Linear Programming –
Cotton Blending and Production Allocation

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INTRODUCTION

The introduction of linear programming (LP) has produced remarkable benefits in a number of industries. The early experimental applications of LP techniques in the petroleum industry as a refinery management tool had such profound effects that LP is now standard in almost every aspect of that industry. More recently LP has been employed in such areas as metal alloy blending, feed blending, ice cream blending, and food products blending — always with dramatic effect.

The first application of LP in the textile industry was designed to produce optimal plant efficiency, that is, allocate plant resources to production problems so as to achieve the highest practical return. The purpose of this manual is to demonstrate the application of LP in the blending of cotton. Because the cotton blending process involves complex quality control, it is particularly responsive to LP techniques. By the use of LP, the mill operator can determine the specific allocation of raw cottons required to produce a given blended yarn at minimum cost — subject to any stated restrictions on yarn quality and raw cotton availabilities. The immediate and more obvious LP results enable the mill operator to:

- Minimize the cost of cotton blends
- Minimize substandard blends
- Maintain accurate inventory records
- Purchase and sell most economically

The basis of the LP technique is the formulation of a mathematical model of the allocation problem. For problems of any practical size, this model is entered into a computer, and the computer LP system rapidly calculates the optimal (least-cost) solution. The system may also produce reports which indicate the effect on the optimal solutions of possible changes in the given prices, availabilities, specifications, etc.

Little mathematical knowledge or skill is required to formulate an LP model. Nor do the operation of the computer and the analysis of computer results require any advanced technical skill. Linear programming does require the expression of all the elements in the process — individual cotton quality analyses, costs, yarn specifications, etc. — in the form of linear equations. The equations will, of course, reflect the blending philosophy of the particular company using LP. The general principles of linear programming are discussed in the IBM data processing application manual An Introduction to Linear Programming (E20-8171), which should be read in conjunction with this manual. (IBM data processing application manual E20-0025 provides a glossary of textile terms.)

To demonstrate the methods and advantages of LP in cotton blending, we shall present a typical production problem as a basis for the development of an LP model which can be solved by an IBM linear programming system.

PROBLEM PROFILE AND ECONOMICS

The fundamental problem is to produce a specified cotton yarn by feeding an appropriate blend of raw cottons into the opening-and-blending machine at the beginning of the yarn producing process. In order to produce a least-cost blend, the producer must consider a complex variety of factors including the prices, grades, and availabilities of a great many raw cottons. The crucial interrelations among the several qualities of raw cotton make it exceedingly difficult to determine a least-cost blend.

In the past, the treatment of the blending problem has been to formulate a relatively simple blend ratio for a specified yarn, which is then empirically tested and corrected, often by the addition of expensive high-quality grades. This method frequently results in production delays and quality giveaway. Further, the initial difficulty of determining a proper blend is vastly compounded by common fluctuations in the availability and price of specific cottons, since the alteration of any one component will alter all the relationships required for least-cost production. Some mills are already profiting from the application of linear programming to this problem, enabling the producer to examine all possible combinations and quickly determine the most economical cotton blend. The LP model can also be applied to "force" overstocked grades into minimum-cost blends, thus contributing to the achievement and maintenance of ideal inventory control.

LP MODEL FORMULATION — SINGLE-BLEND PROBLEM

A linear programming model for cotton blending is a mathematical representation, in the form of linear equations or inequalities, of all known and estimated factors relevant to the production of the specified blend. To demonstrate the method for formulating such a model, we postulate a simple problem — the production of a specified yarn from 20 available grades of raw cotton. In practice, a greater variety of cotton inventory may be available; but regardless of the number, price, and grade of available cotton (the factors which complicate manual calculation), they can easily be included in the LP model, increasing the model's size but not its complexity.

INPUT DATA REQUIREMENTS

The following basic data is required to formulate the LP model:

1. Blend (yarn) specifications
2. Quality analysis of each raw cotton available
3. Price of each raw cotton
4. Inventory level of each raw cotton

This information is available from purchasing, cost accounting, inventory accounting, and in the case of quality analysis, from an appropriate pilot plant or testing laboratory.

SAMPLE PROBLEM

The factors that most appropriately define yarn quality are not generally agreed upon throughout the industry. However, a model based on any set of quality characteristics can be constructed, so long as these characteristics are linear (or can be linearly approximated) and satisfy the manufacturer's needs. For this sample problem, we have taken the work of Waters and Philips (reference 7 in the Bibliography) as a basis and will treat yarn quality in terms of five parameters: fineness, staple length, fiber strength, yarn count, and grade index. These are defined as follows:

Fineness. Fiber fineness is a relative measure of the diameter or linear density of an individual fiber. Cotton fineness is typically measured by an instrument, called a Micronaire, that determines the rate of airflow through a sample of fixed weight. The finer the fiber, the larger the total surface area in a standard sample, and hence, the more resistance to airflow. Micronaire fineness values are read directly from an air gage. Fineness is linear, directly proportional to the number of neps (tangles of fiber) per square inch and also to spinnable limit (the spinnable limit is the yarn count at which a particular blend meets the ends down specification). Since a high nep count must be avoided, the fineness must ordinarily fall within specific maximum and minimum limits depending on the yarn count desired.

Staple length. Staple length is the length of the cotton fiber. It is directly proportional to spinnable limit in a relationship which is linear for all practical purposes. Of the various methods for determining staple length, the Fibrograph, an optical instrument which scans an appropriately prepared sample of cotton fibers, produces satisfactory data for LP use. The Fibrograph aids in determining both the mean length and the upper-half mean length to the nearest 1/32 inch (these lengths are then usually converted to thousandths of an inch). The upper-half mean length is the average length of the fibers that have lengths greater than the sample mean. The mean length is the average length of all fibers in the sample that are longer than 1/4 inch. The fiber length uniformity of the sample may be expressed as the ratio between mean length and upper-half mean length.

Fiber strength. Fiber strength is determined by breaking a carefully combed and straightened sample of 15 to 20 milligrams using a standard Pressley 1/8-inch gage test. Fiber strength is directly proportional to yarn strength and spinnable limit. It is specified by the Pressley Index (P. I.).

Yarn count. The yarn count of in-process samples is determined by weighing, to the nearest grain, a skein of 120 yards of spun yarn. One thousand divided by this weight is the yarn count. The yarn count is directly related to spinnable limit and represents an important factor in the quality of the finished woven goods.

Grade index. The grade of a raw cotton is usually determined by a cotton classer on the basis of source, age, cavitomic damage, color, feel, moisture content, trash content, appearance, and an intangible factor called character. The nine established grades can be converted for use in linear programming to a numerical scale with 100. 0 as the "middling" value. A grade index below 100, then, represents lower quality, and a grade index above 100 represents higher quality.

The yarn to be produced in our sample problem will have the blend specifications listed in Figure 1. These describe the desired yarn quality in terms of the five parameters discussed above.

Waters and Philips have shown that, for all practical purposes, the relationship of each of these cotton properties to spinnable limit is linear; hence these properties are suitable for a linear programming model. We propose to blend the specified cotton from 20 raw cottons of various origin, quality, and price. Figure 2 provides a quality analysis for each of the raw cottons.

We will further assume that current inventory conditions make it desirable to "force" the use of the raw cotton we have designated X8; we will specify that the blend must contain at least 30% of this cotton. Limitations on the use of any particular raw cotton inventory could be introduced similarly by requiring that the blend contain more than some specific percentage of that cotton if inventory must be reduced, or less than some specific percentage if inventory is low.

Property	Minimum	Maximum
Fineness (Micronaire)		4.35
Staple length (Fibrograph — upper-half mean, inches)	1.075	
Fiber strength (1/8-inch gage, P. I.)	3.23	
Yarn count	21.3	
Grade index	100.2	

Figure 1. Blend specifications

Cotton Group	Fineness (Micronaire)	Staple Length (Upper-half Mean)	Fiber Strength (1/8-inch gage, P. I.)	Yarn Count	Grade Index	Price per lb. ($)
X1	4.45	1.05	3.13	21	105	0.3533
X2	4.31	1.09	3.25	23	105	0.3549
X3	4.39	1.06	3.08	20	100	0.3511
X4	4.29	1.10	3.27	23	100	0.3549
X5	4.42	1.05	3.12	21	95	0.3540
X6	4.33	1.11	3.31	24	95	0.3530
X7	4.50	1.07	3.28	23	90	0.3483
X8	4.27	1.09	3.23	22	90	0.3496
X9	4.45	1.05	3.15	22	110	0.3533
X10	4.35	1.10	3.30	24	110	0.3523
X11	4.48	1.06	3.09	20	85	0.3479
X12	4.33	1.09	3.28	23	85	0.3511
X13	4.17	1.05	3.37	26	105	0.3724
X14	4.00	1.08	3.62	32	105	0.3850
X15	4.16	1.07	3.41	27	100	0.3828
X16	3.98	1.10	3.58	31	100	0.3839
X17	4.19	1.06	3.39	26	95	0.3673
X18	4.04	1.08	3.61	31	95	0.3817
X19	4.20	1.07	3.42	27	90	0.3851
X20	4.01	1.09	3.57	31	90	0.3844

Figure 2. Quality analyses of raw cottons for blending

An LP model consists of a set of linear equations or inequalities called constraints. A constraint relates the value of some particular quality factor (or other factor, such as cost) in the final blend to the amounts of each raw material used in the blend, and expresses the final blend specification for this factor. For example, suppose three syrups are to be blended to produce a product which must have 10% sugar content and no more than 15% corn starch. Assuming the three syrups have 5%, 12%, and 20% sugar content and 30%, 3%, and 1% corn starch, respectively, the sugar and corn starch constraints would be expressed as:

$$5\ S1 + 12\ S2 + 20\ S3 = 10 \text{ (Sugar)}$$

$$30\ S1 + 3\ S2 + S3 \leq 15 \text{ (Starch)}$$

where S1, S2, and S3 are variables (unknowns) representing the percentage or proportion of each syrup in the final product. (With suitable changes in the coefficients and right-hand side, the variables could represent actual weights, volumes, etc., of each material used rather than proportions in the final blend.)

In the cotton blending LP model, a constraint similar to the above is formulated for each specification and inventory limitation. A special cost constraint or "objective function" is formulated, using the unit prices of each raw material as coefficients, to express the total unit cost of the final blend. The LP system solves the problem by computing a set of values for the variables which simultaneously satisfies all the constraints and at the same time minimizes the total value of the objective function (cost).

To set up the model in a format convenient for presentation to the computer LP system, the coefficients and right-hand sides of the constraints are arranged in a tabular array of columns and rows called a matrix. Figure 3 shows the completed matrix for our sample problem (we shall trace its formulation in detail below).

Each raw cotton available for use in the blend (symbolized by X1, X2, X3,..., X20) appears at the head of a matrix column, which is called a <u>problem activity</u>. We wish to determine what fraction of the final blend each raw cotton should provide. These fractions are the variables which the LP system will compute. The figures appearing in each column are coefficients of the variable which heads the column. Thus the figure 0.3533 in the column headed X1 signifies that the first term in the COST equation (objective function) is 0.3533 times X1. The second term in the COST equation is 0.3549 times X2, and so on.

This use of column headings in a matrix format avoids the need to repeat the variables in each of the constraint terms.

Each row, called a <u>problem constraint</u>, is a sum of terms which expresses a particular specification or limitation identified by the row's symbolic name, or mnemonic (see Figure 4 for explanation of mnemonics). Thus, in effect, the data given in

	COTTON GROUPS																				
ROW \ COL	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17	X18	X19	X20	RHS
COST	.3533	.3549	.3511	.3549	.3540	.3530	.3483	.3496	.3533	.3523	.3479	.3511	.3724	.3850	.3828	.3839	.3673	.3817	.3851	.3844	= MIN
FIN	4.45	4.31	4.39	4.29	4.42	4.33	4.50	4.27	4.45	4.35	4.48	4.33	4.17	4.00	4.16	3.98	4.19	4.04	4.20	4.01	≤ 4.35
LEN	1.05	1.09	1.06	1.10	1.05	1.11	1.07	1.09	1.05	1.10	1.06	1.09	1.05	1.08	1.07	1.10	1.06	1.08	1.07	1.09	≥ 1.075
STR	3.13	3.25	3.08	3.27	3.12	3.31	3.28	3.23	3.15	3.30	3.09	3.28	3.37	3.62	3.41	3.58	3.39	3.61	3.42	3.57	≥ 3.23
CNT	21.0	23.0	20.0	23.0	21.0	24.0	23.0	22.0	22.0	24.0	20.0	23.0	26.0	32.0	27.0	31.0	26.0	31.0	27.0	31.0	≥ 21.3
GRA	105.0	105.0	100.0	100.0	95.0	95.0	90.0	90.0	110.0	110.0	85.0	85.0	105.0	105.0	100.0	100.0	95.0	95.0	90.0	90.0	≥ 100.2
INVA8								1.0													≥ .30
SUM	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	= 1.0

Figure 3. LP model matrix for single-blend problem

Figures 1 and 2 have been transferred to the model matrix of Figure 3. The formulation of each of the constraint rows is discussed below.

Row Mnemonic	Constraint Expressed
COST	Price per pound (objective function)
FIN	Fineness (Micronaire)
LEN	Staple length (Fibrograph, upper-half mean, inches)
STR	Fiber strength (1/8-inch, P.I.)
CNT	Yarn count
GRA	Grade index
INVA8	Inventory availability of cotton X8
SUM	Material balance

Figure 4. Mnemonics used in model matrix

Cost Constraint (Objective Function)

The objective function to be minimized in this sample problem is the cost per pound of the specified blend. This function may be expressed by the linear equation.

$$0.3533\ X1 + 0.3549\ X2 + \ldots + 0.3844\ X20 = \text{COST} \quad (\text{MINIMIZE})$$

where in each term the coefficient is the price per pound, and the variable is the proportion used in the final blend of the corresponding cotton. "MINIMIZE" signifies that the LP system will compute a set of values for the variables (representing the proportion of each raw cotton in the blend) which satisfies all the constraints and at the same time results in the lowest possible cost per pound for the blend.

Specification Constraints

The specifications for the desired cotton blend (Figure 1) establish five constraints for the model. The first constraint, fineness, is established as a maximum for this blend. All the others — stable length, fiber strength, yarn count, and grade index — are minimum specifications.

Using the quality analysis of each raw cotton for these five factors (Figure 2), linear expressions which constrain the final blend to the desired specifications can be formulated.

The fineness specification may be expressed as:

(fineness of cotton X1) x (proportion of X1 in blend) + (fineness of cotton X2) x (proportion of X2 in blend) + . . . + (fineness of cotton X20) x (proportion of X20 in blend) must be equal to or less than 4.35

The second row (FIN) of Figure 3 incorporates this expression into the model matrix:

$$4.45\ X1 + 4.31\ X2 + 4.39\ X3 + \ldots + 4.01\ X20 \leq 4.35$$

The staple length (LEN) specification (directly proportional to spinnable limit, and hence a minimum specification) is formulated similarly: The staple length of each cotton is multiplied by the proportion of that cotton used in the blend, these products are summed, and the sum is set equal-to-or-greater-than the specified length:

$$1.05\ X1 + 1.09\ X2 + 1.06\ X3 + \ldots + 1.09\ X20 \geq 1.075$$

The specifications for fiber strength (STR), yarn count (CNT), and grade index (GRA) are formulated in precisely the same way:

STR:

$$3.13\ X1 + 3.25\ X2 + 3.08\ X3 + \ldots + 3.57\ X20 \geq 3.23$$

CNT:

$$21\ X1 + 23\ X2 + 20\ X3 + \ldots + 31\ X20 \geq 21.3$$

GRA:

$$105\ X1 + 105\ X2 + 100\ X3 + \ldots + 90\ X20 \geq 100.2$$

These five expressions effectively constrain the final blend to the specifications. Should additional cotton groups become available (say, X21 through X50), they can be introduced as possible components for this blend simply by incorporating their prices and quality characteristics into these expressions — that is, by adding new columns (but no new rows) to the matrix.

Inventory Availability Constraint

In setting up the sample problem, we specified that raw cotton X8 must supply at least 30% of the final blend. Since the variables of the model matrix have been expressed as fractional proportions, we can incorporate this constraint by formulating a new row (INVA8) with the coefficient 1.0 in column X8 and ≥ 0.30 as its right-hand side. The row then expresses the simple relation:

$$X8 \geq 0.30\ .$$

Material Balance Constraint

We have formulated the model matrix with the assumption that the variables representing raw cotton are expressed as fractional proportions of the final blend. This condition must be stated explicitly in the model. To accomplish this we simply establish a row (SUM) with the coefficient 1.0 in every column and 1.0 as its right-hand side. The relation thus expressed is:

$$X1 + X2 + X3 + . . . + X20 = 1.0 .$$

This material balance constraint will result in a solution stating the proportions of raw cottons in the blend as fractions which, when summed, equal 1.

Other Constraint Possibilities

For this cotton blend problem the eight constraint rows formulated above complete a matrix model. The first of these rows is the objective function — cost — which must be minimized within the quality specifications defined by the next five rows and the inventory availability constraint on cotton X8. The eighth row constrains the total cotton blended to one, that is, 100%.

It is also possible to formulate the model matrix with raw cotton activities stated as actual weights and a material balance constraint expressing the total blend quantity required. This formulation would be desirable if inventory of one or more raw cottons were extremely low or high and/or the quantity of yarn needed were critical or very high. Inventory constraints could then be included in the model in terms of actual weights available.

There is another possible formulation in which the anticipated selling price of the blend is included in the objective function, being given a sign opposite to that of the raw cotton terms. In this case the optimal solution is a blend that yields maximum profit rather than minimum cost. If actual weights are used, the solution indicates the maximum quantity of specified yarn that can be spun from available inventory on a maximum-profit basis. Such a formulation can be applied to develop optimal production and marketing plans, evaluate purchasing agreements, etc. This is discussed further in the section on product allocation models.

MULTIBLEND MODEL FORMULATION

Having once formulated a basic single-blend matrix model for the blending of a specified yarn, it is an easy matter to design a multiblend model. Such a model allows the producer to compute minimum-cost blends for a number of mills (or consecutive blends for the same mill), simultaneously, even if different blends are specified for each mill. As Figure 5 indicates, a multiblend model consists of a set of submatrices, each of which has the appropriate constraint rows to express blend specifications, and each of which has a unique designation for its column activities. That is, the raw cotton symbols for blend 1 are prefixed with a numeral 1, those for blend 2 are prefixed with a 2, and so on. Thus, in a single computer run the producer may determine how much of each raw cotton to use in each mill.

Ideally, the model should be solved with no inventory constraints in order to determine optimal solutions. However, such constraints can be introduced, either into the submatrices (reflecting local warehouse stocks), or into the overall matrix (reflecting irremedial inventory limitations). This procedure will then permit computation of the optimum distribution of available stock to each mill for minimum-cost overall blending. The most obvious advantage of the multiblend LP model is that, in a minimum of computer time, it allocates from all available inventory supplies to all the mills at optimal levels. Further, re-solutions based on

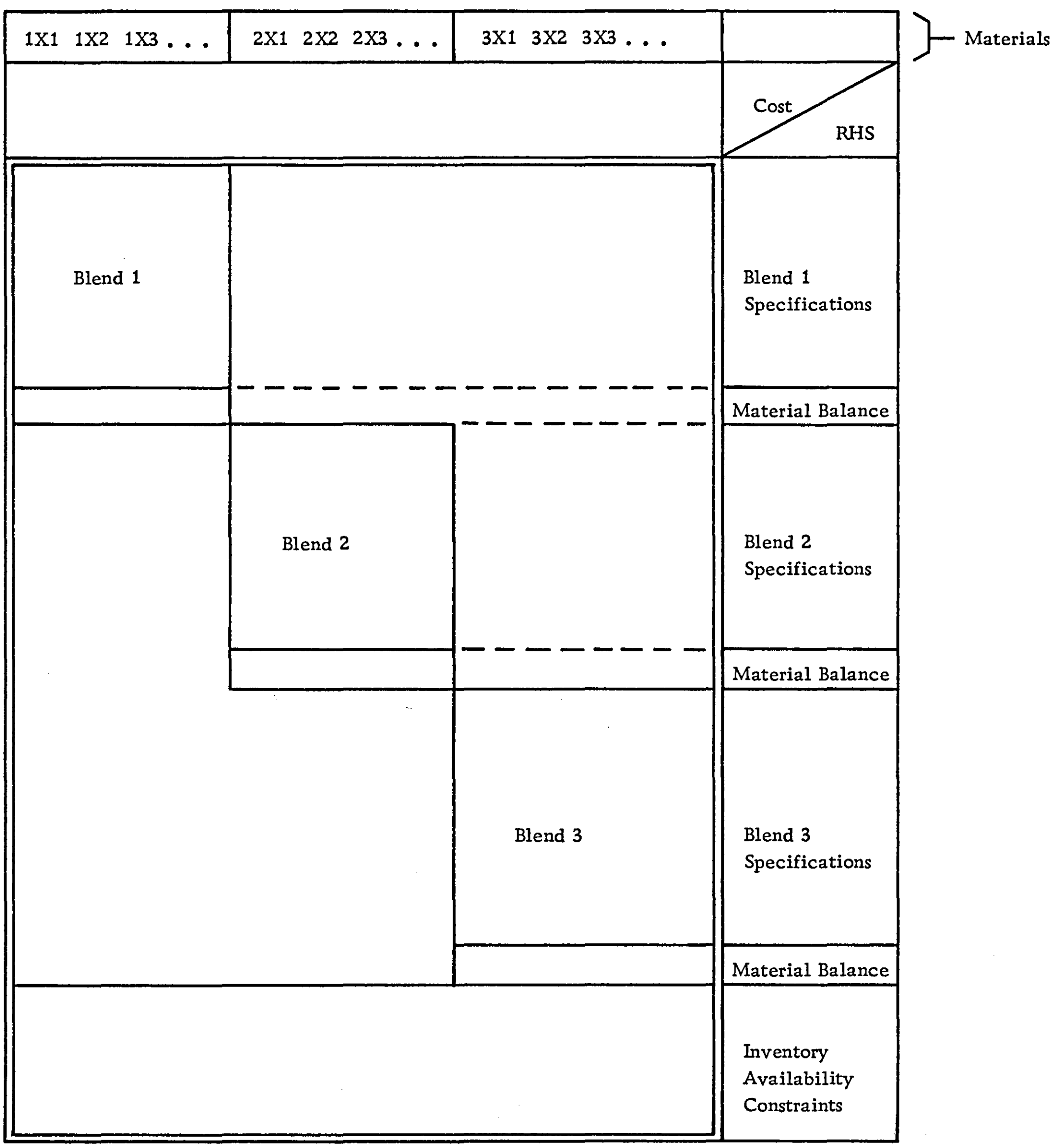

Figure 5. Schematic of multiblend model matrix

output report suggestions will respond to overall considerations of inventory and costs rather than to a single blend problem. It is important, however, to add what may be different handling costs to the basic price of raw cotton which must be moved to a number of different mills from a central warehouse. Quite conceivably, a multiblend model solution may indicate that a nonoptimal solution for one mill will allow the best overall use of inventory and mill capacity.

OUTPUT REPORTS

The linear programming system will employ the input data to compute a variety of output reports. We are here principally concerned with four basic reports which the system produces:

- Basis variables report
- Slacks report
- DO. D/J report
- Cost range report

Each of these reports is discussed and illustrated below.

BASIS VARIABLES REPORT

The basis variables (BASIS. VARBLS) report provides a list of the raw cottons included in the "basis" (that is, the set of all activities appearing at a nonzero level in the optimal blend), and indicates the proportion required of each.

The IBM 1620/1311 LP system provides the optimal blend shown in Figure 6 for the single-blend problem we have formulated. The total cost per pound of the blend (given in the slacks report to be discussed next) is $0. 351. If the solution of Figure 6 is to be implemented without change, it can be disseminated immediately to both the cotton inventory accounting department and the mill. The inventory accounting department employs the record of raw materials consumed to maintain updated inventory records. The mill employs the solution as a work order to be followed in loading the opener, or as a standard blend.

```
BASIS.

VARBLS      NAME ACTIVITY LEVEL
            X07            .170
            X08            .320
            X10            .510
```

Figure 6. Basis variables report — optimal solution

At this point, the model may be adjusted to deal with full-bale quantities as a special raw material restriction. A 100-bale mix of the optimal blend would present no problem, since it requires 17 bales of X7, 32 bales of X8, and 51 bales of X10. But suppose that mix of only 40 bales is required. Since 17% of 40 equals 6. 8 bales, and we do not wish to use fractions of a bale, we can round off X7 to 7 bales. X7 will then comprise 7/40, or 17. 5%, of the final blend. We simply revise the matrix by constraining X7 to exactly 17. 5% of the final blend and re-solve the matrix.

Of course, imposing bale quantity constraints can upset the balance of qualities which meet the specifications for the final blend. It may, consequently, become necessary to obtain several re-solutions, with constraint changes suggested by the previous solution. However, this approach to controlling full-bale allocations has proved highly effective in actual application. As linear programming becomes more widely used in the industry, it may, quite possibly, suggest radical changes in cotton storing practices.

SLACKS REPORT

The slacks report (Figure 7) provides a list of all the right-hand-side (row) mnemonics and indicates for each linear inequality the difference (or "slack"), if any, between its upper or lower bound and the actual value computed in the optimal solution. For each equality, or inequality, solved at a bound, the slacks report provides a figure called the simplex multiplier, which indicates the amount by which the optimal cost per pound of the blend would change if the right-hand side of the constraint were changed by one unit (assuming that no significant changes are made in the other matrix elements).

```
SLACKS          NAME ACTIVITY LEVEL   SIMPLEX MULT.
                COST            .351
                FIN                           .006
                LEN             .017
                STR             .044
                CNT            1.890
                GRA                           .000
                INVA8           .020
```

Figure 7. Slacks report

For example, the FIN figures indicate that the maximum fineness specification was met at its upper bound (there is no slack), and the simplex multiplier indicates that if a slightly higher

Micronaire fineness specification were possible, the final cost per pound of the blend would decrease by 0.6¢. (The same indication will occur more graphically in the DO. D/J report.) The report further reveals that fineness is the limiting specification since four of the remaining constraints were exceeded — length (LEN) has a slack of 0.017; strength (STR), 0.044; yarn count (CNT), 1.89; and X8 availability (INVA8), 0.020 — and grade (GRA) was met exactly, having neither a slack nor a simplex multiplier. The first line in the slacks report (COST) gives the cost per pound of the optimal blend.

DO. D/J REPORT

The DO. D/J report (Figure 8) consists of two parts. The first part (VBLS) lists all the column activities — raw materials in this case — which are solved at a bound. Most often that bound is zero, an indication that the material, at its specific price, is not used in the optimal blend. For each of these materials, the report indicates its current price and the amount this price must drop, as well as the actual price to which it must drop, before the material may be introduced into the basis.

For example, cotton X1, which costs 35.3¢, would tie with cotton X9 for entry into the optimal blend if its price dropped by 0.2¢ (REDUCED COST) to 35.1¢ per pound. Cotton X11 at 34.8¢ has a reduced cost of zero, indicating that it ties with one of the cottons listed in the basis variables report and might be used, if inventory considerations dictated, in this blend. Notice that raw cottons X1 through X12 all have rather low reduced costs (the highest being 0.4¢), while X13 through X20 remain unlikely components for this blend, the reduced costs ranging as high as 3.3¢. This cost data provides valuable information to the purchasing department, either for immediate use or as a guide for future buying when production requirements and cotton prices can be anticipated.

Though we have no illustration here, sometimes an upper bound restrains the use of a blend component. In such a case the material would be listed, and the report would indicate the highest price at which the material would remain in the basis at its bound.

The second part of the DO. D/J report (ROWS, at the bottom of Figure 8) makes graphic some of the information from the slacks report. This report lists all the row mnemonics. For each equation (and each inequality solved at a bound) the report indicates the "cost" of the quality or inventory limitation by giving the reduction in the cost per pound of blend which would be realized (in the neighborhood of the optimal solution) if the right-hand side were changed by one unit. (This figure is the simplex multiplier from the slacks report.)

For example, the first line under ROWS in Figure 8 indicates that if the fineness (FIN) maximum specification could be increased, the cost of the final blend would drop at the rate of 0.6¢ per pound for each unit of increase in the neighborhood of the optimal solution. The report indicates that no other specification affects the cost of this optimal blend. If a constraint solved at a lower bound were limiting the optimal cost, a positive value would be listed under DECR B VALUE for that row. (The listing of zero for GRA indicates that its relaxation would have no effect on the current optimal cost.)

DO.D/J				
VBLS	NAME	CURRENT COST	REDUCED COST	BASIS VALUE
	X01	.353	.002	.351
	X02	.355	.003	.352
	X03	.351	.001	.350
	X04	.355	.004	.351
	X05	.354	.004	.350
	X06	.353	.003	.350
	X09	.353	.002	.352
	X11	.348	.000	.348
	X12	.351	.003	.348
	X13	.372	.020	.353
	X14	.385	.032	.353
	X15	.383	.031	.352
	X16	.384	.031	.353
	X17	.367	.016	.351
	X18	.382	.030	.352
	X19	.385	.035	.350
	X20	.384	.033	.351
ROWS	NAME	INCR B VALUE	DECR B VALUE	
	FIN	.006		
	LEN			
	STR			
	CNT			
	GRA		.000	

Figure 8. DO. D/J report

COST RANGE REPORT

The cost range (COST. R) report shown in Figure 9 indicates for each activity which is included in the basis (optimal blend) the following data: current cost (that is, price per pound), highest cost before its quantity in the optimal solution changes, what other activity would enter the solution at that highest cost, lowest cost before its quantity in the optimal solution changes, what other activity would enter the solution at that lowest cost.

The quantity of each raw cotton in the optimal blend (given by the basis variables report) will remain unchanged within the cost range indicated by the cost range report. For example, 17% of cotton X7 would be required in an optimal blend even if it cost 0.1¢ per pound more. However, if its price rose above 34.9¢, some of it would be replaced by cotton X11. If its price dropped below 32.7¢, probably much more of it would be used, and cotton X16 would enter the optimal blend to provide necessary quality. Similarly, cotton X8 would be to some degree replaced by cotton X11 if its price rose above 35.1¢. Below 34.8¢, however, a slack would appear in the fineness constraint, indicating that the optimal solution would give away fineness quality. Finally, cotton X3 would enter the optimal blend if X10 cost more than 35.4¢, and cotton X11 would enter the solution if X10 cost less than 35.1¢.

The cost range report provides a good measure of sensitivity to price changes since it indicates at what prices the optimal solution will change, and what raw cottons may be used most appropriately to substitute for unavailable or overpriced stock.

PRODUCT ALLOCATION MODEL FORMULATION

Another important application of linear programming techniques enables mill management to determine the optimum product mix (that is, quantity and variety of different textile styles to be produced) in order to maximize profit over a specific production period. This product mix must be formulated within the inherent constraints set by maximum available production capacity at each step in the fabric manufacturing process (carding, drawing, roving, etc.).

To demonstrate the formulation of this problem in linear programming terms we assume a hypothetical (though reasonably complete) mill and derive the LP model matrix for the production of six styles over one production period. The production rates for the styles differ. For example, the time required to draw the sliver used in spinning yarn for weaving one yard of style 1 is not the same as for one yard of style 2. At every step in the process, different production rates exist for each of the styles.

The input data for this matrix consist principally of production rates on each of the machines used in the manufacturing process for each of the styles. In addition, we must know the total production time available for each machine, as well as the anticipated profit per yard for each of the styles and the minimum or maximum demand for each of the styles. The mill machine production time capacity for the subject period is detailed in Figure 10.

Picker capacity	1
Carding capacity	50
Drawing capacity	24
Roving capacity	500
Filling-spindle capacity	10000
Warp-spindle capacity	5000
Spooler capacity	1
Warper capacity	1
Slasher capacity	1
Loom capacity	500

Figure 10. Mill machine availability in machine-weeks

In the schematic model matrix (Figure 11) each of the six possible styles being considered for production is a problem activity, and each machine total capacity (available production time) is a problem constraint. Further, limitations on the amount of specific styles are introduced by bounding the corresponding variables. In other words, the constraint rows limit machine use to plant capacity.

COST.R NAME	CURRENT COST	HIGHEST COST	HI-VAR	LO-VAR	LOWEST COST
X07	.348	.349	X11	X16	.327
X08	.350	.351	X11	FIN	.348
X10	.352	.354	X03	X11	.351

Figure 9. Cost range report

Objective Function	Style 1 Profit 1	Style 2 Profit 2	Style 3 Profit 3	Style 4 Profit 4	Style 5 Profit 5	Style 6 Profit 6	RHS Profit (Maximum)
Picker	Pick. Time 1	Pick. Time 2	Pick. Time 3	Pick. Time 4	Pick. Time 5	Pick. Time 6	≤ 1 (Picker Time Capacity)
Carding	Card. Time 1	Card. Time 2	Card. Time 3	Card. Time 4	Card. Time 5	Card. Time 6	≤ 50 (Carding Capacity)
Drawing	Draw. Time 1	Draw. Time 2	Draw. Time 3	Draw. Time 4	Draw. Time 5	Draw. Time 6	≤ 24
Roving	Rov. Time 1	Rov. Time 2	Rov. Time 3	Rov. Time 4	Rov. Time 5	Rov. Time 6	≤ 500
Filling Spindles	Fill. Spin. Time 1	Fill. Spin. Time 2	Fill. Spin. Time 3	Fill. Spin. Time 4	Fill Spin. Time 5	Fill. Spin. Time 6	≤10000
Warp Spindles	Warp Spin. Time 1	Warp Spin. Time 2	Warp Spin. Time 3	Warp Spin. Time 4	Warp Spin. Time 5	Warp Spin. Time 6	≤ 5000
Spooler	Sp. Time 1	Sp. Time 2	Sp. Time 3	Sp. Time 4	Sp. Time 5	Sp. Time 6	≤ 1
Warper	Warp. Time 1	Warp. Time 2	Warp. Time 3	Warp. Time 4	Warp. Time 5	Warp. Time 6	≤ 1
Slasher	Slash. Time 1	Slash. Time 2	Slash. Time 3	Slash. Time 4	Slash. Time 5	Slash. Time 6	≤ 1
Loom	Loom Time 1	Loom Time 2	Loom Time 3	Loom Time 4	Loom Time 5	Loom Time 6	≤ 500
Bounded Variables	≥ A	≥ B	≤ C	≤ D	≥ E ≤ F		

Figure 11. Schematic matrix for product allocation model

The solution will indicate the product mix which produces the maximum profit. If market conditions require certain levels of production, the quantity of each style produced can be bounded. In the schematic, style 1 must be equal to or greater than A yds., style 2 equal to or greater than B yds., styles 3 and 4 equal to or less than C and D yds., respectively, and style 5 equal to or greater than E yds. and equal to or less than F yds. Style 6 remains unbounded.

The coefficients in the objective function (profit) row are the anticipated profits per yard for each of the styles. These figures are derived from such factors as production costs, distribution costs (where applicable), and selling price. The row expresses a total profit figure in the form:

(profit per yard style 1) x (yards style 1 produced) + (profit per yard style 2) x (yards style 2 produced) + ...

which the LP system will maximize within the constraints imposed by the matrix.

Each coefficient in the body of the matrix is associated with a style and a machine and represents a production rate — the time in weeks required to process the cotton or yarn for one yard of a specific textile style. Thus, in the second row of the schematic matrix, "PICK. TIME 1" is the picker time in weeks required to process the cotton necessary to produce one yard of style 1. The right-hand side is the total picker capacity in machine-weeks available over the specified production period. Hence:

(pick. time 1) x (yds. style 1) + (pick. time 2) x (yds. style 2) + . . . + (pick. time 6) x (yds. style 6) ≤ 1 machine-week.

The remaining constraints are formulated similarly.

The solution output for such a model will indicate not only the most profitable product mix, but the efficiency of the manufacturing chain as well. For instance, the slacks report will reveal how much of each machine's total capacity remains unused, and will also indicate the bottlenecks in the process and provide useful information on the "cost" of insufficient machine capacity which obviously limits production. Such a model is an indispensable aid in planning for plant expansion, since, for any given product mix, it clearly identifies which machine capacities should be enlarged.

The DO. D/J report will indicate for each style which does not enter an optimal product mix the additional profit per yard required before production of that style becomes desirable. The cost range report will reveal, for each style in the optimal mix, the lowest and highest style profit figures which would leave that solution unchanged. Further, this report will reveal which styles would enter the solution if the profit margins on the styles in the optimal mix were to fall outside the limits established by the cost range report.

A product allocation model can be formulated which assumes that a week's production of any style in any quantity can be sold. Though such an unrestricted production model may be applicable for several weeks at a time under certain conditions, sales orders on hand and market forecasts ordinarily will establish certain maximum and minimum levels of production for each style. Such restrictions can easily be introduced into the model by bounding the variables (quantities of style produced) to meet requirements and forecasts, as indicated in Figure 11. LP application in this area allows management to compare rapidly an unrestricted optimal product mix with the product mix dictated by market conditions. Such a comparison may well influence future marketing decisions — it may suggest that certain styles should be curtailed and new markets developed for others, if the two types of solutions differ dramatically.

SUMMARY

The various output reports furnished by the LP system not only provide a detailed listing of the specific optimal solution, but also alert the producer to a variety of relationships, any one of which may profoundly influence the total cost or profit. The computer enables the producer to re-solve the problem rapidly with a number of variations suggested by the output reports. He can, in effect, use the LP model as an aid in the solution of a series of different problems. What if the price of each raw cotton varies? What if certain inventory purchases are possible at specific prices? What if quality controls vary? These factors, together with changes in product market demand, determine the ideal frequency of solution for a specific mill. The LP solutions provide data which enables the producer to make the most judicious policy decisions in matters of cotton blending, quality control, inventory control, product mix, purchasing, and product research. LP techniques make possible continuous management study — resulting in decreased costs, increased efficiency, and maximum profits.

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