5	10071

19	6	10456
19	7	10841
19	8	11226
19	9	11611
19	10	11996
19	11	12381
19	12	12766
19	13	13151
19	14	13536
19	15	13921
19	16	14306
19	17	14691
19	18	15076
19	19	15461
19	20	15846
19	21	16231
19	22	16616
19	23	17001
19	24	17386
19	25	17771
19	26	18156
19	27	18541
19	28	18926
19	29	19311
19	30	19696
19	31	20081
19	32	20466
19	33	20851
19	34	21236
19	35	21621
19	36	22006
19	37	22391
19	38	22776
19	39	23161
19	40	23546
19	41	23931
19	42	24316
19	43	24701
19	44	25086
19	45	25471

N	LENGTH	CORE REQUIREMENT
20	2	9080
20	3	9505
20	4	9930
20	5	10355
20	6	10780
20	7	11205
20	8	11630
20	9	12055
20	10	12480
20	11	12905
20	12	13330
20	13	13755
20	14	14180
20	15	14605

20	16	15030
20	17	15455
20	18	15880
20	19	16305
20	20	16730
20	21	17155
20	22	17580
20	23	18005
20	24	18430
20	25	18855
20	26	19280
20	27	19705
20	28	20130
20	29	20555
20	30	20980
20	31	21405
20	32	21830
20	33	22255
20	34	22680
20	35	23105
20	36	23530
20	37	23955
20	38	24380
20	39	24805
20	40	25230
20	41	25655
20	42	26080
20	43	26505
20	44	26930
20	45	27355

N	LENGTH	CORE REQUIREMENT
21	2	9252
21	3	9719
21	4	10186
21	5	10653
21	6	11120
21	7	11587
21	8	12054
21	9	12521
21	10	12988
21	11	13455
21	12	13922
21	13	14389
21	14	14856
21	15	15323
21	16	15790
21	17	16257
21	18	16724
21	19	17191
21	20	17658
21	21	18125
21	22	18592
21	23	19059
21	24	19526
21	25	19993

21	26	20460
21	27	20927
21	28	21394
21	29	21861
21	30	22328
21	31	22795
21	32	23262
21	33	23729
21	34	24196
21	35	24663
21	36	25130
21	37	25597
21	38	26064
21	39	26531
21	40	26998
21	41	27465
21	42	27932
21	43	28399
21	44	28866
21	45	29333

N	LENGTH	CORE REQUIREMENT
22	2	9432
22	3	9943
22	4	10454
22	5	10965
22	6	11476
22	7	11987
22	8	12498
22	9	13009
22	10	13520
22	11	14031
22	12	14542
22	13	15053
22	14	15564
22	15	16075
22	16	16586
22	17	17097
22	18	17608
22	19	18119
22	20	18630
22	21	19141
22	22	19652
22	23	20163
22	24	20674
22	25	21185
22	26	21696
22	27	22207
22	28	22718
22	29	23229
22	30	23740
22	31	24251
22	32	24762
22	33	25273
22	34	25784
22	35	26295

22	36	26806
22	37	27317
22	38	27828
22	39	28339
22	40	28850
22	41	29361
22	42	29872
22	43	30383
22	44	30894
22	45	31405

N	LENGTH	CORE REQUIREMENT
23	2	9620
23	3	10177
23	4	10734
23	5	11291
23	6	11848
23	7	12405
23	8	12962
23	9	13519
23	10	14076
23	11	14633
23	12	15190
23	13	15747
23	14	16304
23	15	16861
23	16	17418
23	17	17975
23	18	18532
23	19	19089
23	20	19646
23	21	20203
23	22	20760
23	23	21317
23	24	21874
23	25	22431
23	26	22988
23	27	23545
23	28	24102
23	29	24659
23	30	25216
23	31	25773
23	32	26330
23	33	26887
23	34	27444
23	35	28001
23	36	28558
23	37	29115
23	38	29672
23	39	30229
23	40	30786
23	41	31343
23	42	31900
23	43	32457
23	44	33014
23	45	33571

N	LENGTH	CORE REQUIREMENT
24	2	9816
24	3	10421
24	4	11026
24	5	11631
24	6	12236
24	7	12841
24	8	13446
24	9	14051
24	10	14656
24	11	15261
24	12	15866
24	13	16471
24	14	17076
24	15	17681
24	16	18286
24	17	18891
24	18	19496
24	19	20101
24	20	20706
24	21	21311
24	22	21916
24	23	22521
24	24	23126
24	25	23731
24	26	24336
24	27	24941
24	28	25546
24	29	26151
24	30	26756
24	31	27361
24	32	27966
24	33	28571
24	34	29176
24	35	29781
24	36	30386
24	37	30991
24	38	31596
24	39	32201
24	40	32806
24	41	33411
24	42	34016
24	43	34621
24	44	35226
24	45	35831
N	LENGTH	CORE REQUIREMENT
25	2	10020
25	3	10675
25	4	11330
25	5	11985
25	6	12640
25	7	13295
25	8	13950
25	9	14605
25	10	15260

25	11	15915
25	12	16570
25	13	17225
25	14	17880
25	15	18535
25	16	19190
25	17	19845
25	18	20500
25	19	21155
25	20	21810
25	21	22465
25	22	23120
25	23	23775
25	24	24430
25	25	25085
25	26	25740
25	27	26395
25	28	27050
25	29	27705
25	30	28360
25	31	29015
25	32	29670
25	33	30325
25	34	30980
25	35	31635
25	36	32290
25	37	32945
25	38	33600
25	39	34255
25	40	34910
25	41	35565
25	42	36220
25	43	36875
25	44	37530
25	45	38185
N	LENGTH	CORE REQUIREMENT
26	2	10232
26	3	10939
26	4	11646
26	5	12353
26	6	13060
26	7	13767
26	8	14474
26	9	15181
26	10	15888
26	11	16595
26	12	17302
26	13	18009
26	14	18716
26	15	19423
26	16	20130
26	17	20837
26	18	21544
26	19	22251
26	20	22958

26	21	23665
26	22	24372
26	23	25079
26	24	25786
26	25	26493
26	26	27200
26	27	27907
26	28	28614
26	29	29321
26	30	30028
26	31	30735
26	32	31442
26	33	32149
26	34	32856
26	35	33563
26	36	34270
26	37	34977
26	38	35684
26	39	36391
26	40	37098
26	41	37805
26	42	38512
26	43	39219
26	44	39926
26	45	40633

N	LENGTH	CORE REQUIREMENT
27	2	10452
27	3	11213
27	4	11974
27	5	12735
27	6	13496
27	7	14257
27	8	15018
27	9	15779
27	10	16540
27	11	17301
27	12	18062
27	13	18823
27	14	19584
27	15	20345
27	16	21106
27	17	21867
27	18	22628
27	19	23389
27	20	24150
27	21	24911
27	22	25672
27	23	26433
27	24	27194
27	25	27955
27	26	28716
27	27	29477
27	28	30238
27	29	30999
27	30	31760

27	31	32521
27	32	33282
27	33	34043
27	34	34804
27	35	35565
27	36	36326
27	37	37087
27	38	37848
27	39	38609
27	40	39370
27	41	40131
27	42	40892
27	43	41653
27	44	42414
27	45	43175

N	LENGTH	CORE REQUIREMENT
28	2	10680
28	3	11497
28	4	12314
28	5	13131
28	6	13948
28	7	14765
28	8	15582
28	9	16399
28	10	17216
28	11	18033
28	12	18850
28	13	19667
28	14	20484
28	15	21301
28	16	22118
28	17	22935
28	18	23752
28	19	24569
28	20	25386
28	21	26203
28	22	27020
28	23	27837
28	24	28654
28	25	29471
28	26	30288
28	27	31105
28	28	31922
28	29	32739
28	30	33556
28	31	34373
28	32	35190
28	33	36007
28	34	36824
28	35	37641
28	36	38458
28	37	39275
28	38	40092
28	39	40909
28	40	41726

28	41	42543
28	42	43360
28	43	44177
28	44	44994
28	45	45811
N	LENGTH	CORE REQUIREMENT
29	2	10916
29	3	11791
29	4	12666
29	5	13541
29	6	14416
29	7	15291
29	8	16166
29	9	17041
29	10	17916
29	11	18791
29	12	19666
29	13	20541
29	14	21416
29	15	22291
29	16	23166
29	17	24041
29	18	24916
29	19	25791
29	20	26666
29	21	27541
29	22	28416
29	23	29291
29	24	30166
29	25	31041
29	26	31916
29	27	32791
29	28	33666
29	29	34541
29	30	35416
29	31	36291
29	32	37166
29	33	38041
29	34	38916
29	35	39791
29	36	40666
29	37	41541
29	38	42416
29	39	43291
29	40	44166
29	41	45041
29	42	45916
29	43	46791
29	44	47666
29	45	48541
N	LENGTH	CORE REQUIREMENT
30	2	11160
30	3	12095
30	4	13030
30	5	13965

30	6	14900
30	7	15835
30	8	16770
30	9	17705
30	10	18640
30	11	19575
30	12	20510
30	13	21445
30	14	22380
30	15	23315
30	16	24250
30	17	25185
30	18	26120
30	19	27055
30	20	27990
30	21	28925
30	22	29860
30	23	30795
30	24	31730
30	25	32665
30	26	33600
30	27	34535
30	28	35470
30	29	36405
30	30	37340
30	31	38275
30	32	39210
30	33	40145
30	34	41080
30	35	42015
30	36	42950
30	37	43885
30	38	44820
30	39	45755
30	40	46690
30	41	47625
30	42	48560
30	43	49495
30	44	50430
30	45	51365
N	LENGTH	CORE REQUIREMENT
31	2	11412
31	3	12409
31	4	13406
31	5	14403
31	6	15400
31	7	16397
31	8	17394
31	9	18391
31	10	19388
31	11	20385
31	12	21382
31	13	22379
31	14	23376
31	15	24373

31	16	25370
31	17	26367
31	18	27364
31	19	28361
31	20	29358
31	21	30355
31	22	31352
31	23	32349
31	24	33346
31	25	34343
31	26	35340
31	27	36337
31	28	37334
31	29	38331
31	30	39328
31	31	40325
31	32	41322
31	33	42319
31	34	43316
31	35	44313
31	36	45310
31	37	46307
31	38	47304
31	39	48301
31	40	49298
31	41	50295
31	42	51292
31	43	52289
31	44	53286
31	45	54283

N	LENGTH	CORE REQUIREMENT
32	2	11672
32	3	12733
32	4	13794
32	5	14855
32	6	15916
32	7	16977
32	8	18038
32	9	19099
32	10	20160
32	11	21221
32	12	22282
32	13	23343
32	14	24404
32	15	25465
32	16	26526
32	17	27587
32	18	28648
32	19	29709
32	20	30770
32	21	31831
32	22	32892
32	23	33953
32	24	35014
32	25	36075

32	26	37136
32	27	38197
32	28	39258
32	29	40319
32	30	41380
32	31	42441
32	32	43502
32	33	44563
32	34	45624
32	35	46685
32	36	47746
32	37	48807
32	38	49868
32	39	50929
32	40	51990
32	41	53051
32	42	54112
32	43	55173
32	44	56234
32	45	57295

N	LENGTH	CORE REQUIREMENT
33	2	11940
33	3	13067
33	4	14194
33	5	15321
33	6	16448
33	7	17575
33	8	18702
33	9	19829
33	10	20956
33	11	22083
33	12	23210
33	13	24337
33	14	25464
33	15	26591
33	16	27718
33	17	28845
33	18	29972
33	19	31099
33	20	32226
33	21	33353
33	22	34480
33	23	35607
33	24	36734
33	25	37861
33	26	38988
33	27	40115
33	28	41242
33	29	42369
33	30	43496
33	31	44623
33	32	45750
33	33	46877
33	34	48004
33	35	49131

33	36	50258
33	37	51385
33	38	52512
33	39	53639
33	40	54766
33	41	55893
33	42	57020
33	43	58147
33	44	59274
33	45	60401

N	LENGTH	CORE REQUIREMENT
34	2	12216
34	3	13411
34	4	14606
34	5	15801
34	6	16996
34	7	18191
34	8	19386
34	9	20581
34	10	21776
34	11	22971
34	12	24166
34	13	25361
34	14	26556
34	15	27751
34	16	28946
34	17	30141
34	18	31336
34	19	32531
34	20	33726
34	21	34921
34	22	36116
34	23	37311
34	24	38506
34	25	39701
34	26	40896
34	27	42091
34	28	43286
34	29	44481
34	30	45676
34	31	46871
34	32	48066
34	33	49261
34	34	50456
34	35	51651
34	36	52846
34	37	54041
34	38	55236
34	39	56431
34	40	57626
34	41	58821
34	42	60016
34	43	61211
34	44	62406
34	45	63601

N	LENGTH	CORE REQUIREMENT
35	2	12500
35	3	13765
35	4	15030
35	5	16295
35	6	17560
35	7	18825
35	8	20090
35	9	21355
35	10	22620
35	11	23885
35	12	25150
35	13	26415
35	14	27680
35	15	28945
35	16	30210
35	17	31475
35	18	32740
35	19	34005
35	20	35270
35	21	36535
35	22	37800
35	23	39065
35	24	40330
35	25	41595
35	26	42860
35	27	44125
35	28	45390
35	29	46655
35	30	47920
35	31	49185
35	32	50450
35	33	51715
35	34	52980
35	35	54245
35	36	55510
35	37	56775
35	38	58040
35	39	59305
35	40	60570
35	41	61835
35	42	63100
35	43	64365
35	44	65630
35	45	66895

N	LENGTH	CORE REQUIREMENT
36	2	12792
36	3	14129
36	4	15466
36	5	16803
36	6	18140
36	7	19477
36	8	20814
36	9	22151
36	10	23488

36	11	24825
36	12	26162
36	13	27499
36	14	28836
36	15	30173
36	16	31510
36	17	32847
36	18	34184
36	19	35521
36	20	36858
36	21	38195
36	22	39532
36	23	40869
36	24	42206
36	25	43543
36	26	44880
36	27	46217
36	28	47554
36	29	48891
36	30	50228
36	31	51565
36	32	52902
36	33	54239
36	34	55576
36	35	56913
36	36	58250
36	37	59587
36	38	60924
36	39	62261
36	40	63598
36	41	64935
36	42	66272
36	43	67609
36	44	68946
36	45	70283

N	LENGTH	CORE REQUIREMENT
37	2	13092
37	3	14503
37	4	15914
37	5	17325
37	6	18736
37	7	20147
37	8	21558
37	9	22969
37	10	24380
37	11	25791
37	12	27202
37	13	28613
37	14	30024
37	15	31435
37	16	32846
37	17	34257
37	18	35668
37	19	37079
37	20	38490

37	21	39901
37	22	41312
37	23	42723
37	24	44134
37	25	45545
37	26	46956
37	27	48367
37	28	49778
37	29	51189
37	30	52600
37	31	54011
37	32	55422
37	33	56833
37	34	58244
37	35	59655
37	36	61066
37	37	62477
37	38	63888
37	39	65299
37	40	66710
37	41	68121
37	42	69532
37	43	70943
37	44	72354
37	45	73765

N	LENGTH	CORE REQUIREMENT
38	2	13400
38	3	14887
38	4	16374
38	5	17861
38	6	19348
38	7	20835
38	8	22322
38	9	23809
38	10	25296
38	11	26783
38	12	28270
38	13	29757
38	14	31244
38	15	32731
38	16	34218
38	17	35705
38	18	37192
38	19	38679
38	20	40166
38	21	41653
38	22	43140
38	23	44627
38	24	46114
38	25	47601
38	26	49088
38	27	50575
38	28	52062
38	29	53549
38	30	55036

38	31	56523
38	32	58010
38	33	59497
38	34	60984
38	35	62471
38	36	63958
38	37	65445
38	38	66932
38	39	68419
38	40	69906
38	41	71393
38	42	72880
38	43	74367
38	44	75854
38	45	77341

N	LENGTH	CORE REQUIREMENT
39	2	13716
39	3	15281
39	4	16846
39	5	18411
39	6	19976
39	7	21541
39	8	23106
39	9	24671
39	10	26236
39	11	27801
39	12	29366
39	13	30931
39	14	32496
39	15	34061
39	16	35626
39	17	37191
39	18	38756
39	19	40321
39	20	41886
39	21	43451
39	22	45016
39	23	46581
39	24	48146
39	25	49711
39	26	51276
39	27	52841
39	28	54406
39	29	55971
39	30	57536
39	31	59101
39	32	60666
39	33	62231
39	34	63796
39	35	65361
39	36	66926
39	37	68491
39	38	70056
39	39	71621
39	40	73186

39	41	74751
39	42	76316
39	43	77881
39	44	79446
39	45	81011

N	LENGTH	CORE REQUIREMENT
40	2	14040
40	3	15685
40	4	17330
40	5	18975
40	6	20620
40	7	22265
40	8	23910
40	9	25555
40	10	27200
40	11	28845
40	12	30490
40	13	32135
40	14	33780
40	15	35425
40	16	37070
40	17	38715
40	18	40360
40	19	42005
40	20	43650
40	21	45295
40	22	46940
40	23	48585
40	24	50230
40	25	51875
40	26	53520
40	27	55165
40	28	56810
40	29	58455
40	30	60100
40	31	61745
40	32	63390
40	33	65035
40	34	66680
40	35	68325
40	36	69970
40	37	71615
40	38	73260
40	39	74905
40	40	76550
40	41	78195
40	42	79840
40	43	81485
40	44	83130
40	45	84775

N	LENGTH	CORE REQUIREMENT
41	2	14372
41	3	16099
41	4	17826
41	5	19553

41	6	21280
41	7	23007
41	8	24734
41	9	26461
41	10	28188
41	11	29915
41	12	31642
41	13	33369
41	14	35096
41	15	36823
41	16	38550
41	17	40277
41	18	42004
41	19	43731
41	20	45458
41	21	47185
41	22	48912
41	23	50639
41	24	52366
41	25	54093
41	26	55820
41	27	57547
41	28	59274
41	29	61001
41	30	62728
41	31	64455
41	32	66182
41	33	67909
41	34	69636
41	35	71363
41	36	73090
41	37	74817
41	38	76544
41	39	78271
41	40	79998
41	41	81725
41	42	83452
41	43	85179
41	44	86906
41	45	88633

N	LENGTH	CORE REQUIREMENT
42	2	14712
42	3	16523
42	4	18334
42	5	20145
42	6	21956
42	7	23767
42	8	25578
42	9	27389
42	10	29200
42	11	31011
42	12	32822
42	13	34633
42	14	36444
42	15	38255

42	16	40066
42	17	41877
42	18	43688
42	19	45499
42	20	47310
42	21	49121
42	22	50932
42	23	52743
42	24	54554
42	25	56365
42	26	58176
42	27	59987
42	28	61798
42	29	63609
42	30	65420
42	31	67231
42	32	69042
42	33	70853
42	34	72664
42	35	74475
42	36	76286
42	37	78097
42	38	79908
42	39	81719
42	40	83530
42	41	85341
42	42	87152
42	43	88963
42	44	90774
42	45	92585

N	LENGTH	CORE REQUIREMENT
43	2	15060
43	3	16957
43	4	18854
43	5	20751
43	6	22648
43	7	24545
43	8	26442
43	9	28339
43	10	30236
43	11	32133
43	12	34030
43	13	35927
43	14	37824
43	15	39721
43	16	41618
43	17	43515
43	18	45412
43	19	47309
43	20	49206
43	21	51103
43	22	53000
43	23	54897
43	24	56794
43	25	58691

43	26	60588
43	27	62485
43	28	64382
43	29	66279
43	30	68176
43	31	70073
43	32	71970
43	33	73867
43	34	75764
43	35	77661
43	36	79558
43	37	81455
43	38	83352
43	39	85249
43	40	87146
43	41	89043
43	42	90940
43	43	92837
43	44	94734
43	45	96631

N	LENGTH	CORE REQUIREMENT
44	2	15416
44	3	17401
44	4	19386
44	5	21371
44	6	23356
44	7	25341
44	8	27326
44	9	29311
44	10	31296
44	11	33281
44	12	35266
44	13	37251
44	14	39236
44	15	41221
44	16	43206
44	17	45191
44	18	47176
44	19	49161
44	20	51146
44	21	53131
44	22	55116
44	23	57101
44	24	59086
44	25	61071
44	26	63056
44	27	65041
44	28	67026
44	29	69011
44	30	70996
44	31	72981
44	32	74966
44	33	76951
44	34	78936
44	35	80921

44	36	82906
44	37	84891
44	38	86876
44	39	88861
44	40	90846
44	41	92831
44	42	94816
44	43	96801
44	44	98786
44	45	OVERLAP 100K

N	LENGTH	CORE REQUIREMENT
45	2	15780
45	3	17855
45	4	19930
45	5	22005
45	6	24080
45	7	26155
45	8	28230
45	9	30305
45	10	32380
45	11	34455
45	12	36530
45	13	38605
45	14	40680
45	15	42755
45	16	44830
45	17	46905
45	18	48980
45	19	51055
45	20	53130
45	21	55205
45	22	57280
45	23	59355
45	24	61430
45	25	63505
45	26	65580
45	27	67655
45	28	69730
45	29	71805
45	30	73880
45	31	75955
45	32	78030
45	33	80105
45	34	82180
45	35	84255
45	36	86330
45	37	88405
45	38	90480
45	39	92555
45	40	94630
45	41	96705
45	42	98780
45	43	OVERLAP 100K

N	LENGTH	CORE REQUIREMENT
46	2	16152

46	3	18319
46	4	20486
46	5	22653
46	6	24820
46	7	26987
46	8	29154
46	9	31321
46	10	33488
46	11	35655
46	12	37822
46	13	39989
46	14	42156
46	15	44323
46	16	46490
46	17	48657
46	18	50824
46	19	52991
46	20	55158
46	21	57325
46	22	59492
46	23	61659
46	24	63826
46	25	65993
46	26	68160
46	27	70327
46	28	72494
46	29	74661
46	30	76828
46	31	78995
46	32	81162
46	33	83329
46	34	85496
46	35	87663
46	36	89830
46	37	91997
46	38	94164
46	39	96331
46	40	98498
46	41	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
47	2	16432
47	3	18793
47	4	21054
47	5	23315
47	6	25576
47	7	27837
47	8	30098
47	9	32359
47	10	34620
47	11	36981
47	12	39142
47	13	41403
47	14	43664
47	15	45925
47	16	48186

47	17	50447
47	18	52708
47	19	54969
47	20	57230
47	21	59491
47	22	61752
47	23	64013
47	24	66274
47	25	68535
47	26	70796
47	27	73057
47	28	75318
47	29	77579
47	30	79840
47	31	82101
47	32	84362
47	33	86623
47	34	88884
47	35	91145
47	36	93406
47	37	95667
47	38	97928
47	39	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
48	2	16920
48	3	19277
48	4	21634
48	5	23991
48	6	26348
48	7	28705
48	8	31062
48	9	33419
48	10	35776
48	11	38133
48	12	40490
48	13	42847
48	14	45204
48	15	47561
48	16	49918
48	17	52275
48	18	54632
48	19	56989
48	20	59346
48	21	61703
48	22	64060
48	23	66417
48	24	68774
48	25	71131
48	26	73488
48	27	75845
48	28	78202
48	29	80559
48	30	82916
48	31	85273
48	32	87630

48	33	89987
48	34	92344
48	35	94701
48	36	97058
48	37	99415
48	38	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
49	2	17316
49	3	19771
49	4	22226
49	5	24681
49	6	27136
49	7	29591
49	8	32046
49	9	34501
49	10	36956
49	11	39411
49	12	41866
49	13	44321
49	14	46776
49	15	49231
49	16	51686
49	17	54141
49	18	56596
49	19	59051
49	20	61506
49	21	63961
49	22	66416
49	23	68871
49	24	71326
49	25	73781
49	26	76236
49	27	78691
49	28	81146
49	29	83601
49	30	86056
49	31	88511
49	32	90966
49	33	93421
49	34	95876
49	35	98331
49	36	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
50	2	17720
50	3	20275
50	4	22830
50	5	25385
50	6	27940
50	7	30495
50	8	33050
50	9	35605
50	10	38160
50	11	40715
50	12	43270
50	13	45825

50	14	48380
50	15	50935
50	16	53490
50	17	56045
50	18	58600
50	19	61155
50	20	63710
50	21	66265
50	22	68820
50	23	71375
50	24	73930
50	25	76485
50	26	79040
50	27	81595
50	28	84150
50	29	86705
50	30	89260
50	31	91815
50	32	94370
50	33	96925
50	34	99480
50	35	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
51	2	18132
51	3	20789
51	4	23446
51	5	26103
51	6	28760
51	7	31417
51	8	34074
51	9	36731
51	10	39388
51	11	42045
51	12	44702
51	13	47359
51	14	50016
51	15	52673
51	16	55330
51	17	57987
51	18	60644
51	19	63301
51	20	65958
51	21	68615
51	22	71272
51	23	73929
51	24	76586
51	25	79243
51	26	81900
51	27	84557
51	28	87214
51	29	89871
51	30	92528
51	31	95185
51	32	97842
51	33	OVERLAP 100K

N	LENGTH	CORE REQUIREMENT
52	2	18552
52	3	21313
52	4	24074
52	5	26835
52	6	29596
52	7	32357
52	8	35118
52	9	37879
52	10	40640
52	11	43401
52	12	46162
52	13	48923
52	14	51684
52	15	54445
52	16	57206
52	17	59967
52	18	62728
52	19	65489
52	20	68250
52	21	71011
52	22	73772
52	23	76533
52	24	79294
52	25	82055
52	26	84816
52	27	87577
52	28	90338
52	29	93099
52	30	95860
52	31	98621
52	32	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
53	2	18980
53	3	21847
53	4	24714
53	5	27581
53	6	30448
53	7	33315
53	8	36182
53	9	39049
53	10	41916
53	11	44783
53	12	47650
53	13	50517
53	14	53384
53	15	56251
53	16	59118
53	17	61985
53	18	64852
53	19	67719
53	20	70586
53	21	73453
53	22	76320
53	23	79187

53	24	82054
53	25	84921
53	26	87788
53	27	90655
53	28	93522
53	29	96389
53	30	99256
53	31	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
54	2	19416
54	3	22391
54	4	25366
54	5	28341
54	6	31316
54	7	34291
54	8	37266
54	9	40241
54	10	43216
54	11	46191
54	12	49166
54	13	52141
54	14	55116
54	15	58091
54	16	61066
54	17	64041
54	18	67016
54	19	69991
54	20	72966
54	21	75941
54	22	78916
54	23	81891
54	24	84866
54	25	87841
54	26	90816
54	27	93791
54	28	96766
54	29	99741
54	30	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
55	2	19860
55	3	22945
55	4	26030
55	5	29115
55	6	32200
55	7	35285
55	8	38370
55	9	41455
55	10	44540
55	11	47625
55	12	50710
55	13	53795
55	14	56880
55	15	59965
55	16	63050
55	17	66135

55	18	69220
55	19	72305
55	20	75390
55	21	78475
55	22	81560
55	23	84645
55	24	87730
55	25	90815
55	26	93900
55	27	96985
55	28	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
56	2	20312
56	3	23509
56	4	26706
56	5	29903
56	6	33100
56	7	36297
56	8	39494
56	9	42691
56	10	45888
56	11	49085
56	12	52282
56	13	55479
56	14	58676
56	15	61873
56	16	65070
56	17	68267
56	18	71464
56	19	74661
56	20	77858
56	21	81055
56	22	84252
56	23	87449
56	24	90646
56	25	93843
56	26	97040
56	27	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
57	2	20772
57	3	24083
57	4	27394
57	5	30705
57	6	34016
57	7	37327
57	8	40638
57	9	43949
57	10	47260
57	11	50571
57	12	53882
57	13	57193
57	14	60504
57	15	63815
57	16	67126
57	17	70437

57	18	73748
57	19	77059
57	20	80370
57	21	83681
57	22	86992
57	23	90303
57	24	93614
57	25	96925
57	26	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
58	2	21240
58	3	24667
58	4	28094
58	5	31521
58	6	34948
58	7	38375
58	8	41802
58	9	45229
58	10	48656
58	11	52083
58	12	55510
58	13	58937
58	14	62364
58	15	65791
58	16	69218
58	17	72645
58	18	76072
58	19	79499
58	20	82926
58	21	86353
58	22	89780
58	23	93207
58	24	96634
58	25	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
59	2	21716
59	3	25261
59	4	28806
59	5	32351
59	6	35896
59	7	39441
59	8	42986
59	9	46531
59	10	50076
59	11	53621
59	12	57166
59	13	60711
59	14	64256
59	15	67801
59	16	71346
59	17	74891
59	18	78436
59	19	81981
59	20	85526
59	21	89071

59	22	92615
59	23	96161
59	24	99706
59	25	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
60	2	22200
60	3	25865
60	4	29530
60	5	33195
60	6	36860
60	7	40525
60	8	44190
60	9	47855
60	10	51520
60	11	55185
60	12	58850
60	13	62515
60	14	66180
60	15	69845
60	16	73510
60	17	77175
60	18	80840
60	19	84505
60	20	88170
60	21	91835
60	22	95500
60	23	99165
60	24	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
61	2	22692
61	3	26479
61	4	30266
61	5	34053
61	6	37840
61	7	41627
61	8	45414
61	9	49201
61	10	52988
61	11	56775
61	12	60562
61	13	64349
61	14	68136
61	15	71923
61	16	75710
61	17	79497
61	18	83284
61	19	87071
61	20	90858
61	21	94645
61	22	98432
61	23	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
62	2	23192
62	3	27103
62	4	31014

62	5	34925
62	6	38836
62	7	42747
62	8	46658
62	9	50569
62	10	54480
62	11	58391
62	12	62302
62	13	66213
62	14	70124
62	15	74035
62	16	77946
62	17	81857
62	18	85768
62	19	89679
62	20	93590
62	21	97501
62	22	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
63	2	23700
63	3	27737
63	4	31774
63	5	35811
63	6	39848
63	7	43885
63	8	47922
63	9	51959
63	10	55996
63	11	60033
63	12	64070
63	13	68107
63	14	72144
63	15	76181
63	16	80218
63	17	84255
63	18	88292
63	19	92329
63	20	96366
63	21	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
64	2	24216
64	3	28381
64	4	32546
64	5	36711
64	6	40876
64	7	45041
64	8	49206
64	9	53371
64	10	57536
64	11	61701
64	12	65866
64	13	70031
64	14	74196
64	15	78361
64	16	82526

64	17	86691
64	18	90856
64	19	95021
64	20	99186
64	21	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
65	2	24740
65	3	29035
65	4	33330
65	5	37625
65	6	41920
65	7	46215
65	8	50510
65	9	54805
65	10	59100
65	11	63395
65	12	67690
65	13	71985
65	14	76280
65	15	80575
65	16	84870
65	17	89165
65	18	93460
65	19	97755
65	20	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
66	2	25272
66	3	29699
66	4	34126
66	5	38553
66	6	42980
66	7	47407
66	8	51834
66	9	56261
66	10	60688
66	11	65115
66	12	69542
66	13	73969
66	14	78396
66	15	82823
66	16	87250
66	17	91677
66	18	96104
66	19	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
67	2	25812
67	3	30373
67	4	34934
67	5	39495
67	6	44056
67	7	48617
67	8	53178
67	9	57739
67	10	62300
67	11	66861

67	12	71422
67	13	75983
67	14	80544
67	15	85105
67	16	89666
67	17	94227
67	18	98788
67	19	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
68	2	26360
68	3	31057
68	4	35754
68	5	40451
68	6	45148
68	7	49845
68	8	54542
68	9	59239
68	10	63936
68	11	68633
68	12	73330
68	13	78027
68	14	82724
68	15	87421
68	16	92118
68	17	96815
68	18	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
69	2	26916
69	3	31751
69	4	36586
69	5	41421
69	6	46256
69	7	51091
69	8	55926
69	9	60761
69	10	65596
69	11	70431
69	12	75266
69	13	80101
69	14	84936
69	15	89771
69	16	94606
69	17	99441
69	18	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
70	2	27480
70	3	32455
70	4	37430
70	5	42405
70	6	47380
70	7	52355
70	8	57330
70	9	62305
70	10	67280
70	11	72255

70	12	77230
70	13	82205
70	14	87180
70	15	92155
70	16	97130
70	17	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
71	2	28052
71	3	33169
71	4	38286
71	5	43403
71	6	48520
71	7	53637
71	8	58754
71	9	63871
71	10	68988
71	11	74105
71	12	79222
71	13	84339
71	14	89456
71	15	94573
71	16	99690
71	17	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
72	2	28632
72	3	33893
72	4	39154
72	5	44415
72	6	49676
72	7	54937
72	8	60198
72	9	65459
72	10	70720
72	11	75981
72	12	81242
72	13	86503
72	14	91764
72	15	97025
72	16	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
73	2	29220
73	3	34627
73	4	40034
73	5	45441
73	6	50848
73	7	56255
73	8	61662
73	9	67069
73	10	72476
73	11	77883
73	12	83290
73	13	88697
73	14	94104
73	15	99511
73	16	OVERLAP 100K

N	LENGTH	CORE REQUIREMENT
74	2	29816
74	3	35371
74	4	40926
74	5	46481
74	6	52036
74	7	57591
74	8	63146
74	9	68701
74	10	74256
74	11	79811
74	12	85366
74	13	90921
74	14	96476
74	15	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
75	2	30420
75	3	36125
75	4	41830
75	5	47535
75	6	53240
75	7	58945
75	8	64650
75	9	70355
75	10	76060
75	11	81765
75	12	87470
75	13	93175
75	14	98880
75	15	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
76	2	31032
76	3	36889
76	4	42746
76	5	48603
76	6	54460
76	7	60317
76	8	66174
76	9	72031
76	10	77888
76	11	83745
76	12	89602
76	13	95459
76	14	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
77	2	31652
77	3	37663
77	4	43674
77	5	49685
77	6	55696
77	7	61707
77	8	67718
77	9	73729
77	10	79740
77	11	85751

77	12	91762
77	13	97773
77	14	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
78	2	32280
78	3	38447
78	4	44614
78	5	50781
78	6	56948
78	7	63115
78	8	69282
78	9	75449
78	10	81616
78	11	87783
78	12	93950
78	13	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
79	2	32916
79	3	39241
79	4	45566
79	5	51891
79	6	58216
79	7	64541
79	8	70866
79	9	77191
79	10	83516
79	11	89841
79	12	96166
79	13	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
80	2	33560
80	3	40045
80	4	46530
80	5	53015
80	6	59500
80	7	65985
80	8	72470
80	9	78955
80	10	85440
80	11	91925
80	12	98410
80	13	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
81	2	34212
81	3	40859
81	4	47506
81	5	54153
81	6	60800
81	7	67447
81	8	74094
81	9	80741
81	10	87388
81	11	94035
81	12	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT

82	2	34872
82	3	41683
82	4	48494
82	5	55305
82	6	62116
82	7	68927
82	8	75738
82	9	82549
82	10	89360
82	11	96171
82	12	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
83	2	35540
83	3	42517
83	4	49494
83	5	56471
83	6	63448
83	7	70425
83	8	77402
83	9	84379
83	10	91356
83	11	98333
83	12	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
84	2	36216
84	3	43361
84	4	50506
84	5	57651
84	6	64796
84	7	71941
84	8	79086
84	9	86231
84	10	93376
84	11	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
85	2	36900
85	3	44215
85	4	51530
85	5	58845
85	6	66160
85	7	73475
85	8	80790
85	9	88105
85	10	95420
85	11	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
86	2	37592
86	3	45079
86	4	52566
86	5	60053
86	6	67540
86	7	75027
86	8	82514
86	9	90001
86	10	97488

86	11	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
87	2	38292
87	3	45953
87	4	53614
87	5	61275
87	6	68936
87	7	76597
87	8	84258
87	9	91919
87	10	99580
87	11	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
88	2	39000
88	3	46837
88	4	54674
88	5	62511
88	6	70348
88	7	78185
88	8	86022
88	9	93859
88	10	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
89	2	39716
89	3	47731
89	4	55746
89	5	63761
89	6	71776
89	7	79791
89	8	87806
89	9	95821
89	10	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
90	2	40440
90	3	48635
90	4	56830
90	5	65025
90	6	73220
90	7	81415
90	8	89610
90	9	97805
90	10	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
91	2	41172
91	3	49549
91	4	57926
91	5	66303
91	6	74680
91	7	83057
91	8	91434
91	9	99811
91	10	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
92	2	41912
92	3	50473

92	4	59034
92	5	67595
92	6	76156
92	7	84717
92	8	93278
92	9	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
93	2	42660
93	3	51407
93	4	60154
93	5	68901
93	6	77648
93	7	86395
93	8	95142
93	9	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
94	2	43416
94	3	52351
94	4	61286
94	5	70221
94	6	79156
94	7	88091
94	8	97026
94	9	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
95	2	44180
95	3	53305
95	4	62430
95	5	71555
95	6	80680
95	7	89805
95	8	98930
95	9	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
96	2	44952
96	3	54269
96	4	63586
96	5	72903
96	6	82220
96	7	91537
96	8	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
97	2	45732
97	3	55243
97	4	64754
97	5	74265
97	6	83776
97	7	93287
97	8	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
98	2	46520
98	3	56227
98	4	65934
98	5	75641
98	6	85348

98	7	95055
98	8	OVERLAP 100K
N	LENGTH	CORE REQUIREMENT
99	2	47316
99	3	57221
99	4	67126
99	5	77031
99	6	86936
99	7	96841
99	8	OVERLAP 100K

SAMPLE PROBLEMS

The following pages illustrate the inversion of a finite segment of the Hilbert matrix of order 10.* Thus $a_{ij} = \frac{1}{i+j-1}$

This matrix was chosen because of its ill-conditioned nature. During the run pivoting was requested and the noisy mode was not in use. The inverse of the inverse was also requested at the conclusion of the inversion process. Input data was 8 significant digits to all cases and a variety of word sizes was employed ranging from too small (case 1) and too large (case 5).

* The author recognizes that this matrix is not genuinely the Hilbert matrix but is a finite segment of the above. However, its condition number (the ratio of the maximum to the minimum eigenvalue) is probably as high as the Hilbert matrix itself and, without question, it is certainly ill-conditioned. Because of the fact that many of the elements of the Hilbert matrix are non-terminating decimals no program can satisfactorily invert the true Hilbert matrix unless it wishes to assume that each element in the matrix is composed of two elements, p and q where p and q specify the numerator and denominator respectively of the rational number $\frac{1}{i+j-1}$

LISTING OF DATA INPUT TO INVERSION PROGRAM

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1.00000000
 .50000000
 .33333333
 .25000000
 .20000000
 .16666666
 .14285714
 .12500000
 .11111111
 .10000000
 .50000000
 .33333333
 .25000000
 .20000000
 .16666666
 .14285714
 .12500000
 .11111111
 .10000000
 .09090909
 .33333333
 .25000000
 .20000000
 .16666666
 .14285714
 .12500000
 .11111111
 .10000000
 .09090909
 .08333333
 .25000000
 .20000000
 .16666666
 .14285714
 .12500000
 .11111111
 .10000000
 .09090909
 .08333333
 .07692307
 .20000000
 .16666666
 .14285714
 .12500000
 .11111111
 .10000000
 .09090909
 .08333333
 .07692307
 .07142857
 .16666666
 .14285714
 .12500000
 .11111111
 .10000000
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.09090909
.08333333
.07692307
.07142857
.06666666
.14285714
.12500000
.11111111
.10000000
.09090909
.08333333
.07692307
.07142857
.06666666
.06250000
.12500000
.11111111
.10000000
.09090909
.08333333
.07692307
.07142857
.06666666
.06250000
.05882352
.11111111
.10000000
.09090909
.08333333
.07692307
.07142857
.06666666
.06250000
.05882352
.05555555
.10000000
.09090909
.08333333
.07692307
.07142857
.06666666
.06250000
.05882352
.05555555
.05263157

CASE 1

LENGTH: 6

MATRIX

100000	01	01 01
500000	00	01 02
333333	00	01 03
250000	00	01 04
200000	00	01 05
166666	00	01 06
142857	00	01 07
125000	00	01 08
111111	00	01 09
100000	00	01 10
500000	00	02 01
333333	00	02 02
250000	00	02 03
200000	00	02 04
166666	00	02 05
142857	00	02 06
125000	00	02 07
111111	00	02 08
100000	00	02 09
909090	-01	02 10
333333	00	03 01
250000	00	03 02
200000	00	03 03
166666	00	03 04
142857	00	03 05
125000	00	03 06
111111	00	03 07
100000	00	03 08
909090	-01	03 09
833333	-01	03 10
250000	00	04 01
200000	00	04 02
166666	00	04 03
142857	00	04 04
125000	00	04 05
111111	00	04 06
100000	00	04 07
909090	-01	04 08
833333	-01	04 09
769230	-01	04 10
200000	00	05 01
166666	00	05 02
142857	00	05 03
125000	00	05 04
111111	00	05 05
100000	00	05 06
909090	-01	05 07
833333	-01	05 08
769230	-01	05 09
714285	-01	05 10
166666	00	06 01
142857	00	06 02
125000	00	06 03
111111	00	06 04

100000	00	06 05
909090	-01	06 06
833333	-01	06 07
769230	-01	06 08
714285	-01	06 09
666666	-01	06 10
142857	00	07 01
125000	00	07 02
111111	00	07 03
100000	00	07 04
909090	-01	07 05
833333	-01	07 06
769230	-01	07 07
714285	-01	07 08
666666	-01	07 09
625000	-01	07 10
125000	00	08 01
111111	00	08 02
100000	00	08 03
909090	-01	08 04
833333	-01	08 05
769230	-01	08 06
714285	-01	08 07
666666	-01	08 08
625000	-01	08 09
588235	-01	08 10
111111	00	09 01
100000	00	09 02
909090	-01	09 03
833333	-01	09 04
769230	-01	09 05
714285	-01	09 06
666666	-01	09 07
625000	-01	09 08
588235	-01	09 09
555555	-01	09 10
100000	00	10 01
909090	-01	10 02
833333	-01	10 03
769230	-01	10 04
714285	-01	10 05
666666	-01	10 06
625000	-01	10 07
588235	-01	10 08
555555	-01	10 09
526315	-01	10 10

INVERSE

102485	02	01 01
-172284	03	01 02
246111	03	01 03
-133735	03	01 04
101462	04	01 05
-325513	04	01 06
159363	04	01 07
263238	04	01 08
-237583	04	01 09
327622	03	01 10
-162258	03	02 01
220242	04	02 02
-738726	04	02 03
138832	05	02 04
-382651	05	02 05
712102	05	02 06
-223468	05	02 07
-277500	05	02 08
796043	05	02 09
-209356	05	02 10
236266	03	03 01
-598440	04	03 02
410167	05	03 03
-138912	06	03 04
284993	06	03 05
-308135	06	03 06
-504810	05	03 07
553831	06	03 08
-558566	06	03 09
181945	06	03 10
197873	03	04 01
731370	04	04 02
-113345	06	04 03
444970	06	04 04
-622065	06	04 05
442420	05	04 06
804910	06	04 07
-144602	07	04 08
123764	07	04 09
-448458	06	04 10
654987	03	05 01
-272735	05	05 02
212715	06	05 03
-475176	06	05 04
-147960	06	05 05
170564	07	05 06
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CASE 2

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1111111100	00	01 09
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5236700505859337600550656179474690955830	08	06 08
1170121023733978262127098883206504204713	09	06 09
-6867237872421772014403291824810872814260	08	06 10
4801566007515737090258306225626072866028	05	07 01
-2254447567442314180044947447336985235053	07	07 02
2542785413097672491351034144205857074628	08	07 03
-1142185356160917836083684626411515589675	09	07 04
2304820607669843496008431623540624256367	09	07 05
-1724034784979187945329890468816237218268	09	07 06
-6372416041332092938701522090495256670213	08	07 07
1455611298966433584831183800222034661326	09	07 08
-3612499439406156349759777870995110505187	08	07 09
-1282411189999492593829672437291577026891O	08	07 10
-4882355308596132609185072186634320610410	05	08 01
2197346195494799075999666519186992388797	07	08 02
-2336983013194025611477743490792732535G9	08	08 03
9593186357672048176498151082683264661580	08	08 04
-1620504663270345387687797508245955554376	09	08 05
5236700505859337600550656179474676613470	08	08 06
1455611298966433584831183800222035460188	09	08 07
-1205695132130435930077899219056715790317	09	08 08
-2564248765156663999114821138541292080990	08	08 09
3565295004214817696899872367427555000271	08	08 10
-1908053124200800613651984786350090536950	05	09 01
8934127855783550869885933409185158920584	06	09 02
-1027923301714439994603449078715105512751	08	09 03
4859977922629720743825128559307255855054	08	09 04
-1101217405974925212581226612676339012770	09	09 05
1170121023733978262127098883206503394024	09	09 06
-3612499439406156349759777870995110709770	08	09 07
-2564248765156663999114821138541289240178	08	09 08
1513031994574625073904246011550823410536	08	09 09
5604983494592647957003838569073304429745	06	09 10
2093228100527626271075366842091246377405	05	10 01
-9504275837256923426205199643233245133800	06	10 02
1039826931616495093336105666328442773192	08	10 03
-4544592738439005568671285205551293089551	08	10 04
8983079009289793840627525691907755922180	08	10 05
-6867237872421772014403291824810865915990	08	10 06
-1282411189999492593829672437291577200887S	08	10 07
3565295004214817696899872367427553746432	08	10 08
5604983494592647957003838569073368557200	06	10 09
-8579688027615528374506639358435858688294	07	10 10

MATRIX

9999999999999999999999999999999999531850699	-00	01 01
4999999999999999999999999999999595888049	-00	01 02
3333333299999999999999999999999644508663	-00	01 03
2499999999999999999999999999999682663054	-00	01 04
1999999999999999999999999999999713387179	-00	01 05
1666666599999999999999999999999738655624	-00	01 06
1428571399999999999999999999999759804392	-00	01 07
1249999999999999999999999999999777767448	-00	01 08
1111111099999999999999999999999793216265	-00	01 09
9999999999999999999999999999999808369868	-01	01 10
4999999999999999999999999999999596439118	-00	02 01
3333333299999999999999999999999651894055	-00	02 02
2499999999999999999999999999999693917925	-00	02 03
1999999999999999999999999999999726860345	-00	02 04
1666666599999999999999999999999753368520	-00	02 05
1428571399999999999999999999999775158183	-00	02 06
1249999999999999999999999999999793387886	-00	02 07
1111111099999999999999999999999808866480	-00	02 08
9999999999999999999999999999999808221749360	-01	02 09
9090908999999999999999999999999808337412827	-01	02 10
3333333299999999999999999999999645963306	-00	03 01
2499999999999999999999999999999694779116	-00	03 02
1999999999999999999999999999999731718478	-00	03 03
1666666599999999999999999999999760651935	-00	03 04
1428571399999999999999999999999783921698	-00	03 05
1249999999999999999999999999999803041865	-00	03 06
1111111099999999999999999999999819033324	-00	03 07
9999999999999999999999999999999808326081130	-01	03 08
9090908999999999999999999999999808442773035	-01	03 09
8333332999999999999999999999999808544172214	-01	03 10
2499999999999999999999999999999684848810	-00	04 01
1999999999999999999999999999999728417250	-00	04 02
1666666599999999999999999999999761348782	-00	04 03
1428571399999999999999999999999787127024	-00	04 04
1249999999999999999999999999999807850648	-00	04 05
1111111099999999999999999999999824873570	-00	04 06
9999999999999999999999999999999808391076310	-01	04 07
9090908999999999999999999999999808511883316	-01	04 08
8333332999999999999999999999999808615715559	-01	04 09
7692306999999999999999999999999808705928522	-01	04 10
1999999999999999999999999999999716100747	-00	05 01
1666666599999999999999999999999755430879	-00	05 02
1428571399999999999999999999999785131923	-00	05 04
1249999999999999999999999999999808369868	-00	05 04
1111111099999999999999999999999827045163	-00	05 05
9999999999999999999999999999999808423818580	-01	05 06
9090908999999999999999999999999808552035680	-01	05 07
8333332999999999999999999999999808660839650	-01	05 08
7692306999999999999999999999999808754343963	-01	05 09
7142856999999999999999999999999808835575279	-01	05 10
1666666599999999999999999999999741734014	-00	06 01
1428571399999999999999999999999777574229	-00	06 02
1249999999999999999999999999999804619550	-00	06 03
1111111099999999999999999999999825771065	-00	06 04

9999999999999999999999999999999808427650340	-01	06 05
9090908999999999999999999999999808567182440	-01	06 06
8333332999999999999999999999999808683815836	-01	06 07
7692306999999999999999999999999808782778038	-01	06 08
7142856999999999999999999999999808867816055	-01	06 09
6666665999999999999999999999999808941686157	-01	06 10
1428571399999999999999999999999809763126557	-00	07 01
1249999999999999999999999999999809796044997	-00	07 02
1111111099999999999999999999999809820869903	-00	07 03
9999999999999999999999999999999808402782730	-01	07 04
9090908999999999999999999999999808558682380	-01	07 05
8333332999999999999999999999999808686665840	-01	07 06
7692306999999999999999999999999808793632469	-01	07 07
7142856999999999999999999999999808884383439	-01	07 08
6666665999999999999999999999999808962359149	-01	07 09
6249999999999999999999999999999809030089745	-01	07 10
1249999999999999999999999999999809781245337	-00	08 01
1111111099999999999999999999999809811682631	-00	08 02
9999999999999999999999999999999808346240890	-01	08 03
9090908999999999999999999999999808525547610	-01	08 04
8333332999999999999999999999999808669550150	-01	08 05
7692306999999999999999999999999808787750577	-01	08 06
7142856999999999999999999999999808886530230	-01	08 07
6666665999999999999999999999999808970328264	-01	08 08
6249999999999999999999999999999809042324828	-01	08 09
5882351999999999999999999999999809104858231	-01	08 10
1111111099999999999999999999999809796786403	-00	09 01
9999999999999999999999999999999808250908080	-01	09 02
9090908999999999999999999999999808464147260	-01	09 03
8333332999999999999999999999999808630769890	-01	09 04
7692306999999999999999999999999808764563890	-01	09 05
7142856999999999999999999999999808874371914	-01	09 06
6666665999999999999999999999999808966129742	-01	09 07
6249999999999999999999999999999809043965301	-01	09 08
5882351999999999999999999999999809110835132	-01	09 09
5555554999999999999999999999999809168912764	-01	09 10
9999999999999999999999999999999808102625170	-01	10 01
9090908999999999999999999999999808367138990	-01	10 02
8333332999999999999999999999999808566336860	-01	10 03
7692306999999999999999999999999808721953830	-01	10 04
7142856999999999999999999999999808846892750	-01	10 05
6666665999999999999999999999999808949422670	-01	10 06
6249999999999999999999999999999809035092004	-01	10 07
5882351999999999999999999999999809107758313	-01	10 08
5555554999999999999999999999999809170183946	-01	10 09
5263156999999999999999999999999809224399379	-01	10 10

DESCRIPTION OF DECKS RECEIVED WITH THIS PROGRAM

There are three decks received with this program. These are described as follows:

1. The Simultaneous Equations A La King source program. There are 439 cards in this deck which is sequentially numbered in columns 78 through 80. The numbering system is such that the first card is numbered 003 and the last card 441. This is to serve as a constant reminder that cards 001 and 002 are parameter cards which must be supplied at the user's location.

2. A sample object program which was produced using a mantissa length of 12 and a matrix rank of 3. There are 226 cards in this deck sequentially numbered in columns 76-80. These numbers are 00000 - 00225. This deck was produced utilizing the subroutines with hardware divide.

3. Data cards. There are 18 data cards sequentially numbered 001 through 018. The sequence numbers appear in columns 78-80. These cards may be used as input to the sample object program. This data consists of a 3 x 3 matrix (the first nine cards) and 3 constant vectors (the last nine cards).

Items numbered 2 and 3, as specified above, were utilized to produce the next 16 pages in this document. The first eight pages demonstrate the solution of the system of simultaneous equations utilizing the 0 fill operation, as described in the section entitled "NOISY" mode. The remaining eight pages demonstrate the solution of the system utilizing the 9 fill operation as described in the section entitled "NOISY"mode.

MATRIX

100000000000	01	01 01
200000000000	01	01 02
300000000000	01	01 03
000000000000	-99	02 01
200000000000	01	02 02
-300000000000	01	02 03
500000000000	01	03 01
700000000000	01	03 02
100000000000	01	03 03

INVERSE

-621621621611	00	01 01
-513513513508	00	01 02
324324324322	00	01 03
405405405401	00	02 01
378378378376	00	02 02
-810810810801	-01	02 03
270270270270	00	03 01
-810810810810	-01	03 02
-540540540540	-01	03 03

CONSTANT VECTOR

000000000000	-99
100000000000	01
300000000000	01

SOLUTION VECTOR

459459459458	00
135135135136	00
-243243243243	-00

CONSTANT VECTOR

-100000000000	01
200000000000	01
300000000000	01

SOLUTION VECTOR

567567567566	00
108108108111	00
-594594594594	00

CONSTANT VECTOR

800000000000	01
700000000000	01
100000000000	01

SOLUTION VECTOR

-824324324311	01
581081081075	01
154054054055	01

MATRIX

100000000000	01	01 01
200000000000	01	01 02
300000000000	01	01 03
000000000000	-99	02 01
200000000000	01	02 02
-300000000000	01	02 03
500000000000	01	03 01
700000000000	01	03 02
100000000000	01	03 03

INVERSE

-621621621611	00	01 01
-513513513508	00	01 02
324324324322	00	01 03
405405405401	00	02 01
378378378376	00	02 02
-810810810801	-01	02 03
270270270270	00	03 01
-810810810819	-01	03 02
-540540540540	-01	03 03

CONSTANT VECTOR

000000000000	-99
100000000000	01
300000000000	01

SOLUTION VECTOR

459459459458	00
135135135136	00
-243243243243	-00

CONSTANT VECTOR

-100000000000	01
200000000000	01
300000000000	01

SOLUTION VECTOR

567567567557	00
108108108111	00
-594594594595	00

CONSTANT VECTOR

800000000000	01
700000000000	01
100000000000	01

SOLUTION VECTOR

-824324324311	01
581081081075	01
154054054054	01

PROGRAM LISTING

The attached listing was assembled using a "LENGTH" operand of 15 and a "N" of 10. All addresses above location 454 will be altered for different parameter cards. A listing is given here only that the interested used may see the program instructions. Core locations will not correspond for parameters other than those specified above.

```
00015                 00001 LENGTH DS    ,15                                    0002
00010                 00001 N      DS    ,10                                    0004
00017                 00099 L      DS    ,LENGTH62                              0006
00402 46 00434 00400  01025 GO     BC4   A01029                                 0008
00414 15 07515 00000  01026        TDM   NOISE&100                              0010
00426 49 02864 00000  01027        B     A02001                                 0012
00434                 01028        DORG  *-3                                    0014
00434 15 07515 00009  01029 A01029 TDM   NOISE&100,9                            0016
00446 49 02864 00000  01030        B     A02001                                 0018
00454                 01031        DORG  *-3                                    0020
00454 49 03134 00000  01032        B     A05230                                 0022
00461                 01033        DORG  *-4                                    0024
00477    00017        01039 MTX    DS    L                                      0026
02177    01700        01040        DS    N*N*L                                  0028
02179    00002        01050 PIVTBL DS    2                                      0030
02197    00018        01060        DS    N*2-2                                  0032
02199    00002        01061 UPTBLE DS    2                                      0034
02217    00018        01062        DS    N*2-2                                  0036
02222    00005 -0010  01070 TABLE  DSA   N,L                                    0038
02227    00005 -0017
02244    00017        01080        DC    L,0                                    0041
02244    00002        01090 ZERO   DC    2,-99,*                                0044
02261    00017        01091        DC    L,0                                    0047
02261    00002        01092 ONE    DC    2,1,*                                  0050
02220                 01100 MASTER DS    ,TABLE-2                               0053
02225                 01110 TEMP1  DS    ,TABLE63                               0055
02278    00017        01130 TEMP3  DS    L                                      0057
02295    00017        01150 WORK   DS    L                                      0059
02448    00153        01160        DS    N*L-L                                  0061
02498    00050        01170        DNB   50                                     0063
02528    00030        01180 BLANKS DNB   30                                     0066
02529    00001        01190        DC    1,@                                    0069
02569    00040        01191        DS    40                                     0072
02609    00040        01192 KONZ   DC    40,0                                   0074
02570    00041        01193        DC    41,0,KONZ-39                           0077
02611    00080        01200 OUTPUT DAS   80                                     0080
02771    00008        01210 CMNT1  DAC   8,INVERSE@                             0082
02787    00007        01220 CMNT2  DAC   7,MATRIX@                              0085
02801    00016        01230 CMNT3  DAC   16,SOLUTION VECTOR@                    0088
02833    00016        01231 CMNT4  DAC   16,CONSTANT VECTOR@                    0091
02864 15 02245 0000J  02001 A02001 TDM   ONE-L61,1,11                           0094
02876 16 05262 -5974  02003        TFM   A1710366,PUNCH2                        0096
02888 16 03055 -0477  02010        TFM   A05150623,MTX                          0098
02900 32 02221 00000  05010        SF    TABLE-1                                0100
02911                 05011 TEMP69 DS    ,*                                     0102
02912 32 02226 00000  05030        SF    TABLE64                                0104
02924 16 02954 -2199  05031        TFM   A0503366,UPTBLE                        0106
02936 16 02959 000-1  05032        TFM   A05033611,1,10                         0108
02948 16 00000 -0000  05033 A05033 TFM                                         0110
02960 11 02959 000-1  05034        AM    *-1,1,10                               0112
02972 11 02954 000-2  05035        AM    A0503366,2,10                          0114
02984 24 02959 02222  05036        C     A05033611,TABLE                        0116
02996 47 02948 01100  05037        BNH   A05033                                 0118
                      05050 *      READ DATA AND STORE IN ROW ARRAY             0120
```

```
03008 37 07255 00500 05070 A05070 RACD INPUT                           0122
03020 27 06378 06377 05090        BT   FLOAT,FLOAT-1                   0124
03032 16 07527 -3055 05150 A05150 TFLS ,DONE                           0126
03044 49 07496 00000
03051    00005 -0000
03056    00005 -7234
03062 11 03055 -0017 05170        AM   A05150&23,L                     0131
03074 14 03055 -2177 05190        CM   A05150&23,MTX&N*N*L             0133
03086 47 03008 01200 05200        BNF  A05070                          0135
03098 47 03134 00200 05210        BNC2 A05230                          0137
03110 27 05846 05845 05220        BT   ELED,ELED-1                     0139
03122 17 05914 -3134 05225        BTM  PUNCH1,*+12                     0141
03134 26 02220 02222 05230 A05230 TF   MASTER,TABLE                    0143
03146 16 03386 -2197 05231        TFM  A06160&6,PIVTBL&N*2-2           0145
03158 46 03776 00100 05260 A05260 BC1  A09030                          0147
03170 26 02225 02220 06010        TF   TEMP1,MASTER                    0149
03182 12 02225 000-1 06020        SM   TEMP1,1,10                      0151
03191                06021 TEMP4  DS   ,A05260&33                      0153
03194 16 03324 -0477 06030        TFM  A06120&28,MTX                   0155
03206 16 03432 -0477 06040        TFM  A06180&28,MTX                   0157
                     06050 *           SEARCH 1ST ROW FOR A1J MAX      0159
03218 13 02225 -0017 06060        MM   TEMP1,L                         0161
03230 32 00095 00000 06070        SF   95                              0163
03242 11 00099 -0477 06080        AM   99,MTX                          0165
03254 16 07527 -3277 06100        TFLS TEMP3,ZERO                      0167
03266 49 07496 00000
03273    00005 -2278
03278    00005 -2244
03284 16 03191 000-1 06110        TFM  TEMP4,1,10                      0172
03296 16 07527 -3319 06120 A06120 TFLS MTX&N*N*L,                      0174
03308 49 07496 00000
03315    00005 -2177
03320    00005 -0000
03326 33 02175 00000 06130        CF   MTX&N*N*L-2                     0179
03338 16 07527 -3361 06140 A06140 FS   MTX&N*N*L,TEMP3                 0181
03350 49 07416 00000
03357    00005 -2177
03362    00005 -2278
03337                06141 HOLD   DS   ,A06140-1                       0186
03368 47 03434 01100 06150        BNP  A07010                          0188
03380 26 00000 03191 06160 A06160 TF   ,TEMP4                          0190
03392 26 03467 03191 06170        TF   TEMP5,TEMP4                     0192
03404 16 07527 -3427 06180 A06180 TFLS TEMP3,                          0194
03416 49 07496 00000
03423    00005 -2278
03428    00005 -0000
03434 11 03324 -0017 07010 A07010 AM   A06120&28,L                     0199
03446 11 03432 -0017 07020        AM   A06180&28,L                     0201
03458 11 03191 000-1 07030        AM   TEMP4,1,10                      0203
03467                07031 TEMP5  DS   ,*-2                            0205
03470 24 03324 00099 07040        C    A06120&28,99                    0207
03482 47 03296 01100 07050        BNH  A06120                          0209
03494 11 03386 000-K 07060        AM   A06160&6,2,1011                 0211
                     07090 *           PIVOT                           0213
03506 12 03467 000-1 08010        SM   TEMP5,1,10                      0215
```



```
03518 46 03776 01200 08020        BZ   A09030                          0217
03530 13 03467 -0017 08030        MM   TEMP5,L                         0219
03542 32 00095 00000 08040        SF   95                              0221
03554 11 00099 -0477 08045        AM   99,MTX                          0223
03566 16 03642 -0477 08050        TFM  A08090&28,MTX                   0225
03578 16 03667 -0477 08060        TFM  A08100&23,MTX                   0227
03590 26 03672 00099 08070        TF   A08100&28,99                    0229
03602 26 03697 00099 08080        TF   A08110&23,99                    0231
03614 16 07527 -3637 08090 A08090 TFLS MTX&N*N*L,                      0233
03626 49 07496 00000
03633    00005 -2177
03638    00005 -0000
03644 16 07527 -3667 08100 A08100 TFLS ,                               0238
03656 49 07496 00000
03663    00005 -0000
03668    00005 -0000
03674 16 07527 -3697 08110 A08110 TFLS ,MTX&N*N*L                      0243
03686 49 07496 00000
03693    00005 -0000
03698    00005 -2177
03704 11 03642 -0170 08120        AM   A08090&28,N*L                   0248
03716 11 03667 -0170 08130        AM   A08100&23,N*L                   0250
03728 11 03672 -0170 08140        AM   A08100&28,N*L                   0252
03740 11 03697 -0170 08150        AM   A08110&23,N*L                   0254
03752 14 03642 -2177 08160        CM   A08090&28,MTX&N*N*L             0256
03764 47 03614 01200 08170        BNE  A08090                          0258
                     08180 *           PIVOT COMPLETE                  0260
                     09010 *           BEGIN ITFRATION FOR INVERSE     0262
03776 26 00079 02609 09030 A09030 TF   79,KONZ                         0264
03788 16 07527 -3811 09040        TFLS WORK&N*L-L,ONE                  0266
03800 49 07496 00000
03807    00005 -2448
03812    00005 -2261
03818 16 07527 -3841 09050        FD   WORK&N*L-L,MTX                  0271
03830 49 07476 00000
03837    00005 -2448
03842    00005 -0477
03848 16 03919 -2295 09060        TFM  A09120&23,WORK                  0276
03860 16 03924 -0494 09065        TFM  A09120&28,MTX&L                 0278
03872 16 03949 -2295 09070        TFM  A09130&23,WORK                  0280
03884 26 00079 02609 09110 A09110 TF   79,KONZ                         0282
03896 16 07527 -3919 09120 A09120 TFLS ,                               0284
03908 49 07496 00000
03915    00005 -0000
03920    00005 -0000
03926 16 07527 -3949 09130 A09130 FM   ,WORK&N*L-L                     0289
03938 49 07456 00000
03945    00005 -0000
03950    00005 -2448
03956 11 03919 -0017 09140        AM   A09120&23,L                     0294
03968 11 03924 -0017 09150        AM   A09120&28,L                     0296
03980 11 03949 -0017 09155        AM   A09130&23,L                     0298
03992 14 03949 -2448 09160        CM   A09130&23,WORK&N*L-L            0300
04004 47 03884 01200 09170        BNE  A09110                          0302
04016 16 04147 -0477 10010        TFM  A10100&23,MTX                   0304
```

```
04028 16 04152 -0647 10020        TFM  A10100628,MTX&N*L                    0306
04040 16 04177 -0477 10030        TFM  A10110623,MTX                        0308
04052 16 04207 -0664 10040        TFM  A10120623,MTX&N*L&L                  0310
04064 16 04212 -0477 10050        TFM  A10120628,MTX                        0312
04076 16 04237 -0477 10060        TFM  A10125623,MTX                        0314
04088 16 04242 -0664 10065        TFM  A10125628,MTX&N*L&L                  0316
04100 16 04182 -2295 10070 A10070 TFM  A10110628,WORK                       0318
04112 26 00079 02609 10090 A10090 TF   79,KONZ                              0320
04124 16 07527 -4147 10100 A10100 TFLS *                                    0322
04136 49 07496 00000
04143    00005 -0000
04148    00005 -0000
04154 16 07527 -4177 10110 A10110 FM   *                                    0327
04166 49 07456 00000
04173    00005 -0000
04178    00005 -0000
04184 16 07527 -4207 10120 A10120 FS   *                                    0332
04196 49 07416 00000
04203    00005 -0000
04208    00005 -0000
04214 16 07527 -4237 10125 A10125 TFLS *                                    0337
04226 49 07496 00000
04233    00005 -0000
04238    00005 -0000
04244 11 04147 -0017 10130        AM   A10100623,L                          0342
04256 11 04177 -0017 10135        AM   A10110623,L                          0344
04268 11 04182 -0017 10140        AM   A10110628,L                          0346
04280 11 04207 -0017 10145        AM   A10120623,L                          0348
04292 11 04212 -0017 10150        AM   A10120628,L                          0350
04304 11 04237 -0017 10155        AM   A10125623,L                          0352
04316 11 04242 -0017 10160        AM   A10125628,L                          0354
04328 14 04182 -2448 10170        CM   A10110628,WORK&N*L-L                  0356
04340 47 04112 01200 10180        BNF  A10090                               0358
04352 26 04435 04152 10200        TF   A11010623,A10100628                   0360
04364 26 04465 04147 10210        TF   A11020623,A10100623                   0362
04376 26 04495 04147 10215        TF   A11030623,A10100623                   0364
04388 26 04500 04152 10217        TF   A11030628,A10100628                   0366
04400 26 00079 02609 10220        TF   79,KONZ                              0368
04412 16 07527 -4435 11010 A11010 FM   *,WORK&N*L-L                         0370
04424 49 07456 00000
04431    00005 -0000
04436    00005 -2448
04442 16 07527 -4465 11020 A11020 TFLS *ZERO                                0375
04454 49 07496 00000
04461    00005 -0000
04466    00005 -2244
04472 16 07527 -4495 11030 A11030 FS   *                                    0380
04484 49 07416 00000
04491    00005 -0000
04496    00005 -0000
04502 11 04147 -0017 11040        AM   A10100623,L                          0385
04514 11 04177 -0017 11050        AM   A10110623,L                          0387
04526 11 04207 -0017 11060        AM   A10120623,L                          0389
04538 11 04212 -0017 11070        AM   A10120628,L                          0391
04550 11 04237 -0017 11080        AM   A10125623,L                          0393
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04562 11 04242 -0017 11090        AM   A10125628,L                          0395
04574 11 04152 -0170 11100        AM   A10100628,N*L                        0397
04586 14 04152 -2177 11110        CM   A10100628,MTX&N*N*L                  0399
04598 47 04100 01200 11130        BNF  A10070                               0401
04610 16 04657 -2007 11140        TFM  A11170623,MTX&N*N*L-N*L              0403
04622 16 04662 -2295 11150        TFM  A11170628,WORK                       0405
04634 16 07527 -4657 11170 A11170 TFLS *                                    0407
04646 49 07496 00000
04653    00005 -0000
04658    00005 -0000
04664 11 04657 -0017 11180        AM   A11170623,L                          0412
04676 11 04662 -0017 11190        AM   A11170628,L                          0414
04688 14 04662 -2465 11200        CM   A11170628,WORK&N*L                   0416
04700 47 04634 01200 11210        BNE  A11170                               0418
04712 12 02220 000-1 12010        SM   MASTER,1,10                          0420
04724 47 03158 01200 12020        BNZ  A05260                               0422
04736 46 05232 00100 12021        BC1  A17101                               0424
                     12030 *           INVERSE COMPLETE                     0426
                     12040 *           RE-PIVOT                             0428
04748 16 04795 -2179 13010        TFM  A13030611,PIVTBL                     0430
04760 16 02220 000-9 13020        TFM  MASTER,N-1,10                        0432
04772 16 03191 000-1 13021 A13021 TFM  TEMP4,1,10                           0434
04784 26 02911 00000 13030 A13030 TF   TEMP69                              0436
04796 24 02911 03191 13040        C    TEMP69,TEMP4                         0438
04808 46 05184 01200 13050        RE   A17080                               0440
04820 21 02911 02220 13060        A    TEMP69,MASTER                        0442
04832 21 03191 02220 13070        A    TEMP4,MASTER                         0444
04844 27 04926 02911 13080        BT   CALC,TEMP69                          0446
04856 26 05050 00099 13090        TF   A16080628,99                         0448
04868 26 05075 00099 13100        TF   A16090623,99                         0450
04880 27 04926 03191 13110        BT   CALC,TEMP4                           0452
04892 26 05080 00099 13120        TF   A16090628,99                         0454
04904 26 05105 00099 13130        TF   A16100623,99                         0456
04916 49 05010 00000 13131        B    A14180612                            0458
04923                13132        DORG *-4                                 0460
04924    00002       14080        DS   2                                   0462
04936 12 04925 000-1 14090 CALC   SM   CALC-1,1,10                          0464
04938 13 04925 -0010 14100        MM   CALC-1,N                             0466
04950 26 05009 00099 14110        TF   GOLD,99                              0468
04962 13 05009 -0017 14120        MM   GOLD,L                               0470
04974 32 00095 00000 14131        SF   95                                   0472
04986 11 00099 -0477 14140        AM   99,MTX                               0474
04998 42 00000 00000 14180 A14180 RR                                        0476
05009                14200 GOLD   DS   ,A14180611                          0478
05010 26 02911 02222 16060        TF   TEMP69,TABLE                         0480
05022 16 07527 -5045 16080 A16080 TFLS TEMP3,                               0482
05034 49 07496 00000
05041    00005 -2278
05046    00005 -0000
05052 16 07527 -5075 16090 A16090 TFLS *                                    0487
05064 49 07496 00000
05071    00005 -0000
05076    00005 -0000
05082 16 07527 -5105 16100 A16100 TFLS *TEMP3                               0492
05094 49 07496 00000
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05101     00005 -0000
05106     00005 -2278
05112 11 05050 -0017 16120          AM   A16080628,L                                0497
05124 11 05075 -0017 16130          AM   A16090623,L                                0499
05136 11 05080 -0017 16140          AM   A16090628,L                                0501
05148 11 05105 -0017 16150          AM   A16100623,L                                0503
05160 12 02911 000-1 16160          SM   TEMP69,1,10                                 0505
05172 47 05022 01200 16170          BNZ  A16080                                     0507
05184 12 02220 000-1 17080 A17080   SM   MASTER,1,10                                 0509
05196 11 04795 000-2 17090          AM   A13030611,2,10                             0511
05208 14 04795 -2199 17091          CM   A13030611,PIVTBL&N*2                      0513
05220 47 04772 01200 17092          BNF  A13021                                     0515
05232 47 05268 00300 17101 A17101   BNC9 *&36                                       0517
05244 27 05846 05845 17102          BT   CLCD,CLCD-1                                0519
05256 17 00000 -5268 17103 A17103   BTM  ,*612                                      0521
05268 16 05262 -5914 17104          TFM  A1710366,PUNCH1                            0523
                     17110 *             RE-PIVOT COMPLETE                        0525
                     18040 *             IF NO SOLUTION VECTORS                   0527
                     18050 *             PROGRAM HANGS UP WITH                    0529
                     18060 *             READER NO FEED CONDITION                 0531
05280 15 05341 00001 18070 A18070   TDM  A1813061,1                                 0533
05292 16 05435 -2295 18090          TFM  A18180623,WORK                             0535
05304 16 05448 -6268 18100          TFM  A18190666,PUNCH3                           0537
05316 37 07255 00500 18110 A18110   RACD INPUT                                      0539
05328 27 06378 06377 18120          BT   FLOAT,FLOAT-1                              0541
05340 41 05412 00000 18130 A18130   NOP  A18180                                     0543
05352 15 05341 00009 18140          TDM  A1813061,9                                 0545
05364 27 05846 05845 18150          BT   CLCD,CLCD-1                                0547
05376 31 02610 02832 18160          TR   OUTPUT-1,CMNT4-1                           0549
05388 15 02641 00000 18161          TDM  OUTPUT630                                  0551
05400 39 02611 00400 18170          WACD OUTPUT                                     0553
05412 16 07527 -5435 18180 A18180   TFLS ,DONE                                      0555
05424 49 07496 00000
05431     00005 -0000
05436     00005 -7234
05442 17 00000 -5454 18190 A18190   BTM  ,*612                                      0560
05454 16 05448 -6304 18200          TFM  *-6,PUNCH4                                 0562
05466 11 05435 -0017 18210          AM   A18180623,L                                0564
05478 14 05435 -2465 18220          CM   A18180623,WORK&L*N                         0566
05490 47 05316 01200 18230          BNE  A18110                                     0568
                     19010 *             SOLVE SYSTEM BY MATRIX                   0570
                     19020 *             VECTOR MULTIPLICATION                    0572
05502 27 05846 05845 19030          BT   CLCD,CLCD-1                                0574
05514 31 02610 02800 19031          TR   OUTPUT-1,CMNT3-1                           0576
05526 15 02641 00000 19041          TDM  OUTPUT630                                  0578
05538 39 02611 00400 19050          WACD OUTPUT                                     0580
05550 16 05585 -0477 18060          TFM  A19080611,MTX                              0582
05562 16 05784 -6268 19070          TFM  A1918066,PUNCH3                            0584
05574 16 05668 -0000 19080 A19080   TFM  A19120628                                  0586
05586 16 05698 -2295 19090          TFM  A19130628,WORK                             0588
05598 16 07527 -5621 19100          TFLS DONE,ZERO                                  0590
05610 49 07496 00000
05617     00005 -7234
05622     00005 -2244
05628 26 00079 02609 19110          TF   79,KONZ                                    0595
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138

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05640 16 07527 -5663 19120 A19120   TFLS MTX&N*N*L,                                0597
05652 49 07496 00000
05659     00005 -2177
05664     00005 -0000
05670 16 07527 -5693 19130 A19130   FM   MTX&N*N*L,                                0602
05682 49 07456 00000
05689     00005 -2177
05694     00005 -0000
05700 16 07527 -5723 19135          FA   DONE,MTX&N*N*L                             0607
05712 49 07496 00000
05719     00005 -7234
05724     00005 -2177
05730 11 05668 -0017 19140          AM   A19120628,L                                0612
05742 11 05698 -0017 19150          AM   A19130628,L                                0614
05754 14 05698 -2465 19160          CM   A19130628,WORK&N*L                         0616
05766 47 05628 01200 19170          BNF  A19120-12                                  0618
05778 17 00000 -5790 19180 A19180   BTM  ,*612                                      0620
05790 16 05784 -6304 19190          TFM  *-6,PUNCH4                                 0622
05802 11 05585 -0170 19200          AM   A19080611,N*L                              0624
05814 14 05585 -2177 19210          CM   A19080611,MTX&N*N*L                        0626
05826 47 05874 01200 19220          BNF  A19080                                     0628
05838 M9 05280 00000 19230          B    A18070,,0                                  0630
05846                19250          DORG *-3                                        0632
05846 16 05864 -2611 79020 CLCD     TFM  CLCD618,OUTPUT                             0634
05858 16 00000 000-0 79030          TFM  ,,10                                       0636
05870 11 05864 000-2 79040          AM   CLCD618,2,10                               0638
05882 14 05864 -2771 79050          CM   CLCD618,OUTPUT&160                         0640
05894 47 05858 01200 79060          BNF  CLCD612                                    0642
05906 42 00000 00000 79070          RR                                              0644
05908                79080          DORG *-9                                        0646
05912     00005      79081          DS   5                                         0648
05914 31 02610 02786 80020 PUNCH1   TR   OUTPUT-1,CMNT2-1                           0650
05926 15 02623 00000 80021          TDM  OUTPUT612                                  0652
05938 39 02611 00400 80030          WACD OUTPUT                                     0654
05950 26 06262 05913 80031          TF   A8024066,PUNCH1-1                          0656
05962 49 06022 00000 80032          B    A80070                                     0658
05974 31 02610 02770 80050 PUNCH2   TR   OUTPUT-1,CMNT1-1                           0660
05986 15 02625 00000 80051          TDM  OUTPUT614                                  0662
05998 39 02611 00400 80060          WACD OUTPUT                                     0664
06010 26 06262 05973 80061          TF   A8024066,PUNCH2-1                          0666
06022 16 06098 -0477 80070 A80070   TFM  A80110628,MTX                              0668
06034 31 02610 02449 80080          TR   OUTPUT-1,BLANKS-79                         0670
06046 16 02685 000-1 80090          TFM  OUTPUT674,1,10                             0672
06058 16 02689 000-1 80100 A80100   TFM  OUTPUT678,1,10                             0674
06070 16 07527 -6093 80110 A80110   TFLS OUTPUT&L-2,                                0676
06082 49 07496 00000
06089     00005 -2626
06094     00005 -0000
06100 27 07020 07019 80111          BT   SPREAD,SPREAD-1                            0681
06112 33 02684 00000 80120          CF   OUTPUT673                                  0683
06124 33 02688 00000 80130          CF   OUTPUT677                                  0685
06136 38 02610 00400 80140          WNCD OUTPUT-1                                   0687
06148 32 02684 00000 80150          SF   OUTPUT673                                  0689
06160 32 02688 00000 80160          SF   OUTPUT677                                  0691
06171                80161 TEMP98   DS   ,*                                        0693
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139

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06172 11 06098 -0017 80170         AM   A80110&28,L                              0695
06184 11 02689 000-1 80180         AM   OUTPUT&78,1,10                          0697
06196 24 02689 02222 80190         C    OUTPUT&78,TABLE                         0699
06208 47 06070 01100 80200         BNH  A80110                                  0701
06220 11 02685 000-1 80210         AM   OUTPUT&74,1,10                          0703
06232 24 02685 02222 80220         C    OUTPUT&74,TABLE                         0705
06244 47 06058 01100 80230         BNH  A80100                                  0707
06256 49 00000 00000 80240 A80240  B                                            0709
06268 31 07610 02449 81020 PUNCH3  TR   OUTPUT-1,BLANKS-79                      0711
06280 26 06303 06267 81030         TF   PUNCH4-1,PUNCH3-1                       0713
06292 49 06304 00000 81040         B    *&12                                    0715
06304 16 07527 -6327 81050 PUNCH4  TFLS OUTPUT&L-2,DONE                         0717
06316 49 07496 00000
06323     00005 -2626
06328     00005 -7234
06334 27 07020 07019 81055         BT   SPREAD,SPREAD-1                         0722
06346 38 02610 00400 81060         WNCD OUTPUT-1                                0724
06358 26 06376 06303 81065         TF   *&18,PUNCH4-1                           0726
06370 4R 00000 00000 81070         B    ,,1                                     0728
06378                81080         DORG *-3                                     0730
                     85000 *       THIS IS THE FLOAT SUBROUTINE                 0732
06378 16 07527 -6401 85020 FLOAT   TFLS DONE,KON1                               0734
06390 49 07496 00000
06397     00005 -7234
06402     00005 -7251
06408 15 06671 00003 85060         TDM  A8718061,3                              0739
06420 15 06733 00001 85080         TDM  A8809061,1                              0741
06432 15 06893 00001 85100         TDM  DECPT61,1                               0743
06444 16 06810 -7218 85120         TFM  A8901066,DONE-L61                       0745
06456 15 06828 00001 85140         TDM  A89050,1                                0747
06468 15 06893 00001 85160         TDM  DECPT61,1                               0749
06480 16 06498 -7333 86010         TFM  A8603066,INPUT&78                       0751
06492 14 00000 000-0 86030 A86030  CM   ,,10                                    0753
06504 47 06536 01200 86050         BNE  A86140                                  0755
06516 12 06498 000-2 86070         SM   A8603066,2,10                           0757
06525     00002      86090 RECMK   DC   2,*,*-2                                 0759
06528 49 06492 00000 86110         B    A86030                                  0762
06536                86130         DORG *-3                                     0764
06536 11 06498 000-1 86140 A86140  AM   A8603066,1,10                           0766
06548 26 06566 06498 86160         TF   A8618066,A8603066                       0768
06560 31 00000 06524 86160 A86180  TR   ,RECMK-1                               0770
06572 14 07255 000-0 87010 A87010  CM   INPUT,,10                              0772
06584 47 06684 01200 87030         BNE  A88010                                  0774
06596 31 07254 07256 87050 A87050  TR   INPUT-1,INPUT&1                        0776
06608 45 06572 07255 87070         BNR  A87010,INPUT                            0778
06620 32 07218 00000 87090 A87090  SF   DONE-L61                               0780
06632 24 07232 07249 87110         C    DONE-2,KON1-2                           0782
06644 47 06670 01200 87130         BNE  A87180                                  0784
06656 16 07234 000RR 87140         TFM  DONE,99,101I                            0786
06668 42 00000 00000 87160         BB                                           0788
06670                87170         DORG *-9                                     0790
06670 33 07232 00000 87180 A87180  CF   DONE-2                                  0792
06682 42 00000 00000 87190         BB                                           0794
06684                87200         DORG *-9                                     0796
06684 14 07255 000J0 88010 A88010  CM   INPUT,10,10                             0798
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140

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06696 46 06596 01200 88030         BE   A87050                                  0800
06708 14 07255 000K0 88050         CM   INPUT,20,10                             0802
06720 46 06872 01200 88070         BE   SETSW                                   0804
06732 41 06780 00000 88090 A88090  NOP  A88170                                  0806
06744 14 07255 000P0 88110         CM   INPUT,70,10                             0808
06756 46 06596 01200 88130         BE   A87050                                  0810
06768 15 06733 00009 88150         TDM  A8809061,9                              0812
06780 47 06892 01300 88170 A88170  BI   DECPT                                   0814
06792 15 06893 00009 88190         TDM  DECPT61,9                               0816
06804 25 00000 07255 89010 A89010  TD   ,INPUT                                  0818
06816 11 06810 000-1 89030         AM   A8901066,1,10                           0820
06828 11 07234 000-1 89050 A89050  AM   DONE,1,10                               0822
06840 14 06810 -7233 89070         CM   A8901066,DONE-1                         0824
06852 46 06620 01200 89090         BE   A87090                                  0826
06864 49 06596 00000 89110         B    A87050                                  0828
06872                89130         DORG *-3                                     0830
06872 15 06671 00002 89150 SETSW   TDM  A8718061,2                              0832
06884 49 06596 00000 89170         B    A87050                                  0834
06892                89190         DORG *-3                                     0836
06892 41 07000 00000 90010 DECPT   NOP  A90180                                  0838
06904 31 07254 07256 90030         TR   INPUT-1,INPUT&1                         0840
06916 45 06936 07255 90050         BNR  A90090,INPUT                            0842
06928 49 06620 00000 90070         B    A87090                                  0844
06936                90080         DORG *-3                                     0846
06936 14 07255 000P0 90090 A90090  CM   INPUT,70,10                             0848
06948 46 06980 01200 90110         BE   A90150                                  0850
06960 15 06828 00004 90120         TDM  A89050,4                                0852
06972 49 06804 00000 90130         B    A89010                                  0854
06980                90140         DORG *-3                                     0856
06980 12 07234 000-1 90150 A90150  SM   DONE,1,10                               0858
06992 49 06904 00000 90160         B    DECPT&12                                0860
07000                90170         DORG *-3                                     0862
07000 15 06828 00004 90180 A90180  TDM  A89050,4                                0864
07012 49 06586 00000 90190         B    A87050,,5                               0866
07020                90200         DORG *-3                                     0868
07020 25 02659 02528 91010 SPREAD  TD   OUTPUT&48,BLANKS                        0870
07032 26 02661 02626 91020         TF   OUTPUT&50,OUTPUT&L-2                    0872
07044 33 02660 00000 91040         CF   OUTPUT&49                              0874
07056 44 07080 02661 91050         BNF  *&24,OUTPUT&50                          0876
07068 15 02659 0000- 91070         TDM  OUTPUT&48,0,11                          0878
07080 33 02661 00000 91080         CF   OUTPUT&50                              0880
07092 26 02626 02624 91090         TF   OUTPUT&L-2,OUTPUT&L-4                   0882
07104 25 02610 02528 91110         TD   OUTPUT-1,BLANKS                        0884
07116 25 02611 02528 91120         TD   OUTPUT,BLANKS                          0886
07128 44 07152 02626 91140         BNF  *&24,OUTPUT&L-2                         0888
07140 15 02611 0000- 91160         TDM  OUTPUT,0,11                             0890
07152 33 02626 00000 91170         CF   OUTPUT&L-2                              0892
07164 33 02612 00000 91180         CF   OUTPUT&1                               0894
07176 42 00000 00000 91190         BB                                           0896
07234     00047      94070 DONE    DS   47                                     0898
07251     00017      94089         DC   L,0                                    0900
07251     00002      94090 KON1    DC   2,0,*                                  0903
07253     00001      94091         DAS  1                                       0906
07255     00002      94110 INPUT   DC   2,0                                     0908
07257     00002      94120         DC   2,0                                     0911
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141

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07259  00002  94130          DC   2,0                0914
07261  00002  94140          DC   2,0                0917
07263  00002  94150          DC   2,0                0920
07265  00002  94160          DC   2,0                0923
07267  00002  94170          DC   2,0                0926
07269  00002  94180          DC   2,0                0929
07271  00002  94190          DC   2,0                0932
07273  00002  94200          DC   2,0                0935
07275  00002  95010          DC   2,0                0938
07277  00002  95020          DC   2,0                0941
07279  00002  95030          DC   2,0                0944
07281  00002  95040          DC   2,0                0947
07283  00002  95050          DC   2,0                0950
07285  00002  95060          DC   2,0                0953
07287  00002  95070          DC   2,0                0956
07289  00002  95080          DC   2,0                0959
07291  00002  95090          DC   2,0                0962
07293  00002  95100          DC   2,0                0965
07295  00002  95110          DC   2,0                0968
07297  00002  95120          DC   2,0                0971
07299  00002  95130          DC   2,0                0974
07301  00002  95140          DC   2,0                0977
07303  00002  95150          DC   2,0                0980
07305  00002  95160          DC   2,0                0983
07307  00002  95170          DC   2,0                0986
07309  00002  95180          DC   2,0                0989
07311  00002  95190          DC   2,0                0992
07313  00002  95200          DC   2,0                0995
07315  00002  96010          DC   2,0                0998
07317  00002  96020          DC   2,0                1001
07319  00002  96030          DC   2,0                1004
07321  00002  96040          DC   2,0                1007
07323  00002  96050          DC   2,0                1010
07325  00002  96060          DC   2,0                1013
07327  00002  96070          DC   2,0                1016
07329  00002  96080          DC   2,0                1019
07331  00002  96090          DC   2,0                1022
07333  00002  96100          DC   2,0                1025
07335  00040  96110          DAS  40                 1028
07415  00001  99990 NOISE    DAC  1,0                1030
00402         99999          DEND GO                 1033
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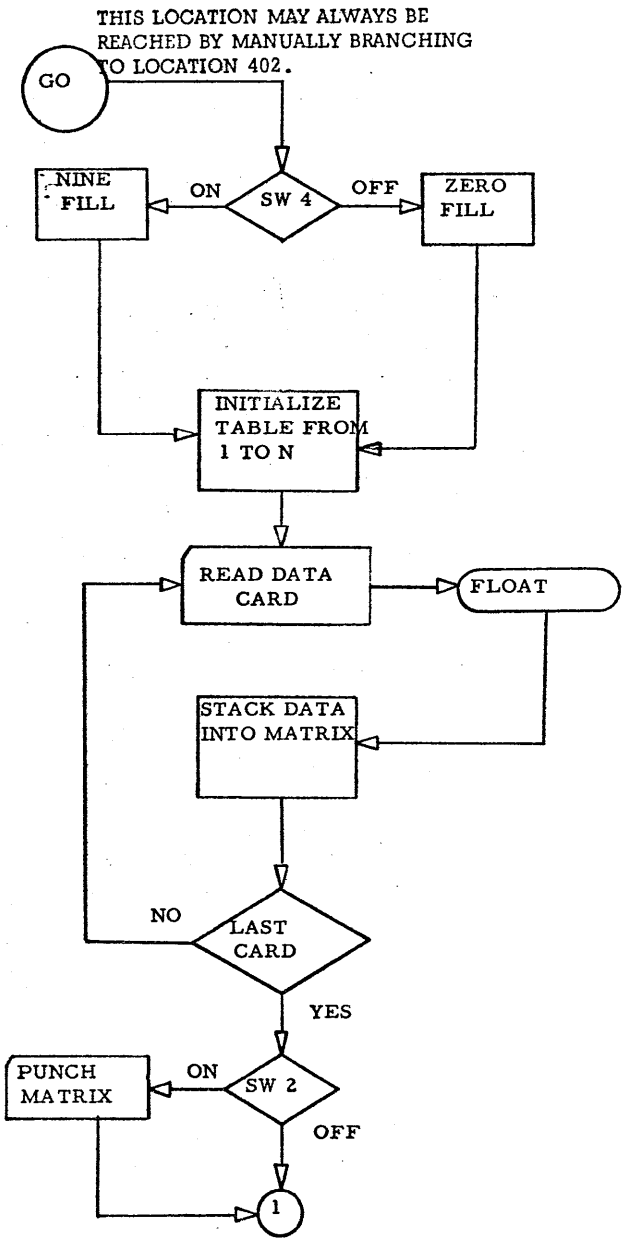
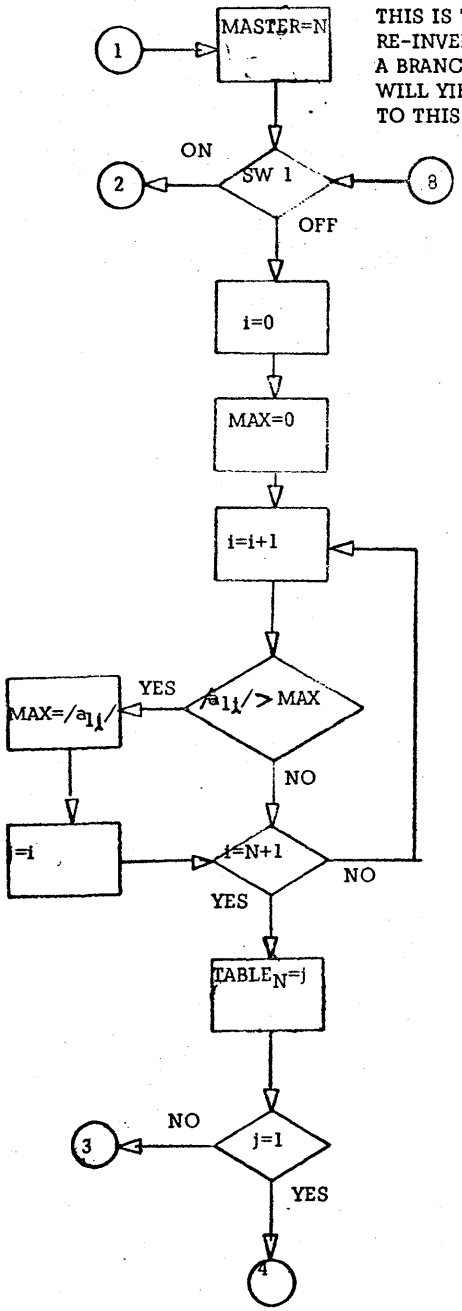
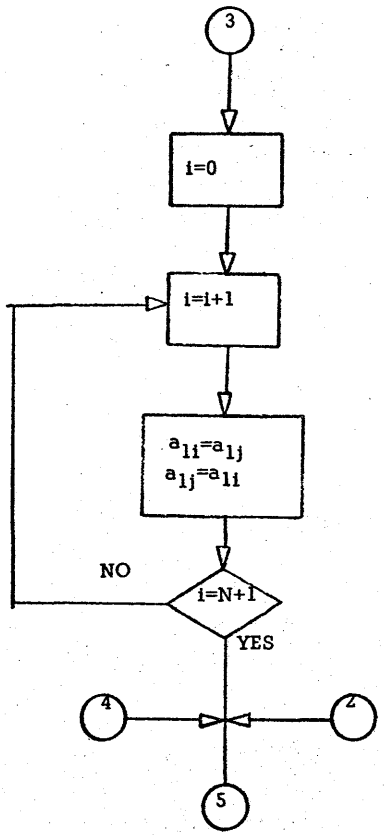
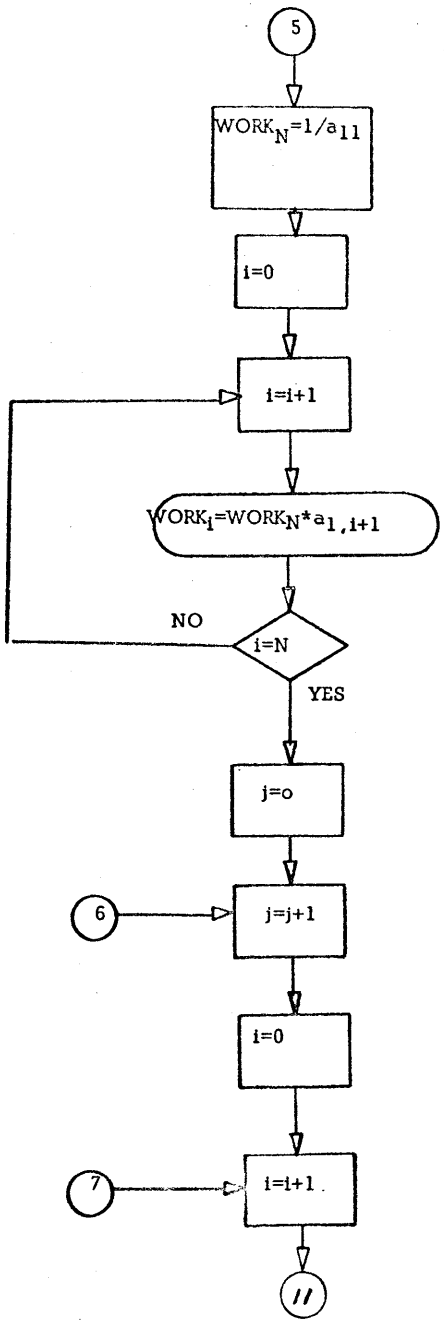
FLOW CHARTS

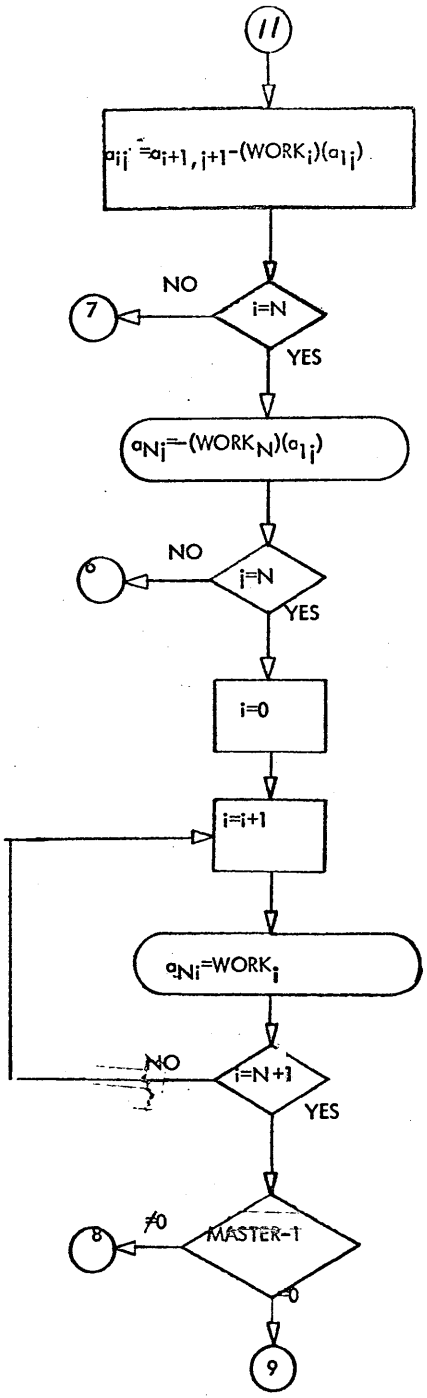

THIS LOCATION MAY ALWAYS BE REACHED BY MANUALLY BRANCHING TO LOCATION 402.
GO
NINE FILL
ON
SW 4
OFF
ZERO FILL
INITIALIZE TABLE FROM 1 TO N
READ DATA CARD
FLOAT
STACK DATA INTO MATRIX
NO
LAST CARD
YES
PUNCH MATRIX
ON
SW 2
OFF
1

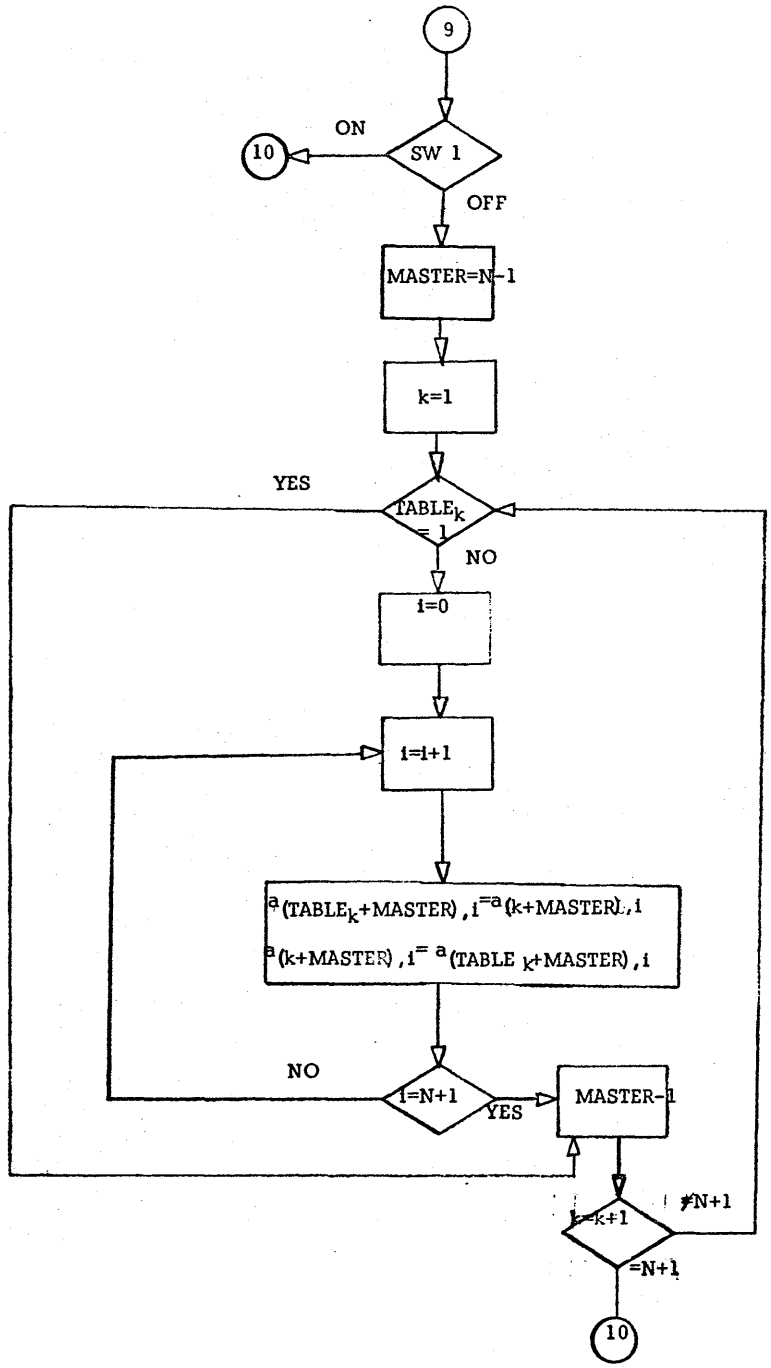


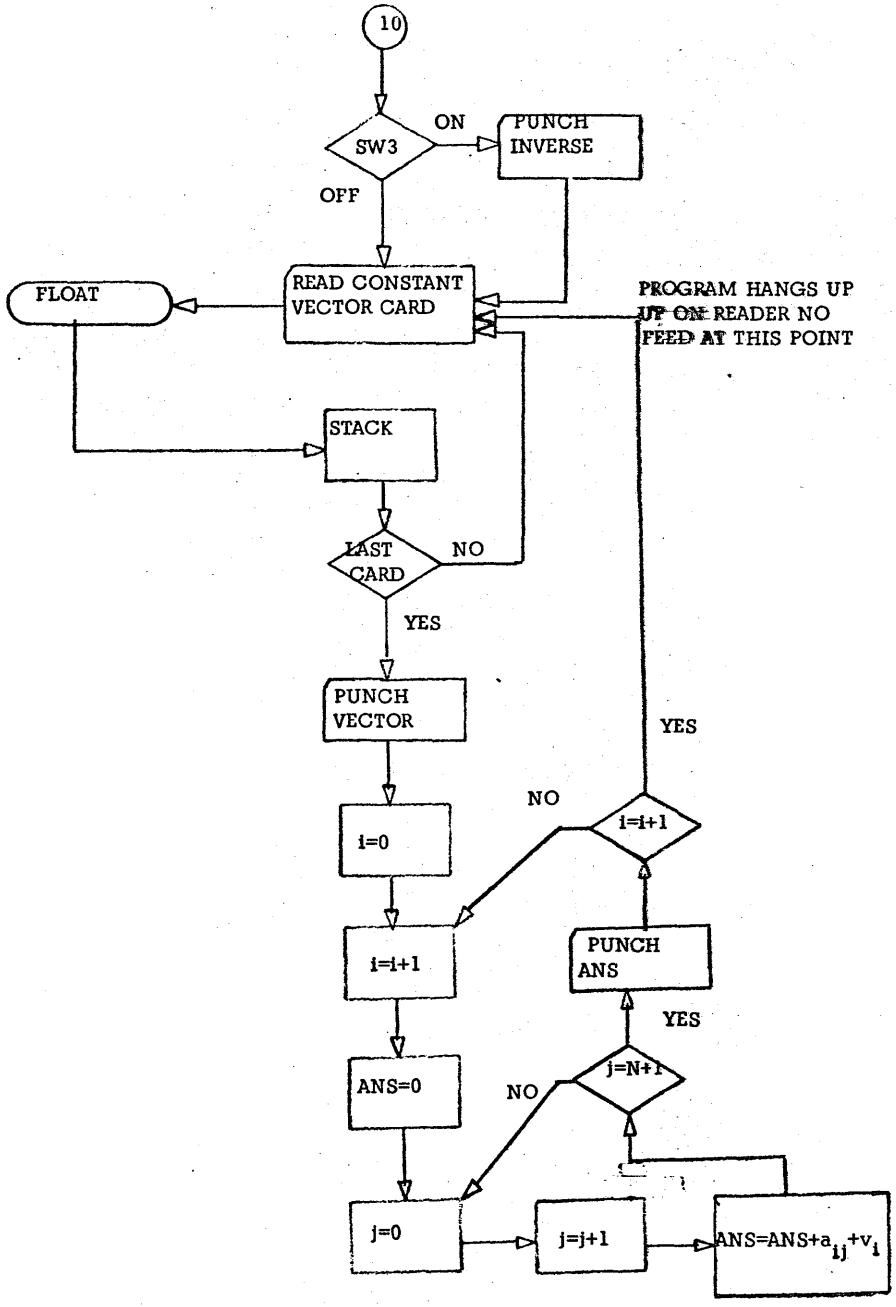
THIS IS THE ENTRY POINT FOR RE-INVERSION OF THE INVERSE. A BRANCH TO LOCATION 454 WILL YIELD A SECONDARY BRANCH TO THIS VARIABLE ADDRESS.




5
WORK$_N$=1/a$_{11}$
i=0
i=i+1
WORK$_i$=WORK$_N$*a$_{1,i+1}$
NO
i=N
YES
j=o
6
j=j+1
i=0
7
i=i+1
11


11
a$_{ij}$=a$_{i+1,j+1}$-(WORK$_i$)(a$_{1j}$)
NO
7
i=N
YES
a$_{Ni}$=-(WORK$_N$)(a$_{1i}$)
NO
6
j=N
YES
i=0
i=i+1
a$_{Ni}$=WORK$_i$
NO
i=N+1
YES
≠0
8
MASTER-1
=0
9


9
ON
10
SW 1
OFF
MASTER=N-1
k=1
YES
TABLE$_k$ = 1
NO
i=0
i=i+1
$a_{(TABLE_k+MASTER),i} = a_{(k+MASTER),i}$
$a_{(k+MASTER),i} = a_{(TABLE_k+MASTER),i}$
NO
i=N+1
YES
MASTER-1
k=k+1
=N+1
10


10
SW3
ON
PUNCH INVERSE
OFF
READ CONSTANT VECTOR CARD
FLOAT
PROGRAM HANGS UP UP ON READER NO FEED AT THIS POINT
STACK
LAST CARD
NO
YES
PUNCH VECTOR
YES
NO
i=0
i=i+1
i=i+1
PUNCH ANS
YES
ANS=0
j=N+1
NO
j=0
j=j+1
ANS=ANS+a_{ij}+v_i