1620 GENERAL PROGRAM LIBRARY CROUT REDUCTION

5.0.021

DISCLAIMER

Although each program has been tested by its contributor, no warranty, express or implied, is made by the contributor or 1620 USERS Group, as to the accuracy and functioning of the program and related program material, nor shall the fact of distribution constitute any such warranty, and no responsibility is assumed by the contributor or 1620 USERS Group, in connection therewith.

1620 USERS GROUP PROGRAM REVIEW AND EVALUATION

(fill out in typewriter or pencil, do not use ink)

Program No. ________________ Date ______________

Program Name: ____

1. Does the abstract adequately describe what the program is and what it does? Yes___ No___
   Comment ______________________________

2. Does the program do what the abstract says? Yes___ No___
   Comment ______________________________

3. Is the Description clear, understandable, and adequate? Yes___ No___
   Comment ______________________________

4. Are the Operating Instructions understandable and in sufficient detail? Yes___ No___
   Comment ______________________________
   Are the Sense Switch options adequately described (if applicable)? Yes___ No___
   Are the mnemonic labels identified or sufficiently understandable? Yes___ No___
   Comment ______________________________

5. Does the source program compile satisfactorily (if applicable)? Yes___ No___
   Comment ______________________________

6. Does the object program run satisfactorily? Yes___ No___
   Comment ______________________________

7. Number of test cases run______. Are any restrictions as to data, size, range, etc. covered adequately in description? Yes___ No___
   Comment ______________________________

8. Does the Program Meet the minimal standards of the 1620 Users Group? Yes___ No___
   Comment ______________________________

9. Were all necessary parts of the program received? Yes___ No___
   Comment ______________________________

10. Please list on the back any suggestions to improve the usefulness of the program. These will be passed onto the author for his consideration.

Please return to:

Mr. Richard L. Pratt
Data Corporation
7500 Old Xenia Pike
Dayton, Ohio 45432

Your Name ______________________

Company ______________________

Address ______________________

User Group Code ______________________

THIS REVIEW FORM IS PART OF THE 1620 USER GROUP ORGANIZATION'S PROGRAM REVIEW AND EVALUATION PROCEDURE. NONMEMBERS ARE CORDIALLY INVITED TO PARTICIPATE IN THIS EVALUATION.

11/09/64

CROUT REDUCTION

S. S. Millwright

Modifications or revisions to this program, as they occur, will be announced in the appropriate Catalog of Programs for IBM Data Processing Systems. When such an announcement occurs, users should order a complete new program from the Program Information Department.

Direct Inquiries to:

Mr. R. C. Wenrick
336 Woodward Road, S.E.
Albuquerque, New Mexico

CROUT REDUCTION

DECK KEY

Deck 1	Fortran Source deck
Deck 2	Object deck
Deck 3	Input to sample problem

1620 USERS Group Library

Program Abstract

Title: Crout Reduction

Author; Organization: S. S. Millwright
ACF Industries, Incorporated
Albuquerque, New Mexico

Direct Inquiries to: Mr. R. C. Wenrick
336 Woodward Road, S. E.
Albuquerque, New Mexico
Telephone: 247-0361, Ext. 507

Purpose/Description: Will evaluate determinants of maximum order 36, solve simultaneous linear systems with an indefinite number of constant vectors, and/or determine the inverse of maximum order 36 matrices. Singular input matrices are detected. Output includes data which indicates accuracy of solution vector.

Mathematical Method: Crout Reduction. See Introduction to Numerical Analysis by F. B. Hildebrand, McGraw-Hill Book Co., Inc., 1956, pp. 429-435

Restrictions, Range: Element in first row and first column of input matrix must not be zero.

Storage Requirements: All of core.

Equipment Specifications:
Memory 20K ___ 40K ___ 60K X K ___ Automatic Divide: Yes ___ No ___

Indirect Addressing: Yes ___ No ___ Other Special Features Required ______

Additional Remarks: This program was written in Fortran language. It was compiled on a machine equipped with the automatic divide/automatic floating point features. Smaller arrays could be specified in order to adapt this program to smaller machines. Successful solutions for systems up to ninth order have been accomplished with this program.

TABLE OF CONTENTS

INTRODUCTION

M-201 is a Fortran language program which employs the Crout Reduction[1] technique in order to evaluate determinants of maximum order 36, to solve simultaneous linear systems consisting of up to 36 equations in 36 unknowns with an indefinite number of constant vectors, and/or to calculate the inverses of maximum order 36 matrices.

The coefficient matrix is tested for singularity by the program. If it is found to be singular, an error message is typed and punched and the program proceeds to the next problem with no programmed stop.

If one or more solution vectors for simultaneous systems are desired, a matrix multiplication of the coefficient matrix times the solution vector will be performed to obtain a calculated constant vector. The difference between the input constant vector and the calculated constant vector for each solution vector will appear in the output. This difference will give an indication of the inaccuracy in the solution vector due to truncation.

If the solution vector accuracy is found to be unsatisfactory, the user may find it desirable to input the above defined "difference" vector as a new constant vector together with the original coefficient matrix. The new solution vector thus obtained may then be added to the original solution vector in order to obtain a better approximation of the true solution vector. This process may be repeated any number of times. The program does not perform this correction automatically; the input must be presented to the program as a separate problem and the "correction" vector added externally if this iteration technique is to be applied.

The user must insure that the element in row 1, column 1 of the coefficient matrix has a value other than zero.

GENERAL ANALYSIS

An augmented matrix of the original system of the form:

(1)
$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & c_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & c_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} & c_n \end{array}\right]$$

1 Hildebrand, F. B.: "Introduction to Numerical Analysis", McGraw - Hill Book Co., Inc., New York (1956), pp 429 - 435.

Where the a's are the coefficients of the unknowns and the c's are the constants can be reduced to an augmented matrix of the form:

(2)
$$\left[\begin{array}{cccc|c} a'_{11} & a'_{12} & \cdots & a'_{1n} & c'_1 \\ a'_{21} & a'_{22} & \cdots & a'_{2n} & c'_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a'_{n1} & a'_{n2} & \cdots & a'_{nn} & c'_n \end{array}\right]$$

By application of the relations

$$a'_{ij} = a_{ij} - \sum_{K=1}^{j-1} a'_{iK} a'_{Kj} \qquad (i \geq j)$$

$$a'_{ij} = (1/a'_{ii}) \left[a_{ij} - \sum_{K=1}^{i-1} a'_{iK} a'_{Kj} \right] \qquad (i < j)$$

$$c'_i = (1/a'_{ii}) \left[c_i - \sum_{K=1}^{i-1} a'_{iK} c'_K \right]$$

First to find the elements of the first column (a'_{11} to a'_{n1}), next to find the remaining elements of the first row (a'_{12} to c'_1), next the remaining elements of the second column (a'_{22} to a'_{n2}), and so on until the augmented matrix has been completely determined.

It can be shown that the value of the determinant of the original coefficient matrix is equal to the product of the n number of elements of the reduced matrix which lie on the main diagonal. If $|A|$ denotes the value of this determinate then, symbolically:

$$|A| = \prod_{i=1}^{n} a'_{ii}$$

The solution vector for the original system can be obtained from the relation:

$$X_i = c'_i - \sum_{k=i+1}^{n} a'_{ik} X_k$$

The solution vector is calculated foot to head i.e., in the order $X_n, X_{n-1}, \ldots X_1$

It can also be shown that if the constant vector in (1) be replaced with an n^{th} order unit matrix and n solution vectors be calculated considering the n columns of the unit matrix to be n constant vectors proceeding from left to right, that the resulting n solution vectors are identically the left-to-right columnwise elements of the inverse of the coefficient matrix from (1).

INPUT

All input is from punched cards and shall consist of the following:

Card No.	Data	CC	Remarks
1	Heading	1-55	Up to 55 Hollerith Characters
2	N	1-5	I5 - Order of matrix
	MTRX	6-10	I5 - >O, invert matrix; ≤O, do not invert matrix
	LSLN	11-15	I5 - >O, solution(s) to simultaneous system desired; ≤O, no solution desired
	KNO	16-20	I5 - number of constant vectors in input
	JVAL	21-25	I5 - >O, evaluate determinant of coeff. matrix; ≤O, do not evaluate determinant.
	INXT	26-30	I5 - >O, read data for next problem; ≤O, exit from M-201
3 to 2 + EN (where E = nearest integer $\geq \frac{N}{6}$)	A(I, J)	1-60	6E10.4 - Values of coefficients of input matrix arranged by rows up to six elements per card. (elements must all be from the same row on any given card)
3 + EN to 2 + E(N + KNO) (where E = nearest integer $\geq \frac{N}{6}$)	C(I)	1-60	6E10.4 - values of constant vector elements up to six per card (Elements must all be from same constant vector on any given card)

OUTPUT

All output is on punched cards except for the singular input matrix error message which will be typed as well as punched.

The output deck is designed to be listed on the IBM 1403 printer using a Fortran output lister program which skips to a new page for each new problem and deletes the sequence numbers in cc 78 - 80 of the output deck.

The output deck will consist of appropriate header cards, and any or all of the following data as called for on input card 2:

1. The value of the determinant of the coefficient matrix complete with an identifying label.

2. The solution vector(s) together with the "difference" vector(s) which is the difference between the input constant vector and a calculated constant vector. These will appear with identifying header cards and labels.

3. The inverse of the coefficient matrix listed by columns in five-column blocks. The column number will appear above the appropriate column; the row number will appear to the left of the appropriate row.

OPERATING INSTRUCTIONS

1. Load M-201 object deck followed by input deck into the read hopper of the IBM 1622.

2. Ready the punch hopper with blank cards.

3. Set console switches:
   I/O to STOP
   Parity to STOP
   OFLO to PROGRAM
   Sense switches -- none tested.

4. Depress RESET and LOAD.

PROGRAM LISTING

The following pages contain the source language listing for M-201.

```
C       M-201 CROUT REDUCTION
C       MAY BE USED TO--
C       1. EVALUATE DETERMINANTS (MAXIMUM ORDER=36)
C       2. SOLVE UP TO 36 SIMULTANEOUS LINEAR EQUATIONS
C       3. FIND THE INVERSE OF A MATRIX
C       ELEMENT IN FIRST ROW AND FIRST COLUMN,A(1,1),MAY NOT BE ZERO
        DIMENSION A(40,40),AA(36,36),X(36),C(36),CC(36),NQ(40)
   99   PUNCH 11
        READ 3
        PUNCH 3
        READ 1,N,MTRX,LSLN,KNO,JVAL,INXT
        DO 100 I=1,N
        DO 100 J=1,N,6
        READ 2,A(I,J),A(I,J+1),A(I,J+2),A(I,J+3),A(I,J+4),A(I,J+5)
  100   CONTINUE
        DO 200 I=1,N
  200   AA(I,1)=A(I,1)
        DO 300 J=2,N
  300   AA(1,J)=A(1,J)/A(1,1)
        DO 400 I=2,N
        DO 400 J=2,N
  400   AA(I,J)=0.
        J=2
  490   II=J
        DO 510 I=II,N
        LIM1=J-1
        DO 500 K=1,LIM1
  500   AA(I,J)=AA(I,J)+(AA(I,K)*AA(K,J))
  510   AA(I,J)=A(I,J)-AA(I,J)
        IF(AA(J,J))520,900,520
  520   IF(N-J)700,700,530
  530   I=J
        J=J+1
        JJ=J
        DO 610 J=JJ,N
        LIM2=I-1
        DO 600 K=1,LIM2
  600   AA(I,J)=AA(I,J)+(AA(I,K)*AA(K,J))
        AA(I,J)=A(I,J)-AA(I,J)
```



```
  610   AA(I,J)=AA(I,J)/AA(I,I)
        J=I+1
        GO TO 490
  700   IF(JVAL)800,800,720
  720   VALUE=AA(1,1)
        DO 710 I=2,N
  710   VALUE=VALUE*AA(I,I)
        PUNCH 4, VALUE
  800   IF(LSLN)730,730,220
  220   DO 230 I=1,N,6
        READ 2,C(I),C(I+1),C(I+2),C(I+3),C(I+4),C(I+5)
  230   CONTINUE
        DO 240 I=1,N
        X(I)=0.
  240   CC(I)=0.
        CC(1)=C(1)/AA(1,1)
        DO 250 I=2,N
        LIM6=I-1
        DO 260 K=1,LIM6
  260   CC(I)=CC(I)+(AA(I,K)*CC(K))
        CC(I)=C(I)-CC(I)
  250   CC(I)=CC(I)/AA(I,I)
        X(N)=CC(N)
        LIM7=N-1
        DO 270 I=1,LIM7
        II=N-I
        LIM8=II+1
        DO 280 K=LIM8,N
  280   X(II)=X(II)+(AA(II,K)*X(K))
  270   X(II)=CC(II)-X(II)
        DO 290 I=1,N
  290   CC(I)=0.
        DO 291 I=1,N
        DO 291 J=1,N
  291   CC(I)=CC(I)+(A(I,J)*X(J))
        DO 292 I=1,N
  292   CC(I)=C(I)-CC(I)
        PUNCH 8
        DO 293 I=1,40
```

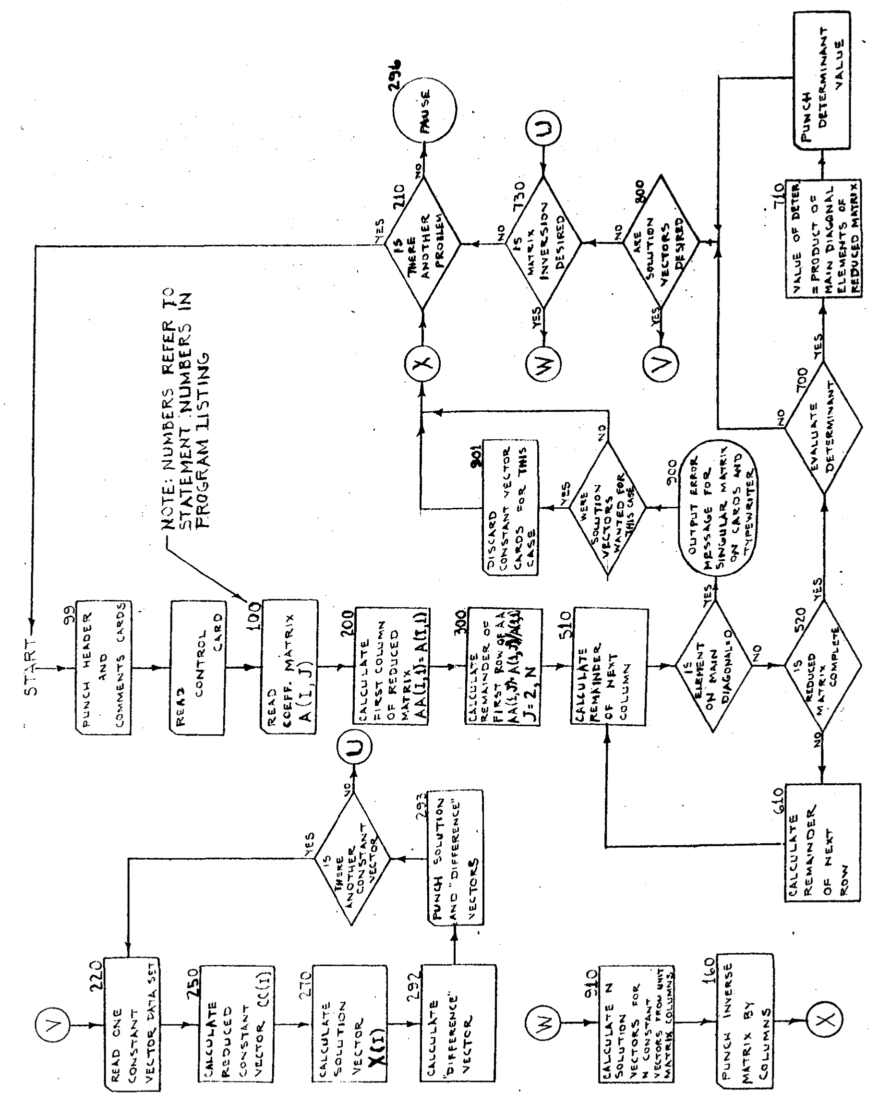
```
  293 NQ(I)=I
      DO 294 I=1,N
      PUNCH 9,NQ(I),X(I),CC(I)
  294 CONTINUE
      KNO=KNO-1
      IF(KNO)730,730,220
  730 IF(MTRX)210,210,810
  810 C(1)=1.
      CC(1)=1./AA(1,1)
      DO 820 I=2,N
      C(I)=0.
  820 CC(I)=0.
      NIJ=N+4
      DO 830 I=1,NIJ
      DO 830 J=1,NIJ
  830 A(I,J)=0.
      J=0
  840 J=J+1
      DO 860 I=2,N
      LIM3=I-1
      DO 890 K=1,LIM3
  890 CC(I)=CC(I)+(AA(I,K)*CC(K))
      CC(I)=C(I)-CC(I)
  860 CC(I)=CC(I)/AA(I,I)
      A(N,J)=CC(N)
      LIM4=N-1
      DO 910 I=1,LIM4
      II=N-I
      LIM5=II+1
      DO 920 K=LIM5,N
  920 A(II,J)=A(II,J)+(AA(II,K)*A(K,J))
  910 A(II,J)=CC(II)-A(II,J)
      IF(N-J)110,110,120
  120 C(J)=0.
      C(J+1)=1.
      DO 130 I=1,N
  130 CC(I)=0.
      GO TO 840
  110 PUNCH 5
```



```
      DO 160 I=1,40
  160 NQ(I)=I
      DO 170 J=1,N,5
      PUNCH 6,NQ(J),NQ(J+1),NQ(J+2),NQ(J+3),NQ(J+4)
      DO 170 I=1,N
      PUNCH 7,NQ(I),A(I,J),A(I,J+1),A(I,J+2),A(I,J+3),A(I,J+4)
  170 CONTINUE
  210 IF(INXT)296,296,99
  296 PAUSE
      DUM=EXITF(1.)
  900 PRINT 3
      PRINT 10
      PUNCH 10
      IF(LSLN)210,210,901
  901 DO 902 I=1,N,6
      READ 2,C(I),C(I+1),C(I+2),C(I+3),C(I+4),C(I+5)
  902 CONTINUE
      KNO=KNO-1
      IF(KNO)210,210,901
    1 FORMAT(I5,I5,I5,I5,I5,I5)
    2 FORMAT(6E10.4)
    3 FORMAT(55H
    4 FORMAT(//43HVALUE OF DETERMINANT OF COEFFICIENT MATRIX=E11.5)
    5 FORMAT(//31HINVERSE OF COEFFICIENT MATRIX--)
    6 FORMAT(//9XI3,10XI3,10XI3,10XI3,10XI3)
    7 FORMAT(I3,2XE11.5,2XE11.5,2XE11.5,2XE11.5,2XE11.5)
    8 FORMAT(//41H        SOLUTION VECTOR    ACTUAL C-CALC. C)
    9 FORMAT(2HX(I3,4H)    E11.5,7XE11.5)
   10 FORMAT(/24HINPUT MATRIX IS SINGULAR)
   11 FORMAT(17H*CROUT REDUCTION*//)
      END
```

FLOW CHART

The following page contains the flow chart for M-201.



SAMPLE PROBLEM

The following pages contain the input to and the output from the solution of an order 4 linear system with one constant vector.


```
INPUT TO SAMPLE PROBLEM

SAMPLE PROBLEM (FOURTH ORDER MATRIX)
 4    1    1    1    0
 2.0        2.0        0.0        0.0
 3.0        2.0       -1.0        0.0
 0.0        3.0       -4.0        1.0
 0.0        0.0       -1.0        4.0
 0.0       -1.0      -13.0      -10.0
```

CROUT REDUCTION

SAMPLE PROBLEM (FOURTH ORDER MATRIX)

VALUE OF DETERMINANT OF COEFFICIENT MATRIX= .54000E+02

	SOLUTION VECTOR	ACTUAL C-CALC. C
X(1)	.10000E+01	.00000E-99
X(2)	-.10000E+01	.00000E-99
X(3)	.20000E+01	.00000E-99
X(4)	-.20000E+01	.00000E-99

INVERSE OF COEFFICIENT MATRIX--

	1	2	3	4	5
1	-.33333E-00	.55555E-00	-.14814E-00	.37037E-01	.00000E-99
2	.83333E-00	-.55555E-00	.14814E-00	-.37037E-01	.00000E-99
3	.66666E-00	-.44444E-00	-.14814E-00	.37037E-01	.00000E-99
4	.16666E-00	-.11111E-00	-.37037E-01	.25925E-00	.00000E-99