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NEW TECHNIQUES AND EQUIPMENT FOR CORRELATION COMPUTATION

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James F. Kaiser and Roy K. Angell

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ABSTRACT

This memorandum, which is in two parts, describes some recent developments in the design of correlation computers.

In the first part, the underlying theory of the correlation function and its calculation are summarized; and the effects of finite averaging time, sampling, and quantization are discussed. Theoretical analysis, based on the work of Widrow, and experimental evidence are introduced to show that for many applications no significant change occurs in the correlograms for quantization above 4 or 8 levels, contrasting sharply with the previous practice of specifying 100- to 200-level quantization. For certain applications, even two-level quantization yields significant results. A classification of types of correlation computers is introduced and is followed by a detailed survey and description of the many components and circuits which may be used to perform the required mathematical operations. Using the component survey as a basis, two correlator designs for a particular requirement - one analog and the other analog-digital - are outlined and discussed in detail. A comprehensive bibliography is included.

The second part of the memorandum concerns the design and operation of a two-level real-time correlator. This correlator, which was constructed to experiment with two-level quantization, is designed around the ability of the magnetic-core shift register to accept and store, in time sequence, information which has been given in the form of the binary digits ONE and ZERO.

Inputs in analog form are sampled and quantized to two levels (one bit) and inserted into a ten-element shift register. Parallel outputs from all cores are multiplied by the output of the last core, thus forming products with different time displacements. The products are averaged in low-pass filters, and the resulting ten-point correlogram is displayed continuously on an oscilloscope or with a voltmeter. The correlator was tested with periodic and random inputs, and the evaluation of the results indicates the practicality of two-level quantization and the possibilities for real-time electronic correlation.

ACKNOWLEDGMENT

In addition to the authors, other laboratory staff members who contributed, through consultations or direct assistance, to the research reported in this document are Mr. Harry Pople, Mr. Douglas Ross, and Professor A. K. Susskind. Their help is gratefully acknowledged. Mr. John E. Ward has served as editor for the completed document.

FOREWORD

The material in this memorandum is the result of two different studies made during 1957 in the Servomechanisms Laboratory.

The material in Part I is adapted from a study, "Correlation Computer Design Study," by J. F. Kaiser performed for the Communication Biophysics Group of the Research Laboratory of Electronics, M.I.T. Funds for this work were provided by the Caspary Research Grant to M.I.T. Although this study was made for a particular correlation application (bio-electric potentials), the excellent summary of basic theory and techniques makes it valuable for any correlation problem. Those portions of the study which are of general interest have been adapted for use in this memorandum, and only the specific design details of a correlator for the bio-electric application have been omitted. Permission of Professor W. A. Rosenblith of the Research Laboratory of Electronics for use of this material is gratefully acknowledged.

The material in Part II is a study performed as a Master's Thesis by Mr. R. K. Angell and supported by the Air Force under Contract AF33(616)-3950. This thesis was a direct outgrowth of the study in Part I and was undertaken in order to demonstrate the practicality of a two-level real-time correlator, sometimes referred to herein as a "one-bit correlator". The thesis was supervised by Mr. J. F. Kaiser. The equipment was constructed solely to demonstrate the principle, and its success indicates that two-level correlators of this type could be designed for many correlation problems and would have great advantages in speed and complexity over existing methods.

This work has been supported as part of the airborne data-processing studies under Task 50678 of Contract AF33(616)-3950 because of the growing significance of correlation techniques in modern information processing. Correlation techniques are appearing in more and more of the literature concerning reconnaissance, intelligence, identification and location of unknown electromagnetic transmissions, and transmission of information at signal-to-noise levels much less than unity. The material should be of interest to anyone performing correlation analysis of data of any kind.

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PART I

CONSIDERATIONS IN THE DESIGN OF CORRELATION COMPUTERS

by

James F. Kaiser

CHAPTER I

INTRODUCTION

This study of schemes for realizing the correlation computation was undertaken to review the present state of the art of computation, both analog and digital, and to review the theory of the basic correlation computation as a statistical tool with the intended goal of arriving at the correlation computer design which will best suit the requirements of its users. The most important design considerations are speed and accuracy of computation, versatility and reliability of its operation, and the economy of construction and operation. In short, what is desired is the fastest, most accurate, simple, and reliable correlation computer.

In choosing a method for correlation computation, it is apparent that the choice must depend heavily upon the form of the input data, the form desired of the output data, and the time scale of the data--considerations which arise in the use of the machine. To permit the most flexibility in the choice of a design or method of computation, we have chosen to separate the overall problem into two parts. The first part concerns that of properly applying the correlation computer to a data reduction task. Considerations are those of accuracy and those of the choice of the length of data to be correlated and the number and time separation of the computed correlation values. The second part concerns the design of a correlator which will perform the single correlation computation as rapidly as possible.

This study is arranged into three sections. The first section deals with the theoretical concepts of correlation. The effects of finite averaging time, sampling, quantization, and averaging method are discussed. Spectral analysis is also considered. In the second section, the discussion centers on the various schemes used for correlation computation with emphasis on their inherent advantages and disadvantages in respect to our problem specification. The components used to realize the individual operations are examined in detail. Conclusions as to design are stated in the third section.

CHAPTER II

THEORETICAL ASPECTS OF CORRELATION ANALYSIS

A. THE CORRELATION FUNCTIONS

The correlation functions that provide the basis for the analysis of random time functions are defined (15)* as

$$\varphi_{12}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} f_1(t) f_2(t+\tau) dt \qquad \text{(crosscorrelation)}$$

$$\varphi_{11}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} f_1(t) f_1(t+\tau) dt \qquad \text{(autocorrelation)}$$

If $f_1(t)$ and $f_2(t)$ are integrable square (which is almost always met in practical situations), then $\varphi_{12}(\tau)$ and $\varphi_{11}(\tau)$ are defined for all τ and possess Fourier transforms that are commonly called power-density spectra. These spectra are respectively

$$\Phi_{12}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \varphi_{12}(\tau)\, e^{-j\omega\tau}\, d\tau \qquad \text{(complex cross-power-density spectrum)}$$

$$\Phi_{11}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \varphi_{11}(\tau) \cos\omega\tau\, d\tau \qquad \text{(power-density spectrum)}$$

In using correlation techniques, some investigators prefer to base their analysis on interpretation of the relevant correlation functions, while others prefer interpretation of the spectral functions. Disregarding for the moment the relative merits of each method, the literature appears to be in general agreement that computation of spectra is most readily accomplished from the correlation functions rather than from the data directly. With this opinion as a basis, we shall now consider the computational problems that exist in the practical calculation of the correlation functions.

* Number in parentheses refers to numbered items in the Bibliography.

The integral expressions for the correlation functions are seen to be based on a time average. This requires that the time function being correlated must be one generated from a stationary process, i.e., a process whose statistical characteristics are time invariant. If the time function is not stationary, interpretation of the correlation function in its simple form becomes difficult if not impossible.

The correlation integration is carried out over limits tending toward infinity. Thus, information over all time is required of the time function. Practically, this is impossible; and one must be satisfied with analysis of a finite amount of data. The questions which arise are what is the effect of a finite averaging time and what is the effect of using an imperfect integrator to perform the integration operation?

Because of the availability of high-speed digital computing machines, the investigator must be acquainted with the problems and effects of the sampling and quantization operations on the time functions, these operations being required on the data for digital computations.

This section will treat these problems in sufficient detail to give the investigator some idea of the magnitude and type of error introduced by each.

B. TIME SERIES CHARACTERISTICS

The use of the time average in the calculation of the correlation functions assumes most heavily that the time functions have statistical characteristics that are time invariant. From the practical point of view, such stationary time functions are not met exactly in most physical processes. By continuously monitoring the data, one often observes both bursts of variation and relative changes in the level of variation. It is then necessary to decide whether to correlate over short periods where the variation level appears to be relatively constant, or to correlate over a very long period and obtain an average correlogram for the process. If the latter correlation is chosen, much longer computation time is required; and it is necessary to interpret the effects of averaging the different levels of variation. On the other hand, if short correlations are taken, a view of how the correlation changes with level of variation is obtained at the sacrifice of slightly higher variance of error.

Provided the length of record of the short runs is at least an order of magnitude (factor of ten) greater than the period of the lowest frequency or time constant of the process other than those corresponding to changes in level of variation, then the average correlation for the process may be obtained by simply adding and averaging the short-time correlations. This second method of looking at several short-time correlations has several important advantages over computation of one long-time correlation. First, it gives one more of a feel for the effects of change in activity on the correlation. Second, it may result in important savings in computation time if it is shown early that the short term correlations differ only slightly from one another. This factor is extremely important from an economic point of view since it is desirable to obtain as much interpretable information as possible from a unit of computation time.

In support of the ideas just discussed, reference is made to Figs. 1 and 2, which show two common types of correlogram (correlation function) characteristics.* In each figure the correlogram at the top was obtained from a 60-second run length. The second and third correlograms correspond to computation over the first 7-1/2 seconds and the last 7/12 seconds, respectively. The change in correlogram characteristics with time is most obvious. For example, in Fig. 1c the apparent modulation

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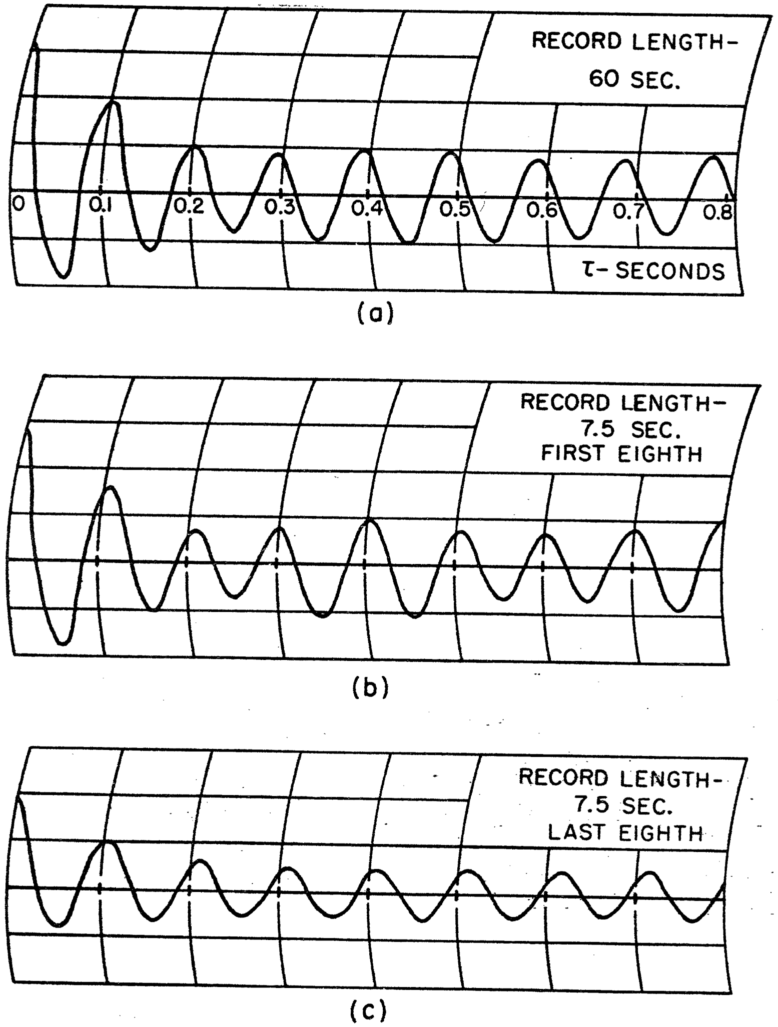
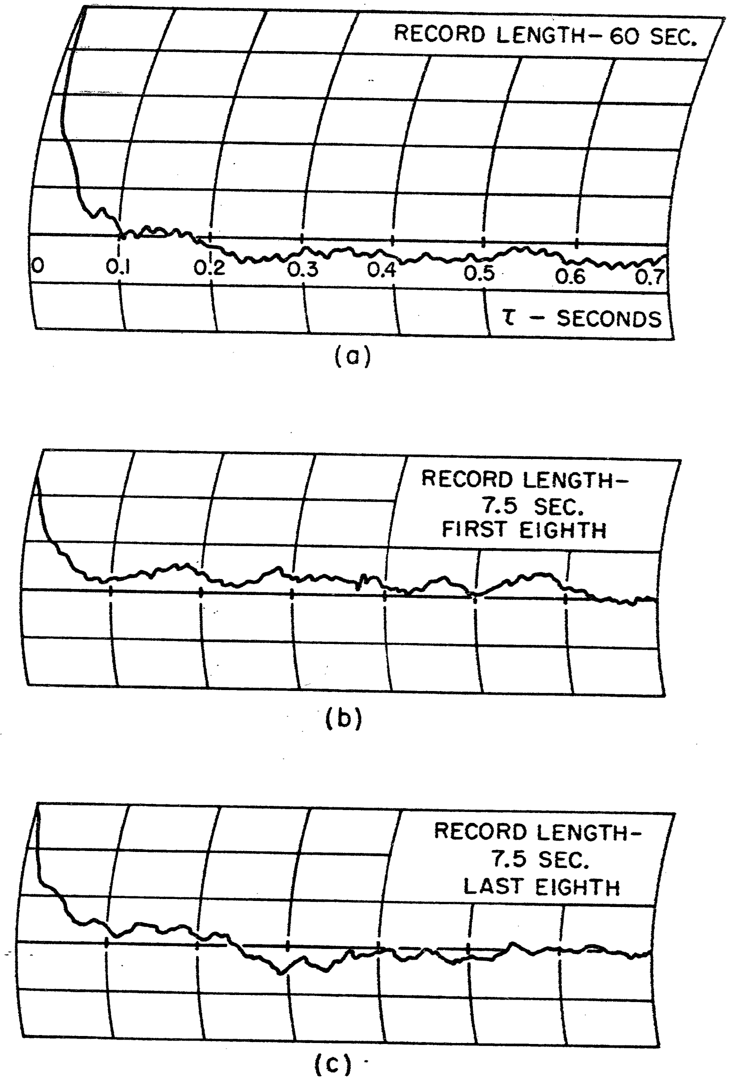
* These particular data, which were obtained from physical experiments on cell potential activity in the human body, are used because they are typical of the data to be correlated in many applications and also because they were readily available. These same sets of data are used throughout this report when recourse to an example is necessary to illustrate a point in discussion.





Figs. 1 and 2 Variation in the Autocorrelation as a Result of of Time Varying Statistics of the periodic component is missing whereas both Figs. 1a and 1b show this phenomenon clearly. Likewise, from Figs. 2a, 2b, and 2c, the first 7-1/2 seconds of data gives a much more variable correlogram than does the last 7-1/2 seconds of data. The last 7-1/2 seconds of data, however, gives a correlogram having much the same features as the long 60-second correlation. These examples point out clearly the advantages to be gained in short-term correlation and what sacrifices are made to obtain them.

One source of error in correlation that results from non-stationary series is caused by drift or very slow change in the value of the mean of the signal. This slow variation can be removed by coupling the signal to the correlator through a high-pass RC filter. The break frequency of this filter is chosen to be lower than the lowest frequency of interest in the correlation by at least a factor of two. Filters of identical characteristics should be used in each of the two input channels to the correlator.

C. EFFECTS OF FINITE AVERAGING TIME, SAMPLING, AND IMPERFECT INTEGRATION ON THE ACCURACY OF THE CORRELATION

The accuracy of the correlation computation depends on the averaging time interval, the number and spacing of samples if sampling techniques are used, and the characteristics of the integrator or averaging device. Much material has appeared in the literature discussing in detail the first three items (14, 15, 16, 17, 19, 20, 21, 24). Of these references, (20) gives perhaps the most complete and readable discussion of these effects.

From the literature, the following general conclusions can be drawn. For random signals the rms error in the correlation determination varies to a first approximation inversely as the square root of the record length. For periodic or deterministic signals the rms error varies inversely as the first power of the record length. Exact determination of the rms error in the correlation computation requires knowledge of not only the precise correlation function for the process but also of the fourth-order probability density distribution. Hence, exact determination of rms error can be made for only the simplest theoretical cases.

If one assumes a fixed data length, calculation of the correlation function by the use of samples and summation results in a higher value of rms error than if the computation were done continuously. This is easy to understand if one looks upon the sampling operation as discarding some of the information in the data. The rms computation error for the sampled case varies inversely as the number of samples for deterministic signals and inversely as the square root of the number of samples for the random signals. These variations are approximate; exact calculation of error again requires knowledge of both the second- and fourth-order probability distributions of the input data.

If the integration required in the correlation computation is to be accomplished approximately by the use of a filter, then in general a longer record length is required for the same accuracy of computation.

We can therefore conclude (neglecting inaccuracies in the components) that for a specified accuracy of correlation computation the speed at which the computation can be carried out is directly dependent on the speed at which the data can enter the computation. For a fixed data speed, the continuous-type correlator utilizing a pure integrator gives the most accurate result.

Because calculation of the variance requires knowledge of both second- and fourth-order probability distribution functions of the signals, the exact evaluation of the variance for a practical case becomes very difficult. To give some standard against which one can approximately assess the variance for practical cases, the signal can be assumed to be band limited noise such as would be obtained if white noise were passed through a low-pass filter.

The autocorrelation function of such a signal is given by

$$\varphi_f(\tau) = \sigma^2 e^{-\omega_1 |\tau|}$$

where σ^2 = mean square value of the signal

ω_1 = bandwidth of the signal in rad/sec

τ = correlation time shift

If we denote by L the record length, then the expression for the variance σ_φ^2 of the correlation determination as a function of τ and L becomes

$$\sigma_\varphi^2 = \sigma \left[\frac{1}{\omega_1 L} \left\{ 1 + e^{-2\omega_1 \tau} (1+2\omega_1 \tau) \right\} - \frac{1}{\omega_1^2 L^2} \left\{ \frac{1}{2} - e^{-2\omega_1 L} + e^{-2\omega_1 \tau} \left(\frac{1}{2} + \omega_1 \tau + \omega_1^2 \tau^2 \right) \right\} \right]$$

In the cases of interest to us

$$\omega_1 L > 1$$

Hence, the variance becomes approximately

$$\sigma_\varphi^2 = \frac{\sigma^4}{\omega_1 L} \left\{ 1 + e^{-2\omega_1 \tau} (1 + 2\omega_1 \tau) \right\}$$

For very small τ shifts

$$\sigma_\varphi^2 = \frac{2\sigma^4}{\omega_1 L} \qquad \text{or} \qquad \frac{\sigma_\varphi}{\sigma^2} = \frac{\sqrt{2}}{\sqrt{\omega_1 L}}$$

whereas for large τ shifts, $\omega_1 \tau > 1$

$$\sigma_\varphi^2 = \frac{\sigma^4}{\omega_1 L} \qquad \text{or} \qquad \frac{\sigma_\varphi}{\sigma^2} = \frac{1}{\sqrt{\omega_1 L}}$$

Figure 3 shows graphically the rms computation error σ_φ as a function of the record length for both $\tau = 0$ and $\omega_1\tau \gg 1$. The rms error value as determined from this graph represents an upper bound on the rms error that is encountered in practical situations. This rms error value refers only to the

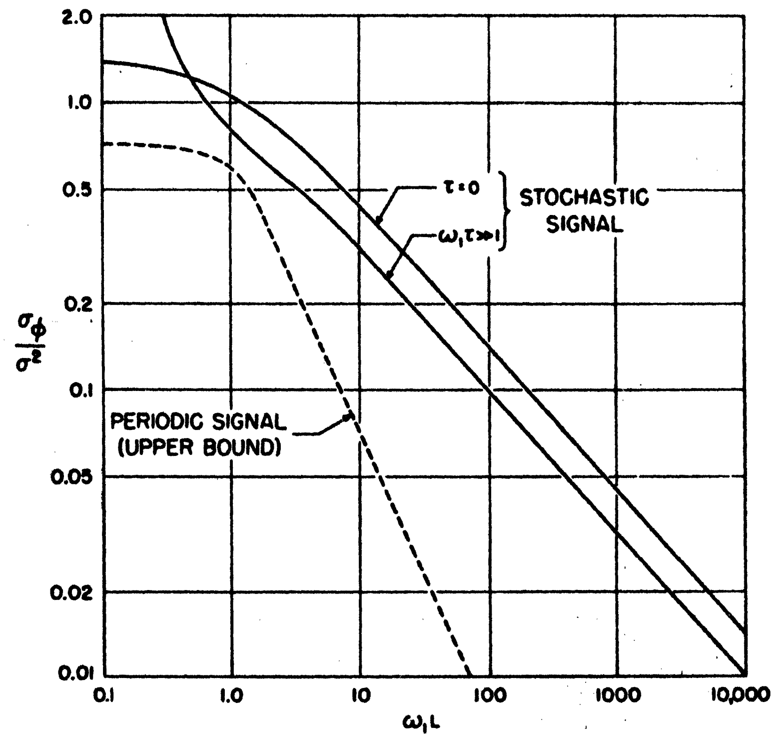


Fig. 3 RMS Computation Error as a Function of Record Length

calculation of single points on the correlation curve. From knowledge of this error value, little can be inferred about how the shape or overall characteristics of the correlation curve for a finite record length deviate from the true correlation characteristic. As a rule, the fit will be closer than that indicated by the value of the variance.

For periodic signals the variance diminishes with the square of the record length. For example, if the signal is assumed to have the form

$$f(t) = A\cos(\omega t + \theta)$$

then

$$\varphi_f(\tau) = \frac{A^2}{2}\cos\omega\tau$$

The variance of φ_f as a function of record length becomes

$$\sigma_{\varphi}^{2} = \frac{A^4}{8}\left(\frac{\sin \omega L}{\omega L}\right)^2$$

thus

$$\sigma_{\varphi} = \frac{A^2}{2\sqrt{2}}\left|\frac{\sin \omega L}{\omega L}\right|$$

but

$$\varphi_{max} = \frac{A^2}{2}$$

whence

$$\frac{\sigma_{\varphi}}{\varphi_{max}} = \left|\frac{\sin \omega L}{\sqrt{2}\,\omega L}\right|$$

Thus, for $\omega L = n\pi$, where n is an integer $\neq 0$, the variance is equal to zero. As an upper bound to the rms error we use

$$\frac{\sigma_{\varphi}}{\varphi_{max}} = \begin{cases} \left|\dfrac{\sin \omega L}{\sqrt{2}\,\omega L}\right| & 0 < \omega L < \dfrac{\pi}{2} \\ \dfrac{1}{\sqrt{2}\,\omega L} & \dfrac{\pi}{2} < \omega L \end{cases}$$

A plot of this function is shown in Fig. 3. Thus, for example, a record length of only 5 periods of the frequency ω can result in a 2.2% rms computation error, in the correlation coefficient as an upper rms limit. If we recall the correlation function of Fig. 1c where a very pronounced periodicity of approximately $\omega = 63$ radians/sec is present, it is easy to understand in one sense why the obtained correlation is so smooth. $L\omega$ for this case is 460 corresponding to a rms error of 0.15% if the periodic component is assumed to be the only component of importance.

One must be very careful in applying the results of these two simple cases to practical situations primarily because the processes usually encountered are nonstationary and of wide variability in statistical makeup. The two cases are presented only to serve as a broad guide.

To give an example of how the results might be misleading, refer again to Figs. 1a and 1b. There appears to be a marked modulation to the envelope characteristic of the periodic component. This could be characterized by two signals of nearly equal frequencies beating with one another. The expression for such a signal component could be written as

$$f_1(t) = \cos \omega_1 t(1 + a \cos \omega_2 t)$$

or

$$f_1(t) = \cos \omega_1 t + \frac{a}{2} \cos(\omega_1 - \omega_2)t + \frac{a}{2} \cos(\omega_1 + \omega_2)t$$

where $\omega_1 \gg \omega_2$. In such a case it can be shown that the value of the variance of the computed correlation function is mostly dependent on the low frequency ω_2 of the modulation. If the modulation is of very low frequency, extremely long record lengths must be used to reduce the theoretical variance to a tolerable value. We know from experience that extremely long records are not required to show the presence of the modulation; yet from calculation of the variance alone, we may be seriously in error with only moderate record lengths. From the experience gained by different members of the Servomechanisms Laboratory, the rule-of-thumb criterion that the record length should be at least 8 to 10 times the maximum time shift has been found to give very satisfactory results.

If sampling techniques are used, the same rule applies to record length with the additional requirement that the number of samples be at least 10^3 and preferably greater than 10^4 for good correlation results (3, 8, 15).

D. SPECTRUM ANALYSIS

Although the general or main characteristics of the autocorrelation function appear to be affected only slightly by the use of finite records of moderate length, the fact that errors or variability are present should caution the investigator in trying to read too much out of small irregularities in the correlation function. We feel that there is much to gain in understanding the information in the correlation if the spectrum or Fourier transform of the correlation is studied. The spectral study would not be intended to replace the correlogram study, but instead, to augment it and thus add greater insight into the problem.

Investigators using spectral graphs (18, 20, 22, 23) have found that the numerical computation of the spectra from the computed autocorrelation function can be carried out in general easier and faster than by the use of schemes involving band pass filtering and averaging of the original time function. This is especially true when use is made of special purpose analog or digital computers. Thus, the computation of spectra would follow logically the present computation of autocorrelation functions.

The spectra, as are the functions from which they are obtained, are affected by the use of a finite record length. The most important effect is that of limiting the frequency resolution to a minimum possible value ω_{res} of

$$\omega_{res} = \frac{1}{L - \frac{3}{4}\tau_{max}} \qquad \text{(Reference 18)}$$

where L is the record length and τ_{max} is the maximum time shift of the correlation function. This resolution is seldom achieved in practical cases because of the excessive amount of calculation required. In the more usual situation, the number of computed points of the spectral function is made equal to the number of computed values of the correlation function--usually in the range of fifty to two hundred points. In this case, the maximum frequency resolution obtainable in the spectrum is equal to the reciprocal of the maximum time shift of the correlation function. To realize this resolution, a special weighting function (23) is introduced in the transformation computation in order to reduce the effects of truncation of the correlation function. The introduction of the special weighting function adds little to the complexity of the Fourier transformation computation. The effects of the addition of this weighting function on the obtained spectral function is discussed in detail by Ross (23).

An example of the use of spectra in the correlation studies will be pointed out in the next section where the effects of quantization are considered in some detail.

E. QUANTIZATION AS APPLIED TO THE COMPUTATION OF CORRELATION FUNCTIONS

1. Theoretical Considerations

If it is first assumed that the data to be correlated is in a continuous analog form, then in order to perform the correlation computation digitally the data must first be sampled at discrete evenly spaced points and then the magnitude of the sample digitized or quantized. The effects of sampling have already been discussed. The problem remains as to what effects does quantization introduce. It would be advantageous to know how coarsely one is permitted to quantize the data without introducing excessive errors in the correlation computation. If a special-purpose digital machine is to be built to perform the computation, it is known that the size and complexity decrease and the speed increases as the size (bit length) of the input number is reduced. It is then to one's advantage to use as small a number (as coarse a quantization) as is tolerable in view of the errors introduced.

The theoretical study of the effects of quantization carried out by Widrow (24) suggest that extremely coarse quantization is permissible in the correlation computation with the resultant statistical errors being calculable. For simple first-order processes, it is shown that the noise introduced by quantization is completely uncorrelated with the statistics of the process if a certain relation exists between the quantization coarseness and the characteristic function of the amplitude distribution of the input. If the relation is satisfied, then the moments of all orders of the unquantized signals are calculable from the moments of the quantized signal and information on the quantization coarseness.

For higher-order processes the correlation between quantization noise and quantized signal is a function of the correlation of the quantizer input signal with itself. For signals with Gaussian distributed amplitudes, Widrow (24) gives the following relation that approximates closely the correlation of quantization noise as a function of the correlation coefficient of the quantizer input signal:

$$\sigma_N = e^{-4\pi^2 \frac{\sigma^2}{q^2}\left(1-\frac{\sigma_{12}}{\sigma^2}\right)}$$

where σ_N = normalized correlation of quantization noise with signal

σ = standard deviation of quantizer input signal

q = quantization box width

σ_{12} = normalized correlation coefficient of the quantizer input signal

A plot of this relation for different values of the correlation of quantized noise is shown in Fig. 4. The use of this figure is as follows. If a box size or quantization coarseness of say $q = 2\sigma$ is used, then one can, by reading off the intersection of the slanting lines with the ordinates determined by $(\frac{q}{\sigma})^2 = 4$, obtain the correlation value of quantization noise to be expected for any normalized correlation coefficient of the signal being quantized. Thus, we see that for $q = 2\sigma$ (extremely coarse quantization) the noise introduced by quantization is less than 1% correlated with the signal for all correlation coefficient of the signal less than 0.1 in magnitude.

Van Vleck* (68) has shown that for two-level quantization the correlation function $R(\tau)$ of the quantized signal is simply related to the correlation function $\varphi(\tau)$ of the continuous signal as

$$R(\tau) = \frac{2}{\pi} \sin^{-1} [\varphi(\tau)]$$

Thus, in the two-level case for correlation values less than 0.54, $R(\tau)$ is within 5% of $\varphi(\tau)$.

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* A derivation of this result is included on page 147 of this memorandum.

The importance of this result is immediate to the problem of correlation. If the noise introduced by the quantization process is completely uncorrelated, then the only effect of quantization is to change the values of the even moments of the input signal an amount calculable directly from the

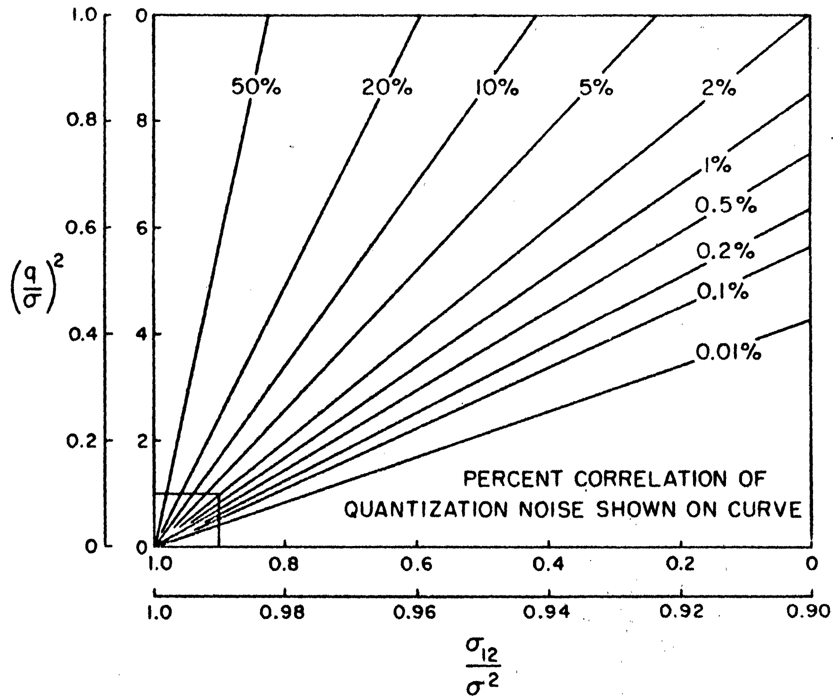


Fig. 4 Quantization Box Size vs. Correlation Coefficient of Quantizer Input Signal with Correlation of Quantization Noise as a Parameter

quantization size. All other obtained points on the correlation curve (i.e., other than at $\tau = 0$) are theoretically unaffected by the quantization, a very interesting and <u>useful</u> result. A series of experimental tests, that will be mentioned subsequently, were carried out which verified this theoretical result quite well.

The errors in the even moments of the correlation for $\tau = 0$ of the quantized signal are found simply by considering the flat-topped distribution of the quantization noise. This distribution is shown in Fig. 5. The moments are given by

$$\overline{y^n} = \int_{-\infty}^{\infty} y^n \, p(y) dy$$

but

$$p(y) = \begin{cases} \frac{1}{q} & -\frac{q}{2} < y < \frac{q}{2} \\ 0 & y < -\frac{q}{2}, \quad y > \frac{q}{2} \end{cases}$$

therefore

$$\overline{y^n} = \begin{cases} 0 & n \text{ odd} \\ \frac{2}{q} \int_0^{\frac{q}{2}} y^n \, dy & n \text{ even} \end{cases}$$

or

$$\overline{y^n} = \frac{\left(\frac{q}{2}\right)^n}{n+1} \qquad n \text{ even}$$

Thus, the mean square and mean fourth values of the noise are, respectively

$$\overline{y^2} = \frac{q^2}{12}$$

$$\overline{y^4} = \frac{q^4}{80}$$

It must be recalled that these theoretical results assume that the distribution of amplitudes of the input signal or signals satisfies the "Nyquist Condition" with respect to quantization size. Signals whose distributions are Gaussian or very nearly Gaussian satisfy this condition approximately up to extremely coarse quantization. Fortunately, most of the data from the physical processes of nature have amplitude distributions that are approximately Gaussian. The distributions which can cause serious errors when quantization is introduced are those possessing discontinuities or large changes in slope. Fig. 6 gives several examples of these types of amplitude distributions.

2. Experimental Results

To obtain a feeling for the implications of quantization and to test the validity of theoretical results, a series of

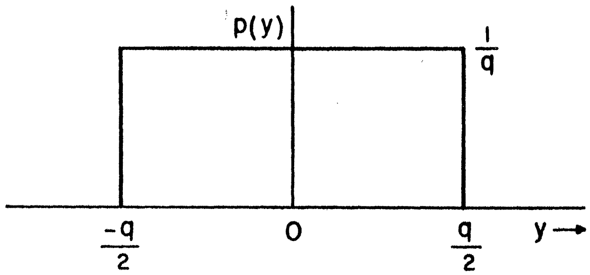


Fig. 5 Probability Distribution Function of Quantization Noise

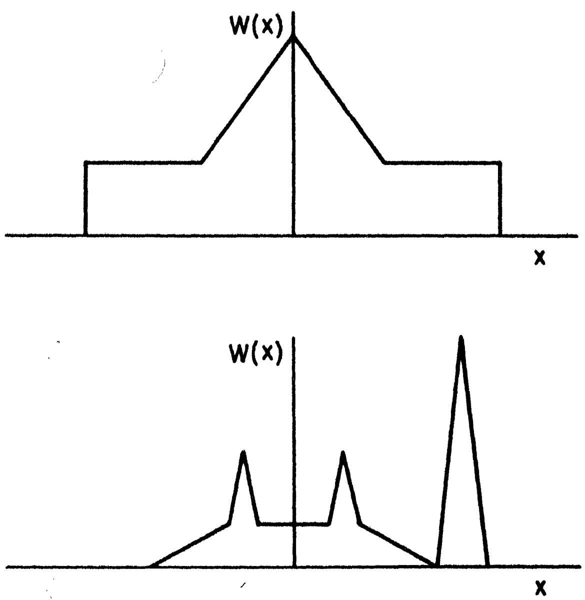


Fig. 6 Amplitude Distributions Not Satisfying the "Nyquist Condition"

correlograms was computed with the aid of the Whirlwind I digital computer. The data were taken from tape recordings of the cell potential activity during two different experiments. The correlograms of the data exemplified the two extremes encountered in this type of study. One, denoted by Run A, represents what appears to be a well-defined behavior; the other, Run B, a much more complex structure.

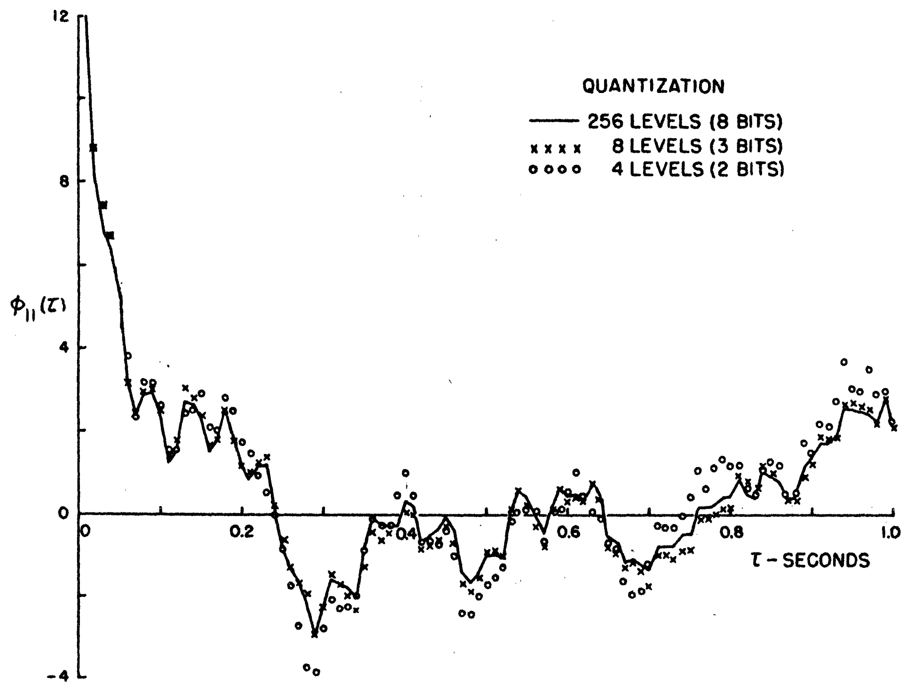
The correlogram computations were aimed at finding the effect on the correlation characteristics of quantization of varying degrees of coarseness and at determining the effects of bias and saturation that would occur in a practical realization of a quantized computation. The correlations were obtained from the summation of 650 products of sample pairs. Each correlation consisted of 100 evenly spaced computed values. The sample size (650 points) is regarded as small for this type computation (15), and hence care must be exercised in the interpretation of the results.

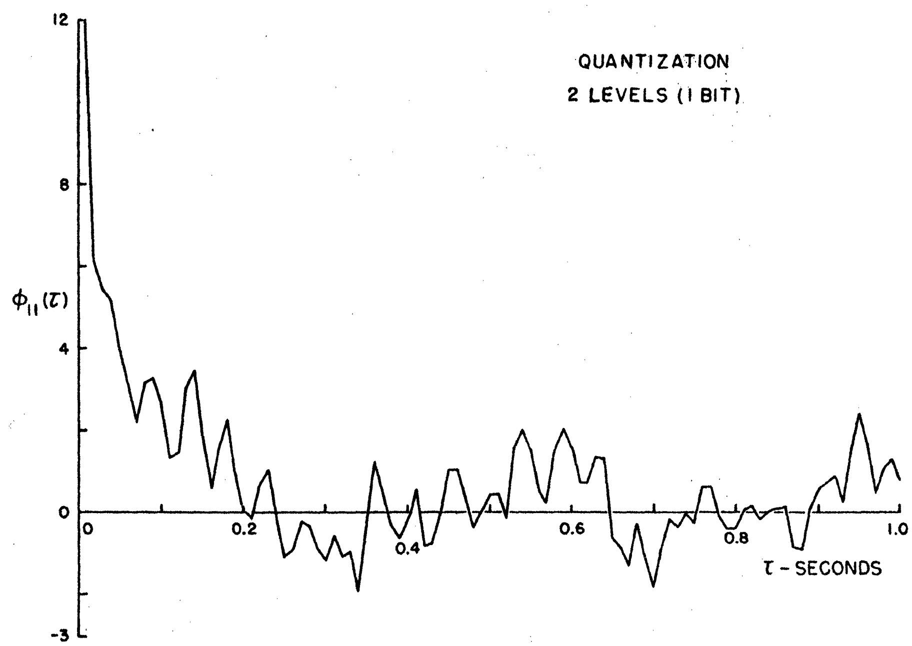
The first graph, Fig. 7, shows the effect of quantization coarseness on the correlation characteristics of the Run B data. It is seen that quantization to 3 bits (8 levels) results in only minor amplitude differences with no apparent changes in overall characteristics. For 2-bit (4 levels) quantization, the variations in amplitude deviation are somewhat greater but still the general characteristics are retained. For the extreme case of one-bit (2 levels) quantization coarseness, Fig. 8 shows that only the major characteristics of correlation are intact. The theoretical results of the previous section indicate that better correlation should be obtained. The discrepancy between theory and experiment in this situation is believed to be due primarily to the small sample size. Aside from this, the results of one-bit quantization are quite startling and interesting.

The comparison between correlations was made on the general basis of how closely the curves conform to one another. An additional check into the effects of the quantization is made by observing the power density spectra of the correlations. As shown in Fig. 9, quantization down to three bits introduces no detectable changes in the frequency characteristics of the spectra. The minima and maxima appear unchanged frequency-wise. One-bit quantization appears to reduce the frequency definition as notable differences are present, especially at the higher frequencies. This, again, is believed due to the small sample size.

The data of Run A was subjected to the same procedure as outlined above. Quantization was adjusted to be 8 bits, 3 bits, 2 bits, and one bit, respectively. The results are shown in Figs. 10 through 12.

It is immediately apparent that the same general conclusions as reached for the Run B data apply here directly. Quantization to three bits causes no changes in the important characteristics of the correlation and only slight difference amplitude variations. The spectra (Fig. 12) show that in all cases the apparent modulation of the cosinusoid is unaffected





Figs. 7 and 8 Correlogram, Run B

by coarseness of quantization. The only perceptible effect in the spectral study was that in the one-bit case the discrepancies were large primarily at the high frequencies. In all cases, the relative magnitudes of the two adjacent maxima at 8 and 10 cps were preserved.

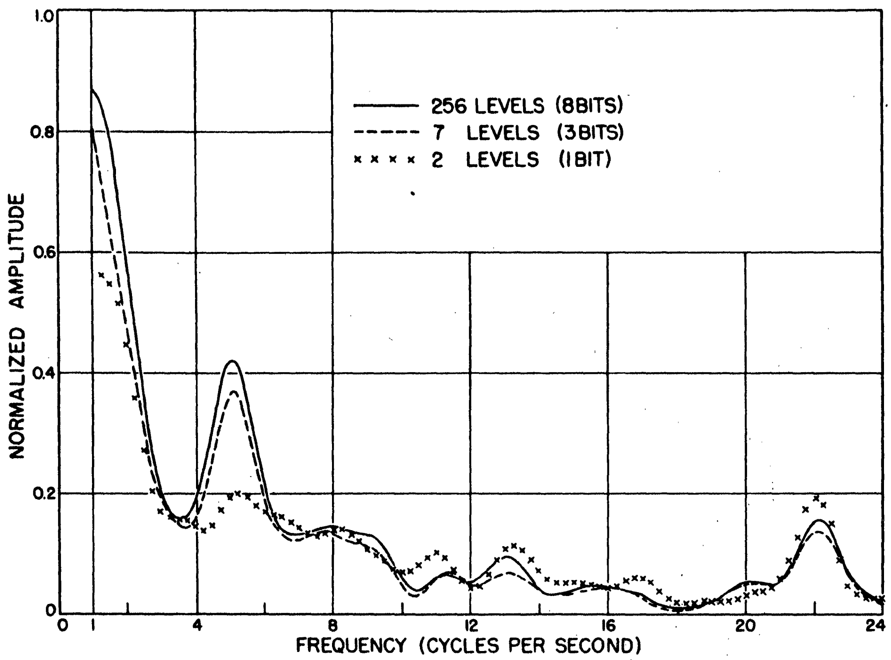
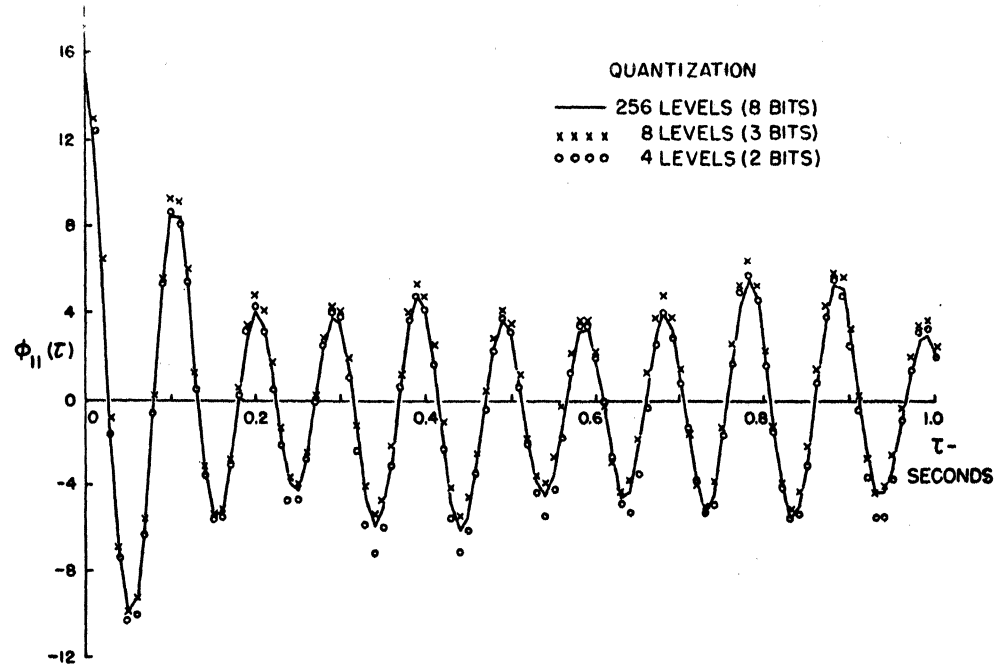
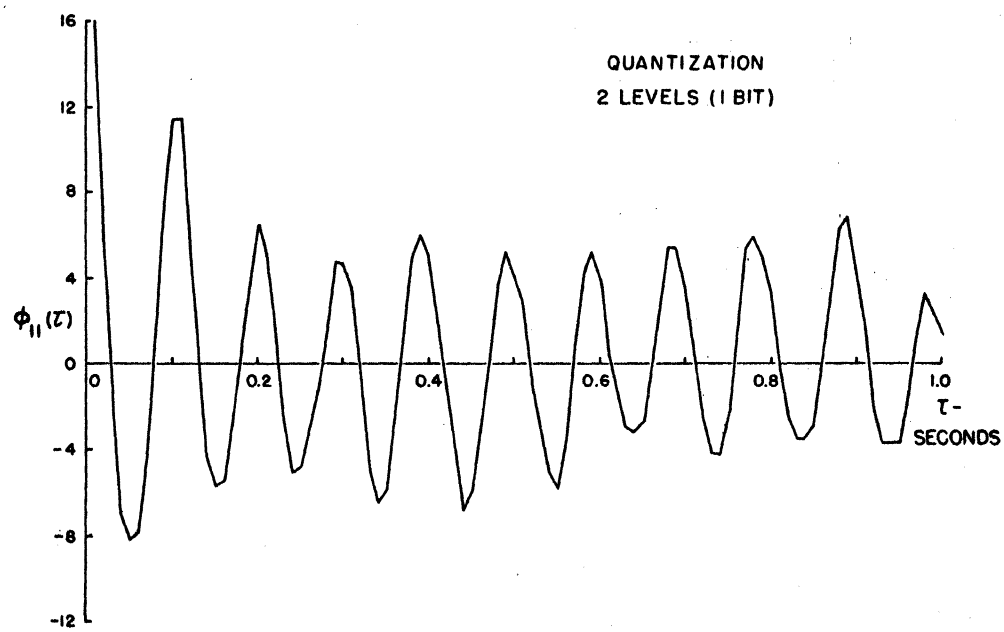


Fig. 9 Power Density Spectra, Run B

From the similarity of results obtained from these tests on two sets of data of quite different correlation characteristics, it can be concluded that very satisfactory correlations can be computed digitally using quantization as coarse as 8 levels (3 bits). Quantization to 4 levels (2 bits) appears to give good results and there is every indication that this result should improve if the sample size is increased to a moderate value.

Because quantization to 2 levels (one bit) gives a surprisingly fair correlation, work in the form of a master's thesis was conducted toward the construction and evaluation of such a device which would compute and display ten values of the correlation function simultaneously. The results of this thesis are included in their entirety in Part II of this





Figs. 10 and 11 Correlogram, Run A

memorandum. The one-bit correlator should prove very valuable as a simple and rather inexpensive device capable of giving good estimations of the correlation functions.

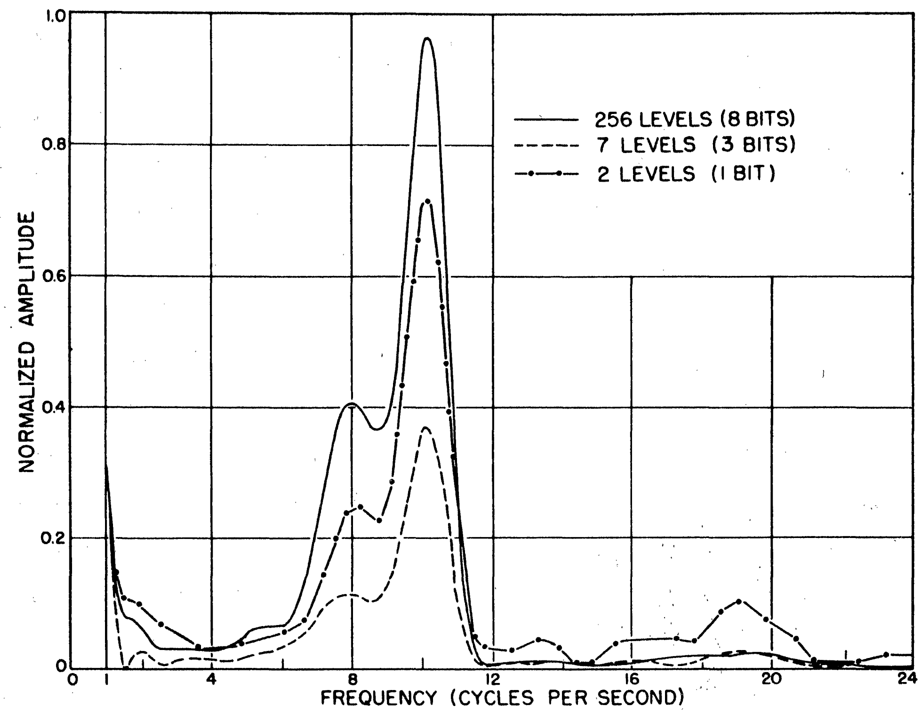


Fig. 12 Power Density Spectra, Run A

Because any practical design must utilize a quantizing element, a study was carried out to determine the effects of data bias shift and data-mean-square-value shift on the correlation characteristic when only a finite number of quantization levels are available. For the first run the "level" size or "box" width was chosen such that 98% of the data points fell within the total number of boxes to be used (3 and 7 for the Run B data and 3 for the Run A data). Any data points falling outside the end boxes (top and bottom levels) were interpreted as being inside the respective end box. This type of run is termed a "straight" run. A graph illustrating this adjustment of box width is shown in Fig. 13a for a nearly Gaussian distribution of signal amplitudes.

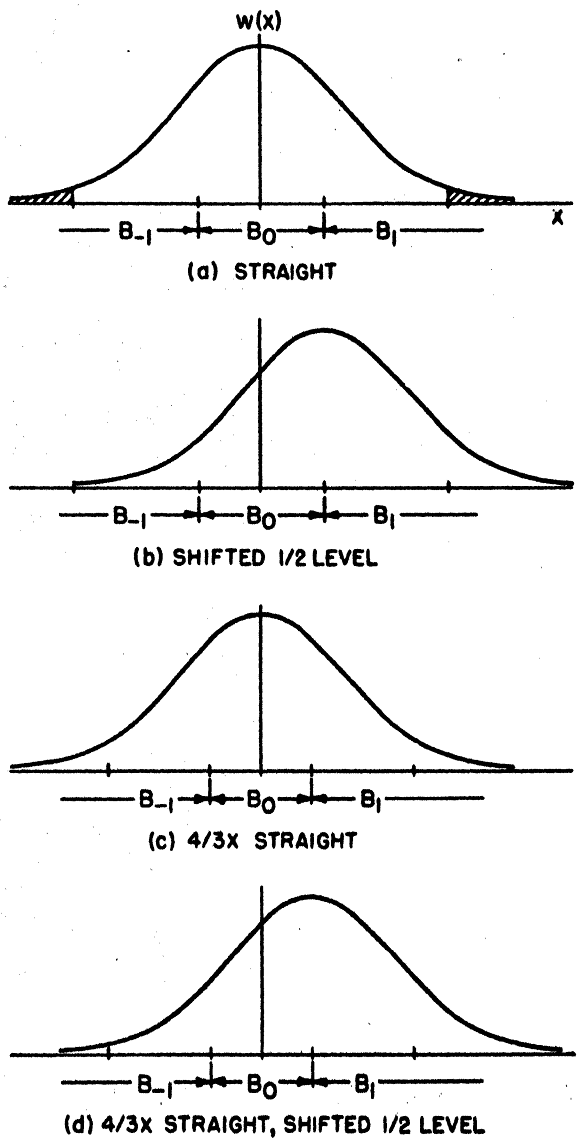


Fig. 13 Changes Made in the Amplitude Distribution to Study the Effects of Bias, Amplification Factor and Saturation

To determine the effect of a shift in bias and the resulting box saturation, a bias of one-half a box width was added to the data as adjusted for a straight run. The resulting amplitude distribution over the boxes is shown in Fig. 13b. The effect being investigated by this maneuver is not only the resultant change in correlogram bias but the shape of the correlogram as determined by saturation in the end box B_1.

In order to try to isolate the effects of end box saturation, the data of the straight run was multiplied by a constant factor such that 98% of data points covered n + 1 boxes where they had covered n boxes before. The test was then run still using only n boxes. Saturation was thus present in both end boxes (the effective end box width thus being 1-1/2 boxes). This operation is shown in Fig. 13c.

For a final run the amplified data of Run 3 was again shifted one-half a box width, thus doubly saturating the right end box and returning the left end box to roughly the unsaturated condition.

This manipulation is shown in Fig. 13d. Figure 13 was prepared using 3 levels for the quantization.

Figure 14 illustrates the correlations obtained by applying the sequence of four manipulations to the Run A data. The first observation is that the general character, including the envelope modulation, of the correlation is retained throughout all the operations. At only three levels of quantization this result is very rewarding because it indicates that the bias and saturation effects are not extreme at all. The shift in correlation level for the second and fourth runs was approximately the value computed from the square of the bias set in. If the data had an initial mean value of $\overline{f}$, then it can easily be shown that the change in correlation value for an added bias $\overline{b}$ is simply $2\overline{fb} + \overline{b}^2$.

It is noticed that the effect of saturating the end boxes as in Run 3 gave a wider variation in the correlogram. This was to be expected since the end boxes are now being used more than in the first run. Despite the heavy saturation effect in the right end box for Run 4, the general correlation characteristic was quite well preserved. In all cases the major frequency information remained fixed in the spectrum.

The same sequence of operations was applied to the Run B data first with quantization at 7 levels and then at 3 levels. The correlograms of Run B at 7 levels, Fig. 15, strongly support all the conclusions reached on the 3-level Run A results. At 7 levels the different correlations remarkably display all the moderate and major rises and falls of straight correlation. Amplifying the data as in runs three and four again results in a slightly increased correlation amplitude due to greater use of the end boxes.

The correlograms of Run B at 3 levels showed much increased variability over that at 7 levels. For this reason, a power spectrum presentation of the results is chosen. Fig. 16 shows these results in detail. It should first be mentioned that the amplitude scale factor for runs two and four differs from that of runs one and three by a factor slightly greater than 2.

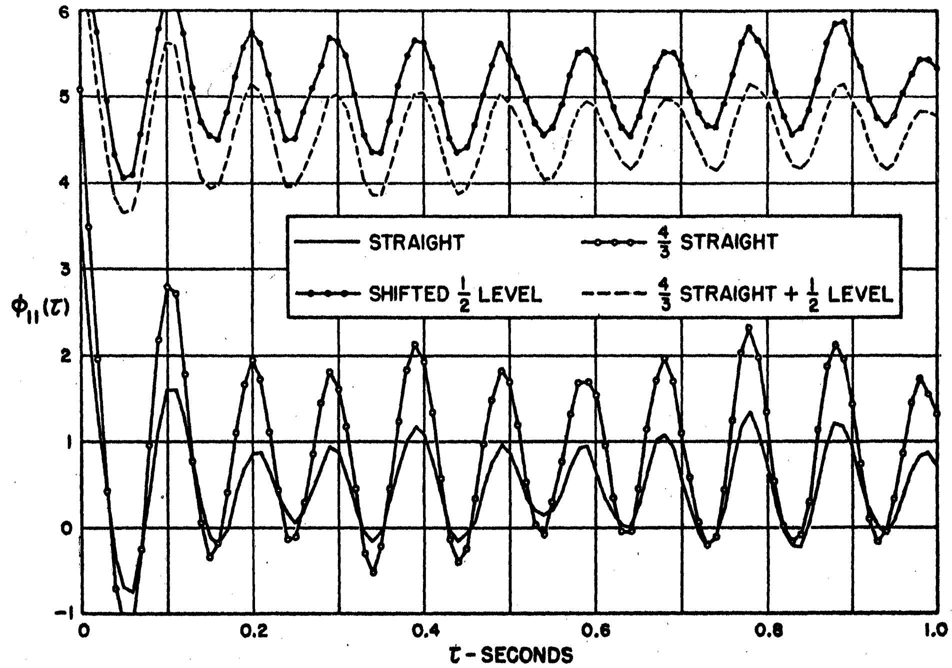


Fig. 14 Correlogram, 3 Levels, Run A

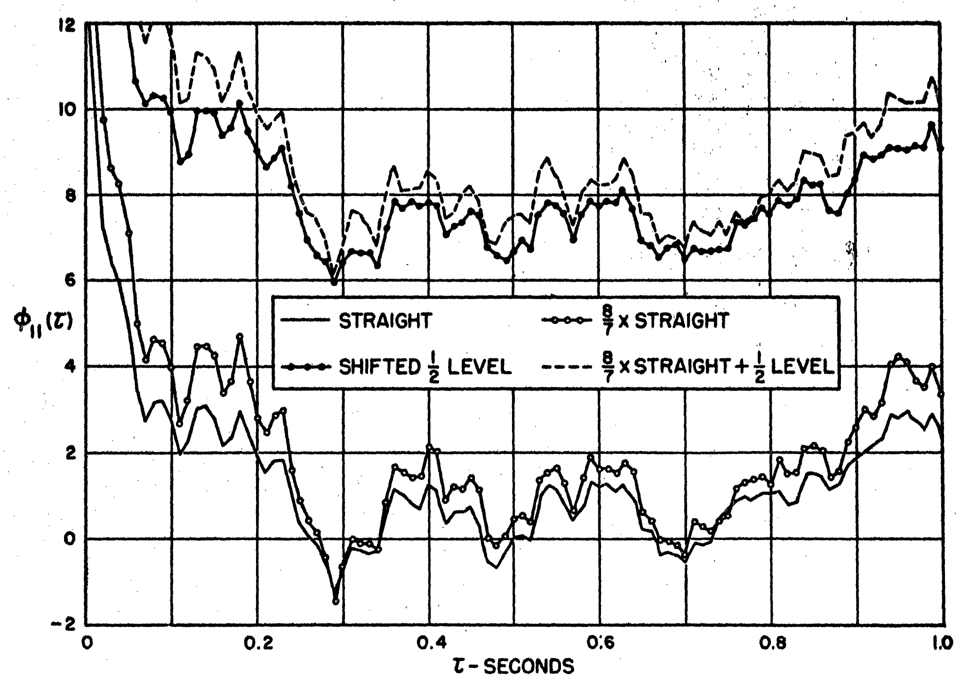


Fig. 15 Correlogram, 7 Levels, Run B

The first general comment is that the power density spectra have roughly the same general shapes. For runs 2, 3, and 4 the spectra are noticeably different from run 1 in regards to location of peaks and valleys in the range of frequencies from 7 cps to 11 cps. The maxima at 5 cps

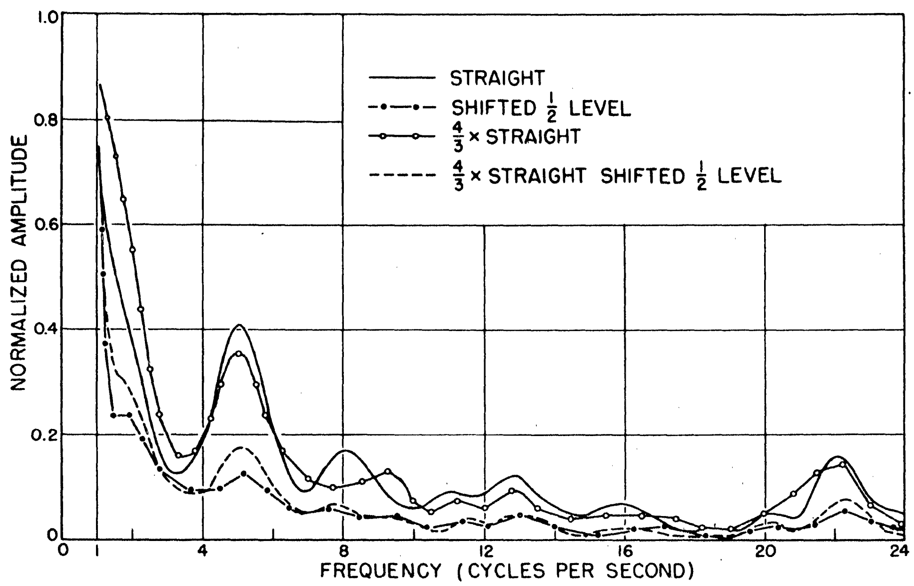


Fig. 16 Power Density Spectra, 3 Levels, Run B

and 22 cps (the major peaks) remained intact throughout the various operations. Little else can be said concerning these results other than it must be recalled that all computed points are the summation of only 650 sample products. This relatively small sample size can account for much of the variability noted in the 3-level results.

In summarizing the results of these experimental studies of quantization effects, it is first pointed out that these results are based on correlations determined from only 650 sample pairs--a fairly small sample size. All conclusions are made in cognizance of this fact.

First, correlations obtained with quantization as coarse as 7 levels (3 bits) were excellent when compared with both the analog results and the 256-level results. There are no detectable changes in the frequency information contained in the data as a result of this quantization. A change in bias or scale factor of $\pm 7\%$ (1/2 level) produces no noticeable or uncomputable change in the correlations, frequency information again being unaffected. The change in mean-square value due to quantization is computable and is relatively small for the 7-level case. As a whole, the 7-level study supports the theoretical results rather closely. It is recalled that the normalized value of the correlation functions were usually considerably less than 0.5, especially in the case of the Run B data.

Second, the quantizations at 4 levels (2 bits) produced correlations whose variability although large could possibly be acceptable in lieu of the fact that this variability could partially, at least, be attributable to the small sample size. Again frequency information appeared to be unaffected by bias shifts and saturation effects.

The results of one-bit quantization were satisfactory enough to merit additional study in the form of a thesis investigation. A simple real-time correlator is brought into the realm of possibility by the encouraging results of these studies. It should also be noted that the variability of the 2-level quantization results over the 256-level results was less than the variability between the sectional correlograms for the first eighth and last eighth of the data!

CHAPTER III

STUDY OF BASIC SYSTEM TYPES FOR CORRELATION COMPUTATION

A. INTRODUCTION

This section begins with a discussion of the dependence of correlator type on the input and desired output information. The various schemes available for realizing each correlator type are discussed as to their advantages and disadvantages. The section closes with a rather comprehensive listing of most of the currently used techniques (that are applicable) for performing the operations of delay and storage, multiplication, integration, and recording.

B. SYSTEM BREAKDOWN

The procedure of obtaining the correlation function of a set of data involves three basic steps. The first step consists of transforming the data into a form that the correlator can use. If automatic analog or digital computers are to be used, the data must be transcribed to the form that each computer can interpret and operate upon. The second step consists of performing the correlation computation. The third and final step is the transformation of the computer results to a form suitable for interpretation or other use. This form may be a plotted curve, a tabular listing, a scope plot, a punched tape, etc.

The choice of a device to perform the actual correlation computation then is seen to depend upon both the form of the input data and on the desired form of the output results. The form and character of the input data impose the more stringent restrictions on the type of correlator than does the output requirement. The restrictions in design due to these specifications are discussed in detail in the pertinent sections which follow.

C. SYSTEM SPECIFICATION

For the purposes of discussion it will be assumed that the correlator is to be used as an investigative tool for the analysis of experimental data from physical processes. A difficulty, then, that is encountered is that the investigator often does not know what part of the correlation function or what part of the power spectrum contains the information he is seeking. He is therefore in a sense forced to look at the correlation function in rather fine detail over a sizable range in time delay. This uncertainty and wide variation requires that the correlator be designed such that the spacing between computed values and the number of computed values can be varied over wide limits. It is not uncommon for a correlation to contain upwards of one hundred computed points. The spacing should be chosen so that there are at least four computed points per the highest frequency to be discerned in the data. Nyquist's sampling theorem requires an absolute minimum of two points per cycle. On this basis the total number of computed points should be equal to the product of the ratio of the highest frequency to lowest frequency of interest and the number of points per cycle of the highest frequency.

To make possible a time scale change, the test data can be recorded on magnetic tape. This permits the investigator to use more efficiently the correlation computer by allowing him to run the data in as fast as the computer can process it. A frequency modulation scheme is usually used in the recording to enable the low frequencies to be recorded, and to help prevent loss of information during subsequent re-recording and playback.

For purposes of discussion, the form desired of the output of the correlator is assumed to be a point-by-point plot of the correlation function. As will be pointed out later, it may also be advantageous if the output is also recorded in a form that will easily permit further computation by a general-purpose digital computer. Such a form might be a punched paper tape or a digital magnetic tape record.

With the input and output specifications thus spelled out literally, it is desired to determine the correlation computer which will perform the single correlation computation as rapidly as possible. The design is to have the versatility, and flexibility previously mentioned, and must be as simple and reliable as possible while retaining a computational accuracy commensurate with data length and data statistics as outlined in Chapter II.

D. BASIC SCHEMES FOR PERFORMING THE CORRELATION COMPUTATION

To the writer's knowledge, there are four principal types of correlation computers. The classification is made on the basis of the mathematical manner in which the correlation coefficient is computed.

The first type is essentially the straight analog correlation computer. Its operation is as follows. A value of time delay τ is chosen and fixed. The data is then continuously multiplied by itself shifted in time by τ. The continuous product is then continuously integrated. After a fixed length of time T the computation is stopped, and the correlation coefficient value is read as the analog of the final value of the integrator. The next point is computed using the same data but a different value of τ. Descriptions of several successful correlators of this type are found in references 1, 6, 7, 9, 10, 11, 12, 13, 26.

The second type of calculation scheme consists of a slight modification of the first type. In this system instead of computing the correlation on a point-by-point basis, the value of time shift τ is slowly and continuously varied. Thus, the correlation function is generated continuously. In this scheme a very long data length is required. For the correlation to be meaningful, the statistics of the data must be reasonably time invariant over the long run. As a general rule, this scheme has no advantage in time saving and accuracy over Scheme 1 for comparable analog components. A correlator of this type is discussed in detail in (5).

The third type of correlation scheme is based on the sampling principle. The data is sampled at a rate such that the adjacent samples can be assumed to have statistics independent of one another. The sampling interval thus chosen is then greater than the maximum time shift of interest. The correlation coefficient is then built up by summing a fixed number of products of the sample of the time function and the sample of the time function shifted by τ units of time. The number of samples required for a fairly stable correlation function result has been discussed in Chapter II and is usually in excess of several thousand. Thus, this scheme requires a record length at least several thousand times the maximum τ shift. Correlators of this type are discussed in (3) and (8), and find their greatest use where the input information is located in a high frequency pass band and where a very long record length is available.

The fourth type consists of a slight variation of the third type, and is aimed toward a more efficient use of the record length. In this scheme the data is sampled at intervals corresponding to the minimum time shift increment. Independence of adjacent samples is not assumed or required. Again the computation consists simply of a summation of a fixed number of products of samples. This scheme is usually employed when the computation is performed on a general-purpose digital computer. The record length required by this scheme is nearly $\frac{1}{N}$ times that required by the third type where N denotes the number of computed points.

Correlator designs of types two and three require very long record lengths and are thus highly dependent on the stationarity of the recorded time signals. Correlators of types one and four, on the other hand, can be used with relatively short record lengths and tend to be much more efficient in the processing of the available data. The remainder of the discussion is confined to correlator designs of type one and type four.

E. DISCUSSION OF POSSIBLE SYSTEM DESIGNS AS TO COMPONENTS

The type-one system is essentially a continuous analog computing system whereas the type-four system is basically a discrete computing system being most likely digital in form, although discrete analog systems are possible. Because the basic correlation computation consists of four fundamental mathematical operations and because these operations can each be accomplished by many components, the possible number of correlator designs is very great. The designs may involve entirely analog components or all digital components, or may be a hybrid type utilizing both analog and digital techniques.

It is necessary to make a few general comments first before discussing the systems and components in detail. It is assumed first that the input to the correlator is to be data recorded on magnetic tape in frequency modulated form. Because of bandwidth limitations in the recording scheme, the highest frequency component of the input data is limited to 5 kilocycles per second for a 30-ips tape speed. Thus, a goal of the correlator design must be a capability of processing input data at this maximum rate. For an analog system this specification sets the bandwidths required of the individual computing components. For a digital system this specification partly establishes the minimum sampling and computation rate. The other factors which affect the sampling rate are the record length and desired minimum spacing between computed correlation values.

Regardless of the sampling rate, any type-four system will not be considered satisfactory unless it can perform about on an equal basis with the type-one system. Performance is measured directly in terms of speed and accuracy of computation. Considerations such as versatility, reliability, cost, and complexity become important in system comparison only after approximate equality of performance is established.

A second point to be brought out is that both type-one and type-four correlators compute the values of the correlation function at discrete points. Because the input data rate is assumed to be limited not by the computation process but by the data recording or storage means the time to compute one point on the correlation function is about the same for both systems. If total computation time is to be reduced by computer design, simultaneous computation of several points must be done. This is termed multiplexing. One consideration therefore in choosing a design must be based on how simply the single point computation method may be multiplexed. The total computation time can be reduced using this means by a factor of N where N is the number of simultaneously computed points.

In discussing analog type elements, attention must be paid to the problems of accuracy, bandwidth, drift, and dependency on tube characteristics. For digital elements there is essentially no drift problem, but attention must be directed now to complexity as a function of the bit size of the data numbers and to the time required to perform the particular digital operations. It should also be recalled that the input data is essentially of analog form. The desired output is an analog plot and possibly a digital tabulation if spectra are to be calculated later. Thus, if digital means are to be used, both analog-to-digital and digital-to-analog converters would be required.

As an aid in evaluating and discussing the different systems, two data runs typical of the near extremes that a correlator for the data discussed in Chapter II might encounter are considered. These typical runs are:

		COMPUTER TIME	
Short Run	Real Time	100:1 Speed up	10:1 Speed up
Highest frequency	50 cps	5,000 cps	500 cps
Maximum time shift	1 sec	0.01 sec	0.1 sec
Record length	10 sec	0.1 sec	1 sec
Number of points desired	200	200	200
Δ	5 m sec	.05 m sec	.5 m sec
Sample rate for 5,000 samples min.	500 s/sec	50,000 s/sec	5,000 s/sec

Long Run	Real Time	COMPUTER TIME 100:1 Speed up	10:1 Speed up
Highest frequency	50 cps	5,000 cps	500 cps
Maximum time shift	2 sec	.02 sec	.2 sec
Record length	2,000 sec	20 sec	200 sec
Number of points desired	400	400	400
Δ	5 m sec	.05 m sec	.5 m sec
Sample rate for 5,000 samples min.	2-1/2 s/sec	250 s/sec	25 s/sec

The important considerations implied by these runs are:

(1) For very short runs the sampling rate may have to exceed the reciprocal of the minimum time shift $\Delta\tau$ in order to obtain a sufficient number of samples.

(2) If a fixed sampling rate is used, then the number of samples will vary directly as the run length. This may be by a factor of 1,000 to 1.

(3) If the correlation function is computed one point at a time, total computation time may be as long as 2.2 hrs or as short as 20 sec.

In review, the type-one system evaluates the integral

$$\varphi_{12}(\tau) = \frac{1}{T} \int_{0}^{T} f_1(t)\ f_2(t + \tau)dt$$

The specific operations that are involved are:

(1) delay of $f_2(t)$ by a fixed time τ

(2) continuous formation of the product of $f_1(t)$ with the delayed signal $f_2(t + \tau)$

(3) integration of the continuous product over the finite time interval $0 < t < T$

(4) division by T to obtain the value of the correlation function $\varphi_{12}(\tau)$

Figure 17 illustrates in block-diagram form this computation sequence.

The type-four system forms the correlation function as

$$\varphi_{12}(\tau) = \frac{1}{N} \sum_{n=1}^{N} f_1(t_n)\ f_2(t_n + \tau)$$

This computation is based on discrete operations with the samples of the two time signals $f_1(t)$ and $f_2(t)$. The integration is replaced by a summation over a finite number of sample-pair products. Again, Fig. 17 illustrates the block diagram of this computation procedure.

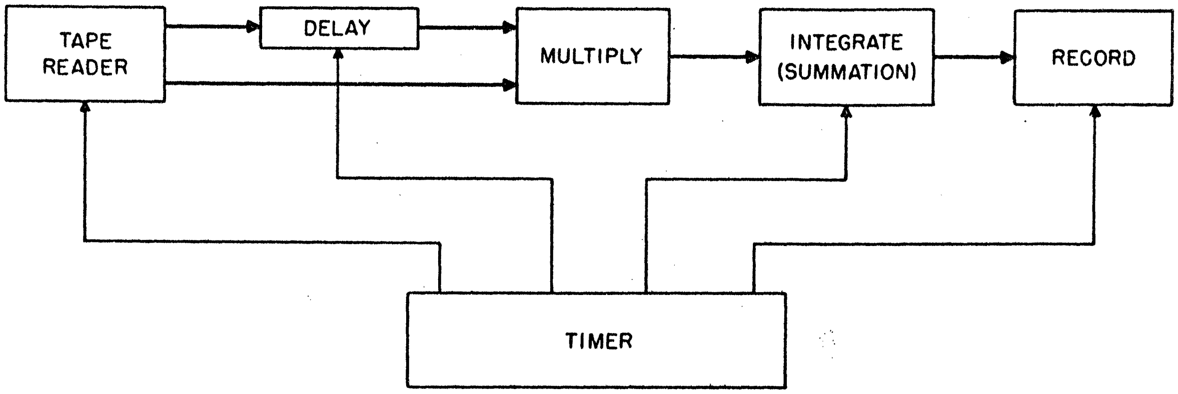


Fig. 17 Correlation Computation Block Diagram

1. Delay System

The first operation to be realized in either system is that of the delay of the input signal. If the data is assumed to be in analog form, then the delay might be accomplished by a tapped artificial delay line (1, 7); or by a set displacement between two reading heads on the data tape; or by a fixed displacement between the reading heads on a rotating magnetic drum.

The use of a delay line is restricted to the situation where the time delay between adjacent points is fixed. Since a maximum of 400 points per correlation is desired, the line would necessarily be very long and complex. The maximum delay of 0.4 sec would be extremely difficult to obtain. Corrections for phase shift and attenuation would have to be included for each tap. Thus, the delay line would be most impractical for the proposed system.

The use of two variably displaced reading heads on the data tape has been used (10) with moderate sucess. The problems encountered are those of change in tape dimensions with temperature, humidity, and tension and those of mechanical interference between reading and recording heads. This latter problem becomes especially trying if several read heads are used with a multiplexing scheme. The tape on such a delay device is operated in contact with the heads and at speeds from 60 to 150 ips--the faster speeds giving greater mechanical distances for identical time delays. The use of a tape delay of this design should be considered when thought is given to multiplexing.

The third type of delay unit is the magnetic drum. This type is currently in use in the present correlator and is thoroughly described in the literature (11, 12, 27). The advantages of this type of drum are excellent dimension or delay stability and a resolution of better than 3000:1 in the head spacing. The drum however is very susceptible to dust and dirt as the spacing between head and surface must be held to under 0.001". This spacing accuracy requirement makes the mechanical design very critical. More will be said of the drum in Chapter IV. The drum thus offers a very flexible medium for the time delay operation.

If a digital scheme is to be used, then the data can either be stored by the means outlined above and converted to digital form on use, or can first be stored in digital form after sampling and quantizing. Storage of data by digital means has been achieved in high-speed digital computers in a large variety of ways(61). The storage size required is equal to the product of the bit size of the numbers and the number of numbers to be simultaneously stored. To keep storage at a minimum, the number of stored data points can be limited to the number of computed points on the correlation curve. As was pointed out in sections C and D of Chapter II, in order to obtain a sufficient number of samples for the short runs, the time between samples should be equal to the time between computed correlation values. Thus, for a storage of this limited type, a mechanism is required either to shift the data from storage element to storage element or to sequentially shift the computation to the correct memory cells. The first mechanism is, in general, much easier to realize. For it a magnetic shift register (28, 29, 32) could be used. For a maximum number of correlation points of 400 and for a data bit size of 3 bits, as substantiated by tests outlined in Chapter II, Sec. E, a total of 1200 memory elements would be required to obtain the greatest delay. Variation in sampling rate is accompanied then by variation in the shifting rate of the register. With such a device outputs are available simultaneously for computation of all the correlation points. No additional heads must be added as in the case of the magnetic drum delay system. The resolution of this type delay system is equal to the total number of computed points--for our example, 400. To achieve a resolution equal to that of a typical magnetic drum system (about 3000:1), a magnetic shift register scheme would require 9000 elements. This large a memory may be too costly and unwieldly from a circuitry point of view.

A more practical system for obtaining high resolution would be to combine a drum delay system with a magnetic shift register system. The drum system would be used to obtain the fixed set of long delays required. Schemes could be devised to insure that the long delays were a fixed integer multiple of the shift time increment.

For applicable magnetic shift register units, the element cost, not including any supporting circuitry, is approximately $10 per bit. Thus, a 1200-cell memory element would have a base cost of approximately $12,000. This figure is considerably greater than the comparable magnetic drum delay system. If quantization to two bits is permitted, the cost would still be around the $8,000 figure.

Thus, it is apparent that the magnetic drum delay system offers the most versatile type of delay properties and is as a whole less expensive to build than a digital storage system of equivalent capability. It should be pointed out, however, that the design of the drum system becomes increasingly involved when the number of read heads is increased as would be required in multiplexing, whereas the multiplexing feature is inherent in the magnetic memory without the addition of any extra equipment insofar as the memory is concerned.

Other-type digital memory systems such as relays, ring counters, hard tube flip-flops, acoustics delay lines, electrostatic systems, ferroelectric cells and capacitor-diode types were investigated but were not considered for this problem because of cost, complexity, speed, and other factors.

2. Multipliers

The classification of multipliers logically breaks down into two parts: analog and digital. For the purposes of discussion, the analog multipliers have been further subdivided into four groups. The first group consists of the straight analog multiplying techniques, such as variable -μ tubes, crossed-field schemes, and FM modulation methods. The second group comprises those devices which use the quarter-square principle. Sampling types, pulse width-height schemes, time division, and time-share multipliers constitute the third group. Miscellaneous methods such as coincidence counting, log-antilog circuits, and electrodynamometers are included in the fourth group. Digital multipliers are then considered in the second part.

Before listing the detailed performance characteristics of the particular multipliers, a summary of the important considerations is given. Because the storage element which furnishes inputs to the multiplier has been assumed to have a bandwidth of approximately 5 kc for $\pm$ 1 db, the analog-type multiplier must have a bandwidth of twice this frequency, or 10 kc.

A second consideration which somewhat eases the requirements on the multiplier and leads to a more simple design is the fact that the input signal is usually passed through a high pass filter first to eliminate any very slowly changing bias levels which could cause serious error in the correlation determination. Thus, the associated amplifiers working with the basic multiplier circuitry need not be pure d-c amplifiers. This, in general, permits a wider bandwidth to be obtained in the amplifier design with a simpler construction. The inputs to the multiplier may be both positive and negative. Thus, four-quadrant operation of the multiplier is desired. One-quadrant operation could be forced if sufficient bias was added to each input signal and if the component of the output that is proportional to the two inputs was subtracted out. The accuracy that can be realized by such a procedure is less than that of a comparable four-quadrant device.

The general specification as to accuracy is set at ±1% or less. Susceptibility to drift, sensitivity to change in parts or environment, number of tubes or transistors required, approximate cost, and relative ease in multiplexing are also considered. It is pointed out that the multiplier is followed by an integrator. Hence, only the time average value of the multiplier output need be accurate. Thus, a multiplier output with considerable ripple would be tolerable if the ripple had zero average value.

General discussions of the many types of multipliers are found in the literature (33, 34, 35, 60). The results of a more detailed literature investigation are set forth in Table I which follows.

Multipliers utilizing the log-antilog principle (54, 55, 56, 57) are not considered suitable to the correlation application because they are restricted to one-quadrant operation and because an inherent accuracy of less than 5% is difficult to obtain. Electrodynamometer types (6) are subject to low frequency limitations and to an inherent low pass filter type integrating characteristic and, thus, are not usable for our application. Multiplication using the method of coincidences (33, 58) is limited primarily to one-quadrant operation. In order to obtain an accuracy of 1% or less with modulating frequencies of the order to 50 kc, at least one minute of record data must be processed (58). At these frequencies the

TABLE I

GROUP	TYPE	QUAD-RANTS	FREQUENCY RESPONSE	ACCURACY % FULL SCALE	DRIFT	SENSITIVITY TO PART REPLACEMENT	NUMBER OF TUBES	MULTIPLEXING PROBLEMS	REFERENCE NUMBERS
A-1	FM-AM DOUBLE MODULATION	4	15 KC - 15°	1%	DEPENDS ON DISCRIMINATOR PARAMETERS	DEPENDS ON DIODE BALANCE	24	10 TUBES FOR EACH ADDITIONAL MULTIPLICAND	36
	FM-AM DOUBLE MODULATION	4	10 KC - 7°	2%	MUST BE ADJUSTED EVERY 2 HOURS	DEPENDS ON DIODE BALANCE	9	3 TUBES AND 4 TRANSFORMERS FOR EACH ADDITIONAL MULTIPLICAND	37
	DOUBLE AMPLITUDE MODULATION	4	30 KC	0.5%	0.15% FULL SCALE	NO DETAILS	20	7 TUBES PER MULTIPLICAND	38
	CROSSED FIELDS - PHOTOCELL	4	5 KC	2%	0.1% FULL SCALE PER HOUR (PRIMARILY IN PHOTOCELL AND OPERATIONS AMPLIFIER)	CHANGES IN PHOTOCELL CHARACTERISTICS	10 PLUS A CATHODE RAY TUBE	NO GAIN	40
	ELECTRON BEAM TUBE	4	70 KC - -3DB	2%	——	——	——	REQUIRES A SPECIAL TUBE FOR EACH PRODUCT	41
A-2	SQUARE LAW VACUUM TUBE	4	10 KC	2%	SLIGHT	CRITICAL MATCH OF 6B8'S	4 - 6B8 2 OTHERS	NO GAIN	10, 11, 12
	SQUARE LAW VACUUM TUBE	4	15 CPS TO 800 KC ± 1 DB	2%	2.5% OVER 165 MINUTES	——	8 - 6AC7 4 - 6AS6	NO GAIN	5
	DIODE NETWORK	4	10 KC (DEPENDS ON BANDW'DH OF OPERAT'L AMPLIFIER)	1%	DUE TO DRIFT IN THE OPERATIONAL AMPLIFIER	DEPENDS ON DIODE CHARACTERISTICS	4 OPERATIONAL AMPLIFIERS 22 DIODES	NO GAIN	42
	BEAM DEFLECTION SQUARE LAW	4	LIMITED BY OPERATIONAL AMPLIFIERS	1%	DUE TO DRIFT IN THE OPERATIONAL AMPLIFIERS	DEPENDS ON TUBE CHARACTERISTICS	3 OPERATIONAL AMPLIFIERS 2 SQUARE-LAW TUBES	NO GAIN	44
	DIODE NETWORK PLUS OPERATIONAL AMPLIFIERS	4	100 CPS FOR 1° PHASE SHIFT	0.1%	VERY LOW	EASY TO CALIBRATE	3 OPERATIONAL AMPLIFIERS 2 SQUARE-LAW NETWORKS	NO GAIN	45
	PADDED THYRITE PLUS OPERATIONAL AMPLIFIERS	4	LIMITED BY OPERATIONAL AMPLIFIERS	2%	——	TEMPERATURE SENSITIVE	3 OPERATIONAL AMPLIFIERS 2 THYRITE NETWORKS	NO GAIN	43
A-3	TIME DIVISION	4	1 KC MAX. 20 KC REPETITION RATE	1%	——	SWITCH CHARACTERISTICS SENSITIVE TO TUBE CHANGES	3 OPERATIONAL AMPLIFIERS 6 TUBES	3 TUBES PER PRODUCT	46
	TIME DIVISION PLUS PRECISION ELECTRONIC SWITCH	4	2 CPS (DUE TO HEAVY FILT'R'G) 2 KC CARRIER	0.1%	DUE TO DRIFT IN THE OPERATIONAL AMPLIFIERS	STABILITY OF SWITCH AND ASSOCIATED RESISTORS	2 OPERATIONAL AMPLIFIERS PLUS SWITCHES	ABOUT 1/3 OF BASIC CIRCUITRY MUST BE REPEATED PER PRODUCT	48
	SWEEP COMPARISON PLUS TIME SHARE	1	1 KC	0.2%	——	CRITICAL IN COMPARATOR	15 TUBES	——	50
	SWEEP COMPARISON	1	200 CPS	1%	——	CRITICAL MATCHING OF DIODES IN COMPARATOR	15 TUBES	——	51
	DISPLACED TRIANGULAR WAVE	4	100 CPS (1 KC REPETITION RATE)	2%	——	TRANSISTOR SWITCHING CHARACTERISTICS SHOULD BE MATCHED	4 TRANSISTORS 4 DIODES PLUS WAVE GENERATOR	NO GAIN - LOW INPUT IMPEDANCE	52
	DISPLACED WAVE USING TRANSISTORS AND CORES	4	400 CPS TIMING WAVE	5%	DUE TO TEMPERATURE CHANGES	CORES AND TRANSISTORS MUST BE MATCHED	8 DIODES 4 TRANSISTORS 2 CORES	SOME SAVING CAN BE OBTAINED	53
	DISPLACED WAVE TIME DIVISION USING TRANSISTORS	2	1 KC REPETITION RATE	0.5%	DUE TO TEMPERATURE CHANGES OF TRANSISTORS AND POWER SUPPLIES	——	1 TUBE 2 TRANSISTORS 1 OPERATIONAL AMPLIFIER	VERY SIMPLE	49

bandwidth of the device would be of the order of 1 kc. Considering the amount of circuitry required, it is felt that this method does not hold much promise for our application.

In comparing the different analog multipliers to the specifications of our problem, it is seen that the multipliers of Group 1, although having wide bandwidth performance, are all more complex and, hence, more expensive to construct. The drift of the devices are very dependent on the tube characteristics. The scheme which shows the most promise is that described in (37) where it appears that accuracy and drift have been sacrificed at the expense of simplicity. Such schemes as (36, 37, 38) do show savings in complexity when multiplexed.

Multiplier schemes, Group 2, using the quarter-square principle can be built with sufficient bandwidth and accuracy and with sufficient simplicity to meet the specifications. However, this type is subject to both drift and variation due to changes in component characteristics. Multipliers of this group offer no advantages in simplicity when multiplexed. To perform ten multiplications, ten individual multipliers are necessary, requiring a total of ten times as many parts.

The multipliers listed in Group 3, those employing pulse width and amplitude modulation, show the most promise when multiplexing is considered. Accuracies in the order of 2% or less can be achieved with a relatively simple design. The performance of this type multiplier becomes dependent on component characteristics only when accuracy much greater than 1% is required. The output of the multiplier is in general a rectangle wave whose mean value is proportional to the product of the two input signals. Since the multiplier is to be followed by an integrator, the output wave need not be filtered first. The time division multipliers described in the literature (46, 48, 49) were followed by filters to reduce ripple, hence the reason for the low bandwidth specification as is shown in Table I. This type multiplier is widely used in commercial analog computers (60). Commercial designs are available which could be adapted for correlator use with a minimum amount of rebuilding.

Summarily, the time-division type analog multiplier holds the most promise for a multiplexed computation scheme. If no multiplexing is to be utilized, then the choice is between the quarter-square multiplier (12) and the application of a modified commercial time division multiplier.

The second part of this section concerns the use of digital schemes for performing the multiplication of the quantized data. The two general types of multiplication schemes are the direct multiplier and the stored multiplication table. In the first scheme, the multiplication is carried out in much the same fashion as one normally does it by hand except that the binary number system is used. The operations are simply those of alternately shifting and adding. The pair of operations must be repeated as many times as there are binary digits of the representative multiplier. It is this type of scheme (or some modification thereof) that is utilized in the general-purpose computers.

The size and complexity of the shift and add-type multiplier design depends on the bit size of the numbers, the speed at which the operations are to be performed, and the nature of the input and desired output information. It shall be assumed that for the correlator design the input information is in the form of 3-bit numbers temporarily stored in a core memory, and that the computation rate is to be 20,000 products per second as an upper limit. This requires that the entire multiplication operation takes place in a period of 50 μ sec minimum.

A survey of general-purpose machines now in operation (61) shows that this figure of 50 μ sec is reached by only the faster and more elaborate machines. It is noted also that these machine times are for multiplication of numbers usually in excess of 12 bits in length. As an example of a more recent development, a group at Lincoln Laboratory has built a transistor multiplier capable of handling 8-bit numbers. For their design about 660 transistors are required to obtain a 11 μ sec operating time. If the complexity of their circuit is reduced to around 400 transistors, the computation time lengthens to about 25 μ sec. By scaling down this type multiplier to handle only 3-bit numbers, the complexity could probably be reduced to about 200 transistors for a computing time of 20 to 30 μ sec. In scaling down a design only the number of flip-flops in the multiplier decreases. The circuits which control the sequence of operations performed during the multiplication are not altered as the bit size of the numbers is decreased.

The estimate given above is felt to be fairly representative of multipliers using flip-flops of either transistor or vacuum tube composition. It is possible to perform the operations with magnetic core elements serving as the flip-flops. Unless great engineering effort is expended toward an optimum digital multiplier for the 3-bit task, the choice between transistors, tubes, and cores appears to be on an even basis and, hence, could be motivated by other considerations in the system design.

Unless more elaborate and necessarily more complex techniques are used, the average multiplication time for the shift and add-type multiplier will lie in the range of 20-40 μ sec. Thus, if a multiplexing scheme is to be used, then it becomes obvious that the digital multiplier cannot be time shared between channels but that a separate multiplier must be provided for each channel.

If the multiplier was fast enough to be time shared, circuitry would have to be added both to switch sequentially the inputs and outputs of the multiplier and to provide a means of restoring the data information back in the core memory after it had been read out as an input. The circuits needed to restore this information, although reasonably simple, require upwards of 12 μ sec to complete the operation. Thus, the modest read out and restore times also tend to limit the extent to which time-sharing can be used. It is hardly conceivable that with current methods one could use a single multiplier to perform more than three or four multiplications during the 50 μ sec minimum interval.

The second type of multiplier consists of a programmed multiplication table in matrix form. Description of matrices used for multiplying and switching applications are discussed briefly in (32). Briefly the matrix consists of a number of input lines equal to twice the total number of bits in both input numbers and a number of output lines equal to the total number of different products. A decoder is required to convert the output line information back into binary form for transfer to the accumulators. The matrix multiplier is the most rapid type of digital multiplier. Multiplication time can be below 1 μ sec with little difficulty.

In order to get an idea of how complex the matrix multiplier is, designs for both 2- and 3-bit multipliers have been worked out. The matrix and decoder for the 2-bit multiplier are shown in Fig. 18. For a complete matrix multiplier, the maximum number of elements necessary are given in Table II. The number of diodes shown in the table could possibly be reduced by improved design to a slightly smaller figure. However, it is felt that the figures given are fairly representative of actual design.

Table II

No. of Bits	Diodes			Flip-Flops	No. of Lines to Switch
	in Matrix	Decoder	Total		
1	2	0	2	3	3
2	42	9	51	8	8
3	337	57	394	12	12
4	2032	269	2301	16	16

The matrix multiplier has the definite advantage of not requiring very much in the way of control circuitry. Since multiplication speeds are so fast, such a multiplier could easily be time shared. The number of lines to be switched per product to perform this time sharing is shown in the last column in Table II. If, for the moment, the switching problem is neglected, it can be seen that the cost of a matrix multiplier for very small numbers is much less than that of a comparable shift and add type multiplier.

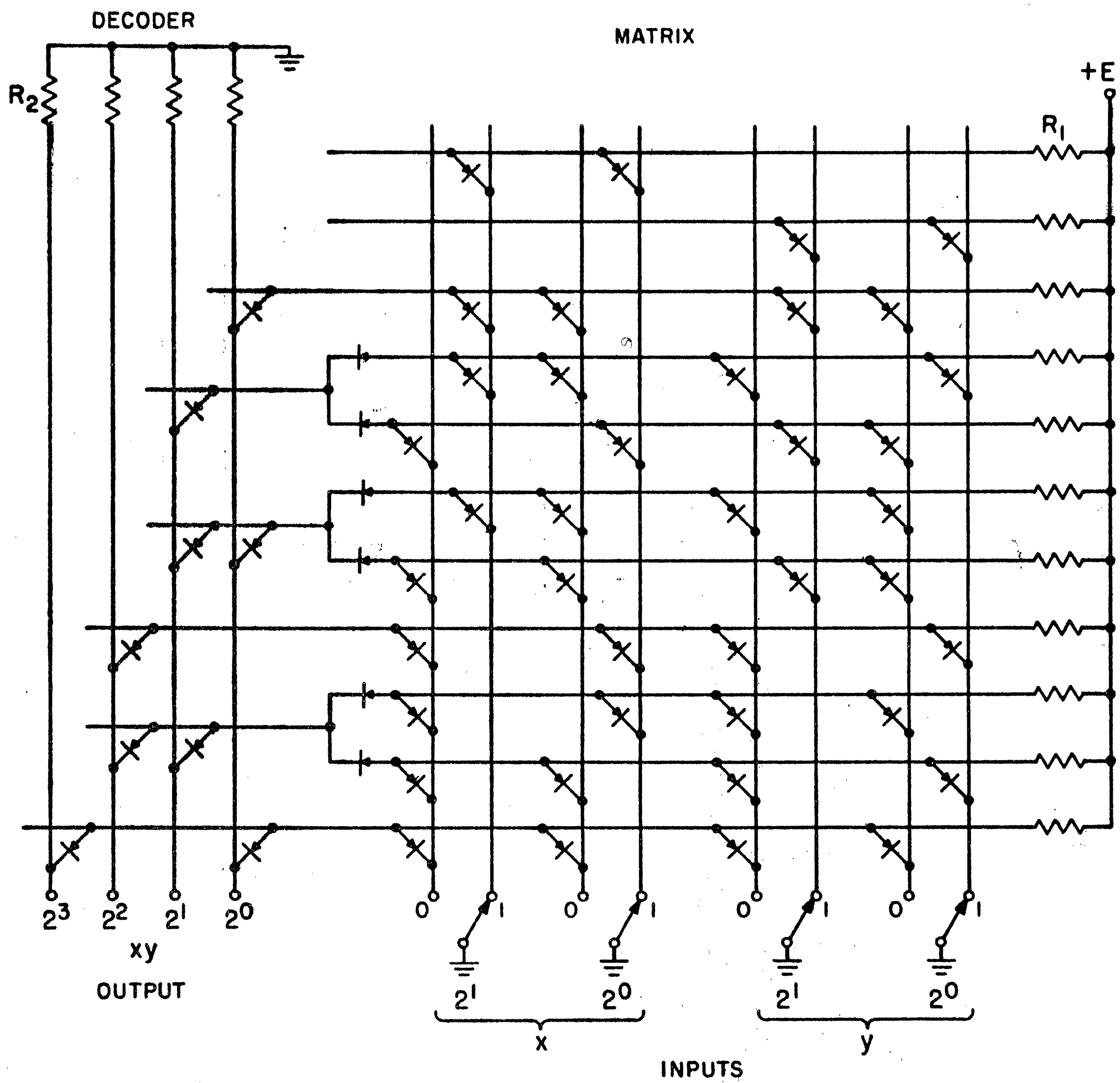


Fig. 18 Two Bit Multiplier

From the previous discussion, it can be concluded that the 2-bit matrix multiplier will, in general, be less expensive and faster than the shift and add type multiplier. In the 3-bit case, the diode matrix type appears to be somewhat simpler to construct and possibly less expensive than the shift and add type. For number sizes greater than 3 bits, the matrix and decoder become extremely complex and costly, and thus impractical. From the multiplexing standpoint, it is felt that for both the 2-bit case and the 3-bit case it would be simpler to build multiple matrices and decoders than to devise a time-sharing system. As mentioned before, if a time-sharing scheme is used in conjunction with a magnetic shift register storage, then circuitry must be provided to write the information back into storage after it has been read out, since read-out at any time other than during the shift pulse destroys the information in the core.

For interest a hybrid type of matrix multiplier is shown in Fig. 19. The multiplier accepts as its inputs the data in digital form from the shift register storage elements. The resistors on the right side are so chosen that the product lines carry a voltage proportional to the product when the line is not grounded. Thus, the voltage across the resistor R_3 will have a magnitude proportional to the product of the digital inputs. With a simple switching circuit, the output voltage can be shaped into a rectangular pulse of constant width. Hence, the area of the output pulse (its average value) would constitute the desired product in analog form. This scheme has the advantages that it is somewhat simpler than the pure matrix type and that the output is directly in analog form. For the 2-bit case, the design is straightforward; whereas for the 3-bit case, the design is somewhat more complicated by the larger number of diodes needed. The finite values of the forward, and especially the back, resistance of the diodes must be taken into account.

3. Integrators and Accumulators

If an analog system is being used, then the operation of continuous integration with respect to time of the correlation product is to be carried out. This operation is most easily realized by the widely used feedback integrator. Some of the existing correlators using this scheme are (4, 8, 10, 11, 12). In present-day analog computers all time integrators are of this form (45, 60). As a result, their design has been developed to a fine degree. Various schemes have been utilized to reduce drift and to extend the usable bandwidth. Two

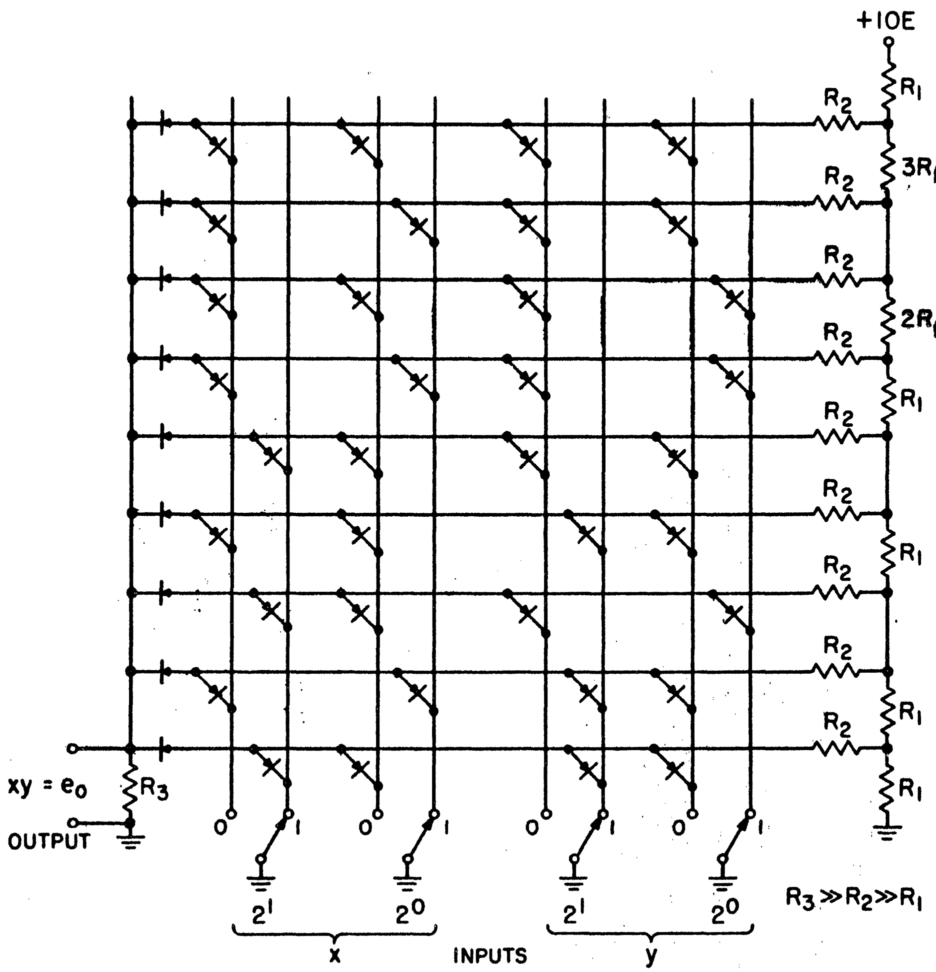


Fig. 19 Digital-Analog Multiplier

particular discussions of these schemes are contained in (64, 65). Drift can be kept below 100 mv per hour and the open loop gain crossover frequency extended to more than 10 kc.

It is obvious that if multiplexing schemes are to be utilized, one integrator or accumulator must be available for each simultaneously computed point. There is nothing to be gained in design simplicity by multiplexing. Five points simply require five separate integrator units.

Low-pass filters are not considered practical as averaging or integrating devices because of the extreme variability of data frequencies and length. Filters become practical only when the integrating time is reasonably small, say less than several seconds.

If a digital multiplier is used whose output is a binary number, then an accumulator or summing register is required to perform the integration (summation). Such devices have been constructed in a variety of forms and are the basic computing blocks in the large general-purpose machines. The accumulator circuitry involves both the simple logic elements and the necessary control elements. These elements can be built using transistors, tubes, and/or magnetic cores.

The important design factors for the accumulator are the total register capacity and the operating speed. The operating speed of 20,000 operations per second imposes no serious limitations in the design. The register capacity is a function of both the bit size of the product and the total number of samples to be summed. For the purposes of estimation, the total number of samples is assumed not to exceed 50,000.

Two possible types of accumulators are envisioned. The first is a simple adder into which all numbers are positive. The second type is a reversible binary adder where the input numbers may be either positive or negative. The second type then may either count up or down, depending on the sign of the product input. In general, the maximum count for the reversible counter will be less than that for the simple adder. Table III relates bit size, sample size and register length for each of the two types of accumulator. The assumptions made in the preparation of this table are that the amplitude of the data is Gaussian distributed and that the quantization range is extended over five times the rms value of the data. For a Gaussian distribution 98.7% of the points will fall within this range.

Thus, from the standpoint of register size there seems to be little difference between the two types--a difference of approximately 3 bits in length. As far as accumulator design

Table III

			Required Register Capacity (Bits)	
Bit Size	Samples	Mean	Simple Adder	Reversible Binary
2	5,000	zero	14	11
		+ 25%	15	12
2	50,000	zero	18	15
		+ 25%	18	15
3	5,000	zero	17	14
		+ 25%	17	15
3	50,000	zero	20	17
		+ 25%	20	18

goes, this difference is insignificant. Likewise, changing the number size from 2 bits to 3 bits raises the accumulator size by 3 bits in each case.

The design of the accumulator is somewhat simplified by the fact that the input number size is limited to either a 4- or 6-bit number corresponding to quantization to 2 or 3 bits. This means the accumulator action need be only 4 or 6 bits long with the remaining summing action performed by a simple binary counter counting the carries from the short accumulator. Such a counter need not be fast acting since carries can occur as fast as 20,000 times per second as an absolute maximum. The counter requirements become much less severe as one proceeds down the counter.

This scheme is most simply accomplished if all input numbers are positive. If some numbers are negative, then the design of both the counter and accumulator necessarily become more complex. For all input numbers positive, the length of the accumulator can be changed to accommodate a greater number of samples simply by adding more elements to the counter chain. The only control circuitry required is a reset at the end of the add cycle.

To obtain some idea of the complexity of an accumulator, reference (66) was used as a basis for estimation. Per bit or order of an accumulator, the logic elements required are 2 flip-flops, 3 AND circuits, 2 OR circuits, and a delay element. Such logic elements if realized by either vacuum tubes or transistors would comprise about 8 tube sections and 4 diodes. The counter is nothing more than a chain of flip-flops in cascade.

It is reasonable to conclude that the cost of a digital accumulator would be perhaps several times that of an analog integrator having comparable performance. The digital accumulator has the advantages that it is free of such ailments as drift; that it is in general a more reliable element whose output characteristics are practically independent of its internal element characteristics; and that the output being in digital form is directly available for recording on a punched tape.

4. Division

In computing the correlation coefficients, the integral of the product must be divided by the length of the integration time in the continuous case, or by the number of samples in the discrete or digital case. Since primarily the relative shape of correlation function is of interest, it is sufficient to obtain the correlation within a constant of proportionality. Thus, in the analog case only the length of integration time must be held fixed for all calculated correlation points. This can be accomplished by incorporating special signals on the data tapes signifying the start and completion of computation.

For the digital system a counter must be incorporated to stop the computation after a fixed number of samples have been operated upon. A 16-bit counter would enable a count up to 65,535. By constructing an AND circuit from the different parts of the counter, the computer could be set to stop on any number of samples up to this number.

5. Output Equipment

For many purposes, the desired form of the output information is a graphical plot capable of being reproduced. As was pointed out in Chapter II, it is also extremely advantageous in many cases to have the output information recorded in such a form that further operations on the computed results such as spectra computations can be accomplished with a minimum of expense and effort. Since in all likelihood these subsequent computations would be carried out on a general-purpose digital machine, the recording medium should be one that the particular computer can accept as an input. For purposes of illustration, the following discussion concerns the input requirements for general-purpose digital computers, such as the IBM 704. For correlators connected directly to special-purpose computers, many of these data handling problems would not be present.

Figures 20 and 21 show the block diagrams of the output equipment for correlators of both the digital and analog varieties. The two systems are almost identical except for the location and type of the converter and the construction of the switch and timer control. The construction of the swtich and storage element and the converter depend upon the operating characteristics of the graphical recorder and the punch.

Figure 20 assumes that the punch is operated from computed data already in digital form--resulting from a digital computation. If, however, the correlation is accomplished by an analog correlator as in Fig. 21, then an analog-to-digital converter must be provided to code the data into an acceptable form for punching. The subject of analog-to-digital conversion is discussed at length in (32). The requirements on the converter are very broad. First, it must convert at a rate that exceeds the speed of the tape punch in accounting for the multiple-character representation of the data numbers. For a four-character representation, conversion must be at a rate exceeding two per second. Since conversions at rates up to 50 kc are rather simply achievable, conversion rate is not a

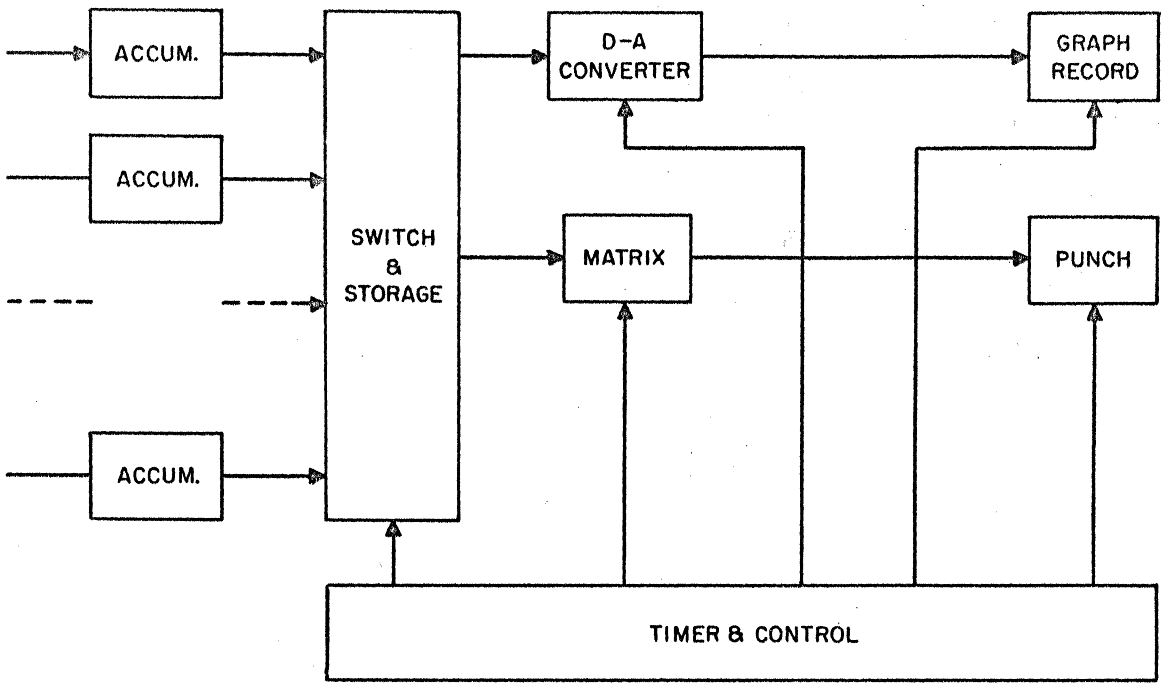


Fig. 20 Output Equipment for Digital Computation

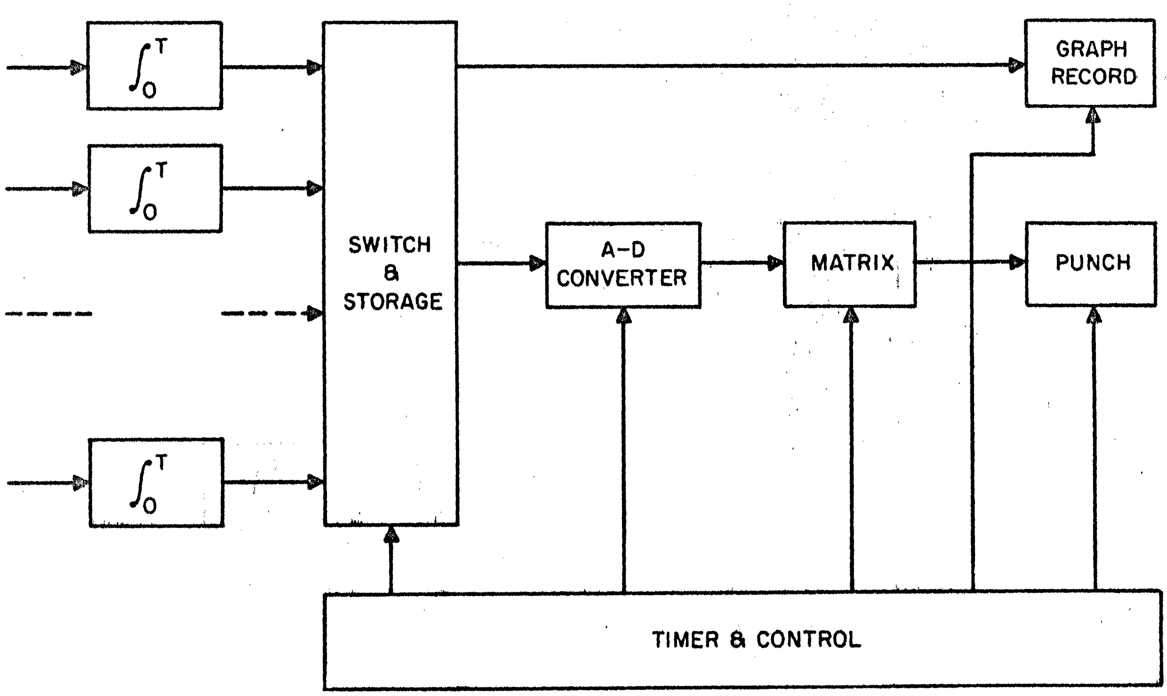


Fig. 21 Output Equipment for Analog Computation

problem. The other requirement on the conversion device is that it be able to convert the analog information to at most a 10-bit binary quantity, or one part in 1024. As far as the eye is concerned, a plotting accuracy of about 0.5% should be sufficient, which corresponds to an 8-bit representation.

In choosing an analog-to-digital converter, it is possible to use to advantage the fact that the voltage being quantized appears as the output voltage of the integrator. A possible scheme is shown in Fig. 22. At the end of the correlation computation, the voltage at the output of the integrator is proportional to the correlation coefficient. To quantize this voltage,

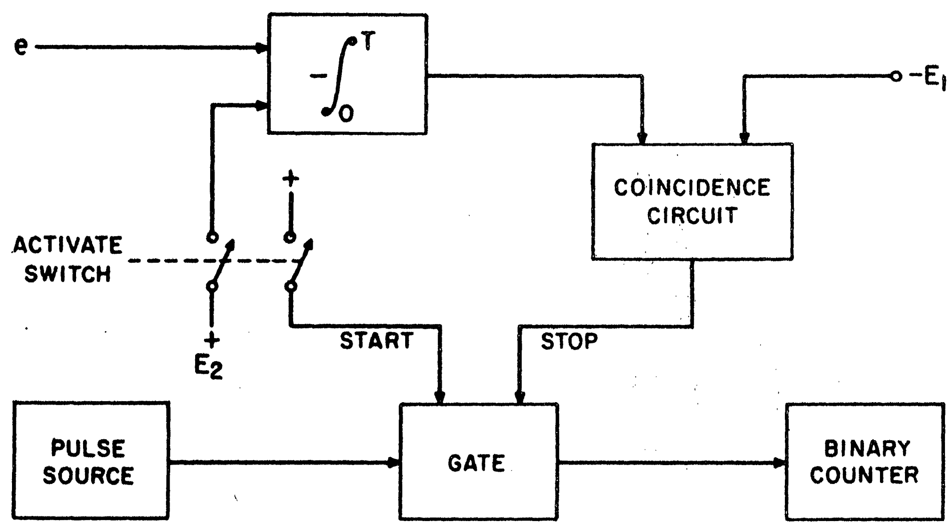


Fig. 22 Analog-Digital Converter

a fixed voltage E_2 is applied to the input to the integrator at the same instant a gate is opened to permit pulses from the pulse source to enter the binary counter. The fixed input voltage will be integrated by the integrator, and the result subtracted from the output of the integrator until the net output voltage of the integrator is equal to the fixed negative value $-E_1$. At this time the coincidence circuit will close the gate, thus stopping the pulse train. The count in the binary counter is then the desired digital representation of the integrator output. The integrator is then reset to have zero output before the next computation is to proceed by using the integrator as the high-quality linear sweep generator, both the sweep generator and sampler are eliminated from conventional converter circuitry. The converter would then require only a pulse source, coincidence circuit, gate, and a binary counter of 10-bit length.

The second type of data display often required is a reproducible graphical plot of the computed correlation points. If an analog system is used, the recorder can be driven directly from the output of the integrators. If a digital computing scheme is used, then a digital-to-analog converter must be added to drive the recorder. This converter need only convert the last 8 bits of the accumulator count to an analog voltage because it is difficult to tell finer differences by eye, and also because recorders are seldom more accurate than $\pm 1\%$. Converters or decoders using multiple sources and weighted resistors are discussed at length in (32). Again the requirement on the operating speed of the converter is very loose; the conversion rate must be greater than two per second.

If a recorder is to be used, the following characteristics are important. For ease of readability the chart scale should be linear and rectangular. The paper should be transparent to make for easy reproduction. An ink line on the chart and a zero center pen motion are desirable. If a single correlation point is computed for each pass of the data, a constant speed drive on the recorder chart appears to be the most simple solution.

If the system is multiplexed, the recording problem becomes much more complicated. It is then necessary to record not one but several points successively for each pass of the data tape. For the purpose of discussion, assume that there are five simultaneous analog correlations being obtained. To minimize the complexity of the switching and storage unit as shown in Figs. 20 and 21, the accumulator or integrator values should be recorded as quickly as possible. The most reasonable scheme appears to be one employing a stepping or ratchet drive on the chart or strip recorder. Since the correlation is computed at discrete, evenly spaced values of time shift, there would be no sacrifice in versatility by using the stepping drive. In fact, the stepping drive should be simpler and cheaper to construct than the stable constant speed drive contained in many strip recorders.

The advantages of a stepping chart drive are positive spacing between recorded points regardless of variations in pulse group spacing and simplicity of drive. Problems that are introduced by this scheme are those of stylus drag errors due to a static chart during recording and false indications given by stylus overshoot. The first problem becomes negligible if the galvanometer torque constant is sufficiently great. The overshoot problem can be eliminated simply by introducing enough series resistances to make dynamics critically damped.

A possible sequence of operations using a stepping chart recorder is:

(1) record output of Integrator No. 1
(2) return stylus to zero position
(3) advance chart one step
(4) repeat sequences 1, 2, 3 for integrators 2 through 5

By returning the stylus to the zero position after each recorded point, a constant check on drift in the recorder drive amplifier is made. Use of a high-performance galvanometer can result in an operating time of approximately 0.5 seconds or less for each recorded point. Thus, a group of five points could be recorded in under three seconds. Keeping this total recording time to such a low figure makes it possible to eliminate any temporary storage of the computed data.

The remaining problem in the output equipment is that of performing the necessary switching functions for the plotting and punching of the output data. The switching required for the digital case, as shown in Fig. 20, is by far the more complex than for the analog case of Fig. 21. The switching schemes available for the digital case are the matrix plus gate tubes, beam switching tubes, and the mechanical commutating switch. For five accumulators of 10-bit capacity each, a total of 50 lines must be gated at the proper times to the correct locations. If a matrix and gate tubes are used, upwards of 20 tubes and 100 diodes would be required. Ten beam switching tubes could do the same task. A mechanical commutating switch is feasible because the switching rates are rather low--about twice per second. If a stepping drive is used on the strip recorder, a commutating switch could possibly be driven from the same drive. For the analog case a commutating switch seems to be the most practical switching scheme.

CHAPTER IV

RECOMMENDATIONS FOR A CORRELATOR DESIGN

A. GENERAL RESULTS

If the data being correlated have frequency components that extend over a very broad range, much time can be saved if two modified correlations are taken instead of one single correlation with a sufficiently large number of points to resolve the highest and lowest frequencies. The first correlation is obtained on the data after it has been passed through a low-pass filter; the second correlation is obtained on the data after it has been passed through a high-pass filter which is the complement of the low-pass filter. (See Appendix.) The data after being passed through the low-pass filter can be re-recorded with an additional speed up in order to utilize more effectively the tape and computer capabilities. The complete correlation function of the original data is then found by simply adding together the two correlations on the modified data.

It is well to mention again that the information contained in the correlation function is only a small part of the total information carried in the stochastic signals being investigated. Actually, correlations of all higher orders are required to completely characterize statistically the stochastic signal. No results more striking than the results of the one- and two-bit correlations are needed to help emphasize this point. Thus, a very precisely determined correlation function may reveal little more information than one obtained by less sophisticated means. The effective and efficient use of the correlator therefore resides in the intelligence and finesse of the investigator.

The most encouraging results of quantization studies prompted a detailed investigation into the design and construction of digital correlators that would meet the requirements of the particular correlation problem. The conclusion arrived at is that the digital correlator offers a practical solution to the problem when it is desired to compute several points simultaneously. The two-bit correlator compares in cost and is more reliable than its equivalent analog machine. For the single point correlation the analog machine appears to be the best answer. The actual construction of a digital correlator would require a rather large amount of engineering time if a reliable and efficient design is to be used. The hardware cost and complexity could be reduced somewhat from the "brute force" design through the ingenuity and application of the broad experience of a digital systems designer. The reliability of a digital correlator would be superior to that of its analog counterpart at least as far as the digital elements are concerned. This is based on the experience of the operation of large general-purpose machines, such as Whirlwind, where reliability is extremely high. The advantages, disadvantages, and problems associated with a digital design are brought out in a subsequent section in this chapter where a possible digital design is considered in some detail. In both the analog and digital designs, the construction of a digital output mechanism is considered. This output would be used to feed a general-purpose digital computer for performing the spectral computation.

B. AN ANALOG CORRELATOR

As was pointed out in Chapter III, a correlator of the analog type represents the most efficient type correlator from the standpoint of processing data obtained from a bandwidth limited source. The discussion in this section therefore is restricted to the description of a possible analog correlator capable of computing several points simultaneously.

As a proposed design the use of a continuous data tape loop and a multiplexing scheme are suggested. The block diagram of such a system is shown in Fig. 23. The operation and design considerations of this system are now considered in detail.

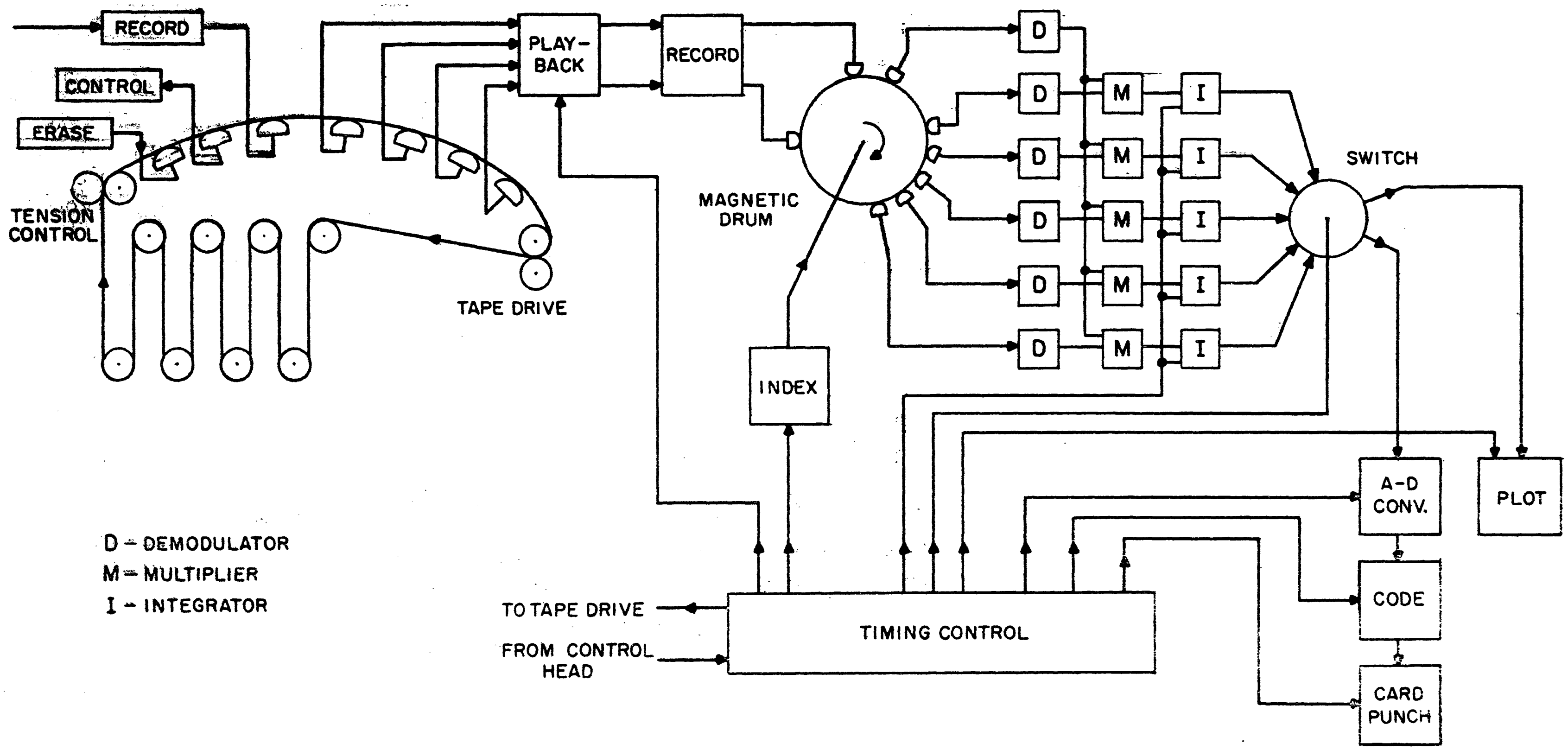


Fig. 23 Analog Correlator

A oontinuous tape loop is used both as a mechanism for storing the input data and for providing for relatively long time shifts in the correlation. The tape loop speed is chosen on the basis of input data rate, of length of tape loop, and of the effect on mechanical dimensions. High tape rates give the possibility of high computation rates. If the usable bandwidth of the tape record playback system is utilized, then the length of tape and hence mechanical dimensions of the tape loop device will be fixed.

The limiting factor in determining tape loop speed then resides not in the tape loop design but in the performance of the associated equipment in the correlation computer. For the analog system these factors are the magnetic drum delay bandwidth, multiplier bandwidth, and the required plotting and resetting time interval of the correlator. For a magnetic drum of the type used in (12), the upper limit is about 50 kc which is adequate for an FM system running at 30 ips but not sufficient for an FM system operating with a 60-ips tape speed. Quarter-square multipliers can be built with bandwidths in the tens to hundreds kc bandwidth. Time-division multipliers are rather difficult to construct to handle frequencies above 10 kc. For the purpose of discussion, it shall be assumed that the total time required for plotting the data and resetting the computing elements will not exceed three seconds. Thus, for a 30-ips tape speed, 90 inches of tape length are required between the end and beginning of the data on the loop; for a 60-ips tape speed, 180 inches, or 15 feet, of tape is required.

Before making a decision as to the tape speed for the loop, a calculation of data tape length should be made. If the typical "long" and "short" runs are used, then one finds tape lengths (for data only) of 600 inches and 3 inches, respectively. The long run has a ratio of data length to maximum time shift of 1000:1--a figure considered excessive for a single run. If this ratio is limited to 200:1, then a tape length of 120 inches is required which is a quite feasible figure. If extremely long records are to be correlated, then it is suggested that the record be broken down into shorter sections which a tape loop could handle. This point will be returned to shortly. Thus, it is noted that for tape speeds of 30 ips or 60 ips, as much time is required to plot computed results and reset the system as is required to perform the computation. This suggests that a tape speed of 30 ips and a tape loop length of 120 inches be chosen; the computation can be accomplished as the data tape completes one circuit and the readout of computed information performed during a second pass of the data, thus conserving tape length.

The control for the "compute" and "readout" cycles could be accomplished from a recorded signal on a third channel of the tape loop. A tape loop of this type is shown in Fig. 23. If much longer record lengths are to be processed, a scheme such as is shown in Fig. 24 could be used. Methods of this type are utilized in the tape storage elements of several large general-purpose digital computers. Tape transports with continuous tape loops are now commercially available.

The use of a tape loop gives several major advantages: no rewind time is required; the tape drive design need not be compromised for fast start-stop operation; and by adding several read heads, the tape loop may be used to obtain delays longer than those obtainable from the magnetic drum. The tape drive consists simply of a powered capstan and a tension control. It can be designed for single-speed operation or for two-speed operation if the loop is to be used to gain an additional time scale speed up. Two-speed operation permits much more versatility in the recording and playback speed ups. For example, if the basic tape transport can operate at 0.3 ips, 3 ips, and 30 ips and the tape loop can operate at 10 ips and 30 ips, then data speed-ups of 1:3, 1:1, 10:3, 10:1, 100:3, and 100:1 are possible. The tape used for the tape loop should be chosen to have good dimensional stability in the presence of variations in tension, temperature, and humidity.

The next element to be considered is the magnetic drum storage. The drum used in (12) was built after a very satisfactory design by Goff (11). If multiplexing is to be employed, the mechanical design of the record and playback head carrier becomes rather complex and involved. If it is assumed that five simultaneous delays of different values are required, one has a choice of several different schemes of head spacing as represented by the time shifts shown in Fig. 25. From the standpoint of mechanical interference, the first scheme is the easier one to realize because only one carrier need be used. From the standpoint of the plotting and read-out problem, the second scheme is by far the simpler one. To perform the plot of data computed as in the first scheme would require either the chart to exhibit large excursions very rapidly in both the forward and reverse directions, or the use of several recorders--one for each set of delays. Such a scheme thus appears impractical.

A mechanical design to implement the second method requires a separate head carrier for each separate delay desired. Figure 26 shows one crude design that would have the flexibility required. Rotation of the control shaft θ_2 adjusts the initial stagger spacing of the heads; rotation of shaft θ_1 advances all heads together. Mechanically, the problem is difficult because the spacing between the heads and the drum must be kept below 0.001 inches and above the contact point. Such spacing will be difficult to achieve for the several head carriers moving simultaneously. The key to the design of a satisfactory analog correlator rests in the solution of this mechanical problem.

The application of the multiplexing scheme requires that each computation channel have its own playback amplifier and demodulator, multiplier, and integrator. The demodulator would be of a standard design to recover the signal from the recorded frequency modulated signal. For a multiplexed system a multiplier utilizing the time division principle is preferred because of the possible saving of components through

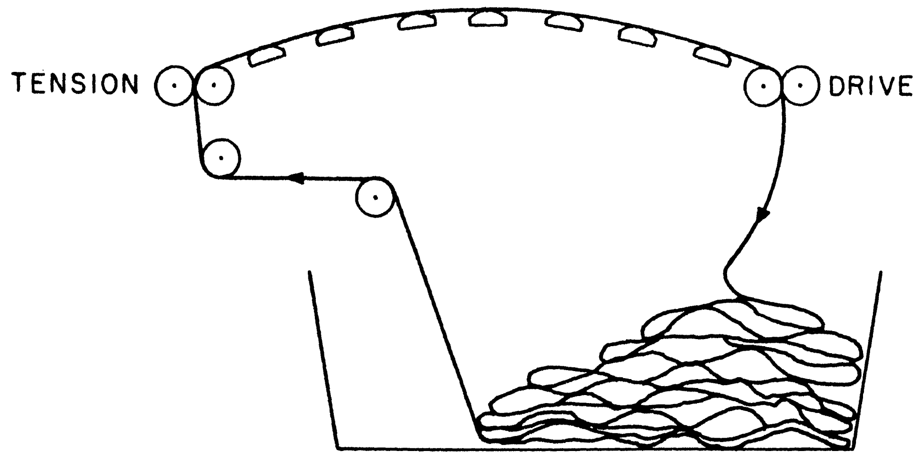


Fig. 24 Tape Loop for Long Tapes

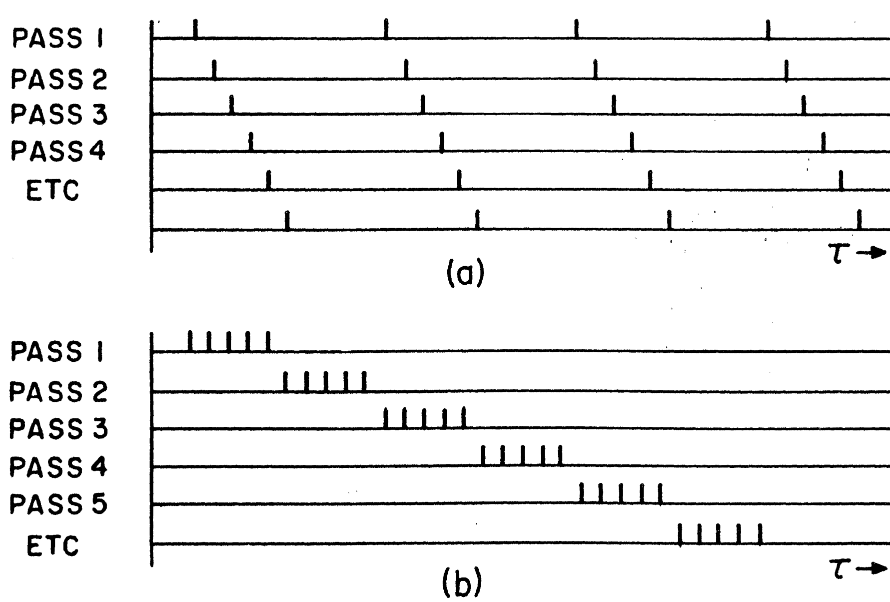


Fig. 25 Data Spacing Schemes

multiplexing and because of its relative insensitiveness to component characteristics. The integrator would be a simple feedback type integrator as previously described.

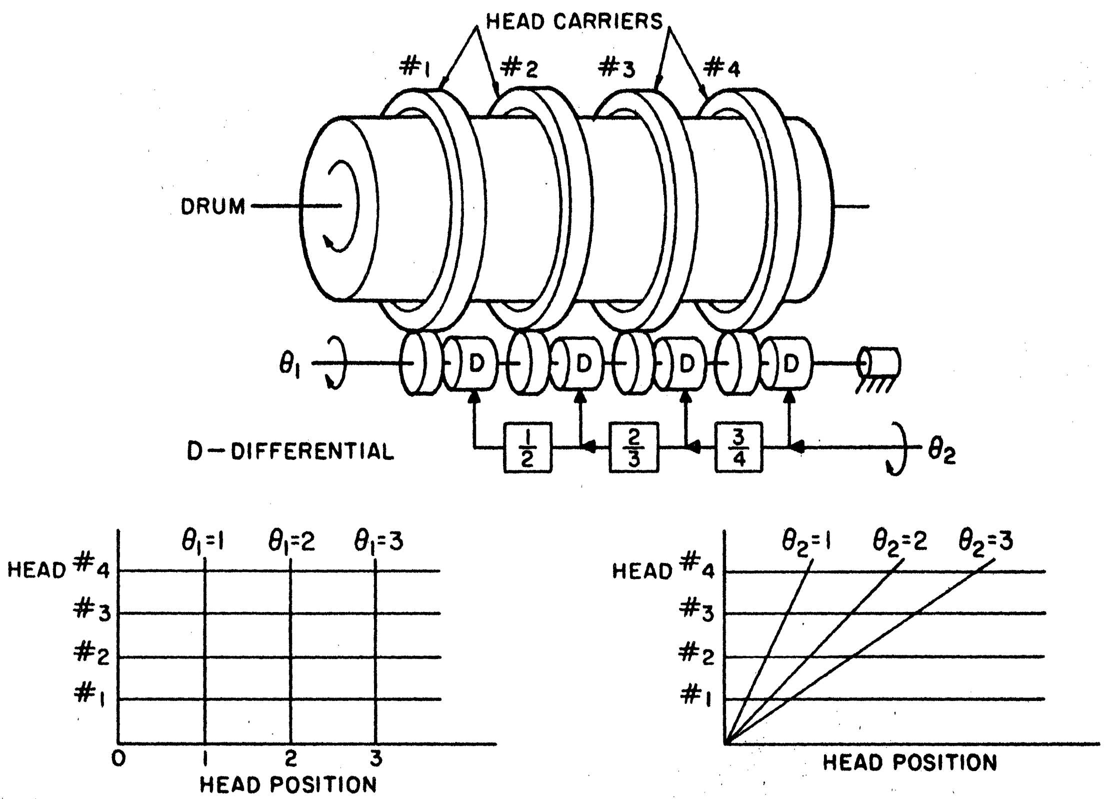


Fig. 26 Drum Head Control

A problem that arises with the application of multiplexing is that of calibration and drift in components. With the single-point correlator the exact calibration factor need not be known as long as it remains constant over the entire run. In the multiplexed system, each channel has its own demodulator and multiplier; and extreme care must be taken to ensure that each channel has identical operating characteristics. The procedure of calibration will then be much more critical and, unless automatic balancing schemes are used, will require correspondingly longer setup time. This fact is a disadvantage for the analog system.

C. AN ANALOG-DIGITAL CORRELATOR

The hybrid design proposed to overcome the difficulties mentioned previously is shown in Fig. 27. The input and output sections of the correlator are identical to those of the analog

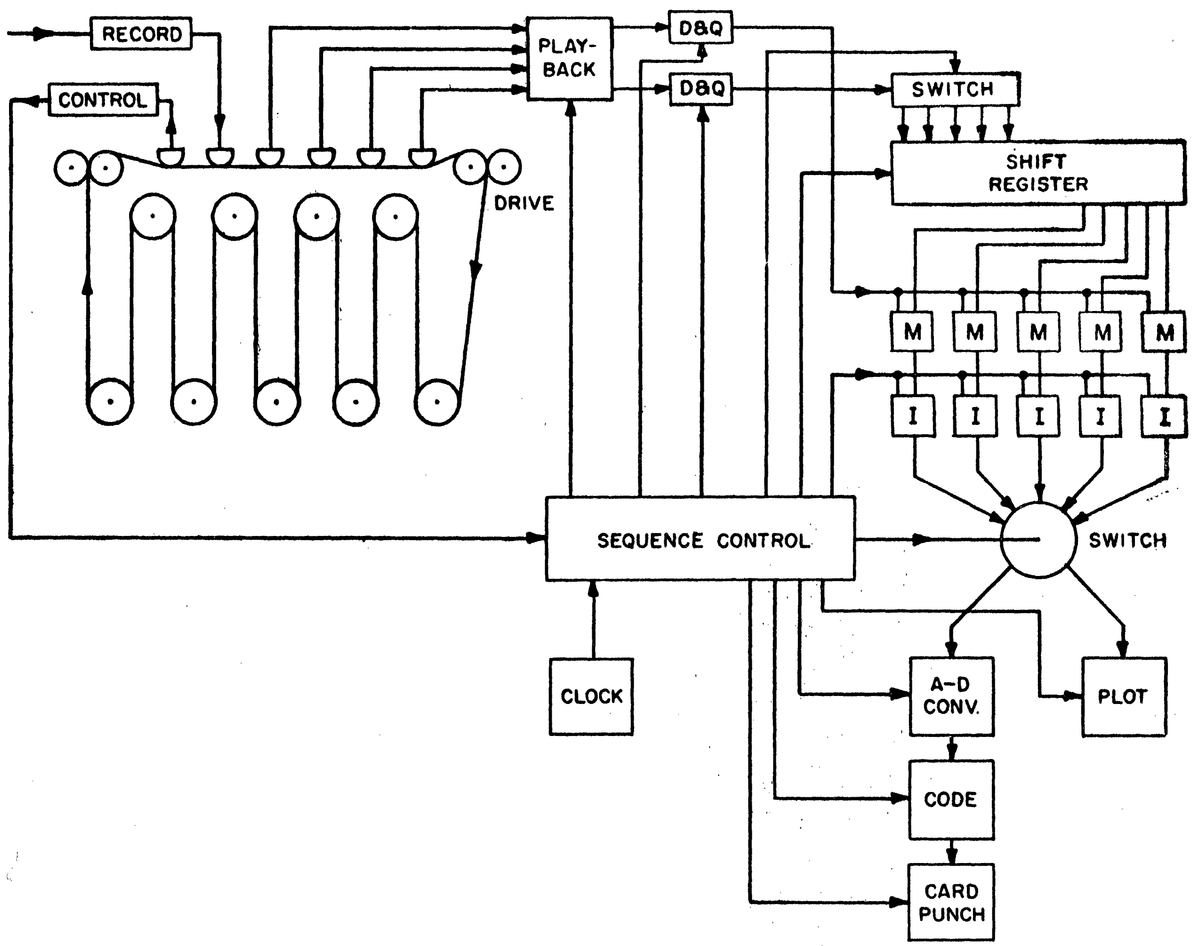


Fig. 27 Digital-Analog Correlator

design. The tape loop is used both as a continuous storage for the data and as the coarse time delay element. The reading or playback heads will number up to four or six, depending on the total number of points desired in the correlation computation. During operation only two of the playback heads are used at any one time. The signals from these two heads are demodulated and then quantized to 2- or 3-bit numbers.

The delayed quantized signal is then fed to the shift register through a switch. Rather than switch the five outputs of the shift register after each data pass, the single input line to the shift register is switched to obtain the proper set of delays. Because the shift register is a rather expensive item, it is most desirable to limit the length to a reasonable value and operate it in conjunction with a tape loop rather than to build the line long enough to accommodate the maximum time shift in the correlation. As a result the line length was chosen to be 100 units long. Thus, with the five adjacent output lines, a total of twenty possible input lines are available from the switch.

Because of the even function property of the autocorrelation function, only one signal needs to be delayed. For crosscorrelation, the negative values of time shift can be had with one register by simply interchanging the two input signals. The total number of elements in the shift register is equal to the bit size of the input data multiplied by the register length. For 2- and 3-bit numbers, the number of elements is 200 and 300, respectively. The cost of such a line is about the same as that of the basic magnetic drum; but the shift register has <u>no</u> moving parts, and hence, tends to be a much more reliable device.

Although the shift-register-tape-loop combination does not have the versatility of available time shift values as does the magnetic drum system, considerable increase in the range of time shift values can be obtained if the read heads on the tape loop are spaced nonuniformly, as shown in Fig. 28. Thus, with six playback heads on the loop and a little ingenuity in manipulation, a correlation containing better than 2200 points is obtainable.

For the 2- and 3-bit situations, a multiplier of the matrix type is chosen from the standpoint of economy and simplicity of construction. The correlator scheme shown in Fig. 27 utilizes the hybrid matrix multiplier as described previously (Fig. 19) because the resultant analog output is more easily integrated. Accumulators tend to be more costly than the simple feedback integrators. The remaining part of the system is identical to the analog system, and attention is therefore directed to the discussion in the pertinent sections of Chapter III for details of component operation. Had an accumulator and straight matrix multiplier been used, the output equipment would be arranged as in Fig. 20. It must be pointed out that the hybrid multiplier, electronic integrator system has a calibration problem, since all channels must have identical gain and dynamic characteristics. The problem here is of much lower magnitude than in the all-analog correlator. The all-digital system is subject to drift only as it affects the demodulator and quantizer. The digital system in general will tend to be a more reliable system with fewer maintenance and calibration problems than the analog systems. The digital system has no moving parts other than the tape loop, commutating switches, and recorder. The operation of the digital system is much less dependent on tube characteristics than the analog system.

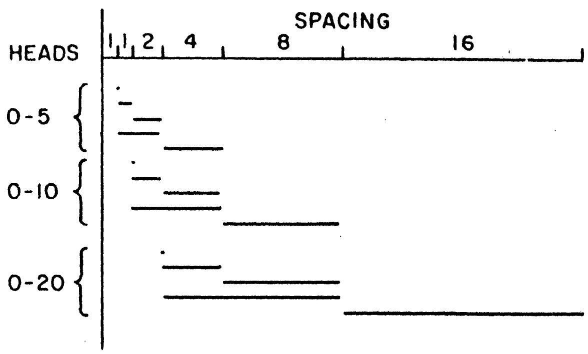


Fig. 28 Head Stagger Scheme

D. COMPUTATION OF SPECTRA

As has been suggested elsewhere in this report, it is felt that spectrum analysis should be used to support the correlation function interpretation. In its study of many physical processes, the Servomechanisms Laboratory has had the occasion to develop a series of Fourier transform programs required to compute the spectrum from the correlation function. The programs were written to operate on the Whirlwind I computer. Spectral studies require a Fourier transform computer program of moderate variability. Because the correlation functions obtained may vary in length, the program must be able to handle correlations of say 100, 200, 300, or 400 pts.

A problem that arises in the computation of spectra is that of minimizing the errors that result from truncation (finite correlation function length). This problem is considered in detail in a Servomechanisms Laboratory report (23), and new weighting functions are introduced in the computation procedure to reduce the errors due to truncation. The new weighting functions suppress the spurious oscillations due to truncation, but also tend to slightly broaden sharp peaks occurring in the spectra. The weighting function does not cause frequency shift of the major frequencies in the spectrum. Relative amplitudes between peaks in the spectra are preserved to an interpreable degree. By varying the weighting function used and by carefully measuring the amount the spectrum grows, one can not only tell that the frequency components are densely packed but can also make very good estimates of the precise distribution and amplitudes of the component frequencies. Thus, it would be highly desirable to be able to change the weighting function in the program.

APPENDIX

CORRELATION DETERMINATION USING FILTERS

This appendix contains a simple derivation of the relation that must exist between the filters in order that the correlation function of the original data be simply recovered. Figure A-1 shows the filtering scheme used.

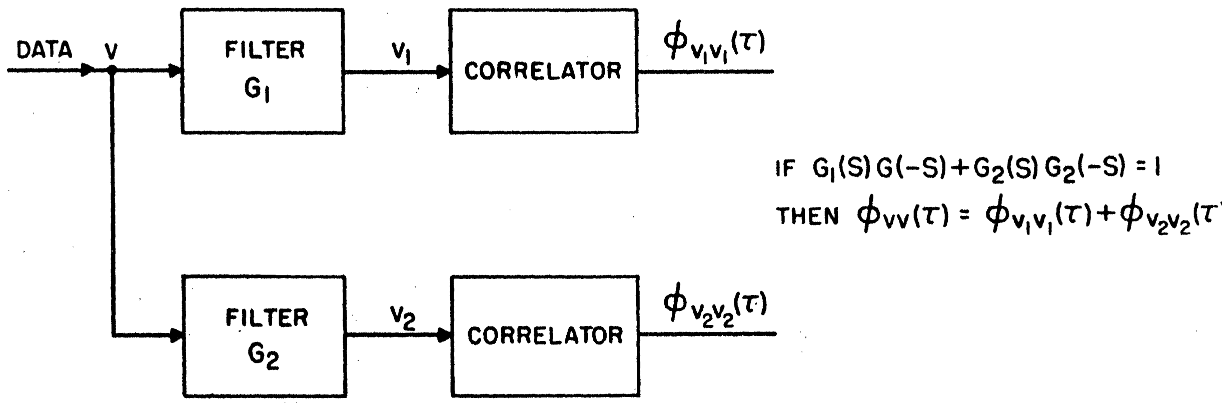


Fig. A-1 Possible Filter Scheme to Aid in Correlation Determination

The scheme is first worked out for autocorrelation. The power-density spectrum of the input is given by $\Phi_{vv}(s)$ -- the Fourier transform of the data correlation function. The correlation computation is carried out on the filtered signals v_1 and v_2. The transforms of these correlations are given in terms of $\Phi_{vv}(s)$ as

$$\Phi_{v_1v_1}(s) = G_1(s)\, G_1(-s)\, \Phi_{vv}(s)$$

$$\Phi_{v_2v_2}(s) = G_2(s)\, G_2(-s)\, \Phi_{vv}(s)$$

Adding these two equations gives

$$\Phi_{v_1v_1}(s) + \Phi_{v_2v_2}(s) = [G_1(s)\, G_1(-s) + G_2(s)\, G_2(-s)]\ \Phi_{vv}(s)$$

If the filters are chosen such that

$$G_1(s)\ G_1(-s) + G_2(s)\ G_2(-s) \doteq 1$$

then the correlation function of the data can be found exactly by adding the correlation functions of v_1 and v_2 because of the additive property of the Fourier transforms. A simple example of a pair of such filters is

$$G_1(s) = \frac{1}{\tau s+1} \qquad \text{(low-pass filter)}$$

$$G_2(s) = \frac{\tau s}{\tau s+1} \qquad \text{(high-pass filter)}$$

If this scheme is to be applied to cross-correlation determination, identical filters must be placed in each channel.

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PART II

A TWO-LEVEL REAL-TIME CORRELATOR

by

Roy K. Angell

INTRODUCTION

This thesis covers an investigation of computation of correlation functions by a small special-purpose electronic computer. The heart of this correlator is a ten-element magnetic core shift register which acts as a variable delay device. Analog information, quantized to two levels, is fed serially into the shift register. The parallel outputs of the ten cores are multiplied by the output of the last core, and the products are averaged by a low-pass filter. Thus, ten points on a correlogram are being formed simultaneously. A greater number of points could be computed by adding more cores to the shift register. The increments and total range of τ are determined by the sampling rate which can be varied through the range 4 kcs to 10 kcs in this correlator.

The thesis required a study of two important questions. First, the quality of correlograms which result from the ultimate in quantization coarseness (two levels), was studied. Secondly, the problem of building a simple real-time correlator was investigated. As a vehicle for this study, a correlator was constructed using vacuum tube pulse circuits and the magnetic core shift register mentioned above.

The correlator was used to obtain correlograms of both periodic and random time functions. The results show that a one-bit correlator will furnish the principal characteristics of a correlation function. These tests are described, and an evaluation of the results is made in Chapter IV.

In Chapter I brief descriptions are given of the two basic forms of correlators, analog and digital, together with a little history of their origin. This chapter attempts to summarize briefly the theoretical background for the problem of performing electronic computation of correlation functions and, in particular, the theory as it is related to the correlator described in this thesis. Source material is abundant, but no single treatment of all aspects was found. Therefore, the first chapter is devoted to a synopsis of the pertinent theory.

Chapter II shows the approach that was made in translating theory to circuitry. The problem considered is exactly what, when, and how is information to be placed into, and taken out of, the magnetic core shift register. To obtain the answer to this question required considerable study of the shift register. Lastly, the problem of how to handle the information obtained from the register is discussed.

The circuit descriptions of Chapter III have been kept as brief as possible. As constructed, the apparatus performs autocorrelation, with the modifications required for cross-correlation having been pointed out in Chapter II. The correlator was constructed as a "breadboard" design, all mechanical and electrical work being done by the author. No discussion is made of the operation of basic circuits, such as multivibrators, coincidence gates, clippers, and amplifiers, since this information is available in many texts. The material presented here is related to the manner in which these circuits operate in this correlator.

As noted previously, a discussion of the tests performed on the completed correlator is contained in the last chapter, together with some general conclusions.

CHAPTER I

HISTORICAL AND THEORETICAL BACKGROUND

Analysis of periodic and aperiodic functions in the frequency domain by Fourier methods has been used by communications engineers for a long time. By these methods a signal can be described by its power-frequency spectrum. This spectrum can be obtained experimentally by averaging the output of a selective filter at adjacent points throughout the bandwidth of the signal. As the filter bandwidth is narrowed indefinitely (the number of adjacent points increased indefinitely) and the averaging time is lengthened, the signal is specified more completely. Fourier series or Fourier integrals are not, however, adequate for the mathematical analysis of random time functions. Analysis in the time domain, by means of the correlation functions of statistics, furnishes another means of attacking the latter problem. Such analysis of time functions has been developed rather recently.* The initial work of Wiener was developed for practical use in communication engineering by Lee (2). A brief chronology of early papers developing statistical theory may be found in (19). Considerable discussion of statistical analysis as applied to practical problems begins to appear in the Research Laboratory for Electronics (M.I.T.) Quarterly Progress Reports of 1948 and continues from that time forward. It was during that period that work on an electronic correlator was begun (18).

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* Reference 1 follows an earlier report (1942), by Wiener, having the same title.

A. CORRELATION FUNCTIONS

The crosscorrelation function of the time functions $f_1(t)$ and $f_2(t)$ is defined by the relation

$$R_{12}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f_1(t)\; f_2(t+\tau)dt^{*} \qquad 1\text{-}1$$

It is assumed that the time functions $f_1(t)$ and $f_2(t)$ are stationary; i.e., their statistical characteristics are independent of the time at which these characteristics are observed. When $f_1(t) = f_2(t)$, the above relation becomes the autocorrelation function of either signal. The quantity τ represents a displacement in time.

Another method of describing correlation functions is in terms of probability distribution functions. In this case the autocorrelation function is described in terms of the characteristics of any member of an ensemble of random functions. An ensemble is a large set (theoretically an infinite set) of member functions emanating from identical generators. The resulting correlation function is referred to as an ensemble average whereas the previous relation is called a time average. According to the ergodic hypothesis, the two are equal. The following expression shows the autocorrelation function of a member function of an ensemble.

$$R_{11}(\tau) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y_1 y_2\; p(y_1, y_2, \tau)\; dy_1\; dy_2 \qquad 1\text{-}2$$

$p(y_1, y_2, \tau)$ is the probability that at any time a member of the ensemble will have a value in the range y_1 and $y_1 + dy_1$, and at a time τ seconds later will have a value in the range y_2 and $y_2 + dy_2$. The foregoing expression becomes a cross-correlation function between related random functions if y_1

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* In much of the literature the symbol $\varphi(\tau)$ is used to represent correlation functions. The symbol $R(\tau)$ is used in some of the literature and is used here for ease of manuscript preparation.

and y_2 are values for two different time functions separated by a time τ. If these two signals are taken from independent processes, the crosscorrelation function reduces to the expression

$$R_{12}(\tau) = \overline{y}_1 \, \overline{y}_2 \qquad 1\text{-}3$$

which is simply the product of the mean values of the two functions. If the signals are not independent and there are no hidden periodic components, $R_{12}(\tau)$ will be asymptotic to the value given by 1-3.

Certain properties of the autocorrelation function in relation to the time function from which it is derived are listed below:

(1) It is an even function of τ.

(2) The value at $\tau = 0$ is the mean square of the time function and is the maximum value of the autocorrelation function.

(3) It has periodic components of the same periods as the time function from which it is derived.

(4) It discards phase information contained in the time function.

(5) As $\tau \rightarrow \infty$, it tends to the square of the average value of the time function.

(6) The autocorrelation function and the power density spectrum are Fourier transforms of one another (Weiner's Theorem).

Some comparative properties of the crosscorrelation function are as follows:

(1) It is generally not an even function of τ.

(2) It does not necessarily have a maximum at $\tau = 0$.

(3) $R_{12}(\tau) = R_{21}(-\tau)$.

(4) Refer to comments associated with equation 1-3.

B. COMPUTATION OF CORRELATION FUNCTIONS

Manual calculation of correlation functions is extremely laborious, therefore, many systems have been devised to perform the process electronically. (6, 7, 8, 13, 18.) These schemes, which implement modified forms of equation 1-1, are of two basic types, the analog (continuous), and the digital (sampling). Brief descriptions of an example of each form of correlator are given in the paragraphs which follow.

1. An Analog Correlator

An analog correlator for studying brain potentials is described in (8). It performs the operations necessary to furnish the result in equation 1-4. The signals are recorded

$$R_{12}(\tau) = \frac{1}{T} \int_0^T f_1(t)\ f_2(t-\tau)dt \qquad 1\text{-}4$$

on magnetic tape; and during analysis the value of τ is obtained by the spacing, which is variable, of two playback heads. The maximum value of τ is limited by mechanical design and drum speed, unless modifications are made. The correlation function is not available until the signals have been recorded and played back once for each value of delay. The tape recorders, amplifiers, switching unit, playback heads, plotter, etc. make a rather large collection of equipment.

2. A Digital Correlator

H. E. Singleton describes a digital, or sampling, correlator in (6). This type forms the correlations in accordance with the following relation which approximates equation 1-1 if the number of samples N is large.

$$R_{12}(\tau) = \frac{1}{N} \sum_{j=1}^{N} a_j b_j \qquad 1\text{-}5$$

This is a summation of N products of samples of $f_1(t)$ and $f_2(t + \tau)$ of amplitudes a_j and b_j, respectively. Singleton, although working on one continuous time function, is essentially using an ensemble of time functions because samples are taken far enough apart so that the samples are statistically independent of one another. A process of this sort is very wasteful of information, and the result is an abnormally long time to compute one point on the correlation curve (order of 20 seconds). In an effort to reduce the time required to obtain a correlation function, a five-channel correlator was constructed. (7) This latter equipment still required a long time to obtain a complete curve of 20 to 100 points. Additional channels might be utilized if there is no objection to the resulting increased equipment complexity. The foregoing processes could be speeded up if samples were spaced closely together although their independence would no longer be assured. This would require temporary storage of sections of the input signal with a resultant increase in equipment complexity. The resulting time average would be approximately equivalent to the average obtained by Singleton. According to the ergodic hypothesis, the two would be exactly equivalent if the process were carried out over an infinite length of time.

C. LIMITATIONS ON MEASUREMENT OF CORRELATION FUNCTIONS

In a practical correlator neither the averaging time indicated in equation 1-4 nor the number of samples indicated in equation 1-5 can be made infinite. To determine the usefulness of a correlator as a tool, it becomes important to determine the errors incurred by utilizing finite observation intervals for calculation of the correlation functions. The following discussion is a condensation of the results shown in (3, 4, and 5). Intermediate steps in the derivation may be found in the references cited; therefore, in the interest of brevity they will not be repeated.

1. Continuous Correlator

The following notation will be used in the discussion

$$R_{12}(\tau_0) \equiv \overline{f_1(t)\ f_2(t+\tau_0)} \equiv \overline{x(t)} \qquad 1\text{-}6$$

where the bars indicate an infinite time average. In a practical correlator x(t) the instantaneous product of the two inputs is formed, and this is fed into an averaging or integrating device which usually has the form of a low pass filter. The output, $y(t)$, of the filter is the convolution of the filter's impulse response and the input, $x(t)$. It is assumed that the filter is passive, having an impulse response which is $h(t)$ for $0 < t < T$, and is zero elsewhere. The resulting time average reduces to the expression

$$\overline{y(t)} = R_{12}(\tau_0) \int_{-\infty}^{\infty} h(\tau,T)d\tau = R_{12}(\tau_0) \int_{0}^{T} h(\tau)d\tau \qquad 1\text{-}7$$

Now the root mean square (RMS) of the variations of the correlator output about its mean value gives a measurement of correlator error or noise. It is indicated by the symbol σ_y and is defined by the relation

$$\sigma_y^{\,2} \equiv \overline{[y(t) - \overline{y(t)}]^2} \equiv \overline{y^2(t)} - \overline{y(t)}^2 \qquad 1\text{-}8$$

It is desired to minimize σ_y in relation to $\overline{y(t)}$ or to maximize the following signal-to-noise ratio which gives an indication of the correlator accuracy.

$$\frac{S}{N} = \frac{\overline{y(t)}}{y} \qquad 1\text{-}9$$

where $\overline{y(t)}$ is given by equation 1-7.

Let us define a new variable, $Z(t)$, as the variation of the product function $x(t)$, about its mean.

$$Z(t) = x(t) - \overline{x(t)} \qquad 1\text{-}10$$

then

$$R_Z(\tau) = R_x(\tau) - \overline{x(t)}^2 \qquad 1\text{-}11$$

Now if the input to the averaging filter were white noise with a spectral density of unity, the autocorrelation function of the output would be

$$R_h(\tau) \equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} H(w,T)^2 \, e^{jw\tau} \, dw \qquad 1\text{-}12$$

The mean square error of the output may be expressed in terms of correlation functions defined in equations 1-11 and 1-12 as follows:

$$\sigma^2 = \int_{-\infty}^{\infty} R_Z(\tau)\, R_h(\tau)\, d\tau \qquad 1\text{-}13$$

Remembering that the impulse response of the filter was non-zero only over the interval 0 < t < T and remembering that autocorrelation functions are even functions, then equation 1-9 becomes:

$$\frac{S}{N} = \frac{R_{12}(\tau_0) \int_0^T h(\tau)\, d\tau}{\left[2 \int_0^T R_Z(\tau)\, R_h(\tau)\, d\tau\right]^{1/2}} \qquad 1\text{-}14$$

Thus, it is seen that the signal-to-noise ratio depends on the statistical properties of the input signals as well as the characteristics of the averaging filter. Interpretations of equation 1-14 for certain specific cases are made in the discussion that follows.

2. Continuous Averaging with an Ideal Integrator

If the averaging device is an ideal integrator having an impulse response equal to one for 0 < t < T and zero elsewhere, equation 1-14 will take the following forms:

$$\text{As } T \to 0 \quad \frac{S}{N} \to \frac{R_{12}(\tau_0)}{\left[R_Z(0)\right]^{1/2}} = \frac{x(t)}{\sigma_x} \qquad 1\text{-}15$$

This is simply the signal-to-noise ratio at the output of the multiplier and is independent of T.

$$\text{As } T \to \infty \quad \frac{S}{N} \to \frac{R_{12}(\tau_0)\sqrt{T}}{\left[2 \int_0^T (1-\frac{\tau}{T})\, R_Z(\tau)\, d\tau\right]^{1/2}} \qquad 1\text{-}16$$

3. Continuous Averaging with a Low-Pass Filter

If the averaging device is a low-pass RC filter with an impulse response ae^{-at} for $0 < t < T$ and zero elsewhere, equation 1-14 will take the following forms:

As T becomes small compared to the filter time constant, $\frac{1}{a}$, the signal-to-noise ratio approaches that of equation 1-15. (This is equivalent to saying that a low-pass RC filter acts as an ideal integrator for periods of time short in comparison with the filter time constant.

$$\text{As } T \rightarrow \infty \quad \frac{S}{N} \rightarrow \frac{R_{12}(\tau_0)\sqrt{\frac{1}{a}}}{\left[\int_0^{\infty} e^{-a\tau} R_Z(\tau)d\tau\right]^{1/2}} \qquad 1\text{-}17$$

Thus, for the extreme cases (very short or very long averaging times), the signal-to-noise ratio is independent of T. For intermediate ranges of T, equation 1-16 will hold for the RC filter case as well as for the ideal integrator.

The foregoing relationships are summarized in Fig. 1-1 which shows noise-to-signal ratio $\sigma_y / \overline{y(t)}$, for power

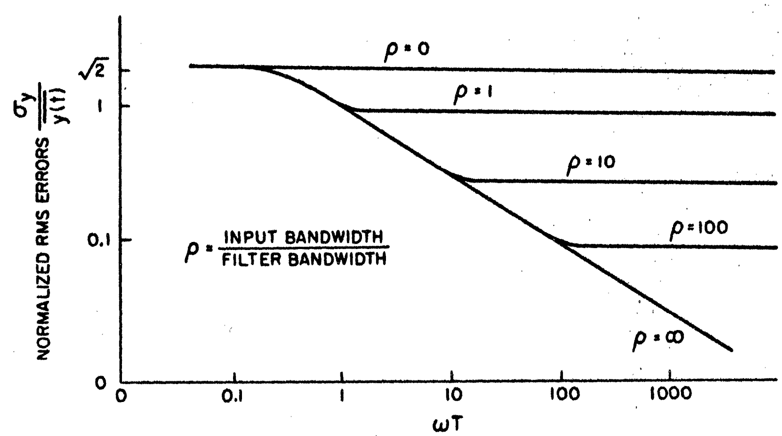

Fig. 1-1 Measurement Errors of Low Pass RC Filter

measurements, $R_y(0)$, on a random signal of bandwidth W. The curves are similar to those found in (5). The horizontal axis is a dimensionless integrator "time" obtained by multiplying signal bandwidth and real time, necessary to obtain a specific measurement accuracy on a signal of known bandwidth.

4. Correlation on Sampled Inputs

In most practical cases sampling is done periodically rather than at random. Therefore, only the periodic case will be treated here. For equal averaging times the errors in correlating a sampled input will be greater than errors resulting from the use of a continuous input. This is to be expected since sampling discards information between samples.

Costas (3) develops the following formula for the mean square error of a sampled function (variance of the sampled mean).

$$\sigma_{\dot{Z}} = \frac{\sigma^2}{n} + \frac{1}{n^2}\sum_{k=1}^{n-1} 2(n-k)\ R(kt_0) + \frac{m^2(1-n)}{n}$$

σ^2 = variance of unsampled function (1-18)

m = mean of unsampled function

n = number of samples

t_0 = sampling interval ($kt_0 \rightarrow \tau$)

If each sample is statistically independent in equation 1-18, this reduces to the expression

$$\sigma_{\dot{Z}}^2 = \frac{\sigma^2}{n} \qquad 1\text{-}19$$

Davenport (4) derives a similar expression and incorporates it into several formulas for signal-to-noise ratio simplified to describe a number of special cases. First, if a given number of samples are taken in an interval $T \rightarrow 0$, then the result approaches equation 1-15. This is intuitively obvious since the above procedure approaches the continuous sampling case. There are two important results for cases where the averaging time becomes large. First, if $R_Z(T)$--see equation 1-11--contains a periodic component whose period is the same as the sampling period, then the signal-to-noise ratio becomes independent of the averaging period and the number of samples. This case should be avoided since the periodic signal will be translated as a d-c level. The second case, where the sampling period is not related to any periodic components of the input signal, is given by equation 1-20.

$$\text{As} \quad T \rightarrow \infty \quad \frac{S}{N} \rightarrow \frac{R_{12}(\tau_0)\sqrt{M}}{\left[R_Z(0)\right]^{1/2}} \qquad \text{1-20}$$

This result shows that the number of samples must be increased by a factor of four to double the signal-to-noise ratio. In practice orders of magnitude of 10^3 to 10^4 samples could be accumulated until the correlation function shows that further accumulation is not changing the results materially.

D. EFFECTS OF SAMPLING AND QUANTIZATION (9, 20)

In forming the correlation function, the operations of time delay, multiplication, and averaging must be performed. In using a digital machine to perform these functions, the inputs are first sampled and quantized. What effect do these operations, made necessary by the nature of the machine, have on the resulting correlogram?

1. Sampling

The process of sampling a time function may be described as modulation by a train or sequence of unit impulses spaced at some time interval T. (20) The output of the sampler, or impulse modulator, is a train of impulses, each having an area equal to the amplitude of the input function at the sampling instants. In forming a correlation function, there is a time delay and multiplication involved. In order to form non-zero products, the time delay must be an integer multiple of T. This means that the resulting correlogram has discrete values at intervals of T and is zero in the intervening spaces. The complete correlogram is an envelope that passes through the discrete values. As T is made shorter, the envelope is more completely specified.

It is helpful to look at the process of sampling in the frequency domain as well as in the time domain. If the input time function has a transform denoted by F(s), then the sampled transform F*(s) is related to it by the following equation:

$$F^*(s) = \frac{1}{T} \sum_{n=-\infty}^{\infty} F(s + jnW) \qquad 1\text{-}21$$

where n is an integer and W is the sampling frequency $(W=2\pi/T)$. It can be seen that if F(s) has frequency components greater than W/2, then the sampled spectrum components will overlap and F(s) cannot be recovered. This is the basis for the Nyquist sampling criterion which says that the sampling frequency must be at least twice the highest frequency present in the unsampled function. Of course, the problem here is not the recovery of the original function. The correlation function is the Fourier cosine transform of the power density spectrum which is related to the frequency spectrum as follows:

$$\Phi_{11}(w) = \lim_{T \to \infty} \frac{\pi}{T} \left| F_1(w) \right|^2 \qquad 1\text{-}22$$

The Nyquist sampling criterion still applies if the form of the power density spectrum is to be preserved. Therefore, the sampling requirements for recovery of time functions apply to the computation of their correlation functions.

2. Quantization

Quantization is the process of analog-to-digital conversion required when the correlation function is to be computed digitally in accordance with equation 1-5. All amplitudes in a given range, or box size q, are given a discrete value. The reciprocal of the box size (1/q) is a measure of the fineness of quantization. Widrow (9) describes this process as area sampling and relates it to the Nyquist sampling theory. He shows that the Fourier transform (or characteristic function) of the probability distribution function of a signal is periodic if the signal is quantized. The radian fineness of quantization

$2\pi/q$ must be large enough to separate the components of the periodic characteristic function. This form of a Nyquist criterion is in terms of the statistical characteristics of a signal rather than of the signal itself. This suits our interest in restrictions on the recovery of the correlation function which is a statistical property of the input signal.

Quantization introduces a signal distortion referred to as quantization noise. For Gaussian distributed inputs Widrow relates the correlation of the quantization noise to the normalized correlation of the quantizer input signals, with the box size q as a parameter. The normalized correlation of quantizer input signals is the mean of their product divided by the mean square $(X_1 X_2/\sigma^2)$. The case of particular interest here is for $q=2\sigma$ where σ is the standard deviation of the input signal. This corresponds to a box size approximately one third the dynamic range of the input signal. When the input correlation coefficient for this box size is less than 0.6, then the quantization noise is practically uncorrelated. Even when the input correlation coefficient is 0.9, that of the quantization noise is only 0.3. This suggests that three-level quantization would be adequate to obtain correlograms of reasonable accuracy.

The foregoing theory has been verified experimentally and was carried a step further by Kaiser. (21) He compared the results of two-level (one-bit) quantization with the results of eight-level (3-bit) quantization on the correlation function obtained from human brain potentials. With the extreme quantization coarseness, the major characteristics of the correlation function were retained.* This was a strong motivation for the initiation of this thesis investigation.

Some interesting results related to two-level quantization were published in 1943. (25) Van Vleck showed that when Gaussian noise signals are clipped (quantized) to two levels,

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* It is interesting to note in relation to the statements of Section C-4 of this chapter, that only 650 products were used to obtain the points of these correlograms.

the resulting correlation function is proportional to the arc sine of the correlation function of the input without clipping. His derivation is summarized in Appendix V.

CHAPTER II

GENERAL DESIGN CONSIDERATIONS

To perform autocorrelation or crosscorrelation electronically, the required mathematical manipulations must be translated into suitable circuitry. Provision must be made to perform the following functions:

(1) furnish time delays of various magnitudes corresponding to values of τ

(2) perform multiplication

(3) sum and average the results of the above multiplications

(4) record or display the resulting correlation function

The first function, that of obtaining time delays, may be accomplished in a variety of ways. Lumped parameter electrical networks, phantastron delay circuits, magnetic tape, storage tubes, and various mechanical methods might be used for this purpose. Each has certain advantages and disadvantages as can be seen by reference to other writings. See, for example, (6, 7, and 8). The magnetic core shift register offers an inviting method of obtaining all desired values of τ simultaneously. To understand the problems involved, a study of the magnetic core shift register is necessary. This is done in the next section of this chapter.

The second function, that of multiplication, can be performed on the digital output of the shift register by some form of AND circuit or coincidence gate. In the synthesis of a correlator the details of circuit construction are reasonably straightforward. It is important, however, to have a knowledge of where the various circuits are to be used and the time sequence of their operation. These latter topics are the subject of the second section of this chapter.

The third function involves the process of integration. Electronic integration brings to mind the voltage current relationship of capacitors; i.e., $e = \frac{1}{C}\int i\,dt$. The discussion of averaging circuits, which will be made later, rests heavily on this "integrating" property of capacitors.

After the correlation function has been computed, it is equally important that the results be made available in a useable form. Many schemes have been worked out for recording and displaying information. The last section of this chapter will discuss briefly the problem of a display circuit for this correlator.

A. THE MAGNETIC CORE SHIFT REGISTER

The correlator described in this thesis is built around the ability of the magnetic core shift register to accept binary information in time sequence, store this information, then read it out in parallel form at the occurrence of a pulse acting on all cores simultaneously. Although in this case a series-parallel transformation is made, it should be noted that these shift registers may be used for parallel-series operation should this be desirable.

Magnetic core shift registers are of two basic types: the single-line register using one magnetic core per binary digit and the double-line register using two magnetic cores per digit. To appreciate the differences in operation of these two types of registers, the following points should be kept in mind. First, interrogation of the magnetic core is destructive of the information in that core; and second, information cannot be placed into the core while interrogation is in process. These characteristics of the cores results in the following restrictions on their use. Interrogation (or reading) of a core and shifting information toward the next core <u>must</u> be done simultaneously*, whereas interrogation and writing into a core <u>must not</u> be done simultaneously. In practice this means that the

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* It is possible to make an exception to this statement. With the use of additional circuitry, the information in a core could be temporarily stored during interrogation or shifting and then written back into the same core. Special schemes have been devised for non-destructive interrogation of magnetic cores, but they have not been made practical as yet.

shift pulse should also cause the core contents to be transmitted to the output circuits. In other words, the shift and read pulses are one and the same. Since a core can store only one digit at a time, it must be cleared before another digit is written into it. This is most easily done by reading and shifting all cores in a line simultaneously. The information shifted out of each core must be stored temporarily until read out is finished; then, it may be written into any of the cleared cores (including itself if suitable connections are made). The single-line register accomplishes storage by means of an RLC or RC delay network between cores. The double-line register uses another core for storage. This means that the latter register requires twice as many cores as the single-line register to perform the same functions. Furthermore, the single-line register, with its single-shift pulse line, utilizes simpler driving circuitry than the double-line register. A single-line register, with an RLC intercore network was used for this thesis work. Binary digits ZERO and ONE advance serially through the register, all cores being interrogated in parallel, at periodic intervals, by a shift pulse. Experience as far back as 1953 showed that as many as fifty stages of a single-line register could be cascaded, and that they could be operated at frequencies up to 100 kc/second. (10)

Each magnetic core in the register has three essential windings which control, or are controlled by, the core's rectangular hysteresis loop*. See Fig. 2-1. The cores are controlled by a shift pulse which operates to simultaneously place all cores of the register in the ZERO state of magnetization. If a core is in the ONE state, the shift pulse causes a sudden reversal of flux with a resultant large voltage across the output winding. This corresponds to a flux change $\Delta B = B_m - B_r$ in Fig. 2-1. If the core had been in the ZERO state, the shift pulse would cause a small, but not negligible, output voltage

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* Some dimensioned hysteresis loops for actual cores may be found in Chapter II of Reference 11.

since the hysteresis loop is not perfectly rectangular (the residual flux is less than the maximum flux). This voltage results from the flux change $\Delta B = B_m + B_r$. As will be discussed later, there must be absolute discrimination against ZERO's in the output circuits of this correlator. The write pulse, when present, occurs between shift pulses and always places the core in the ONE state. Absence of a write pulse corresponds to the binary digit ZERO.

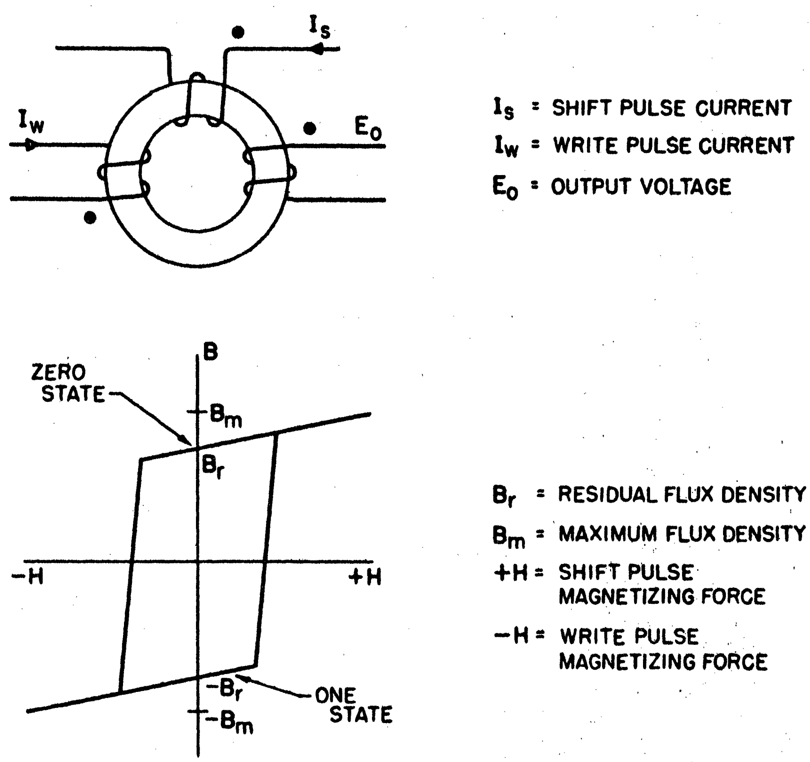


Fig. 2-1 Idealized Magnetic Core and Its Hysteresis Loop

It should be noted that each core acts like a magnetic amplifier with the shift pulses furnishing the added energy necessary to make up for losses as information passes along the register. As will be explained later, the write pulse sees a virtually unloaded core whereas the shift pulse is heavily loaded by the intercore network.

As noted previously, an RLC intercore network prevents simultaneous occurrence of a write and a shift pulse in any core except the first one of the register. External circuitry must insure that write-in to this first core does not coincide with the shift or read-out pulse. The diode of Fig. 2-2 prevents discharge of the capacitor back into the output winding. This diode is back biased during the write pulse, hence the

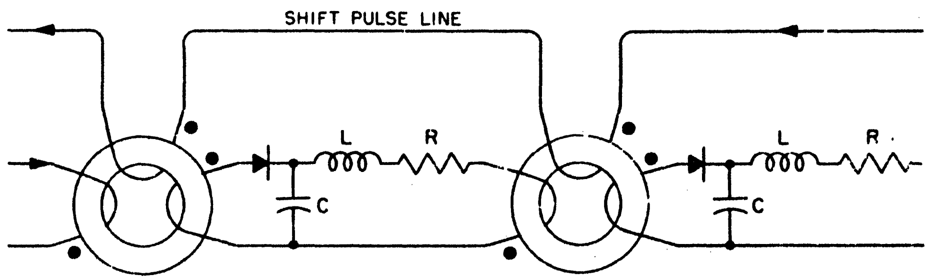


Fig. 2-2 Two Stages of a Single Line Shift Register

output circuit does not load the core during write-in. The shift pulse causes only a negligible output back into the write winding because of the turns ratio and high impedance looking into this winding. The low impedance looking toward the output winding strongly favors transfer of energy to this winding.

In the design of an instrument utilizing a magnetic core shift register, it is important to know the required characteristics of the shift and write pulses. If a core is unloaded, the energy supplied by the shift pulse to switch the core from the ONE state to the ZERO state is

$$W_s = (\text{volume}) \int_{-B_r}^{B_m} H\, dB = \int_0^T E(t)\, I(t)\, dt \qquad 2\text{-}1$$

Where H and B have the meaning shown in Fig. 2-1, T is the duration of the shift pulse and E(t) and I(t) are the voltage and current associated with the core winding during the pulse. The flux density is related to the magnetizing force by the characteristics of the core which are represented by the hysteresis curve, and the magnetizing force is a product of current and the number of shift winding turns. All of this may be summarized in the basic fact that the switching energy is related to a time integral of shift pulse current. This means that for a given core there is a minimum shift pulse current and duration required to switch the core. Additional energy, not accounted for in the above discussion, must be furnished to supply losses and the core load.

The time required to switch the core varies with the applied magnetizing force. It has been found empirically that the product of switching time and magnetizing force is approximately constant over a wide range of operation. For purposes of this thesis, the switching time (which is short compared to planned operating frequencies) is not critical. Therefore, design will not be concerned with providing additional energy in the shift pulse source for the purpose of reducing switching times. It is important, however, that sufficient energy be supplied to insure complete switching every time a shift pulse occurs. Partial transition may result in subsequent erroneous transitions of core state. For example, if a core in the ONE state is only partially switched and the next input is a ZERO, the next shift pulse may add sufficient energy to create a spurious ONE in the output. More detailed discussion of this last possibility, as well as a more complete discussion of magnetic core energy relationships, may be found in (11 and 14).

In addition to the minimum pulse duration requirement, which is dictated by energy considerations, there is also a maximum allowable pulse duration for single-line registers. This requirement is dictated by the intercore network. The shift pulse must end while the intercore storage capacitor still has the energy to make a complete write-in on the following core. A shift pulse of excessive length will actually erase all ONE's from the register. One further point: the shift pulse source should appear to be an open circuit between pulses in order not to load the write winding during the write pulse. As previously noted, a back biased diode keeps the output winding open circuited during write-in.

The manufacturer built the intercore network into the register available for this thesis, as well as having fixed the core material and turns ratios. Design is, therefore, limited to providing for a shift pulse current of proper magnitude and duration so that the cores are switched completely and write pulses are not destroyed during write-in. A write pulse source, activated by the input time function, must supply a current pulse of proper magnitude and duration to write into the first core. Although nothing has been said about pulse shape, the rise and fall times of the pulses may have an effect on the stability of operation; and this possibility must not be overlooked.

In summary, the foregoing discussion points up the following design considerations necessary to properly incorporate the magnetic core shift register into the correlator:

(1) There must be a shift pulse source which supplies a current pulse of proper amplitude and duration so that complete switching is accomplished every time.

(2) Analog input signals must be converted to digital current pulse of proper amplitude and duration to write into the first core in the line. Because quantization is to be confined to two levels, this should be relatively easy to do.

(3) Attention should be given to the optimum rise and fall times of the pulse by observing the effect of varying these quantities on the operation of the register.

(4) Provision must be made to insure that the write pulse going into the first core does not coincide with the shift pulse. The intercore network takes care of this function for other cores provided the shift pulse is not too long.

(5) The cores must not be too heavily loaded by output circuitry.

B. OPERATIONAL TIME SEQUENCES

Assuming that the basic sampling frequency has been determined in accordance with the principles outlined in the previous chapter, it next becomes necessary to plan for the sequence of events dictated by the magnetic core properties and the associated circuitry. This sequence must properly implement the mathematical expression for correlation without violating conditions imposed by the "hardware."

Let us assume a sampling rate of 8 kc/second is being used and the magnetic cores require a shift pulse of approximately 8 microseconds' duration. Thus, all cores are receiving an 8-microsecond shift pulse every 125 microseconds. Sampling of the input must occur at the same frequency as the shift pulse to provide information to the register at the same rate that it advances through the register. It might be helpful to think of the process as quantization in time. All the information in a 125-microsecond section of the input function is quantized to one discrete value. In this correlator, two-level quantization in amplitude takes place at the input; hence, the discrete value is a binary digit, ONE or ZERO, and only one core is required to store all the information that is accepted in the 125-microsecond section of time. A ten-core register contains a representation of 1250 microseconds of the input function, provided an 8 kc/second sampling rate is being used. The entire register acts as though it were a 1250-microsecond "window" that moves back 125 microseconds along the input function with each shift pulse.

Before following through a sequence of operations, one point should be stressed. Information cannot be written into a core during a shift pulse. This means that sampling must occur between shift pulses. The easiest method of insuring proper write-in is to control the sampler by the trailing edge of the shift pulse. The discussion that follows is related to the flow chart in Fig. 2-3. The shift pulse period is given the general value T. For purposes of illustration, only four cores are shown and all are assumed to be in the ZERO state initially. At time $t = 0$, the correlator is started and the first shift pulse occurs. Delta microseconds later (width of shift pulse = Δ) the sampler operates to place a binary digit into the first core. At the occurrence of the fifth shift pulse ($t = 4T$), this digit is read out of the last core. It is then used to gate (multiply) parallel outputs from all cores

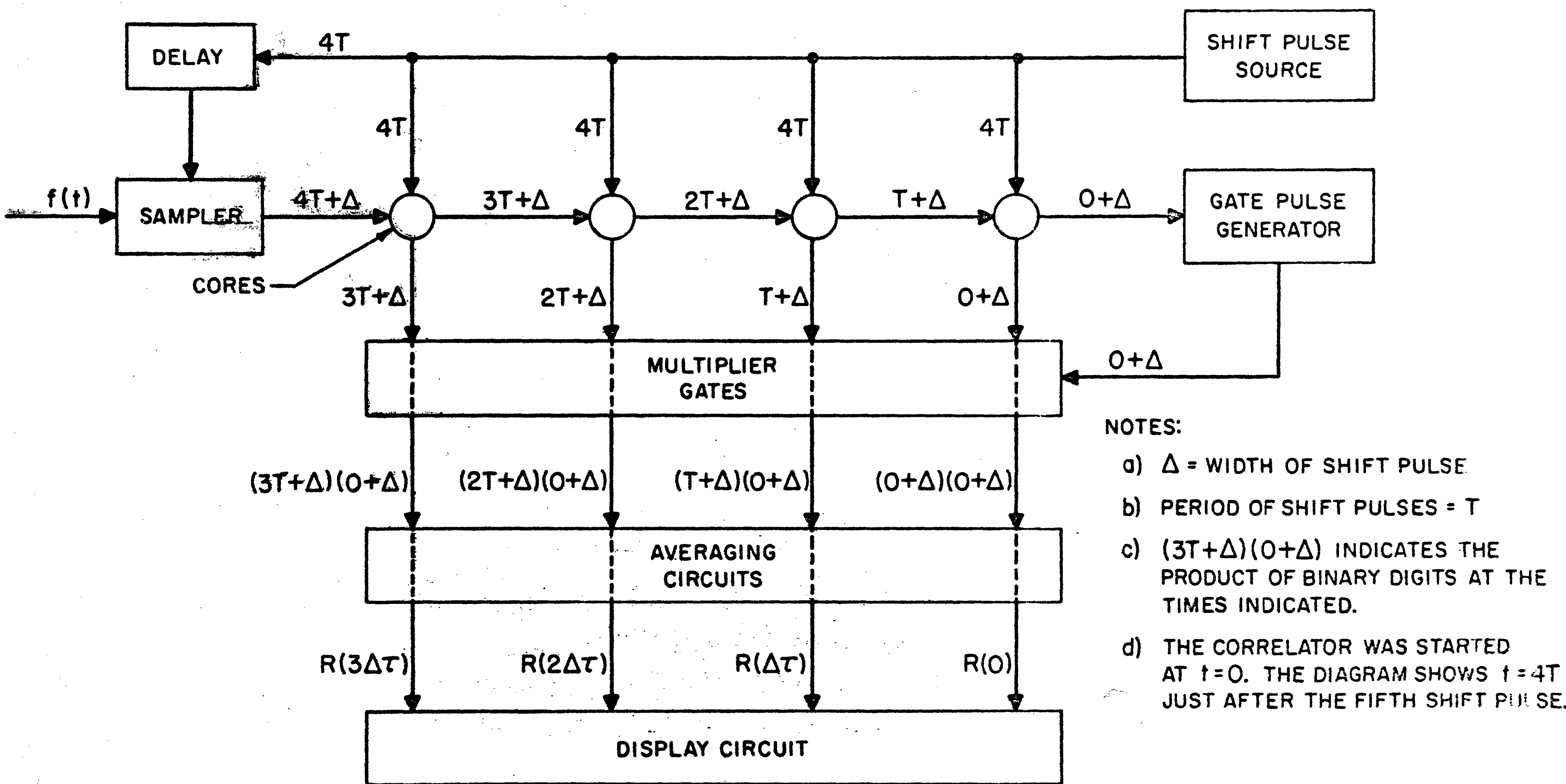


Fig. 2-3 Time Sequence Flow Chart

including its own. From this time forward, every shift pulse results in four products (in this example). The products are summed, averaged, and displayed as four points on a correlation curve $R(\tau)$, where $\tau = 0$, T, 2T, and 3T microseconds.

It should be remembered that the increments of τ are equal to the sampling period which is also the period of the shift pulse. A high sampling frequency will result in small increments of τ and hence a smaller total range of τ for any given number of cores. If it were necessary to increase the range of τ without adding cores or decreasing the sampling frequency, it might be possible to use the same cores for an extended range. For example, suppose that in the above system the output from the last core were continuously recorded on a magnetic tape and then read by a pickup spaced so that a delay of 4T microseconds was sustained. The output from this pickup could then be used to gate the four cores with a resulting correlation curve for $\tau = 4T$, 5T, 6T, and 7T microseconds. Thus, it is possible to obtain a complete correlation curve for any range of τ by taking it in sections. Obviously, this procedure would lengthen the time required to obtain a complete correlation function. However, there is a limit to the number of cores that may be cascaded; therefore, the latter procedure might be necessary in cases where the nature of the input function necessitated a high sampling frequency (corresponding to a short total range of τ for a given number of cores).

Although in the previous discussion the last core was used for gating the output of all the cores, it could just as well be done by the first core. As a matter of fact, gating could be done by a binary sample from the input function. In this case, however, the sample must not be delayed in the manner required for proper write-in, but must occur at the time of the shift pulse in order to coincide with the outputs from all cores. Use of the last core appears to be simpler, and it does insure that the register has a complete section of the input when the first multiplying pulse occurs at the output gates.

To perform crosscorrelation, the second function, after being quantized, could be used to do the gating to the averaging circuits.

If the input function is removed before the correlator is turned off, a few shift pulses will soon clear all cores to the ZERO state.

C. SUMMATION AND AVERAGING

Summation and averaging will be the function of an integrating circuit. Discussion in the previous chapter indicated some of the factors to be considered in determining the proper number of samples or the averaging time required to obtain a correlation function with small errors. In practical correlators various provisions have been made for these factors. For example, the analog correlator of (8) makes provision for integration with one per cent accuracy for periods as long as 50 seconds but also makes use of a 10-second period. Cowley's correlator (13) makes use of a low pass averaging filter which integrates signals of frequencies above 15 cycles per second. Levin (7) makes provision for obtaining from 4,000 samples to 16,000 samples for each point on the correlation curve. The time or the number of samples required is related to the statistical characteristics of the input signals, and these may not be known. The exact requirements for the integrator may be found by experimentation with the class of signals being investigated. This was discussed in the first chapter. Here the discussion relates to the type of circuits that may be useful.

The step response of an ideal integrator is a ramp. The low pass RC filter approximates this for a limited time if it has a long time constant. Consider the filter of Fig. 2-4a where the following relation applies:

$$E_0 = \frac{1}{C}\int i\,dt = \frac{1}{RC}\int (E-E_0)\,dt \qquad 2\text{-}2$$

The solution to this equation is:

$$E_0 = E\,(1 - e^{-t/RC}) \qquad 2\text{-}3$$

To increase the linearity of integration, it appears only necessary to increase the values of resistance and capacitance. However, in a simple RC integrator this will result in an extremely low gain E_0/E for a given integration time; and, in any event, this method is limited by the values obtainable with practical components. Capacitor leakage resistance and

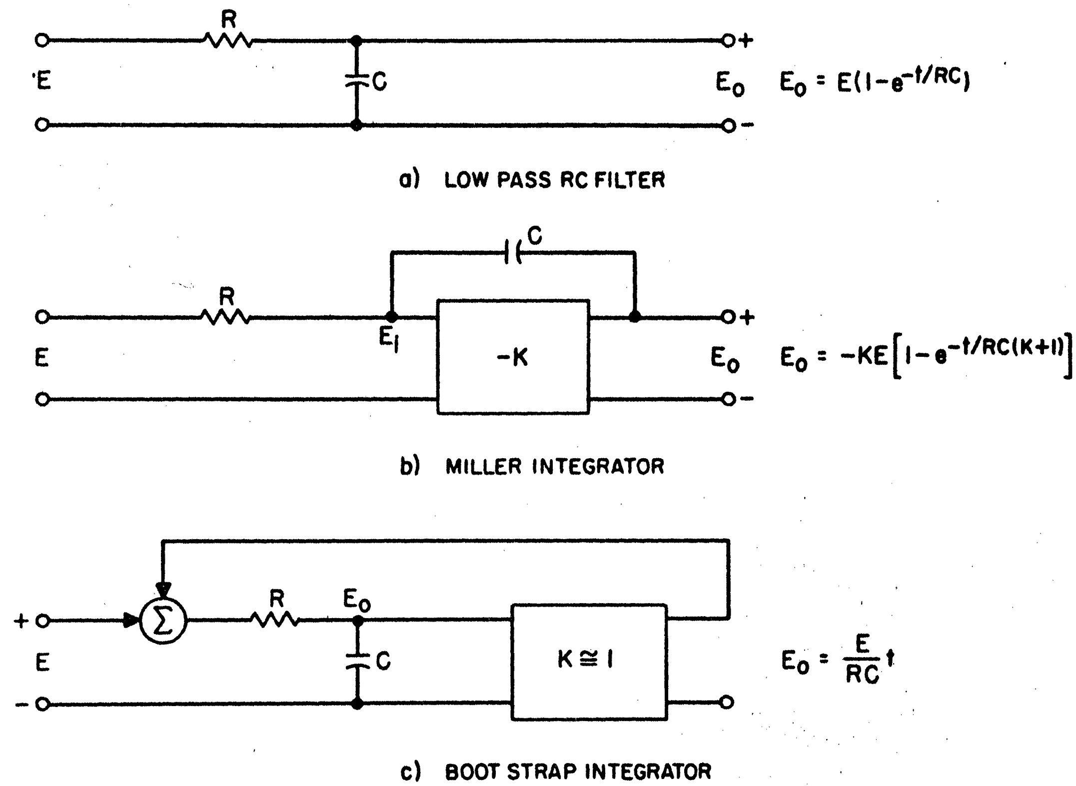


Fig. 2-4 Basic Integrator Forms

external loading limit the time constants available in a simple RC circuit. One method of obtaining a large effective capacitance is to use a Miller integrator. See Fig. 2-4b. Here a capacitance between grid and plate of an amplifier stage appears to be multiplied in size by one plus the amplifier gain. Also, the ratio E_0/E is multiplied by the gain of the amplifier.

If the current into the capacitor of the low-pass RC filter were constant, the integration would be ideal. This current is $i = \frac{E - E_0}{R}$. A constant current input could be obtained by raising the input voltage at the same rate as the capacitor voltage. This may be accomplished with a unity-gain feedback amplifier as shown in Fig. 2-4c. The result is called a bootstrap integrator. Derivations of the gain relations shown in Fig. 2-4 are presented in Appendix I. One more method will be described after a few introductory remarks.

The simplified process being carried out in a one-bit correlator gives rise to other possibilities. The mathematical expression for the correlation function is repeated here for ready reference in the discussion that follows.

$$R(\tau) = \frac{1}{N} \sum_{j} a_j \, b_j \qquad \text{2-4}$$

Since quantization is carried to only two levels, the values of a_j and b_j are 1 or 0. The following products are defined:

$1 \times 0 = 0$ $\qquad$ $1 \times 1 = 1$

$0 \times 1 = 0$ $\qquad$ $0 \times 0 = 0$

Therefore, the product $a_j b_j$ is always a 1 or 0.* This means that the correlation function can be obtained by simply averaging the long time rate at which ONE products are obtained. This is equivalent to making an average frequency measurement. A device for measuring the frequency of voltage pulses is the counting rate meter used in atomic radiation instruments. (15)

Consider the circuit shown in Fig. 2-5 which shows the basic configuration for a counting rate meter circuit.

At the occurrence of a negative pulse, the capacitor C_1 charges through diode 1 and the output resistance R_1 of the pulse circuit. The voltage across C_1 at the end of the pulse is

$$e_c = E\,(1 - e^{-T/RC}) \qquad \text{2-5}$$

---

* On a basis of like products equal ONE and unlike products equal ZERO, the product 0 x 0 would also be equal to one. The form taken here will result in reduced amplitude of the entire correlation function but will not alter its form. It is felt that this procedure is justified because of the simplification in the gate circuits where the multiplication is formed.

and is approximately E since T is chosen to be over 5 times constants in duration. At the end of the pulse, discharge occurs through diode 2 and C_2R_2. Now the charge which had accumulated on C_1 starts to distribute itself between the two capacitators so that

$$e_{c1} C_1 + e_{c2} C_2 = EC_1 = Q \qquad 2\text{-}6$$

But, when $e_{c1} = e_{c2}$, diode 2 opens. Then

$$e_{c2} = \frac{EC_1}{C_1 + C_2} \qquad 2\text{-}7$$

Since $C_2 \gg C_1$, this reduces to $e_{c2} \simeq E \frac{C_1}{C_2}$ or $e_{c2} C_2 \simeq EC_1 = Q$.

The average current through R_2 is then $\frac{Q}{y}$ where $\frac{1}{y} = N$, the average rate of the pulses. Then

$$V = IR_2 = NQR_2 = NEC_1R_2 \qquad 2\text{-}8$$

Naturally this derivation holds true only if $V \ll E$. Notice that the output voltage does not depend on the duration of the pulses as long as they are long enough to give C_1 a full charge. A similar analysis could be carried out for positive applied pulses resulting in the same expression for V.

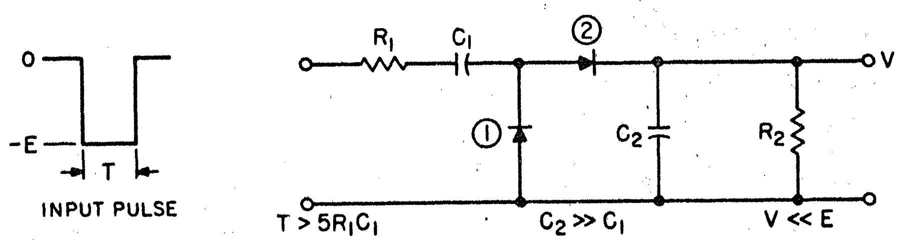


Fig.2-5 Counting Rate Meter Circuit

Another very simple averaging circuit would be a parallel RC circuit placed in the plate circuit of a pentode gate. Each current pulse would place a charge Q = IT on the capacitator. If the average number of pulses is N per second, the rate at which charge is deposited on the capacitator is NQ = NIT. The average voltage across R is NITR. It is assumed that RC $\gg$ T.

Some form of the circuits which have been discussed above should be adequate for the averaging circuit.

D. DISPLAY OF RESULTS

This problem is largely one of designing a commutator to take the outputs from each integrator and convey them sequentially to the display unit. This method, rather than trying to take parallel outputs from all channels simultaneously, is used in the interest of not requiring an unduly complicated display unit.

The commutator may be electronic or mechanical. One possible mechanical system would employ a rotary selector switch such as has been used in some telephone systems. A set of wipers are rotated over a semi-circular bank of contacts by the action of a stepping magnet. Each output channel would be connected to one of the contacts in a single layer of the bank. The wiper for this layer would be connected to the vertical axis of an oscilloscope or other indicator. The contacts of another layer would be connected to discrete adjacent increments of voltage. The wiper of this layer would cause displacement along the horizontal axis of the indicator. As both wipers move together, each output channel would have its value displayed sequentially at adjacent horizontal positions. A discussion of the rotary selector switch may be found in (17).

An electronic commutator is preferable for high speed, clean commutation of many outputs. Petree (16) describes a system which has been used with five channels. Its liberal use of tubes (3 per channel) is a serious disadvantage in cases where many channels are being handled. The diode matrix system described by Kilpatric (12) appears to be the most suitable for this correlator and its use is planned. Although synchronization with the timing of the other correlator circuits is unnecessary, the shift pulse source may be used as a convenient source of timing pulses to time the operation of the circuits controlling the diode matrix system.

CHAPTER III

DESCRIPTION OF CIRCUITS

In the previous chapter, Fig. 2-3 summarized the necessary timing of operations to perform autocorrelation. From these time relationships it was necessary to synthesize suitable circuitry; therefore, the block diagram of Fig. 3-1 was evolved. Having a general picture of requirements, the first step was to provide a shift pulse source for the magnetic core shift register. Next, a circuit for sampling and converting the input signal from analog-to-digital form was constructed. Finally, circuits for taking the information out of the shift register and forming the correlogram were designed and built. The descriptions that follow are presented in the order just mentioned.

A. MAGNETIC CORE SHIFT REGISTER

To determine the requirements for the shift pulse source, and also the input circuits, tests were conducted on the magnetic core shift register. Ten magnetic cores, manufactured by Sprague Electric Company, were available to make up this register. Each core package was complete with a built-in RLC delay network but required the addition of a diode (see Fig. 3-2). With apparatus available in the Servomechanisms Laboratory, tests were conducted to determine the inputs required to properly operate the assembled ten core shift register. It was found that, with a shift pulse having an amplitude of 175 milliamperes, core switching would occur with pulse durations of 3.7 to 12 microseconds. An intermediate value of 7.5 microseconds was selected as being the optimum value for the shift pulse width. The entire shift register operated satisfactorily for frequencies up to 12 kilocycles per second.

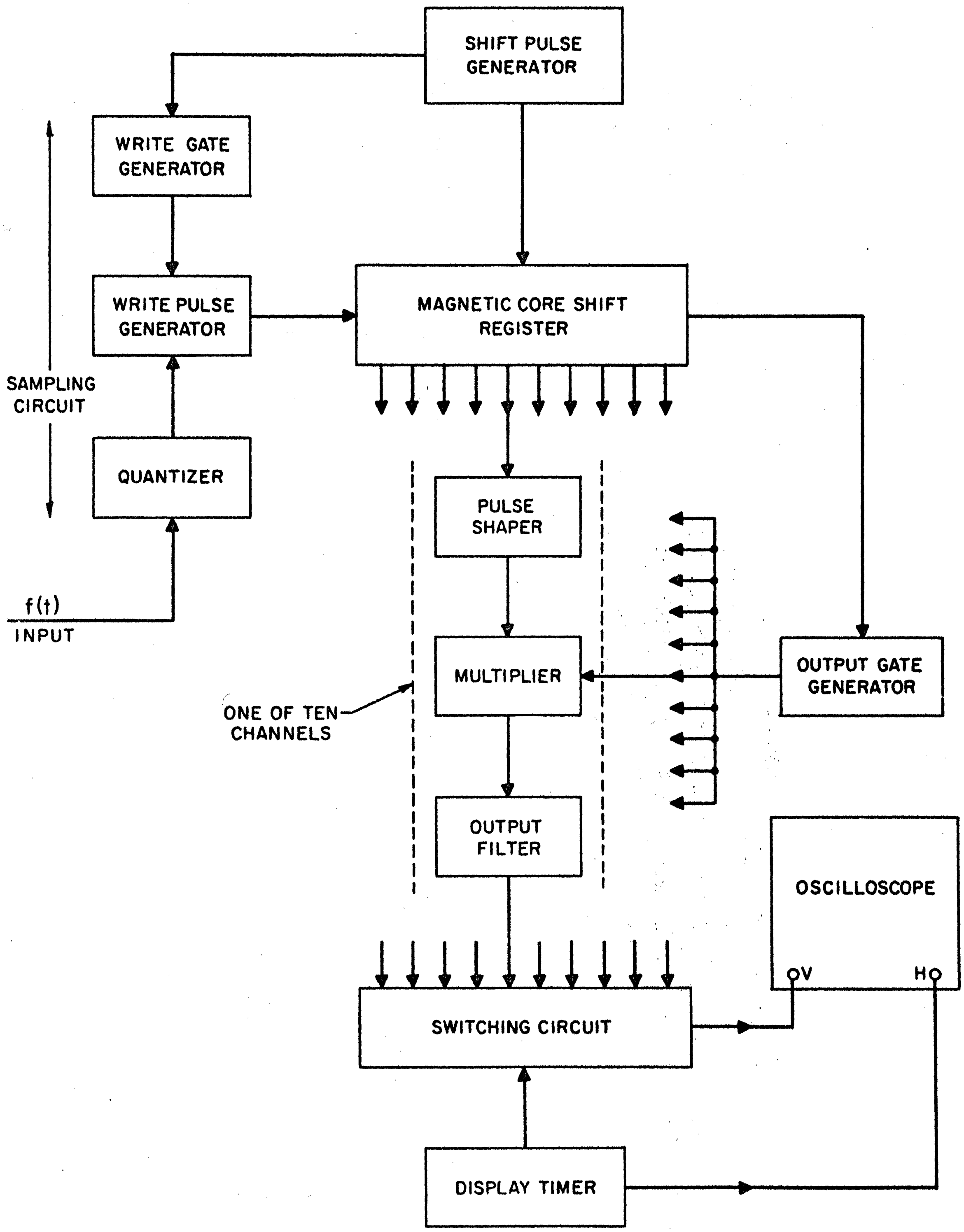
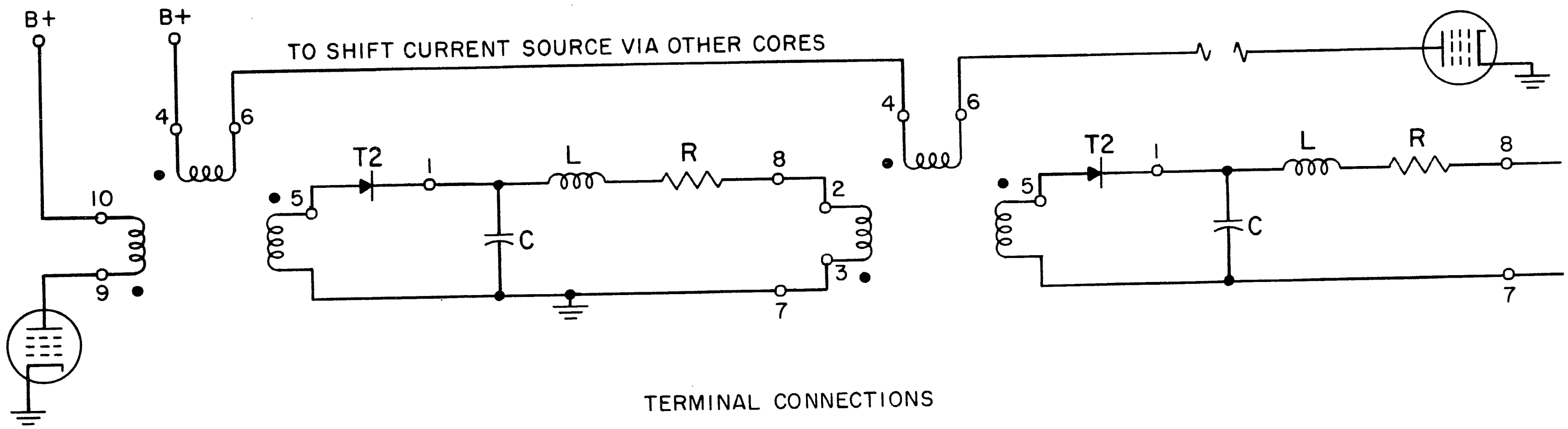
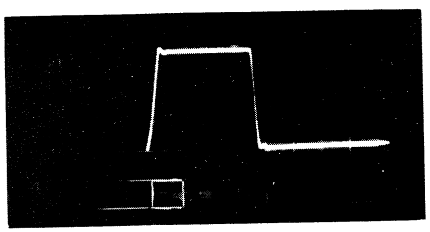


Fig. 3-1 Correlator Block Diagram (Autocorrelation Arrangement)

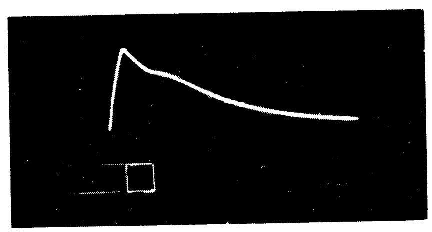


TERMINAL CONNECTIONS



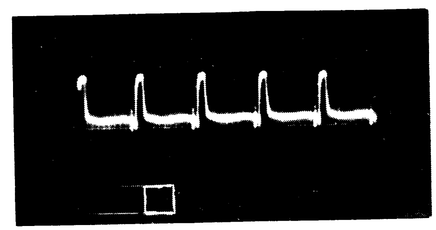
SHIFT PULSE

TERMINALS: 4;6
SQUARE SIZE: 50 MILLIAMPS
2 μSEC.



OUTPUT PULSE

TERMINALS: 1; GROUND
SQUARE SIZE: 5 VOLTS
5 μSEC.



WRITE PULSE

TERMINALS: 9; 10
SQUARE SIZE: 2 MILLIAMPS
50 μSEC.

Fig. 3-2 Shift Register Connections and Waveforms

Two sections of the shift register are shown schematically in Fig. 3-2. A photograph of the output voltage at terminal 1 is shown in the same figure. It has an amplitude of 15 volts, rise time of 4 microseconds, and decays exponentially with a time constant of approximately 18 microseconds. The shift pulse and write pulse are those actually obtained with the shift-pulse and write-pulse generators constructed for the correlator.

It was found that, with a write pulse width of 15 to 20 microseconds into terminals 9 and 10 of the first core, a minimum current of 2 milliamperes was needed for write-in. A value of 3 to 4 milliamperes was chosen as a design criterion to insure complete write-in every time. The foregoing measurements furnished the information needed to design the circuits to be used with the register.

B. SHIFT PULSE GENERATOR

This circuit is shown in Fig. 3-3. A free running multivibrator using a type 5965 twin triode is the basic timing device for the shift pulse generator. By adjusting the positive grid voltage level, the frequency of the multivibrator may be adjusted from 2 kcs to above the 12-kcs operating limit of the shift register. The square wave at the cathode of one side of the multivibrator tube is differentiated, with the resulting positive spikes being used to trigger a one-shot multivibrator. A diode in the grid circuit of the one-shot multivibrator prevents preliminary quenching of this circuit by the negative input spikes. Actually, this precaution is not essential for the present application since the period of the one-shot multivibrator is much less than one half the shortest period of the free-running multivibrator.

The one-shot multivibrator controls the shift pulse width and is set for 7.5 microseconds. The diodes in the grid circuit of the cathode follower, that forms the next state, accurately control the pulse height. The grid of V3A swings during the shift pulse (this latter value may be adjusted from approximately 0 to -40 volts by the amplitude control). The 180 uuf capacitor was added for more stable core switching. This capacitor increased the rise time by a factor of about 3 and reduced overshoot by a factor of almost 10. The trailing

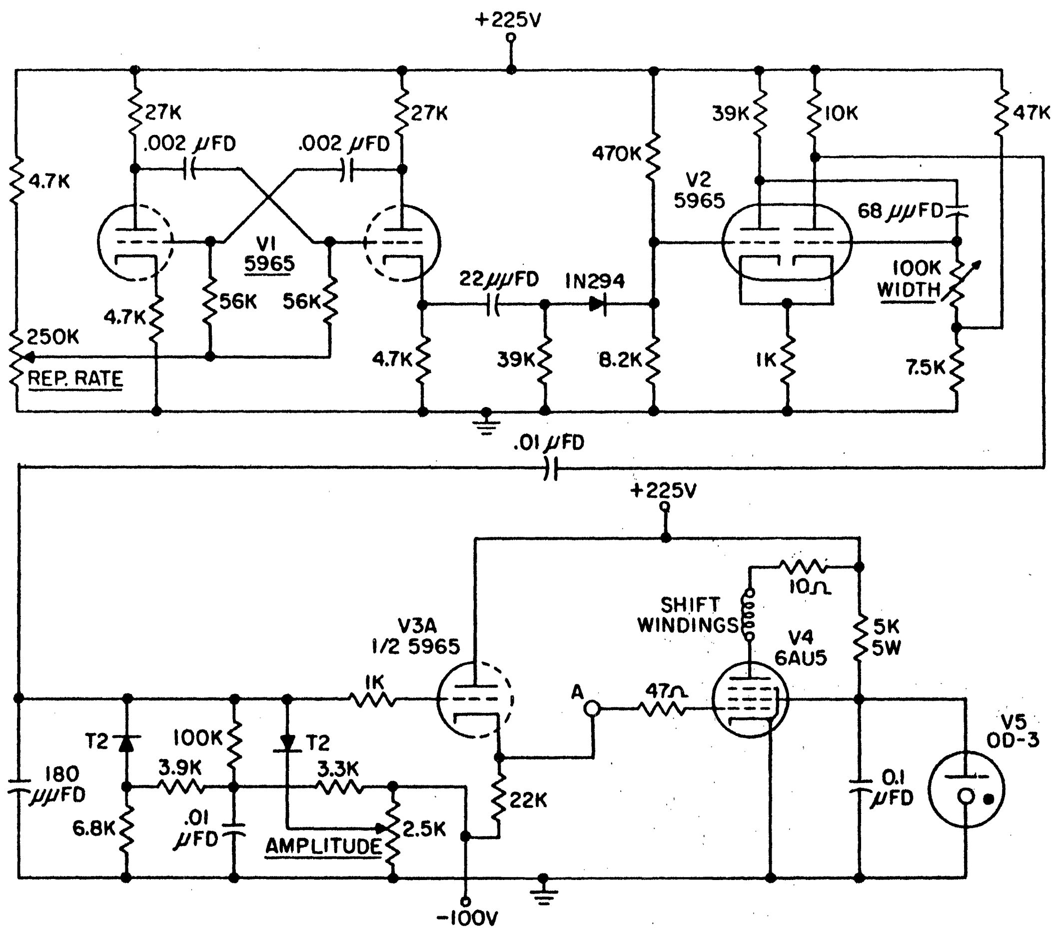


Fig. 3-3 Shift Pulse Generator

edge was modified to a lesser extent. The resultant pulse shape may be seen in Fig. 3-2. Before this step had been taken, there was considerable oscillation at the leading and trailing edges of the shift pulse.

The output of the cathode follower controls a pentode current source which drives the shift pulse windings. It will be noted that with this arrangement a grounded core winding may not be used. The 10-ohm resistor in the plate circuit is used for measuring purposes.

C. SAMPLING CIRCUIT

As was mentioned in Chapter II, write-in must not occur during the shift pulse. For this reason, it was decided to use the trailing edge of the shift pulse to initiate a gate control pulse for the write pulse generator. The output from V3A of the shift pulse source is differentiated, and the positive spike from the leading edge is clipped in the V3B grid circuit (Fig. 3-4). The negative spike from the trailing edge is

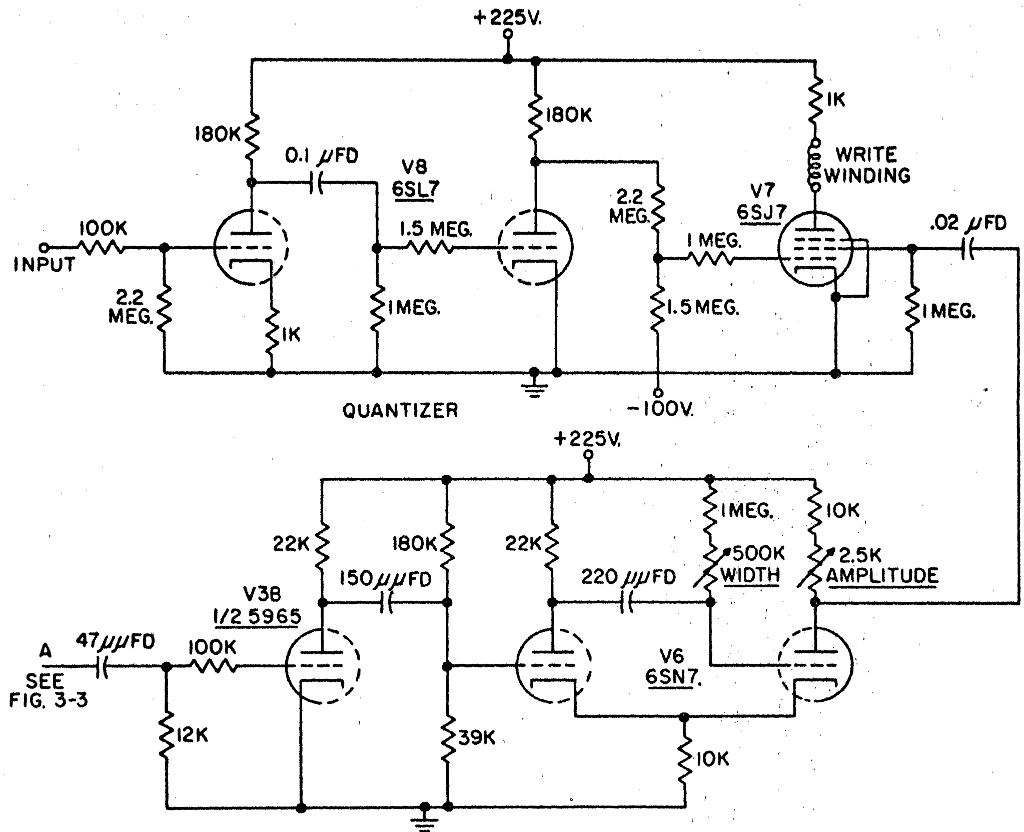


Fig. 3-4 Sampling Circuit

amplified and inverted. It is then used to trigger a one-shot multivibrator (V8) which controls the screen grid of the write pulse generator. The write pulse generator is not a standard pulse circuit. It was found that, if the control grid was raised to ground potential and the screen grid raised to 60 to 70 volts, then a current pulse of 4 to 4.5 milliamperes would be obtained. If this circuit were to be designed again, a coincidence gate of the type used for the multiplier circuits described in Section D of this chapter would be used. The present circuit works satisfactorily, however.

The quantizing circuit is used as one control for the write pulse generator, the previously described one-shot multivibrator being the other control. The quantizer takes all positive input voltages of any amplitude and makes them of sufficient amplitude to raise the control grid of the write pulse generator from -19 volts (below cut-off) to ground potential. Negative input voltages have no effect since they work to keep the write pulse generator cut off. Consideration was given to the use of a Schmitt trigger circuit (15, 22) to accomplish quantization. The Schmitt trigger tends to switch at different values, depending upon the direction from which the cross-over value is approached. This hysteresis affect makes very careful design necessary to reduce it to an acceptable amount; therefore, a simpler method was evolved. The high gain saturating amplifier (V8), shown in Fig. 3-4, will raise the pentode control grid (V7) to 0 volts, with inputs as small as 0.1 volt positive at frequencies as low as 4 cycles per second. The voltage at the pentode grid does not go above 0 volts because of clipping by grid current through a 1 meg-ohm resistor. Adjustment of circuit parameters can be made to cause the circuit to be more sensitive if desired. The quantizing circuit is d.c. coupled to the write pulse generator control grid.

D. OUTPUT CIRCUITS

The previous circuits take analog information; convert it to digital form (one bit or two levels); pass it into the magnetic core shift register; and control the advance of information through the register. Now this information must be taken out of the register and used to form the correlation function. Figure 3-5 shows one of the ten output channels which form points on the correlogram. The pulse available at terminal 1 of each core is pictured in Fig. 3-2. The output from each core is RC coupled to a 12AX7 twin-triode amplifier. The first stage of this amplifier is biased 4 volts below the -3.5 volt cut-off voltage. This is necessary to furnish absolute discrimination against ZERO's when they are present in the core. A ZERO

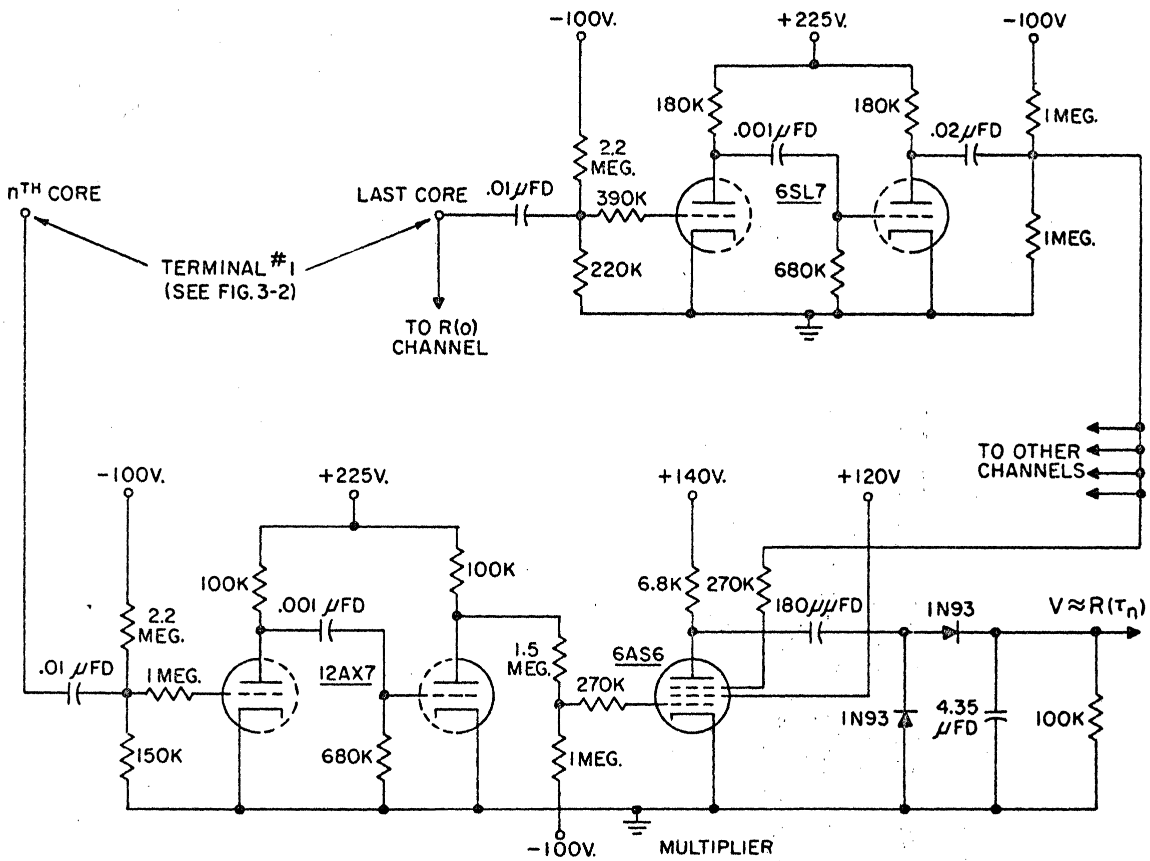
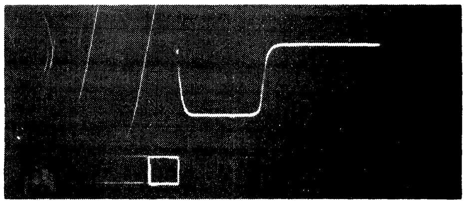


Fig. 3-5 Output Circuit (One Channel)

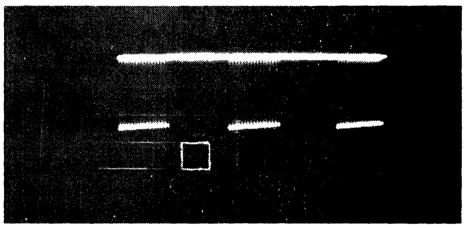
actually results in an output voltage of 2 or 3 volts at terminal 1 of each core (see Fig. 3-6c). This voltage will not bring the first stage of the amplifier out of cut-off. A ONE results in an output of 15 volts which not only brings the tube into conduction but will go beyond, to a point where grid current is drawn. The large resistor in the grid circuit results in sharp clipping and keeps loading of the core at a minimum when the grid reaches zero volts. After another stage of amplification, the triangular core output pulse has been



(a)

Pulse into averaging circuit

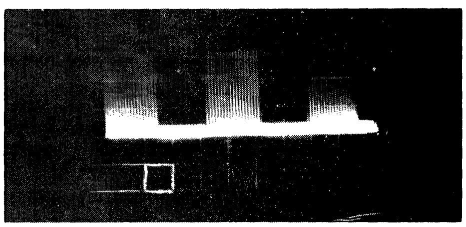
square size: 20 volts
1 microsecond



(b)

Train of pulses into averaging circuit (channel 3)

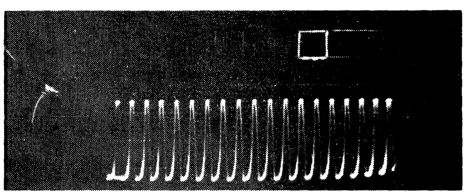
square size: 20 volts
1 millisecond



(c)

Train of output pulses at core # 0

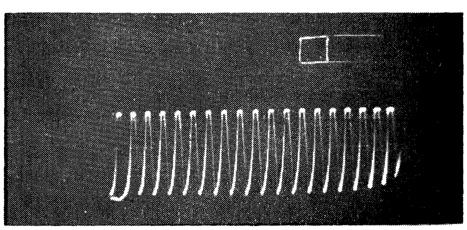
square size: 5 volts
1 millisecond



(d)

Input pulses to grid #3 of multiplier

square size: 20 volts
200 microseconds



(e)

Input pulses to grid #1 of multiplier

square size: 5 volts
200 microseconds

Fig. 3-6 Waveforms for Sinusoidal Input

shaped into a fairly rectangular pulse (Fig. 3-6e).* This pulse is d.c. coupled to the control (first) grid of a 6AS6 pentode coincidence gate.

The last core is connected to another two stage high gain amplifier in addition to the one described in the previous paragraph. The first stage of this amplifier is biased 4.5 volts below the cut-off value which for this tube (6SL7) is approximately -4 volts. The result is a slightly narrower pulse at the output of this amplifier than was obtained from the 12AX7 amplifier since it was obtained closer to the top of the triangular input pulse (Fig. 3-6d). The rectangular pulses are approximately 35 microseconds wide for the first described amplifier and 45 microseconds for the latter amplifier. The 6SL7 amplifier is coupled to the third (suppressor) grid of the 6AS6 coincidence gate in each of the ten channels.

Pulses at the two controlling grids of the 6AS6 coincidence gates raise both of these grids from a large negative voltage up to zero volts. When this occurs at both grids simultaneously, an output pulse occurs at the plate of the coincidence gate. This is equivalent to a 1 x 1 = 1 multiplication. Under any other circumstances (1 x 0 or 0 x 0), no output will occur. The width of the output pulse is controlled by the width of the narrower of the two pulses. This is the pulse from the 6SL7 amplifier whose output is common to all channels. The result is uniformity in the width of pulses from all ten multipliers (coincidence gates). The negative output pulses from the multipliers have an amplitude of approximately 55 volts. (See Figs. 3-6a and 3-6b.) It will be noted that careful selection of tubes and operating points insures that no output occurs for zeros. Compare Figs. 3-6b and 3-6c.

The output pulses from each multiplier are fed to an averaging circuit in the same channel. The circuit is the counting rate meter discussed in Chapter II. A derivation for

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* In Figs. 3-6d and 3-6e the time scale is reversed, increasing time going to the left.

the response of this circuit to a train of input pulses is given in Appendix II. Parameters (R_2 and C_1) for this circuit were chosen so that, at sampling frequencies up to 10 kc, output voltage would be less than one tenth the magnitude of the input voltage pulses. This is necessary for linearity of integration. The time constant of the averaging filter may be altered by varying the effective size of C_2. It was noted previously that the quantizer acted as a high pass filter with a break point at approximately 12.5 radians per second. This has the effect of eliminating very slow signal variations such as might occur in a non-stationary random input signal, but it also makes necessary the measurement of the average value by other means such as a d.c. voltmeter. An averaging filter time constant of 1 second would show a noise-to-signal ratio of approximately 0.3 for <u>continuous</u> averaging. Samples are being used here indicating the need for a longer averaging time. For an input with a zero mean, a sampling rate of 8 kcs would cause approximately 4,000 samples to be accumulated every second at $\tau = 0$. Lesser numbers would be accumulated for other values of τ. In the last analysis it becomes a matter of trying out different averaging times and looking at the results to determine what is satisfactory.

A measurement of the back resistance of the IN93 germanium diodes was made and indicated that shunting effect on the averaging circuit is negligible (with a back voltage of 100 volts, the diodes pass only about 30 microamperes).

E. DISPLAY CIRCUIT

A few possible arrangements for displaying the correlogram were discussed in Chapter II. More detailed information on a diode matrix display is given in Appendix III. For the present thesis work, only two output channels were constructed. These channels were connected to various magnetic cores with a probe, and the output was read with a vacuum tube voltmeter.

The construction of ten duplicate channels and an associated switching and display circuit was not vital to this investigation. Schemes for doing this have been devised. If simplicity was the most important factor, each core could be connected through a hand-operated selector switch to a single output channel. The value for each channel could be obtained from a meter reading in about ten seconds. It would be necessary to maintain contact long enough for the average to be formed. Two dozen points could be recorded in about four minutes. If it is desired to make the display automatic, and faster, the equipment becomes comparatively complicated. A separate channel must be constructed for each point on the correlogram so that all averages are continuously ready for display. As mentioned in Chapter II, there is now a choice between a mechanical and electronic commutator for switching, with the decision resting heavily on economic considerations.

Reserving one channel for horizontal sweep synchronization, a 63-point correlogram using a diode matrix would require from 176 to 384 diodes, depending on the matrix configuration used (23). In addition, a total of at least 76 vacuum tubes or transistors would be required. Thus, it might be well to consider using a motor-driven selector switch. Reference (17) describes a 22-point rotary selector switch driven by a stepping magnet. This switch may readily be modified for use with 44 channels. High-speed mechanical commutators have been developed to sample 60 to 100 circuits at speeds up to 2,000 rpm; and (24) describes a mercury jet switch which will sample 120 circuits, 60 times a second.

Special-purpose commutating tubes, such as the cyclophon and radial beam tube, are not considered practical for this correlator and are mentioned here only in the interest of indicating their existence.

CHAPTER IV

TESTS AND CONCLUSIONS

This chapter discusses some of the tests made with the correlator to determine its performance. Certain aspects of the circuits described in the previous chapter are mentioned in relation to their effect on the results. Comparisons are made between expected results and the actual correlograms obtained for inputs such as sinusoids, triangular waves, square waves, white noise, and noise shaped by filters.

The chapter ends with a few comments on the direction that circuit development for a one-bit correlator might take, and some general conclusions.

A. CORRELOGRAMS OF PERIODIC FUNCTIONS

The first correlograms taken were for a sinusoidal input from an H.P. Model 202B audio oscillator. Readings were made with an RCA voltohmyst junior. Various input frequencies and sampling rates were tried. The circuit parameters for the output filter were varied and the effects noted. It is believed that Figs. 4-1 and 4-2 can be used to summarize the important characteristics of the correlogram for this type of input. The output filter was made up of a 100,000 ohm resistor in parallel with 4.35 microfarads of capacitance. Increasing the size of the resistor raises the amplitude of the output, thus reducing the linearity of integration. To increase the time constant, the value of capacitance must be increased. This is discussed at more length in Chapter II. The correlograms for a sinusoid can be seen to have certain properties which are listed below:

(1) The frequency of the input is accurately indicated.

(2) The correlogram has a triangular form.

(3) The amplitude increases with increased sampling rate.

(4) The output filter voltage slowly fluctuates about an average value.

(5) The minimums are flattened.

A brief discussion of the properties is made in the paragraphs that follow.

There is nothing startling about the fact that the correlator reports the frequency of the input signal accurately. Of course, the use of two level quantization on the input means

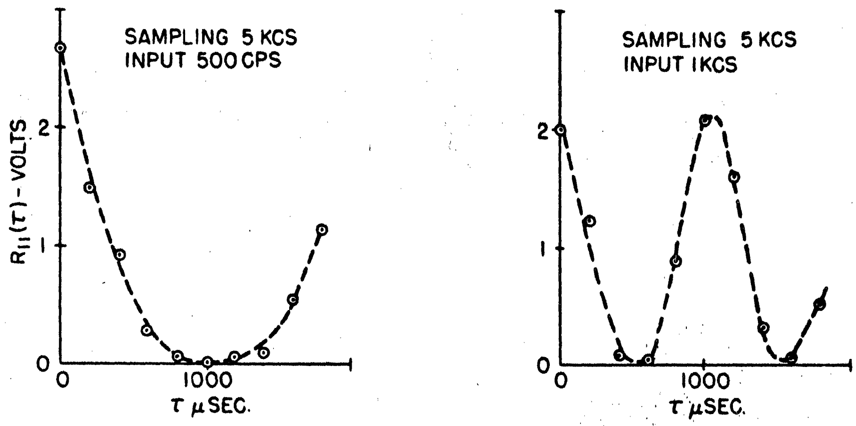

Fig. 4-1 Correlograms of Sinusoidal Inputs

that triangular waves, sinusoids, and square waves all look like square waves after quantization. Hence, all of the waves will show the triangular autocorrelation function of a square wave. This was verified by using a Hewlett Packard function generator to furnish all of these inputs. Of course, these inputs could be distinguished by the correlator if the crossover point of the quantizer were varied or if a d-c level were added to the input. In this case the square wave should not be affected; but the correlation functions of triangular and sinusoidal waveforms will be flattened at the top if a positive level is added, and at the bottom if a negative level is added.

The amplitude of the output is dependent upon properties of the counting rate meter, as discussed in Chapter II and summarized by equation 2-8.

There are several things which might cause the output amplitude to fluctuate. Drift in the clock frequency would cause any given channel to be computing for fluctuating values of τ. Drift in amplitude or zero value of the input would also cause the output to fluctuate since the crossover point in the quantizer would be affected. The two stages of the quantizer

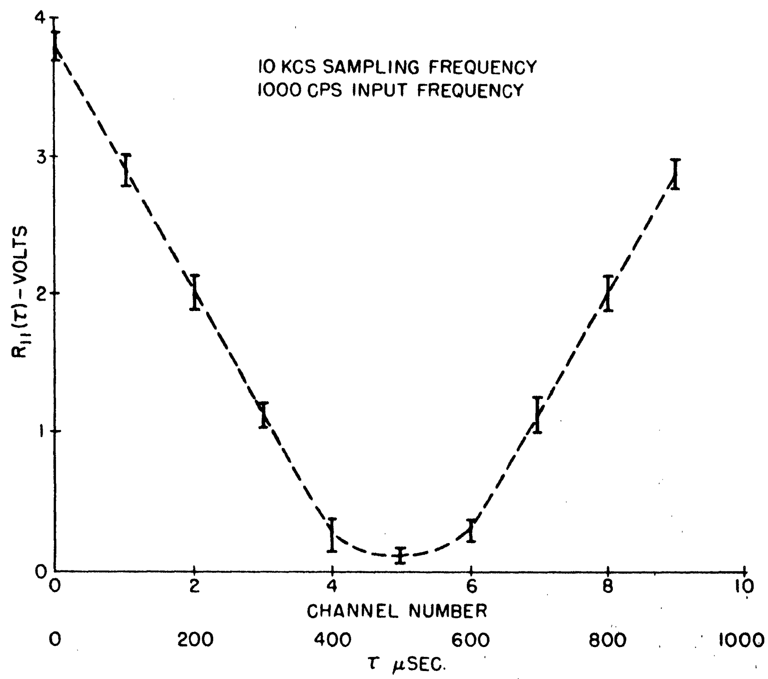

Fig. 4-2 Correlogram Fluctuations

have gains of approximately 43 and 57, or a total gain of almost 2,500. Thus, by nature it is extremely sensitive to input noise, changes in signal strength, and drift in supply voltage. Another source of output fluctuation is drift in the coincidence gates used for write-in. This may be caused by shifts in the clock frequency (thus varying the number of samples taken when the input is positive). Any drift in the amplitude of pulses from the output multiplier gate will also cause the correlation function to fluctuate. Thus, it seems that refinement of the circuitry would be the first step in reducing the fluctuations. It might be noted that it is fairly easy to average, by eye, the output variations indicated on a voltmeter. This was done for the correlograms presented herein.

The cause of flattening of the minimums is more difficult to explain. The fact that the quantizer shifts at a voltage slightly higher than zero volts means that, for a sinusoidal input with zero mean, the shift register receives a few more ONE's than ZERO's. The output of the quantizer is a train of pulses which do not quite form a square wave. The correlation function of such a wave is flattened slightly at the bottom. This effect should be less noticeable when the input actually is a square wave. This was checked, but flattening at the bottom of the correlogram was not reduced. Therefore, another explanation is needed.

A more likely cause of flattening of a correlogram having a triangular shape is inherent in the use of two-level quantization. The input is two-valued. Van Vleck (25) and Bussgang (26) discuss the effect on a correlation function of extreme clipping of an input signal so that it is two-valued. They show that the actual correlogram is proportional to the arc sine of the theoretical correlation function.* This relationship will translate a straight slope into a curve that is steep at maximums and somewhat flattened at minimums. This effect is quite apparent in Fig. 4-1.

B. CORRELOGRAMS OF RANDOM FUNCTIONS

White noise from a Scott noise generator, Type 810-A, was used to determine the response of the correlator to random functions. The test equipment arrangement is shown in the block diagram of Fig. 4-3a. The noise generator furnished noise in a range 20 cps to 20 kcs. The McIntosh 20 W-2 amplifier has a response within ± 0.2 db over the same range. The filter circuit and its measured transfer function are shown in Fig. 4-3b and 4-3c, respectively. The filter characteristic was measured with input and output impedance connections, the same as those present in the test circuit.

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* See Appendix V.

Correlograms were made with a white noise input and with a band limited noise input.

1. <u>White Noise</u>

The circuit for this test is shown in Fig. 4-3a where the filter bypass switch is closed. The results are summarized by the curves on the lower half of Fig. 4-4. Variation in the gain setting of the McIntosh amplifier results in a change of

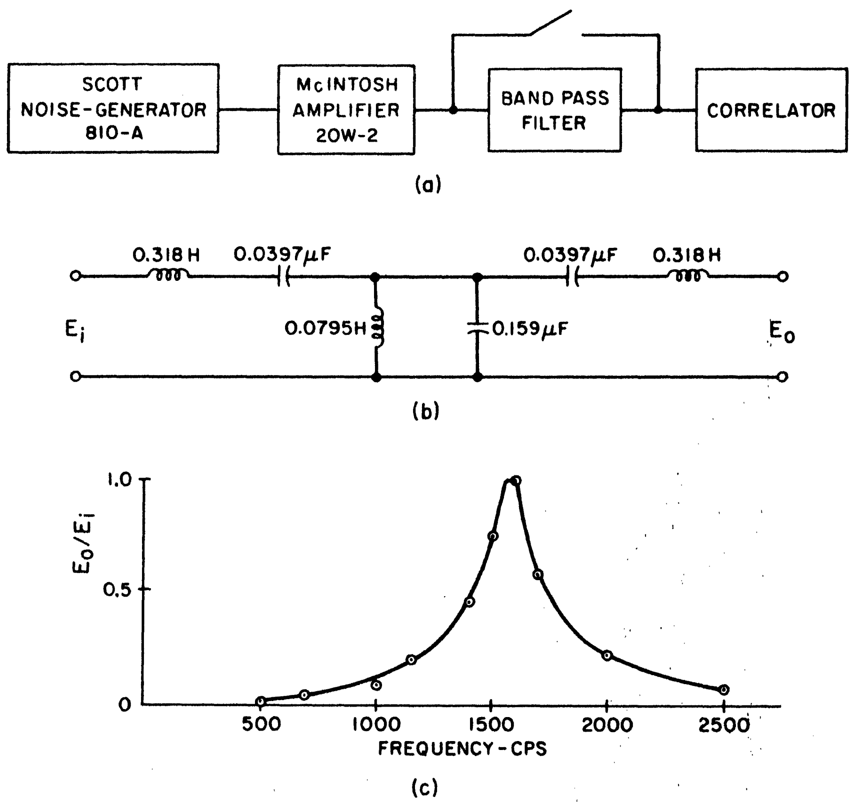

Fig. 4-3 Test Equipment Arrangement and Filter Characteristics

the mean square value of the input but in no change in the form of the correlation function. This is as expected. The autocorrelation function of true white noise of infinite bandwidth is an impulse. The correlogram shows the noise here to be virtually uncorrelated for τ shifts greater than 150 microseconds.

2. <u>Band-limited Noise</u>

According to Wiener's theorem, the autocorrelation function of a random signal is the Fourier cosine transform of its power density spectrum. Therefore, by shaping the power density spectrum with filters of known characteristics, the theoretical autocorrelation function may be computed. Thus,

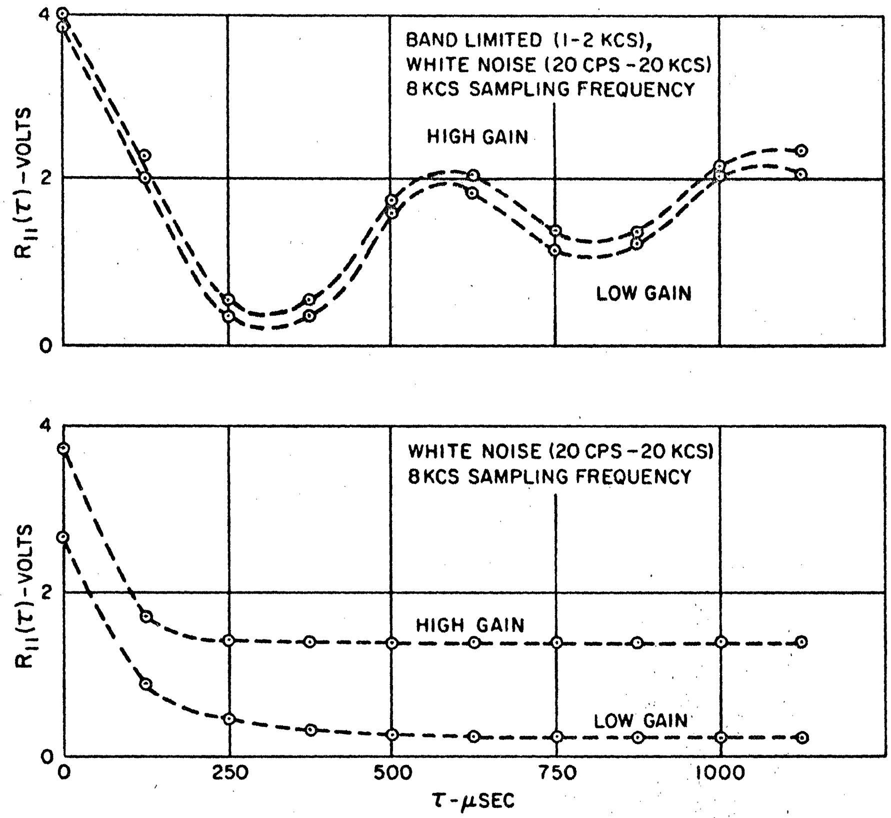

Fig. 4-4 Amplifier Gain Effects on Correlograms of Noise Inputs

if white noise were band limited by an ideal rectangular filter, the resulting correlation function would have the form*

$$R_{11}(\tau) = K \left(\frac{w_2 \sin w_2 \tau}{w_2 \tau} - \frac{w_1 \sin w_1 \tau}{w_1 \tau}\right) \qquad 4\text{-}1$$

where w_2 is the higher of the two frequency limits and K is a constant. Since the above filter is not realizable, a more practical formula is the following one which is developed in (27).

$$R_{11}(\tau) = e^{-\frac{w_0 |\tau|}{w_q}} \cos w_0 \tau \qquad 4\text{-}2$$

* See Appendix IV.

where w_0 is the resonant frequency of a high Q circuit. Both formulas describe curves which are similar in general shape.

Reference to the filter characteristic of Fig. 4-3c show that the filter used for these tests had a Q of approximately 8. Thus, according to equation 4-2, $R_{11}(\tau)$ is a decaying cosine wave which should decrease to half value in a little over 1,100 microseconds.

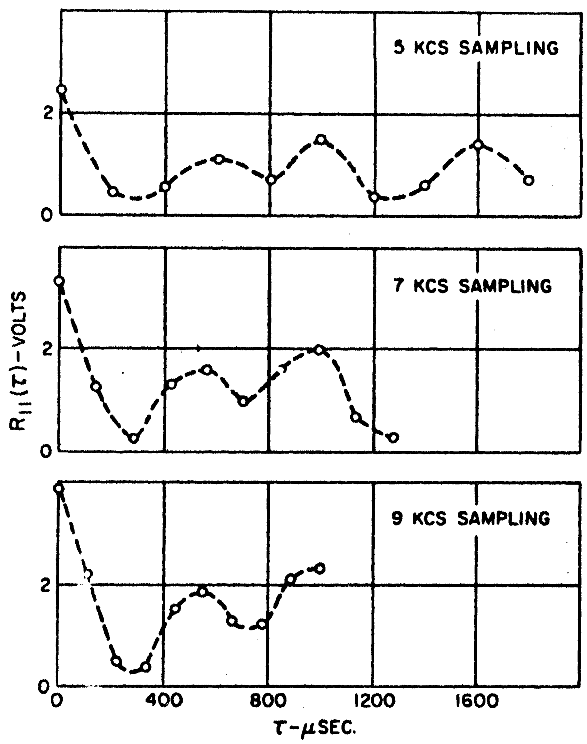

Fig. 4-5 Effects of Sampling Rate on Correlograms of Band Limited Noise

Figure 4-5 shows the correlograms obtained using the circuit arrangement of Fig. 4-3a for various correlator sampling rates. Increasing the sampling rate increases the gain and the detail, while shortening the total range of τ that is obtained. There appear to be frequency components of 1 kcs and 2 kcs superimposed in the correlogram. The frequency expected accord-to equation 4-2 is approximately 1,600 cps. The maximum value occurs for $\tau = 0$, as anticipated, but the curves do not decay as predicted. It is felt that a longer shift register, furnishing a greater range of τ, is needed to make a more detailed comparison of experimental and theoretical results.

C. COMMENTS ON POSSIBLE CIRCUIT IMPROVEMENTS

Previous comments showed the possibility of correlogram errors due to the quantizer. The coincidence gate used for write-in may only do a partial job if the input changes polarity during the time of the sampling pulse. A more stable clock would be desirable to make free from drift the values of τ

which each magnetic core represents. The output multiplier seems to be satisfactory although pulses of higher amplitude would be desirable. These comments do not get to the real center of the problem, however. A one-bit multichannel correlator shows promise of furnishing useful correlograms with a minimum of equipment and time. The most important step seems to be the minimization of equipment bulk by use of transistors.

It is believed that all of the circuits could be readily transistorized with the possible exception of the shift pulse generator which must furnish large currents. This problem might be lessened by proper selection of the magnetic cores which make up the shift register.

The magnetic core shift register operates very well as a delay line. It does not have the attenuation present in artificial lines, and it is almost as accurate in time as the clock that regulates it (small time variations in the output from individual cores are possible).

D. CONCLUSIONS

The principal conclusion is that a two-level correlator is a useful device. It is the ultimate in simplicity and furnishes recognizable correlograms in a minimum of time. With the circuits used in this investigation, only 10 vacuum tubes, exclusive of power supply requirements, are needed to furnish one channel. Two additional tubes are required for each additional channel. A true real time correlator would require a channel for each value of τ and would also require a switching circuit. These requirements necessarily add to the bulk and cost of the equipment, but it would still be simple compared to correlators presently in existence.

If one were dealing with signals which were stationary over a period of a few minutes, a simple one-channel correlator with a hand-operated switch to move from core to core in the shift register could be used to quickly obtain a dozen or more points on a correlogram (see Chapter III). One then has a portable correlator that can be wheeled around as easily as an oscilloscope or wave analyzer. Such a correlator could be useful in obtaining immediate results from bioelectric signals, music, speech; or it might be used as an inexpensive classroom or laboratory demonstrator in the study of statistical functions.

As experience with a two-level correlator is gained, it should be possible to obtain more information with this correlator than might at first appear possible. For example, it was pointed out earlier that by varying the crossover point of the quantizer, square waves, sinusoids, and triangular waves may be distinguished. This would not be possible with two-level quantization without this simple artifice.

It is felt that the results indicated in this thesis warrant construction of two-level (one-bit) correlators. The magnetic core shift register is an excellent delay and temporary storage device for such correlators.

APPENDIX I

DERIVATION OF INPUT-OUTPUT RELATIONSHIPS FOR TWO BASIC INTEGRATORS

Notation follows that in Fig. 2-4.

A. MILLER INTEGRATOR

$$E_o = -KE_i \quad ; \quad i_C = i_K = \frac{E - E_i}{R} = \frac{E + \frac{E_o}{K}}{R}$$

$$i_C = C\,\frac{d(E_i - E_o)}{dt}$$

$$E_i - E_o = -\frac{E_o}{K} - E_o = -E_o\left(\frac{1 + K}{K}\right)$$

Combining the above relations gives the following expression:

$$-\frac{C(1 + K)}{K}\,\frac{d\,E_o}{dt} = \frac{E}{R} + \frac{E_o}{RK}$$

$$\frac{d\,E_o}{dt} + \frac{1}{RC(1 + K)}\,E_o = -\frac{KE}{RC(1 + K)}$$

$$E_o = -KE\left[1 - e^{-t/RC(1 + K)}\right]$$

B. BOOTSTRAP INTEGRATOR

$$C\,\frac{d\,E_o}{dt} = i \quad ; \quad i = \frac{E + KE_o - E_o}{R} = \left.\frac{E}{R}\right|_{K = 1}$$

then

$$\frac{d\,E_o}{dt} = \frac{E}{RC} \quad \text{and} \quad E_o = \frac{E}{RC}\,t$$

If a series expansion were made of the expression for the Miller Integrator where $-K \rightarrow \infty$, the result would approach the expression for the Bootstrap Integrator.

APPENDIX II

A DERIVATION FOR THE IMPULSE RESPONSE OF THE COUNTING RATE METER

Notation follows that of Fig. 2-5.

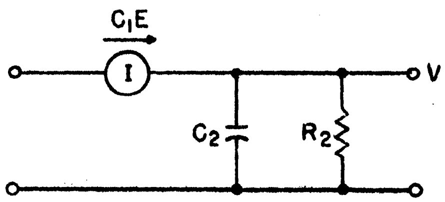

Fig. A-1 Low-Pass Filter

Each pulse charges C, to a value E, therefore the following equivalent circuit applies provided R $\gg$ V. The width of the input pulses is assumed to be negligible in this derivation. For a single input pulse

$$V(s) = IZ = C_1E \frac{1/C_2}{S + 1/RC_2}$$

The input train of pulses is $\dfrac{C_1E}{1 - e^{-sT}}$

where T is the sampling period.

Then,

$$V(s) = \frac{C_1E}{C_2}\left(\frac{1}{1 - e^{-sT}} \cdot \frac{1}{s + 1/\tau}\right) \quad \text{where } \tau = R_2C_2.$$

The residue in the pole at $-1/\tau$ is

$$\frac{C_1E}{C_2}\left(\frac{1}{1 - e^{T/\tau}}\right)$$

Removing the pole at $-1/\tau$ from the previous expression, we have

$$\frac{C_1E}{C_2}\left[\frac{1}{1 - e^{-sT}} \cdot \frac{1}{S + \frac{1}{\tau}} - \frac{\frac{1}{1 - e^{T/\tau}}}{S + \frac{1}{\tau}}\right] = \left(\frac{1 - \frac{1 - e^{-sT}}{1 - e^{T/\tau}}}{S + \frac{1}{\tau}}\right)\left(\frac{1}{1 - e^{-sT}}\right)\frac{C_1E}{C_2}$$

The first term in the resulting expression represents a pulse, and the second term represents repetition of this pulse. The form of the first term may be rearranged as follows:

$$\mathcal{L}^{-1}\left(\frac{\dfrac{-e^{T/\tau} + e^{-sT}}{1 - e^{T/\tau}}}{s + \dfrac{1}{\tau}}\right) = \frac{1}{e^{T/\tau} - 1}\left[e^{T/\tau} e^{-t/\tau} u(t) - e^{-(t-T)/\tau} u(t-T)\right]$$

This pulse has the following form (see Fig. A-2a) after reinserting the amplitude factor C_1E/C_2. The total response V(t) is then a string of the above pulses added to the response for the pole which was previously removed. The final result has the following form (see Fig. A-2b)

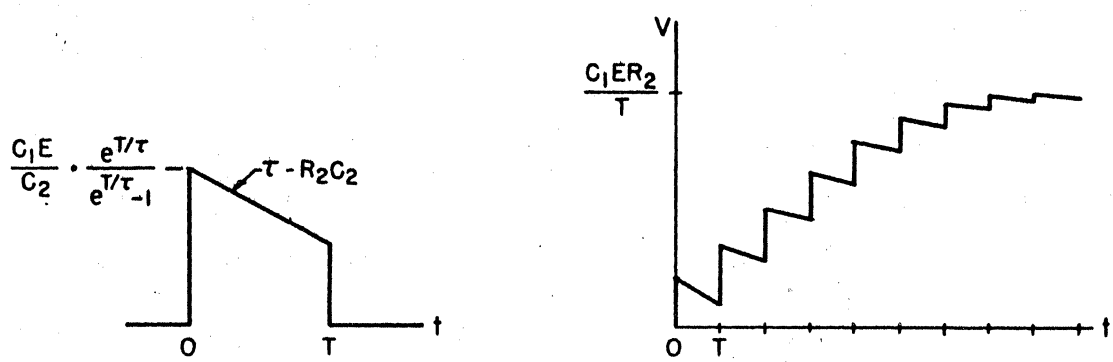

Fig. A-2 Counting Rate Meter Output Voltage Waveform

The foregoing discussion shows the manner in which pulses occurring periodically would charge the output capacitor C_2. If the input pulses occurred randomly with an average rate n, then Campbell's theorem may be used to show the fluctuation of V(t) about its final value $V = nC_1ER_2$. (15)

$$(\Delta V)^2 = \overline{\left(V(t) - V\right)^2} = n \int_{-\infty}^{\infty} V^2(t)dt - V^2$$

where V is the average value of V(t). The response to a single pulse at t = 0

$$V(t) = \frac{C_1}{C_2} E\, e^{-t/R_2C_2} \qquad \text{for} \quad t \geq 0$$

Substituting in the previous expression, the result is

$$(\Delta V)^2 = \frac{n\ C_1^{\,2} E^2 R_2}{2\ C_2} - V^2 \quad *$$

The relative error is then

$$\frac{\Delta V}{V} = \frac{1}{\sqrt{2\ R_2\ C_2\ n}}$$

* This corrects an error in Reference (15) at this step.

APPENDIX III

A DIODE MATRIX SWITCHING CIRCUIT

A block diagram of this circuit, simplified for seven values of τ and a channel for oscilloscope synchronization, is shown in Fig. A-3. A complete description of a similar

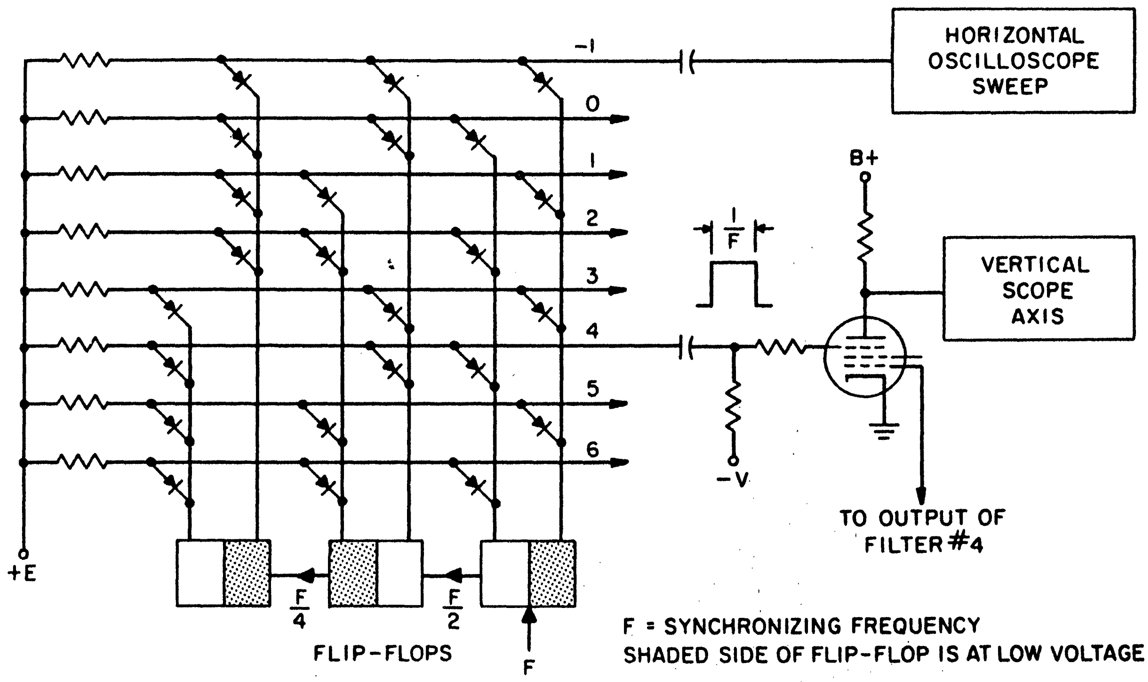

Fig. A-3 Block Diagram of Diode Matrix Switching System

switching system may be found in (12) and (13). Details concerning the operation of a diode matrix may be found in (23). The coincidence gates are biased to cut-off until the diode matrix places a zero or slightly positive voltage on the suppressor grid. This is done sequentially by action of a chain of flip-flop multivibrators on the matrix. The amount of conduction from each coincidence gate is then controlled by the voltage which one of the output filters maintains on the control grid. Thus, the output to the oscilloscope takes on voltages, in time sequence, which are determined by the various output filter voltages. Figure A-3 shows some of the detail for channel 4.

One of the outputs from the matrix is used to synchronize the horizontal sweep of the oscilloscope. The entire switching circuit may be synchronized by the clock in the shift pulse source although this is not essential.

APPENDIX IV

CALCULATION OF AUTOCORRELATION FUNCTION OF BAND LIMITED NOISE

If an ideal band pass filter were realizable, the power spectrum of white noise might be shaped as shown below. Now, according to Wiener's theorem

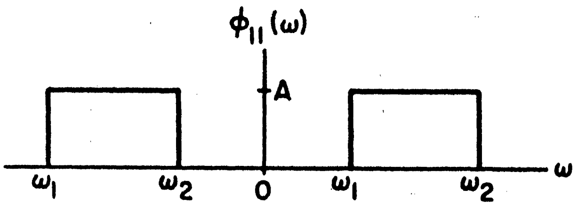

Fig. A-4 Power Spectrum of White Noise Passed Through an Ideal Band-Pass Filter

$$R_{11}(\tau) = \int_{-\infty}^{\infty} \Phi_{11}(w) \cos \omega\tau \, dw.$$

In this case then

$$R_{11}(\tau) = 2A \int_{w_1}^{w_2} \cos \omega\tau \, dw$$

$$= \frac{2A}{\tau} \sin \omega\tau \Big|_{w_1}^{w_2}$$

$$= 2A \left(w_2 \frac{\sin w_2 \tau}{w_2 \tau} - w_1 \frac{\sin w_1 \tau}{w_1 \tau} \right)$$

Actually of course such a filter is not realizable. However, it can be shown (2) that

$$\Phi_{11}(w) = \left| H(w) \right|^2 \Phi_{ii}(w)$$

This takes into account the filter frequency characteristic H(w). For white noise $\Phi_{ii}(w)$ is a constant. Equation 4-2 is derived from this last relationship.

APPENDIX V

CORRELATIONS OF TWO LEVEL NOISE SIGNALS (25)

Given two signals X and Y having Gaussian amplitude distributions, the joint probability that X lies between X and X + dX and that Y lies between Y and Y + dY is

$$\frac{1}{2\pi\sqrt{1-r^2}}\, e^{-(X^2 + Y^2 - 2\,rXY)\,/\,2(1-r^2)}\, dX\, dY$$

where $r = \overline{XY}$. The correlation function is then found by using this expression in formula 1-2 of the first chapter.

If X and Y are two valued ($\pm$ 1), the correlation function is given by

$$R(\tau) = \frac{1}{2\pi\sqrt{1-r^2}} \left[\int_0^\infty \int_0^\infty e^{-a} dX\, dY + \int_{-\infty}^0 \int_{-\infty}^0 e^{-a} dX\, dY - \int_0^\infty \int_{-\infty}^0 e^{-a} dX\, dY - \int_{-\infty}^0 \int_0^\infty e^{-a} dX\, dY \right]$$

where

$$a = \frac{(X^2 + Y^2 - 2\,rXY)}{2\,(1 - r^2)}$$

If polar coordinates $X = \rho \cos \varphi$ and $Y = \rho \sin \varphi$ are introduced, the above expression can be written as

$$R(\tau) = 4 \int_0^{\pi/2} d\varphi \int_0^\infty \frac{1}{2\pi\sqrt{1-r^2}}\, e^{\frac{-\rho^2\,(1-r\,\sin 2\varphi)}{2\,(1-r^2)}}\, \rho\, d\rho - 1$$

$$= \frac{2\sqrt{1-r^2}}{\pi} \int_0^{\pi/2} \frac{d\varphi}{1-r\,\sin 2\varphi} - 1 = \frac{2}{\pi} \sin^{-1} r$$

which says that the correlation function of the two-valued signal is $2/\pi$ times the arc sine of the continuous (unclipped) correlation function.

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