Electricity and magnetism

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C O N N E X I O N S

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Chapter 1

Special theory of relativity[1]

The Newtonian mechanics is considered to be valid in all inertial frames of reference, which are moving at a constant relative velocity with respect to each other. Einstein broadened the scope of this theorem and extended the validity of all physical laws including electromagnetic theory to all inertial frames of reference. Now, constancy of speed of light in vacuum is a core consideration in the electromagnetic theory. Therefore, Einstein postulated that speed of light is a constant in all inertial frames of reference. The speed of light does not depend upon the motion of either the source emitting it or the receiver of the light. This simple assertion about the constancy of the speed of light in vacuum is an epoch making assertion as it contradicts one of the equally fundamental assertion that speed (velocity) is a relative concept and that it essentially depends on the state of motion of observer.

We can comprehend the import of special theory of relativity by a simple example. Let a light pulse is moving in x-direction with its speed "c" and let a space craft is also moving ahead in the same direction with a speed "v". These motions are observed from a position on the ground. Let us also assume that there is no atmosphere and we are observing motions in vacuum. Now, the speed with which light reaches spacecraft should be the relative speed "c-v". This is what we deduce classically. Special theory of relativity, however, asserts that the relative speed of light with respect to spacecraft is "c" only – notwithstanding the speed of spacecraft (v).

[1]This content is available online at <http://cnx.org/content/m32527/1.36/>.

Motion of a light pulse and a spacecraft

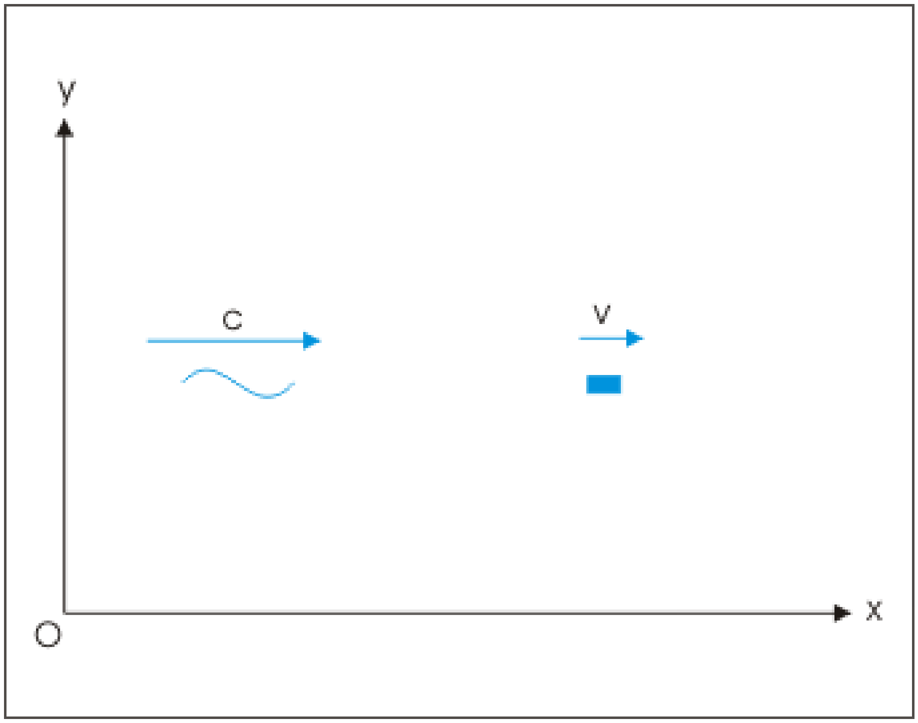


Figure 1.1: Motion of a light pulse and a spacecraft

The physical interpretation of the assertion of special theory of relativity is quite unthinkable classically. The constant relative speed of approach by light in the above example is possible only if the constituents of speed (distance and time) are different for observers having different motions. In the instant example, both “distance” and “time” as measured by spacecraft are different than the corresponding measurements by a ground observer which is observing motions of both light and spacecraft. The measurements of “distance” and “time” in two different frames of reference need to be different such that speed ratio for light in vacuum i.e. “x/t” or "x’/t’" in two inertial references (parameters in one reference is denoted by unprimed varibales whereas parameters in other reference is denoted be primed variables) remains a constant.

Motion of a light pulse and a spacecraft

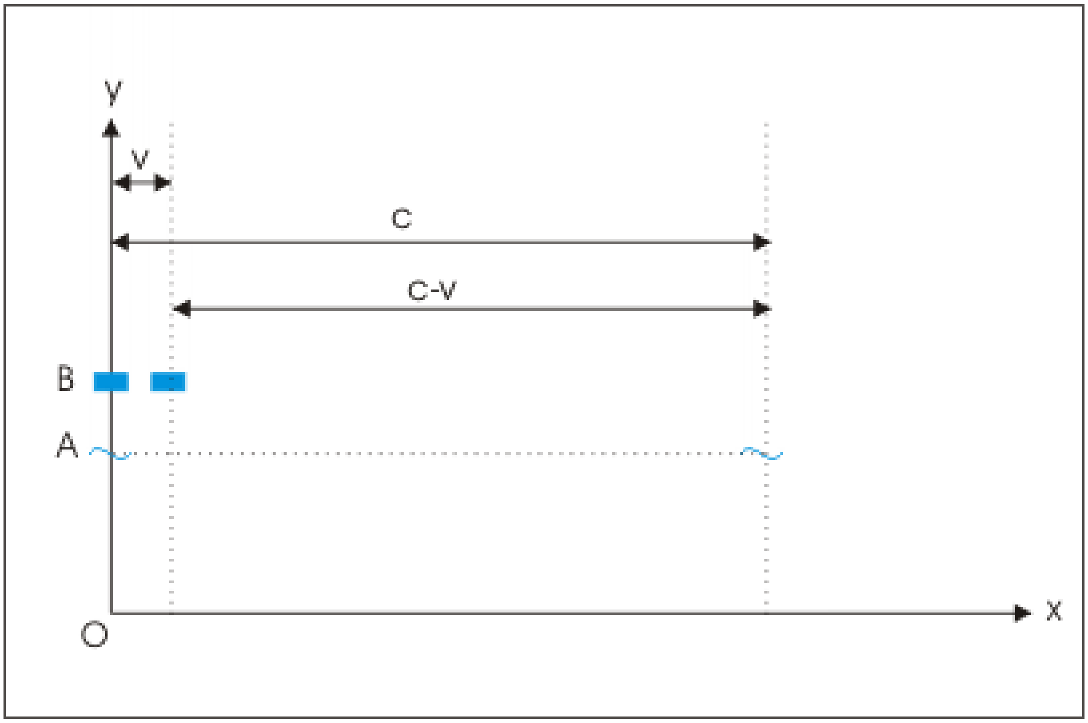


Figure 1.2: Motion of a light pulse and a spacecraft

In the figure above, we consider motion of a light pulse and spacecraft which are moving with speed "c" and "v" respectively in x-direction. They are initially at x=0 when t =0. The positions of light pulse and spacecraft are also shown after 1 second. As seen from the reference of ground (coordinate system), pulse and spacecraft travel "c" and "v" meters respectively. The linear distance between spacecraft and pulse after 1 second is "c-v" in ground reference. But according to special relativity, the linear distance between light pulse and spacecraft after 1 second should be "c" in the reference of spacecraft. As "c-v" can not be "c", it is deduced that measurements of distance and time in two references are different. A part of discripancy is due to difference in the measurement of distance and the remaining due to difference in the measurement of time. These diffferences need to be such that ratio of ditance and time is a constant for the pulse of light in all inertial references.

In essence, special theory of relativity removes "relativity" from "speed of light" and attaches "relativity" to "space (distance)" and "time". This is the difficult part. Classically, we have considered both these elements as universally invariants with respect to all frames of reference which are moving with constant relative velocity (inertial frames of reference). We shall try to come to terms with these new ideas in subsequent modules. But the essence of special theory of relativity is captured as follows : The speed of light in vacuum is invariant whereas distance between two points and time intervals are variant in the system of inertial frames of reference.

The ideas of classical relativity, where in space and time are invarinat and speed of light is variant, is captured by Galilean transformation which enables us to measure motion in two inertial frames of reference. The speed of an object is modified by the relative speed of the frames of reference.

$u' = u \pm v$

where "u" and "u"' are the speeds of an object as measured in two frames of reference which themselves move with a speed "v" with respect to each other.

Einstein employed a different transform called "Lorentz transformation" to capture the idea of invariant speed of light and variant distance and time measurements. The Lorentz tansformation provides the exact relation between coordinates (space and time) of inertial references. We shall discuss these transformations separately in the module.

Further, since we are considering constancy of speed of light in relation to inertial references only, the special theory of relativity is "restricted" to inertial frames of reference and therefore is "special" not "general".

1.1 Postulates of Special Theory of relativity

There are many versions of postulates. The essence of special theory of relativity is finally agreed to be captured by following two principles/ postulates :

1. The principle of relativity : The laws of physics are the same for all observers in uniform motion relative to one another (inertial frames of reference).

2. The principle of constancy of speed of light in vacuum : Light in vacuum propagates with the constant speed through all systems of inertial coordinates, regardless of the state of motion of the light source.

Few scholars consider either of above postulates sufficient to describe special theory of relativity. They are supplementary to each other. As a matter of fact, one can be deduced from other and vice –versa with certain extrapolation.

Proceeding from the principle of relativity, we can arrive at the principle of constancy of speed of light in vacuum. The principle of relativity considers validity of all physical laws across all inertial frames of reference. This means that law of propagation of light (electromagnetic theory) is same across coordinates systems in uniform translatory motion. But, the law of propagation of light says that light moves at a constant speed in vacuum and is independent of the motion of source. Thus, speed of light is constant in terms of any system of inertial coordinates, regardless of the state of motion of the light source. This is exactly the the principle of constancy of speed of light in vacuum.

Similarly, we can proceed from the principle of constancy of speed of light in vacuum to the principle of relativity. If we accept constancy of speed of light in vacuum across all inertial references, then we consider that law of propagation of light in vacuum (electromagnetic theory) is valid in them. Now, the laws of motion are already considered to be independent of inertial frames of reference. Addition of electromagnetic theory to this class of invariants suggests that other physical laws in their simplest form are also valid in all inertial references. This is exactly the principle of relativity.

Clearly, two principles are deducible from each other. Yet, we require to state special theory of relativity in terms of two principles. We see that though we are able to deduce second principle from first, but in the process we have narrowed the scope of principle of relativity. The principle of relativity is a very general principle extending to all physical laws - not only to laws of motion and propagation of light. Similarly, the deduction of first principle from second is not direct deduction - rather an extension. For these reasons, it is generally prudent to state both the principles of special theory of relativity.

The important consideration of special theory of relativity is the inclusion of Maxwell's electromagnetic theory being valid in inertial references. Earlier we limited the scope of validity only to Newton's laws of motion. We should understand that Newton's laws of motion are special case of a more general special theory of relativity. Let us have a look at the validity of the Newton's laws motion in inertial references involving relativistic consideration at higher speed :

1: Newton's first law of motion of motion is valid in inertial references.

2: Newton's second law of motion which defines force in terms of time rate of change of linear momentum is valid in inertial references.

3: Newton's second law of motion which defines force in terms of the product of mass and acceleration is not valid in inertial references, because mass is not a constant in relativistic mechanics. The more general relativistic or modified form of Newton's second law valid in all inertial references, however, reduces to classical Newton's second law of motion at lower speed.

4: Newton's third law of motion as stated in the form of equal and opposite action and reaction is not valid in inertial references.

5: The conservation of linear momentum, which is the consequence of Newton's third law, is valid in inertial references.

We shall return to these aspects in detail subsequently.

1.2 Studying special theory of relativity

The idea of constancy of speed of light in all inertial references shakes up well rooted concepts about distance (space) and time. It raises many questions and makes the study of special theory of relativity a bit difficult. Generally the explanations appear to be inadequate or not very convincing. As a matter of fact, there is a temptation to view the theory with a sense of disbelief. But the fact of life is that relativity (we shall use this term to mean "special theory of relativity" for brevity), there is not even a single "exception of" or "departure from" the predictions of special relativity as on date (spanning a period of more than a century).

After many readings of relativity theories, it emerges that it would be futile to follow the conventional approach of studying relativity by explaining the "unthinkable" first and then derive conclusions. No description, however good, satisfies a reader that incidence of time for an event or length of a rod is different in two inertial references. Keeping this aspect of study in mind, we shall attempt a slightly different approach here. Upfront, we shall accept relativistic assertions about distance (space) and time. This is a better approach as it allows us to proceed with the theory and come back to the lingering thoughts when we are equipped with the basic or working knowledge of the theory. After all, electromagnetic theory of light (and hence constancy of speed of light in vacuum) is such an elegant and complete theory that we can only be more than willing to accept assertions which are based on it.

Yet another aspect of the study of relativity is that it relates phenomena which occur over a very large spatial extent. The consideration of motion of light even for 0.1 second involves a linear extent of 30000000 meters. Clearly, there is limitation to pick examples or illustration from our real world. Most of the experiments or illustrations cited in the study of relativity are reasonable as imagined. Conception of special theory of relativity is more an outcome of "experiments in head" than the actual ones, but such experiments are rigorous and subject to direct or indirect scientific verification. This process of performing mental experiments is known by a German term "Gedankenexperimenten". Einstein used this process often to reach conclusions. Clearly, we shall also be required to do a bit of "Gedankenexperimenten" to understand his theories. In a nutshell, we should be ready to imagine spacecrafts or space objects moving at very high speeds without any inhibition. We may even imagine ourselves sitting in those high speed spacecraft. Similarly, we may imagine a train which is moving at a speed of say 100000 km/hr. Apart form the scientific validity of reasoning, there is no constraint in imagining experiments or examples which otherwise can not be realized in our small world.

1.3 What is time ?

We do not know exactly what is time. But we know some of its properties. The closest that we come to define time is about the manner in which we measure it. This measurement is essentially an outcome of the characteristic of time known as "simultaneity". Einstein wrote "That train arrives here at 7 o'clock", I mean something like this : "The pointing of the small hand of my watch to 7 and the arrival of the train are simultaneous events." Thus, we measure time of an event by way of the simultaneity of two events – one belonging to measuring device and other belonging to an arbitrary event like arrival of a train. This argument clinches the issue of time from the relativistic perspective. If we are able to prove that two events which are simultaneous in one inertial frame of reference but "not" simultaneous in another, then we can be sure that measurements of time in two inertial references could indeed be different.

In special theory of relativity, time and time rate in two inertial references are treated as different. If t and t' be the time recorded for a given event in two inertial references, then t may not equal to t'. We shall return to this topic again.

1.4 Absolute and stationary reference

There is no preferred inertial reference frame. This idea predates special relativity. It means that there is no absolute reference frame. Had there been an absolute reference, then we would have a fixed universal space in which all other objects (references) would be considered to be either in rest or moving. But the concept of space is a variant. In other words, the perception of space changes from one reference (say ground) to other reference (say moving train). If we drop a stone from a train, then the trajectory of the stone is a straight vertical line for an observer on the train. The same stone, however, is seen to follow a parabolic trajectory for an observer on the ground. Referring to these trajectories for a single motion of a stone, Einstein questioned the very concept of fixed space.

Trajectory of motion

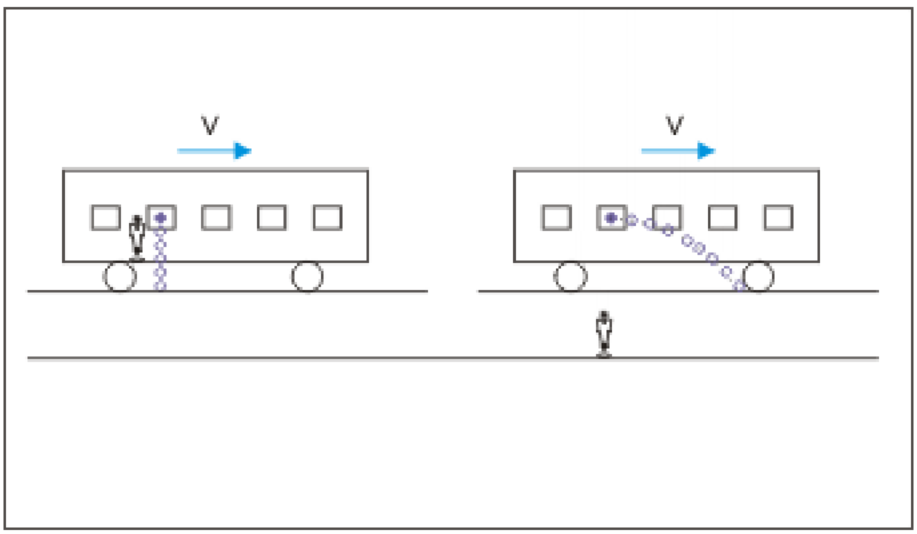


Figure 1.3: Trajectory of motion

Though there is no absolute reference, but the notion of a stationary reference is a powerful idea which stems from our life long perception of stationary objects in Earth's reference. Despite the fact that Earth is moving at about 107,278 km/hr (29.8 km/s) around Sun, we are generally not aware about it unless we make detailed observations about celestial objects. But as the concept of stationary system is ingrained in our perception, we employ this concept with great effect in the study of relative motion. We refer either of the moving systems as stationary in which an observer is making the measurements. Consider for example two spacecrafts moving with uniform velocities. We can refer either of spacecrafts as stationary and other as moving with a velocity which is measured from the referred stationary reference (spacecraft). There is no preference. In the case of a motion of a train, for example, we consider Earth as stationary and train as moving reference. There is no bar though that we consider moving train as stationary reference with the observer and Earth as moving reference in the opposite direction to the moving train.

The "rest" is local concept. The object like a house is at rest in Earth's reference. But the same object as seen from a spaceship is not stationary.

1.5 Constancy of speed of light

Constancy of speed of light has two different considerations in the study of relativity. In the first place, it is the central idea of special relativity. But besides this consideration, the constancy of light has other important consideration in that it is one of the measuring standards which can not be challenged in any inertial reference. This aspect is important as meaning of space and time in different inertial frames is not very explicit. We shall see subsequently that they are in fact entangled. Further, the distance (space) and time are relative and are therefore very subjective in conception and measurement.

On the other hand, speed of light in vacuum is invariant in inertial references (though it is not invariant in accelerated references). As such, it can be used as a parameter to measure "time" and "distance". A linear distance, for example, can be expressed in terms of "time" taken by light to cover a given distance. Alternatively, a particular time interval can be expressed in terms of "linear distance" covered by the light in a given time.

The official measure of speed of light in vacuum is as given here :

c = 299,792,458 meters/second

1.6 Galilean transformation

The transforms are mathematical constructs which allow us to convert one set of spatial (x,y,z) and time (t) measurements in one frame of reference to another frame of reference based on certain physical principle or law. Our current context is limited to inertial frames of reference. Therefore, we shall study transforms which refer to inertial frames of reference. Here, we shall study Galilean and Lorentz transforms. The Galilean transform encapsulates the ideas of non-relativistic mechanics whereas Lorentz transform encapsulates the ideas of relativistic mechanics.

The concepts of a transform, physical laws and inertial frames of reference are entangled with each other. The physical laws are required to be valid across all inertial frames of references.

Galilean transform gives the relation between two inertial systems which are moving at a constant velocity with respect to each other. If space (co-ordinates) and time values in one reference are known, then we can find out space and time values using Galilean transform in another reference which is moving at a constant velocity "v' with respect to first in x-direction. Let two inertial reference systems are denoted by unprimed and primed variables and their spatial origins coincide at t = t' = 0. Then, space (x',y',z') and time (t') co-ordinates of a "single arbitrary event" in primed inertial reference is related to space (x,y,z) and time (t) of unprimed inertial reference as :

Galilean transformation

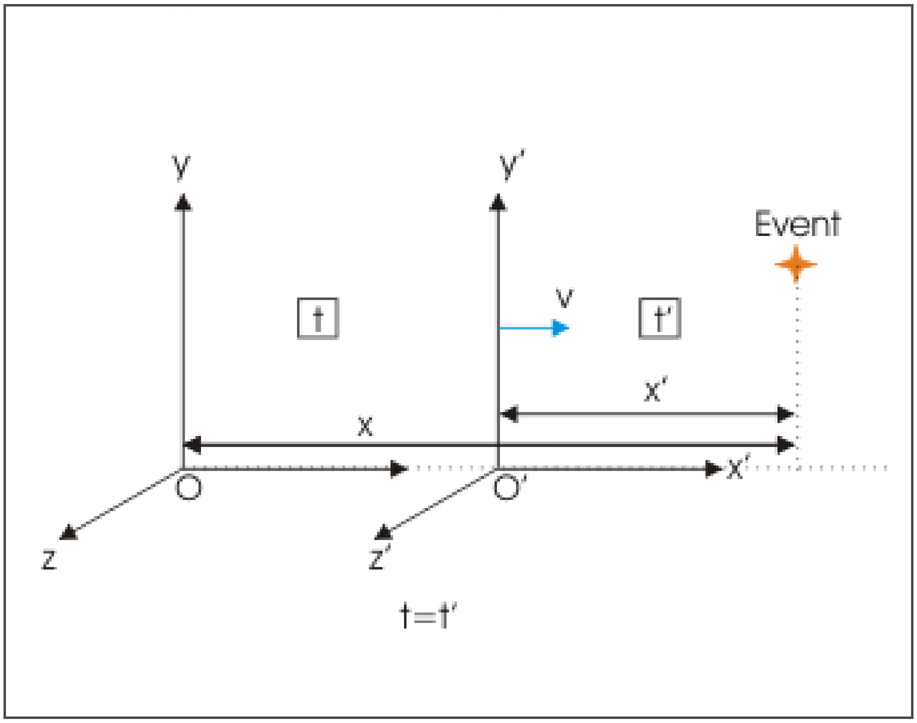


Figure 1.4: Time is same in two inertial references.

$$x\prime = x - vt$$

$$y\prime = y$$

$$z\prime = z$$

$$t\prime = t$$

We can also express unprimed variables in terms of primed variable by solving for unprimed variable as :

$$x = x\prime + vt$$

$$y = y\prime$$

$$z = z\prime$$

$$t = t\prime$$

The most important aspect of Galilean transform is the last equation, t' = t, denoting that time is an invariant for inertial frames of references. The constancy of time across inertial frames of reference is the key consideration here. With the advent of special theory of relativity, however, this transform is considered as a restricted case as it is valid for small relative speed,v, only. At higher values of relative speed “v”, we need to employ Lorentz transform in accordance with special theory of relativity such that speed of light in vacuum is constant in all inertial references.

Further, we get the equation for the velocities of a particle or object at position "x" or "x’" in the unprimed and primed references respectively by differentiating first equation of the transform,

u’ = u - v

where “u” and “u”’ are the speeds of a particle or object as measured in two frames of reference which themselves move with a speed “v” with respect to each other.

1.7 Lorentz transformation

Like Galilean transform, Lorentz transform provides relation for space and time between inertial systems for all possible range of relative velocity. Importantly, it satisfies the postulate of special theory of relativity that speed of light in vacuum is a constant. The derivation of Lorentz transform has elaborate historical perspectives and is also the subject of insight into the relativistic space and time concepts. For this reason, we shall keep the derivation of this separate to be dealt later. Here, we shall restrict our consideration to the final form of Lorentz transform only. Let two inertial reference systems are denoted by unprimed and primed variables and their spatial origins coincide at t = t’ = 0. Then, space (x’,y’,z’) and time (t’) co-ordinates of a "single arbitrary event" in primed inertial reference is related to space (x,y,z) and time (t) of primed inertial reference as :

Lorentz transformation

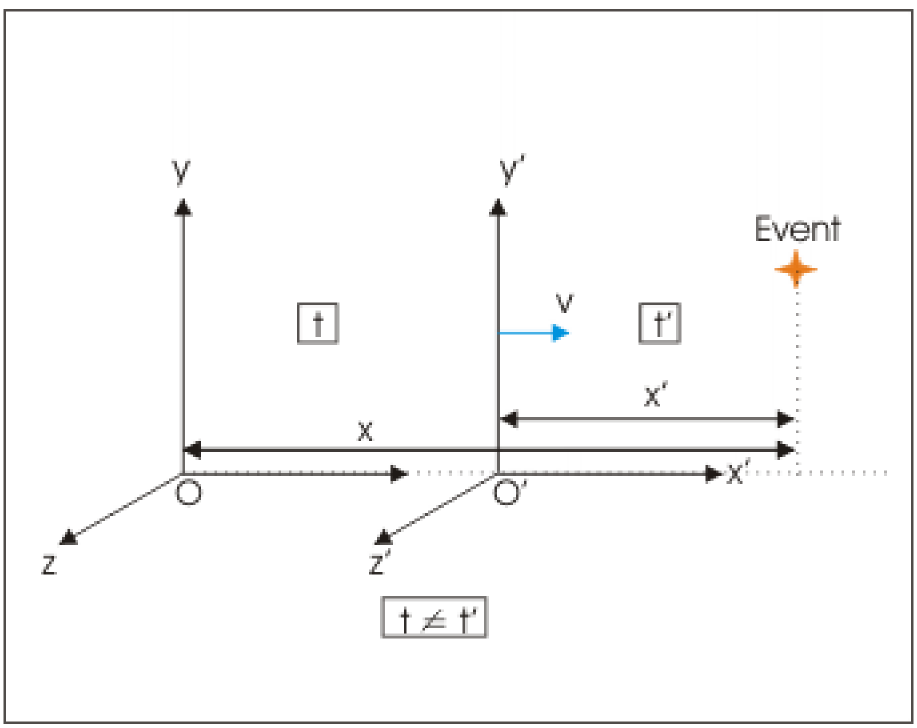


Figure 1.5: Time is not same in two inertial references.

$$x\prime = \gamma\,(x - vt)$$

$$y\prime = y$$

$$z\prime = z$$

$$t\prime = \gamma\left(t - \frac{vx}{c^2}\right)$$

where,

$$\gamma = \frac{1}{\sqrt{(1-\beta^2)}} = \frac{1}{\sqrt{(1-\frac{v^2}{c^2})}}$$

The dimensionless γ is called Lorentz factor and dimensionless β is called speed factor. For small relative velocity, v, the terms $v^2/c^2 \rightarrow 0$, $v/c^2 \rightarrow 0$ and $\gamma \rightarrow 1$. In this case, the Lorentz transform is reduced to Galilean transform as expected. Further, we can write transformation in the direction from primed to unprimed reference as :

$$x = \gamma\,(x\prime + vt\prime)$$

$$t = \gamma\left(t\prime + \frac{vx\prime}{c^2}\right)$$

Note the change of the sign between terms on right hand side.

1.7.1 Transformation involving two events

If two events, separated by a distance, occur along x axis at two instants, then we can write Lorentz transformations of space and time differences using following notations :

$$\Delta x = x_2 - x_1; \quad \Delta t = t_2 - t_1; \quad \Delta x\prime = x_2\prime - x_1\prime; \quad \Delta t\prime = t_2\prime - t_1\prime$$

The subscripts 1 and 2 denote two events respectively. The transformations in the direction from unprimed to primed references are :

$$\Delta x\prime = \gamma\,(\Delta x - v\Delta t)$$

$$\Delta t\prime = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$$

The transformations in the direction from primed to unprimed references are :

$$\Delta x = \gamma\,(\Delta x\prime + v\Delta t\prime)$$

$$\Delta t\prime = \gamma\left(\Delta t\prime + \frac{v\Delta x\prime}{c^2}\right)$$

1.7.2 Constancy of speed of light in inertial references

We can test Lorentz transform against the basic assumption that speed of light in vacuum is constant. Let a light pulse moves along x-axis. Then, consideration in unprimed reference gives speed of light as :

$$c = \frac{x}{t}$$

If Lorentz transform satisfies special theory of relativity for constancy of speed of light, then the propagation of light as seen from the primed reference should also yield the ratio x'/t' equal to c i.e. speed of light in vacuum. Now,

$$\frac{x\prime}{t\prime} = \frac{\gamma\left(x - vt\right)}{\gamma\left(t - \frac{vx}{c^2}\right)} = \frac{\left(x - vt\right)}{\left(t - \frac{vx}{c^2}\right)}$$

Dividing numerator and denominator by “t” and substituting x/t by c, we have :

$$\Rightarrow \frac{x\prime}{t\prime} = \frac{\left(\frac{x}{t} - v\right)}{\left(1 - \frac{vx}{tc^2}\right)}$$

$$\Rightarrow \frac{x\prime}{t\prime} = \frac{\left(c - v\right)}{\left(1 - \frac{vc}{c^2}\right)} = \frac{\left(c - v\right)}{\left(1 - \frac{v}{c}\right)}$$

$$\Rightarrow \frac{x\prime}{t\prime} = \frac{c\left(c - v\right)}{\left(c - v\right)} = c$$

Clearly, Lorentz transform meets the requirement of special theory of relativity in so far as to guarantee that speed of light in vacuum is indeed a constant.

1.8 Lorentz factor

Lorentz factor ,γ, is the multiplicative factor in the transformation equations for x-coordinate and time. It is a dimensionless number whose value depends on the relative speed “v”. Note that the relativistic transformation for x-coordinate is just Lorentz factor times the non-relativistic or Galilean transformation.

$$x\prime = \gamma\left(x - vt\right)$$

Lorentz factor appears in most of the relativistic equations including the calculation of relativistic effects like time dilation, length contraction, mass etc. An understanding of the beahviour of this factor at different relative velocity is intuitive for assessing the extent of relativistic effect. Few values of Lorentz factor are tabulated here.

Lorentz factors

Speed (v)	**0 0.1c 0.2c 0.3c 0.4c 0.5c 0.6c 0.7c 0.8c 0.9c 0.99c 0.999c**
Lorentz factor (γ)	1.000 1.005 1.021 1.048 1.091 1.115 1.250 1.400 1.667 2.294 7.089 22.366

Table 1.1

Lorentz factor begins at 1 and as v->c, y-> infinity. It is either equal to 1 or greater than 1. In other words, it is never less than 1. A plot of Lorentz factor .vs. relative speed is shown here.

Lorentz factor .vs. speed plot

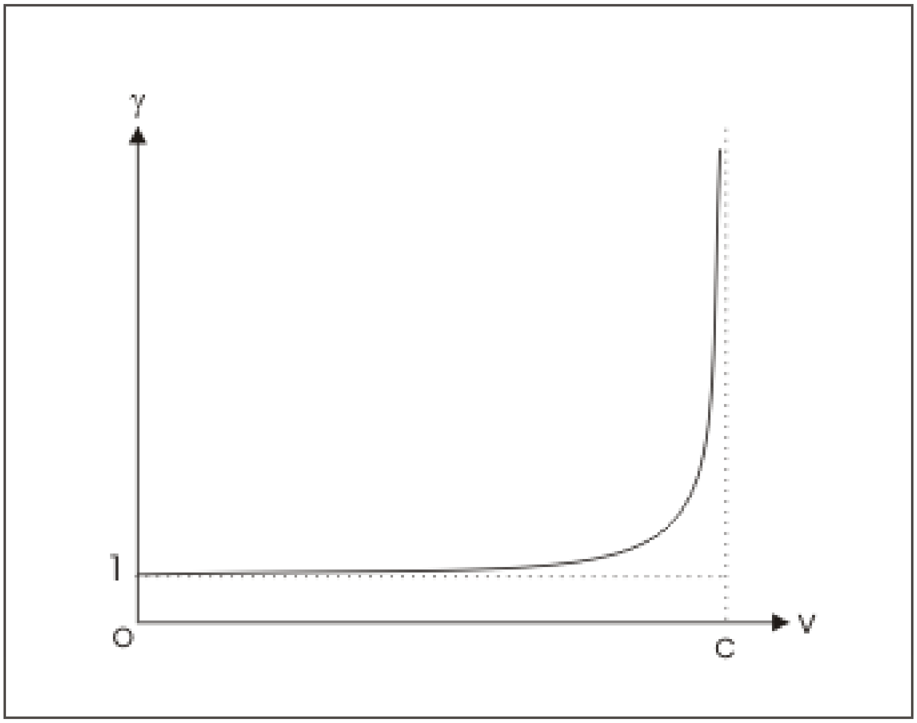


Figure 1.6: Lorentz factor .vs. speed plot

1.9 Space-time interpretation

We identify an event with spatial (x,y,z) and temporal (t) coordinates. Important point is that an event does not belong to any reference. It is described by different coordinates in different reference system. In classical description, spatial and temporal parameters are essentially independent of each other. The time t of an event can not be dependent on spatial specification (x,y,z,). Now, this independence is not there in relativistic kinematics. In order to imbibe the nature of space time relation, we shall work with few Lorentz transformations here.

We interpret an event in two inertial references which are moving with respect to each other at a velocity say 0.3c in x-direction. We shall consider very small time interval like 0.000005 second so that distance involved is easy to visualize. For convenience, we consider the approximate value of speed of light 300000000 m/s. In time 0.000005 s, the separation of two reference frame at the speed 0.3c works out to be 0.3 X 300000000 X 0.000005 = 450 m.

Lorentz transformation

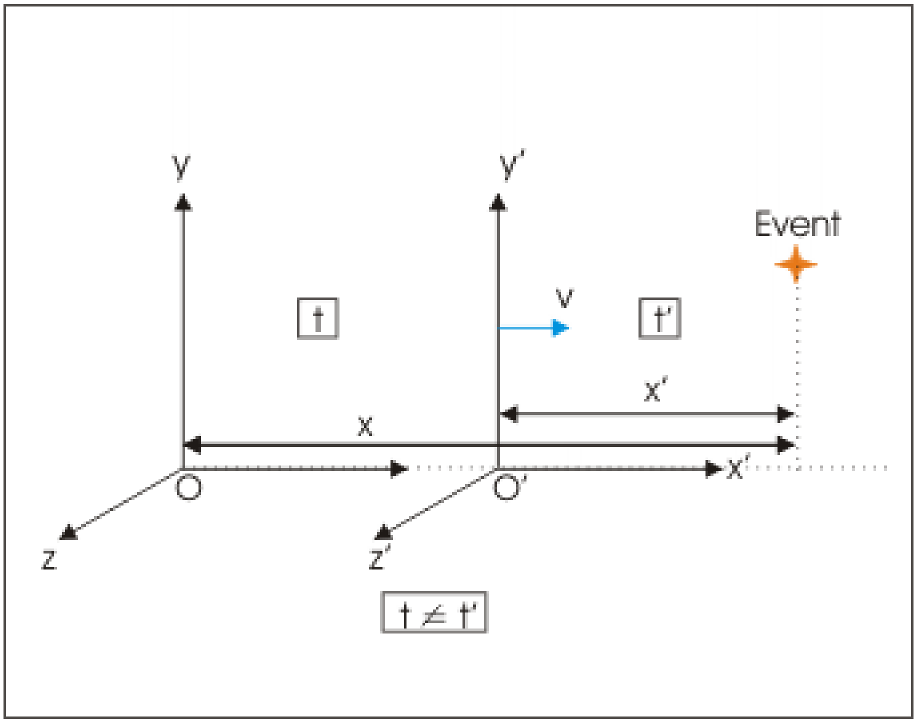


Figure 1.7: Time and space are entangled.

Here, we calculate both Galilean and Lorentz distance and time of events in two references for events identified in first reference by x and t values. Unprimed values refer to stationary reference, whereas primed values refer to moving reference which is moving right in x-direction with a relative velocity 0.3c. The calculations have been done using Excel worksheet (Reader can also try and verify the results) where distance is in meters and time in seconds.

Lorentz factors

x	t	x'(Galilean)	t'(Galilean)	x'(Lorentz)	t'(Lorentz)
0	0 0	0	0	0	
2	0.000005 -448	0.000005	-469.6317	0.0000052393	
100	0.000005 -350	0.000005	-366.8998	0.0000051366	

Table 1.2

Since the origins of two references coincide for both Galilean and Lorentz transformations at t= t'=0, the space and time values are all zero as shown in the first row of the table.

Let us now consider the second row of the table. Here, position of event is x=2 m at time, t = 0.000005 s. In this time, primed reference has moved 450 m. According to Galilean transform, the event takes place at -450+2 = -448 m (to the left of origin) in the moving reference. Since time is invariant in Galilean transformation, the time of event is same in moving reference for non-relativistic Galilean transformation.

However, when we employ relativistic Lorentz transformation, the event occurs at -469.6317 m (to the left of origin) in the moving reference. Here, the measurement of distance in moving reference is different than that calculated with Galilean transformation. Also, time is not invariant. The event occurs at 0.0000052393 s in this reference instead of 0.000005 s in the unprimed stationary reference. Thus, we see that both space and time are not invariant in Lorentz transformation.

Now, we set out to compare the values of second and third row to see the effect of change in the position of event while keeping the time of event same. In the relativistic transformation, we see that time value changes just because there is spatial change in stationary reference. The time values are 0.0000052393 s and 0.0000051366 s for x = 2 m and x = 100 m respectively. This is yet another dimension of relativistic kinematics. This suggests that space and time are entangled. Individual measurements of event parameters do not only change because of relative speed (It is a foregone conclusion in relativistic kinematics). The additional point here is that time value changes simply because of change in space value (x) even when the relative velocity is kept constant.

We conclude thus :

1: Spatial and temporal values in the inertial references are different on account of relative velocity.

2: The temporal (t) values are dependent on spatial values (x,y,z). The space and time specifications of an event are entangled.

The spatial dependence of temporal parameter, as a matter of fact, is also evident from the relativistic time relation :

$$t\prime = \gamma \left(t - \frac{vx}{c^2} \right)$$

Note the presence of spatial parameter (x) on the right hand side of the equation. Clearly, spatial and temporal values of an event are entangled.

1.10 Velocity addition

Let us consider a scenario of police car chase. The police personnel fires a shot in the direction of the criminal's car speeding ahead. What is the velocity of the bullet? It depends on the observer. The bullet moves from the gun, which is stationary in the reference of police car. The velocity of bullet will be as per the specification of the gun. This will be the same velocity as when fired from ground. Let this velocity be u'. Clearly, this velocity is inherent to the gun irrespective of its motion. If "v" be the velocity of the police car, then according to Galilean transformation, the velocity of bullet in the ground reference, "u", is obtained by addition of two velocities :

u = v + u'

When a javelin is thrown, the velocity of javelin is sum of the velocities of the javelin thrower and the javelin itself with respect to thrower. The very idea of thrower to throw javelin while running is to leverage his/her velocity towards increasing the velocity of javelin in ground reference. Also, consider motion of two cars which are moving towards each other with velocities u and w along a straight line. The velocity of approach for any of two cars is u+w. These are well established results which are outcome of non-relativistic Newtonian kinematics.

The seemingly well defined algorithm about algebraic addition of velocity runs into serious problem when we extend this concept to high speed cases. Let us do a bit of "Gedankenexperimenten". Let two spaceships are approaching with a velocity c/2 and 3c/4. What is the velocity of approach? Clearly, it is c/2+3c/4 = 5c/4. This is a speed which exceeds the speed of light. Take another example. Let us consider that we move to a light source like a bulb in our house. Of course, we imagine that there is no atmosphere. The speed of approach here is speed of light plus our speed of approach – again exceeding the speed of light and also rendering it a variable dependent on our motion.

On the other hand, electromagnetic theory specifies the speed of light in vacuum to be exactly c. Not only that, Michelson's experiment concluded that speed of light is a constant in all directions. Lorentz experiment proved that speed of light in vacuum is independent of motion of source emitting it. We, therefore, deduce

Galilean or non-relativistic addition of Velocity is not true at high speed or in the relativistic context. This is one aspect of relativistic consideration. The other aspect emerges from special theory of relativity which embodies Lorentz transformation. Let us explore Lorentz factor :

$$\gamma = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

When v->c, γ -> ∞. For v=0.999999999c, γ= 22360. When v=c, γ is undefined (a ratio with zero as denominator). When v>c, γ is imaginary. Clearly, speed of light (electromagnetic wave) in vacuum is the highest speed in nature. Matter can not achieve the speed of light. There is no question of exceeding it. This aspect of speed limit for matter has been verified experimentally, wherein it is found that a particle acquires greater relativistic mass instead of gaining speed even when it is accelerated to achieve or exceed speed of light (by continuously imparting energy). We shall describe this experiment after we have discussed about relativistic mass. We should, however, treat this violation of addition of velocity as one of the key experimental evidence that gave strong credence to special theory of relativity in the initial years.

Chapter 2

Kirchhoff's circuit laws[1]

Kirchhoff's circuit laws are facilitating rules for analyzing electrical circuits. These rules are handy where circuits are more complex beyond the scope of series and parallel combination of resistances and where circuits involve intermixing of electrical sources and resistances (appliances or resistors). Kirchhoff's laws are brilliant reflection of fundamental laws like conservation of charge and energy in the context of electrical circuits.

There are two Kirchhoff's laws which are known by different names :

1: Kirchhoff's current law (KCL) : It is also referred as Junction or point or Kirchhoff's first rule.

2: Kirchhoff's voltage law (KVL) : It is also referred as Loop or Mesh or Kirchhoff's second rule.

2.1 Kirchhoff's current law (KCL)

No point in the circuit accumulates charge. This is the basic consideration here. Then, the principle of conservation of charge implies that the amount of current flowing towards a point should be equal to the amount of current flowing away from that point. In other words, net current at a point in the circuit is zero. We follow the convention whereby incoming current is treated as positive and outgoing current as negative. Mathematically,

$$\sum I = 0$$

There is one exception to this law. A point on a capacitor plate is a point of accumulation of charge.

Example 2.1

Problem : Consider the network of resistors as shown here :

[1] This content is available online at <http://cnx.org/content/m30943/1.8/>.

Network of resistors

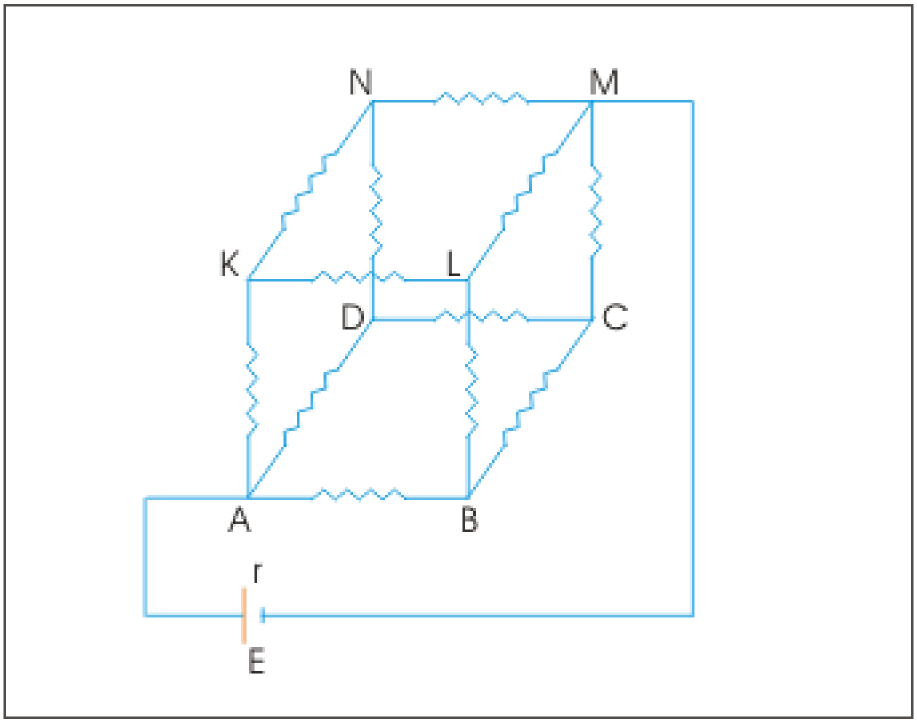


Figure 2.1: Each resistor in the network has resistance R.

Each resistor in the network has resistance R. The EMF of battery is E having internal resistance r. If I be the current that flows into the network at point A, then find current in each resistor.

Solution :

It would be very difficult to reduce this network and obtain effective or equivalent resistance using theorems on series and parallel combination. Here, we shall use the property of symmetric distribution of current at each node and apply KCL. The current is equally distributed to the branches AB, AD and AK due to symmetry of each branch meeting at A. We should be very careful about symmetry. The mere fact that resistors in each of three arms are equal is not sufficient. Consider branch AB. The end point B is connected to a network BCML, which in turn is connected to other networks. In this case, however, the branch like AK is also connected to exactly similar networks. Thus, we deduce that current is equally split in three parts at the node A. If I be the current entering the network at A, then applying KCL :

Current flowing away from A = Current flowing towards A

As currents are equal in three branches, each of them is equal to one-third of current entering the circuit at A :

$$I_{AB} = I_{AD} = I_{AK} = \frac{I}{3}$$

Currents are split at other nodes like B, D and K symmetrically. Applying KCL at all these nodes, we have :

$$I_{BC} = I_{BL} = I_{DC} = I_{DN} = I_{KN} = I_{KL} = \frac{I}{6}$$

Network of resistors

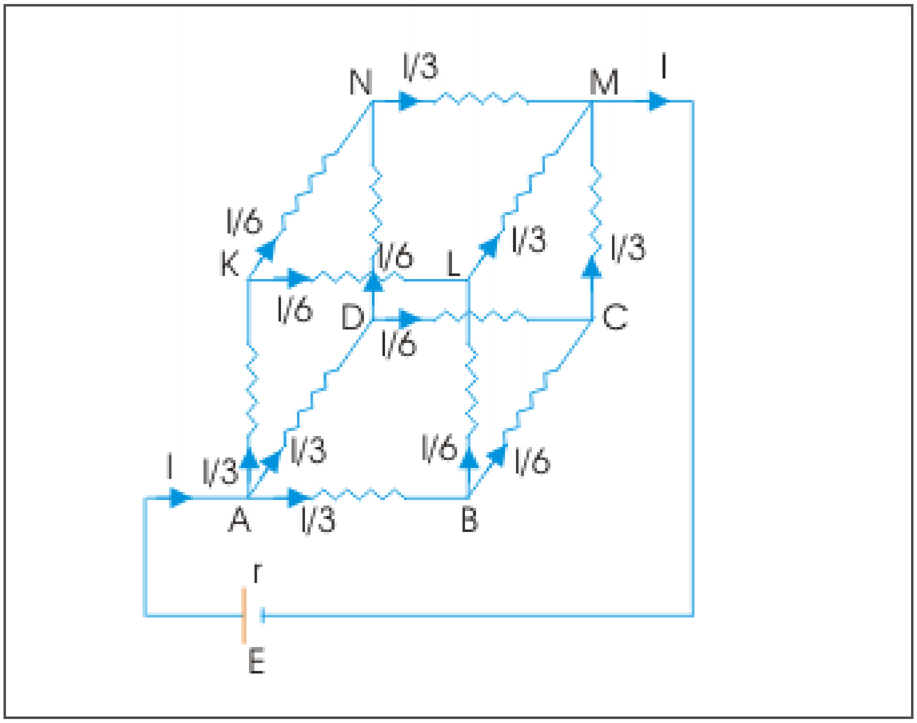


Figure 2.2: Current in resistors

On the other hand, currents recombine at points C, L, N and M. Applying KCL at C,L and N, we have :

$$I_{LM} = I_{CM} = I_{NM} = \frac{I}{3}$$

These three currents regroup at M and finally current I emerges from the network.

2.2 Kirchoff's Voltage law (KVL)

This law is based on conservation of energy. Sum of potential difference (drop or gain) in a closed circuit is zero. It follows from the fact that if we start from a point and travel along the closed path to the same point, then the potential difference is zero. Recall that electrical work done in carrying electrical charge in a closed path is zero and hence potential difference is also zero :

$$\sum V = 0$$

where V stands for potential difference across an element of the circuit.

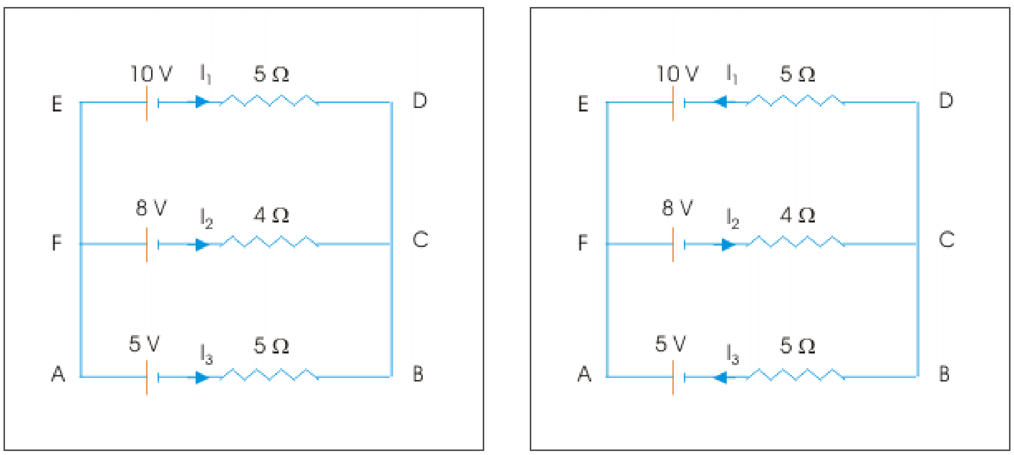
2.3 Applications

Kirchhoff's laws are extremely helpful in analyzing complex circuits. Their application requires a bit of practice and handful of methods i.e. techniques. Many people like to use a set of procedures which yield results, but they are not intuitive. We shall take a midway approach. We shall rely mostly on the laws as defined and few additional techniques. Some of the useful techniques or procedures are discussed here with examples illustrating the application. The basic idea is to generate as many equations as there are unknowns (current, voltage etc.) to analyze the circuit.

2.3.1 Direction of current (DOC)

We assign current direction between two nodes i.e. in the arm in any manner we wish. The solution of the problem will eventually yield either positive or negative current value. A positive value indicates that the assumed direction of current is correct. On the other hand, a negative value simply means that current in that particular arm flows in a direction opposite to assumed direction. See the manner in which current directions are indicated for the same circuit in two different ways :

Direction of currents



(a) (b)

Figure 2.3: (a) Direction of current can be arbitrarily chosen. (b) Direction of current can be arbitrarily chosen.

Application of KCL to the current assignments in first figure at node C yields :

$$\sum I = I_1 + I_2 + I_3 = 0$$

Application of KCL to the current assignments in first figure at node C yields :

$$\sum I = -I_1 + I_2 - I_3 = 0$$

Alternatively, we denote currents in different branches such that numbers of unknowns are minimized. We can use KCL to reduce number of variables in the circuit using first figure as :

$$I_3 = -\left(I_1 + I_2\right)$$

Current in resistors

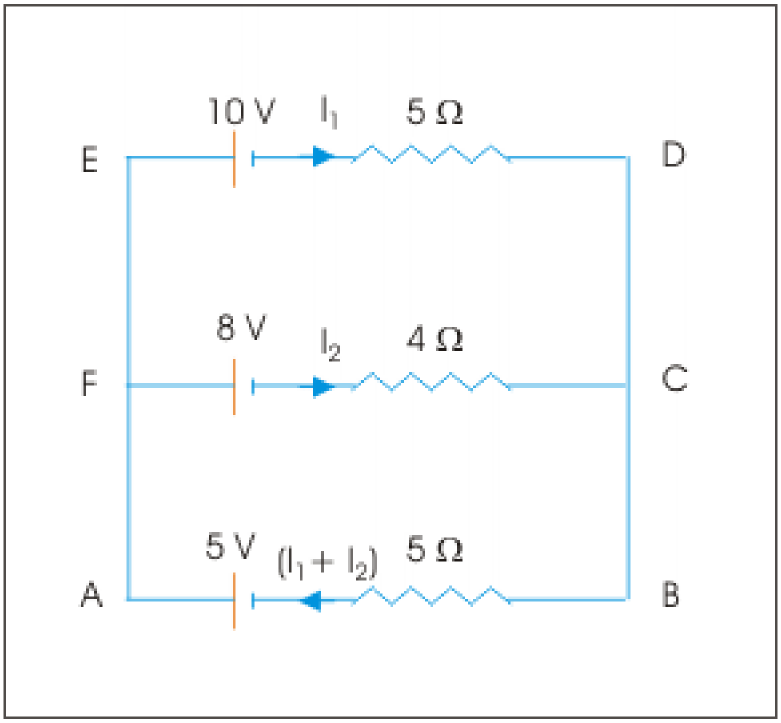


Figure 2.4: Network of resistors

Further, we should also clearly understand that direction of current (DOC) in a closed loop need not be cyclic. Consider the loop EDCFE in the figure above. Here I_1 is clockwise whereas I_2 is anticlockwise.

2.3.2 Direction of travel (DOT)

We apply Kirchhoff's voltage law to each of the closed loop. In the figure below, there are two loops ABCFA and EDCFE. We arbitrarily select direction of travel (DOT) either clockwise or counterclockwise. There is no restriction on the choice because a change in the direction changes the sign of voltage drop for all elements, which is equated to zero. Hence, choice of direction of travel does not effect the final equation. We write down voltage drop across various circuit elements moving from a node following DOT till we return to the starting node.

Direction of travel

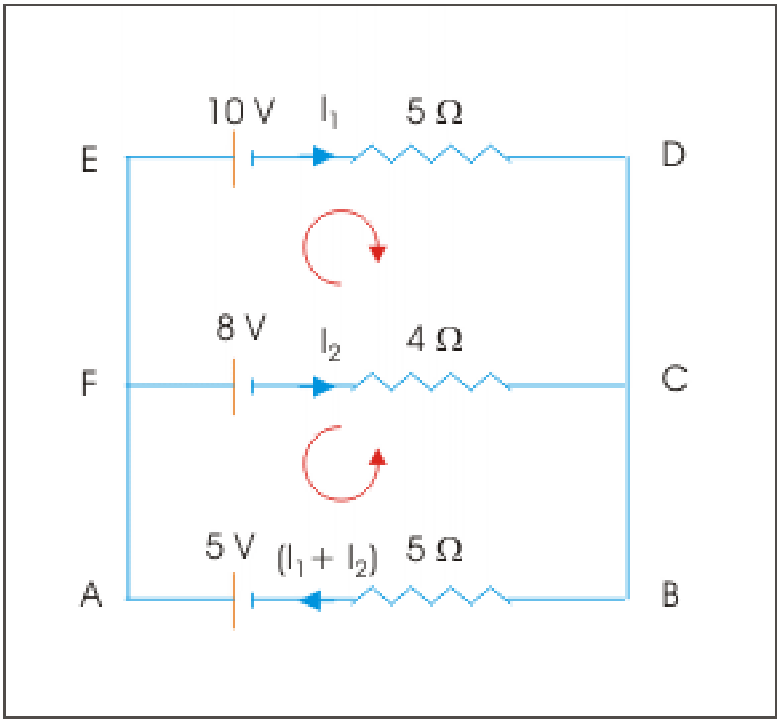


Figure 2.5: Network of resistors

2.3.3 Voltage across resistor and power source

The sign of voltage drop across a resistor depends on the relative direction of DOC and DOT. Consider the loop EDCFE. Starting from node E (say), we move toward D following clockwise DOT (Direction of Travel). From the direction of current (DOC), it is clear that the end of resistor 5 Ω where current enters is at higher potential than at the end where current exits the resistor. Hence, there is a potential drop, which is indicated by a negative sign. On the other hand, we move in the arm CF from C to F in the opposite direction of the current (COD). Here again, the end of resistor 4 Ω where current enters is at higher potential than at the end where current exits the resistor. Hence, there is a potential gain as we move across resistor from C to F, which is indicated by a positive sign.

We conclude that if DOT and DOC are same then potential difference across resistor is negative and if they are opposite then the potential difference across resistor is positive.

The sign of power source is easier to decide. It merely depends on the direction of travel (DOT). Moving across a EMF source from negative to positive terminal is like moving from a point of lower to point of higher potential. Thus, if traveling across a source, we move from negative to positive terminal then potential difference is positive otherwise negative.

Combining above considerations, we write KVL equations for loops ABCFA and EDCFE as :

Voltage across circuit elements

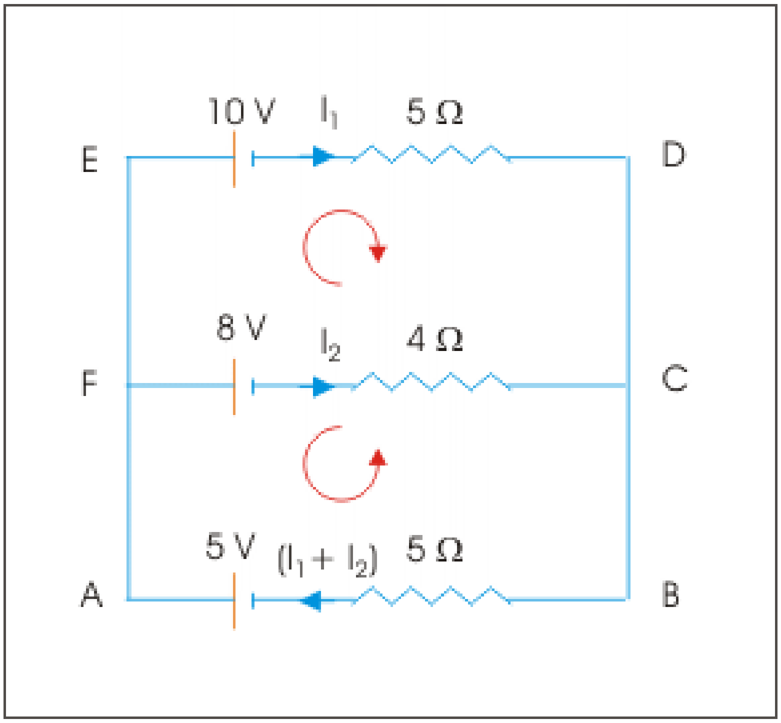


Figure 2.6: Network of resistors

Loop EDCFE (Starting from E) :

$$\sum V = -10 - 5I_1 + 4I_2 + 8 = 0$$

$$5I_1 - 4I_2 = -2$$

Loop ABCFA (Starting from A) :

$$\sum V = -5 + 5\left(I_1 + I_2\right) + 4I_2 + 8 = 0$$

$$\Rightarrow 5\left(I_1 + I_2\right) + 4I_2 = -3$$

$$\Rightarrow 5I_1 + 9I_2 = -3$$

Subtracting first from second we eliminate I_1 and we have :

$$\Rightarrow 13I_2 = -1$$

$$\Rightarrow I_2 = -\frac{1}{13} \quad A$$

Current in ED,

$$\Rightarrow I_1 = \frac{(-2+4I_2)}{5} = \frac{\left(-2+4X\frac{-1}{13}\right)}{5} = -\frac{6}{13} \quad A$$

Current in BA,

$$\Rightarrow (I_1 + I_2) = -\frac{1}{13} - \frac{6}{13} = -\frac{7}{13} \quad A$$

Clearly, direction of current in each of the branch are opposite to the ones assumed.

Example 2.2

Problem : Consider the network of resistors as shown here :

Network of resistors

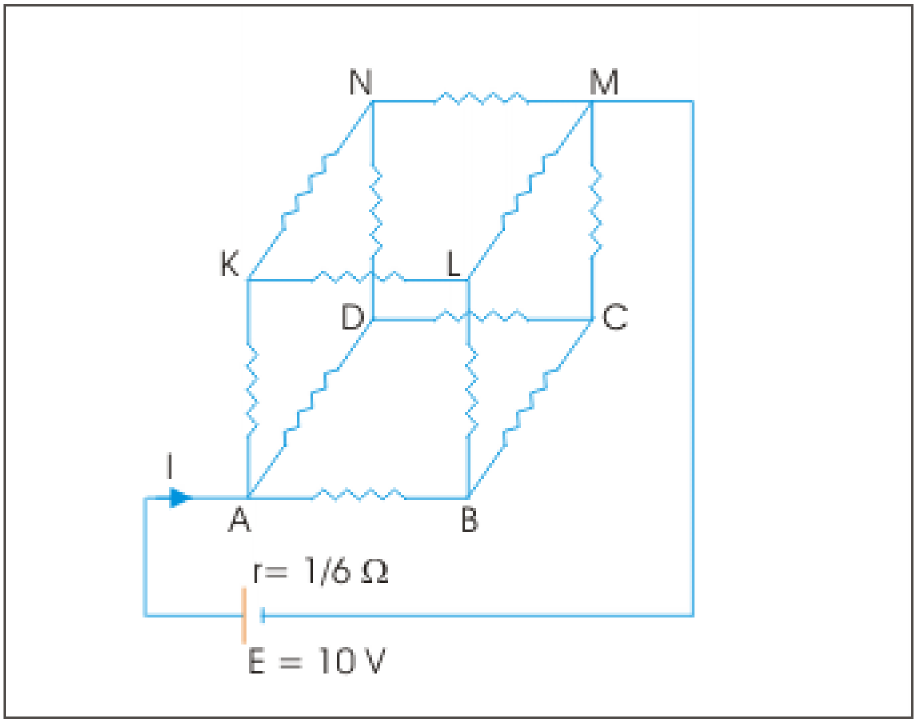


Figure 2.7: Network of resistors connected to EMF source

Each resistor in the network has resistance 2 Ω. The EMF of battery is 10 V having internal resistance 1/6 Ω. Determine the equivalent resistance of the network.

Solution : We have seen in the earlier example that if I be the current, then current is distributed in different branches of the network as shown in the figure.

Network of resistors

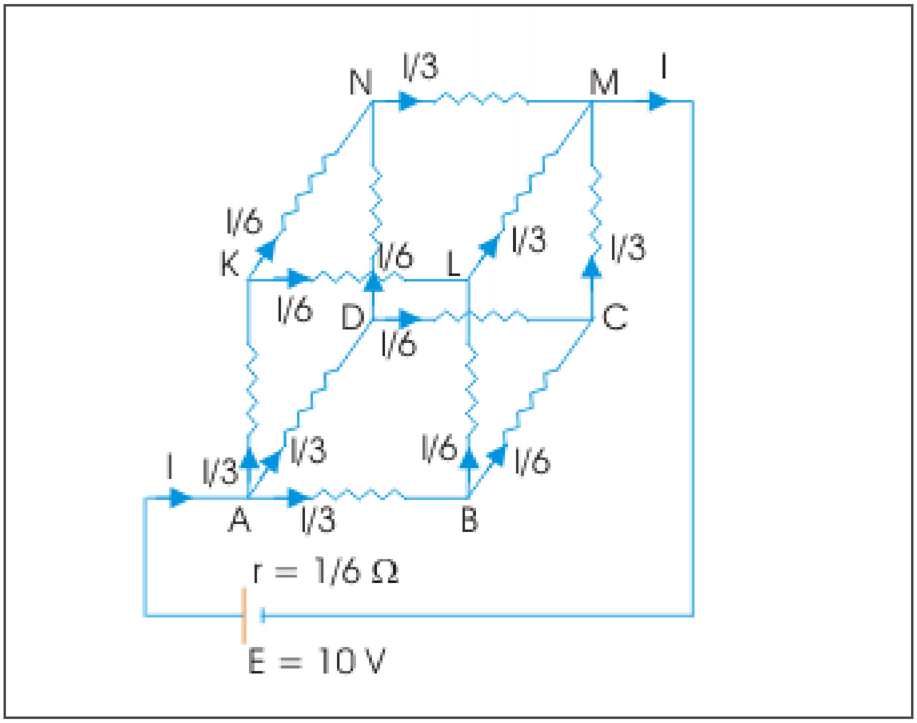


Figure 2.8: Network of resistors connected to EMF source

Clearly, we need to determine current I in order to calculate equivalent resistance of the network. For this, we consider the loop ABCMA in clockwise direction. Applying KVL :

$$\sum V = -\frac{I}{3} - \frac{I}{6} - \frac{I}{3} - IX\frac{1}{6} + 10 = 0$$

$$\Rightarrow \frac{5I}{6} + \frac{I}{6} = 10$$

$$\Rightarrow I = 10 \quad A$$

Let R_{eq} be the equivalent resistance of the network. Reducing given circuit and applying KVL in clockwise direction, we have :

Equivalent resistance

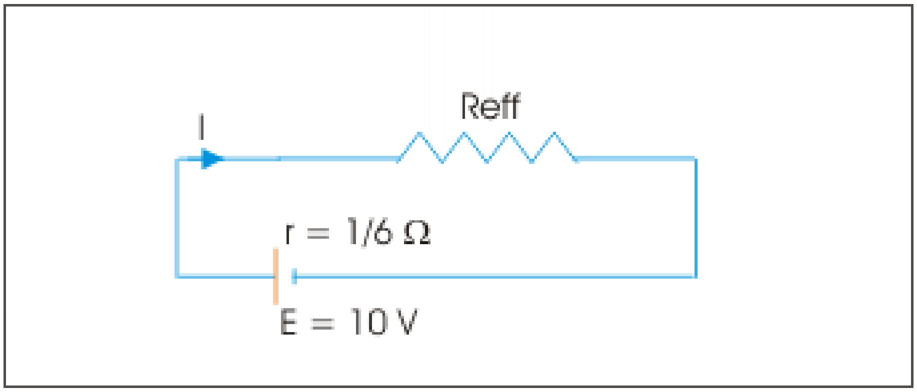


Figure 2.9: Network of resistors connected to EMF source

$$\sum V = -10XR_{eq} - 10X\frac{1}{6} + 10 = 0$$

$$\Rightarrow R_{eq} = \frac{50}{60} = \frac{5}{6} \quad \Omega$$

2.4 Exercise

Exercise 2.1 *(Solution on p. 29.)*

Consider the network of resistors as shown here :

Electrical Network

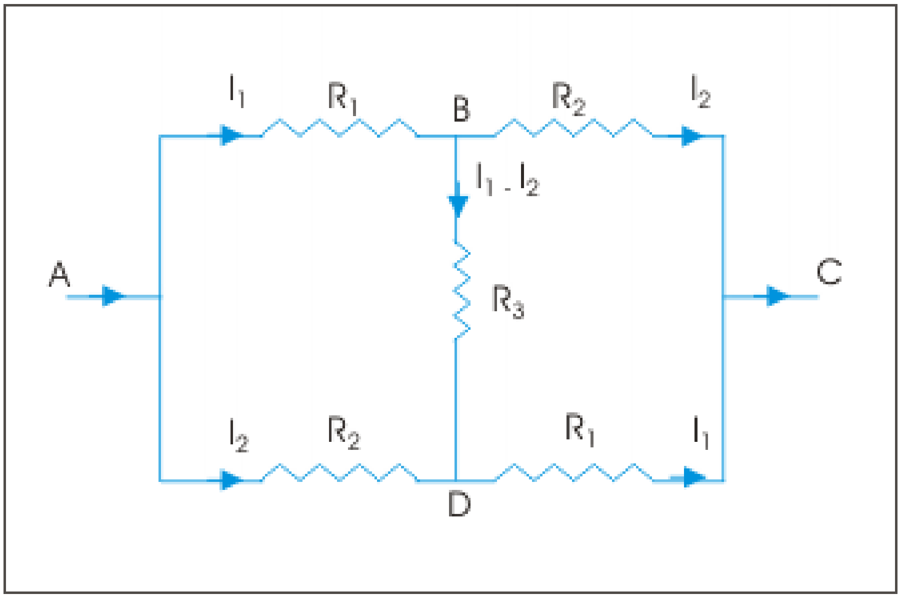


Figure 2.10: Electrical Network

Determine the equivalent resistance of the network between A and C.

Exercise 2.2 ***(Solution on p. 30.)***

Consider the network of resistors and batteries as shown here :

Electrical Network

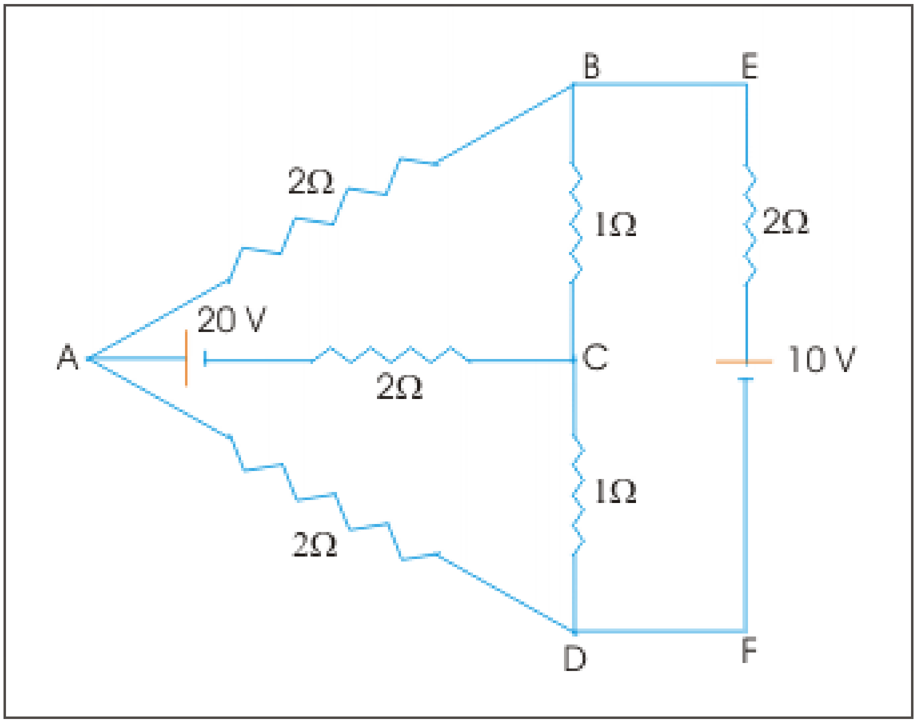


Figure 2.11: Electrical Network

Find the currents in different braches of the network.

Solutions to Exercises in Chapter 2

Solution to Exercise 2.1 (p. 26)

In order to determine equivalent resistance, we assume that given network is connected to an external source of EMF equal to E. Now, the external EMF is related to effective resistance as :

$$E = IR_{eq}$$

Once this relation is known, we can determine equivalent resistance of the given network. It is important to note that current distribution is already given in the problem figure.

Electrical Network

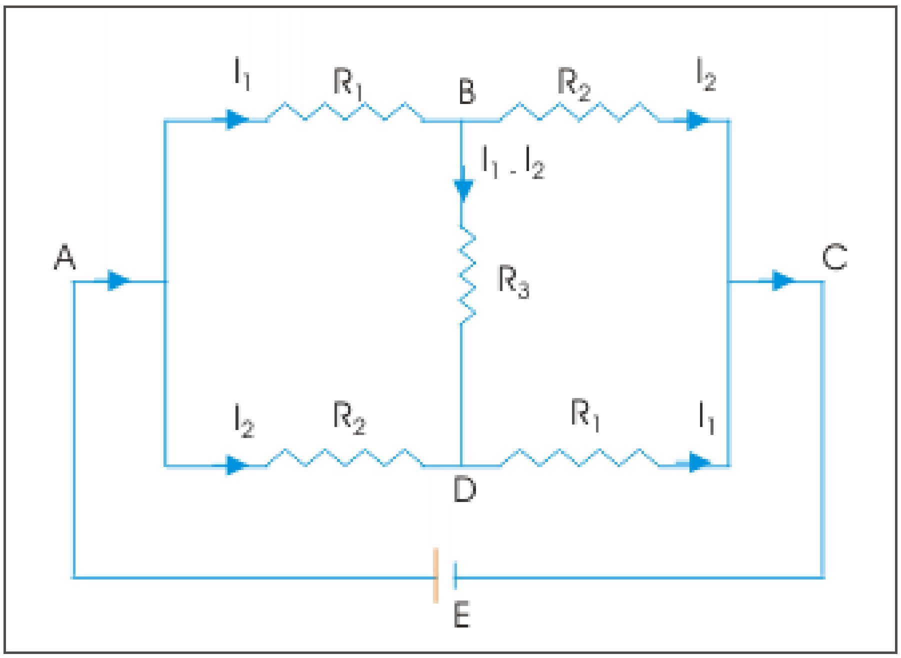


Figure 2.12: Electrical Network

Considering loop ABCEA in clockwise travel, we have KVL equation as :

$$\sum V = -I_1R_1 - I_2R_2 + E = 0$$

$$\Rightarrow E = I_1R_1 + I_2R_2$$

Considering loop ABDA in clockwise travel, we have KVL equation as :

$$\sum V = -I_1R_1 - (I_1 - I_2)\,R_3 + I_2R_2 = 0$$

We solve for I_2 to get an expression for it in terms of I_1 as :

$$\Rightarrow I_2 = \frac{I_1\,(R_1 + R_3)}{(R_2 + R_3)}$$

Substituting above expression of I_2 in the equation obtained earlier for E, we have :

$$\Rightarrow E = I_1 R_1 + \frac{I_1 \left(R_1 + R_3\right) R_2}{\left(R_2 + R_3\right)}$$

$$\Rightarrow I_1 = \frac{\left(R_2 + R_3\right) E}{\left[R_3 \left(R_1 + R_2\right) + 2R_1 R_2\right]}$$

Putting this expression for I_1 in the expression obtained earlier for I_2, we have :

$$\Rightarrow I_2 = \frac{\left(R_1 + R_3\right) E}{\left[R_3 \left(R_1 + R_2\right) + 2R_1 R_2\right]}$$

But, we know that :

$$I = I_1 + I_2$$

$$\Rightarrow I = \frac{\left(R_1 + R_2 + 2R_3\right) E}{\left[R_3 \left(R_1 + R_2\right) + 2R_1 R_2\right]}$$

Thus,

$$\Rightarrow R_{eq} = \frac{E}{I} = \frac{\left[R_3 \left(R_1 + R_2\right) + 2R_1 R_2\right]}{\left(R_1 + R_2 + 2R_3\right)}$$

Solution to Exercise 2.2 (p. 27)

We assign currents with directions in different branches as shown in the figure. We can, however, assign current directions in any other manner we wish. Here, starting from I_1 and I_2 in branches CA and AB respectively and applying KCL at A, the current in AD is $I_1 - I_2$. Let the current in FE is I_3. Applying KCL at B, current in BC is $I_2 + I_3$. Again applying KCL at C, current in CD is $I_2 + I_3 - I_1$.

Electrical Network

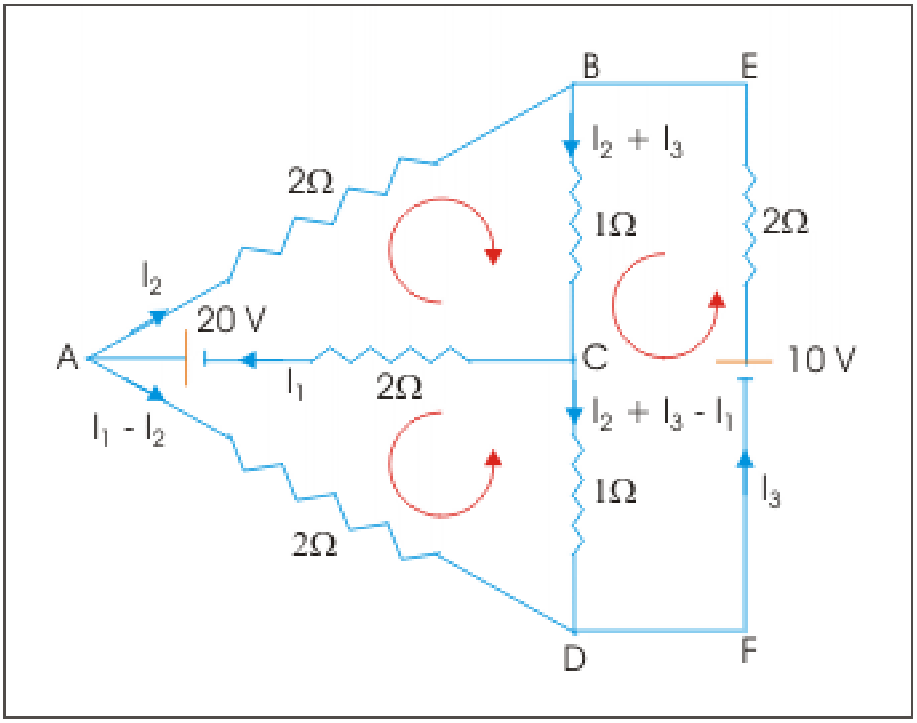


Figure 2.13: Electrical Network

Considering loop ABCA in clockwise travel, we have KVL equation as :

$$\sum V = -2I_2 - 1\left(I_2 + I_3\right) - 2I_1 + 20 = 0$$

$$\Rightarrow 2I_1 + 3I_2 + I_3 = 20$$

Considering loop ADCA in anticlockwise travel, we have KVL equation as :

$$\sum V = -2\left(I_1 - I_2\right) + \left(I_2 + I_3 - I_1\right) - 2I_1 + 20 = 0$$

$$\Rightarrow 5I_1 - 3I_2 - I_3 = 20$$

Considering loop BCDFEB in anticlockwise travel, we have KVL equation as :

$$\sum V = -\left(I_2 + I_3\right) - \left(I_2 + I_3 - I_1\right) + 10 - 2I_3 = 0$$

$$\Rightarrow -I_1 + 2I_2 + 4I_3 = 10$$

We have three equations with three variables. Solving, we have :

$$I_1 = 40/7 \quad A; \quad I_2 = 13/7 \quad A; \quad I_3 = 3 \quad A$$

Currents in different branches are :

$$I_{CA} = I_1 = \frac{40}{7} \quad A; \quad I_{AB} = I_2 = \frac{13}{7} \quad A$$

$$I_{AD} = I_1 - I_2 = \frac{40}{7} - \frac{13}{7} = \frac{27}{7} \quad A$$

$$I_{BC} = I_2 + I_3 = \frac{13}{7} + 3 = \frac{34}{7} \quad A$$

$$I_{CD} = I_2 + I_3 - I_1 = \frac{34}{7} - \frac{40}{7} = -\frac{6}{7} \quad A$$

$$I_{DFEB} = I_3 = 3 \quad A$$

Note that current in branch CD is negative. It means that current in the branch is opposite to the assumed direction.

Chapter 3

Biot - Savart Law[1]

Biot – Savart law is the basic law providing a relation between cause and effect in electromagnetism. In electrostatics, Coulomb's law tells us the relation between point charge (cause) and electric field (effect) that the charge produces in its surrounding. Similarly, Biot-Savart's law tells us the relation between current element or moving charge (cause) and magnetic field (effect) that the current element or moving charge produces in its surrounding. Besides, Biot – Savart law is an empirical law (a result of experimental observations) just like Coulomb's law.

Clearly, there is a strong evidence of parallelism in the study of electrostatics and electromagnetism. There is, however, one important distinction between them. The electric field is along the straight line joining charge and the position in space i.e. along displacement vector. The relationship here is linear. The magnetic field, on the other hand, is along the perpendicular direction of the plane constituted by the small current element and displacement vector. This feature of magnetic field introduces a new dimension to the formulation of Biot-Savart law. We have to compulsorily rely on vector notations and operations. In a nutshell, we are required to be a bit conscious of the direction of magnetic field, which often requires visualization in three dimensional space.

3.1 Magnetic field due to small thin current element

The Biot – Savart law is formulated in a restricted context. This law is true for (i) a small element "**dl** " of a thin wire carrying current and (ii) steady current i.e. flow of charge per unit time through the wire is constant. Biot – Savart's law for free space is given by :

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I\mathbf{l}X\mathbf{r}}{r^3}$$

[1]This content is available online at <http://cnx.org/content/m31057/1.17/>.

Magnetic field due to small thin current element

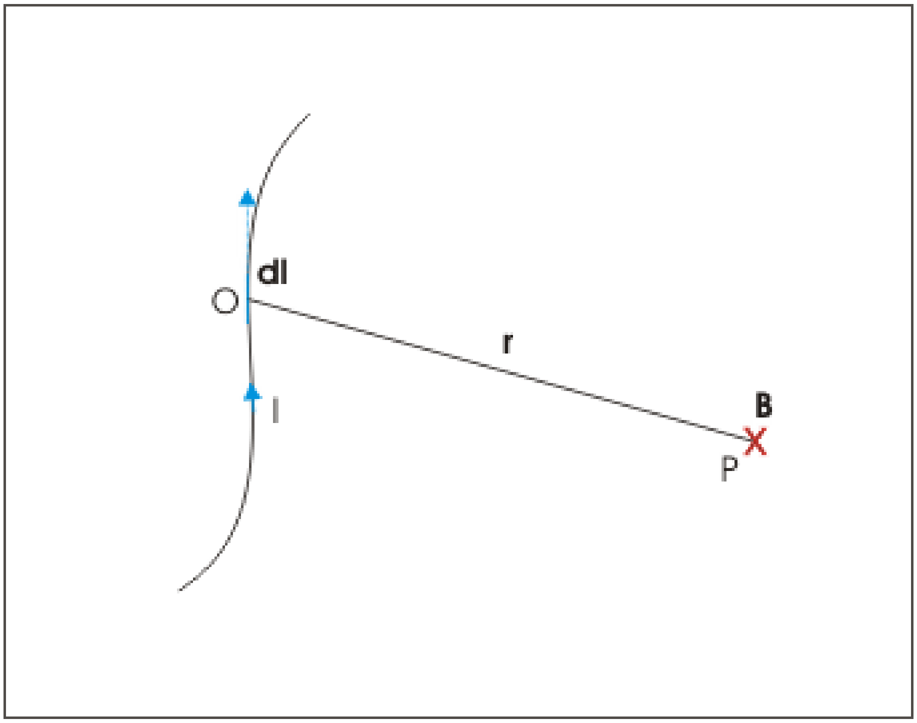


Figure 3.1: Magnetic field acts perpendicular to the drawing plane containing wire.

The ratio $\mu_0/4\pi$ is the proportionality constant and has the value of 10^{-7}Tm/A. The constant μ_0 is known as permeability of free space. The SI unit of magnetic field is Tesla (T), which is defined in the context of magnetic force on a moving charge in magnetic field (See module Lorentz force (Section 7.3: Magnetic field (B))). It is expressed as 1 Newton per Ampere - meter. Further, the vector representation of small length element of wire "d**l**" in the expression is referred as "current length element" and the vector "Id**l**" is referred simply as "current element". The direction of current length vector "d**l**" is the direction of tangent drawn to it in the direction of current in the wire.

Note the vector cross product in the numerator. Direction of magnetic field produced is given by the direction of vector cross product d**lXr** . Further, it is also clear that as far as magnitude of magnetic field is concerned it is inversely proportional to the square of the linear distance i.e. $1/r^2$ (one of r in the numerator cancels with that in the denominator). This means that Biot-Savart Law is also inverse square law like Coulomb's law.

Now, the unit vector in the direction of line joining current element and point is given by :

$$\hat{\mathbf{r}} = \frac{\mathbf{r}}{r}$$

$$\Rightarrow \mathbf{r} = r\,\hat{\mathbf{r}}$$

Substituting in the Biot-Savart expression for **r**, we have :

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I\mathbf{l}X\,\hat{\mathbf{r}}}{r^2}$$

Some important deductions arising from Biot-Savart law are given in the following subsections.

3.1.1 Direction of magnetic field and superposition principle

The direction of magnetic field is the direction of vector cross product **dl X r**. In the figure shown below, the wire and displacement vector are considered to be in the plane of drawing (xy plane). Clearly, direction of magnetic field is perpendicular to the plane of drawing. In order to know the orientation, we align or curl the fingers of right hand as we travel from vector **dl** to vector **r** as shown in the figure. The extended thumb indicates that magnetic field is into the plane of drawing (-z direction), which is shown by a cross (X) symbol at point P.

Magnetic field due to small thin current element

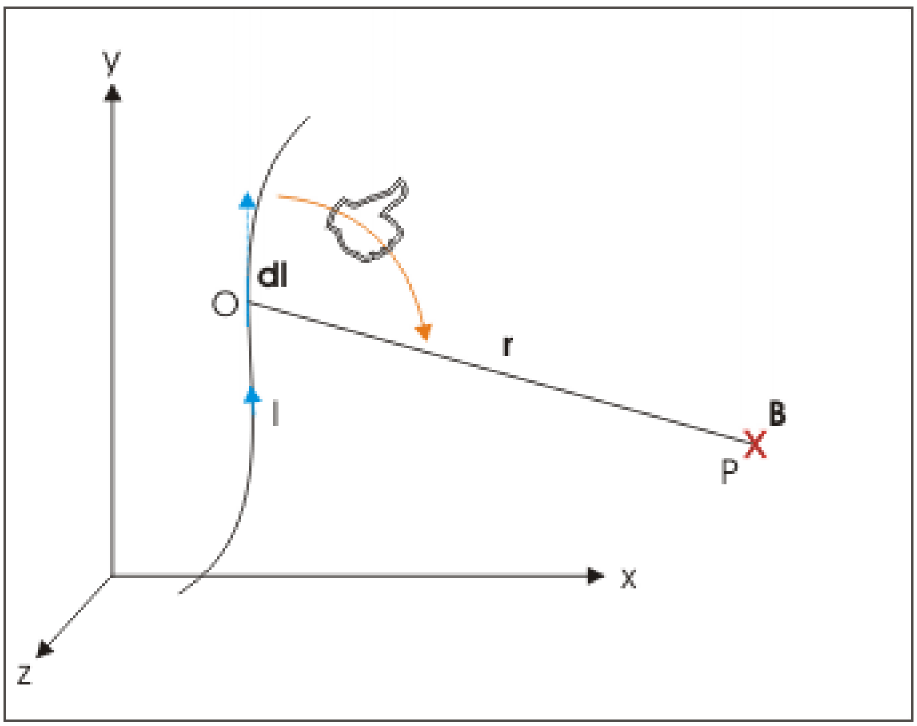


Figure 3.2: Magnetic field acts perpendicular to the drawing plane

This was a simplified situation. What if wire lies in three dimensional space (not in xy plane of reference shown in figure) such that different parts of the wire form different planes with displacement vectors. In such situations, magnetic fields due to different current elements of the current carrying wire are in different directions as shown here.

Magnetic field due to small thin current element

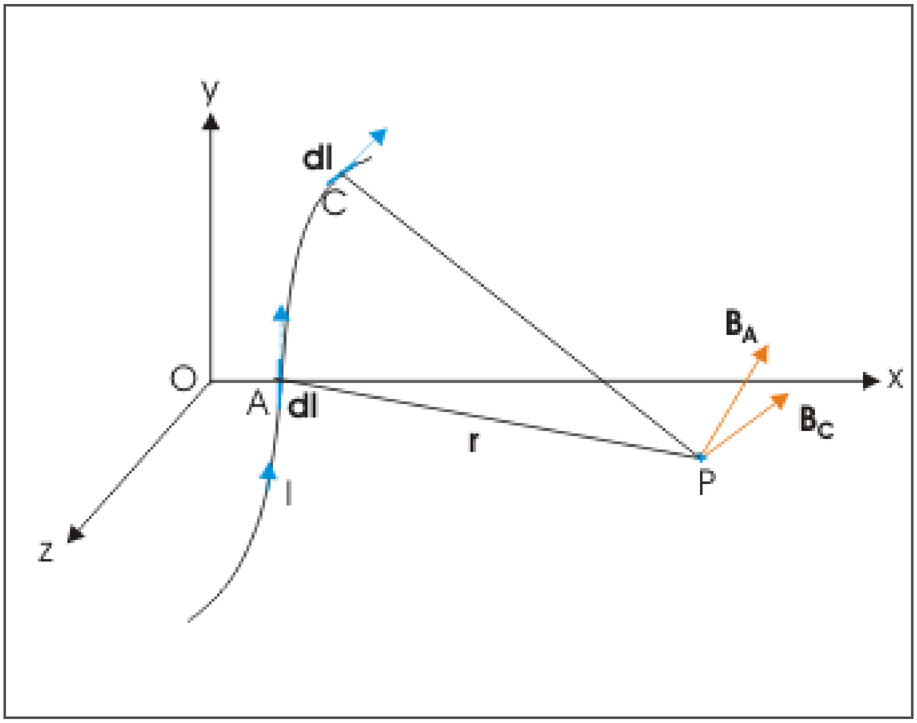


Figure 3.3: Magnetic field acts perpendicular to the plane formed by current element and displacement vectors

It is clear that directions of magnetic field due to different elements of the wire may not be along the same line. On the other hand, a single mathematical expression such as that of Biot-Savart can not denote multiple directions. For this reason, Biot-Savart's law is stated for a small element of wire carrying current – not for the extended wire carrying current. However, we can find magnetic field due to extended wire carrying current by using superposition principle i.e. by using vector additions of the individual magnetic fields due to various current elements. We shall see subsequently that as a matter of fact we can integrate Biot-Savart's vector expression for certain situations like straight wire or circular coil etc as :

$$\mathbf{B} = \int \mathbf{B} = \int \frac{\mu_0}{4\pi} \frac{I \mathrm{l} X \hat{\mathbf{r}}}{r^2}$$

For better appreciation of directional property of magnetic field, yet another visualization of three dimensional representation of magnetic field due to a small element of current is shown here :

Magnetic field due to small thin current element

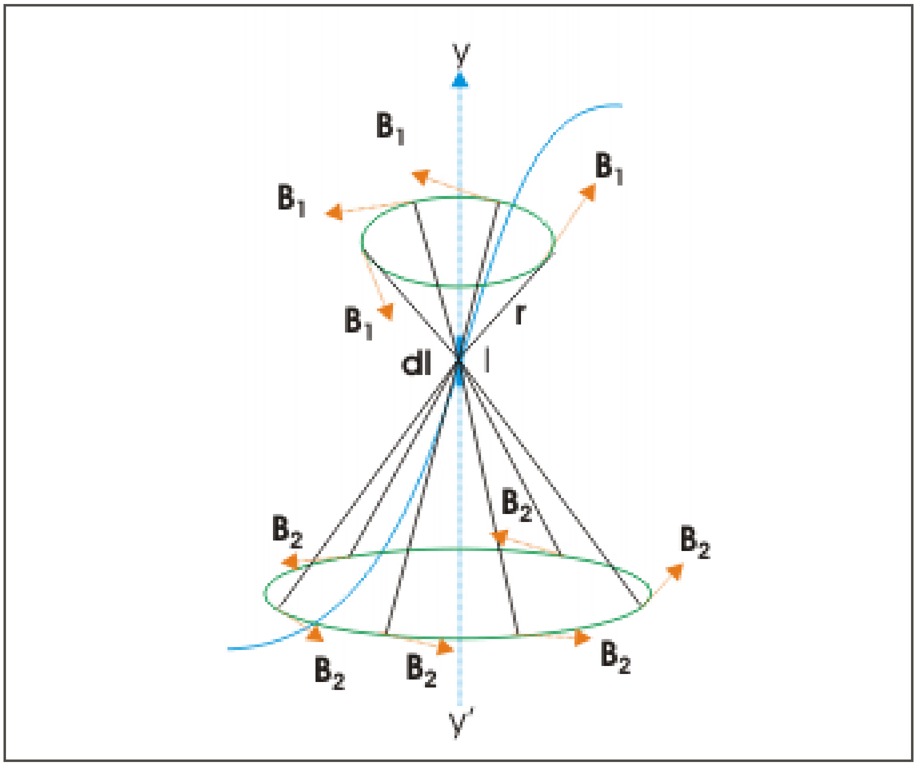


Figure 3.4: Magnetic field acts perpendicular to the plane formed by current element and displacement vectors

The circles have been drawn such that their centers lie on the tangent YY' drawn along the current length element **dl** and the planes of circles are perpendicular to it as shown in the figure. Note that magnetic field being perpendicular to the plane formed by vectors **dl** and **r** are tangential to the circles drawn. Also, each point on the circle is equidistant from the current element. As such, magnitudes of magnetic field along the circumference are having same value. Note, however, that they have shown as different vectors $\mathbf{B}_1$, $\mathbf{B}_2$ etc. as their directions are different.

For the time being we shall use the right hand rule for the vector cross product to determine the direction of magnetic field for each current element. There are, however, few elegant direction finding rules for cases of extended wires carrying current like straight wire or circular coil. These rules will be described in separate modules on the respective topics.

3.1.2 Magnitude of magnetic field

The magnitude of magnetic field is given by :

$$B = \frac{\mu_0}{4\pi}\frac{Il\sin\theta}{r^2}$$

The magnitude depends on angle (θ) between two vector elements "d**l**" and "**r**". For a point on the wire element or on the tangent drawn to it, the angle $\theta = 0°$ or $180°$ and the trigonometric sine ratio of the angle is zero i.e. $\sin\theta = 0$. Thus, magnetic field at a point on the extended line passing through vector "d**l**" is zero.

Further magnetic field is very small due to small value of proportionality constant, which is equal to 10^{-7} SI unit. The relative weakness of magnetic field is evident from the fact that proportionality constant for Coulomb's law has the value $9X10^9$ in SI unit.

3.1.3 Other form of Biot – Savart's law

We have stated earlier that source of magnetic field is a small element of current or a moving charge. After all, current is nothing but passage of charge. Clearly, there needs to be an alternative expression for the Biot-Savart's law in terms of charge and its velocity. Now, for steady current :

$$Il = \frac{q}{t}l = q\frac{l}{t} = qv$$

This equivalence for current with moving charge with respect to production of magnetic field helps us to formulate Biot – Savart' law for a charge q, which is moving with constant speed v as :

$$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{q\mathbf{v}X\hat{\mathbf{r}}}{r^2}$$

The equivalence noted for current and moving charge is quite interesting for sub-atomic situations. An electron moving around nucleus can be considered to be equivalent to current. In Bohr's atom,

$$I = \frac{-e}{T}$$

where T is time period of revolution. Now,

$$T = \frac{2\pi r}{v}$$

where v is the speed of electron moving around. Combining above two equations, we have :

$$\Rightarrow I = -\frac{ev}{2\pi r}$$

Thus, an electron moving in circular path is equivalent to a steady current I. Negative sign here indicates that the equivalent current is opposite to the direction of motion of electron around nucleus.

3.1.4 The source (cause) of magnetic field

The basic source (cause) of electric field is a scalar point charge. What is the correspondence here? Is current (I) the corresponding basic source for the magnetic field? An examination of the Biot – Savart's law reveals that it is not "I" alone which is basic source (cause) – rather it is the vector Id**l**, referred as "current element". This means that the source responsible for magnetic field is identified by current (I) and length of element (dl) together. Equivalently, the basic source of magnetism is a moving charge represented by the vector q**v**.

3.2 Experimental verification of Biot-Savart's law

Current flows through a closed circuit. As such, it would be difficult to determine magnetic field due to a small current element as required for verification of Biot-Savart's law. There is, however, a cleverly designed circuit arrangement which allows us to approximate requirements of determining magnetic field due to small current element. Look at the circuit arrangement shown in the figure. The parts of the wire along AB and CD when extended meet at point P. We arrange the layout in such a manner that the segment AD represents a small current element. The direction of magnetic field produced at P due to this small current element is out of the plane of drawing (shown by a filled circle i.e. dot) as current flows upward in the arm AD (apply right hand vector product rule).

Magnetic field due to small thin current element

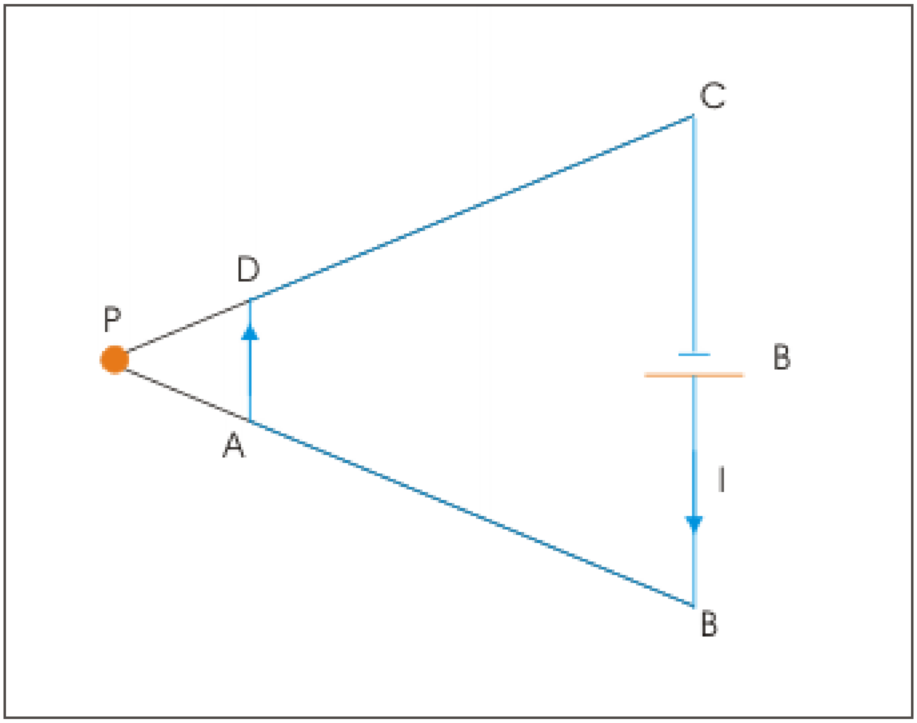


Figure 3.5: Magnetic field acts perpendicular to the plane formed by current element and displacement vectors

The current in the arm AB and CD do not produce magnetic field at point P as the point lies on the extended line of the current length element. Recall that θ=0, sinθ=0, hence B=0. On the other hand the wire segment BC is designed to be far off from point P in comparison to small wire segment AD. Since magnetic field due to individual current element of segment AD is inversely proportional to the square of linear distance, the magnetic field at P due to AC is relatively negligible with respect to magnetic field due to small wire element AD. Clearly, magnetic field at P is nearly equal to magnetic field due to small current element AD. The measurement of magnetic field at P with this arrangement allows us to determine magnetic field due to small current element AD and thus, allows us to verify the law.

3.3 Electromagnetism

We study magnetism under the nomenclature "electromagnetism" to emphasize that magnetism is actually a specific facet of electrical phenomenon. This is not farther from the reality as well. Let us see what happens when charge flows through the wire. Every particle carrying charge is capable of producing electrical field. In this case of a wire carrying steady current, however, charge is moving with certain velocity through the wire (conductor). Though, there is net velocity associated with the charge, the net electric charge in any infinitesimal volume element is zero. This means that the "charge density" at any point is zero but the "current density" at that point is non-zero for a conductor carrying current.

Since there is no charge density, there is no electric field. Recall that a net charge stationary or moving produces electric field. On the other hand, since there is net motion of charge, there is magnetic field.

Subsequently, we shall learn that a varying or changing magnetic field sets up an electric field. This aspect is brought out by Faraday's induction law. The electromagnetic induction sets up the basis of interlinking of electrical and magnetic phenomena. The production of electric field (and hence current in a conductor) due to varying magnetic field suggests that its inverse should also be true. As a matter of fact, this is so. Maxwell discovered that a varying electric field sets up a magnetic field. Thus, two phenomena are reciprocal of each other and prove the strong connection between electricity and magnetism.

In general, we consider electrical property to be the precursor of magnetic property. One of the most important arguments that advances this thinking is the existence of electrical monople i.e. a charge of specific polarity. There is no such magnetic monopole as yet. Magnetic polarities exist in pair (recall a magnet has a pair of north and south pole).

The connection between electric and magnetic field is futher verified by the fact that a stationary charge in one frame of reference sets up only electric field in that reference. But the same stationary charge in one frame of reference sets up both electric and magnetic fields in a frame of reference, which is moving at certain relative velcoity with respect to first reference. Similarly, a moving charge in one frame of reference sets up both electric and magnetic fields in that frame of reference, but it sets up only electric field in a reference in which the moving charge is stationary (we can always imagine one such frame to exist).

The above discussion also draws an important distinction between “current in wire” and “moving charge”, which have been said to be equivalent in earlier text. Current in wire sets up only magnetic field. Moving charge, on the other hand, sets up magnetic field in addition to electric field as there is net charge – unlike the case of current in wire in which there is no net charge. Clearly, equivalence of "current in small element of wire" and "moving charge" is limited to production of magnetic field only.

Chapter 4

Magnetic field due to current in straight wire[1]

The Biot-Savart law allows us to calculate magnetic field due to steady current through a small element of wire. Since direction of magnetic field due to different current elements of an extended wire carrying current is not unique, we need to add individual magnetic vectors to obtain resultant or net magnetic field at a point. This method of determining the net magnetic field follows superposition principle, which says that magnetic fields due to individual small current element are independent of each other and that the net magnetic field at a point is obtained by vector sum of individual magnetic field vectors :

$$\mathbf{B} = \sum \mathbf{B}_i = \mathbf{B}_1 + \mathbf{B}_2 + \mathbf{B}_3 + \ldots\ldots$$

We calculate magnetic field due to individual current element (I d**l**) using Biot-Savart law :

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\mathbf{l}X\mathbf{r}}{r^3}$$

where "d**l**" is referred as "current length element" and "I d**l**" as "current element".

In the case of a straight wire, the task of vector addition is simplified to a great extent because direction of magnetic field at a point due to all current elements comprising the straight wire is same.

4.1 Direction of magnetic field (Right hand thumb rule)

A straight line and a point constitute an unique plane. This is true for all points in three dimensional rectangular space (x,y,z). For convenience, let us consider that the point of observation (P) lies in xy plane as shown in the figure below. We can say that the straight wire along y-axis also lie in xy plane. Clearly, this plane is the plane of current length element d**l** and displacement vector **r**, which appear in the Biot-Savart expression. The direction of magnetic field is vector cross product d**l**X **r**, which is clearly perpendicular to the plane xy. This means that the magnetic field is along z-axis. This conclusion is independent of the relative positions of current length elements of the wire with respect to observation point P.

In a nutshell, we conclude that the directions of magnetic fields due to all current elements constituting straight wire at a point P are same. Though, magnitudes of magnetic fields are different as different current elements are located at different linear distance from the point i.e. displacement vectors (**r**) are different for different current length elements (d**l**).

[1]This content is available online at <http://cnx.org/content/m31103/1.10/>.

Magnetic field due to current in straight wire

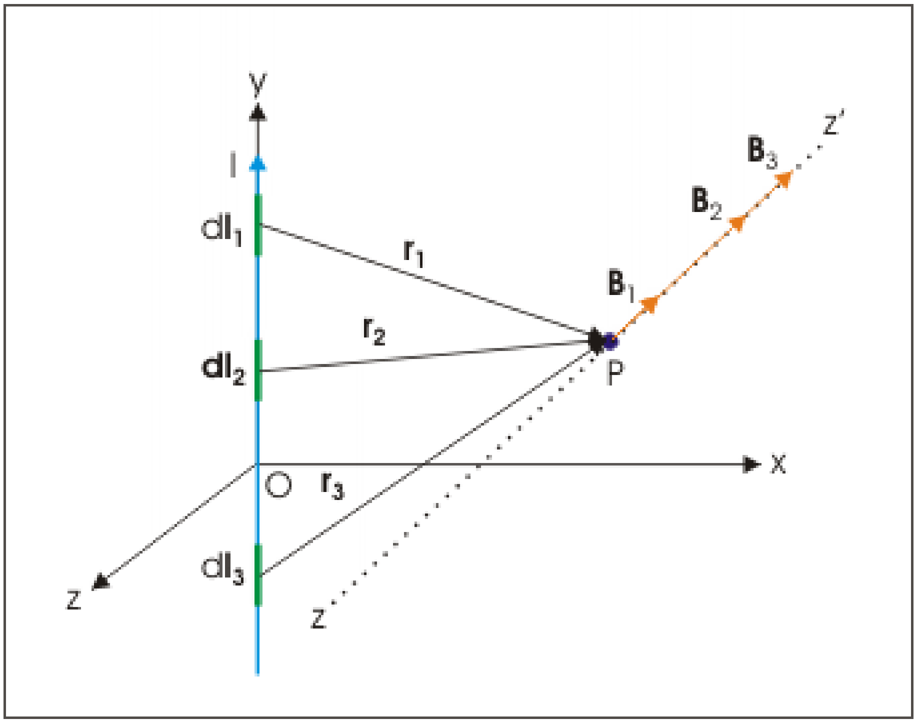


Figure 4.1: Magnetic fields due to all current elements constituting straight wire at a point P are same.

See in the figure how magnetic fields due to three current elements in positive y-direction are acting in negative z-direction. The magnetic fields due to different current elements are $\mathbf{B}_1$, $\mathbf{B}_2$ and $\mathbf{B}_3$ acting along PZ' as shown in the figure. Note that magnitudes of magnetic fields are not equal as current elements are positioned at different linear distance.

The magnetic field is along z-axis either in positive or negative z direction depending on the direction of current and whether observation point is on right or left of the current carrying straight wire. By convention, magnetic field vector into the plane of drawing is denoted by a cross (X) and magnetic field vector out of the plane of drawing is denoted by a dot (.). Following this convention, magnetic field depicted on either side of a current carrying straight wire is as shown here :

Magnetic field due to current in straight wire

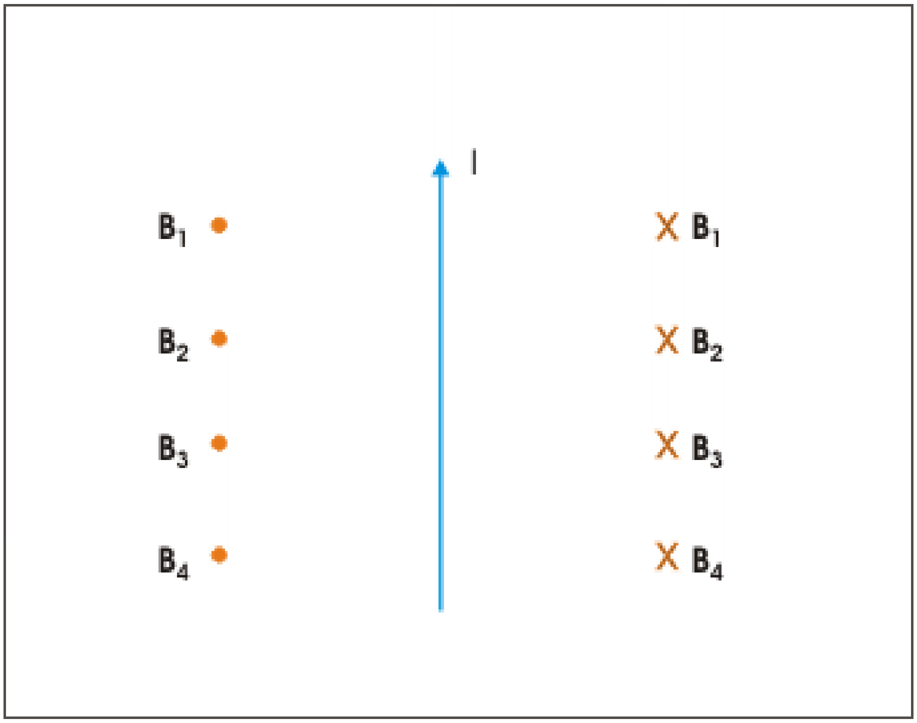


Figure 4.2: Representation of magnetic field in terms of cross and dot.

Here $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$ and $\mathbf{B}_4$ are the net magnetic fields at four different positions due to all current elements of the wire. If we draw a circular path around the straight wire such that its plane is perpendicular to the wire and its center lies on it, then each point on the perimeter is equidistant from the center. As such magnitudes of magnetic field on all points on the circle are equal. The direction of magnetic field as determined by right hand vector cross product rule is tangential to the circle.

Magnetic field due to current in straight wire

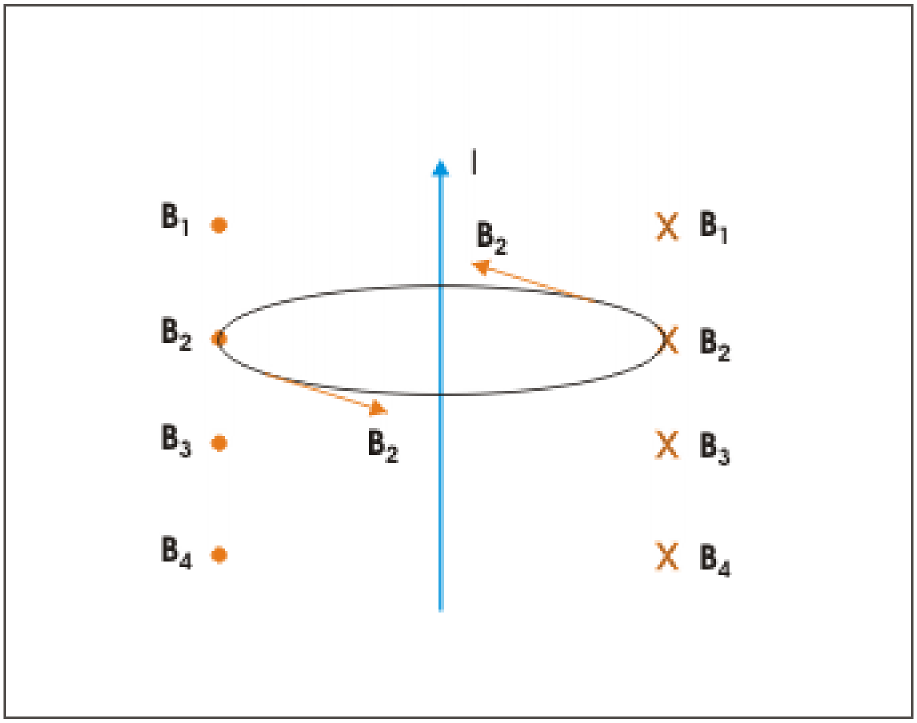


Figure 4.3: Magnetic field on the perimeter of circle is tangential.

The observations as above are the basis of **Right hand thumb rule** for finding direction of magnetic field due to current in straight wire. If holding straight wire with right hand so that the extended thumb points in the direction of current, then curl of the fingers gives the direction of magnetic field around the straight wire.

Right hand thumb rule

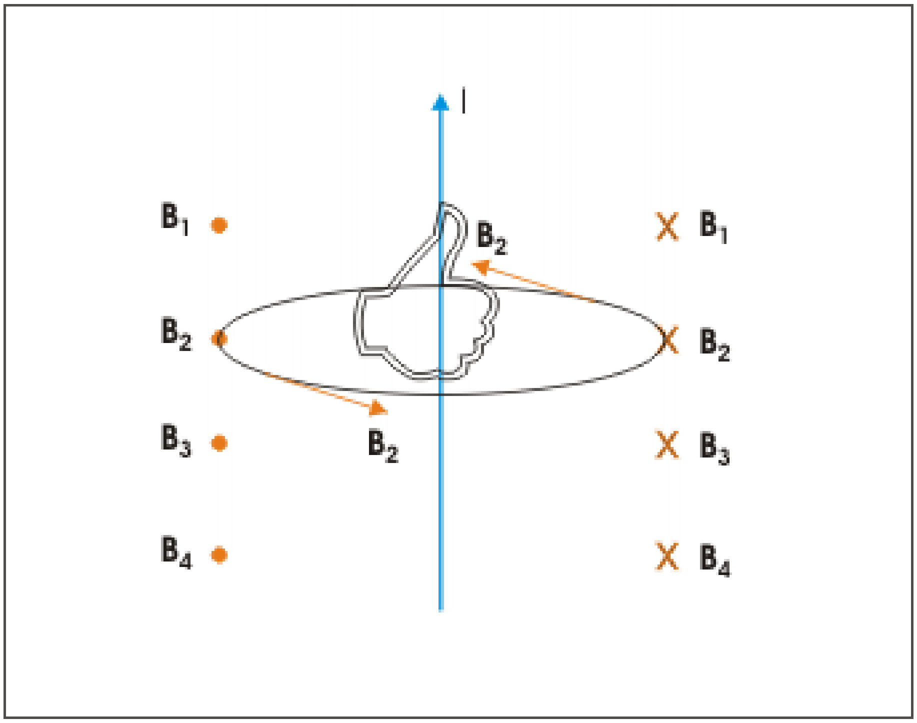


Figure 4.4: If holding straight wire with right hand so that the extended thumb points in the direction of current, then curl of the fingers gives the direction of magnetic field around the straight wire.

4.2 Magnetic field due to current in finite straight wire

Since directions of magnetic fields due to all current elements are same, we can integrate the expression of magnitude as given by Biot-Savart law for the small current element (we have replaced dl by dy in accordance with notation in the figure) :

$$B = \int \mathrm{B} = \frac{\mu_0}{4\pi} \int \frac{Iy\sin\theta}{r^2}$$

Magnetic field due current in finite straight wire

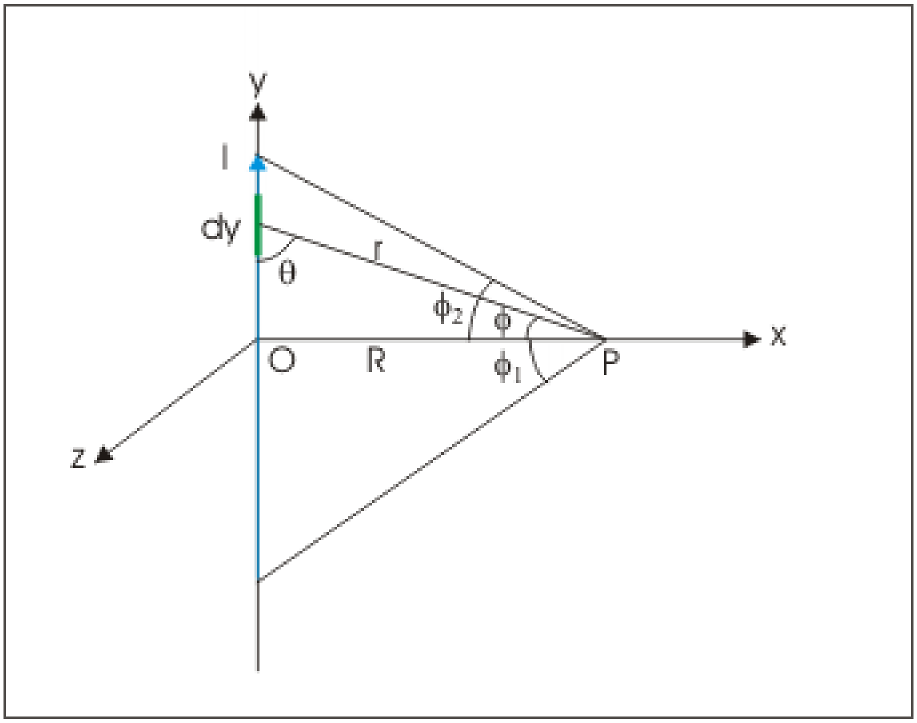


Figure 4.5: Magnitude of magnetic field is obtained by integration of elemental magnetic field.

In order to evaluate this integral in terms of angle ϕ, we determine đy, r and θ in terms of perpendicular distance "R" (which is a constant for a given point) and angle "ϕ". Here,

$$y = R\tan\phi$$

$$dy = R\sec^2\phi\phi$$

$$r = R\sec\phi$$

$$\theta = \frac{\pi}{2} - \phi$$

Substituting in the integral, we have :

$$\Rightarrow B = \frac{\mu_0}{4\pi}\int \frac{IR\sec^2\phi\phi\sin\left(\frac{\pi}{2}-\phi\right)}{R^2\sec^2\phi} = \frac{\mu_0}{4\pi}\int \frac{I\cos\phi\phi}{R}$$

Taking out I and R out of the integral as they are constant :

$$\Rightarrow B = \frac{\mu_0 I}{4\pi R}\int \cos\phi\phi$$

Integrating between angle "ϕ_1" and "ϕ_2", we have :

$$\Rightarrow B = \frac{\mu_0 I}{4\pi R} \int_{\phi_1}^{\phi_2} I \cos\phi\phi$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi R} (\sin\phi_2 - \sin\phi_1)$$

We follow the convention whereby angle is measured from perpendicular line. The angle below perpendicular line is treated negative and angle above perpendicular line is positive. In case, we want to do away with the sign of angle, we put $\phi_1 = -\phi_1$ in above equation :

$$\Rightarrow B = \frac{\mu_0 I}{4\pi R} (\sin\phi_1 + \sin\phi_2)$$

Note that angles being used with this expression are positive numbers only. Also note that the magnitude of magnetic field depends on where the point of observation P lies with respect to straight wire, which is reflected in the value of angle ϕ.

We can also express magnetic field due to current in a straight wire at a perpendicular distance "R" in terms of angles between straight wire and line joining point of observation and end points.

Magnetic field due to current in wire

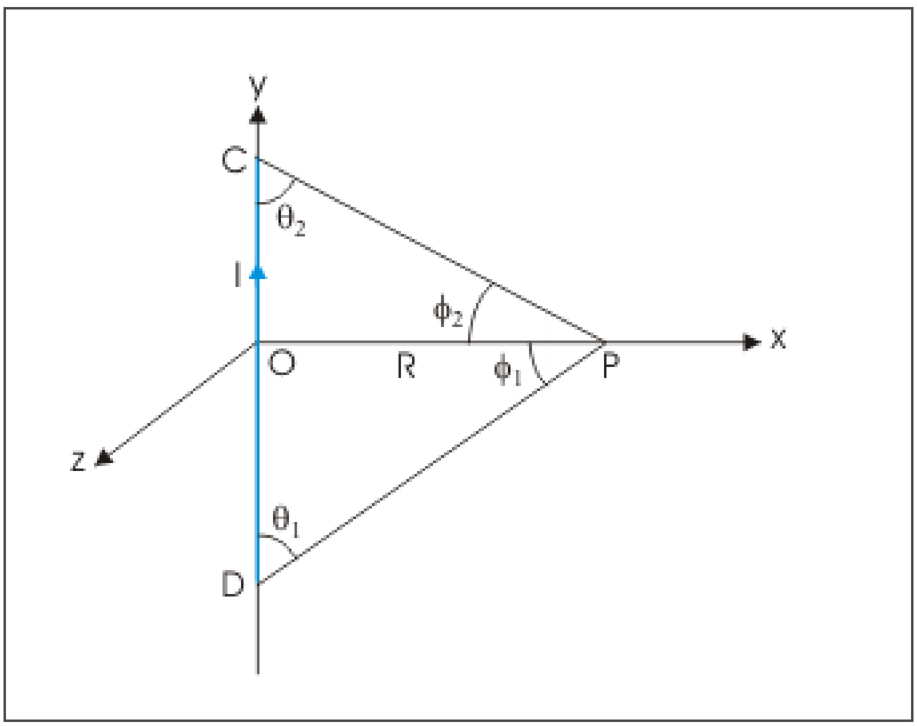


Figure 4.6: Magnetic field due current in finite straight wire

$$B = \frac{\mu_0 I}{4\pi R} X \left[\sin\left(\frac{\pi}{2} - \theta_1\right) + \sin\left(\frac{\pi}{2} - \theta_2\right)\right]$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi R} X \left[\cos\theta_1 + \cos\theta_2\right]$$

4.2.1 Magnetic field at a point on perpendicular bisector

In this case, angles on either side of the bisector are equal :

$$\phi_1 = \phi_2 = \phi$$

Magnetic field at a point on perpendicular bisector

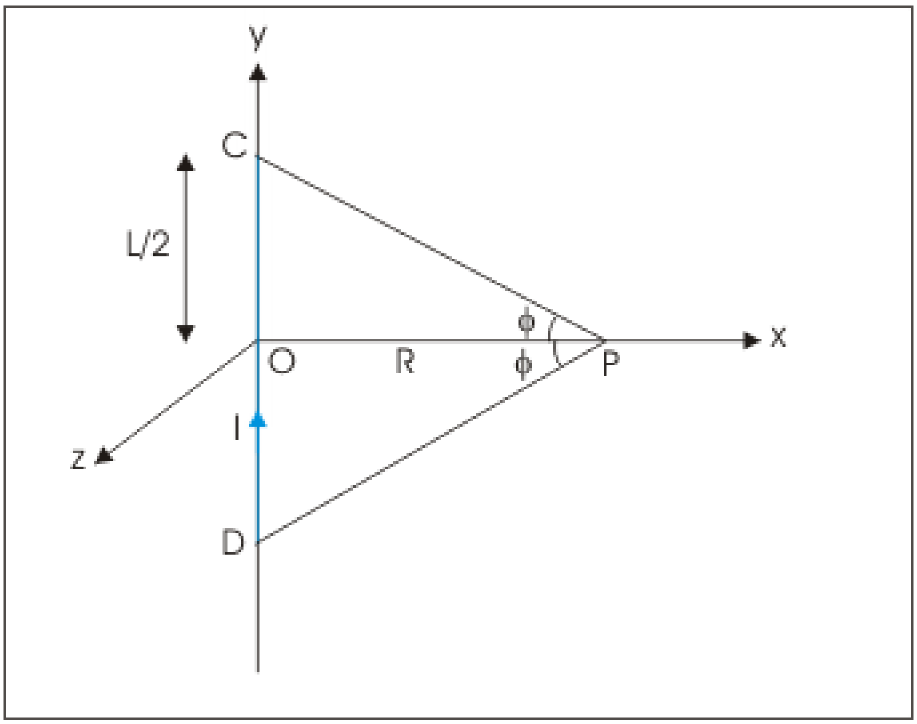


Figure 4.7: Magnetic field at a point on perpendicular bisector

Magnetic field at a point on perpendicular bisector is :

$$B = \frac{\mu_0 I}{4\pi R} X \left[\sin\phi_1 + \sin\phi_2\right] = \frac{\mu_0 I}{4\pi R} X 2\sin\phi$$

$$\Rightarrow B = \frac{\mu_0 I \sin\phi}{2\pi R}$$

Let "L" be the length of wire. Then,

$$\sin\phi = \frac{OC}{PC} = \frac{\frac{L}{2}}{\sqrt{\left\{\left(\frac{L}{2}\right)^2 + R^2\right\}}} = \frac{L}{\sqrt{(L^2 + 4R^2)}}$$

Putting in the equation of magnetic field,

$$\Rightarrow B = \frac{\mu_0 I \sin\phi}{2\pi R} = \frac{\mu_0 I L}{2\pi R\sqrt{(L^2+4R^2)}}$$

Example 4.1

Problem : A square loop of side "L" carries a current "I". Determine the magnetic field at the center of loop.

Solution : The magnetic field due to each side of the square here is same as Magnetic field due to current in straight wire at a distance L/2 on the perpendicular bisector. The magnetic fields due to current in the four sides are in the same direction. Hence, magnitude of magnetic field due to current in loop is four times the magnetic field due to current in one side :

$$\Rightarrow B = 4X\frac{\mu_0 I L}{2\pi R\sqrt{(L^2+4R^2)}}$$

Here, R = L/2

Magnetic field at the center of square loop

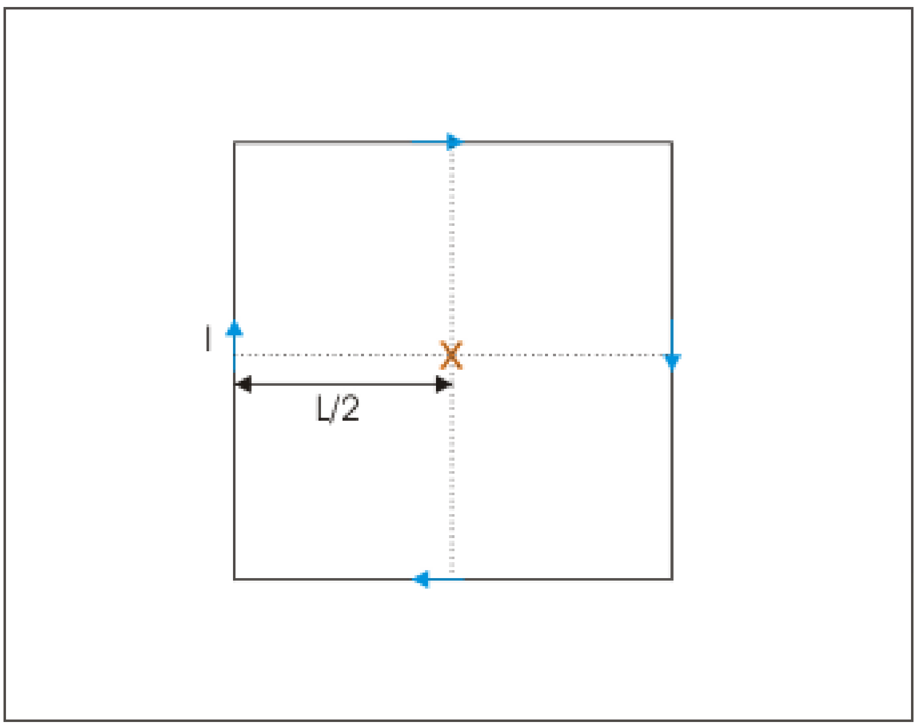


Figure 4.8: Magnetic field at the center of square loop

$$\Rightarrow B = 4X\frac{\mu_0 I L}{2\pi\frac{L}{2}\sqrt{\{L^2+4\left(\frac{L}{2}\right)^2\}}} = \frac{4\mu_0 I L}{\pi L\sqrt{2}L}$$

$$\Rightarrow B = \frac{2\sqrt{2}\mu_0 I}{\pi L}$$

4.2.2 Magnetic field at a point near the end of current carrying finite straight wire

In this case, the angles involved are :

Magnetic field due current in finite straight wire

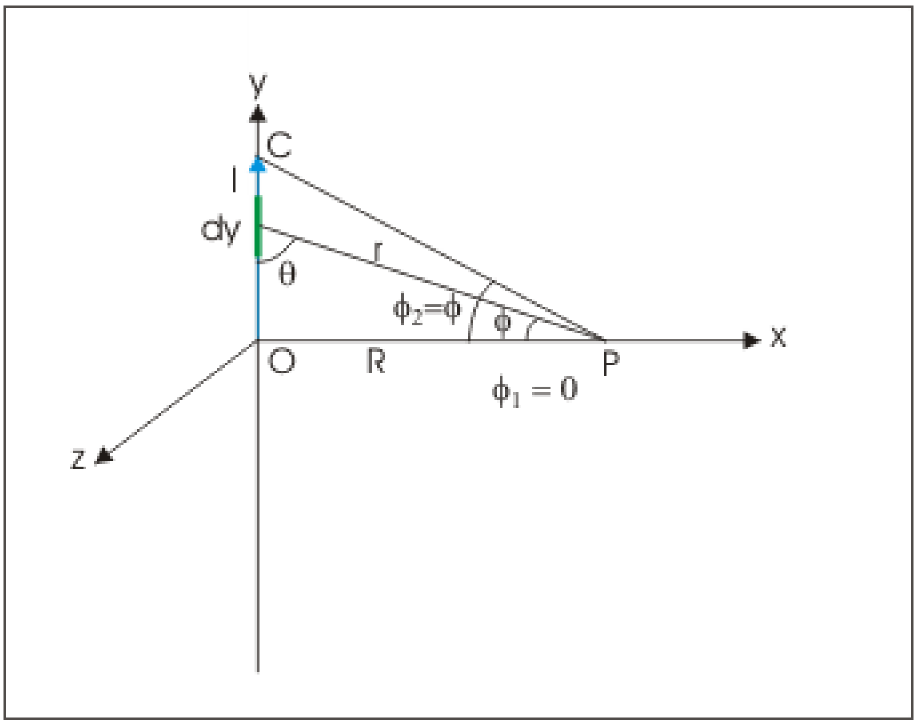


Figure 4.9: Magnitude of magnetic field is obtained by integration of elemental magnetic field.

$$\phi_1 = 0; \quad \phi_2 = \phi$$

and

$$\Rightarrow B = \frac{\mu_0 I}{4\pi R}(\sin 0 + \sin\phi) = \frac{\mu_0 I \sin\phi}{4\pi R}$$

We can also get the expression for magnetic field in terms of length of wire. Here,

$$\sin\phi = \frac{OC}{PC} = \frac{L}{\sqrt{(L^2 + R^2)}}$$

Putting in the expression of magnetic field, we have :

$$B = \frac{\mu_0 I L}{4\pi R\sqrt{(L^2 + R^2)}}$$

In case, R = L, then,

$$\Rightarrow B = \frac{\mu_0 I L}{4\pi L\sqrt{(L^2 + L^2)}} = \frac{\mu_0 I}{4\pi L\sqrt{2}} = \frac{\sqrt{2}\mu_0 I}{8\pi L}$$

Example 4.2
Problem : A current 10 ampere flows through the wire having configuration as shown in the figure. Determine magnetic field at P.

Magnetic field due to current in the arrangment

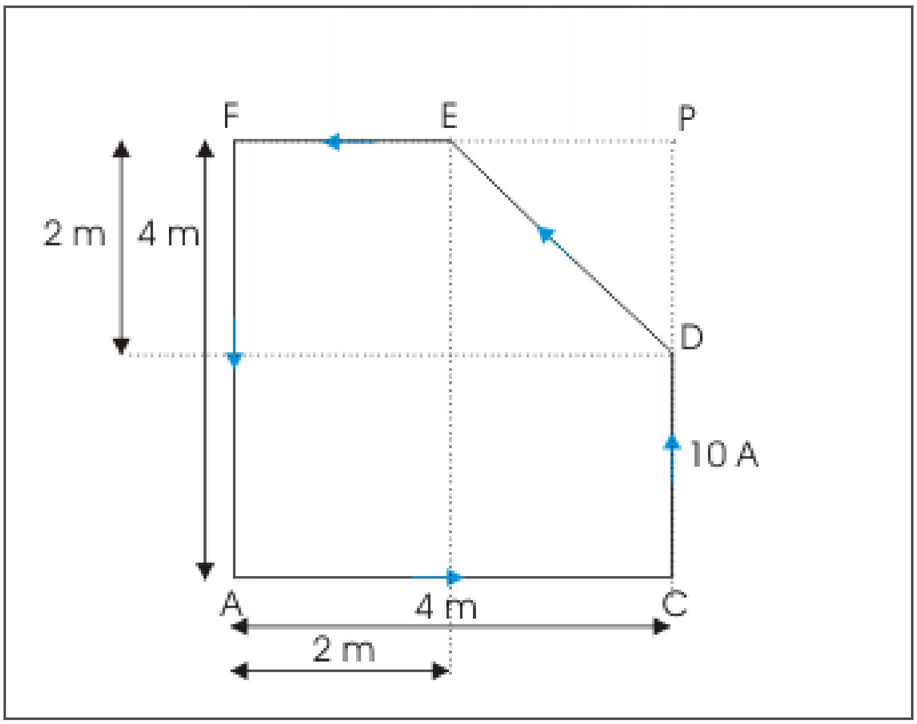


Figure 4.10: Magnetic field due current in the arrangment

Solution : We shall determine magnetic field to different straight segments of wires. Let us consider the out of plane orientation as positive. Now, for wire segment AC, the point P is at the end of straight wire of length 4 m and is at a perpendicular linear distance of 4 m. The magnetic field at P due to segment AC is perpendicular and out of the plane of drawing. The magnetic field due to segment AC is :

$$B_{\text{AC}} = \frac{\sqrt{2}\mu_0 I}{8\pi L} = \frac{\sqrt{2}\mu_0 I}{8\pi X 4} = \frac{\sqrt{2}\mu_0 I}{32\pi}$$

For the wire segment CD, the point P lies on the extended line passing through the wire. The magnetic field due to this segment, therefore, is zero.

$$B_{\mathrm{CD}} = 0$$

For the wire segment DE, the angles between the line segment and line joining the point P with end points are known by geometry of the figure. Hence, Magnetic field due to this segment is :

$$B_{\mathrm{DE}} = -\frac{\mu_0 I}{4\pi R}\left(\cos\theta_1 + \cos\theta_2\right) = -\frac{\mu_0 I}{4\pi\sqrt{2}} X \left(\cos 45^0 + \cos 45^0\right)$$

$$\Rightarrow B_{\mathrm{DE}} = -\frac{\mu_0 I}{4\pi\sqrt{2}} X \frac{2}{\sqrt{2}} = -\frac{\mu_0 I}{4\pi}$$

For the wire segment EF, the point P lies on the extended line passing through the wire. The magnetic field due to this segment, therefore, is zero.

$$B_{\mathrm{EF}} = 0$$

For the wire segment GA, the point P is at the end of straight wire of length 4 m and is at a perpendicular linear distance of 4 m. The magnetic field at P due to segment GA is perpendicular and out of the plane of drawing. The magnetic field is :

$$B_{\mathrm{FA}} = \frac{\sqrt{2}\mu_0 I}{32\pi}$$

The net magnetic field at P is :

$$B = B_{\mathrm{AC}} + B_{\mathrm{CD}} + B_{\mathrm{DE}} + B_{\mathrm{EF}} + B_{\mathrm{FA}}$$

$$\Rightarrow B = \frac{\sqrt{2}\mu_0 I}{32\pi} + 0 - \frac{\mu_0 I}{4\pi} + 0 + \frac{\sqrt{2}\mu_0 I}{32\pi}$$

$$\Rightarrow B = \frac{\mu_0 I\left(\sqrt{2} - 4\right)}{16\pi}$$

$$\Rightarrow B = -\frac{2.59 X 4\pi X 10^{-7} X 10}{16\pi} = -\frac{2.59 X 10^{-7} X 10}{4}$$

$$\Rightarrow B = -0.65 X 10^{-6} = -6.5 X 10^{-5} \quad T$$

The net magnetic field is into the plane of drawing.

4.3 Magnetic field due to current in infinite (long) straight wire

The expression for the magnitude of magnetic field due to infinite wire can be obtained by suitably putting appropriate values of angles in the expression of magnetic field due to finite wire. Here,

$$\phi_1 = \frac{\pi}{2}; \quad \phi_2 = \frac{\pi}{2}$$

and

$$B = \frac{\mu_0 I}{4\pi R}\left(\sin\phi_1 + \sin\phi_2\right)$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi R}\left(\sin\frac{\pi}{2} + \sin\frac{\pi}{2}\right)$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi R}$$

The important point to note here is that magnetic field is independent of the relative angular position of point of observation with respect to infinite wire. Magnetic field, however, depends on the perpendicular distance from the wire.

In reality, however, we always work with finite wire or at the most with long wire. A finite length wire is approximated as infinite or long wire for at least for close points around the wire.

4.3.1 Magnetic field at a point near the end of current carrying long wire

The wire here extends from an identified position to infinity in only one direction. In this case, the angles involved are :

$$\phi_1 = 0; \quad \phi_2 = \frac{\pi}{2}$$

and

$$\Rightarrow B = \frac{\mu_0 I}{4\pi R}\left(\sin 0 + \sin\frac{\pi}{2}\right)$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi R}$$

Example 4.3

Problem : Calculate magnetic field at point P due to current 5 A flowing through a long wire bent at right angle as shown in the figure. The point P lies at a linear distance 1 m from the corner.

Magnetic field due to current in wire

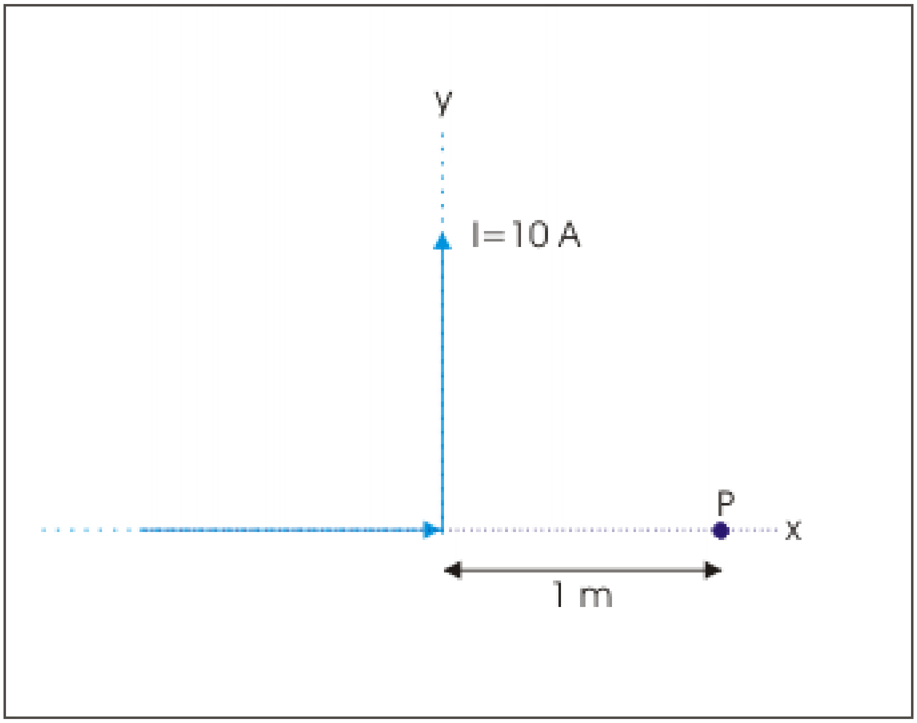


Figure 4.11: Magnetic field due to current in wire.

The point P lies on the extension of wire segment in x-direction. Here angle between current element and displacement vectors is zero i.e. θ =0 and sinθ . As such this segment does not produce any magnetic field at point P. On the other hand, the point P lies near one of the end of the segment of wire in y-direction. The wire being long, the magnetic field due to wire segment in y-direction is :

$$B = \frac{\mu_0 I}{4\pi R}$$

Putting values,

$$\Rightarrow B = \frac{\mu_0 I}{4\pi R} = \frac{10^{-7} X5}{1} = 5X10^{-7} \quad T$$

Applying Right hand thumb rule, the magnetic field at P is perpendicular to xy plane and into the plane of drawing (i.e. negative z-direction).

$$\Rightarrow \mathbf{B} = -5X10^{-7}\mathbf{k}$$

4.4 Exercises

Exercise 4.1 *(Solution on p. 57.)*

Two long straight wires at A and C, perpendicular to the plane of drawing, carry currents such that point D is a null point. The wires are placed at a linear distance of 10 m. If the current in the wire at A is 10 A and its direction is out of the plane of drawing, then find (i) the direction of current and (ii) magnitude of current in the second wire.

Two straight wires carrying current

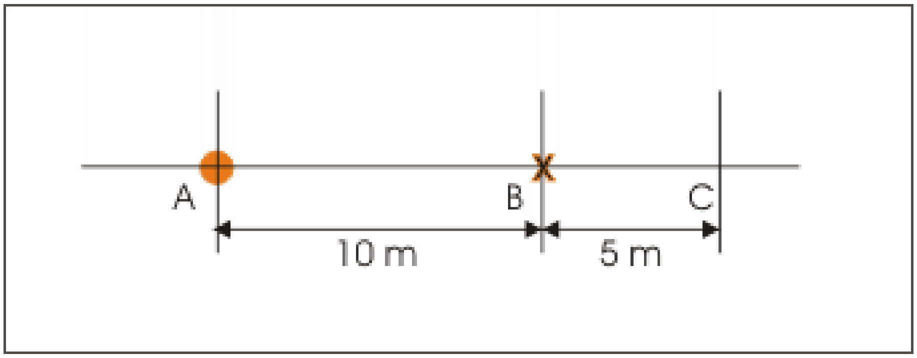


Figure 4.12: Two straight wires carrying current

Exercise 4.2 *(Solution on p. 57.)*

Calculate magnetic field at the center due to current flowing in clockwise direction through a wire in the shape of regular hexagon. The arm of hexagon measures 0.2 m and current through the wire is 10 A.

Exercise 4.3 *(Solution on p. 58.)*

A current $10\sqrt{2}$ ampere flows through the wire having configuration as shown in the figure. Determine magnetic field at P.

Magnetic field due to current in wire

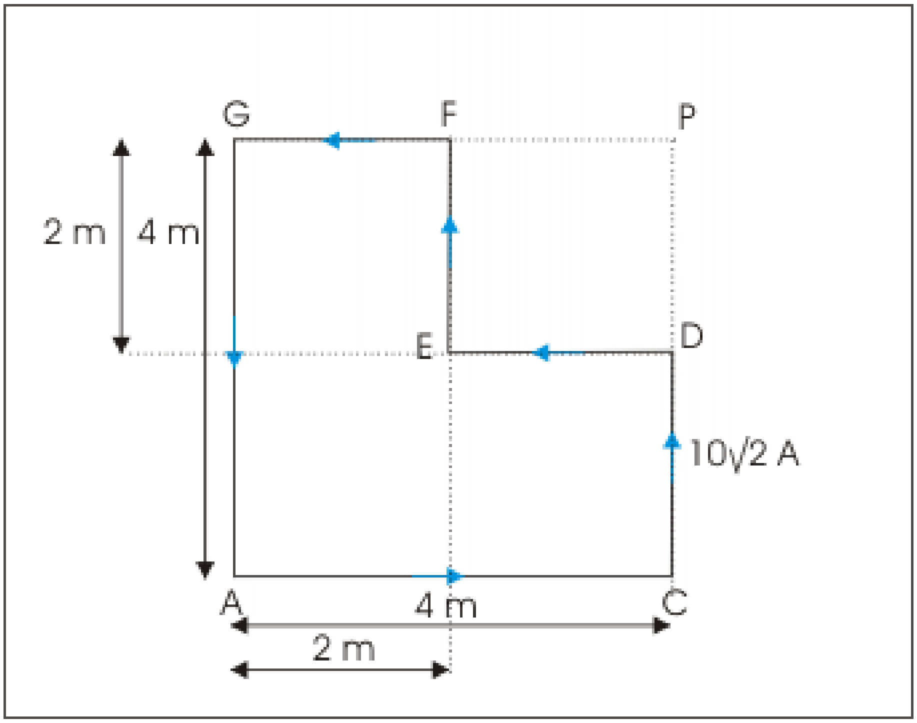


Figure 4.13: Magnetic field due to current in wire

Solutions to Exercises in Chapter 4

Solution to Exercise 4.1 (p. 54)
In order to nullify the magnetic field at D due to current in wire at A, the direction of magnetic field due to current in the wire at C should be equal and opposite. This means that the current in the wire at C should be opposite that of wire at A. Hence, the direction of current in the wire at C should be into the plane of drawing. Now, the magnitudes of magnetic fields due to currents are equal. Let the current in second wire be I, then:

Two straight wires carrying current

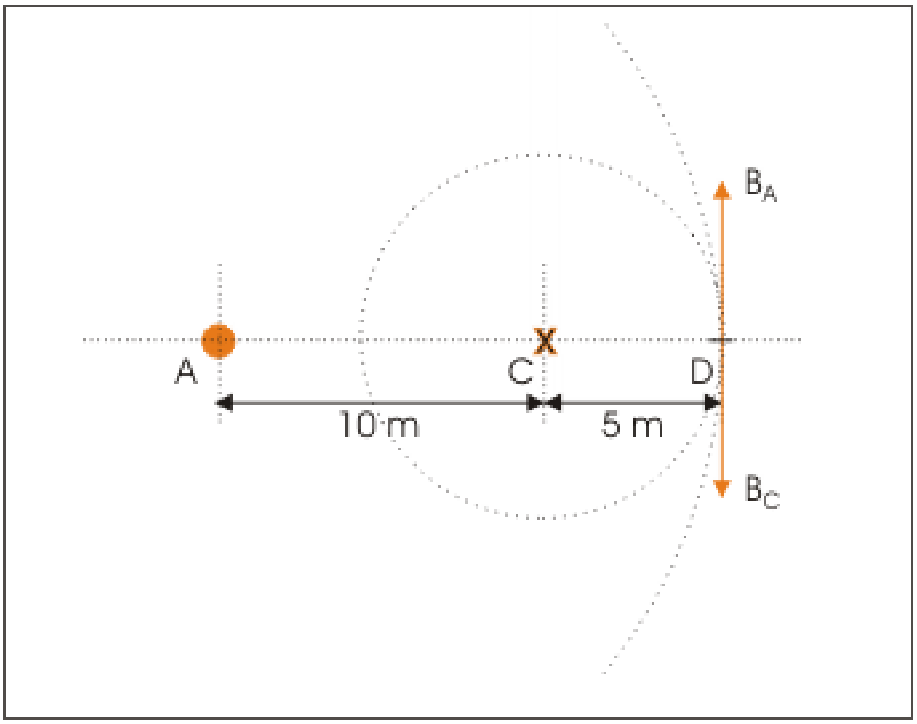


Figure 4.14: Two straight wires carrying current

$$\frac{\mu_0 I}{2\pi X 5} = \frac{\mu_0 X 10}{2\pi X 15}$$

$$\Rightarrow I = \frac{50}{15} = 3.34 \quad A$$

Solution to Exercise 4.2 (p. 55)
Applying right hand rule for vector cross product, we realize that magnetic field due to each arm of the hexagon for given current direction (clockwise) is into the plane of hexagon. As such, we can algebraically add magnetic field due to each arm to obtain net magnetic field.

$$B = 6B_a$$

where B_a is magnetic field due to current in one of the arms. Now, we consider one of the arms of hexagon as shown in the figure. Here,

Magnetic field at the center due current

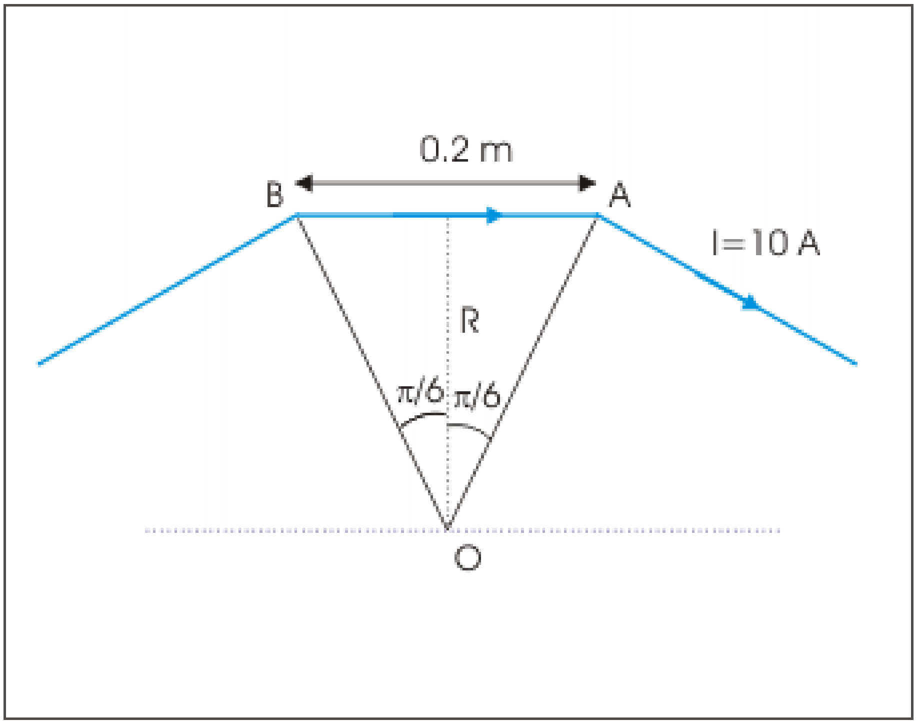


Figure 4.15: Magnitude of magnetic field is six times the magnetic field due to one arm.

$$\phi_1 = \frac{\pi}{6}; \quad \phi_2 = \frac{\pi}{6}$$

$$R = \frac{a}{2}\cot\phi_1 = \frac{0.2}{2}\cot\frac{\phi}{6} = 0.1X\sqrt{3} = 0.1732 \quad m$$

and

$$\Rightarrow B = 6B_a = \frac{6\mu_0 I}{4\pi R}\left(\sin\frac{\pi}{6} + \sin\frac{\pi}{6}\right)$$

Putting values, we have :

$$\Rightarrow B = \frac{6X10^{-7}X10X1}{0.1732} == 3.462X10^{-5} \quad T$$

Solution to Exercise 4.3 (p. 55)
We shall determine magnetic field to different straight segments of wires. Let us consider the out of plane orientation as positive. Now, for wire segment AC, the point P is at the end of straight wire of length 4 m and is at a perpendicular linear distance of 4 m. The magnetic field at P due to segment AC is perpendicular and out of the plane of drawing. The magnetic field due to segment AC is :

$$B_{AC} = \frac{\sqrt{2}\mu_0 I}{8\pi L} = \frac{\sqrt{2}\mu_0 I}{8\pi X4} = \frac{\sqrt{2}\mu_0 I}{32\pi}$$

For the wire segment CD, the point P lies on the extended line passing through the wire. The magnetic field due to this segment, therefore, is zero.

$$B_{CD} = 0$$

For the wire segment DE, the point P is at the end of straight wire of length 2 m and is at a perpendicular linear distance of 2 m. The magnetic field at P due to segment DE is perpendicular and into the plane of drawing. The magnetic field is :

$$B_{DE} = -\frac{\sqrt{2}\mu_0 I}{8\pi L} = -\frac{\sqrt{2}\mu_0 I}{8\pi X2} = -\frac{\sqrt{2}\mu_0 I}{16\pi}$$

For the wire segment EF, the point P is at the end of straight wire of length 2 m and is at a perpendicular linear distance of 2 m. The magnetic field at P due to segment DE is perpendicular and into the plane of drawing. The magnetic field is :

$$B_{EF} = -\frac{\sqrt{2}\mu_0 I}{16\pi}$$

For the wire segment FG, the point P lies on the extended line passing through the wire. The magnetic field due to this segment, therefore, is zero.

$$B_{FG} = 0$$

For the wire segment GA, the point P is at the end of straight wire of length 4 m and is at a perpendicular linear distance of 4 m. The magnetic field at P due to segment GA is perpendicular and out of the plane of drawing. The magnetic field is :

$$B_{GA} = \frac{\sqrt{2}\mu_0 I}{32\pi}$$

The net magnetic field at P is :

$$B = B_{AC} + B_{CD} + B_{DE} + B_{EF} + B_{FG} + B_{GA}$$

$$\Rightarrow B = \frac{\sqrt{2}\mu_0 I}{32\pi} + 0 - \frac{\sqrt{2}\mu_0 I}{16\pi} - \frac{\sqrt{2}\mu_0 I}{16\pi} + 0 + \frac{\sqrt{2}\mu_0 I}{32\pi}$$

$$\Rightarrow B = -\frac{\sqrt{2}\mu_0 I}{16\pi}$$

$$\Rightarrow B = -\frac{\sqrt{2}X4\pi X10^{-7}X10\sqrt{2}}{16\pi} = -\frac{2X10^{-7}X10}{4}$$

$$\Rightarrow B = -5X10^{-7} \quad T$$

The net magnetic field is into the plane of drawing.

Chapter 5

Magnetic field due to current in a circular wire[1]

Magnetic field due to current in circular wire is largely axial. It means that we need to concentrate our investigation of magnetic field on axial positions. One of the important axial positions is center of the circular wire itself. We shall limit our discussion in this module to this case of magnetic field at the center of circular coil. The procedure for deriving expression for the magnetic field due to current in circular wire is same as that of current carrying straight wire. Here also, we make use of superposition principle whereby we combine the small magnetic fields due to each of the small current elements composing the circular coil.

Circular wire is considered to be composed of small linear current elements. We determine magnetic field due to each of the linear current elements applying Biot-Savart law. Finally, we determine net magnetic field using superposition principle (i.e. by determining vector sum of magnetic fields due to all current elements).

In general, the bending of current carrying wire in circular shape has the effect of strengthening or localizing magnetic field in narrower region about the axis.

5.1 Direction of magnetic field (Right hand thumb rule)

Let us consider two diametrically opposite small current elements on the circular wire. The magnetic field lines are compressed inside the circle as it accommodates all the circular closed lines drawn outside. This compression of magnetic field lines is maximum at the center. In the figure here, we consider the circular coil in horizontal plane. The magnetic field lines being perpendicular to current elements are in the plane of drawing.

[1] This content is available online at <http://cnx.org/content/m31199/1.11/>.

Magnetic field due to current in circular wire

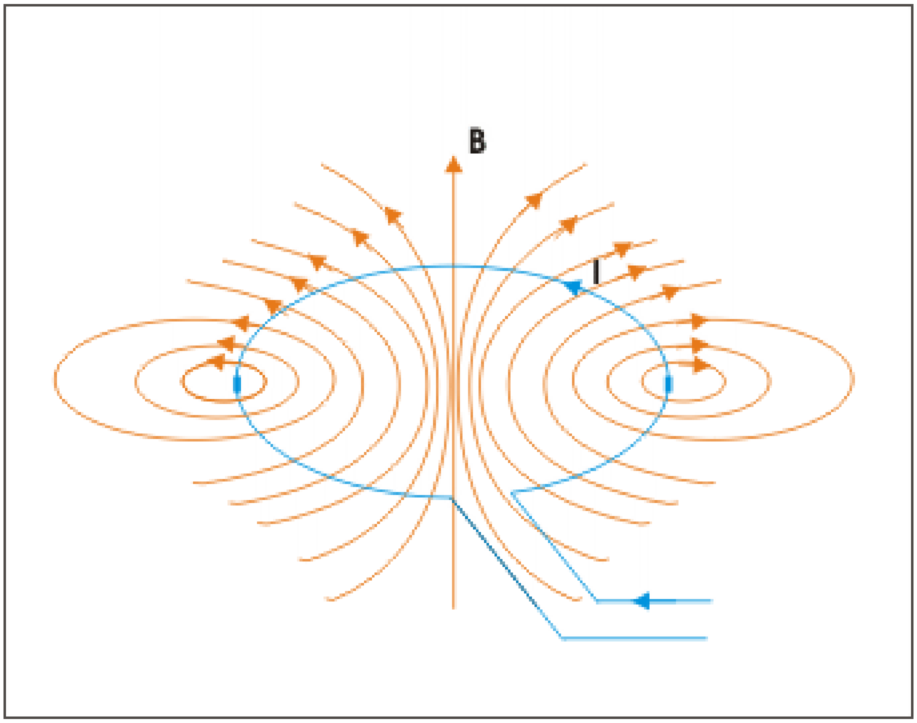


Figure 5.1: Magnetic field lines due to oppositely placed current elements

Such is the case with any other pair of current elements as well. This means that magnetic field line passing though axis is reinforced by all such diametrically opposite pairs of current element. The magnetic field due to current in circular wire, therefore, is nearly axial.

The observations as above are the basis of Right hand thumb rule for current in circular wire. If we orient right hand such that curl of fingers follows the direction of current in the circular wire, then extended thumb points in the direction of magnetic field at its center.

Right hand thumb rule

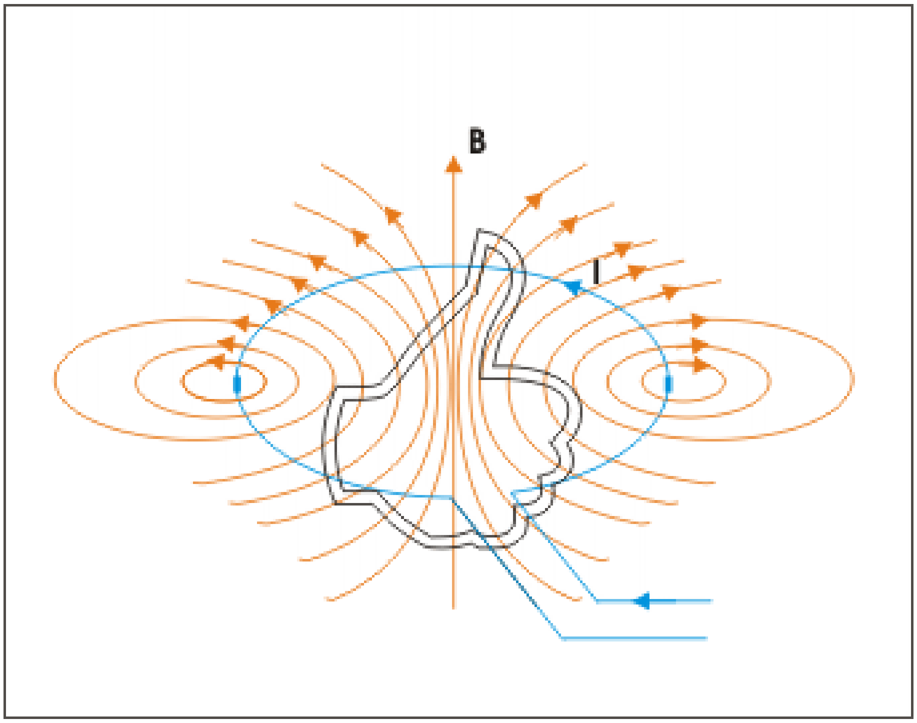


Figure 5.2: If we orient right hand such that curl of fingers follows the direction of current in the circular wire, then extended thumb points in the direction of magnetic field at its center.

Right hand thumb rules for straight wire and circular wire are opposite in the notations. The curl of hand represents magnetic field in the case of straight wire, whereas it represents current in the case of circular wire. Similarly, the extended thumb represents current in the case of straight wire, whereas it represents magnetic field in the case of circular wire.

There is yet another simple way to find the direction of axial magnetic field at the center. Just look at the circular loop facing it. If the current is clockwise, then magnetic field is away from you and if the current is anticlockwise, then magnetic field is towards you.

5.1.1 Current in circular wire and magnet

The directional attributes of the magnetic field due to current in circular wire have an important deduction. If the current in a circular loop is anticlockwise when we look from one end (face), then the same current is clockwise when we look from opposite end (face). What it means that if direction of magnetic field is towards you from one face, then the direction of magnetic field is away from you from the other end and vice versa.

Directions of current in circular wire

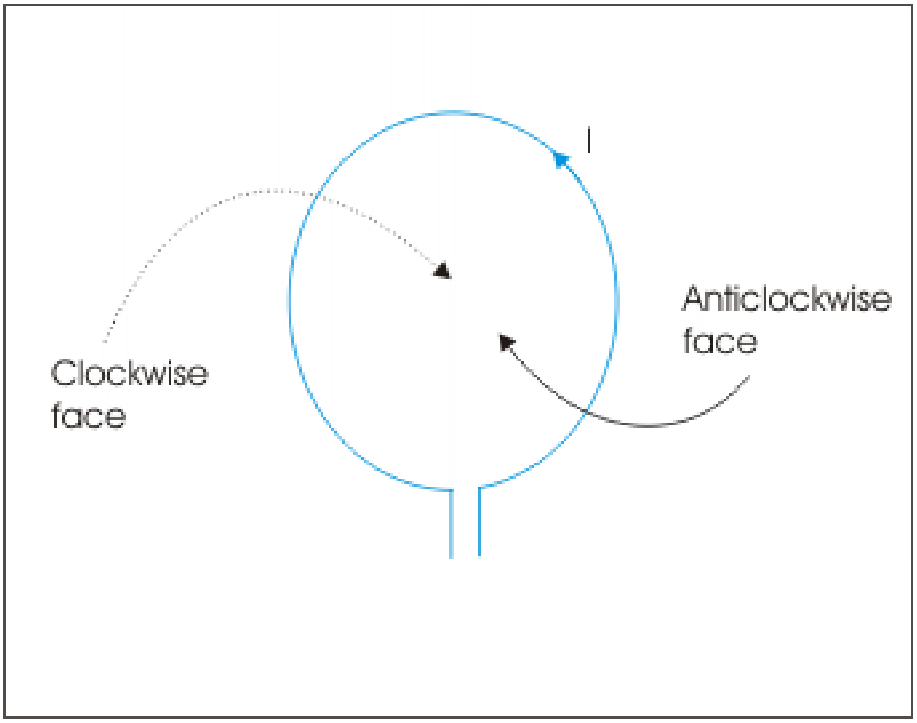


Figure 5.3: Directions of current in circular wire

The magnetic lines of force enters from the face in which current is clockwise and exits from the face in which current is anticlockwise. This is exactly the configuration with real magnet. The anticlockwise face of the circular wire is equivalent to north pole and clockwise face is equivalent to south pole of the physical magnet. For this reason, a current in a circular wire is approximately equivalent to a tiny bar magnet.

Equivalence of current in circular wire with magnet

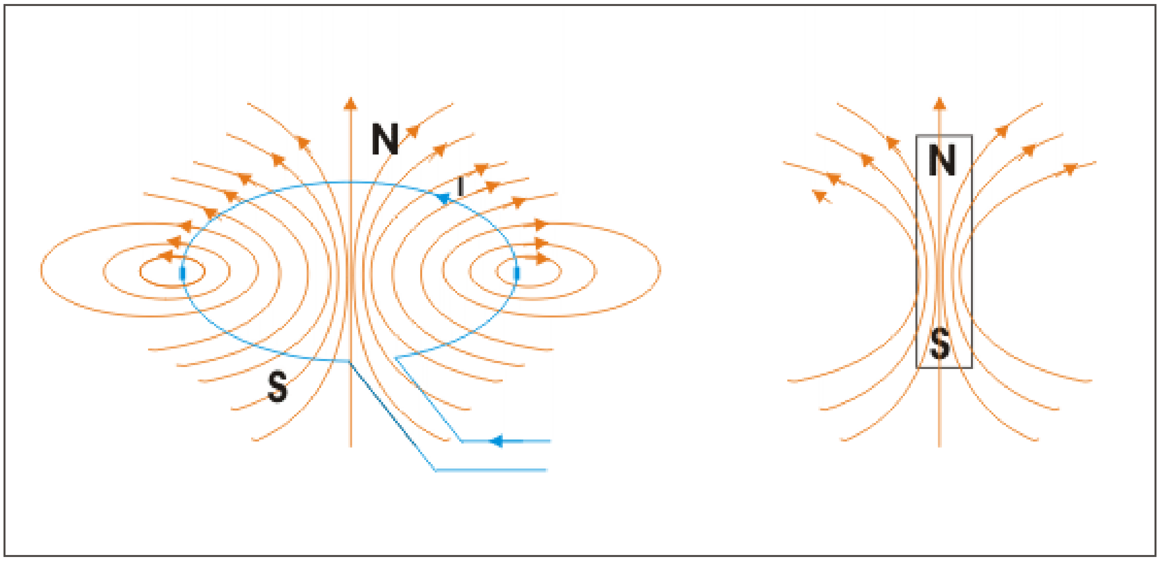


Figure 5.4: The magnetic field lines are similar in two cases

We shall learn more about this aspect when we study magnetic moment and physical magnets.

5.2 Magnitude of magnetic field due to current in circular wire

Evaluation of Biot-Savart expression at the center of circle for current in circular wire is greatly simplified. There are threefold reasons :

1: The directions of magnetic fields due to all current elements at the center are same just as in the case of straight wire.

2: The linear distance (r) between current length element (d**l**) and the point of observation (center of circular wire) is same for all current elements.

3: The angle between current length element vector (d**l**) and displacement vector (**R**) is right angle for all current elements. Recall that angle between tangent and radius of a circle is right angle at all positions on the perimeter of a circle.

Magnetic field due to current in circular wire

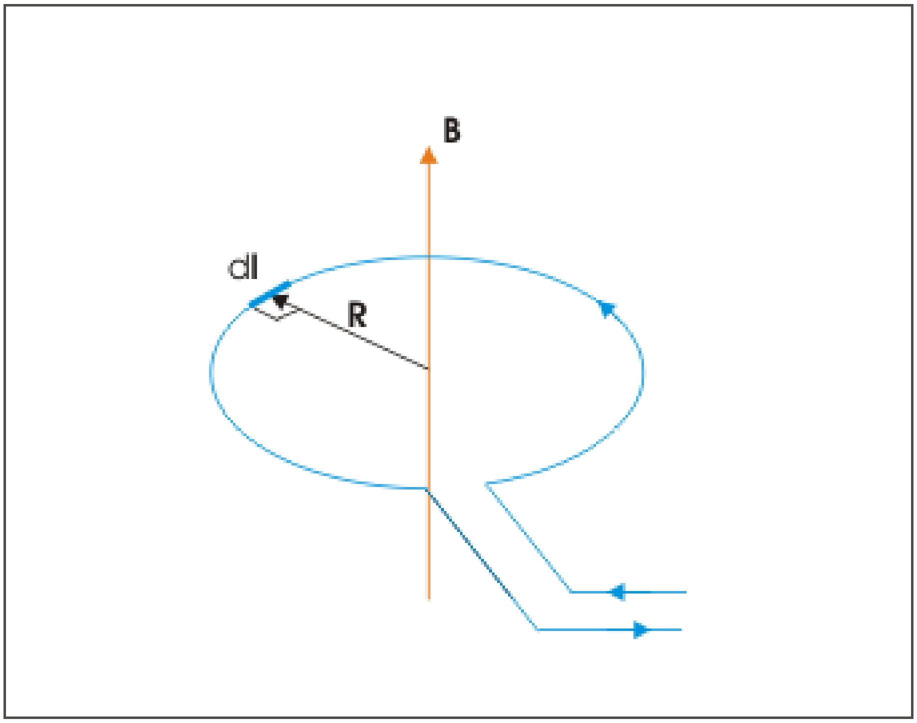


Figure 5.5: Magnetic field due to current in circular wire

The magnitude of magnetic field due to a current element according to Biot-Savart law (Section 3.1.2: Magnitude of magnetic field) is given by :

$$\mathrm{B} = \frac{\mu_0}{4\pi}\frac{Il\sin\theta}{r^2}$$

But, θ=90° and sin90°=1. Also, r = R = Radius of circular wire.

$$\Rightarrow \mathrm{B} = \frac{\mu_0}{4\pi}\frac{Il}{R^2}$$

All parameters except "đl" in the right hand expression of the equation are constants and as such they can be taken out of the integral.

$$B = \int \mathrm{B} = \frac{\mu_0 I}{4\pi R^2}\int l$$

The integration of dl over the complete circle is equal to its perimeter i.e. 2πR.

$$\Rightarrow B = \frac{\mu_0 I}{4\pi R^2} X 2\pi R = \frac{\mu_0 I}{2R}$$

If the wire is a coil having N circular turns, then magnetic filed at the center of coil is reinforced N times :

$$B = \frac{\mu_0 N I}{2R}$$

Example 5.1

Problem : A thin ring of radius "R" has uniform distribution of charge, q, on it. The ring is made to rotate at an angular velocity "ω" about an axis passing through its center and perpendicular to its plane. Determine the magnitude of magnetic field at the center.

Solution : A charged ring rotating at constant angular velocity is equivalent to a steady current in circular wire. We need to determine this current in order to calculate magnetic field. For this, let us concentrate at any cross section of the ring. All the charge passes through this cross section in one time period of revolution. Thus, equivalent current is :

$$I = \frac{q}{T} = \frac{q\omega}{2\pi}$$

Now, magnetic field due to steady current in circular wire is :

$$B = \frac{\mu_0 I}{2R}$$

Substituting for current, we have :

$$\Rightarrow B = \frac{\mu_0 q\omega}{4\pi R}$$

Example 5.2

Problem : Calculate magnetic field at the center O for the current flowing through wire segment as shown in the figure. Here, current through wire is 10 A and radius of the circular part is 0.1 m.

Magnetic field due to current in wire

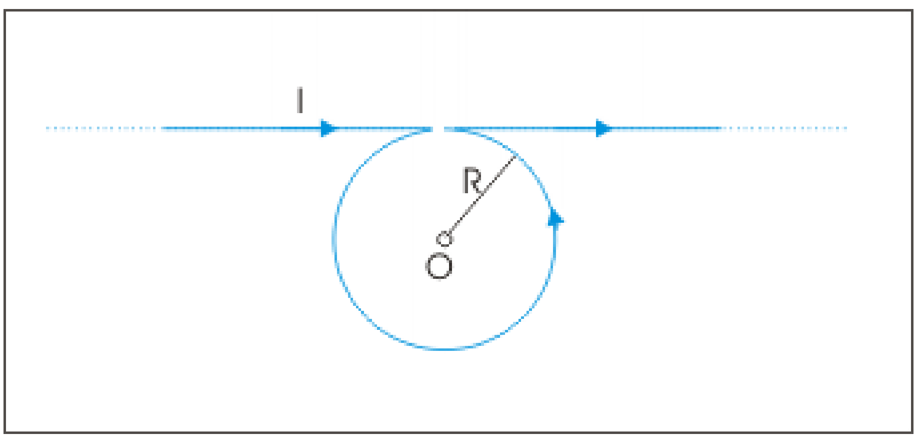


Figure 5.6: Magnetic field due to current in wire

Solution : Magnetic field at O is contributed by long straight wire and circular wire. The direction of magnetic field at O due to straight part of the wire is into the plane of drawing as obtained by applying Right hand thumb rule for straight wire. The direction of current in the circular part is anticlockwise and hence magnetic field due to this part is out of the plane of drawing as obtained by applying Right hand thumb rule for circular wire.

The magnitude of magnetic field due to circular wire is :

$$B_C = \frac{\mu_0 I}{2R}$$

The magnitude of magnetic field due to straight wire is :

$$B_S = \frac{\mu_0 I}{2\pi R}$$

Hence, magnitude of magnetic field at O is algebraic sum of two magnetic fields (we consider outward direction as positive) :

$$B = B_C - B_S = \frac{\mu_0 I}{2R} - \frac{\mu_0 I}{2\pi R}$$

Putting values :

$$\Rightarrow B = \frac{4\pi 10^{-7} X 10}{2X0.1} - \frac{4\pi 10^{-7} X 10}{2\pi X0.1}$$

$$\Rightarrow B = 62.9X10^{-6} - 20X10^{-6} = 42.9 \quad \mu T$$

The net magnetic field is acting out of the plane of paper.

5.3 Magnitude of magnetic field due to current in circular arc

The magnitude of magnetic field due to current in arc shaped wire can be obtained by integrating Biot-Savart expression in an appropriate range. Now, the integral set up for current in circular wire is :

$$B = \int B = \frac{\mu_0 I}{4\pi R^2} \int l$$

Circular arc is generally referred in terms of the angle θ, it subtends at the center of the circle. From geometry, we know that :

$$l = R\theta$$

Substituting in the integral and taking the constant R out of the integral, we have :

$$B = \frac{\mu_0 I}{4\pi R} \int \theta$$

$$\Rightarrow B = \frac{\mu_0 I \theta}{4\pi R}$$

This is the expression for the magnitude of magnetic field due to current in an arc which subtends an angle θ at the center. Note that the expression is true for the circle for which $\theta = 2\pi$ and magnetic field is :

$$\Rightarrow B = \frac{\mu_0 I X 2\pi}{4\pi R} = \frac{\mu_0 I}{2R}$$

Example 5.3

Problem : Find the magnetic field at the corner O due to current in the wire as shown in the figure. Here, radius of curvature is 0.1 m for the quarter circle arc and current is 10 A.

Magnetic field due to current in wire

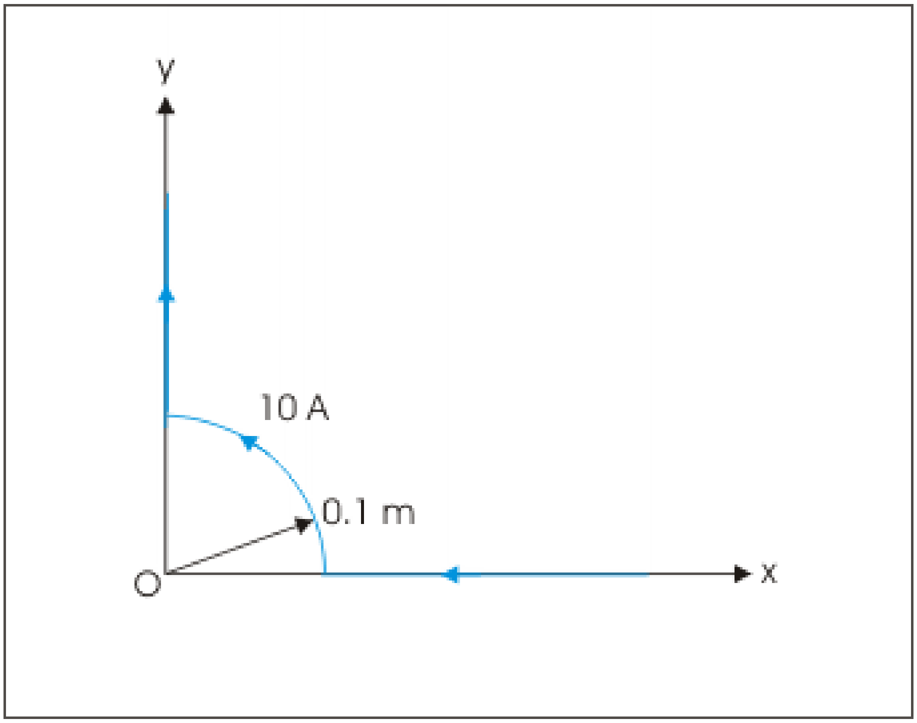


Figure 5.7: Magnetic field due to current in wire

Solution :

Here the straight line wire segment AB and CD when extended meet at O. As such, there is no magnetic field due to current in these segments. The magnetic field at O is, therefore, solely due to magnetic field due to quarter arc AC. The arc subtends an angle $\pi/2$ at its center i.e O. Now,

$$B = \frac{\mu_0 I \theta}{4\pi R}$$

Putting values, we have :

$$\Rightarrow B = \frac{10^{-7} X 10 X \pi}{0.1 X 2} = 0.157 X 10^{-6} = 0.157 \quad \mu T$$

Since current in the arc is anticlockwise, magnetic field is perpendicular and out of the plane of drawing.

5.4 Current in straight wire .vs. current in circular wire

A length of wire, say L, is given and it is asked to maximize magnetic field in a region due to a current I in the wire. Which configuration would we consider – a straight wire or a circular wire? Let us examine the magnetic fields produced by these two configurations.

If we bend the wire in the circle, then the radius of the circle is :

$$R = \frac{L}{2\pi}$$

The magnetic field due to current I in the circular wire is :

$$B_C = \frac{\mu_0 I}{2R} = \frac{2\pi\mu_0 I}{2L} = \frac{\pi\mu_0 I}{L} = \frac{3.14\mu_0 I}{L}$$

In the case of straight wire, let us consider that wire is long enough for a point around middle of the wire. For comparison purpose, we assume that perpendicular linear distance used for calculating magnetic field due to current in straight wire is equal to the radius of circle. The magnetic field at a perpendicular distance "R" due to current in long straight wire is given as :

$$B_L = \frac{\mu_0 I}{4\pi R} = \frac{\mu_0 I X 2\pi}{4\pi L} = \frac{\mu_0 I}{2L} = \frac{0.5\mu_0 I}{L}$$

Clearly, the magnetic field due to current in circular wire is 6.28 times greater than that due to current in straight wire at comparable points of observations. Note that this is so even though we have given advantage to straight wire configuration by assuming it to be long wire. In a nutshell, a circular configuration tends to concentrate magnetic field along axial direction which is otherwise spread over the whole length of wire.

Example 5.4

Problem : A current 10 A flowing through a straight wire is split at point A in two semicircular wires of radius 0.1 m. The resistances of upper and lower semicircular wires are 10 Ω and 20 Ω respectively. The currents rejoin to flow in the straight wire again as shown in the figure. Determine the magnetic field at the center "O".

Magnetic field due to current in wire

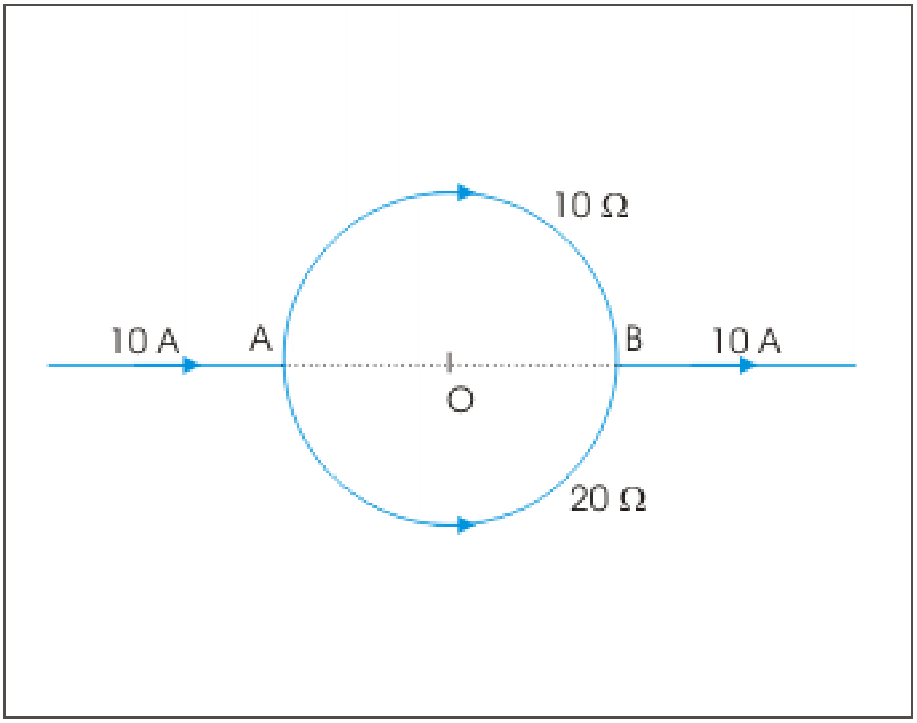


Figure 5.8: Magnetic field due to current in wire

Solution : The straight wire sections on extension pass through the center. Hence, magnetic field due to straight wires is zero. Here, the incoming current at A is distributed in the inverse proportion of resistances. Let the subscripts "1" and "2' denote upper and lower semicircular sections respectively. The two sections are equivalent to two resistances in parallel combination as shown in the figure. Here, potential difference between "A" and "B" is :

Currents in semicircular segments

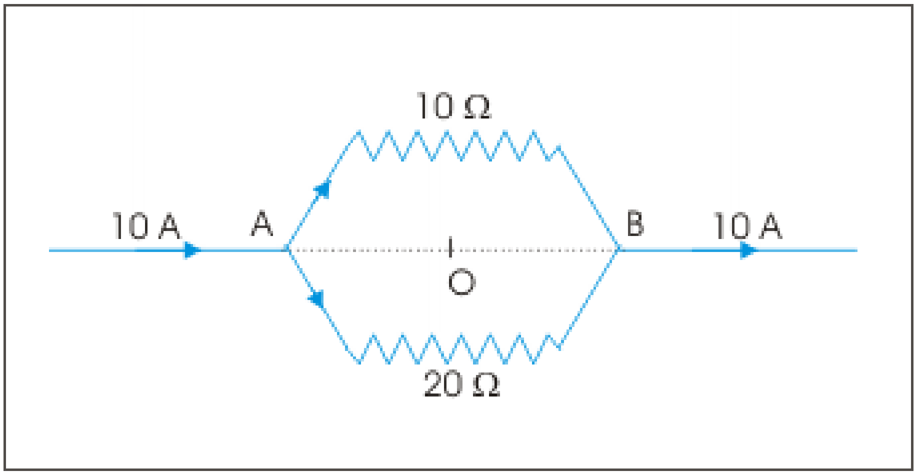


Figure 5.9: Currents in semicircular segments

$$V_{AB} = \frac{IXR_1XR_2}{(R_1+R_2)} = I_1R_1 = I_2R_2$$

$$\Rightarrow I_1 = \frac{IXR_2}{(R_1+R_2)} = \frac{10X20}{30} = \frac{20}{3} \quad A$$

$$\Rightarrow I_2 = \frac{IXR_1}{(R_1+R_2)} = \frac{10X10}{30} = \frac{10}{3} \quad A$$

We see that current in the upper section is twice that in the lower section i.e. $I_1 = 2I_2$. Also, the magnetic field is perpendicular to the plane of semicircular section (plane of drawing). The current in the upper semicircular wire is clockwise. Thus, the magnetic field due to upper section is into the plane of drawing. However, the current in the lower semicircular is anticlockwise. Thus, the magnetic field due to lower section is out of the plane of drawing. Putting $\theta = \pi$ for each semicircular section, the net magnetic field due to semicircular sections at "O" is:

$$B = \frac{\mu_0 I_1 \pi}{4\pi R} - \frac{\mu_0 I_2 \pi}{4\pi R}$$

$$\Rightarrow B = \frac{\mu_0 I_1 \pi}{4\pi R} - \frac{\mu_0 I_1 \pi}{8\pi R} = \frac{\mu_0 I_1 \pi}{8\pi R}$$

$$\Rightarrow B = \frac{10^{-7}X20}{3X8X0.1} = 8.3X10^{-7} \quad T$$

The net magnetic field is into the plane of drawing.

5.5 Exercises

Exercise 5.1 *(Solution on p. 76.)*

An electron circles a single proton nucleus of radius $3.2X10^{-11}$ m with a frequency of 10^{16} Hz. The charge on the electron is $1.6X10^{-19}$ Coulomb. What is the magnitude of magnetic field due to orbiting electron at the nucleus?

Exercise 5.2 *(Solution on p. 76.)*

Calculate the magnetic field at O for the current loop shown in the figure.

Magnetic field due to current in wire

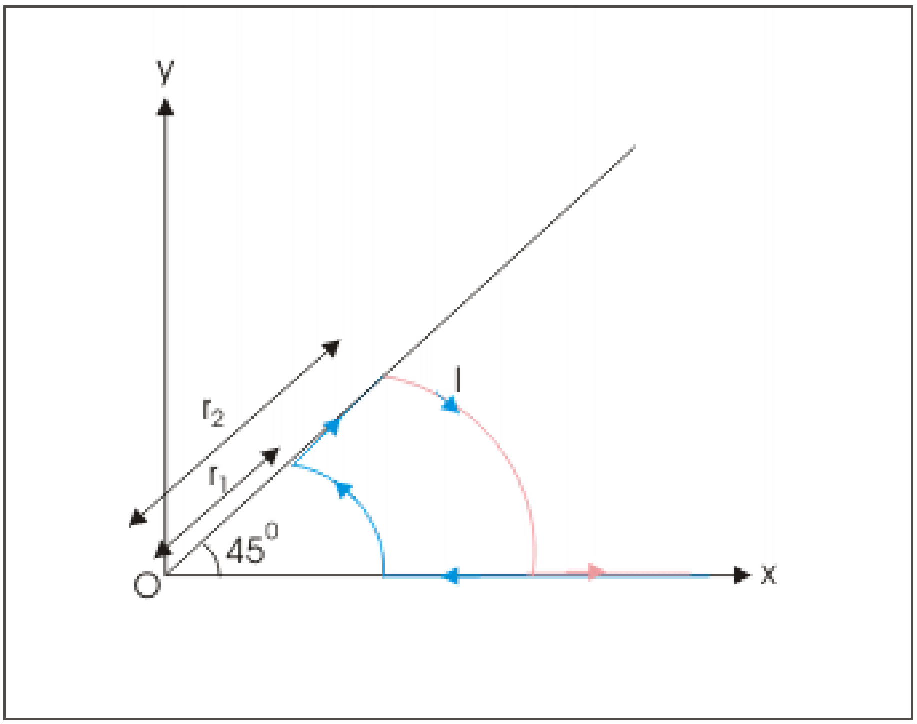


Figure 5.10: Magnetic field due to current in wire

Exercise 5.3 *(Solution on p. 76.)*

A current of 10 ampere flows in anticlockwise direction through the arrangement shown in the figure. Determine the magnetic field at the center "O".

Magnetic field due to current in wire

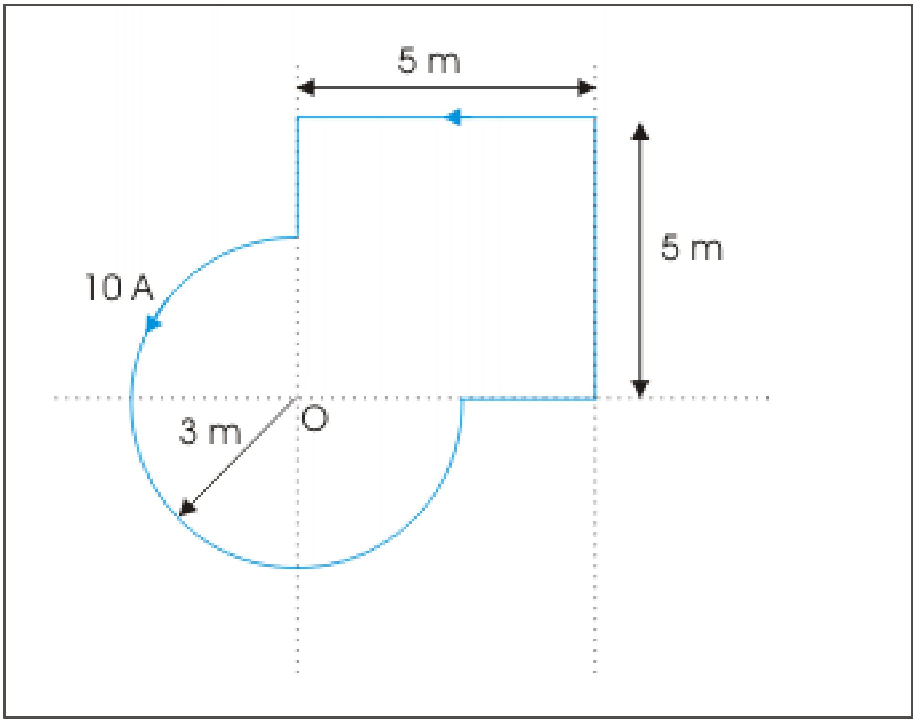


Figure 5.11: Magnetic field due to current in wire

Exercise 5.4 *(Solution on p. 77.)*

A current of 10 ampere flows in anticlockwise direction through the arrangement shown in the figure. The curved part is a semicircular arc. Determine the magnetic field at the center "O".

Magnetic field due to current in wire

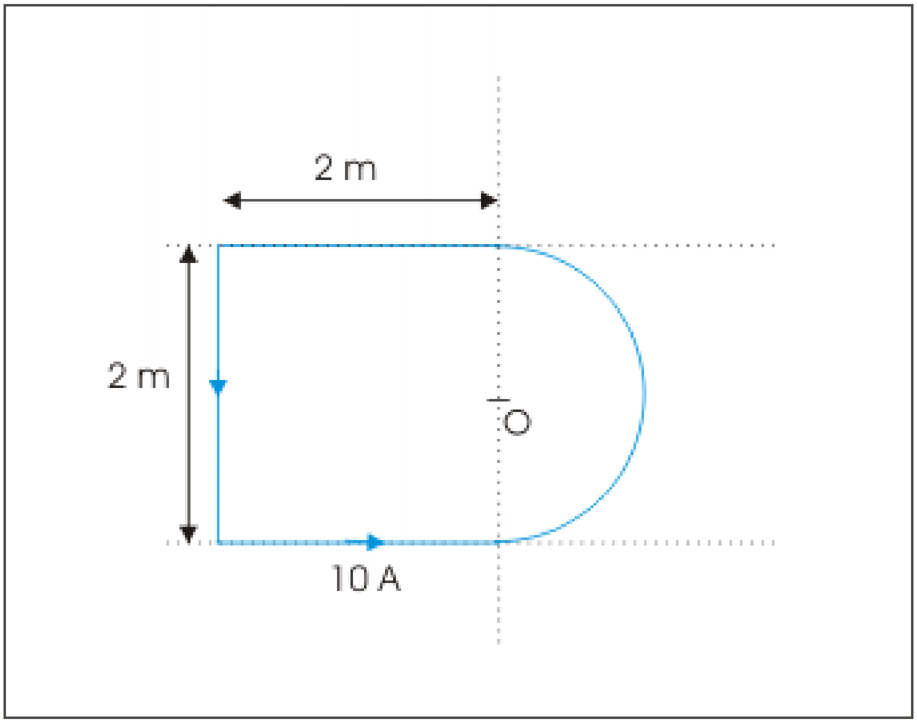


Figure 5.12: Magnetic field due to current in wire

Exercise 5.5 *(Solution on p. 77.)*

A thin disc of radius "R" has uniform distribution of charge, q, on it. The ring is made to rotate at an angular velocity "ω" about an axis passing through its center and perpendicular to its plane. Determine the magnitude of magnetic field at the center of the disc.

Solutions to Exercises in Chapter 5

Solution to Exercise 5.1 (p. 73)
The equivalent current is given by :

$$I = \frac{q}{T} = q\nu$$

where ν and T are frequency and time period of revolutions respectively. The magnitude of magnetic field due to circular wire is given by :

$$B = \frac{\mu_0 I}{2R}$$

Substituting for I, we have :

$$\Rightarrow B = \frac{\mu_0 I}{2R} = \frac{\mu_0 q\nu}{2R}$$

Putting values,

$$\Rightarrow B = \frac{4\pi 10^{-7} X 1.6 X 10^{-19} X 10^{16}}{2X3.2X10^{-11}}$$

$$\Rightarrow B = 31.4 \quad T$$

Solution to Exercise 5.2 (p. 73)
The magnetic field due to linear part of the wire is zero as they pass through O when extended. The magnetic field due to inner arc is greater than outer arc. Further, magnetic field due to anticlockwise current in the inner arc is out of the plane of drawing and magnetic field due to clockwise current in the outer arc is into the plane of drawing. Net magnetic field due to the current in the wire is out of the plane of drawing, whose magnitude is :

$$B = \frac{\mu_0 I\theta}{4\pi r_1} - \frac{\mu_0 I\theta}{4\pi r_2}$$

$$\Rightarrow B = \frac{\mu_0 I\pi}{4\pi r_1 X4} - \frac{\mu_0 I\pi}{4\pi r_2 X4}$$

$$\Rightarrow B = \frac{\mu_0 I}{16}\frac{(r_2 - r_1)}{r_1 r_2}$$

Solution to Exercise 5.3 (p. 73)
The magnetic field at "O" due to $\frac{3}{4}$ th of the circular arc is :

$$B_C = \frac{\mu_0 I X 3\pi}{4\pi R X 2} = \frac{3\mu_0 I}{8R} = \frac{3X4\pi X10^{-7}X10}{8X3}$$

$$\Rightarrow B_C = 5\pi X10^{-7} = 15.7X10^{-7} \quad T$$

Two linear part segments when extended pass through "O" and as such do not contribute to magnetic field. The magnetic field at "O" due to one 5 m segment is :

$$B_{L_1} = \frac{\sqrt{2}\mu_0 I}{8\pi R} = \frac{\sqrt{2}X4\pi X10^{-7}X10}{8\pi X5}$$

$$B_{L_1} = \sqrt{2}X10^{-7} \quad T$$

The magnetic field at "O" due to two 5 m segments is :

$$B_L = 2XB_{L_1} = 2X\sqrt{2}X10^{-7} = 2.83X10^{-7} \quad T$$

Magnetic fields due to both circular arc and linear segments are acting out of the plane of drawing, the net magnetic field at "O" is :

$$B = B_C + B_L = 15.7X10^{-7} + 2.83X10^{-7} = 18.53XX10^{-7} = 1.853X10^{-6}T$$

Solution to Exercise 5.4 (p. 74)

The magnetic field at "O" due to semicircular arc acts upward and its magnitude is :

$$B_C = \frac{\mu_0 IX\pi}{4\pi R} = \frac{\mu_0 I}{4X1} = \frac{4\pi X10^{-7}X10}{4} = 31.4X10^{-7} \quad T$$

The magnetic field due to lower straight conductor acts upward and its magnitude is :

$$B_{L_1} = \frac{\sqrt{2}\mu_0 I}{8\pi R} = \frac{\sqrt{2}X4\pi X10^{-7}X10}{8\pi X1}$$

$$\Rightarrow BL_1 = \sqrt{2}X5X10^{-7} = 7.07X10^{-7} \quad T$$

The magnetic field due to upper straight conductor acts upward and its magnitude is equal to that due to lower straight conductor :

$$\Rightarrow B_{L_2} = \sqrt{2}X5X10^{-7} = 7.07X10^{-7} \quad T$$

For the straight conductor at the far end, the center "O" lies on the bisector. The magnetic field acts upward and its magnitude is :

$$B_{L_3} = \frac{\mu_0 IL}{4\pi R\sqrt{(L^2 + 4R^2)}}$$

Here, R = 2 m, L = 2 m. Putting values in the equation, we have :

$$\Rightarrow B_{L_3} = \frac{\mu_0 IL}{4\pi R\sqrt{(L^2 + 4R^2)}} = \frac{4\pi X10^{-7}X10X2}{4\pi X2\sqrt{(2^2 + 4X2^2)}}$$

$$\Rightarrow B_{L_3} = \sqrt{5}X10^{-7} = 2.24X10^{-7} \quad T$$

The net magnetic field at "O" is :

$$B = B_C + B_{L_1} + B_{L_2} + B_{L_3}$$

$$\Rightarrow B = 31.4X10^{-7} + 7.07X10^{-7} + 7.07X10^{-7} + 2.24X10^{-7}$$

$$\Rightarrow B = 4.78X10^{-6} \quad T$$

Solution to Exercise 5.5 (p. 75)

We consider disc to be composed of infinite numbers of thin ring. We consider one such ring of thickness dr at a distance "r" from the center carrying charge dq. This ring carrying charge "dq" and rotating is equivalent to a current. The magnetic field at the center to this thin ring is (as obtained earlier in the example problem) :

Magnetic field due to rotating charged disc

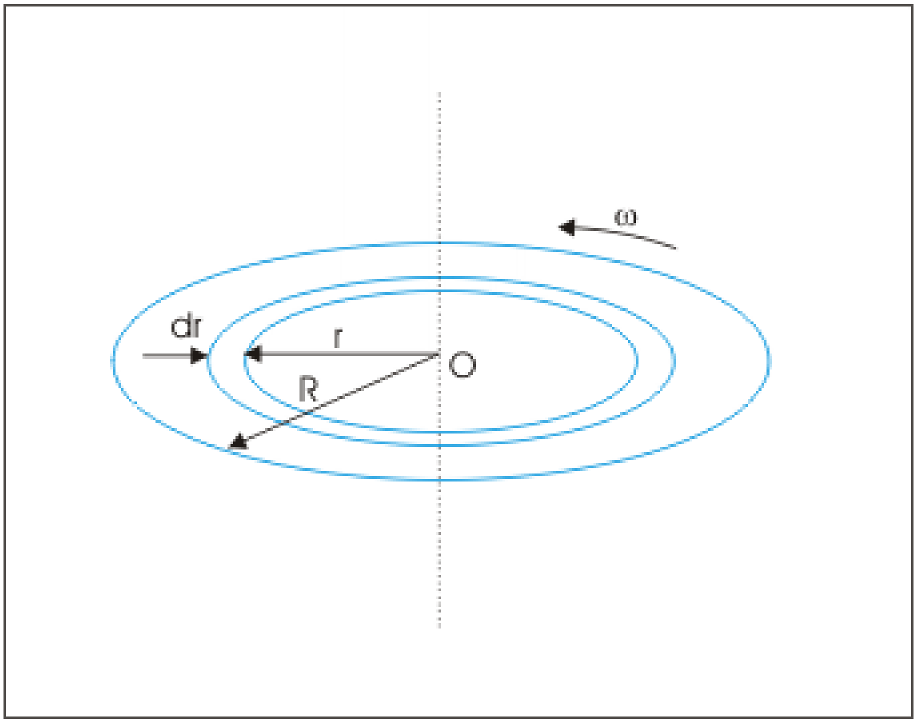


Figure 5.13: Magnetic field due to rotating charged disc

$$B = \frac{\mu_0 q\omega}{4\pi r}$$

We need to determine "dq" in terms of given parameters. The current surface density, σ, is :

$$\sigma = \frac{q}{\pi R^2}$$

The area of the thin ring is :

$$A = 2\pi r r$$

Hence, charge on the ring is :

$$q = \sigma A = \frac{2\pi r q r}{\pi R^2} = \frac{2rqr}{R^2}$$

Putting this espression for "dq", the expression of magnetic field at the center due to rotating ring is :

$$B = \frac{\mu_0 2rq\omega r}{4\pi r R^2} = \frac{\mu_0 q\omega r}{2\pi R^2} = \frac{\mu_0 \omega q r}{2\pi R^2}$$

In order to obtain magnetic field due to the rotating disc, we integrate the expression of magnetic field due to ring from r = 0 to r =R.

$$B = \int \mathrm{B} = \int_0^R \frac{\mu_0 \omega q r}{2\pi R^2}$$

Taking out constants out of the integration sign, we have :

$$\Rightarrow B = \frac{\mu_0 w q}{2\pi R^2} \int_0^R r$$

$$\Rightarrow B = \frac{\mu_0 w q}{2\pi R}$$

Chapter 6

Magnetic field at an axial point due to current in circular wire[1]

We have already determined magnetic field due to current in circular wire at its center. The approach to determine magnetic field at an axial point is similar. We begin with magnetic field due to small current element and then try to integrate the Biot-Savart expression for the small magnetic field for the entire circle following superposition principle.

This extension of earlier procedure, however, demands a bit of extra three dimensional imagination to arrive at the correct result. In this module, we shall attempt to grasp three dimensional elements as clearly as possible with figures. Let us have a look at the differential Biot-Savart expression :

$$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\mathbf{l}Xr}{r^3}$$

There are three vector quantities d$\mathbf{B}$, d$\mathbf{l}$ and $\mathbf{r}$. We investigate the spatial relation among these quantities for magnetic field at an axial point.

6.1 Magnetic field on an axial point

The magnitude of magnetic field due to current in a current element is given by :

$$B = \frac{\mu_0}{4\pi}\frac{Il\sin\theta}{r^2}$$

In order to evaluate magnetic field due to complete circular wire, we need to set up corresponding integral properly with respect to various elements constituting the expression. In following subsections, we study these elements in which point of observation is a point on axial line.

6.1.1 The angle between current length element and displacement vectors

The angle (θ) as appearing in the Biot-Savart expression between current length element vector d$\mathbf{l}$ and displacement vector $\mathbf{r}$ is right angle. See figure. This right angle should be distinguished with acute angle ϕ, which is the angle between OA and AP as shown in the figure.

[1]This content is available online at <http://cnx.org/content/m31277/1.3/>.

The angle between current length element and displacement vectors

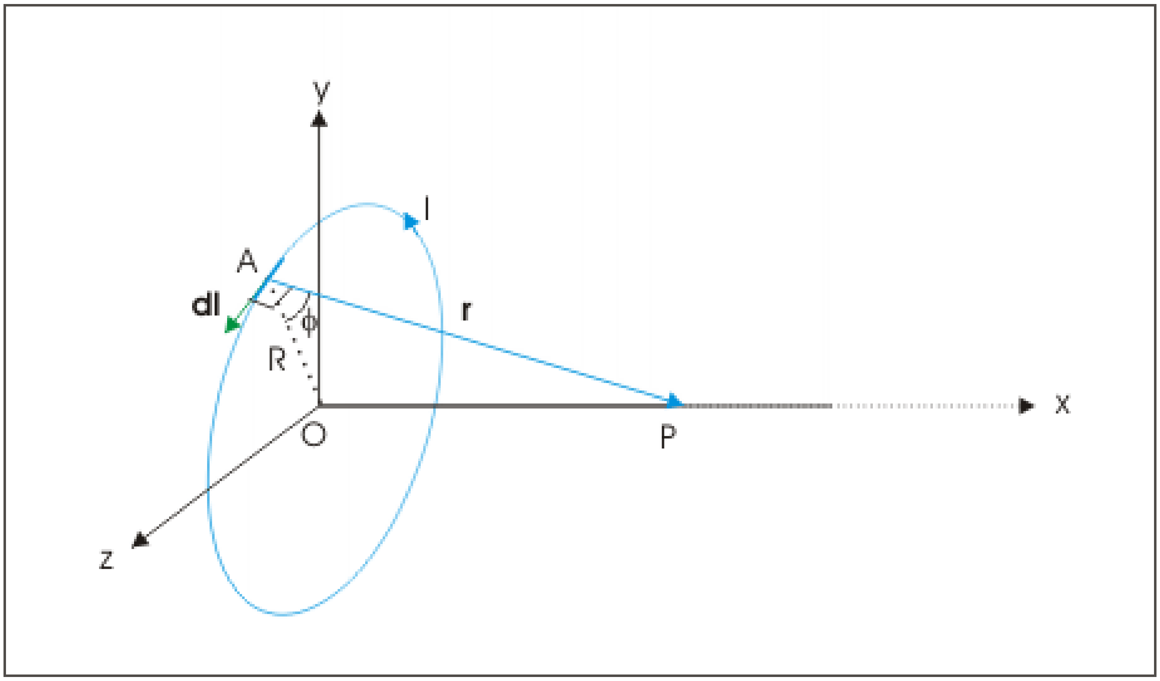


Figure 6.1: The angle between current length element and displacement vectors is right angle.

The above fact reduces Biot-Savart expression to :

$$B = \frac{\mu_0}{4\pi}\frac{Il\sin 90}{r^2} = \frac{\mu_0}{4\pi}\frac{Il}{r^2}$$

This simplification due to enclosed angle being right angle is true for all points on the circle.

6.1.2 Magnitude of magnetic field

All current elements are at equal linear distance from point P. As a result, the magnitude of magnetic field at P due to any of the equal current elements is same.

$$B_1 = B_2 = \ldots\ldots.$$

6.1.3 Direction of elemental magnetic field

Unlike enclosed angle (θ), linear distance (r) and magnitude of magnetic field, the direction of magnetic field due to current elements are not same. As such, we can not integrate Biot-Savart differential expression to determine net magnetic field at P. Let us investigate the direction of magnetic fields due to two diametrically opposite current elements. Let the circular wire lie in yz plane as shown in the figure.

Direction of elemental magnetic field

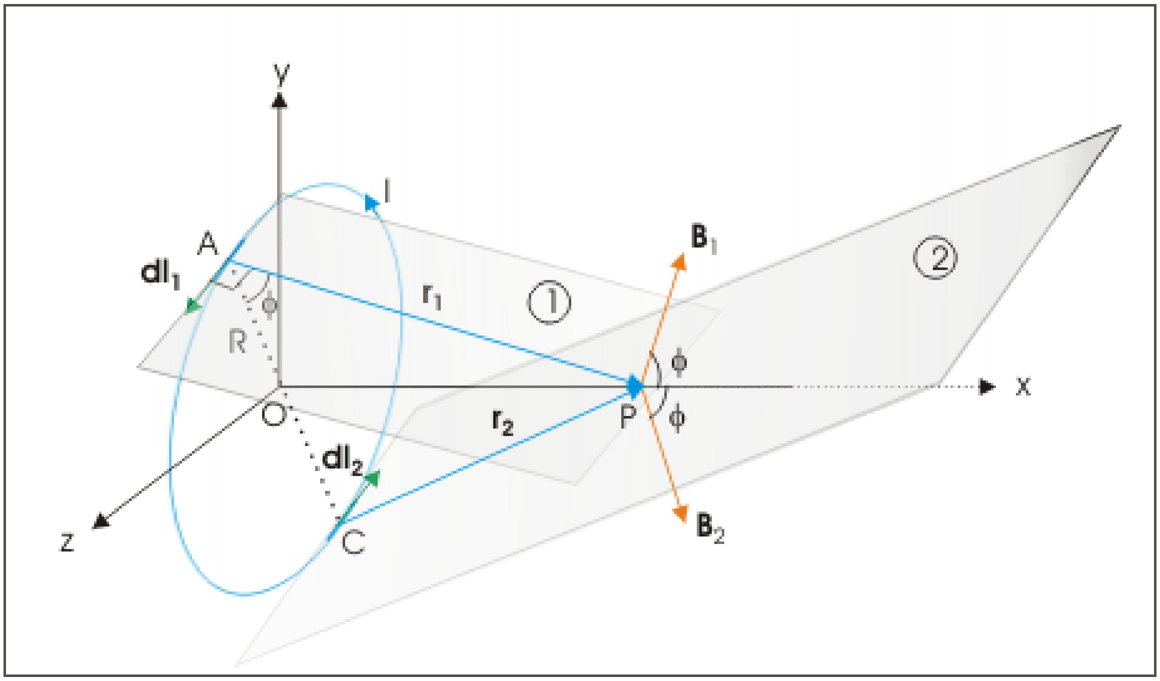


Figure 6.2: Magnetic field is perpendicular to plane formed by current length element and displacement vectors.

The current length vector $d\mathbf{l}_1$ and displacement vector $\mathbf{r}_1$ form a plane shown as plane 1 and the magnetic field due to current element, $\mathbf{B}_1$, is perpendicular to plane 1. Similarly, the current length vector $d\mathbf{l}_2$ and displacement vector $\mathbf{r}_2$ form a plane shown as plane 2 and the magnetic field due to current element, $\mathbf{B}_2$), is perpendicular to plane 2. Clearly, these magnetic fields are directed in three dimensional space. If we imagine magnetic fields due to other current elements of the circular wire, then it is not difficult to imagine that these elemental magnetic fields are aligned on the outer surface of a conic section and that they are not in same plane.

Direction of elemental magnetic field

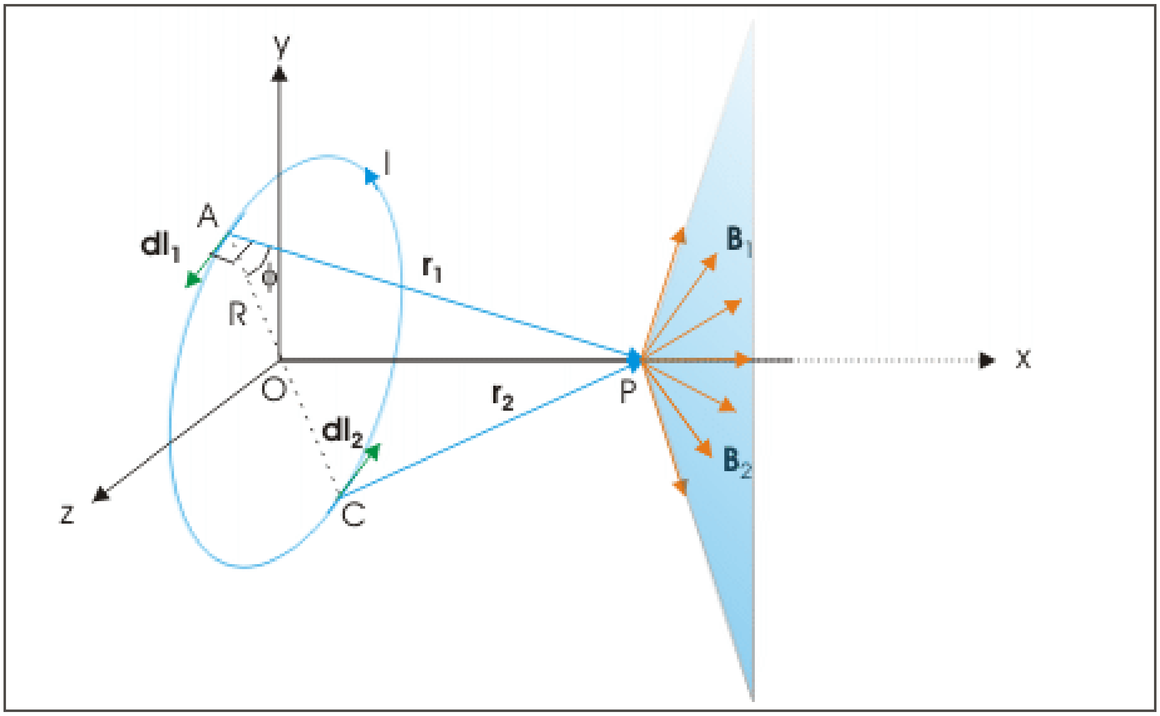


Figure 6.3: Magnetic fields are aligned on the outer surface of a conic section.

Another important point to observe is that all elemental magnetic field vectors form same angle ϕ. This can be verified from the fact that B_1 is perpendicular to AP and Px is perpendicular to OA. Hence, angle between B_1 and Px is equal to angle between OA and AP i.e. ϕ with x-axis. By symmetry, we can see that all elemental magnetic field vectors form the same angle with x- axis.

6.1.4 Resolution of elemental magnetic field vectors and net magnetic field

We resolve magnetic field vectors along x-axis and perpendicular to it, which lies on a plane perpendicular to axis i.e a plane parallel to the plane of circular coil (yz plane) as shown in the figure. We have shown two pairs of diametrically opposite current elements. See that axial components are in positive x-direction. The perpendicular components, however, cancels each other for a diametrically opposite pair.

Resolution of elemental magnetic field vectors and net magnetic field

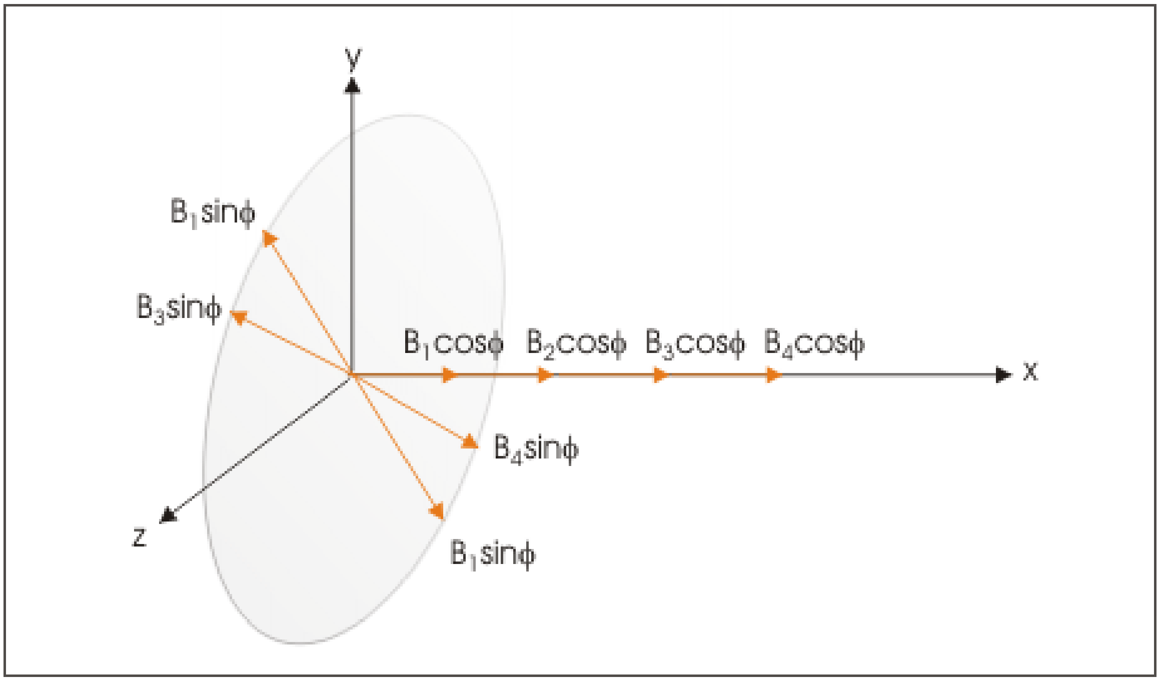


Figure 6.4: Net magnetic field is axial.

This situation greatly simplifies the integration process. We need only to algebraically add axial components. Since all are in same direction, we integrate the axial component of differential Biot-Savart expression :

$$B = \int B = \frac{\mu_0 I}{4\pi} \int \frac{l}{r^2} \cos\phi$$

Note that both r and cos ϕ are constants and they can be taken out of integral,

$$\Rightarrow B = \frac{\mu_0 I \cos\phi}{4\pi r^2} \int l$$

$$\Rightarrow B = \frac{\mu_0 I \cos\phi}{4\pi r^2} X 2\pi R = \frac{\mu_0 I R \cos\phi}{2r^2}$$

Now,

$$r = \left(x^2 + R^2\right)^{\frac{1}{2}}$$

In triangle OAP,

$$\cos\phi = \frac{R}{r} = \frac{R}{\left(x^2 + R^2\right)^{\frac{1}{2}}}$$

Putting these values in the expression of magnetic field, we have :

$$\Rightarrow B = \frac{\mu_0 IR\cos\phi}{2r^2} = \frac{\mu_0 IR^2}{2(x^2+R^2)^{\frac{3}{2}}}$$

This is the expression of magnitude of magnetic field on axial line. Note that we have derived this expression for anticlockwise current. For clockwise current, the magnetic field will have same magnitude but oriented towards the circular wire. Clearly, direction of axial magnetic field follows Right hand thumb rule.

If there are N turns of circular wires stacked, then magnetic field is reinforced N times and magnetic field is :

$$\Rightarrow B = \frac{\mu_0 NIR^2}{2(x^2+R^2)^{\frac{3}{2}}}$$

In order to show the direction, we may write the expression for magnetic field vector using unit vector in the axial direction as :

$$\Rightarrow \mathbf{B} = \frac{\mu_0 NIR^2}{2(x^2+R^2)^{\frac{3}{2}}}\mathbf{i}$$

Recall that one of the faces of circular wire has clockwise direction of current, whereas other face of the same circular wire has anticlockwise direction of current. The magnetic field lines enter from the face where current is clockwise and exit from the face where current is anticlockwise.

Example 6.1

Problem : Two identical circular coils of radius R are placed face to face with their centers on a straight line at a distance $2\sqrt{3}$ R apart. If the current in each coil is I flowing in same direction, then determine the magnetic field at a point "O" midway between them on the straight line.

Two identical circular coils at a distance

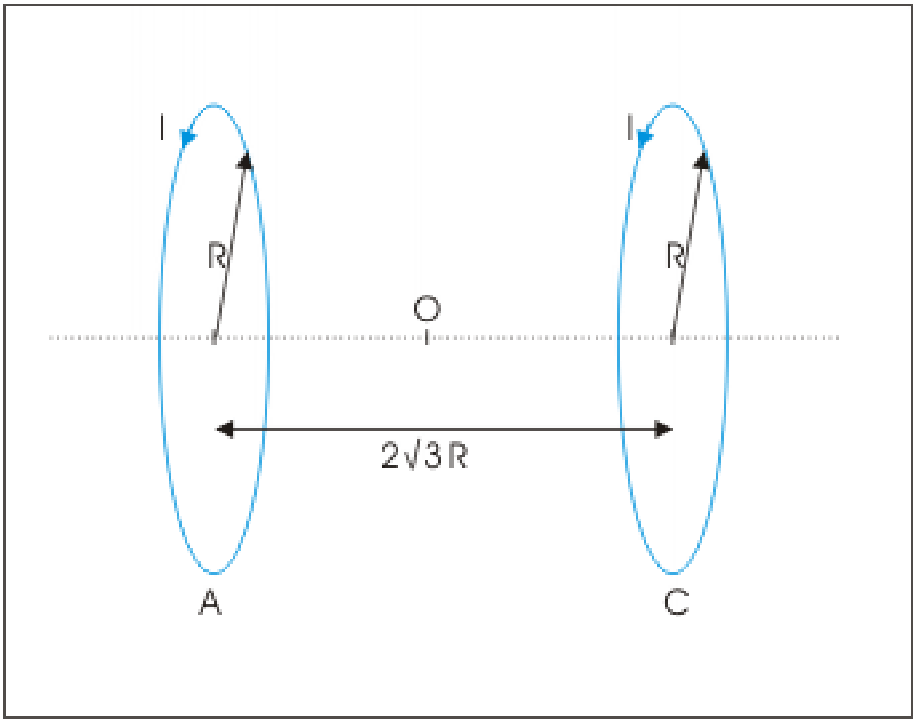


Figure 6.5: Two identical circular coils at a distance

Solution : For an observer at "O", the current in coil A is anticlockwise. The magnetic field due to this coil is towards the observer i.e. towards right. On the other hand, the current in coil C is clockwise for an observer at "O". The magnetic field due to this coil is away from the observer i.e. again towards right. The magnitude of magnetic field due to either coil is :

$$B\prime = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$$

Here, x = $\sqrt{3}$ R,

$$\Rightarrow B\prime = \frac{\mu_0 I R^2}{2(3R^2 + R^2)^{3/2}} = \frac{\mu_0 I R^2}{2(4R^2)^{3/2}} = \frac{\mu_0 I}{16R}$$

The net magnetic field is twice the magnetic field due to one coil,

$$\Rightarrow B = 2B\prime$$

$$\Rightarrow B = 2B\prime = 2X\frac{\mu_0 I}{16R} = \frac{\mu_0 I}{8R}$$

The net magnetic field is directed towards right.

6.2 Variation of magnetic field along the axis

As far as magnitude of magnetic field is concerned, it decreases away from the circular wire. It is maximum when point of observation is center. In this case,

x = 0 and magnetic field, B is :

$$B = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{\frac{3}{2}}} = \frac{\mu_0 I R^2}{2R^3} = \frac{\mu_0 I}{2R}$$

This result is consistent with the one derived for this case in earlier module. For magnetic field at a far off point on the axis,

$$x^2 \gg R^2$$

$$x^2 + R^2 \approx x^2$$

Putting in the expression of magnetic field, we have :

$$B = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{\frac{3}{2}}} = \frac{\mu_0 I R^2}{2x^3}$$

Clearly, magnetic field falls off rapidly i.e. inversely with the cube of linear distance x along the axis. A plot of the magnitude of current is shown here in the figure :

Variation of magnetic field along the axis

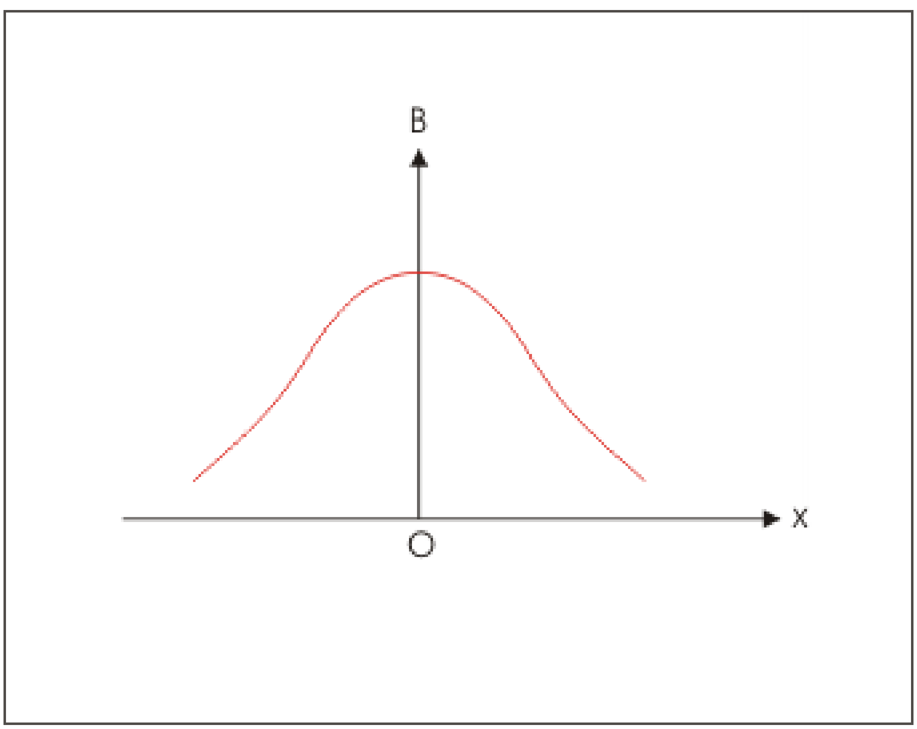


Figure 6.6: Variation of magnetic field along the axis

6.3 Magnetic moment

The concept of moment is a very helpful concept for describing magnetic properties. The description of circular coil as magnetic source in terms of magnetic moment, as a matter of fact, underlines yet another parallelism that runs between electrostatics and electromagnetism.

Magnetic moment of a closed shaped wire is given by :

$$\mathbf{M} = NI\mathbf{A}$$

For a single turn of circular wire :

$$\mathbf{M} = I\mathbf{A}$$

The magnetic moment is a vector obtained by multiplying area vector with current. The direction of area vector is perpendicular to the plane of wire. For circular wire shown in the module,

$$\mathbf{A} = \pi R^2\mathbf{i}$$

Now, axial magnetic field vector is given by :

$$\mathbf{B} = \frac{\mu_0 I R^2}{2(x^2+R^2)^{\frac{3}{2}}}\mathbf{i}$$

But,

$$\mathbf{M} = I\mathbf{A} = I\pi R^2\mathbf{i}$$

Substituting in the expression of magnetic field,

$$\Rightarrow \mathbf{B} = \frac{\mu_0 \mathbf{M}}{2\pi(x^2+R^2)^{\frac{3}{2}}}$$

For a far off axial point $(x^2 + R^2 \approx x^2)$:

$$\Rightarrow \mathbf{B} = \frac{\mu_0 \mathbf{M}}{2\pi x^3} = \frac{\mu_0 2\mathbf{M}}{4\pi x^3}$$

See the resemblance; it has the same form as that for electrical field due to an electrical dipole having dipole moment **p**on an axial point :

$$\mathbf{E} = \frac{2\mathbf{p}}{4\pi\epsilon_0 x^3}$$

Chapter 7

Lorentz force[1]

Lorentz force is the electromagnetic force on a point or test charge. The corresponding force law for electromagnetic force is an empirical law providing the combined expression for electrical and magnetic forces experienced by the test charge. Lorentz force for a point charge comes into existence under certain conditions. The existence of either electrical or magnetic or both fields is primary requirement.

The force law sets up the framework under which two force types (electrical and magnetic) operate. The law is fundamental to the study of electromagnetic interactions in terms of field concepts. For the consideration of force(s) on the test charge, the important deduction is that electrical field interacts only with electrical field and magnetic field interacts only with magnetic field. In our context of electromagnetic force, we can say that electrical force results from interaction of two electrical fields and magnetic force results from interaction of two magnetic fields.

7.1 Lorentz force expression

The law is stated in vector form as :

$$\mathbf{F} = q\left[\mathbf{E} + (\mathbf{v}X\mathbf{B})\right]$$

We may recognize that Lorentz force is actually vector sum of two forces :

$$\Rightarrow \mathbf{F} = q\mathbf{E} + q\left(\mathbf{v}X\mathbf{B}\right)$$

For convenience, we refer the first force as Lorentz electrical force and second force as Lorentz magnetic force. The Lorentz electrical force is given by first part as :

$$\mathbf{F}_E = q\mathbf{E}$$

The electrical part of law is actually the relation we have already studied in the context of Coulomb's law and Electrical field. Electrical force on the point charge "q" acts in the direction of electrical field (**E**) and as such the particle carrying the charge is accelerated in the direction of **E**. If "m" be the mass of the particle carrying charge, then acceleration of the particle is :

$$\mathbf{a}_E = \frac{\mathbf{F}_E}{m} = \frac{q\mathbf{E}}{m}$$

Lorentz magnetic force is given by second part as :

$$\mathbf{F}_M = q\left(\mathbf{v}X\mathbf{B}\right)$$

[1] This content is available online at <http://cnx.org/content/m31327/1.10/>.

Magnetic force on the point charge "q" acts in the direction perpendicular to the plane formed by **v** and **B** vectors. The direction of vector cross product is the direction of magnetic field, provided test charge is positive. The orientation of vector cross product is determined using Right hand thumb rule. If the curl of right hand follows the direction from vector **v** to **B**, then extended thumb points in the direction of vector cross product.

Direction of vector cross product

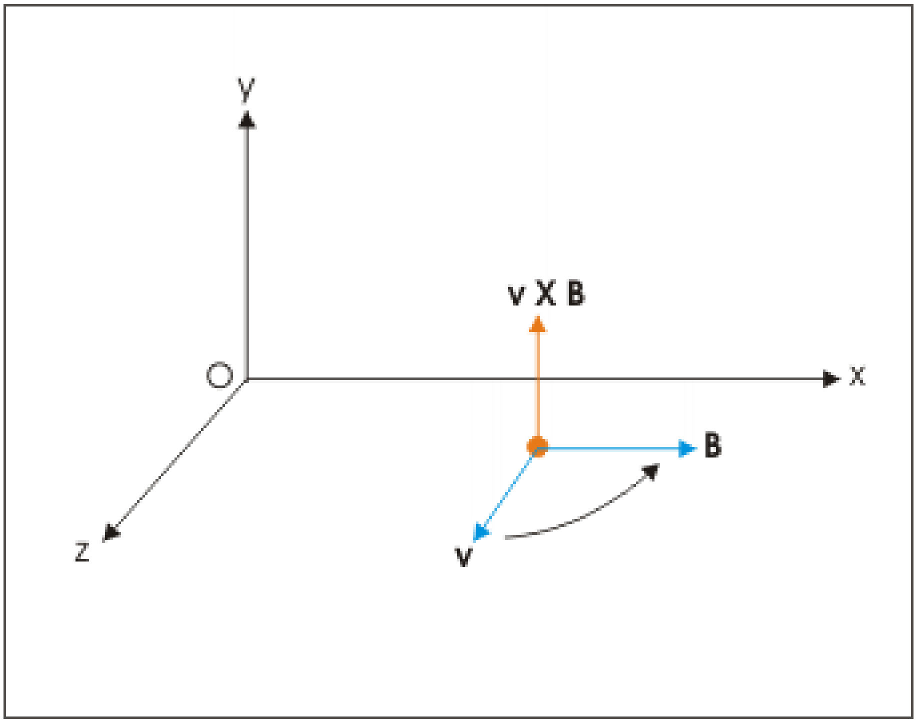


Figure 7.1: The direction of vector cross product is given by Right hand thumb rule.

We should understand an important point that direction of magnetic field is determined **not** by the direction of vector cross product **vXB** alone, but by the direction of expression "q(**vXB**)". What it means that if charge is negative, then direction of force is opposite to that determined by vector cross product "**vXB**". The figure below shows the opposite orientations of vector cross product "**vXB**" and the magnetic force.

Lorentz magnetic force

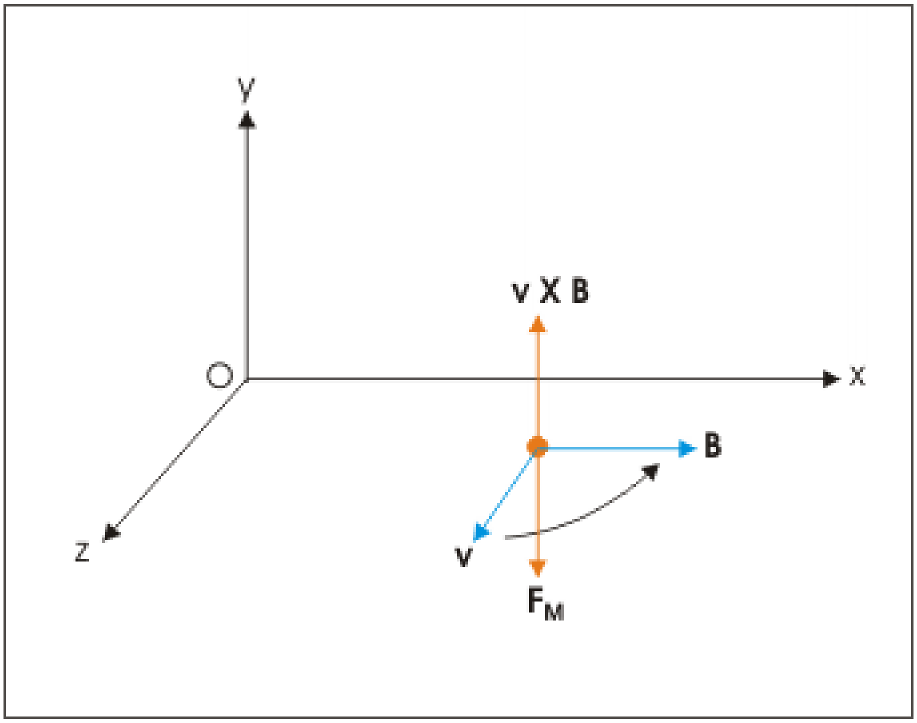


Figure 7.2: Directions of vector cross product and magnetic force are opposite when charge is negative

The acceleration of the particle is given by :

$$\mathbf{a}_M = \frac{\mathbf{F}_M}{m} = \frac{q\left(\mathbf{v}X\mathbf{B}\right)}{m}$$

The magnitude of magnetic force is given by :

$$F_M = qvB\sin\theta$$

where θ is the smaller angle between **v** and **B** vectors. The magnitude of magnetic field is maximum when $\theta = 90$ or 270 and the maximum value of magnetic field is qvB. It is also clear from the expression of magnitude that magnetic force is zero even when magnetic field exists for following cases :

1: charge is stationary i.e. v=0

2: when charge is moving in the direction of magnetic field or in opposite direction i.e. θ=0 or 180 and sin θ =0.

Further, if only electrical field exists, then only electrical force applies on the point charge and the point charge is accelerated in the direction of electrical field (**E**). If only magnetic field exists, then only magnetic force applies on the point charge except for the cases mentioned above (when magnetic force is zero) and the point charge is accelerated in the direction of vector expression q(**vXB**). If both electrical and magnetic field exist, then charge is subjected to both kinds of force provided conditions for zero magnetic force are not met. In the last case, acceleration of the point charge is in the direction of resultant force :

$$\mathbf{a} = \frac{\mathbf{F}}{m} = \frac{q\left[\mathbf{E} + \left(\mathbf{v}X\mathbf{B}\right)\right]}{m}$$

Example 7.1

Problem : An electron, moving along x-axis in an uniform magnetic field **B**, experiences maximum magnetic force along z-axis. Find the direction of magnetic field.

Solution : Since the particle experiences maximum magnetic force, the angle between velocity and magnetic field vector is right angle. Now, magnetic force in z-direction is also perpendicular to the magnetic field. Hence, magnetic field is either in positive or negative y-direction. By applying Right hand rule of vector cross product, we find that it is oriented in positive y-direction if the charge is positive. But, charge on electron is negative.

Lorentz magnetic force

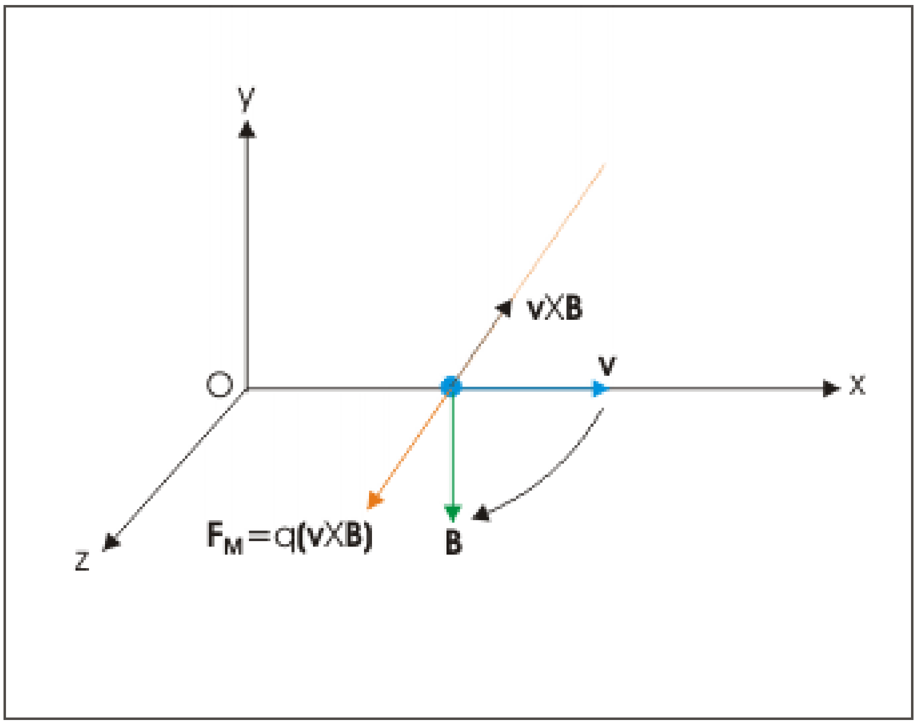


Figure 7.3: Orientations of direction of vector cross product and magnetic force

Hence, magnetic field is oriented along negative y-direction.

7.1.1 Nature of magnetic force

The nature of magnetic force is different to electrical force. First, it is not linear in the sense that it does not operate in the direction of magnetic field. This is unlike electric force which acts in the direction of applied electric field. The magnetic force, as we have seen in the preceding section, acts in the side-way direction following vector cross product rule. Also, magnetic force is relatively weaker as magnetic field is a weaker field in comparison with electric field.

The first of the two distinguishing characteristics as described above has important implications. Since magnetic force is perpendicular to the direction of velocity, it can only change the direction of motion –

not its magnitude. The magnetic force can not change the magnitude of velocity i.e. speed of the charged particle. In turn, we can say that magnetic force can not bring about a change in the kinetic energy of the charged particle as speed remains same due to magnetic field.

An immediate fall out of the magnetic force is very interesting. This force does no work. We know work is scalar dot product of force and displacement. Now, velocity is time rate of displacement. It means velocity and displacement have same direction. Since magnetic force is perpendicular to velocity, it is also perpendicular to small elemental displacement. What it means that magnetic force is always perpendicular to displacement. Thus, work done by magnetic force is zero.

Yet another important consequence of the nature of magnetic force is that a charged particle in magnetic field keeps changing direction of motion of the particle all the time. Since direction of velocity is changed every instant, direction of magnetic force being perpendicular to it is also changed all the time. Note that direction of magnetic force is automatically adjusted or changed with the motion. If the particle does not escape out of the magnetic field, the implication is that the particle may approximate a circular path. At any moment – whether particle completes a circular path or not – the magnetic force acts in radial direction to the motion. On a comparison note, we can see that the electric force is independent of the direction of motion. It is along electric field. It does not change with motion.

Lorentz magnetic force

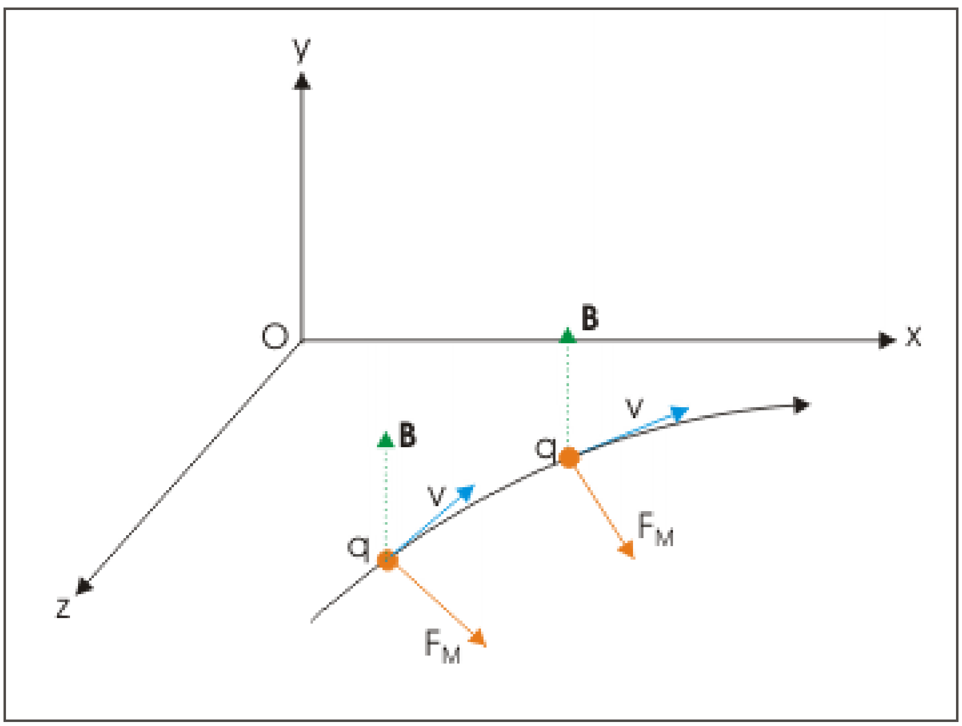


Figure 7.4: Magnetic force changes direction as direction of motion changes.

We make use of this feature in many important applications like cyclotron to accelerate particle or entrapping plasma etc. But we should be aware of its role in these applications. The effect of Magnetic force is limited to change in direction only. Change in speed is effected by electric field.

7.1.2 Magnitude of magnetic force

The magnetic field is a weak field and so is the magnetic force. Let us consider an electron moving with a velocity $3X10^7$ m/s in a magnetic field of $5X10^{-3}$ T. If velocity and magnetic field are perpendicular to each other, then magnetic force on the electron is :

$$F_M = qvB = 1.6X10^{-19}X3X10^7X5X10^{-3} = 2.4X10^{-14} \quad N$$

Clearly, magnetic force is really very weak. However, even this weak force is great enough for subatomic particle like electron. For example, the acceleration of electron due to this magnetic force is :

$$a = \frac{F_M}{m} = \frac{2.4X10^{-14}}{9.1X10^{-31}} = 2.6X10^{16} \quad m/s^2$$

Indeed this is an extraordinary acceleration.

7.1.3 Context of Lorentz force law

Lorentz magnetic force law completes the picture on "effect side" in the study of electromagnetism. The "cause side" i.e. generation of magnetic field is described by Biot-Savart law. Thus, Lorentz force law describes the effect of electric and magnetic fields on a test charge – but not the cause of these fields. This is a serious limitation because test charge on its own is also the cause of electric and magnetic fields. These fields, in turn, would modify the fields operating on the test charge.

Also, the electromagnetic force causes acceleration of test charge. An accelerated charge, in turn, radiates. As such, application of Lorentz force law by itself would not be sufficient to describe motion of test charge. A charged electron which is expected to describe a circular motion under magnetic field without consideration of radiation would actually spiral down with radiation as shown in the figure and expected motion might simply be not there.

Motion of charge under magnetic field

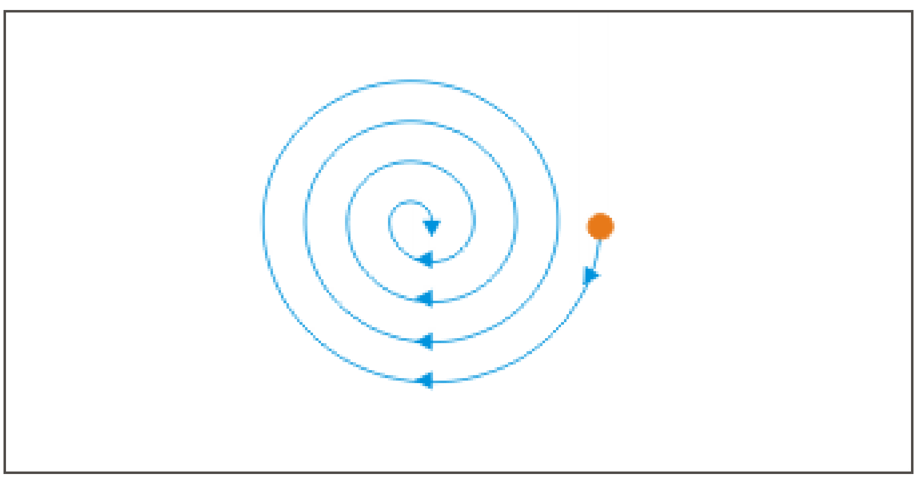


Figure 7.5: Motion of charge under magnetic field

Recall that this was the reason for which Rutherford's atomic model was eventually rejected and Bohr's model was accepted. We shall, however, ignore radiation while studying motion of charged particles under electromagnetic fields – unless state specifically to consider radiation.

Example 7.2

Problem : A particle carrying a charge 1μC is moving with velocity **3i** – **3k** in a uniform field -5 **k**. If units are SI units, then determine the angle between velocity and magnetic field vectors. Also determine the magnetic force.

Solution : The cosine of the enclosed angle is :

$$\cos\theta = \frac{\mathbf{v}.\mathbf{B}}{|\mathbf{v}||\mathbf{B}|} = \frac{(3\mathbf{i} - 3\mathbf{k}) \,.\, (-5\mathbf{k})}{|\,(3\mathbf{i} - 3\mathbf{k})\,||-5\mathbf{k}|}$$

$$\Rightarrow \cos\theta = \frac{15}{15\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = 45^\circ$$

Magnetic force

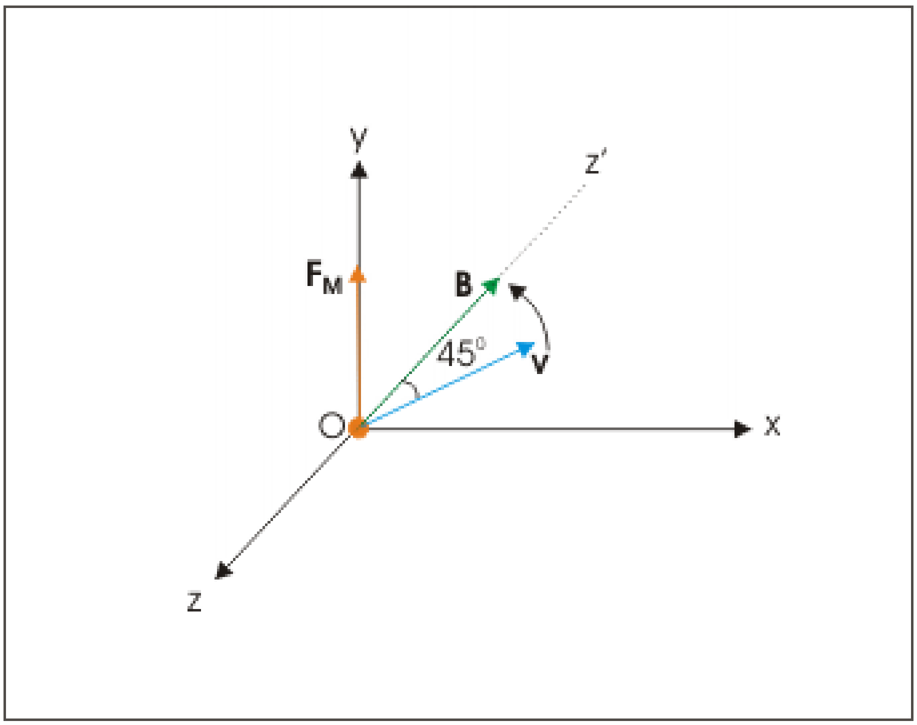


Figure 7.6: Magnetic force is perpendicular to plane formed by velocity and magnetic field vectors.

The velocity and magnetic field vectors lie in x-z plane. The magnetic force is :

$$\mathbf{F}_M = q\,(\mathbf{v}X\mathbf{B}) = 1X10^{-6}\,[(3\mathbf{i} - 3\mathbf{k})\,X - 5\mathbf{k}]$$

$$\Rightarrow \mathbf{F}_M = 1X10^{-6}X15\mathbf{j} = 15X10^{-6}\mathbf{j}$$

Magnetic force is along positive y – direction, which is perpendicular to the x-z plane of velocity and magnetic field vectors.

7.2 Context of electromagnetic interactions

In the discussion so far, we have assumed existence of electrical and magnetic fields. Here, we shall consider about the manner in which electrical and magnetic fields are set up by a source like charge or current and then investigate forces being experienced by the test charge. We shall consider three important cases in which (i) a stationary charge sets up an electrical field (ii) a moving charge sets up both electrical and magnetic fields and (iii) a current carrying wire sets up magnetic field. For each case, we shall discuss two states of test charge (i) it is stationary and (ii) it is moving. Also, note that we shall be deliberately concentrating on the forces experienced by the test charge. It is, however, implied that source charge or conductor carrying current also experiences the same amount of force in accordance with Newton's third law of motion.

7.2.1 Force due to stationary charge

A stationary point source charge changes electrical properties of space around it. This property is quantified by the electrical field **E** at a particular point. If another point test charge is brought at that point, then it experiences electrical force, which is given by electrical part of the Lorentz force.

What happens when the test charge is moving also? It still experiences only the electrical force. No magnetic force is in play. See here that stationary source charge produces only electrical field around it. On the other hand, moving charge brought in its field sets up both electric and magnetic fields. The electric field is set up because moving test charge represents a net charge. But since it is also moving, magnetic field is set up by it in its surrounding in accordance with Biot-Savart Law.

We can easily see that two electrical fields (one due to stationary source charge and other due to moving test charge) interact to result in electrical force. However, there is only one magnetic field due to moving test charge without other magnetic field to interact with. As such, moving charge experiences only Lorentz electrical force in the presence of a stationary source charge.

7.2.2 Force due to moving charge

We now consider a moving charge, which acts as the source for setting up the fields. A moving charge produces both electrical and magnetic fields. If we bring another charge in its surrounding, then it experiences only electrical force. No magnetic force is in play. A stationary test charge only produces electrical field. There is no magnetic field to interact with the magnetic field produced by the moving source charge.

However, if we introduce moving test charge in the surrounding of source moving charge, then the moving test charge experiences both electrical and magnetic fields except for the situation when motion of the charge is neither parallel or anti-parallel to the magnetic field. However, if the motion of test charge is either parallel or anti-parallel to magnetic field produced by moving source charge, then the test charge only experiences electrical force.

7.2.3 Force due to current in wire

The current in wire sets up magnetic field in accordance with Biot-Savart law. Importantly, it does not set up electric field around it. Current through conductor is equivalent to passage of charge. Though, there is net transfer of electrons across a cross section of wire, but there is no accumulation of charge anywhere. As such, the wire carrying current is charge neutral even though there is flow of charge through it.

Now when a test charge is brought at a point in the surrounding of wire, the test charge does not experience any force. The wire sets up a magnetic field whereas charge sets up electrical field. These two different field types do not interact and there is no force on the test charge. On the other hand, if test charge is moving with certain velocity then it sets up electrical as well as magnetic fields. Two magnetic fields interact and as a result, the test charge experiences magnetic force except for the situation when motion of the charge is either parallel or anti-parallel to the magnetic field of the current in wire.

7.3 Magnetic field (B)

Strangely we have discussed and used the concept of magnetic field quite frequently, but without even defining it. There are certain difficulties involved here. There is no magnetic monopole like electrical monopole i.e. point charge. The smallest unit considered to be the source of magnetic field is a small current element. The Biot-Savart law gives relation for magnetic field due to a small current element. But, it is not quantifiable. How much is the "small" magnetic field or the "small" current length element?

As a matter of fact, the expression of Lorentz magnetic force provides us a measurable set up which can be used to define magnetic field. We have noted that magnitude of magnetic force is maximum when angle between velocity and magnetic field vectors is right angle.

$$F_{\max} = qvB$$

$$B = \frac{F_{\max}}{qv}$$

Thus we can define magnetic field (B) as a vector whose magnitude is equal to the maximum force experienced by a charge q divided the product "qv". The direction of magnetic field is given by vector expression q(**vXB**). The SI unit of magnetic field is Tesla, which is written in abbreviated form as T. One Tesla (T), therefore, is defined as the magnetic field under which 1 coulomb test charge moving in perpendicular direction to it at a velocity 1 m/s experiences a force of 1 Newton.

7.4 Exercise

Exercise 7.1 ***(Solution on p. 100.)***

A proton is projected in positive x-direction with a speed of 3 m/s in a magnetic field of (2**i**+3**j**) X 10^{-6} T . Determine the force experienced by the particle.

Solutions to Exercises in Chapter 7

Solution to Exercise 7.1 (p. 99)
Here,

$$\mathbf{v} = 3i \quad m/s$$

$$\mathbf{B} = (2\mathbf{i} + 3\mathbf{j})\, X10^{-6} \quad T$$

$$q = 1.6X10^{-19} \quad C$$

The magnetic force is given by :

$$\mathbf{F}_M = q\,(\mathbf{v}X\mathbf{B})$$

$$\Rightarrow \mathbf{F}_M = 1.6X10^{-19}\left[3\mathbf{i}X\,(2\mathbf{i} + 3\mathbf{j})\,10^{-6}\right]$$

$$\Rightarrow \mathbf{F}_M = 1.6X10^{-19}X9X10^{-6}\mathbf{k}$$

$$\Rightarrow \mathbf{F}_M = 1.44X10^{-24}\mathbf{k} \quad \text{Newton}$$

Chapter 8

Motion of a charged particle in magnetic field[1]

Motion of a charged particle in magnetic field is characterized by the change in the direction of motion. It is expected also as magnetic field is capable of only changing direction of motion. In order to keep the context of study simplified, we assume magnetic field to be uniform. This assumption greatly simplifies the description and lets us easily visualize the motion of a charged particle in magnetic field.

Lorentz magnetic force law is the basic consideration here. Hence, we shall first take a look at the Lorentz magnetic force expression :

$$\mathbf{F} = q\left(\mathbf{v}X\mathbf{B}\right)$$

We briefly describe following important points about this expression :

1: There is no magnetic force on a stationary charge (v=0). As such, our study here refers to situations in which charge is moving with certain velocity in the magnetic field. This condition is met when the charge is released with certian velocity in the magnetic field.

2: The magnetic field (**B**) is an uniform stationary magnetic field for our consideration in the module. It means that the magnitude and direction of magnetic field do not change during motion. The charged particle, however, is subjected to magnetic force acting side way. The direction of motion of charged particle, therefore, changes. In turn, the direction of magnetic force being perpendicular to velocity also changes. Important point to underline here is that this loop of changing directions of velocity and magnetic force is continuous. In other words, the directions of both velocity and magnetic force keeps changing continuously with the progress of motion.

This aspect of continuous change is shown in the figure below. Note that direction of magnetic field is fixed in y-direction. Initially, the charged particle is at the origin of coordinate reference with a velocity **v** in x-direction. Applying right hand rule for vector cross product and considering a point positive charge, we see that magnetic force is directed in z-direction. As a result, the particle is drawn to move along a curved path with velocity (having same speed) directed tangential to it. The magnetic force vector also changes sign being perpendicular to velocity vector. In this manner, we see that the directions of both velocity and magnetic force keeps changing continuously as pointed out.

[1] This content is available online at <http://cnx.org/content/m31345/1.9/>.

Motions of a charged particle in magnetic field

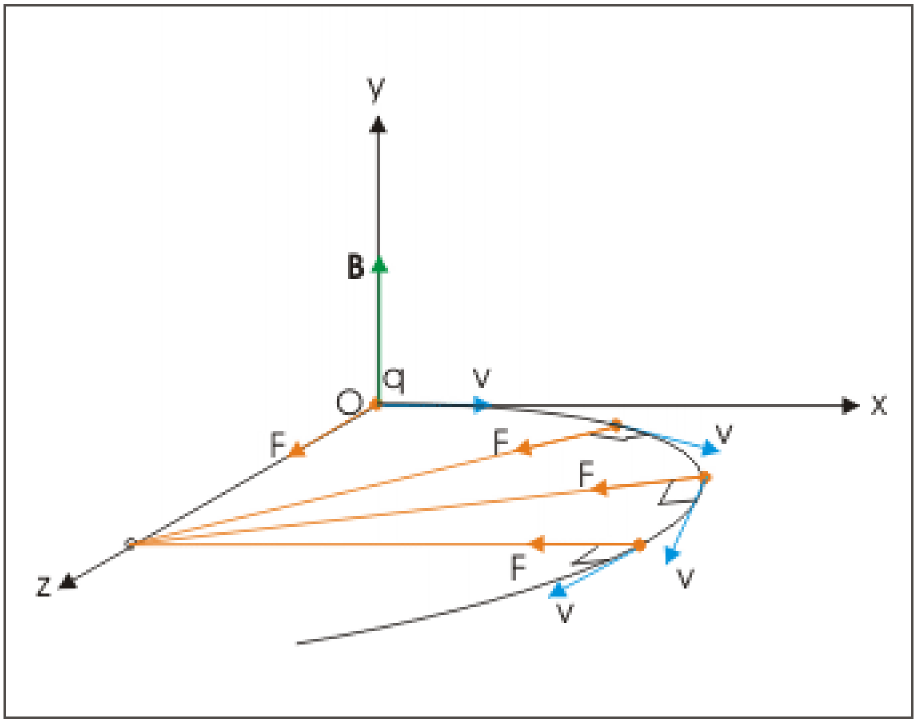


Figure 8.1: Motions of a charged particle in magnetic field

3: The nature of motion depends on the initial directions of both velocity and magnetic field. The initial angle between velocity and magnetic field ultimately determines the outcome i.e. nature of motion.

We shall, therefore, discuss motion of charged particular on the basis of the enclosed angle (θ) between velocity and magnetic field vectors. There are following three cases :

- The motion of the charged particle is along the direction of magnetic field.
- The motion of the charged particle is perpendicular to the direction of magnetic field.
- The motion of the charged particle is neither along nor perpendicular to the direction of magnetic field.

8.1 Motion of the charged particle along magnetic field

There are two possibilities. The enclosed angle (θ) is either 0° or 180°. In either case, sine of the angle is zero. Therefore, magnetic force is zero and the motion of particle remains unaffected (of course here we assume that there is no other force field present).

8.2 Motion of the charged particle perpendicular to magnetic field

This is the case in which charged particle experiences maximum magnetic force. It is given by :

$$F = qBv\sin 90^\circ = qvB$$

If the span of magnetic field is sufficient around the charged particle, then it will describe a circular path as magnetic force is always perpendicular to the motion. The magnetic force provides the centripetal force required for circular motion. The span of magnetic field around charged particle is important. Here, we consider some interesting cases as shown in the figure. For all cases, we assume that motion of charged particle is in the plane of the drawing and magnetic field is perpendicular and into the plane of drawing. Magnetic field is shown by evenly distributed X sign indicating that it is an uniform magnetic field directed into the plane of drawing.

Motions of a charged particle in magnetic field

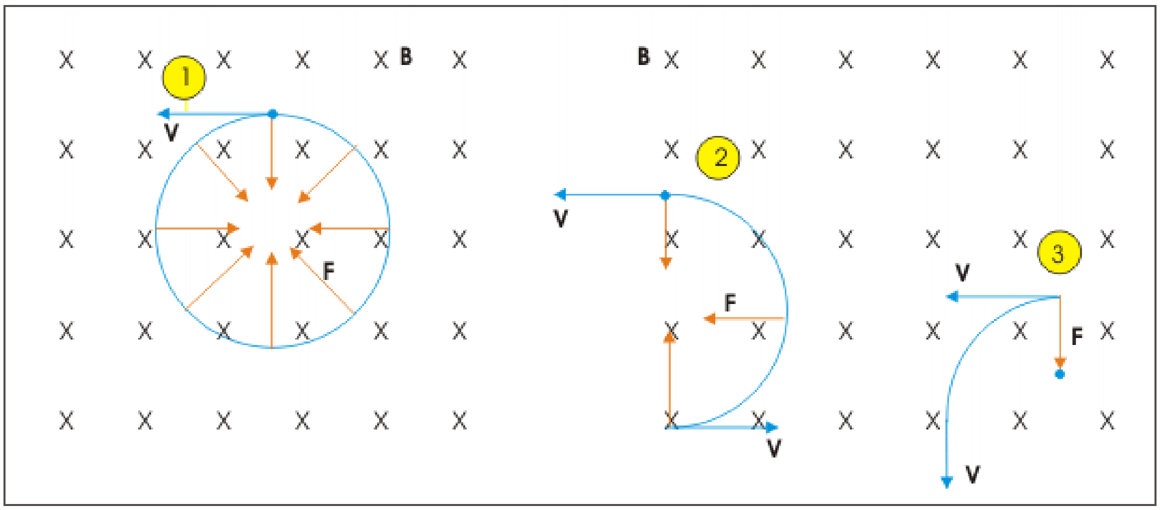


Figure 8.2: Motions of a charged particle in magnetic field

In the first case, there is sufficient span of magnetic field around charged particle and it is able to describe circular path. In second case, the charged particle enters the region of magnetic field and never completes the circular trajectory. Similarly, the charged particle in third case also does not complete the circular path as it comes out of the region of magnetic field even before completing half circle.

Now, we consider the first case in which the charged particle is able to complete circular path. Let the mass of the particle carrying charge is m. Then, magnetic force is equal to centripetal force,

$$\frac{mv^2}{R} = qvB$$

$$\frac{mv}{R} = qB$$

The radius of circular path, R, is given as :

$$R = \frac{mv}{qB}$$

We can easily interpret the effects of various parameters in determining the radius of circular path. Greater charge and magnetic field result in smaller radius. On the other hand, greater mass and speed result in greater radius. Now, the time period of revolution is :

$$T = \frac{2\pi R}{v} = \frac{2\pi m}{qB}$$

Frequency of revolution is :

$$\nu = \frac{1}{T} = \frac{qB}{2\pi m}$$

Angular speed is :

$$\omega = 2\pi\nu = \frac{2\pi qB}{2\pi m} = \frac{qB}{m}$$

Important aspect of these results is that properties related to periodicity of revolutions i.e. time period, frequency and angular velocity all are independent of the speed of the particle. It is a very important result which is used in cyclotron (to accelerate charged particle) to synchronize with the frequency of application of electric field. We shall learn about this in another module.

8.2.1 Specific charge

The ratio of charge and mass of the particle is known as specific charge and is denoted by α. Evidently, its unit is Coulomb/kg. This quantity is important in describing motion of charged particle in magnetic field. We observe that magnetic force is proportional to charge q, whereas acceleration of the particle carrying charge is inversely proportional to mass m. Clearly, the effects of these two quantities are opposite and hence they appear as the ratio q/m in most of the formula describing motion. Recasting formulas with specific charge, we have :

$$R = \frac{v}{\alpha B}; \quad T = \frac{2\pi}{\alpha B}; \quad \nu = \frac{\alpha B}{2\pi}; \quad \omega = \alpha B$$

8.2.2 Angular deviation

Having known the time period, it is easy to know the angle subtended at the center by the arc of travel during the motion in a particular time interval. Since time period T corresponds to a angular travel of 2π, the angular travel or deviation (ϕ) corresponding to any time travel, t, is :

$$\phi = \frac{2\pi}{T} X t = \frac{2\pi qBt}{2\pi m} = \frac{qBt}{m}$$

Alternatively,

$$\phi = \omega t = \frac{qBt}{m}$$

8.2.3 Equations of motion

We consider circular motion of a particle carrying a positive charge q moving in x-direction with velocity $\mathbf{v}_0$ in a uniform magnetic field $\mathbf{B}$, which is perpendicular and into the plane of drawing. Let xy be the plane of drawing and -z be the direction of magnetic field. Here,

$$\mathbf{v}_0 = v_0\mathbf{i}; \quad \mathbf{B} = -B\mathbf{k}$$

where v_0 is the magnitude of velocity. Applying Right hand rule of vector cross product, we see that magnetic force $\mathbf{F}$ is directed in y-direction. These initial orientations are shown in the figure assuming that we begin our observation of motion when particle is at the origin.

Motion of particle carrying charge

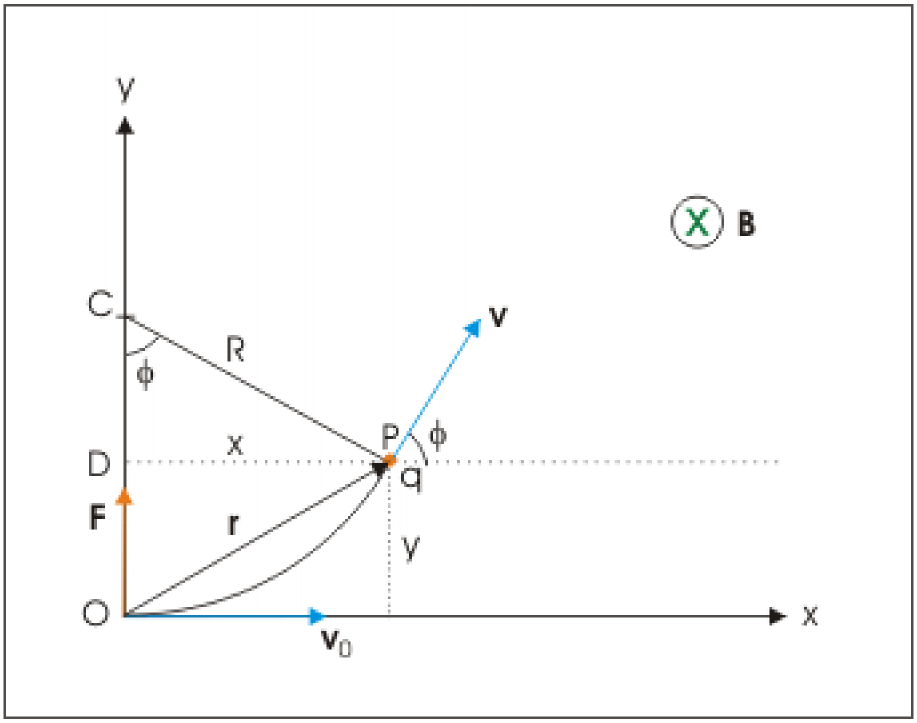


Figure 8.3: Motion of particle carrying charge

The magnetic force **F** provides the necessary centripetal force for the particle to execute circular motion in xy plane in anticlockwise direction with center of circle lying on y-axis. Let the particle be at a point P after time t. Expressing velocity vector in components :

$$\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j}$$

Let the velocity vector makes an angle ϕ with the x-axis. As the magnitude of velocity does not change due to magnetic force, we have :

$$\Rightarrow \mathbf{v} = v_0\cos\phi\mathbf{i} + v_0\sin\phi\mathbf{j}$$

Since particle is executing a uniform circular motion with a constant angular speed,

$$\phi = \omega t$$

Substituting this in the expression of velocity,

$$\Rightarrow \mathbf{v} = v_0\cos\omega t\mathbf{i} + v_0\sin\omega t\mathbf{j}$$

Again substituting for angular speed,

$$\Rightarrow \mathbf{v} = v_0\cos\frac{qBt}{m}\mathbf{i} + v_0\sin\frac{qBt}{m}\mathbf{j}$$

This is the expression of velocity at any time "t" after the start of motion. Let the displacement vector of the particle from the origin is $\mathbf{r}$. Then :

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j}$$

$$\Rightarrow \mathbf{r} = R\sin\phi\mathbf{i} + (R - R\cos\phi)\,\mathbf{j}$$

Substituting for R and ϕ,

$$\Rightarrow \mathbf{r} = \frac{mv_0}{Bq}\left[\sin\frac{qBt}{m}\phi\mathbf{i} + \left(1 - \cos\frac{qBt}{m}\right)\mathbf{j}\right]$$

8.2.4 Motion of charged particle entering a magnetic field

The charged particle entering a magnetic field describes an arc which is at most a semicircle. If the span of magnetic field is limited, then there is no further bending of path due to magnetic force. Let us consider a case in which a particle traveling in the plane of drawing enters a region of magnetic field at angle α.

Motion of charged particle entering a magnetic field

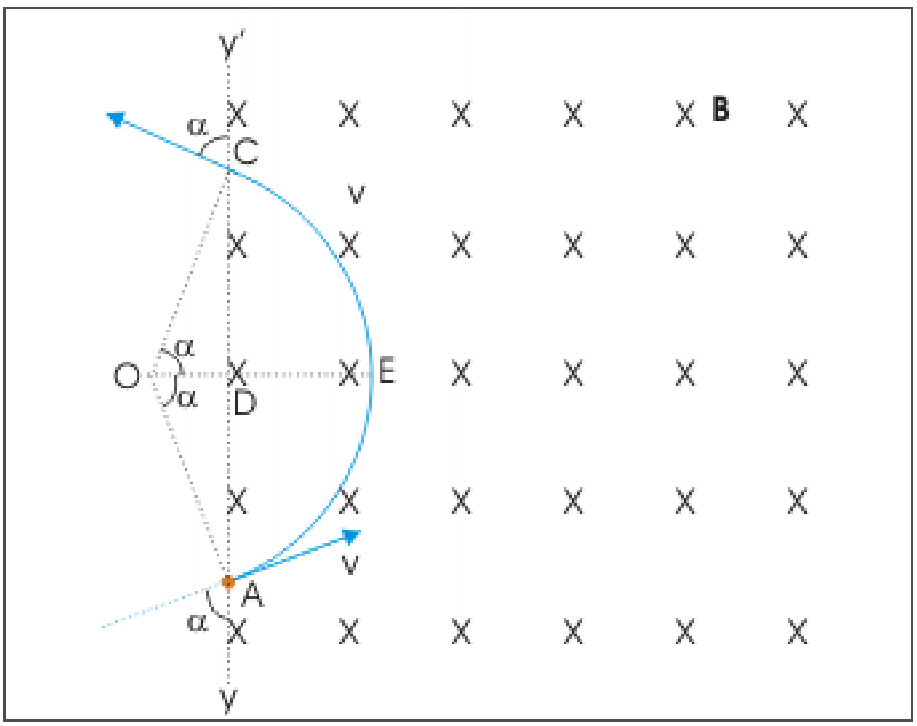


Figure 8.4: Motion of charged particle entering a magnetic field

We should realize here that even though the charged particle enters magnetic region obliquely (i.e at an angle) in the plane of motion, the directions of velocity and magnetic field are still perpendicular to each other. The particle, in turn, follows a circular path. However, the particle needs to move in the region behind the boundary YY' in order to complete the circular path. But, there is no magnetic field behind the boundary. Therefore, the charged particle is unable to complete the circular path. From geometry, it is clear that point of entry and point of exit are points on the circle which is intersected by the boundary YY'. By symmetry, the angle that the velocity vector makes with the boundary YY' at the point of entry is same as the angle that velocity vector makes with the boundary YY' at the point of exit.

By geometry, the angle between pair of lines is same as the angle between the lines perpendicular to them. Hence,

$$\angle OAD = \angle COD = \alpha$$

and

$$\Rightarrow \angle AOC = 2\alpha$$

The length of arc, AEC is :

$$l = AEC = 2\alpha R$$

Substituting for R, we have :

$$\Rightarrow l = \frac{2\alpha mv}{qB}$$

The time of travel in the magnetic field is :

$$\Rightarrow t = \frac{l}{v} = \frac{2\alpha m}{qB}$$

When charged particle enters magnetic field at right angle, velocity vector is perpendicular to the boundary of magnetic field. We know that a tangent can be drawn on a circle in this direction only at the points obtained by the intersection of the circle by the boundary line which divides the circle in two equal sections. A charged particle can, therefore, travel a semicircular path when it enters into the region magnetic field at right angle, provided of course the span of magnetic is sufficient.

Motion of charged particle entering a magnetic field at right angle

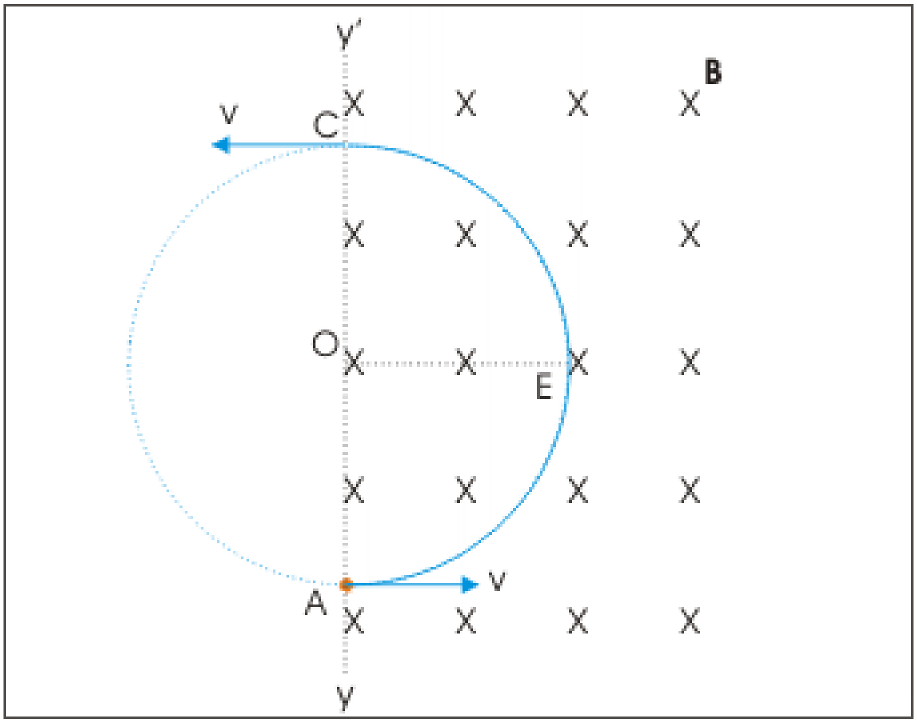


Figure 8.5: Motion of charged particle entering a magnetic field at right angle

We should understand that circular arc path as obtained by the analysis above can be subject to availability of magnetic field till the charged particle begins to move backwards. For a smaller extent of the magnetic field, we find that the particle emerges out of the magnetic field without being further deviated. If the extent of magnetic field is greater than or equal to R, then charged particle describes up to a semicircle depending on the angle at which it enters magnetic region. However, if the extent of magnetic field is less than R, then particle emerges out of the magnetic field without being further deviated.

8.3 Motion of the charged particle oblique to magnetic field

This is the general case of motion of a charged particle in magnetic field. Here, velocity and magnetic field vectors are at an acute angle θ. In order to study the motion, we resolve the velocity vector such that one of the components is parallel and other is perpendicular to the magnetic field.

$$v_{||} = v\cos\theta$$

$$v_{\perp} = v\cos\theta$$

The velocity component perpendicular to the magnetic field results in a magnetic force which provides the necessary centripetal force for the particle to move along a circular path as discussed in previous section. On the other hand, the velocity component parallel to magnetic field results in zero magnetic force and motion in this direction is unaffected due to this component of velocity. The charged particle moves in this direction without being accelerated.

We can visualize superimposition of two motions. The initial conditions of set up are shown in the figure in which particle is shown to have velocity **v** at the origin of coordinate system. The magnetic field is directed in x-direction. The magnetic force (F) due to perpendicular component of velocity and magnetic field is directed in negative z-direction.

Helical motion

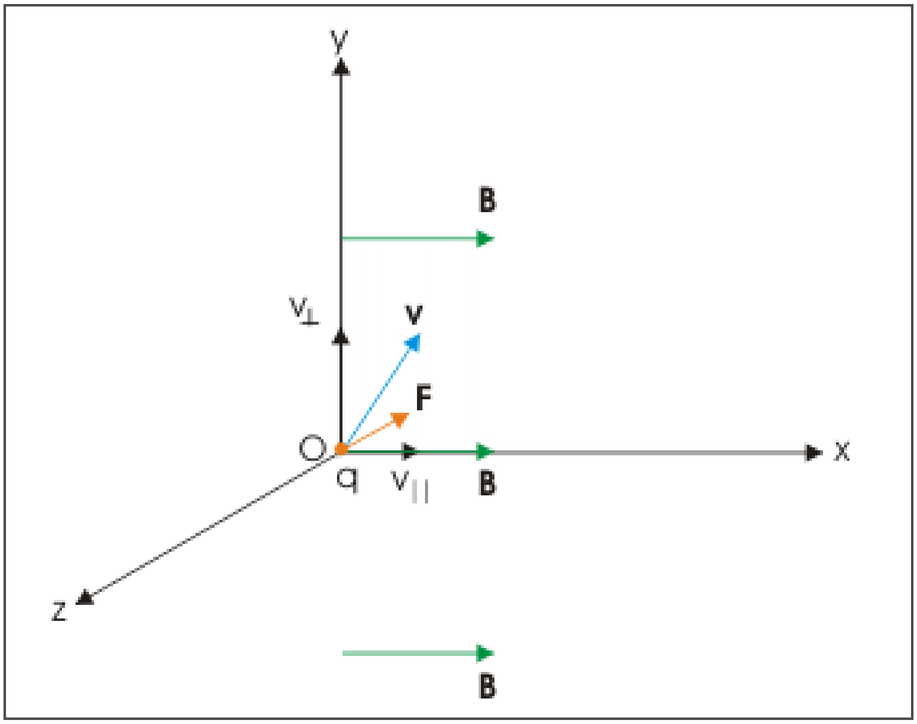


Figure 8.6: Motion of charged particle oblique to magnetic field

If we ignore the parallel component of velocity, then particle will follow circular path due to perpendicular component of velocity in y-z plane as shown here :

Helical motion

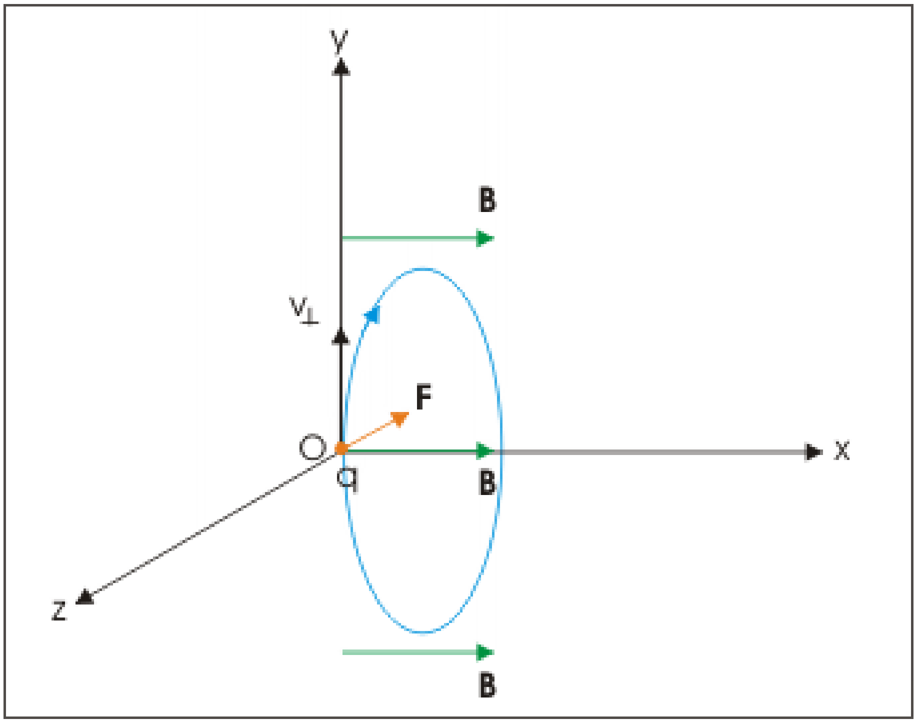


Figure 8.7: Motion of charged particle oblique to magnetic field

But, there is a component of velocity in x-direction. The charged particle still completes a revolution, but not in the circular plane because charged particle also moves in the direction perpendicular to the circular plane. The net result is that the path of revolution is a stretched out series of circles in the form of a helix.

Helical motion

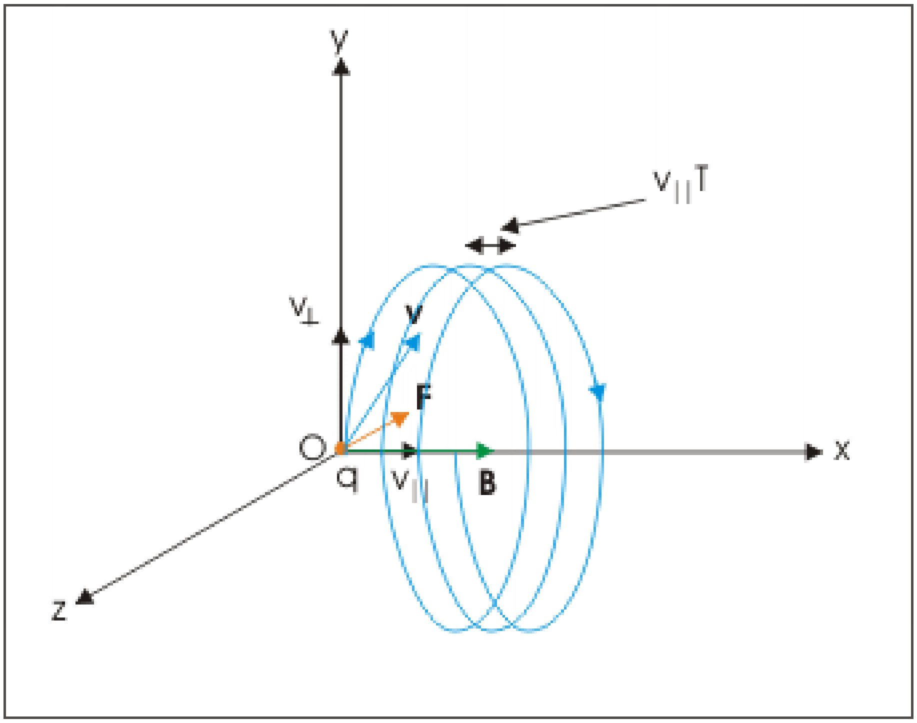


Figure 8.8: Motion of charged particle oblique to magnetic field

The expression for radius is similar as that for the circular motion under magnetic field (earlier case). The only change is that v is exchanged by $v_{\perp}$.

$$R = \frac{mv_{\perp}}{qB} = \frac{mv\sin\theta}{qB}$$

The expressions for time period, frequency and angular velocity etc do not change as these parameters are independent of velocity.

The distance between two consecutive points in x-direction determines the pitch of the helical path. This distance in x-direction is traveled by the particle with the parallel component of velocity in the time in which particle completes a revolution. If T be the time period of revolution, then pitch, p, of the helical path is :

$$p = v_{||}T = vT\cos\theta = \frac{2\pi mv\cos\theta}{qB}$$

Example 8.1

Problem : An electron with a kinetic energy of 10 eV moves into a region of uniform magnetic field of $5X10^{-4}$ T. The initial angle between velocity and magnetic field vectors is 60 degree. Determine the pitch of resulting helical motion.

Solution : The expression of pitch of helical path is :

$$p = \frac{2\pi mv\cos\theta}{qB}$$

We notice here that speed is not directly given. However, kinetic energy is given in electron volt unit. By definition, an electron volt is equal to kinetic energy gained by an electron while passing through a potential difference of 1 V. We get kinetic energy in Joule by multiplying electron-volt value by $1.6X10^{-19}$.

$$K = \frac{mv^2}{2} = 10eV = 10X1.6X10^{-19}J = 16X^{-19}J$$

$$\Rightarrow v = \sqrt{\left(\frac{2K}{m}\right)} = \sqrt{\left(\frac{2X16X10^{-19}}{9.1X10^{-31}}\right)} = \sqrt{\left(3.52X10^{12}\right)} = 1.88X10^6 \quad m/s$$

Putting values in the expression of pitch :

$$\Rightarrow p = \frac{2\pi X9.1X10^{-31}X1.88X10^6X0.5}{1.6X10^{-19}X5X10^{-4}}$$

$$\Rightarrow p = 6.71X10^{-2} \quad m = 6.71 \quad cm$$

8.4 Magnetic bottle

In plasma research, one of the main tasks is to contain plasma (ions or charged elementary particles). Plasma particles can not be restrained in any material confinement because of extraordinarily high temperature associated with them. A magnetic bottle is an arrangement of two magnetic sources (solenoids or any other magnetic source) which produce magnetic fields. The arrangement is such that direction of magnetic field is from one solenoid to another. The magnetic field between two solenoids is non-uniform. It is stronger near the solenoid and weaker in the middle. See that lines of force are denser near the solenoids and rarer in the middle.

A charged particle is in the helical motion in this magnetic region. As it moves in stronger magnetic region near the solenoid, the radius of helical path is smaller. On the other hand, the radius of helical path is greater in the middle as magnetic field is weaker there.

$$R = \frac{mv}{qB}$$

Magnetic bottle

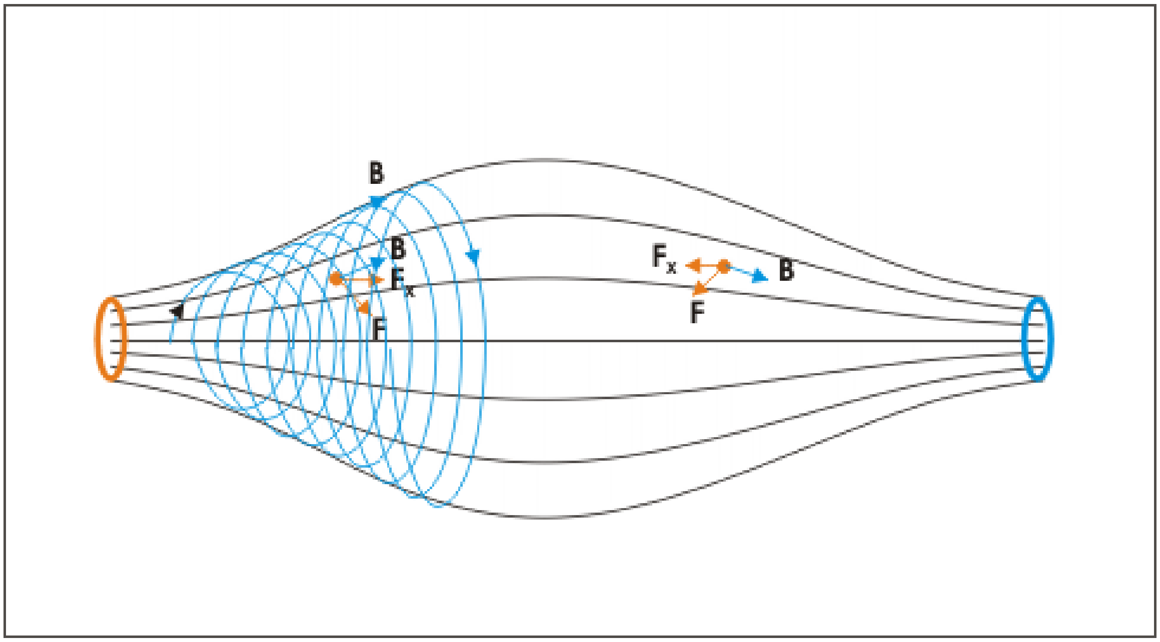


Figure 8.9: The charged particle is trapped in magnetic field

As the particle reaches towards the solenoid i.e. end of the arrangement, it is rebounded because there is a component of magnetic force pointing towards the central part of the arrangement. See figure that how force components point toward middle. This component decelerates the particle till it stops and starts moving in opposite direction. The stronger magnetic region near the solenoid, therefore, functions as reflector of charged particles.

In this manner, plasma particles are confined within a region due to suitably designed magnetic field. The whole arrangement works like a bottle for the charged particles and hence is called magnetic bottle.

Chapter 9

Motion of a charged particle in electric and magnetic fields[1]

Motion of a charged particle in the simultaneous presence of both electric and magnetic fields has variety of manifestations ranging from straight line motion to the cycloid and other complex motion. Both electric and magnetic fields impart acceleration to the charged particle. But, there is a qualification for magnetic field as acceleration due to magnetic field relates only to the change of direction of motion. Magnetic force being always normal to the velocity of the particle tends to move the particle about a circular trajectory. On the other hand, electric force is along electric field and is capable to bring about change in both direction and magnitude depending upon the initial direction of velocity of the charged particle with respect to electric field. If velocity and electric vectors are at an angle then the particle follows a parabolic path.

One of the important orientations of electric and magnetic fields is referred as "crossed fields". We use the term "crossed fields" to mean simultaneous presence of electric and magnetic fields at right angle. The behavior of charged particles such as electrons under crossed fields has important significance in the study of electromagnetic measurement and application (determination of specific charge of electron, cyclotron etc.).

Before we proceed, we should understand that elementary charged particles have mass of the order of 10^{-28} kg or less. Therefore, even small electric or magnetic force is capable to generate very high acceleration of the order of 10^{12} m/s^2 or more. Under proper set up, these particles achieve velocity comparable to speed of light. In order to keep our discussion in the simple classical context, however, we shall confine our discussion limited to the cases which are less complicated and which neglect relativistic effects.

Some of the important applications or phenomena associated with simultaneous presence of two fields include :

- Motion of a charged particle in electric and magnetic fields
- Measurement of specific charge of an electron (J.J.Thomson experiment)
- Acceleration of charged particles (cyclotron)

In this module, we shall study first two of the listed application or phenomena. The third one i.e. cyclotron will be discussed in a separate module.

9.1 Motion of a charged particle in electric and magnetic fields

We have already studied motion of charged particle in individual fields. Here, we shall combine the effects of two fields. Few of the interesting cases are discussed here.

[1]This content is available online at <http://cnx.org/content/m31547/1.2/>.

9.1.1 Charged particle is moving along parallel electric and magnetic field

The velocity, electric and magnetic vectors are in in the same direction. Let they are aligned along x-axis. Since magnetic field and velocity vectors are parallel, there is no magnetic force.

$$F_M = v_0 qB\sin 0^\circ = 0$$

where v_0 is initial speed of the particle. The charged particle is, however, acted upon by electric field. It is accelerated or decelerated depending on the polarity of charge and direction of electric field. Considering positive charge, the electric force on the charge is given as :

$$F_E = qE$$

The acceleration of particle carrying charge in x-direction is :

$$\Rightarrow a_y = \frac{F_E}{m} = \frac{qE}{m}$$

The displacement along x-axis after time “t” is given by :

$$x = v_0 t + \frac{1}{2} a_y t^2$$

$$\Rightarrow x = v_0 t + \frac{qEt^2}{2m}$$

9.1.2 Charge is moving perpendicular to parallel electric and magnetic fields

Let electric and magnetic fields align along y-direction and velocity vector is aligned along positive x-direction. Let the charge be positive and initial velocity be v_0 .In this case, velocity and magnetic field vectors are perpendicular to each other. Applying Right hand vector cross product rule, we determine that magnetic force is acting in positive z-direction. If electric field is not present, then the particle revolves along a circle in xz plane as shown in the figure below.

Motion of a charged particle in magnetic field

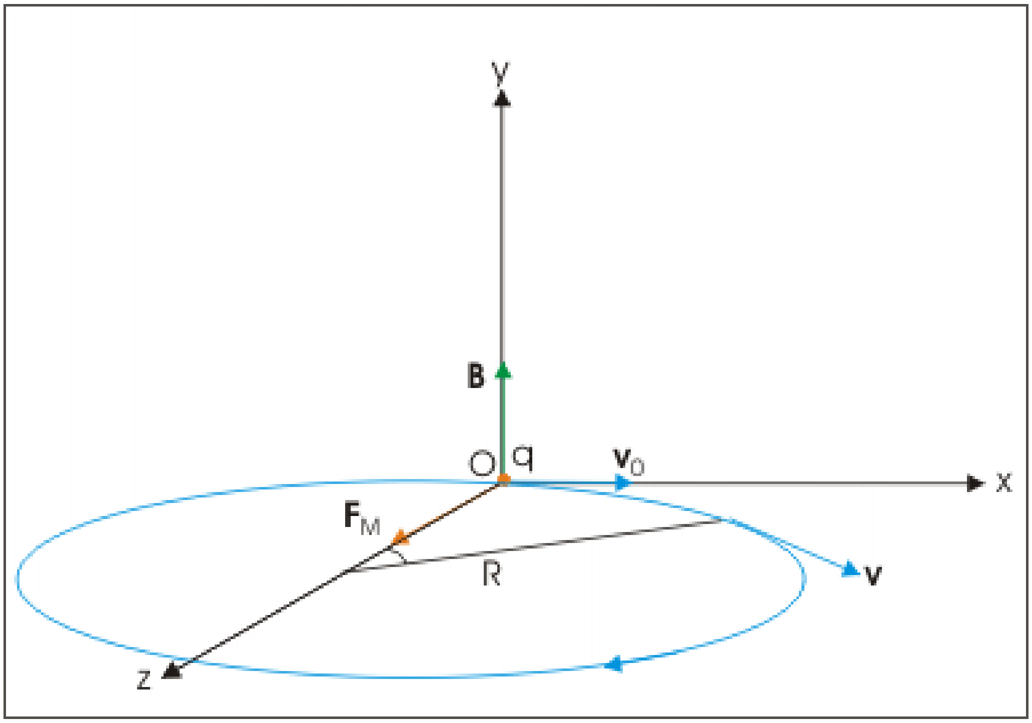


Figure 9.1: Motion of a charged particle in magnetic field

However, electric field in y-direction imparts acceleration in that direction. The particle, therefore, acquires velocity in y-direction and resulting motion is a helical motion. But since particle is accelerated in y –direction, the linear distance between consecutive circular elements of helix increases. In other words, the resulting motion is a helical motion with increasing pitch.

Motion of a charged particle in electric and magnetic fields

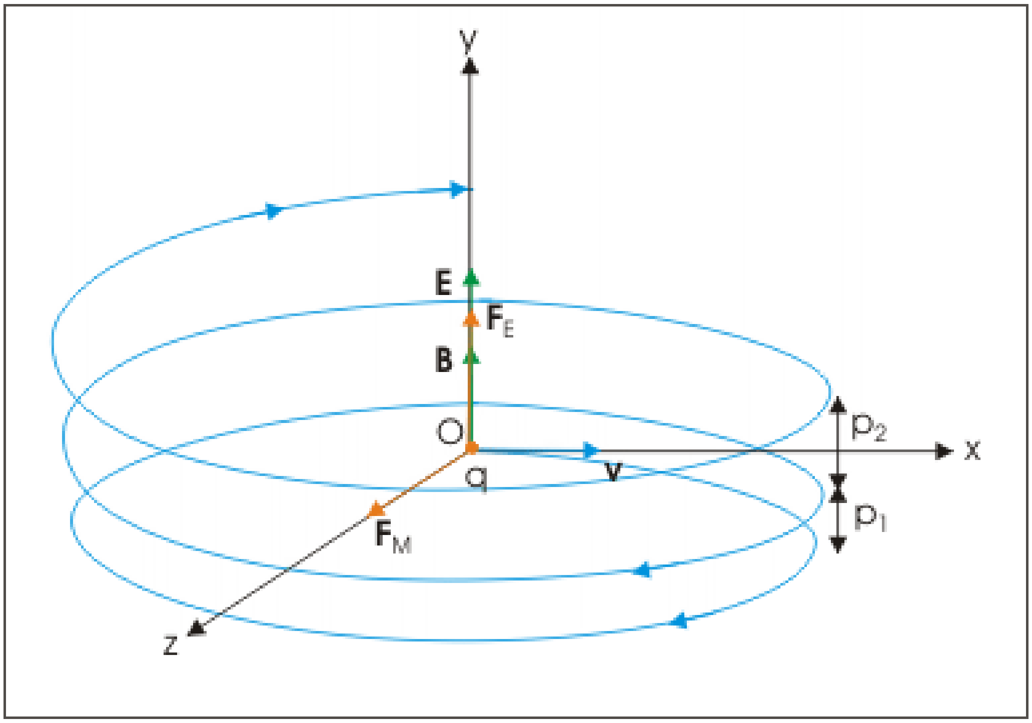


Figure 9.2: Resulting motion is a helical motion with increasing pitch.

The radius of each of the circular element and other periodic attributes like time period, frequency and angular frequency are same as for the case of circular motion of charged particle in perpendicular to magnetic field.

$$R = \frac{v}{\alpha B}; \quad T = \frac{2\pi}{\alpha B}; \quad \nu = \alpha B / 2\pi; \quad \omega = \alpha B$$

9.1.2.1 Velocity of the charged particle

The velocity of the particle in xz plane (as also derived in the module Motion of a charged particle in magnetic field (Section 8.2.3: Equations of motion)) is :

$$\mathbf{v} = v_x \mathbf{i} + v_z \mathbf{j} = v_0 \cos\omega t \mathbf{i} + v_o \sin\omega t \mathbf{k}$$

$$\Rightarrow \mathbf{v} = v_0 \cos(\alpha B t)\, \mathbf{i} + v_0 \sin(\alpha B t)\, \mathbf{k}$$

where α is specific charge. We know that magnetic force does not change the magnitude of velocity. It follows then that magnitude of velocity is xy plane is a constant given as :

$$v_x^2 + v_z^2 = {v_{\text{xy}}}^2$$

But, there is electric field in y-direction. This imparts linear acceleration to the charged particle. As such, the particle which was initially having no component in y direction gains velocity with time as electric field imparts acceleration to the particle in y direction. The velocity components in xz plane, however, remain same. The acceleration in y-direction due to electric field is :

$$\Rightarrow a_y = \frac{F_E}{m} = \frac{qE}{m} = \alpha E$$

Since initial velocity in y-direction is zero, the velocity after time t is :

$$\Rightarrow v_y = a_y t = \alpha E t$$

The velocity of the particle at a time t, therefore, is given in terms of component velocities as :

$$\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} + v_j\mathbf{k}$$

$$\Rightarrow \mathbf{v} = v_0\cos\left(\alpha Bt\right)\mathbf{i} + \alpha Et\mathbf{j} + v_0\sin\left(\alpha Bt\right)\mathbf{k}$$

9.1.2.2 Displacement of the charged particle

Component of displacement of the charged particle in xz plane is given (see module Motion of a charged particle in magnetic field (Section 8.2.3: Equations of motion)) as :

Displacement of the charged particle in xz plane

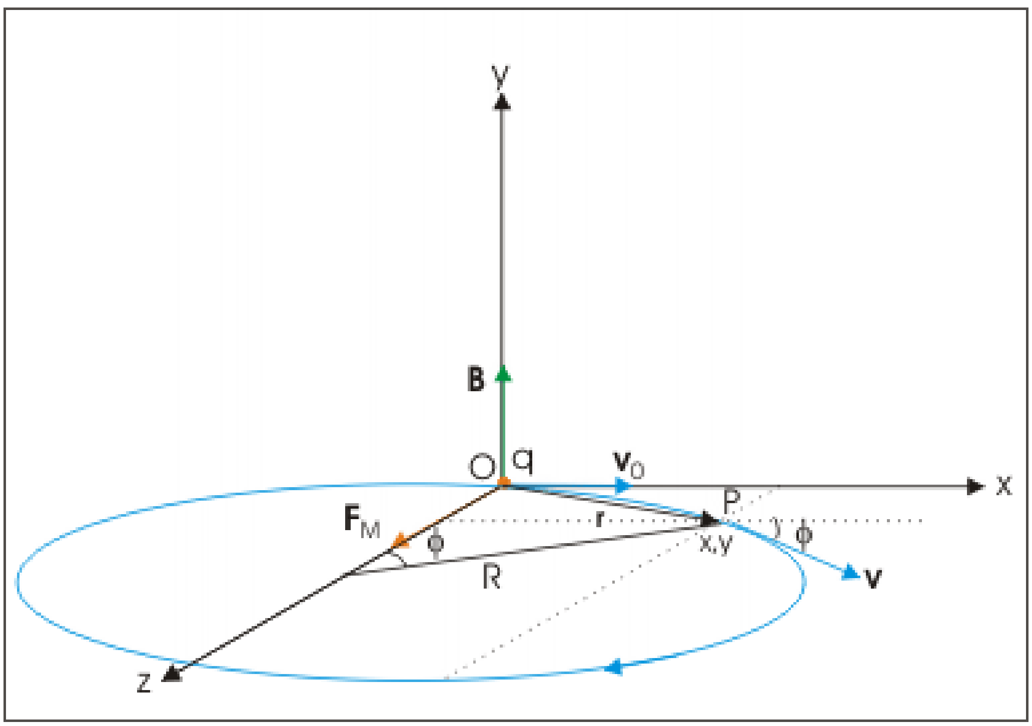


Figure 9.3: Displacement of the charged particle in xz plane

$$x = R\sin\left(\alpha Bt\right) = \frac{v_0}{\alpha B}\sin\left(\alpha Bt\right)$$

$$z = R\left[1-\cos\left(\alpha Bt\right)\right] = \frac{v_0}{\alpha B}\left[1-\cos\left(\alpha Bt\right)\right]$$

The motion in y-direction is due to electric force. Let the displacement in this direction be y after time t. Then :

$$y = \frac{1}{2}a_y t^2 = \frac{1}{2}\alpha E t^2$$

The position vector of the particle after time t is :

$$\mathbf{r} = x\mathbf{i} + yj + z\mathbf{k}$$

$$\Rightarrow \mathbf{r} = \frac{v_0}{\alpha B}\sin\left(\alpha Bt\right)\mathbf{i} + \frac{1}{2}\alpha Et^2\mathbf{j} + \frac{v_0}{\alpha B}\left[1-\cos\left(\alpha Bt\right)\right]\mathbf{k}$$

9.1.3 Charge is placed at rest in crossed electric and magnetic fields

Let electric and magnetic fields are aligned along z and x directions and charge is placed at the origin of coordinate system. Initially, there is no magnetic force as charge is at rest. However, there is electric force, which accelerates the charge in z-direction. As the particle acquires velocity in z-direction, the magnetic force comes into play and tries to rotate the particle in xz plane about a center on x-axis.

Cycloid motion

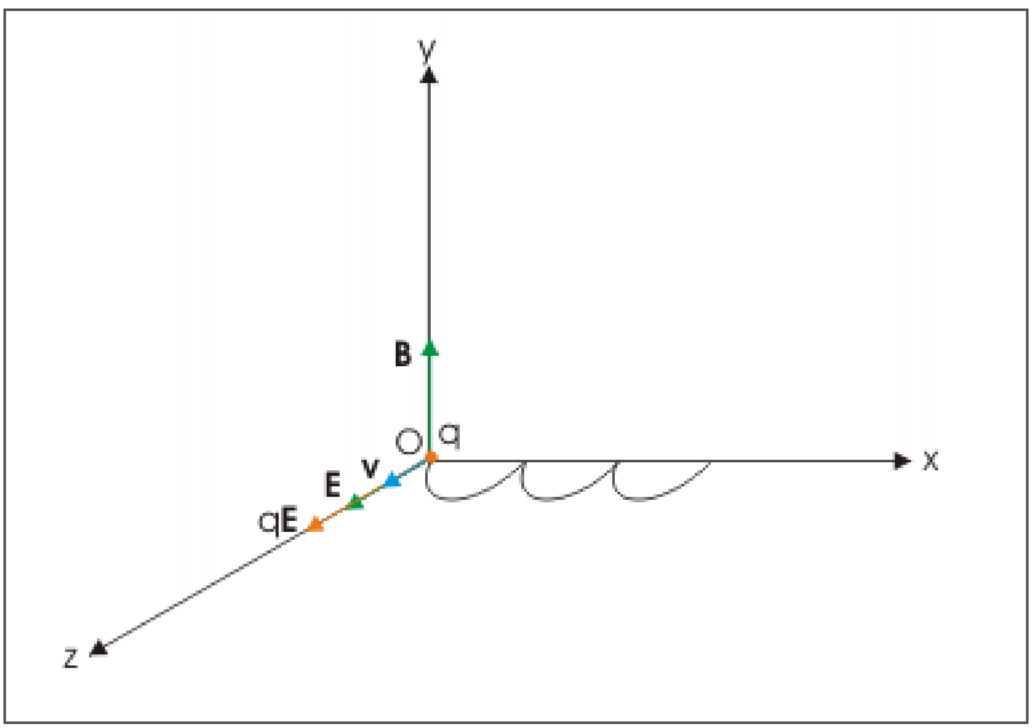


Figure 9.4: Cycloid motion

However, z-component of velocity keeps increasing with time due to electric force in that direction. The magnetic force though draws the charged particle away from z-axis along a curved path. This action of magnetic force is countered by electric force in z-direction. The velocity of charged particle ultimately reduces to zero at x-axis. This cycle repeats itself forming cycloid motion. The cycloid path is generated by a point on the circumference of a rolling wheel. Here, we shall skip the mathematical derivation and limit ourselves to a descriptive analysis only.

9.2 Determination of specific charge of electron (J.J.Thomson's experiment)

The specific charge of an electron is ratio of charge and mass of electron. The specific charge (α) of electron is measured employing crossed fields on a beam of electrons. The beam of electrons emerging from cathode plate passes through a very narrow slit in anode plate. The electrons are accelerated between cathode and anode due to applied electrical potential V. The kinetic energy of the electron emerging from the slit is given by :

$$\frac{1}{2}mv^2 = eV$$

$$\Rightarrow mv^2 = 2eV$$

where v is the velocity of electron moving into the region of force fields.

Two parallel plates connected to an electric source produce a uniform electric field **E** from positive plate to negative plate. The electrical force works in the direction opposite to the direction of field **E** as charge on electron is negative. In the figure, electric field is directed in downward direction. Hence, electric force acts in upward direction.

On the other hand, the magnetic field is produced by a solenoid in a circular region covering the plate as shown in the figure. Its direction is chosen such that it applies a force in the opposite direction to that applied by the electrical field. For a magnetic field into the plane of drawing as shown by uniformly distributed cross signs, the magnetic field applies a upward magnetic force on a positive charge. However, as the charge on the electron is negative, the magnetic force acts in downward direction.

J.J.Thomson's experimental set up

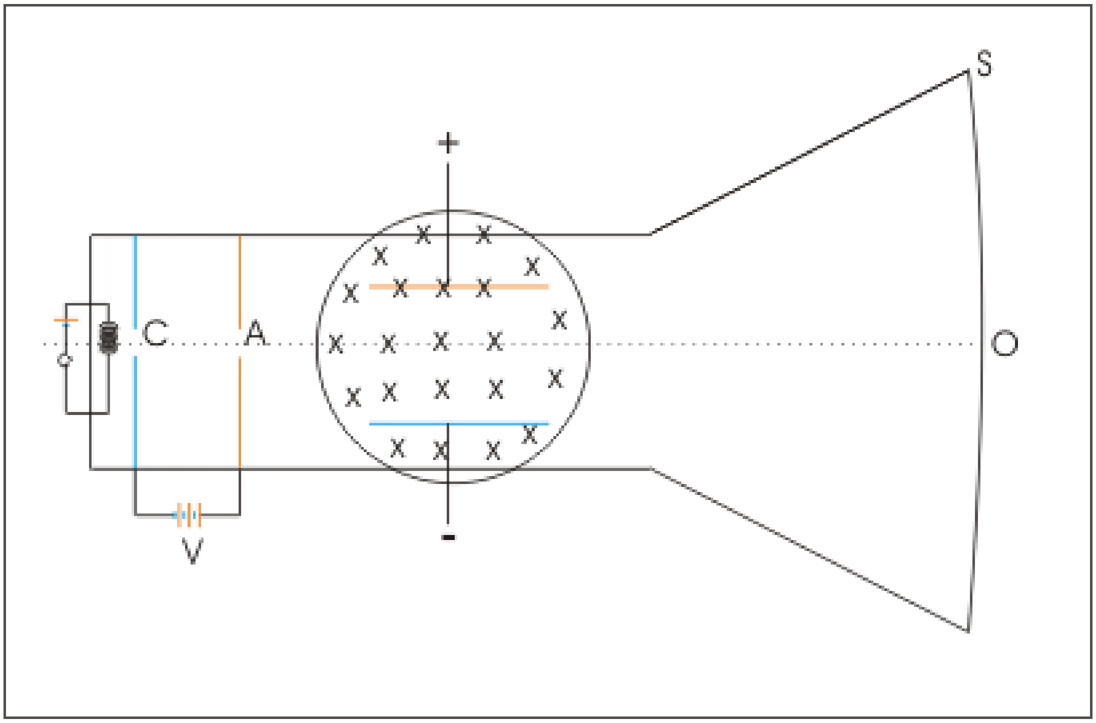


Figure 9.5: Measurement of specific charge of electron

The beam of electrons hit the center of fluorescent screen, producing light as electrons collide with it when electric and magnetic fields are switched off. The point on the fluorescent screen is noted. Then, the electric field is switched on which moves the electron beam in upward direction following a parabolic path. Finally, magnetic field is turned and its magnitude is adjusted such that electric and magnetic forces acting in opposite directions balance each other and the electron is brought to hit original spot as noted earlier for the fields in switched off condition. In this situation:

$$eE = evB$$

$$\Rightarrow v = \frac{E}{B}$$

Note that maximum magnetic force applies as velocity and magnetic field vectors are perpendicular to each other. Substituting expression of v in the kinetic energy equation obtained earlier, we have :

$$\frac{mE^2}{B^2} = 2eV$$

$$\Rightarrow \alpha = \frac{e}{m} = \frac{E^2}{2VB^2}$$

All the quantities on the right hand side of the equation are measurable, allowing us to measure the specific charge of electron. As a matter of fact, the determination of specific charge of particles composing cathode ray by J.J.Thomson is considered to be the discovery of electron. It can also be easily inferred that he could determine the nature of charge of an electron by studying direction of deviation (upward or downward) when only either of the fields operate. In the derivation above, we measure potential difference applied to accelerate particle between cathode and anode. We should, however, realize that we can determine specific charge measuring some other quantities as well. We can measure the deflection of electron beam when either of two fields operates and use the data to determine specific charge of an electron.

Example 9.1

Problem : The d.c. voltage applied to accelerate particle between cathode and anode and the d.c. voltage applied to the plates to produce electric field perpendicular to electrons beam are equal in the Thomson's experimental set up. If each of the two d.c. voltages as applied are doubled, then by what factor should the magnetic field be changed to keep the electron beam un-deflected.

Solution : Let V_1 , E_1 and B_1 be the potential difference, electric field and magnetic field for un-deflected condition. Then, the specific charge is given by :

$$\alpha = \frac{e}{m} = \frac{E_1^2}{2V_1 B_1^2}$$

Here, the electric field can be expressed in terms of potential difference provided we know the separation between plates. Let the separation be d.

$$E_1 = \frac{V_1}{d}$$

Putting in the equation above, we have :

$$\Rightarrow \alpha = \frac{V_1^2}{2d^2 V_1 B_1^2} = \frac{V_1}{2d^2 B_1^2}$$

Let B_2 be the new magnetic field when two potential differences as applied are doubled. Here,

$$V_2 = 2V_1$$

Putting new values in the expression for specific charge (note that specific charge of electron is a constant),

$$\alpha = \frac{2V_1}{2d^2 B_2^2}$$

Combining two equations,

$$\frac{2V_1}{2d^2 B_2^2} = \frac{V_1}{2d^2 B_1^2}$$

$$\Rightarrow 2V_1 d^2 B_2^2 = 4V_1 d^2 B_1^2$$

$$\Rightarrow \frac{B_2^2}{B_1^2} = 2$$

$$\Rightarrow \frac{B_2}{B_1} = \sqrt{2}$$

9.2.1 Measurement of deflection by magnetic field

Once the magnetic and electric forces are balanced, electric field is switched off and electron beam is allowed to be deviated due to magnetic field. The magnetic force acts always perpendicular to the direction of motion. The particle, therefore, moves along a circular path inside the region of magnetic field. When electron moves out of the magnetic field, it moves along the straight line and hits the fluorescent screen. If R be the radius of curvature, then :

$$\frac{mv^2}{R} = evB$$

$$\Rightarrow mv = eRB$$

Substituting $v = E/B$ as obtained earlier

$$\alpha = \frac{e}{m} = \frac{E}{RB^2}$$

We measure R using geometry. We see that the angles enclosed between pairs of two perpendicular lines are equal. Hence,

J.J.Thomson's experimental set up

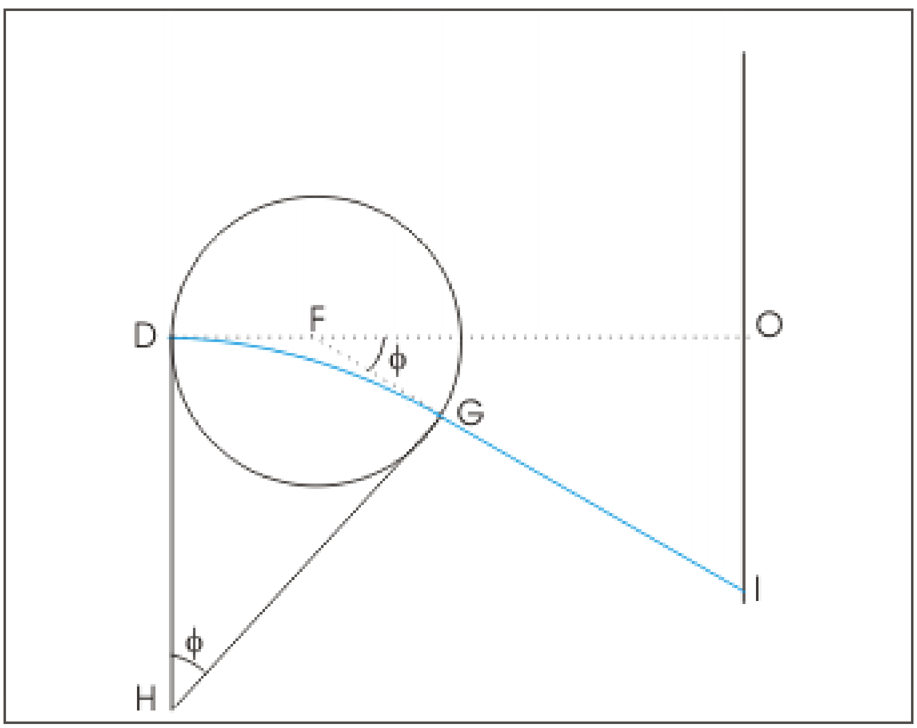


Figure 9.6: Deviation due to only magnetic field

$$\phi = \frac{DG}{R} = \frac{OI}{FO}$$

$$\Rightarrow R = \frac{FOXDG}{OI}$$

We approximate DG to be equal to the width of magnetic region.

9.2.2 Measurement of deflection by electric field

In this case, once the magnetic and electric forces are balanced, electric field is switched off and electron beam is allowed to be deviated due to electric field. The electron beam moving into the region of electric field experiences an upward force. The force in upward (y-direction) imparts acceleration in y-direction. The particle, however, moves with same velocity in x-direction. As a result, path of motion is parabolic. Let the length of plate be L and y be the deflection inside the plate. Then, time to travel through the plate is :

J.J.Thomson's experimental set up

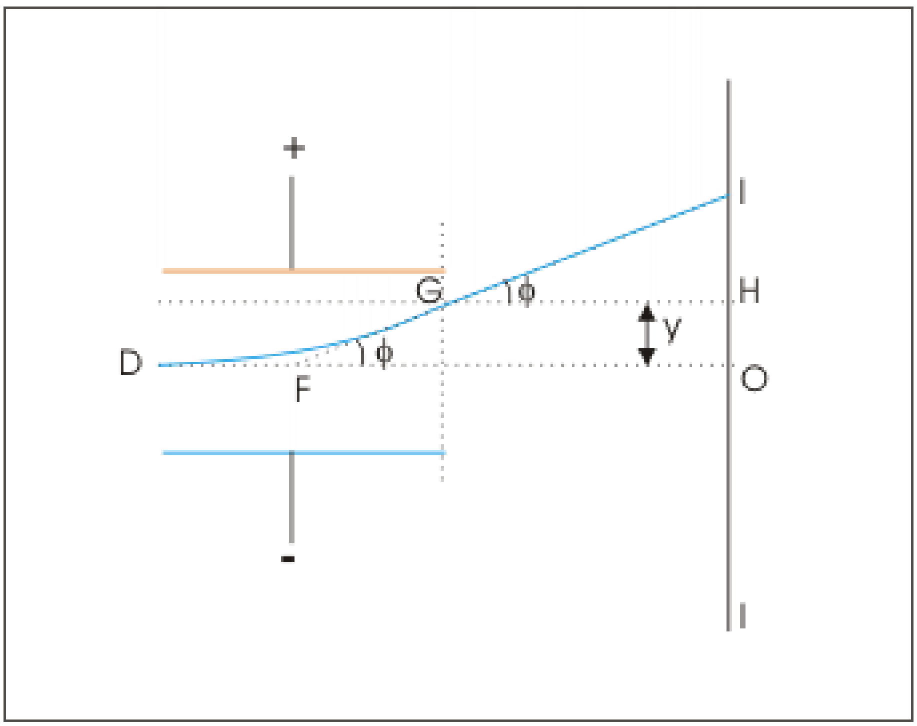


Figure 9.7: Deviation due to only electric field

$$t = \frac{L}{v}$$

and acceleration of the particle in y-direction is :

$$a_y = \frac{F_E}{m} = \frac{eE}{m}$$

The vertical displacement is :

$$y = \frac{1}{2} a_y t^2$$

Substituting for time and acceleration, we have :

$$y = \frac{1}{2} X \frac{eE}{m} X \frac{L^2}{v^2} = \frac{eEL^2}{2mv^2}$$

Substituting v=E/B as obtained earlier,

$$y = \frac{eEL^2}{2m} X \left(\frac{B}{E}\right)^2 = \frac{\alpha EL^2}{2} X \left(\frac{B}{E}\right)^2$$

$$\Rightarrow \alpha = \frac{e}{m} = \frac{2yE}{B^2 L^2}$$

It is clear that measuring GH and HI, we can determine angle ϕ and then y as required.

Chapter 10

Cyclotron[1]

High speed charged particles are required for nuclear and atomic investigations. Cyclotron is one of the devices popularly known as "particle accelerator" to accelerate charged particle to a very high speed. It uses "crossed" magnetic and electric fields at right angles to achieve the objective. The chief role of magnetic field is to make the process of acceleration confined to a small and manageable region. As far as the change in speed is concerned, it is affected only by the electric field. Recall that magnetic field can not change magnitude of velocity i.e. speed.

But we should be careful in extrapolating above facts to obvious conclusions. As a matter of fact, we shall find that magnetic field actually affects the speed attained by the charged particle indirectly by controlling number of revolutions in the cyclotron. On the other hand, the speed acquired by the charged particle is independent of applied voltage. We shall explore all these aspects in detail in this module.

10.1 Acceleration due to electric field

Electric force accelerates particle only to change its speed if motion of the charged particle is in the direction of electric field. A potential difference V accelerates particle to achieve a speed as given by :

$$\frac{1}{2}mv^2 = eV$$

$$\Rightarrow v = \sqrt{\left(\frac{2eV}{m}\right)}$$

There is, however, difficulty in generating potential difference greater than 10^6 V. For this limiting value, the speed attained by a proton would be :

$$\Rightarrow v = \sqrt{\left(2X1.6X10^{-19}X10^6/1.66X10^{-27}\right)}$$

$$\Rightarrow v = \sqrt{\left(1.928X10^{14}\right)} = 1.39X10^7 \quad m/s$$

This is just 4.63 % of the speed of light and is not good enough. This speed of the particle is thus required to be subjected to repeated application of electric force. This is done linearly by electric force in what is known as "linear accelerator". Else, we use magnetic field to bend the path of motion and present the charged particle repeatedly to electric field for acceleration as in cyclotron. We should know that there is relative size and cost comparison and advantages between linear accelerator and cyclotron. Sometimes though, the two types of accelerators are used in conjunction where cyclotron functions as the initial accelerator for the system of particle accelerators.

[1] This content is available online at <http://cnx.org/content/m31761/1.2/>.

10.2 Working of cyclotron

10.2.1 The particles accelerated by cyclotron

The accelerators are used for accelerating charged elementary particles or charged ions. It can not accelerate a neutral particle. Besides, the cyclotron as described generally, is not used for accelerating light mass particles like electron or positron. The reason is that electron having negligible mass accelerates rather too quickly for repeated acceleration within the given size of cyclotron. Instead, the light mass charged particles are accelerated by a device known as "betatron" which uses torus shaped vacuum tube as secondary coil. The tube is a hollow cylinder shaped in a circle. The varying magnetic field, produced by secondary coil, sets up electric field which, in turn, accelerates electron through the tube. On the other hand, the magnetic field due to primary coil, spins the electron and keeps it in the center of the path.

We shall, therefore, refer cyclotron with acceleration of charged particles such as proton, ionized deuteron, alpha particle and similar other ions.

10.2.2 Construction of cyclotron

It consists of two hollow semicircular Dees so named because of their D-shape. The plane of Dees is the plane of revolution of charged particle, preferably a plane midway in the Dees. The Dees are constructed of conducting material like copper in order (i) to function as electrodes for applying alternating electrical potential using electrical source known as "electrical oscillator" and (ii) to shield moving charged particle from electric field within the Dees. The Dees are kept face to face diametrically opposite at a small distance known as the "gap". Electric field operates only in the gap to change speed of the charged particle. We should note that electric field does not accelerate charged particle when it is moving along semicircular path within the Dees as it is shielded from electrical field.

Cyclotron

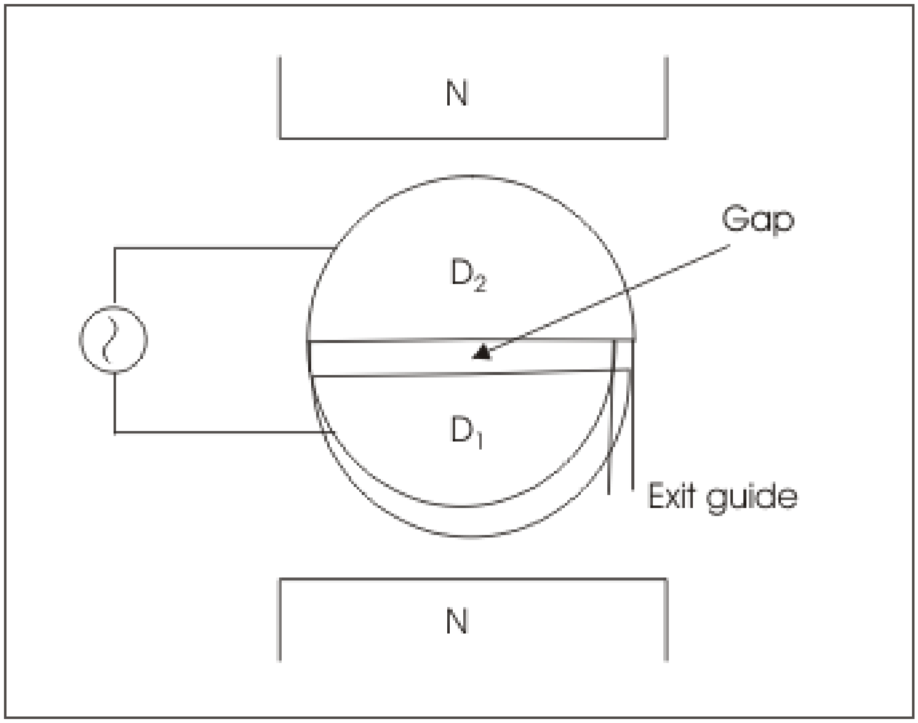


Figure 10.1: Cyclotron

There is an exit channel at the perimeter of one of Dees which finally guides the accelerated charged particle towards a target. The whole set up of Dees is placed between two poles of a powerful magnetic such that its field is perpendicular to the plane of Dees and hence perpendicular to the plane of motion.

This system of Dees is placed in evacuated confinement so that the charged particle moves unhindered.

10.2.3 Working principle

The charged particle (say a positively charged proton) is released near mid point of the face of one of the Dees. Being in the electric field from one Dee to another, it is accelerated by the electric force in the direction of electric field. As the particle enters the adjoining Dee, the magnetic force, being perpendicular to it, renders the charged particle to move along a semicircular path within the Dee. By the time, it emerges again in the narrow gap separating the two Dees, the electrical polarity of Dees changes so that the particle is again accelerated again with an increase in speed.

Working of cyclotron

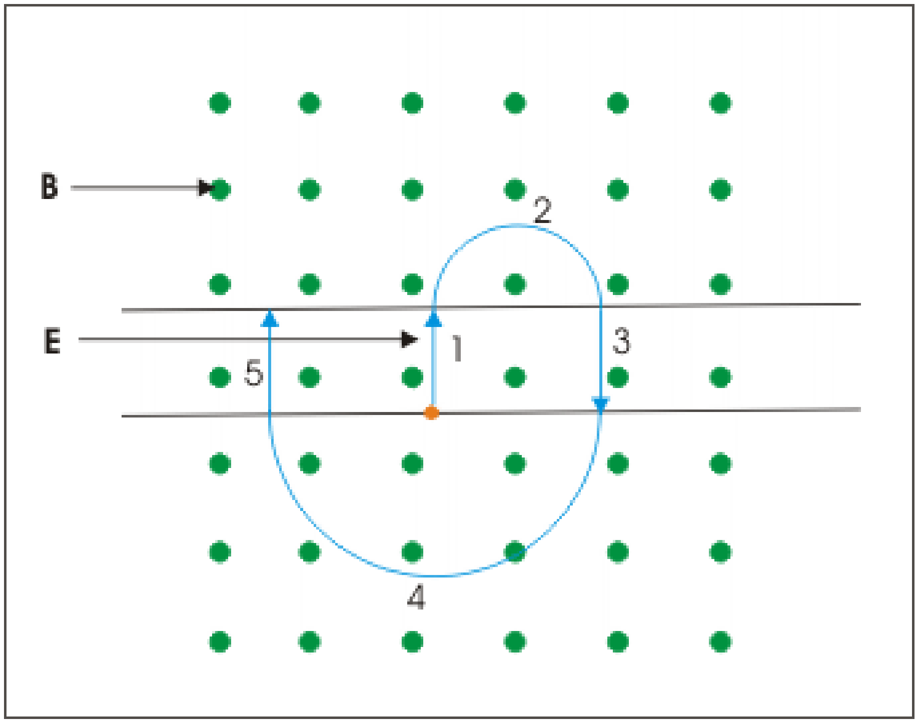


Figure 10.2: Working of cyclotron

But as the speed of the particle has increased, the radius of curvature of the semicircular path increases in accordance with the formula :

$$r = \frac{mv}{qB}$$

For given charge, mass and magnetic field, the radius is proportional to the speed. Clearly, the charged particle begins to move in a larger semicircular path after every passage through the gap. By the time particle reaches the gap successively, electric polarity of Dees keeps changing ensuring that the charged particle is accelerated with an increase in speed. This process continues till the charged particle reaches the periphery and exits through the guide with high energy and bombards a given target being investigated. The description of different segments of the path of accelerated particle is given here :

1: Path is a straight line. Particle is accelerated due to electric force. Speed and kinetic energy of the particle increase.

2: Path is a semicircular curve. Particle is accelerated due to magnetic force. This acceleration is centripetal acceleration without any change in speed and kinetic energy of the particle.

3: Path is a straight line. Particle is accelerated due to electric force in the direction opposite to the direction as in case 1. Speed and kinetic energy of the particle increase by same amount as in the case 1.

4: Path is a semicircular curve of greater radius of curvature due to increased speed. Particle is accelerated due to magnetic force. This acceleration is centripetal acceleration without any change in speed and kinetic energy of the particle.

5: Path is a straight line. Particle is accelerated due to electric force in the direction opposite to the direction as in case 1. Speed and kinetic energy of the particle increase by same amount as in the case 1 or

3.

We see that the particle follows consecutive larger semicircular path due to increase in the speed at the end of semicircular journey. The resulting path of charged particle, therefore, is a spiral path – not circular.

10.2.4 Frequency of alternating voltage supply

What should be the frequency at which the electrical oscillator changes sign? As per the account given in the previous section, the particle is required to be accelerated after completion of every semicircular journey of charged particle in the Dee. Does it mean that electrical polarity should be changed twice for one revolution in the magnetic field? Answer is no. Though particle is speeded up twice in a cycle, it requires change of direction of electric field only once. One of the directions is the existing direction and other is the reversed or changed direction. See the figure. Count the numbers of "change of directions" involved and numbers of revolutions. There are 7 occasions each when electric field has one of two possible directions. On the other hand, there are 7 revolutions counted from the beginning. Clearly, numbers of changes in directions are equal to numbers of revolutions. This means that frequency of electric oscillator should be equal to frequency of revolutions.

Frequency of oscillator

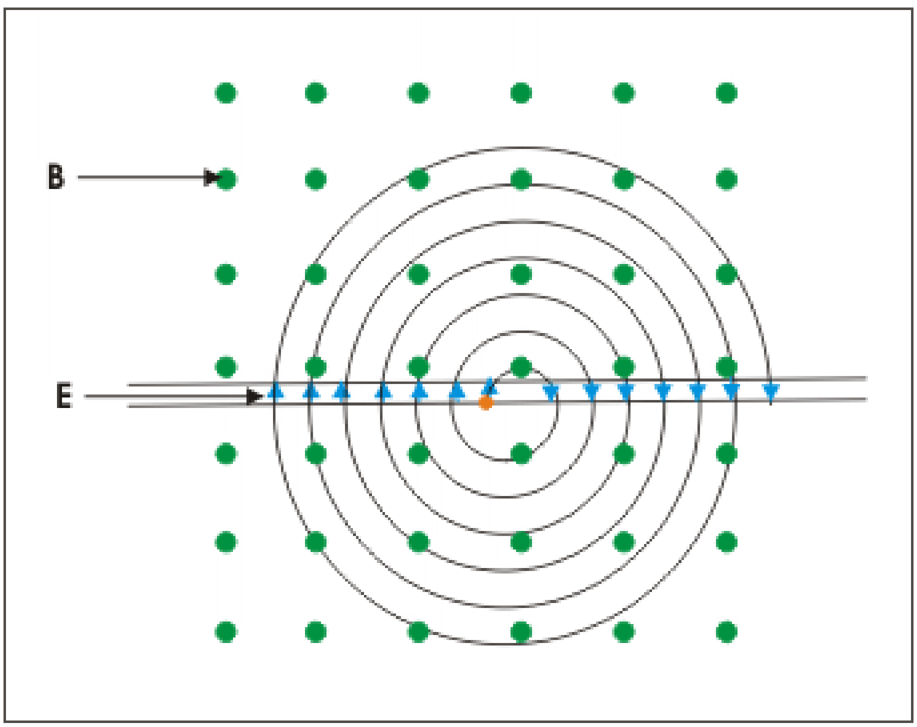


Figure 10.3: Frequency of oscillator

From the perspective of energy also, it is required that energy is added up to the moving charge at its natural frequency. This is the principle involved in resonance phenomena. We can pump energy to a periodic or oscillating system by supplying energy in small quantity at the natural frequency of the system. Hence, frequency of electrical oscillator is :

$$\nu = \frac{qB}{2\pi m}$$

Note that periodic properties of spiral motion are exactly same as that of circular motion of a charged particle in magnetic field. The frequency at which the charged particle completes spiral revolution is independent of the velocity. It is a very important feature of motion of charged particle in magnetic field. So even if the speed of the particle is increased with every passage through the gap, the time taken to reach the gap consecutively is same. It is the core consideration here allowing us to have a fixed frequency of electrical oscillator for a given magnetic field or conversely allowing us to have a constant magnetic field for a given frequency of electric oscillator. Of course, these constant values are determined keeping in mind the specific charge (charge and mass ratio) and size of the cyclotron.

Example 10.1

Problem : A frequency of an electric oscillator is 10 MHz. What should be the magnitude of magnetic field for accelerating doubly ionized alpha particle? Assume mass of alpha particle 4 times that of proton.

Solution : The frequency of cyclotron is :

$$\nu = \frac{qB}{2\pi m}$$

$$\Rightarrow B = \frac{2\pi m\nu}{q}$$

Putting values,

$$\Rightarrow B = \frac{2X3.14X4X1.66X10^{-27}X10X10^{6}}{2X1.6X10^{-19}}$$

$$\Rightarrow B = 1.3.T$$

10.2.5 Energy of charged particle

The energy of the finally accelerated particle corresponds to the speed when it travels in the outermost semicircular path having radius equal to that of Dees.

$$R = \frac{mv_{\max}}{qB}$$

$$\Rightarrow v_{\max} = \frac{qBR}{m}$$

$$\Rightarrow K_{\max} = \frac{1}{2}m{v_{\max}}^2 = \frac{q^2B^2R^2}{2m}$$

Example 10.2

Problem : Compare the final velocities of a proton particle and ionized deuteron when accelerated by a cyclotron. It is given that radius of cyclotron is 0.3 m and magnetic field is 2 T. Assume mass of deuteron twice that of the proton.

Solution : Let subscripts 1 and 2 correspond to proton and deuteron respectively. Note that deuteron is an isotope of hydrogen comprising of 1 proton and 1 neutron in the nucleus. The ionized deuteron thus carries one electronic positive charge same as proton. Now, final velocity of the charged particle accelerated by cyclotron is given as :

$$v_{\max} = \frac{qBR}{m}$$

Hence,

$$\frac{v_{\max 1}}{v_{\max 2}} = \frac{q_1 B_1 R_1 m_2}{q_2 B_2 R_2 m_1}$$

But $R_1 = R_2$, $q_1 = q_2$, $B_1 = B_2$ and $m_2 = 2m_1$. Thus,

$$\Rightarrow \frac{v_{\max 1}}{v_{\max 2}} = 2$$

10.2.6 Numbers of revolutions

The kinetic energy of the charged particle is increased every time it comes in the gap between the Dees. The energy is imparted to the charged particle in "lumps". By design of the equipment of cyclotron, it is also evident that the amount of energy imparted to the particle is equal at every instance it crosses the gap between Dees.

Since particle is imparted energy twice in a revolution, the increase in energy corresponding to one revolution is :

$$\Delta E = 2qV$$

Let there be N completed revolutions. Then total energy,

$$\Rightarrow E = N\Delta E = 2qNV$$

Equating this with the expression obtained earlier for energy, we have :

$$\Rightarrow 2qNV = \frac{q^2 B^2 R^2}{2m}$$

$$\Rightarrow N = \frac{q^2 B^2 R^2}{4mqV}$$

10.2.7 Magnetic field and energy

From the expression of kinetic energy of the accelerated particle, it is clear that kinetic energy of the charged particle increases with the magnitude of magnetic field – even though magnetic field is incapable to bring about change in speed of the particle being always perpendicular to the motion. It is so because increasing magnetic field reduces the radius of curved motion inside Dees. Therefore, there are greater numbers of revolutions before reaching to the periphery. See that numbers of completed revolutions are directly proportional to the square of magnetic field.

$$N = \frac{q^2 B^2 R^2}{4mqV}$$

Greater numbers of revolutions result in greater numbers of times electrons are subjected to electrical potential difference in the gap between Dees. The maximum kinetic energy of the particle is :

$$K_{\max} = 2qNV$$

As such, energy of the emerging particle increases for a given construction of cyclotron when magnetic field increases.

10.2.8 Potential difference and energy

Again, it is clear from the expression of kinetic energy of the accelerated particle that the energy of emerging particle from the cyclotron is independent of potential applied in the gap. It appears to contradict the fact that it is the electric force which accelerates the charged particle in the gap. No doubt, the greater potential difference results in greater electric force on the charged particle. This, in turn, results in greater acceleration of the particle and hence velocity. But then, particle begins to rotate in greater semicircle. This results in lesser numbers of rotations possible within the fixed extent of Dees. In other words, the greater potential difference results in greater acceleration but lesser numbers of opportunities for acceleration. Now,

$$N = \frac{q^2 B^2 R^2}{4mqV}$$

and

$$K_{\text{max}} = 2qNV$$

Clearly, the numbers of revolutions is inversely proportional to the potential difference applied in the gap. On the other hand, maximum energy of the particle is directly proportional to the product "NV". Combining two facts, we find that energy of the particle is indeed independent of the applied voltage in the gap.

10.3 Limitations of cyclotron

We have already noted two limitations of cyclotron as accelerator. One limitation is that it can not accelerate neutral particle. Second limitation is that lighter elementary particles like electrons or positrons can not be accelerated and requires important changes or modifications of the device. In addition to these, there are two other important limitations as described here.

10.3.1 Relativistic effect

The relativistic effect becomes significant enough to be neglected when particle achieve 10 % of the speed of light. The energy corresponding to this speed for a proton is about 5MeV. Initially, the small relativistic effect is accommodated by an standard cyclotron, but it begins to fail to accelerate charged particle at higher energy level of 50 MeV or so.

At higher speed, the mass of the particle increases in accordance with following equation :

$$m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

where mo is rest mass and c is the speed of light in vacuum. The particle becomes heavier at higher speed. Putting this in the expression of frequency, we have :

$$\Rightarrow \nu = \frac{qB\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}{2\pi m_0}$$

$$\Rightarrow \nu = \nu_0 \sqrt{\left(1 - \frac{v^2}{c^2}\right)}$$

where ν_0 is classical frequency. Clearly, the frequency of revolution decreases with increasing velocity whereas frequency of applied electrical oscillator is fixed. The particle, therefore, gets out of step with the alternating electrical field. As a result, speed of the particle does not increase beyond a certain value.

10.3.2 High energy particle

The cyclotron is also limited by the mere requirement of magnet size as radius of Dees increases with increasing speed of the particle being accelerated. Let us calculate speed corresponding of a 100 GeV particle in a magnetic field of 1 T. The radius of revolution is related to kinetic energy :

$$K_{\max} = \frac{q^2B^2R^2}{2m}$$

$$\Rightarrow R = \sqrt{\left(\frac{2mK_{\max}}{q^2B^2}\right)}$$

The given kinetic energy is :

$$\Rightarrow K_{\max} = 100X10^9 \quad eV = 10^{11}X1.6X10^{-19} = 1.6X10^{-8} \quad J$$

Now, putting values assuming particle to be a proton,

$$\Rightarrow R = \sqrt{\left(\frac{2X1.66X10^{-27}X1.6X10^{-8}}{\left(1.6X10^{-19}\right)^2X1}\right)}$$

$$\Rightarrow R = 0.144X10^2 = 14.4m$$

We can imagine how costly it would be to create magnet of such an extent. For higher energy, the required radius could be in kilometers.

10.3.3 Synchrocyclotron and Synchrotron

The synchrocyclotron is a device that addresses the limitation due to relativistic effect. The frequency of oscillator is reduced gradually in order to maintain the resonance with the spiral motion of charged particle. Note that magnetic field remains constant as in the case of cyclotron.

In synchrotron as against synchrocyclotron, both magnetic field and electric field are variable. It aims to address both the limitations due to relativistic effect as well as due to the requirement of large cross section of magnets. The particle is accelerated along a fixed large circular path inside a torus shaped tunnel. The magnetic field here bends the particle, where as electric field changes speed. Clearly, the requirement of a large cross section of magnet is converted into multiple bending magnets along a large radius fixed circular path.

Chapter 11

Ampere's law[1]

Ampere law supplements Biot-Savart law (Chapter 3) in providing relation between current and magnetic field. Biot-Savart law provides expression of magnetic field for a small current element. If we need to find magnetic field due to any extended conductor carrying current, then we are required to use techniques like integration and superposition principle. Ampere law is another law that relates magnetic field and current that produces it. This law provides some elegant and simple derivation of magnetic field where derivation using Biot-Savart law would be a difficult proposition. This advantage of Ampere law lies with the geometric symmetry, which is also its disadvantage. If the conductor or circuit lacks symmetry, then integration involving Ampere's law is difficult.

Ampere law as modified by Maxwell for displacement current is one of four electromagnetic equations.

11.1 Basis of Ampere law

In order to understand the basis of Ampere law, we investigate here the magnetic field produced by a straight conductor carrying current. The expression of magnetic field due to long straight (infinite) conductor carrying current as obtained by applying Biot-Savart law (Chapter 3) is :

$$B = \frac{\mu_0 I}{2\pi R}$$

where R is the perpendicular distance between straight conductor and point of observation. Rearranging, we have :

$$\Rightarrow 2\pi RB = \mu_0 I$$

If we carefully examine the left hand expression, then we find that it is an integral of the scalar product of magnetic field and length element about the perimeter of a circle drawn with center on the straight conductor and point of observation lying on it.

$$\int \mathbf{B}.\mathbf{l} = \mu_0 I$$

Now, we evaluate the left hand integral to see whether our observation is correct or not? For the imaginary circular path, the direction of length element and magnetic field are tangential to the circle. The angle between two vector quantities is zero. Hence, left hand side integral is :

$$\int \mathbf{B}.\mathbf{l} = \int Bl\cos 0^\circ = \int Bl$$

[1]This content is available online at <http://cnx.org/content/m31895/1.8/>.

Integration along circular path

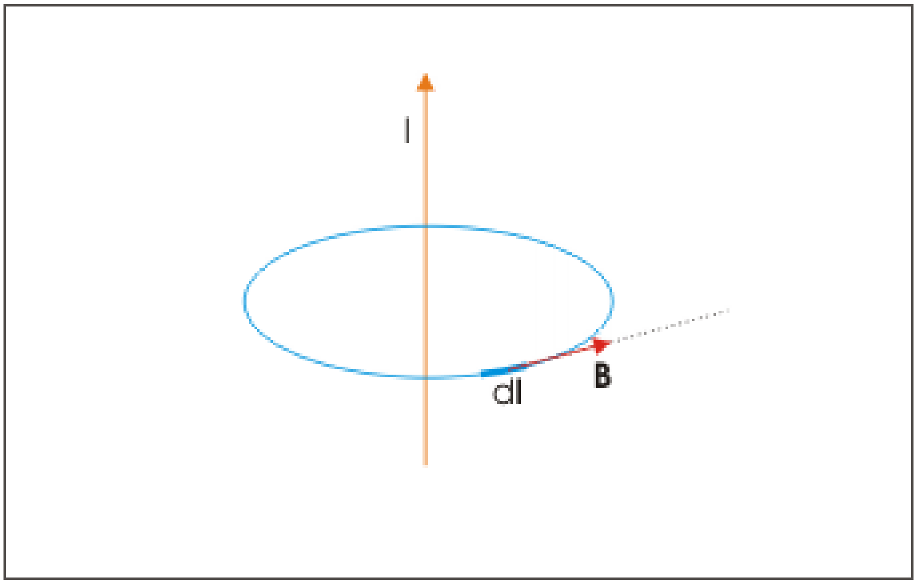


Figure 11.1: The angle between magnetic and line element vectors is zero.

Since magnitude of magnetic field due to current in straight wire are same at all points on the circular path - being at equal distance from the center, we take magnetic field out of the integral,

$$\Rightarrow \int \mathbf{B.l} = B \int l = 2\pi RB$$

Substituting in the equation of line integral of magnetic field as formulated earlier, we have the same expression of magnetic field for long straight conductor as obtained by applying Biot-Savart law :

$$B = \frac{\mu_0 I}{2\pi R}$$

It is clear here that the left hand side integration should be carried out over a "closed" path. This closed path is termed as "closed imaginary line" or "Ampere loop". Hence, we write the equation as :

$$\oint \mathbf{B.l} = \mu_0 I$$

Note the circle in the middle of integration sign which indicates a closed path of integration. This formulation is evidently an alternative to Biot-Savart law in the instant case. Now the question is whether this relation is valid for any "closed imaginary line"? The answer is yes. Though the above equation involving closed line integral is valid for any closed imaginary path, but only few of these closed paths allow us to use the equation for determining magnetic field. For instance, if we consider a square path around the straight wire, then we face the problem that points on the path are not equidistant from the wire and as such magnetic field is not same as in the case of a circular path. It is also evident that we need to choose a loop which passes through the point of observation. After all, we are interested to know magnetic field due to currents at a particular point in a region. See Ampere's law(exercise) : Problem 1 (Section 12.1.2:) which illustrates this aspect of application of Ampere's law.

Further, considering our ability or constraints for integration around any path, we look for a contour which passes through points where magnetic field is same or where certain simplifying relation between magnetic field and line element vectors exists. This issue is important as it renders integration derivable. Clearly, this is where symmetry of object carrying current comes into play.

Thus, symmetry of object carrying current and selection of path for the integration are two important requirements for putting Ampere law to use though the law itself is true for all closed path and any configuration of conductor.

11.2 Statement of Ampere law

There are few variants of this law. We shall begin with the simplest form. There is one precondition as well. This law in the form discussed here is true for steady current and is not valid for time varying current. In the simplest form, it states that the line integral of scalar product of magnetic field and length element vectors along a closed imaginary line is equal to the product of absolute permeability of free space and the net "free" current through the imaginary closed line. Mathematically,

$$\oint \mathbf{B}.\mathbf{l} = \mu_0 I$$

The "free" current represents the current owing to moving electrons or ions. This law is modified by Maxwell for time dependent varying current using the concept of "displacement" current. We shall briefly discuss displacement current and the Maxwell modification in the next section.

The sign of current through the loop is determined by the direction in which line integral is executed. We curl fingers of right hand such that it is aligned with the direction of integration along the closed path. The extended thumb, then, points in the direction of positive current. Alternatively, if the direction of integration is counterclockwise, then current coming toward the viewer of closed path is positive and the current going away is negative. The net current through the loop is the algebraic sum of positive and negative currents. See Ampere's law(exercise) : Problem 2 and 4 (Section 12.1.3:) for illustrations.

Sign of current

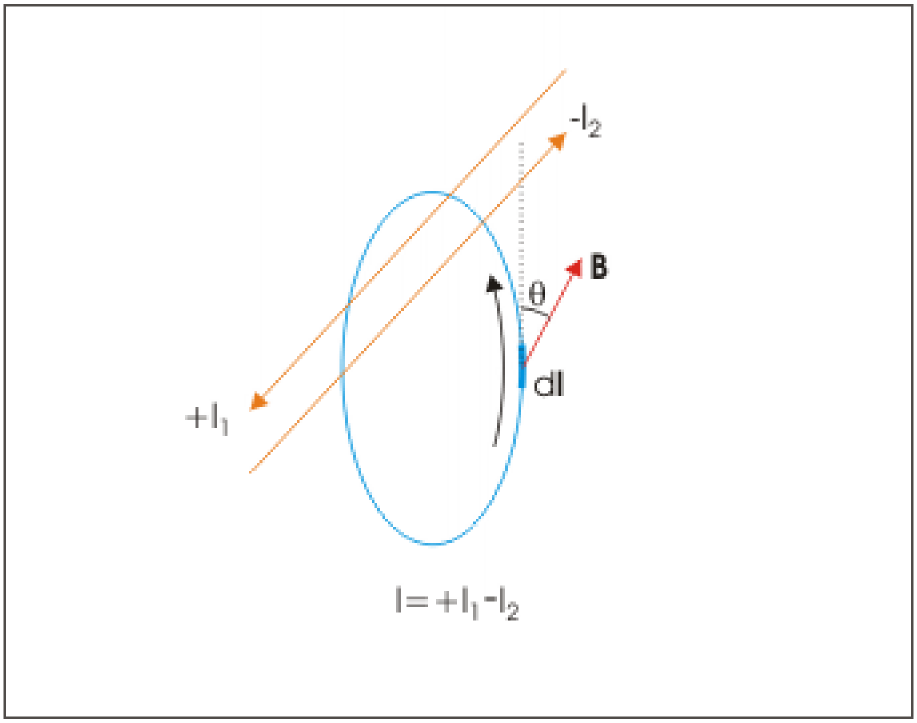


Figure 11.2: We curl fingers of right hand such that it is aligned with the direction of integration along the closed path. The extended thumb, then, points in the direction of positive current.

In the second form of the law, the right hand side of the equation is substituted with a surface integral as given here :

$$\oint \mathbf{B}.\mathbf{l} = \mu_0 \oint \mathbf{J}.\mathbf{S}$$

Here **J** is current density through surface **S**. The **S** is the surface for which imaginary closed line serves as boundary. Note that we consider surface area element (đ**S**) as a vector. The surface area element vector is normal to the surface and its orientation across the surface is determined in the same manner as we determine the sign of the current. We curl fingers of right hand such that it is aligned with the direction of integration along the closed path. The extended thumb, then, points in the direction of surface area element vector. Alternatively, if the direction of integration along the Ampere loop is anticlockwise, then surface area element vector is directed toward the viewer of closed path and if the direction of integration is clockwise, then surface area element vector is directed away from the viewer of closed path.

Direction of surface

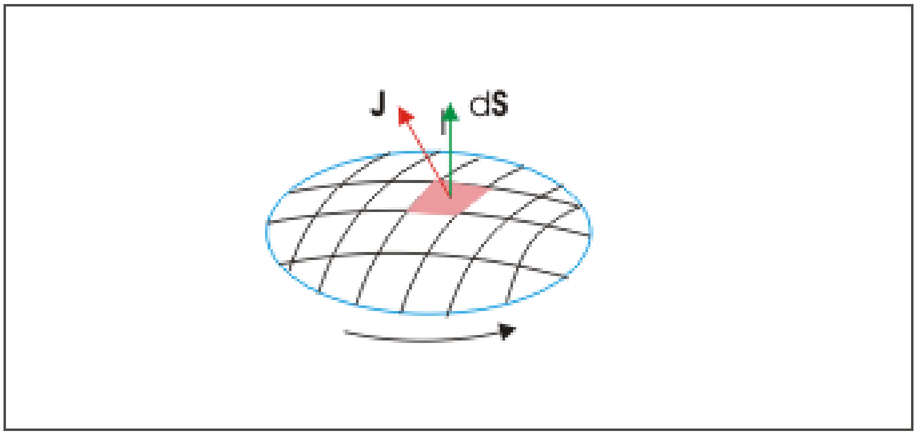


Figure 11.3: We curl fingers of right hand such that it is aligned with the direction of integration along the closed path. The extended thumb, then, points in the direction of surface area vector.

Since surface area vector is always normal, we may use the concept of normal unit vector **n** and denote surface vector as :

$$\mathbf{S} = \widehat{n}S \quad \text{and} \quad \mathbf{S} = \widehat{n}S$$

Now, there can be infinite numbers of surfaces which can be drawn for a given closed boundary line. The choice of surface is easier to make if the imaginary closed line (loop) is in one plane. The surface in the same plane is generally chosen in that case. However, if the loop is not in one plane, then there is no simple choice. It does not matter then. The law is valid for all surfaces which are bounded by the loop.

Ampere loop and surfaces

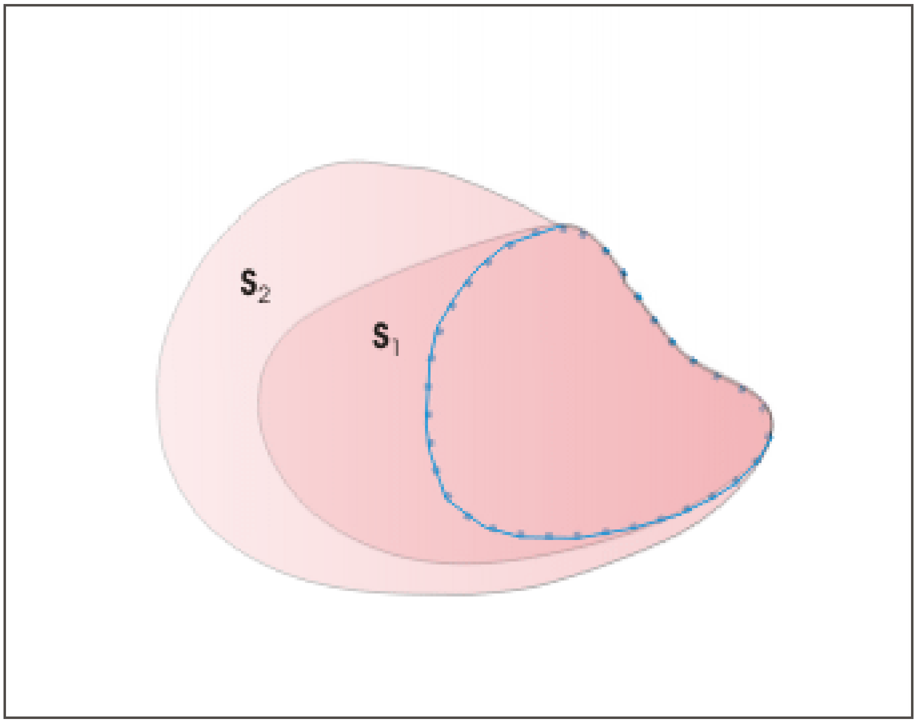


Figure 11.4: Surfaces may be drawn in three dimensions with Ampere loop as boundary.

If the consideration of magnetic field and current is done in a medium, then we need to substitute "μ_0" by "$\mu_0\mu_r$"or "μ" representing permeability of the medium.

11.2.1 Ampere loop and enclosed current

One important consequence of freedom to draw imaginary loop is that it is our choice to keep a current inside or outside the loop. This appears to be a perplexing situation as we know that magnetic field at a point results due to magnetic fields due to each current. For illustration, let us consider five current carrying long conductors as shown in the figure, two of which are into the plane (shown by cross signs) and three are out of the plane of drawing (shown by filled circles). Now, we can draw valid Ampere loop in different ways to determine magnetic field at a point P in the plane of drawing as shown in the figure here.

Magnetic field at a point

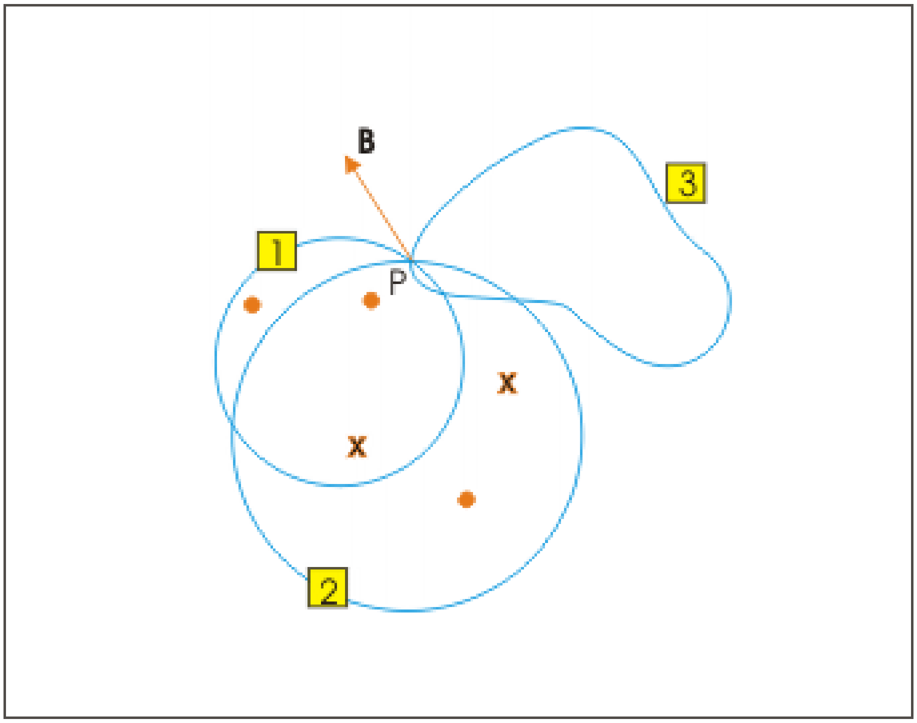


Figure 11.5: The currents are flowing perpendicular to the plane of drawing.

Here, the currents are flowing perpendicular to the plane of drawing. The magnetic fields due to currents in long wires are in the plane of drawing as we can check by applying Right hand thumb rule. Since point P is not equidistant from the length elements of the loops drawn, the actual integration would be very rigorous and difficult. We shall, therefore, make only qualitative assertions here which are consistent with Ampere's law. Further, we also make the simplifying assumptions that current in each wire is "I" and that we carry out integration in anticlockwise direction in each case. Let the magnetic field at point P is **B** as shown.

For the loop 1, there are two currents out of the page and one into the page. Thus, the net current is "I" flowing out of the page. For the loop 2, there are two currents out of the page and two into the page. Thus, the net current is zero. For the loop 3, there is no current at all. Thus, the net current is again zero. Now, how is it possible that integration of magnetic fields in three cases yields an unique value of magnetic field at P? The point to understand here is that when we integrate along a path, the sum of vector dot product "**B**.dl" for the complete closed path, due to currents lying outside the loop, cancels out. However, it does not cancel out for the currents inside. This is the reason Ampere's law considers only currents enclosed within the imaginary boundary.

This fact underlines an important fact that absence of current across Ampere loop does not ensures that magnetic field in a region is zero. We can verify this by using a square loop inside a solenoid. A solenoid, as we shall study, produces a uniform magnetic field within it. Let the magnetic field be **B** as shown. Clearly, there is no current passing through the enclosure of the square loop as current in solenoid flows through the helical coil covering the region under consideration. Let us now carry out the integration in clockwise direction along ACDEA.

Magnetic field at a point

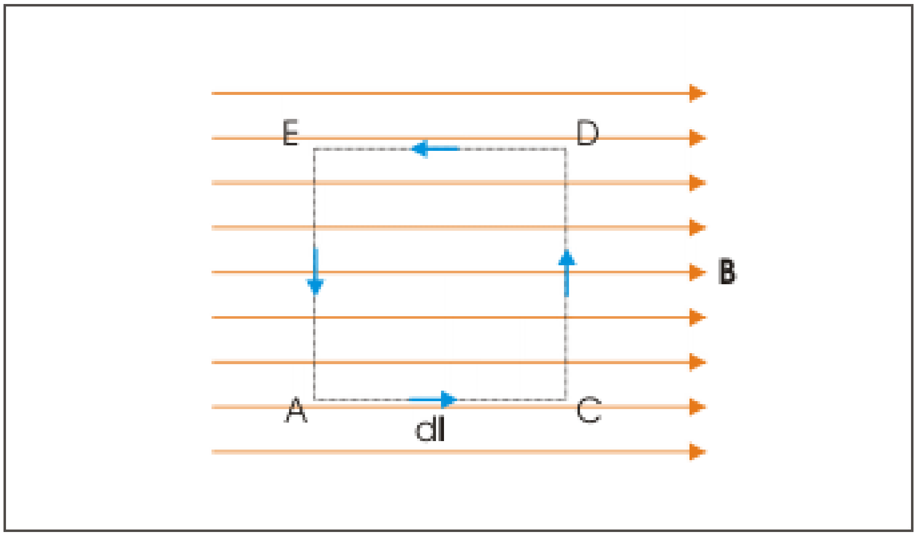


Figure 11.6: The currents are flowing perpendicular to the plane of drawing.

$$\oint \mathbf{B.l} = \int_{AC} \mathbf{B.l} + \int_{CD} \mathbf{B.l} + \int_{DE} \mathbf{B.l} + \int_{EA} \mathbf{B.l}$$

$$\oint \mathbf{B.l} = \int_{AC} Bl\cos 0^\circ + \int_{CD} Bl\cos 90^\circ + \int_{DE} Bl\cos 180^\circ + \int_{EA} Bl\cos 90^\circ$$

$$\oint \mathbf{B.l} = Ba + 0 - Ba + 0 = 0$$

Clearly, existence of magnetic field does not require net current through the loop. For another example, see Ampere's law(exercise) : Problem 3 (Section 12.1.4:)

11.2.2 Maxwell modification

The basic assertion of Maxwell electromagnetic theory is that changing electric filed sets up magnetic field in the same manner in which a varying magnetic field sets up electric field as given by Farady's induction law. The Maxwell equation is complementary to Farady's induction law and is given as :

$$\oint \mathbf{B.l} = \mu_0 \epsilon_0 \frac{\phi_E}{t}$$

Note how the time rate of change of electric field $\frac{\phi_E}{t}$ is related to magnetic field ($\mathbf{B}$) by this equation. In order to account for this additional cause of magnetic field resulting from varying electric field, a more generalized form of Ampere law including the term given by Maxwell is :

$$\oint \mathbf{B.l} = \mu_0 I + \mu_0 \epsilon_0 \frac{\phi_E}{t}$$

Of course for situation involving only steady current, the form of Ampere's law is reduced to its original form.

The presence of magnetic field between capacitor plates during charging of a capacitor confirms Maxwell law. As the charge builds up on the capacitor plate, there is varying electric field in the gap between plates. This varying electric field, in turn, sets up magnetic field. We can, therefore, suggest that the varying electric field is equivalent to a current. After all, a current also produces magnetic field. But we know there is no actual current between two plates. Hence, this equivalent current is a sort of pseudo current and is known as "displacement" current, which when present would have produced the same magnetic field in the gap as actually produced by the varying electric field.

We should understand that this assertion about displacement current or setting up of magnetic field due to varying electric field is an important step in explaining electromagnetic propagation. In a nutshell, it says that the presence or propagation of magnetic or electric field do not require either a charge or a current. That is exactly what we see with the propagation of electromagnetic field which is known to be composed of time varying electric and magnetic components. The changing electric field sets up magnetic field and changing magnetic field sets up electric field in a complementary manner. This is how electromagnetic field is continuously driven to propagate electromagnetic wave without presence of either charge or current. In other words, the two varying fields drive each other without the conventional source like charge or current.

11.3 Application of Ampere's law

Ampere's law is a powerful tool for calculating magnetic field for certain geometric forms of conductors carrying current. It was, however, pointed out that this law may be limited as well for many other situations where left hand side integral can not be evaluated easily. Though there are no specific rules for selecting a closed Ampere loop, but there are certain guidelines which can be helpful in applying this law. These guidelines are :

- Draw closed loop such that the point of observation lies on the loop.
- If required, draw closed loop such that magnetic field is constant along the path of integration.
- If required, draw closed loop such that magnetic field and line vectors are along the same direction or are perpendicular to each other.
- If required, draw closed loop such that there is no magnetic field. This may appear bizarre but we draw such segment of Ampere loop as in the case of solenoid (we shall see this consideration subsequently in this module).
- If required, draw closed loop as a combination of segments (like a rectangular path with four arms) in a manner which takes advantages of the situations enumerated at 2, 3 and 4.

11.3.1 Magnetic field due to a long cylindrical conductor

We consider three points of observation (i) A, inside the conductor (ii) C, just outside the conductor and (iii) D, outside conductor for applying Ampere's law. One important consideration here is that magnetic field due to infinite conductor is independent of the elevations of observation points with respect to the straight cylindrical conductor. The magnetic field only depends on the perpendicular linear distance (r) of the observation point from the axis of cylindrical conductor. This situation is approximately valid for long conductor as well. If the conductor is not long enough then also we can meet the requirement of independence for observation points at those points, which are close to the conductor and the ones which are not near the ends of the conductor.

In order to apply Ampere's law, we consider three imaginary circles containing these points separately with their centers lying on the axis of cylinder such that their planes are at right angles to the cylinder. Let the current through the conductor is I. We note here that current in the conductor is confined only to the surface of cylinder of radius R.

Magnetic field due to current in cylindrical conductor

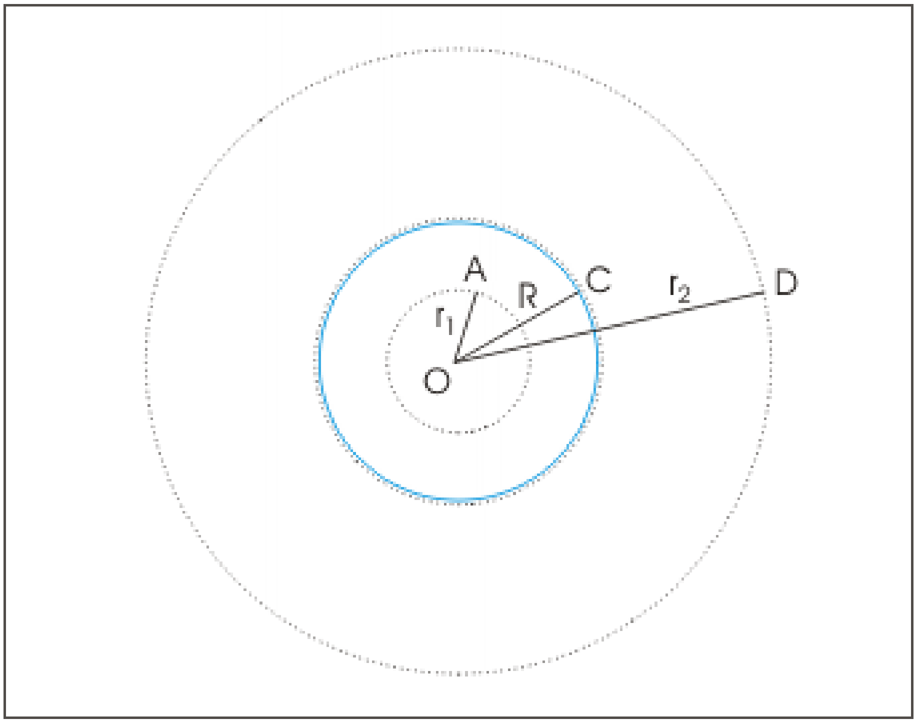


Figure 11.7: The currents are flowing perpendicular to the plane of drawing.

For the point A inside the conductor, the current inside the loop is zero.

$$\oint \mathbf{B.l} = \mu_0 I = 0$$

$$\Rightarrow BX2\pi r_1 = 0$$

$$\Rightarrow B = 0$$

Note that absence of current here is used to deduce that magnetic field is also absent. We can do this with the circular symmetry having constant magnetic field along the path as circle is a continuous curve without any possibility that integral values in different segments of imaginary loop cancel out along the circular path. Thus, if I = 0, then B=0.

Now, for the point B just outside the conductor, the current inside the loop is I.

$$\oint \mathbf{B.l} = \mu_0 I$$

$$\Rightarrow BX2\pi R = I$$

$$B = \frac{\mu_0 I}{2\pi R}$$

For the point C outside the conductor, the current inside the loop is I.

$$\oint \mathbf{B.l} = \mu_0 I$$

$$\Rightarrow BX2\pi r_2 = I$$

$$B = \frac{\mu_0 I}{2\pi r_2}$$

11.3.2 Magnetic field due to a long cylindrical conductor with uniform current density

In this case, current is distributed across the cross section uniformly. In order to apply Ampere's law, we consider three imaginary circles containing these points separately with their centers lying on the axis of cylinder such that their planes are at right angles to the cylinder. Let the total current through the conductor is I.

Magnetic field due to a long cylindrical conductor with uniform current density

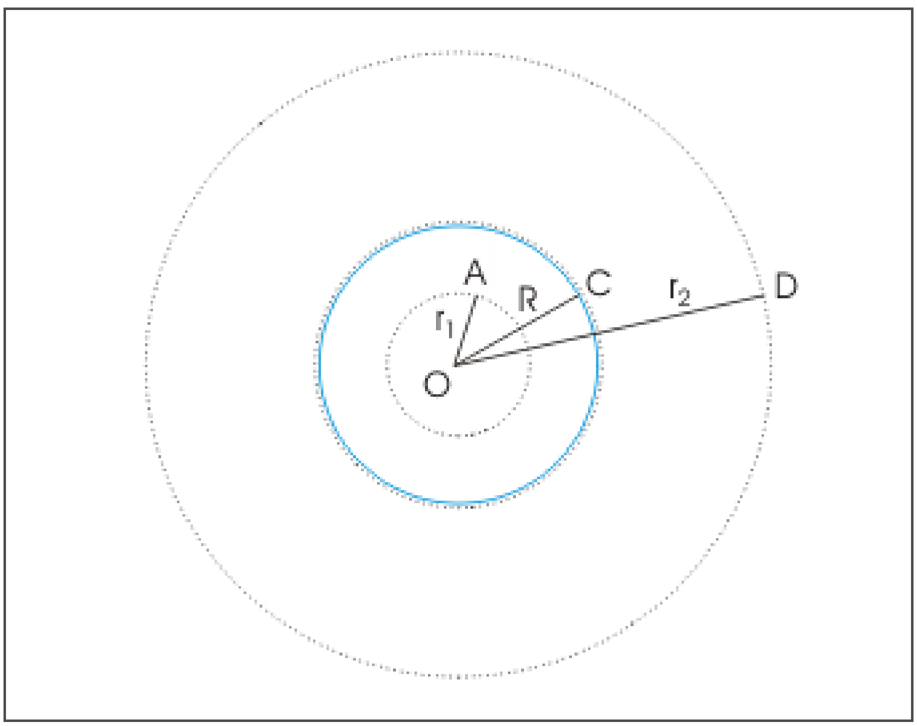


Figure 11.8: The currents are flowing perpendicular to the plane of drawing.

For the point A inside the conductor, the current inside the loop is not zero. Since current is distributed over the cross section area uniformly, the current through the loop area is proportionately smaller and is given by :

$$I\prime = \frac{\pi r_1^2 I}{\pi R^2} = \frac{r_1^2 I}{R^2}$$

Now,

$$\oint \mathbf{B.l} = \mu_0 I\prime$$

$$\Rightarrow BX2\pi r_1 = \frac{\mu_0 r_1^2 I}{R^2}$$

$$\Rightarrow B = \frac{\mu_0 r_1 I}{2\pi R^2}$$

For the point B just outside the conductor, the current inside the loop is I.

$$\oint \mathbf{B.l} = \mu_0 I$$

$$\Rightarrow BX2\pi R = I$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi R}$$

For the point C outside the conductor, the current inside the loop is I.

$$\oint \mathbf{B.l} = \mu_0 I$$

$$\Rightarrow BX2\pi r_2 = I$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r_2}$$

Example 11.1

Problem : The current density varies within a long cylindrical wire of radius "R" as J=kr where "r" is linear distance from the center in the perpendicular cross section of wire. Find the magnetic field at a distance r= R/2 and at a point outside the wire.

Solution : In order to find the current within the conductor, we consider an annular ring of infinitesimally small thickness "dr". The current through the small cross section of annular ring is :

Magnetic field due to a long cylindrical conductor with non-uniform current density

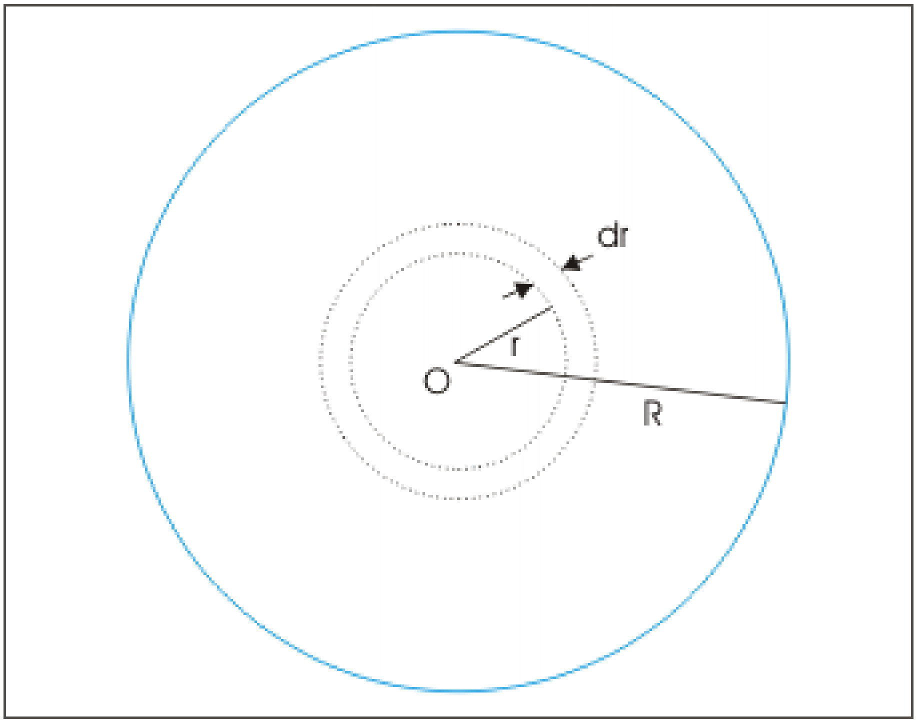


Figure 11.9: The currents are flowing perpendicular to the plane of drawing.

$$I = JA = JX2\pi rr = krX2\pi rr = 2\pi kr^2r$$

Integrating between r = 0 and r =R/2, the current inside the circular loop of radius R/2 is,

$$I = \int_0^{R/2} 2\pi kr^2r$$

$$\Rightarrow I = 2\pi k\left[\frac{r^3}{3}\right]_0^{R/2}$$

$$\Rightarrow I = 2\pi k\left[\frac{R^3}{24}\right] = \frac{\pi kR^3}{12}$$

Applying Ampere's law about a loop of radius R/2,

$$\oint \mathbf{B}.\mathbf{l} = \mu_0 I$$

$$\Rightarrow BX\frac{2\pi R}{2} = \frac{\mu_0\pi kR^3}{12}$$

$$\Rightarrow B = \frac{\mu_0 kR^2}{12}$$

For additional examples, see Ampere's law(exercise) : Problem 5,6,7 and 9 (Section 12.1.6:)

11.3.3 Solenoid

A solenoid is a tightly wound helical coil. It works as a magnet when current is passed through the coil. We may treat a solenoid as the aggregation of large numbers of circular current aligned about a common axis. It tends to reinforce magnetic field due to each of the circular coil, resulting into a device to produce magnetic field. An ideal solenoid has infinite length. A long coil approximates an ideal solenoid. The consideration here is valid for even short solenoid for points which are well inside the coil.

Solenoid

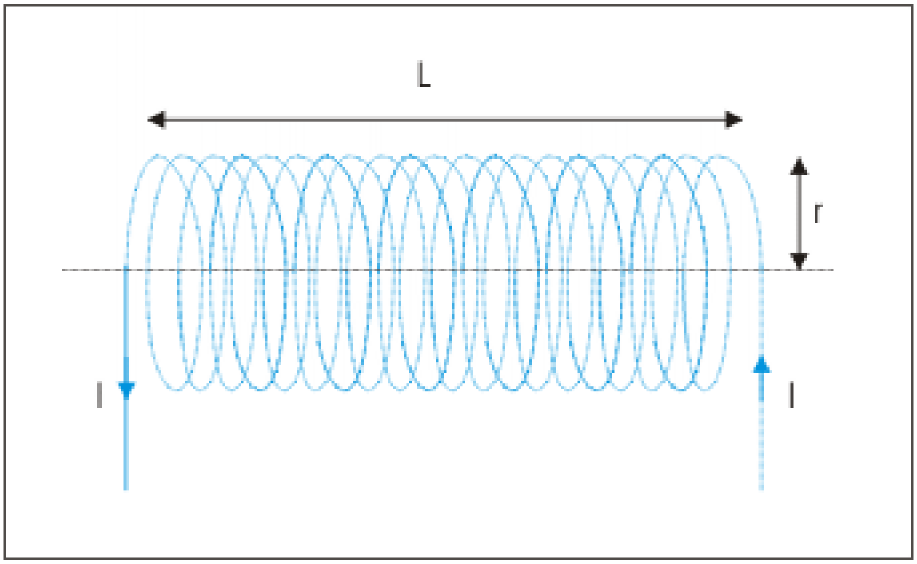


Figure 11.10: A solenoid is a tightly wound helical coil.

11.3.3.1 Nature of magnetic field

The current in left end coil is clockwise and serves as south end of solenoid i.e. end through which magnetic field enters the solenoid. On the other hand, the current in the right end coil is anticlockwise and serves as north end of solenoid i.e. end through which magnetic field exits the solenoid. The magnetic fields between two adjacent coils at the periphery (edge) cancel each other. The magnetic field outside solenoid is nearly zero or comparatively much weaker to be considered to be zero. The field inside the solenoid is uniform. The magnetic field at the ends of solenoid, however, spreads out. The nature of magnetic field of a solenoid is similar to magnetic field due to a bar magnet.

Magnetic field due to a solenoid

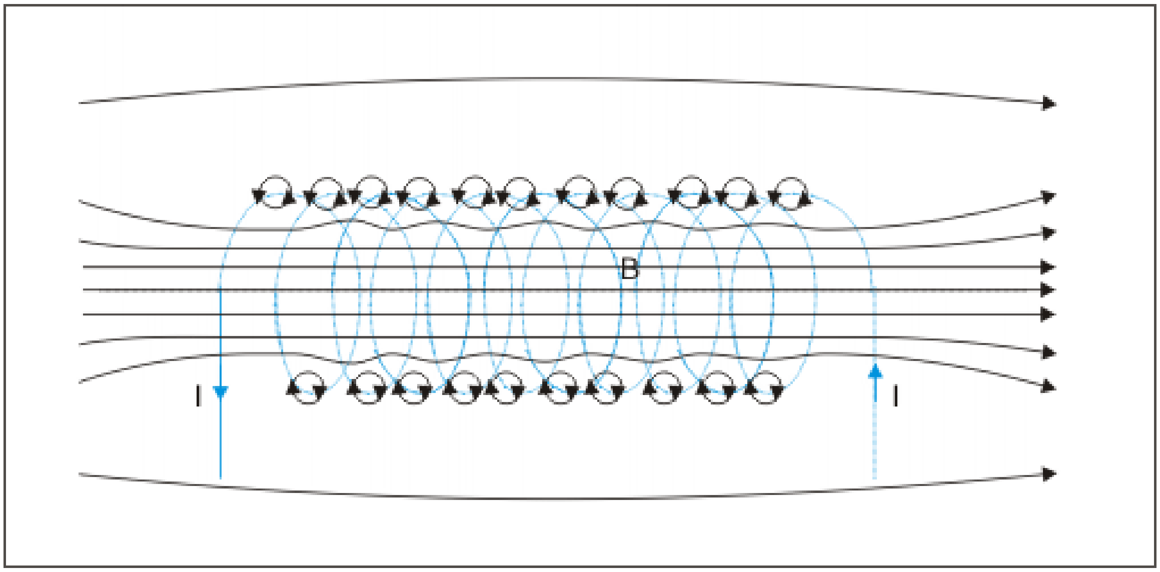


Figure 11.11: A solenoid is a tightly wound helical coil.

11.3.3.2 Magnitude of magnetic field

We draw a rectangular Ampere loop ACDEA as shown in the figure. The directions of currents at the edges are shown by filled circle for currents coming out of the plane of drawing and by cross for currents going into the plane of drawing. We carry out the integration in anticlockwise direction such that currents coming out of the plane of drawing are considered positive.

Magnetic field due to a solenoid

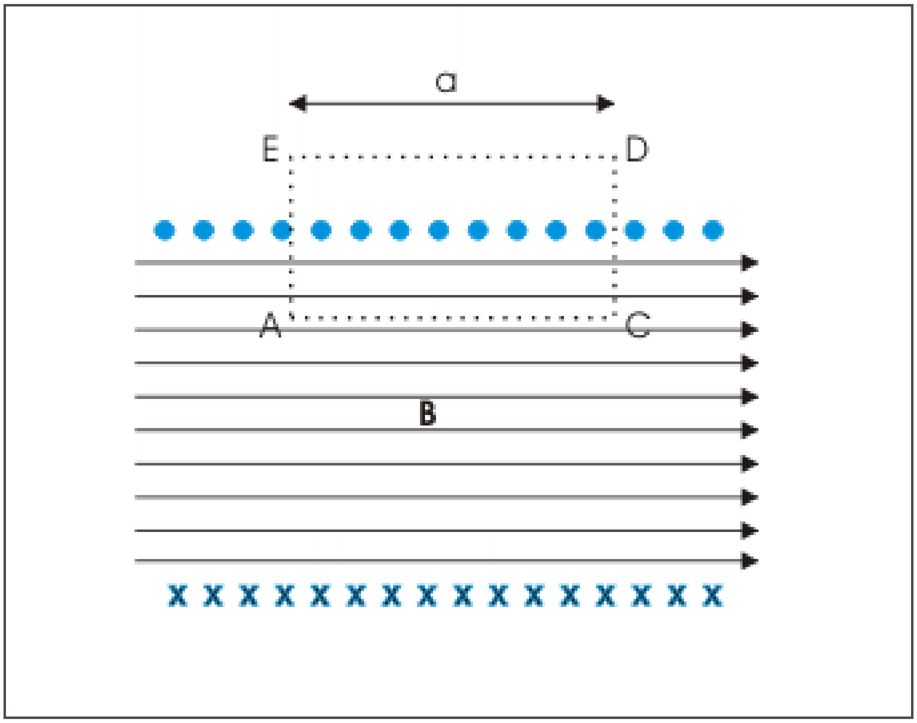


Figure 11.12: A solenoid is a tightly wound helical coil.

Applying Ampere's law,

$$\oint \mathbf{B}.\mathbf{l} = \int_{AC} \mathbf{B}.\mathbf{l} + \int_{CD} \mathbf{B}.\mathbf{l} + \int_{DE} \mathbf{B}.\mathbf{l} + \int_{EA} \mathbf{B}.\mathbf{l}$$

We see that magnetic filed is either perpendicular or there is no magnetic field in transverse directions from C to D and from E to A. For these conditions, the integral along these paths are zero. Further, the line segment DE falls in the region where magnetic field is zero. Thus, all three integrals except the first on the right hand side are equal to zero.

$$\oint \mathbf{B}.\mathbf{l} = \int_{AC} Bdl\cos 0^\circ = Ba$$

The total current through the loop is numbers of times the wire crosses the plane of drawing. If "n" be the numbers of turns per unit length, then total current is "na". Hence,

$$\Rightarrow Ba = \mu_0 naI$$

$$\Rightarrow B = \mu_0 nI$$

The magnetic field is proportional to the current and numbers of turns per unit length of solenoid. Importantly, it does not depend on the radius of coil.

For illustration, see Ampere's law(exercise) : Problem 8 (Section 12.1.9:).

11.3.4 Toroid

A toroid is solenoid bent along a circular path in the shape of a doughnut. By symmetry, the magnetic field is circular inside the toroid and is zero outside it. It is also constant on a circular loop of radius “r” drawn inside the toroid being equidistant from the center of doughnut. The total current passing through Ampere loop is NI where N is the total numbers of turns. Applying Ampere’s law, we have :

Magnetic field due to a toroid

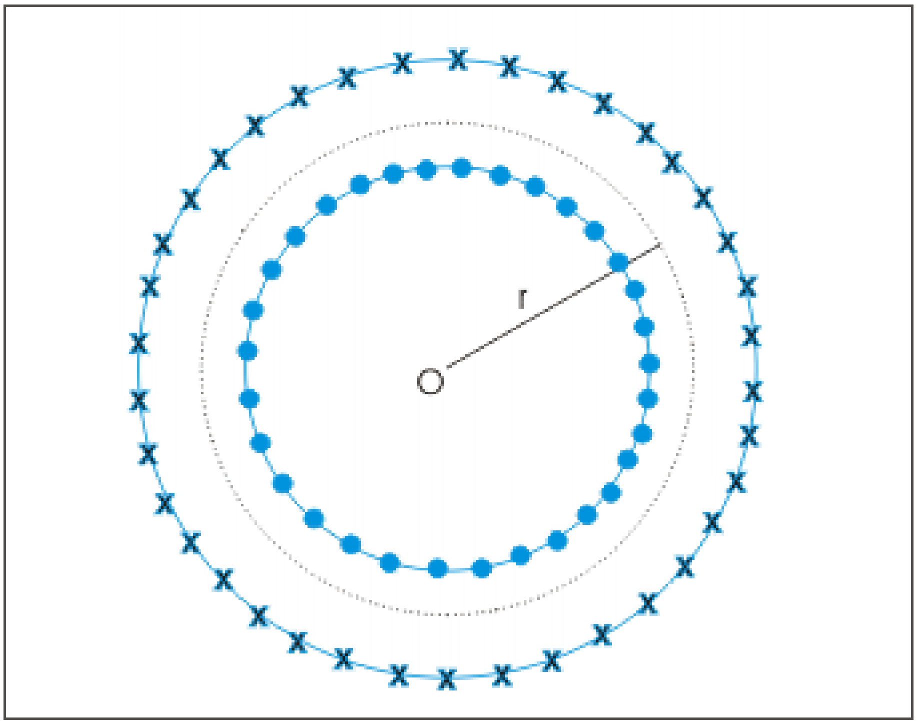


Figure 11.13: A toroid is solenoid bent along a circular path in the shape of a doughnut.

$$\oint B.l = \mu_0 NI$$

The magnetic field and line element vectors are in the same direction. Hence,

$$\Rightarrow BX2\pi r = \mu_0 NI$$

$$\Rightarrow B = \frac{\mu_0 NI}{2\pi r}$$

It is important to observe that magnetic field inside the toroid is not constant across the cross-section. It is inversely proportional to “r”. It depends upon the linear distance as we move from the interior side to exterior side. We may also write this expression in terms of numbers of turns per unit length as :

$$n = \frac{N}{2\pi r}$$

and

$$\Rightarrow B = \mu_0 n I$$

But this form is not advisable as it conceals the non-uniform nature of magnetic field inside the toroid. It is easy to find the direction of magnetic field. We orient the fingers of right hand in the direction of current along the turn of coil. Then, the extended thumb gives the direction of magnetic field.

Chapter 12

Ampere's law (Exercise)[1]

12.1 Worked out exercises

12.1.1

Problem 1: Two wires each carrying current I are perpendicular to xy plane. The current in one of them is into the plane denoted by a cross sign and the current in the other wire is out of the plane denoted by a filled circle. If the linear distance between the positions of two wires is "2a", then find the net magnetic field at a distance "b" on the perpendicular bisector of the line joining the positions of two wires.

[1]This content is available online at <http://cnx.org/content/m31927/1.3/>.

Magnetic field at perpendicular bisector

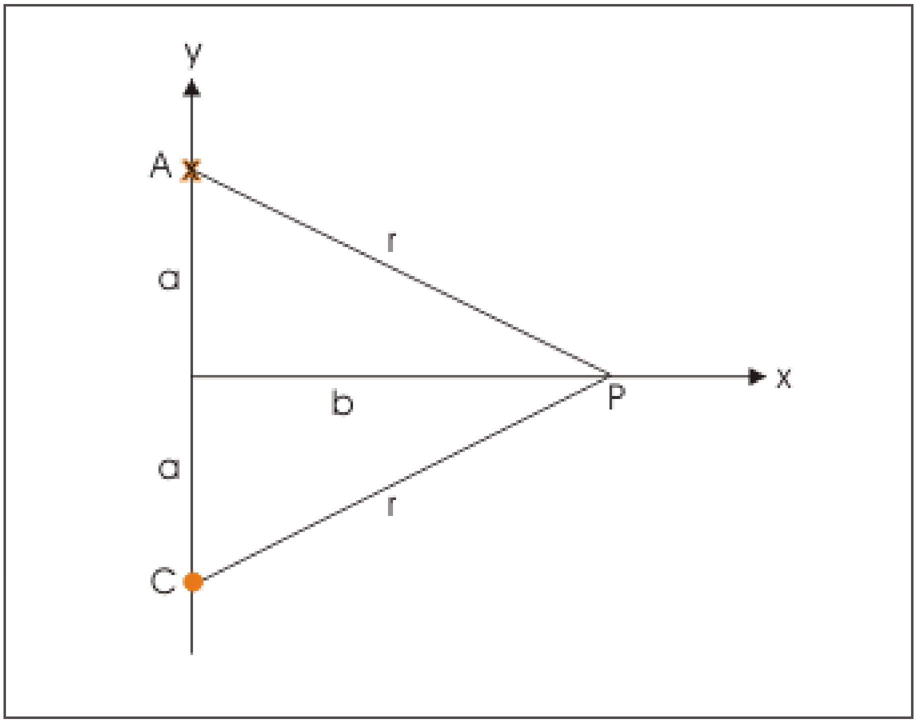


Figure 12.1: Magnetic field at perpendicular bisector

Solution : The magnitudes of magnetic fields due to wires at A and B are equal. Applying Ampere's law, the magnetic field due to each wire is :

$$B = \frac{\mu_0 I}{2\pi r}$$

The magnetic fields are directed tangential to the circle drawn containing point "P" with centers "A" and "B" as shown in the figure. Each magnetic field makes an angle say "θ" with the bisector. The components in y-direction cancel out, whereas x-components add up. Clearly, the net magnetic field is directed in negative x – direction. The magnitude of net magnetic field is :

Magnetic field at perpendicular bisector

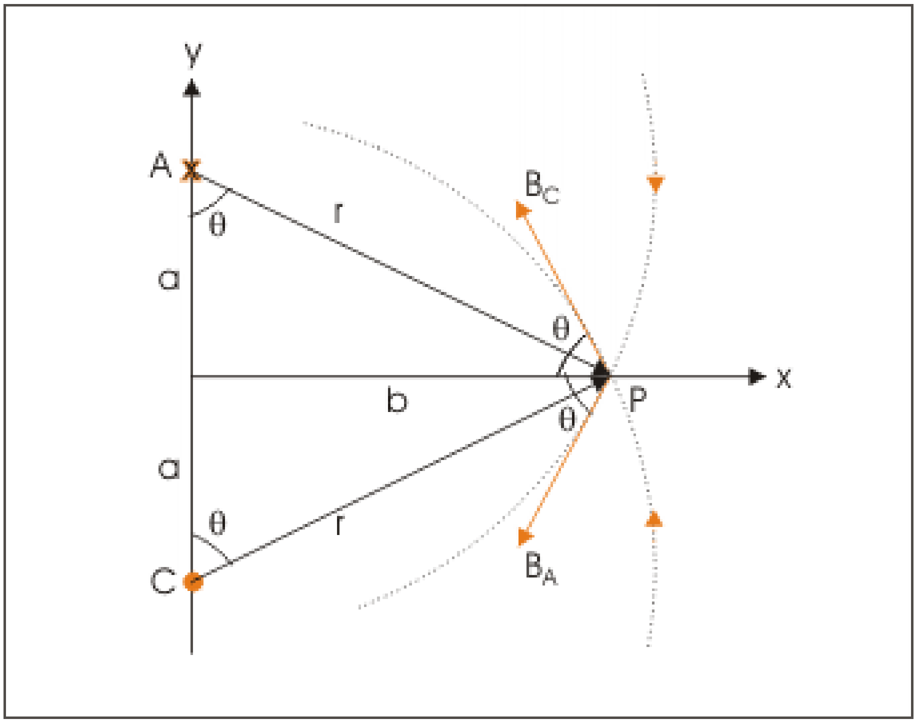


Figure 12.2: Magnetic field at perpendicular bisector

$$\Rightarrow B = 2X\frac{\mu_0 I\cos\theta}{2\pi r} = \frac{\mu_0 I\cos\theta}{\pi r}$$

Now,

$$\cos\theta = \frac{a}{r} = \frac{a}{\sqrt{(a^2+b^2)}}$$

and

$$r = \sqrt{(a^2+b^2)}$$

Putting these expressions in the equation for the magnetic field at "P", we have :

$$\Rightarrow B = \frac{\mu_0 I\cos\theta}{\pi r} = \frac{\mu_0 I a}{\pi\sqrt{(a^2+b^2)}\sqrt{(a^2+b^2)}}$$

$$\Rightarrow B = \frac{\mu_0 I a}{\pi\,(a^2+b^2)}$$

12.1.2

Problem 2: Five straight wires, carrying current I, are perpendicular to the plane of drawing. Four of them are situated at the corners and fifth wire is situated at the center of a square of side "a". Two of the wires at the corners are flowing into the plane whereas the remaining three are flowing out of the plane. Find the net magnetic field at the center of square.

Five straight wires, carrying current I

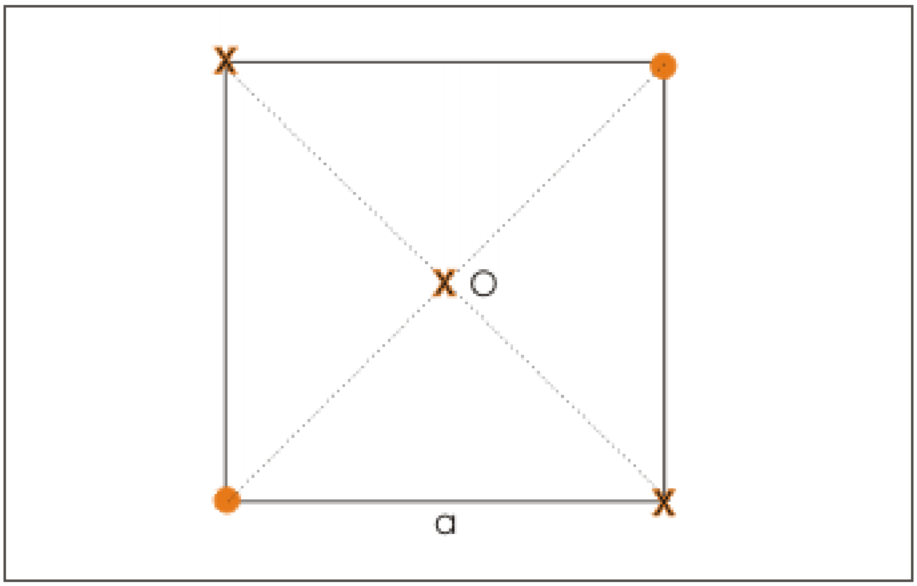


Figure 12.3: Five straight wires, carrying current I

Solution : According to Ampere's law (Section 11.1: Basis of Ampere law), the magnetic field due to a straight wire carrying current "I" at a perpendicular distance "r" is given as :

$$B = \frac{\mu_0 I}{2\pi R}$$

The wires at the corners carry equal currents and the center "O" is equidistant from these wires. Thus, magnetic fields due to these four wires have equal magnitude. In order to find the directions of magnetic fields, we draw circles containing point of observation "O". The direction of magnetic field is tangential to the circle. Applying Right hand thumb rule for straight wire, we determine the orientation of magnetic field as shown in the figure. Clearly, the net magnetic field due to these four wires at the center is zero.

Directions of magnetic fields

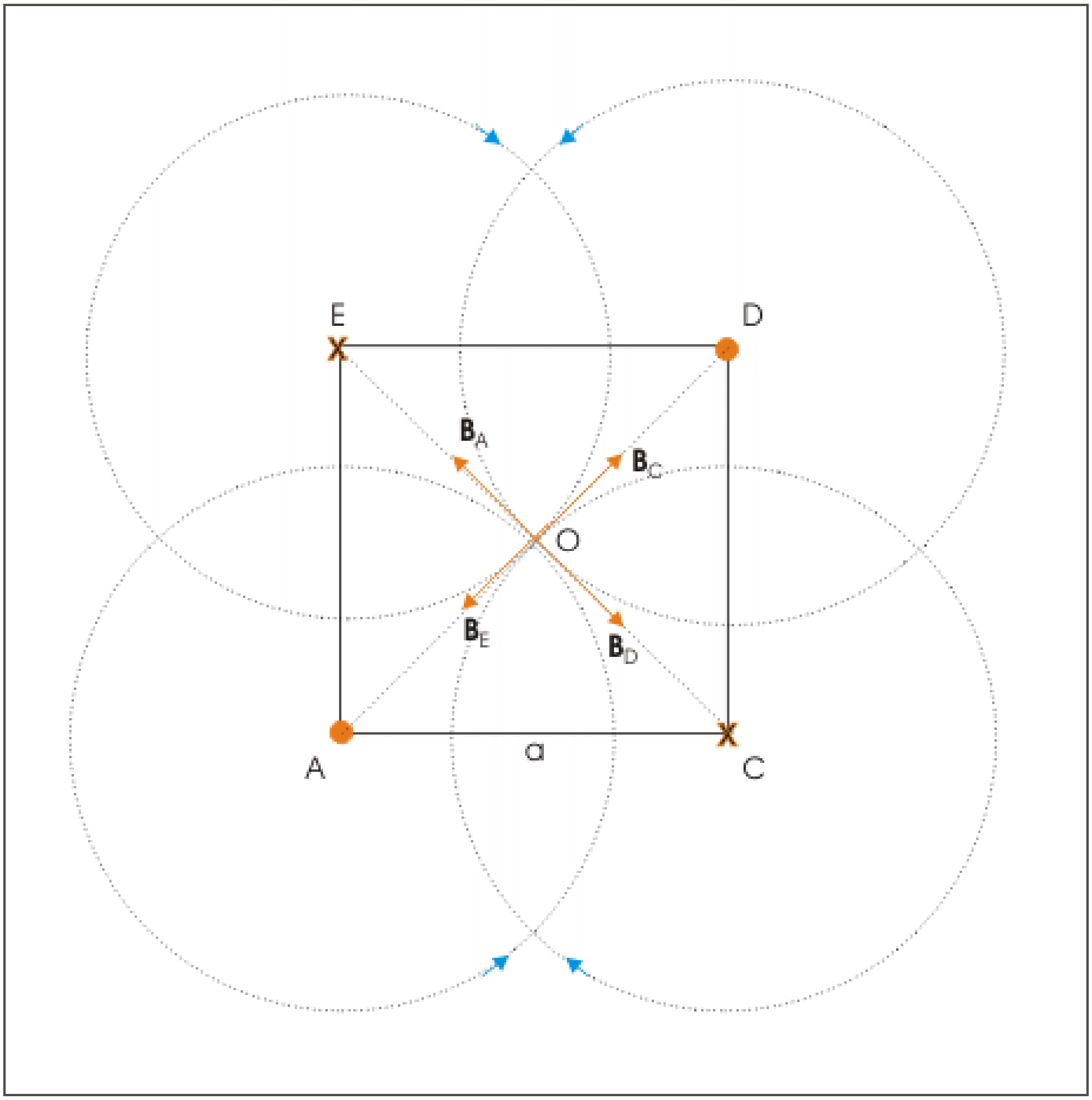


Figure 12.4: Directions of magnetic fields

Now, magnetic field at a point on the wire itself is zero. Thus, magnetic fields due to all the five wires at the center "O" is zero.

It is interesting to note that if straight wires with currents are arranged differently, for example, two currents out of the plane at A and C respectively and the other two currents into the plane at D and E respectively are arranged, then magnetic fields do not cancel and there is net non-zero magnetic field at "O" due to currents in four wires.

12.1.3

Problem 3: There are five long wires perpendicular to the plane of drawing, each carrying current I as shown by filled circles (out of plane) and crosses (into the plane) in the figure below. Determine closed line integrals $\oint \mathbf{B}.d\mathbf{l}$ for each of the four contours in the direction of integration shown.

Currents and Ampere's loops

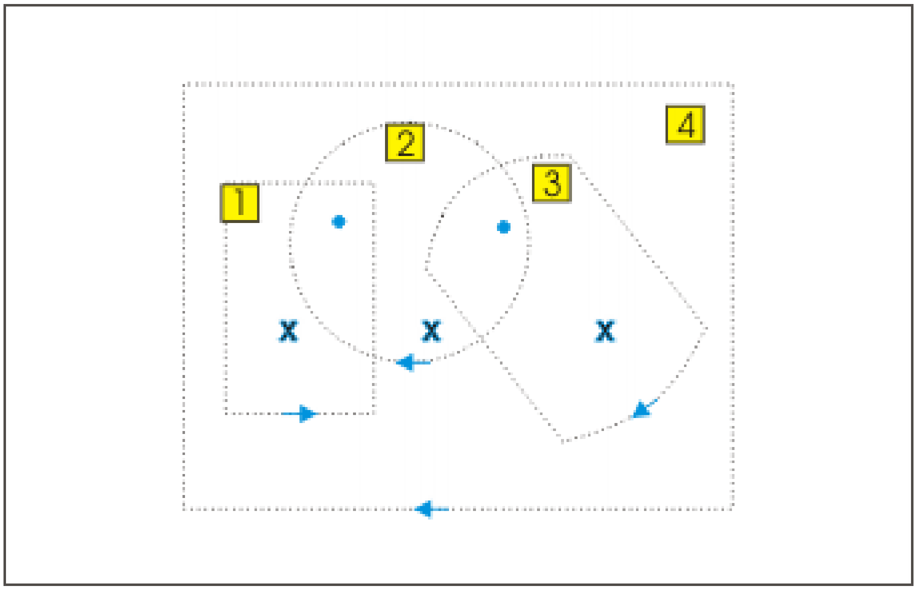


Figure 12.5: Currents and Ampere's loops

Solution :

According to Ampere's law (Section 11.2: Statement of Ampere law), the closed line integral is related to enclosed current as :

$$\oint \mathbf{B}.\mathbf{l} = \mu_0 I$$

For loop 1, the integration direction is anticlockwise. The current out of the page is positive and current into the page is negative. There are one in and one out current here. The net current is zero. Hence,

$$\Rightarrow \oint \mathbf{B}.\mathbf{l} = 0$$

For loop 2, the integration direction is clockwise. The current out of the page is negative and current into the page is positive. There are one in and two out current here. The net current is one out current i.e. "-I". Hence,

$$\oint \mathbf{B}.\mathbf{l} = -\mu_0 I$$

For loop 3, the integration direction is clockwise. The current out of the page is negative and current into the page is positive. There are one in and one out current here. The net current is zero. Hence,

$$\oint \mathbf{B}.d\mathbf{l} = 0$$

For loop 4, the integration direction is clockwise. The current out of the page is negative and current into the page is positive. There are three in and two out current here. The net current is one in current i.e. "I". Hence,

$$\oint \mathbf{B}.\mathbf{l} = \mu_0 I$$

12.1.4

Problem 4: The magnetic field in a region is given by relation :

$$\mathbf{B} = 5\hat{\mathbf{l}}$$

Closed line integral of magnetic field

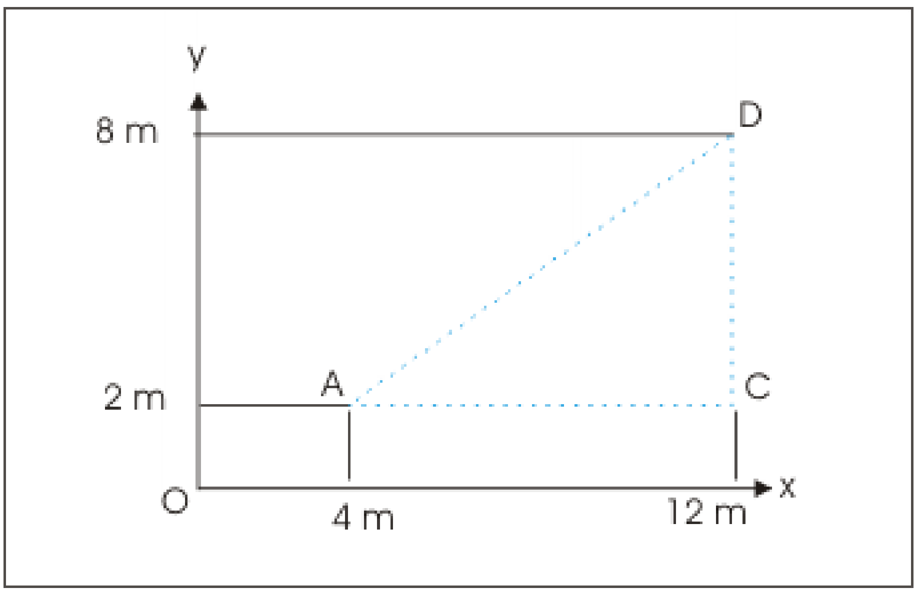


Figure 12.6: Closed line integral of magnetic field

where **i** is the unit vector in x direction. Determine closed line integral of magnetic field along the triangle ACD.

Solution :

For line segment AC, magnetic field and length element are in the same direction. Applying Ampere's law :

$$\int_{AC} \mathbf{B}.\mathbf{l} = \int_{AC} 5x\cos 0^\circ = \int_{AC} 5x$$

$$\Rightarrow \int_{AC} \mathbf{B.l} = 5\int_{AC} x = 5X8 = 40 \quad Tm$$

We see that magnetic field is perpendicular to line segment CD. Therefore, magnetic line integral for this segment is equal to zero.

For the line segment DA, the length is $\sqrt{(8^2 + 6^2)} = 10$ m.

Closed line integral of magnetic field

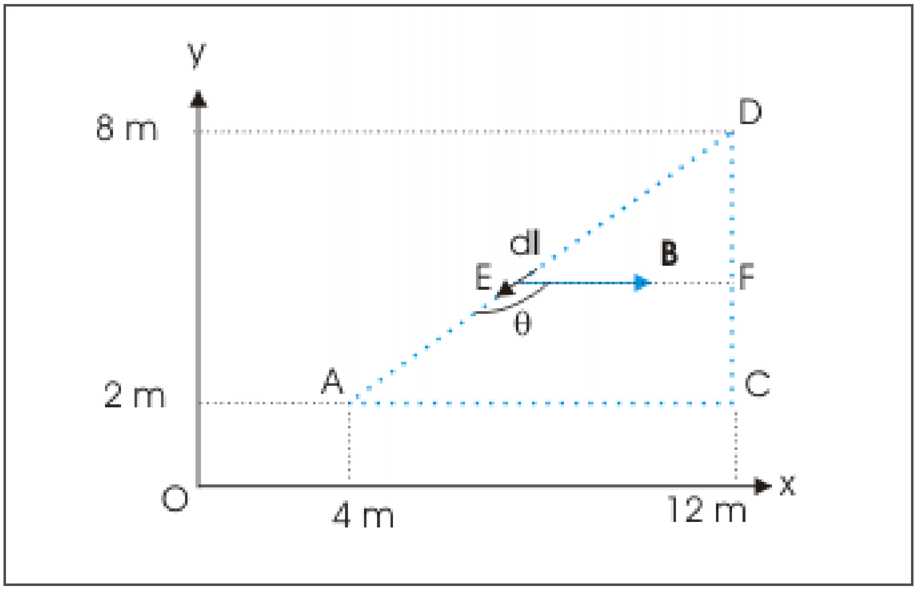


Figure 12.7: Closed line integral of magnetic field

$$\int_{DA} \mathbf{B.l} = \int_{DA} 5x\cos\angle AEF = 5\int_{DA} x\cos(\pi - \angle DEF) = -5\int_{DA} x\cos\angle DEF$$

$$\Rightarrow \int_{DA} \mathbf{B.l} = -5\int_{DA} x\cos\angle DAC = -5\int_{DA} \frac{AC}{AD}x = -5\int_{DA} \frac{8}{10}x$$

$$\Rightarrow \int_{DA} \mathbf{B.l} == -4\int_{DA} x = -4XDA = -40 \quad Tm$$

Adding two values, the value of closed line integral is zero.

$$\oint \mathbf{B.l} = 0$$

Thus, we see that current though the region is zero even though there exists magnetic field in the region.

12.1.5

Problem 5: Straight wires are mounted tightly over a long hollow cylinder of radius "R" such that they are parallel to the axis of cylinder. The perpendicular cross-section of the arrangement is shown in the figure below. If there are N such wires each carrying a current I, then determine magnetic field inside and outside the cylinder.

Magnetic field due to current in tightly packed straight wires

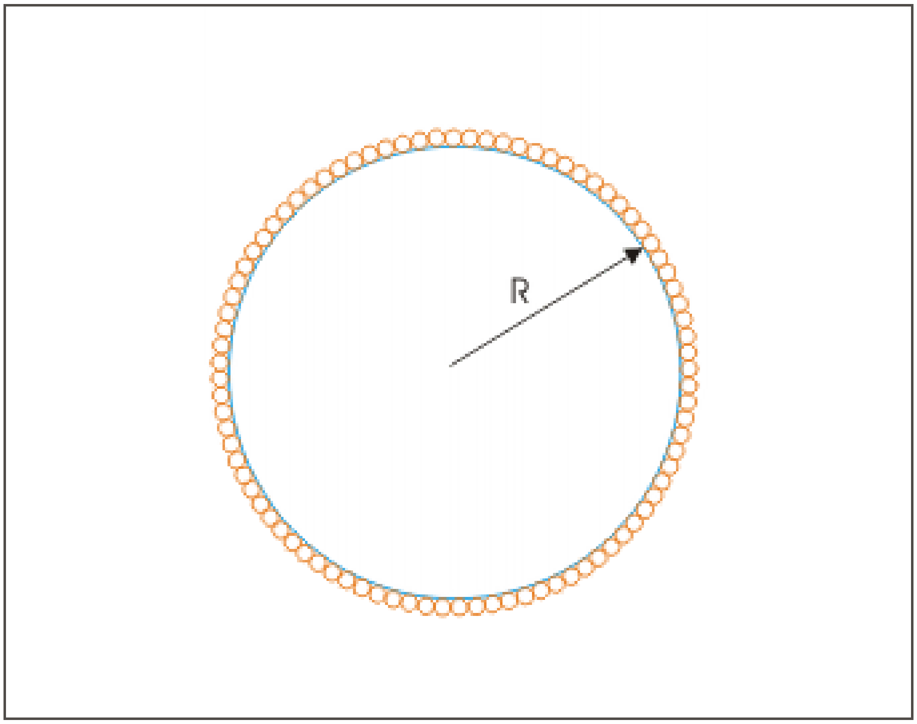


Figure 12.8: Magnetic field due to current in tightly packed straight wires

Solution : We draw an Ampere loop of radius "r" for applying Ampere's law (Section 11.2: Statement of Ampere law) at a point inside the cylinder. But there is no current inside. Hence,

Magnetic field due to current in tightly packed straight wires

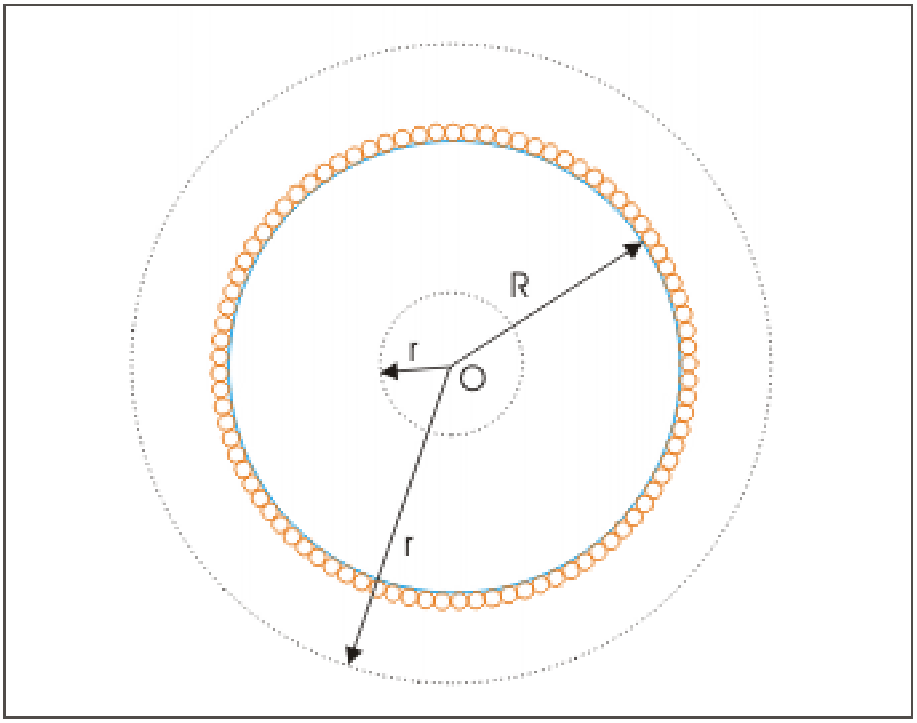


Figure 12.9: Magnetic field due to current in tightly packed straight wires

$$\oint \mathbf{B}.\mathbf{l} = 0$$

Here integration of dl along the loop is equal to perimeter of loop i.e. 2πr. Hence, B = 0. For determining magnetic field at an outside point, we draw an Ampere loop of radius "r". Here, the total current is NI. Hence,

$$\oint \mathbf{B}.\mathbf{l} = \mu_0 NI$$

$$\Rightarrow 2\pi r B = \mu_0 NI$$

$$\Rightarrow B = \frac{\mu_0 NI}{2\pi r}$$

12.1.6

Problem 6: A cylindrical conductor of radius R carries current I distributed uniformly across the cross-section. Draw the curve showing variation of magnetic field as we move away from the axis of conductor in perpendicular direction.

Solution : Let the perpendicular direction to the axis be x-axis. The magnetic field at a point inside the conductor is given by :

$$B = \frac{\mu_0 I x}{2\pi R^2}$$

Clearly, magnetic field increases linearly as move away from the axis towards the edge of conductor and attains the maximum at the surface, when x=R and magnetic field is given as:

$$B = \frac{\mu_0 I R}{2\pi R^2} = \frac{\mu_0 I}{2\pi R}$$

The magnetic field at a point outside the conductor is :

$$B = \frac{\mu_0 I}{2\pi x}$$

The magnetic field is inversely proportional to the linear distance "x". The required plot of magnetic field .vs. x is as shown in the figure below :

Variation of magnetic field

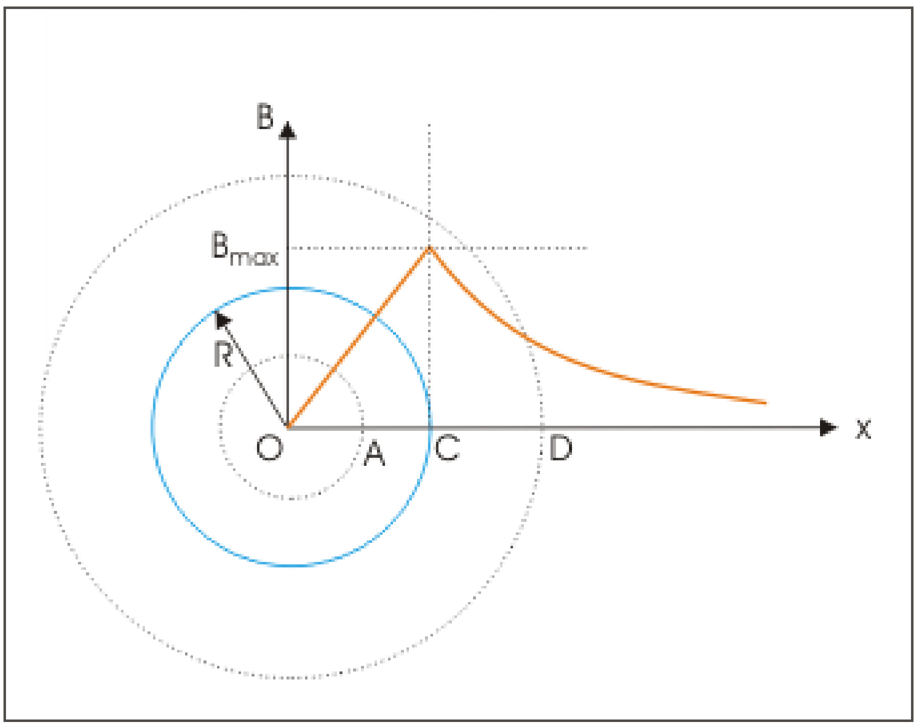


Figure 12.10: Variation of magnetic field

12.1.7

Problem 7: A long annular cylindrical conductor of radii "a" and "b" carries current I. The perpendicular cross section of annular cylinder is shown in the figure below. If the current distribution in the annular region is uniform, determine magnetic field at a point in the annular region at a radial distance "r" from the axis.

Magnetic field due to current in annular cylindrical conductor

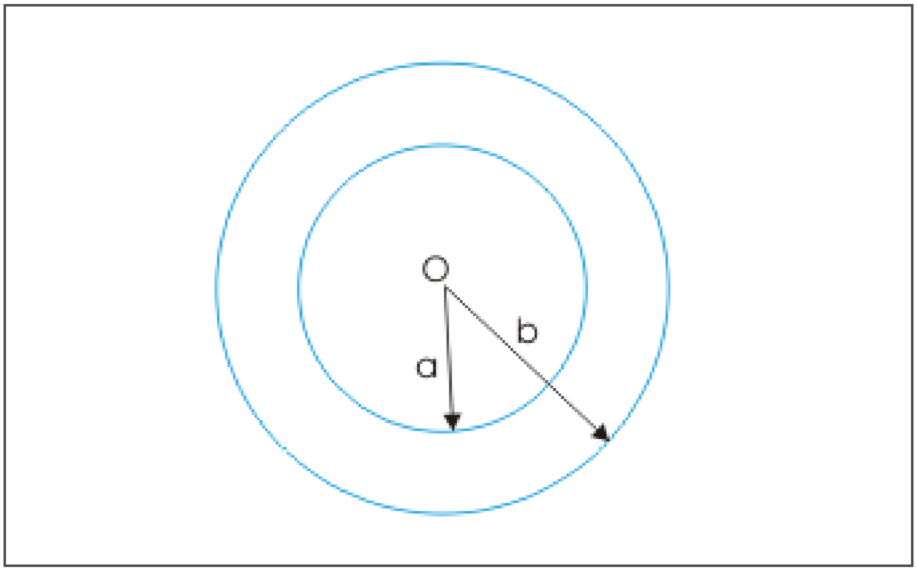


Figure 12.11: Magnetic field due to current in annular cylindrical conductor

Solution : According to Ampere's law (Section 3.2: Experimental verification of Biot-Savart's law),

$$\oint \mathbf{B}.\mathbf{l} = \mu_0 I$$

In order to evaluate this equation, we need to know the current in the annular region from r=a to r=r. For this we need the value of current density. Here, total current is given. Dividing this by the total area of the region gives us the current density,

$$J = \frac{I}{\pi \left(b^2 - a^2\right)}$$

The net current through the Ampere loop of radius "r" falling in the annular region is given by multiplying current density with the annular area between r=a and r=r. Applying Ampere's law for a loop of radius r,

Magnetic field due to current in annular cylindrical conductor

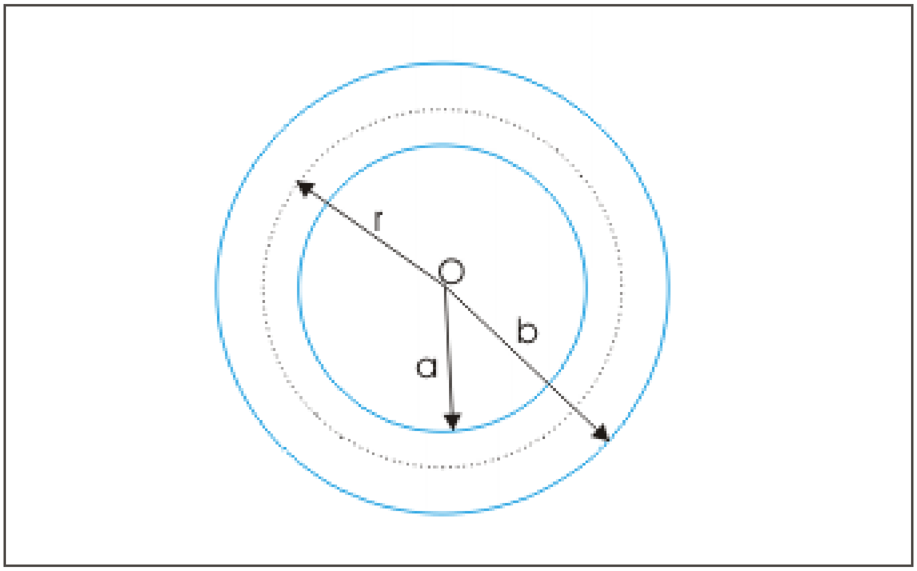


Figure 12.12: Magnetic field due to current in annular cylindrical conductor

$$\oint \mathbf{B}.\mathbf{l} = \mu_0 \pi \left(r^2 - a^2\right) J = \frac{\mu_0 \pi \left(r^2 - a^2\right) I}{\pi \left(b^2 - a^2\right)}$$

$$\Rightarrow 2\pi r B = \frac{\mu_0 I \left(r^2 - a^2\right)}{\left(b^2 - a^2\right)}$$

$$\Rightarrow B = \frac{\mu_0 I \left(r^2 - a^2\right)}{2\pi r \left(b^2 - a^2\right)}; \quad a < r < b$$

12.1.8

Problem 8: A long annular cylindrical conductor of radii “a” and “b” carries a current. If the current distribution in the annular region is given as J = kr, where k is a constant, then determine magnetic field at a point in the annular region at a radial distance “r” from the axis.

Solution : This question is similar to earlier question with one difference that areal current density is not uniform. We see here that the current distribution in the annular region is given as J=kr. Clearly, current density increases as we move from inner edge to the outer edge of the annular cylinder. The current in the small strip đr is :

$$I = 2\pi r r J = 2\pi r r k r = 2\pi k r^2 r$$

Magnetic field due to current in annular cylindrical conductor

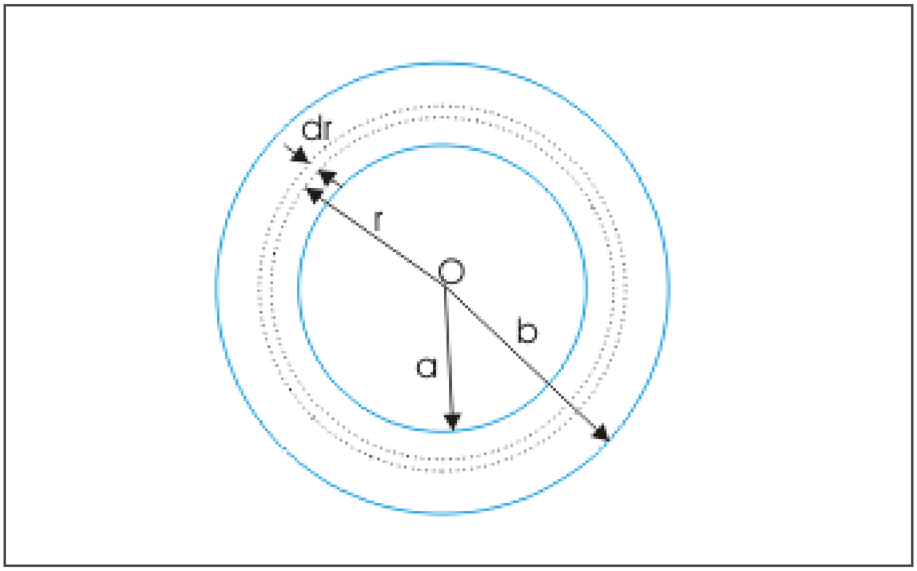


Figure 12.13: Magnetic field due to current in annular cylindrical conductor

Applying Ampere's law for a loop of radius r and considering that current is distributed from r=a to r=r,

$$\oint \mathbf{B}.d\mathbf{l} = \mu_0 \oint dI = \mu_0 \int_a^r 2\pi k r^2 dr = 2\pi \mu_0 k \int_a^r r^2 dr$$

$$\Rightarrow 2\pi r B = 2\pi \mu_0 k \left[\frac{r^3}{3}\right]_a^r = \frac{2\pi \mu_0 k \left(r^3 - a^3\right)}{3}$$

$$\Rightarrow B = \frac{\mu_0 k \left(r^3 - a^3\right)}{3r}; \quad a < r < b$$

12.1.9

Problem 9: A long solenoid having 1000 turns per meter carries a current of 1 A. A long straight conductor of radius 0.5 cm and carrying a current of 10π A is placed coaxially along the axis of solenoid. Compare magnetic fields due to two currents at that point. Also determine magnetic field at a point on the surface of straight conductor.

Solution : The magnetic field due to solenoid is uniform inside the solenoid and is given as :

$$B_S = \mu_0 n I = 4\pi X 10^{-7} X 1000 X 1 = 4\pi X 10^{-4} \quad T$$

The magnetic field due to straight conductor on its surface is :

$$B_C = \frac{\mu_0 I}{2\pi R} = \frac{4\pi X 10^{-7} X 10\pi}{2\pi X 0.5 X 10^{-2}} = 4\pi X 10^{-4} \quad T$$

Magnetic field due solenoid and straight conductor

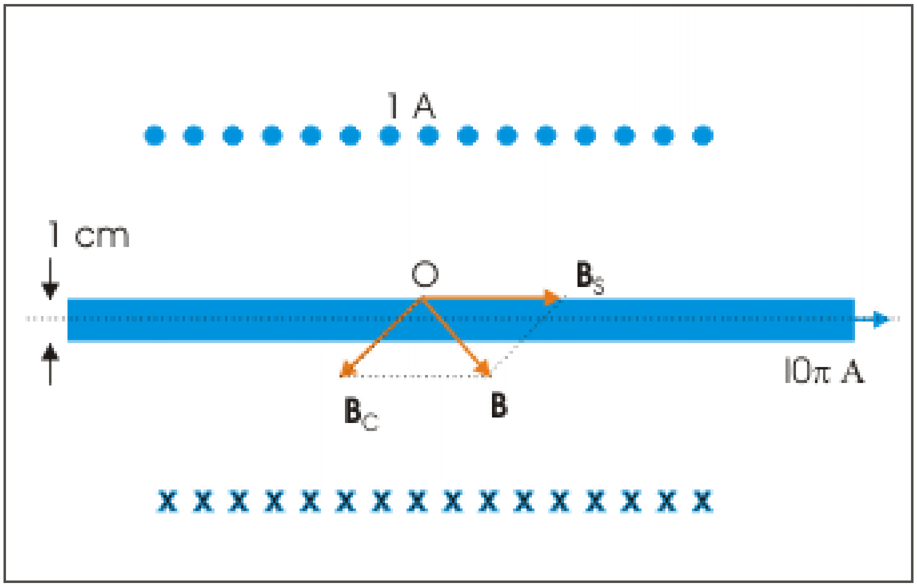


Figure 12.14: Magnetic field due solenoid and straight conductor

The magnetic field due to straight conductor is tangential to the circumference and hence is perpendicular to magnetic field due to solenoid. The resultant magnetic field is, therefore,

$$B = \sqrt{\left(2X16\pi^2 X10^{-8}\right)} = \sqrt{2}X4\pi X10^{-4} = 1.778X10^{-3} \quad T$$

Both solenoid and straight conductor produces equal magnetic field at the surface of conductor. It is interesting to observe that a straight conductor requires a current of magnitude which is 10π i.e. 31.4 times the current in solenoid. This illustrates the effectiveness of solenoid over a straight conductor in setting up a magnetic field with respect to straight conductor. For this reason, a solenoid is generally used as a magnet in application situations.

12.1.10

Problem 10: A long cylindrical conductor of radii "a" is coaxially placed inside an annular cylindrical conductor of radii "b" and "c". The perpendicular cross section of the coaxial annular cylinders is shown in the figure below. If currents in two conductors are I each but in opposite direction, then find magnetic field at a point (i) inside the inner conductor (ii) region between two cylinders (iii) inside annular cylinder and (iv) outside the annular cylinder. Assume current density to be uniform in both cylinders.

Magnetic field due to current in coaxial cylindrical conductors

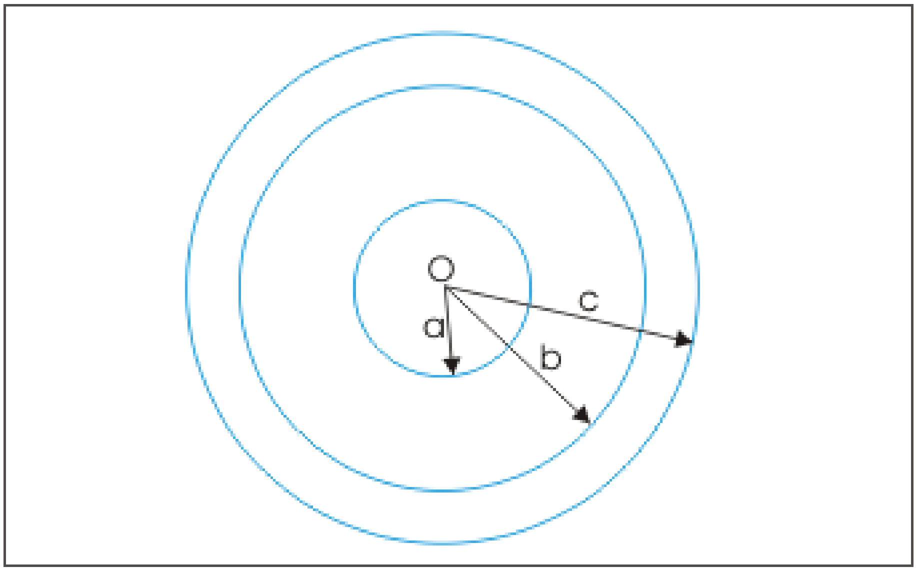


Figure 12.15: Magnetic field due to current in coaxial cylindrical conductors

Solution : We note that current densities in two cylinders are uniform. To find magnetic field at a point inside the inner cylinder, we first determine its current density.

$$J_i = \frac{I}{\pi a^2}$$

Note that current outside the Ampere loop in the inner cylinder and current in the outer conductor do not contribute towards enclosed current. Applying Ampere's law for a loop of radius r inside the inner cylinder,

$$\oint \mathbf{B}.\mathbf{l} = \mu_0 \pi r^2 J = \frac{\mu_0 \pi r^2 I}{\pi a^2} = \frac{\mu_0 I r^2}{a^2}$$

$$\Rightarrow 2\pi r B = \frac{\mu_0 I r^2}{a^2}$$

$$\Rightarrow B = \frac{\mu_0 I r^2}{2\pi r a^2}; \quad r < a$$

To find magnetic field at a point between inner and outer cylinders, we apply Ampere's law for a loop of radius r between the region (a<r<b). Note that outer conductor does not contribute towards enclosed current. Applying Ampere's law (Section 11.2: Statement of Ampere law) for a loop of radius r between inner and outer cylinders,

$$\oint \mathbf{B}.\mathbf{l} = \mu_0 I$$

$$\Rightarrow 2\pi r B = \mu_0 I$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

To find magnetic field at a point inside the outer cylinder, we apply Ampere's law for a loop of radius r between the region (b<r<c). Note that current in the inner conductor and annular region defined by b<r<c contribute towards enclosed current. In order to find the enclosed current in the outer cylinder, we first determine its current density.

$$J_o = \frac{I}{\pi\left(c^2 - b^2\right)}$$

Further the current in inner and outer cylinders are opposite in direction. We observe here that current density of inner cylinder is greater as current I is divided by smaller area. Thus, we shall deduct the current through the annular region of outer cylinder from the current in inner cylinder. Applying Ampere's law for a loop of radius r inside the outer cylinder,

$$\oint \mathbf{B.l} = \mu_0 I - \mu_0 X \frac{\pi\left(r^2 - b^2\right) XI}{\pi\left(c^2 - b^2\right)} = \mu_0 \left[I - \frac{\left(r^2 - b^2\right) XI}{\left(c^2 - b^2\right)}\right]$$

$$\Rightarrow 2\pi r B = \frac{\mu_0 I\left(c^2 - b^2 - r^2 + b^2\right)}{\left(c^2 - b^2\right)} = \frac{\mu_0 I\left(c^2 - r^2\right)}{\left(c^2 - b^2\right)}$$

$$\Rightarrow B = \frac{\mu_0 I\left(c^2 - r^2\right)}{2\pi r\left(c^2 - b^2\right)}$$

To find magnetic field at a point outside the outer cylinder, we apply Ampere's law for a loop of radius r (r>c). The net current the loop is zero. Hence,

$$\oint \mathbf{B.l} = 0$$

$$\Rightarrow 2\pi r B = 0$$

$$\Rightarrow B = 0$$

Chapter 13

Magnetic force on a conductor[1]

We have seen that a moving charge experiences force in the presence of magnetic field. Now, current in a wire or a conductor results from the motion of negatively charged "free" or "conduction" electrons. It is, therefore, imperative that these moving electrons will experience "magnetic force" due to the presence of magnetic field.

When a straight conductor carrying current is placed in a magnetic field, then conduction electrons in the conductor are under the influence of both electric and magnetic fields. The presence of electric field results in "net drift of charge (electrons)" in the conductor and it is the cause of current in the conductor. The presence of magnetic field, on the other hand, results in side way force on individual electrons (perpendicular to the conductor) resulting in the development of electrical potential across the width of the conductor or a force on the conductor itself depending on whether we are considering current through a wide conductor strip or a thin wire.

The difference in the effect of applications of two field types lies in the difference of nature of force they apply. Electrical force is linear force i.e. in the direction of electric field and is responsible for current in conductor. Magnetic force is non-linear side way force perpendicular to the direction of velocity of moving charge. The magnetic force acts to deflect electrons to the edge of a conductor. If we are considering a wide strip of conductor, then there is scope for electrons to move laterally across the width of the strip. In this case, we observe development of electrical potential difference between the edges of the conductor (known as Hall's effect). However, if we are considering current through a thin wire, electrons have no scope for transverse motion and they are also not allowed to move out of the body of wire due to electric attractive force. The side way magnetic force, therefore, results in a transverse magnetic force on the wire itself.

[1] This content is available online at <http://cnx.org/content/m32246/1.3/>.

Magnetic force and its effect

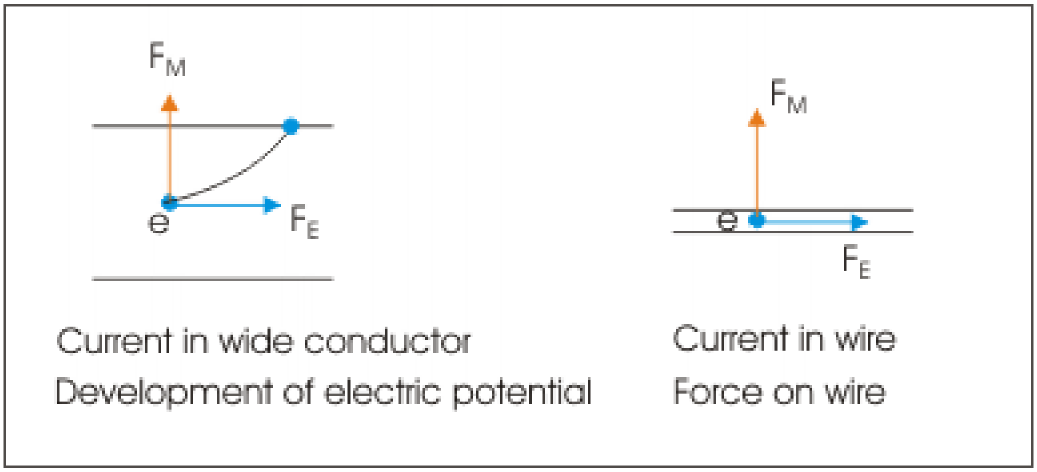


Figure 13.1: Magnetic force and its effect

The conductor can have any orientation with respect to magnetic field. Irrespective of the orientations of conductor and magnetic field, the magnetic force is always perpendicular to both conductor length and magnetic field vectors. This fact simplifies our investigation a great deal as we need to consider only transverse magnetic force which is always perpendicular to the direction of current or the conductor length vector. This aspect is illustrated in the figure below in which conductor length vector (in the direction of current) and the magnetic field vector are oriented at an arbitrary angle "θ", but magnetic force is perpendicular to the conductor.

Direction of Magnetic force

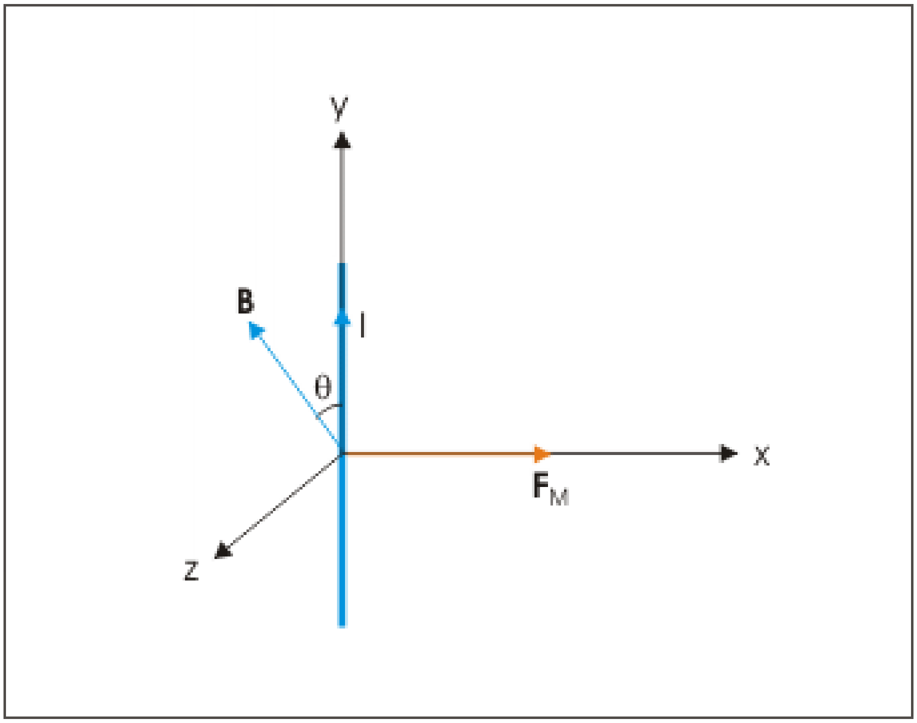


Figure 13.2: Direction of Magnetic force

13.1 Hall's effect

Here, we consider a wide strip of a conductor of width "a" and thickness "b", which is carrying a current "I".

Wide strip of a conductor

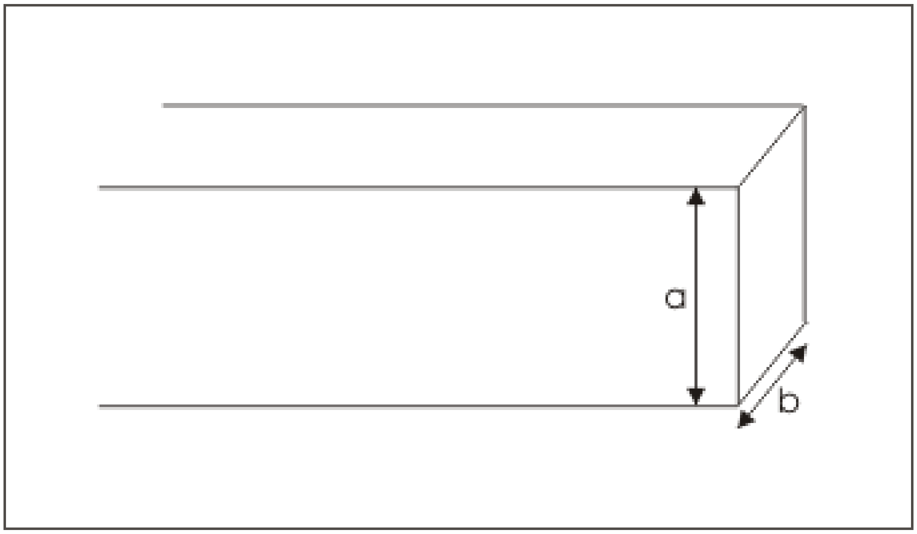


Figure 13.3: Wide strip of a conductor

Let the direction of conventional current be from right to left so that charge carrier electrons are moving from left to right. Also, let magnetic field be directed in to the plane of drawing. The direction of magnetic force is direction of vector expression " $-e\left(\mathbf{v}_d X\mathbf{B}\right)$ ". Applying Right hand thumb rule, the direction of vector cross product "$\mathbf{v}_d X\mathbf{B}$ " is upward direction. Hence, the direction of magnetic force i.e. direction of vector "$-e\left(\mathbf{v}_d X\mathbf{B}\right)$ " is downward as shown in the figure.

Direction of magnetic force

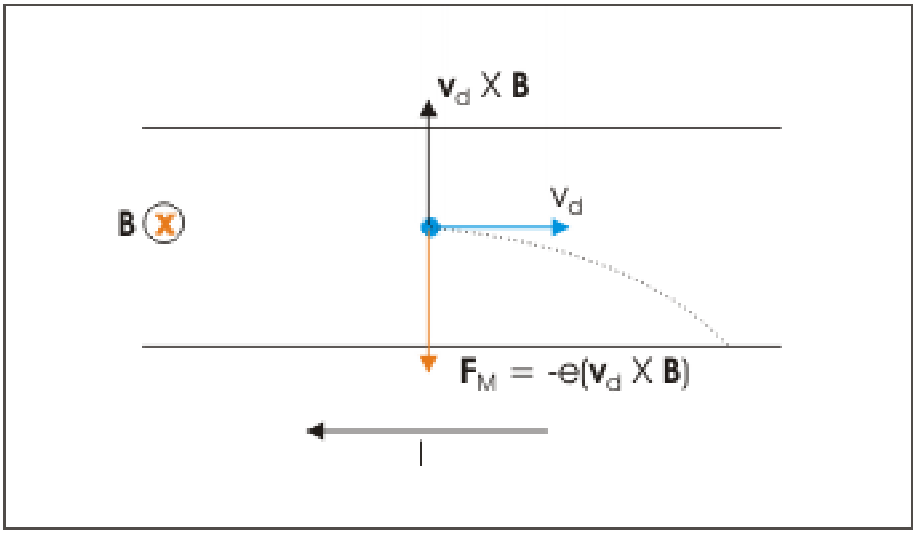


Figure 13.4: Direction of magnetic force

The magnetic force in the downward direction tends to drift electron in downward direction following a parabolic path. This drifting polarizes the conductor strip electrically. We know that each infinitesimally small element of the conductor is electrically neutral. But, there is accumulation of negative charge at lower edge as electrons drift down due to magnetic force. Correspondingly, there is accumulation of positive charge at the upper edge as there is depletion of electrons exposing immobile positive atoms in that region. The process of polarization, however, continues only momentarily. At any moment, the opposite polarity of charges at the edges sets up an electric field. In this case, the electric field is directed from upper (positive edge) to lower edge (negative edge). This electric field, in turn, pulls electron upward.

Polarization of charges and electric field

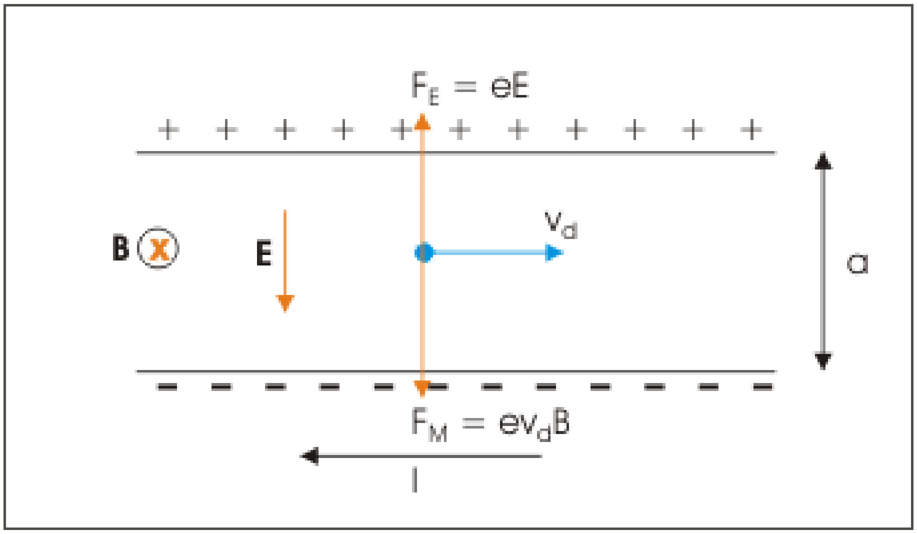


Figure 13.5: Polarization of charges and electric field

The dynamic condition is brought under equilibrium when electric force equals magnetic force. Let "E" be the electric field at equilibrium,

$$eE = ev_d B$$

$$\Rightarrow v_d = \frac{E}{B}$$

where v_d is the drift velocity. Once the equilibrium is reached, electrons keep moving with the drift velocity as they would have moved in the absence of magnetic field. Here, the opposite edges of the conductor strip function as infinite charged plates. The electric field, E, is given as :

$$E = \frac{V}{a}$$

where, "a" is the width of the conductor strip and "V" is the electrical potential difference between the edges of conductor strip. This potential difference between the edges is known as Hall's potential. We can measure it by connecting a voltmeter to the edges of the conductor strip.

13.1.1 Numbers of free electrons per unit volume

The "Hall effect" can be used to measure numbers of electrons per unit volume in a conductor. We know that the drift velocity of an electron is :

$$v_d = \frac{I}{neA}$$

where "n" is numbers of free electrons per unit volume and "A" is the cross section area of the strip. Substituting in the equation of equilibrium, we have :

$$v_d = \frac{E}{B}$$

$$\frac{I}{neA} = \frac{E}{B}$$

Substituting for "E", we have :

$$\Rightarrow \frac{I}{neA} = \frac{V}{aB}$$

$$\Rightarrow n = \frac{IaB}{eAV}$$

Also, the area A is product of width and thickness, A = ab. Hence,

$$\Rightarrow n = \frac{IB}{ebV}$$

The quantities in the right hand expression are either known or measurable. Thus, we are able to measure the numbers of free (conduction) electrons per unit volume using Hall's effect.

13.1.2 Drift Velocity

Use of Hall's effect allows measurement of drift velocity as well. The magnitude of drift velocity is about 0.0003 m/s, which is quite a small value that can be measured in the laboratory. The determination of drift velocity uses a very simple technique based on the detection of Hall's effect.

The idea here is to move the conductor strip carrying current in the direction opposite to the direction of drift velocity i.e. in the direction of conventional current in the presence of uniform magnetic field. The motion of conductor is adjusted such that the relative drift velocity of electron with respect to stationary magnetic field is zero. In this case, speed of conductor strip is equal to the drift speed of electron. Also, the magnetic force is zero as relative velocity of electrons with respect to stationary magnetic field is zero. In turn, there is no drifting of electron towards the edge of the conductor and the Hall potential is zero. Thus, we are able to detect when the velocity of conductor strip equals drift velocity of electron.

13.1.3 Motion of a conductor strip in magnetic field

The net drift velocity in a conductor is zero unless an electric potential difference is applied to the conductor. If we move the conductor strip in a uniform magnetic field, then free or conduction electrons acquire relative velocity with respect to stationary magnetic field. This, in turn, would set up a magnetic force on the conduction electrons. Clearly, the action to move conductor strip in the magnetic field is equivalent to imparting a net drift velocity to the conduction electrons.

Motion of a conductor strip in magnetic field

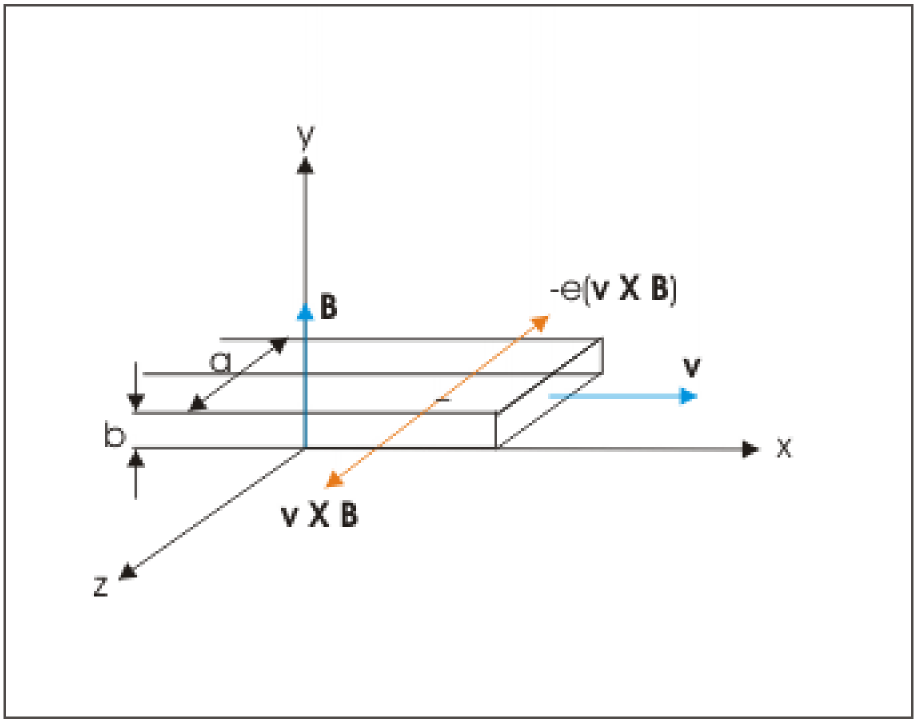


Figure 13.6: Motion of a conductor strip in magnetic field

Let us consider a metallic strip of width "a" and thickness "b" moving in x-direction as shown in the figure with a velocity "v". Also let the magnetic field is in the y-direction. Applying Right hand rule, we see that "**v**X**B**" is directed in z-direction and "-e(**v**X**B**)" is directed in negative z-direction. As a result, one edge is negatively charged and the other edge is positively charged. At equilibrium,

Polarization of charges and electric field

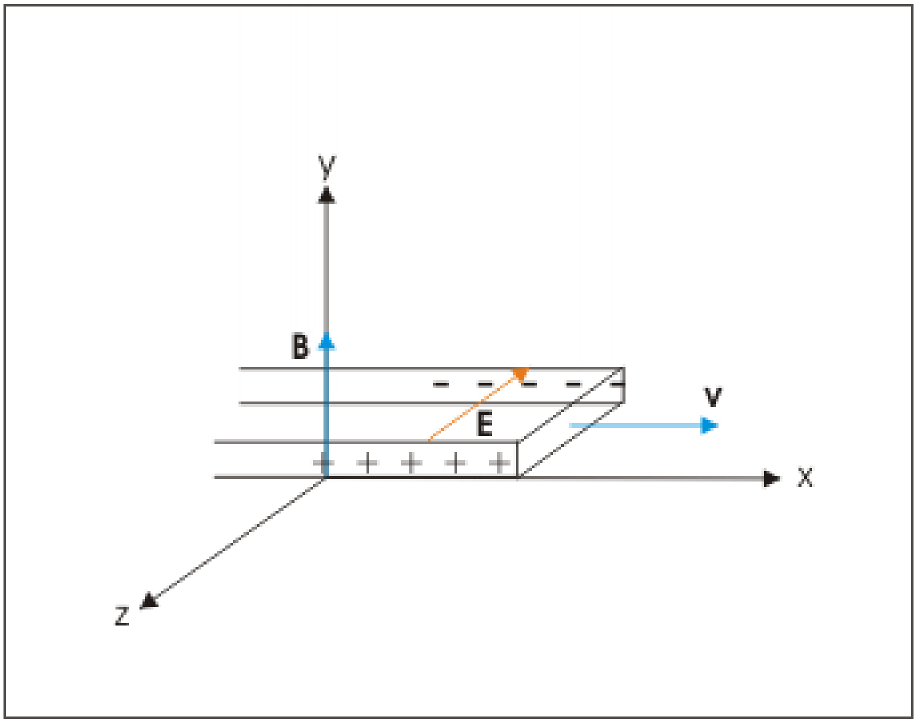


Figure 13.7: Polarization of charges and electric field

$$v = \frac{E}{B}$$

and potential difference across the edge is :

$$\Rightarrow V = Ea = vBa$$

13.2 Magnetic force on a straight wire

In the case of a thin wire, there is no room for electrons to move sideways as in the case of wide strip of conductor. The sideway motion thus produces a thrust on the wire and there is a net magnetic force on the wire. It is evident that we need to account for magnetic force on each of the free conduction electrons. Since each of these forces is transverse to the straight wire, the direction of net force is same as that of the magnetic force working on any of the conduction electrons. This fact allows us to simply add individual forces arithmetically to determine the resultant force. Further, the net force on wire will also depend on the length of wire being considered as the numbers of free electrons is proportional to the length of wire.

According to Lorentz law (Chapter 7) the magnetic force on a single electron :

$$F_i = ev_d B\sin\theta$$

Net magnetic force on the wire

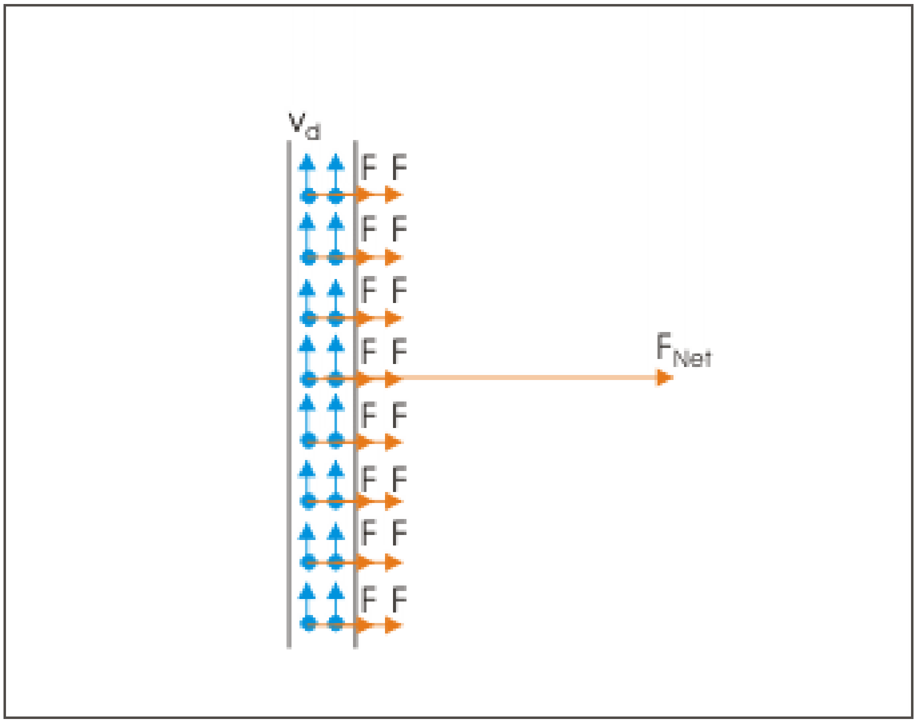


Figure 13.8: Net magnetic force on the wire is arithmetic sum of individual magnetic forces on conduction electrons.

where θ is the angle between magnetic field and drift velocity. Let there be "n" electrons per unit volume. Also, let "L" and "A" be the length and cross section respectively of the wire under consideration. Clearly, the total numbers of electrons in the length "L" of the wire is :

$$N = nAL$$

Hence, total magnetic force on the wire of length "L" is :

$$F = \sum F_i = nALXF_i = nALev_dB\sin\theta$$

But we know that :

$$v_d = \frac{I}{neA}$$

Substituting, we have :

$$\Rightarrow F = \frac{nALeXIXB\sin\theta}{neA} = ILB\sin\theta$$

In vector form, this is written using concept of cross vector product as :

$$\Rightarrow \mathbf{F} = I\mathbf{L}X\mathbf{B}$$

The direction of length vector is same as that of the direction of current in wire.

13.2.1 Magnetic force on a non-linear wire

If the wire under consideration is not a straight wire, then we can not use the expression formulated above. It is important to understand here that the above expression is valid for a straight wire. This is the basic assumption which allowed us to carry out arithmetic sum of individual forces as directions of magnetic forces on individual electrons were same. However, if the wire is not straight, then it would not be possible to do the arithmetic sum for obtaining the resultant force as the directions of magnetic force would be different.

For such situation involving nonlinear wire, we prefer to have an expression for a infinitesimally small length of wire. This consideration of very small length of wire guarantees that the wire element is straight. Following the similar argument as for a straight wire, the magnetic force on an infinitesimally small length of wire is :

$$\mathbf{F} = I\mathbf{L}X\mathbf{B}$$

We can, then, use this expression and integrate along non-linear wire. Of course, such calculation will depend on the possibility to divide the given wire into segments for which integration of this expression is possible.

13.2.2 Current element and moving charge

We have pointed out the equivalent role of current element and moving charge in the context of production or setting up of magnetic field. An inspection of the expression of magnetic force on a charge and a current element indicate that the equivalence is true also in the case of experiencing magnetic force. In the case of moving charge, the magnetic force is given by :

$$F = q\left(\mathbf{v}X\mathbf{B}\right)$$

On the other hand, the magnetic force on a small current carrying wire element is :

$$\mathbf{F} = I\mathbf{L}X\mathbf{B}$$

Clearly, the term "q**v**" and "Id**L**" play the equivalent role in two cases.

Example 13.1

Problem : An irregular shaped flexible wire loop of length "L" is placed in a perpendicular and uniform magnetic field "B" as shown in the figure below (The magnetic force represented by filled circle is perpendicular and out of the plane of drawing). Determine the tension in the loop if a current "I" is passed through it in anticlockwise direction.

An irregular shaped flexible wire loop in magnetic field

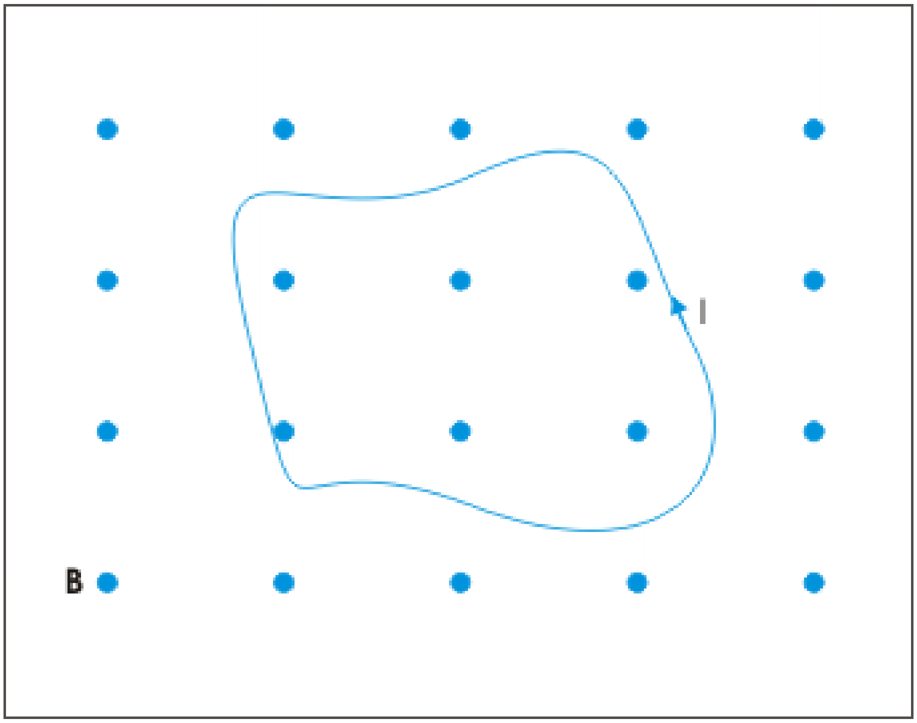


Figure 13.9: An irregular shaped flexible wire loop in magnetic field

Solution : The wire loop is flexible. There would be tension, provided the loop elements experience magnetic force in outward direction at all points on it. Applying Right hand thumb rule for any small segment of the loop, we find that the wire is indeed subjected to outward magnetic force. Clearly, the loop expands to become a circular loop. The radius of the circle is given by :

An irregular shaped flexible wire loop in magnetic field

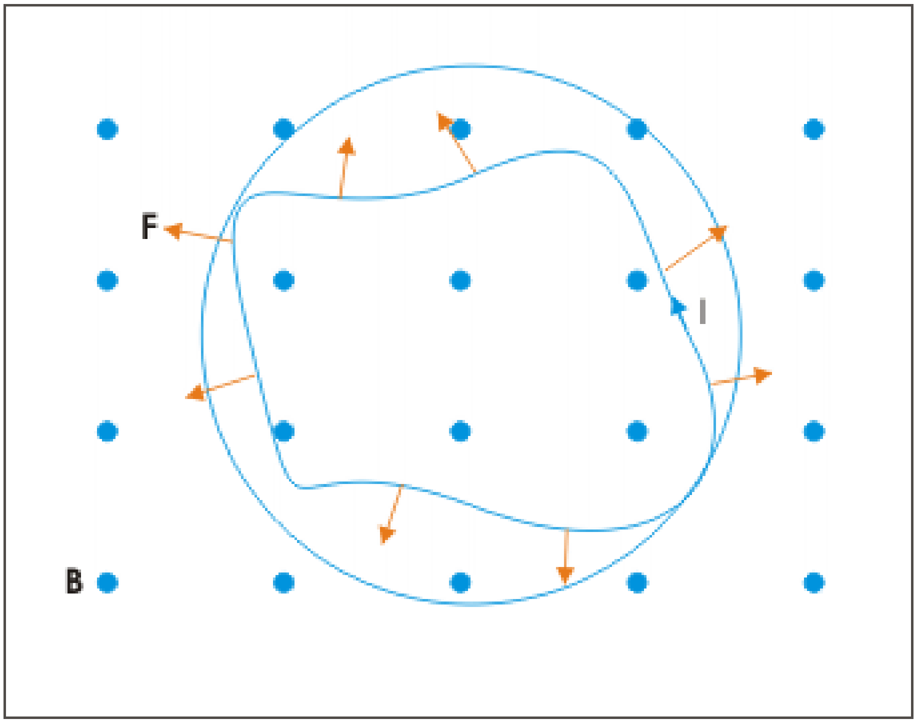


Figure 13.10: An irregular shaped flexible wire loop straightens up to acquire a circular shape due to magnetic force.

$$2\pi r = L$$

$$\Rightarrow r = \frac{L}{2\pi}$$

In order to determine tension in the wire, we consider a very small element of the circular loop. Let the loop element subtends an angle dθ at the center. Let "T" be the tension in the wire. It is clear that components of tension in the downward direction should be equal to magnetic force on the small wire element.

The tension in the circular loop carrying current

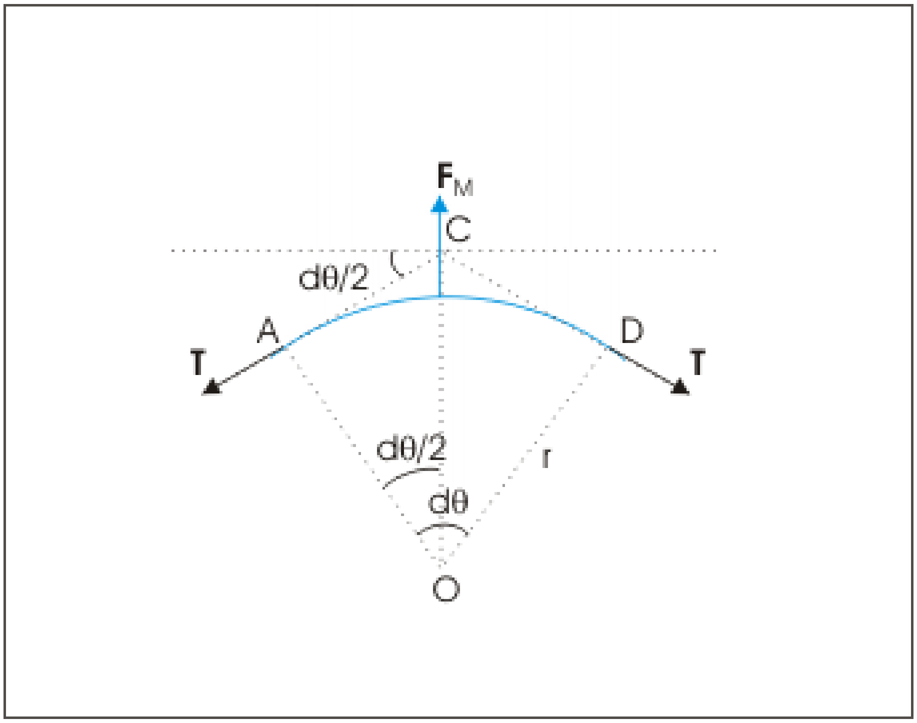


Figure 13.11: The tension in the circular loop carrying current

$$2T\sin\frac{\theta}{2} = F_M$$

Since loop element is very small, we approximate as :

$$\sin\frac{\theta}{2} \approx \frac{\theta}{2}$$

Further, we can consider the small loop element to be a straight wire for the calculation of magnetic force. Now, the magnetic force on the loop element is :

$$F_M = IBL = IBr\theta$$

Substituting in the equilibrium equation,

$$\Rightarrow 2T\frac{\theta}{2} = IBrd\theta$$

$$\Rightarrow T = IBr$$

Again substituting for the radius of circle, we have :

$$\Rightarrow T = \frac{ILB}{2\pi}$$

13.3 Magnetic force between parallel wires carrying current

The situation here is just an extension of the study of the magnetic force on a current carrying wire. The basic consideration here is that a wire carrying current can function in either of following two roles : (i) it produces magnetic field and (ii) it experiences magnetic force.

In the case of two parallel wires, one of the wires works as the producer of magnetic field whereas the other wire is considered to experience the magnetic force due to magnetic field produced by the first wire. This role is completely exchangeable. It only depends on what we want to observe. If we want to observe the magnetic force on the first wire, then the second wire works as the producer of magnetic field and vice-versa.

Let us consider here two long straight wires carrying currents I_1 and I_2 in the same direction. It is important to note here that one of two wires is a long straight wire. It ensures that magnetic field due to one of them is same at equal perpendicular distance. Otherwise, it would be difficult to determine magnetic force as they will be different at different points of the other wire. According to Ampere's law (Chapter 11), the magnetic field due to first long wire at a perpendicular distance "r" is :

$$B = \frac{\mu_0 I_1}{2\pi r}$$

Magnetic force between two parallel wires carrying current

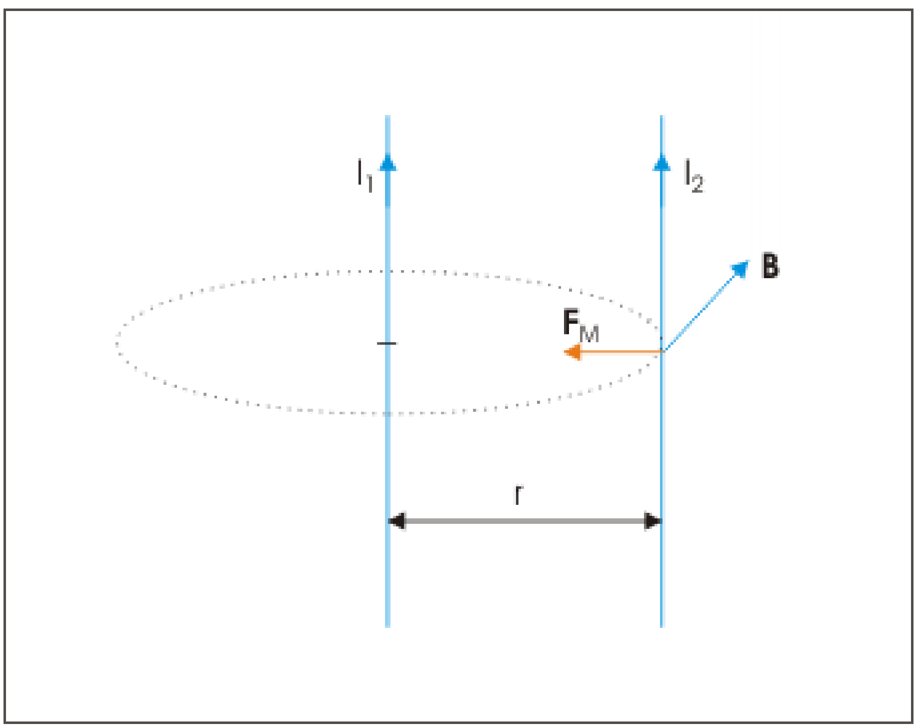


Figure 13.12: Magnetic force between two parallel wires carrying current

Applying Right hand thumb rule, we see that magnetic field is perpendicular and into the plane of drawing. Thus, angle between length and magnetic field vector is right angle. The magnetic force on the second wire is:

$$F = I_2 LB\sin\theta = I_2 LB\sin 90^0 = I_2 LB$$

The direction of magnetic force is obtained again by applying Right hand thumb rule. We curl fingers of right hand such that it follows the curve as we move from the length vector to the magnetic field vector. The extended thumb, then, points in the direction of magnetic force. In this case, magnetic force acts towards right as shown in the figure. The magnetic force on unit length of second wire is obtained by putting L=1 m,

$$F = I_2 B$$

Substituting for B, we have :

$$\Rightarrow F = \frac{\mu_0 I_1 I_2}{2\pi r}$$

The above expression gives the magnetic force on second wire due to first wire. We should here understand that second wire also applies equal and opposite force on the first wire in accordance with Newton's third law. Thus, two parallel wires carrying current in the same direction attract each other. If the currents are in the opposite directions, then two wires repel each other.

If one of two wires is a finite wire of length "L", then magnetic force on either of the parallel wires is given by multiplying the force per unit length with the length of finite wire,

$$\Rightarrow F = \frac{\mu_0 I_1 I_2 L}{2\pi r}$$

13.3.1 Definition of an Ampere

The SI unit of current i.e. Ampere is defined in terms of magnetic force between two parallel wires carrying current. Significantly, this unit is not defined in terms of charge per unit time as measuring the same is difficult.

Putting, $I_1 = I_2 = 1A, r = 1m$, the magnetic force per unit length is :

$$\Rightarrow F = \frac{\mu_0 I_1 I_2}{2\pi r} = \frac{4\pi X 10^{-7} X 1 X 1}{2\pi X 1} = 2X10^{-7} \quad N$$

Thus, one Ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 m apart in vacuum, would produce between these conductors a force equal to $2X10^{-7}$ Newton per meter of length.

Example 13.2

Problem : Two horizontal copper wires are parallel to each other in a vertical plane with a separation of 0.5 cm. The wires carry equal magnitude of current such that the lower wire becomes weightless. The mass per unit length of wires is 0.05 kg/m. Determine the currents in the wire and their relative directions.

Solution : The lower wire has its weight due to its mass and gravity. If it becomes weightless on passage of currents in the wire, then it means that the lower wire is attracted by the upper wire. Clearly, currents in two parallel wires are flowing in the same direction. Now, magnetic force per unit length on the wire should be equal to weight of the wire per unit length.

$$F = mg$$

Where "F" is magnetic force per unit length and "m" is mass per unit length. Putting expression of magnetic field in the equation, we have :

$$\Rightarrow \frac{\mu_0 I_1 I_2}{2\pi r} = mg$$

Since $I_1 = I_2 = I$, we have:

$$\Rightarrow \frac{\mu_0 I^2}{2\pi r} = mg$$

$$I = \sqrt{\left(\frac{2\pi rmg}{\mu_0}\right)}$$

Putting values,

$$\Rightarrow I = \sqrt{\left(\frac{2\pi X 0.5 X 10^{-2} X 0.05 X 10}{4\pi X 10^{-7}}\right)}$$

$$\Rightarrow I = \sqrt{(1.25 X 10^4)}$$

$$\Rightarrow I = 110 \quad A$$

13.4 Magnetic force between two charges moving parallel to each other

Let two charge carrying particles are at a linear distance "r" at a given instant. The initial state of motions of two charges is shown in the figure.

Magnetic force between two charges moving parallel to each other

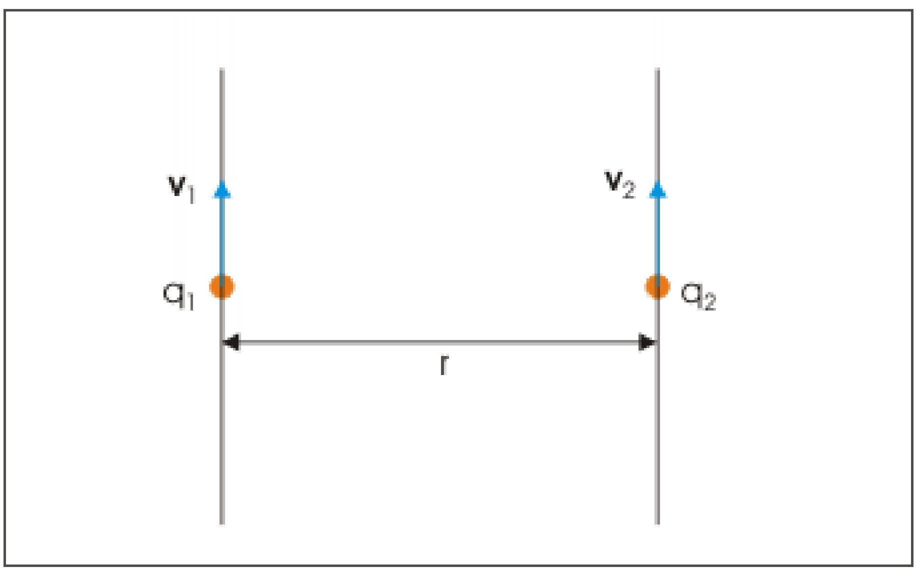


Figure 13.13: Magnetic force between two charges moving parallel to each other

The magnetic field at the position of second charge due to first charge is given by Biot-Savart law (Chapter 3) as expressed for moving charge is:

Magnetic force between two charges moving parallel to each other

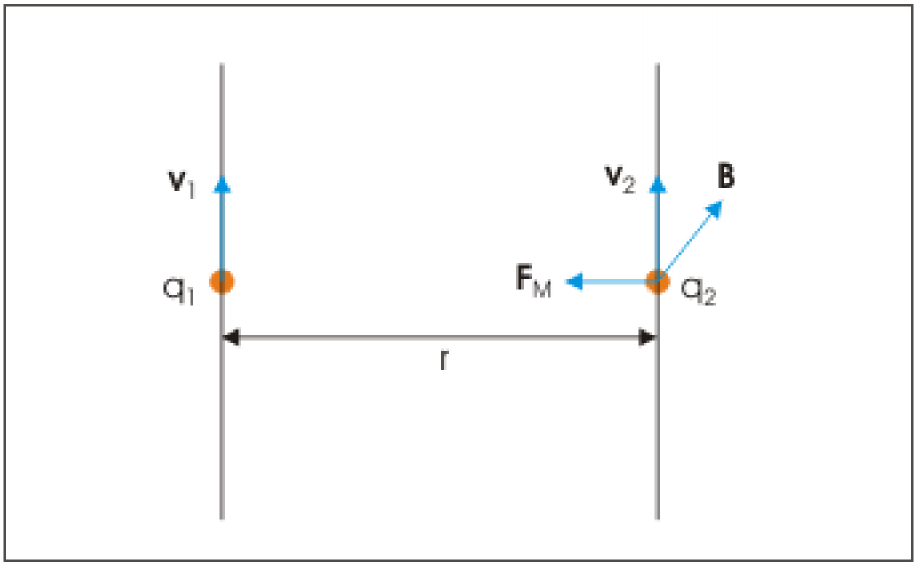


Figure 13.14: Magnetic force between two charges moving parallel to each other

$$B = \frac{\mu_0 q_1 v_1}{4\pi r^2}$$

The direction of magnetic field is "**vXr**", which is into the plane of drawing. Now, magnetic force on the charge is given by Lorentz force law (Chapter 7) as :

$$F_M = q_2 v_2 B$$

Substituting for magnetic field, we have :

$$\Rightarrow F_M = \frac{\mu_0 q_1 q_2 v_1 v_2}{4\pi r^2}$$

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V

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