UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

LIBRARY ROUTINE J 3 - 289

TITLE: Roots of a Polynomial (SADOI Only)

TYPE: Closed subroutine

NUMBER OF WORDS: 85

DESCRIPTION: This subroutine calculates the roots of the polynomial

$$a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots + a_{n-1} x + a_n = 0$$

METHOD OF USE: The coefficients $a_0, a_1, \ldots, a_n$ are to be placed in the Williams Memory beginning at (COEFF). Enter the subroutine with Q = 50 n 50 q where n is the degree of the polynomial. This will cause complex roots to be found in conjugate pairs with a saving in time. The entry Q = J0 n 50 q will cause the roots to be found individually. The latter entry should be used when the coefficients are not all real.

STORAGE AND SYMBOLIC ADDRESSES:

The n + 1 coefficients should be placed with a_0 in (COEFF), a_1 in 2(COEFF), a_2 in 4(COEFF), etc. Thus, 2n + 2 words are necessary at (COEFF).

The routine may put an extraneous root in 2n(ROOTS) and 2n + 1(ROOTS). Thus 2n + 2 words of storage are necessary at (ROOTS).

24 words of temporary storage are required. This block is to be specified by S4.

(A5) must define the complex number floating point arithmetic routine. See this routine for number format, etc.

S3 is the location of the floating accumulator used with A5.

0, 1, 2, and 3 are used by A5 while this routine is operating.

DURATION: The time is approximately 3 seconds per root. This is reduced if complex roots are found in conjugate pairs. The time also depends on the condition of the polynomial. In any event, the time will be no greater than that of Library Routine J 2.

ACCURACY: Usually about 9 decimal places. Multiple roots are not found as accurately.

MATHEMATICAL METHOD: A root R_i is found using the same method as J 2. If $|\frac{Im R_i}{Re R_i}| \geq \simeq 10^{-6}$ the root is classified as complex. For the 50 n 50 q entry, the conjugate of R_i is placed in the block of roots and divided out. For the J0 n 50 q entry, the procedure is exactly that of J 2.

B-LINES USED: B-lines 0 and 1 are used during the operation of this routine and are not reset.

DATE February 15, 1960
PROGRAMMED BY Maurice E. Suhre
APPROVED BY J N Snyder

ns

LOCATION	ORDER	NOTES PAGE 1 J 3
	00K(J3)	
0	K5 13L	
	42 67L	link
1	40 84L	entry info.
	46 6L	n
2	10 20F	
	42 41L	n
3	42 63L	n
	F5 41L	
4	42 6L	n + 1
	50 4L	
5	26 (A5)	
	8K F	
6	1K F	no. of roots
	0K F	set b line zero
7	03 7L	
	05 1022(COEFF)	a_n
8	8S 4S4	
	04 1018(COEFF)	
9	04 1020(COEFF)	Form initial f_0, f_1, f_2
	8S 2S4	
10	00 1020(COEFF)	
	00 1020(COEFF)	
11	8S S4	
	85 69L	
12	0K 4F	
	0S 6S4	form initial δ, x, λ, h
13	80 6S4	
	02 12L	
14	85 S4	
	87 10S4	
15	8S 14S4	
	87 10S4	
16	8S 16S4	
	81 2S4	

LOCATION	ORDER	NOTES PAGE 2 J 3
17	87 6S4	
	8S 18S4	
18	87 6S4	
	8S 20S4	
19	85 10S4	
	84 6S4	
20	87 4S4	
	84 16S4	
21	84 20S4	
	8S 16S4	
22	87 16S4	
	8S 20S4	
23	8K 2F	
	87 4S4	
24	87 6S4	
	8S 22S4	compute λ by
25	81 14S4	the formula
	80 18S4	
26	80 4S4	
	87 10S4	
27	87 22S4	
	8S 14S4	
28	84 14S4	
	84 20S4	
29	8J 225(A5)	
	8S 14S4	
30	8J 42(A5)	
	87 16S4	
31	82 32L	
	81 14S4	
32	8S 14S4	
	85 16S4	
33	84 14S4	
	8S 14S4	
34	81 22S4	

LOCATION	ORDER	NOTES PAGE 3 J 3
	86 14S4	
35	8S 10S4	
	87 12S4	
36	8S 12S4	
	84 8S4	
37	8S 8S4	
	8J 57L	convergence test
38	8K 1F	form new h, x, δ, f_0, f_1
	84 10S4	
39	8S 6S4	
	85 2S4	
40	8S S4	
	85 4S4	
41	8S 2S4	
	0K F	
42	85 (COEFF)	
	87 8S4	form new f_2
43	04 2(COEFF)	
	02 42L	
44	8S 4S4	
	8J 52L	test $f_2/f_1 > 10$
45	85 10S4	
	87 69L	
46	8S 10S4	
	85 12S4	
47	87 69L	Replace λ by $1/2\lambda$, and
	8S 12S4	recompute h, x, and δ.
48	85 8S4	
	80 12S4	
49	8S 8S4	
	8K 1F	
50	84 10S4	
	8S 6S4	
51	8K 1F	
	82 41L	
52	L5 2S4	

LOCATION	ORDER	NOTES PAGE 4 J 3
	50 3S4	
53	10 5F	
	01 10F	
54	L0 1S3	
	F4 85L	Test $f_2/f_1 > 10$
55	36 29(A5)	
	L5 L	
56	46 2(A5)	
	26 29(A5)	
57	L5 12S4	
	50 13S4	
58	10 5F	
	01 10F	convergence test
59	L0 1S3	
	L4 68L	
60	36 29(A5)	
	50 60L	
61	26 (A5)	
	1S (ROOTS)	
62	8J 42(A5)	
	1S 2(ROOTS)	
63	8J 71L	
	0K F	
64	05 (COEFF)	
	17 (ROOTS)	divide out root
65	04 2(COEFF)	
	0S 2(COEFF)	
66	02 64L	
	8N 75L	
67	12 7L	
	8N F	exit
68	00 F	
	00 8F	
69	20 F	
	00 16F	

LOCATION	ORDER	NOTES PAGE 5 J 3
70	00 F	
	00 F	
71	L1 84L	
	36 29(A5)	
72	L7 2S3	
	10 23F	
73	40 F	check for complex
	L3 F	root.
74	36 29(A5)	
	49 82L	
75	26 29(A5)	
	L5 63L	
76	L0 205(A5)	step down addresses
	42 63L	
77	42 41L	
	L3 82L	check complex root
78	36 29(A5)	indicator
	L5 192(A5)	
79	L4 205(A5)	
	40 192(A5)	step B 1
80	41 82L	clear indicator
	L5 83L	
81	40 2(A5)	Reset A5. Enter RH side
	26 29(A5)	of 63L, divide out new root.
82	00 F	
	00 F	complex root indicator
83	50 63L	
	S5 20F	
84	00 F	
	00 F	entry information
85	80 F	
	00 F	constant