Polynomial Regression Program For The IBM 1620

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COMMON USERS GROUP PROGRAM REVIEW AND EVALUATION

(fill out in typewriter, ink or pencil)

Program No._____ Date_____

Program Name:_____

1. Does the abstract adequately describe what the program is and what it does? Yes____ No____
   Comment_____

2. Does the program do what the abstract says? Yes____ No____
   Comment_____

3. Is the description clear, understandable, and adequate? Yes____ No____
   Comment_____

4. Are the Operating Instructions understandable and in sufficient detail? Yes____ No____
   Comment_____
   Are the Sense Switch options adequately described (if applicable)? Yes____ No____
   Are the mnemonic labels identified or sufficiently understandable? Yes____ No____
   Comment_____

5. Does the source program compile satisfactorily (if applicable)? Yes____ No____
   Comment_____

6. Does the object program run satisfactorily? Yes____ No____
   Comment_____

7. Number of test cases run_____. Are any restrictions as to data, size, range, etc. covered adequately in description? Yes____ No____
   Comment_____

8. Does the Program meet the minimal standards of COMMON? Yes____ No____
   Comment_____

9. Were all necessary parts of the program received? Yes____ No____
   Comment_____

10. Please list on the back any suggestions to improve the usefulness of the program. These will be passed onto the author for his consideration.

Please return to:

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Attn: PREP FORM COORDINATOR

Your Name _____
Company _____
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2/2/67

POLYNOMIAL REGRESSION PROGRAM FOR THE I. B. M. CARD 1620

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Modifications or revisions to this program, as they occur, will be announced in the appropriate Catalog of Programs for IBM Data Processing Systems. When such an announcement occurs, users should order a complete new program from the Program Information Department.

DESCRIPTION

This program obtains by least squares the coefficients of a polynomial $\hat{y} = b_o + \Sigma_i b_i (x-\bar{x})^i$, where i may range from 1 to at most 9, for as many as 250 pairs (y, x). Program switches provide the option of obtaining predictions $\hat{y}$ at each of the data points, and the option of obtaining predictions for a specified range of x's. Output may be either on the console typewriter (fixed point) or from the card punch (floating point). Either or both x and y may be transformed. Logarithm, exponential, and reciprocal transformations are included.

CARD PREPARATION

Data input consists of 1 header card, N cards each containing a y and an x, and, if predictions are called for, a card containing the lowest x for which predictions are wanted, an increment, and the highest x.

HEADER CARD (NO FLAGS)

Cols. 1 - 3	xxx = number of pairs (x, y) = N
Col. 4 - 5	xx = highest order wanted = 01 through 09
Col. 6	x = x, y reversal code
	0 if y precedes x
	1 if x precedes y
Col. 7 - 8	xx = transformation code for y
Col. 9 - 10	xx = transformation code for x
	00 = no transformation
	01 = replace y or x with $\log_{10} y$ or $\log_{10} x$
	02 = replace y or x with e^y or e^x
	03 = replace y or x with 1/y or 1/x
Col. 11 - 12	xx = order at which residuals should start
Col. 13 - 14	xx = order at which predictions should start

DATA CARDS (NO FLAGS)

Either x or y may appear first on the card. Column 6 of the HEADER CARD will specify the order. Both must be entered in fixed point form. The format is described in the attached description of the CARD CONVERSION SUBROUTINE.

PREDICTION DEFINITION CARD (NO FLAGS)

The format requirement for the prediction definition card is the same as for the data cards. The sequence on the card is (1) the lowest x for which predictions are wanted, (2) the increment for x, and (3) the highest x for which predictions are wanted.

These x's will be in the same units as the original x inputs. All x's will be transformed in the same manner.

SWITCH SETTINGS

With all four switches in the off position, the output consists only of the coefficients, their associated sums of squares, and the F ratios using the residual mean square as the estimate of error. These F ratios change as the residual mean square changes. This output will be in fixed point form (6 decimals) from the console typewriter. This output will be punched in floating point form if Switch 3 is on.

If Switch 1 is on and if the order of the polynomial is not less than the digits in columns 11 and 12 of the HEADER CARD, the residuals program will be entered. The output will consist of the run identification (i.e., the sequence of entry), the transformed y, the corresponding prediction $\hat{y}$, and the residual, $y - \hat{y}$, divided by the standard deviation of an observation at the x input. The output will be from the typewriter in fixed point form if Switch 4 is off and from the card punch in floating point form if Switch 4 is on (in this case, the transformed x is also included).

If Switch 2 is on and if the order of the polynomial is not less than the digits in columns 13 and 14 of the HEADER CARD, the output will include predictions for x's specified by the Prediction Definition Card. This output medium is controlled by Switch 4, as for the residuals. The output will consist of the untransformed x, the prediction, and the standard deviation of an observation at the specified x.

All switch settings may be changed during the running of the program. If Switch 2 is off immediately following the data entry, the Prediction Definition Card will not be read. If at some subsequent time predictions are wanted, the instruction "READ PREDICTION DEFINITIONS NOW" will be typed and the card will then be read.

COMPUTATIONAL PROCEDURE

It is necessary to find the coefficients of the polynomial of order D given by $\hat{y} = b_0 + \Sigma b_i x^i$, where i ranges from 1 to D. A disadvantage in such a solution is that the computational accuracy becomes increasingly worse, because of the loss of significant leading digits. In this program an alternative but equivalent polynomial, $\hat{y} = b_0 + \Sigma b_i (x-\bar{x})^i$ is fit to the data, in order to decrease the loss in accuracy.

The equations to be solved are of the form

$$\Sigma y = b_0 N + b_1 \Sigma(x-\bar{x}) + b_2 \Sigma(x-\bar{x})^2 + \ldots + b_D \Sigma(x-\bar{x})^D$$

$$\Sigma y(x-\bar{x}) = b_0 \Sigma(x-\bar{x}) + b_1 \Sigma(x-\bar{x})^2 + b_2 \Sigma(x-\bar{x})^3 + \ldots + b_D \Sigma(x-\bar{x})^{D+1}$$

$$\Sigma y(x-\bar{x})^2 = b_0 \Sigma(x-\bar{x})^2 + b_1 \Sigma(x-\bar{x})^3 + b_2 \Sigma(x-\bar{x})^4 + \ldots + b_D \Sigma(x-\bar{x})^{D+2}$$

$$\vdots$$

$$\Sigma y(x-\bar{x})^D = b_0 \Sigma(x-\bar{x})^D + b_1 \Sigma(x-\bar{x})^{D+1} + b_2 \Sigma(x-\bar{x})^{D+2} + \ldots + b_D \Sigma(x-\bar{x})^{2D}$$

In order to determine the error terms it is also necessary to have $\Sigma(y-\bar{y})^2$. All summations above extend from 1 to N. $\Sigma(x-\bar{x})$ is, of course, zero, within rounding error.

The data are transformed (if transformations are specified) and then stored in memory. At the same time Σx and Σy are being obtained. A statement of the analysis being performed is printed. If the logarithm of y and the reciprocal of x are specified, the statement "LOGARITHM OF Y VERSUS RECIPROCAL OF X PLUS" will appear, followed by the average of the reciprocals of x.

The sums of the powers of $x-\bar{x}$ are obtained up to the 2D power, and the sums of the products of y and powers of $x-\bar{x}$ are obtained up to the D power. $\Sigma(y-\bar{y})^2$ is also obtained.

The matrix of sums of powers is obtained in triangular form (the matrix is symmetric), and a 1 x 1 inverse is formed. The inverse is increased in size as the order of the polynomial is increased. The inverse matrix is obtained for use in the residuals and predictions parts of the program.

The inversion procedure follows the bordering method described by Faddeeva (Computational Methods of Linear Algebra, by V. N. Faddeeva, Dover Publications). The method is described below, with some notational changes.

MATRIX INVERSION METHOD

The system of equations can be written in matrix form as $X'Y = X'XB$. Letting $A = X'X$, $C = X'Y$, and $D = A^{-1}$, the solution is $B = A^{-1}C = DC$

Let the subscript n denote the size of the matrices at some point in the computation. Then

$$A_n = \begin{bmatrix} A_{n-1} & U_{n-1} \\ U'_{n-1} & A_{nn} \end{bmatrix} \qquad B_n = \begin{bmatrix} B_{n-1} \\ b_n \end{bmatrix}$$

$$D_n = \begin{bmatrix} D_{n-1} & r_{n-1} \\ r'_{n-1} & d_{nn} \end{bmatrix} \qquad C_n = \begin{bmatrix} C_{n-1} \\ c_n \end{bmatrix}$$

In this notation A_n and D_n are matrices of size n by n, A_{n-1} and D_{n-1} are matrices of size n-1 by n-1, U_{n-1} and r_{n-1} are vectors of size n-1, B_n and C_n are vectors of size n, B_{n-1} and C_{n-1} are vectors of size n-1, and A_{nn}, d_{nn}, b_n, and C_n are scalars.

The memory required by this method consists of the matrix A_n in triangular form, the vector B_n, and an auxiliary vector V_{n-1} of size n-1, and a few temporary locations. The equations to form the new inverse are:

$$V_{n-1} = A^{-1}_{n-1} u_{n-1}$$

$$1/d_{nn} = \alpha_n = a_{nn} - V'_{n-1} u_{n-1}$$

$$r_{n-1} = -V_{n-1}/\alpha_n = -d_{nn} V_{n-1}$$

$$D_{n-1} = A^{-1}_{n-1} + \alpha_n V_{n-1} V'_{n-1}$$

The first equation forms the auxiliary vector, the second forms the new diagonal, the third provides a new row (or column) for the expanded inverse, and the fourth equation corrects the previous inverse. The vector r_{n-1} replaces u_{n-1}, and d_{nn} replaces a_{nn}, so that D_n then is the A^{-1} for the increased n.

The equations for the coefficients are not given by Faadeeva, but are found to be:

$$b_n = d_{nn} (C_n - B'_{n-1} u_{n-1})$$

$$SSb_n = b_n^2/d_{nn} = (C_n - B'_{n-1} u_{n-1} u_{n-1})$$

$$\text{new } B_{n-1} = \text{previous } B_{n-1} + (C_n - B'_{n-1} u_{n-1}) r_{n-1}$$

The first equation gives the new coefficient, the second gives the sum of squares attributable to this coefficient, and the third equation corrects the previous coefficients.

RESIDUALS AND PREDICTIONS

The coefficients provide a basis for obtaining predictions for specified x's, whether they be the input x's or those defined by the Prediction Definition Card. The equation is $\hat{y} = b_o + \Sigma_{i=1}^{D} b_i (x_o - \bar{x})^i$, where x_o is the value of x at which the prediction is made.

The prediction equation can be written in matrix form as $\hat{y} = X_o' B$, where B is the vector of coefficients, $B = [b_o b_1 \ldots b_D]$ and X_o is the vector of powers of $x_o - \bar{x}$, $X_o' = [1, x_o - \bar{x} \ldots (x_o - \bar{x})^D]$. The estimated variance of the prediction is $X_o'(X'X)^{-1} X_o s^2$, where s^2 is the residual mean square, which is the only estimate of error at this point. The observations at x_o are assumed to be distributed with estimated variance s^2 around the prediction, so that the variance of an observation is $[1 + X_o'(X'X)^{-1} X_o] s^2$. The square root of this quantity is the "ERROR ON Y" in the predictions part of this program and the divisor for the residual, $y - \hat{y}$, in the residuals part.

OPERATING INSTRUCTIONS

Put the 1620 in the manual mode and press Reset. Load the polynomial regression program. The instructions as to the switch settings will be typed, and then the rest of the program is loaded.

Make the switch settings, and then put the header card followed by the (y,x) data cards into the reader hopper. Press Reader Start on the 1622 and Start on the 1620.

If Switch 2 is on initially, the Prediction Definition Card must follow the last data. If Switch 2 is off initially and turned on after the data entry, the computer will halt for the Prediction Definition Card, unless this card has been loaded into buffer.

After the data entry the average x will be printed (if transformations of the x's are specified, this will be the average of the transformed x's). The 1620 will seem to have stopped after this, but the sums of powers and cross products are being obtained (the Multiplier neons will be moving).

If no transformations are specified, and if more than 250 pairs (y,x) are read, then the program must be re-loaded if transformations are wanted in the next polynomial regression.

EXAMPLE

An example consisting of 9 pairs of data was set up to illustrate the program. The instructions are illustrated at the top of the following page 1. The header, data, and prediction definition cards are listed at the bottom of the page.

The header specifies 9 pairs of data, that at most a cubic is wanted, i.e., $y = b_0+b_1(x-\bar{x}) + b_2(x-\bar{x})^2 + b_3(x-\bar{x})^3$, that the y precedes x on the input cards, that neither y nor x should be transformed, that residuals should start with the quadratic, and that predictions should start with the cubic. The predictions will be obtained for x = 5, 10, and 15.

The complete typewriter output is shown on page 2 and the complete card output on page 3.

ORDER	COEFFICIENT	SUM OF SQUARES	F RATIO	0 0 X - 5188888889
00	5126888887	4964623000	5170000000	
01	5020895083	5213224265	5414324598	
ORDER	COEFFICIENT	SUM OF SQUARES	F RATIO	0 0 X - 5188888889
00	5126839220	4964415538	5160000005	
01	5020851915	5213224265	5412317772	
02	4714750124	4720746232	4919324126	
RUN	TRANSFORMED Y	PREDICTION	RESID./ERROR	
001	5111000000	5110481222	5039623094	5110000000
002	5115000000	5114610938	5033443777	5130000000
003	5117000000	5118752461	511566036M	5150000000
004	5120000000	5120827650	507389455M	5160000000
005	5124000000	5122905790	5096961178	5170000000
006	5130000000	5129157922	5072726522	5210000000
007	5135000000	5135436618	503763203M	5213000000
008	5140000000	5139637173	5031251165	5215000000
009	5150000000	5150190215	501344837M	5220000000
ORDER	COEFFICIENT	SUM OF SQUARES	F RATIO	0 0 X - 5188888889
00	5126746004	4957944257	5150000001	
01	5021760314	5213224265	5411411195	
02	4786911376	4720746232	4917901888	
03	471519789M	4864712819	5055840582	
RUN	TRANSFORMED Y	PREDICTION	RESID./ERROR	
001	5111000000	5110866582	4991710950	5110000000
002	5115000000	5114543369	5037674665	5130000000
003	5117000000	5118504484	511244325M	5150000000
004	5120000000	5120568866	504685417M	5160000000
005	5124000000	5122676974	5110917863	5170000000
006	5130000000	5129172463	5068781324	5210000000
007	5135000000	513573304	505777374M	5213000000
008	5140000000	5140021700	491654702M	5215000000
009	5150000000	5149912353	4957820963	5220000000
DEFINED X		PREDICTION	ERROR ON Y	
	5150000000	5118504484	5012090763	
	5210000000	5129172463	5012031420	
	5215000000	5140021700	5013114138	


```
01010*  POLYNOMIAL REGRESSION
01020       DORG2178
01030       NOP ,6,10
01040XYZA   TF  XYZB+6,XYZ
01050       AM  *-1,10,10
01060XYZB   WATY
01070       RCTY
01080       TF  XYZC+6,XYZ+5
01090       AM  *-1,10,10
01100XYZC   WATY
01110       RCTY
01120       RCTY
01130       SM  XYZA-1,1,10
01140       BNZ XYZA
01150       RCTY
01160       RNCDO
01170       B   0
01180A1     DAC 30,POLYNOMIAL REGRESSION PROGRAM@
01190M2     DAC 41,WRITTEN BY GENERAL FOODS RESEARCH CENTER@
01200M3     DAC 28, SWITCH 1 ON   DO RESIDUALS@
01210M4     DAC 29,          OFF   NO RESIDUALS@
01220A5     DAC 30, SWITCH 2 ON   DO PREDICTIONS@
01230A6     DAC 31,          OFF   NO PREDICTIONS@
01240A7     DAC 34, SWITCH 3 ON   PUNCH COEFFICIENTS@
01250A8     DAC 35,          OFF   PRINT COEFFICIENTS@
01260A9     DAC 44, SWITCH 4 ON   PUNCH RESIDUALS (SWITCH 1 ON)
01270       DAC 31, AND PREDICTIONS (SWITCH 2 ON)@
01280A10    DAC 45,          OFF   PRINT RESIDUALS (SWITCH 1 ON)
01290       DAC 31, AND PREDICTIONS (SWITCH 2 ON)@
01300A11    DAC 40,PUT ONE CONTROL CARD AND DATA CARDS INTO
01310       DAC 15, READER HOPPER@
01320A12    DAC 26,PRESS READER START, START@
01330XYZ    DSA A1,M2,M3,M4,A5,A6,A7,A8,A9,A10
01340       DSA A11,A12
01350       DEND2178
```

POLYNOMIAL REGRESSION PROGRAM
WRITTEN BY GENERAL FOODS RESEARCH CENTER

SWITCH 1 ON DO RESIDUALS
OFF NO RESIDUALS

SWITCH 2 ON DO PREDICTIONS
OFF NO PREDICTIONS

SWITCH 3 ON PUNCH COEFFICIENTS
OFF PRINT COEFFICIENTS

SWITCH 4 ON PUNCH RESIDUALS (SWITCH 1 ON) AND PREDICTIONS (SWITCH 2 ON)
OFF PRINT RESIDUALS (SWITCH 1 ON) AND PREDICTIONS (SWITCH 2 ON)

PUT ONE CONTROL CARD AND DATA CARDS INTO READER HOPPER
PRESS READER START, START

```
00903000000203
1.1  1
1.5  3
1.7  5
2.0  6
2.4  7
3.0  10
3.5  13
4.0  15
5.0  20
5  5  15
```


Y VERSUS X MINUS 8.888888

B 0 =	2.688888	SSE=	.064623	DF =	7.000000
B 1 =	.208950	SS =	13.224265	F =	1432.459800

B 0 =	2.683922	SSE=	.064415	DF =	6.000000
B 1 =	.208519	SS =	13.224265	F =	1231.777200
B 2 =	.000147	SS =	.000207	F =	.019324

RUN	TRANSFORMED Y	PREDICTION	RESID./ERROR
001	1.100000	1.048122	.396230
002	1.500000	1.461093	.334437
003	1.700000	1.875246	1.566036-
004	2.000000	2.082765	.738945-
005	2.400000	2.290579	.969611
006	3.000000	2.915792	.727265
007	3.500000	3.543661	.376320-
008	4.000000	3.963717	.312511
009	5.000000	5.019021	.134483-

B 0 =	2.674600	SSE=	.057944	DF =	5.000000
B 1 =	.217603	SS =	13.224265	F =	1141.119500
B 2 =	.000869	SS =	.000207	F =	.017901
B 3 =	.000151-	SS =	.006471	F =	.558405

RUN	TRANSFORMED Y	PREDICTION	RESID./ERROR
001	1.100000	1.086658	.091710
002	1.500000	1.454336	.376746
003	1.700000	1.850448	1.244325-
004	2.000000	2.056886	.468541-
005	2.400000	2.267697	1.091786
006	3.000000	2.917246	.687813
007	3.500000	3.573320	.577737-
008	4.000000	4.002170	.016547-
009	5.000000	4.991235	.057820

DEFINED X	PREDICTION	ERROR ON Y
5.000000	1.850448	.120907
10.000000	2.917246	.120314
15.000000	4.002170	.131141

READY FOR NEXT SET

```
02010* SYMBOLS AND CONSTANTS
02020       DORG2178
02030       DS  2
02040OUTPUTDS  1
02050INPUT DS  1
02060I      DS  2
02070OUT1   DS  15
02080OUT2   DS  15
02090OUT3   DS  15
02100       DS  111
02110T1     DS  10
02120T2     DS  10
02130T3     DS  10
02140T4     DS  10
02150T5     DS  10
02160T6     DS  10
02170T7     DS  10
02180Y      DS  10
02190X      DS  10
02200N      DS  3
02210D      DS  2
02220REVYX  DS  1
02230TRY    DS  2
02240TRX    DS  2
02250STARTRDS  2
02260STARTPDS  2
02270CROSS  DSB 10,10
022808      DS  1,CROSS
02290TOTAL  DS  10
02300DFE    DS  1,TOTAL
02310SUMS   DSB 10,19
02320VECTORDS  1,SUMS
02330SSB    DS  1,SUMS+110
02340TOTSS  DS  10
02350SSE    DS  1,SUMS+100
02360MM     DSB 10,55
02370AVGY   DS  10
02380MSE    DS  1,AVGY
02390AVGX   DS  10
02400DVAR   DS  2
02410FLZERODC   10,0
0242081     DAC 10,Y VERSUS @
02430B2     DAC 9,X MINUS @
0244083     DAC 7,B 1 = @
0245084     DAC 8,  SS = @
02460844    DAC 8,  SSE= @
0247085     DAC 7,  F = @
02480855    DAC 7, DF = @
0249086     DAC 4,111@
0250088     DAC 19,READY FOR NEXT SET@
02510HEAD1  DAC 49,ORDER  COEFFICIENT   SUM OF SQUARES       F RATIO
02520HEAD1ADAC 7,   1  1
02530HEAD1BDAC 5,  X +
02540HEAD1CDAC 20, 1111111111       @
02550HEAD2  DAC 50,RUN    TRANSFORMED Y    PREDICTION   RESID./ERROR@
02560       DAC 31,                              @
02570HEAD3  DAC 47,    DEFINED X        PREDICTION     ERROR ON Y@
02580       DAC 34,                                  @
02590ROUND  DC  10,1
```



```
03010* TRANSFORMATION SUBROUTINE
03020       DAS 5
03030TRANS  B
03040       DORG*-3
03050LOG    FLOGTRANS-1,TRANS-1
03060       B   ASIS
03070       DORG*-3
03080EXP    FEX TRANS-1,TRANS-1
03090       B   ASIS
03100       DORG*-3
03110RECIP  FD  FLONE,TRANS-1
03120       TF  TRANS-1,99
03130ASIS   BB
03140       DORG*-9
03150ADDR   DS  5,TRANS+6

04010* SET UP TRANSFORMATION BRANCH. ADDR CONTAINS LABEL TO BRANCH TO.
04020       DAS 1
04030SET    TFM ADDR,ASIS
04040       CM  SET-1,1,10
04050       BN  SET3
04060       BH  SET1
04070       TFM ADDR,LOG
04080       WATYA2
04090       B   SET3
04100       DORG*-3
04110SET1   CM  SET-1,2,10
04120       BH  SET2
04130       TFM ADDR,EXP
04140       WATYA3
04150       B   SET3
04160       DORG*-3
04170SET2   TFM ADDR,RECIP
04180       WATYA4
04190SET3   BB
04200       DORG*-9
04210A2     DAC 14,LOGARITHM OF @
04220A3     DAC 10,E TO THE @
04230A4     DAC 15,RECIPROCAL OF @

05010* MATRIX ADDRESS COMPUTATION. 99 IS DESTROYED.
05020V1     DAS 1
05030V2     DAS 1
05040MAC    TFM ADDR,MM
05050       TF  V3,V1
05060       C   V2,V1
05070       BNN MAC1
05080       TF  V3,V2
05090       TF  V2,V1
05100MAC1   A   ADDR-1,V2
05110       TFM V2,19,10
05120       SM  V2
05130       MM  V3,5,10
05140       SF  98
05150       TF  *+23,99
05160       MM  V2
05170       A   ADDR,99
05180       BB
05190       DORG*-9
```

```
05200V3     DS   1,MAC1+35

06010* WORK SUBROUTINE USED FOR RESIDUALS AND PREDICTIONS
06020WORK   FS   X,AVGX
06030       TF   T1,FLONE
06040       TFM  WORK2+23,B
06050       TFM  J,0,10
06060       TF   OUT2,FLZERO
06070       TF   T2,FLONE
06080WORK1  TF   V1,J
06090       TF   T3,T1
06100WORK2  FM   B,T1
06110       FA   OUT2,99
06120       AM   WORK2+23,10,10
06130       TF   K,J
06140WORK3  BT   MAC,K
06150       TF   WORK4+35,ADDR
06160       FM   T1,T3
06170WORK4  FM   99,MM
06180       FA   T2,99
06190       C    J,K
06200       BE   WORK5
06210       FA   T2,99
06220WORK5  FM   T3,X
06230       TF   T3,99
06240       AM   K,1,10
06250       C    K,DVAR
06260       BNH  WORK3
06270       FM   T1,X
06280       TF   T1,99
06290       AM   J,1,10
06300       C    J,DVAR
06310       BNH  WORK1
06320       FM   T2,MSE
06330       TF   T2,99
06340       FSQROUT3,T2
06350WORK7  B    DDD
06360       DORG*-4
06370       DNB  50
06380       DNB  30
06390       DC   1,@
06400BLANK  DS   1,WORK7+7

07010* BEGINNING OF MAIN PROGRAM
07020START  RCTY
07030       RNCDN-2
07040       SF   N-2
07050       SF   D-1
07060       SF   TRY-1
07070       SF   TRX-1
07080       SF   STARTR-1
07090       SF   STARTP-1

08010* SET BRANCHES FOR Y AND X TRANSFORMATIONS
08020       BT   SET,TRY
08030       TF   ABB+11,ADDR
08040       WATY B1
08050       BT   SET,TRX
08060       TF   ACC+11,ADDR
08070       WATY B2
```



```
09010* CLEAR AND SET UP ACCUMULATORS
09020       TFM  CLEAR+6,CROSS
09030CLEAR  TF   ,FLZERO
09040       AM   *-6,10,10
09050       CM   CLEAR+6,CROSS+330
09060       BN   CLEAR

10010* DATA INPUT LOOP
10020       TF   C1,N
10030       TFM  AC+6,DATA
10040       TFM  AD+6,DATA+10
10050AB     TFM  CCCC2+6,Y,,SET STORE
10060       BTM  CCCC,2,10,TWO INPUTS
10070       BD   DOREV,REVYX
10080       B    ABB
10090       DORG*-3
10100DOREV  TF   T5,X
10110       TF   X,Y
10120       TF   Y,T5
10130ABB    TFM  ADDR
10140       BT   TRANS,Y
10150AC     TF   ,TRANS-1,,STORE TRANSFORMED Y
10160       AM   *-6,20,10
10170       FA   MM,TRANS-1
10180ACC    TFM  ADDR
10190       BT   TRANS,X
10200AD     TF   ,TRANS-1,,STORE TRANSFORMED X
10210       AM   *-6,20,10
10220       FA   MM+10,TRANS-1
10230       FA   TOTAL,FLONE
10240       SM   C1,1,10
10250       BNZ  AB

11010* GET AVERAGES. COMPLETE STATEMENT ON CODING OF X
11020       FD   MM+10,TOTAL
11030       TF   AVGX,99,,STORE AVERAGE X
11040       TF   ROUND-8,91
11050       FA   AVGX,ROUND
11060AD2    BT   WWWW,AVGX,,PRINT AVERAGE X
11070       FD   MM,TOTAL
11080       TF   AVGY,99,STORE AVERAGE Y
11090       RCTY

12010* READ X LOWER,DELTA X, AND X UPPER FOR PREDICTIONS
12020       TDM  EE1+6,0
12030       BNC2AD4
12040       TDM  EE1+6,1
12050AD3    TFM  CCCC2+6,T5,,SET STORAGE LOCATION
12060       BTM  CCCC,3,10,READ X LOWER, DELTA, UPPER
12070       BD   AD4,EE1+6
12080       TDM  EE1+6,1
12090       B    EE1
12100       DORG*-3

13010* SUM POWERS OF X. SUM CROSS-PRODUCTS OF Y AND POWERS OF X
13020AD4    TF   C1,N
13030       TFM  AF2+11,SUMS+10
13040       A    AF2+10,D
13050       A    AF2+10,D
```

```
13060       TFM AH2+11,CROSS+10
13070       A   AH2+10,D
13080       TFM AE+11,DATA+10
13090       TFM AG+11,DATA
13100AE     TF  X,,,PICK UP X
13110       AM  *-1,20,10
13120       FS  X,AVGX
13130       TFM AF+23,SUMS
13140       TF  99,FLONE
13150AF     FA  SUMS,99
13160       AM  AF+23,10,10
13170       FM  99,X
13180AF2    CM  AF+23
13190       BN  AF
13200       TFM AH+23,CROSS
13210AG     TF  Y,,,PICK UP Y
13220       AM  *-1,20,10
13230       TF  99,Y
13240AH     FA  CROSS,99
13250       AM  AH+23,10,10
13260       FM  99,X
13270AH2    CM  AH+23
13280       BN  AH
13290       FS  Y,AVGY
13300       FM  Y,Y
13310       FA  TOTSS,99
13320       SM  C1,1,10
13330       BNZ AE

14010* SET UP X PRIME X MATRIX
14020       TFM AJ+11,SUMS,,INITIALIZE PICKUP
14030       TFM AK+6,MM,,INITIALIZE STORE
14040       TFM C1,10,10,SET BASIC COUNTER
14050AJ     TFM AK+11,,,SET PICKUP FOR DIAGONAL
14060       TF  C2,C1,,SET SECONDARY COUNTER
14070AK     TF
14080       AM  AK+6,10,10
14090       AM  AK+11,10,10
14100       SM  C2,1,10
14110       BNZ AK
14120       AM  AJ+11,20,10
14130       SM  C1,1,10
14140       BNZ AJ

15010* SET UP LOOP FOR INCREASING ORDER OF POLYNOMIAL
15020       TFM DVAR,1,10,LIMIT IS INPUT D
15030       FD  CROSS,MM
15040       TF  B,99,,SET FIRST B
15050       TFM BD+11,CROSS+10
15060       TF  SSE,TOTSS
15070       TFM BN+6,SSB
15080       FS  TOTAL,FLONE
15090       FD  FLONE,MM
15100       TF  MM,99,,SET 1 BY 1 INVERSE
15110       TD  HEAD1A+6,TRY
15120       TD  HEAD1A+12,TRX
15130       TFM BBB1+6,HEAD1C+2
15140       TFM BBB1+11,AVGX-9
15150BBB1   TD
15160       AM  BBB1+6,2,10
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15170       AM  BBB1+11,1,10
15180       CM  BBB1+11,AVGX+1
15190       BN  BBB1
15200       TFM HEAD1B+8,20,10
15210       BNF *+24,AVGX
15220       TFM HEAD1B+8,10,10
15230BBBBBBBNC3BB-24
15240       TR  OUTPUT,BLANK
15250       WACDHEAD1
15260       B   BB
15270       DORG*-3

16010* GET AUXILIARY VECTOR, START NEW DIAGONAL AND NEW B
16020       RCTY
16030       RCTY
16040BB     TFM I,0,10
16050       TF  V1,DVAR
16060       BT  MAC,DVAR
16070       TF  BC+11,ADDR,,DIAG
16080       TF  BL+6,ADDR,,DIAG
16090BC     TF  T1,MM,,PICK UP DIAG
16100BD     TF  T2,B+10,,PICK UP XY CROSS-PRODUCT
16110       TFM BK+23,B,,NEW B
16120       TFM BH+6,VECTOR
16130BE     TF  T3,FLZERO,,VECTOR
16140       TFM J,0,10
16150BF     TF  V1,I
16160       BT  MAC,J
16170       TF  BG+23,ADDR
16180       TF  V1,J
16190       BT  MAC,DVAR
16200       TF  BG+35,ADDR
16210BG     FM  MM,MM
16220       FA  T3,99
16230       AM  J,1,10
16240       C   J,DVAR
16250       BN  BF
16260BH     TF  VECTOR,T3,,STORE AUX VECTOR
16270       AM  *-6,10,10
16280       TF  V1,I
16290       BT  MAC,DVAR
16300       TF  BJ+35,ADDR
16310       TF  BK+35,ADDR
16320BJ     FM  T3,MM
16330       FS  T1,99
16340BK     FM  B,MM
16350       AM  BK+23,10,10
16360       FS  T2,99
16370       AM  I,1,10
16380       C   I,DVAR
16390       BN  BE
16400       FD  FLONE,T1
16410BL     TF  MM,99,,STORE NEW DIAGONAL
16420       TF  T3,99,,SAVE NEW DIAGONAL
16430       TF  T4,99
16440       SF  T4
16450       TF  BM+6,BD+11
16460       AM  BD+11,10,10
16470       FM  T2,T3
16480BM     TF  B+10,99,,STORE NEW B
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16490       FM  99,T2
16500BN     TF  SSB,99,,STORE SUM OF SQUARES
16510       AM  *-6,10,10

17010* FORM ERROR D.F. AND SUM OF SQUARES
17020       FS  SSE,99
17030       FS  DFE,FLONE
17040       FD  SSE,DFE
17050       TF  MSE,99,,STORE ERROR VARIANCE

18010* ENLARGE INVERSE, CORRECT PRIOR INVERSE, OUTPUT
18020       TFM CJ+23,8
18030       TFM CE+23,VECTOR
18040       TFM CLL+11,SSE
18050       TFM 1,0,10

19010* CORRECT PRIOR INVERSE
19020CC     TF  J,1
19030       TF  CE+35,CE+23
19040       TF  V1,1
19050CD     BT  MAC,J
19060       TF  CF+23,ADDR
19070CE     FM  VECTOR,VECTOR
19080       AM  CE+35,10,10
19090       FM  99,T3
19100CF     FA  MM,99
19110       AM  J,1,10
19120       C   J,DVAR
19130       BN  CD

20010* ENLARGE INVERSE
20020       BT  MAC,DVAR
20030       TF  CH+6,ADDR
20040       TF  CG+23,CE+23
20050       AM  CE+23,10,10
20060CG     FM  VECTOR,T4
20070CH     TF  MM,99,,STORE NEW COLUMN OF INVERSE

21010* CORRECT PRIOR B VALUES
21020       FM  99,T2
21030CJ     FA  B,99
21040CK     TF  CL+11,CJ+23
21050CL     TF  OUT1,B,,PUT B INTO OUTPUT AREA
21060       AM  CJ+23,10,10
21070CLL    TF  OUT2,SSE,,PUT SUM OF SQUARES INTO OUTPUT AREA
21080       AM  *-1,10,10
21090       FD  OUT2,MSE
21100       TF  OUT3,99,,PUT F RATIO INTO OUTPUT AREA
21110       BNC3CM
21120       CF  OUT1-9
21130       CF  OUT2-9
21140       CF  OUT3-9
21150       CF  1-1
21160       BNF CLL2,OUT1
21170       TDM OUT1,4,11
21180CLL2   WNCDOUTPUT
21190       SF  1-1
21200       B   CN
21210       DORG*-3
21220CM     RCTY
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21230       TD  B3+4,1
21240       WATYB3
21250       BT  WWWW,OUT1
21260       CM    1,0,10
21270       BNE CM1-12
21280       WATYB44
21290       B   CM1
21300       DORG*-3
21310       WATYB4
21320CM1    BT  WWWW,OUT2
21330       CM    1,0,10
21340       BNE CM2-12
21350       WATYB55
21360       B   CM2
21370       DORG*-3
21380       WATYB5
21390CM2    BT  WWWW,OUT3
21400CN     AM  1,1,10
21410       C   1,DVAR
21420       BN  CC
21430       TF  V1,DVAR
21440       BE  CK
21450       RCTY
21460       C   DVAR,STARTR
21470       BN  EE-24
21480       BNC1EE-24

22010* RESIDUALS PROGRAM
22020       TFM WORK7+6,DDD
22030       TFM DD+23,DATA
22040       TFM DD+35,DATA+10
22050       TFM 1,1,9
22060       BNC4DD1
22070       WACDHEAD2
22080       B   DD
22090       DORG*-3
22100DD1    RCTY
22110       RCTY
22120       WATYHEAD2
22130       RCTY
22140DD     BNC1EE-24
22150       TF  OUT1,,,PICK UP Y
22160       TF  X
22170       TF  OUT3+12,X
22180       AM  DD+23,20,10
22190       AM  DD+35,20,10
22200       B   WORK
22210       DORG*-3
22220DDD    TF  99,OUT1
22230       FS  99,OUT2
22240       FD  99,OUT3
22250       TF  OUT3,99,,STORE STANDARDIZED RESIDUAL
22260       BNC4DE
22270       CF  OUT1-9
22280       CF  OUT2-9
22290       CF  OUT3-9
22300       CF  OUT3+3
22310       CF  1-2
22320       BNF DDD1,OUT1
22330       TDM OUT1,4,11
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2234CDDD1  BNF DDD2,OUT2
22350      TDM OUT2,4,11
2236ODDD2  BNF DDD3,OUT3
22370      TDM OUT3,4,11
2238ODDD3  WNCDOUTPUT
22390      SF  1-2
22400      B   DF
22410      DORG*-3
2242ODE    TD  B6,1-2
22430      TD  B6+2,1-1
22440      TD  B6+4,1
22450      RCTY
22460      WATYB6
22470      BT  WWWW,OUT1
22480      BT  WWWW,OUT2
22490      BT  WWWW,OUT3
2250ODF    AM  1,1,10
22510      C   1,N
22520      BNH DD

23010* PREDICTIONS PROGRAM
23020      C   DVAR,STARTP
23030      BN  FF
2304OEE    BNC2FF
23050      BD  EE2,EE1+6
23060      RCTY
23070      RCTY
23080      WATYNOGO
23090      RCTY
23100      B   AD3
23110      DORG*-3
23120NOGO  DAC 32,READ PREDICTION DEFINITIONS NOW@
23130EE2   BNC4EE1
23140      TR  OUTPUT,BLANK
23150      WACDHEAD3
23160      B   EF-24
23170      DORG*-3
23180EE1   RCTY
23190      RCTY
23200      WATYHEAD3
23210      RCTY
23220      TF  T4,T5,,INITIALIZE X LOWER
23230      TFM WORK7+6,EG
2324OEF    BNC2FF
23250      TF  ADDR,ACC+11
23260      BT  TRANS,T4,,TRANSFORM X
23270      TF  X,TRANS-1
23280      B   WORK
23290      DORG*-3
2330OEG    BNC4EF1
23310      TF  OUT1,T4
23320      CF  OUT1-9
23330      CF  OUT2-9
23340      CF  OUT3-9
23350      BNF *+24,OUT1
23360      TDM OUT1,4,11
23370      BNF *+24,OUT2
23380      TDM OUT2,4,11
23390      WNCDOUTPUT
23400      B   EG2
```



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23410      DORG*-3
2342OEF1   RCTY
23430      BT  WWWW,T4,,PRINT X
23440      BT  WWWW,OUT2,,OUTPUT PREDICTION
23450      BT  WWWW,OUT3,,OUTPUT STD. ERROR
2346OEG2   FA  T4,T6
23470      C   T4,T7
23480      BNH EF
2349OFF    AM  DVAR,1,10
23500      C   DVAR,D
23510      BNH BBBBBB
23520      RCTY
23530      WATYB8
2354OFG    H
23550      B   START
23560      DORG*-3
2357OJ     DS  2,START+35
2358OK     DS  2,START+47
2359OC1    DS  5,START+59
2360OC2    DS  5,START+71
2361OFLONE DC  10,5110000000,FG+11

24010*  SUBROUTINE FOR FLOAT TO FIXED TO PRINT     484 DIGITS
2402OWWWWA DAC 8,00000000
24030      DAC 8,000000 @
2404OWWWWB DAS 1
2405OWWWWD DS  16
2406OWWWWE DS  12
24070      DS  4
2408OWWWWC DS  2
24090      DS  8
24100WWWW  TR  WWWWB-1,WWWWA-1
24110      CM  WWWWC,59,10
24120      BN  WWWW1
24130      S   WWWWE,WWWWE
24140      WATYWWWWD+2
24150      TF  WWWWE+1,WWWW-1
24160      WNTYWWWWD+4
24170      BB
24180      DORG*-9
2419OWWWW1 TFM WWWW3+11,WWWWC+1
24200      TFM WWWW3+6,WWWWD
24210      SM  WWWWC,50,10
24220      S   WWWW3+6,WWWWC
24230      S   WWWW3+6,WWWWC
24240      TD  WWWW8+7,WWWW-1
24250      CF  WWWW-1
2426OWWWW2 CM  WWWW3+6,WWWWE
24270      BNN WWWW4
24280      CM  WWWW3+11,WWWW
24290      BE  WWWW4
2430OWWWW3 TD
24310      AM  WWWW3+11,1,10
24320      AM  WWWW3+6,2,10
24330      B   WWWW2
24340      DORG*-3
2435OWWWW4 TF  WWWWE+4,WWWWE+2
24360      TFM WWWWD,3,10
24370      BNF WWWW5,WWWW8+7
24380      TFM WWWWE+2,20,10
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24390WWWW5 TFM WWWW6+11,WWWWB
24400WWWW6 BD  WWWW8
24410      TF  WWWW7+6,WWWW6+11
24420WWWW7 TFM ,0,10
24430      AM  WWWW6+11,2,10
24440      CM  WWWW6+11,WWWWD
24450      BN  WWWW6
24460WWWW8 WATYWWWWB
24470      BB
24480      DORG*-9

25010* CONVERSION OF VARIABLE LENGTH FIXED POINT DATA FROM CARDS
25020      DAS 1
25030CCCC  TFM CCC1+6,INPUT
25040CCC1  TFM ,0,10
25050      AM  CCC1+6,2,10
25060      CM  CCC1+6,INPUT+144
25070      BNZ CCC1
25080CCC2  RACDINPUT
25090      TR  INPUT+143,CCCCCC+1
25100CCC3  TR  INPUT-3,INPUT-1
25110      BNR CCC4,INPUT-2
25120      B   CCC2
25130      DORG*-3
25140CCC4  CM  INPUT-2,0,10
25150      BE  CCC3
25160      TR  CCCCH-1,CCCCCC-11
25170      B   CCC6
25180      DORG*-3
25190CCC5  TR  INPUT-3,INPUT-1
25200CCC6  CM  INPUT-2,70,10
25210      BN  CCCC3
25220      TR  CCCD3,CCCD3+1
25230      TD  CCCDDD,INPUT-2
25240      BD  CCC7,CCCD2
25250      AM  CCCCH,1,10
25260CCC7  BNF CCC5,CCCD3
25270      CM  INPUT,45,10
25280      BNE CCC8
25290      TD  CCC777+1,INPUT+1
25300      TD  INPUT+5, INPUT+4
25310      SF  INPUT+5
25320CCC777A   CCCCH,INPUT+6
25330      TR  INPUT-3,INPUT+5
25340CCC8  C   CCCDDD,FLZERO
25350      BE  CCCC2
25360CCC9  TF  CCCD3+1,CCCCH
25370      BD  CCCC1,CCCD3+2
25380      SM  CCCCH,1,10
25390      TR  CCCD3,CCCD3+1
25400      TDM CCCDDD,0
25410      B   CCC9
25420      DORG*-3
25430CCCC1 BD  CCCC2,CCCD1
25440      SF  CCCDDD
25450CCCC2 TF  ,CCCDDD
25460      AM  CCCC2+6,10,10
25470      SM  CCCC-1,1,10
25480      BNZ CCC3
25490      BB
```



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25500      DORG*-9
25510CCCC3 CM  INPUT-2,20,10
25520      BNE CCCC4
25530      TDM CCCD1,0
25540      B   CCC5
25550      DORG*-3
25560CCCC4 CM  INPUT-2,3,10
25570      BNE CCCC5
25580      TDM CCCD2,1
25590      B   CCC5
25600      DORG*-3
25610CCCC5 CM  INPUT-2,0,10
25620      BNE CCCC8
25630CCCC7 TR  CCCD3,CCCD3+1
25640      TDM CCCDDD,0
25650CCCC6 BNF CCCC7,CCCD3
25660      B   CCC8
25670      DORG*-3
25680CCCC8 CM  INPUT-2,10,10
25690      BE  CCC5
25700      WATYINPUT-2
25710      H
25720      DORG*-9
25730CCCCCCDC  12,501000000000
25740      DC  2,0
25750      DAC 1,@
25760CCCCH DS  2
25770CCCD1 DS  1
25780CCCD2 DS  1
25790CCCD3 DS  1
25800CCCDDDDS  9
25810CCCD4 DS  2

26010*
26020DATA  DSB 10,500
26030      DEND2178
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COMPUTER TECHNOLOGY