"Rational"ity

By:
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Online:
< http://cnx.org/content/col10350/1.2/ >

C O N N E X I O N S

Rice University, Houston, Texas

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Chapter 1

Prolegomena

1.1 Using Interval Notation[1]

Interval notation is another method for writing domain and range.

In set builder notation braces (curly parentheses {}) and variables are used to express the domain and range. Interval notation is often considered more efficient.

In interval notation, there are only 5 symbols to know:

- Open parentheses ( )
- Closed parentheses [ ]
- Infinity ∞
- Negative Infinity $-\infty$
- Union Sign $\cup$

To use interval notation:

Use the open parentheses ( ) if the value is not included in the graph. (i.e. the graph is undefined at that point... there's a hole or asymptote, or a jump)

If the graph goes on forever to the left, the domain will start with ($-\infty$. If the graph travels downward forever, the range will start with ($-\infty$. Similarly, if the graph goes on forever at the right or up, end with ∞)

Use the brackets [ ] if the value is part of the graph.

Whenever there is a break in the graph, write the interval up to the point. Then write another interval for the section of the graph after that part. Put a union sign between each interval to "join" them together.

Now for some practice so you can see if any of this makes sense.

Write the following using interval notation:

Exercise 1.1 ***(Solution on p. 18.)***

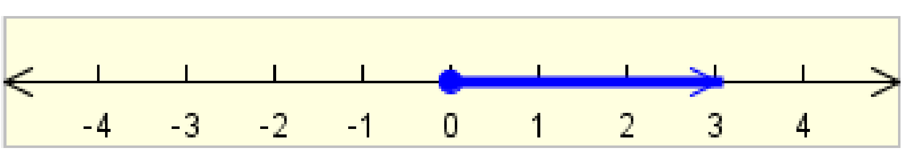

Figure 1.1

[1] This content is available online at <http://cnx.org/content/m13596/1.2/>.

Exercise 1.2 *(Solution on p. 18.)*

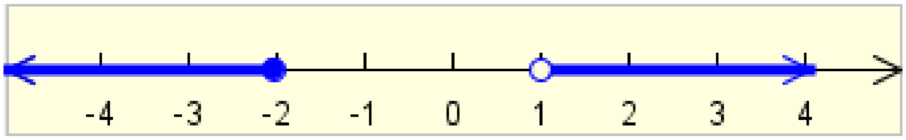

Figure 1.2

Exercise 1.3 *(Solution on p. 18.)*

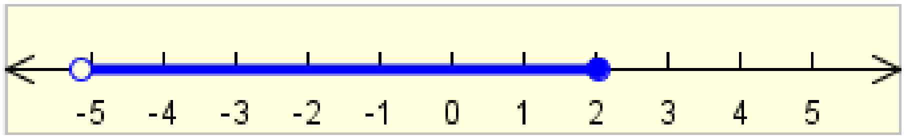

Figure 1.3

Exercise 1.4 *(Solution on p. 18.)*

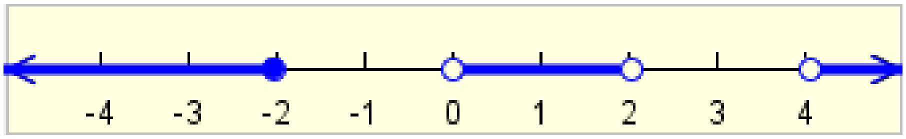

Figure 1.4

Exercise 1.5 *(Solution on p. 18.)*

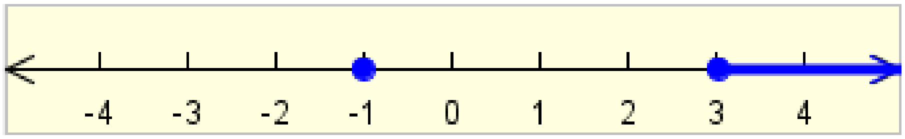

Figure 1.5

Exercise 1.6 *(Solution on p. 18.)*

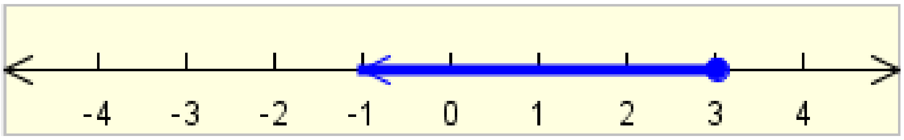

Figure 1.6

Write the domain and range of the following in interval notation:

Exercise 1.7 *(Solution on p. 18.)*

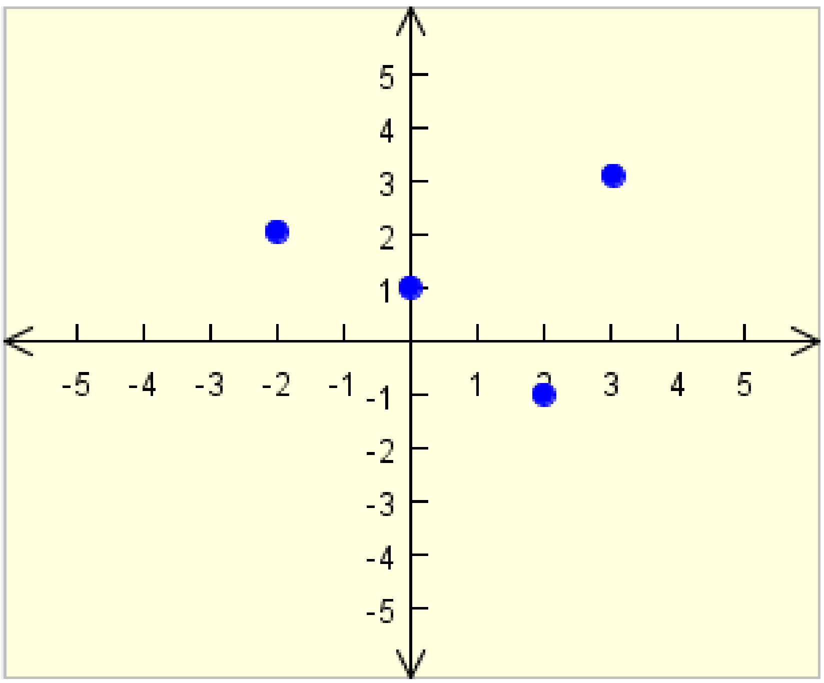

Figure 1.7

Exercise 1.8 *(Solution on p. 18.)*

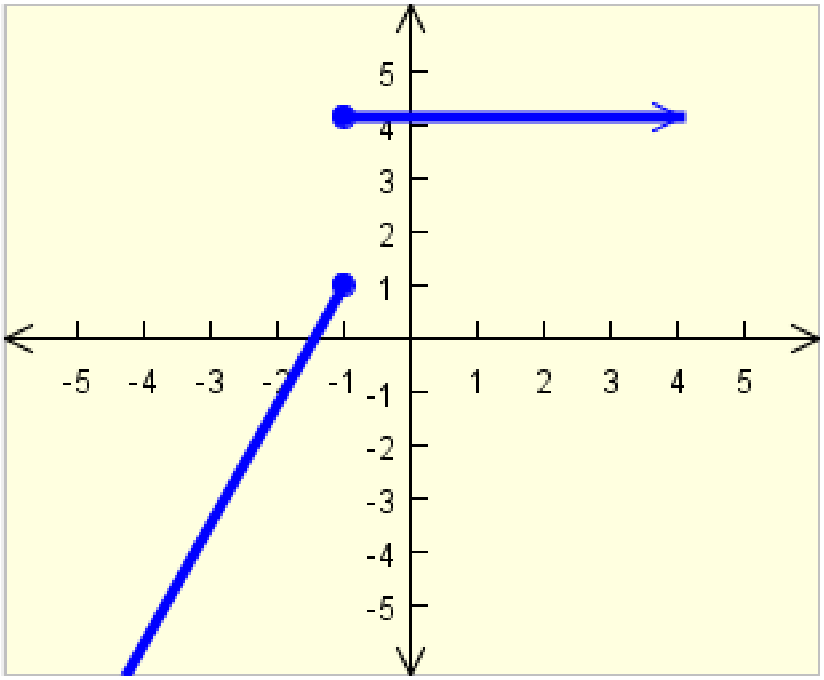

Figure 1.8

Exercise 1.9 *(Solution on p. 18.)*

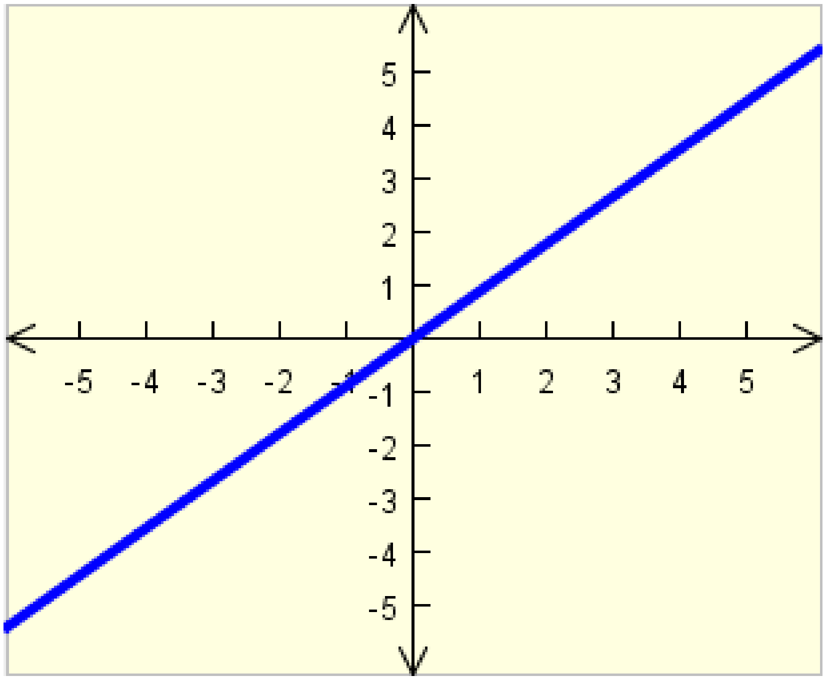

Figure 1.9

Exercise 1.10 *(Solution on p. 18.)*

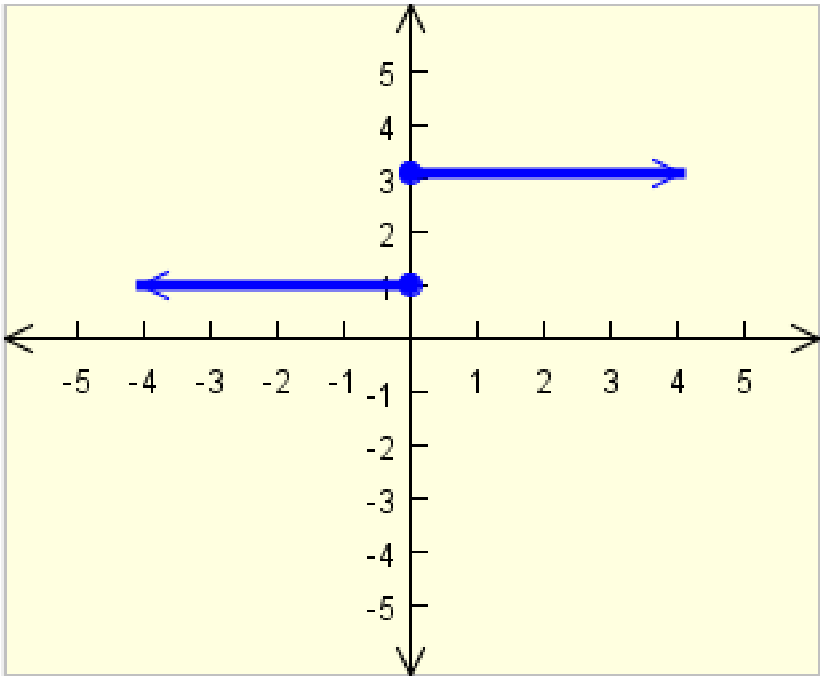

Figure 1.10

Exercise 1.11 *(Solution on p. 18.)*

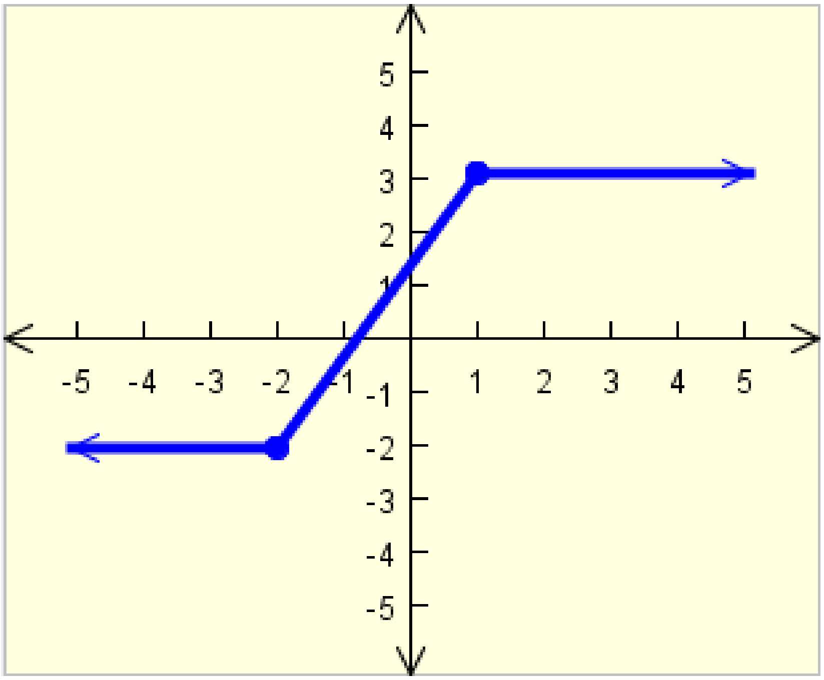

Figure 1.11

Exercise 1.12 *(Solution on p. 18.)*

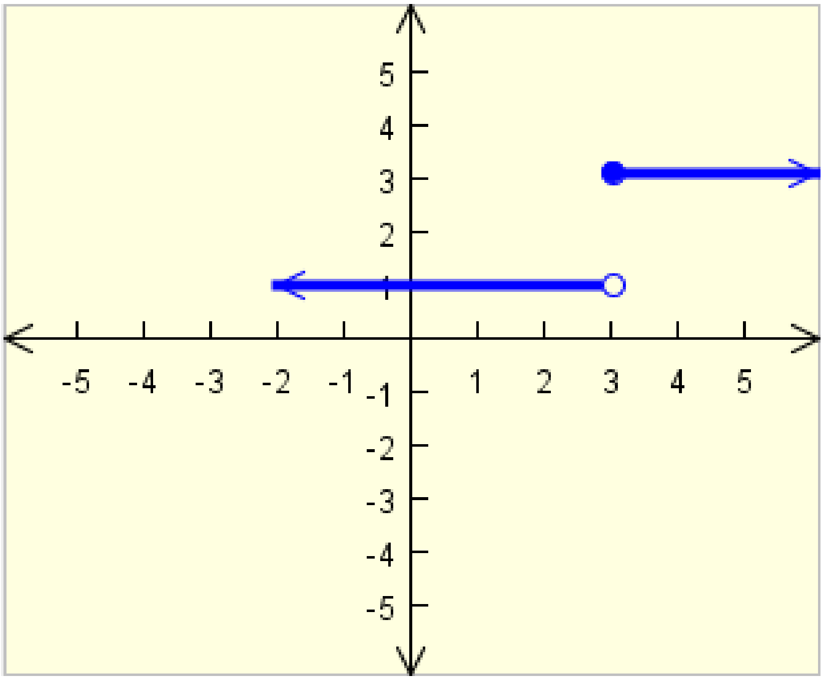

Figure 1.12

Exercise 1.13 ***(Solution on p. 18.)***

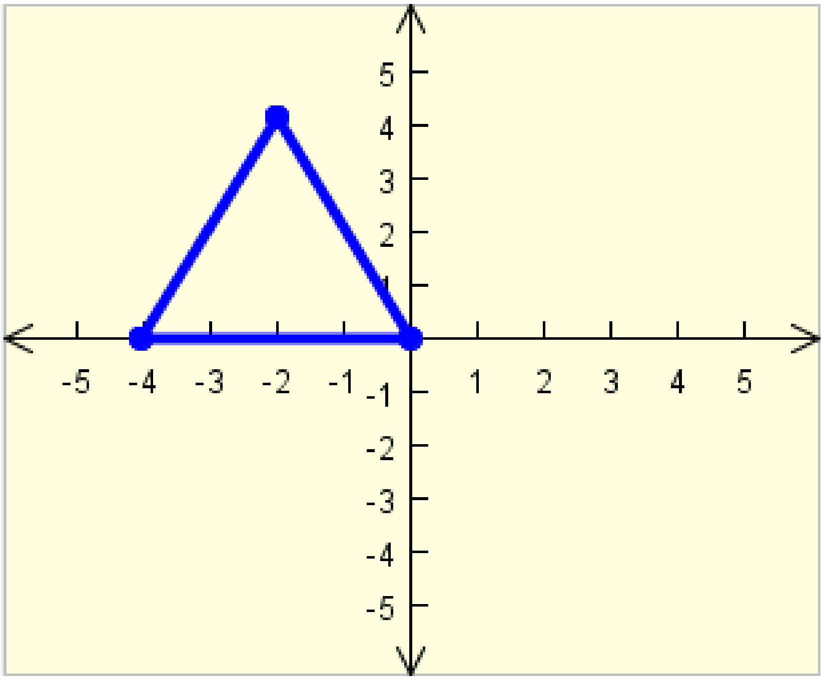

Figure 1.13

Exercise 1.14 *(Solution on p. 18.)*

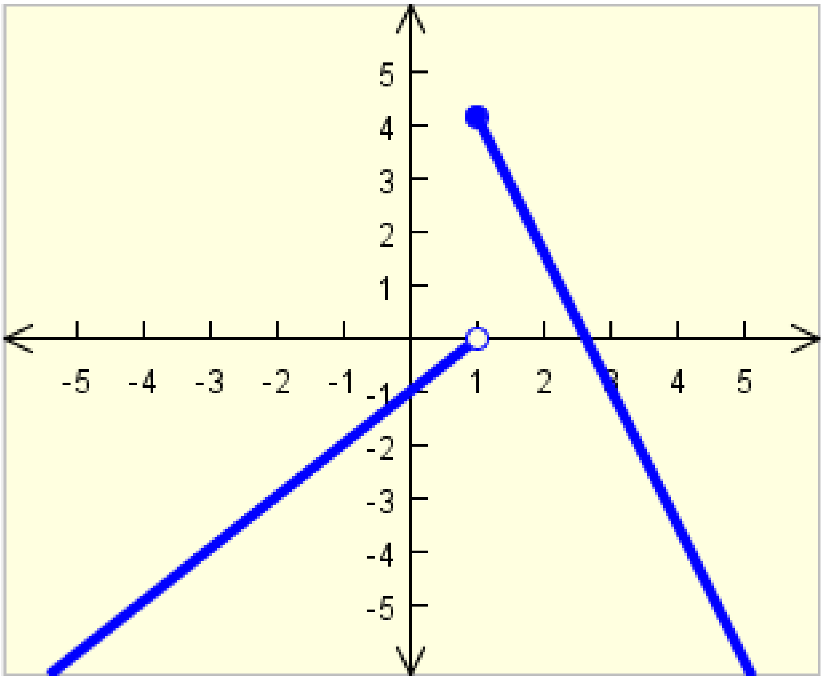

Figure 1.14

Exercise 1.15 *(Solution on p. 18.)*

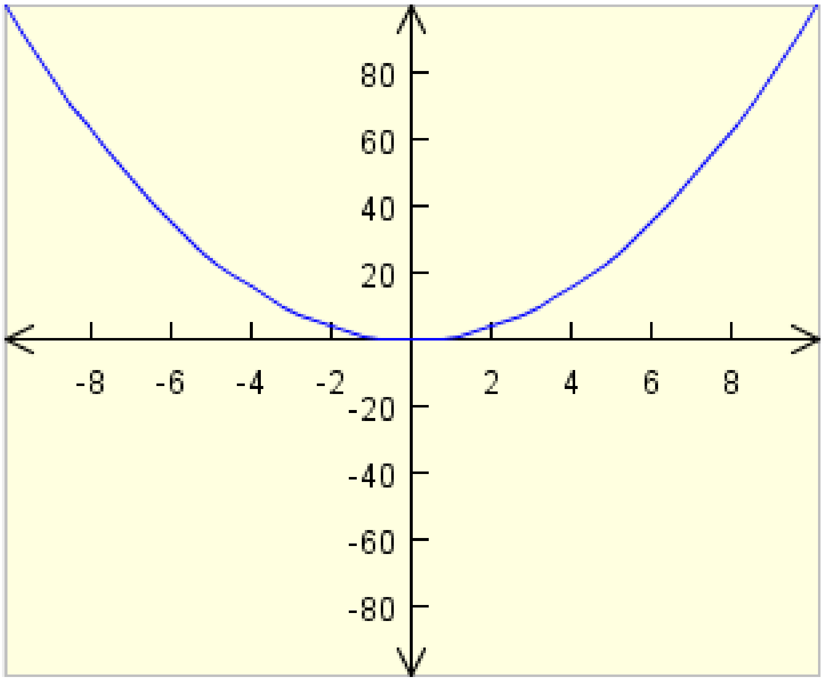

Figure 1.15

Exercise 1.16 *(Solution on p. 18.)*

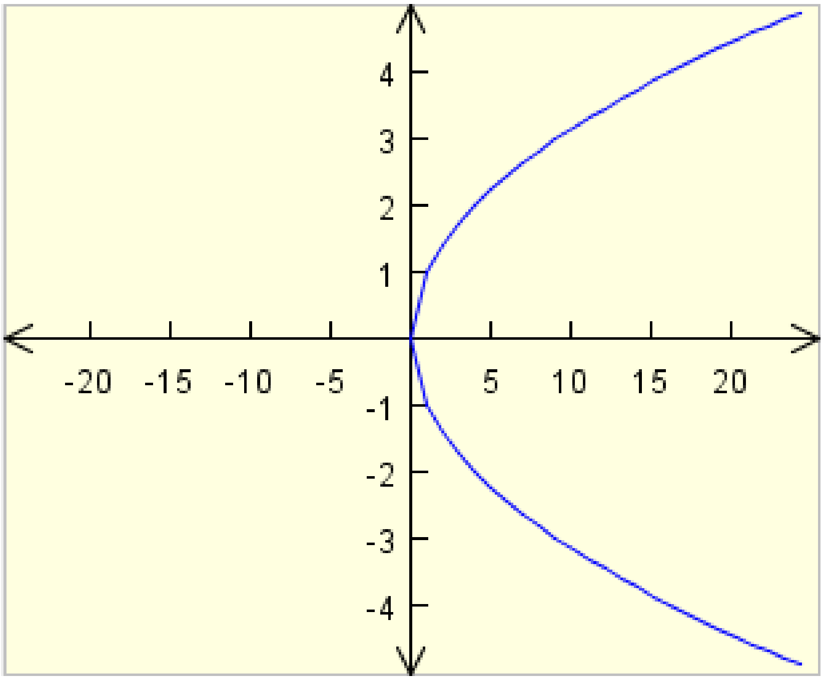

Figure 1.16

Exercise 1.17 *(Solution on p. 18.)*

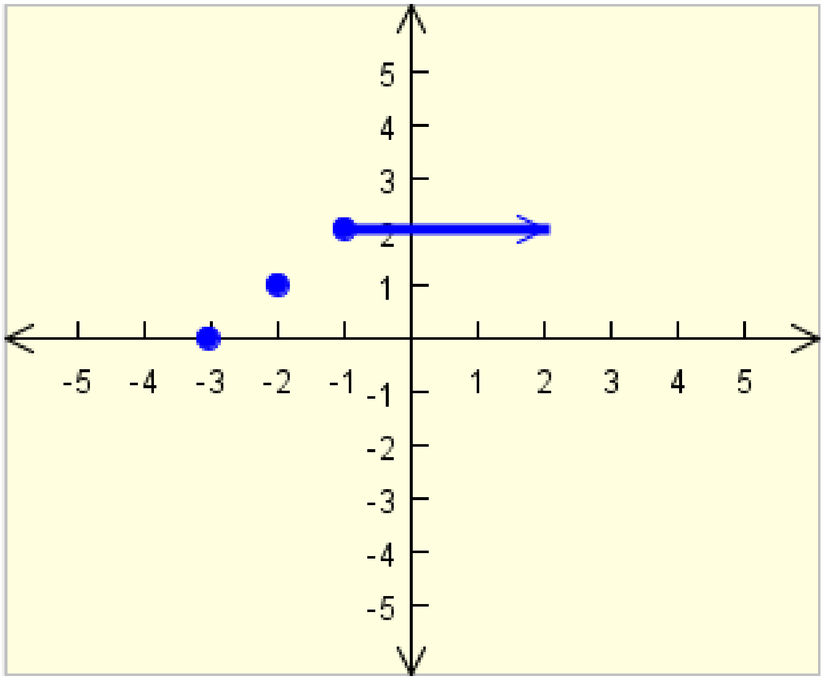

Figure 1.17

Exercise 1.18 *(Solution on p. 18.)*

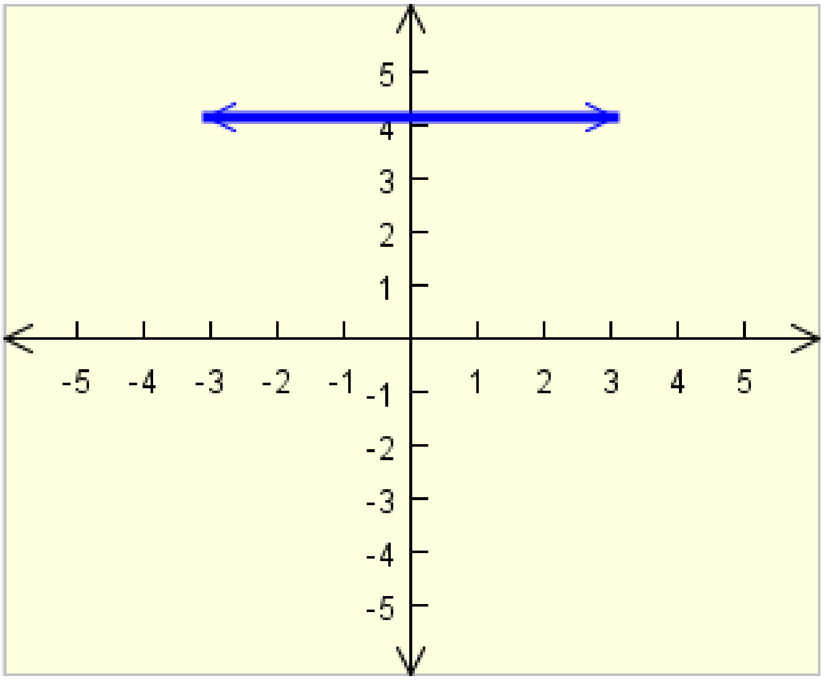

Figure 1.18

Exercise 1.19 *(Solution on p. 18.)*

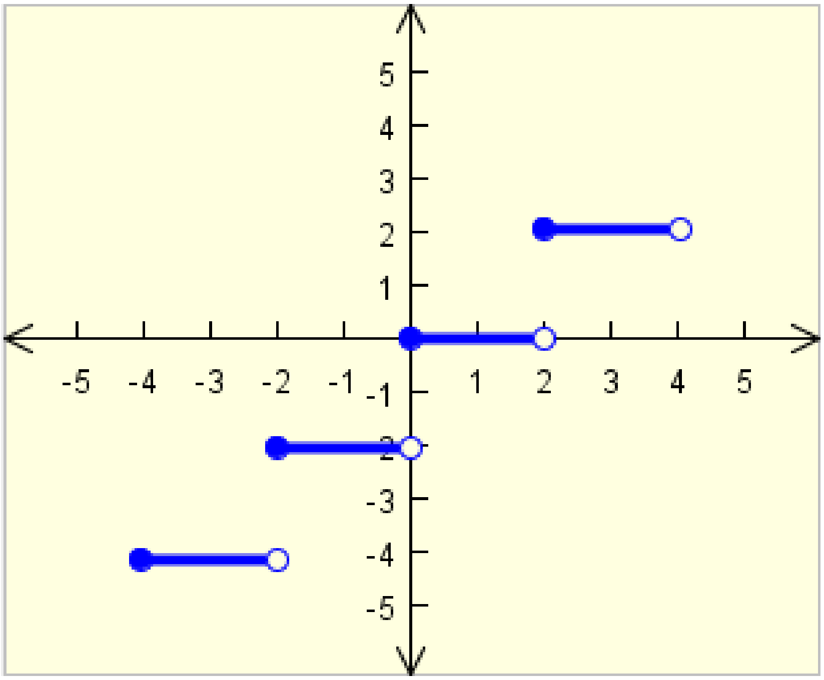

Figure 1.19

Exercise 1.20 *(Solution on p. 19.)*

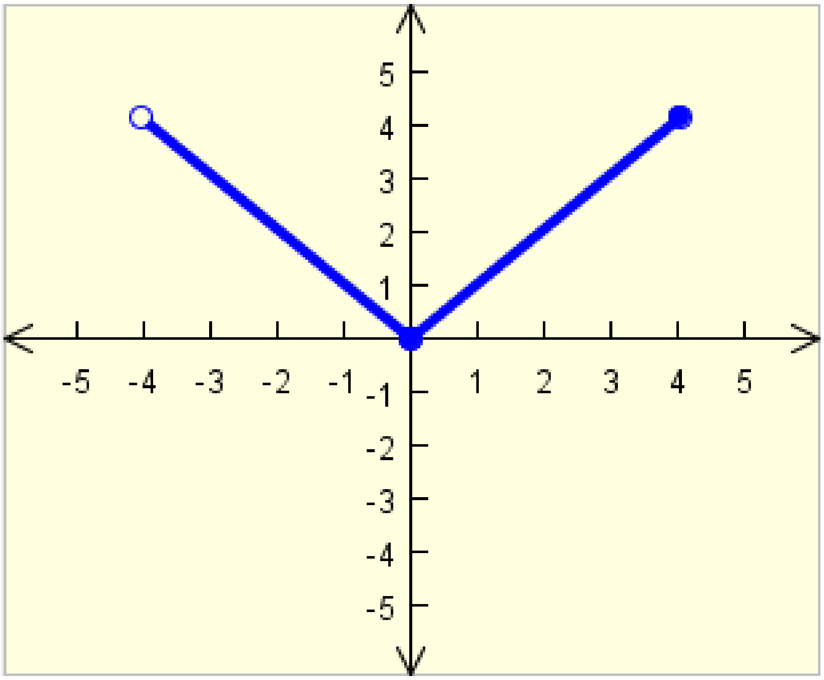

Figure 1.20

1.2 x and y-intercepts[2]

A **rational function** is a function of the form $R(x) = \frac{p(x)}{q(x)}$, where p and q are polynomial functions and $q \neq 0$.

The domain is all real numbers except for numbers that make the denominator = 0.

x-intercepts are the points at which the graph crosses the x-axis. They are also known as roots, zeros, or solutions.

To find x-intercepts, let y (or f(x)) = 0 and solve for x. In rational functions, this means that you are multiplying by 0 so to find the x-intercept, just set the numerator (the top of the fraction) equal to 0 and solve for x.

Remember: x-intercepts are points that look like (x,0)

Example 1.1
For $y = \frac{x-1}{x-2}$ find the x-intercept
The x-intercept is (1,0) since $x - 1 = 0$, $x = 1$

The **y-intercept** is the point where the graph crosses the y-axis. If the graph is a function, there is only one y-intercept (and it only has ONE name)

To find the y-intercept (this is easier than the x-intercept), let x = 0. Plug in 0 for x in the equation and simplify.

Remember: y-intercepts are points that look like (0,y)

[2]This content is available online at <http://cnx.org/content/m13602/1.2/>.

Example 1.2
For $y = \frac{x+1}{x-2}$ find the y-intercept

The y-intercept is $(0, \frac{-1}{2})$ since $\frac{0+1}{0-2} = \frac{-1}{2}$

Find the x- and y-intercepts of the following:

Exercise 1.21 *(Solution on p. 19.)*
$y = \frac{1}{x+2}$

Exercise 1.22 *(Solution on p. 19.)*
$y = \frac{1-3x}{1-x}$

Exercise 1.23 *(Solution on p. 19.)*
$y = \frac{x^2}{x^2+9}$

Exercise 1.24 *(Solution on p. 19.)*
$y = \frac{\sqrt{x+1}}{(x-2)^2}$

Exercise 1.25 *(Solution on p. 19.)*
$y = \frac{3x}{x^2-x-2}$

Exercise 1.26 *(Solution on p. 19.)*
$y = \frac{1}{x-3} + 1$

Exercise 1.27 *(Solution on p. 19.)*
$y = \frac{x^2-4}{\sqrt{x+1}}$

Exercise 1.28 *(Solution on p. 19.)*
$y = 4 + \frac{5}{x^2+2}$

Exercise 1.29 *(Solution on p. 19.)*
$y = \frac{\sqrt{5x-2}}{x-3}$

Exercise 1.30 *(Solution on p. 19.)*
$y = \frac{x^3-8}{x^2+1}$

Solutions to Exercises in Chapter 1

Solution to Exercise 1.1 (p. 1)
$[0, \infty)$
Solution to Exercise 1.2 (p. 2)
$(-\infty, -2] \cup (1, \infty)$
Solution to Exercise 1.3 (p. 2)
$(-5, 2]$
Solution to Exercise 1.4 (p. 2)
$(-\infty, -2] \cup (0, 2) \cup (4, \infty)$
Solution to Exercise 1.5 (p. 2)
$[-1] \cup [3, \infty)$
Solution to Exercise 1.6 (p. 2)
$(-\infty, 3]$
Solution to Exercise 1.7 (p. 3)
Domain: $[-2] \cup [0] \cup [2] \cup [3]$
Range: $[-1] \cup [1] \cup [2] \cup [3]$
Solution to Exercise 1.8 (p. 3)
Domain: $(-\infty, \infty)$
Range: $(\infty, 1] \cup [4]$
Solution to Exercise 1.9 (p. 4)
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Solution to Exercise 1.10 (p. 5)
Domain: $(-\infty, \infty)$
Range: $[1] \cup [3]$
Solution to Exercise 1.11 (p. 6)
Domain: $(-\infty, \infty)$
Range: $[-2, 3]$
Solution to Exercise 1.12 (p. 7)
Domain: $(-\infty, \infty)$
Range: $[1] \cup [3]$
Solution to Exercise 1.13 (p. 8)
Domain: $[-4, 0]$
Range: $[0, 4]$
Solution to Exercise 1.14 (p. 9)
Domain: $(-\infty, \infty)$
Range: $(-\infty, 4]$
Solution to Exercise 1.15 (p. 10)
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Solution to Exercise 1.16 (p. 11)
Domain: $[0, \infty)$
Range: $(-\infty, \infty)$
Solution to Exercise 1.17 (p. 12)
Domain: $[-3] \cup [-2] \cup [-1, \infty)$
Range: $[0] \cup [1] \cup [2]$
Solution to Exercise 1.18 (p. 13)
Domain: $(-\infty, \infty)$
Range: $[4]$
Solution to Exercise 1.19 (p. 14)
Domain: $[-4, 4)$

Range: $[-4] \cup [-2] \cup [0] \cup [2]$

Solution to Exercise 1.20 (p. 15)

Domain: $(-4, 4]$

Range: $[0, 4]$

Solution to Exercise 1.21 (p. 17)

x-intercept: None since $1 \neq 0$

y-intercept: (0, $\frac{1}{2}$) since $\frac{1}{0+2} = \frac{1}{2}$

Solution to Exercise 1.22 (p. 17)

x-intercept: ($\frac{1}{3}$,0) since $1 - 3x = 0$, $-3x = -1$, $x = \frac{1}{3}$

y-intercept: (0,1) since $\frac{1-3\times 0}{1-0} = 1$

Solution to Exercise 1.23 (p. 17)

x-intercept: (0,0) since $x^2 = 0$, $x = 0$

y-intercept: (0,0) since $\frac{0^2}{0^2+9} = \frac{0}{9} = 0$ or because the x-intercept is (0,0)

Solution to Exercise 1.24 (p. 17)

x-intercept: (-1,0) since $\sqrt{x+1} = 0$, $x + 1 = 0$, $x = -1$

y-intercept: (0, $\frac{1}{4}$) since $\frac{\sqrt{0+1}}{(0-2)^2} = \frac{\sqrt{1}}{(-2)^2} = \frac{1}{4}$

Solution to Exercise 1.25 (p. 17)

x-intercept: (0,0) since $3x = 0$, $x = 0$

y-intercept: (0,0) since the x-intercept is (0,0)

Solution to Exercise 1.26 (p. 17)

x-intercept: (2,0) since $\frac{1}{x-3} + 1 = 0$, $\frac{1}{x-3} = -1$, $-x + 3 = 1$, $-x = -2$, $x = 2$

y-intercept: (0, $\frac{2}{3}$) since $\frac{1}{0-3} + 1 = \frac{-1}{3} + 1 = \frac{2}{3}$

Solution to Exercise 1.27 (p. 17)

x-intercepts: (-2,0), (2,0) since $x^2 - 4 = 0$, $x^2 = 4$, $x = -2$, $x = 2$

y-intercept: (0, -4) since $\frac{0^2-4}{\sqrt{0+1}} = \frac{-4}{\sqrt{1}} = -4$

Solution to Exercise 1.28 (p. 17)

x-intercept: None since $4 + \frac{5}{x^2+2} = 0$, $\frac{5}{x^2+2} = -4$, $-4x^2 - 8 = 5$, $-4x^2 = 13$, $x^2 = \frac{13}{4}$, a number squared will never be a negative number, so there is no x-intercept

y-intercept: (0, $\frac{13}{2}$) since $4 + \frac{5}{0^2+2} = 4 + \frac{5}{2} = \frac{13}{2}$

Solution to Exercise 1.29 (p. 17)

x-intercept: ($\frac{2}{5}$, 0) since $\sqrt{5x-2} = 0$, $5x - 2 = 0$, $5x = 2$, $x = \frac{2}{5}$

y-intercept: None since $y = \frac{\sqrt{5\times 0-2}}{0-3}$ takes the square root of a negative number.

Solution to Exercise 1.30 (p. 17)

x-intercept: (2,0) since $x^3 - 8 = 0$, $(x-2)\left(x^2 + 2x + 4\right) = 0$, $x = 2$

y-intercept: (0,-8) since $\frac{0^3-8}{0^2+1} = \frac{-8}{1} = -8$

Chapter 2

Domain Knowledge

2.1 Simple Rational Functions[1]

For fractions, the denominator (the bottom) of the fraction cannot equal 0. Determine **domain** restrictions by setting the denominator equal to 0 and solving.

Example 2.1
Find the domain of $y = \frac{1}{x}$
$\{x \mid x \neq 0\}$

Exercise 2.1 *(Solution on p. 24.)*
Find the domain of $y = \frac{1}{x-5}$

Exercise 2.2 *(Solution on p. 24.)*
Find the domain of $y = \frac{4x+3}{x-7}$

Exercise 2.3 *(Solution on p. 24.)*
Find the domain of $y = \frac{7x}{5-2x}$

Exercise 2.4 *(Solution on p. 24.)*
Find the domain of $y = \frac{2}{(x-3)(x+7)}$

Exercise 2.5 *(Solution on p. 24.)*
Find the domain of $y = \frac{7x}{2x^2-7x+3}$

Exercise 2.6 *(Solution on p. 24.)*
$y = \frac{2x+1}{(x+5)^2}$

Exercise 2.7 *(Solution on p. 24.)*
Find the domain of $y = \frac{x+3}{x^2+25}$

Exercise 2.8 *(Solution on p. 24.)*
Find the domain of $y = \frac{x-7}{x^2+2}$

Exercise 2.9 *(Solution on p. 24.)*
Find the domain of $y = \frac{5}{|x-3|}$

Exercise 2.10 *(Solution on p. 24.)*
Find the domain of $y = \frac{4}{|x|-4}$

[1]This content is available online at <http://cnx.org/content/m13352/1.7/>.

2.2 Radical Functions[2]

When finding the domain of even-degree roots, the expression under the radical must be greater than or equal to 0.

Example 2.2
Find the domain of $y = \sqrt{x}$
$\{x \mid x \geq 0\}$

PRACTICE - Find the Domain of the following:

Exercise 2.11 *(Solution on p. 24.)*
$y = \sqrt{2x - 5}$

Exercise 2.12 *(Solution on p. 24.)*
$y = \sqrt[4]{7 - x}$

The rest of the answers will be expressed in interval notation since that is a simpler way to express answers.

Exercise 2.13 *(Solution on p. 24.)*
$y = \sqrt[4]{4x^2 - 16}$

Exercise 2.14 *(Solution on p. 24.)*
$y = \sqrt{16 - 25x^2}$

Exercise 2.15 *(Solution on p. 24.)*
$y = \sqrt{(x - 7)(x + 1)}$

Exercise 2.16 *(Solution on p. 24.)*
$y = \sqrt{2x^2 - 7x + 3}$

Exercise 2.17 *(Solution on p. 24.)*
$y = x\left(\sqrt{x^2 + 4}\right)$

Exercise 2.18 *(Solution on p. 24.)*
$y = x + \sqrt{-x + 8}$

Exercise 2.19 *(Solution on p. 24.)*
$y = \sqrt{6x^2 + 8}$

Exercise 2.20 *(Solution on p. 24.)*
$y = \sqrt{(-8) - 6x^2}$

2.3 Algebraic Functions[3]

When finding domain consider the following:

- In rational functions, the denominator cannot equal 0
- When even-degreed roots are in the numerator, the expression under the radical must be greater than or equal to 0
- When even-degreed roots are in the denominator, the expression under the radical must be greater than 0

[2] This content is available online at <http://cnx.org/content/m13583/1.3/>.
[3] This content is available online at <http://cnx.org/content/m13607/1.3/>.

Exercise 2.21 *(Solution on p. 24.)*

$y = \sqrt{12 - x}$

Exercise 2.22 *(Solution on p. 24.)*

$y = x^2 + 9x - 20$

Exercise 2.23 *(Solution on p. 24.)*

$y = \sqrt{x^2 + 6x + 5}$

Exercise 2.24 *(Solution on p. 24.)*

$y = \frac{x-2}{\sqrt{x+4}}$

Exercise 2.25 *(Solution on p. 25.)*

$y = \frac{\sqrt{7-x}}{x}$

Exercise 2.26 *(Solution on p. 25.)*

$y = \frac{x-1}{\sqrt{x^2-4x}}$

Exercise 2.27 *(Solution on p. 25.)*

$y = \frac{\sqrt{x^2-1}}{x^2-4}$

Exercise 2.28 *(Solution on p. 25.)*

$y = \frac{3x-1}{\sqrt{x}+5}$

Exercise 2.29 *(Solution on p. 25.)*

$\frac{1}{|\sqrt{x+1}|}$

Solutions to Exercises in Chapter 2

Solution to Exercise 2.1 (p. 21)
$\{x \mid x \neq 5\}$ since $x-5 \neq 0$, $x \neq 5$
Solution to Exercise 2.2 (p. 21)
$\{x \mid x \neq 7\}$ since $x-7 \neq 0$, $x \neq 7$
Solution to Exercise 2.3 (p. 21)
$\left\{x \mid x \neq \frac{5}{2}\right\}$ since $5-2x \neq 0$, $x \neq \frac{5}{2}$
Solution to Exercise 2.4 (p. 21)
$\{x \mid x \neq 3 \;\; or \;\; -7\}$ since $x \neq 3$ and $x \neq -7$
Solution to Exercise 2.5 (p. 21)
$\left\{x \mid x \neq \frac{1}{2} \;\; or \;\; 3\right\}$ since $2x^2-7x+3 \neq 0$, $(2x-1)(x-3) \neq 0$, $2x-1 \neq 0$ and $x-3 \neq 0$, $x \neq \frac{1}{2}$ and $x \neq 3$
Solution to Exercise 2.6 (p. 21)
$\{x \mid x \neq -5\}$ since $(x+5)^2 \neq 0$, $x+5 \neq 0$, $x \neq -5$
Solution to Exercise 2.7 (p. 21)
$\{x \mid x \in \mathbb{R}\}$ since $x^2+25 \neq 0$, $x^2 \neq -25$, $x \in \mathbb{R}$
Solution to Exercise 2.8 (p. 21)
$\{x \mid x \in \mathbb{R}\}$ since $x^2+2 \neq 0$, $x^2 \neq -2$, $x \in \mathbb{R}$
Solution to Exercise 2.9 (p. 21)
$\{x \mid x \neq 3\}$ since $|x-3| \neq 0$, $x-3 \neq 0$, $x \neq 3$
Solution to Exercise 2.10 (p. 21)
$\{x \mid x \neq -4 \;\; or \;\; 4\}$ since $|x|-4 \neq 0$, $|x| \neq 4$, $x \neq -4$ and $x \neq 4$
Solution to Exercise 2.11 (p. 22)
$\left\{x \mid x \geq \frac{5}{2}\right\}$ since $2x-5 \geq 0$, $2x \geq 5$, $x \geq \frac{5}{2}$
Solution to Exercise 2.12 (p. 22)
$\{x \mid x \leq 7\}$ since $7-x \geq 0$, $-x \geq -7$, $x \leq 7$
Solution to Exercise 2.13 (p. 22)
$(-\infty,-2] \cup [2,\infty)$ since $4x^2-16 \geq 0$, $4x^2 \geq 16$, $x^2 \geq 4$, $(x \leq -2) \;\; or \;\; (x \geq 2)$
Solution to Exercise 2.14 (p. 22)
$\left[\frac{-4}{5}, \frac{4}{5}\right]$ since $16-25x^2 \geq 0$, $-25x^2 \geq -16$, $x^2 \leq \frac{16}{25}$, $\left(x \geq \frac{-4}{5}\right) \;\; and \;\; \left(x \leq \frac{4}{5}\right)$
Solution to Exercise 2.15 (p. 22)
$(-\infty,-1] \cup [7,\infty)$, $\sqrt{(x-7)(x+1)} \geq 0$
Solution to Exercise 2.16 (p. 22)
$(-\infty,1/2] \cup [3,\infty)$, $2x^2-7x+3 \geq 0$, $(2x-1)(x-3) \geq 0$, $\left(x \leq \frac{1}{2}\right) \;\; or \;\; (x \geq 3)$
Solution to Exercise 2.17 (p. 22)
$(-\infty,\infty)$, since $x^2+4 \geq 0$, $x^2 \geq -4$ This will always be true, for all real numbers, any number squared is always positive
Solution to Exercise 2.18 (p. 22)
$(-\infty,8]$ since $-x+8 \geq 0$, $-x \geq -8$, $x \leq 8$
Solution to Exercise 2.19 (p. 22)
$(-\infty,\infty)$, since $6x^2+8 \geq 0$, $6x^2 \geq -8$, $x^2 \geq \frac{-8}{6}$ This will always be true, for all real numbers, any number squared is always positive
Solution to Exercise 2.20 (p. 22)
No solution since $(-8)-6x^2 \geq 0$, $-6x^2 \geq 8$, $x^2 \geq \frac{-8}{6}$ This will never be true, so there is no solution, since any number squared is always positive, so it will never be less than 0.
Solution to Exercise 2.21 (p. 23)
$(-\infty,12]$ since $12-x \geq 0$
Solution to Exercise 2.22 (p. 23)
$(-\infty,\infty)$ since there are no even-degreed roots and it is not a rational function
Solution to Exercise 2.23 (p. 23)
$(-\infty,-5] \cup [-1,\infty)$ since $x^2+6x+5 \geq 0$

Solution to Exercise 2.24 (p. 23)
$(-4, \infty)$ since $x + 4 > 0$
Solution to Exercise 2.25 (p. 23)
$(-\infty, 0) \cup (0, 7]$ since $7 - x \geq 0$ and $x \neq 0$
Solution to Exercise 2.26 (p. 23)
$(-\infty, 0) \cup (4, \infty)$ since $x^2 - 4x > 0$
Solution to Exercise 2.27 (p. 23)
$(-\infty, -2) \cup (-2, -1] \cup [1, 2) \cup (2, \infty)$ since $x^2 - 1 \geq 0$ and $x^2 - 4 \neq 0$
Solution to Exercise 2.28 (p. 23)
$[0, 25) \cup (25, \infty)$ since $\sqrt{x} + 5 \neq 0$ and $x \geq 0$
Solution to Exercise 2.29 (p. 23)
$(-1, \infty)$ since $x + 1 > 0$

Chapter 3

Astounding Analysis

3.1 Discontinuities[1]

Vertical Asymptotes occur when factors in the denominator = 0 and do not cancel with factors in the numerator

- **Vertical asymptotes** are vertical lines the graph approaches
- The equation of the vertical asymptote is x = (that number which makes the denominator = 0)

Holes (**Removable Discontinuities**) occur when the factor in the denominator = 0 and it cancels with like factors in the numerator.

- Holes are open "points" so they have an x and y coordinate
- The x-value is the number that makes the cancelled factor = 0.
- The y-value is found by substituting x into the "reduced" equation (**after** cancelling) like factors.

Find the vertical asymptotes and holes (if any) for the following. Don't forget that vertical asymptotes are equations and holes are points!

Example 3.1
$y = \frac{1}{x}$
Vertical Asymptote: $x = 0$
Hole: None

Example 3.2
$y = \frac{x(x-1)}{x-1}$
Vertical Asymptote: None
Hole: (1,1) since (x-1) was cancelled, the hole is at x=1. To find the y-coordinate, plug 1 into the reduced equation: $\frac{x(x-1)}{x-1} = x = 1$

Exercise 3.1 ***(Solution on p. 31.)***
$y = \frac{4x+3}{x-7}$

Exercise 3.2 ***(Solution on p. 31.)***
$y = \frac{9x}{3-2x}$

Exercise 3.3 ***(Solution on p. 31.)***
$y = \frac{7}{(x-9)(x+1)}$

Exercise 3.4 ***(Solution on p. 31.)***
$y = \frac{7x}{2x^2-7x+3}$

[1]This content is available online at <http://cnx.org/content/m13605/1.3/>.

Exercise 3.5 *(Solution on p. 31.)*

$y = \frac{2x+1}{(x+5)^2}$

Exercise 3.6 *(Solution on p. 31.)*

$y = \frac{x+3}{x^2+25}$

Exercise 3.7 *(Solution on p. 31.)*

$y = \frac{x-7}{x^2+2}$

Exercise 3.8 *(Solution on p. 31.)*

$y = \frac{5}{|x-3|}$

Exercise 3.9 *(Solution on p. 31.)*

$y = \frac{4}{|x|-4}$

Exercise 3.10 *(Solution on p. 31.)*

$y = \frac{3\left(x^2-x-6\right)}{4(x^2-9)}$

Exercise 3.11 *(Solution on p. 31.)*

$y = \frac{-2\left(x^2-4\right)}{3(x^2+4x+4)}$

Exercise 3.12 *(Solution on p. 31.)*

$y = \frac{x^2-4}{x+2}$

Exercise 3.13 *(Solution on p. 31.)*

$y = \frac{x^2(x-3)}{x^2-3x}$

Exercise 3.14 *(Solution on p. 31.)*

$y = \frac{x^3-1}{x-1}$

Exercise 3.15 *(Solution on p. 32.)*

$y = \frac{2x^2-3x-5}{x^2-1}$

3.2 Horizontal Asymptotes[2]

Horizontal asymptotes are horizontal lines the graph approaches.

Horizontal Asymptotes CAN be crossed.

To find horizontal asymptotes:

- If the degree (the largest exponent) of the denominator is **bigger than** the degree of the numerator, the horizontal asymptote is the x-axis (y = 0).
- If the degree of the numerator is **bigger than** the denominator, there is no horizontal asymptote.
- If the degrees of the numerator and denominator are the **same**, the horizontal asymptote equals the leading coefficient (the coefficient of the largest exponent) of the numerator divided by the leading coefficient of the denominator

One way to remember this is the following pnemonic device: BOBO BOTN EATS DC

- BOBO - Bigger on bottom, y=0
- BOTN - Bigger on top, none
- EATS DC - Exponents are the same, divide coefficients

[2]This content is available online at <http://cnx.org/content/m13606/1.8/>.

Find the Horizontal Asymptotes of the following:

Exercise 3.16 *(Solution on p. 32.)*

$f(x) = \frac{4x}{x-3}$

Exercise 3.17 *(Solution on p. 32.)*

$g(x) = \frac{5x^2}{3+x}$

Exercise 3.18 *(Solution on p. 32.)*

$h(x) = \frac{-4x^2}{(x-2)(x+4)}$

Exercise 3.19 *(Solution on p. 32.)*

$g(x) = \frac{6}{(x+3)(4-x)}$

Exercise 3.20 *(Solution on p. 32.)*

$f(x) = \frac{(3x)(x-1)}{2x^2-5x-3}$

Exercise 3.21 *(Solution on p. 32.)*

$q(x) = \frac{(-x)(1-x)}{3x^2+5x-2}$

Exercise 3.22 *(Solution on p. 32.)*

$r(x) = \frac{x}{(x-8)^2}$

Exercise 3.23 *(Solution on p. 32.)*

$r(x) = \frac{x}{x^4-1}$

Exercise 3.24 *(Solution on p. 32.)*

$g(x) = \frac{x-3}{x^2+1}$

Exercise 3.25 *(Solution on p. 32.)*

$r(x) = \frac{3x^2+x}{x^2+4}$

3.3 Slant Asymptotes[3]

Just like vertical and horizontal asymptotes, **slant asymptotes** are lines the graph approaches. They are also called **oblique asymptotes**.

A graph has a slant asymptote if the degree of the numerator is bigger than the degree of the denominator (there is no horizontal asymptote).

To find slant asymptotes, divide the numerator by the denominator and keep only the quotient (the answer, throw away the remainder). Don't forget that these are still lines, so they are written as y =

To divide, you either have to use long division or synthetic division (if possible).

PRACTICE - Find the Slant Asymptotes:

Exercise 3.26 *(Solution on p. 32.)*

$y = \frac{3x^3}{x^2-1}$

Exercise 3.27 *(Solution on p. 32.)*

$y = \frac{2x^2}{x+1}$

Exercise 3.28 *(Solution on p. 32.)*

$y = \frac{x^2-9x+2}{x+4}$

Exercise 3.29 *(Solution on p. 32.)*

$y = \frac{x^3-27}{x^2+3}$

Exercise 3.30 *(Solution on p. 32.)*

$y = \frac{2x^3+7x^2-4}{(x+3)(x-1)}$

[3]This content is available online at <http://cnx.org/content/m13608/1.1/>.

Exercise 3.31 *(Solution on p. 32.)*

$y = \frac{x^2+5x+8}{x+3}$

Exercise 3.32 *(Solution on p. 32.)*

$y = \frac{2x^2+x}{x+1}$

Exercise 3.33 *(Solution on p. 32.)*

$y = \frac{(2x)(x+11)}{x-4}$

Exercise 3.34 *(Solution on p. 32.)*

$y = \frac{x^4}{(x-1)^3}$

Exercise 3.35 *(Solution on p. 32.)*

$y = \frac{x^3-x+3}{x^2+x-2}$

Solutions to Exercises in Chapter 3

Solution to Exercise 3.1 (p. 27)
Vertical Asymptote: $x = 7$ since $x - 7 = 0$
Hole: None
Solution to Exercise 3.2 (p. 27)
Vertical Asymptote: $x = \frac{3}{2}$ since $3 - 2x = 0$, $x = \frac{3}{2}$
Hole: None
Solution to Exercise 3.3 (p. 27)
Vertical Asymptote: $x = 9$, $x = -1$ since $x = 9$ and $x = -1$
Hole: None
Solution to Exercise 3.4 (p. 27)
Vertical Asymptote: $x = \frac{1}{2}$, $x = 3$ since $2x^2 - 7x + 3 = 0$, $(2x - 1)(x - 3) = 0$, $2x - 1 = 0$ and $x - 3 = 0$, $x = \frac{1}{2}$ and $x = 3$
Hole: None
Solution to Exercise 3.5 (p. 27)
Vertical Asymptote: $x = -5$ since $(x + 5)^2 = 0$, $x + 5 = 0$, $x = -5$
Hole: None
Solution to Exercise 3.6 (p. 28)
Vertical Asymptote: None since $x^2 + 25 = 0$, $x^2 = -25$, a number squared will never be negative
Hole: None
Solution to Exercise 3.7 (p. 28)
Vertical Asymptote: None since $x^2 + 2 = 0$, $x^2 = -2$ and any number squared will never be a negative number
Hole: None
Solution to Exercise 3.8 (p. 28)
Vertical Asymptote: $x = 3$ since $|x - 3| = 0$, $x - 3 = 0$, $x = 3$
Hole: None
Solution to Exercise 3.9 (p. 28)
Vertical asymptotes: $x = -4$ and $x = 4$ since $|x| - 4 = 0$, $|x| = 4$, $x = -4$ and $x = 4$
Hole: None
Solution to Exercise 3.10 (p. 28)
Vertical Asymptote: $x = -3$
Hole: (3, $\frac{5}{8}$) since $\frac{3\left(x^2-x-6\right)}{4(x^2-9)} = \frac{3((x-3)(x+2))}{4((x+3)(x-3))} = \frac{3((x+2))}{4((x+3))}$, (x-3) was cancelled, so the hole is at x=3. To find the y-coordinate, plug 3 into the reduced equation: $\frac{3((3+2))}{4((3+3))} = \frac{3\times5}{4\times6} = \frac{15}{24} = \frac{5}{8}$
Solution to Exercise 3.11 (p. 28)
$\frac{-2\left(x^2-4\right)}{3(x^2+4x+4)} = \frac{-2(x+2)(x-2)}{3(x+2)^2} = \frac{-2(x-2)}{3(x+2)}$
Vertical Asymptote: $x = -2$
Hole: None since the vertical asymptote takes care of the hole.
Solution to Exercise 3.12 (p. 28)
Vertical Asymptote: None
Hole: (-2,-4) since $\frac{x^2-4}{x+2} = \frac{(x+2)(x-2)}{x+2} = x - 2$, (x+2) was cancelled, so the hole is at x = -2. To find the y-coordinate, plug -2 into the reduced equation: $-2 - 2 = -4$
Solution to Exercise 3.13 (p. 28)
Vertical Asymptotes: None
Holes: (3,3), (0,0) since $\frac{x^2(x-3)}{x^2-3x} = \frac{x^2(x-3)}{x(x-3)} = x$, x and (x-3) were cancelled, so the holes are at x=0 and x=3. To find the y-coordinate, plug 0 and 3 into the reduced equation: 0, 3
Solution to Exercise 3.14 (p. 28)
Vertical Asymptote: None

Hole: (1,3) since $\frac{x^3-1}{x-1} = \frac{(x-1)\left(x^2+x+1\right)}{x-1} = x^2 + x + 1$, (x-1) was cancelled, so the hole is at x=1. To find the y-coordinate, plug 1 into the reduced equation: $1^2 + 1 + 1 = 3$

Solution to Exercise 3.15 (p. 28)

$\frac{2x^2-3x-5}{x^2-1} = \frac{(2(x-5))(x+1)}{(x+1)(x-1)} = \frac{2(x-5)}{x-1}$

Vertical asymptote: $x = 1$ since $x - 1 = 0$

Hole: (-1, $\frac{7}{2}$) Since (x+1) was cancelled, the hole is at x= -1. To find the y-coordinate, plug -1 into the reduced equation: $\frac{2\times(-1-5)}{-1-1} = \frac{7}{2}$

Solution to Exercise 3.16 (p. 29)

$y = 4$ since the degrees are the same, divide the leading coefficients of the numerator and denominator = $\frac{4}{1} = 4$

Solution to Exercise 3.17 (p. 29)

None since the degree of the numerator is greater than the degree of the denominator.

Solution to Exercise 3.18 (p. 29)

$y = -4$

Solution to Exercise 3.19 (p. 29)

$y = 0$

Solution to Exercise 3.20 (p. 29)

$y = \frac{3}{2}$

Solution to Exercise 3.21 (p. 29)

$y = \frac{1}{3}$

Solution to Exercise 3.22 (p. 29)

$y = 0$

Solution to Exercise 3.23 (p. 29)

y = 0

Solution to Exercise 3.24 (p. 29)

$y = 0$

Solution to Exercise 3.25 (p. 29)

$y = 3$

Solution to Exercise 3.26 (p. 29)

$y = 3x$

Solution to Exercise 3.27 (p. 29)

$y = 2x - 2$

Solution to Exercise 3.28 (p. 29)

$y = x - 13$

Solution to Exercise 3.29 (p. 29)

$y = x$

Solution to Exercise 3.30 (p. 29)

$y = 2x + 3$

Solution to Exercise 3.31 (p. 29)

$y = x + 2$

Solution to Exercise 3.32 (p. 30)

$y = 2x - 1$

Solution to Exercise 3.33 (p. 30)

$y = 2x + 30$

Solution to Exercise 3.34 (p. 30)

$y = x + 3$

Solution to Exercise 3.35 (p. 30)

$y = x - 1$

Chapter 4

Synthesize This

4.1 Putting It All Together - Graphing Rational Functions[1]

When graphing rational functions, find the domain, vertical asymptotes, slant asymptotes, holes (if any), horizontal asymptotes, vertical asymptotes, zeros, and y-intercept.

To practice, graph each rational function. State the domain, hole(s), VA (vertical asyptote(s)), HA(horizontal asymptote), SA(slant asymptote), zeros, and y-intercept(y-int).

Use graph paper[2] .

Exercise 4.1 *(Solution on p. 35.)*

$r(x) = \frac{x+1}{x(x+4)}$

Exercise 4.2 *(Solution on p. 35.)*

$h(x) = \frac{(2x^2)(x-3)}{(x-1)(x+2)}$

Exercise 4.3 *(Solution on p. 36.)*

$f(x) = \frac{3x+3}{2x+4}$

Exercise 4.4 *(Solution on p. 37.)*

$g(x) = \frac{6}{x^2-x-6}$

Exercise 4.5 *(Solution on p. 38.)*

$h(x) = \frac{2x+4}{x-1}$

Exercise 4.6 *(Solution on p. 39.)*

$t(x) = \frac{3x}{x^2+4}$

Exercise 4.7 *(Solution on p. 40.)*

$f(x) = \frac{x^2+4}{x^2-4}$

Exercise 4.8 *(Solution on p. 41.)*

$f(x) = \frac{x}{(x+2)^2}$

Exercise 4.9 *(Solution on p. 42.)*

$f(x) = \frac{5x^2}{x+3}$

Exercise 4.10 *(Solution on p. 43.)*

$f(x) = \frac{x-3}{x^2+1}$

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[1]This content is available online at <http://cnx.org/content/m13604/1.2/>.
[2]http://www.incompetech.com/beta/plainGraphPaper/graph.pdf

4.2 Interesting Graphs!!![3]

Although it is always useful to calculate all the good stuff - domain, vertical asymptotes, horizontal asymptotes, slant asymptotes, holes, x- and y-intercepts, there are some graphs that are just different. This lesson is to show you some unique, yet useful graphs. Try graphing them first on the paper provided and then check your answers. The best way to find the pattern that these graphs follow is to plug in points.

Use graph paper[4] .

Exercise 4.11 *(Solution on p. 44.)*

$y = \frac{|x|}{x}$

Exercise 4.12 *(Solution on p. 45.)*

$y = \frac{|x-2|}{x-2}$

Exercise 4.13 *(Solution on p. 46.)*

$y = \frac{|x+3|}{x+3}$

Exercise 4.14 *(Solution on p. 47.)*

$y = \frac{|x+2|}{x}$

Exercise 4.15 *(Solution on p. 48.)*

$y = \frac{x}{\sqrt{x^2}}$

Exercise 4.16 *(Solution on p. 49.)*

$y = \frac{x}{\sqrt{x^2+2}}$

Exercise 4.17 *(Solution on p. 50.)*

$y = \frac{-6x}{\sqrt{4x^2+5}}$

Exercise 4.18 *(Solution on p. 51.)*

$y = \frac{2x}{\sqrt{x^2+5}}$

[3] This content is available online at <http://cnx.org/content/m13595/1.1/>.
[4] http://www.incompetech.com/beta/plainGraphPaper/graph.pdf

Solutions to Exercises in Chapter 4

Solution to Exercise 4.1 (p. 33)

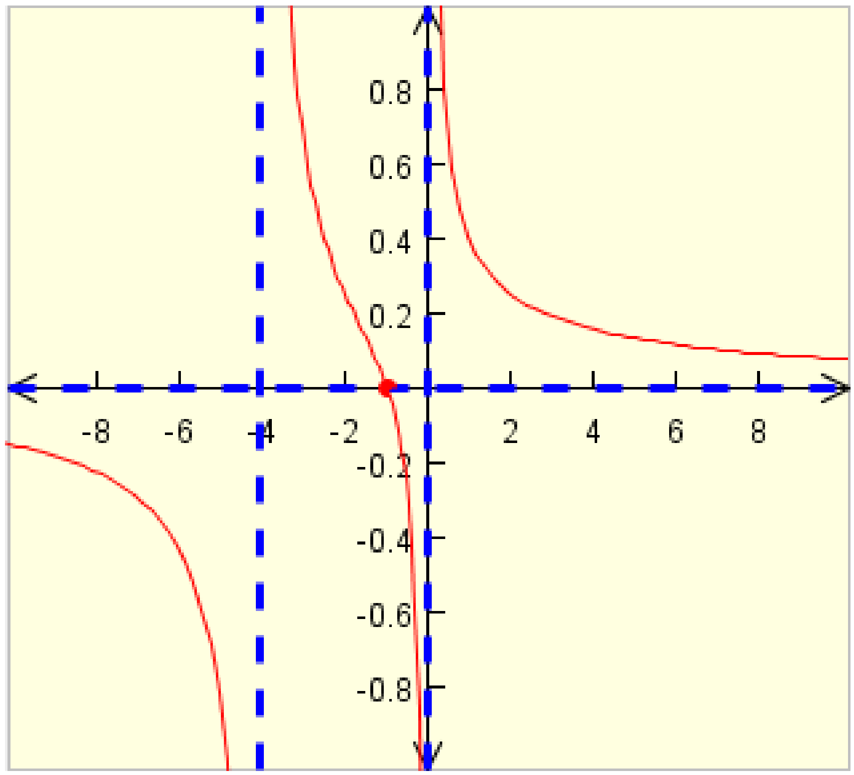

Figure 4.1

Domain: $(-\infty, -4) \cup (-4, 0) \cup (0, \infty)$
Hole: None
VA: $x = 0$, $x = -4$
HA: $y = 0$
SA:None
Zero: (-1,0)
Y-int: None

Solution to Exercise 4.2 (p. 33)

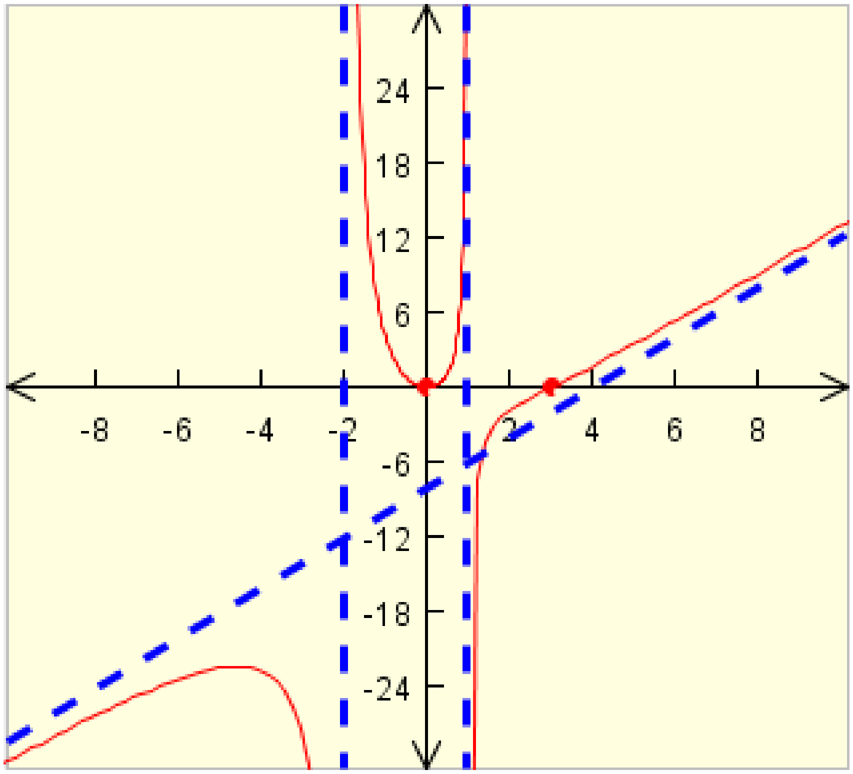

Figure 4.2

Domain: $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$
Hole: None
VA: $x = -2$, $x = 1$
HA:None
SA: $y = 2x - 8$
Zeros: (0,0), (3,0)
Y-int: (0,0)

Solution to Exercise 4.3 (p. 33)

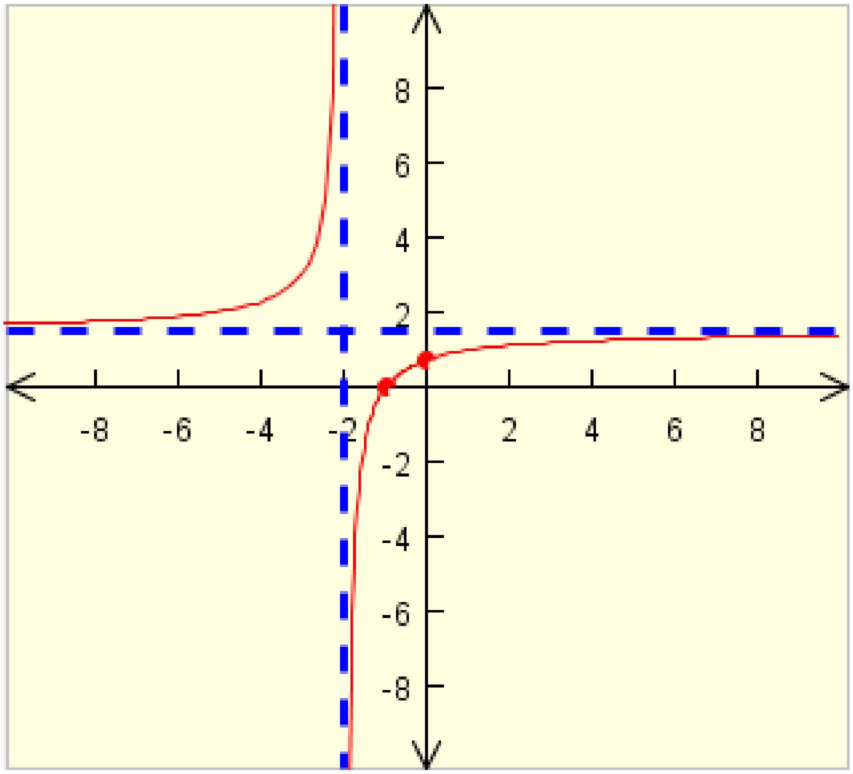

Figure 4.3

Domain: $(-\infty, -2) \cup (-2, \infty)$
Hole: None
VA: $x = -2$
HA: $y = \frac{3}{2}$
SA: None
Zero: (-1,0)
Y-int: $(0, \frac{3}{4})$

Solution to Exercise 4.4 (p. 33)

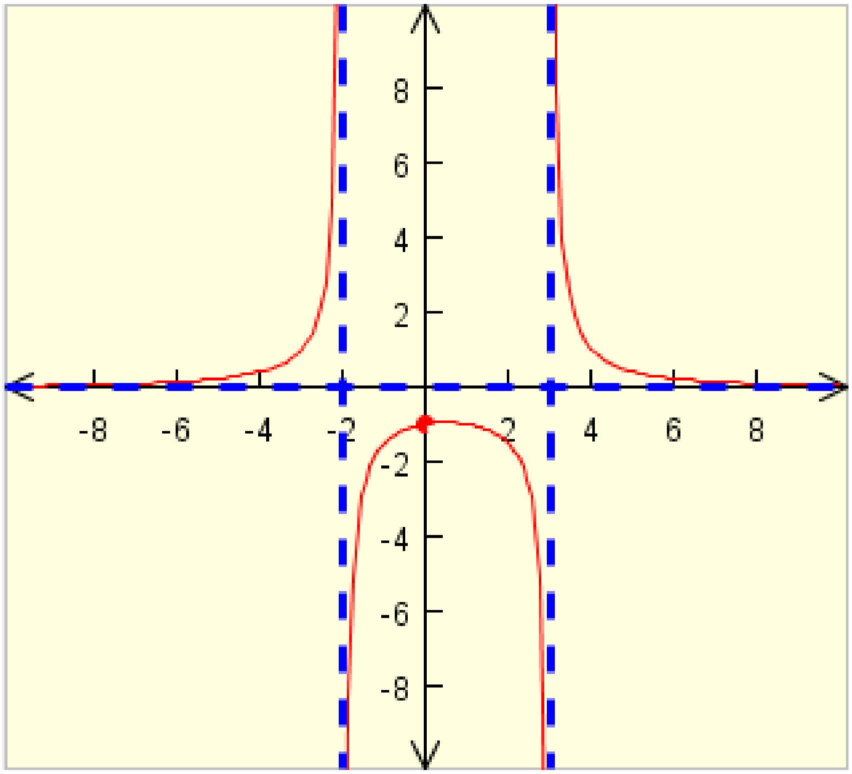

Figure 4.4

Domain: $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$
Hole: None
VA: $x = -2$, $x = 3$
HA: $y = 0$
SA: None
Zeros: None
Y-int: (0,-1)

Solution to Exercise 4.5 (p. 33)

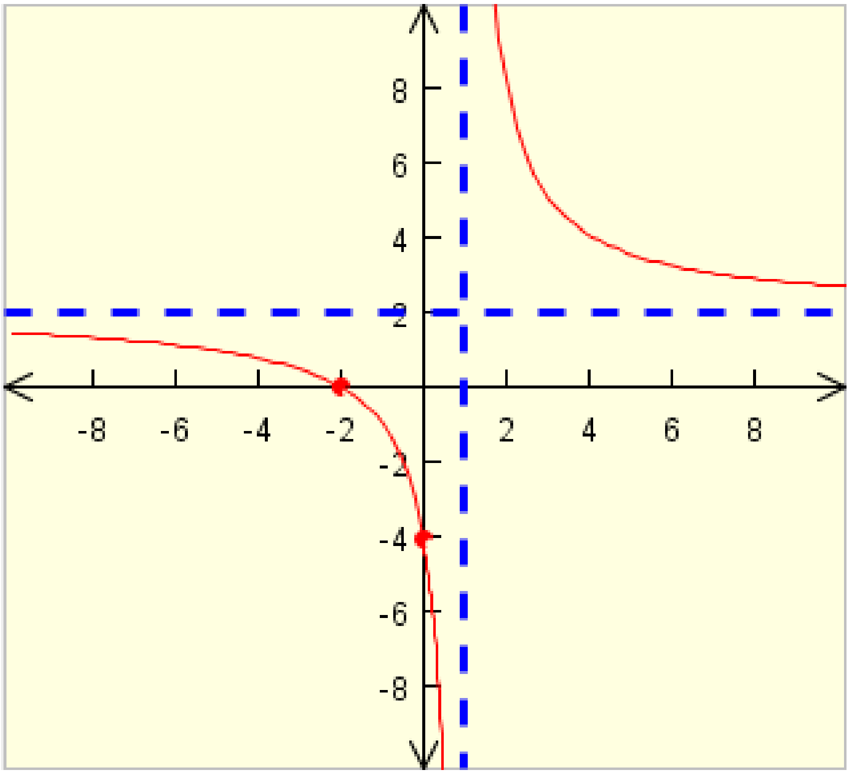

Figure 4.5

Domain: $(-\infty, 1) \cup (1, \infty)$
Hole: None
VA: $x = 1$
HA: $y = 2$
SA: None
Zero: (-2,0)
Y-int: (-2,0)

Solution to Exercise 4.6 (p. 33)

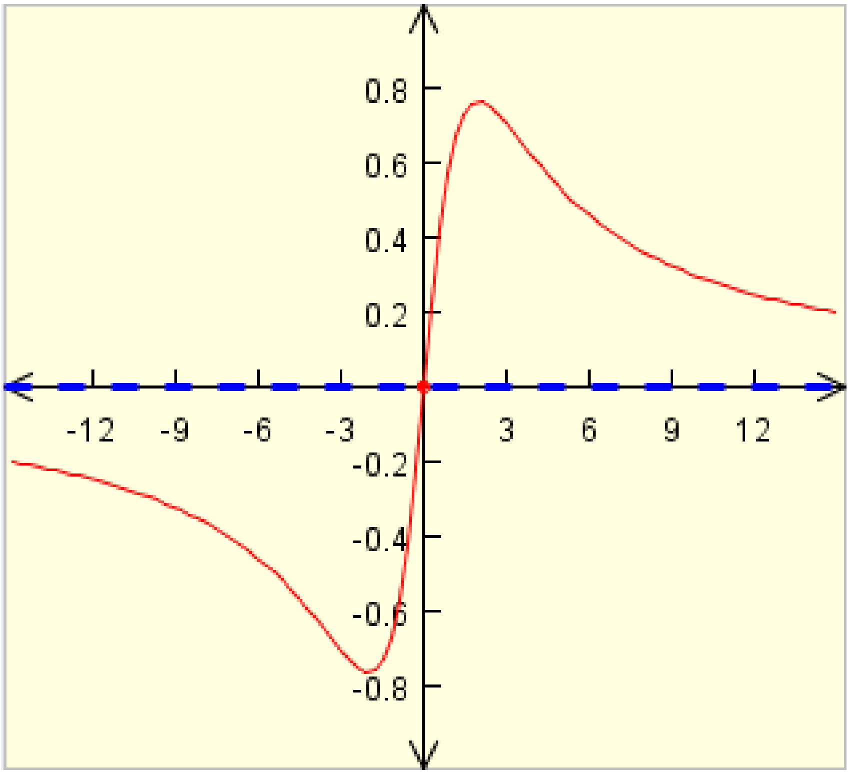

Figure 4.6

Domain: $(-\infty, \infty)$
Hole: None
VA: None
HA: $y = 0$
SA: None
Zero: (0,0)
Y-int: (0,0)

Solution to Exercise 4.7 **(p. 33)**

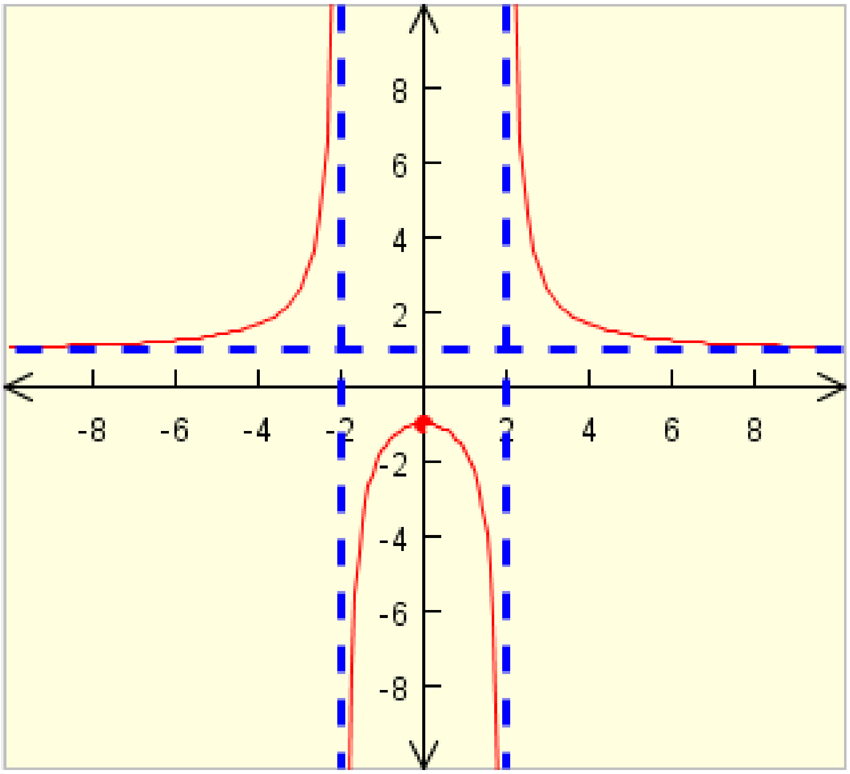

Figure 4.7

Domain: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$
Hole: None
VA: $x = -2$, $x = 2$
HA: $y = 1$
SA: None
Zeros: None
Y-int: (0,-1)

Solution to Exercise 4.8 (p. 33)

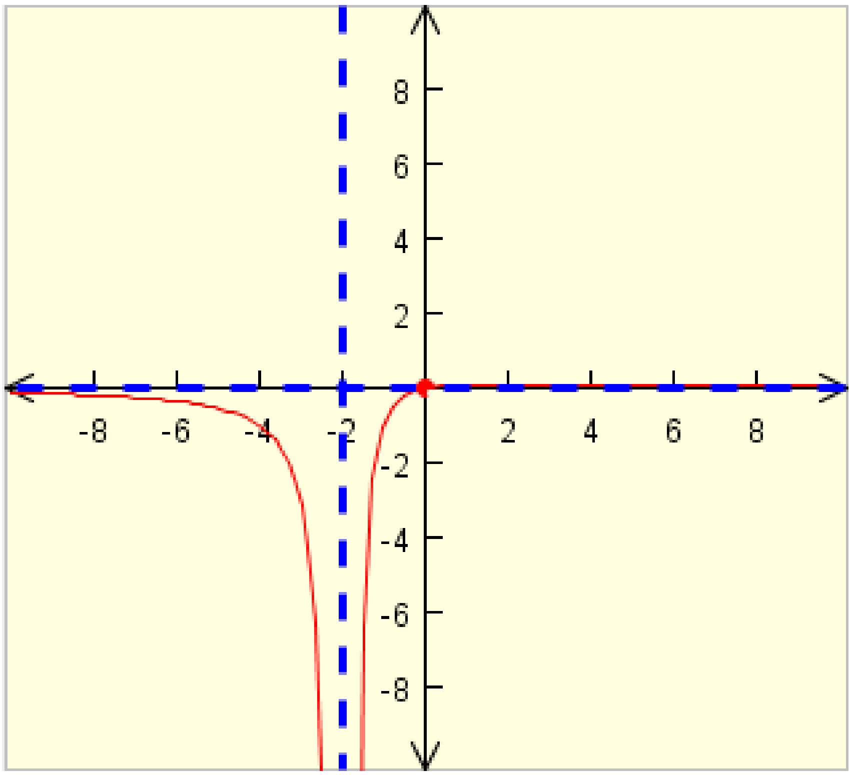

Figure 4.8

Domain: $(-\infty, -2) \cup (-2, \infty)$
Hole: None
VA: $x = -2$
HA: $y = 0$
SA: None
Zeros: (0,0)
Y-int: (0,0)

Solution to Exercise 4.9 (p. 33)

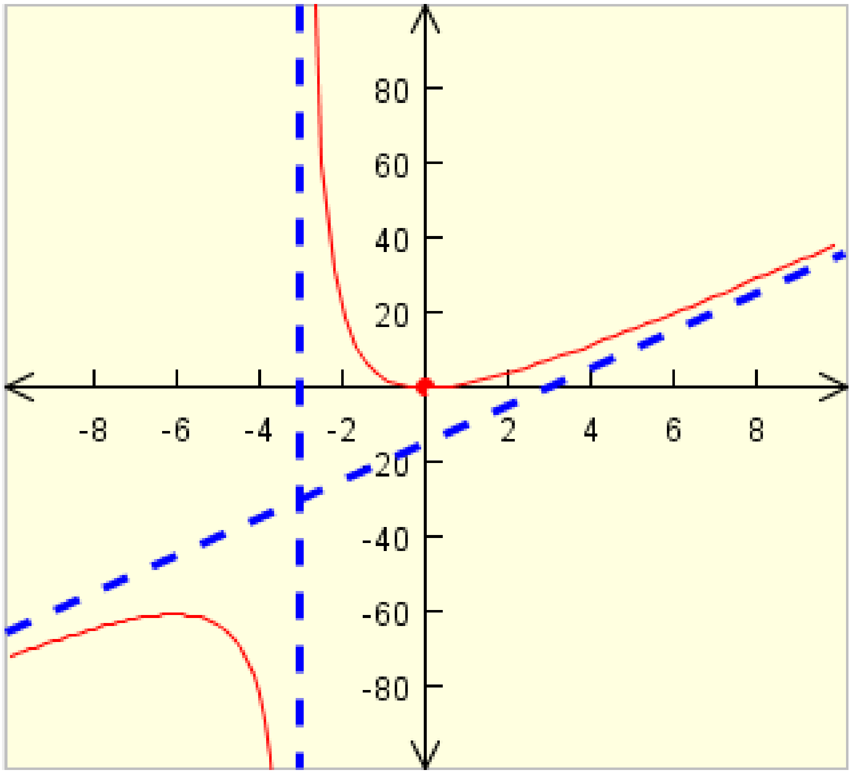

Figure 4.9

Domain: $(-\infty, -3) \cup (-3, \infty)$
Hole: None
VA: $x = -3$
HA: None
SA: $y = 5x - 15$
Zeros: (0,0)
Y-int: (0,0)

Solution to Exercise 4.10 (p. 33)

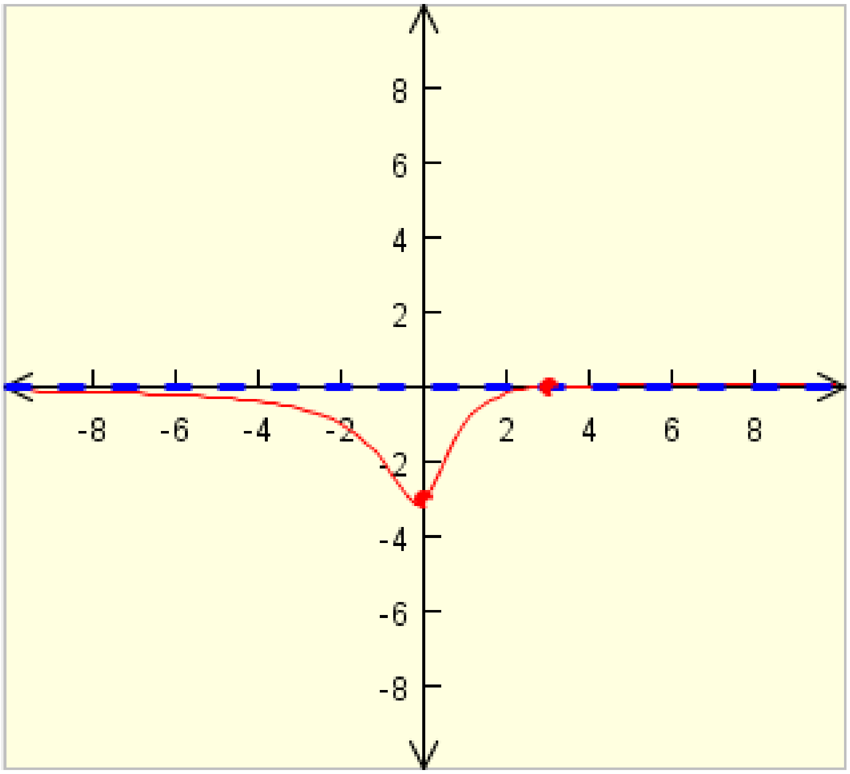

Figure 4.10

Domain: $(-\infty, \infty)$
Hole: None
VA: None
HA: $y = 0$
SA:None
Zeros:(3,0)
Y-int: (0,-3)

Solution to Exercise 4.11 (p. 34)

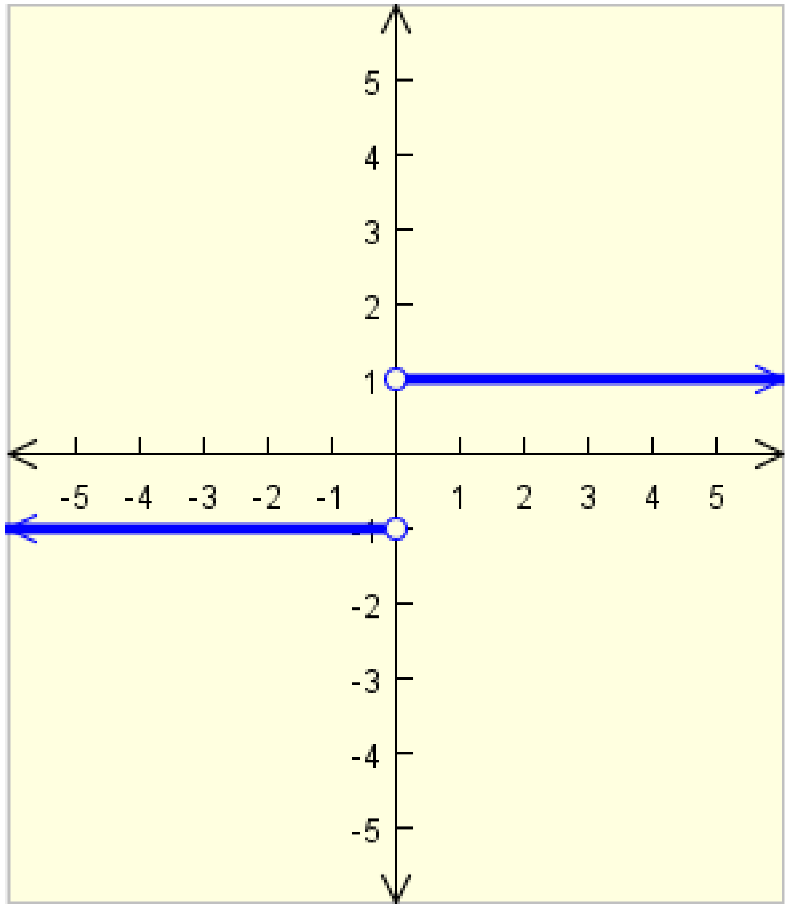

Figure 4.11

Solution to Exercise 4.12 (p. 34)

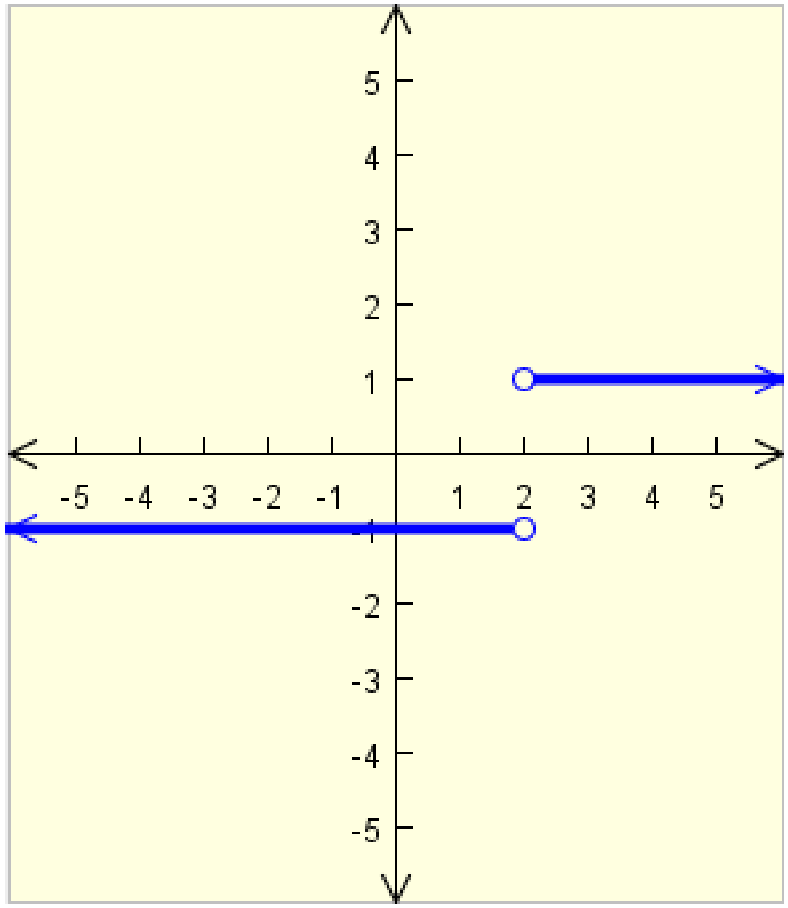

Figure 4.12

Solution to Exercise 4.13 (p. 34)

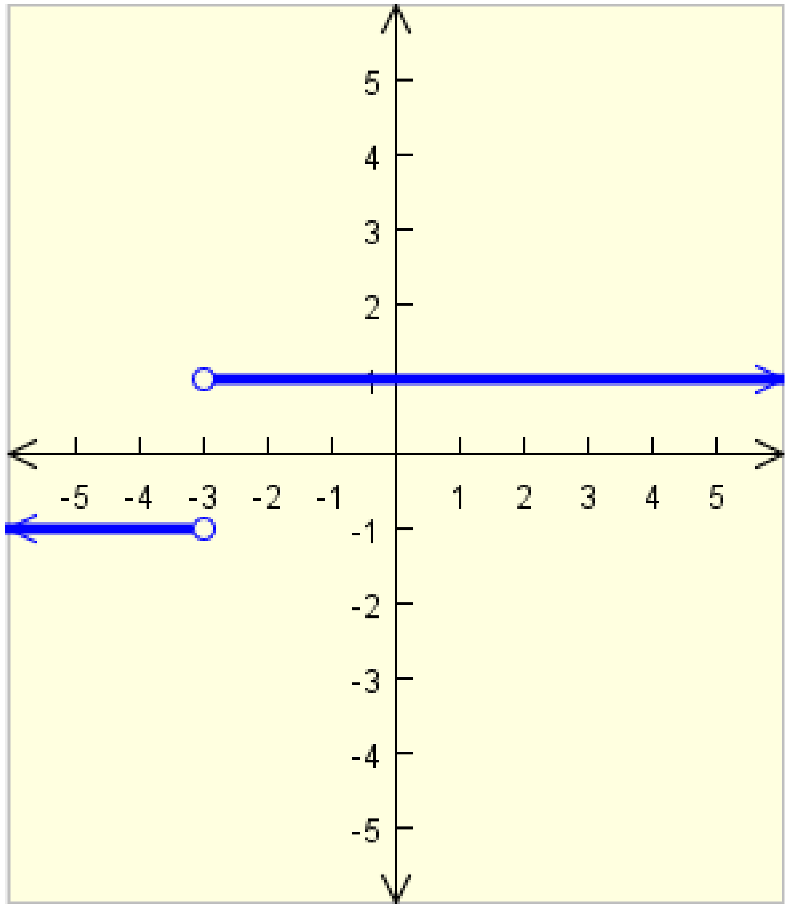

Figure 4.13

Solution to Exercise 4.14 (p. 34)

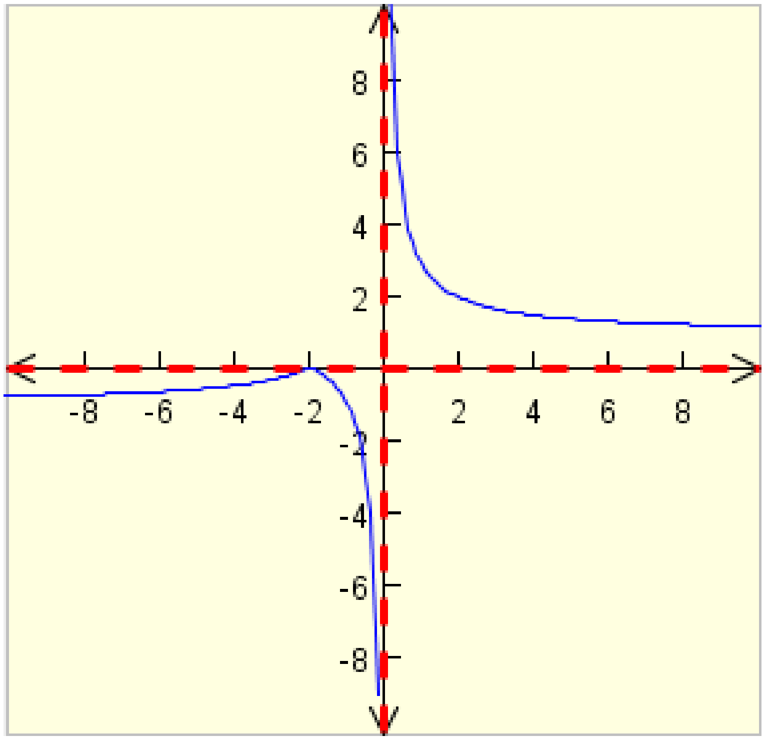

Figure 4.14

Solution to Exercise 4.15 (p. 34)

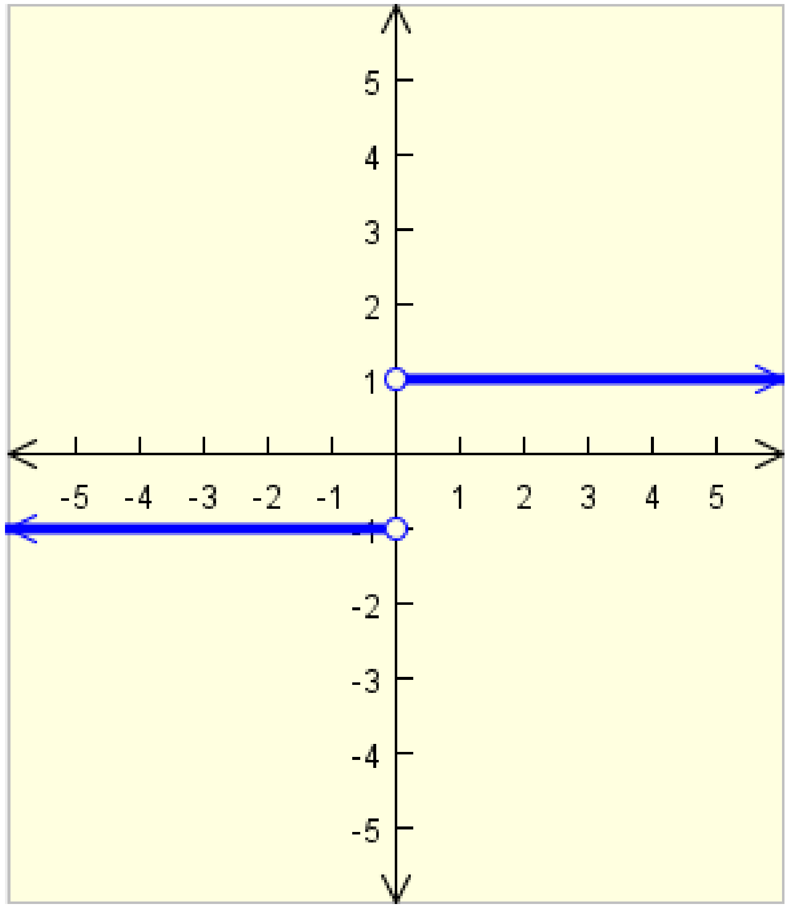

Figure 4.15

Solution to Exercise 4.16 (p. 34)

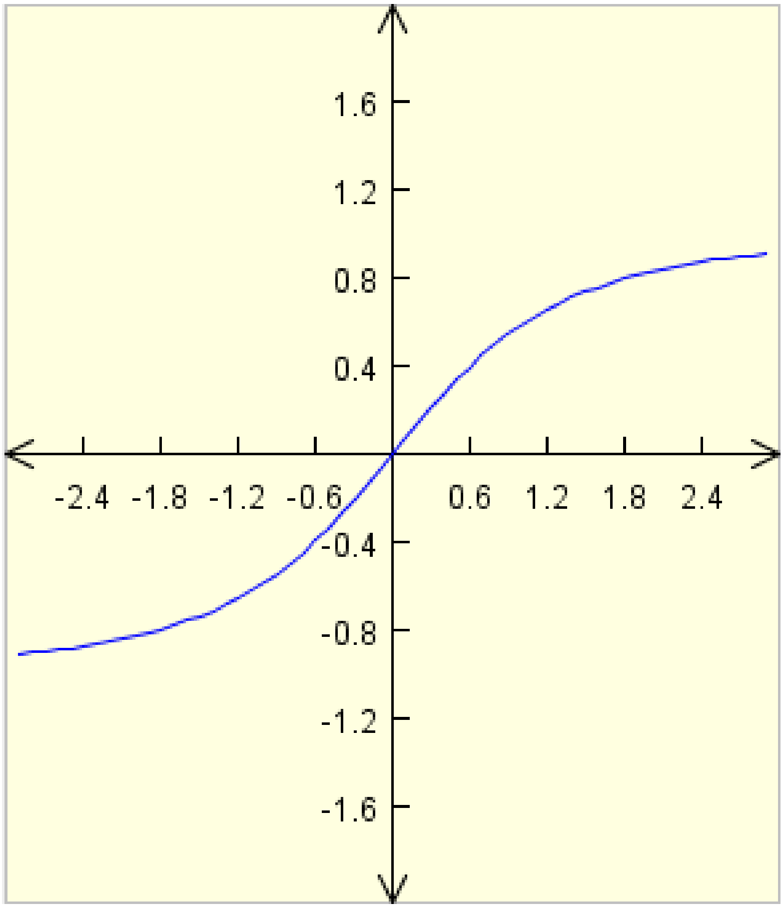

Figure 4.16

Solution to Exercise 4.17 (p. 34)

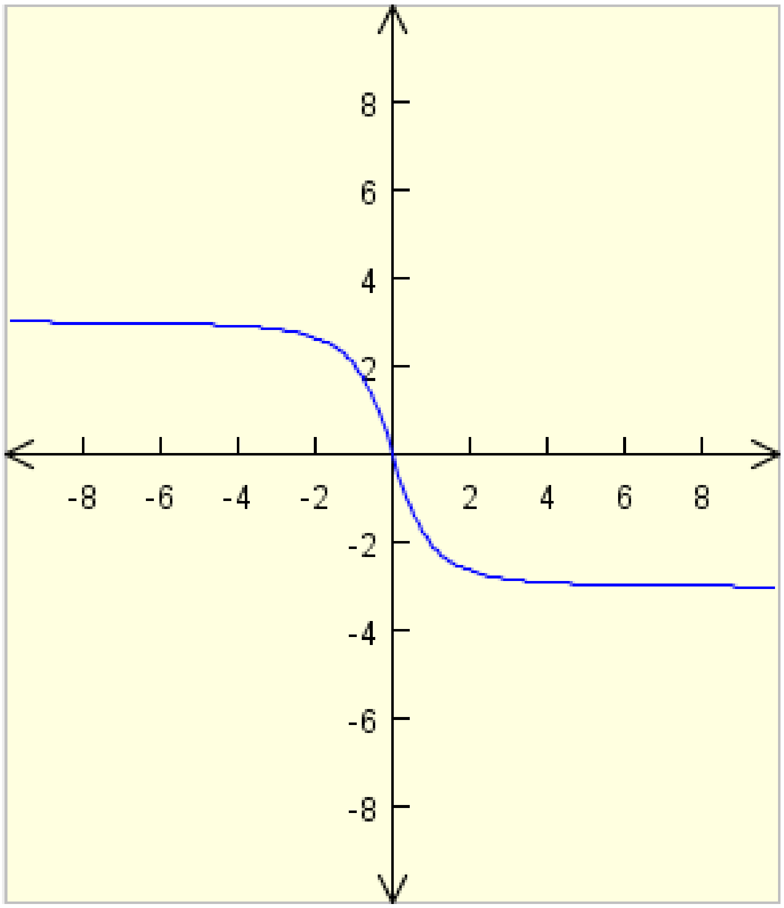

Figure 4.17

Solution to Exercise 4.18 (p. 34)

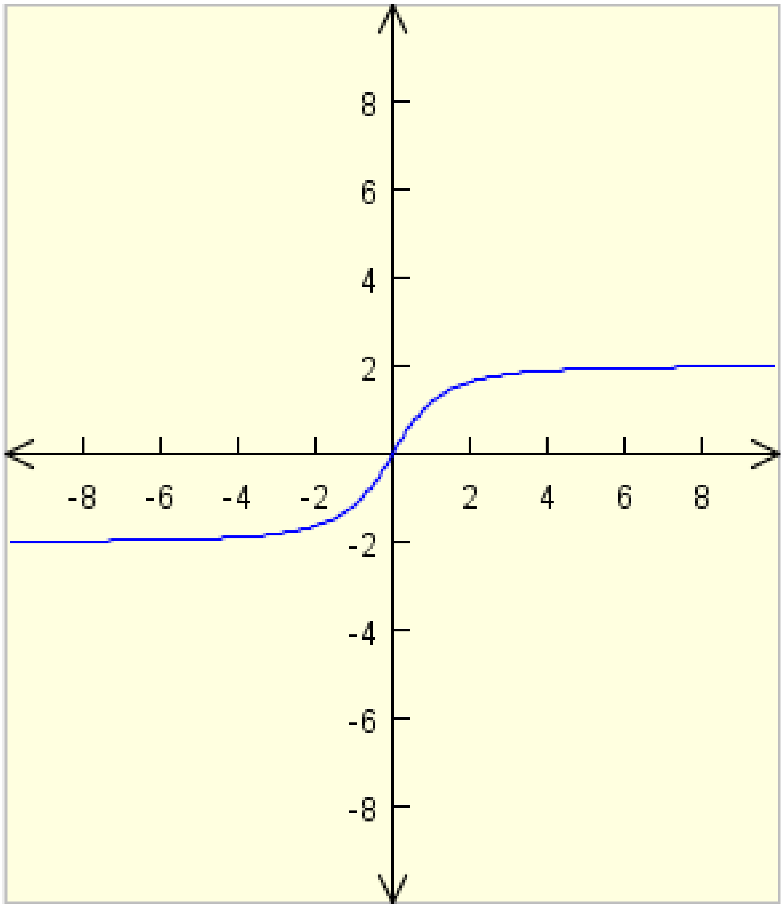

Figure 4.18

Index of Keywords and Terms

Keywords are listed by the section with that keyword (page numbers are in parentheses). Keywords do not necessarily appear in the text of the page. They are merely associated with that section. *Ex.* apples, § 1.1 (1) **Terms** are referenced by the page they appear on. *Ex.* apples, 1

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