UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

LIBRARY ROUTINE H 5 - 85

| | |
|---|---|
| TITLE | Minimization of a Function of n Variables (DOI or SADOI) |
| TYPE | Closed |
| NUMBER OF WORDS | 89 |
| TEMPORARY STORAGE | n + 1 words at S3<br>n + 1 words at S4<br>n + 1 words at S5 |
| ACCURACY | Depends upon the condition of the function |
| DURATION | A "minor" cycle takes T = (.5 + 1.4n + t) ms where n is the number of variables and t = time to compute $f(x_1, \ldots, x_n)$. A "major" cycle takes (1.8 + 4.5n + 2nt + NT) where N is the number of "minor cycles" in a major cycle. |
| PRESET PARAMETER | S3 - SN |

| | |
|---|---|
| 3 - 00 F 00 aF | where a, a+1, ..., a+n are the addresses of $f(x_1, \ldots, x_n)$, $x_1$, $x_2$, ..., $x_n$ on entry and exit; |
| 4 - 00 F 00 bF | b, b+1, ..., b+x are the addresses of $\delta$, $\delta_{x_1}$, ..., $\delta_{x_n}$; |
| 5 - 00 F 00 cF | c, c+1, ..., c+n are working spaces; |
| 6 - 00 F 00 αJ | α is the factor by which the mesh size is decreased; |
| 7 - 00 F 00 ϵJ | ϵ is an end constant such that only mesh sizes <u>larger</u> than ϵ will be used; |
| 8 - 00 F 00 sF | s is the address of a <u>closed</u> subroutine which takes $x_1$ from c+1 (1S5), $x_2$ from c+2 (2S5), ..., $x_n$ from c+n (nS5) and places $f(x_1, x_2, \ldots, x_n)$ in $R_1$; |
| 9 - 00 F 00 (a+n)F | n is the number of variables |
| K - 00 F 00 (b+n)F | |
| S - 00 F 00 (c+n)F | |
| N - 00 F 00 rF or 00 F 40 F | The preset parameter in N allows the programmer some choice in scale; if |

it is 00 F 00 rF

$\delta_{x_i}$ will be computed as:

$$\delta_{x_i} = \frac{2^{-r} f_{x_i}}{\sum_i |2^{-r} f_{x_i}|}$$

whereas if it is 00 F 40 F we set r = 0 in the above expression. In the above:

$$f_{x_i} = f(\ldots x_i - \delta \ldots) - f(\ldots x_i + \delta \ldots)$$

In either case if the denominator exceeds scale it is replaced by $1 - 2^{-39}$.

ENTRY

$x_1^0, x_2^0, \ldots x_n^0$ the initial values of $x_1, x_2, \ldots, x_n$ are to be loaded in 1S3, 2S3, ..., nS3. $\delta_0$, the initial value of $\delta$ is to be in $R_1$, and the main program should contain:

| | |
|---|---|
| p | 50 rF |
| | 50 pF |
| p+1 | 26 qF |

where q is the address of this routine and r is the address to the left hand side of which control will be transferred before every decrease in mesh size. At this address the programmer may place a routine to assess or print intermediate results. This routine should return control to the left hand side of (q+71). This can be done automatically since when control is transferred to r(L.H.) the appropriate return address will be in the right hand address position of $R_2$ and can be utilized by S5, 42 into a 26 at the end of the programmer's interlude routine. The best current values of $f(x_1, \ldots, x_n)$, $x_1, \ldots, x_n$ are always in S3, 1S3, ..., nS3. If it is not desired to leave this code, set r = q + 71. The routine is finally left with the best values of $f(x_1, x_2, \ldots, x_n)$, $x_1, x_2, \ldots, x_n$ in S3, 1S3, 2S3, ..., nS3 respectively.

DESCRIPTION OF METHOD

1 - $f_{x_i}$ are computed

2 - $\delta_{x_i}$ are computed

3 - $f(x_1 + \delta_{x_1}, \ldots, x_n + \delta_{x_n})$ is computed and tested against previous $f(x_1, \ldots, x_n)$. If the function has decreased the old values of function and argument are replaced by the new. The process is repeated with the same $\delta_{x_i}$. This step comprises a "minor" cycle.

4 - When the above process ceases to improve the function, we return to step 1 and compute new $\delta_{x_i}$. Steps 1-4 comprise a "major" cycle.

5 - When the first "minor" cycle of a given "major" cycle fails to improve the function, indicating that no further improvement is to be expected using the current mesh size, control is transferred to the interlude as explained above.

6 - Upon returning to the code, $\delta$, the mesh size will be replaced by $\alpha\delta$ where $\alpha$ is entered in S6 during read-in. The steps 1-5 are then repeated.

7 - Steps 1-6 will be repeated (N+1) times where N is the largest integer for which $\alpha^N\delta_0 > \epsilon$. $\epsilon$ is to be entered in S7 during read-in.

NOTE

A function, poorly conditioned in having a very small gradient with respect to some argument, can deceive this routine in that this coordinate of the minimum will be very poorly found. If the programmer suspects that such a condition exists, this routine can be used for roughing and Routine H 6 - 86, a brute force approach to the minimum by varying one argument at a time, may be used for finishing. All parameters (except SK and SN), entry and contents of storage location of this code and Routine H 6 are identical, so that they may be used at the same time in the machine for the above purpose.

Routines H 3, H 4, H 5, and H 6 constitute a set sufficient (perhaps not necessary) to minimize any function of n variables. Due to economy of storage, Routine H 3 should be used for n = 1 or 2, Routine H 4 for n = 3 or 4, and Routine H 5 for $n \geq 5$.

DATE July 21, 1954 rt. 3/5/59
CODED BY J. N. Snyder
APPROVED BY J. P. Nash

lgr

| LOCATION | ORDER | | NOTES | PAGE 1 | K 5 |
|---|---|---|---|---|---|
| | 00 K(H5) | | | | |
| 0 | 40 S4 | | Store $\delta_0$ | | |
| | S5 1F | | Move in link | | |
| 1 | 46 60L | | Plant interlude address | | |
| | L4 L | | | | |
| 2 | 42 73L | | Plant link | | |
| | L5 3L | | Set 10L for initial entry | | |
| 3 | 50 S8 | | | | |
| | 46 10L | | | | |
| 4 | L5 77L | From 12, 73 | Set initial addresses in transfer orders | | |
| | 40 5L | | | | |
| 5 | L5 F | From 8 | $x_1^0, \ldots, x_n^0$ to 1S5, ..., nS5 | | |
| | 40 F | By 4, 7 | | | |
| 6 | L5 5L | | | | |
| | L4 76L | | Advance addresses in transfer orders | | |
| 7 | 40 5L | | | | |
| | L5 80L | | | | |
| 8 | L0 5L | | Test for i = n | | |
| | 36 5L | | | | |
| 9 | 50 131 | | | | |
| | 50 9L | | | | |
| 10 | 26 F | By 3, 11 | $f(x_1, \ldots, x_n)$ to S3 | | |
| | 40 S3 | | | | |
| 11 | L5 9L | | Set 10L for subsequent entry | | |
| | 47 10L | | | | |
| 12 | 26 4L | | | | |
| | 00 F | | | | |
| 13 | L5 60L | From 10 | Set switch for failure on first try | | |
| | 46 63L | | | | |
| 14 | L5 77L | | | | |
| | 46 19L | | | | |
| 15 | 46 23L | | Set i = 1 in 19 - 27 | | |
| | 40 27L | | | | |
| 16 | L5 22L | | | | |
| | 46 20L | | | | |

| LOCATION | ORDER | | NOTES | PAGE 2 |
|---|---|---|---|---|
| 17 | 46 24L | | | |
| | L5 79L | | | |
| 18 | 42 22L | | | |
| | 40 26L | | | |
| 19 | L5 F | By 14, 29 | | |
| | L4 S4 | From 35 | | |
| 20 | 40 F | By 16, 31 | | |
| | 50 20L | | | |
| 21 | 26 S8 | | | |
| | 10 SN | | Compute $f_{x_i}$ | |
| 22 | 50 1S5 | | | |
| | 40 F | By 18, 34 | | |
| 23 | L5 F | By 15, 30 | | |
| | L0 S4 | | | |
| 24 | 40 F | By 17, 32 | | |
| | 50 24L | | | |
| 25 | 26 S8 | | | |
| | 10 SN | | | |
| 26 | L0 F | By 18, 33 | | |
| | 40 F | By 18, 33 | | |
| 27 | L5 F | By 15, 29 | | |
| | 40 F | By 15, 29 | | |
| 28 | L5 27L | | | |
| | L4 76L | | | |
| 29 | 40 27L | | | |
| | 46 19L | | Advance i to i+1 in 19-27 | |
| 30 | 46 23L | | | |
| | L5 20L | | | |
| 31 | L4 76L | | | |
| | 46 20L | | | |
| 32 | 46 24L | | | |
| | L5 26L | | | |
| 33 | L5 76L | | | |
| | 40 26L | | | |

| LOCATION | ORDER | | NOTES PAGE 3 |
|---|---|---|---|
| 34 | 42 22L | | |
| | L5 80L | | Test for i = n |
| 35 | L0 27L | | |
| | 36 19L | | |
| 36 | L5 79L | | |
| | 46 38L | | Set i = 1 in 38 |
| 37 | 19 38F | | |
| | 40 S5 | | |
| 38 | L7 F | By 36, 40 | |
| | | From 42 | |
| | L4 S5 | | |
| 39 | 40 S5 | | |
| | L5 38L | | |
| 40 | L4 76L | | Compute $\sum_i \lvert f_{x_i} \rvert$ |
| | 46 38L | | |
| 41 | L5 81L | | |
| | L0 38L | | |
| 42 | 36 38L | | |
| | 26 85L | | |
| 43 | 46 44L | From 87 | Set i = 1 in 44-45 |
| | 42 45L | | |
| 44 | L5 F | By 43, 47 | |
| | 66 S5 | | Compute $\delta_{x_i}$ |
| 45 | 7J S4 | | |
| | 40 F | By 43, 48 | |
| 46 | L5 44L | | |
| | L4 76L | | Advance i to i+1 in 44-45 |
| 47 | 46 44L | | |
| | L5 45L | | |
| 48 | L4 76L | | |
| | 42 45L | | |
| 49 | L5 82L | | |
| | L0 45L | | Test for i = n |
| 50 | 36 44L | | |
| | L5 77L | From 70 | |

| LOCATION | ORDER | | NOTES | PAGE 4 |
|---|---|---|---|---|
| 51 | 46 54L | | Set i = 1 in 54-55 | |
| | L5 79L | | | |
| 52 | 42 54L | | | |
| | L5 22L | | | |
| 53 | 46 55L | | | |
| | 50 71L | | | |
| 54 | L5 F | By 51, 56 | | |
| | | From 59 | | |
| | L4 F | By 52, 56 | Set $x_i + \delta_{x_i}$ in 55 | |
| 55 | 40 F | By 53, 58 | | |
| | L5 54L | | | |
| 56 | L4 76L | | | |
| | 40 54L | | Advance i to i+1 in 54-55 | |
| 57 | L5 55L | | | |
| | L4 76L | | | |
| 58 | 46 55L | | | |
| | L5 83L | | Test for i = n | |
| 59 | L0 55L | | | |
| | 36 54L | | | |
| 60 | 50 F | By 1 | Interlude address | |
| | 50 60L | | $f(\ldots x_i + \delta_{x_i} \ldots)$ | |
| 61 | 26 S8 | | | |
| | 40 S5 | | Form f = f (before) | |
| 62 | L0 S3 | | | |
| | 50 53L | | | |
| 63 | 36 F | By 13, 69 | To interlude | |
| | L5 78L | | | |
| 64 | 42 65L | | Set i = 1 in 65 | |
| | 46 65L | | | |
| 65 | L5 F | By 64, 67 | | |
| | | From 68 | | |
| | 40 F | By 64, 67 | Replace old values by new | |
| 66 | L5 65L | | | |
| | L4 76L | | Advance i to i+1 in 65 | |

| LOCATION | ORDER | | NOTES PAGE 5 |
|---|---|---|---|
| 67 | 40 65L | | |
| | L5 84L | | Test for i = n |
| 68 | L0 65L | | |
| | 36 65L | | |
| 69 | L5 70L | | Open failure switch and repeat |
| | 46 63L | | |
| 70 | 50 4L | | Minor cycle |
| | 22 50L | | |
| 71 | 50 74L | | Form $\delta_{i+1} = c\delta_i$ and test against $\epsilon$ |
| | 7J S4 | | |
| 72 | 40 S4 | | |
| | L0 75L | | |
| 73 | 36 4L | | |
| | 22 F | By 2. | Link |
| 74 | 00 F | | |
| | 00 S6 | | = $\alpha$ |
| 75 | 00 F | | |
| | 00 S7 | | = $\epsilon$ |
| 76 | 00 1F | | Address advance |
| | 00 1F | | |
| 77 | L5 1S3 | | |
| | 40 1S5 | | |
| 78 | 00 S5 | | Starting constants |
| | 00 S3 | | |
| 79 | L0 1S4 | | |
| | 40 1S4 | | |
| 80 | L5 S9 | | |
| | 40 SS | | |
| 81 | L7 SK | | |
| | L4 S5 | | End constants |
| 82 | 7J S4 | | |
| | 40 SK | | |
| 83 | 40 SS | | |
| | L5 54L | | |

| LOCATION | ORDER | | NOTES PAGE 6 |
|---|---|---|---|
| 84 | L5 SS | | |
| | 40 S9 | | |
| 85 | L5 S5 | From 42 | |
| | 36 87L | | Test $\sum_i \lvert f_{x_i} \rvert$ |
| 86 | L5 88L | | |
| | 40 S5 | | If > 1, replace by $1-2^{-39}$ |
| 87 | L5 79L | | |
| | 26 43L | | |
| 88 | 7L 4095F | | |
| | LL 4095F | | |