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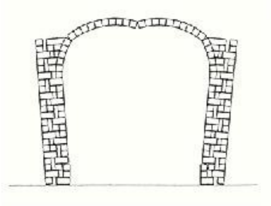
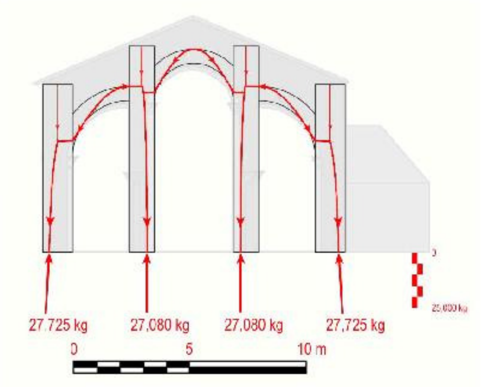
Analysis of Historic Structures





John Ochsendorf, Assistant Professor
MIT

Building
Technology
Program

Hagia Sophia, Istanbul

Image courtesy of Ozgu Bayrak, structurae.de

Completed in 537 AD, dome span of 32 m

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Hagia Sophia, Istanbul

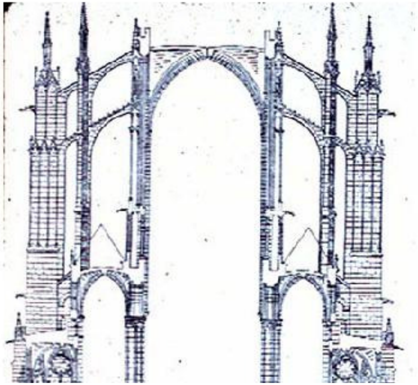
Partial collapse of dome, due to earthquakes:

558 AD (east quadrant)
869 AD (west quadrant)
1346 AD (east quadrant)

Image courtesy of Adrien Mortini, structurae.de

Beauvais Cathedral

Partial collapse in 1284



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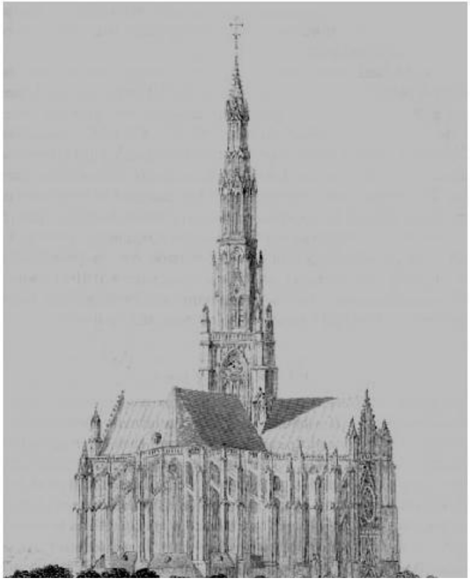
Beauvais Cathedral



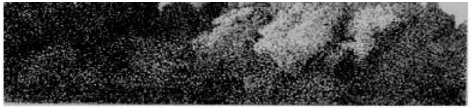
Tower built in 1569

Height of 153 m

Supported on piers

Tower

Tower
collapsed
1573



Basilica of St. Francis in Assisi, Italy

13th C construction

Frescoes by Giotto

Image courtesy of Rob Jaffe, structurae.de

1871 Fire in Chicago

Boston Public Library, 1889-1890

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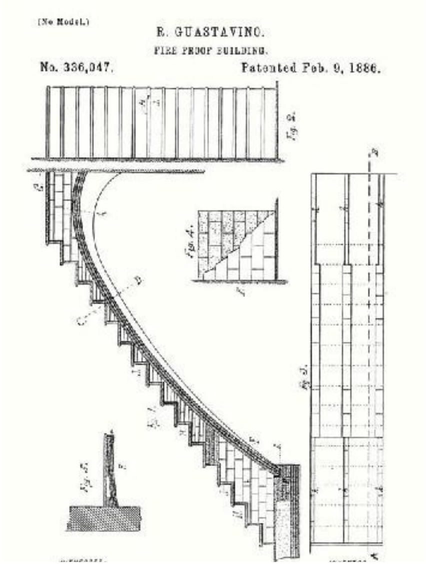
1886 Patent for Fireproof Building



WITNESSES:

INVENTOR

Rafael Guastavino

BY

ATTORNEY

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R. Guastavino Co. (1889-1962)

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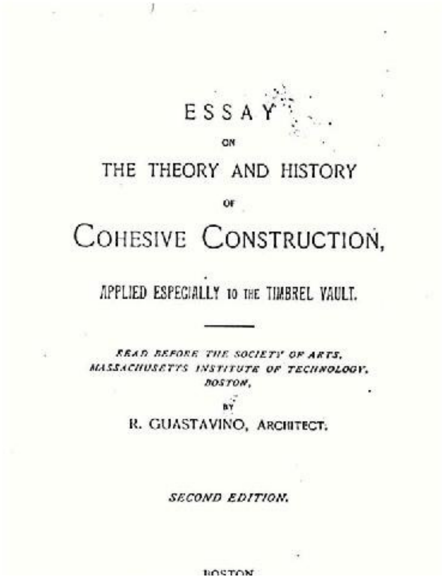
Essay of 1893



ESSAY

ON

THE THEORY AND HISTORY

OF

COHESIVE CONSTRUCTION,

APPLIED ESPECIALLY TO THE TIMBREL VAULT.

READ BEFORE THE SOCIETY OF ARTS,
MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
BOSTON,

BY

R. GUASTAVINO, ARCHITECT.

SECOND EDITION.

BOSTON

BOSTON
TICKNOR AND COMPANY
211 Tremont Street
1893

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Load Testing by Guastavino Sr.


140 lbs.
May 15/0
A.E. Ernst
Photo.

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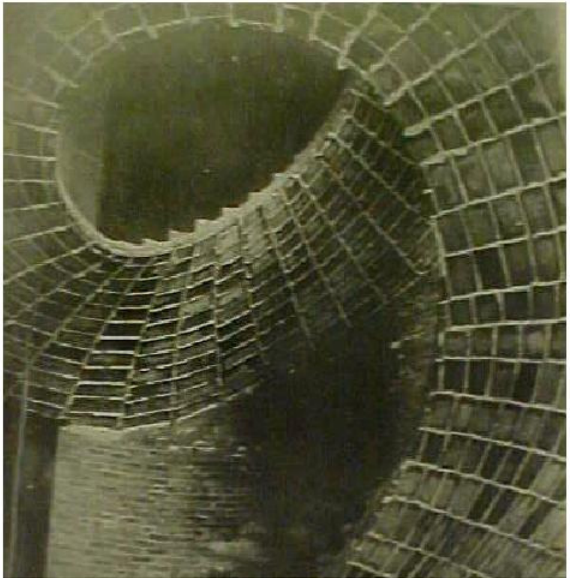
Load Testing by Guastavino Sr.

Spiral Staircases in Compression



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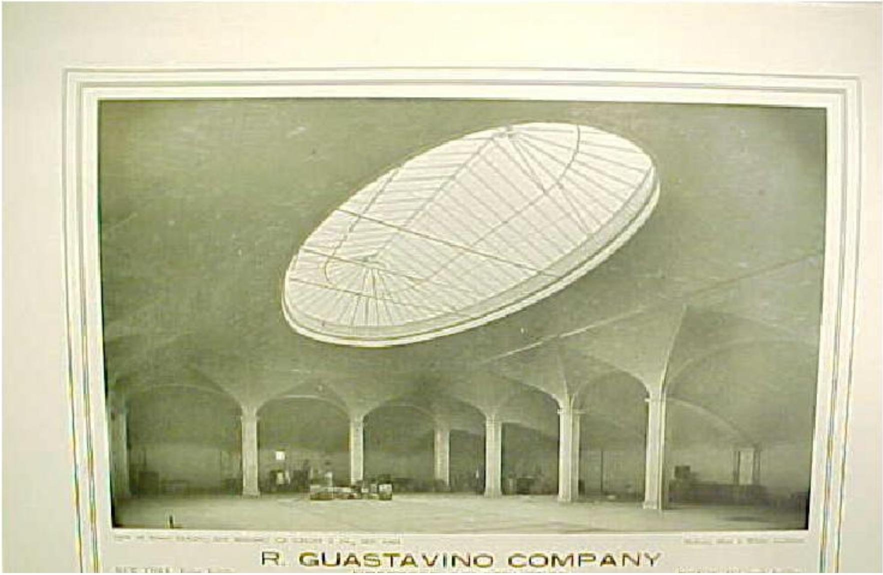
Tiffany Building, NY, 1906



FIREPROOF CONSTRUCTION

Grand Central Station, NY, 1913

Image courtesy of Adrien Mortini, structurae.de

Guastavino Vaulting

Research questions

Mechanics of tile vaults

Calculation methods used by Guastavino

Analysis of complex forms, like spiral staircases

etc

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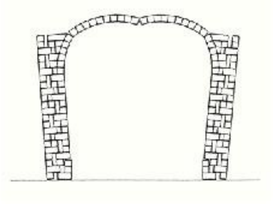
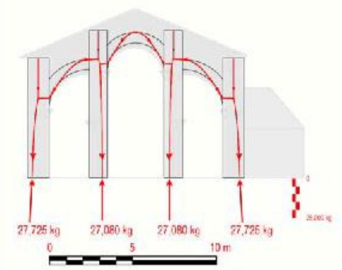
Analysis of Historic Structures



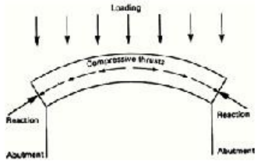


Lecture 2: Intro to Masonry Structure

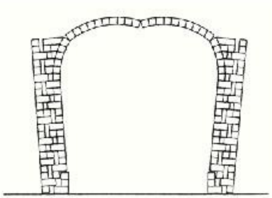
Review of Last Meeting

Principles of historic structure

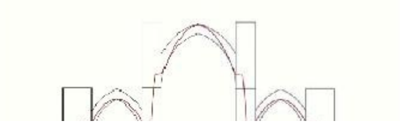
EQUILIBRIUM

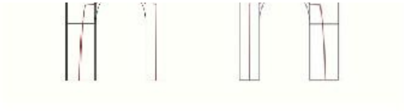


Lower bound and upper bound theorems

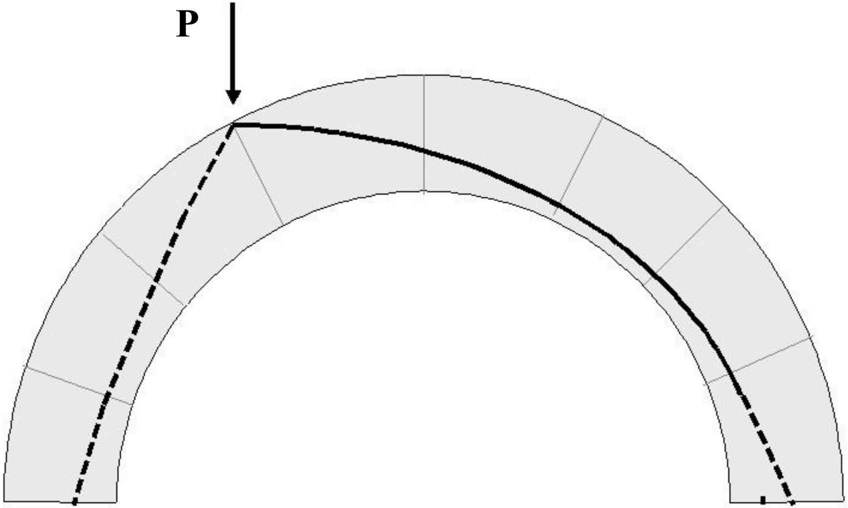


Possible research topics

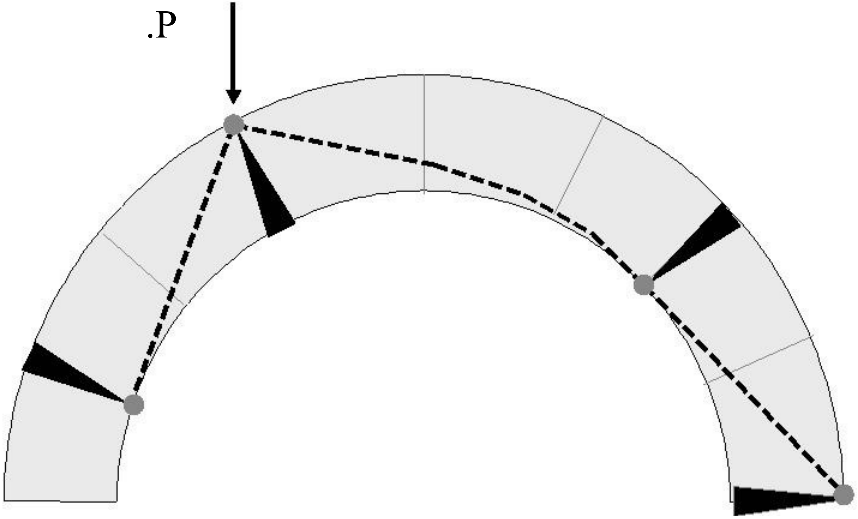




Lower Bound Theorem



Upper Bound Theorem



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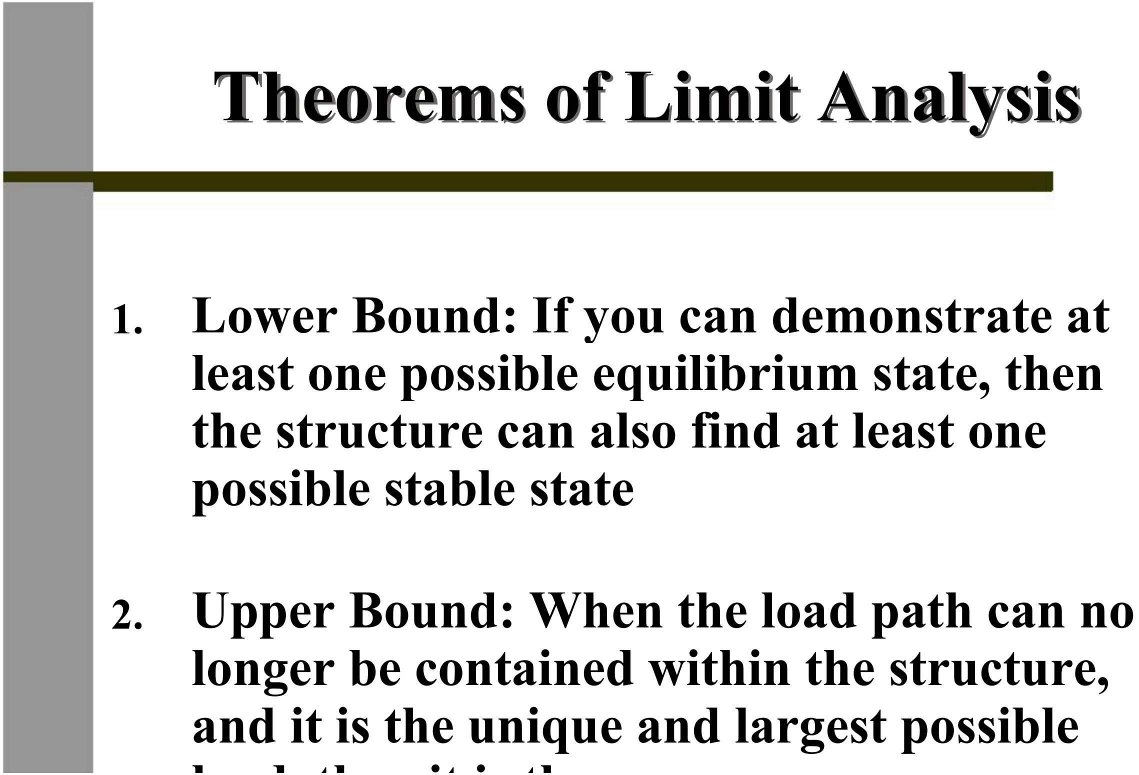

Theorems of Limit Analysis

1. **Lower Bound: If you can demonstrate at least one possible equilibrium state, then the structure can also find at least one possible stable state**

2. **Upper Bound: When the load path can no longer be contained within the structure, and it is the unique and largest possible**

load, then it is the collapse load

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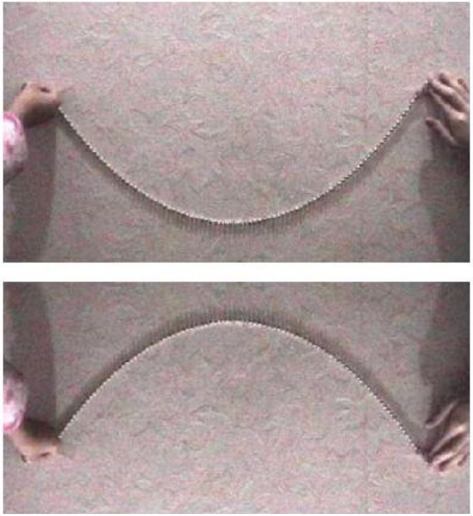
Hookes 2nd Law (1675)

ut pendet continuum *flexile, sic stabit contiguum rigidum inversum*

As hangs the flexible line, so but inverted will stand the rigid arch



arch.

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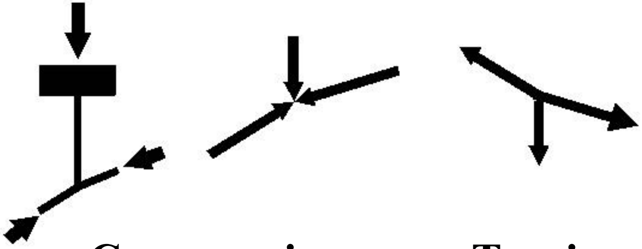
Compression vs. Tension



Compression **Tension**

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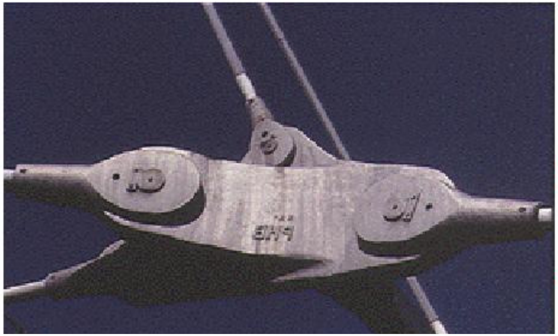
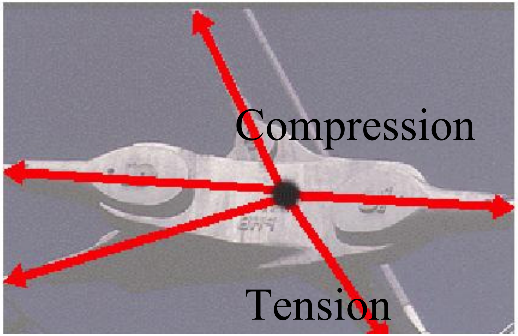
Equilibriumat a Point

SF = 0 (sum of forces is zero)





Structural Equations

Only three types of equations:

1) Equilibrium
2) Material properties (elasticity, etc)
3) Compatibility (geometry)

We will focus on equilibrium equations because they are the most important.

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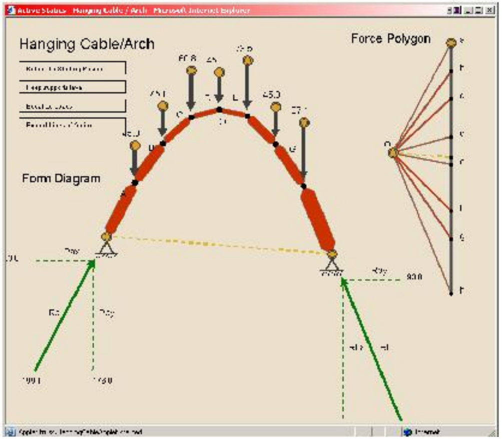
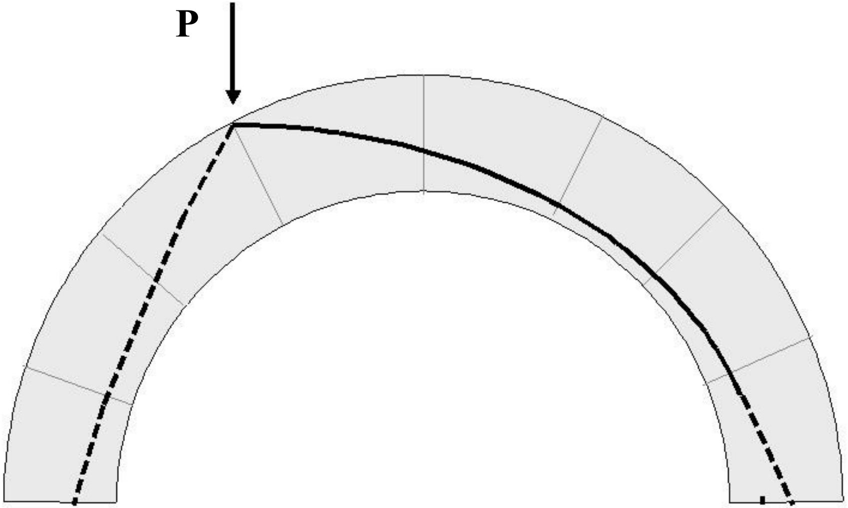
Graphic Statics

Applet by Simon Greenwold

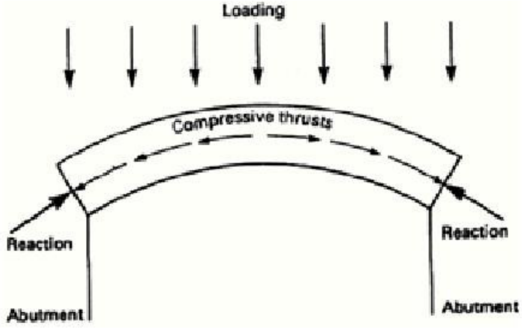


Hookes Hanging Chain



Design of Masonry

Main principle: must be kept in compression



Also applies to cast iron, *unreinforced concrete, and*

■ **other brittle materials**

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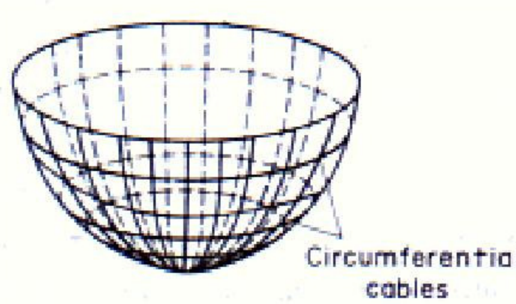
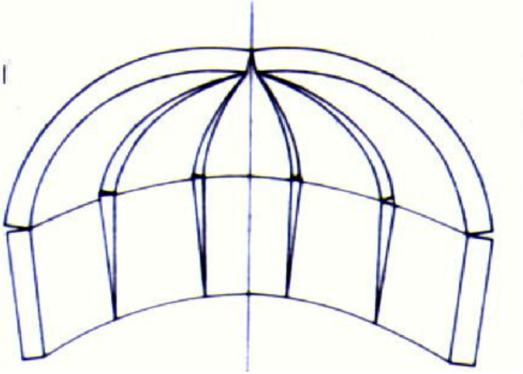
3D Vaults: Slice into arches





(a) Collapse of dome and drum in "orange-slice" segments

Structural Analysis of Masonry

The Stone Skeleton

by Jacques Heyman

Three main assumptions:

No tensile strength

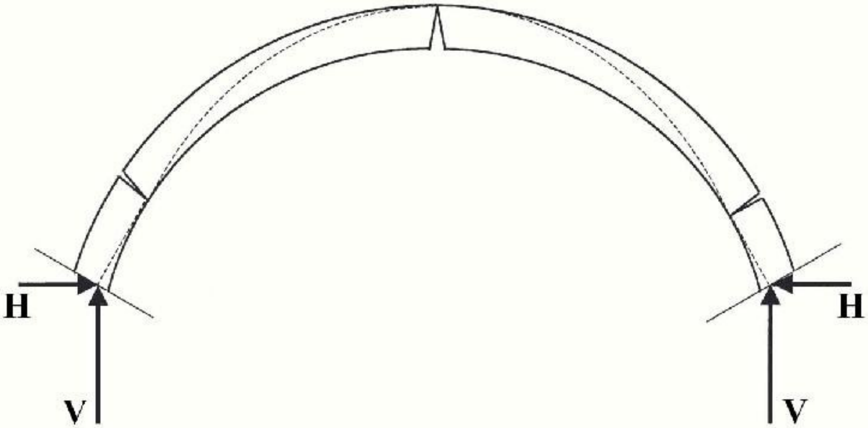
Infinite compressive strength (rigid)

Sliding does not occur

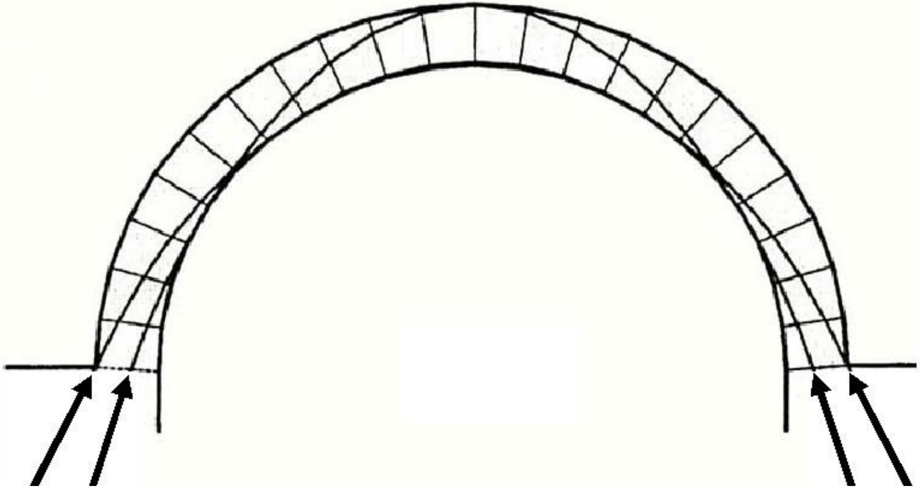
(a)


(h)

Arch on Spreading Supports



Range of Arch Thrust



Internal thrust lines due to self weight of arch

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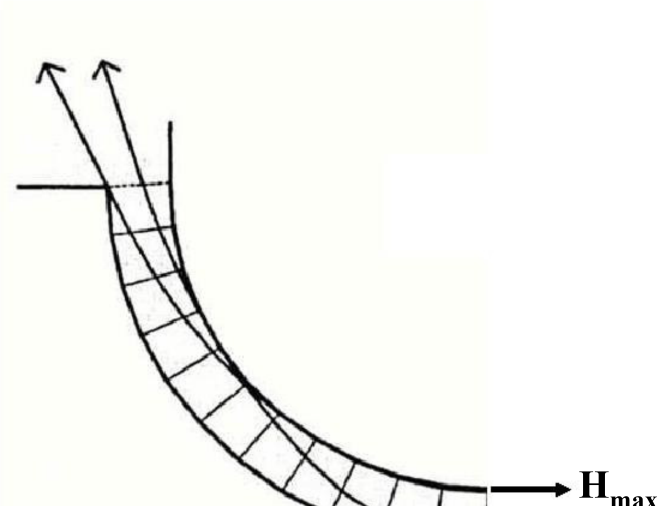
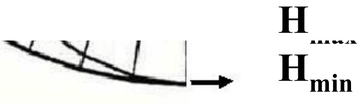
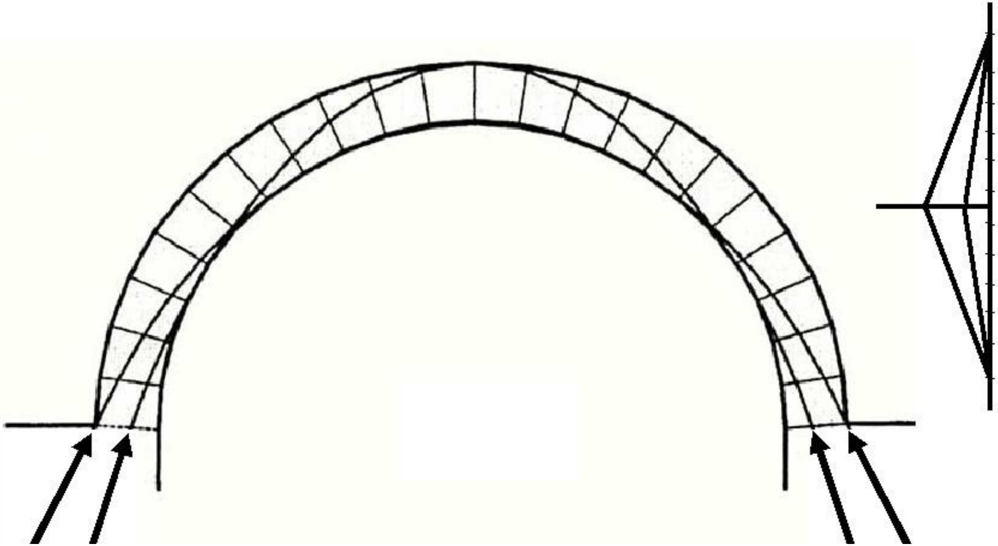
Range of Arch Thrust





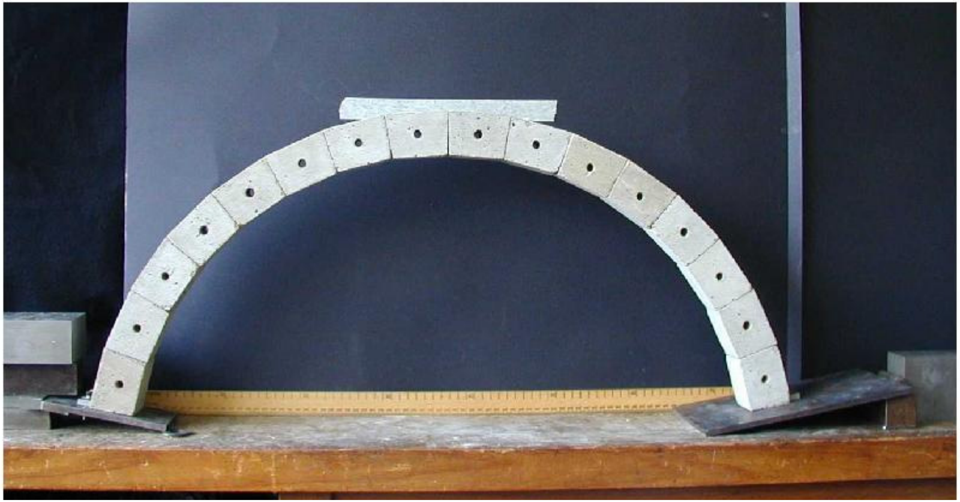
TENSION

Range of Arch Thrust

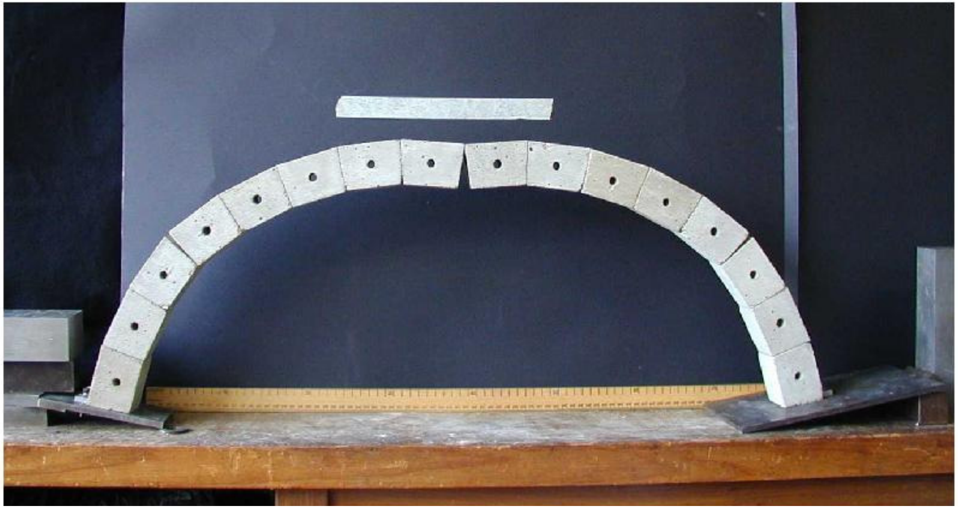


COMPRESSION

Model Arch Experiment



Model Arch at Collapse State



Understanding cracks in masonry

1. Why do cracks occur.
2. What do they tell us.
3. Are they a cause for concern.

Understanding cracks in masonry

1. **Why do cracks occur.**

   -Small movements of supports

2. **What do they tell us.**

   -Where forces are NOT acting

3. **Are they a cause for concern.**

-Usually not, but they can be

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Understanding of Collapse

Causes of collapse:

1. **Displacements**
   -Foundation movements, mortar creep over time
2. **Overloading (truck on a bridge)**
   -Water on vaults, collapsing roof on vault
3. **Accelerations**

3. Accelerations

-Vibrations, earthquakes

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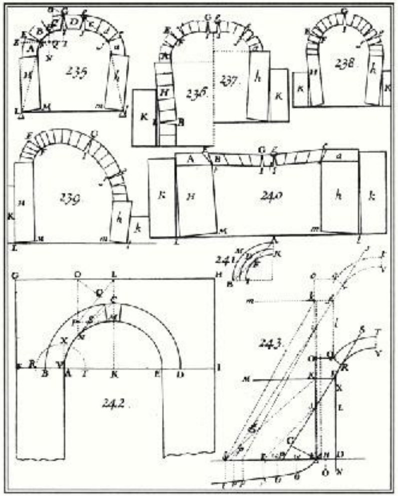
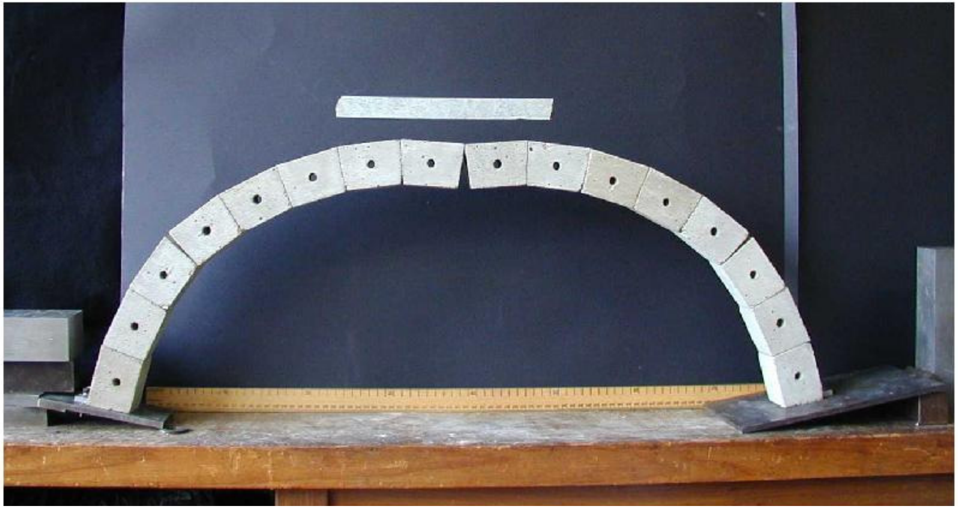
Design and Analysis of Unreinforced Masonry

Stability rather than failure of the material is the dominant concern

Collapse occurs when the load path can no longer be contained within the masonry



Model Arch at Collapse State



Single Span Stone Arch

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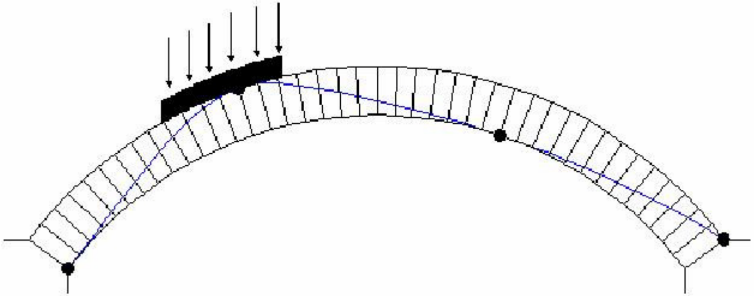
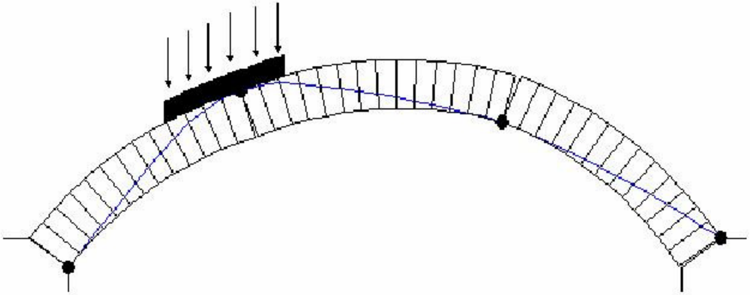
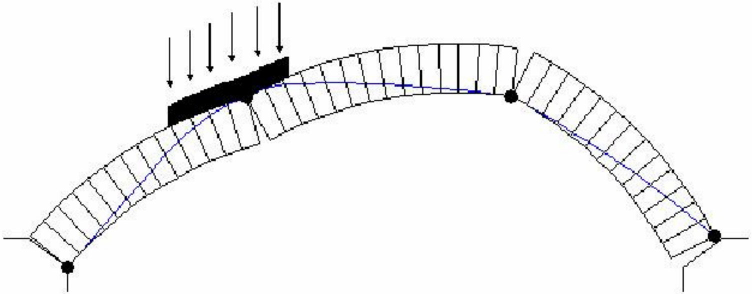
Single Span Arch



Single Span Arch

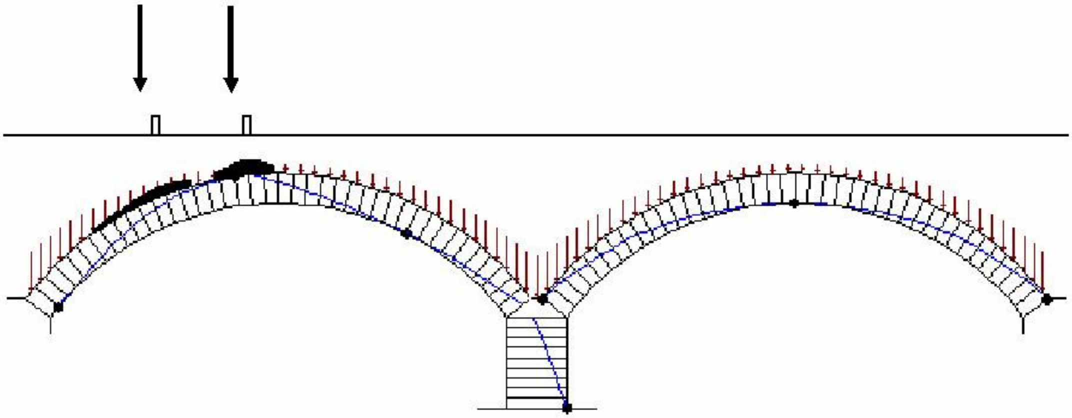


Single Span Arch

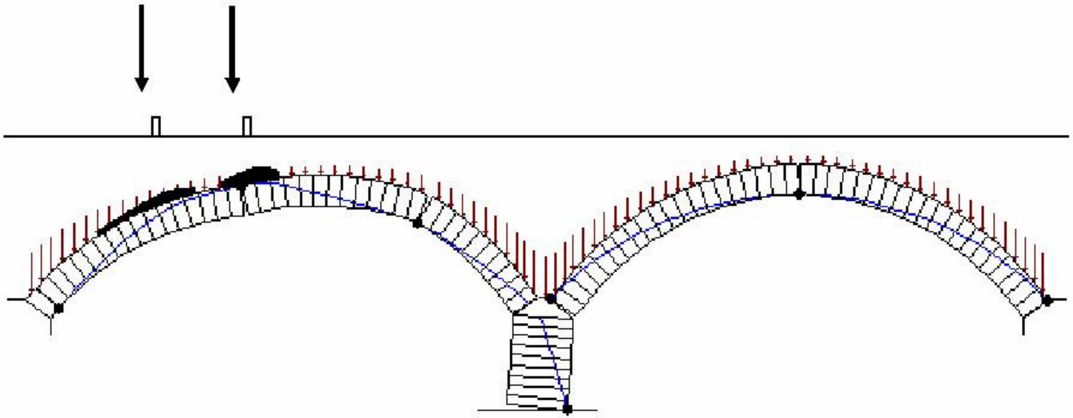


Double Span Stone Arch

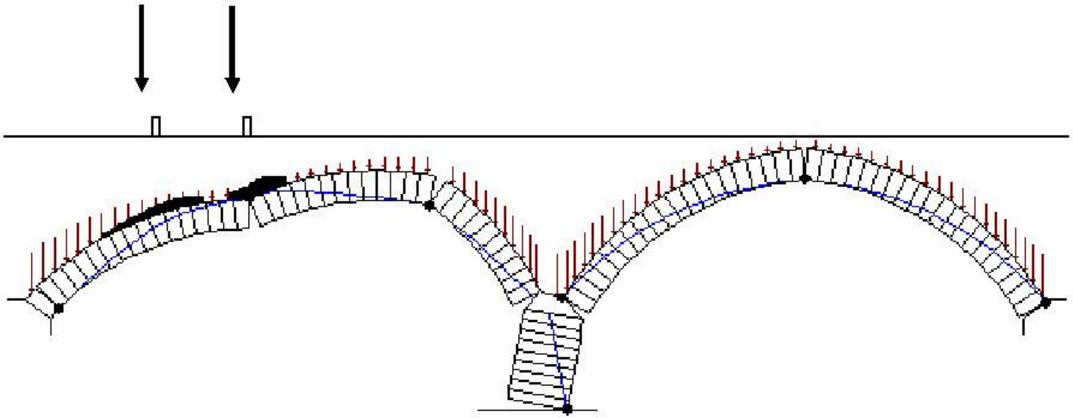
Double Span Arch with Fill



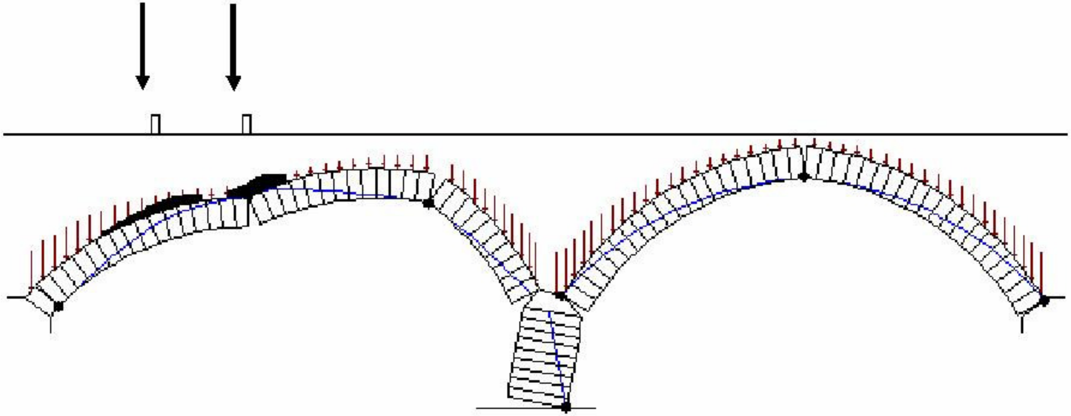
Double Span Arch with Fill



Double Span Arch with Fill



Double Span Arch with Fill



This makes sense for bridges, but buildings dont usually **have trucks driving on top of the vaults**

Deformation over time can cause collapse in buildings

Conclusions

Unreinforced masonry structures have very low stress levels: stability, not strength, governs the safety

Determine collapse states based on thrust line analysis using graphic statics

Equilibrium equations are most important when analyzing

historical structures

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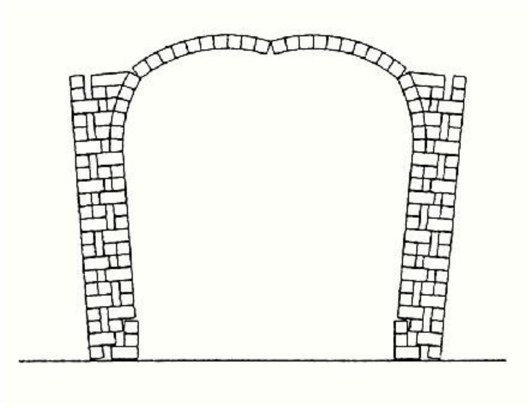
Lecture 3



Analysis of Masonry Structures: Arches, Vaults,

Arches, Vaults, and Buttresses

Limit Analysis of Masonry

Lower Bound Theorem

Seeks permissible line of compressive force **for the given loading**

Upper Bound Theorem

Seeks critical load which results in a failure **mechanism**

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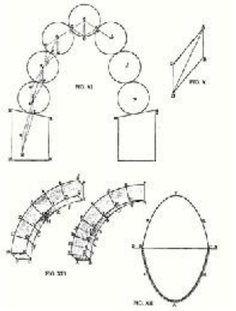
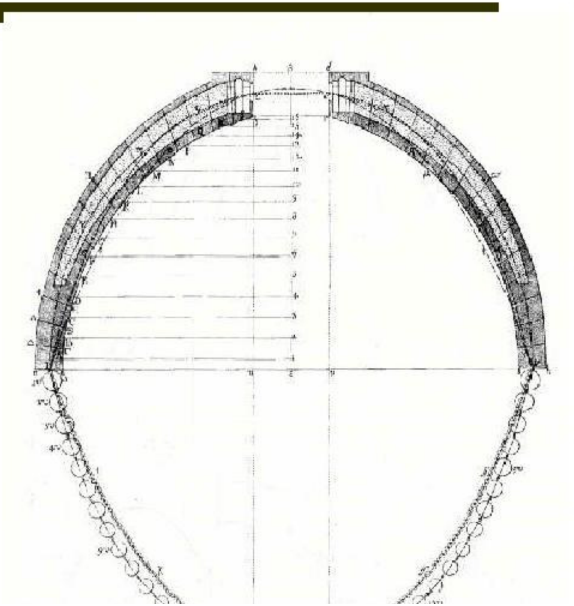
Poleni (1748) applies lower bound to St. Peters of Rome






λ

Selby Abbey, 12th C, England

Tower construction punches through

Arches deform to accommodate support movements

Stable because a line of thrust can be found within the deformed arch

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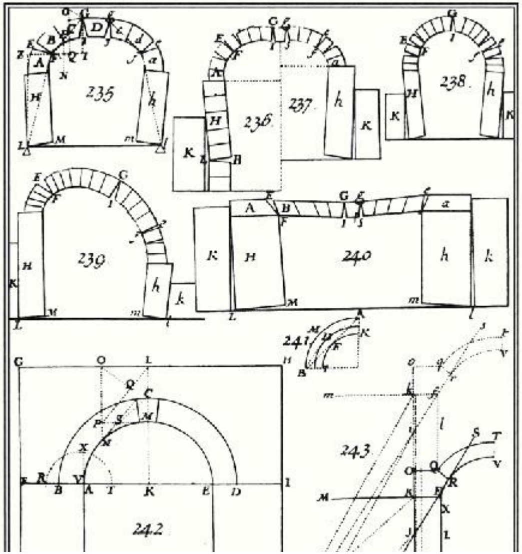
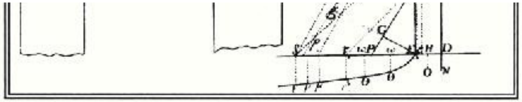
Experiments by Danyzy (1732)



Collapse occurs by hinging between blocks, when a load path can not be contained within the masonry

Safety is a question of geometry and stability, not crushing of stone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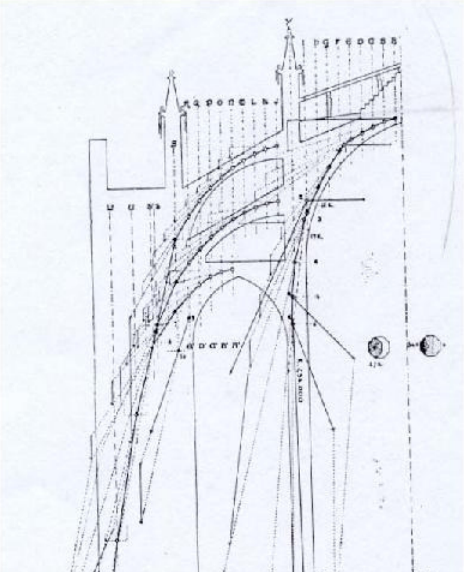
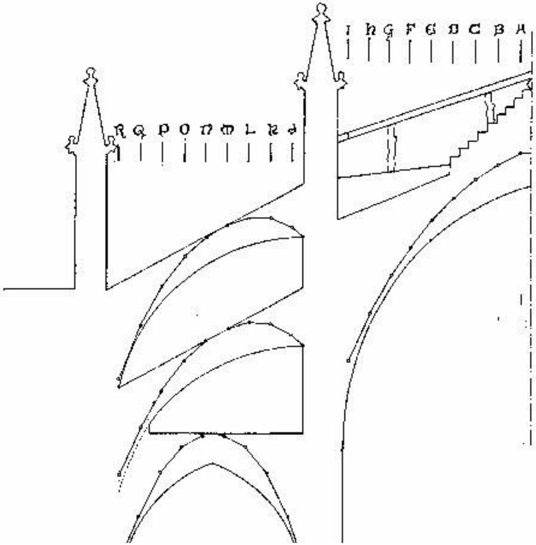
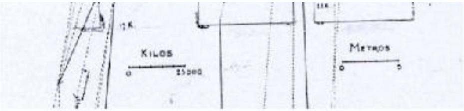
Cathedral in Palma de Mallorca, Analysis by Joan Rubio (1912)






1K
11K
KILOS
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METROS
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Analysis of a Flat Arch

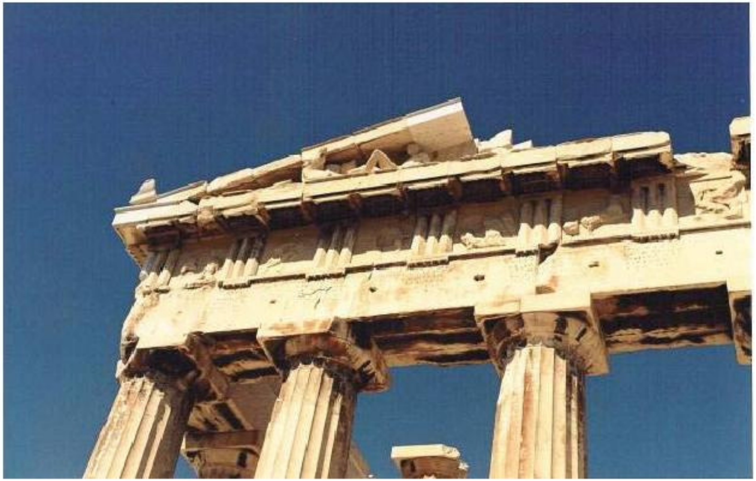
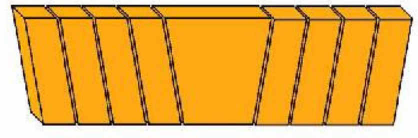


Image courtesy of Denis
Y. Yu, structurae.de

Analysis of a Flat Arch



Thrust of a flat arch.

Flat Arch

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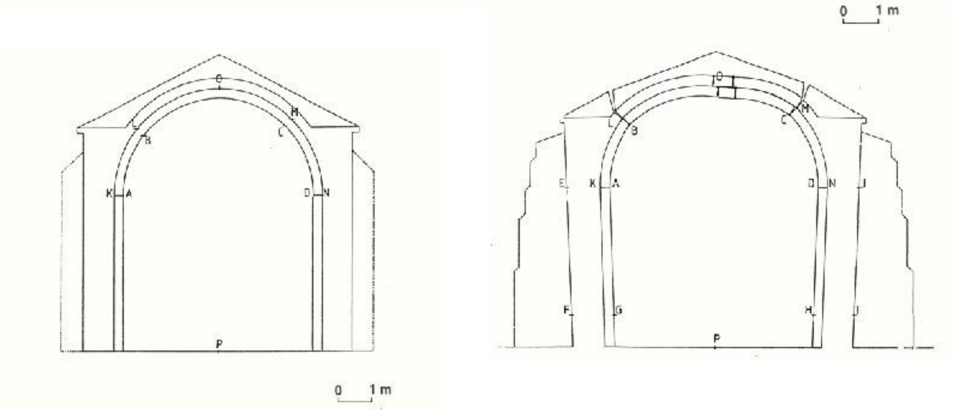
Safety of an Arch on Buttresses



Must solve three problems:

1. Load capacity of buttress (and influence of lean)

2. Collapse state of arch on spreading supports

3. Analysis of arch supported on leaning buttresses

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Buttress Analysis, DuPuit (1870)

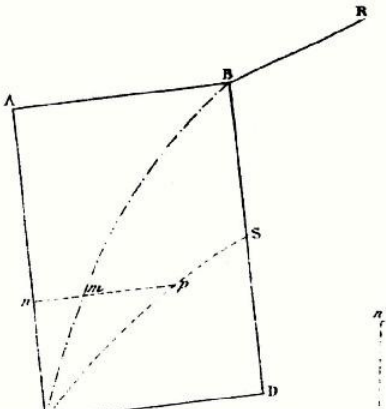


Fig. 86.

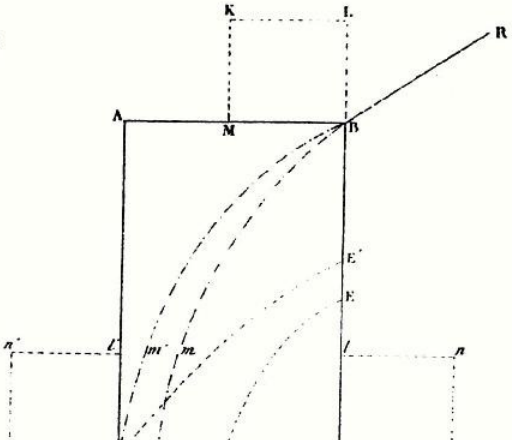
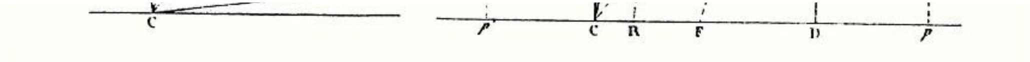


Fig. 87.


C
p
C
R
F
D
p

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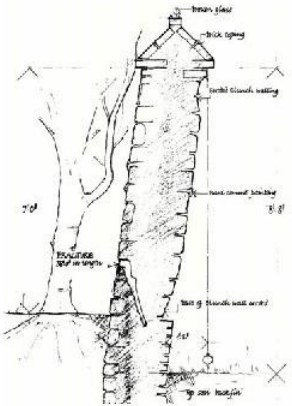
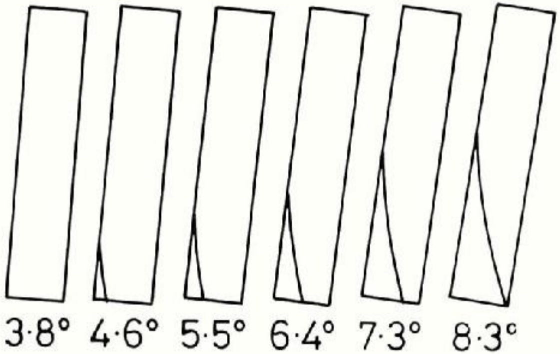
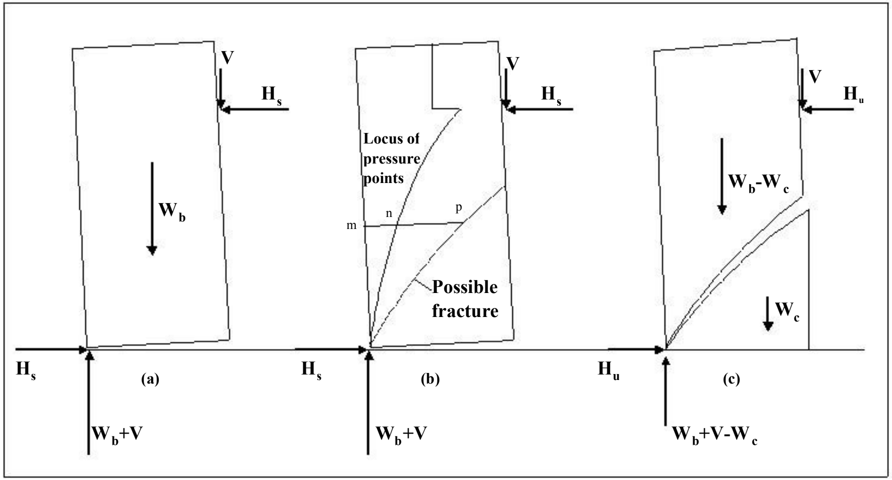
Heyman on Leaning Walls (1992)





Buttress Collapse



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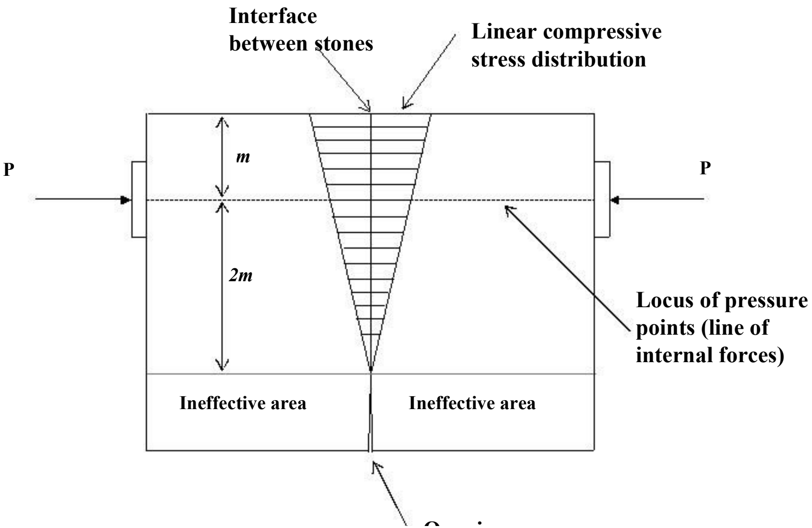
Assumed Compressive Stress Distribution



Interface between stones

Linear compressive stress distribution

Locus of pressure points (line of internal forces)

Opening
between stones

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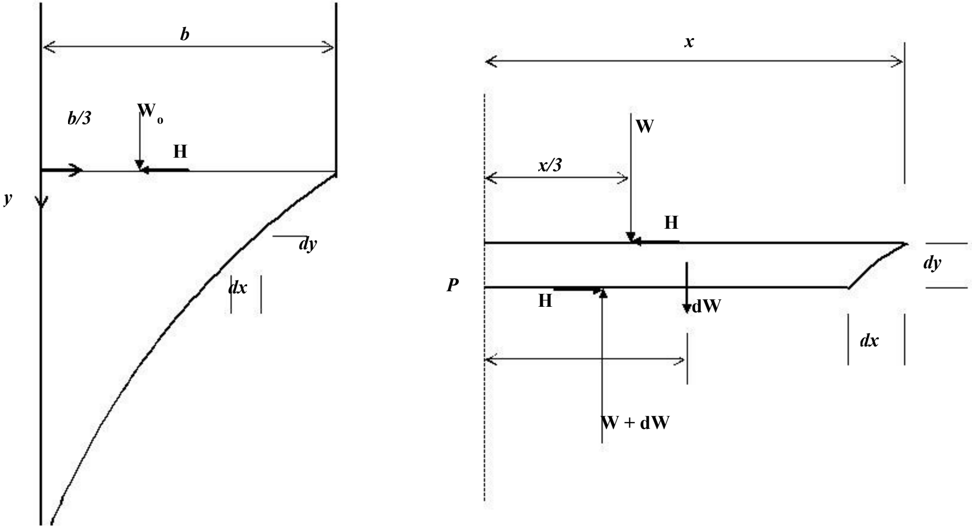
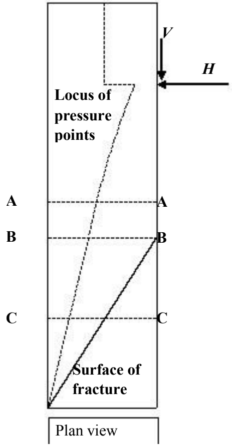
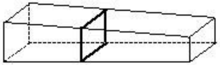
Determine Shape of Fracture

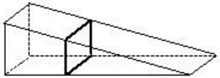


Assumed Buttress Stress Distribution at Collapse

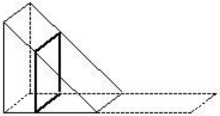




A-A: Locus of pressure points acts near the centroid and the entire section is in compression.



B-B: Locus of pressure points reaches kern point of rectangular section (1/3 point).



C-C: Section properties change as fracture occurs and locus moves to 1/3 of the new section.

Fracture reduces thrust capacity by >30% in many cases

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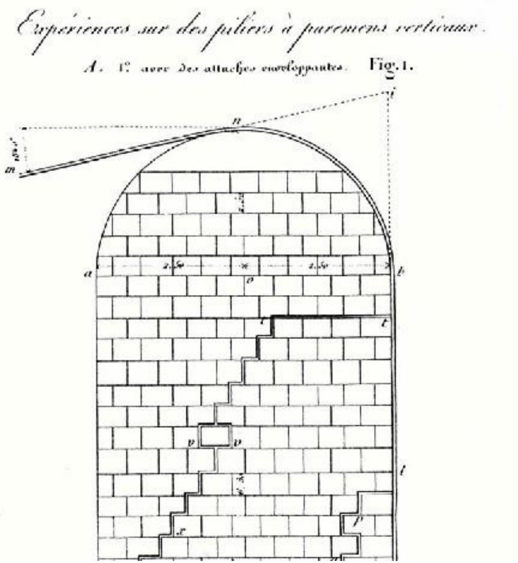
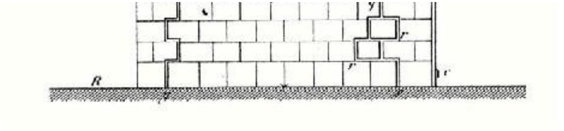
Vicat Experiments on Suspension Bridge Towers (1832)




y
r
r
R
r

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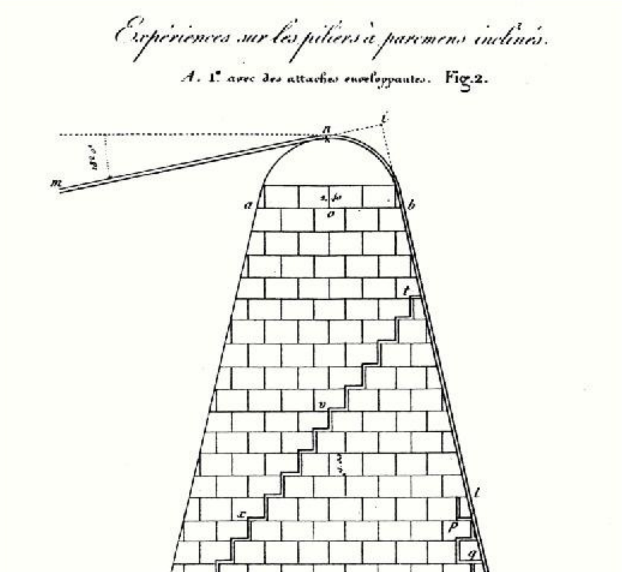
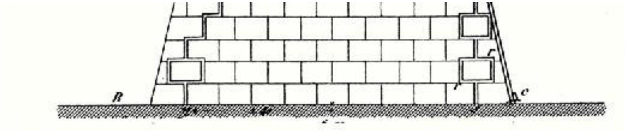
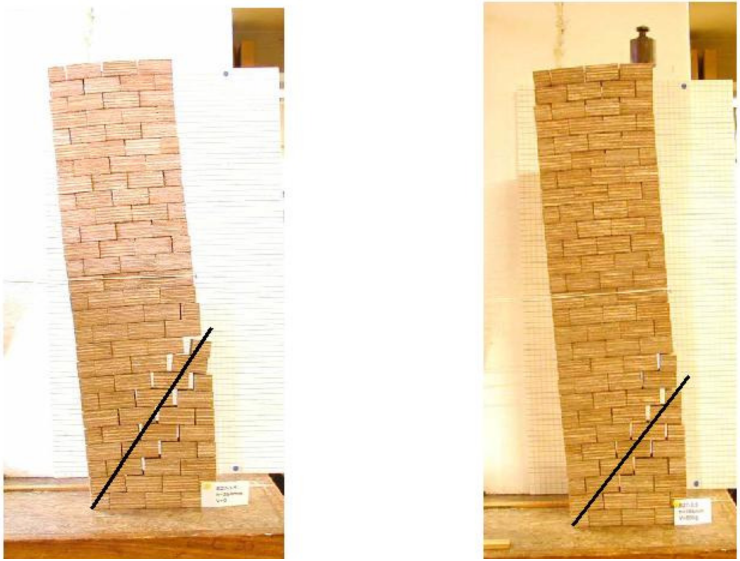
Vicat Experiments on Suspension Bridge Towers (1832)




R
f
r
c

Model Buttress Experiments



Fracture reduces thrust capacity by 20% to 30%

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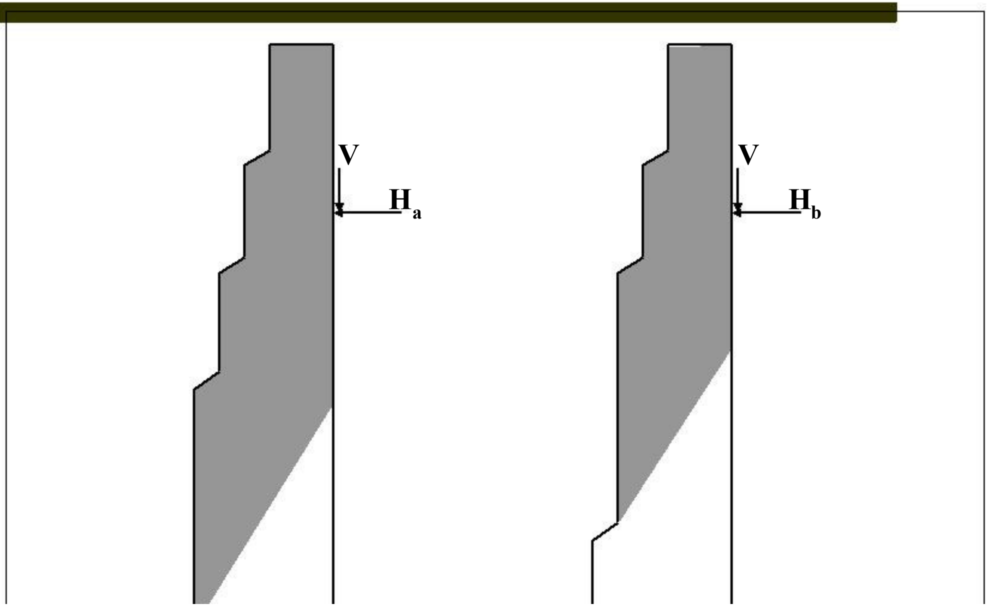
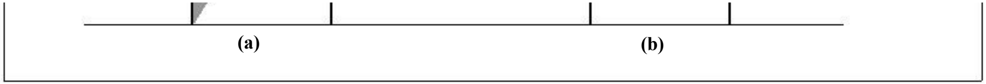
Gothic Buttress Failure State





(a) (b)

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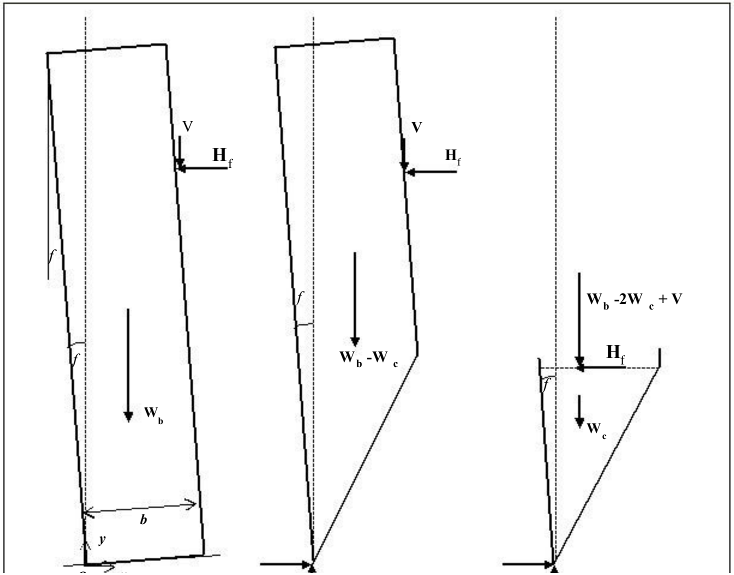
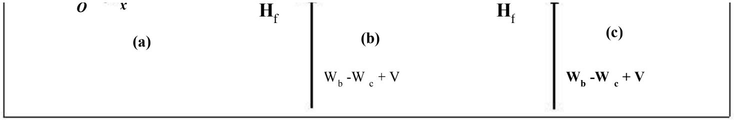
Leaning Buttress




O
x
(a)
H_f
(b)
$W_b - W_c + V$
H_f
(c)
$W_b - W_c + V$

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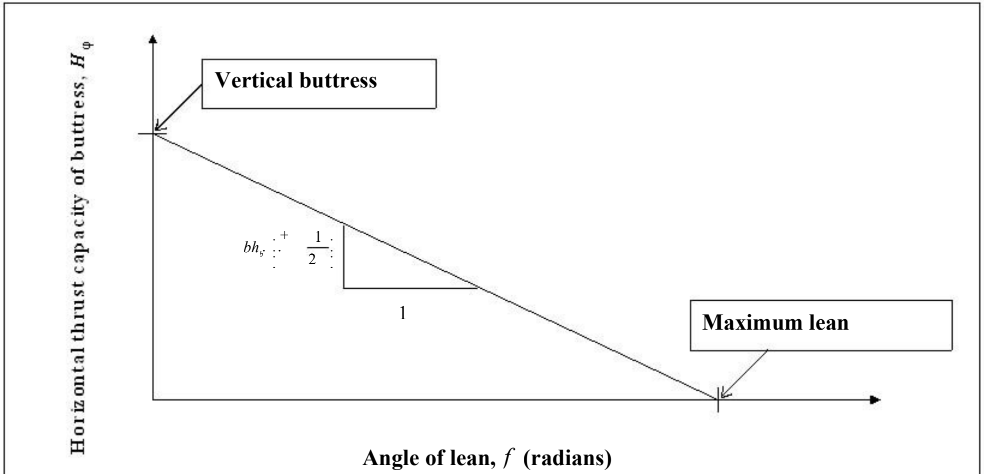
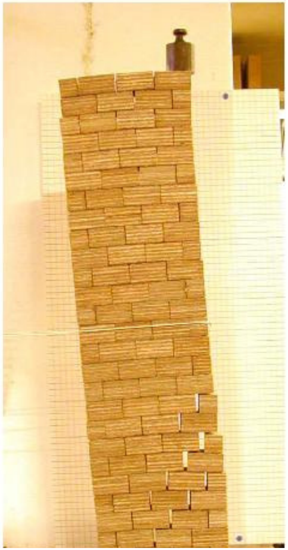
Leaning Buttress Capacity



Load Capacity of a Buttress



A masonry buttress will **fracture at collapse, reducing its load capacity.**

A leaning buttress has a linear **reduction in capacity, based on a small angle approximation as the centroid shifts horizontally.**

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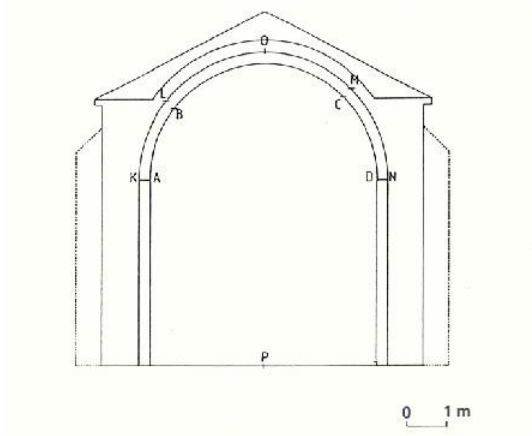
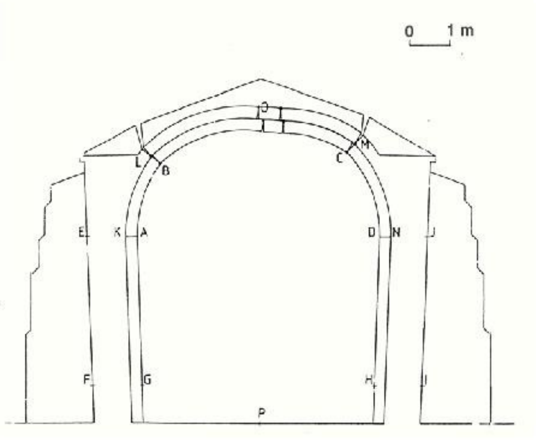
Church in Guimarei, Spain





Geometry changes may threaten stability of the structure

Huerta and Lopez (1997)

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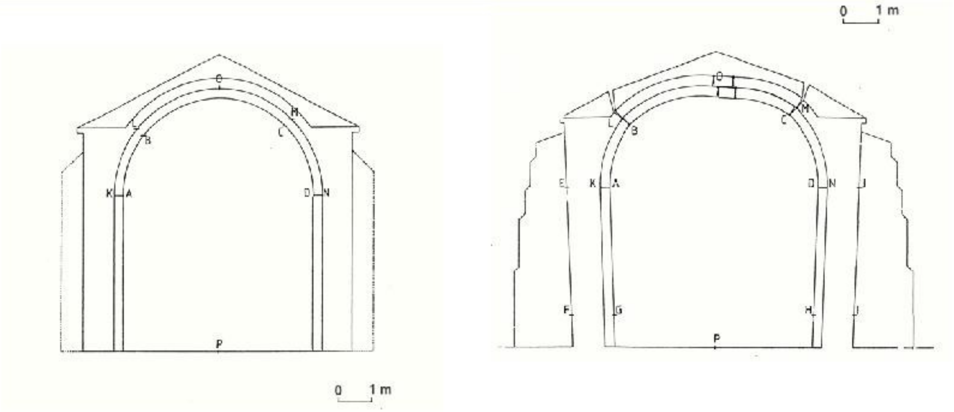
Church in Guimarei, Spain



1. Buttress leans outward (e.g. foundation deforms)
2. Arch deforms and thrust increases

3. Buttress leans further and thrust increases further.

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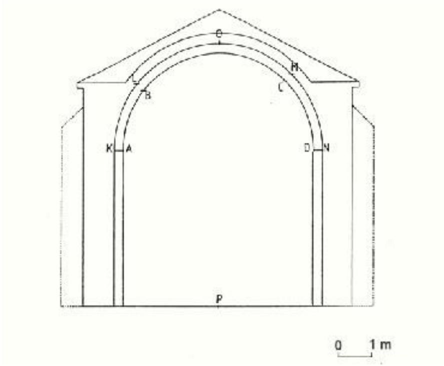
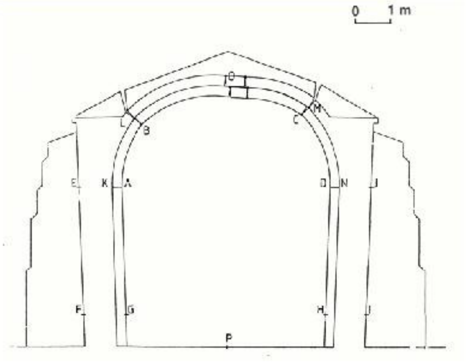
Church in Guimarei, Spain





Must solve three problems:

1. **Load capacity of buttress (and influence of lean)**

2. Collapse state of arch on spreading supports

3. Analysis of arch supported on leaning buttresses

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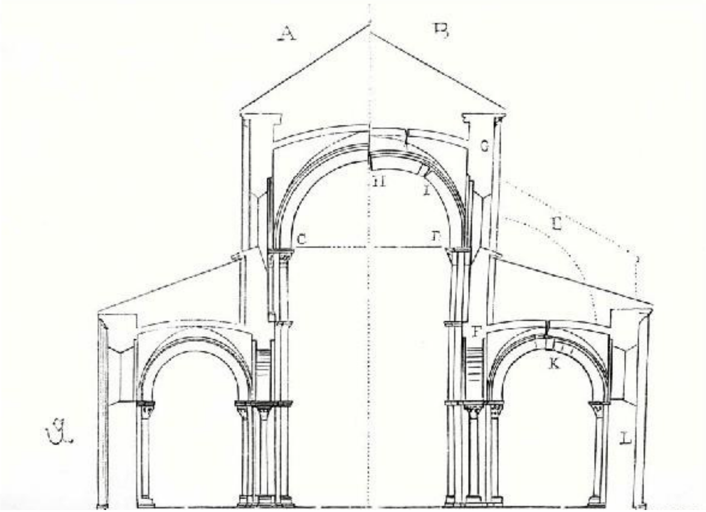
Spreading Arches: Viollets study of Vezelay (1854)



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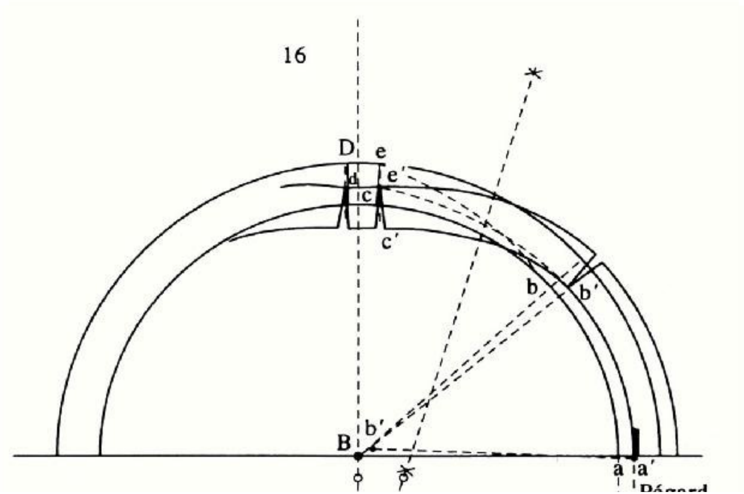
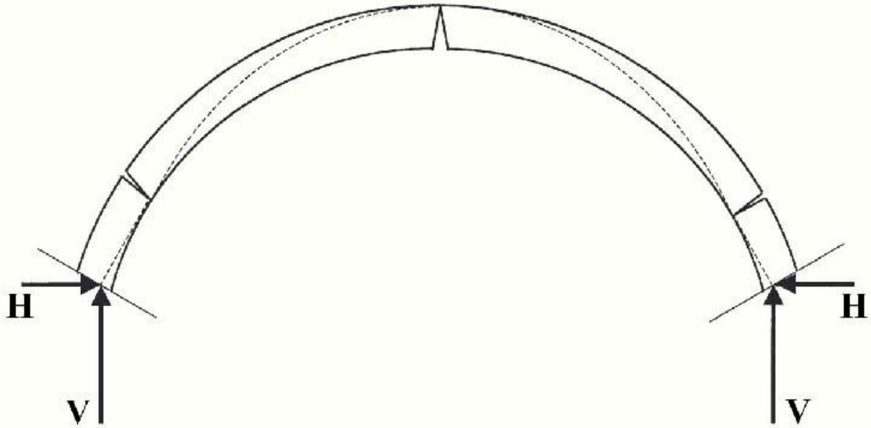
Viollet-le-Duc on Spreading Arches



regard.

Arch on Spreading Supports



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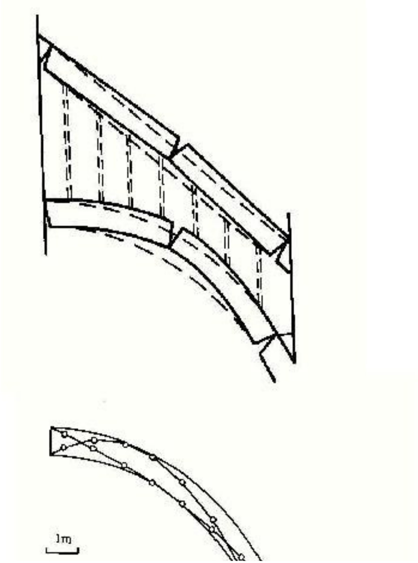
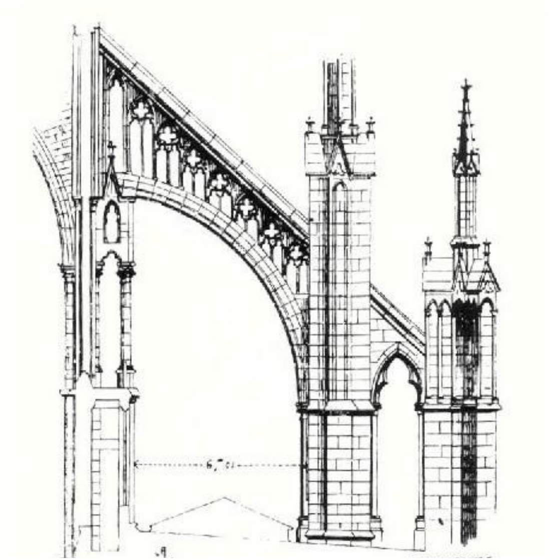
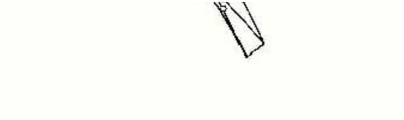
Amiens Possible Collapse Mode








E.GUILLAUMET.

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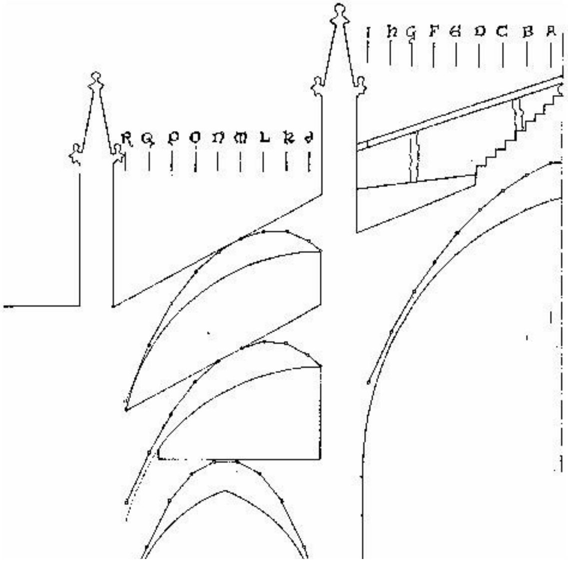
Flying Buttresses at Palma de Mallorca



Conclusions

Unreinforced masonry structures have very low stress levels: stability, not strength, governs the safety

Limit analysis can be used to determine collapse states based on thrust line analysis

Capacity for displacements may be more important than load capacity (particularly for historic buildings)

For high vaulted buildings, the arch will collapse and the

buttress will remain standing in most cases.

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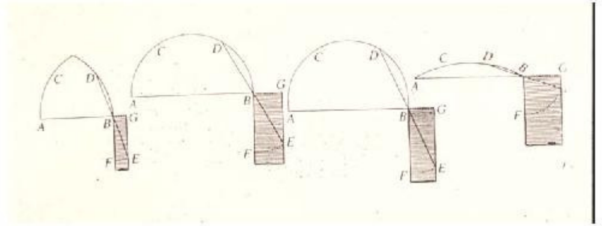
Research Papers on Masonry



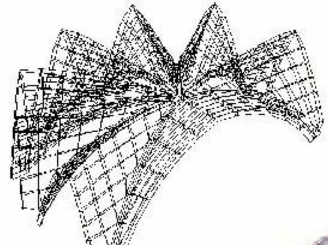
Comparative studies

Arches

Vaulting

Buttresses

Individual structures



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Research Papers on Masonry

Tile vaulting (Guastavino)

Gothic

Romanesque

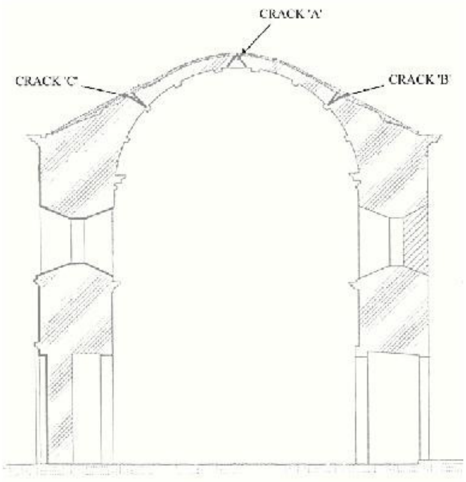
Mamluk

Maya/Aztec

Mycenaean tholos tombs

Individual structures

Lecture 4

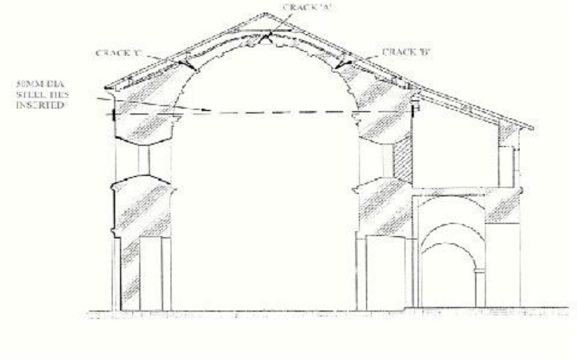


Case

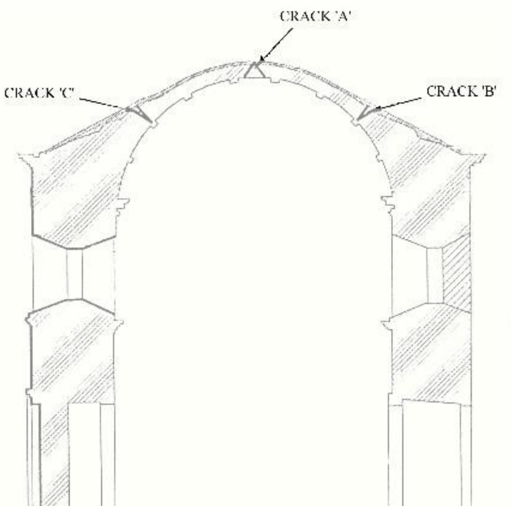
Case Study of

16th C Church in Old Goa

Case Study:
16th C. Church in Goa, India



16th C. Church in Goa, India



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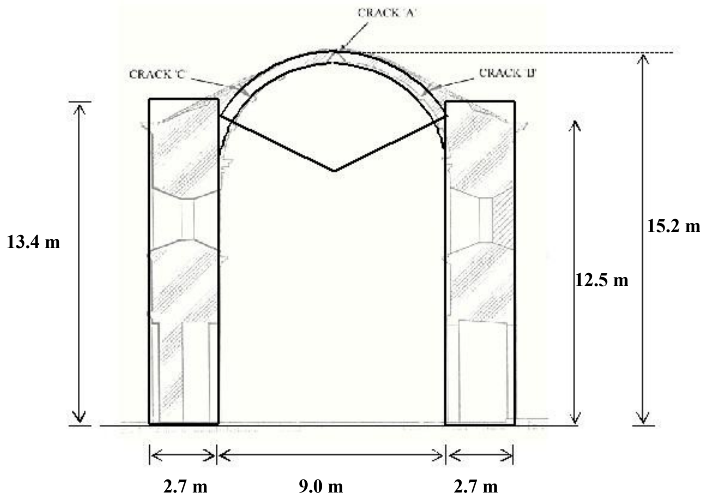
16th C. Church in Goa, India



Unit weight of material 25 kN/m^3 analyze 1 m strip

Solid Buttress at Goa

Weight of the buttress is 905 kN (per metre width)

Vertical reaction of the arch, V =64 kN

Overturning of solid buttress, H_s = 112 kN

The reduction in thrust capacity as the buttress leans is approximately 10 kN per degree of leaning.

For the buttress leaning by 0.4 degrees, the overturning force is reduced to 108 kN

Fractured Buttress at Goa

Weight of the buttress is 905 kN (per metre width)

Vertical reaction of the arch, V =64 kN

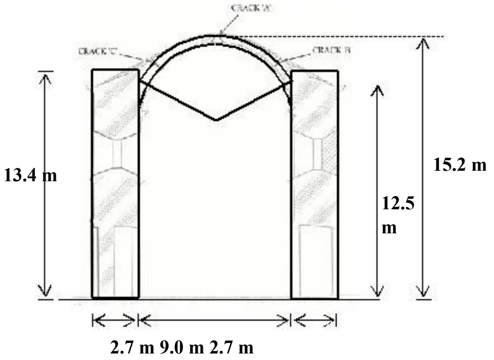
Overturning of solid buttress, H_s = 112 kN

The maximum horizontal thrust for the vertical buttress is approximately H_u=69 kN, corresponding to a fracture height of e =8.7 m. (nearly a 40% reduction)

Therefore, in the existing state with a lean of 0.4, the buttress capacity has been reduced to H_f =65 kN from 69 kN.

To prevent failure of the buttress by sliding, the weight of the buttress above the springing is 61 kN and the weight of the arch provides a vertical force of 64 kN. Assuming a static coefficient of friction of 0.7, sliding will occur for horizontal forces greater than 88 kN. Therefore the buttress will fail by overturning before sliding.

Arch at Goa



Exists in state of minimum thrust

Horizontal force 39 kN and vertical reaction is 64 kN

Span increases by 0.1 m from 9.0 m to 9.1 m

Arch thrust increases to 41 kN

Initial thrust capacity is 69 kN for horizontal thrust, so safety factor is 69 kN/39 kN = 1.8 initially

With leaning, it is reduced to

65 kN/41 kN =

1.6

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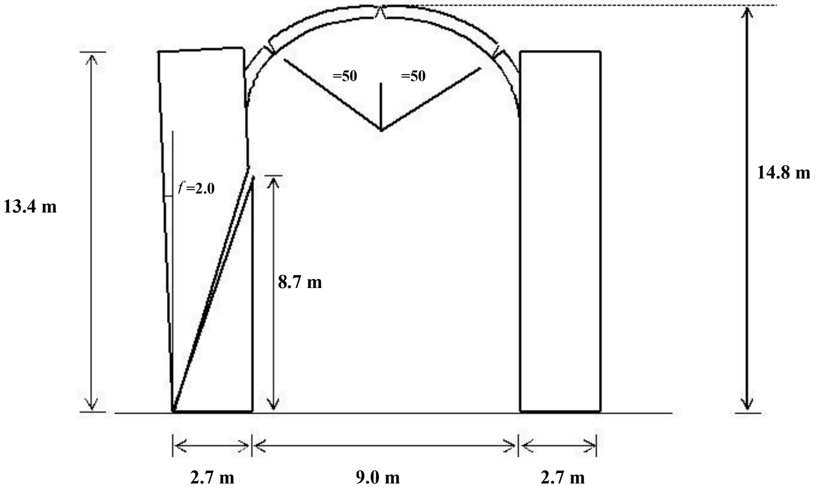
16th C. Church in Goa, India



Collapse state of the church at Goa due to leaning of the South buttress. When the buttress has leaned outward by 2, the thrust from the distorted vault will exceed the thrust capacity of the buttress and the vault will collapse. At this point, the crown of the vault has descended by 0.4 m and the thrust of the arch will have increased from 41 kN to 52 kN.

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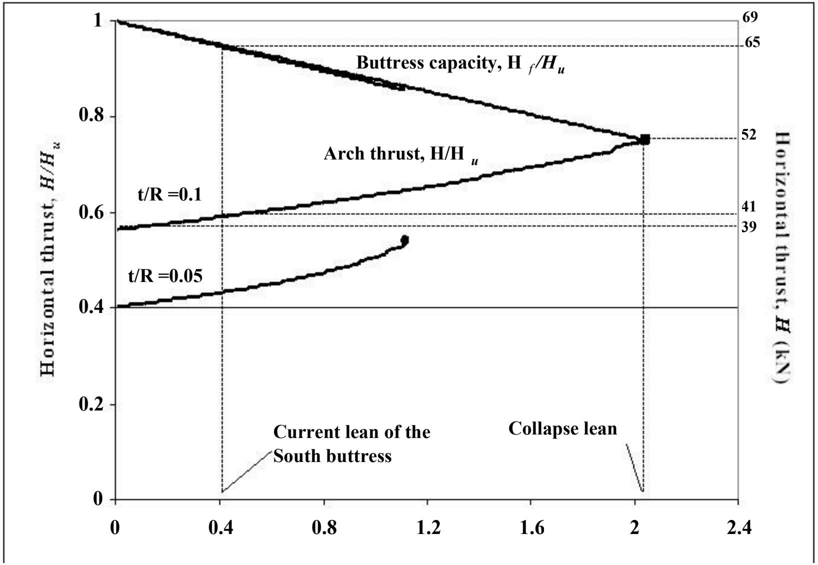
16th C. Church in Goa, India




Angle of South buttress lean, f (degrees)

Arch at Goa

Exists in state of minimum thrust

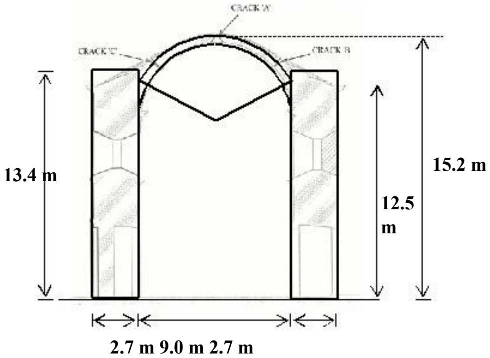
Horizontal force 39 kN and vertical reaction is 64 kN

Span increases by 0.1 m from 9.0 m to 9.1 m

Arch thrust increases to 41 kN

Initial thrust capacity is 69 kN for horizontal thrust, so safety factor is 69 kN/39 kN = 1.8 initially

With leaning, it is reduced to



65 kN/41 kN =

1.6

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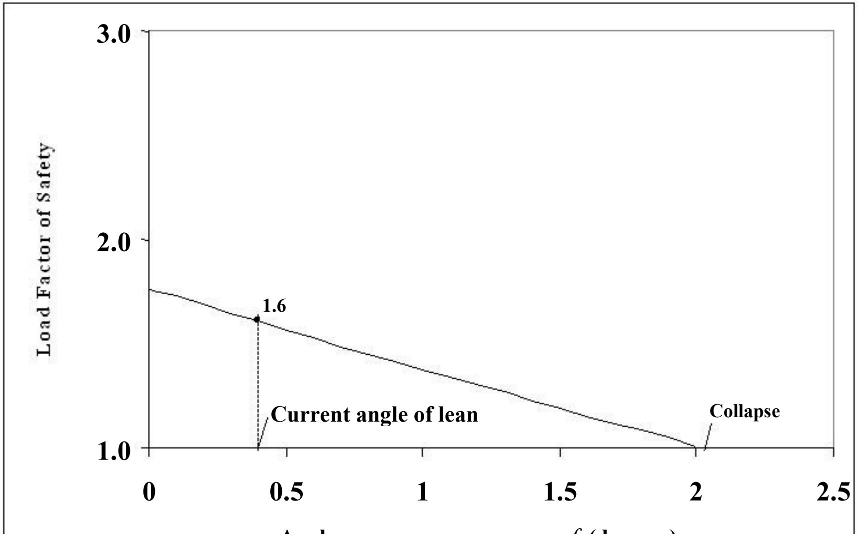
Safety Factor of Goa Church

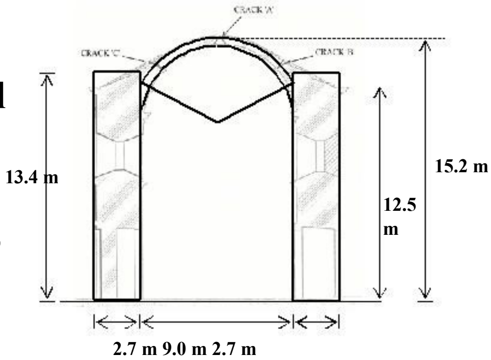


Angle of South buttress lean,	**ƒ (degrees)**

Conclusions from Case Study of Church at Goa, India

Overturning failure of buttress is more critical than a sliding failure

Safety factor against buttress failure is 1.6, compared to original value of 1.8

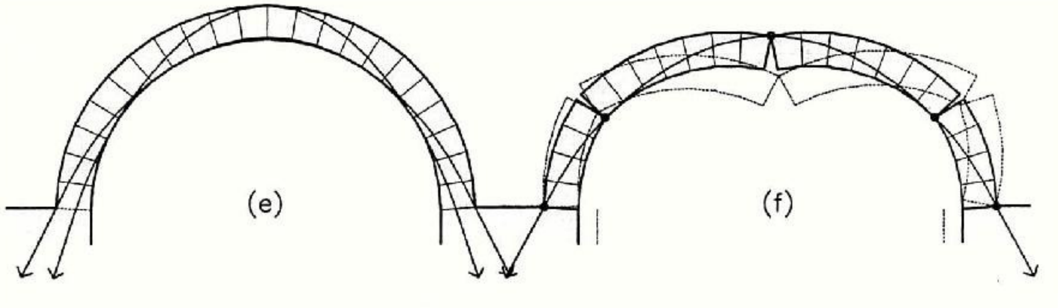


Leaning of buttress was leading to the collapse

Installed steel tie rod to carry

horizontal thrust and to prevent collapse

Range of Arch Thrust



Smars (2000)

Arch on Spreading Supports

Arch on Closing Supports

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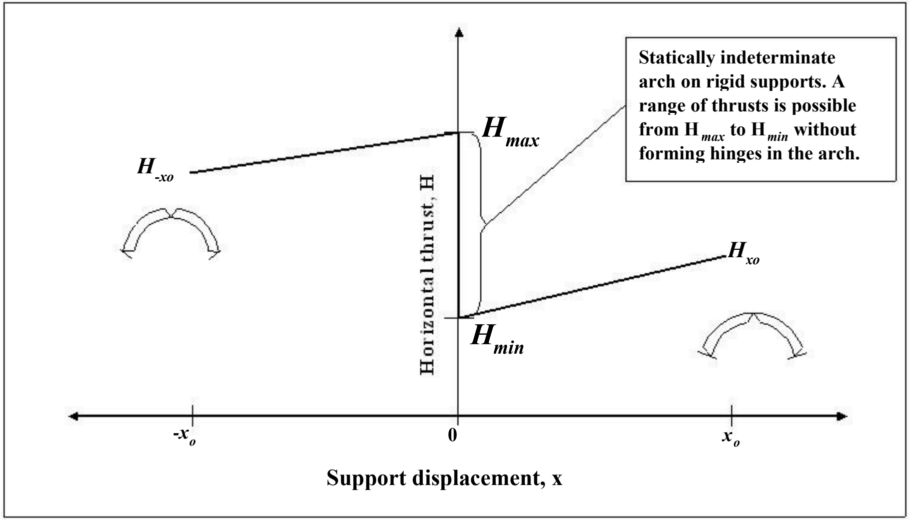
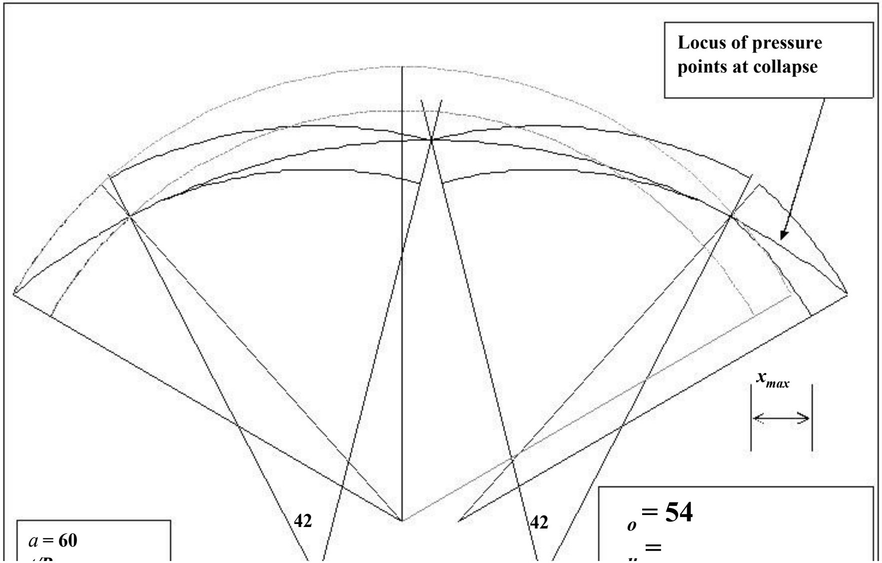
Range of Arch Thrust



Statically indeterminate arch on rigid supports. A range of thrusts is possible from H_{max} to H_{min} without forming hinges in the arch.

Arch at Collapse State




t/R
1 voussoirs
0.10
u
42
Span increase= 8.0%
Thrust increase= 2.17H min
Dip = 1.69t

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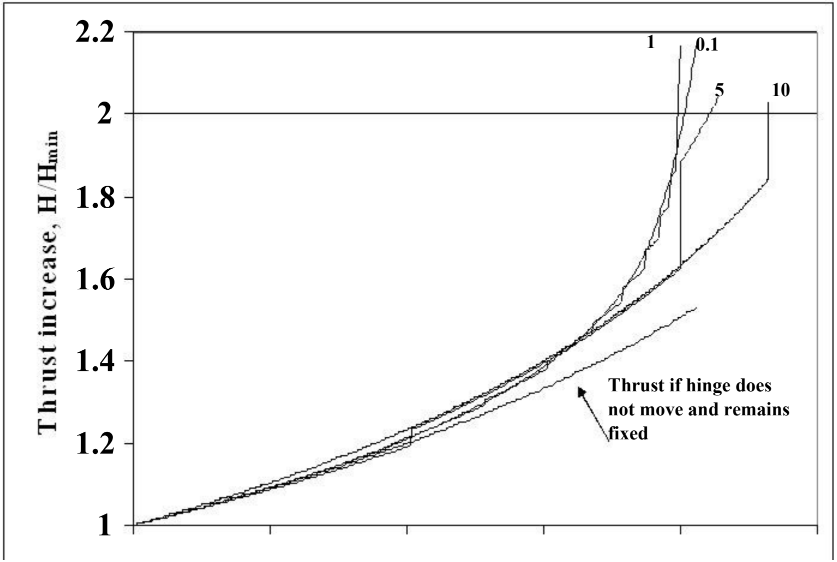
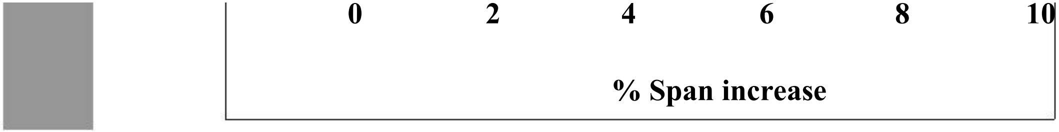
Increase in Arch Thrust



Thrust if hinge does not move and remains fixed


0
2
4
6
8
10
% Span increase

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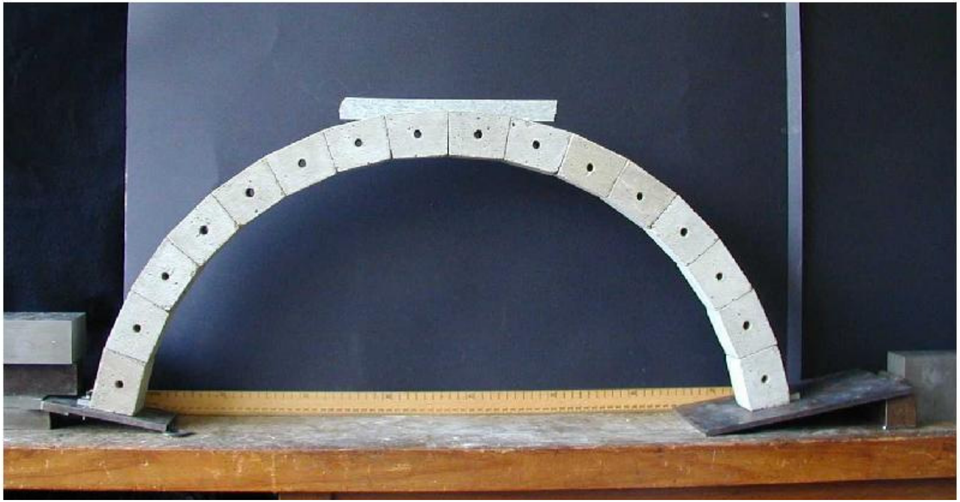
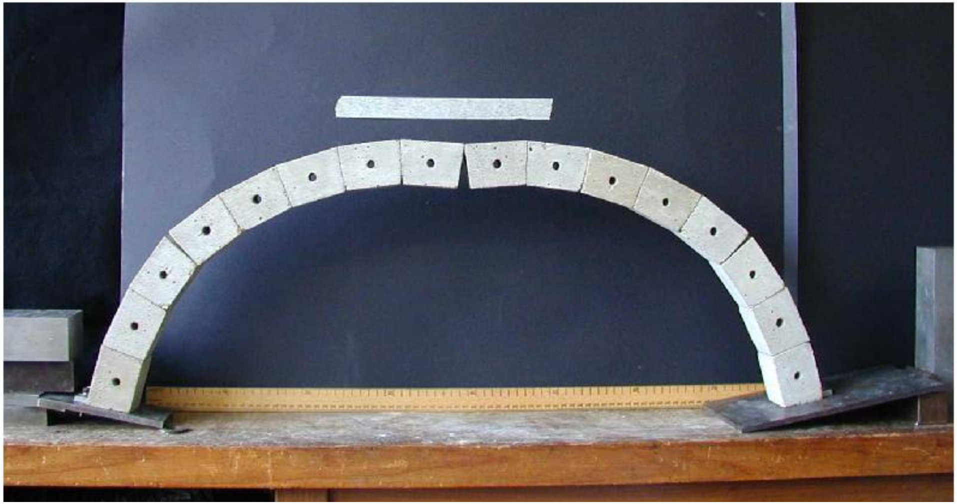
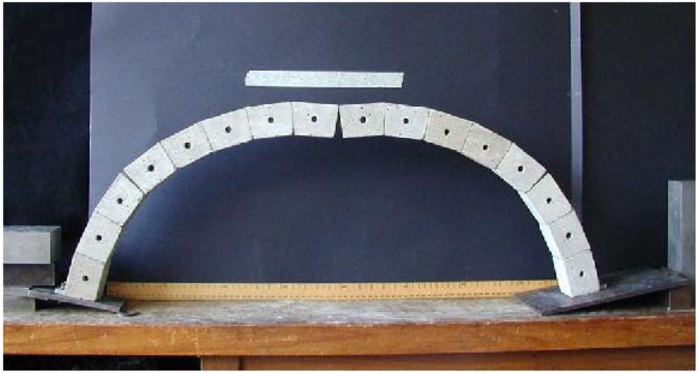
Model Arch Experiment



Model Arch Experiment



Arch on Spreading Supports



For a given span increase, multiple equilibrium **states are possible (with different hinges).**

The collapse state can only be found by **beginning from an assumed equilibrium state.**

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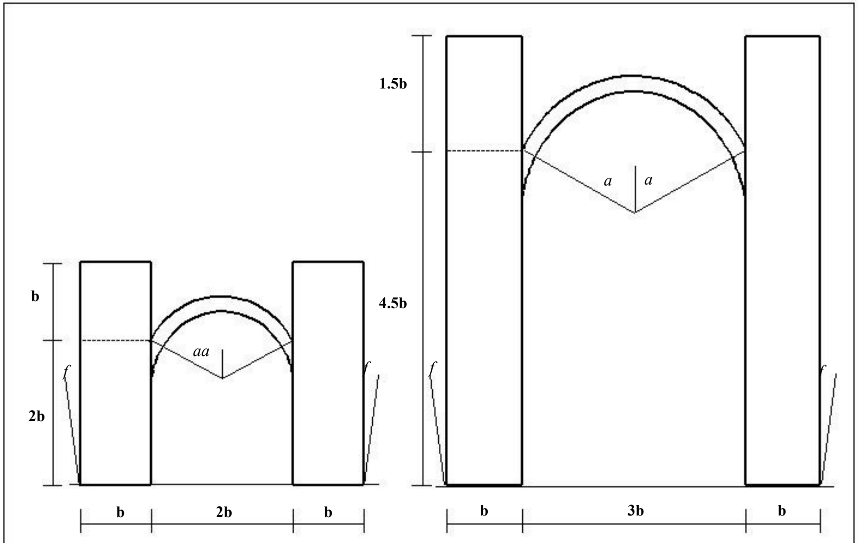
Arch on Buttresses: Case A and B



a) Buttressed arch Case A

b) Buttressed arch Case B

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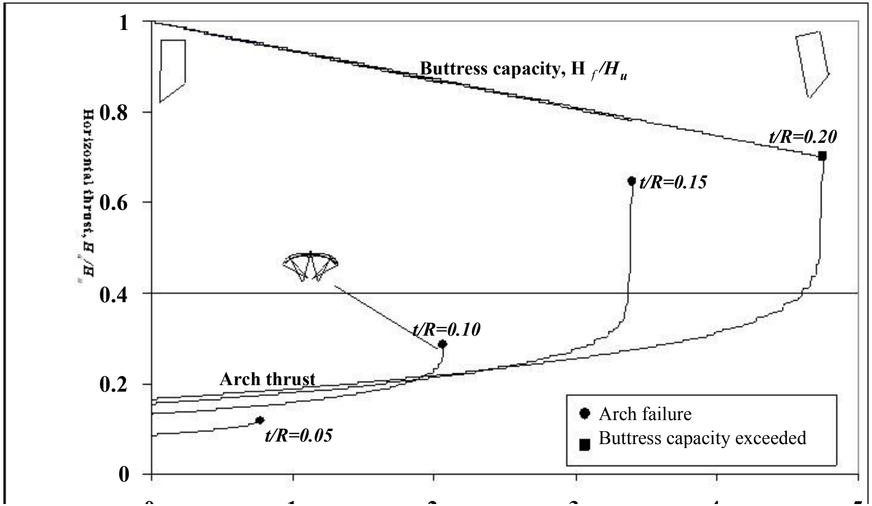
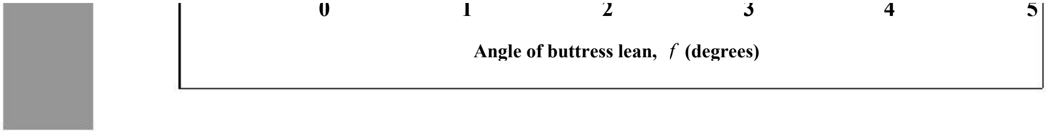
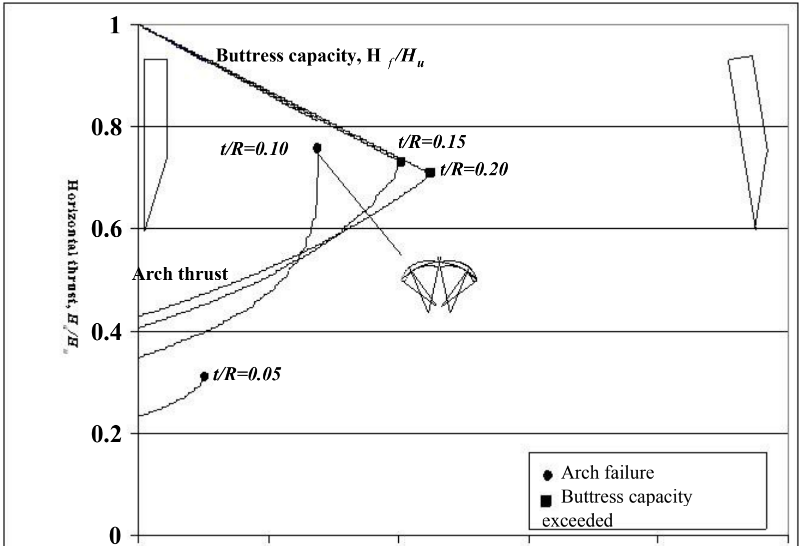
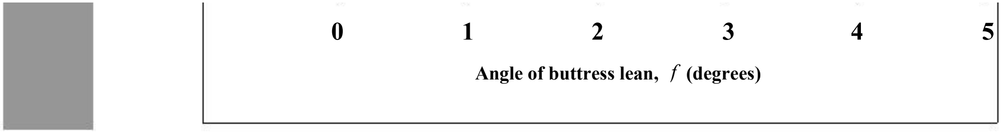
Case A at Collapse




0
1
2
3
4
5
Angle of buttress lean, f (degrees)

Case B at Collapse




0
1
2
3
4
5
Angle of buttress lean, f (degrees)

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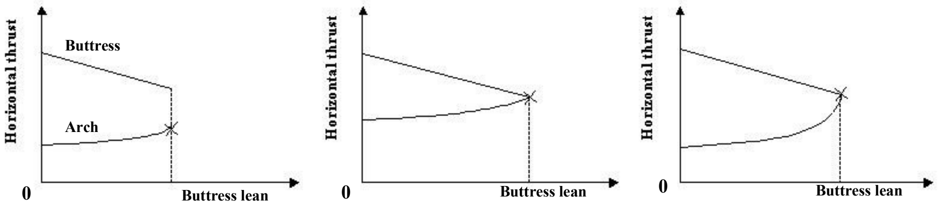
Collapse of Arch on Leaning Buttresses

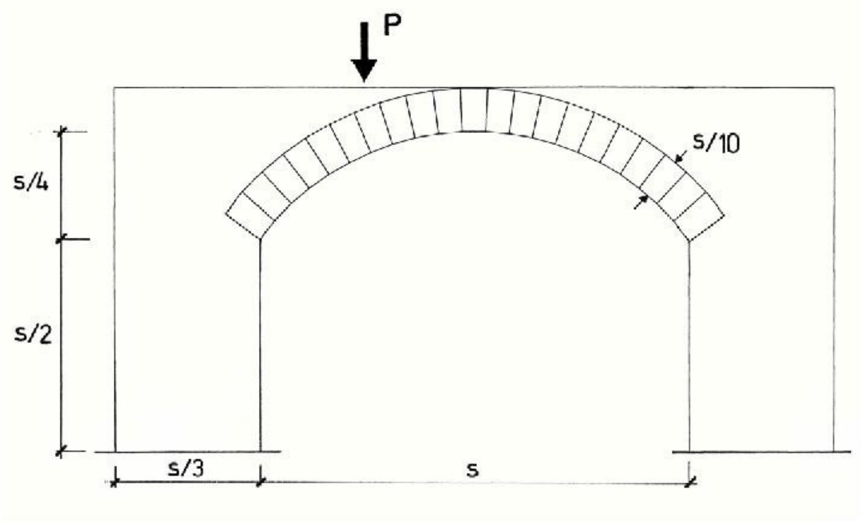


(a) Strong-buttress failure (b) Weak-buttress failure (c) Intermediate failure

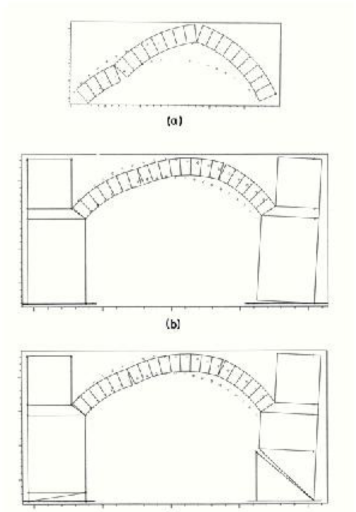
Three types of failure for an arch supported on leaning buttresses. The horizontal axis presents the inclination of the buttress and the vertical axis presents the change in horizontal thrust as the buttresses lean outwards. The arch collapse state is marked by "x".

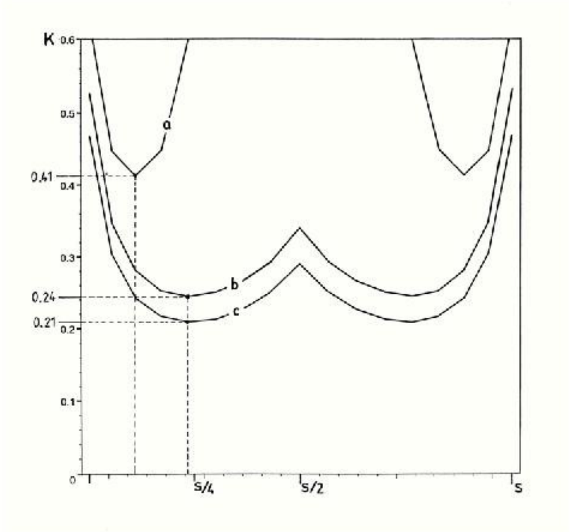
In all cases, the arch collapses and the buttresses remain standing.

Bridge Collapse Under Point Load



Bridge Collapse: Influence of Buttress Fracture





(c)

Problems with Elastic Analysis

- Tells us nothing about collapse state
- Stresses are low, so material failure is not a concern
- Actual displacements are an order of magnitude greater than elastic displacements
- Problem of stability not strength


(e)
(f)

Discussion

Masonry analysis

HW #1

Final projects

Future Challenges

Assessment of structures is growing in importance and guidelines are required

New analysis tools are necessary (elastic FEM are not appropriate)

Static problems are not solved, and dynamic problems present a greater challenge

Specific Problems

What is the true thrust of an arch.

What is the load capacity of a buttress.

How to assess the safety of an ancient structure.

What is the influence of displacements on safety.

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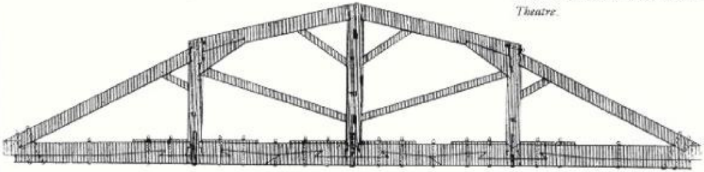
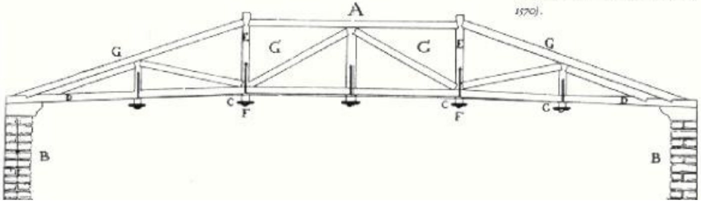
Historic Timber Structures



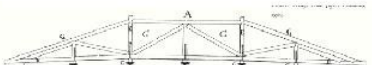
5.5
Wren's roof truss for the Sheldonian Theatre.



5.6
Timber bridge truss (after Palladio, 1570).

Todays Lecture

1. History of Timber Structures
2. Potential Paper Topics
3. Properties of Timber



4. Case study

Historical Development of Timber Structures

- Roman theatres
- Gothic roof systems
- 16th C bridges Palladio
- 17th C roof trusses Wren
- 18th C bridges Grubenmann
- 19th C bridges USA

Roman Timber Structures

Trajans column details of a trussed arch bridge

Roman Roof at Orange (France today)

Timber cantilevers supported a lightweight roof

Spanned greater than 60 feet (20m)

Research questions.

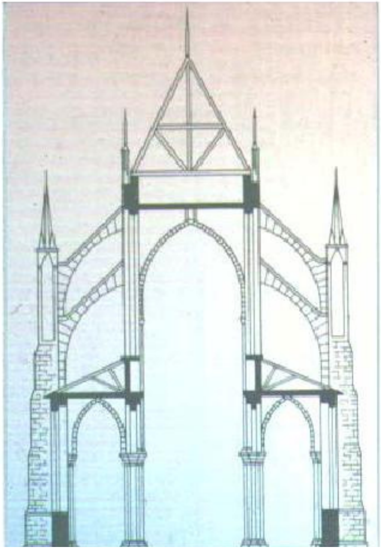
- Support conditions
- Size of timbers
- Geometry of timber trusses

Gothic Roof systems

Timber roof systems span above the vaults

Typical spans of 30-60 ft (10-20 m)

May have been built prior to the vaults to protect and



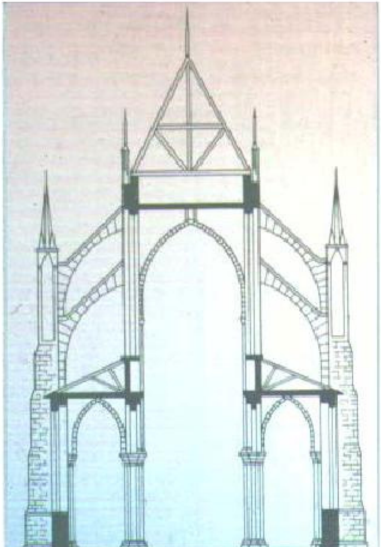
aid the works

Gothic Roof systems

Paper topics

Comparison of timber roof **systems for Gothic cathedrals**

Analysis of various **geometries for roofs**



Hammer-BeamRoof system s

Typical in England

Case study next week

Used to help span longer **distances**

Limit to span for a single beam

Diameter of trees

Length

of
Consistency of materials
elements

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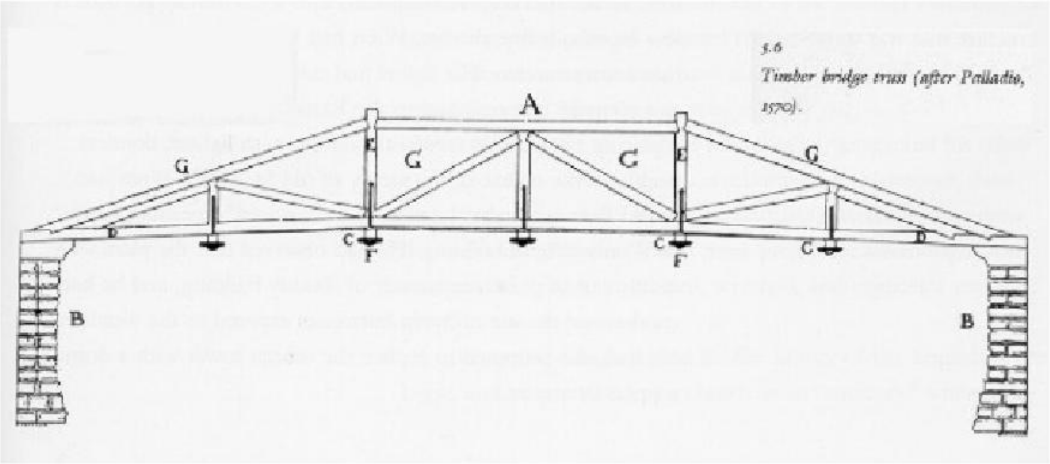
Palladio
Timber Truss Bridge, 1570



5.6

Timber bridge truss (after Palladio, 1570).

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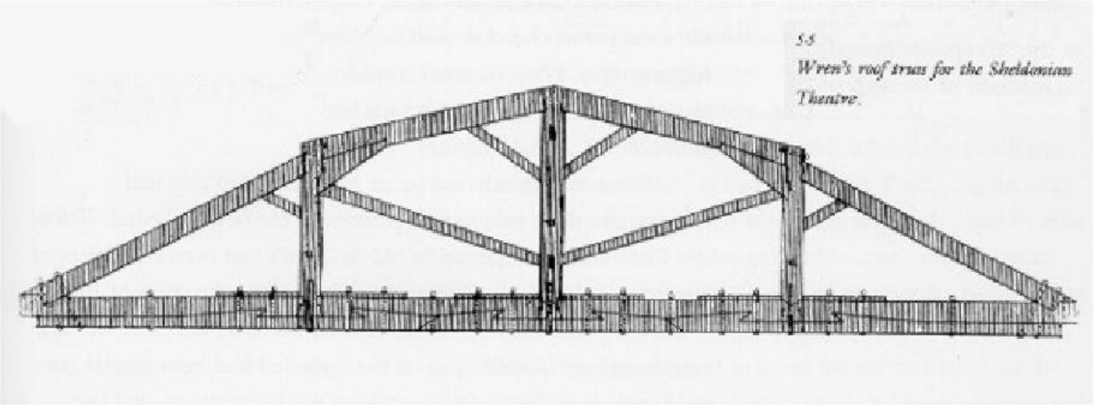
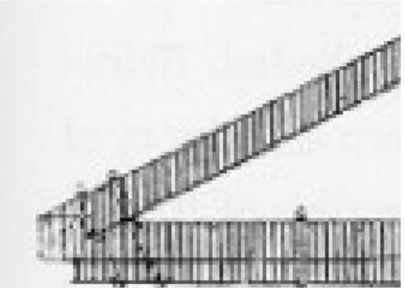
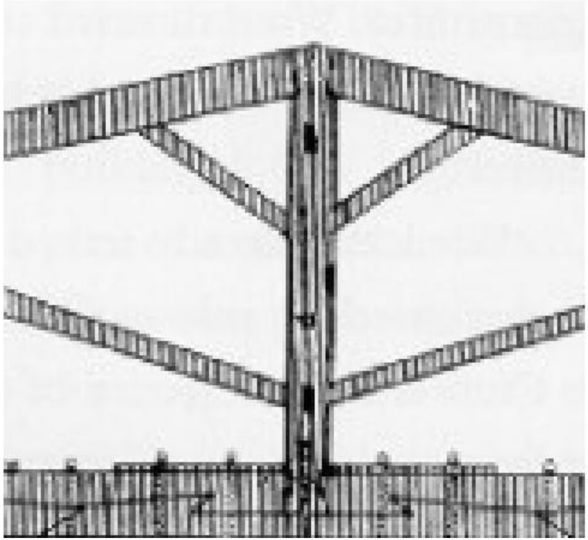
Sheldonian Theatre, Oxford
Christopher Wren, 1669



5-5
Wren's roof truss for the Sheldonian Theatre.

Connection Details





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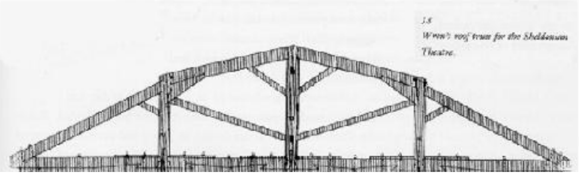
Sheldonian Theatre, Oxford
Christopher Wren, 1669

Paper topic:

Comparison of Wren trusses

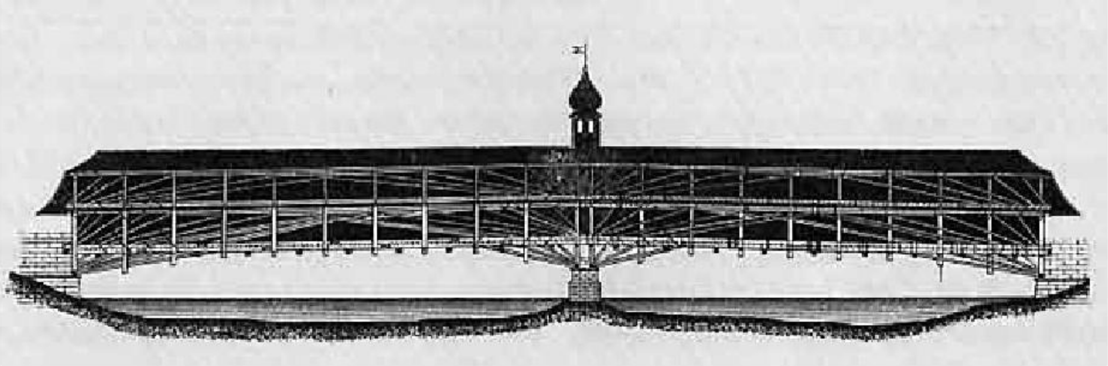
How much did he understand.

How efficient are the truss designs.

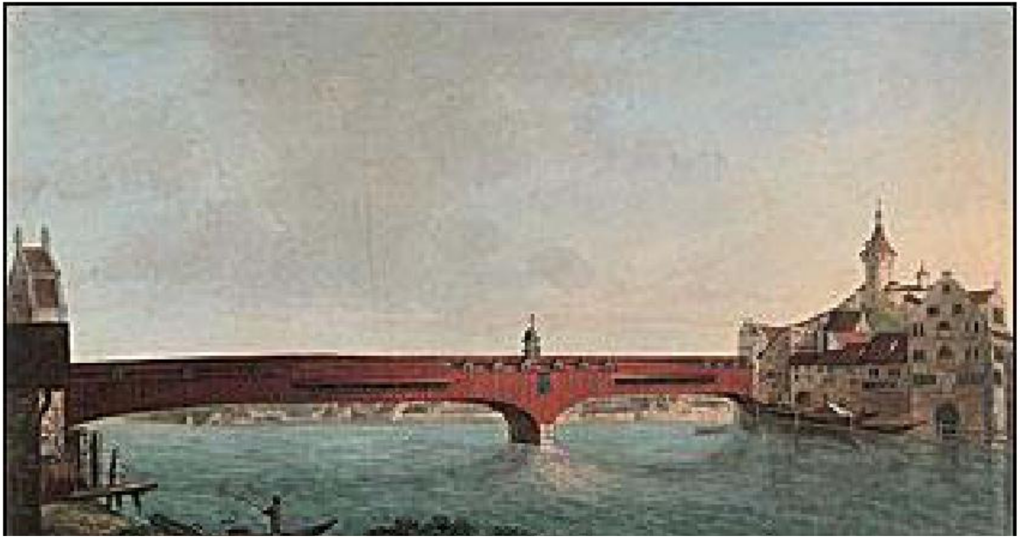


18
Wren's roof truss for the Sheldonian Theatre.

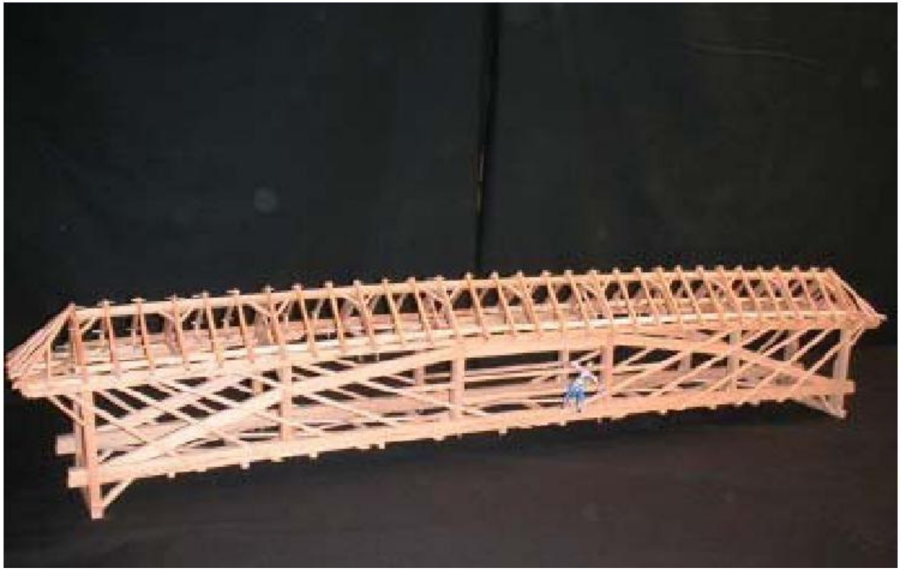
18th C covered bridges in Switzerland



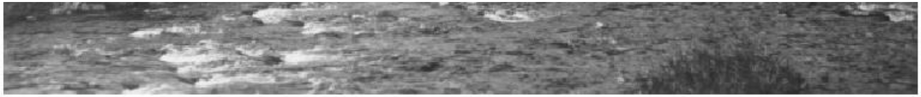
Schaffhausen Bridge, 1755



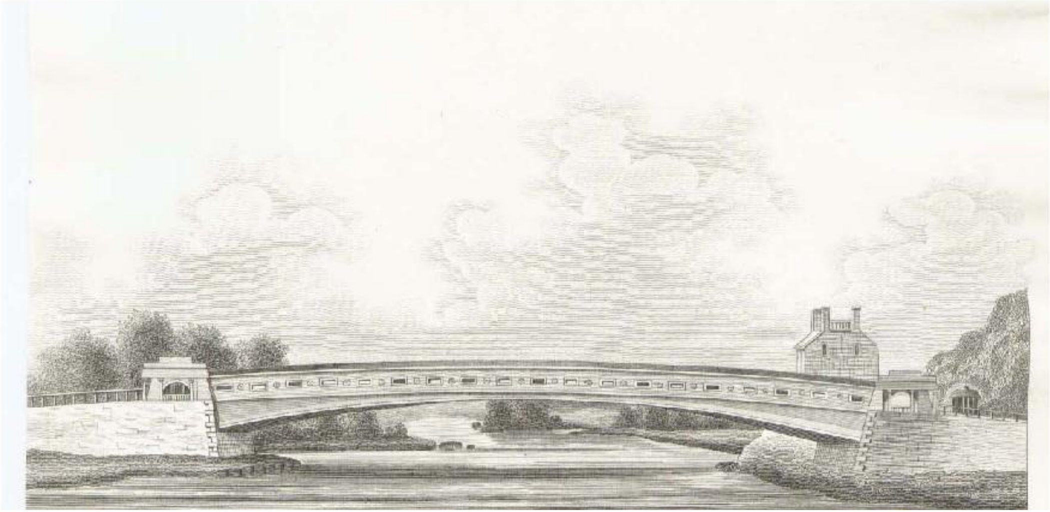
Schaffhausen Bridge, 1755

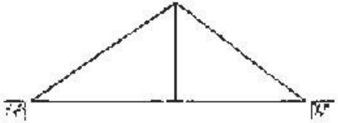


Craft traditions of timber bridges

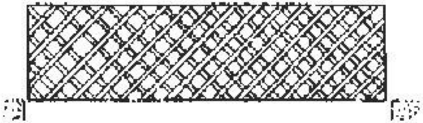


Colossus over Schuylkill River in Philadelphia, 1812, 340 ft span

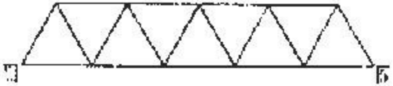




King Post Truss

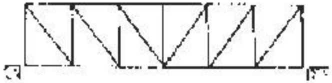


Lattice Truss



James Warren Truss

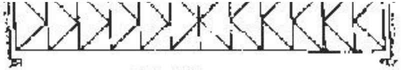
Squire Whipple Truss



Pratt Truss

William Howe Truss

Albert Fink Truss



K-Truss

US Covered Bridges

US Covered Bridges

Taftsville Bridge in NH, 1836

US Covered Bridges

Bamboo Suspension Bridges

From Himalaya and China

Spans of 600 feet (200 m)

Longest spans in the world

Barely studied at all
great paper topic!

Inca Woven Bridge Construction: An Annual Festival

Day 1: Ropes made from local grass or plant fibers

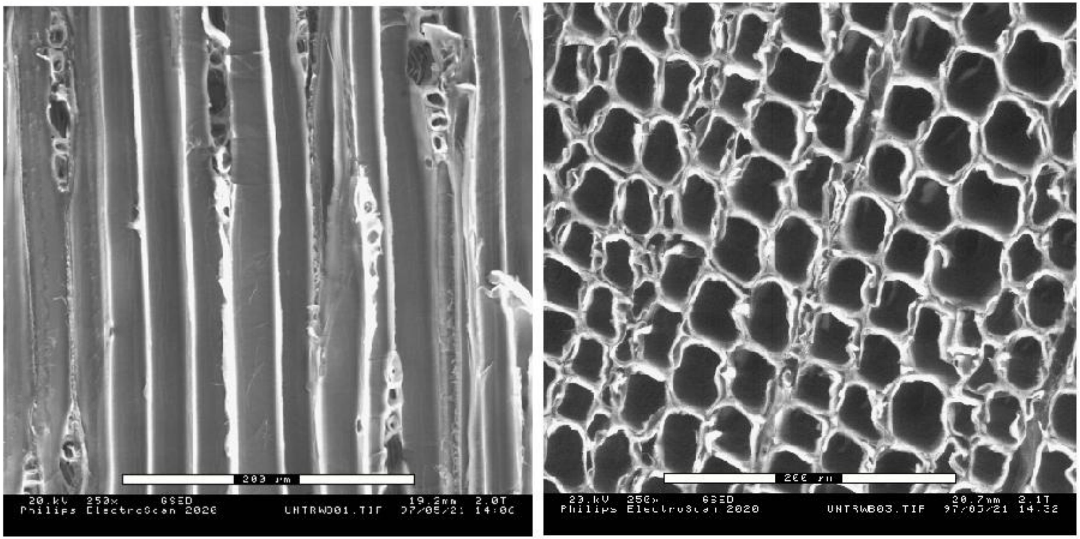
Day 2: Old bridge is cut and new ropes are installed

Day 3: Roadway and handrails are added and bridge is complete

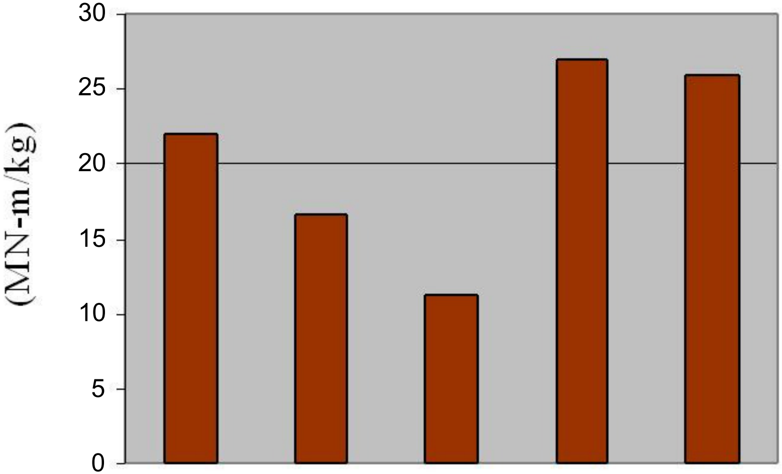
Rebuilt ever

▨	**year for 500 years**

Microstructure of Wood



Stiffness (E) per unit weight



Wood

Brick

[Compression only]

Concrete

Steel

Aluminum

Source: Biggs (1991)

Properties of Timber

- Cellular structure is very efficient
- Handles both compression and tension well
- Different strengths with and against the grain
- Inhomogeneous material with imperfections

Enemies of Timber

Fire

Water

Insects

Conclusions

The distanced spanned by wood is limited by the size of trees

Trusses allow for longer spans

Many subjects of historic timber construction have not been studied

- **Apply simple truss analysis in most cases**

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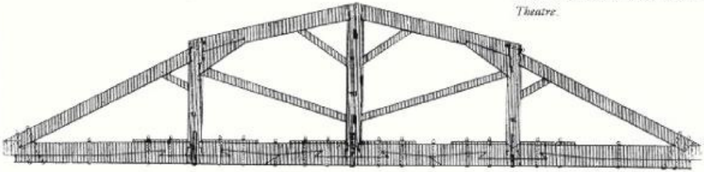
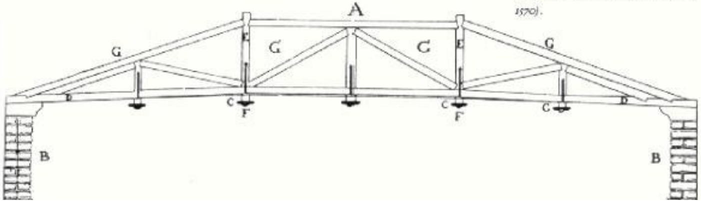
Historic Timber Structures II



5.5
Wren's roof truss for the Sheldanian Theatre.



5.6
Timber bridge truss (after Palladio, 1570).

Todays Lecture

1. **Assumptions for Analysis of Timber**
2. **Static Indeterminacy**
3. **Two case studies**
   - **-Wooden stool**
   - **-Hammerbeam roof**

4. Conclusions

Historical Development of Timber Structures

Roman theatres

Gothic roof systems

16th C bridges Palladio

17th C roof trusses Wren

18th C bridges Grubenmann

19th C bridges USA

Analysis of Timber Structures

Static equilibrium is the guiding principle (stresses are low)

Assumptions greatly influence the results (joints and supports)

Statically determinate or indeterminate structures behave in fundamentally different ways. Be clear about which type of structure you are dealing with.

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Forces in the Legs of a Stool

Three-Legged Stool

Statically determinate

One solution for the axial force in each leg

Why.

- 3 unknowns
- 3 equations of equilibrium

Uneven

floor has no effect

Four-Legged Stool

Statically indeterminate

A four legged table on an uneven surface will rock back and forth

Why.

It is hyperstatic:

4 unknowns

3 equations of equilibrium

(or statically indeterminate)

Four-Legged Stool

Infinite solutions exist

Depends on unknowable support conditions

A four legged table on an uneven surface will rock back and forth

The forces in each leg are constantly changing

Fundamental difference between hyperstatic (indeterminate) and static structures

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Forces in the Leg of a Stool

Statically

Statically Indeterminate

determinate

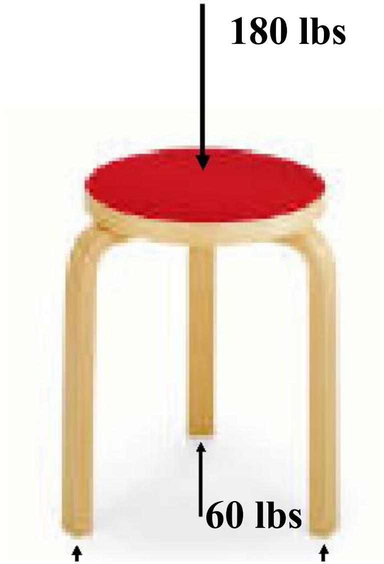
indeterminate (hyperstatic)

Three-Legged Stool

Design for a person weighing 180 pounds

60 pounds/leg

Regardless of uneven floor




60 lbs
60 lbs

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Collapse of a Three-Legged Stool

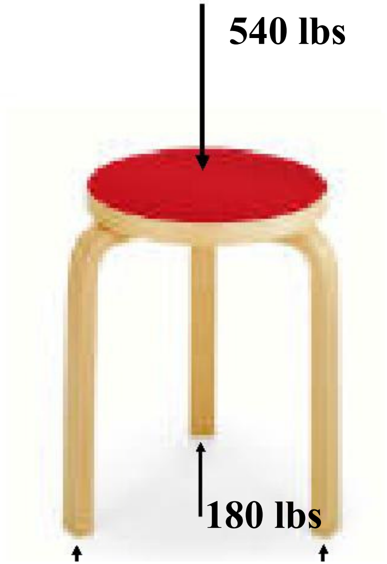
Design for a person weighing 180 pounds

If the safety factor is 3:

$P_{cr} = 3(60) = 180$ lbs

And each leg would be designed to fail at a load of 180 pounds

The stool would carry a




total load
of 540
pounds

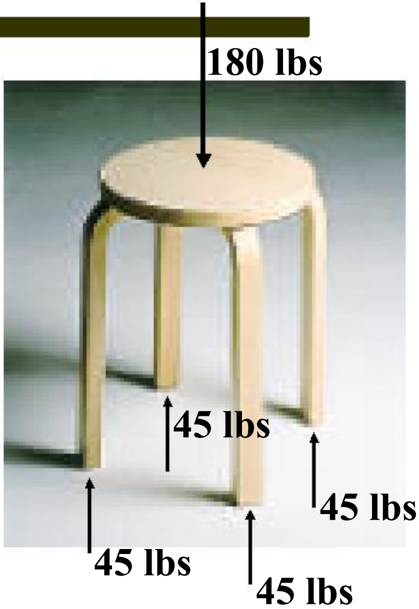

180 lbs
180 lbs

Elastic Solution for 4-Legged Stool

Design for a person weighing 180 pounds

45 pounds/leg

But if one leg does not touch the floor



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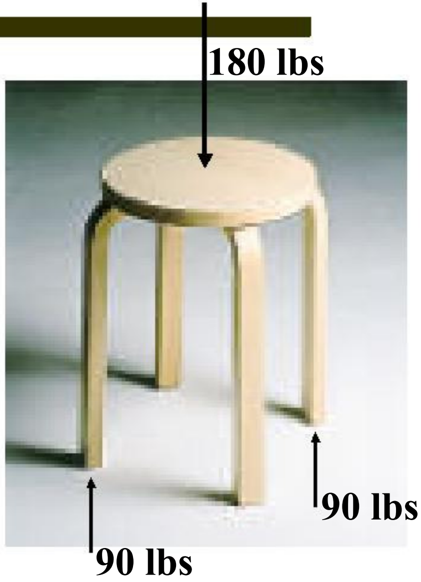
Four-Legged Stool

If one leg doesnt touch the floor, the force in it is zero.

If one leg is zero, then the opposite leg is also zero by moment equilibrium.

The two remaining legs carry all of the load:



90 pounds/leg

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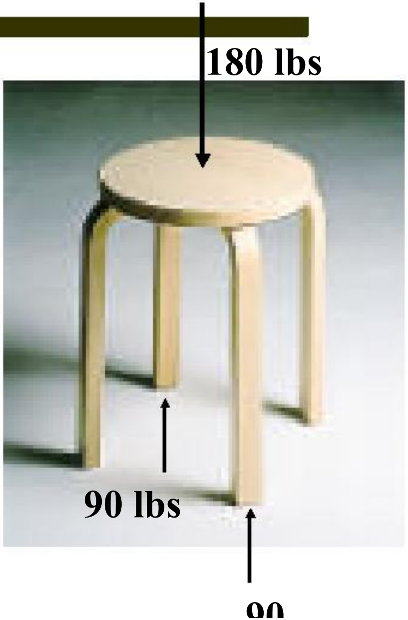
Four-Legged Stool



Therefore

All four legs must be *designed to carry the 90* pounds (since any two legs could be loaded)

90
lbs

Four-Legged Stool

If the elastic solution is accepted, with a load in each leg of 45 pounds, then assuming a safety factor of 3 gives:

P_{cr} = 3(45 lbs) = <u>135 lbs</u>

And each leg would be designed to fail

designed to fall

at a load of

135 pounds

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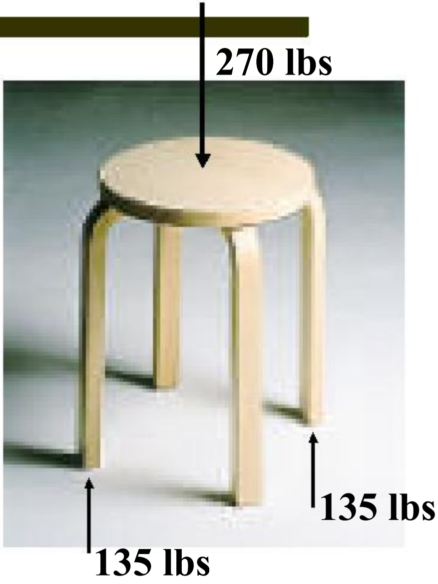
Four-Legged Stool



Now imagine the load is increased to cause failure

When load is 270 lbs, the two legs will begin to fail

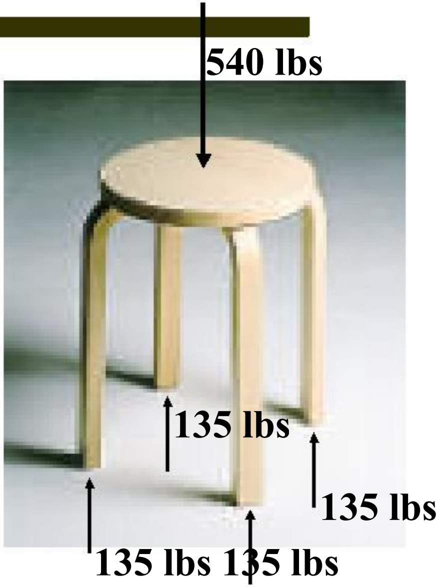
As they squash, the other two legs will start to carry

carry
load
also

Collapse of a 4-Legged Stool

At final collapse state, all four legs carry 135 pounds and the stool carries 540 pounds.

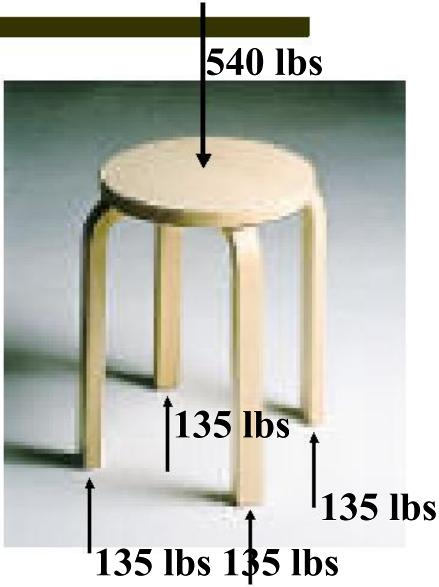
This occurs only if the structure is ductile (ie, if the legs can squash)



Ductile Collapse

So small imperfections do not matter, as long as the structural elements are ductile

The forces in a hyperstatic structure cannot be known exactly, and the solutions depend on the assumptions for the supports



Internal forces are unknowable
(only the structure knows)

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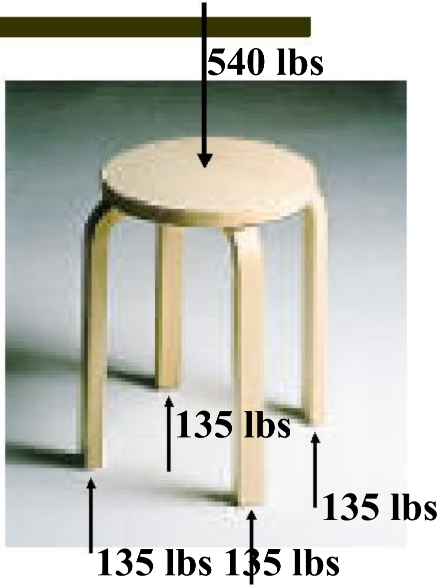
Lower Bound Theorem of Plasticity



If you can find one possible set of forces, then the structure can find a possible set of forces

It does not have to be correct, as long as the structure has capacity for displacements (ductility)

For indeterminate structures, we

cannot be certain of the internal state of the forces

Examples of Statically Determinate Structures

Unstressed by support movements or temperature changes

Three-legged stool

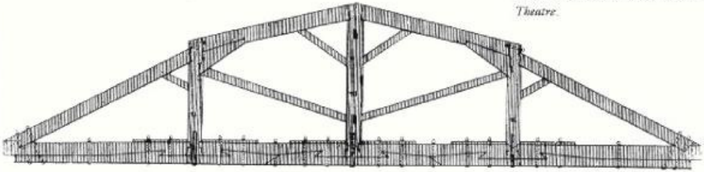
Simply supported beam

Cantilever beam

Three-hinged arch

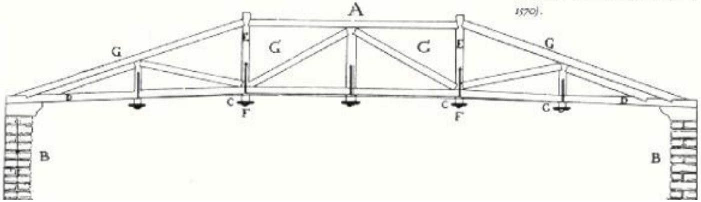
Triangulated truss

Determinate or indeterminate.



5.5

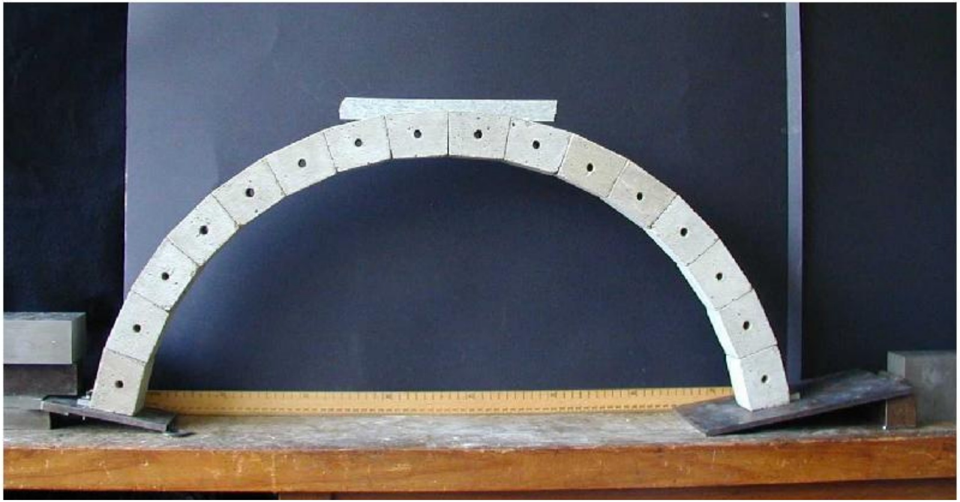
Wren's roof truss for the Sheldonian Theatre.



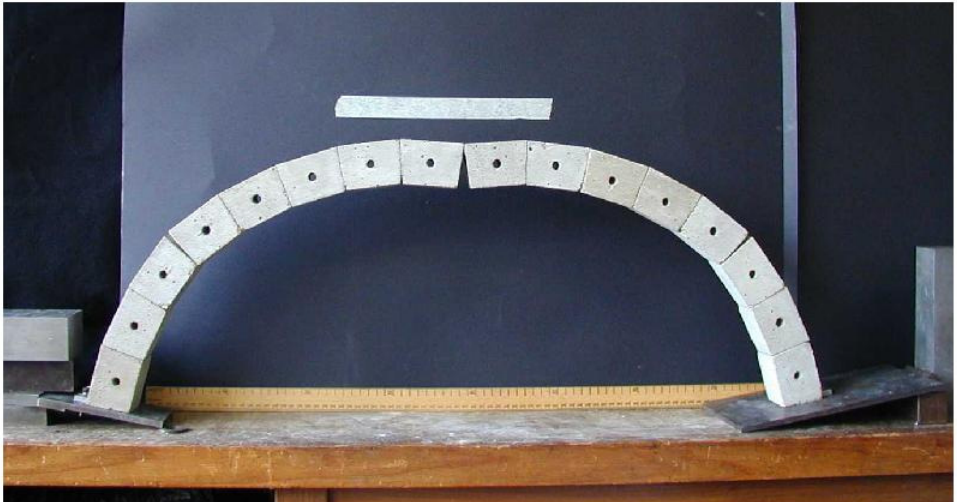
5.6

Timber bridge truss (after Palladio, 1570).

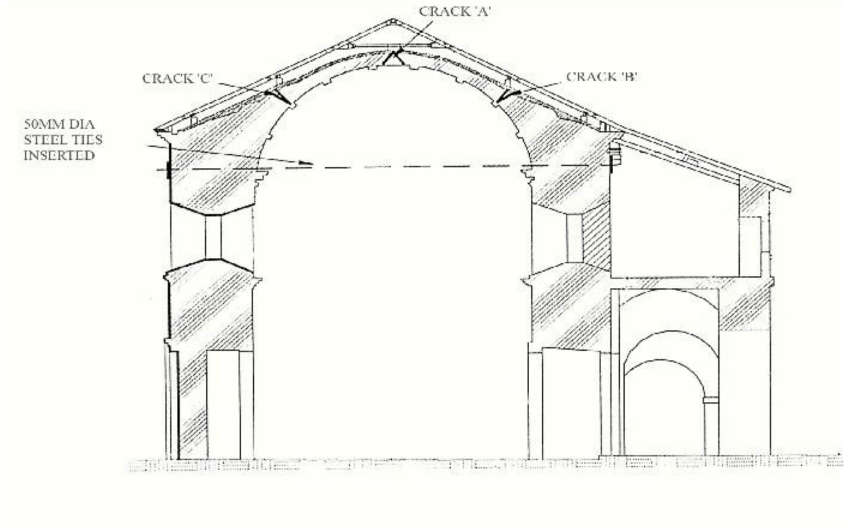
Model Arch Experiment



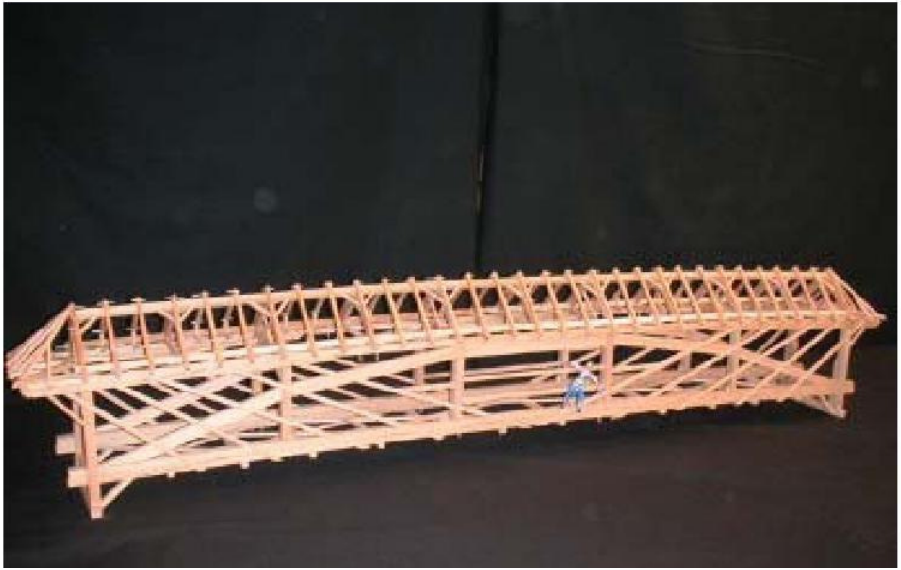
Model Arch Experiment



Case Study:
16th C. Church in Goa, India



Determinate or indeterminate.



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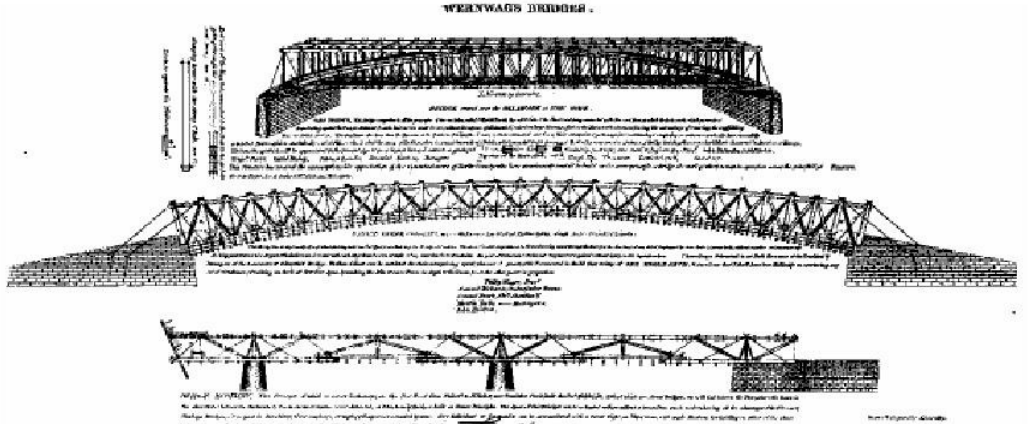
Colossus over Schuylkill River in Philadelphia



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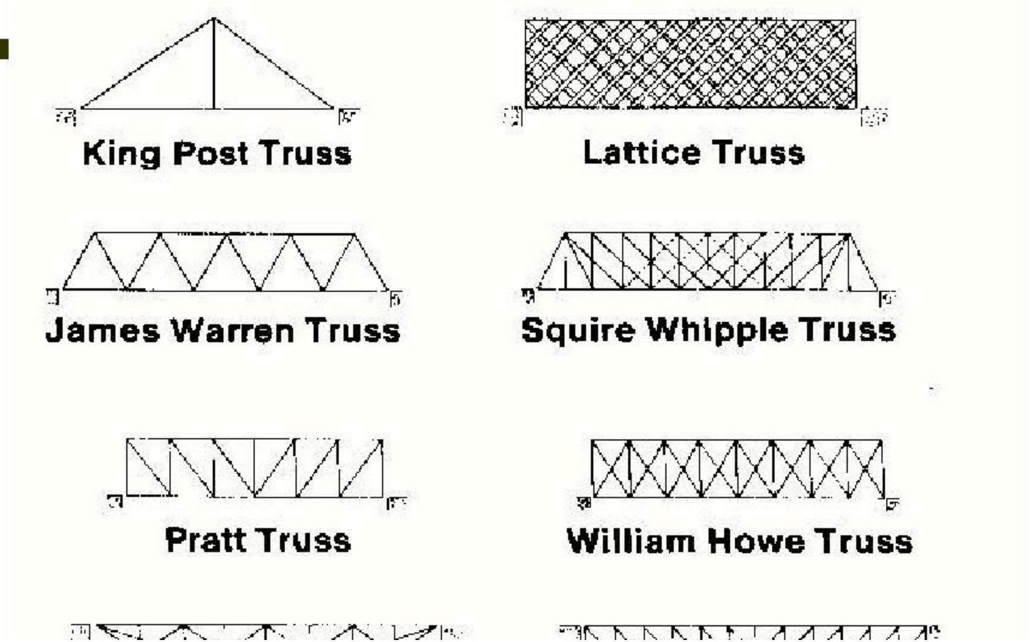
Determinate or indeterminate.




Albert Fink Truss
K-Truss

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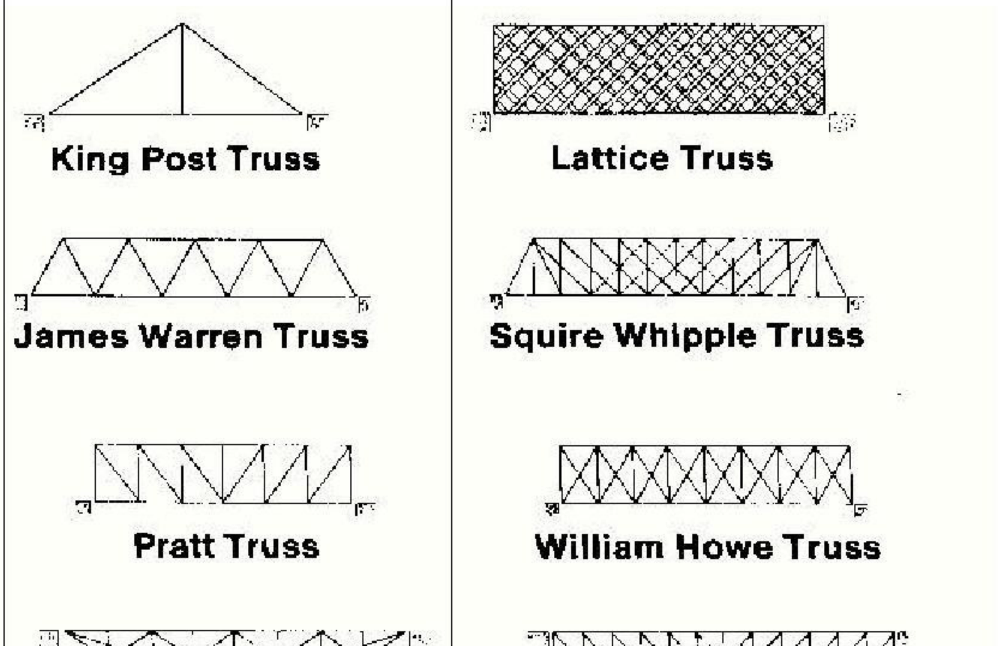
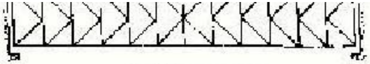
Statically determinate

Statically indeterminate



Albert Fink Truss



K-Truss

Hammer-BeamRoof system s

- **Typical in England**
  - Case study of Westminster **Hall**
  - Used to help span longer **distances**
- **Limit to span for a single beam**
  - Diameter of trees

Length of elements
Consistency of materials

Conclusions

Like traditional masonry structures

Timber has low stresses

Most are statically indeterminate. There is not **one answer for internal forces; depends on supports and assumptions.**

For indeterminate structures, you must explore various possibilities (support conditions are most important)

Equilibrium is the bedrock of our analysis

Analysis of Timber Structures

Static equilibrium is the guiding principle (stresses are low)

Assumptions greatly influence the results (joints and supports)

Statically determinate or indeterminate structures behave in fundamentally different ways. Be clear about which type of structure you are dealing with.

Conclusions

The distanced spanned by wood is limited by the size of trees

Trusses allow for longer spans

Many subjects of historic timber construction have not been studied

Apply simple truss analysis in most cases

Metallic Structures

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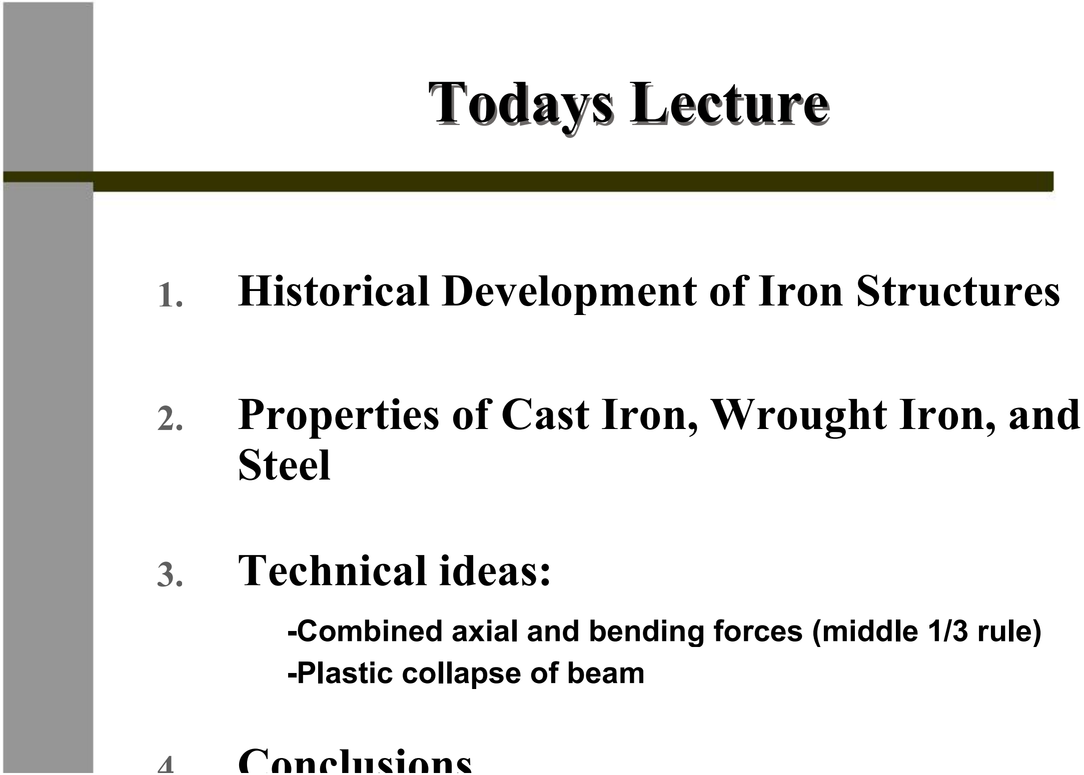


Todays Lecture

1. **Historical Development of Iron Structures**

2. **Properties of Cast Iron, Wrought Iron, and Steel**

3. **Technical ideas:**
    - **Combined axial and bending forces (middle 1/3 rule)**
    - **Plastic collapse of beam**

4 **Conclusions**

4. Conclusions

Metal in Construction

2000 yrs ago: Iron chain bridges in China

Ancient Greece/Tiwanaku: lead/copper **clamps**

Middle Ages: Iron ties in masonry buildings

1779: Iron Bridge at Coalbrookdale

19 th C: Age of iron construction, beginning of steel structures

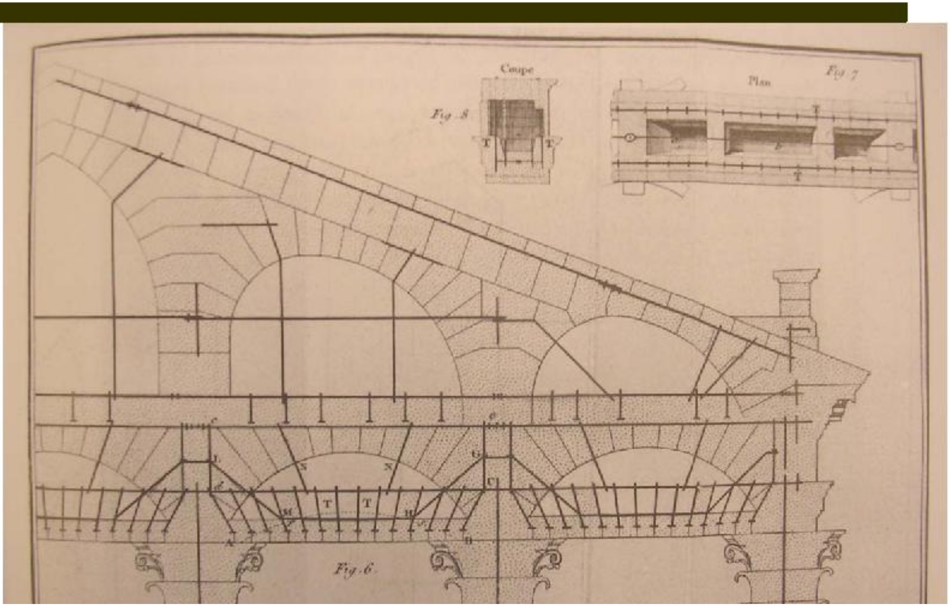
Iron Chain Suspension Bridges in China

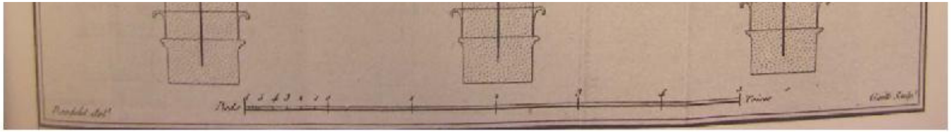
From at least 3rd C AD and
possibly from 3rd C BC

Wrought iron links.

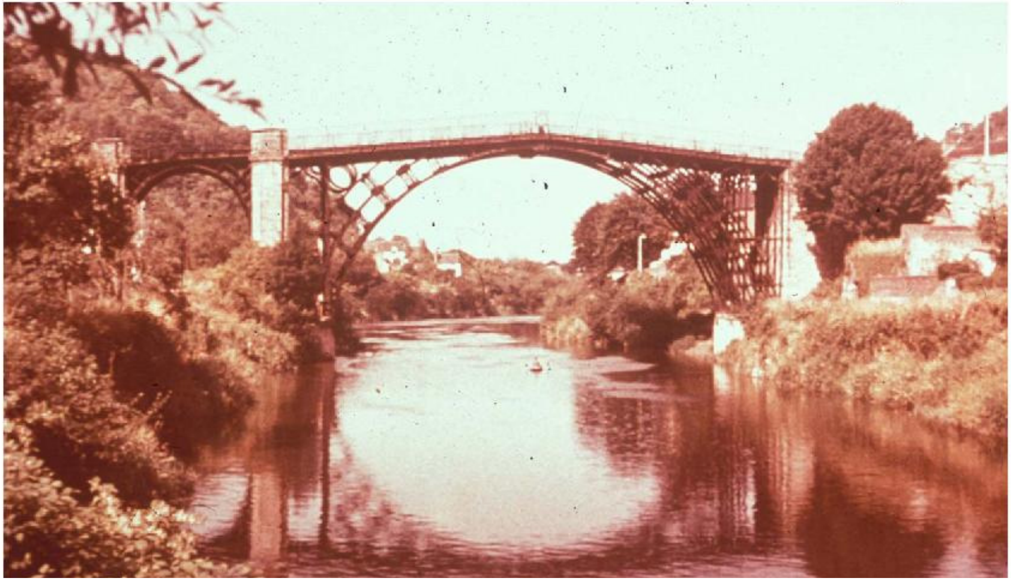
7th C bridge with oxen abutments

French Pantheon, ~1770, Iron reinforcing




Rondelet del.
Pieds
Toises

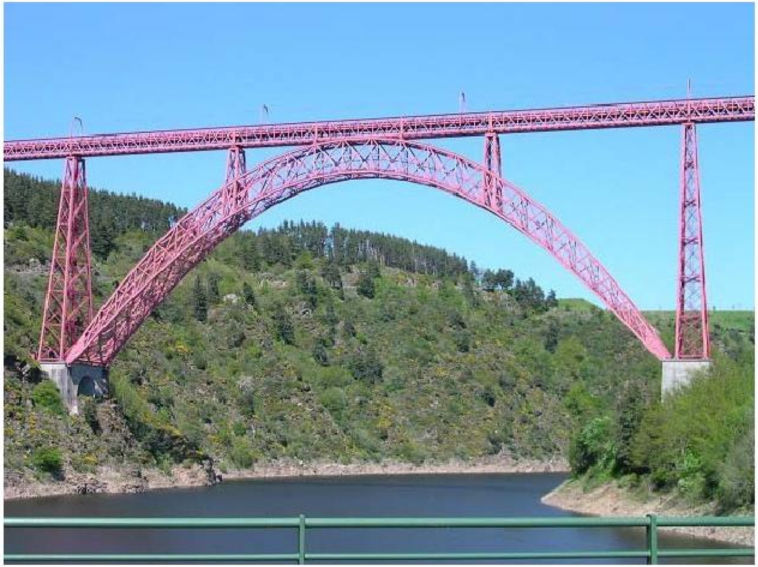
Iron Bridge at Coalbrookdale, 1779



England/Wales, 1779

Iron Bridge Details

Arch Bridge by Gustave Eiffel



Garabit Viaduct, 1884, France

Image courtesy of Erich Baumgartner, structurae.de

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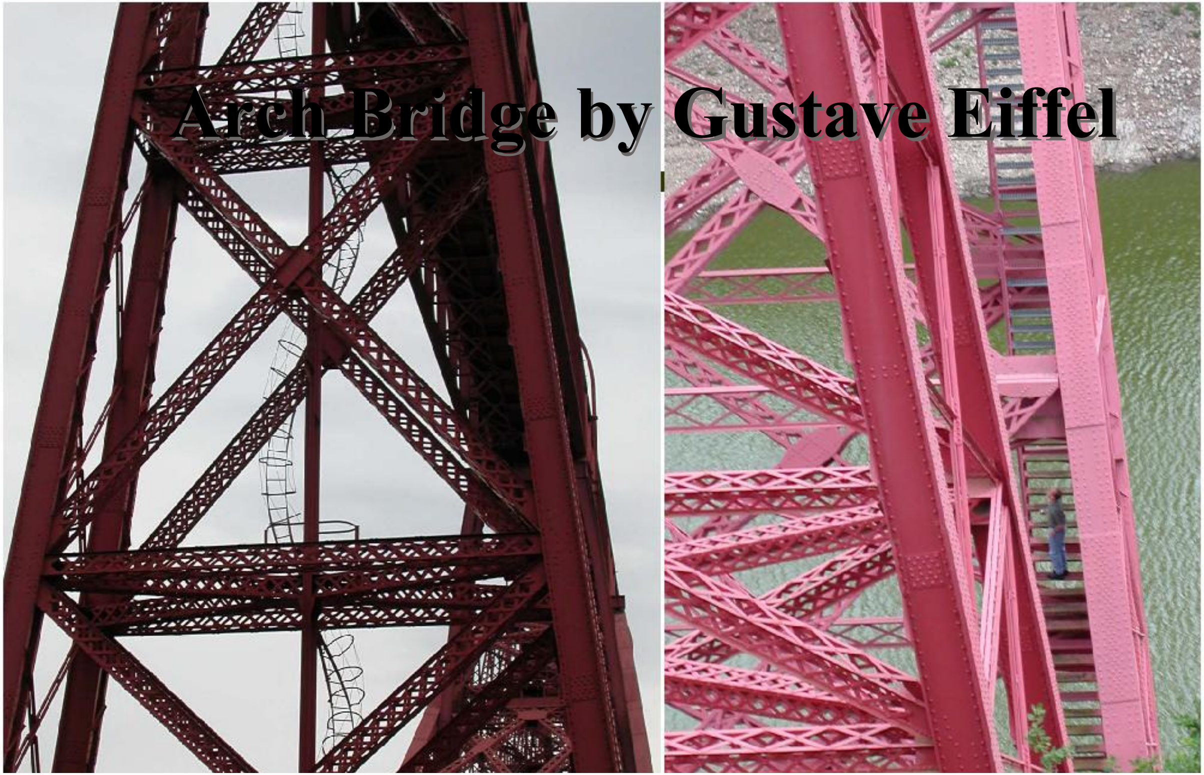
Arch Bridge by Gustave Eiffel



Garabit Viaduct, 1884, France

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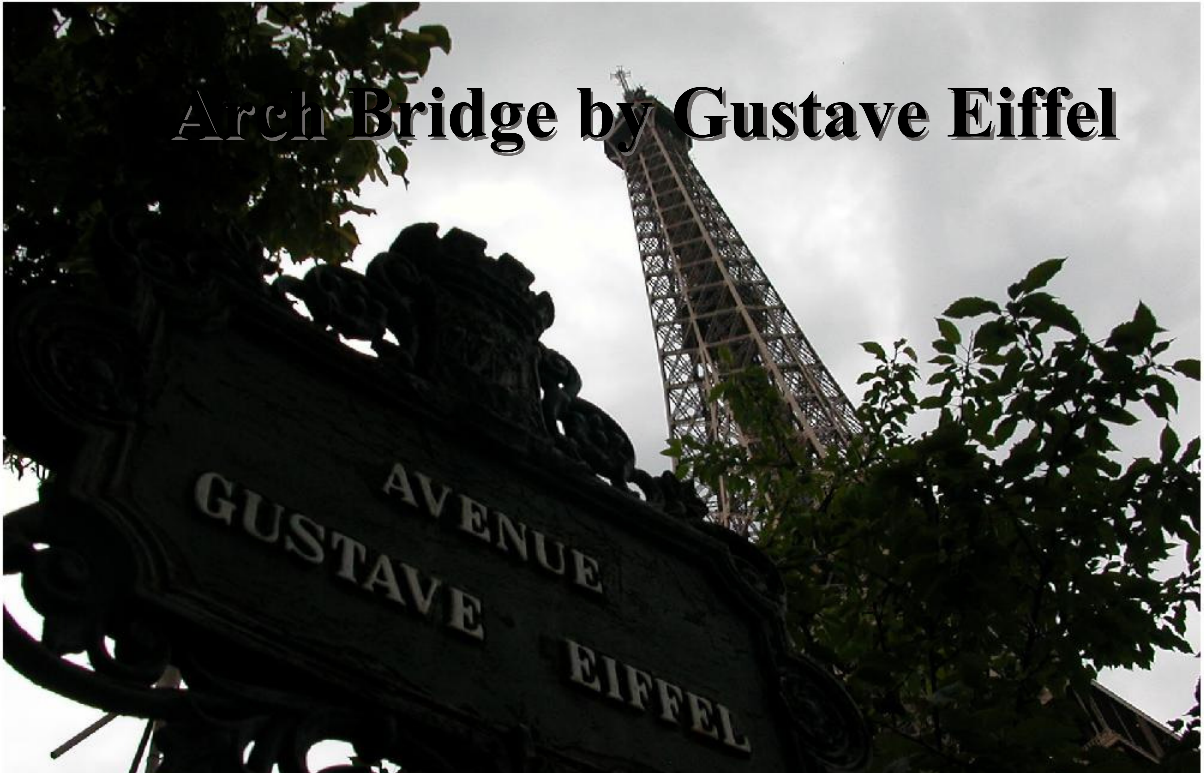
Arch Bridge by Gustave Eiffel



Eiffel Tower, 1889, wrought iron

Cast Iron, Wrought Iron, and Steel

Cast iron: high compression, lower tensile **strength, brittle, like masonry**

Wrought iron: tension and compression, **weaker than steel, like timber**

Steel: post-1850s, tension and compression, **problems with corrosion**

- All are iron + carbon + other impurities

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Cast Iron: 95-98%Iron

Produced commercially after the introduction of cupola blast furnaces in 1794, was smelted at much higher temperatures in the liquid state, and so became saturated with carbon from the furnace fuel, up to about 5%.

It was then poured out (ie cast) into a mold to produce blocks traditionally known as pigs, because the line of individual blocks connected to a channel looked like a litter of suckling pigs, hence the name "pig iron".

The high carbon content makes cast iron very strong in compression, but weak and brittle in tension, even when red hot, so it cannot be forged or rolled

red hot, so it cannot be forged or rolled.

However it also lowers the melting point so it can be easily poured into molds to produce complex shapes.

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Wrought Iron: 99.5-99.9%Iron

Wrought iron was traditionally smelted at a relatively low temperature in the solid state to produce a spongy mass of metal called a bloom, from which the impurities were driven off as liquid slag by hammering, hence the term "wrought" ie "worked" iron.

Wrought iron is very pure, with a carbon content of less than 1%, which makes it resistant to corrosion, strong in tension and malleable. It was much used for gates and railings, and for structural members, but the early methods of production, (the iron cooled quickly and could only be worked in small quantities) limited the size of parts which could be easily made.

Long thin sections were impossible until the development of the rolling mill, which is still used in steel production, during the

eighteenth century. The rollers could be shaped to produce bars of different sections.

Steel: 98.5-99.9%Iron + Carbon and other impurities (Si, Mn, etc)

Iron ore is a common element, which requires processing before it becomes a recognizable metal. It has an ability to combine with other elements and so can occur in a number of forms, but the three major types are wrought iron, cast iron and steel.

Steel is now the most important, but its production dates only from the 1850s. Cast and wrought iron on the other hand are processes which have been known for thousands of years, but it is only comparatively recently that technology advanced to allow widespread production.

The early iron working sites are among our most important industrial monuments. Good cast and wrought iron are much more resistant to corrosion and rusting than steel.

■ **Steel: Up to 1.5% Carbon, plus other impurities that determine properties**

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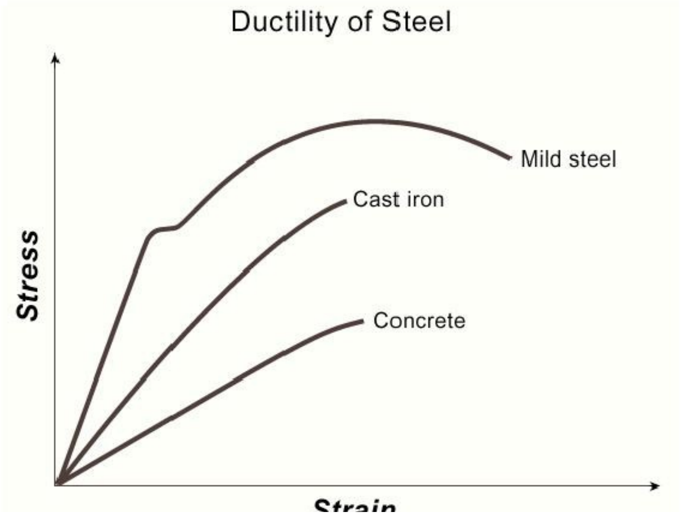
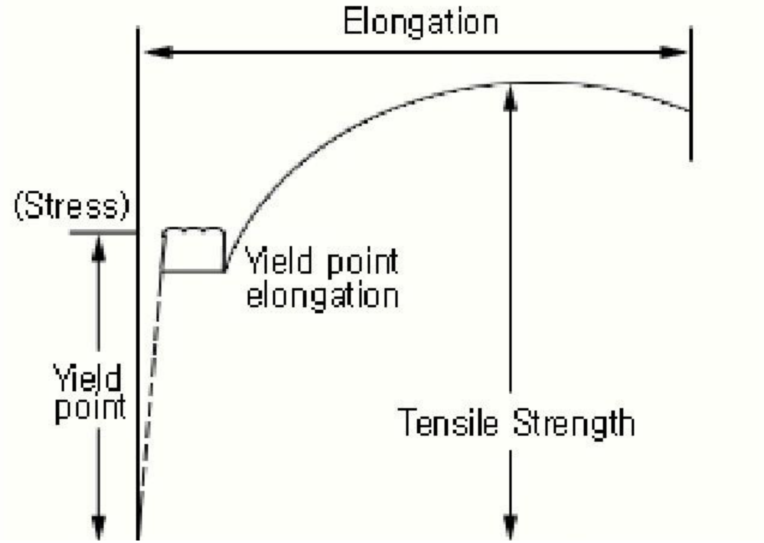
Ductility of Iron and Steel




Strain

Ductility of Steel




c (Strain)

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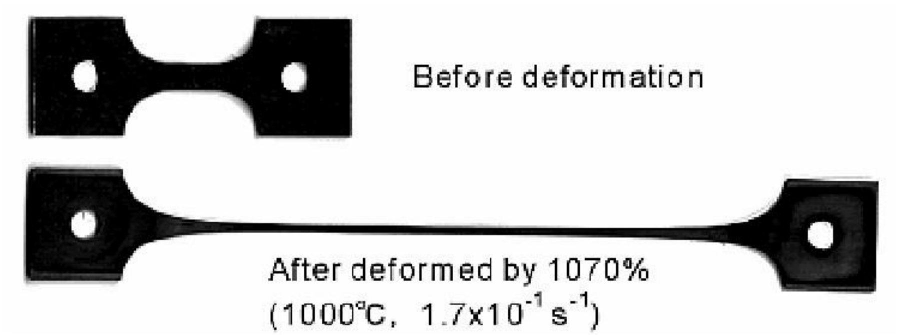
Hi-Ductility Steel



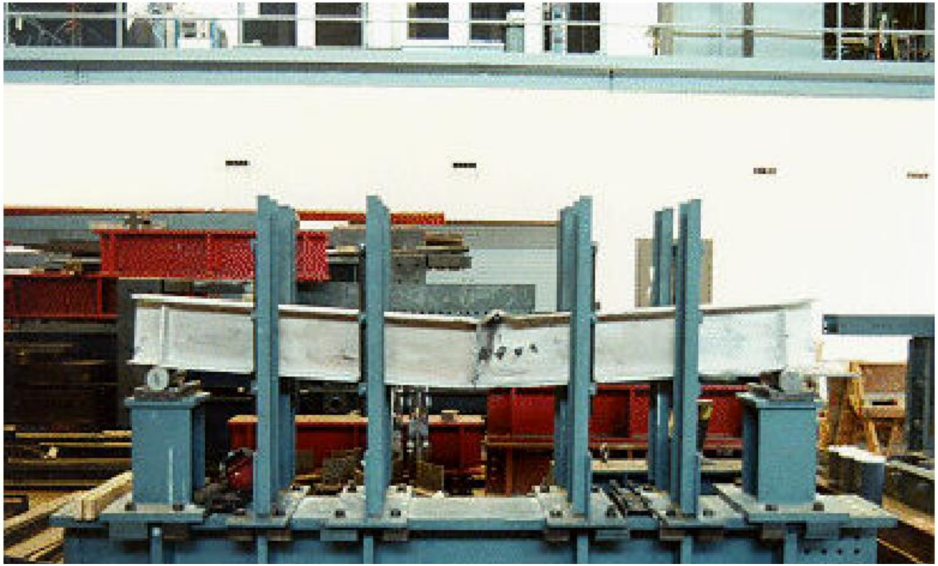
Superplasticity of an Fe-Cr-Ni-Mo stainless steel

Importance of Ductility

Large displacements before collapse (as **opposed to a brittle material, which fails suddenly)**

Energy dissipation as the steel yields **(important for resisting overloading)**

Ductility and Plasticity



Design vs. Assessment of Indeterminate Structures

Design: find one possible set of forces and the structure can find one (Lower bound theorem)

The design values of stresses cannot be observed in practice (only the structure knows)

Assessment: Same is true, but you can look for clues that give you an idea where the forces are (or not), ie cracks,

displacements, etc.

Design vs. Assessment of Indeterminate Structures

Always ask:

WHAT WOULD CAUSE THE COLLAPSE OF THE STRUCTURE.

Use the UPPER BOUND THEOREM to find the collapse load.

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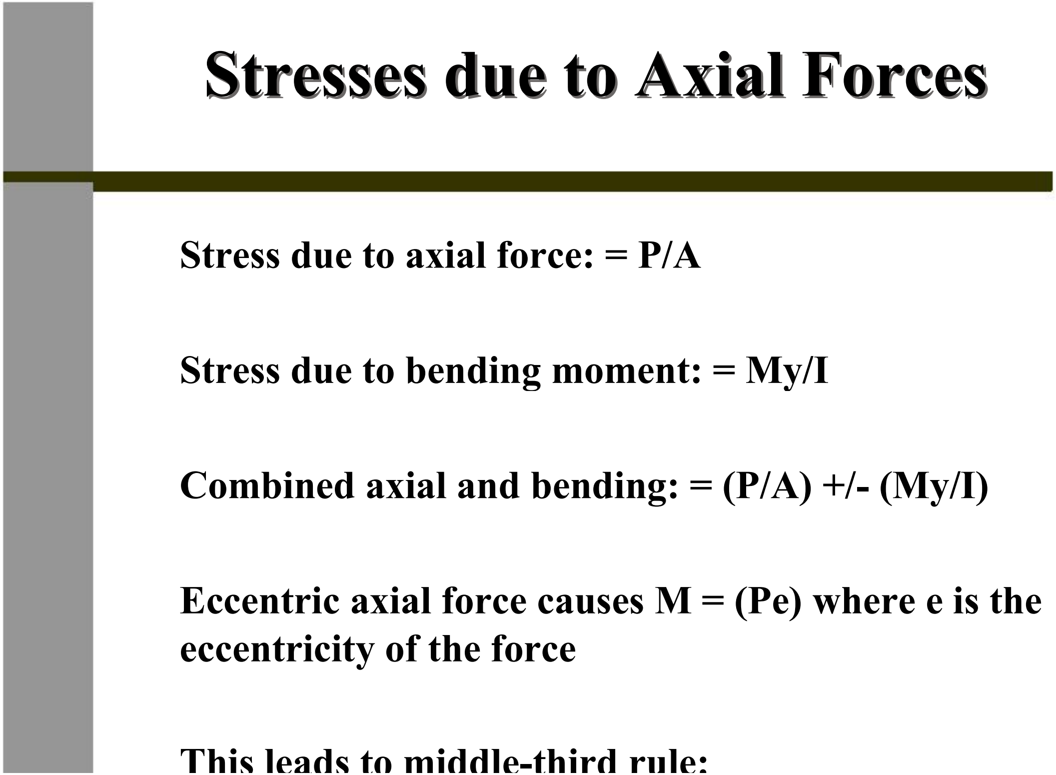

Stresses due to Axial Forces

Stress due to axial force: = P/A

Stress due to bending moment: = My/I

Combined axial and bending: = (P/A) +/- (My/I)

Eccentric axial force causes M = (Pe) where e is the eccentricity of the force

This leads to middle-third rule:

This leads to middle third rule:

eccentricity greater than

1/3 of section causes tension

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Metallic Structures: Conclusions

The type of metal is crucially important

Ductility of metal determines type of analysis

- Brittle (cast iron) can be calculated with thrust lines
- Ductile (wrought iron/steel) can be calculated using **plastic collapse mechanisms**

Determine stresses and possible failure mechanism

Thrust lines can be used to calculate stresses by considering the eccentricity of the thrust

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Historic Concrete Structures

Historic Concrete Structures

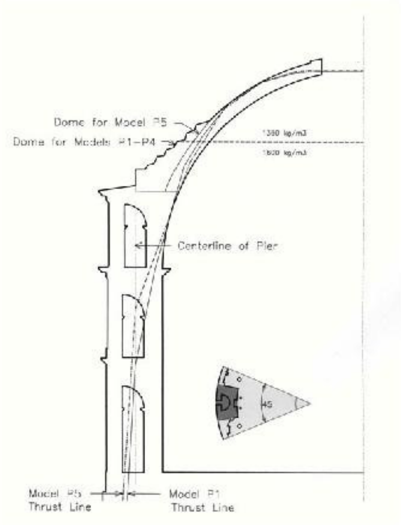
History

Types of concrete structures

Non-destructive testing (NDT)

Analysis Methods

Roman Pantheon, 2nd C AD



Unreinforced concrete

No tension, acts like masonry

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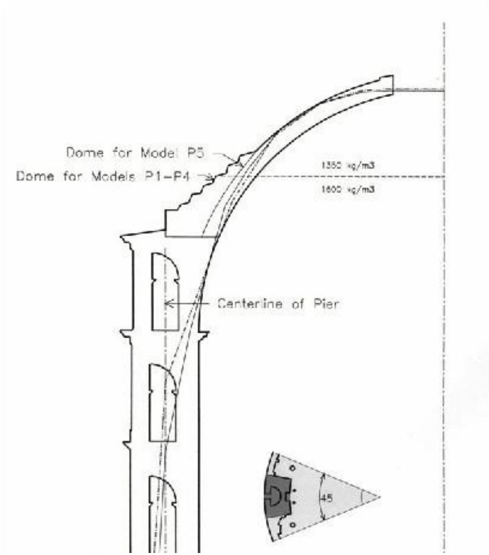
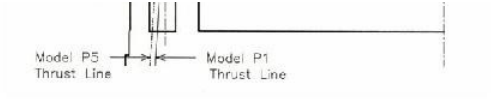
New book on Roman
Concrete from OUP
by Lynn Lancaster

Analysis of Pantheon and
other Roman structures
using thrust lines




Model P5
Thrust Line
Model P1
Thrust Line

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Eddystone Lighthouse
John Smeaton, 1759

English Engineer Smeaton experiments with cement mortars

Finds mortar that can set underwater

Uses for lighthouse foundations

Structure of stone

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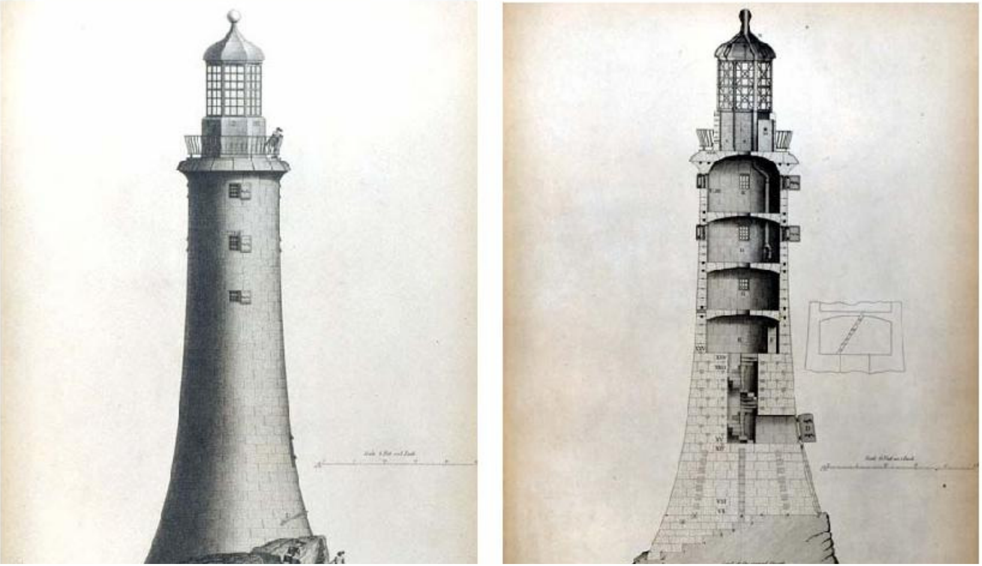
Eddystone Lighthouse John Smeaton, 1759

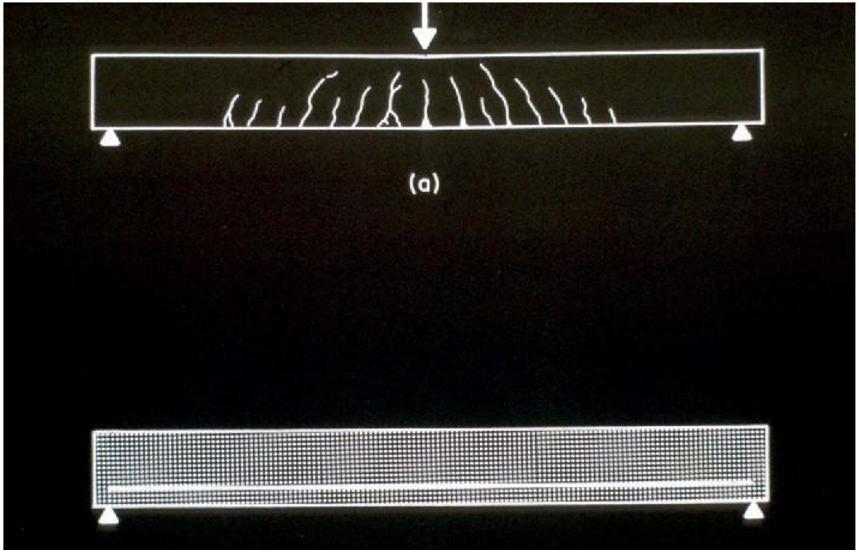



ROCK

Early 19th C Concrete Rail Bridge

Unreinforced, used as liquid masonry

Historic Concrete Structures



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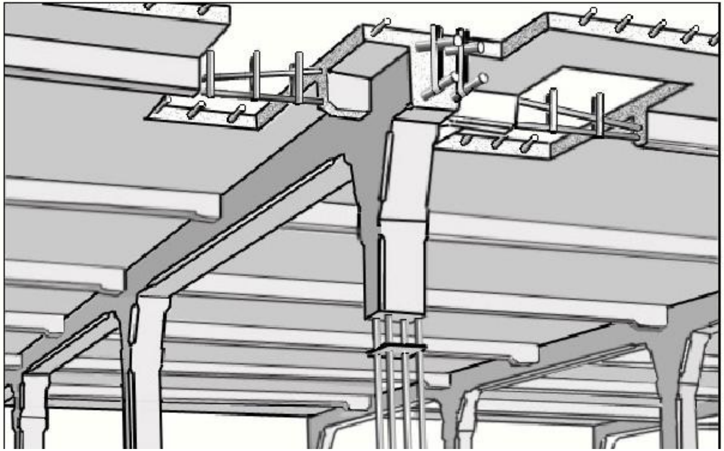
Hennibique System 1890s

Beginnings of Reinforced Concrete



Hennebique system patented in France

Types of Concrete Structures

Unreinforced (mass concrete)

Roman Pantheon

Analyze like masonry

Early reinforced concrete

19th and early 20th C

Square, spiral, and round bars, non-standard

Often followed patents (like Hennebique system)

Analysis is difficult

Modern

reinforced concrete

Standardized rebar patterns and sizes

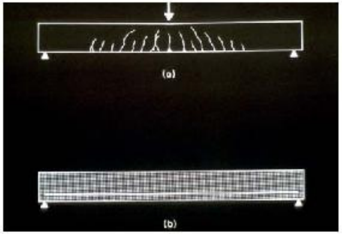
Use concrete codes to assess

Analysis of RC Slab

Unreinforced (mass concrete)

Arches within depth (flat arch)

Determine extent of horizontal forces



Early reinforced concrete

Bond of bars

Shear reinforcement

Modern reinforced concrete

Use a lower bound analysis to redistribute moments in different **directions**

Use yield line analysis for an upper bound approach

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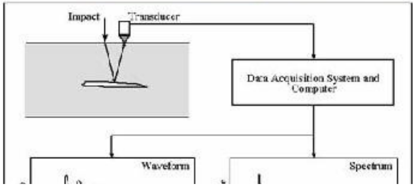
NDT of Reinforced Concrete

Can use non-destructive testing (NDT) to find:

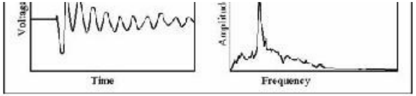
- Extent of corrosion
- Location and size of reinforcement
- Voids and areas of poor concrete
- Delamination of sections

Methods



- Impact-Echo (sound waves)
- Radar
- Magnetometers

Etc.



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Conclusions

Material properties of concrete depend on the time period of construction

Assessment methods are different from design methods

Use Upper Bound, such as yield line analysis for slabs

In order to use plastic theory (Upper Bound Method) the structure must offer ductile behavior (lightly reinforced) rather than brittle behavior (shear failure or over-reinforced

▨ reinforced in bending)