UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

AUXILIARY
LIBRARY ROUTINE V10 - 290

TITLE: Generate a Random Normal Deviate (SADOI Only)

NUMBER OF WORDS: 26 plus 10 words of permanent storage at S3. Routine V 9 must be located at symbolic address (V9). Routine S 5 must be located at symbolic address (S5)

DURATION: About 45 milliseconds

ENTRY:

p	F5 pF
p + 1	26 qF

q = address of this program

Control is returned to the right hand side of p + 1 with X/4 in the accumulator. X is the random normal deviate, i.e. $P[X \leq a] = \int_{-\infty}^{a} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$.

It is suggested that the user enter routine (V9) about 100 times before entering this routine for the first time, so as to discard the first 500 numbers generated by (V9). On the average, 6 random numbers generated by (V9) are used to get one random normal deviate. At the end of a given run using this routine the user can print out as sexadecimal characters (with 82 40F orders) the last numbers generated by V9. These are in S3 to 4S3. When this routine is next used these numbers can be input into S3 to 4S3 (with 81 40F orders) and used as starting numbers. In this way a different sequence of numbers can be obtained on each **run**.

METHOD: Let y be a random variable which is uniformly distributed on the interval (-4,4). Let Z be a random variable which is uniformly distributed on (0,1). The random variable $X = (y \mid e^{-y^2/2} > Z)$ is approximately normally distributed for,

$$P[X < t] = P[y < t \mid e^{-y^2/2} > Z] = \frac{P[y < t,\ e^{-y^2/2} > Z]}{P[e^{-y^2/2} > Z]}$$

$$= \frac{\int_{-4}^{t} 1/8 \int_{0}^{e^{-y^2/2}} dZdy}{\int_{-4}^{4} 1/8 \int_{0}^{e^{-y^2/2}} dZdy} = \frac{\int_{-4}^{t} 1/8\ e^{-y^2/2} dy}{\int_{-4}^{4} 1/8\ e^{-y^2/2} dy} = \frac{\int_{-4}^{t} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy}{\int_{-4}^{4} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy} \cong$$

$$\cong \int_{-4}^{t} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$$

Hence X is approximately normally distributed. The approximation being due to the truncation of the normal distribution to (-4,4). In the machine a random number (y) is generated on (-4,4) and a random number (Z) is generated on (0,1). If $e^{-y^2/2} > Z$, y is accepted as a random normal deviate; otherwise y is rejected, a new y and Z are generated, and the comparison tried again. Actually y is accepted if $y^2 + 2\ln Z < 0$. The probability of acceptance is $\int_{-4}^{4} 1/8\ e^{-y^2/2} dy \cong \frac{\sqrt{2\pi}}{8} \cong .3125$.

REFERENCE: von Neumann, "Monte Carlo Methods".

DATE March 2, 1960
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ns

LOCATION	ORDER		NOTES PAGE 1 V 10
	00K(V10)		
0	42 12L		Plant link
	F5 3L		
1	42 3L		
	F5 6L		
2	42 6L		
	L0 23L		
3	32 13L		+ → go get 10 more random numbers
	L5 F		n = 0, 1, 2, 3, 4
4	40 24L		x/4
	50 24L		
5	75 24L		$2^{-4} x^2$
	40 25L		
6	00 1F		Waste
	L7 10S3	by 22L, 2L	$0 \leq y < 1$
7	50 7L		Waste
	50 7L		
8	26 (S5)		$2^{-5} \ln y < 0$
	00 1F		$2^{-4} \ln y$
9	36 12L		+ → accept, $\lvert 2 \ln y \rvert \geq 32$
	00 1F		$2^{-3} \ln y$
10	36 12L		+ → accept $\lvert 2 \ln y \rvert \geq 16 > x^2$
	L4 25L		$2^{-4}(x^2 + 2 \ln y)$
11	32 L		+ → reject $x^2 \geq 2 \ln y$ - accept
	36 12L		waste
12	L5 24L		x/4 in A
	22 F		link
13	00 F		
	50 13L		get 5 random numbers
14	26 (V9)		random on (-1,1)
	L5 S3		
15	40 5S3		put them in 5S3
	L5 1S3		to 9S3
16	40 6S3		
	L5 2S3		

LOCATION	ORDER		NOTES PAGE 2 V 10
17	40 7S3		
	L5 3S3		
18	40 8S3		
	L5 4S3		accept if $x^2 + 2\ell n\ y < 0$
19	40 9S3		
	50 19L		get 5 more i.e. if $x^2 < -2\ell n\ y$
20	26 (V9)		random numbers.
	L5 14L		
21	42 3L		x: R(-4,4)
	F5 18L		y: R(0,1)
22	42 6L		
	22 3L		$z=(x \mid e^{-x^2/2} > y)$
23	00 1F		
	L7 10S3		
24	Temp. store for x/4		
25	" " " $x^2/16$		