# IIT-Bombay 

By:<br>sagar markal

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## C O N N EXIONS

Rice University, Houston, Texas

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## Chapter 1

## Department

### 1.1 Department ${ }^{1}$

## Select Department

- Aerospace Engineering (Section 1.2.1)
- Biosciences and Bioengineering
- Chemical Engineering
- Chemistry
- Civil Engineering
- Computer Science and Engineering
- Earth Sciences
- Electrical Engineering (Section 1.3.1)
- Energy Science and Engineering
- Humanities and Social Science
- Industrial Design Center
- Mathematics
- Mechanical Engineering
- Metallurgical Engineering and Materials Science
- Physics


### 1.2 Aerospace Engineering

### 1.2.1 Course List ${ }^{2}$

## List of Courses

| Course Code | Course Name | Faculty |
| :--- | :--- | :--- |
| AE651 | Aerodynamics of Compressors and Turbines | Prof Bhaskar Roy |

Table 1.1

[^0]
### 1.3 Electrical Engineering

### 1.3.1 Course List for Electrical Engineering ${ }^{3}$

## Cours list for Electrical Engineering

| Course Code | Course Name | Faculty |
| :--- | :--- | :--- |
| EE111 (Section 1.3.2.1) | Introduction to Electrical Systems | Prof.B.G.Fernandes |
| EE603 | Digital Signal Processing and its Applications | Prof. Vikram M. Gadre |

Table 1.2

### 1.3.2 EE111

### 1.3.2.1 EE111-Syllabus ${ }^{4}$

### 1.3.2.1.1 Syllabus

- Lecture $1^{5}$
- Lecture 2
- Lecture
- Lecture 4
- Lecture 5
- Lecture 6
- Lecture 7
- Lecture 8
- Lecture 9
- Lecture 10
- Lecture 11
- Lecture 12
- Lecture 13
- Lecture 14
- Lecture 15
- Lecture 16 (Section 1.3.2.2)


### 1.3.2.2 EE111-Lecture16 ${ }^{6}$

I intend to complete this chapter Mutually Coupled Circuits.. (Refer slide time: 00:17-00:18)

[^1]

Figure 1.1

Slide 1 Mutually coupled circuits (Refer slide time: 00:28-01:35)


Figure 1.2

Slide 2 Mutual Inductance
In the last class we defined the term mutual inductance. It is defined as or it relates the voltage induced in one coil due to the time varying current flow in the other circuit. Mutual inductance is given by, $\mathrm{M}=\mathrm{N}_{2}$ $\times \mathrm{d} \phi_{12} / \mathrm{di}_{1,}, \phi_{12}$ is produced by $\mathrm{i}_{1}$ that is flowing in coil 1 . This also can be defined as $\mathrm{N}_{1} \times \mathrm{d} \phi_{21} / \mathrm{di}_{2}, \phi_{21}$ is the part of the flux linking coil 1 and the current that flowing in coil 2 is $\mathrm{i}_{2}$. Now what is the relationship between the mutual inductance and the self inductance of two coils? This is given by $\mathrm{k} \sqrt{L_{1} L_{2}}$, where k is the coefficient of coupling. If coils are tightly coupled, $\mathrm{k} \leq 1$. (Refer slide time: 01:35- 02:58)


Figure 1.3

## Slide 3 Sign Convention

Sign Convention. There are two coils, flux produced by these two coils can either add or oppose. What happens if they add? What is the net flux link in the coil 1 , if the fluxes are additive, $\phi_{1}$ is the flux produced by coil $1, \phi_{2}$ is the flux produced by coil 2 . They are additive so net flux link by coil 1 is $\phi_{1}+\phi_{2}$. What is the voltage equation if I neglect the winding terminal voltage? It should be equal to the induced voltage and that induced voltage is equal to the rate of change of flux $\mathrm{v}_{1}=\mathrm{L}_{1} \times \mathrm{di}_{1} / \mathrm{dt}+\mathrm{M} \times \mathrm{di}_{2} / \mathrm{dt}$. So in frequency domain the RMS value of $\mathrm{v}_{1}=\mathrm{j} \omega\left(\mathrm{L}_{1} \mathrm{I}_{1}+\mathrm{MI}_{2}\right)$. Similarly $\mathrm{v}_{2}=\mathrm{j} \omega\left(\mathrm{L}_{2} \mathrm{I}_{2}+\mathrm{MI}_{1}\right)$. If they are opposing then what happens, it is $\mathrm{L}_{1} \mathrm{I}_{1}-\mathrm{MI}_{2}$ ).
(Refer slide time: 02:59-03:14)


Figure 1.4

Slide 4 Observations
So what are the observations, the sign of mutually induced voltage $\mathrm{v}_{12}$ or $\mathrm{v}_{21}$ depends on weather fluxes are additive or subtractive. But then having chosen the direction of current how do I determine the flux if I do not know the way that it is wound, how do I determine the direction of flux? I have to assume the direction of current, then I need to know the way the coil is wound, who knows about the way that it is wound. The winder in the shop floor. Having designed the coil, having fabricated you get a black box can I know how the coil is wound? It is just not possible, if that is the case how do I write the voltage equations? (Refer slide time: 03:50- 04:30)

The sign of mutual induced voltage $\mathrm{v}_{1}$ and $\mathrm{v}_{2}$ depends on weather fluxes are additive or subtractive. (Refer slide time: 03:57-04:23)


Figure 1.5

## Slide 5 Sign convention

How do I determine that the fluxes are additive or subtractive? I can assume the direction of current, either you can enter the terminal or leave. Having assumed the direction of current I need to know the way that the coil is wound, that information was only known to the person who winds the coil, if that information is not known how do I write the voltage equations? Using dot conventions. (Refer slide time: 04:24-04:45)


Figure 1.6

Slide 6 Observations
For each coil the dot is placed at the terminals which are instantaneously of the same polarity. The dot convention, the two dots are placed at two different or two different coils. What did they imply? At any given time these two terminals are of the same polarity. The cursor question now is how to place this dot? ( Refer slide time: 04:54-05:04)


Figure 1.7

## Slide 7 Observations

Two mutually coupled coil, assume that the physical arrangement and mode of each winding is known. The first time it is known, so take any terminal, say A and I will place the dot there and will assign the current into the dot, use the right hand rule to determine the direction of the flux. Pick one terminal of the second coil, (Refer slide time: 05:20-05:43)
any one of the terminal and assign the current into the terminal. I have taken C and assigned the current into the terminal determine the direction of the flux. ( Refer slide time: 05:44- 06:33)


Figure 1.8

Slide 8 Sign of $V$ of $M$ in mesh current eqations
If the flux are ready to place the dot on the terminal of the second coil where current $i_{c}$ enters else place the dot in the other terminal. (Refer slide time: 06:34-06:40)


Figure 1.9

## Slide 9 Observations

Take any one coil, any terminal place, a dot there assign the current determine the direction of current. Take the second coil, any one terminal don't place the dot now you assign the current into the terminal, determine the direction of flux. If the fluxes are additive, place the dot at the terminal where the current enters else place the dot in the other terminal. So at any given time these two terminals are of the same polarity. $i_{A}$ plus then $i_{C}$ also plus, why only if the flux are additive this should be positive that is the question being asked. I will answer this question now. ( Refer slide time: 06:51- 07:59)


Figure 1.10

## Slide 10 Coil

$\mathrm{i}_{1}$ is AC so, the flux produced by $\mathrm{i}_{1}$ is alternating. It will link the coil to the voltage induced. If I connect a load, current will flow, that current will produce its own flux in the coil 2 . This is the positive terminal current entering the node, at any given time. I assume that this is positive, current is entering that node. If I assume this to be positive at a given time, what should be the direction of the current? I am connecting the load across the coil, the voltage is induced in the coil 2 . At some time the polarity of the terminal will be positive now what should be the direction of that current at that time, it should enter this terminal, what should be the direction of the flux produced by the second coil? (Refer slide time: 08:05- 08:18)


Figure 1.11

## Slide 11 Coil

If this is the direction of the flux forced by $\phi_{1}$ and this is also the flux produced by the $\phi_{2}$. What did I do here, I assigned I took one terminal and in the second coil I assigned the current entering the terminal and I saw if that fluxes are additive then dot remains there. If the flux are opposing, the dot goes to second terminal, both are matching isn't it. (Refer slide time: 08:41- 08:52)


Figure 1.12


Figure 1.13

Slide 12 Sign of V of M in mesh current eqations
Writing the voltage equation in the mutually coupled coil what should be the sign of $L$ and M? (Refer slide time: 08:52- 09:05)


Figure 1.14

Slide 13 Sign convention
We assume the direction of current and we knew the way the coil is wound. We determine the direction of flux if they are additive, we find a sign of given $v_{12}$ and $v_{21}$ are the same. (Refer slide time: 09:05- 09:17)


Figure 1.15

Slide 14 Mutual Inductance
Now the second problem is that how you know the way the coil is wound? How did you solve that problem, by placing a dot? ( Refer slide time: 09:17-09:56)


Figure 1.16

Slide 15 Sign of V of M in mesh current equations
Now we know that here is a mutually coupled circuit, dots have been placed. What should be the sign of $L$ and $M$ terms, if the fluxes are additive, sign of $L$ is same as sign of $M$. When the fluxes are additive, current entering the dot terms or the current leaving they are opposite. (Refer slide time: 09:57-10:09)


Figure 1.17

Slide 16 equations
If one coil current enters the dot and the other coil leaves the dot, if one current enters at a dotted terminal and one leaves by a dotted terminal sign of $M$ terms are opposite to the sign of $L$ terms. Fluxes are additive, then total flux linkage is $\mathrm{N}_{1} \times \phi_{1}+\phi_{21}$. ( Refer slide time: 10:20-10:42)


Figure 1.18

Slide 17 equations
Now let us see about the mutually coupled coil, let us see the voltage equation you assign the current the way you own either clock wise or anti clock wise. Now, what is the voltage equation? see what happens in coil 1, current enters the dot what about in coil 2 it leaves the dot terminal or what is the voltage induced in $L_{1}$ there are two mutually coupled coil, both of them are carrying the current. So, voltage induced in one coil, it has two components what are they? One is due to its own flux and the other one is due to the flux produced by the second coil. Now, the sign of $L$ and $M$ depends upon the direction of current the we found that in this case, in one case enters the dot in second loop it leaves the dot. (Refer slide time:11:12 to 12:12)


Figure 1.19

Slide 18 equations
So, what is the voltage induced in $\mathrm{L}_{1}=\mathrm{j} \omega\left[\mathrm{L}_{1} \overline{I_{1}} \mathrm{I}_{1}-\mathrm{M} \overline{I_{2}}\right]$ Voltage induced in second coil is $\mathrm{j} \omega\left[\mathrm{L}_{2} \overline{I_{2}}-\right.$ $\left.\underline{\mathrm{M}} \overline{I_{1}}\right]$ therefore $\mathrm{V}_{1}=\mathrm{R}_{1} \overline{I_{2}}+\mathrm{j} \omega\left[\mathrm{L}_{2} \overline{I_{2}}-\mathrm{M} \overline{I_{2}}\right]$. The second coil voltage equation $\overline{V_{2}}=\mathrm{R}_{2} \overline{I_{2}}+\mathrm{j} \omega\left[\mathrm{L}_{2} \overline{I_{2}}-\mathrm{M}\right.$ $\left.\overline{I_{1}}\right]$. First step determine the currents and find out whether it is entering the dot or leaving the dot that is the step number one assign the mesh current or mesh current in the loops find out whether the currents are current in respective business entering the dot or leaving dot. First step that has to be done then you write the voltage induce in individual coils and finally you write mesh equation. These are the three steps to be done. (Refer slide time:12:34 to 12.57)


Figure 1.20

Slide 19 equations
So $\mathrm{v}_{1}$ is equal to $\mathrm{v}_{1}$ is given by this equation $\mathrm{v}_{2}$ and write in matrix form, first one is $\left(\mathrm{R}_{1}+\mathrm{j} \omega \mathrm{L}_{1}\right) \overline{I_{1}}-$ $\mathrm{j} \omega \mathrm{M} \overline{I_{2}}$. Second term, second row, first element, $-\mathrm{j} \omega \mathrm{MI}_{2}+\left(\mathrm{R}_{2}+\mathrm{j} \omega \mathrm{L}_{2}\right) \overline{I_{2}}$. Magnetic coupling exist between these two coils are electrically they are isolated by seeing this equations can I write (Refer slide time:13:11 to $14: 37$ )


Figure 1.21

Slide 20 Conductively coupled equivalent circuit
voltage equation for can I draw the circuit which is conductively couple this is the voltage and current equation from these equations can I draw conductively couple circuit. See what is this term; second term first row first term. sum of all, impedance of the mesh 1 second row second element sum of all impedance connect to mesh 2, these two are common. Write the voltage equation here what do I get $R_{1}$ ej $\omega 1 \mathrm{~m}+1 \mathrm{~m}$ this is the mutual term common for 1 and 2 , this is sum of all what do $I$ call this asl $1_{1}$ is the self inductance, $m$ is the mutual inductance what could be the difference $l_{1}$ and $m$, what is $l_{1}-m$, what it could be? Self flux minus the mutual, what is left now? Leakage. $5_{1}=51_{1}+51_{2} .51_{2}$ is the mutual, $51_{1}$ means either part off coil one leakage. (Refer slide time:14:44 to 16:07)


Figure 1.22

Slide 21 Conductively coupled equivalent circuit
This is common for mutual. I will just solve another problem, say the mutually coupled circuit. coefficient of coupling is given as $.8, \mathrm{l}_{1}$ is $5, \mathrm{j}$ is $10 . \mathrm{j} \mathrm{X}_{\mathrm{M}}=\sqrt{ } \mathrm{j} 5.66$. Now, I will assign the current $\mathrm{i}_{1}$ in mesh 1 , $\mathrm{i}_{2}$ in mesh 2. $\mathrm{i}_{1}$ is the current flowing in j 5 entering the nod, $\mathrm{i}_{2}$ is the current flowing in j 10 leaving the dot. In first, it enters the dot in j 10 it leaves the dot. What should be the voltage induce in j 5 if there is no coupling, $\mathrm{j} 5 \mathrm{i}_{1}-\mathrm{j} 5.66 \mathrm{i}_{2}$, voltage induce in $\mathrm{j} 10, \mathrm{j} 10 \mathrm{i}_{2}-\mathrm{j} 5.66 \mathrm{i}_{1}$. (Refer slide time: $16: 08$ to $17: 05$ )


Figure 1.23

Slide 22 Mesh 1 and Mesh 2 quations
Voltage equation 50 is equal to voltage induce in j 5 plus voltage across this branch. What is the voltage drop in this branch $\left(3-\mathrm{j}_{4}\right)\left(\mathrm{i}_{1}-\mathrm{i}_{2}\right)+\mathrm{j} 5 \mathrm{i}_{1}-\mathrm{j} 5.66 \mathrm{i}_{2}$. Similarly in mesh $2,5 \mathrm{i}_{2}+\left(3-\mathrm{j}_{4}\right)\left(\mathrm{i}_{2}-\mathrm{i}_{1}\right)$ plus the drop across j 10 so there are two equation solve for $\mathrm{i}_{1}$ and $\mathrm{i}_{2}$. (Refer slide time:17:06 to 18:55)


Figure 1.24

Slide 23 Replacing series connected mutually coupled coils
Now I will connect to mutually coupled coils in series, what is the equivalent inductance that I can get? For this combination, current is entering the dot coil 1 and in coil 2 also it is entering the dot. What is the voltage induced in coil1 $j \omega \mathrm{~L}_{1} \mathrm{I}_{\mathrm{f}}+\mathrm{j} \omega \mathrm{MI}_{\mathrm{f}}$, this is the voltage across in $\mathrm{L}_{1}$ in coil 1. Voltage across in coil 2 $j \omega L_{2} I_{f}+j \omega \mathrm{MI}_{2}$. Finally the external voltage equation $\mathrm{v}=\mathrm{I}_{\mathrm{f}} \mathrm{R}+\mathrm{j} \omega \mathrm{I}_{\mathrm{f}}\left(\mathrm{L}_{1}+\mathrm{M}\right)+\mathrm{j} \omega \mathrm{I}_{\mathrm{f}}\left(\mathrm{L}_{2}+\mathrm{M}\right)$, plus voltage drop across coil1 and voltage drop across coil2. Voltage across coil1 is $j \omega I_{f} 1+m$. combine it we get the net inductance is $\mathrm{I}_{\mathrm{f}} \mathrm{R}+\mathrm{j} \omega \mathrm{I}_{\mathrm{f}}\left(\mathrm{L}_{1}+\mathrm{L}_{2}+2 \mathrm{M}\right)$. (Refer slide time:18:56 to 19:19)


Figure 1.25

Slide 24 Coils connected in parallel
If I enters the dot in coil 1 and leaves the dot in coil 2 , the voltage inducing coil 1 is $j \omega L_{1} I_{f}-j \omega M_{f}$, So $\mathrm{j} \omega \mathrm{I}_{\mathrm{f}}\left(\mathrm{L}_{1}-\mathrm{M}\right)$. (Refer slide time:19.20 to $19: 45$ )


Figure 1.26

Slide 25 Place the dots
For the coil $2, \mathrm{j} \omega \mathrm{L}_{2} \mathrm{I}_{\mathrm{f}}-\mathrm{j} \omega \mathrm{MI}_{\mathrm{f}}$, so $\mathrm{j} \omega \mathrm{I}_{\mathrm{f}}\left(\mathrm{L}_{2}-\mathrm{M}\right)$. (Refer slide time:19:46 to 20:13)


Figure 1.27

Slide 26 Equations
The net inductance is $\mathrm{I}_{\mathrm{f}} \mathrm{R}+\mathrm{j} \omega \mathrm{I}_{\mathrm{f}}\left(\mathrm{L}_{1}+\mathrm{L}_{2}-2 \mathrm{M}\right)$.If I have two coils which are mutually coupled and if I connect them in series the two coupled values are ( $\mathrm{L}_{1}+\mathrm{L}_{2} \pm 2 \mathrm{M}$ ), series adding, it is $\left(\mathrm{L}_{1}+\mathrm{L}_{2}+2 \mathrm{M}\right)$, if they oppose ( $\left.\mathrm{L}_{1}+\mathrm{L}_{2}-2 \mathrm{M}\right)$.

End of the lecture16

## Index of Keywords and Terms

Keywords are listed by the section with that keyword (page numbers are in parentheses). Keywords do not necessarily appear in the text of the page. They are merely associated with that section. Ex. apples, § 1.1 (1) Terms are referenced by the page they appear on. Ex. apples, 1

E EE111, § 1.3.2.1(2)
L Lecture 16, § 1.3.2.2(2)
S Syllabys, § 1.3.2.1(2)

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Text-Book


#### Abstract

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