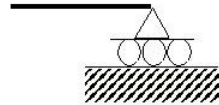


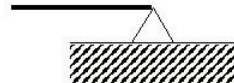
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Designing Connections

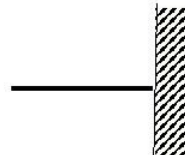
Roller



Pin (hinge)



Rigid connection





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Connection Design

Be clear in the function of the connection

What loads does it have to resist.

How could it fail.

Will it be easy to maintain in the future.



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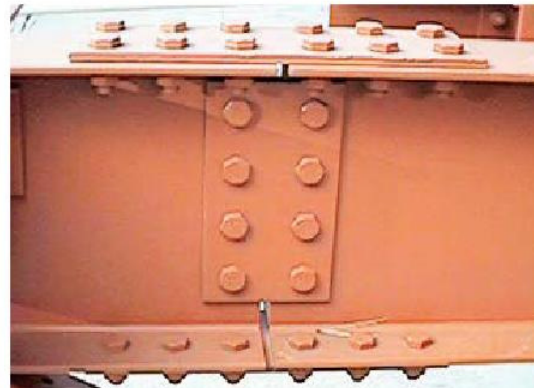
Connection Design for Strength

Must Resist:

Axial forces

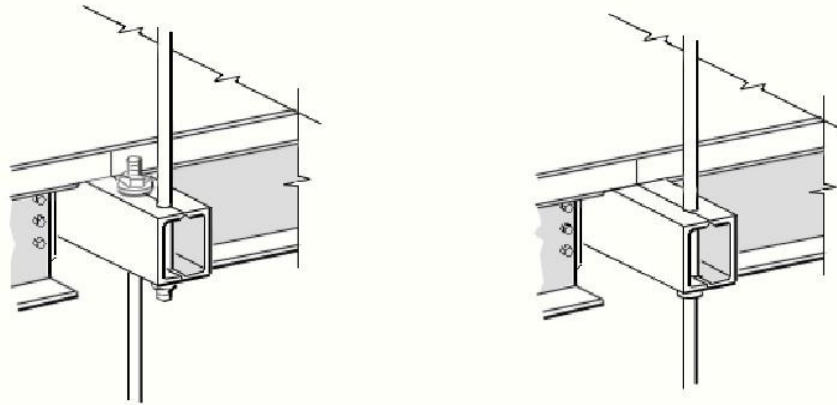
Moment forces

Shear forces





Suspended Walkway of the Kansas City Hyatt Regency,
1981



Is there a difference between these two connections?

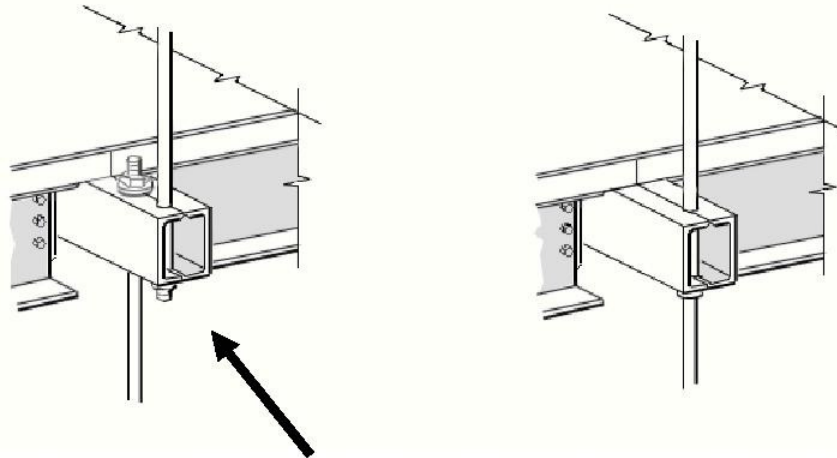
-Technically?

-Aesthetically?



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Suspended Walkway of the Kansas City Hyatt Regency, 1981

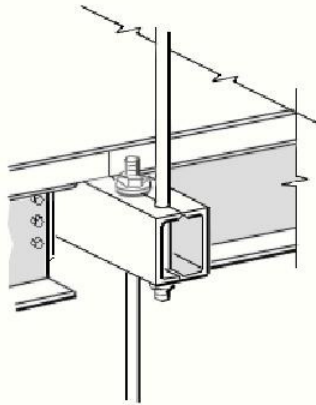


What is the force on this bolt.

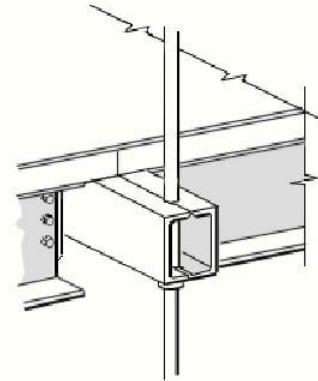


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Suspended Walkway of the Kansas City Hyatt Regency, 1981



As-built connection



Initial connection design

**Everyone agreed to the design change
without thinking of the**



implication

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Lessons from the Kansas City Collapse, 1981

Imagine that you are the structural element or the connection: how could the forces be transferred from one member to the other.

For axial force members, align each member so the connection reduces to a



**REDUCES TO A
single point**

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Connection Geometry



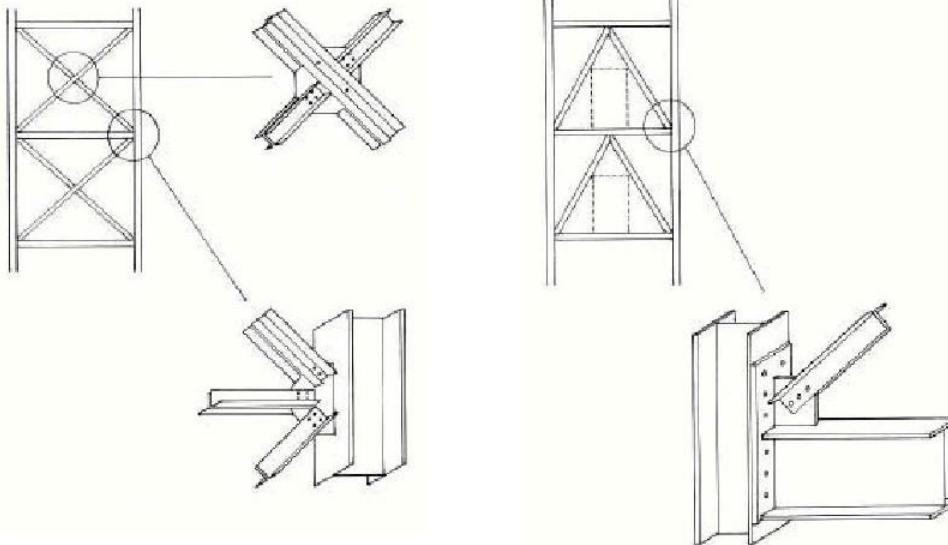
Centroid axis of each member should pass through the same point



through the same point
(practical for structures like trusses.)

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Steel Bracing Connections





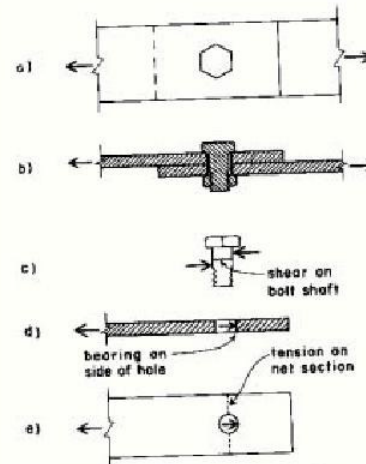
**Centroid axis of each
pass through the same point**

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Bolt in Single Shear

Shear stresses try
to slice the bolt

Stress equals shear
force divided by
the cross-sectional
area of the bolt

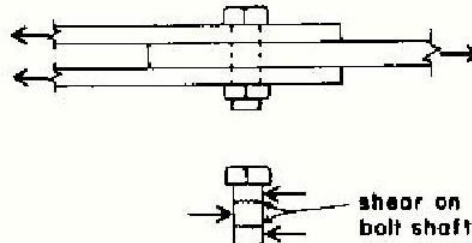


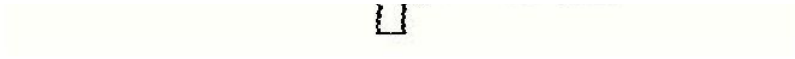


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Bolt in Double Shear

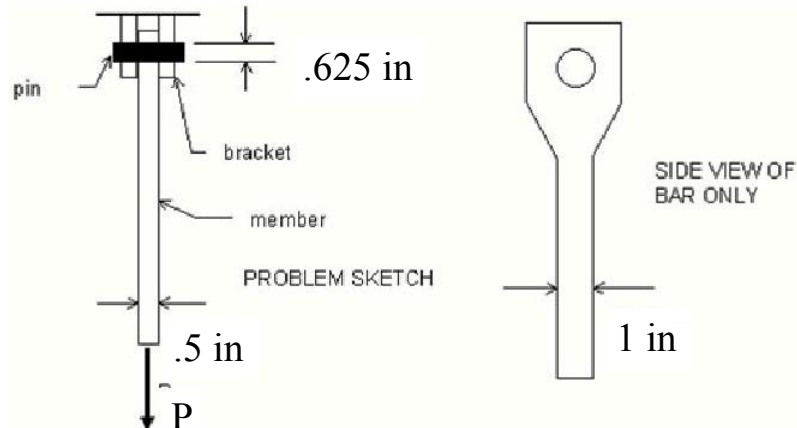
Shear stress is one
half the value of
the applied load





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Example problem



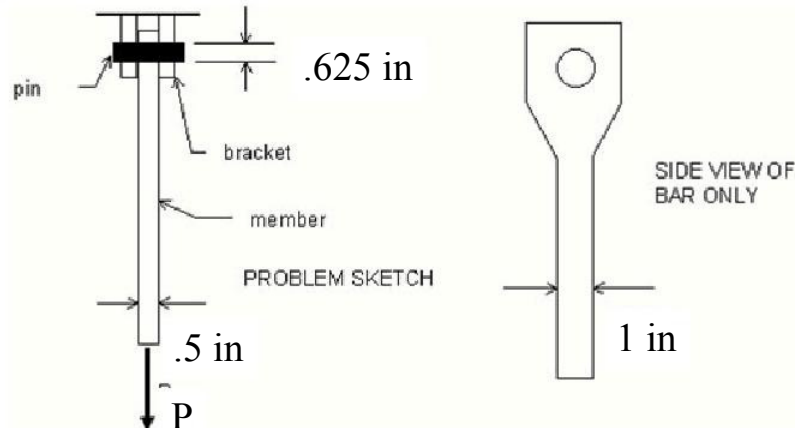
What are the ways this connection



**could fail. (allowable
stress = 10 ksi)**

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Example problem



1. Maximum axial stress on the bar:

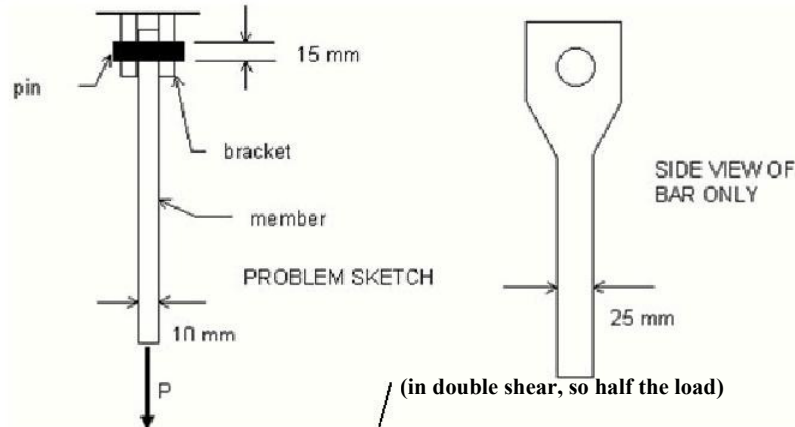
$$F = (\text{Stress})(\text{Area}) = (10 \text{ ksi})$$



$F = (\text{Stress})(\text{Area}) = (10 \text{ ksi})$
 $(1 \text{ in} \times 5 \text{ in}) = 50 \text{ kips} = 50,000 \text{ pounds}$

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Example problem



2. Maximum shear across the pin: (area = 0.31 in²)

$$F = (\text{Stress})(\text{Area}) = (2) \quad (2)$$

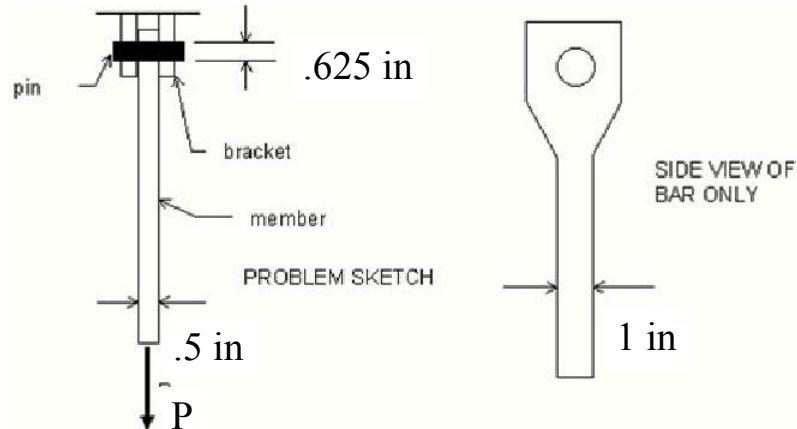


(10 kpsi) (6,210 pounds)

' —
=
6.2

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Example problem

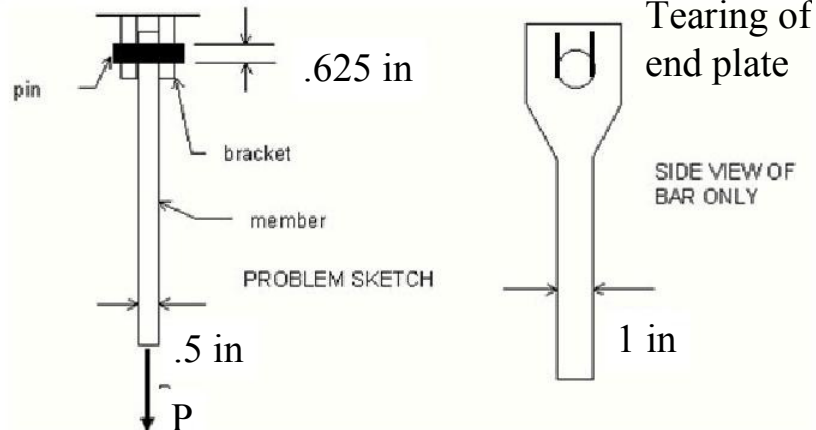


Other modes of failure. (at least 3)



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Example problem

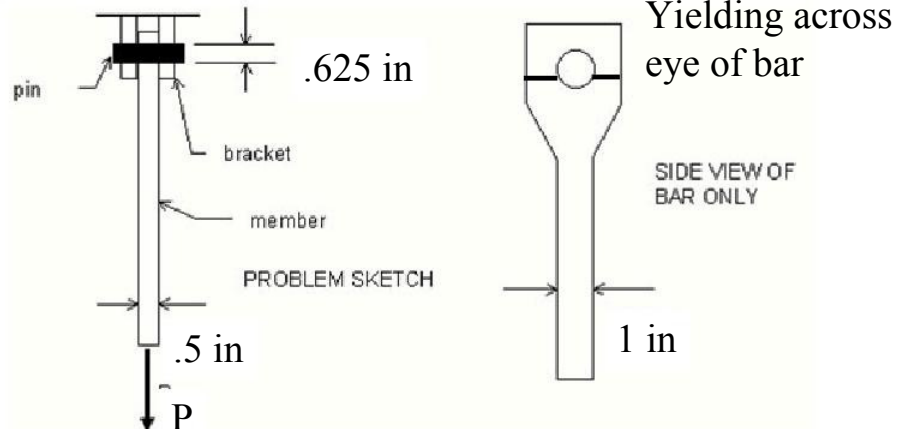


Other modes of failure.



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Example problem

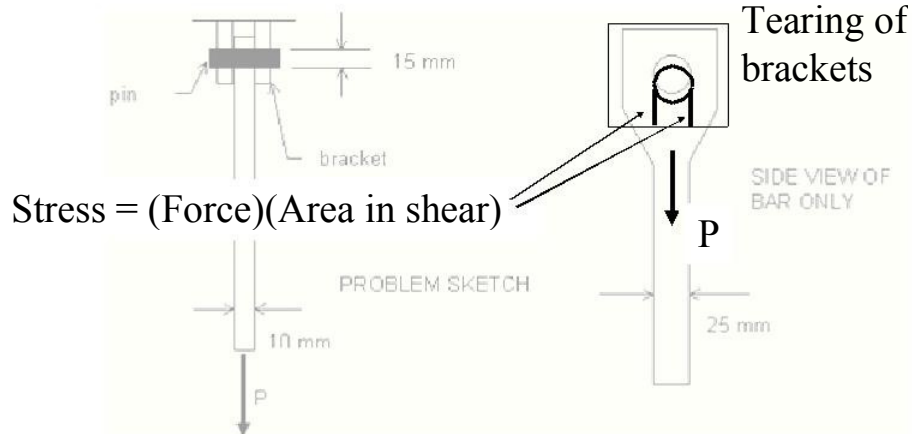


Other modes of failure.



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Example problem

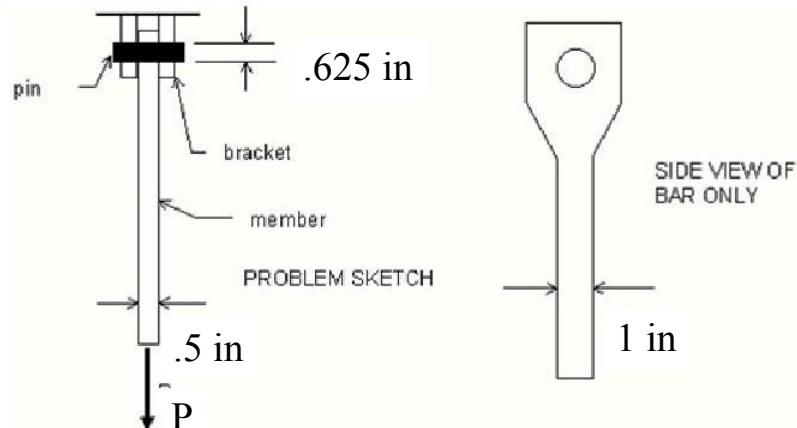


Other modes of failure.



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Example problem



Conclusion: Even the simplest



**connections can fail
in many ways**

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Axial Force Connections

Consider all sections of material where failure could occur

Compare allowable force for each section, and the lowest force value governs the design load capacity

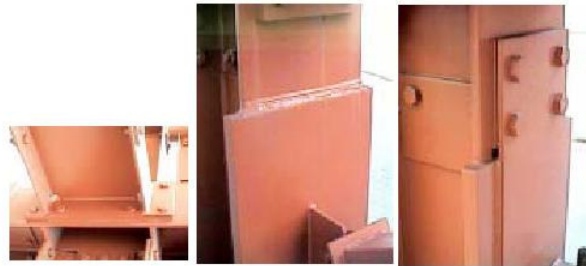
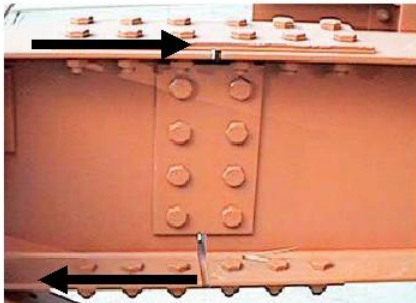
If the joint acts in



of buckling (typically in plates)

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Moment Connections

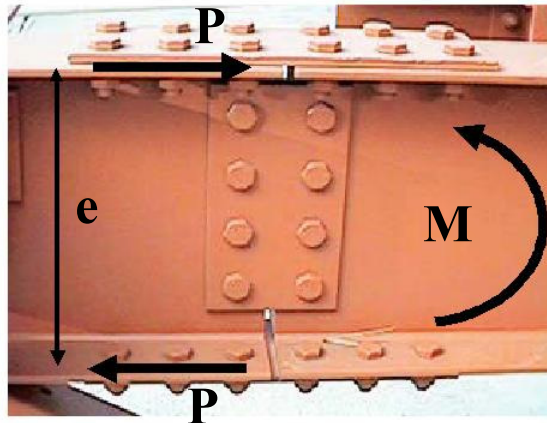




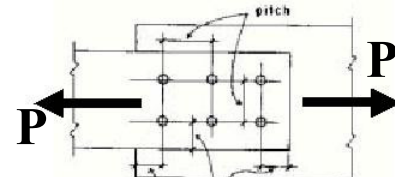
Tie flanges together to transfer moment

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Moment Connections

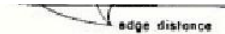


Moment, $M = Pe$





Design for axial force, P



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Shear Connections

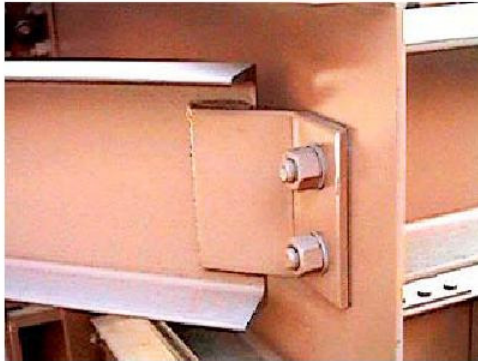




Use the web of the beam to transfer shear

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Shear Connections



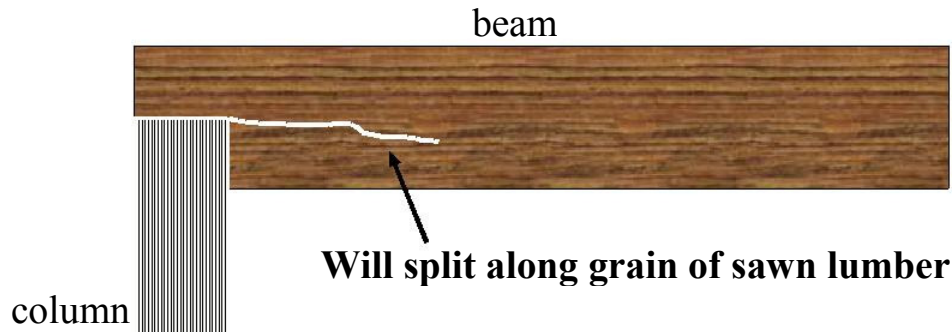


Use the web of the beam to transfer shear

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Connections: Beware!

2. **Wood has different properties with and against the grain: beware of splitting**





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Properties of Timber

Cellular structure is very efficient

Handles both compression and tension well

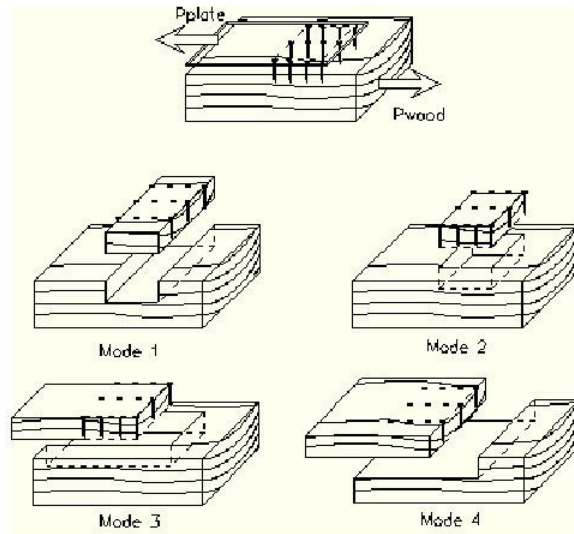
Different strengths with and against the grain

Inhomogeneous material with imperfections



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Metal Shear Plate on Wood





**Must consider various
possible failure**

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Connections: Beware!

- 4. Someone will have to disassemble your connection in the future: your construction today will be somebody's problem in the future**

Case study:

Williamsburg Bridge



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Williamsburg Bridge

**Carried traffic and
trains throughout the
20th century**

**But maintenance was
neglected badly for
decades**

**In 1988 the poor
condition of the bridge
became**



~~became~~
an
emergency

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Decay of Williamsburg Bridge

**Main cables had
corroded badly (were
not galvanized)**

**Pin joints in the main
trusses were corroded**

Rusted girders



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1990-2005: Rebuilding the Williamsburg Bridge

New cables, new girders, new roadways, new bearings, new paint, etc

Original designers didnt consider how to repair many elements



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Designing for Maintenance and Deconstruction

Develop a maintenance plan for your structure

Design components which are accessible and replaceable

Avoid toxic materials which are hazardous for future repairs or demolition



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Connection Conclusions

Design for strength: how could it fail.

Design for serviceability: can it be maintained easily.

To design a good connection you must know exactly what it has to do: seek clarity in design



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Steps in Finite Element Analysis

1. **Define geometry**
2. **Connect nodes with members**
3. **Assign section properties (A, E, and I)**
4. **Define fixity of nodes and connections**
5. **Apply loads**



6. **Run analysis and examine output**

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Structural Failure

Be clear in the function of the connection

What loads does it have to resist.

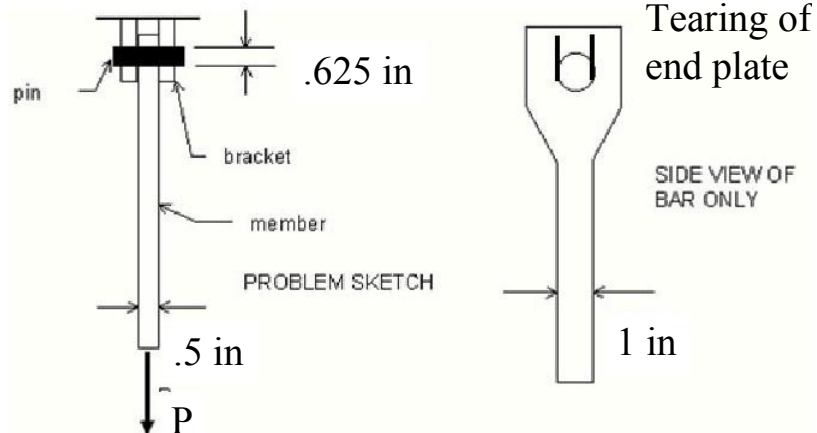
How could it fail.

Will it be easy to maintain in the future.



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Example problem

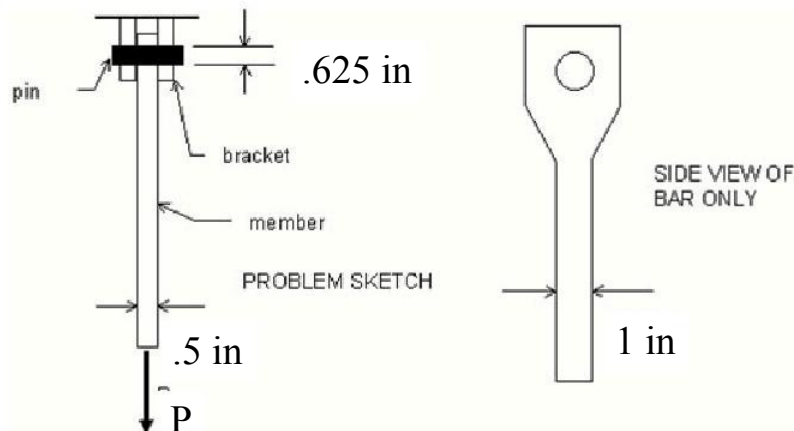


Other modes of failure.



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Example problem



Conclusion: Even the simplest



**connections can fail
in many ways**

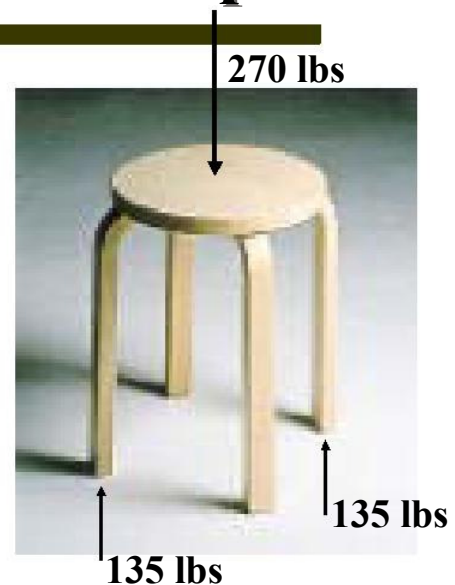
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Four-Legged Stool Example

Now imagine the load is increased to cause failure

When load is 270 lbs, the two legs will begin to fail

As they squash, the other two legs will start to carry





carry
load
also

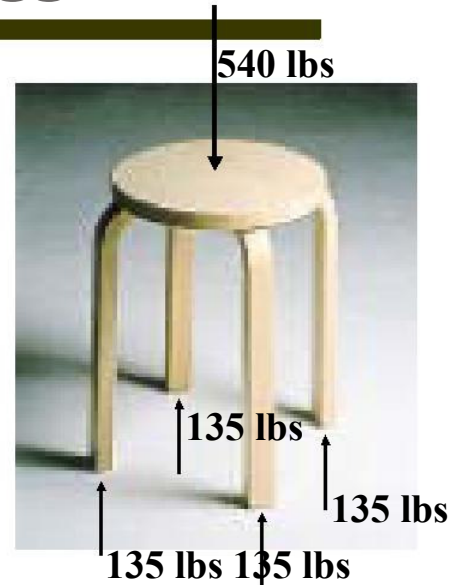
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Collapse of a 4-Legged Stool

At final collapse state, all four legs carry 135 pounds and the stool carries 540 pounds.

This occurs only if the structure is ductile (ie, if the legs can squash)

Is NOT valid if





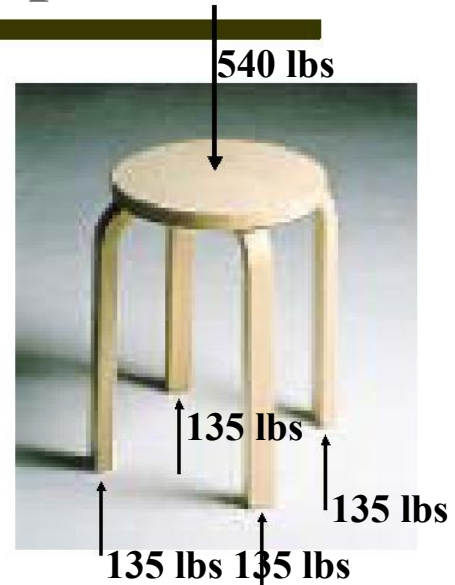
**IS NOT valid if
buckling occurs, will fail suddenly**

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Ductile Collapse

So small imperfections do not matter, as long as the structural elements are ductile

The forces in a hyperstatic structure cannot be known exactly, but this is not important as long as we can predict a ductile collapse state





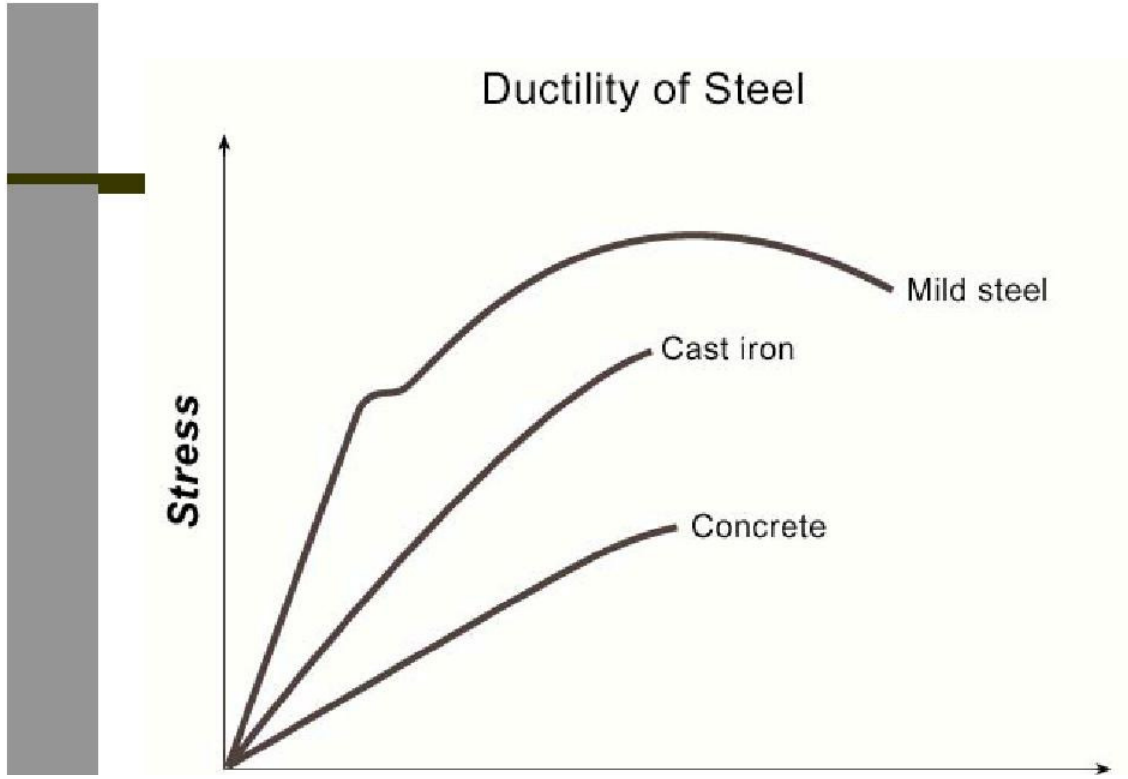
[home.cd3wd.ar.cn.de.en.es.fr.id.it.ph.po.ru.sw](#)

Why is steel a good structural material.

High strength

Ductile material



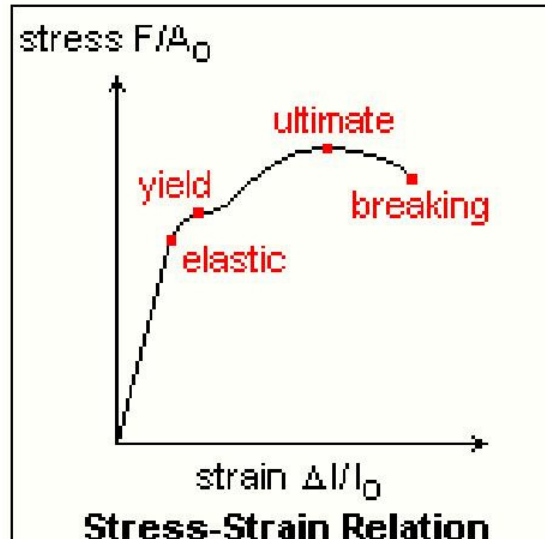




Strain

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Yield Stress of Steel





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Failure of Independent Elements

Tension element: breaking stress exceeded

Compression element:

Crushing stress exceeded

Buckling occurs

Truss:

Statically determinate: one element fails

Indeterminate to n degrees: n elements fail

Beam:

Either flange fails in compression or tension will form a
hinge (Indeterminate to n)



degree: n hinges form)
Shear failure

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Parallel Axis Theorem

To calculate the moment of inertia, I , of a built-up section with respect to an axis other than its centroid:

$$I = SI + SA d^2$$

Where:

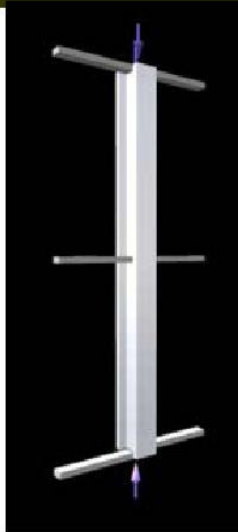
A is the area of the segment



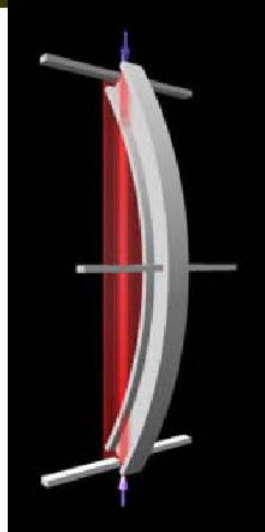
**d is the distance from the
center of the axis being considered**

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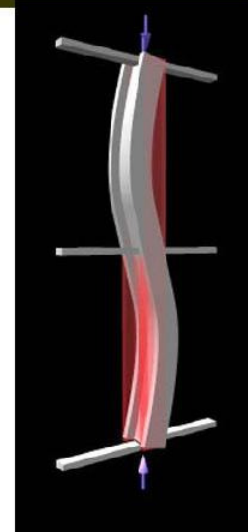
Buckling About a Different Axis



Unbuckled



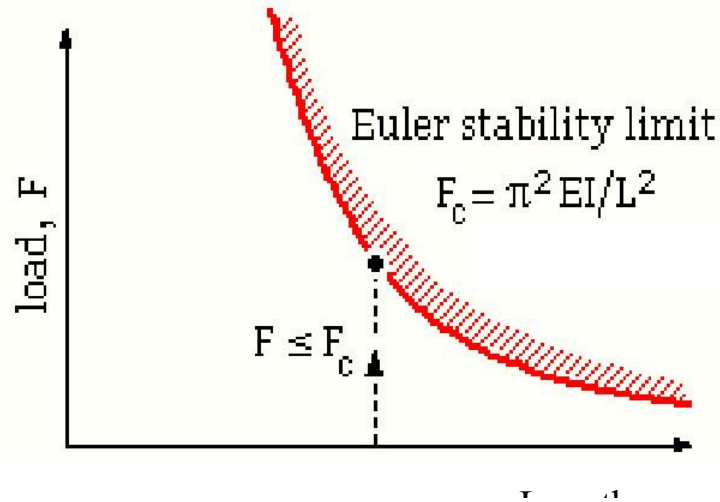
Strong



Weak

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Buckling Load vs. Length of Column

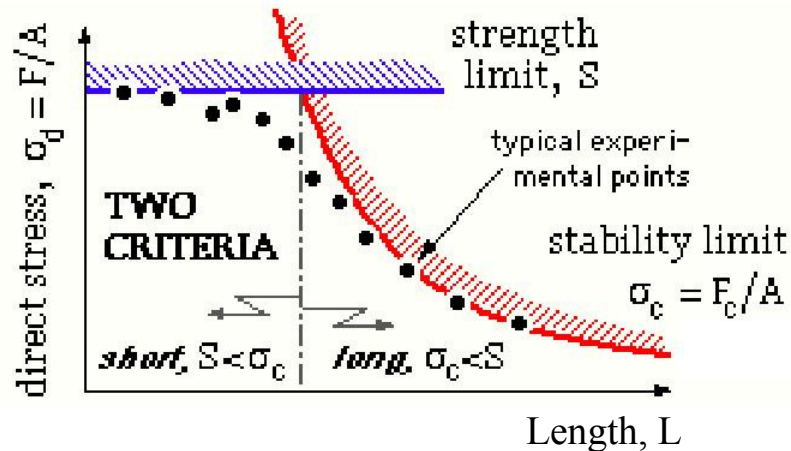




Length,
L

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Failure of Compression Members





Failure by buckling or crushing
Reality may be in between the two modes

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Importance of Ductility

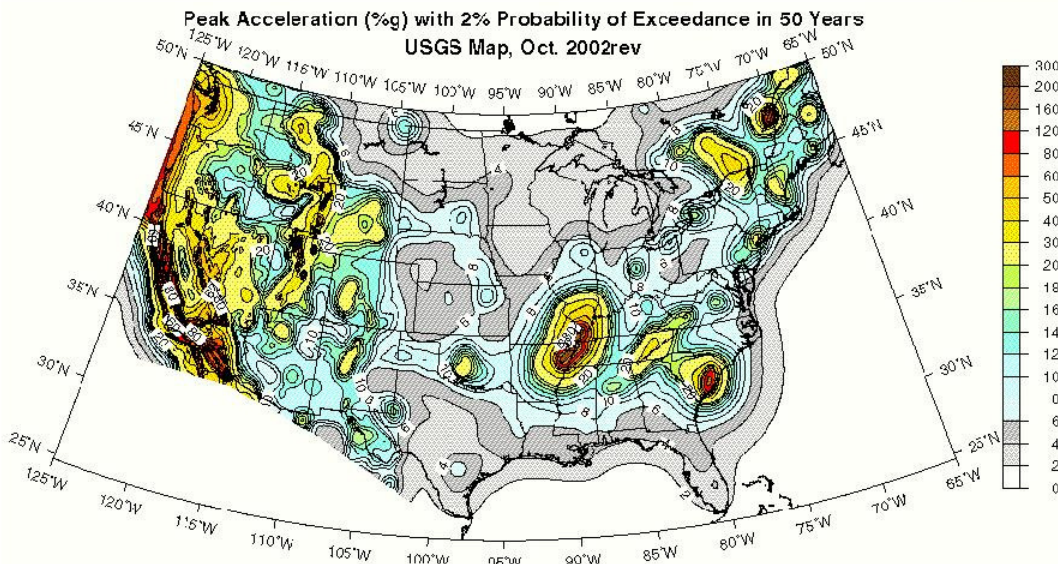
Large displacements before collapse (as **opposed to a brittle material, which fails suddenly**)

Energy dissipation as the steel yields (**important for resisting earthquakes and other overloading**)



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Is there a danger of earthquakes in Boston and other East Coast cities.





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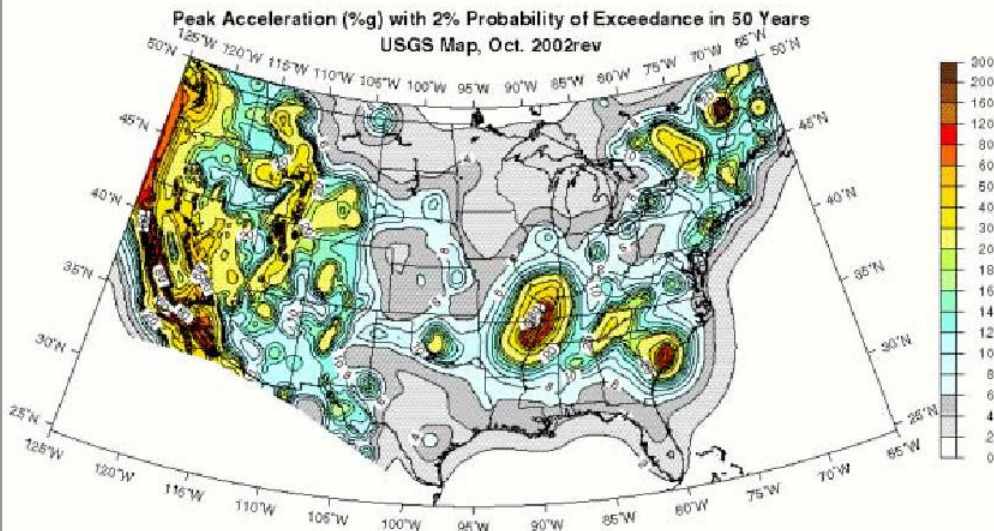
US Seismicity

Which state in the mainland US has had the largest earthquake.



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US Seismicity





Missouri: 8.2 in New Madrid, MO, 1812

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Increased Earthquake Risk

Growing cities and infrastructure

Many existing buildings are untested in a major earthquake

Earthquakes are not dangerous, but our infrastructure is dangerous



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Structural Failures

How could it fail. What is the weak link in the system.

Buckling is difficult to predict due to sensitivity of the parameters

Some failure modes are combined modes, i.e., local crushing can

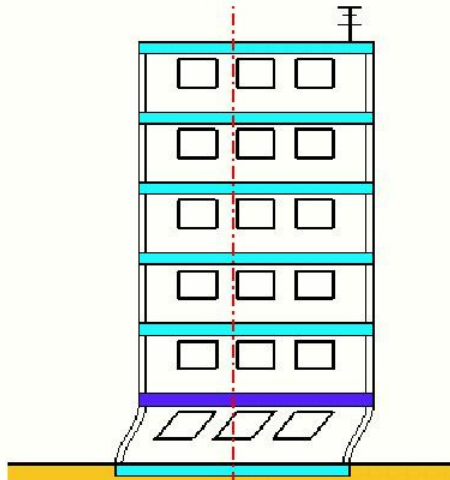


... local crashing can
back to global

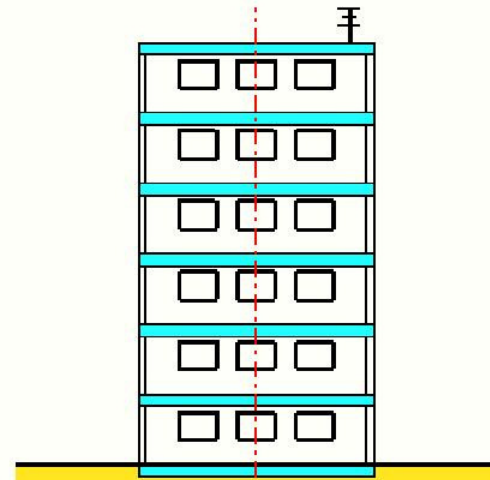
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Loading + Failure

Wind load hitting Skyscraper



Earthquake hitting Skyscraper



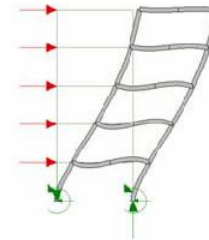


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Lateral Load Systems

1. Frame

rigid joints



2. Bracing

diagonal elements



3. Shear wall

diaphragm





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Good Practice in Wind Design

Provide greater resistance along shorter dimension

Careful attention to detail

Continuity of structure

Provide ties for each element

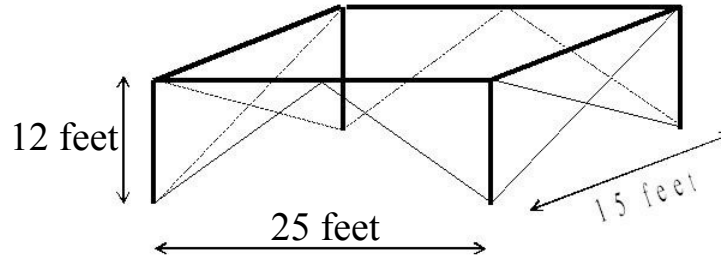
Greater weight can provide greater wind resistance

Protect inhabitants from flying projectiles



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Wind Bracing Example



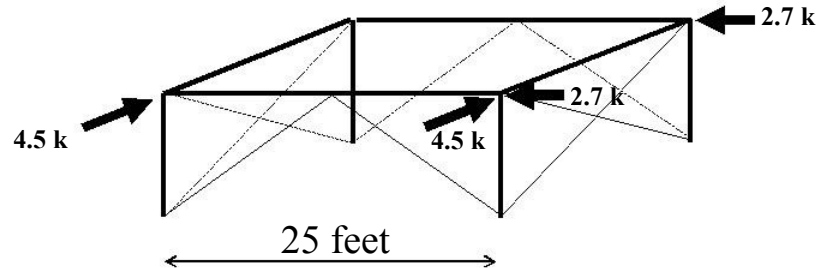
Wind loading of 30 pounds per square foot applied on



equal force applied on
each side

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Wind Bracing Example



$$(25 \text{ ft})(12 \text{ ft})(30 \text{ psf}) = 9,000 \text{ lbs}$$

Applied at top of wall, 4,500 lbs/frame (4.5 k)

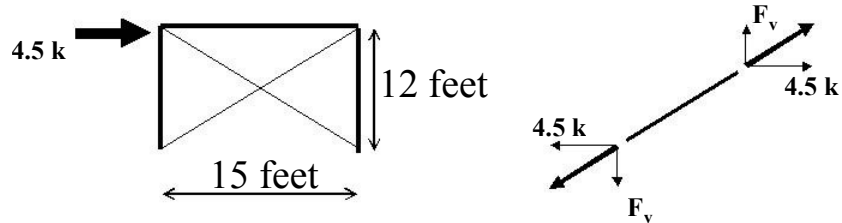


(15 ft)(12 ft)(30 psf) = 5,400 lbs

Applied at top of wall, 2,700 lbs/frame (2.7 k)

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Wind Bracing Example



If the cable carries 100% of wind load, then the horizontal component of force in the cable must be 4.5 k

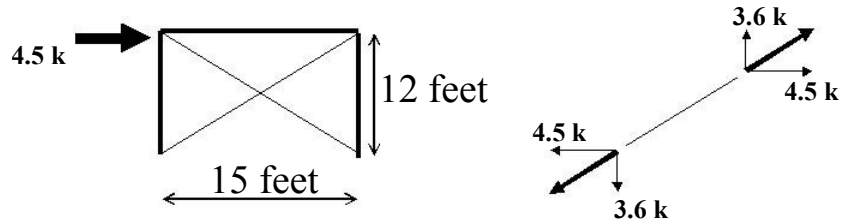
$$F_v / (4.5 \text{ k}) = (12 \text{ ft}) / (15 \text{ ft})$$



$$F_v = 3.6 \text{ k}$$

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Wind Bracing Example



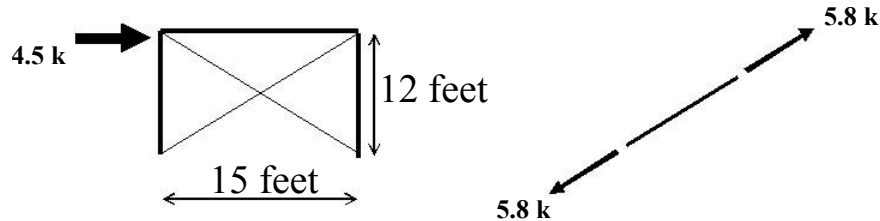
**Internal force in the cable is found from
Pythagorean Theorem**

$$F_{\text{cable}} = \sqrt{(4.5 \text{ k})^2 + (3.6 \text{ k})^2} = 5.8 \text{ k}$$



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Required Area of Steel Cable



Maximum allowable stress in cable is 15 ksi (given)

Stress = Force/Area

$$A_{\text{reqd}} = \text{Force}/\text{Stress} = 5.8 \text{ k}/15 \text{ ksi} = 0.38 \text{ in}^2$$

Cable diameter = 1.7 in

2/4

2)



Cable diameter, d
 = 0.7 inches ($A = p$

Specify steel cable ($A = 0.44 \text{ in}$

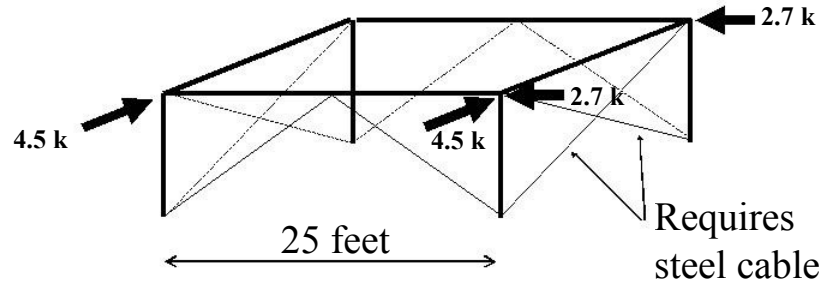
$$\frac{1}{4} \quad \text{in})$$

$$=$$

$$2 \frac{0.38}{\text{in}}$$

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Wind Bracing Example

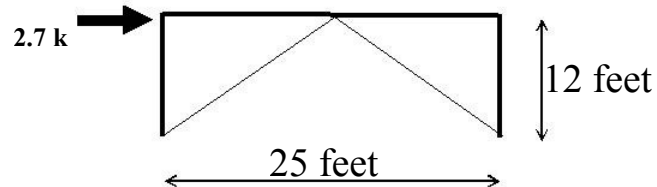


What about bracing in other direction.



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Wind Bracing Example



Assume bracing carries 100% of wind load, then the horizontal component of force in the cable must be 2.7 k

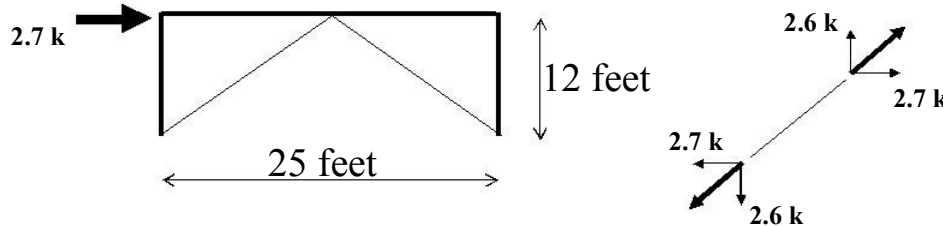
$$F_v / (2.7 \text{ k}) = (12 \text{ ft}) / (12.5 \text{ ft})$$



$$F_v = 2.6 \text{ k}$$

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Wind Bracing Example



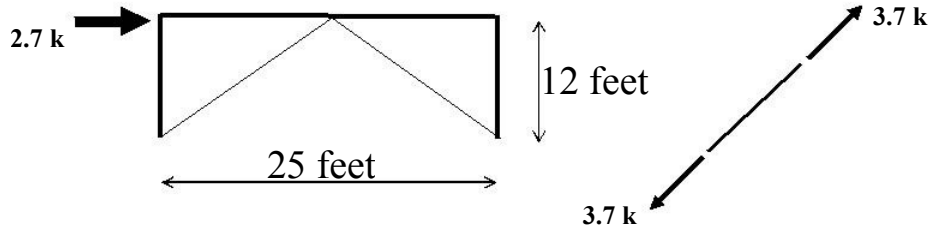
**Internal force in the cable is found from
Pythagorean Theorem**

$$F_{\text{cable}} = \sqrt{(2.7 \text{ k})^2 + (2.6 \text{ k})^2} = 3.7 \text{ k}$$



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Required Area of Steel Cable



Maximum allowable stress in cable is 15 ksi (given)

Stress = Force/Area

$$A_{\text{reqd}} = \text{Force}/\text{Stress} = 3.7 \text{ k}/15 \text{ ksi} = 0.25 \text{ in}^2$$

Cable diameter =

2/4

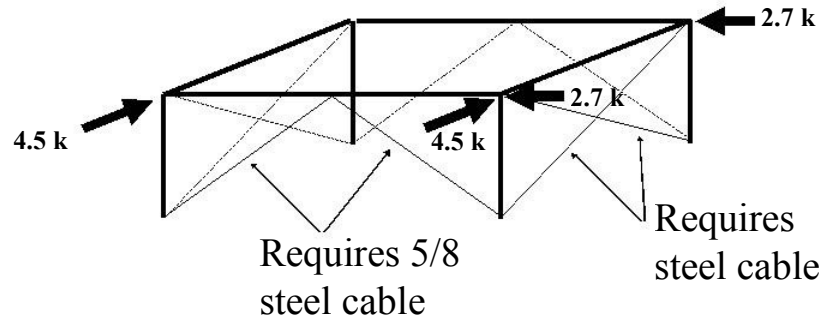
2)



Cable diameter, $d =$ $\frac{1}{4}$ in
 0.56 inches ($A = p$)
 Specify 5/8 steel cable ($A = 0.31 \text{ in}^2$)

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Wind Bracing Example

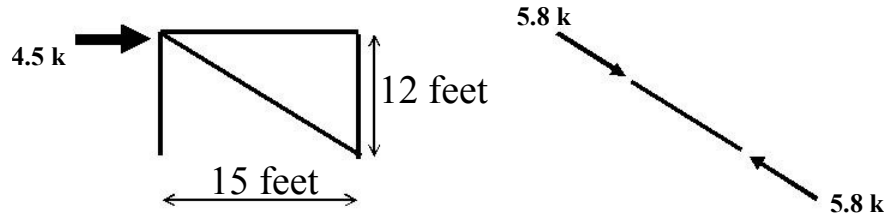


Design of wind bracing for cables



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Compression Element as Wind Bracing



Now one element must carry 5.8 k in both tension and compression

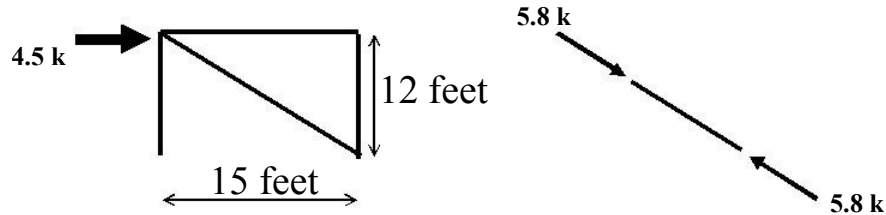
For tension, must have an area of 0.38 in^2 ($3/4$ in diameter bar)



So cross-sectional area must be greater than 0.38 in^2

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Compression Element as Wind Bracing



**Compression element requires about 6 times the material
(to resist buckling)**



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Design Considerations

Tension braces use less material than compression braces for lateral loads

Compression members must resist buckling which requires extra material

**Must consider buckling about either axis,
and should brace**



**and should brace
the weak axis**

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Design Considerations

Want to have ductility in our structures so that sudden collapse does not occur

Failure by buckling may be sudden and unexpected not a ductile failure

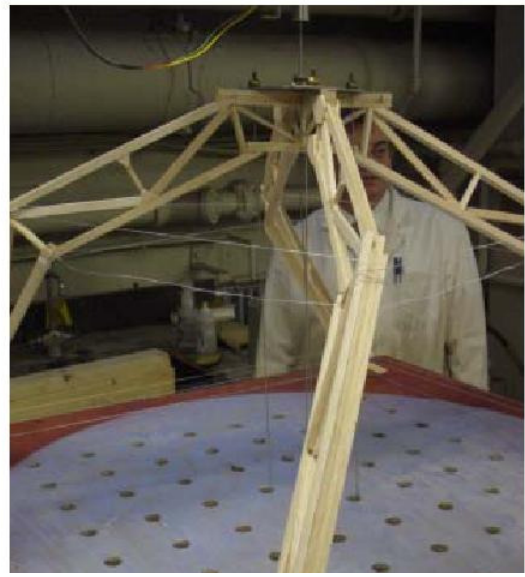
Statically determinate structures are easier to design because you can know the



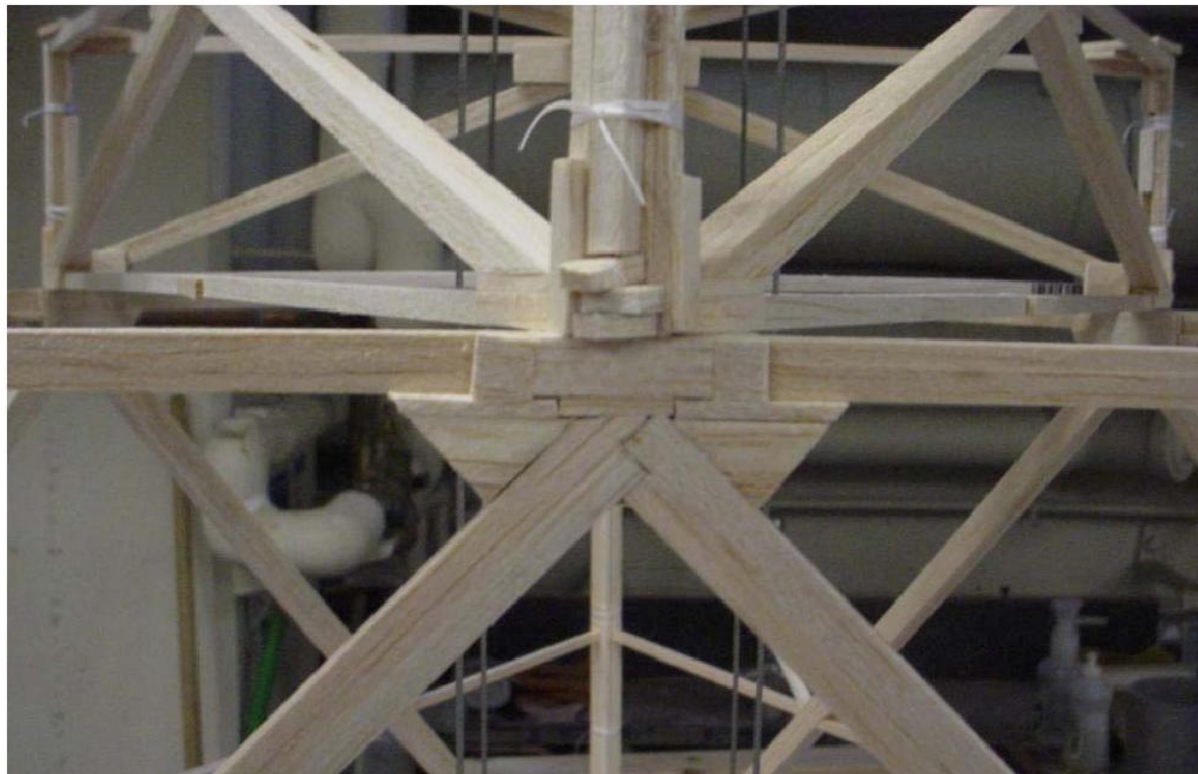
**internal
forces
exactly**

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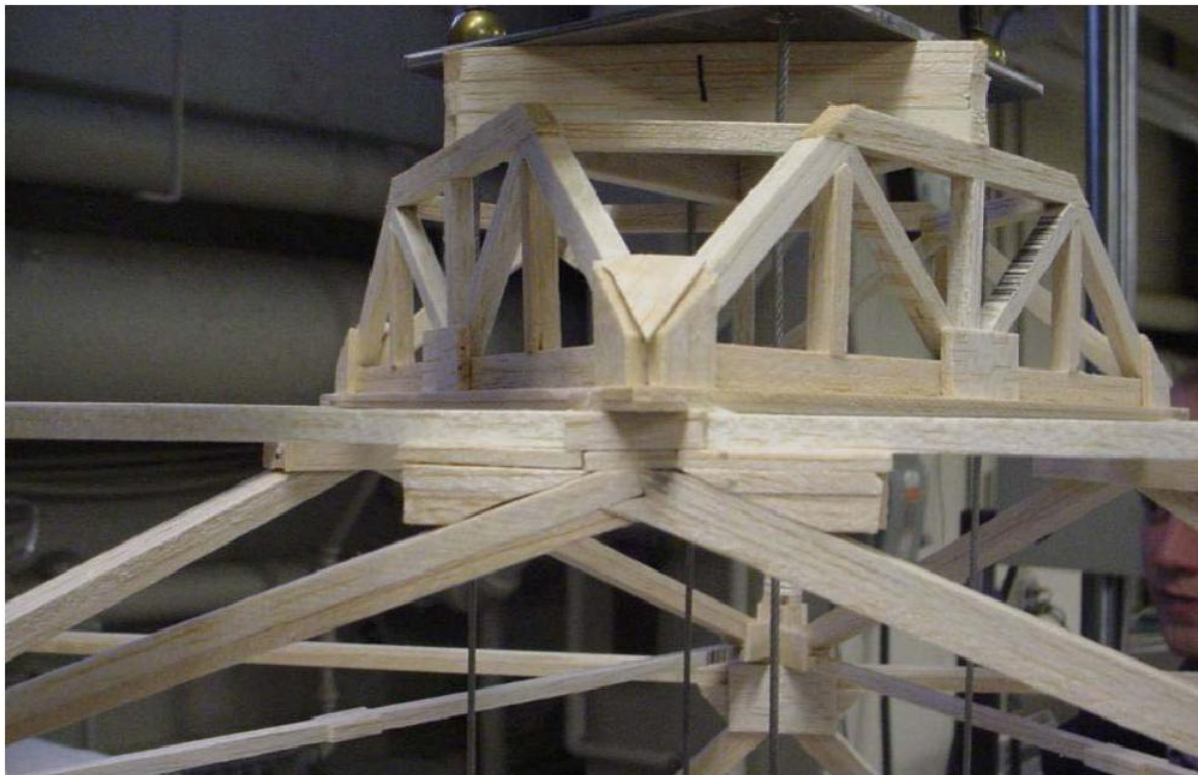
Balsa Dancing

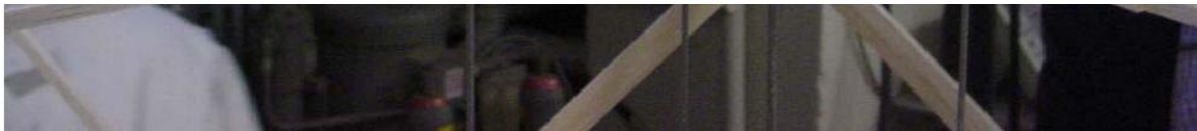


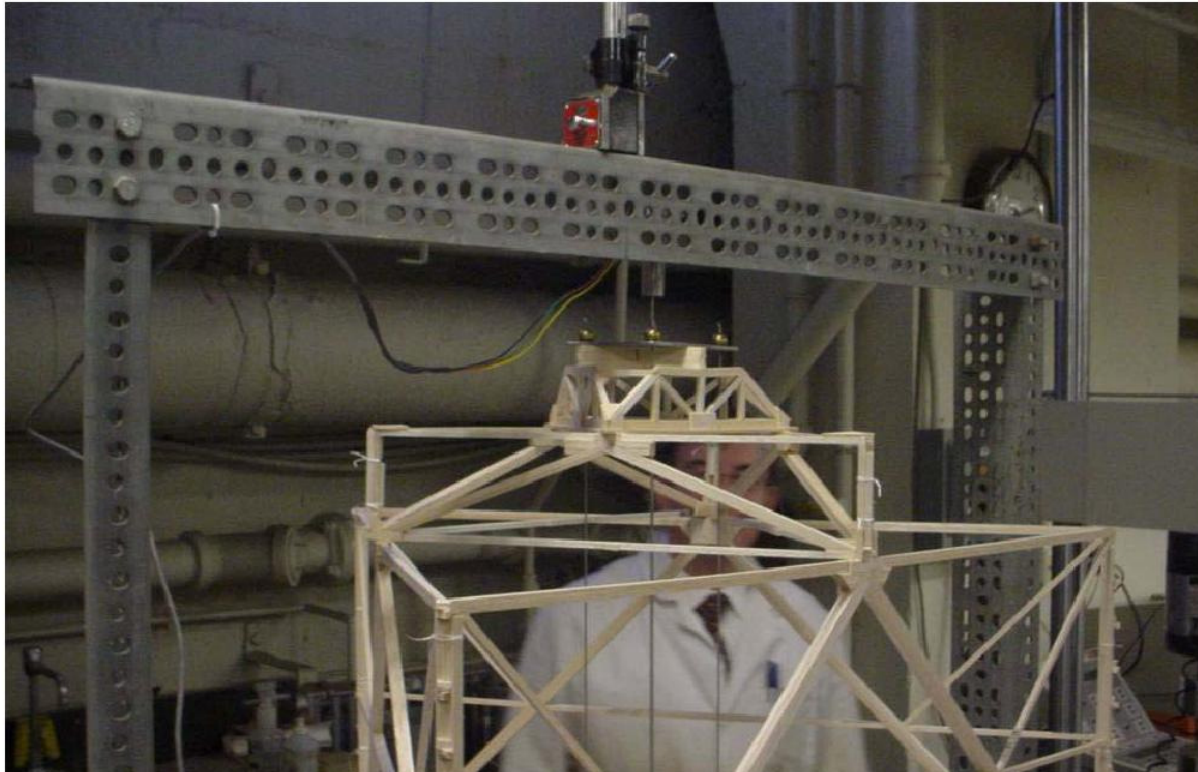




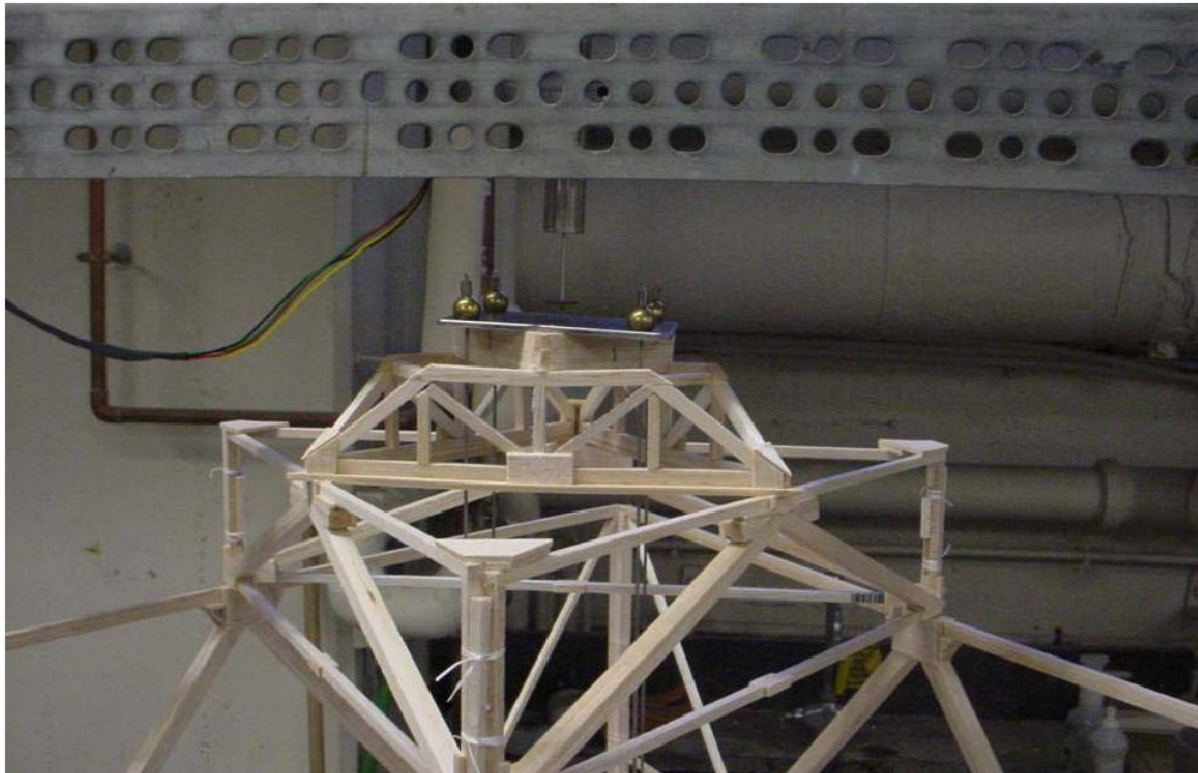




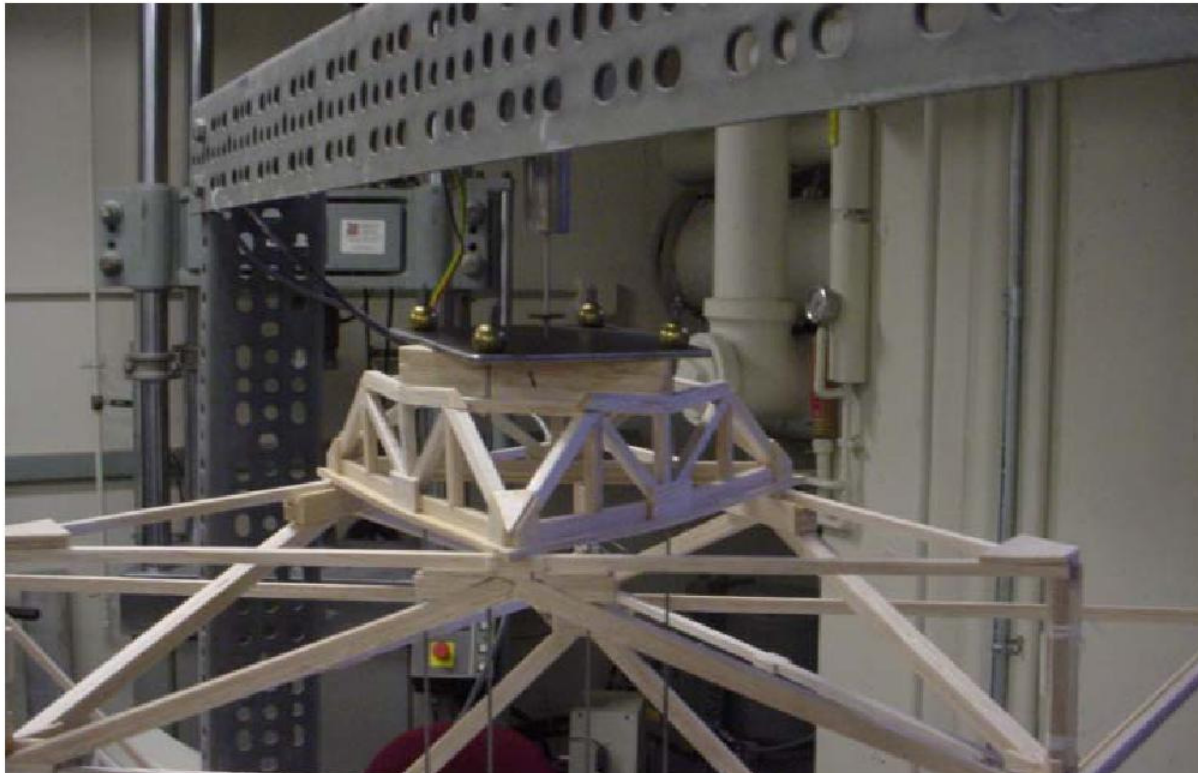




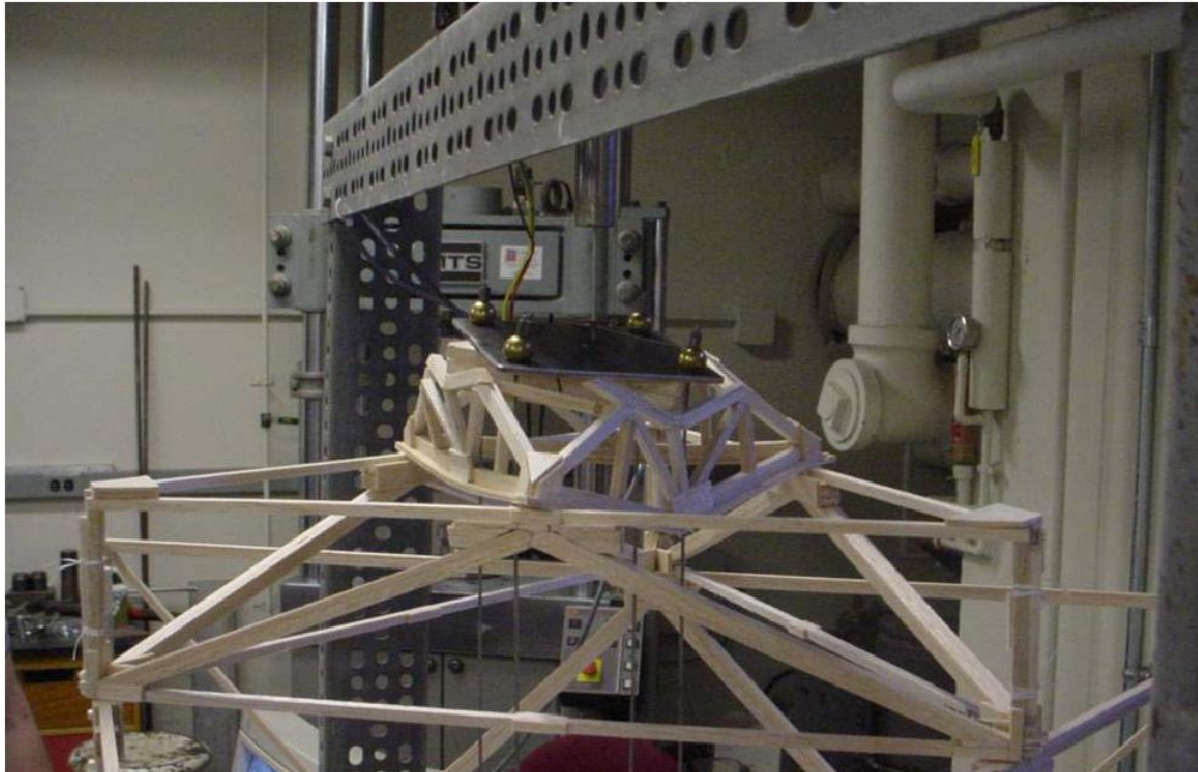




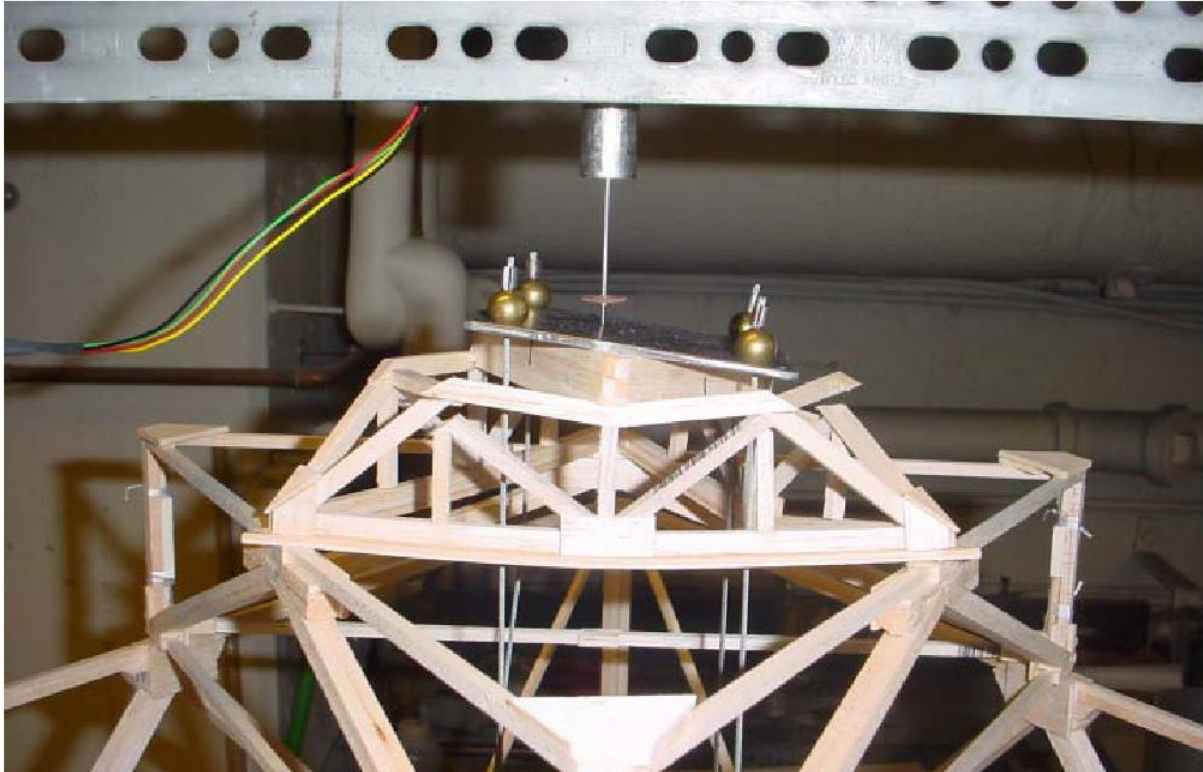




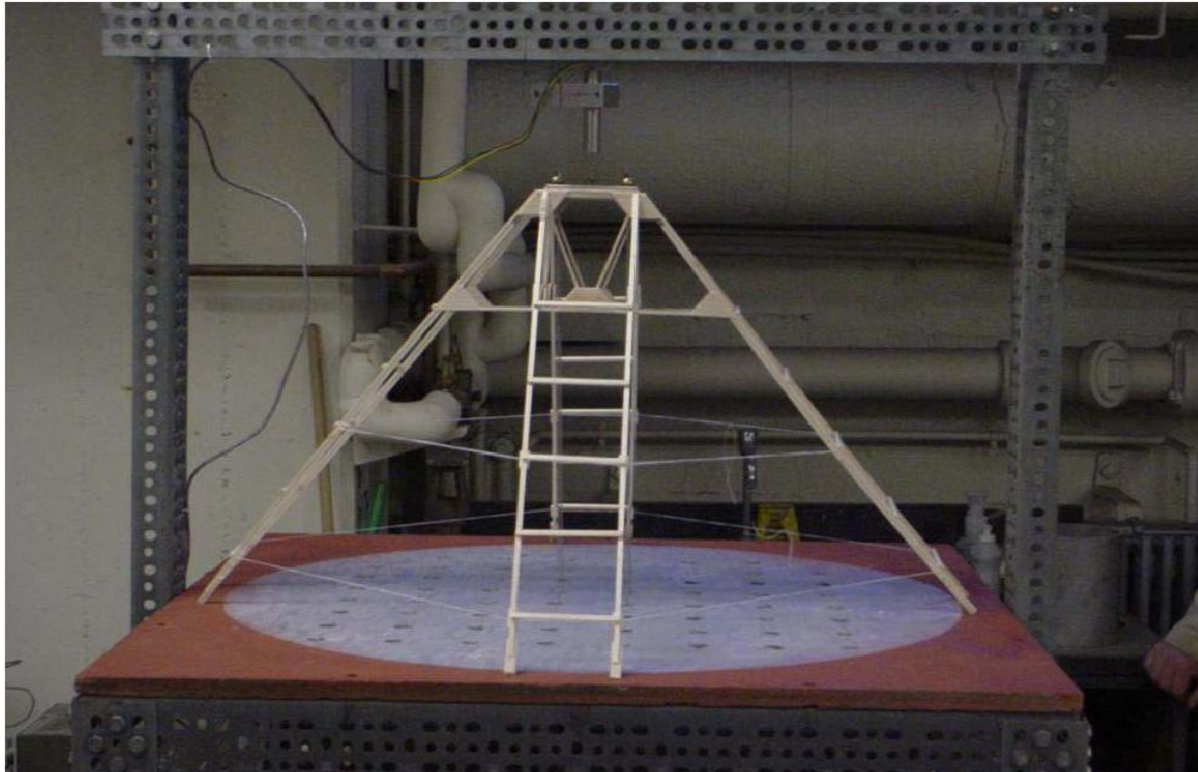








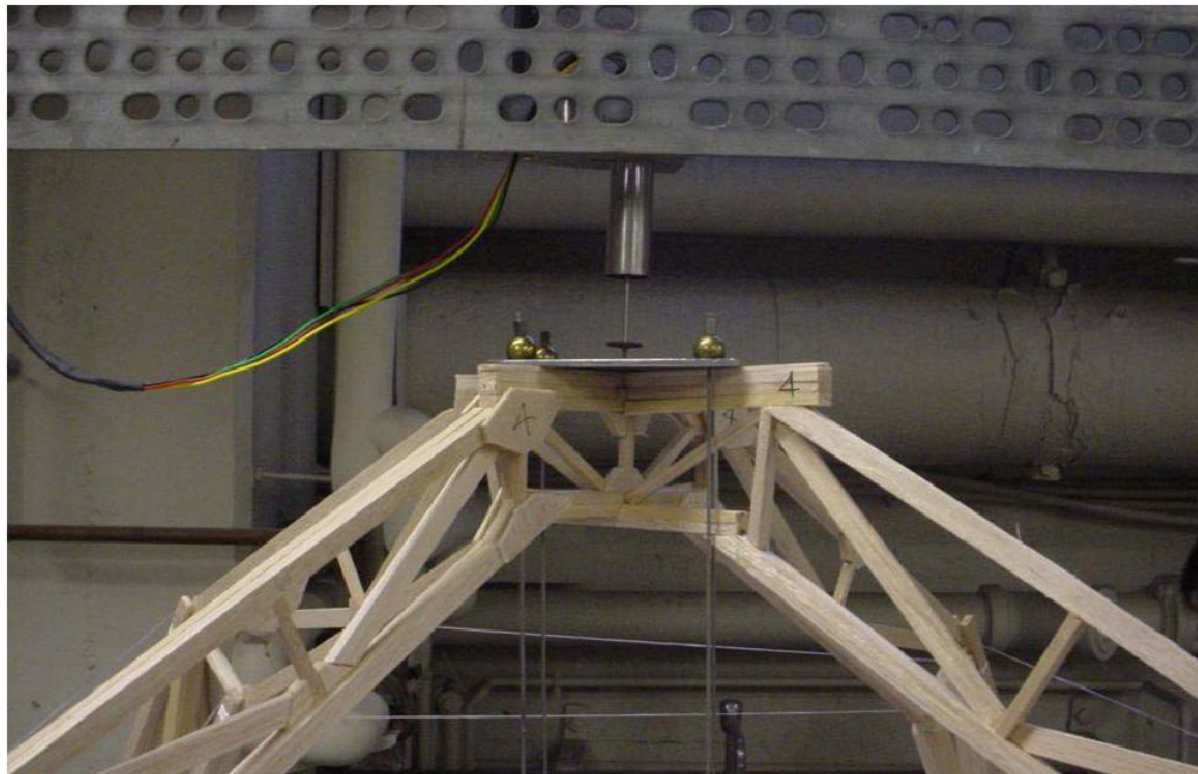




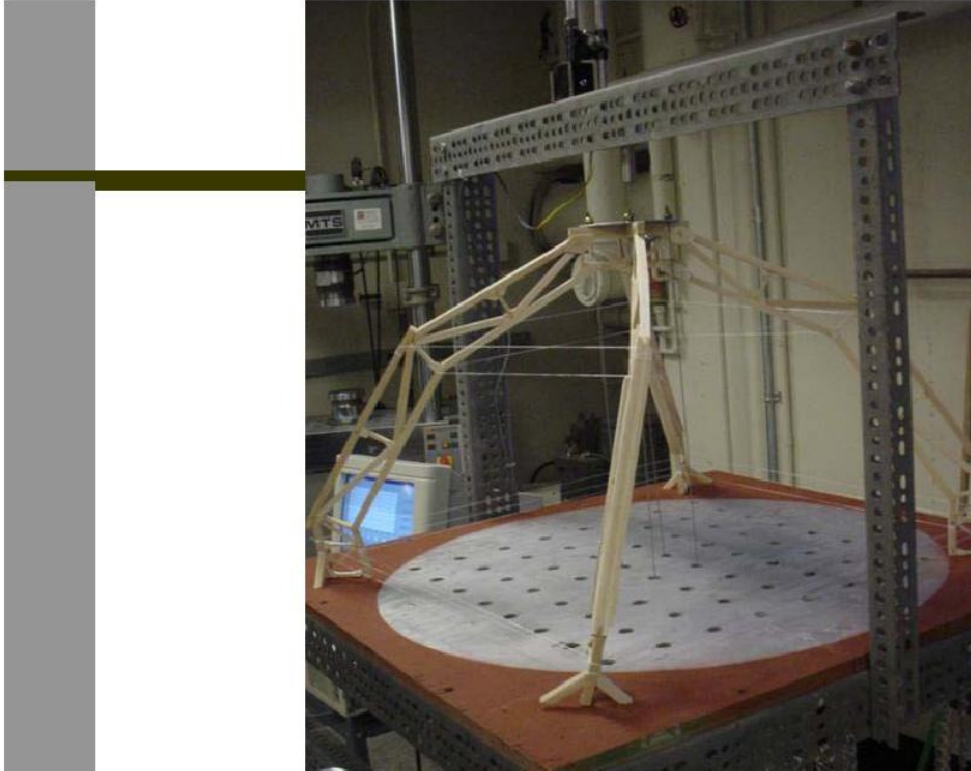




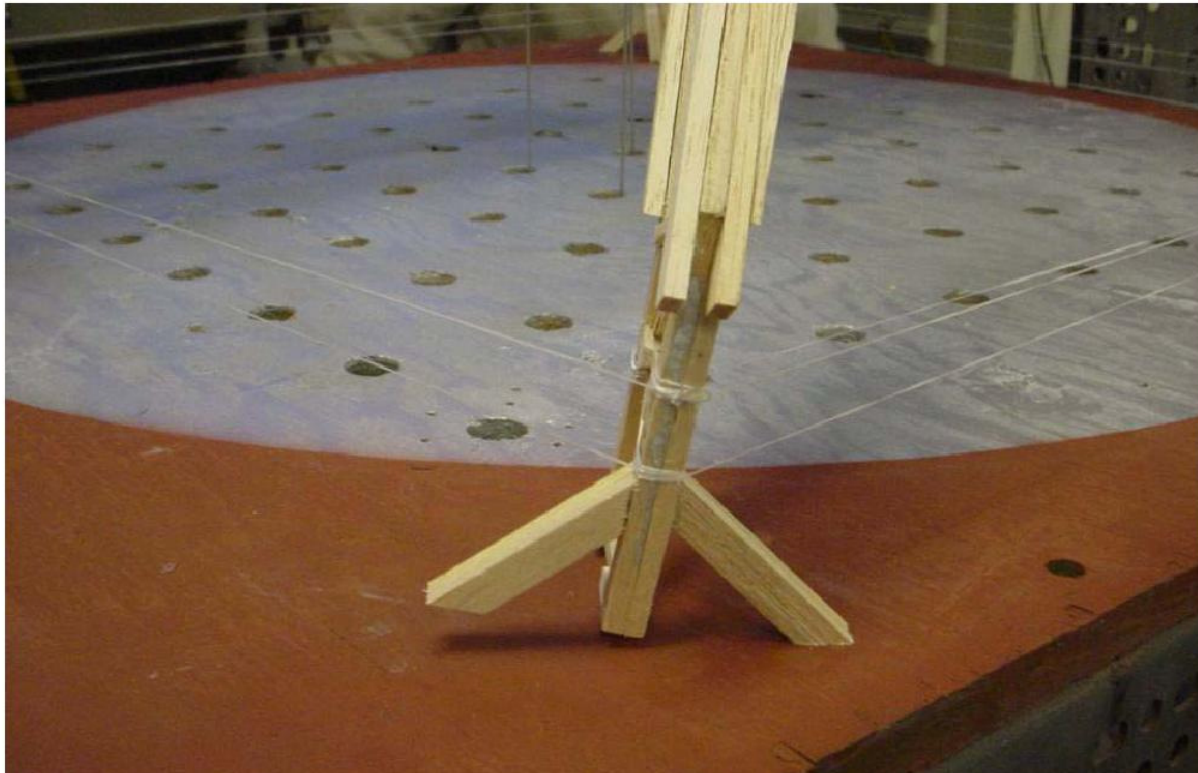




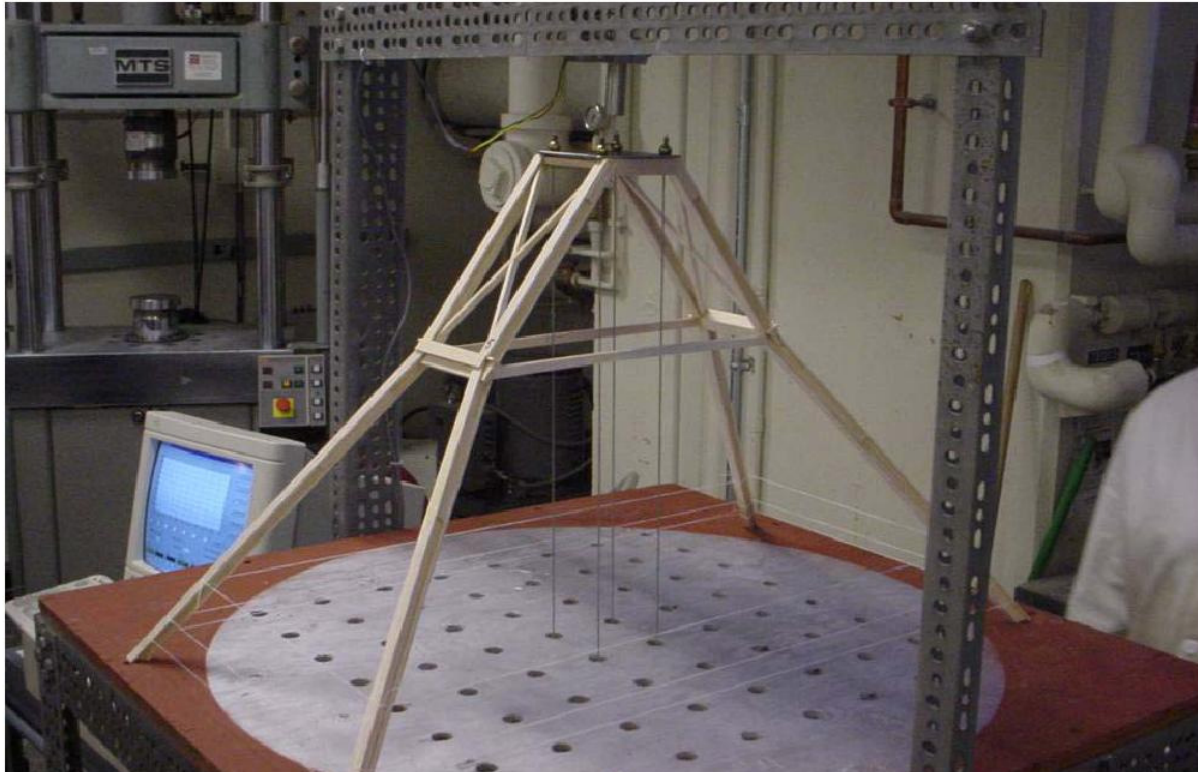




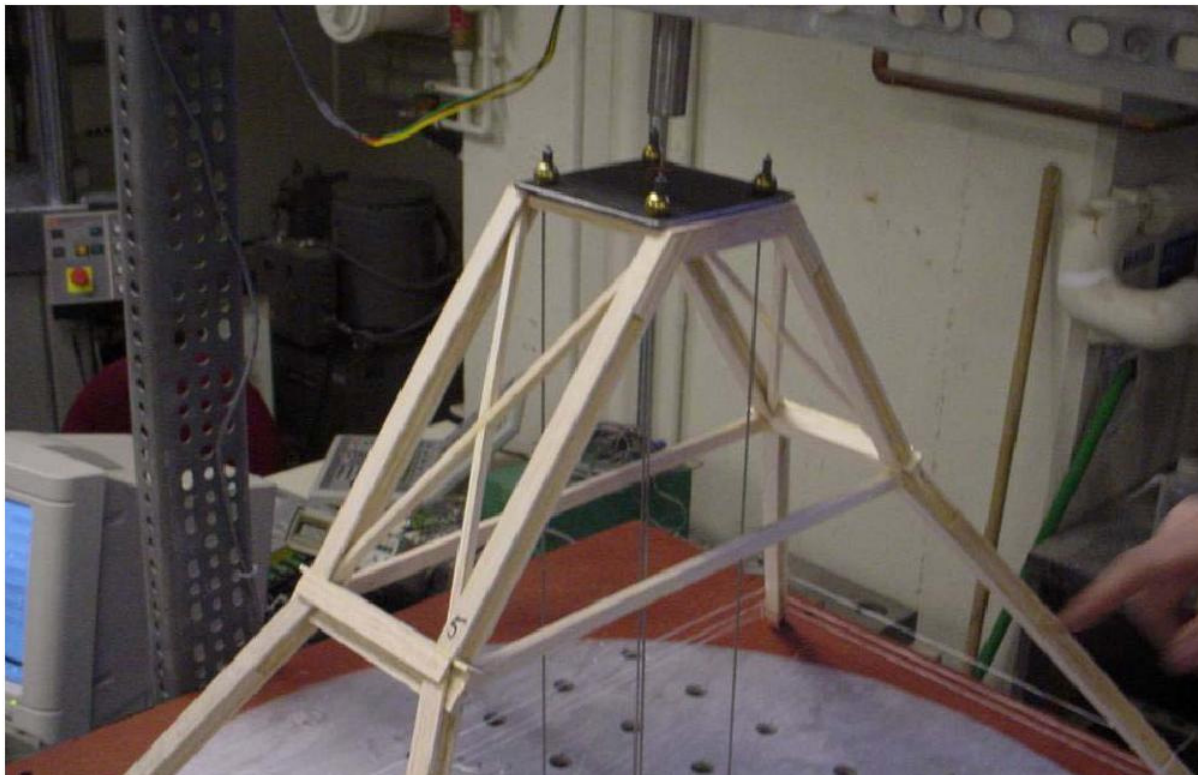




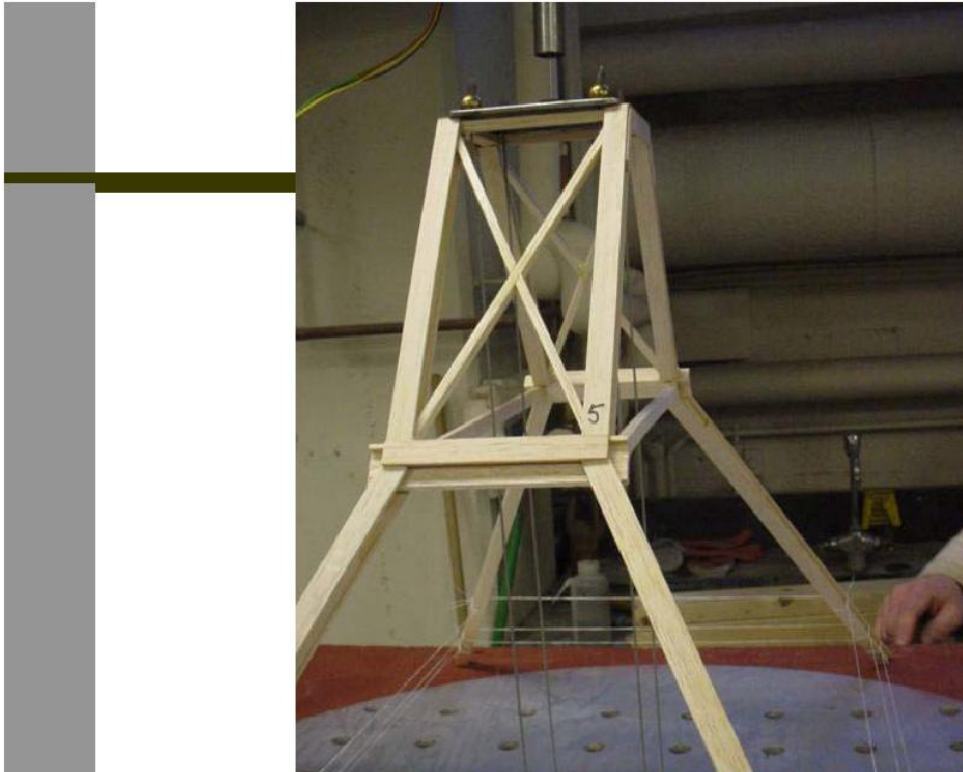




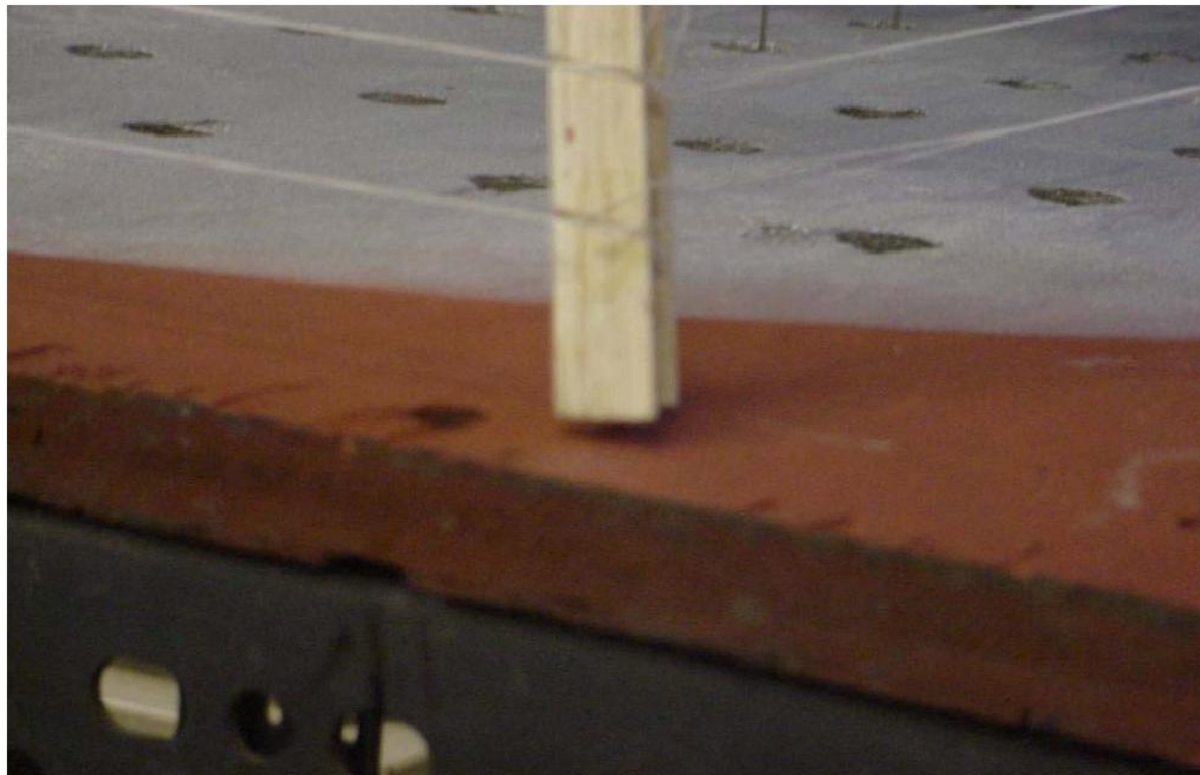


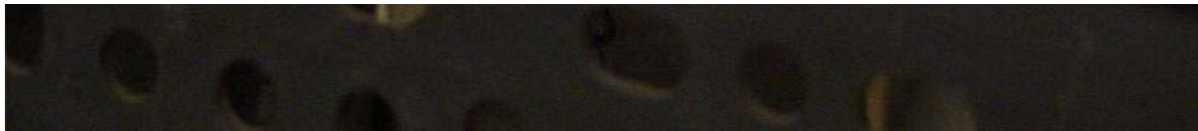


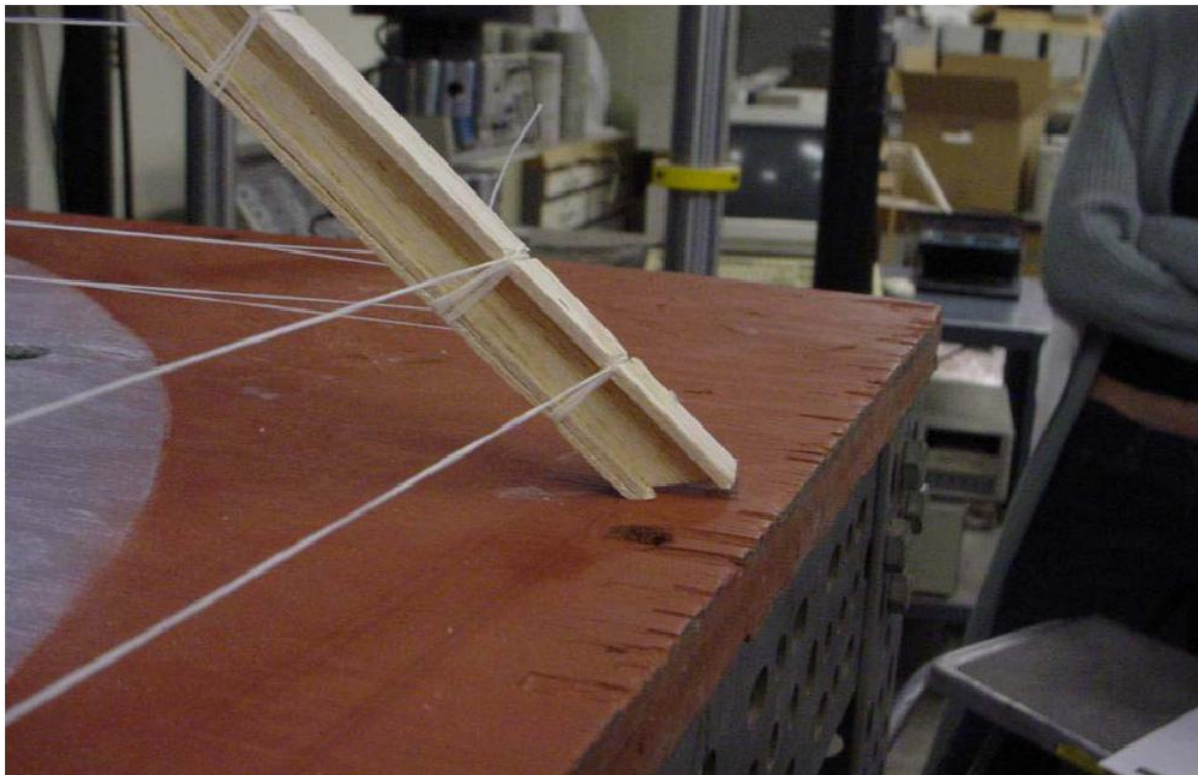




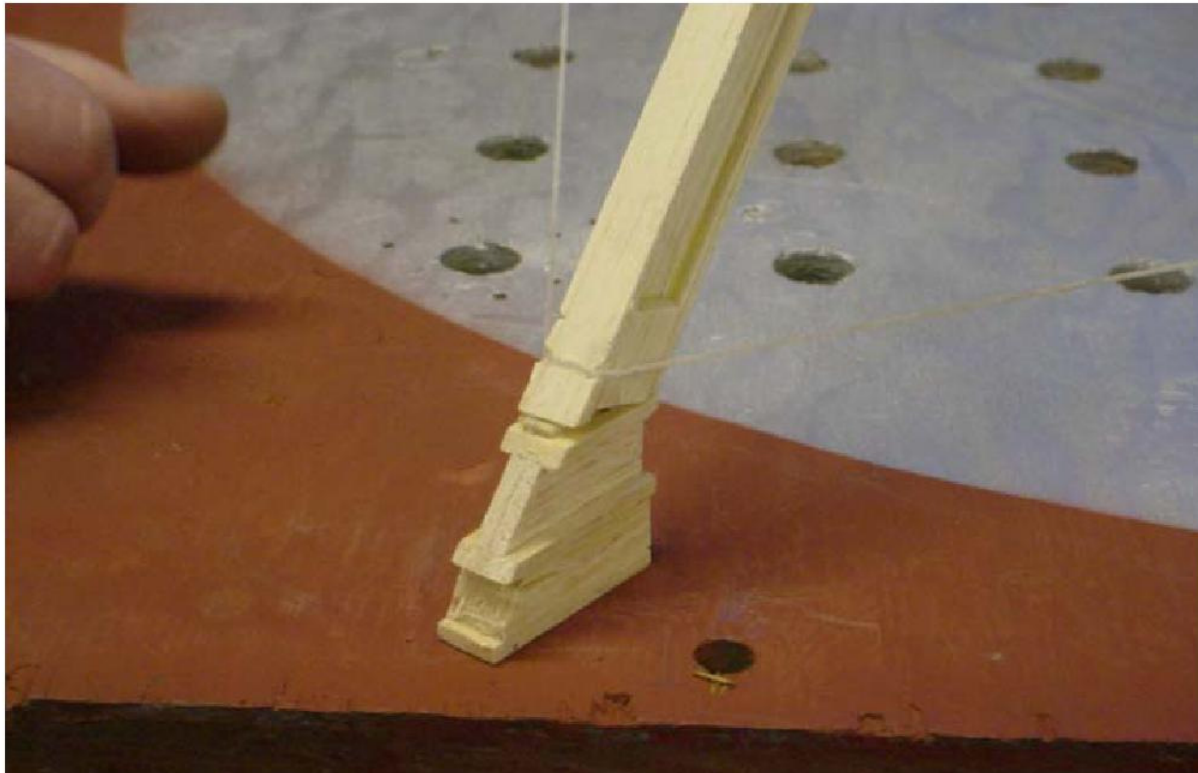




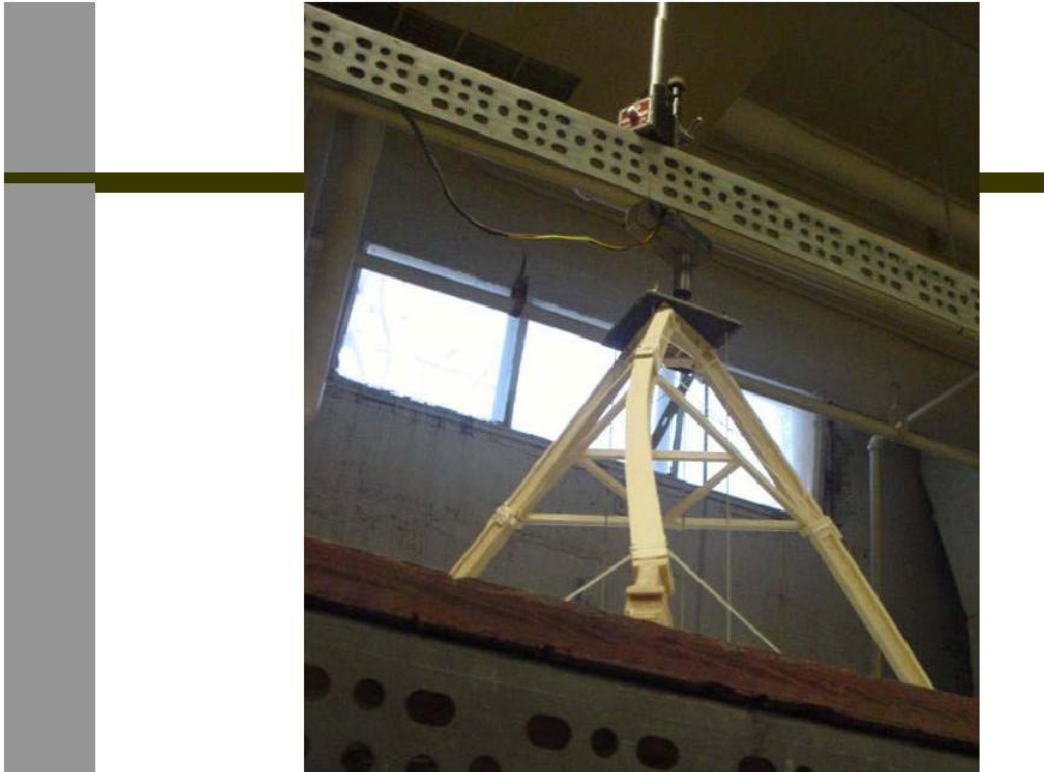




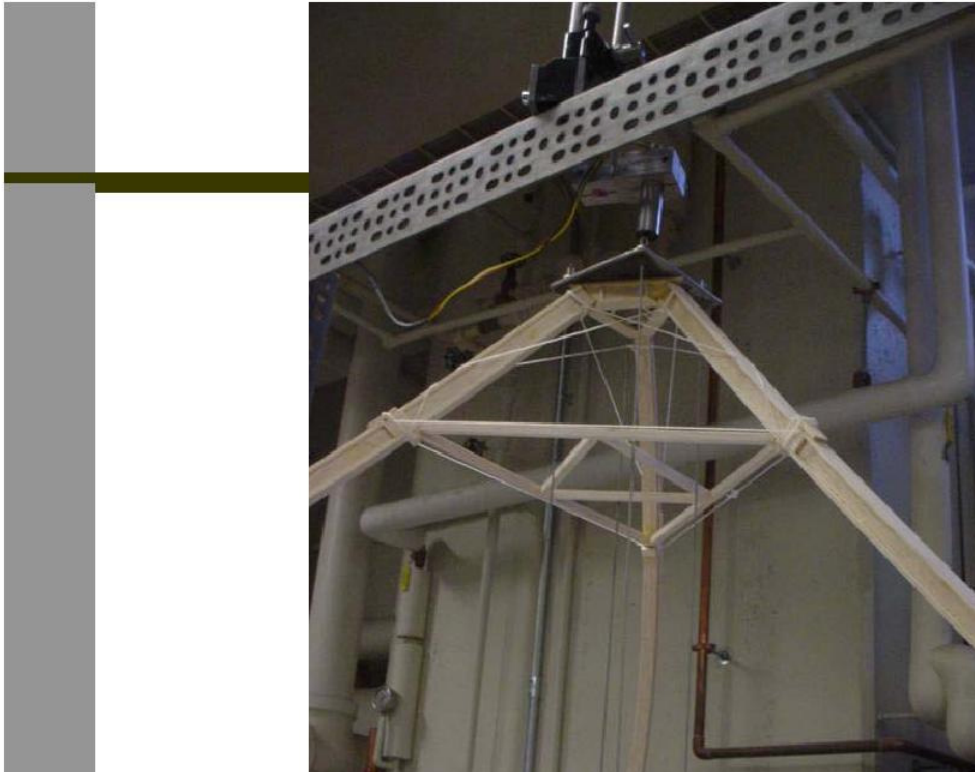












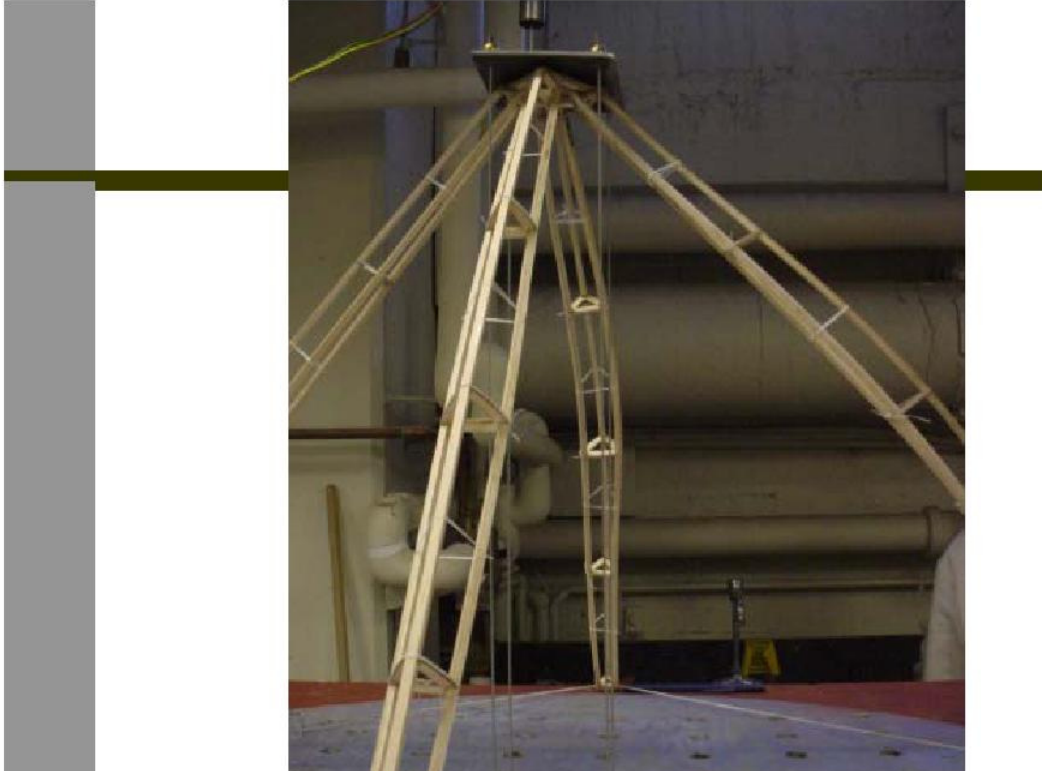








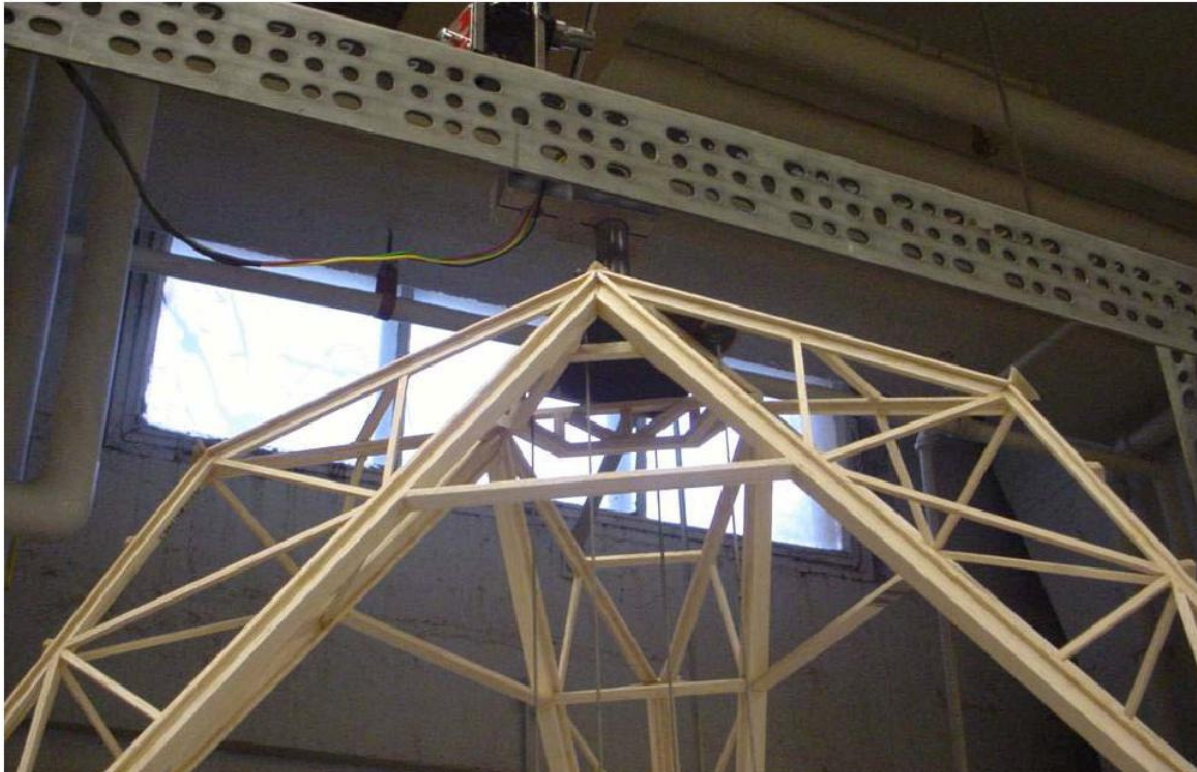




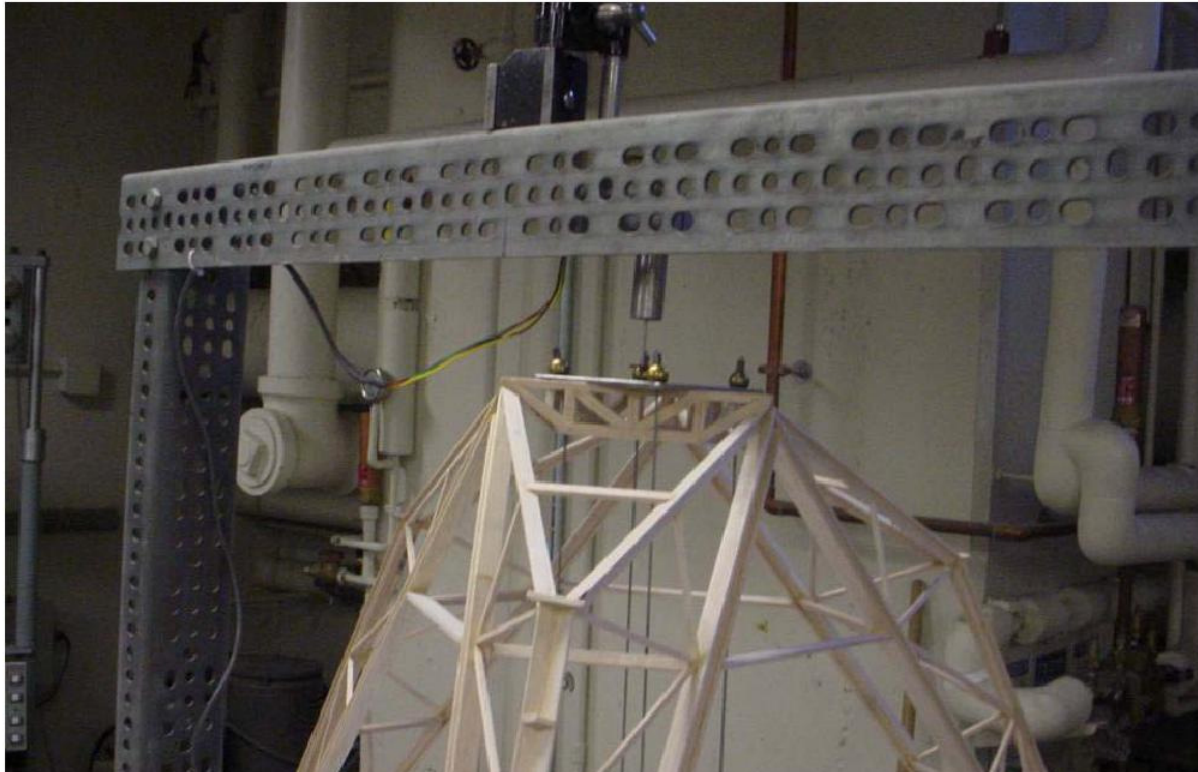




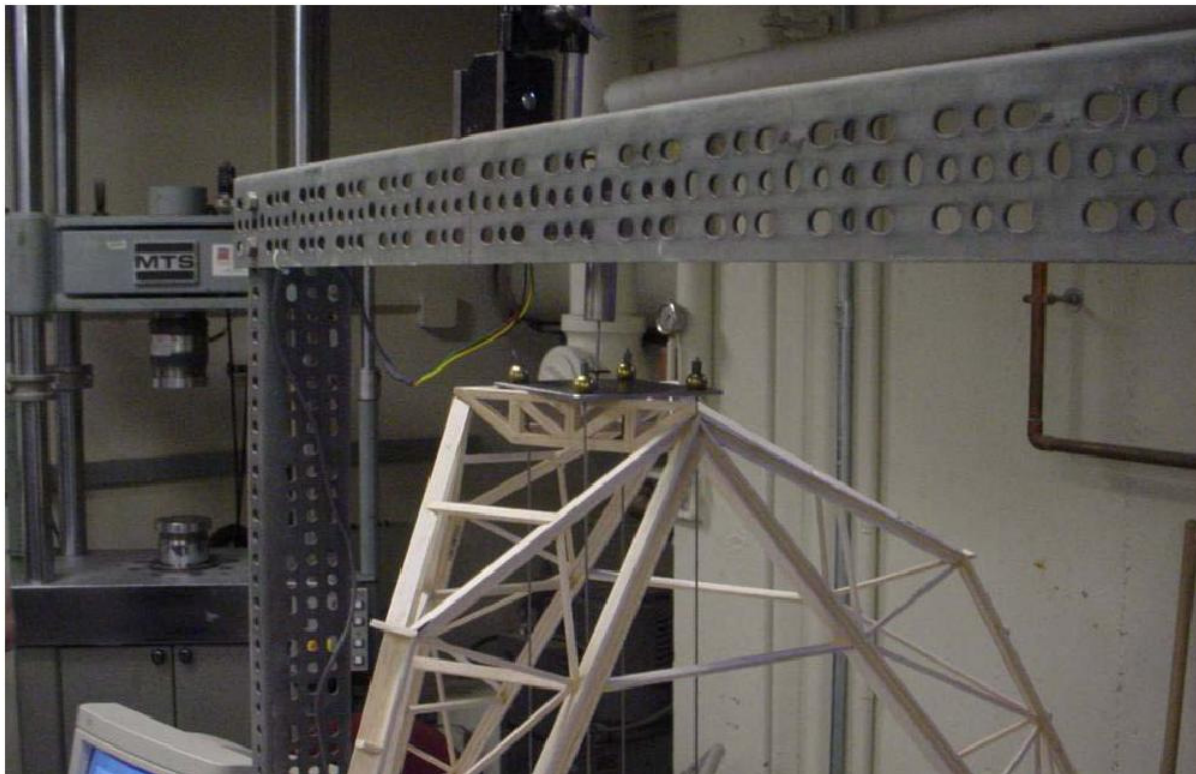




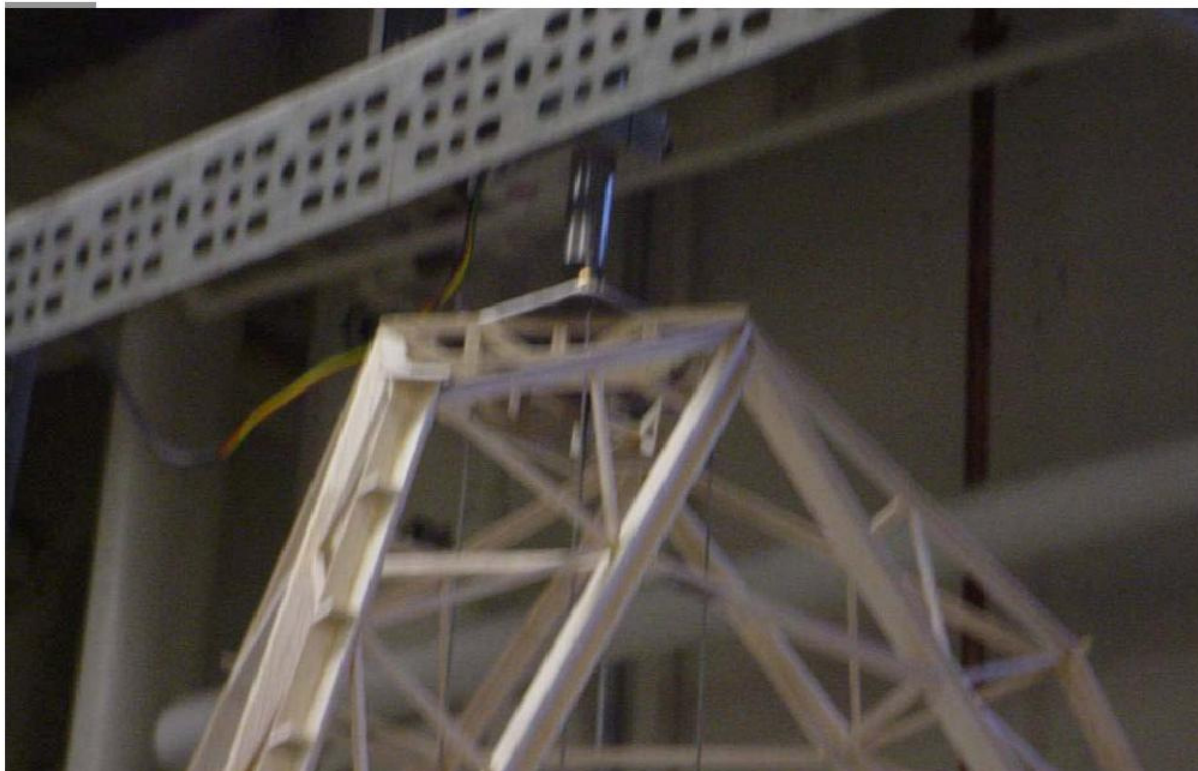




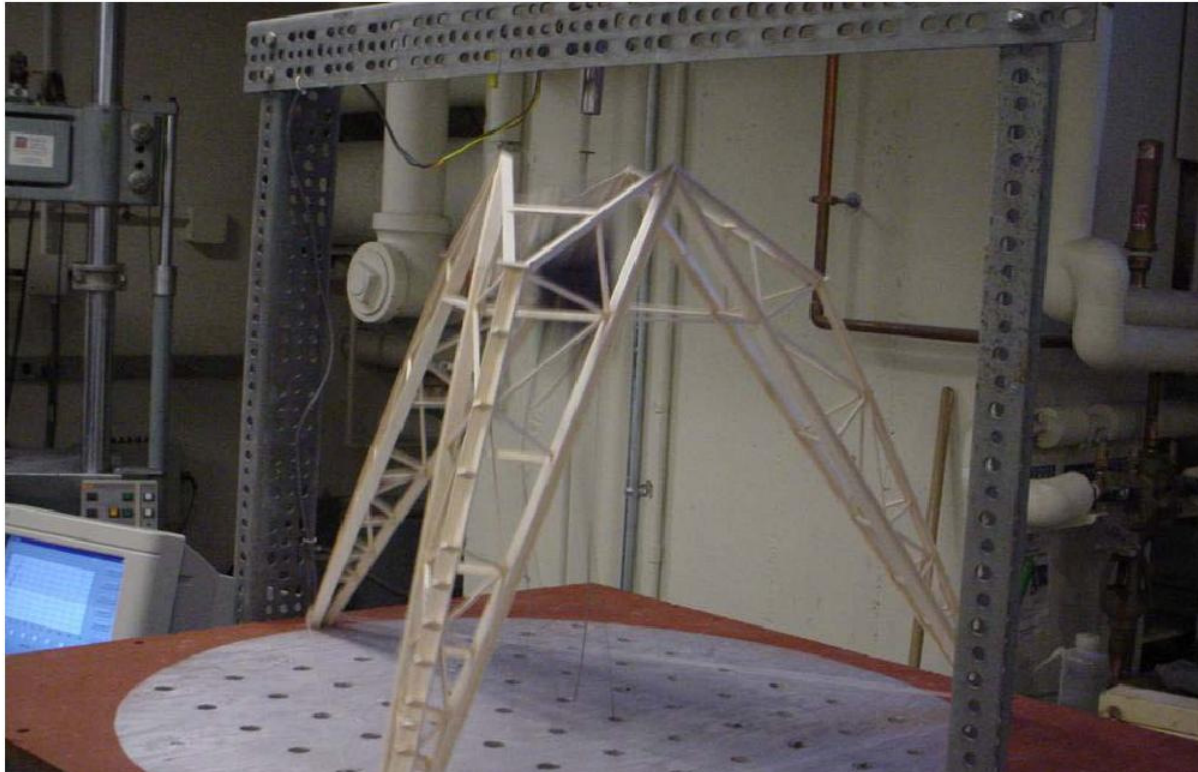






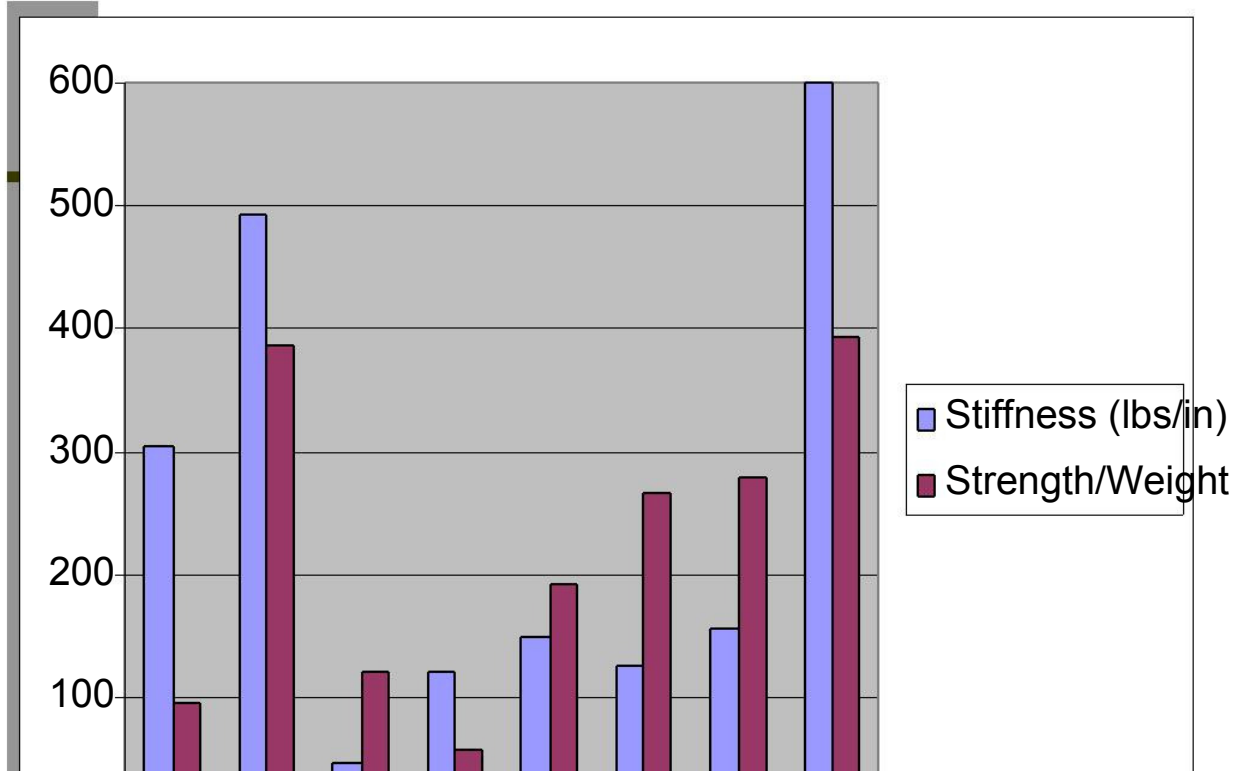


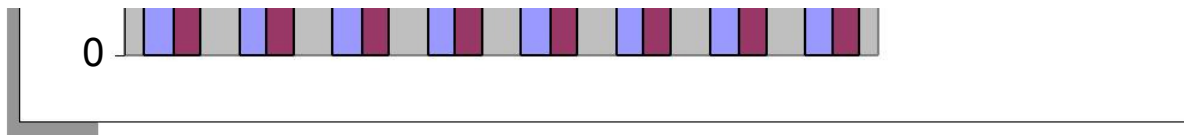




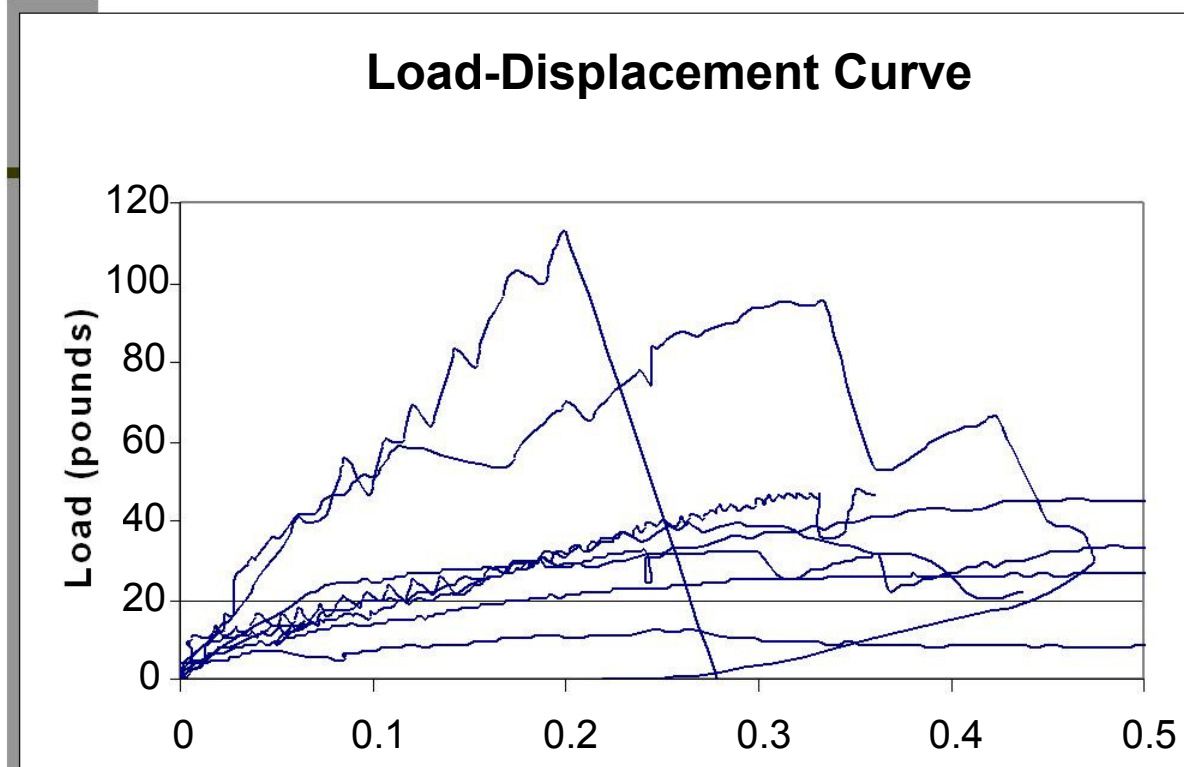


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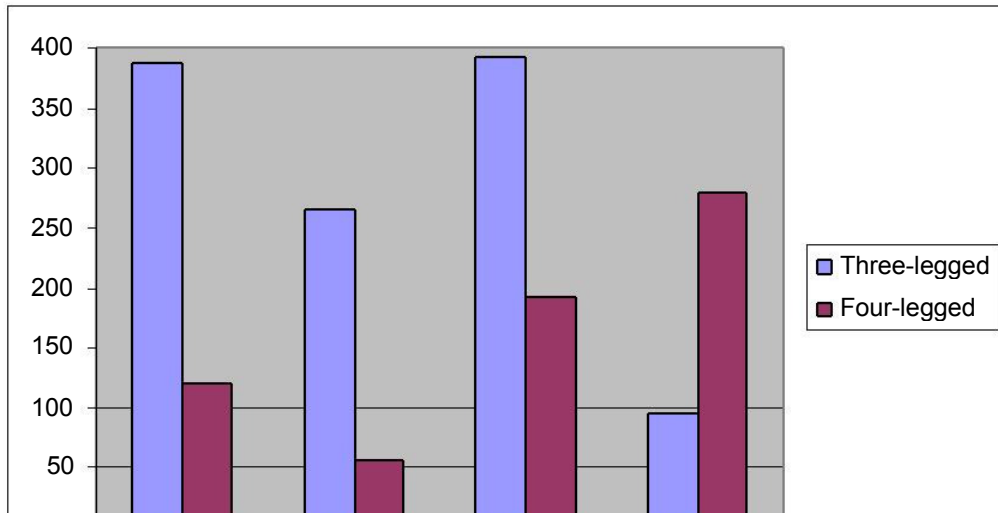
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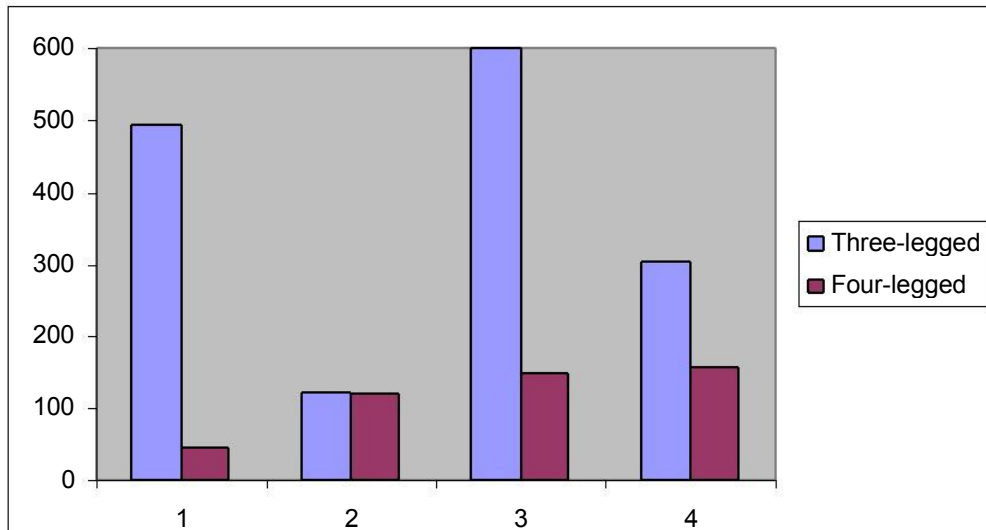
Strength to Weight Ratio

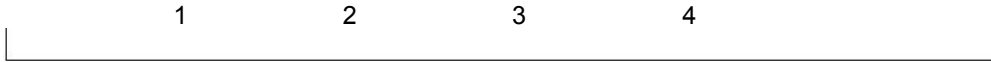




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Stiffness (slope of load-displ curve)



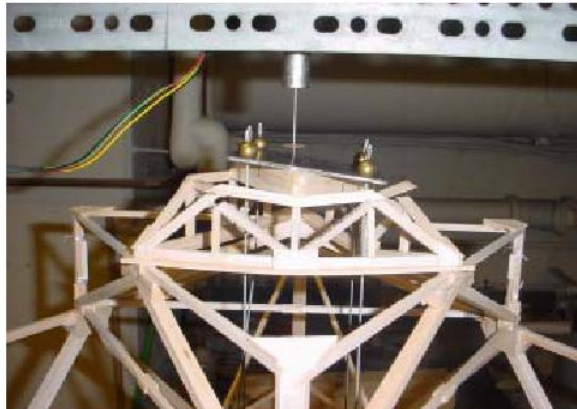


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Lessons: Concept

Systems with many unknowns are difficult to predict

Structures which are simple and clear in their load paths are easier to design





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Lessons: Construction

Details are critical
Repetition helps
Small differences add up to big differences
3D joints are not easy!





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Lessons: Buckling

Failure occurred significantly below calculated values:

- Sensitive to **imperfections**
- Support conditions
- Insufficient bracing
- Lower load in post-**buckled state (not a ductile failure)**
- Effective buckling **length was typically much longer than assumed**





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Sustainable Engineering: The Importance of Structures

Professor John Ochsendorf



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End of Life Design

30 million computers are thrown away each year (only ~14% are recycled now)

Tackling waste flows can reduce environmental impact and save money

The electronics and automobile industry are beginning to design for the end of life (MIT



**Materials Systems
Laboratory)**

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Outline of Lecture

Introduction

Materials Selection

Case Studies

Sustainable Structural Design

Conclusions

Future Challenges



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Construction and the Environment

In the United States, buildings account for:

37% of total energy use

(65% of electricity consumption)

30% of greenhouse gas emissions

30% of raw materials use

30% of waste output (136 million tons/year)

12% of potable water consumption

Source: US Green Building Council (2001)



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Construction and the Environment

US Primary Energy Consumption:

Buildings	37%
Industry	36%
Transportation	28%

Source: US Dept. of Energy



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Spending on Construction

In industrialized nations, construction contributes more than 10% of the Gross Domestic Product (GDP)

An estimated 47% of total spending on construction is for renovation.



**Source:
Daratech
(2001)**

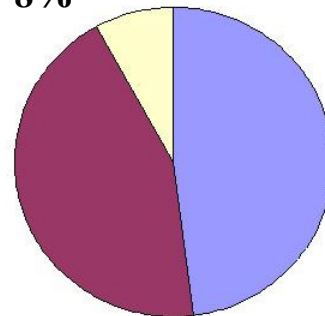
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Construction Waste

US Environmental Protection Agency (EPA) estimates 136 million tons of waste generated by construction each year

Most from demolition or renovation and

**New construction:
8%**



**Renovation:
44%**

**Demolition:
48%**



**nearly half
the weight**

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Goals of Structural Design

Efficiency

Economy

Elegance

**But all must consider
the environmental
impact as well**



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19th Century Design Concern

EFFICIENCY IS IMPORTANT: New materials in construction, such as wrought iron and steel, lead to greater concern for efficiency



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20th Century Design Concern

**MAINTENANCE IS IMPORTANT:
The initial design is important, though
we must also design for maintenance
throughout operating life**



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21st Century Design Concern

**END OF LIFE IS IMPORTANT:
Waste from the construction industry
is a vast consumer of natural resources
on a global scale**



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Buildings are Not Permanent

**Stone pinnacles
of cathedrals are
replaced ~200
years**

**Buildings are
*waste in transit***



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Choosing Materials

Environmental Impact



Durability

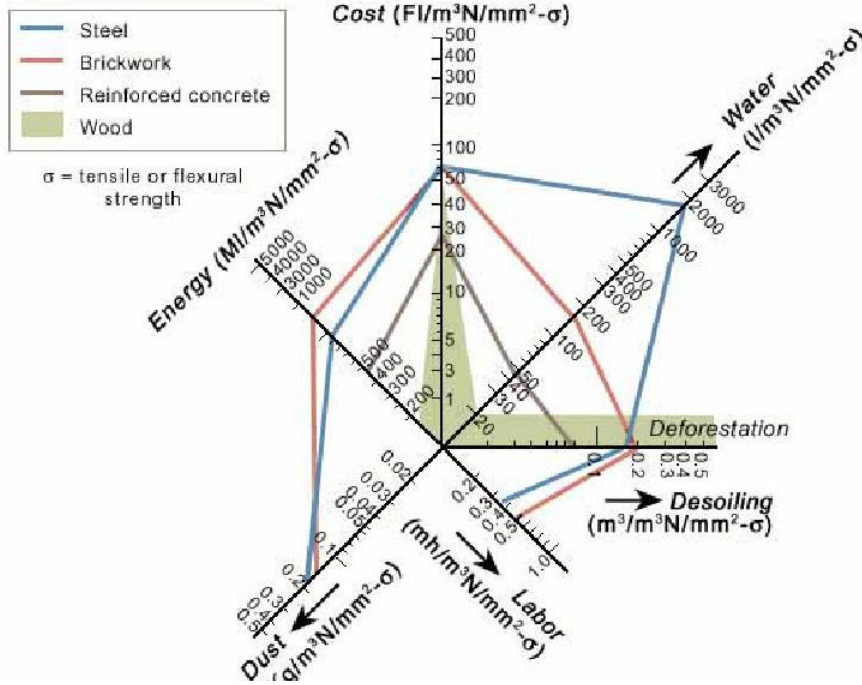


End of Life





Ecological Profile of Materials





Ecological profile of various material properties expressed per unit strength.

The Institution of Structural Engineering

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Is concrete a green material.

Concrete is made from local materials.

Concrete can be made with recycled waste or industrial byproducts (fly ash, slag, glass, etc).

Concrete offers significant energy savings over the lifetime of a building. Concretes high thermal mass moderates temperature swings by storing and releasing energy needed for heating and cooling.



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Energy Required for Concrete

Component	Percent by weight	Energy %
Portland cement	12%	92%
Sand	34%	2%
Crushed stone	48%	6%
Water	6%	0%



Each ton of cement produces ~ 1 ton of CO₂

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Environmental Advantages of Steel

Lower weight reduces foundation requirements

Highly recycled and can continue to be recycled indefinitely

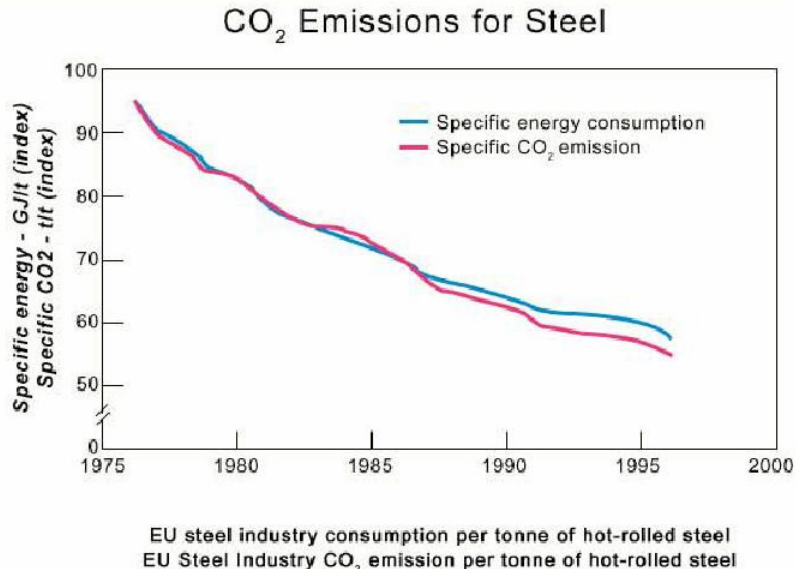
Durable, if protected from corrosion

Can be salvaged for reuse



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Energy Consumption for Steel





(3 year moving averages)

Source: Eurostat

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Environmental Disadvantages of Steel

Very high energy use, predominantly from fossil fuels produces pollution

Lightweight, so lower thermal mass compared to concrete requires more insulation

Is susceptible to corrosion



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CO₂ Emissions in the US

US: 5% of world population, 25% of greenhouse gases

UK: commitment to cut CO₂ emissions 60% by 2050 (well beyond the goals of the Kyoto Protocol)



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Kyoto Protocol and CO₂

**To meet Kyoto Protocol: ~33,000 lbs
of CO₂/year/person (-7% from 1990)**

**But individual contributions are only 1/3 of per capita
contributions rest is industry, agriculture, etc.**

**So individuals annual goal would be 11,000 lbs (though
many scientists are calling for much greater
reductions)**



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Kyoto Protocol

Aims to reduce CO2 emissions by 7% over 1990 levels (though the UK has just committed to going much further 60% reductions of current emissions)

Would limit personal carbon emissions to 11,000 pounds of CO2/year

This quantity of CO2 is produced by:

- Two coast-coast flights (economy class)
- Driving 11,000 miles (with 20 mpg fuel efficiency)
- Casting 16 cubic yards of concrete



About 14 cubic
feet of structural
steel

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Kyoto Protocol

Aims to reduce CO2 emissions by 7% over 1990 levels (though the UK has just committed to going much further)

This requires approximate CO2 emissions of 33,000 lbs/year for each person in the US

Only about 1/3 comes from personal decisions, the rest is due to industry and services

Architects and engineers



~~Architects and Engineers~~
contribute to the industry and services

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The Greenest of ThemAll.

Only one primary building material:

- comes from a renewable resource;**
- cleans the air and water;**
- utilizes nearly 100% of its resource for products;**
- is the lowest in energy requirements;**
- creates fewer air and water emissions; and is**
- totally reusable, recyclable and biodegradable.**

And it has been increasing in US net reserves since 1952, with growth exceeding harvest in the US by more than



30%.

-American Wood Council

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Planting trees.

A healthy tree stores about 13 pounds of CO₂ per year -- NOT MUCH!

Would require nearly 3,000 trees per person to offset CO₂ emissions

Specifying timber reduces CO₂ emissions compared to steel and concrete, but carbon sequestration is a small contribution to this reduction

Main advantage is that wood does not produce nearly



**as much CO₂ as
steel and concrete**

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High vs. Low Embodied Energy.

Materials with the lowest embodied energy intensities, such as concrete, bricks and timber, are usually consumed in large quantities.

Materials with high energy content such as stainless steel are often used in much smaller amounts.

As a result, the greatest amount of embodied energy in a building can be either from low embodied energy materials such as concrete, or high embodied energy materials such as steel.



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Steel and Concrete

Energy intensive materials

High associated CO2 emissions

Dominant structural materials

Industry standards

Many engineers have not designed with other materials

Economies of scale

Steel provides ductility, the ability to absorb energy before
failing

Many other materials can serve in



Many other materials can serve in
place of steel and
concrete.

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Energy Savings from Recycling

	Energy required to produce from virgin material (million Btu/ton)	Energy saved by using recycled materials (percentage)
Aluminum	250	95
Plastics	98	88
Newsprint	29.8	34
Corrugated Cardboard	26.5	24
Glass	15.6	5



STAFF

1989

✓

Source:Roberta Forsell Stauffer of National Technical Assistance Service (NATAS), published in Resource Recycling, Jan/Feb 1989).

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Electronics Industry

**30 million computers thrown away
annually in the US**

Only 14% recycled

Initial design must consider end of life



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Embodied Energy vs. Operating Energy (3,750 sf home)

Home type, location	Heating Energy MM Btu/year (Gj/year)	Embodied Energy MM Btu (Gj)	Embodied Energy in years of heating energy
Conventional, Vancouver	101 (107)	948 (1,000)	9.4
Energy-efficient, Vancouver	57 (60)	1019 (1,075)	17.9
Conventional, Toronto	136 (143)	948 (1,000)	7.0



**Energy
Toronto
efficient,**

78 (82)

1019 (1,075)

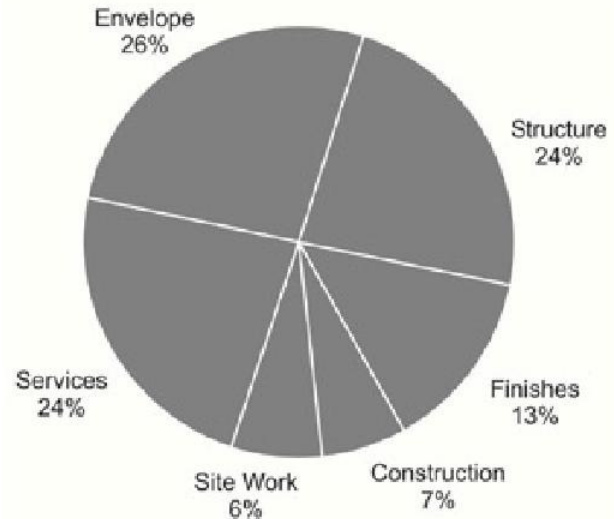
13.1

Source: Ray Cole, Univ. of British Columbia

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Typical Building Embodied Energy

Breakdown of Initial Embodied Energy by Typical Office Building Components Averaged Over Wood, Steel and Concrete Structures [Cole and Kernan, 1996].





Average Total Initial Embodied Energy 4.82 GJ/m²

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Range in Embodied Energy

Material	Density	Low value	High value
	kg/m ³	GJ/m ³	GJ/m ³
Natural aggregates	1500	0.05	0.93
Cement	1500	6.5	11.7
Bricks	~1700	1.7	16
Timber (prepared softwood)	~500	0.26	3.6
Glass	2600	34	81
Steel (sections)	7800	190	460



Plaster	~1200	1.3	8.0
----------------	--------------	------------	------------

Source: BRE, UK, 1994

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Reducing Waste

Design for Less Material Use

Use materials efficiently and maximize program use by combining spaces. (i.e., build smaller)

Design Building for Adaptability

Design multipurpose areas or flexible floor plans which can be adapted for use changes.

Recycle Construction Waste

Wood, metal, glass, cardboard etc. can be salvaged in the construction process. Materials should be used and ordered conservatively.



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Use Recycled Content Products and Materials

High recycled content:

Paper on both the face and the back of all drywall is a 100% recycled product.

Structural steel uses mostly recycled material (though it is still energy-intensive and responsible for harmful pollutants.)

Example of an item that you can specify:

Armstrong ceiling tiles contain 79% recycled material (cornstarch, newsprint, mineral wool, recycled tiles). Both the ceiling tiles and the suspension systems can also be reclaimed and recycled rather than dumped



**in a
landfill.**

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Separating Waste





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Australia: Waste Avoidance and Resource Recovery Act (2001)

**Web site dedicated to Construction &
Demolition waste minimization: onSITE**

<http://onsite.rmit.edu.au/>



(Source of material for this lecture.)

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Steel and Concrete

Can be designed for reuse (concrete pavers)

Can reduce required quantities through efficient design

We will return to the design of indeterminate structures and the importance of structural form

Designers can use the constraints of economics, efficiency, and the environment to



find new forms

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Ecological Comparison of Materials

Each material has environmental advantages and disadvantages

Choice of material will depend on the site and design problem

Embodied energy is only one of many



considerations

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Design Matters

19th Century: Efficient use of materials

20th Century: Maintenance matters

21st Century: End of life matters





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Designing for Maintenance

**Develop a maintenance
plan for your structure**

**Design components
which are accessible and
replaceable**

**Avoid toxic materials
which are hazardous for
future maintenance**

..



operations

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Demolition: Lessons from History

**Sustainable structures
must consider the end of
life of the structure**

**~24% of solid landfill
waste in the US is
generated by the
construction industry**

**Up to 95% of construction
waste is recyclable, and
most**



**most
is
clean
and
unmixed**

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Two Extreme Approaches to Sustainable Structures

- 1. Permanence:** Very high quality construction, with materials which can be reused in future construction

- 2. Temporary:** Less expensive construction, with a short life span. Materials must be low-impact.

impact.



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Two Sustainable Bridge Types

Inca suspension bridge

High stresses
High maintenance
Short lifetime
Low initial cost
Renewable materials
Low load capacity



Roman arch bridge

Low stresses
Low maintenance
Long lifetime
High initial cost
Reusable materials
High load capacity





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Conclusions

Each material has environmental advantages and disadvantages: good design is local

Recycle or reuse materials to decrease waste

Consider end of life in the initial design

History suggests sustainable solutions: Inca structures (temporary) and Roman structures



(permanent) can both be sustainable

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Conclusions

Construction industry generates enormous waste annually

Individual designers can reduce this waste significantly

Energy intensive materials like steel and concrete can be used more efficiently



Alternative materials should be explored

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Future Challenges

Education of architects and engineers

Teaching design and analysis

Assessment of existing structures

Environment as a design constraint, not an opponent

Maintenance and disposal plan for new structures

Code improvements for the reuse of salvaged structures and new uses of traditional materials

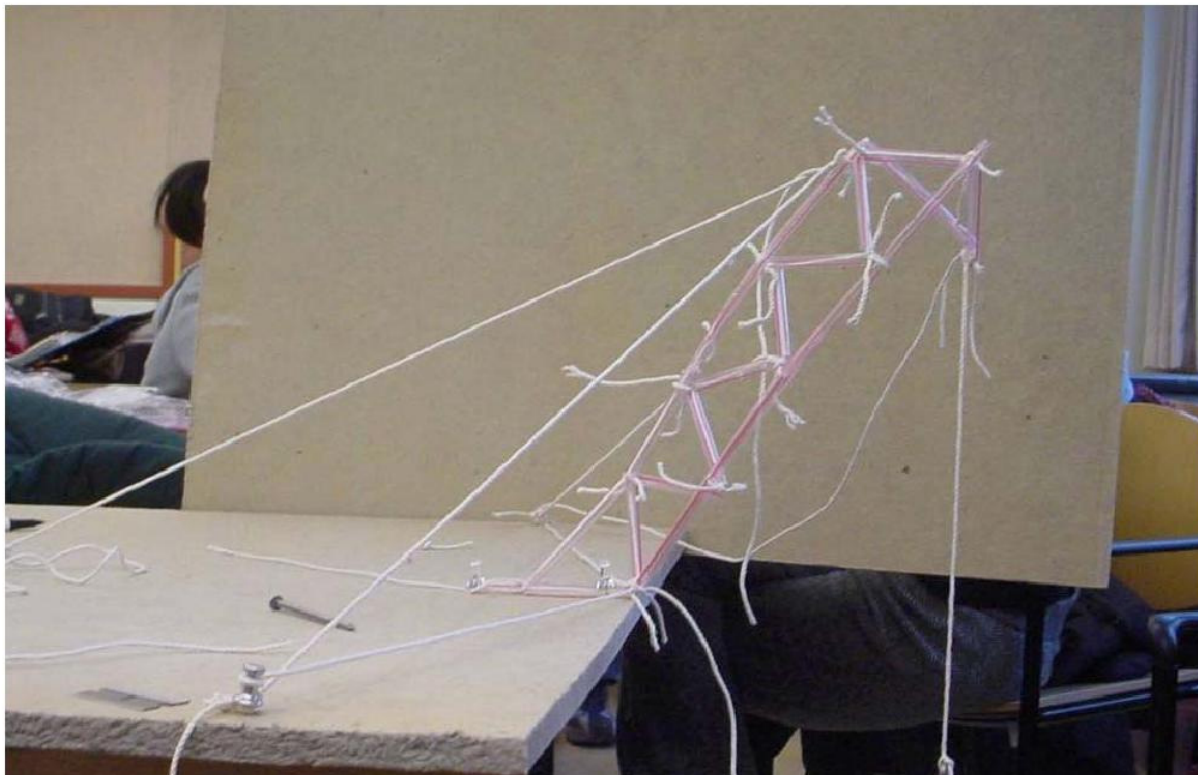


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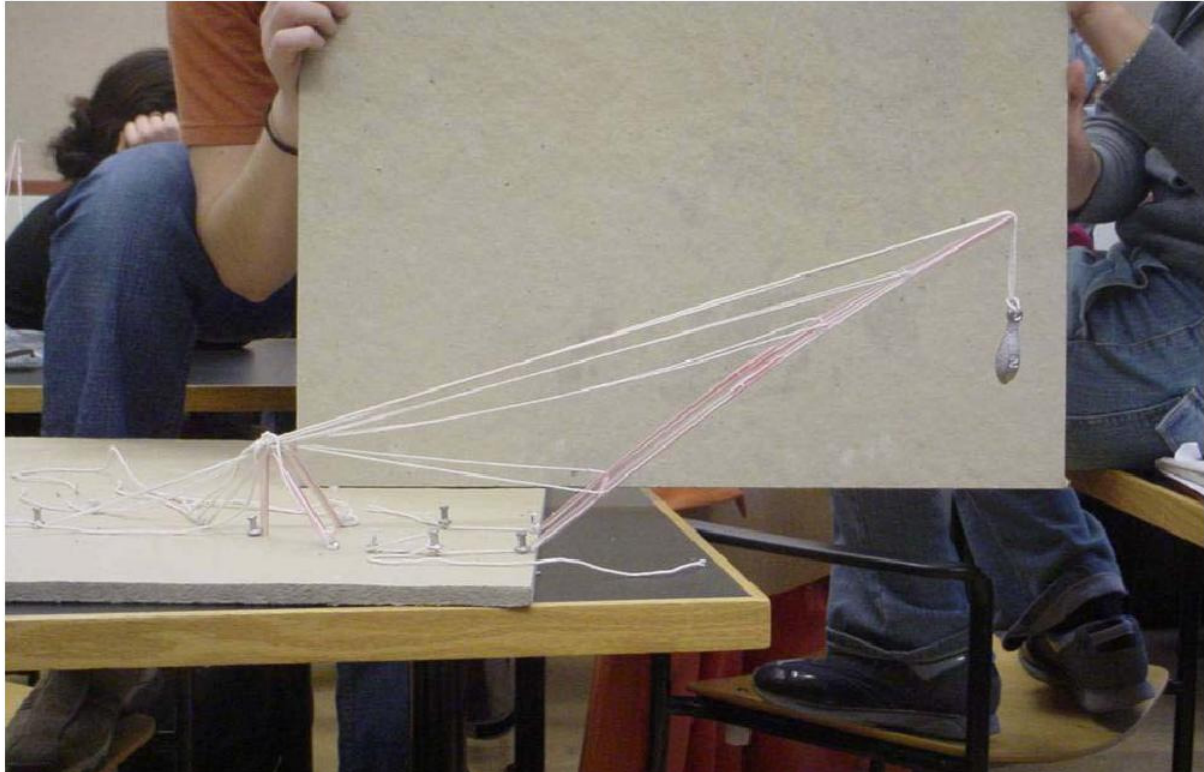
Tension Structures

4.440 Basic Structural Theory

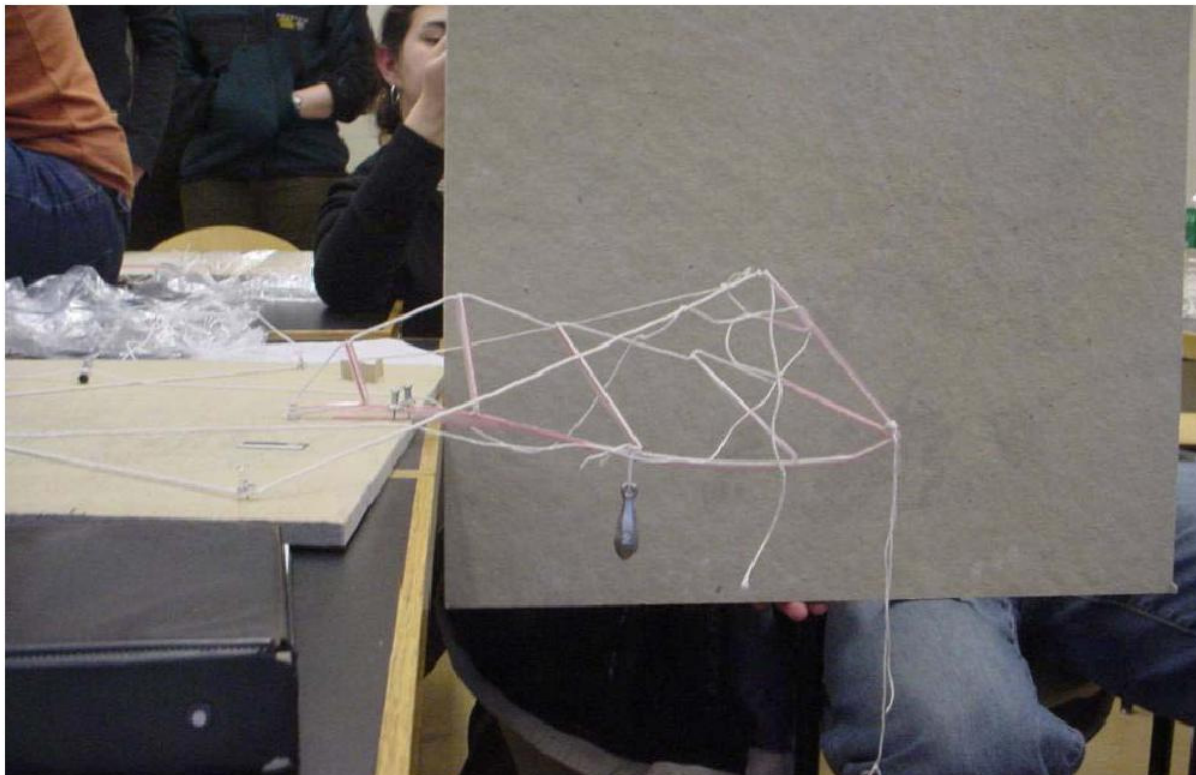


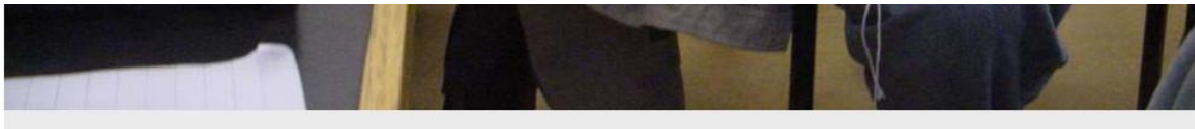


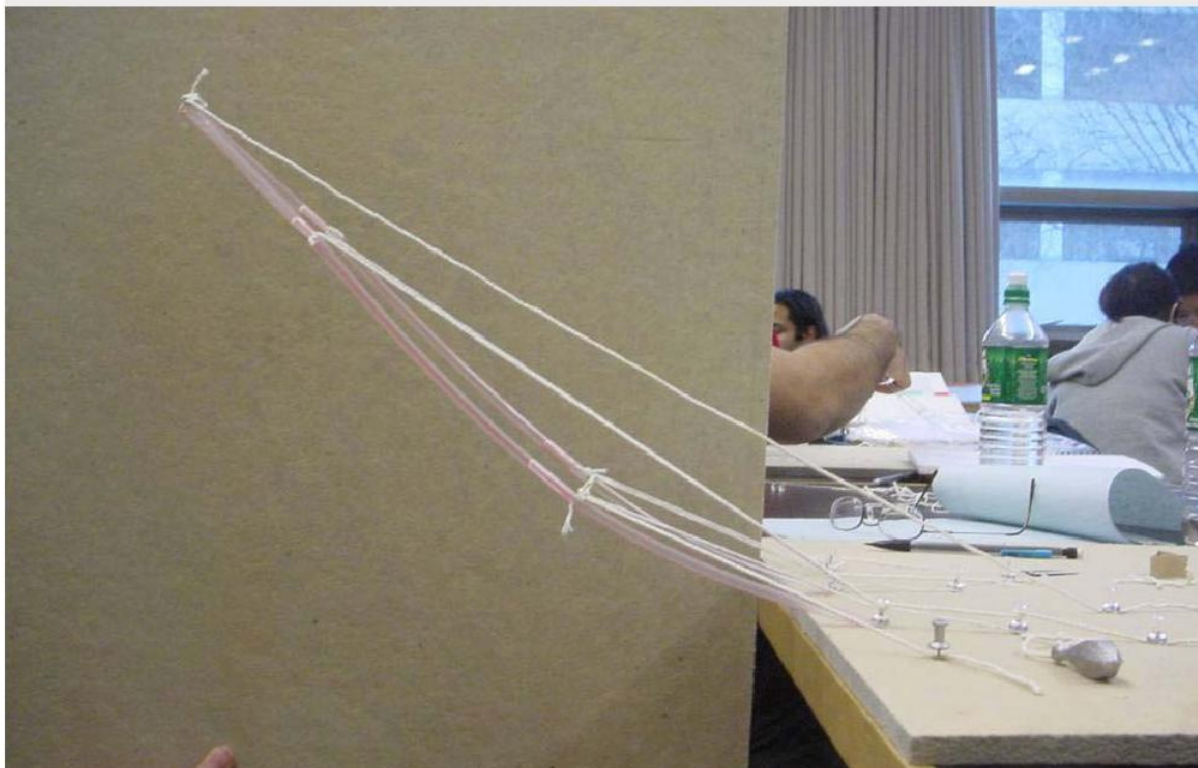


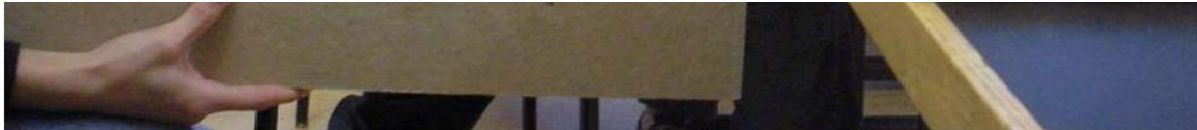


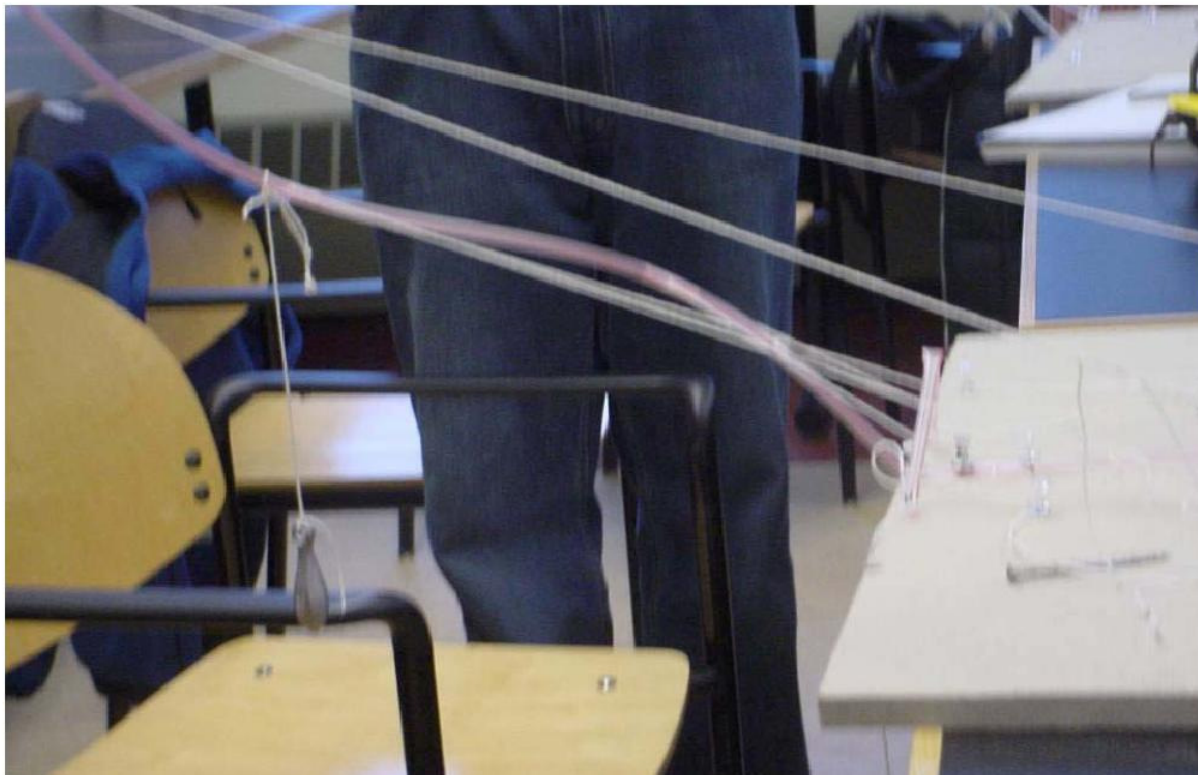


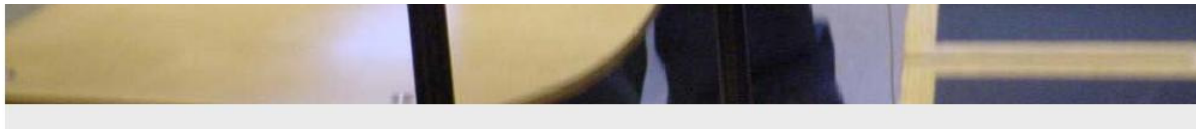


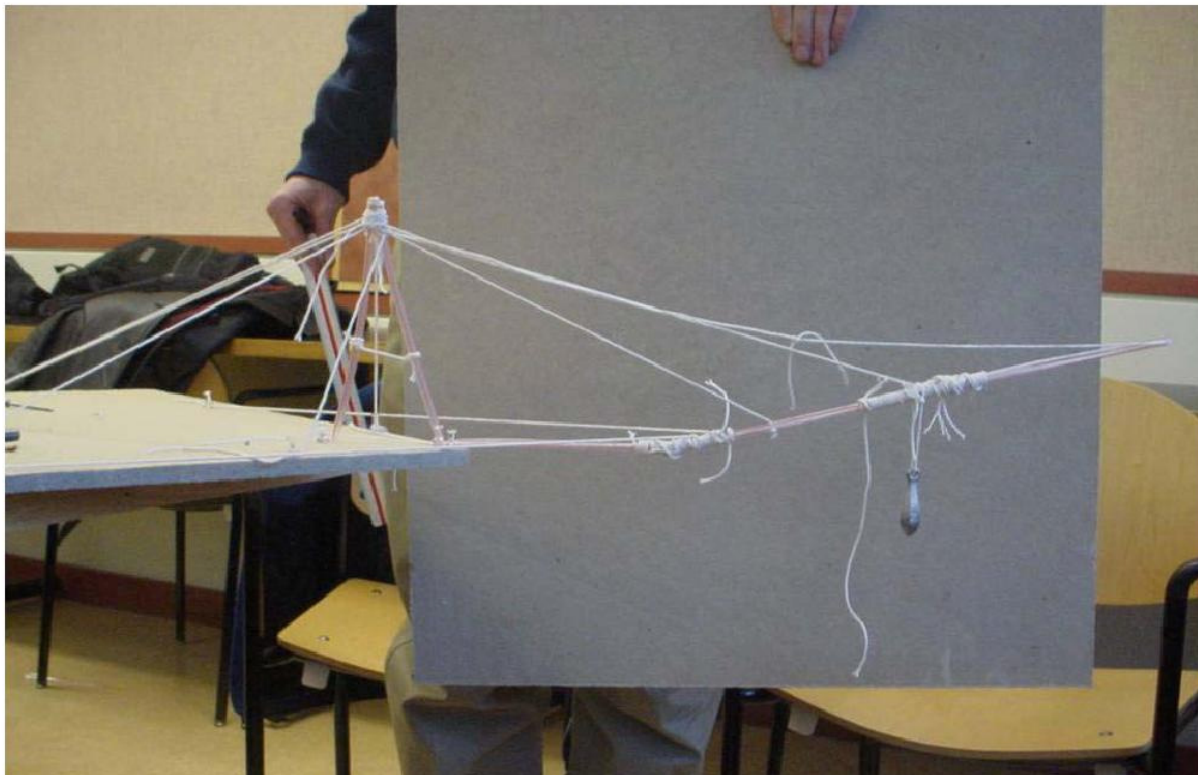










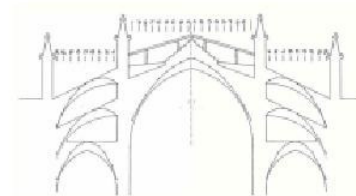




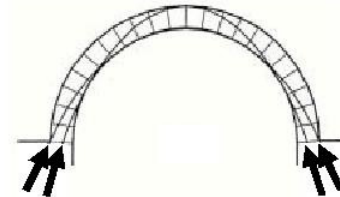
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Review of Compression Lecture

Masonry structures must contain lines of compression within the material



Arches can provide a range of thrust values



Efficient



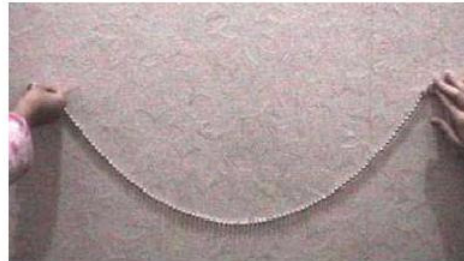
structures follow a

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Hookes 2nd Law (1675)

**ut pendet continuum
*flexile, sic stabit
contiguum rigidum
inversum***

**As hangs the flexible
line, so but inverted
will stand the rigid
arch**



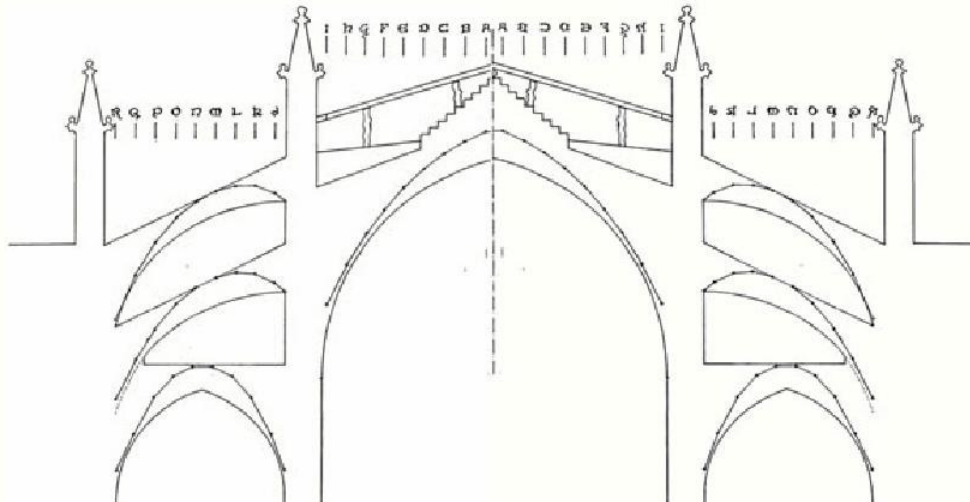


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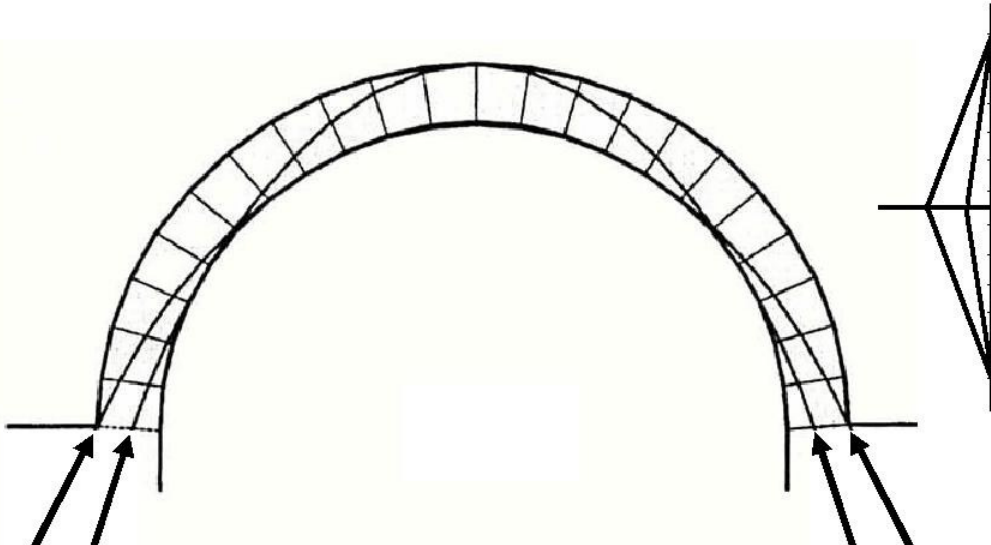
Cathedral of Palma de Mallorca, 16th C Spain





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Range of Arch Thrust





COMPRESSION

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Tension Structures

Tension structures must have a structural form: cables dont lie

Due to their light weight and long spans, tension structures are susceptible to vibration and other dynamic problems

For really long spans (like suspension



**bridges), the self weight of
the structure is the dominant load**

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Truss Analysis





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Review: Member Design

Compression elements have lower stresses due to buckling

$$P_{cr} = p^2EI/(kL)^2$$

When using graphic statics for truss analysis, use clockwise convention to determine if an element is in tension or compression.



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Truss Lecture

Moments of forces for finding reactions

Method of joints for analyzing trusses

Graphical analysis for analyzing trusses



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Three Equations of Equilibrium

1. Sum of vertical forces must be zero

$$\sum F_y = 0$$

2. Sum of horizontal forces must be zero

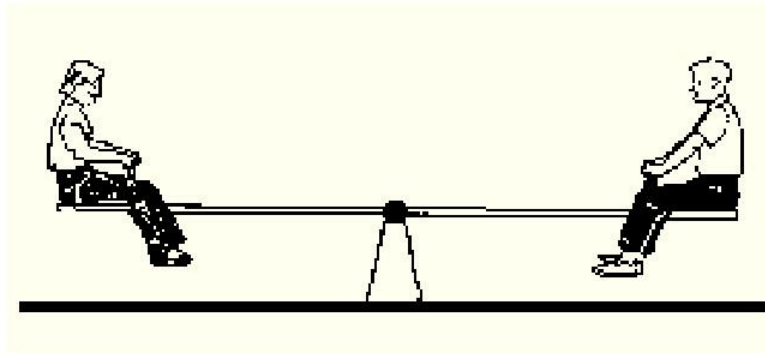
$$\sum F_x = 0$$

3. Sum of moments must be zero


$$SM = 0$$

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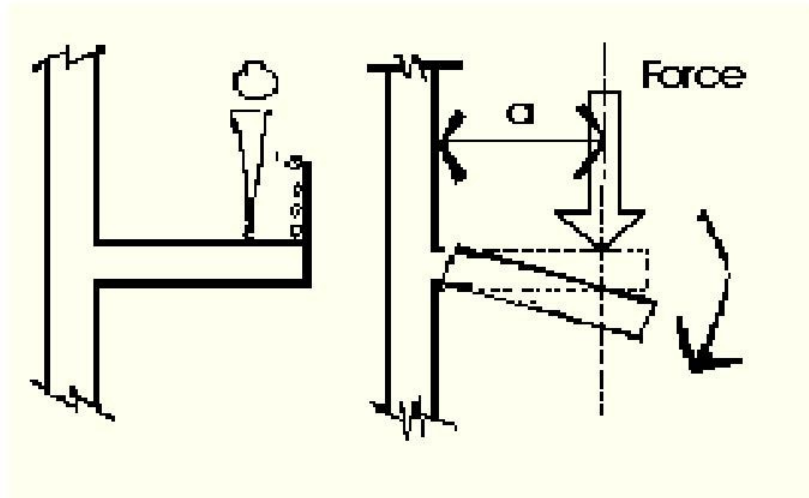
Moment Equilibrium





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Moment of Force

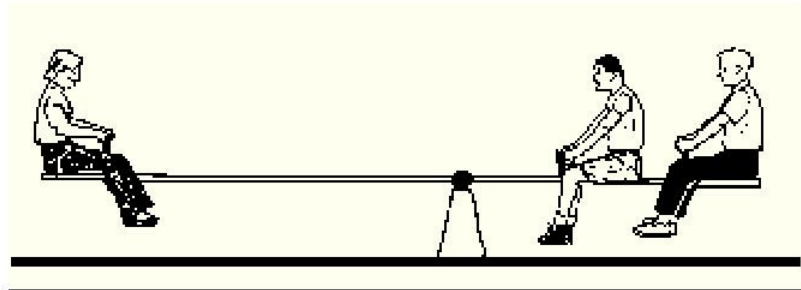




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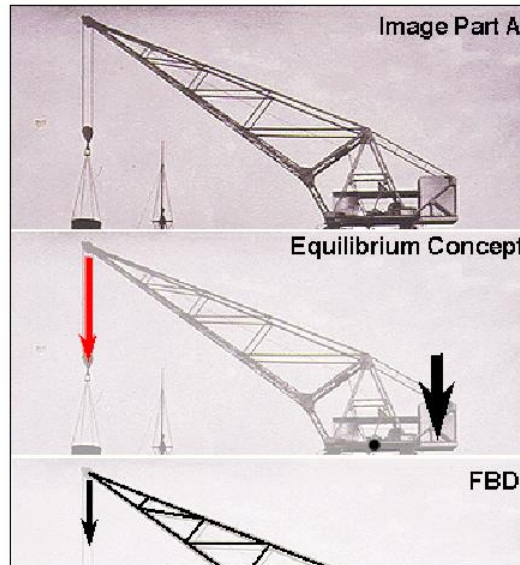
Moment Equilibrium





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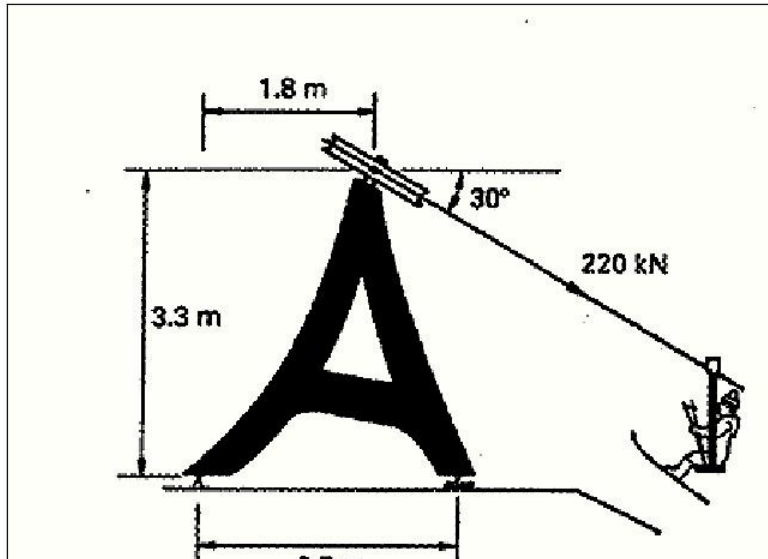
Crane Counterweight





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Equilibrium of Ski Lift Tower





| 2.7 m |

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Eiffel Tower





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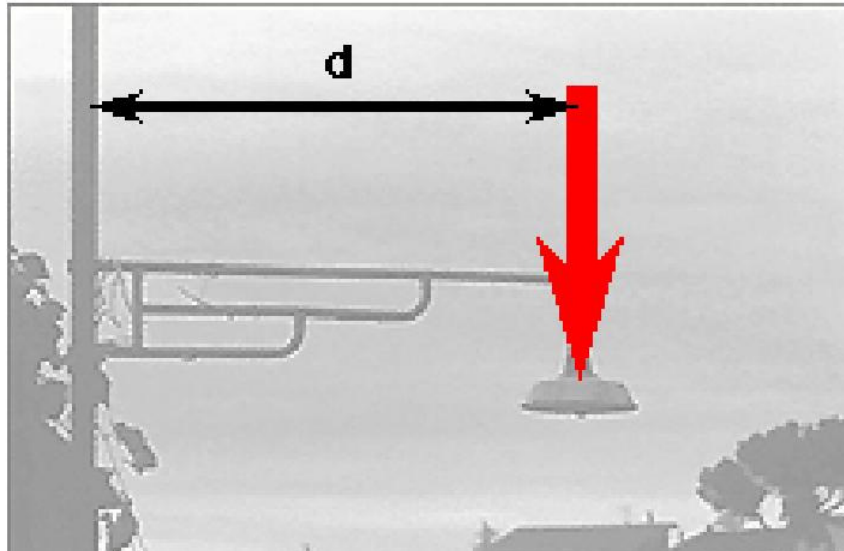
Cantilevered Streetlight





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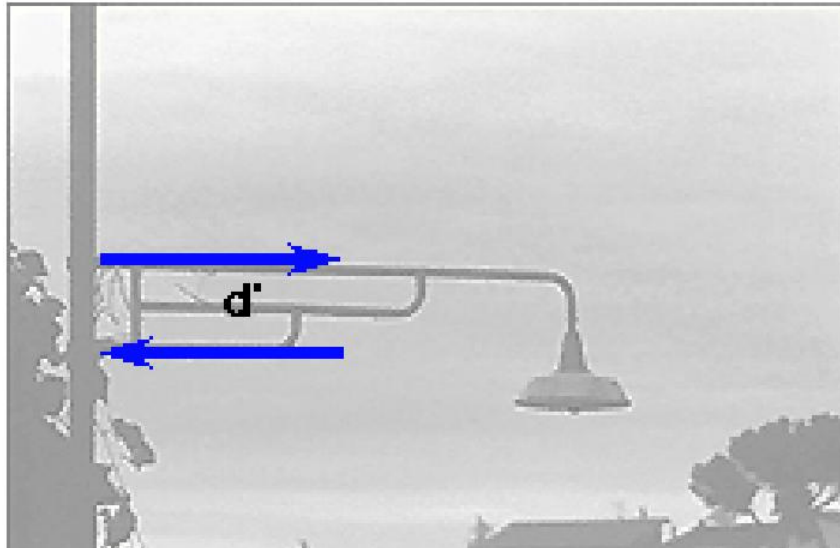
Weight of Lamp





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Reactions at Supports





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Trusses

Rotational Equilibrium

Sum of the moments must equal zero.

Use to calculate reactions at
supports and to find internal forces



Trusses are an efficient way to carry loads with minimal material

Look for examples of moment





equilibrium around you



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Beam Design





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Review of Lecture 5: Trusses

Rotational Equilibrium

Sum of the moments must equal zero.

Use to calculate reactions at
supports and to find internal forces



Trusses are an efficient way to carry loads with minimal material

Look for examples of moment



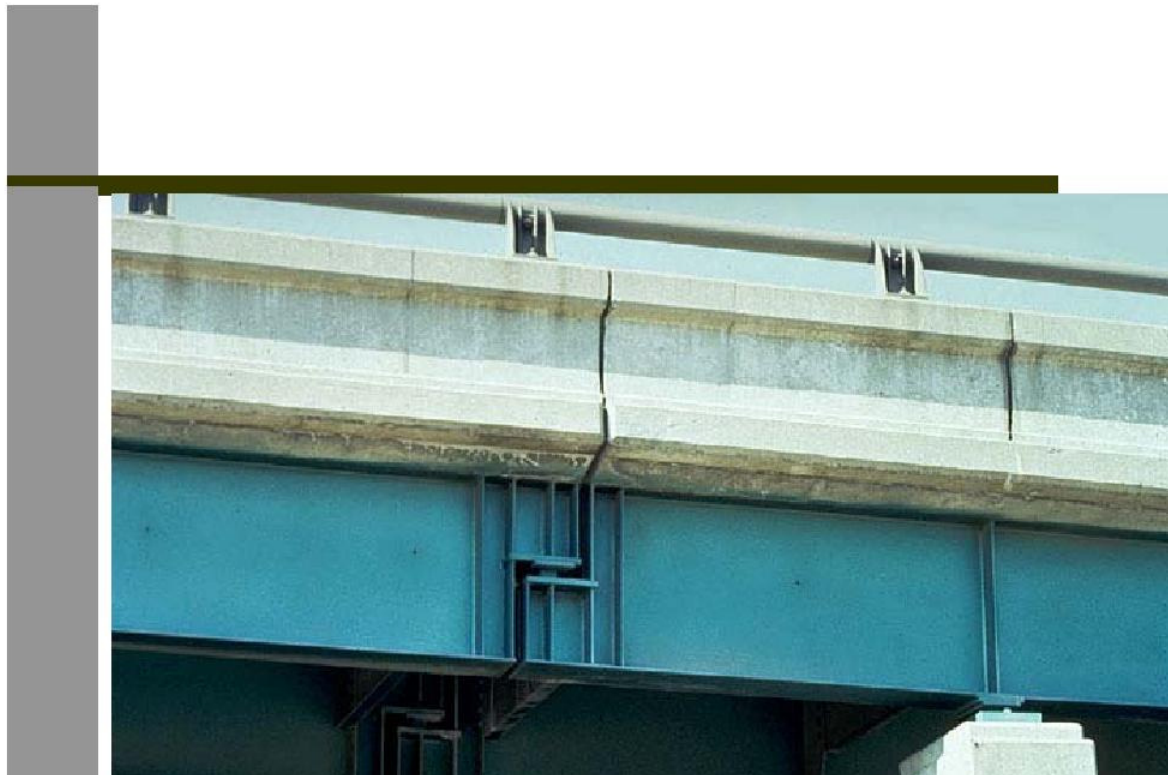


equilibrium around you



















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Concrete beams





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Concrete connections





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Reinforcing rod connections





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Concrete Y-beams





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Interior View





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Castellated Beams





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Timber Cantilever Beams



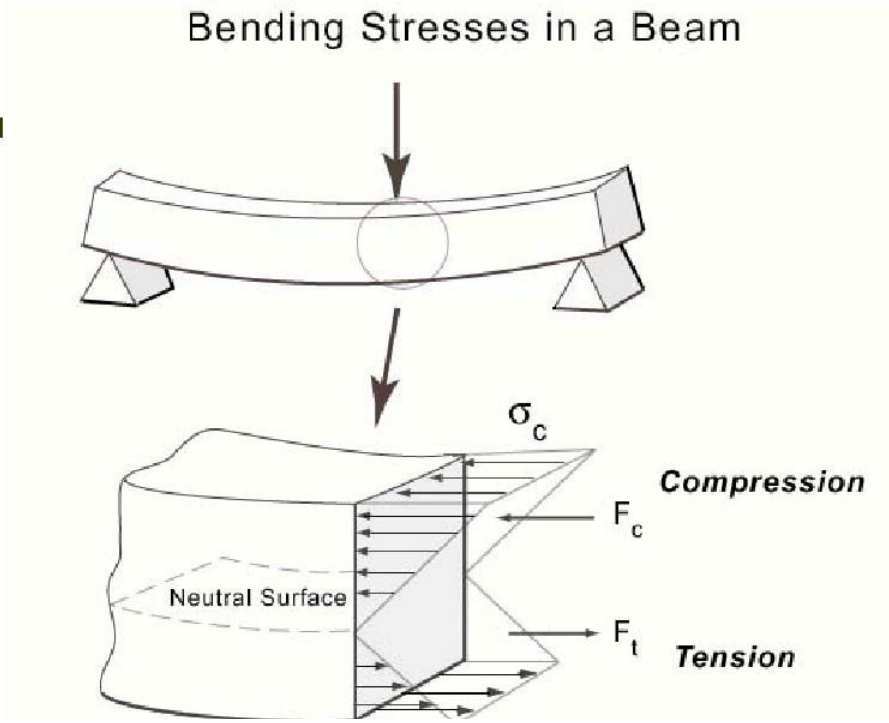










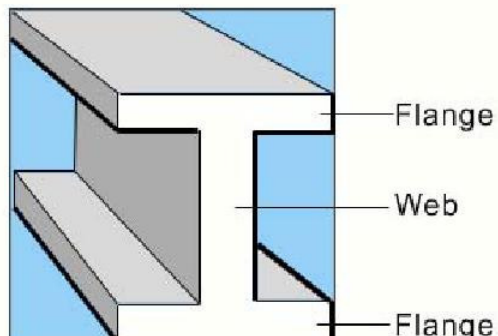


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Beam Terminology

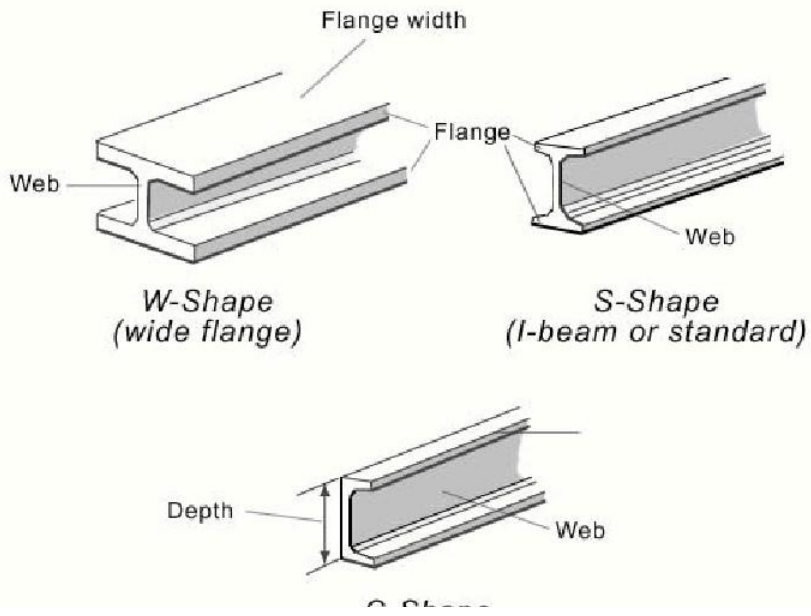
Beam Technology







Steel Section Terminology



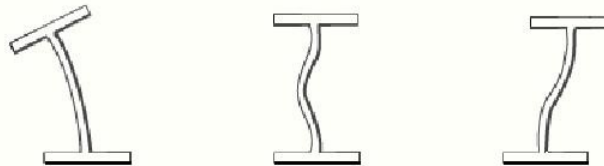


*U-snap
(channel)*

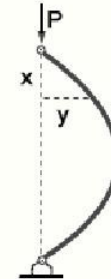
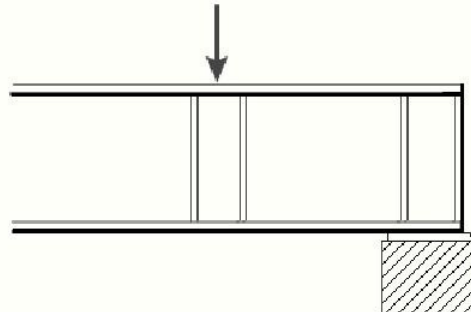


Steel Buckling

Must be stiffened with plates in regions of high shear



Web buckling



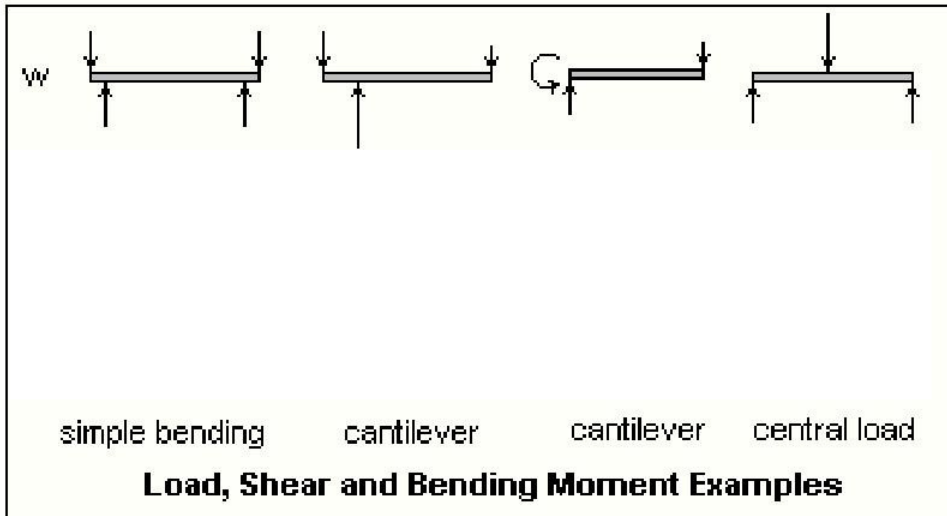
Buckling strut



Bearing plate and web stiffening

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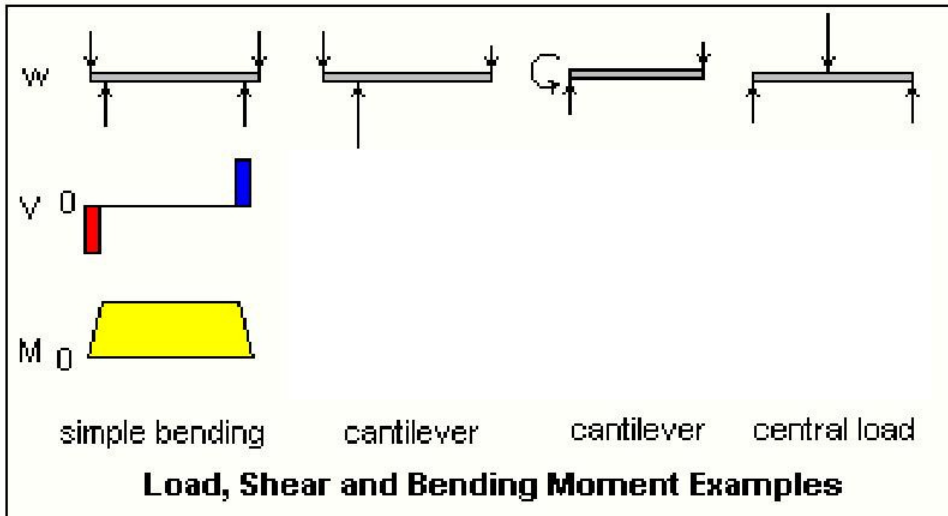
Shear Diagrams





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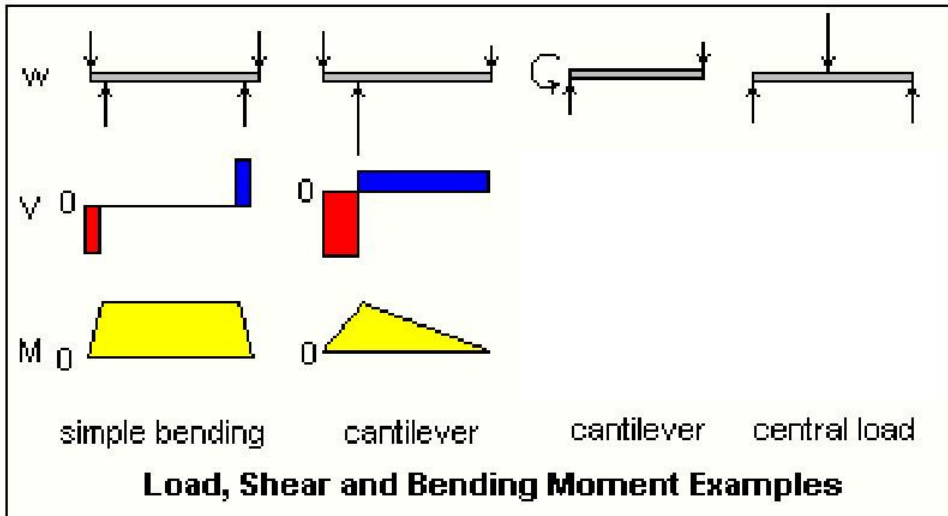
Shear Diagrams





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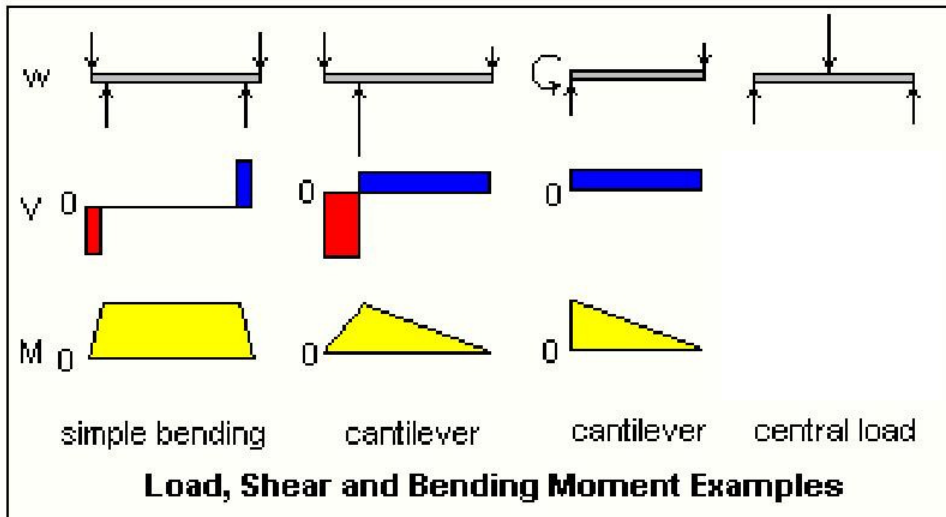
Shear Diagrams





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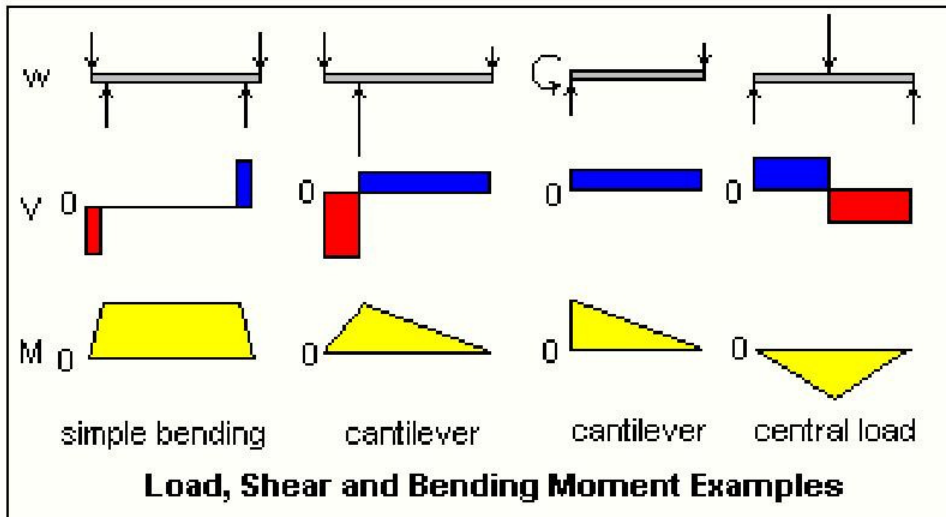
Shear Diagrams





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Shear Diagrams





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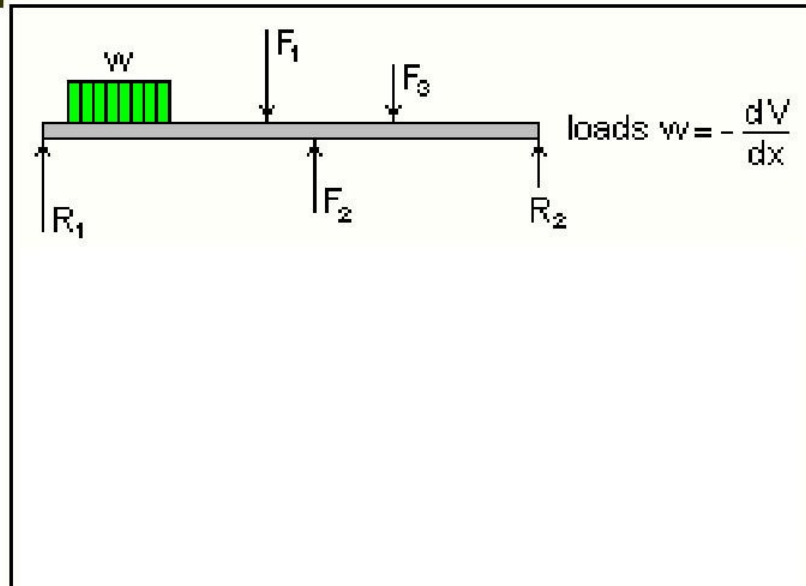
How to draw a shear diagram

- 1) Determine external reactions on beam**
- 2) Walk along beam with your pen**
- 3) Pen goes up and down with the loads**
- 4) Must close diagram at the ends of the beam**



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Shear Diagrams

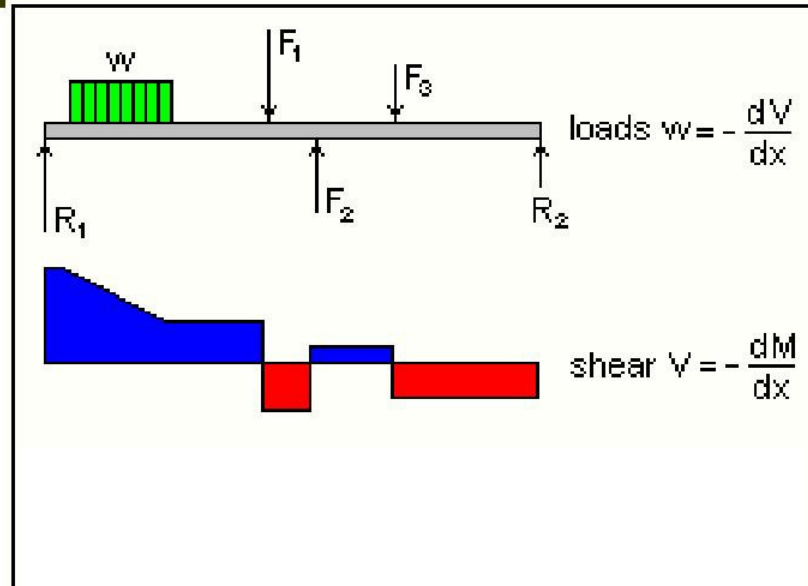




Load, Shear and Moment Diagrams

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Shear Diagrams

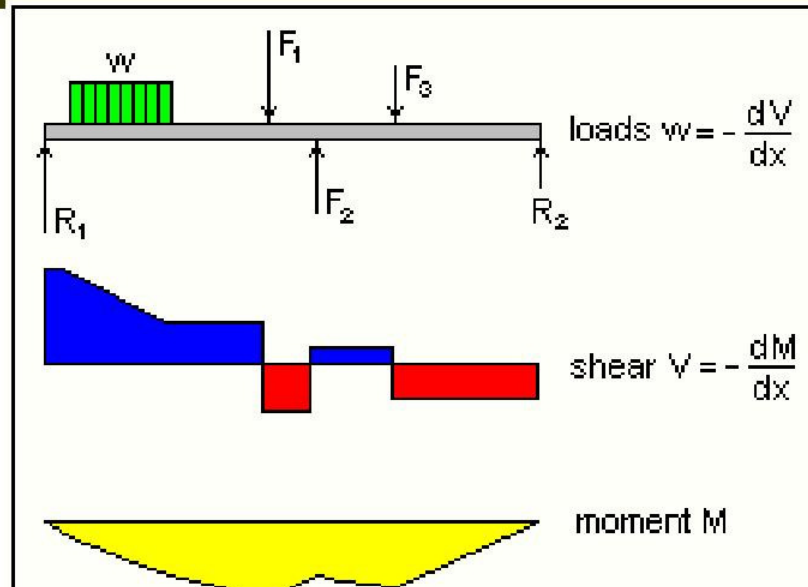




Load, Shear and Moment Diagrams

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Shear Diagrams

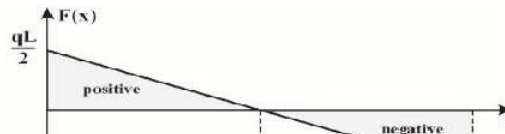
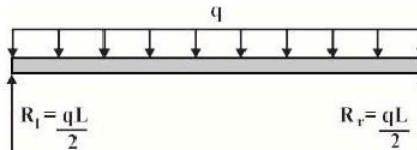
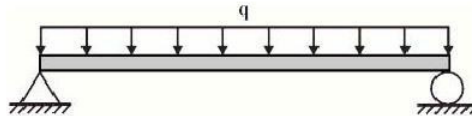


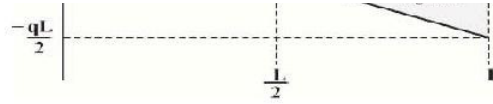


Load, Shear and Moment Diagrams

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Shear Diagram for Uniform Load





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Lecture 6: Beams

Beams carry loads in bending, with compression and tension on opposite sides

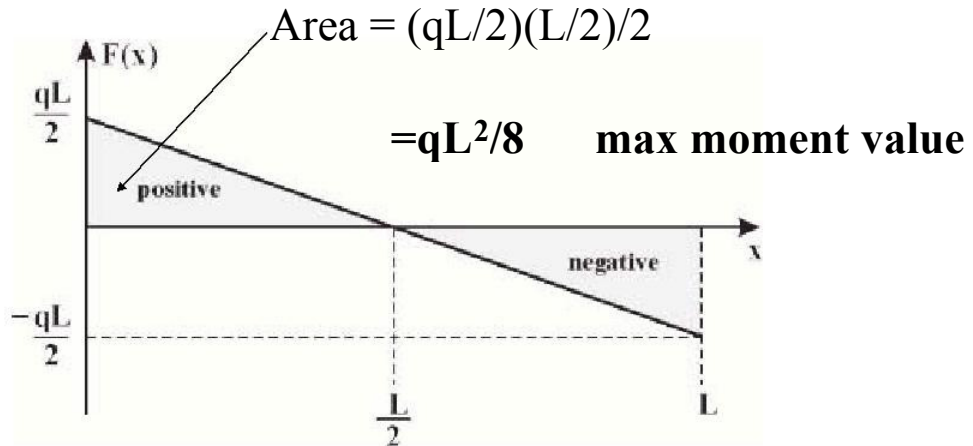
Visualize trusses within the depth of a beam

Shear and moment diagrams are used to illustrate internal forces in beams



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Shear Diagrams





Shear diagram equals the slope of moment diagram

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Indeterminate Structures





Architecture

4.440

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Outline

Introduction

Static Indeterminacy

Support Conditions

Degrees of Static Indeterminacy

Design Considerations

Conclusions



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Forces in the Legs of a Stool





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Three-Legged Stool

Statically determinate

**One solution for the axial force
in each leg**

Why.

3 unknowns

3 equations of equilibrium

Uneven





**floor has no
effect**

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Four-Legged Stool

Statically indeterminate

A four legged table on an uneven surface will rock back and forth

Why.

It is hyperstatic:

4 unknowns

?





3
**equations
of
equilibrium**

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Four-Legged Stool

Infinite solutions exist

Depends on unknowable support conditions

A four legged table on an uneven surface will rock back and forth

The forces in each leg are constantly changing





Fundamental difference between hyperstatic and static structures

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Forces in the Leg of a Stool



Statically



**Statically
Indeterminate**



determinate

**indeterminate
(hyperstatic)**

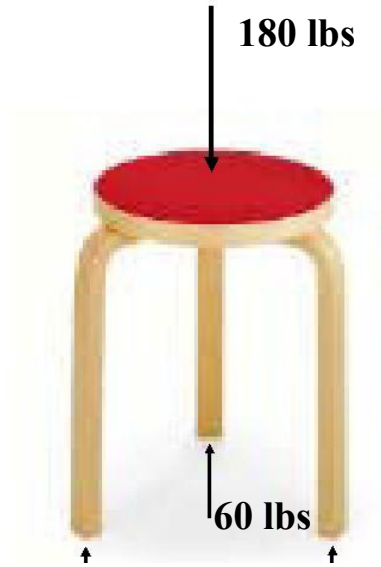
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Three-Legged Stool

**Design for a person
weighing 180 pounds**

60 pounds/leg

**Regardless of uneven
floor**





| 60 lbs | 60 lbs

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Collapse of a Three-Legged Stool

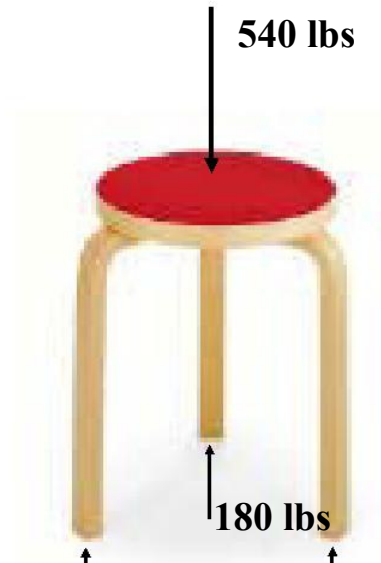
Design for a person
weighing 180 pounds

If the safety factor is 3:

$$P_{cr} = 3(60) = 180 \text{ lbs}$$

And each leg would be
designed to fail at a load of
180 pounds

The stool would carry a





**total load
of 540
pounds**



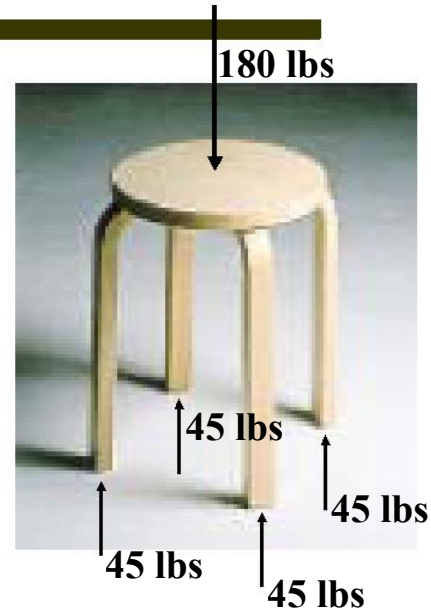
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Elastic Solution for 4-Legged Stool

Design for a person
weighing 180 pounds

45 pounds/leg

But if one leg does not
touch the floor





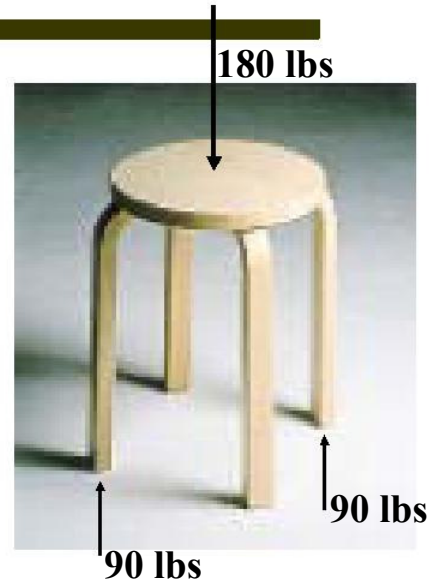
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Four-Legged Stool

If one leg doesn't touch the floor, the force in it is zero.

If one leg is zero, then the opposite leg is also zero by moment equilibrium.

The two remaining legs carry all of the load:





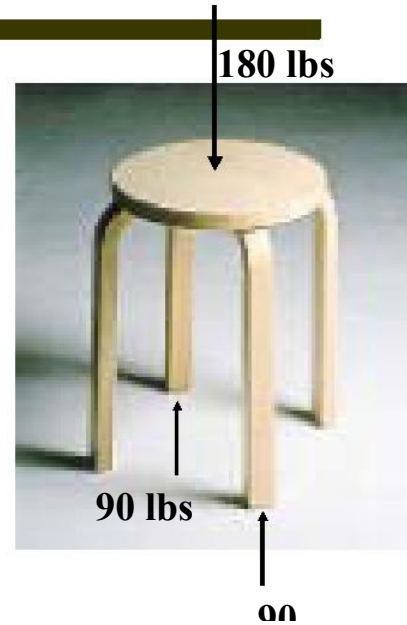
90 pounds/leg

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Four-Legged Stool

Therefore

All four legs must be designed to carry the 90 pounds (since any two legs could be loaded)





20
lbs

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Four-Legged Stool

If the elastic solution is accepted, with a load in each leg of 45 pounds, then assuming a safety factor of 3 gives:

$$P_{cr} = 3(45 \text{ lbs}) = \underline{135 \text{ lbs}}$$

And each leg would be designed to fail





designed to run
185 poachdf

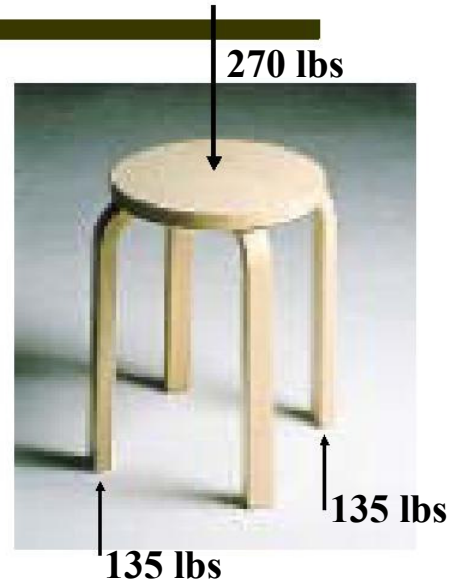
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Four-Legged Stool

Now imagine the load is increased to cause failure

When load is 270 lbs, the two legs will begin to fail

As they squash, the other two legs will start to carry





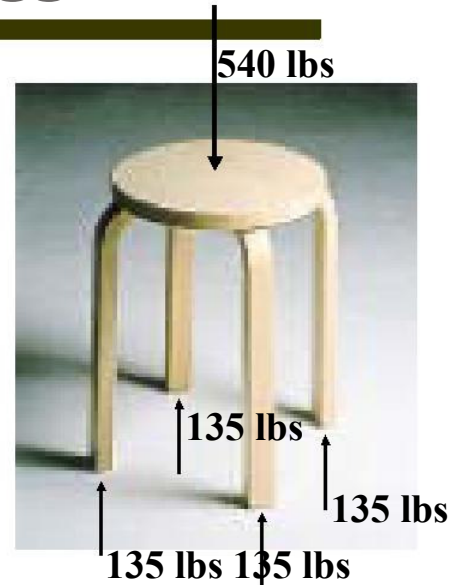
carry
load
also

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Collapse of a 4-Legged Stool

At final collapse state, all four legs carry 135 pounds and the stool carries 540 pounds.

This occurs only if the structure is ductile (ie, if the legs can squash)



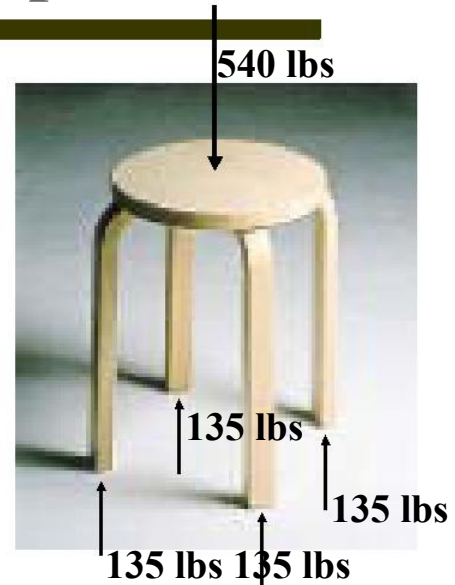


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Ductile Collapse

So small imperfections do not matter, as long as the structural elements are ductile

The forces in a hyperstatic structure cannot be known exactly, but this is not important as long as we can predict the collapse state





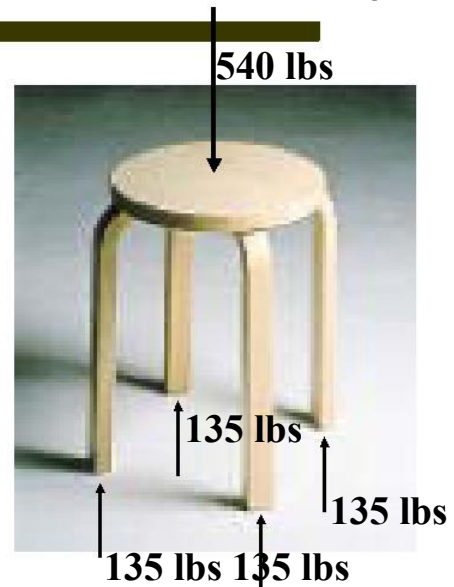
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Lower Bound Theorem of Plasticity

If you can find one possible set of forces, then the structure can find a possible set of forces

It does not have to be correct, as long as the structure has capacity for displacements (ductility)

For indeterminate structures, we





cannot be certain of
state of the forces

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Examples of Statically Determinate Structures

**Unstressed by support movements
or temperature changes**

Three-legged stool

Simply supported beam

Cantilever beam

Three-hinged arch





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Simply Supported Bridge



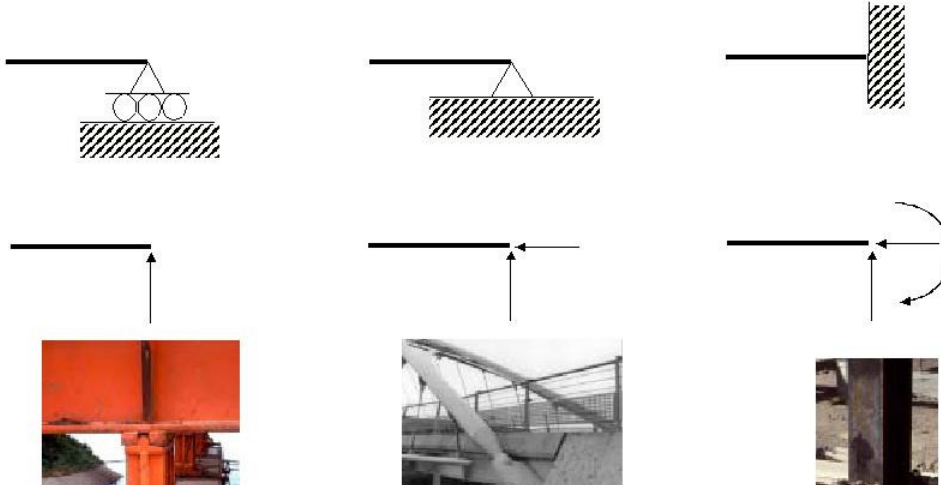


**Can adjust to support
movements and changes
temperature**

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Support Conditions

Roller Pin (hinge) Fixed





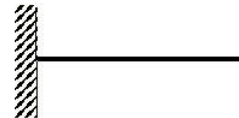
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Statically Determinate Structures

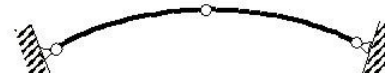
Simply supported beam



Cantilever beam



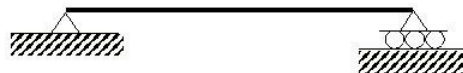
Three-hinged arch





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Simply Supported Beam





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Statically Determinate

Simply supported
beam

Cantilever beam

Three-hinged arch

Three-

Indeterminate (hyperstatic)

Continuous beam

Propped cantilever beam

Fixed end arch

Rigid



**hinged
frame**

frame

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Continuous Beam



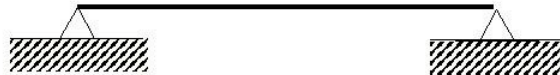
How many unnecessary supports.

What is the degree of static indeterminacy.



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Pin-Ended Beam



Will temperature changes cause forces in the beam.

How many unnecessary supports.

What is the degree of static



What is the degree of static indeterminacy.

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Fixed-End Beam



Will temperature changes cause forces in the beam.

How many unnecessary supports.

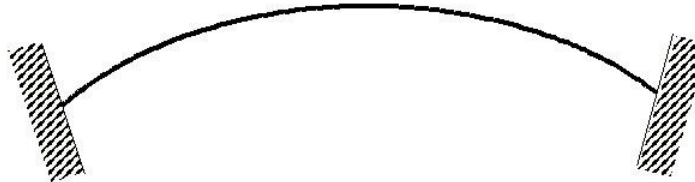
What is the degree of static



What is the degree of static indeterminacy.

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Fixed-End Arch



Will temperature changes or support movements cause forces in the arch.

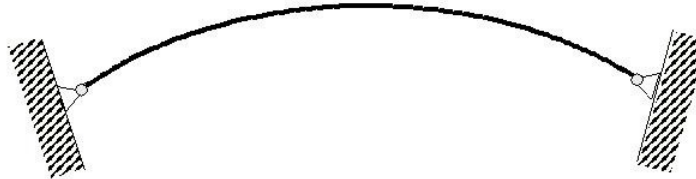
How would you make this structure statically determinate.

What is the degree of static indeterminacy.



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Two-Hinged Arch



Will temperature changes or support movements cause forces in the arch.

How would you make this structure statically determinate.



What is the degree of static indeterminacy.

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Pinned Frame



Will temperature changes or support movements cause forces in the frame.

How would you make this structure



statically determinate.

What is the degree of static indeterminacy.

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Fixed Frame



Will temperature changes or support movements cause forces in the frame.

How would you make this structure



statically determinate.

What is the degree of static indeterminacy.

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Fixed Frame



Will temperature changes or support movements cause forces in the frame.

How would you make this structure



statically determinate.

What is the degree of static indeterminacy.

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How to find forces in statically indeterminate structures

Approximate hand calculations

Make simplifying assumptions

Computer: Finite Element Methods

Solve for internal forces based on relative **stiffness of each element and many other assumptions (elastic analysis)**

Analyze limiting cases to determine one possible state of internal forces



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Finite Element Analysis

**Divide structure into a
mesh of finite
elements**

**Solves for internal
forces based on
relative stiffness of
each element**



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Finite Element Analysis

**But cant account for
imperfections in supports and
construction**

**Like a four-legged stool, it is
impossible to know the exact
forces**

**Finite element analysis is more
sophisticated, but is not
necessarily better**



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Design Considerations

Statically indeterminate structures offer greater redundancy, i.e. more possible load paths

But are less clear in their structural action

More complicated to design and assess

May be more difficult to repair



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Static Indeterminacy

For a given set of applied loads, any possible equilibrium state is acceptable (internal forces in the legs of the stool)

Find extreme equilibrium cases by releasing the extra supports (i.e., assume two legs don't touch the ground)

You can choose any internal equilibrium state as long as buckling



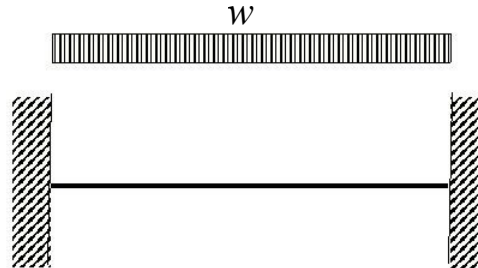


**does not occur (lower
bound theorem)**

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Statically Indeterminate Beams

What is the moment diagram for this beam under a uniform load, w .

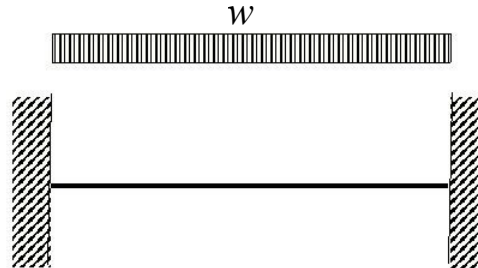




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Statically Indeterminate Beams

What is the moment diagram for this beam under a uniform load, w .



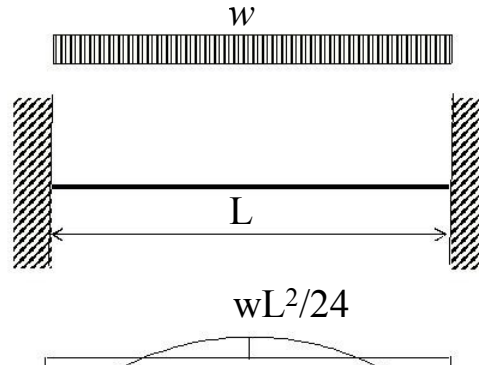


**Is there
one
answer.**

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Statically Indeterminate Beams

What is the moment diagram for this beam under a uniform load, w .



Elastic solution
(perfect world)



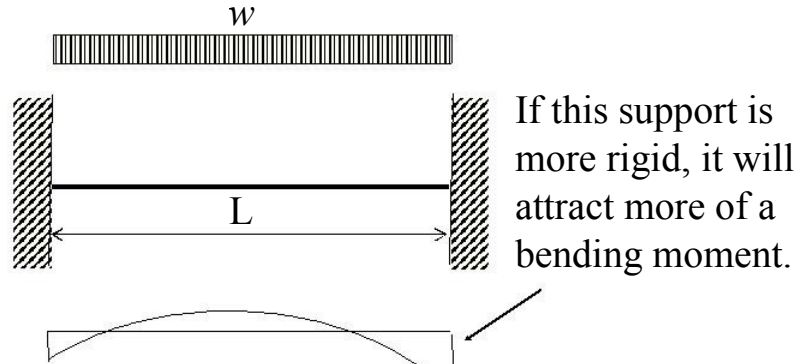
$$wL^2/12 \quad \checkmark$$

$$\checkmark \quad wL^2/12$$

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Statically Indeterminate Beams

But what did we learn from the 4-legged stool.

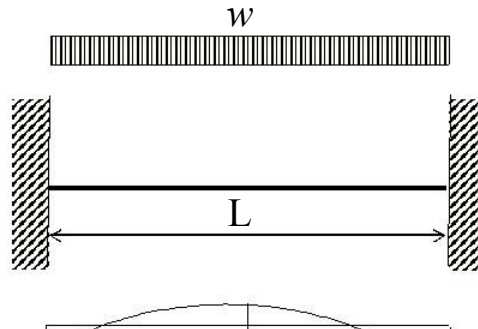




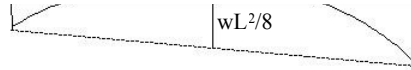
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Statically Indeterminate Beams

The difference between the midspan moment and the closing line is always $wL^2/8$ due to a uniform load.



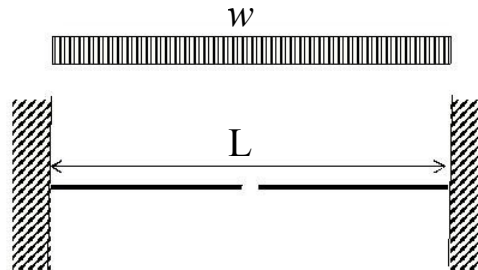
If this support is more rigid, it will attract more of a bending moment.



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Statically Indeterminate Beams

What is the moment diagram for this beam under a uniform load, w , if we make a cut at midspan.

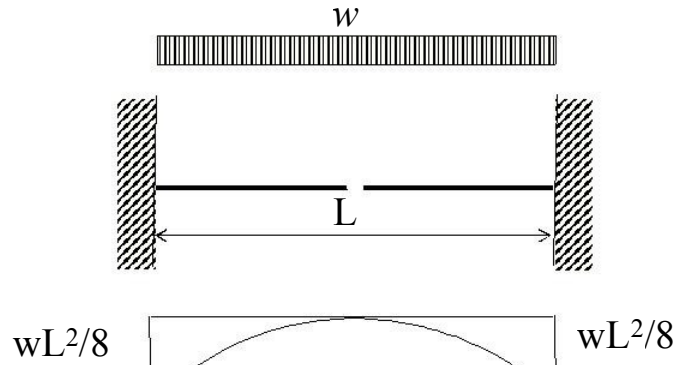




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Statically Indeterminate Beams

What is the moment diagram for this beam under a uniform load, w , if we make a cut at centerspan.

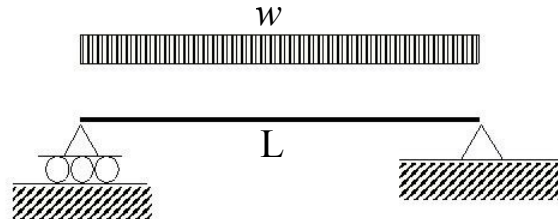




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Statically Indeterminate Beams

What is the moment diagram for this beam under a uniform load, w , if it is simply supported.

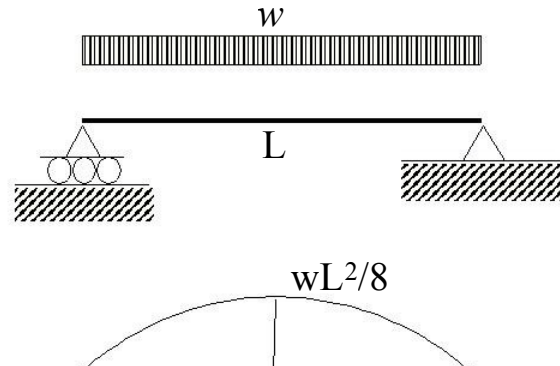


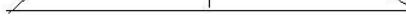


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Statically Indeterminate Beams

What is the moment diagram for this beam under a uniform load, w , if it is simply supported.

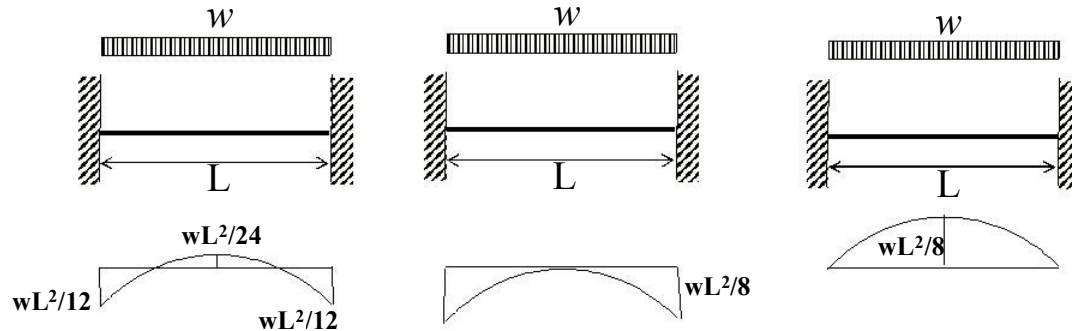




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Statically Indeterminate Beams

Various possible bending moment configurations for a beam under uniform load





Moment diagram shifts up and down as the supports change their degree of fixity

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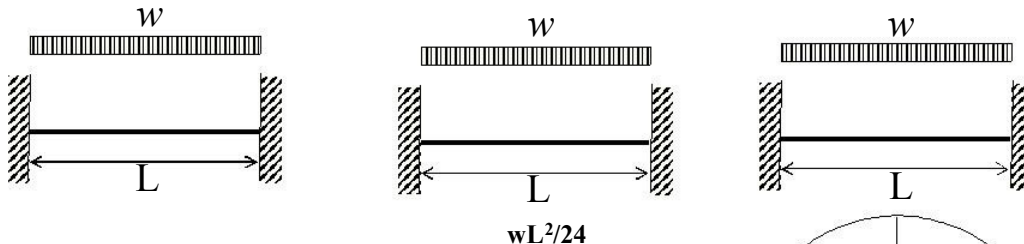
Statically Indeterminate Beams

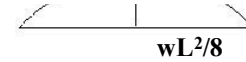
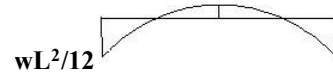
Which is correct. All of them!

As a designer, you choose the function by choosing the form

Shape the structure to reflect the load acting on it

Articulate the role of each structural connection

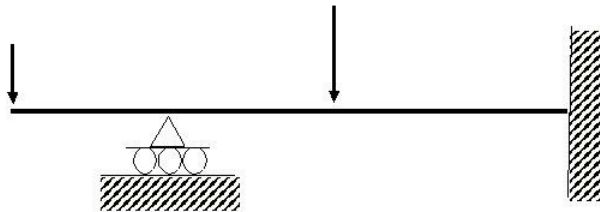




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Statically Indeterminate Beams

What is the moment diagram for this beam under two point loads.



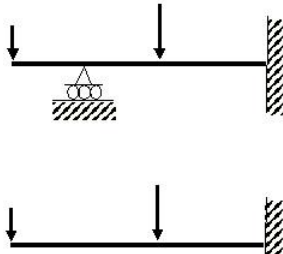


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Statically Indeterminate Beams

Release unknown reactions until the structure becomes statically determinate.

Draw moment diagram for statically determinate structure.



Remove roller support to make it a cantilever beam

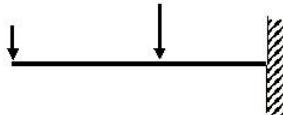


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Statically Indeterminate Beams

Release unknown reactions until the structure becomes statically determinate.

Draw moment diagram for statically determinate structure.





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Statically Indeterminate Beams

Release unknown reactions until the structure becomes statically determinate.

Draw moment diagram for statically determinate structure.



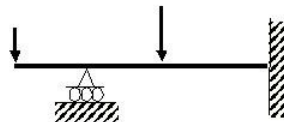


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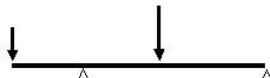
Statically Indeterminate Beams

Release unknown reactions until the structure becomes statically determinate.

Draw moment diagram for statically determinate structure.



Remove fixed support to make it a simply-supported beam.



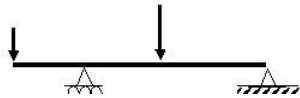
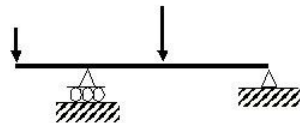


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Statically Indeterminate Beams

Release unknown reactions until the structure becomes statically determinate.

Draw moment diagram for statically determinate structure.



What shape is the moment diagram here.

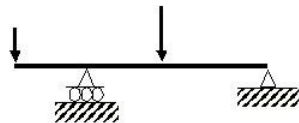


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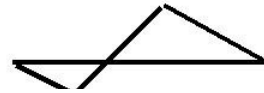
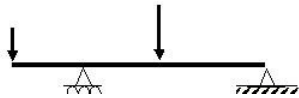
Statically Indeterminate Beams

Release unknown reactions until the structure becomes statically determinate.

Draw moment diagram for statically determinate structure.



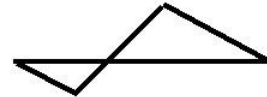
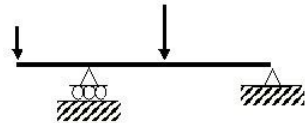
A: The shape of the hanging cable



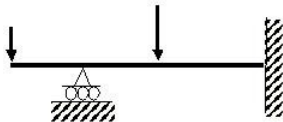


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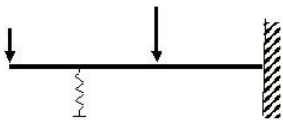
Statically Indeterminate Beams



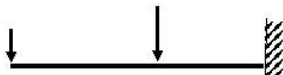
Simply-supported



Indeterminate



Indeterminate



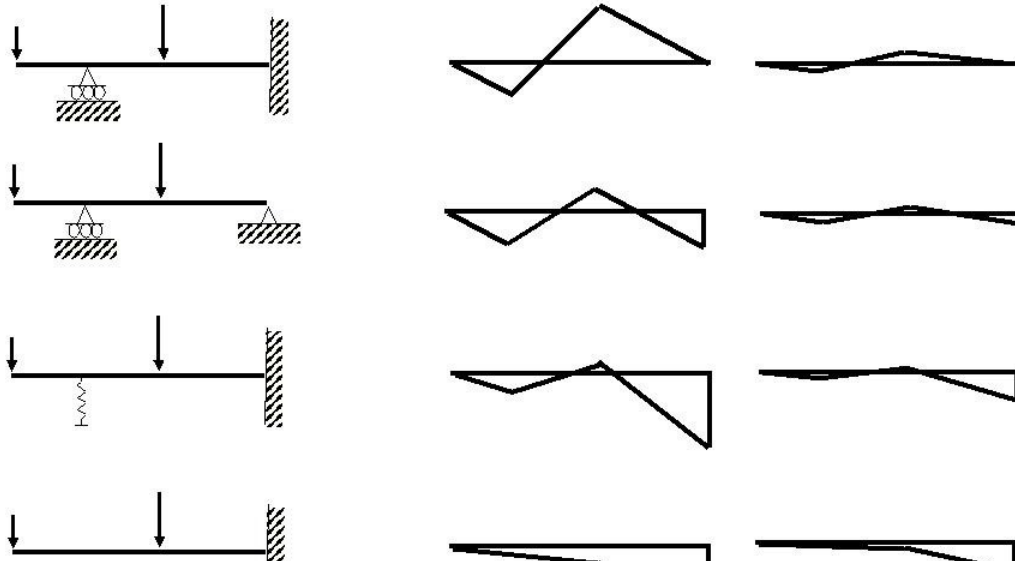
Cantilever



Again, moment diagram shifts up and down

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Statically Indeterminate Beams

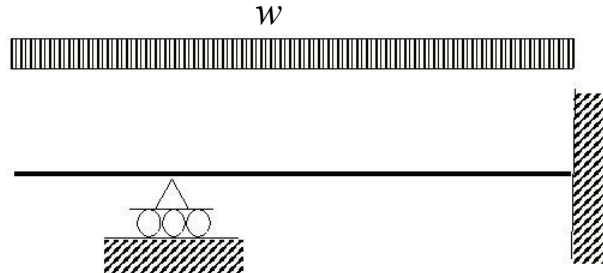




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Statically Indeterminate Beams

What is the moment diagram for this beam under a uniform load, w .

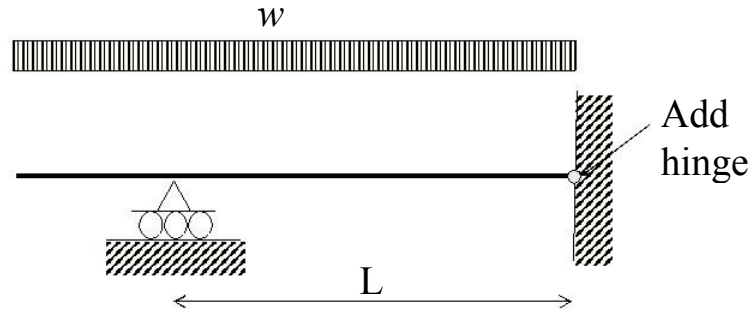




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Statically Indeterminate Beams

Release the right hand support by adding a hinge

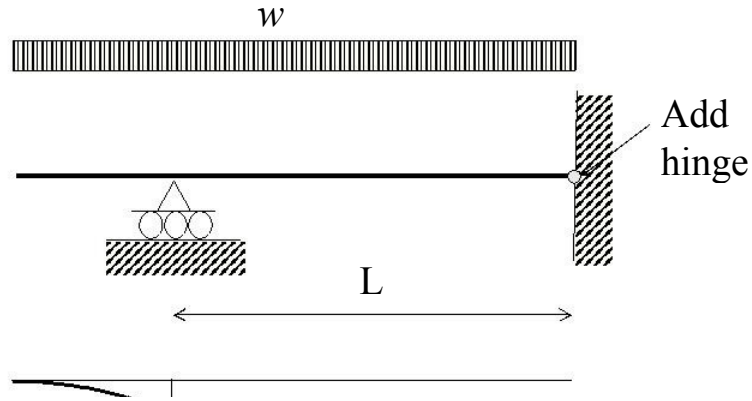




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Statically Indeterminate Beams

Release the right hand support by adding a hinge



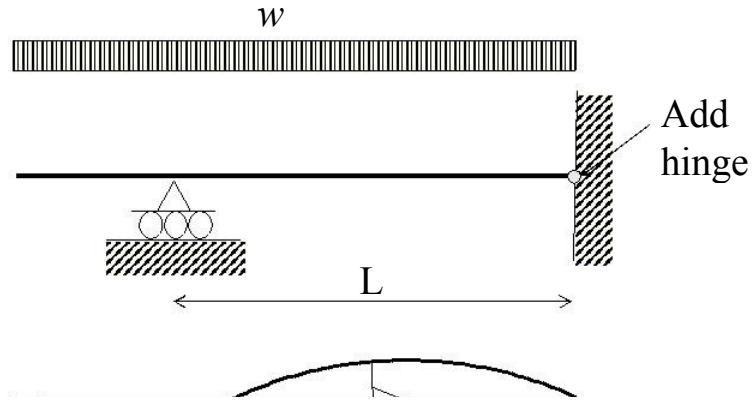


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Statically Indeterminate Beams

Release the right hand support by adding a hinge

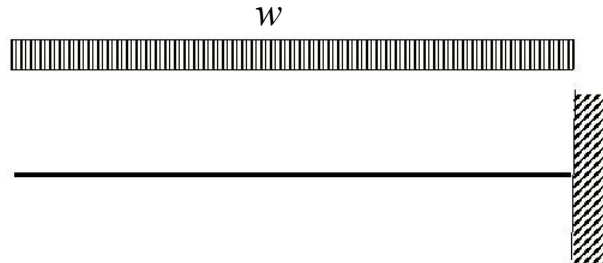




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Statically Indeterminate Beams

**Make statically determinate by
removing the roller support**

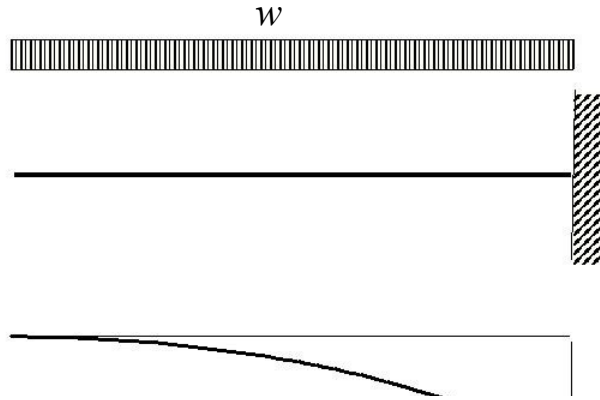




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Statically Indeterminate Beams

What is the moment diagram for this beam under a uniform load, w .

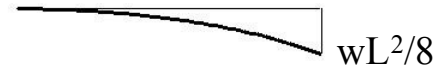
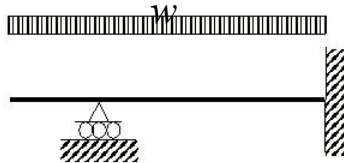




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Statically Indeterminate Beams

What is the moment diagram for this beam under a uniform load, w .





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Conclusions

You choose the function by choosing the form function follows form

For a given loading, the moment diagram simply moves up and down as you change the support conditions

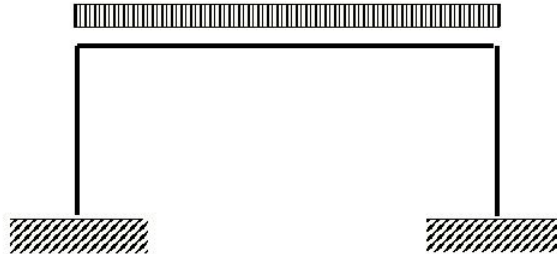
M--64



**MUST
prevent
buckling**

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Fixed Frame Under Uniform Load

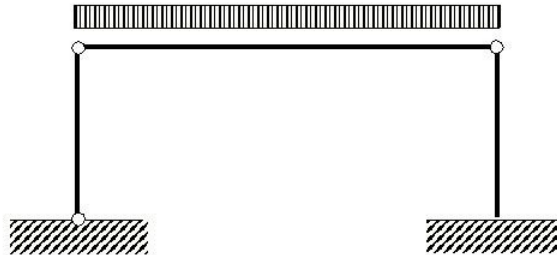


Propose three possible moment diagrams for this frame



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Fixed Frame Under Uniform Load

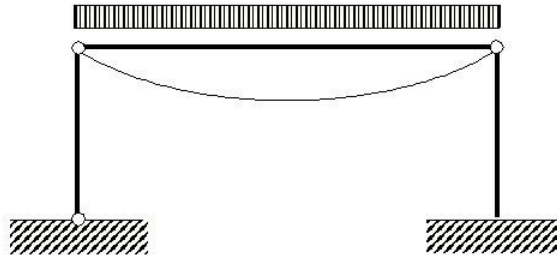


Simply-supported beam on posts



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Fixed Frame Under Uniform Load

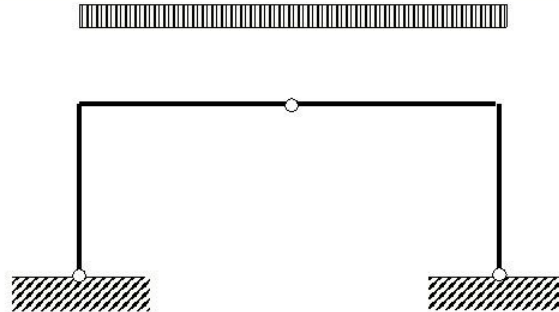


Simply-supported beam on posts



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Fixed Frame Under Uniform Load

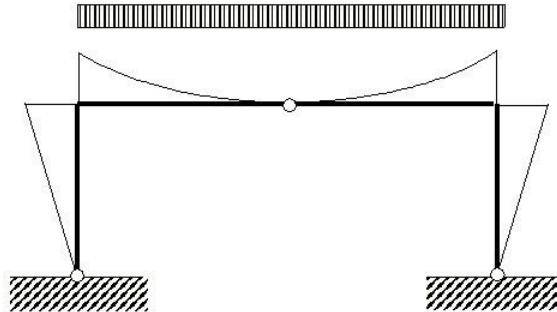


Three-hinged frame



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Fixed Frame Under Uniform Load

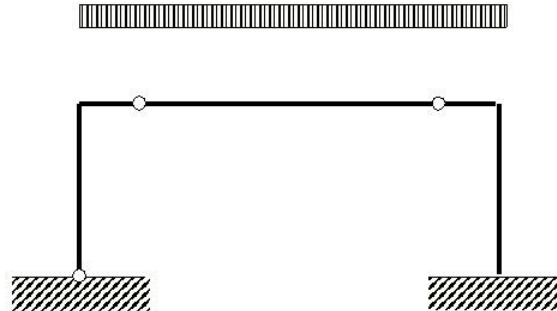


Three-hinged frame



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Fixed Frame Under Uniform Load

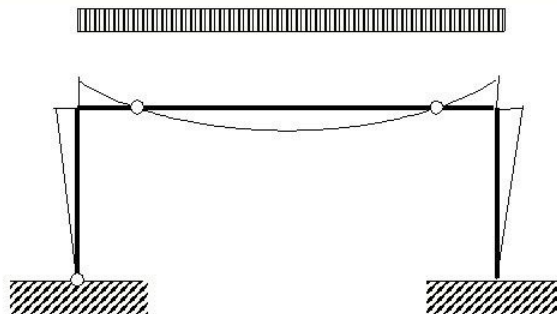


Alternative three-hinged frame



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Fixed Frame Under Uniform Load

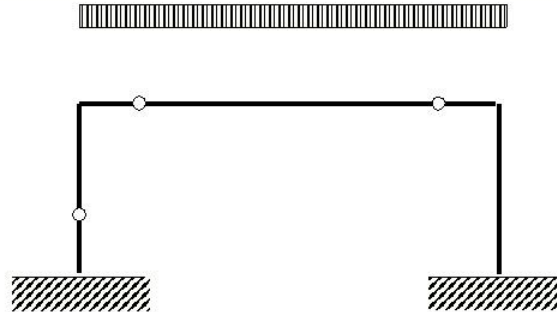


Alternative three-hinged frame



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Fixed Frame Under Uniform Load

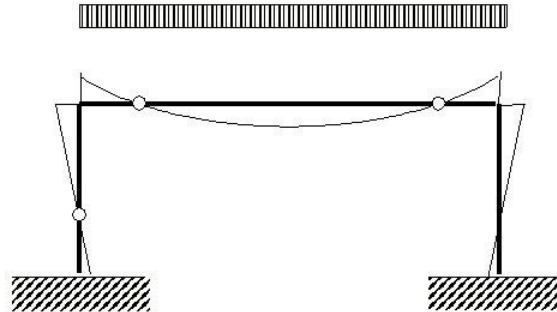


Alternative three-hinged frame



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Fixed Frame Under Uniform Load

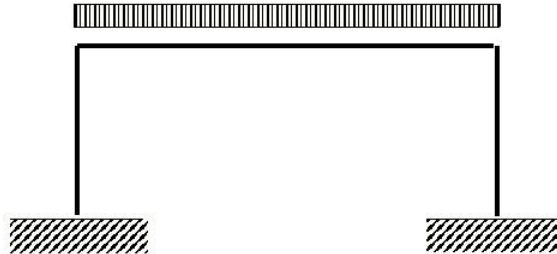


Alternative three-hinged frame



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Fixed Frame Under Uniform Load



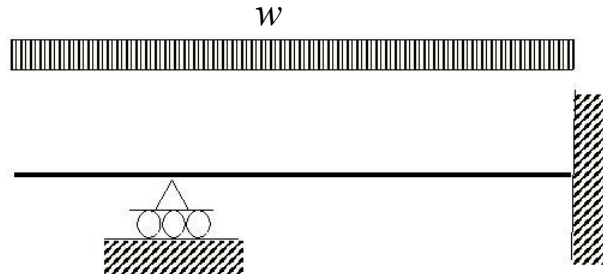
What type of structural forms would work for this load case.



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Statically Indeterminate Beams

What is the moment diagram for this beam under a uniform load, w .

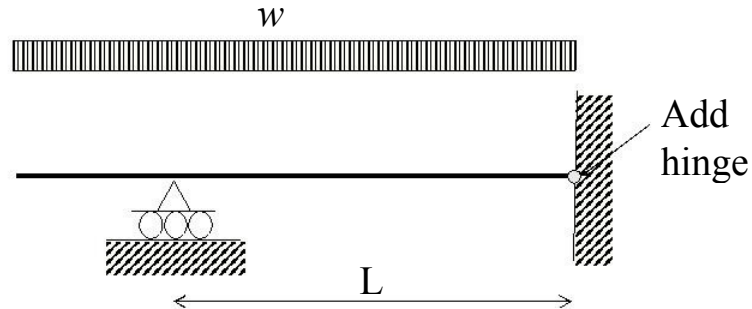




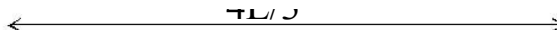
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Statically Indeterminate Beams

Release the right hand support by adding a hinge



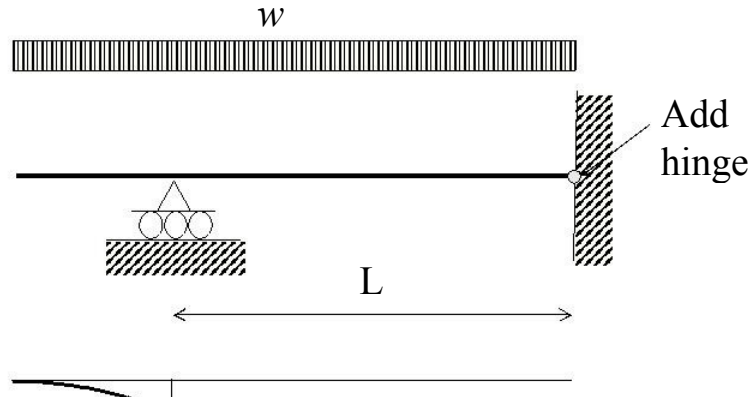
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Statically Indeterminate Beams

Release the right hand support by adding a hinge



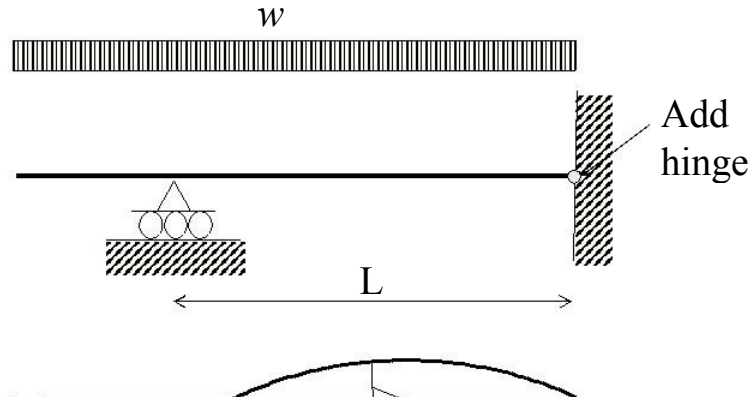


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Statically Indeterminate Beams

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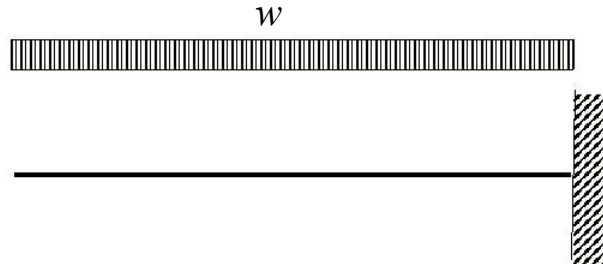




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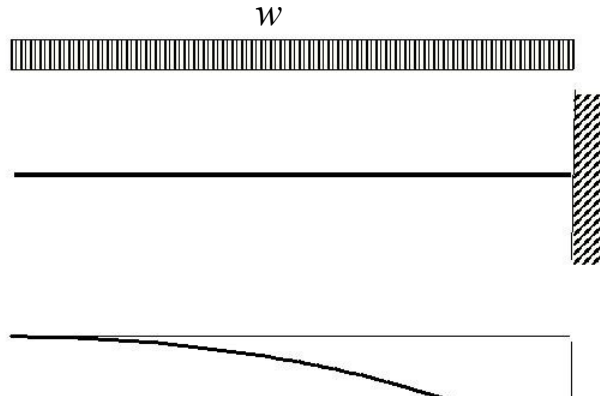




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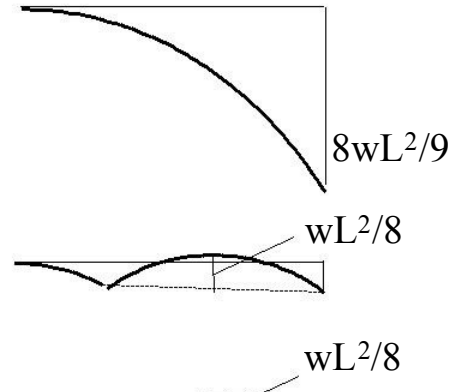
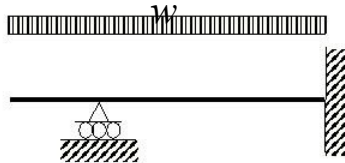




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Statically Indeterminate Beams

What is the moment diagram for this beam under a uniform load, w .





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Review: Indeterminate Structures

For a given loading on a beam, the moment diagram simply moves up and down as you change the support conditions

**You choose the function by choosing the form
function follows form**

Must prevent buckling (think of three-legged stool example)

