Engineering and Scientific
Subroutine Library for AIX

# Guide and Reference

SA22-7272-02

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**Notes!**

- Before using this information and the product it supports, be sure to read the general information under “Special Notices” on page xix.
- For a summary of changes for ESSL Version 3 Release 1.2, see page xxxiii.

## Third Edition (October 1999)

This edition applies to Version 3 Release 1.2 of the IBM* Engineering and Scientific Subroutine Library (ESSL) for Advanced Interactive Executive (AIX)* licensed program, program number 5765-C42 and to all subsequent releases and modifications until otherwise indicated by new editions. Significant changes or additions to the text and illustrations are indicated by a vertical line (|) to the left of the change.

In this document, ESSL refers to the above version of ESSL for AIX. Changes are periodically made to the information herein.

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# Special Notices

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## Programming Interfaces

This *Engineering and Scientific Subroutine Library (ESSL) Version 3 Guide and Reference* manual is intended to help the customer to do application programming. This *ESSL Version 3 Guide and Reference* manual documents General-use Programming Interface and Associated Guidance Information provided by ESSL Version 3.

General-use programming interfaces allow the customer to write programs that obtain the services of ESSL Version 3.

# About This Book

The Engineering and Scientific Subroutine Library (ESSL) for AIX is a set of high-performance mathematical subroutines. ESSL is provided as five run-time libraries, running on RS/6000* POWER, PowerPC, POWER2, and POWER3 processors. ESSL can be used with Fortran, C, C++, and Programming Language/I (PL/I) programs operating under the AIX operating system.

This book is a guide and reference manual for using ESSL in doing application programming. It includes:

- An overview of ESSL and guidance information for designing, coding, and processing your program, as well as migrating existing programs, and diagnosing problems
- Reference information for coding each ESSL calling sequence

This book is written for a wide class of ESSL users: scientists, mathematicians, engineers, statisticians, computer scientists, and system programmers. It assumes a basic knowledge of mathematics in the areas of ESSL computation. It also assumes that users are familiar with Fortran, C, C++, and PL/I programming.

## How to Use This Book

**Front Matter** consists of the Table of Contents, Special Notices, and Preface. Use these to find or interpret information in the book.

**Part 1. “Guide Information”** provides guidance information for using ESSL. It covers the user-oriented tasks of learning, designing, coding, migrating, processing, and diagnosing. Use the following chapters when performing any of these tasks:

- **Chapter 1, “Learning about ESSL”** gives an introduction to ESSL, providing highlights and general information. Read this chapter first to determine the aspects of ESSL you want to use.
- **Chapter 2, “Designing Your Program”** provides ESSL-specific information that helps you design your program. Read this chapter before designing your program.
- **Chapter 3, “Setting Up Your Data Structures”** describes all types of data structures, such as vectors, matrices, and sequences. Use this information when designing and coding your program.
- **Chapter 4, “Coding Your Program”** tells you how to code your scalar and array data, how to code calls to ESSL in Fortran, C, C++, and PL/I programs, and how to do the coding necessary to handle errors. Use this information when coding your program.
- **Chapter 5, “Processing Your Program”** describes how to process your program under your particular operating system on your hardware. Use this information after you have coded your program and are ready to run it.
- **Chapter 6, “Migrating Your Programs”** explains all aspects of migration to ESSL, to this version of ESSL, to different processors, and to future releases and future processors. Read this chapter before starting to design your program.

- **Chapter 7, “Handling Problems”** provides diagnostic procedures for analyzing all ESSL problems. When you encounter a problem, use the symptom indexes at the beginning of this chapter to guide you to the appropriate diagnostic procedure.

**Part 2. “Reference Information”** provides reference information you need to code the ESSL calling sequences. It covers each of the mathematical areas of ESSL, and the utility subroutines. Each chapter begins with an introduction, followed by the subroutine descriptions. Each introduction applies to all the subroutines in that chapter and is especially important in planning your use of the subroutines and avoiding problems. To understand the information in the subroutine descriptions, see “How to Interpret the Subroutine Descriptions” on page xxx. Use the appropriate chapter when coding your program:

- **Chapter 8, “Linear Algebra Subprograms”**
- **Chapter 9, “Matrix Operations”**
- **Chapter 10, “Linear Algebraic Equations”**
- **Chapter 11, “Eigensystem Analysis”**
- **Chapter 12, “Fourier Transforms, Convolutions and Correlations, and Related Computations”**
- **Chapter 13, “Sorting and Searching”**
- **Chapter 14, “Interpolation”**
- **Chapter 15, “Numerical Quadrature”**
- **Chapter 16, “Random Number Generation”**
- **Chapter 17, “Utilities”**

**Appendix A. Basic Linear Algebra Subprograms** provides a list of the Level 1, 2, and 3 Basic Linear Algebra Subprograms (BLAS) included in ESSL.

**Appendix B. LAPACK** provides a list of the LAPACK subroutines included in ESSL.

**Glossary** contains definitions of terms used in this book.

**Bibliography** provides information about publications related to ESSL. Use it when you need more information than this book provides.

## How to Find a Subroutine Description

If you want to locate a subroutine description and you know the subroutine name, you can use the “Looking for a Subroutine?” on page xvii, following the Table of Contents. You can also find them listed individually or under the entry “subroutines, ESSL” in the Index.

## Where to Find Related Publications

If you have a question about the SP, PSSP, or a related product, the following online information resources make it easy to find the information you are looking for:

- If you have installed the RS/6000 SP Resource Center available with Parallel System Support Programs (PSSP) Version 3 Release 1 or later, you can access the SP Resource Center by issuing the command:

  **/usr/lpp/ssp/bin/resource_center**

  If you have the SP Resource Center on CD ROM, see the readme.txt file for information on how to run it.

- Access the RS/6000 Web site at:

  **http://www.rs6000.ibm.com**

A list of all ESSL publications, as well as related programming and hardware publications, are listed in the bibliography. Also included is a list of math background publications you may find helpful, along with the necessary information for ordering them from independent sources. See “Bibliography” on page BIB-1.

## How to Look Up a Bibliography Reference

Special references are made throughout this book to mathematical background publications and software libraries, available through IBM, publishers, or other companies. All of these are described in detail in the bibliography. A reference to one of these is made by using a bracketed number. The number refers to the item listed under that number in the bibliography. For example, reference [1] cites the first item listed in the bibliography.

## Special Terms

Standard data processing and mathematical terms are used in this book. Terminology is generally consistent with that used for Fortran. See the Glossary for definitions of terms used in this book.

### Short and Long Precision

Because ESSL can be used with more than one programming language, the terms **short precision** and **long precision** are used in place of the Fortran terms **single precision** and **double precision**.

### Subroutines and Subprograms

An ESSL **subroutine** is a named sequence of instructions within the ESSL product library whose execution is invoked by a call. A subroutine can be called in one or more user programs and at one or more times within each program. The ESSL subroutines are referred to as **subprograms** in the area of linear algebra subprograms. The term subprograms is used because it is consistent with the BLAS. Many of the linear algebra subprograms correspond to the BLAS; these are listed in Appendix A on page APA-1.

## How to Interpret the Subroutine Names with a Prefix Underscore

A name specified in this book with an underscore (_) prefix, such as _GEMUL, refers to all the versions of the subroutine with that name. To get the entire list of subroutines that name refers to, substitute the first letter for each version of the subroutine. For example, _GEMUL, refers to all versions of the matrix multiplication subroutine: SGEMUL, DGEMUL, CGEMUL, and ZGEMUL. You do **not** use the underscore in coding the names of the ESSL subroutines in your program. You code a complete name, such as SGEMUL. For details about these names, see “The Variety of Mathematical Functions” on page 4.

## Abbreviated Names

The abbreviated names used in this book are defined below.

| Short Name | Full Name |
|---|---|
| AIX | Advanced Interactive Executive |
| BLAS | Basic Linear Algebra Subprograms |
| ESSL | IBM Engineering and Scientific Subroutine Library |
| HTML | Hypertext Markup Language |
| LAPACK | Linear Algebra Package |
| PL/I | Programming Language/I |
| POWER, POWER2, POWER3, and PowerPC processors | RS/6000 processors |
| SL MATH | Subroutine Library—Mathematics |
| SMP | Symmetric Multi-Processing |
| SSP | Scientific Subroutine Package |

## Fonts

This book uses a variety of special fonts to distinguish between many mathematical and programming items. These are defined below.

| Special Font | Example | Description |
|---|---|---|
| Italic with no subscripts | *m*, *inc1x*, *aux*, *iopt* | Calling sequence argument or mathematical variable |
| Italic with subscripts | $x_1$, $a_{mn}$, $x_{j1,j2}$ | Element of a vector, matrix, or sequence |
| Bold italic lowercase | ***x***, ***y***, ***z*** | Vector or sequence |
| Bold italic uppercase | ***A***, ***B***, ***C*** | Matrix |
| Gothic uppercase | `A`, `B`, `C`, `AGB`<br>`IM=ISMAX(4,X,2)` | Array<br>Fortran statement |

# Special Notations and Conventions

This section explains the special notations and conventions used in this book to describe various types of data.

## Scalar Data

Following are the special notations used in the examples in this book for scalar data items. These notations are used to simplify the examples, and they do not imply usage of any precision. For a definition of scalar data in Fortran, C, C++, and PL/I, see Chapter 4 on page 111.

| **Data Item** | **Example** | **Description** |
|---|---|---|
| Character item | `'T'` | Character(s) in single quotation marks |
| Hexadecimal string | `X'97FA00C1'` | String of 4-bit hexadecimal characters |
| Logical item | `.TRUE.`<br>`.FALSE.` | True or false logical value, as indicated |
| Integer data | `1` | Number with no decimal point |
| Real data | `1.6` | Number with a decimal point |
| Complex data | `(1.0,−2.9)` | Real part followed by the imaginary part |
| Continuation | $1.666\overline{6}$ | Continue the last digit<br>(1.6666666... and so forth) |

## Vectors

A vector is represented as a single row or column of subscripted elements enclosed in square brackets. The subscripts refer to the element positions within the vector:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ . \\ x_n \end{bmatrix} \qquad \begin{bmatrix} x_1 & x_2 & x_3 & \ldots & x_n \end{bmatrix}$$

For a definition of vector, see “Vectors” on page 55.

## Matrices

A matrix is represented as a block of elements enclosed in square brackets. Subscripts refer to the row and column positions, respectively:

$$\begin{bmatrix} a_{11} \;.\;\;.\;\;.\; a_{1n} \\ . \qquad\qquad . \\ . \qquad\qquad . \\ . \qquad\qquad . \\ a_{m1} \;.\;\;.\;\;.\; a_{mn} \end{bmatrix}$$

For a definition of matrix, see “Matrices” on page 62.

## Sequences

Sequences are used in the areas of sorting, searching, Fourier transforms, convolutions, and correlations. For a definition of sequences, see “Sequences” on page 105.

### One-Dimensional Sequences

A one-dimensional sequence is represented as a series of elements enclosed in parentheses. Subscripts refer to the element position within the sequence:

$(x_1, x_2, x_3, ..., x_n)$

### Two-Dimensional Sequences

A two-dimensional sequence is represented as a series of columns of elements. (They are represented in the same way as a matrix without the square brackets.) Subscripts refer to the element positions within the first and second dimensions, respectively:

$$\begin{matrix} a_{11} & a_{12} \;.\;\;.\;\;. & a_{1n} \\ a_{21} & a_{22} & a_{2n} \\ . & . & . \\ . & . & . \\ . & . & . \\ a_{m1} & a_{m2} \;.\;\;.\;\;. & a_{mn} \end{matrix}$$

### Three-Dimensional Sequences

A three-dimensional sequence is represented as a series of blocks of elements. Subscripts refer to the elements positions within the first, second, and third dimensions, respectively:

$$\begin{array}{ccc} a_{111} & a_{121}\ .\ .\ . & a_{1n1} \\ a_{211} & a_{221} & a_{2n1} \\ . & . & . \\ . & . & . \\ . & . & . \\ a_{m11} & a_{m21}\ .\ .\ . & a_{mn1} \end{array} \qquad \begin{array}{ccc} a_{112} & a_{122}\ .\ .\ . & a_{1n2} \\ a_{212} & a_{222} & a_{2n2} \\ . & . & . \\ . & . & . \\ . & . & . \\ a_{m12} & a_{m22}\ .\ .\ . & a_{mn2} \end{array} \quad \cdots \quad \begin{array}{ccc} a_{11p} & a_{12p}\ .\ .\ . & a_{1np} \\ a_{21p} & a_{22p} & a_{2np} \\ . & . & . \\ . & . & . \\ . & . & . \\ a_{m1p} & a_{m2p}\ .\ .\ . & a_{mnp} \end{array}$$

# Arrays

Arrays contain vectors, matrices, or sequences. For a definition of array, see “How Do You Set Up Your Arrays?” on page 29.

## One-Dimensional Arrays

A one-dimensional array is represented as a single row of numeric elements enclosed in parentheses:

```
(1.0, 2.0, 3.0, 4.0, 5.0)
```

Elements not significant to the computation are usually not shown in the array. One dot appears for each element not shown. In the following array, five elements are significant to the computation, and two elements not used in the computation exist between each of the elements shown:

```
(1.0, . , . ,2.0, . , . ,3.0, . , . ,4.0, . , . ,5.0)
```

This notation is used to show vector elements inside an array.

## Two-Dimensional Arrays

A two-dimensional array is represented as a block of numeric elements enclosed in square brackets:

```
┌                         ┐
│ 1.0  11.0  5.0  25.0 │
│ 2.0  12.0  6.0  26.0 │
│ 3.0  13.0  7.0  27.0 │
│ 4.0  14.0  8.0  28.0 │
└                         ┘
```

Elements not significant to the computation are usually not shown in the array. One dot appears for each element not shown. The following array contains three rows and two columns not used in the computation:

```
┌                                 ┐
│ .    .    .    .    .    . │
│ .    .    .    .    .    . │
│ .   1.0  2.0  5.0  4.0   . │
│ .   2.0  3.0  6.0  3.0   . │
│ .   3.0  4.0  7.0  2.0   . │
│ .   4.0  5.0  8.0  1.0   . │
│ .    .    .    .    .    . │
└                                 ┘
```

This notation is used to show matrix elements inside an array.

### Three-Dimensional Arrays

A three-dimensional array is represented as a series of blocks of elements separated by ellipses. Each block appears like a two-dimensional array:

$$\begin{bmatrix} 1.0 & 11.0 & 5.0 & 25.0 \\ 2.0 & 12.0 & 6.0 & 26.0 \\ 3.0 & 13.0 & 7.0 & 27.0 \\ 4.0 & 14.0 & 8.0 & 28.0 \end{bmatrix} \begin{bmatrix} 10.0 & 111.0 & 15.0 & 125.0 \\ 20.0 & 112.0 & 16.0 & 126.0 \\ 30.0 & 113.0 & 17.0 & 127.0 \\ 40.0 & 114.0 & 18.0 & 128.0 \end{bmatrix} \ \ldots \ \begin{bmatrix} 100.0 & 11.0 & 15.0 & 25.0 \\ 200.0 & 12.0 & 16.0 & 26.0 \\ 300.0 & 13.0 & 17.0 & 27.0 \\ 400.0 & 14.0 & 18.0 & 28.0 \end{bmatrix}$$

Elements not significant to the computation are usually not shown in the array. One dot appears for each element not shown, just as for two-dimensional arrays.

## Special Characters, Symbols, Expressions, and Abbreviations

The mathematical and programming notations used in this book are consistent with traditional mathematical and programming usage. These conventions are explained below, along with special abbreviations that are associated with specific values.

| Item | Description |
|---|---|
| Greek letters: $\alpha$, $\sigma$, $\omega$, $\Omega$ | Symbolic scalar values |
| $\lvert a \rvert$ | The absolute value of *a* |
| $\boldsymbol{a} \bullet \boldsymbol{b}$ | The dot product of ***a*** and ***b*** |
| $x_i$ | The *i*-th element of vector ***x*** |
| $c_{ij}$ | The element in matrix ***C*** at row *i* and column *j* |
| $x_1 \ldots x_n$ | Elements from $x_1$ to $x_n$ |
| $i = 1, n$ | *i* is assigned the values 1 to *n* |
| $\boldsymbol{y} \leftarrow \boldsymbol{x}$ | Vector ***y*** is replaced by vector ***x*** |
| $\boldsymbol{xy}$ | Vector ***x*** times vector ***y*** |
| $\boldsymbol{AX} \cong \boldsymbol{B}$ | ***AX*** is congruent to ***B*** |
| $a^k$ | *a* raised to the *k* power |
| $e^x$ | Exponential function of x |
| $\boldsymbol{A}^{\mathrm{T}}$; $\boldsymbol{x}^{\mathrm{T}}$ | The transpose of matrix ***A***; the transpose of vector ***x*** |
| $\bar{\boldsymbol{x}}$; $\bar{A}$ | The complex conjugate of vector ***x***; the complex conjugate of matrix ***A*** |
| $\bar{x}_i$; $\bar{c}_{jk}$ | The complex conjugate of the complex vector element $x_i$, where:<br>$\text{if } x_i = (a_i, b_i), \quad \text{then } \bar{x}_i = (a_i, -b_i)$<br>The complex conjugate of the complex matrix element $c_{jk}$ |
| $\boldsymbol{x}^{\mathrm{H}}$; $\boldsymbol{A}^{\mathrm{H}}$ | The complex conjugate transpose of vector ***x***; the complex conjugate transpose of matrix ***A*** |

| Item | Description |
|---|---|
| $\sum_{i=1}^{n} x_i$ | The sum of elements $x_1$ to $x_n$ |
| $\sqrt{a+b}$ | The square root of *a*+*b* |
| $\int_a^b f(x)\,dx$ | The integral from *a* to *b* of *f(x) dx* |
| $\|\boldsymbol{x}\|_2$ | The Euclidean norm of vector ***x***, defined as: $$\sqrt{\sum_{j=1}^{n} \lvert x_j \rvert^2}$$ |
| $\|\boldsymbol{A}\|_1$ | The one norm of matrix ***A***, defined as: $$\max\left\{\sum_{i=1}^{m} \lvert a_{ij} \rvert,\ 1 \le j \le n\right\}$$ |
| $\|\boldsymbol{A}\|_2$ | The spectral norm of matrix ***A***, defined as: $$\max\{\|\boldsymbol{Ax}\|_2 : \|\boldsymbol{x}\|_2 = 1\}$$ |
| $\|\boldsymbol{A}\|_F$ | The Frobenius norm of matrix ***A***, defined as: $$\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^2}$$ |
| $\boldsymbol{A}^{-1}$ | The inverse of matrix ***A*** |
| $\boldsymbol{A}^{-T}$ | The transpose of ***A*** inverse |
| $\lvert\boldsymbol{A}\rvert$ | The determinant of matrix ***A*** |
| *m* by *n* matrix ***A*** | Matrix ***A*** has *m* rows and *n* columns |
| sin *a* | The sine of *a* |
| cos *b* | The cosine of *b* |
| SIGN (*a*) | The sign of *a*; the result is either + or – |
| address {*a*} | The storage address of *a* |
| max(***x***) | The maximum element in vector ***x*** |
| min(***x***) | The minimum element in vector ***x*** |
| ceiling(*x*) | The smallest integer that is greater than or equal to *x* |
| floor(*x*) | The largest integer that is not greater than *x* |
| int(*x*) | The largest integer that is less than or equal to *x* |
| *x* mod(*m*) | *x* modulo *m*; the remainder when *x* is divided by *m* |
| ∞ | Infinity |
| π | Pi, 3.14159265... |

## How to Interpret the Subroutine Descriptions

This section explains how to interpret the information in the subroutine descriptions in Part 2 of this book.

## Description

Each subroutine description begins with a brief explanation of what the subroutine does. When we combine the description of multiple versions of a subroutine, we give enough information to enable you to easily tell the differences among the subroutines. Differences usually occur in either the function performed or the data types required for each subroutine.

## Syntax

This shows the syntax for the Fortran, C, C++, and PL/I calling statements:

| | |
|---|---|
| **Fortran** | CALL NAME-1 \| NAME-2 \| ... \| NAME-n (*arg-1*, *arg-2*, ...  ,*arg-m*, ... ) |
| **C and C++** | name-1 \| name-2 \| ... \| name-n (*arg-1*, ...  ,*arg-m*); |
| **PL/I** | CALL NAME-1 \| NAME-2 \| ... \| NAME-n (*arg-1*, *arg-2*, ...  ,*arg-m*, ... ); |

The syntax indicates:

- The programming language (Fortran, C, C++, or PL/I)
- Each possible subroutine name that you can code in the calling sequence. Each name is separated by the **|** (or) symbol. You specify only one of these names in your calling sequence. (You do not code the **|** in the calling sequence.)
- The arguments, listed in the order in which you code them in the calling sequence. You must code them all in your calling sequence.

  You can distinguish between input arguments and output arguments by looking at the “On Entry” and “On Return” sections, respectively. An argument used for both input and output is described in both the “On Entry” and “On Return” sections. In this case, the input value for the argument is overlaid with the output value.

  The names of the arguments give an indication of the type of data that you should specify for the argument; for example:

  – Names beginning with the letters *i* through *n*, such as *m, incx, iopt*, and *isign*, indicate that you specify integer data.

  – Names beginning with the letters *a* through *h* and *o* through *z*, such as *b*, *t*, *alpha*, *sigma*, and *omega*, indicate that you specify real or complex data.

## On Entry

This lists the input arguments, which are the arguments you pass to the ESSL subroutine. Each argument description first gives the meaning of the argument, and then gives the form of data required for the argument. (To help you avoid errors, output arguments are also listed, along with a reference to the On Return section.)

## On Return

This lists the output arguments, which are the arguments passed back to your program from the ESSL subroutine. Each argument description first gives the meaning of the argument, and then gives the form of data passed back to your program for the argument.

## Notes

The notes describe any programming considerations and restrictions that apply to the arguments or the data for the arguments. There may be references to other parts of the book for further information.

## Function

This is a functional, or mathematical, description of the function performed by this subroutine. **It explains what computation is performed, not the implementation.** It explains the variations in the computation depending on the input arguments. References are made, where appropriate, to mathematical background books listed in the bibliography. References appear as a number enclosed in square brackets, where the number refers to the item listed under that number in the bibliography. For example, reference [1] cites the first item listed.

## Special Usage

These are unique ways you can use the subroutine in your application. In most cases, this book does not address applications of the ESSL subroutines; however, in special situations where the functional capability of the subroutine can be extended by following certain rules for its use, these rules are described in this section.

## Error Conditions

These are all the ESSL run-time errors that can occur in the subroutine. They are organized under three headings; Computational Errors, Input-Argument Errors, and Resource Errors. The return code values resulting from these errors are also explained.

## Examples

**The examples show how you would call the subroutine from a Fortran program.** They show a variety of uses of the subroutine. Except where it is important to show differences in use between the various versions of the subroutine, the simplest version of the subroutine is used in the examples. In most cases, this is the short-precision real version of the subroutine. Each example provides a description of the important features of the example, followed by the Fortran calling sequence, the input data, and the resulting output data.

# What's New for ESSL for AIX

This section summarizes all the changes made to ESSL for AIX.

## What's New for ESSL Version 3 Release 1.2

- The **ESSL POWER Library**, the **ESSL Thread-Safe Library**, and the **ESSL Symmetric Multi-Processing (SMP) Library** are tuned for the RS/6000 POWER3 SMP thin, wide, and high nodes.
- The serial and SMP performance of the packed Dense Linear Algebraic Equation subroutines, _PPF, _PPFCD, and _PPICD have been improved in many cases by utilizing new algorithms based on recursive packed storage format (See references [50], [63 ], and [64 ]).
- The SMP performance of the Dense Linear Algebraic Equation subroutine, DPOF, has been improved for a positive definite symmetric matrix stored in upper storage mode.
- Additional multi-threaded ESSL SMP subroutines are provided (see Table 21 on page 26).

## Changes for ESSL Version 3 Release 1.1

- The **ESSL POWER Library**, the **ESSL Thread-Safe Library**, and the **ESSL SMP Library** are tuned for the RS/6000 POWER3.
- The **ESSL POWER Library**, the **ESSL Thread-Safe Library**, and the **ESSL SMP Library** now support both 32-bit environment and 64-bit environment applications. For details on creating 64-bit environment applications see Chapter 4 on page 111 and Chapter 5 on page 163. If you are migrating to a 64-bit environment, you may need to make changes to your calls to **ERRSET**. For details see “ERRSET—ESSL ERRSET Subroutine for ESSL” on page 966.
- ESSL for AIX provides distinct libraries for AIX 4.2.1 and AIX 4.3.2:
  - The AIX 4.2.1 **ESSL Thread-Safe Library**, the **ESSL Thread-Safe POWER2 Library**, and the **ESSL SMP Library** were built using the pthreads draft 7 library supplied on AIX 4.2.1. This is the same as ESSL Version 3.1.
  - The AIX 4.3.2 **ESSL Thread-Safe Library**, the **ESSL Thread-Safe POWER2 Library**, and the **ESSL SMP Library** were built using the pthreads library that conforms to the IEEE POSIX 1003.1-1996 specification supplied on AIX 4.3.

## Changes for ESSL Version 3

- ESSL for AIX provides two new run-time libraries:
  - The **ESSL SMP Library** provides thread-safe versions of the ESSL subroutines for use on the RS/6000 SMP (for example 604e) processors. In addition, a subset of these subroutines are also multithreaded versions; that is, they support the shared memory parallel processing programming model. You do not have to change your existing application programs that

call ESSL to take advantage of the increased performance of using the SMP processors. You can simply re-link your existing programs. For a list of the multithreaded subroutines in the ESSL SMP Library, see Table 21 on page 26.

  - The **ESSL Thread-Safe Library** provides thread-safe versions of the ESSL subroutines for use on all RS/6000 processors. You may use this library to develop your own multithreaded applications.

    If your existing application program calls ESSL, you only need to re-link your program to take advantage of the increased performance of the ESSL SMP Library or to use the ESSL Thread-Safe Library.

- ESSL provides new subroutines (_GETRF and _GETRS), bringing the total number of subroutines to 458.
- For those ESSL subroutines that require extra working storage to perform computations, ESSL now provides a way to dynamically allocate storage when it does not need to persist after the subroutine call. See “Using Auxiliary Storage in ESSL” on page 32.
- The files for the Hypertext Markup Language (HTML) version of the *ESSL Version 3 Guide and Reference* are packaged with the ESSL product.
- All the ESSL messages are provided in an ESSL message catalog.

## Future Migration

If you are concerned with migration to possible future releases of ESSL or possible future hardware, you should read “Planning for Future Migration” on page 170. **That section explains what you can do now to prevent future migration problems.**

# In Brief—What's Provided in ESSL for AIX

- ESSL provides five run-time libraries:
  - The **ESSL SMP Library** provides thread-safe versions of the ESSL subroutines for use on the RS/6000 SMP (for example, 604e or 630) processors. In addition, a subset of these subroutines are also multithreaded versions; that is, they support the shared memory parallel processing programming model. You do not have to change your existing application programs that call ESSL to take advantage of the increased performance of using the SMP processors. You can simply re-link your existing application programs. For a list of the multithreaded subroutines in the ESSL SMP Library, see Table 21 on page 26.
  - The **ESSL Thread-Safe Library** provides thread-safe versions of the ESSL subroutines for use on all RS/6000 processors. You may use this library to develop your own multithreaded applications.
  - The **ESSL Thread-Safe POWER2 Library** provides thread-safe versions of the ESSL subroutines and is tuned for the RS/6000 POWER2 uniprocessors. You may use this library to develop your own multithreaded applications.
  - The **ESSL POWER Library** is tuned for the RS/6000 POWER, POWER3, and PowerPC uniprocessors.
  - The **ESSL POWER2 Library** is tuned for the RS/6000 POWER2 uniprocessors.

  All libraries are designed to provide high levels of performance for numerically intensive computing jobs on these respective processors. All versions provide mathematically equivalent results.

  The **ESSL POWER Library**, the **ESSL Thread-Safe Library**, and the **ESSL SMP Library** support both 32-bit environment and 64-bit environment applications.

- Callable from Fortran, C, C++, and PL/I programs.

- For a list of subroutines, refer to “List of ESSL Subroutines” on page 10.

# Part 1. Guide Information

This part of the book is organized into seven chapters, providing guidance information on how to use ESSL. It is organized as follows:

- Learning about ESSL
- Designing your program
- Setting up your data structures
- Coding your program
- Processing your program
- Migrating your programs
- Handling problems

# Chapter 1. Introduction and Requirements

This chapter introduces you to the Engineering and Scientific Subroutine Library (ESSL) for Advanced Interactive Executive (AIX*).

## Overview of ESSL

This section gives an overview of the ESSL capabilities and requirements.

ESSL is a state-of-the-art collection of subroutines providing a wide range of mathematical functions for many different scientific and engineering applications. Its primary characteristics are performance, functional capability, and usability.

## Performance and Functional Capability

The mathematical subroutines, in nine computational areas, are tuned for performance on the RS/6000*. The computational areas are:

- Linear Algebra Subprograms
- Matrix Operations
- Linear Algebraic Equations
- Eigensystem Analysis
- Fourier Transforms, Convolutions and Correlations, and Related Computations
- Sorting and Searching
- Interpolation
- Numerical Quadrature
- Random Number Generation

ESSL provides five run-time libraries:

- The **ESSL Symmetric Multi-Processing (SMP) Library** provides thread-safe versions of the ESSL subroutines for use on the RS/6000 SMP (for example, 604e or 630) processors. In addition, a subset of these subroutines are also multithreaded versions; that is, they support the shared memory parallel processing programming model. For a list of the multithreaded subroutines in the ESSL SMP Library, see Table 21 on page 26.
- The **ESSL Thread-Safe Library** provides thread-safe versions of the ESSL subroutines for use on all RS/6000 processors. You may use this library to develop your own multithreaded applications.
- The **ESSL Thread-Safe POWER2 Library** provides thread-safe versions of the ESSL subroutines and is tuned for the RS/6000 POWER2 processors. You may use this library to develop your own multithreaded applications.
- The **ESSL POWER Library** is tuned for the RS/6000 POWER, POWER3, and PowerPC uniprocessors.
- The **ESSL POWER2 Library** is tuned for the RS/6000 POWER2 uniprocessors.

All libraries are designed to provide high levels of performance for numerically intensive computing jobs on these respective processors. All versions provide mathematically equivalent results.

The **ESSL POWER Library**, the **ESSL Thread-Safe Library**, and the **ESSL SMP Library** support both 32-bit environment and 64-bit environment applications.

The ESSL subroutines can be called from application programs written in Fortran, C, C++, and Programming Language/I (PL/I). ESSL runs under the AIX operating system.

## Usability

ESSL is designed for usability:

- It has an easy-to-use call interface.
- If your existing application programs call ESSL, you only need to re-link your program to take advantage of the increased performance of the ESSL SMP Library or to use the ESSL Thread-Safe Library.
- It supports a 64-bit environment.

  64-bit applications can be created on any AIX 4.3.2 system, but can only run on 64-bit hardware.

  The data model used for a 64-bit environment is referred to as LP64. This data model supports 32-bit integers and 64-bit pointers. In accordance with the LP64 data model, all ESSL integer arguments remain 32-bits except for the **iusadr** argument for ERRSET. See “ERRSET—ESSL ERRSET Subroutine for ESSL” on page 966.
- It has informative error-handling capabilities, enabling you to calculate auxiliary storage sizes and transform lengths.
- An online book that can be displayed using an Hypertext Markup Language (HTML) document browser, is available for use with ESSL.

## The Variety of Mathematical Functions

This section describes the mathematical functions included in ESSL.

### Areas of Application

ESSL provides a variety of mathematical functions for many different types of scientific and engineering applications. Some of the industries using these applications are: Aerospace, Automotive, Electronics, Petroleum, Finance, Utilities, and Research. Examples of applications in these industries are:

Structural Analysis
Computational Chemistry
Fluid Dynamics Analysis
Seismic Analysis
Reservoir Modeling
Quantitative Analysis
Time Series Analysis
Computational Techniques
Mathematical Analysis
Dynamic Systems Simulation
Nuclear Engineering
Electronic Circuit Design

### What ESSL Provides

The subroutines provided in ESSL, summarized in Table 1, fall into the following groups:

- Nine major areas of mathematical computation, providing the computations commonly used by the industry applications listed above
- Utilities, performing general-purpose functions

**To help you select the ESSL subroutines that fulfill your needs for performance, accuracy, storage, and so forth, see “Selecting an ESSL Subroutine” on page 25.**

*Table 1. Summary of ESSL Subroutines*

| ESSL Area of Computation | Integer Subroutines | Short-Precision Subroutines | Long-Precision Subroutines |
|---|---|---|---|
| **Linear Algebra Subprograms**: | | | |
| • Vector-scalar | 0 | 41 | 41 |
| • Sparse vector-scalar | 0 | 11 | 11 |
| • Matrix-vector | 0 | 32 | 32 |
| • Sparse matrix-vector | 0 | 0 | 3 |
| **Matrix Operations**: Addition, subtraction, multiplications, rank-k updates, rank-2k updates, and matrix transposes | 0 | 25 | 26 |
| **Linear Algebraic Equations**: | | | |
| • Dense linear algebraic equations | 0 | 30 | 32 |
| • Banded linear algebraic equations | 0 | 18 | 18 |
| • Sparse linear algebraic equations | 0 | 0 | 11 |
| • Linear least squares | 0 | 3 | 3 |
| **Eigensystem Analysis**: Solutions to the algebraic eigensystem analysis problem and the generalized eigensystem analysis problem | 0 | 8 | 8 |
| **(Signal Processing) Computations**: | | | |
| • Fourier transforms | 0 | 15 | 11 |
| • Convolutions and correlations | 0 | 10 | 2 |
| • Related computations | 0 | 6 | 6 |
| **Sorting and Searching**: Sorting, sorting with index, and binary and sequential searching | 5 | 5 | 5 |
| **Interpolation**: Polynomial and cubic spline interpolation | 0 | 4 | 4 |
| **Numerical Quadrature**: Numerical quadrature on a set of points or on a function | 0 | 6 | 6 |
| **Random Number Generation**: Generating vectors of uniformly distributed and normally distributed random numbers | 0 | 3 | 3 |
| **Utilities**: General service operations | 11 | 0 | 3 |
| **Total ESSL Subroutines** | 16 | 217 | 225 |

## ESSL—Processing Capabilities

ESSL provides five run-time libraries, the **ESSL SMP Library**, the **ESSL Thread-Safe Library**, the **ESSL Thread-Safe POWER2 Library**, the **ESSL POWER2 Library**, and the **ESSL POWER Library**. These libraries are designed to provide high levels of performance for numerically intensive computing jobs on the RS/6000 processors. To order the IBM Engineering and Scientific Subroutine Library for AIX, Version 3 Release 1.1 , specify program number 5765-C42. Most of the subroutine calls are compatible with those in the ESSL/370 product. See the

*Engineering and Scientific Subroutine Library Version 2.2 Guide and Reference* for details.

## Accuracy of the Computations

ESSL provides accuracy comparable to libraries using equivalent algorithms with identical precision formats. Both short- and long-precision real versions of the subroutines are provided in most areas of ESSL. In some areas, short- and long-precision complex versions are also provided, and, occasionally, an integer version is provided. The data types operated on by the short-precision, long-precision, and integer versions of the subroutines are RS/6000 architecture precisions: ANSI/IEEE 32-bit and 64-bit binary floating-point format, and 32-bit integer. See the *ANSI/IEEE Standard for Binary Floating-Point Arithmetic, ANSI/IEEE Standard 754-1985*, for more detail. (There are ESSL-specific rules that apply to the results of computations on workstation processors using the ANSI/IEEE standards. For details, see “What Data Type Standards Are Used by ESSL, and What Exceptions Should You Know About?” on page 45.)

For more information on accuracy, see “Getting the Best Accuracy” on page 44.

## High Performance of ESSL

**Algorithms:** The ESSL subroutines have been designed to provide high performance. (See references [30], [41 ], and [42 ].) To achieve this performance, the subroutines use state-of-the-art algorithms tailored to specific operational characteristics of the hardware, such as cache size, Translation Lookaside Buffer (TLB) size, and page size.

Most subroutines use the following techniques to optimize performance:

- Managing the cache and TLB efficiently so the hit ratios are maximized; that is, data is blocked so it stays in the cache or TLB for its computation.
- Accessing data stored contiguously—that is, using stride-1 computations.
- Exploiting the large number of available floating-point registers.
- Using algorithms that minimize paging.
- On the PowerPC SMP processor:
  - The ESSL SMP Library is designed to exploit the processing power and shared memory of the SMP processor. In addition, a subset of the ESSL SMP subroutines have been coded to take advantage of increased performance from multithreaded (parallel) programming techniques. For a list of the multithreaded subroutines in the ESSL SMP Library, see Table 21 on page 26.
  - Choosing the number of threads depends on the problem size, the specific subroutine being called, and the number of physical processors you are running on. To achieve optimal performance, experimentation is necessary; however, picking the number of threads equal to the number of online processors generally provides good performance in most cases. In some cases, performance may increase if you choose the number of threads to be less than the number of online processors.

    You should use the the XL Fortran XLSMPOPTS environment variable to specify the number of threads you want to create.
- On the POWER3 processor:

  - Structuring the ESSL subroutines so, where applicable, the compiled code fully utilizes the dual floating-point execution units. Because two Multiply-Add instructions can be executed each cycle, neglecting overhead, this allows four floating-point operations per cycle to be performed.
  - Structuring the ESSL subroutines so, where applicable, the compiled code takes full advantage of the hardware data prefetching.
- On the POWER2 processor:
  - Structuring the ESSL subroutines so, where applicable, the compiled code fully utilizes the dual fixed-point and floating-point execution units. Because two Multiply-Add instructions can be executed each cycle, neglecting overhead, this allows four floating-point operations per cycle to be performed.
  - Structuring the ESSL subroutines so, where applicable, the compiled code uses the POWER2 Load and Store Floating Point Quad instructions. For example, in one cycle, two Load Floating Point Quad instructions can be executed. Neglecting overhead, this allows four doublewords to be loaded per cycle.
- On the POWER processor:
  - Using algorithms that balance floating-point operations with loads in the innermost loop.
  - Using algorithms that minimize stores in the innermost loops.
  - Structuring the ESSL subroutines so, where applicable, the compiled code uses the Multiply-Add instructions. Neglecting overhead, these instructions perform two floating-point operations per cycle.

**Mathematical Techniques:** All areas of ESSL use state-of-the-art mathematical techniques to achieve high performance. For example, the matrix-vector linear algebra subprograms operate on a higher-level data structure, matrix-vector rather than vector-scalar. As a result, they optimize performance directly for your program and indirectly through those ESSL subroutines using them.

### The Fortran Language Interface to the Subroutines

The ESSL subroutines follow standard Fortran calling conventions and must run in the Fortran run-time environment. When ESSL subroutines are called from a program in a language other than Fortran, such as C, C++, or PL/I, the Fortran conventions must be used. This applies to all aspects of the interface, such as the linkage conventions and the data conventions. For example, array ordering must be consistent with Fortran array ordering techniques. Data and linkage conventions for each language are given in Chapter 4 on page 111.

## Software and Hardware Products That Can Be Used with ESSL

This section describes the hardware and software products you can use with ESSL, as well as those products for installing ESSL and displaying the online documentation.

## For ESSL—Hardware

ESSL runs on the IBM* RS/6000 processors supported by the AIX operating systems.

64-bit applications require 64-bit hardware.

## ESSL—Operating Systems

ESSL is supported in the following operating system environments:

- AIX Version 4.2.1 or later modification levels (either product number 5765-655 or 5765-C34)
- AIX Version 4.3.2 or later modification levels (product number 5765-C34)

## ESSL—Software Products

ESSL requires the software products shown in Table 2 for compiling and running.

To assist C and C++ users, an ESSL header file is provided. Use of this file is described in “C Programs” on page 129 and “C++ Programs” on page 145.

*Table 2. Software Products Required for Use with ESSL*

| For Compiling | For Linking, Loading, or Running |
|---|---|
| XL Fortran for AIX, Version 5.1.1 or later (program number 5808-AAR part number 04L2110) *–or–*<br>XL High Performance Fortran for AIX, Version 1.3.1 or later (program number 5765-613) *–or–*<br>IBM C, C++ compilers Version 3.6.4 or later [2] *–or–*<br>C for AIX, Version 4.4 or later (program number 5765-C64 part number 31L0497 with feature number 0002) *–or–*<br>PL/I Set for AIX, Version 1.1 or later (program number 5765-549) | XL Fortran Run-Time Environment for AIX, Version 5.1.1 or later (program number 5808-AAR part number 04L2123) *–or–*<br>XL High Performance Run-Time Environment for AIX, Version 1.3.1 or later (program number 5765-612) **–and–**<br>C libraries[1] |

[1] The AIX Version 4 product includes the C and math libraries in the Application Development Toolkit.

[2] Available as a component of the VisualAge C++ Professional for AIX, Version 4, product.

## Installation and Customization Products

The ESSL licensed program is distributed on a 4-millimeter or an 8-millimeter cartridge. The *ESSL Version 3 Installation Memo* provides the detailed information you need to install ESSL.

The ESSL product is packaged according to the AIX guidelines, as described in the *IBM AIX Version 4 General Programming Concepts: Writing and Debugging Programs* manual. The product can be installed using the `smit` command, as described in the *IBM AIX Version 4 System Management Guide: Operating System and Devices* manual.

## Software Products for Displaying ESSL Online Information

The *ESSL Guide and Reference Version 3* is available in PostScript and HTML format on the product media.

To view the online publications shipped on the product media, you need the following:

- Access to a common HTML document browser (such as Netscape Navigator).
- The location of the HTML index file provided with the file sets. Contact your system administrator or installer for this location.

## ESSL Internet Resources

This section describes how you can use the ESSL resources available over the Internet.

### Obtaining Documentation

The *ESSL Version 3 Guide and Reference* is available in PDF and HTML format at the IBM RS/6000 Web site at:

**http://www.rs6000.ibm.com/resource/aix_resource/sp_books**

To view the ESSL PDF publication, you need access to the Adobe Acrobat Reader 3.0.1. The Acrobat Reader is shipped with the AIX Version 4.3 Bonus Pack and is also freely available for downloading from the Adobe web site at:

**http://www.adobe.com**.

### Accessing ESSL's Product Home Pages

The following home pages contain information on ESSL and Parallel ESSL:

- For ESSL for AIX, use:

  **http://www.rs6000.ibm.com/software/Apps/essl.html**
- For Parallel ESSL, use:

  **http://www.rs6000.ibm.com/software/sp_products/esslpara.html**

## Getting on the ESSL Mailing List

Information concerning ESSL's home pages and other home pages available for the RS/6000 family of products, plus late breaking information about ESSL, can be obtained by being placed on the ESSL mailing list. In addition, users on the mailing list will receive information about new ESSL function and may receive customer satisfaction surveys and requirements surveys, to provide feedback to ESSL Development on the product and user requirements.

You can be placed on the mailing list by sending a request to either of the following, asking to be placed on the ESSL mailing list:

International Business Machines Corporation
ESSL Development
Department LQJA / MS P963
522 South Rd.

Poughkeepsie, N.Y. 12601-5400

e-mail: essl@us.ibm.com

**Note:** You should send us e-mail if you would like to be withdrawn from the ESSL mailing list.

When requesting to be placed on the mailing list or asking any questions, please provide the following information:

- Your name
- The name of your company
- Your mailing address
- Your Internet address
- Your phone number

## List of ESSL Subroutines

This section provides an overview of the subroutines in each of the areas of ESSL.

Appendix A on page APA-1 contains a list of Level 1, 2, and 3 Basic Linear Algebra Subprograms (BLAS) included in ESSL.

Appendix B on page APB-1 contains a list of Linear Algebra Package (LAPACK) subroutines included in ESSL.

## Linear Algebra Subprograms

The linear algebra subprograms consist of:

- Vector-scalar linear algebra subprograms (Table 3 )
- Sparse vector-scalar linear algebra subprograms (Table 4 )
- Matrix-vector linear algebra subprograms (Table 5 )
- Sparse matrix-vector linear algebra subprograms (Table 6 )

**Notes:**

1. The term **subprograms** is used to be consistent with the Basic Linear Algebra Subprograms (BLAS), because many of these subprograms correspond to the BLAS.
2. Some of the linear algebra subprograms were designed in accordance with the Level 1 and Level 2 BLAS de facto standard. If these subprograms do not comply with the standard as approved, IBM will consider updating them to do so. If IBM updates these subprograms, the updates could require modifications of the calling application program.

### Vector-Scalar Linear Algebra Subprograms

The vector-scalar linear algebra subprograms include a subset of the standard set of Level 1 BLAS. For details on the BLAS, see reference [76]. The remainder of the vector-scalar linear algebra subprograms are commonly used computations provided for your applications. Both real and complex versions of the subprograms are provided.

*Table 3 (Page 1 of 2). List of Vector-Scalar Linear Algebra Subprograms*

| Descriptive Name | Short-Precision Subprogram | Long-Precision Subprogram | Page |
|---|---|---|---|
| Position of the First or Last Occurrence of the Vector Element Having the Largest Magnitude | ISAMAX†■<br>ICAMAX†■ | IDAMAX†■<br>IZAMAX†■ | 201 |
| Position of the First or Last Occurrence of the Vector Element Having Minimum Absolute Value | ISAMIN† | IDAMIN† | 204 |
| Position of the First or Last Occurrence of the Vector Element Having Maximum Value | ISMAX† | IDMAX† | 207 |
| Position of the First or Last Occurrence of the Vector Element Having Minimum Value | ISMIN† | IDMIN† | 210 |
| Sum of the Magnitudes of the Elements in a Vector | SASUM†■<br>SCASUM†■ | DASUM†■<br>DZASUM†■ | 213 |
| Multiply a Vector X by a Scalar, Add to a Vector Y, and Store in the Vector Y | SAXPY■<br>CAXPY■ | DAXPY■<br>ZAXPY■ | 216 |
| Copy a Vector | SCOPY■<br>CCOPY■ | DCOPY■<br>ZCOPY■ | 219 |
| Dot Product of Two Vectors | SDOT†■<br>CDOTU†■<br>CDOTC†■ | DDOT†■<br>ZDOTU†■<br>ZDOTC†■ | 222 |
| Compute SAXPY or DAXPY N Times | SNAXPY | DNAXPY | 226 |
| Compute Special Dot Products N Times | SNDOT | DNDOT | 231 |
| Euclidean Length of a Vector with Scaling of Input to Avoid Destructive Underflow and Overflow | SNRM2†■<br>SCNRM2†■ | DNRM2†■<br>DZNRM2†■ | 236 |
| Euclidean Length of a Vector with No Scaling of Input | SNORM2†<br>CNORM2† | DNORM2†<br>ZNORM2† | 239 |
| Construct a Givens Plane Rotation | SROTG■<br>CROTG■ | DROTG■<br>ZROTG■ | 242 |
| Apply a Plane Rotation | SROT■<br>CROT■<br>CSROT■ | DROT■<br>ZROT■<br>ZDROT■ | 249 |
| Multiply a Vector X by a Scalar and Store in the Vector X | SSCAL■<br>CSCAL■<br>CSSCAL■ | DSCAL■<br>ZSCAL■<br>ZDSCAL■ | 253 |
| Interchange the Elements of Two Vectors | SSWAP■<br>CSWAP■ | DSWAP■<br>ZSWAP■ | 256 |
| Add a Vector X to a Vector Y and Store in a Vector Z | SVEA<br>CVEA | DVEA<br>ZVEA | 259 |
| Subtract a Vector Y from a Vector X and Store in a Vector Z | SVES<br>CVES | DVES<br>ZVES | 263 |
| Multiply a Vector X by a Vector Y and Store in a Vector Z | SVEM<br>CVEM | DVEM<br>ZVEM | 267 |
| Multiply a Vector X by a Scalar and Store in a Vector Y | SYAX<br>CYAX<br>CSYAX | DYAX<br>ZYAX<br>ZDYAX | 271 |
| Multiply a Vector X by a Scalar, Add to a Vector Y, and Store in a Vector Z | SZAXPY<br>CZAXPY | DZAXPY<br>ZZAXPY | 274 |

*Table 3 (Page 2 of 2). List of Vector-Scalar Linear Algebra Subprograms*

| Descriptive Name | Short-Precision Subprogram | Long-Precision Subprogram | Page |
|---|---|---|---|
| † This subprogram is invoked as a function in a Fortran program.<br>▪ Level 1 BLAS | | | |

## Sparse Vector-Scalar Linear Algebra Subprograms

The sparse vector-scalar linear algebra subprograms operate on sparse vectors; that is, only the nonzero elements of the vector are stored. These subprograms provide similar functions to the vector-scalar subprograms. These subprograms represent a subset of the sparse extensions to the Level 1 BLAS described in reference [29]. Both real and complex versions of the subprograms are provided.

*Table 4. List of Sparse Vector-Scalar Linear Algebra Subprograms*

| Descriptive Name | Short-Precision Subprogram | Long-Precision Subprogram | Page |
|---|---|---|---|
| Scatter the Elements of a Sparse Vector X in Compressed-Vector Storage Mode into Specified Elements of a Sparse Vector Y in Full-Vector Storage Mode | SSCTR<br>CSCTR | DSCTR<br>ZSCTR | 279 |
| Gather Specified Elements of a Sparse Vector Y in Full-Vector Storage Mode into a Sparse Vector X in Compressed-Vector Storage Mode | SGTHR<br>CGTHR | DGTHR<br>ZGTHR | 282 |
| Gather Specified Elements of a Sparse Vector Y in Full-Vector Mode into a Sparse Vector X in Compressed-Vector Mode, and Zero the Same Specified Elements of Y | SGTHRZ<br>CGTHRZ | DGTHRZ<br>ZGTHRZ | 285 |
| Multiply a Sparse Vector X in Compressed-Vector Storage Mode by a Scalar, Add to a Sparse Vector Y in Full-Vector Storage Mode, and Store in the Vector Y | SAXPYI<br>CAXPYI | DAXPYI<br>ZAXPYI | 288 |
| Dot Product of a Sparse Vector X in Compressed-Vector Storage Mode and a Sparse Vector Y in Full-Vector Storage Mode | SDOTI†<br>CDOTCI†<br>CDOTUI† | DDOTI†<br>ZDOTCI†<br>ZDOTUI† | 291 |
| † This subprogram is invoked as a function in a Fortran program. | | | |

## Matrix-Vector Linear Algebra Subprograms

The matrix-vector linear algebra subprograms operate on a higher-level data structure—matrix-vector rather than vector-scalar—using optimized algorithms to improve performance. These subprograms include a subset of the standard set of Level 2 BLAS. For details on the Level 2 BLAS, see [34] and [35 ]. Both real and complex versions of the subprograms are provided.

*Table 5. List of Matrix-Vector Linear Algebra Subprograms*

| Descriptive Name | Short-Precision Subprogram | Long-Precision Subprogram | Page |
|---|---|---|---|
| Matrix-Vector Product for a General Matrix, Its Transpose, or Its Conjugate Transpose | SGEMV◂<br>CGEMV◂<br>SGEMX§<br>SGEMTX§ | DGEMV◂<br>ZGEMV◂<br>DGEMX§<br>DGEMTX§ | 296 |
| Rank-One Update of a General Matrix | SGER◂<br>CGERU◂<br>CGERC◂ | DGER◂<br>ZGERU◂<br>ZGERC◂ | 307 |
| Matrix-Vector Product for a Real Symmetric or Complex Hermitian Matrix | SSPMV◂<br>CHPMV◂<br>SSYMV◂<br>CHEMV◂<br>SSLMX§ | DSPMV◂<br>ZHPMV◂<br>DSYMV◂<br>ZHEMV◂<br>DSLMX§ | 315 |
| Rank-One Update of a Real Symmetric or Complex Hermitian Matrix | SSPR◂<br>CHPR◂<br>SSYR◂<br>CHER◂<br>SSLR1§ | DSPR◂<br>ZHPR◂<br>DSYR◂<br>ZHER◂<br>DSLR1§ | 323 |
| Rank-Two Update of a Real Symmetric or Complex Hermitian Matrix | SSPR2◂<br>CHPR2◂<br>SSYR2◂<br>CHER2◂<br>SSLR2§ | DSPR2◂<br>ZHPR2◂<br>DSYR2◂<br>ZHER2◂<br>DSLR2§ | 331 |
| Matrix-Vector Product for a General Band Matrix, Its Transpose, or Its Conjugate Transpose | SGBMV◂<br>CGBMV◂ | DGBMV◂<br>ZGBMV◂ | 340 |
| Matrix-Vector Product for a Real Symmetric or Complex Hermitian Band Matrix | SSBMV◂<br>CHBMV◂ | DSBMV◂<br>ZHBMV◂ | 347 |
| Matrix-Vector Product for a Triangular Matrix, Its Transpose, or Its Conjugate Transpose | STRMV◂<br>CTRMV◂<br>STPMV◂<br>CTPMV◂ | DTRMV◂<br>ZTRMV◂<br>DTPMV◂<br>ZTPMV◂ | 352 |
| Matrix-Vector Product for a Triangular Band Matrix, Its Transpose, or Its Conjugate Transpose | STBMV◂<br>CTBMV◂ | DTBMV◂<br>ZTBMV◂ | 358 |

◂ Level 2 BLAS

§ This subroutine is provided only for migration from earlier releases of ESSL and is not intended for use in new programs.

## Sparse Matrix-Vector Linear Algebra Subprograms

The sparse matrix-vector linear algebra subprograms operate on sparse matrices; that is, only the nonzero elements of the matrix are stored. These subprograms provide similar functions to the matrix-vector subprograms.

*Table 6 (Page 1 of 2). List of Sparse Matrix-Vector Linear Algebra Subprograms*

| Descriptive Name | Long- Precision Subprogram | Page |
|---|---|---|
| Matrix-Vector Product for a Sparse Matrix in Compressed-Matrix Storage Mode | DSMMX | 365 |

*Table 6 (Page 2 of 2). List of Sparse Matrix-Vector Linear Algebra Subprograms*

| Descriptive Name | Long- Precision Subprogram | Page |
|---|---|---|
| Transpose a Sparse Matrix in Compressed-Matrix Storage Mode | DSMTM | 368 |
| Matrix-Vector Product for a Sparse Matrix or Its Transpose in Compressed-Diagonal Storage Mode | DSDMX | 372 |

## Matrix Operations

Some of the matrix operation subroutines were designed in accordance with the Level 3 BLAS de facto standard. If these subroutines do not comply with the standard as approved, IBM will consider updating them to do so. If IBM updates these subroutines, the updates could require modifications of the calling application program. For details on the Level 3 BLAS, see reference [32]. The matrix operation subroutines also include the commonly used matrix operations: addition, subtraction, multiplication, and transposition.

*Table 7 (Page 1 of 2). List of Matrix Operation Subroutines*

| Descriptive Name | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|
| Matrix Addition for General Matrices or Their Transposes | SGEADD<br>CGEADD | DGEADD<br>ZGEADD | 381 |
| Matrix Subtraction for General Matrices or Their Transposes | SGESUB<br>CGESUB | DGESUB<br>ZGESUB | 388 |
| Matrix Multiplication for General Matrices, Their Transposes, or Conjugate Transposes | SGEMUL<br>CGEMUL | DGEMUL<br>ZGEMUL<br>DGEMLP§ | 395 |
| Matrix Multiplication for General Matrices, Their Transposes, or Conjugate Transposes Using Winograd's Variation of Strassen's Algorithm | SGEMMS<br>CGEMMS | DGEMMS<br>ZGEMMS | 405 |
| Combined Matrix Multiplication and Addition for General Matrices, Their Transposes, or Conjugate Transposes | SGEMM♦<br>CGEMM♦ | DGEMM♦<br>ZGEMM♦ | 411 |
| Matrix-Matrix Product Where One Matrix is Real or Complex Symmetric or Complex Hermitian | SSYMM♦<br>CSYMM♦<br>CHEMM♦ | DSYMM♦<br>ZSYMM♦<br>ZHEMM♦ | 420 |
| Triangular Matrix-Matrix Product | STRMM♦<br>CTRMM♦ | DTRMM♦<br>ZTRMM♦ | 428 |
| Rank-K Update of a Real or Complex Symmetric or a Complex Hermitian Matrix | SSYRK♦<br>CSYRK♦<br>CHERK♦ | DSYRK♦<br>ZSYRK♦<br>ZHERK♦ | 435 |
| Rank-2K Update of a Real or Complex Symmetric or a Complex Hermitian Matrix | SSYR2K♦<br>CSYR2K♦<br>CHER2K♦ | DSYR2K♦<br>ZSYR2K♦<br>ZHER2K♦ | 442 |
| General Matrix Transpose (In-Place) | SGETMI<br>CGETMI | DGETMI<br>ZGETMI | 450 |
| General Matrix Transpose (Out-of-Place) | SGETMO<br>CGETMO | DGETMO<br>ZGETMO | 453 |

*Table 7 (Page 2 of 2). List of Matrix Operation Subroutines*

| Descriptive Name | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|
| [♦] Level 3 BLAS | | | |
| [§] This subroutine is provided only for migration from earlier release of ESSL and is not intended for use in new programs. | | | |

## Linear Algebraic Equations

The linear algebraic equations consist of:

- Dense linear algebraic equations (Table 8 )
- Banded linear algebraic equations (Table 9 )
- Sparse linear algebraic equations (Table 10 )
- Linear least squares (Table 11 )

**Notes:**

1. Some of the linear algebraic equations were designed in accordance with the Level 2 BLAS, Level 3 BLAS, and LAPACK de facto standard. If these subprograms do not comply with the standard as approved, IBM will consider updating them to do so. If IBM updates these subprograms, the updates could require modifications of the calling application program. For details on the Level 2 and 3 BLAS, see [32] and [34 ]. For details on LAPACK, see [8].

### Dense Linear Algebraic Equations

The dense linear algebraic equation subroutines provide solutions to linear systems of equations for both real and complex general matrices and their transposes, positive definite real symmetric and complex Hermitian matrices, and triangular matrices. Some of these subroutines correspond to the Level 2 BLAS, Level 3 BLAS, and LAPACK routines described in references [32], [34 ], and [8 ].

*Table 8 (Page 1 of 2). List of Dense Linear Algebraic Equation Subroutines*

| Descriptive Name | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|
| General Matrix Factorization | SGEF<br>CGEF<br>SGETRF[Δ]<br>CGETRF[Δ] | DGEF<br>ZGEF<br>DGETRF[Δ]<br>ZGETRF[Δ]<br>DGEFP[§] | 466<br><br>479 |
| General Matrix, Its Transpose, or Its Conjugate Transpose Solve | SGES<br>CGES | DGES<br>ZGES | 469 |
| General Matrix, Its Transpose, or Its Conjugate Transpose Multiple Right-Hand Side Solve | SGESM<br>CGESM<br>SGETRS[Δ]<br>CGETRS[Δ] | DGESM<br>ZGESM<br>DGETRS[Δ]<br>ZGETRS[Δ] | 473<br>483 |
| General Matrix Factorization, Condition Number Reciprocal, and Determinant | SGEFCD | DGEFCD | 488 |

*Table 8 (Page 2 of 2). List of Dense Linear Algebraic Equation Subroutines*

| Descriptive Name | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|
| Positive Definite Real Symmetric or Complex Hermitian Matrix Factorization | SPPF<br>SPOF<br>CPOF | DPPF<br>DPOF<br>ZPOF<br>DPPFP§ | 492 |
| Positive Definite Real Symmetric Matrix Solve | SPPS | DPPS | 500 |
| Positive Definite Real Symmetric or Complex Hermitian Matrix Multiple Right-Hand Side Solve | SPOSM<br>CPOSM | DPOSM<br>ZPOSM | 503 |
| Positive Definite Real Symmetric Matrix Factorization, Condition Number Reciprocal, and Determinant | SPPFCD<br>SPOFCD | DPPFCD<br>DPOFCD | 508 |
| General Matrix Inverse, Condition Number Reciprocal, and Determinant | SGEICD | DGEICD | 515 |
| Positive Definite Real Symmetric Matrix Inverse, Condition Number Reciprocal, and Determinant | SPPICD<br>SPOICD | DPPICD<br>DPOICD | 520 |
| Solution of a Triangular System of Equations with a Single Right-Hand Side | STRSV◂<br>CTRSV◂<br>STPSV◂<br>CTPSV◂ | DTRSV◂<br>ZTRSV◂<br>DTPSV◂<br>ZTPSV◂ | 527 |
| Solution of Triangular Systems of Equations with Multiple Right-Hand Sides | STRSM♦<br>CTRSM♦ | DTRSM♦<br>ZTRSM♦ | 533 |
| Triangular Matrix Inverse | STRI<br>STPI | DTRI<br>DTPI | 541 |

◂ Level 2 BLAS

♦ Level 3 BLAS

Δ LAPACK

§ This subroutine is provided only for migration from earlier releases of ESSL and is not intended for use in new programs. Documentation for this subroutine is no longer provided.

## Banded Linear Algebraic Equations

The banded linear algebraic equation subroutines provide solutions to linear systems of equations for real general band matrices, real positive definite symmetric band matrices, real or complex general tridiagonal matrices, real positive definite symmetric tridiagonal matrices, and real or complex triangular band matrices.

*Table 9 (Page 1 of 2). List of Banded Linear Algebraic Equation Subroutines*

| Descriptive Name | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|
| General Band Matrix Factorization | SGBF | DGBF | 547 |
| General Band Matrix Solve | SGBS | DGBS | 551 |
| Positive Definite Symmetric Band Matrix Factorization | SPBF<br>SPBCHF | DPBF<br>DPBCHF | 554 |

*Table 9 (Page 2 of 2). List of Banded Linear Algebraic Equation Subroutines*

| Descriptive Name | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|
| Positive Definite Symmetric Band Matrix Solve | SPBS<br>SPBCHS | DPBS<br>DPBCHS | 558 |
| General Tridiagonal Matrix Factorization | SGTF | DGTF | 561 |
| General Tridiagonal Matrix Solve | SGTS | DGTS | 564 |
| General Tridiagonal Matrix Combined Factorization and Solve with No Pivoting | SGTNP<br>CGTNP | DGTNP<br>ZGTNP | 566 |
| General Tridiagonal Matrix Factorization with No Pivoting | SGTNPF<br>CGTNPF | DGTNPF<br>ZGTNPF | 569 |
| General Tridiagonal Matrix Solve with No Pivoting | SGTNPS<br>CGTNPS | DGTNPS<br>ZGTNPS | 572 |
| Positive Definite Symmetric Tridiagonal Matrix Factorization | SPTF | DPTF | 575 |
| Positive Definite Symmetric Tridiagonal Matrix Solve | SPTS | DPTS | 577 |
| Triangular Band Equation Solve | STBSV◂<br>CTBSV◂ | DTBSV◂<br>ZTBSV◂ | 579 |
| ◂ Level 2 BLAS | | | |

## Sparse Linear Algebraic Equations

The sparse linear algebraic equation subroutines provide direct and iterative solutions to linear systems of equations both for general sparse matrices and their transposes and for sparse symmetric matrices.

*Table 10 (Page 1 of 2). List of Sparse Linear Algebraic Equation Subroutines*

| Descriptive Name | Long- Precision Subroutine | Page |
|---|---|---|
| General Sparse Matrix Factorization Using Storage by Indices, Rows, or Columns | DGSF | 586 |
| General Sparse Matrix or Its Transpose Solve Using Storage by Indices, Rows, or Columns | DGSS | 592 |
| General Sparse Matrix or Its Transpose Factorization, Determinant, and Solve Using Skyline Storage Mode | DGKFS<br>DGKFSP§ | 596 |
| Symmetric Sparse Matrix Factorization, Determinant, and Solve Using Skyline Storage Mode | DSKFS<br>DSKFSP§ | 614 |
| Iterative Linear System Solver for a General or Symmetric Sparse Matrix Stored by Rows | DSRIS | 633 |
| Sparse Positive Definite or Negative Definite Symmetric Matrix Iterative Solve Using Compressed-Matrix Storage Mode | DSMCG‡ | 644 |
| Sparse Positive Definite or Negative Definite Symmetric Matrix Iterative Solve Using Compressed-Diagonal Storage Mode | DSDCG | 652 |
| General Sparse Matrix Iterative Solve Using Compressed-Matrix Storage Mode | DSMGCG‡ | 660 |
| General Sparse Matrix Iterative Solve Using Compressed-Diagonal Storage Mode | DSDGCG | 667 |

*Table 10 (Page 2 of 2). List of Sparse Linear Algebraic Equation Subroutines*

| Descriptive Name | Long- Precision Subroutine | Page |
|---|---|---|
| § This subroutine is provided only for migration from earlier releases of ESSL and is not intended for use in new programs. Documentation for this subroutine is no longer provided.<br>‡ This subroutine is provided only for migration from earlier releases of ESSL and is not intended for use in new programs. Use DSRIS instead. | | |

### Linear Least Squares

The linear least squares subroutines provide least squares solutions to linear systems of equations for real general matrices. Two methods are provided: one that uses the singular value decomposition and another that uses a QR decomposition with column pivoting.

*Table 11. List of Linear Least Squares Subroutines*

| Descriptive Name | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|
| Singular Value Decomposition for a General Matrix | SGESVF | DGESVF | 675 |
| Linear Least Squares Solution for a General Matrix Using the Singular Value Decomposition | SGESVS | DGESVS | 683 |
| Linear Least Squares Solution for a General Matrix Using a QR Decomposition with Column Pivoting | SGELLS | DGELLS | 688 |

## Eigensystem Analysis

The eigensystem analysis subroutines provide solutions to the algebraic eigensystem analysis problem $\boldsymbol{Az} = w\boldsymbol{z}$ and the generalized eigensystem analysis problem $\boldsymbol{Az} = w\boldsymbol{Bz}$ (Table 12 ). Many of the eigensystem analysis subroutines use the algorithms presented in *Linear Algebra* by Wilkinson and Reinsch [96 ] or use adaptations of EISPACK routines, as described in the *EISPACK Guide Lecture Notes in Computer Science* in reference [84 ] or in the *EISPACK Guide Extension Lecture Notes in Computer Science* in reference [56 ]. (EISPACK is available from the sources listed in reference [49].)

*Table 12 (Page 1 of 2). List of Eigensystem Analysis Subroutines*

| Descriptive Name | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|
| Eigenvalues and, Optionally, All or Selected Eigenvectors of a General Matrix | SGEEV<br>CGEEV | DGEEV<br>ZGEEV | 698 |
| Eigenvalues and, Optionally, the Eigenvectors of a Real Symmetric Matrix or a Complex Hermitian Matrix | SSPEV<br>CHPEV | DSPEV<br>ZHPEV | 709 |
| Extreme Eigenvalues and, Optionally, the Eigenvectors of a Real Symmetric Matrix or a Complex Hermitian Matrix | SSPSV<br>CHPSV | DSPSV<br>ZHPSV | 718 |
| Eigenvalues and, Optionally, the Eigenvectors of a Generalized Real Eigensystem, Az=wBz, where A and B Are Real General Matrices | SGEGV | DGEGV | 726 |

*Table 12 (Page 2 of 2). List of Eigensystem Analysis Subroutines*

| Descriptive Name | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|
| Eigenvalues and, Optionally, the Eigenvectors of a Generalized Real Symmetric Eigensystem, Az=wBz, where A Is Real Symmetric and B Is Real Symmetric Positive Definite | SSYGV | DSYGV | 732 |

## Fourier Transforms, Convolutions and Correlations, and Related Computations

This signal processing area provides:

- Fourier transform subroutines (Table 13 )
- Convolution and correlation subroutines (Table 14 )
- Related-computation subroutines (Table 15 )

### Fourier Transforms

The Fourier transform subroutines perform mixed-radix transforms in one, two, and three dimensions.

*Table 13. List of Fourier Transform Subroutines*

| Descriptive Name | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|
| Complex Fourier Transform | SCFT<br>SCFTP§ | DCFT | 749 |
| Real-to-Complex Fourier Transform | SRCFT | DRCFT | 757 |
| Complex-to-Real Fourier Transform | SCRFT | DCRFT | 765 |
| Cosine Transform | SCOSF<br>SCOSFT§ | DCOSF | 773 |
| Sine Transform | SSINF | DSINF | 780 |
| Complex Fourier Transform in Two Dimensions | SCFT2<br>SCFT2P§ | DCFT2 | 787 |
| Real-to-Complex Fourier Transform in Two Dimensions | SRCFT2 | DRCFT2 | 794 |
| Complex-to-Real Fourier Transform in Two Dimensions | SCRFT2 | DCRFT2 | 801 |
| Complex Fourier Transform in Three Dimensions | SCFT3<br>SCFT3P§ | DCFT3 | 809 |
| Real-to-Complex Fourier Transform in Three Dimensions | SRCFT3 | DRCFT3 | 815 |
| Complex-to-Real Fourier Transform in Three Dimensions | SCRFT3 | DCRFT3 | 821 |

§ This subroutine is provided only for migration from earlier releases of ESSL and is not intended for use in new programs. Documentation for this subroutine is no longer provided.

## Convolutions and Correlations

The convolution and correlation subroutines provide the choice of using Fourier methods or direct methods. The Fourier-method subroutines contain a high-performance mixed-radix capability. There are also several direct-method subroutines that provide decimated output.

*Table 14. List of Convolution and Correlation Subroutines*

| Descriptive Name | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|
| Convolution or Correlation of One Sequence with One or More Sequences | SCON§<br>SCOR§ | | 828 |
| Convolution or Correlation of One Sequence with Another Sequence Using a Direct Method | SCOND<br>SCORD | | 834 |
| Convolution or Correlation of One Sequence with One or More Sequences Using the Mixed-Radix Fourier Method | SCONF<br>SCORF | | 840 |
| Convolution or Correlation with Decimated Output Using a Direct Method | SDCON<br>SDCOR | DDCON<br>DDCOR | 849 |
| Autocorrelation of One or More Sequences | SACOR§ | | 853 |
| Autocorrelation of One or More Sequences Using the Mixed-Radix Fourier Method | SACORF | | 857 |
| § These subroutines are provided only for migration from earlier releases of ESSL and are not intended for use in new programs. | | | |

## Related Computations

The related-computation subroutines consist of a group of computations that can be used in general signal processing applications. They are similar to those provided on the IBM 3838 Array Processor; however, the ESSL subroutines generally solve a wider range of problems.

*Table 15. List of Related-Computation Subroutines*

| Descriptive Name | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|
| Polynomial Evaluation | SPOLY | DPOLY | 863 |
| I-th Zero Crossing | SIZC | DIZC | 866 |
| Time-Varying Recursive Filter | STREC | DTREC | 869 |
| Quadratic Interpolation | SQINT | DQINT | 872 |
| Wiener-Levinson Filter Coefficients | SWLEV<br>CWLEV | DWLEV<br>ZWLEV | 876 |

## Sorting and Searching

The sorting and searching subroutines operate on three types of data: integer, short-precision real, and long-precision real (Table 16). The sorting subroutines perform sorts with or without index designations. The searching subroutines perform either a binary or sequential search.

*Table 16. List of Sorting and Searching Subroutines*

| Descriptive Name | Integer Subroutine | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|---|
| Sort the Elements of a Sequence | ISORT | SSORT | DSORT | 884 |
| Sort the Elements of a Sequence and Note the Original Element Positions | ISORTX | SSORTX | DSORTX | 886 |
| Sort the Elements of a Sequence Using a Stable Sort and Note the Original Element Positions | ISORTS | SSORTS | DSORTS | 889 |
| Binary Search for Elements of a Sequence X in a Sorted Sequence Y | IBSRCH | SBSRCH | DBSRCH | 892 |
| Sequential Search for Elements of a Sequence X in the Sequence Y | ISSRCH | SSSRCH | DSSRCH | 896 |

## Interpolation

The interpolation subroutines provide the capabilities of doing polynomial interpolation, local polynomial interpolation, and both one- and two-dimensional cubic spline interpolation (Table 17).

*Table 17. List of Interpolation Subroutines*

| Descriptive Name | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|
| Polynomial Interpolation | SPINT | DPINT | 903 |
| Local Polynomial Interpolation | STPINT | DTPINT | 908 |
| Cubic Spline Interpolation | SCSINT | DCSINT | 911 |
| Two-Dimensional Cubic Spline Interpolation | SCSIN2 | DCSIN2 | 917 |

## Numerical Quadrature

The numerical quadrature subroutines provide Gaussian quadrature methods for integrating a tabulated function and a user-supplied function over a finite, semi-infinite, or infinite region of integration (Table 18).

*Table 18 (Page 1 of 2). List of Numerical Quadrature Subroutines*

| Descriptive Name | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|
| Numerical Quadrature Performed on a Set of Points | SPTNQ | DPTNQ | 925 |

*Table 18 (Page 2 of 2). List of Numerical Quadrature Subroutines*

| Descriptive Name | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|
| Numerical Quadrature Performed on a Function Using Gauss-Legendre Quadrature | SGLNQ† | DGLNQ† | 928 |
| Numerical Quadrature Performed on a Function Over a Rectangle Using Two-Dimensional Gauss-Legendre Quadrature | SGLNQ2† | DGLNQ2† | 931 |
| Numerical Quadrature Performed on a Function Using Gauss-Laguerre Quadrature | SGLGQ† | DGLGQ† | 937 |
| Numerical Quadrature Performed on a Function Using Gauss-Rational Quadrature | SGRAQ† | DGRAQ† | 940 |
| Numerical Quadrature Performed on a Function Using Gauss-Hermite Quadrature | SGHMQ† | DGHMQ† | 944 |

† This subprogram is invoked as a function in a Fortran program.

## Random Number Generation

Random number generation subroutines generate uniformly distributed random numbers or normally distributed random numbers (Table 19).

*Table 19. List of Random Number Generation Subroutines*

| Descriptive Name | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|
| Generate a Vector of Uniformly Distributed Random Numbers | SURAND | DURAND | 948 |
| Generate a Vector of Normally Distributed Random Numbers | SNRAND | DNRAND | 951 |
| Generate a Vector of Long Period Uniformly Distributed Random Numbers | SURXOR | DURXOR | 955 |

## Utilities

The utility subroutines perform general service functions that support ESSL, rather than mathematical computations (Table 20).

*Table 20 (Page 1 of 2). List of Utility Subroutines*

| Descriptive Name | Subroutine | Page |
|---|---|---|
| ESSL Error Information-Handler Subroutine | EINFO | 962 |
| ESSL ERRSAV Subroutine for ESSL | ERRSAV | 965 |
| ESSL ERRSET Subroutine for ESSL | ERRSET | 966 |
| ESSL ERRSTR Subroutine for ESSL | ERRSTR | 968 |
| Set the Vector Section Size (VSS) for the ESSL/370 Scalar Library | IVSSET§ | |
| Set the Extended Vector Operations Indicator for the ESSL/370 Scalar Library | IEVOPS§ | |
| Determine the Level of ESSL Installed | IESSL | 969 |

*Table 20 (Page 2 of 2). List of Utility Subroutines*

| Descriptive Name | Subroutine | Page |
|---|---|---|
| Determine the Stride Value for Optimal Performance in Specified Fourier Transform Subroutines | STRIDE | 971 |
| Convert a Sparse Matrix from Storage-by-Rows to Compressed-Matrix Storage Mode | DSRSM | 981 |
| For a General Sparse Matrix, Convert Between Diagonal-Out and Profile-In Skyline Storage Mode | DGKTRN | 985 |
| For a Symmetric Sparse Matrix, Convert Between Diagonal-Out and Profile-In Skyline Storage Mode | DSKTRN | 991 |
| § This subroutine is provided for migration from earlier releases of ESSL and is not intended for use in new programs. Documentation for this subroutine is no longer provided. | | |

# Chapter 2. Planning Your Program

This chapter provides information about ESSL that you need when planning your program. Its purpose is to help you in performing the following tasks:

- Selecting an ESSL subroutine
- Avoiding Conflicts with Internal ESSL Routine Names That are Exported
- Setting up your data
- Setting up your ESSL calling sequences
- Using auxiliary storage in ESSL
- Providing a correct transform length to ESSL
- Getting the best accuracy
- Getting the best performance
- Dealing with errors when using ESSL

If you are using ESSL with PL/I Set for AIX, Version 1, see the PL/I publications for details on calling subroutines and functions.

## Selecting an ESSL Subroutine

Your choice of which ESSL subroutine to use is based mainly on the functional needs of your program. However, you have a choice of several variations of many of the subroutines. In addition, there are instances where certain subroutines cannot be used. This section describes these variations and limitations. See the answers to each question below that applies to you.

## Which ESSL Library Do You Want to Use?

ESSL provides five run-time libraries:

- The **ESSL SMP Library** provides thread-safe versions of the ESSL subroutines for use on the RS/6000 SMP processors. In addition, a subset of these subroutines are also multithreaded versions; that is, they support the shared memory parallel processing programming model. For a list of the multithreaded subroutines in the ESSL SMP Library, see Table 21 on page 26.
- The **ESSL Thread-Safe Library** provides thread-safe versions of the ESSL subroutines for use on all RS/6000 processors. You may choose to use this library if you decide to develop your own multithreaded programs that call the thread-safe ESSL subroutines.

  The number of threads you choose to use depends on the problem size, the specific subroutine being called, and the number of physical processors you are running on. To achieve optimal performance, experimentation is necessary; however, picking the number of threads equal to the number of online processors generally provides good performance in most cases. In a few cases, performance may increase if you choose the number of threads to be less than the number of online processors. For more information about thread concepts, see *IBM AIX Version 4 General Programming Concepts: Writing and Debugging Programs*.
- The **ESSL Thread-Safe POWER2 Library** provides thread-safe versions of the ESSL subroutines and is tuned for the RS/6000 POWER2 uniprocessors. You may use this library to develop your own multithreaded applications.
- The **ESSL POWER2 Library** is tuned for the RS/6000 POWER2 uniprocessors.

- The **ESSL POWER Library** is tuned for the RS/6000 POWER, POWER3, and PowerPC uniprocessors.

The **ESSL POWER Library**, the **ESSL Thread-Safe Library**, and the **ESSL SMP Library** supports both 32-bit environment and 64-bit environment applications. For details see Chapter 4 on page 111 and Chapter 5 on page 163.

*Table 21 (Page 1 of 2). Multithreaded ESSL SMP Subroutines*

| **Subroutine Names** |
|---|
| Vector-Scalar Linear Algebra Subprograms:<br>SASUM, DASUM, SCASUM, DZASUM<br>SAXPY, DAXPY, CAXPY, ZAXPY<br>SCOPY, DCOPY, CCOPY, ZCOPY<br>SDOT, DDOT, CDOTU, ZDOTU, CDOTC, ZDOTC<br>SNDOT, DNDOT<br>SNORM2, DNORM2, CNORM2, ZNORM2<br>SROT, DROT, CROT, ZROT, CSROT, ZDROT<br>SSCAL, DSCAL, CSCAL, ZSCAL, CSSCAL, ZDSCAL<br>SSWAP, DSWAP, CSWAP, ZSWAP<br>SVEA, DVEA, CVEA, ZVEA<br>SVES, DVES, CVES, ZVES<br>SVEM, DVEM, CVEM, ZVEM<br>SYAX, DYAX, CYAX, ZYAX, CSYAX, ZDYAX<br>SZAXPY, DZAXPY, CZAXPY, ZZAXPY |
| Matrix-Vector Linear Algebra Subprograms:<br>SGEMV, DGEMV, CGEMV, ZGEMV<br>SGER, DGER, CGERU, ZGERU, CGERC, ZGERC<br>SSPMV, DSPMV, CHPMV, ZHPMV<br>SSYMV, DSYMV, CHEMV, ZHEMV<br>SSPR, DSPR, CHPR, ZHPR<br>SSYR, DSYR, CHER, ZHER<br>SSPR2, DSPR2, CHPR2, ZHPR2<br>SSYR2, DSYR2, CHER2, ZHER2<br>SGBMV♦, DGBMV♦<br>CGBMV♦, ZGBMV♦<br>SSBMV♦, DSBMV♦<br>CHBMV♦, ZHBMV♦<br>STRMV, DTRMV, CTRMV, ZTRMV<br>STPMV, DTPMV, CTPMV, ZTPMV<br>STBMV♦, DTBMV♦<br>CTBMV♦, ZTBMV♦ |
| Matrix Operations:<br>SGEADD, DGEADD, CGEADD, ZGEADD<br>SGESUB, DGESUB, CGESUB, ZGESUB<br>SGEMUL, DGEMUL, CGEMUL, ZGEMUL<br>SGEMM, DGEMM, CGEMM, ZGEMM<br>SSYMM, DSYMM, CSYMM, ZSYMM, CHEMM, ZHEMM<br>STRMM, DTRMM, CTRMM, ZTRMM<br>SSYRK, DSYRK, CSYRK, ZSYRK, CHERK, ZHERK<br>SSYR2K, DSYR2K, CSYR2K, ZSYR2K, CHER2K, ZHER2K<br>SGETMI, DGETMI, CGETMI, ZGETMI<br>SGETMO, DGETMO, CGETMO, ZGETMO |

*Table 21 (Page 2 of 2). Multithreaded ESSL SMP Subroutines*

| **Subroutine Names** |
|---|
| Dense Linear Algebraic Equations:<br>SGEF, DGEF, CGEF, ZGEF<br>SGETRF, DGETRF, CGETRF, ZGETRF<br>SPPF, DPPF, DPOF<br>SPPFCD*, DPPFCD*, DPOFCD*<br>SPPICD*, DPPICD*, DPOICD*<br>STRSV, DTRSV, CTRSV, ZTRSV<br>STPSV, DTPSV, CTPSV, ZTPSV<br>STRSM, DTRSM, CTRSM, ZTRSM<br>STRI, DTRI |
| Sparse Linear Algebraic Equations:<br>DSRIS† |
| Fourier Transforms:<br>SCFT, DCFT<br>SRCFT, DRCFT<br>SCRFT, DCRFT<br>SCFT2, DCFT2<br>SRCFT2, DRCFT2<br>SCRFT2, DCRFT2<br>SCFT3, DCFT3<br>SRCFT3, DRCFT3<br>DCRFT3, DCRFT3 |
| Convolution and Correlation:<br>SCOND, SCORD<br>SDCON, SDCOR, DDCON, DDCOR |
| Many of the dense linear algebraic equations and eigensystem analysis subroutines make one or more calls to the multithreaded versions of the matrix-vector linear algebra and matrix operation subroutines shown in this table. SCOSF, DCOSF, SSINF, and DSINF make one or more calls to the multithreaded versions of the Fourier Transform subroutines shown in this table. These subroutines benefit from the increased performance of the multithreaded versions of the ESSL SMP subroutines.<br>Your performance may be improved by setting the AIX Environment variables SPINLOOPTIME to 500 and YIELDLOOPTIME to 500. For additional information, see the *AIX Performance Monitoring and Tuning Guide*, SC23-2365.<br>† DSRIS only uses multiple threads when `IPARM(4) = 1` or 2.<br>♦ The Level 2 Banded BLAS use multiple threads only when the bandwidth is sufficiently large.<br>* Multiple threads are used for the factor or inverse computation. |

## What Type of Data Are You Processing in Your Program?

The version of the ESSL subroutine you select should agree with the data you are using. ESSL provides a short- and long-precision version of most of its subroutines processing short- and long-precision data, respectively. In a few cases, it also provides an integer version processing integer data or returning just integer data. The subroutine names are distinguished by a one- or two-letter prefix based on the following letters:

S for short-precision real
D for long-precision real
C for short-precision complex
Z for long-precision complex

I for integer

The precision of your data affects the accuracy of your results. This is discussed in “Getting the Best Accuracy” on page 44. For a description of these data types, see “How Do You Set Up Your Scalar Data?.”

### How Is Your Data Structured? And What Storage Technique Are You Using?

Some subroutines process specific data structures, such as sparse vectors and matrices or dense and banded matrices. In addition, these data structures can be stored using various storage techniques. You should select the proper subroutine on the basis of the type of data structure you have and the storage technique you want to use. If possible, you should use a storage technique that conserves storage and potentially improves performance. For more about storage techniques, see “Setting Up Your Data.”

### What about Performance and Accuracy?

ESSL provides variations among some of its subroutines. You should consider performance and accuracy when deciding which subroutine is the best to use. Study the “Function” section in each subroutine description. It helps you understand exactly what each subroutine does, and helps you determine which subroutine is best for you. For example, some subroutines perform multiple computations of a certain type. This might give you better performance than a subroutine that does each computation individually. In other cases, one subroutine may do scaling while another does not. If scaling is not necessary for your data, you get better performance by using the subroutine without scaling.

## Avoiding Conflicts with Internal ESSL Routine Names That are Exported

Do not use names for your own subroutines, functions, and global variables that are the same as the ESSL exported names. All internal ESSL routine names that are exported begin with the **ESV** prefix, so you should avoid using this prefix for your own names.

## Setting Up Your Data

This section explains how to set up your scalar and array data and points you to where you can find more detail.

### How Do You Set Up Your Scalar Data?

A scalar item is a single item of data, whether it is a constant, a variable, or an element of an array. ESSL assumes that your scalar data conforms to the appropriate standards, as described below. The scalar data types and how you should code them for each programming language are listed under “Coding Your Scalar Data” in each language section in Chapter 4 on page 111.

### Internal Representation

Scalar data passed to ESSL from all types of programs, including Fortran, C, and C++, should conform to the ANSI/IEEE 32-bit and 64-bit binary floating-point format, as described in the *ANSI/IEEE Standard for Binary Floating-Point Arithmetic, ANSI/IEEE Standard 754-1985*

## How Do You Set Up Your Arrays?

An array represents an area of storage in your program, containing data stored in a series of locations. An array has a single name. It is made up of one or more pieces of scalar data, all the same type. These are the elements of the array. It can be passed to the ESSL subroutine as input, returned to your program as output, or used for both input and output, in which case the original contents are overwritten.

Arrays can contain conceptual (mathematical) data structures, such as vectors, matrices, or sequences. There are many different types of data structures. Each type of data structure requires a unique arrangement of data in an array and does not necessarily have to include all the elements of the array. In addition, the elements of these data structures are not always contiguous in storage within an array. Stride and leading dimension arguments passed to ESSL subroutines define the separations in array storage for the elements of the vector, matrix, and sequence. All these aspects of data structures are described in Chapter 3 on page 55. You must first understand array storage techniques to fully understand the concepts of data structures, stride, and leading dimension, especially if you are using them in unconventional ways.

ESSL subroutines assume that all arrays passed to them are stored using the Fortran array storage techniques (in column-major order), and they process your data accordingly. For details, see “Setting Up Arrays in Fortran” on page 112. On the other hand, C, C++, and PL/I programs store arrays in row-major order. For details on what you can do, see:

- For C, see page “Setting Up Arrays in C” on page 133.
- For C++, see page “Setting Up Arrays in C++” on page 149.
- For PL/I, see the PL/I publications.

## How Should Your Array Data Be Aligned?

All arrays, regardless of the type of data, should be aligned on a doubleword boundary to ensure optimal performance; however, when running on a POWER2 processor, it is best to align your long-precision arrays on a quadword boundary. For information on how your programming language aligns data, see your programming language manuals.

## What Storage Mode Should You Use for Your Data?

The amount of storage used by arrays and the storage arrangement of data in the arrays can affect overall program performance. As a result, ESSL provides subroutines that operate on different types of data structures, stored using various storage modes. You should chose a storage mode that conserves storage and potentially improves performance. For definitions of the various data structures and their corresponding storage modes, see Chapter 3 on page 55. You can also find special storage considerations, where applicable, in the “Notes” section of each subroutine description.

## How Do You Convert from One Storage Mode to Another?

This section describes how you can convert from one storage mode to another.

### Conversion Subroutines

ESSL provides several subroutines that help you convert from one storage mode to another:

- DSRSM is used to migrate your existing program from sparse matrices stored by rows to sparse matrices stored in compressed-matrix storage mode. This converts the matrices into a storage format that is compatible with the input requirements for some ESSL sparse matrix subroutines, such as DSMMX.
- DGKTRN and DSKTRN are used to convert your sparse matrix from one skyline storage mode to another, if necessary, before calling the subroutines DGKFS/DGKFSP or DSKFS/DSKFSP, respectively.

### Sample Programs

In addition, sample programs are provided with many of the storage mode descriptions in Chapter 3 on page 55. You can use these sample programs to convert your data to the desired storage mode by adapting them to your application program.

# Setting Up Your ESSL Calling Sequences

This section gives the general rules for setting up the ESSL calling sequences. The information given here applies to all types of programs, running in all environments. For a description and examples of how to code the ESSL calling sequences in your particular programming language, see the following sections:

- “Fortran Programs” on page 111
- “C Programs” on page 129
- “C++ Programs” on page 145

For details on the conventions used in this book to describe the calling sequence syntax, see “How to Interpret the Subroutine Descriptions” on page xxx. It describes how required and optional arguments are indicated in the calling sequence and the naming conventions used for different data types.

## What Is an Input-Output Argument?

Some arguments are used for both input and output. The contents of the input argument are overlaid with the output value(s) on return to your program. Be careful that you save any data you need to preserve before calling the ESSL subroutine.

## What Are the General Rules to Follow when Specifying Data for the Arguments?

You should follow the syntax rules given for each argument in “On Entry” in the subroutine description. Input-argument error messages may be issued, and your program may terminate when you make an error specifying the input arguments. For example:

- Data passed to ESSL must be of the correct type: integer, character, real, complex, short-precision, or long-precision. There is no conversion of data.

Assuming you are using the ESSL header file with your C and C++ programs, you first need to define the following:

  - Complex and logical data in C programs, using the guidelines given on page 132.
  - Short-precision complex and logical data in C++ programs, using the guidelines given on page 148.

- Character values must be one of the specified values. For example, it may have to be `'N'`, `'T'`, or `'C'`.
- Numeric values must fall within the correct range for that argument. For example, a numeric value may need to be greater than or equal to 0, or it may have to be a nonzero value.
- Arrays must be defined correctly; that is, they must have the correct dimensions, or the dimensions must fall within the correct range. For example, input and output matrices may need to be conformable, or the number of rows in the matrix must be less than or equal to the leading dimension specified. (ESSL assumes all arrays are stored in column-major order.)

## What Happens When a Value of 0 Is Specified for N?

For most ESSL subroutines, if you specify 0 for the number of elements to be processed in a vector or the order of a matrix (usually argument *n*), no computation is performed. After checking for input-argument errors, the subroutine returns immediately and no result is returned. In the other subroutines, an error message may be issued.

## How Do You Specify the Beginning of the Data Structure in the ESSL Calling Sequence?

**When you specify a vector, matrix, or sequence in your calling sequence, it does not necessarily have to start at the beginning of the array.** It can begin at any point in the array. For example, if you want vector ***x*** to start at element 3 in array `A`, which is declared `A(1:12)`, specify `A(3)` in your calling sequence for argument *x*, such as in the following SASUM calling sequence in your Fortran program:

```
             N    X    INCX
             |    |     |
X = SASUM( 4 , A(3) , 2 )
```

Also, for example, if you want matrix ***A*** to start at the second row and third column of array `A`, which is declared `A(0:10,2:8)`, specify `A(1,4)` in your calling sequence for argument *a*, such as in the following SGEADD calling sequence in your Fortran program:

```
                A     LDA  TRANSA  B  LDB TRANSB  C  LDC  M   N
                |      |     |     |   |    |     |   |   |   |
CALL  SGEADD( A(1,4) , 11 , 'N'  , B , 4 , 'N'  , C , 4 , 4 , 3 )
```

For more examples of specifying vectors and matrices, see Chapter 3 on page 55.

## Using Auxiliary Storage in ESSL

For the ESSL subroutines listed in Table 22, you need to provide extra working storage to perform the computation. This section describes the use of dynamic allocation for providing auxiliary storage in ESSL and how to calculate the amount of auxiliary storage you need by use of formulas or error-handling capabilities provided in ESSL, if dynamic allocation is not an option.

Auxiliary storage, or working storage, is supplied through one or more arguments, such as *aux*, in the calling sequence for the ESSL subroutine. **If the working storage does not need to persist after the subroutine call, it is suggested you use dynamic allocation.** For example, in the Fourier Transforms subroutines, you may allocate *aux2* dynamically, but not *aux1*. See the subroutine descriptions in Part 2 of this book for details and variations.

*Table 22. ESSL Subroutines Requiring Auxiliary Working Storage*

| **Subroutine Names** |
|---|
| Linear Algebra Subprograms:<br>DSMTM |
| Matrix Operations:<br>_GEMMS |
| Dense Linear Algebraic Equations:<br>_GEFCD _PPFCD _GEICD _PPICD _POFCD<br>_POICD DGEFPΔ DPPFPΔ |
| Sparse Linear Algebraic Equations:<br>DGSF DGSS DGKFS DGKFSPΔ DSKFS DSKFSPΔ<br>DSRIS DSMCG DSDCG DSMGCG DSDGCG |
| Linear Least Squares:<br>_GESVF _GELLS |
| Eigensystem Analysis:<br>_GEEV _SPEV _HPEV _SPSV _HPSV<br>_GEGV _SYGV |
| Fourier Transforms:<br>_CFT _RCFT _CRFT _COSF _SINF<br>SCOSFTΔ _CFT2 _RCFT2 _CRFT2 _CFT3<br>_RCFT3 _CRFT3 SCFTPΔ SCFT2PΔ SCFT3PΔ |
| Convolutions and Correlations:<br>SCONF SCORF SACORF |
| Related Computations:<br>_WLEV |
| Interpolation:<br>_TPINT _CSIN2 |
| Random Number Generation:<br>_NRAND |
| Utilities:<br>DGKTRN DSKTRN |
| Δ Documentation for this subroutine is no longer provided. The *aux* and *naux* arguments for the subroutine are specified the same as for the corresponding serial ESSL subroutine. |

## Dynamic Allocation of Auxiliary Storage

Dynamic allocation for the auxiliary storage is performed when error 2015 is unrecoverable and *naux*=0. For details on which *aux* arguments allow dynamic allocation, see the subroutine descriptions in Part 2 of this book.

## Setting Up Auxiliary Storage When Dynamic Allocation Is Not Used

You set up the storage area in your program and pass it to ESSL through arguments, specifying the size of the *aux* work area in the *naux* argument.

## Who Do You Want to Calculate the Size? You or ESSL?

You have a choice of two methods for determining how much auxiliary storage you should specify:

- Use the formulas provided in the subroutine description to derive **sufficient values** for your current and future needs. Use them if **ease of migration** to future machines and future releases of ESSL is your primary concern. For details, see "How Do You Calculate the Size Using the Formulas?."
- Use the ESSL error-handling facilities to return to you a **minimum value** for the particular processor you are currently running on. (Values vary by platform.) Use this approach if **conserving storage** is your primary concern. For details, see "How Do You Get ESSL to Calculate the Size Using ESSL Error Handling?" on page 34.

## How Do You Calculate the Size Using the Formulas?

The formulas provided for calculating *naux* indicate a **sufficient** amount of auxiliary storage required, which, in most cases, is larger than the minimum amount, returned by ESSL error handling. There are two types of formulas:

- **Simple formulas**

  These are given in the *naux* argument syntax descriptions. In general, these formulas result in the minimum required value, but, in a few cases, they provide overestimates.

- **Processor-independent formulas**

  These are given in separate sections in the subroutine description. In general, these provide overestimates.

Both types of formulas provide values that are sufficient for all processors. As a result, you can migrate to any other processor and to future releases of ESSL without being concerned about having to increase the amount of storage for *aux*. You do, of course, need to weigh your storage requirements against the convenience of using this larger value.

To calculate the amount of storage using the formulas, you must substitute values for specific variables, such as *n*, *m*, *n1*, or *n2.* These variables are arguments specified in the ESSL calling sequence or derived from the arguments in the calling sequence.

## How Do You Get ESSL to Calculate the Size Using ESSL Error Handling?

This section describes how you can get ESSL to calculate auxiliary storage.

### Here Are the Two Ways You Can Do It

Ask yourself which of the following ways you prefer to obtain the information from ESSL:

- **By leaving error 2015 unrecoverable,** you can obtain the minimum required value of *naux* from the input-argument error message, but your program terminates.
- **By making error 2015 recoverable,** you can obtain the minimum required value of *naux* from the input-argument error message and have the updated *naux* argument returned to your program.

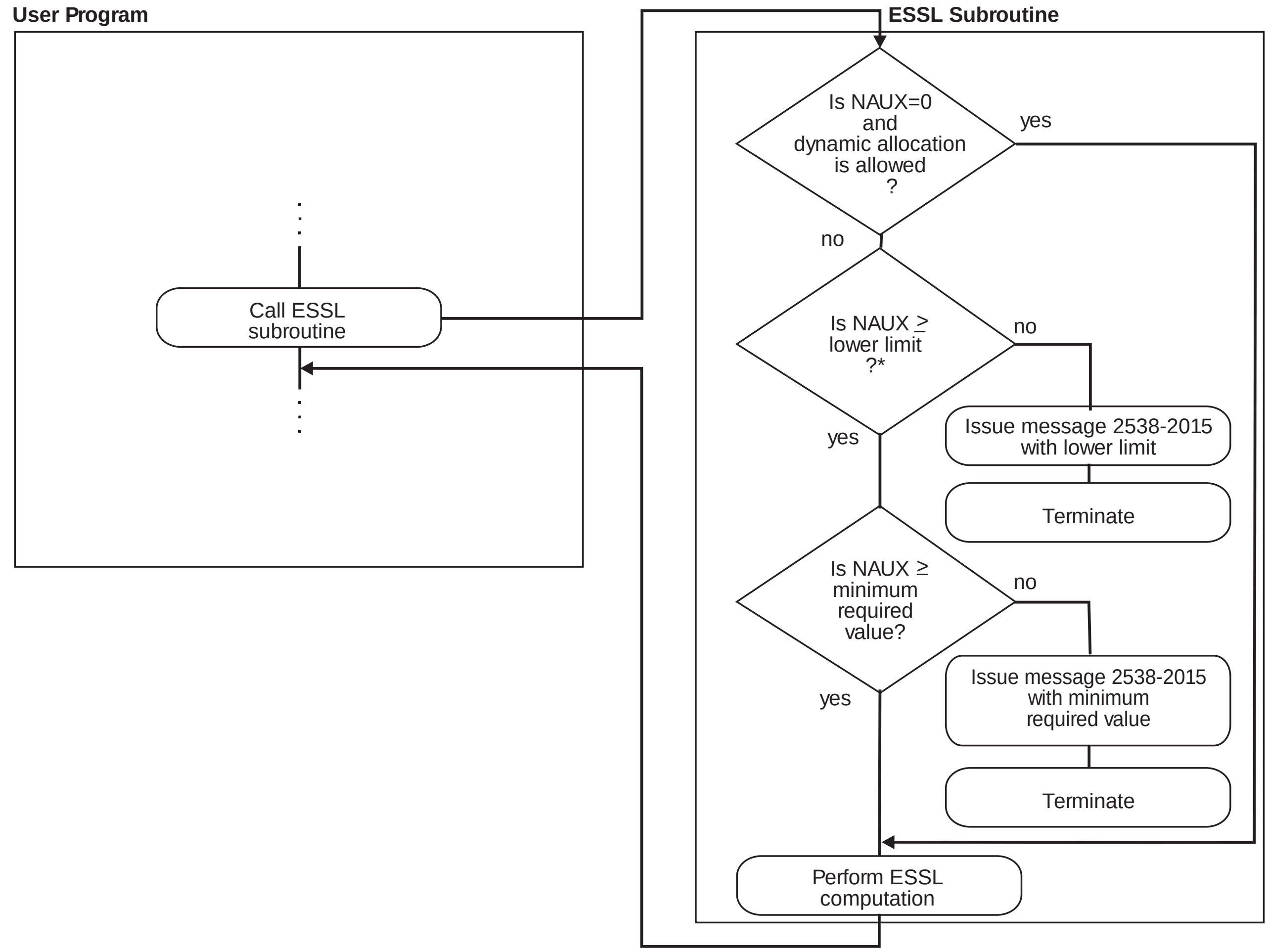
For both techniques, the amount returned by the ESSL error-handling facility is the **minimum** amount of auxiliary storage required to run your program successfully **on the particular processor you are currently running on.** The ESSL error-handling capability usually returns a smaller value than you derive by using the formulas listed for the subroutine. This is because the formulas provide a good estimate, but ESSL can calculate exactly what is needed on the basis of your data.

The values returned by ESSL error handling **may not apply to future processors.** You should not use them if you plan to run your program on a future processor. You should use them only if you are concerned with minimizing the amount of auxiliary storage used by your program.

### The First Way

In this case, you obtain the minimum required value of *naux* from the error message, but your program terminates. The following description assumes that dynamic allocation is not selected as an option.

Leave error 2015 as unrecoverable, without calls to EINFO and ERRSET. Run your program with the *naux* values smaller than required by the subroutine for the particular processor you are running on. As a general guideline, specify values smaller than those listed in the formulas. However, if a lower limit is specified in the syntax (only for several *naux1* arguments in the Fourier transform, convolution, and correlation subroutines), you should not go below that limit. The ESSL error monitor returns the necessary sizes of the *aux* storage areas in the input-argument error message. This does, however, terminate your program when the error is encountered. (If you accidentally specify a sufficient amount of storage for the ESSL subroutine to perform the computation, error handling does not issue an error message and processing continues normally.) Figure 1 on page 35 illustrates what happens when error 2015 is unrecoverable.


User Program
ESSL Subroutine
Call ESSL subroutine
Is NAUX=0 and dynamic allocation is allowed ?
yes
no
Is NAUX ≥ lower limit ?*
no
yes
Issue message 2538-2015 with lower limit
Terminate
Is NAUX ≥ minimum required value?
no
yes
Issue message 2538-2015 with minimum required value
Terminate
Perform ESSL computation

* This check applies only to several NAUX1 arguments in the Fourier transform, convolution, and correlation subroutines.

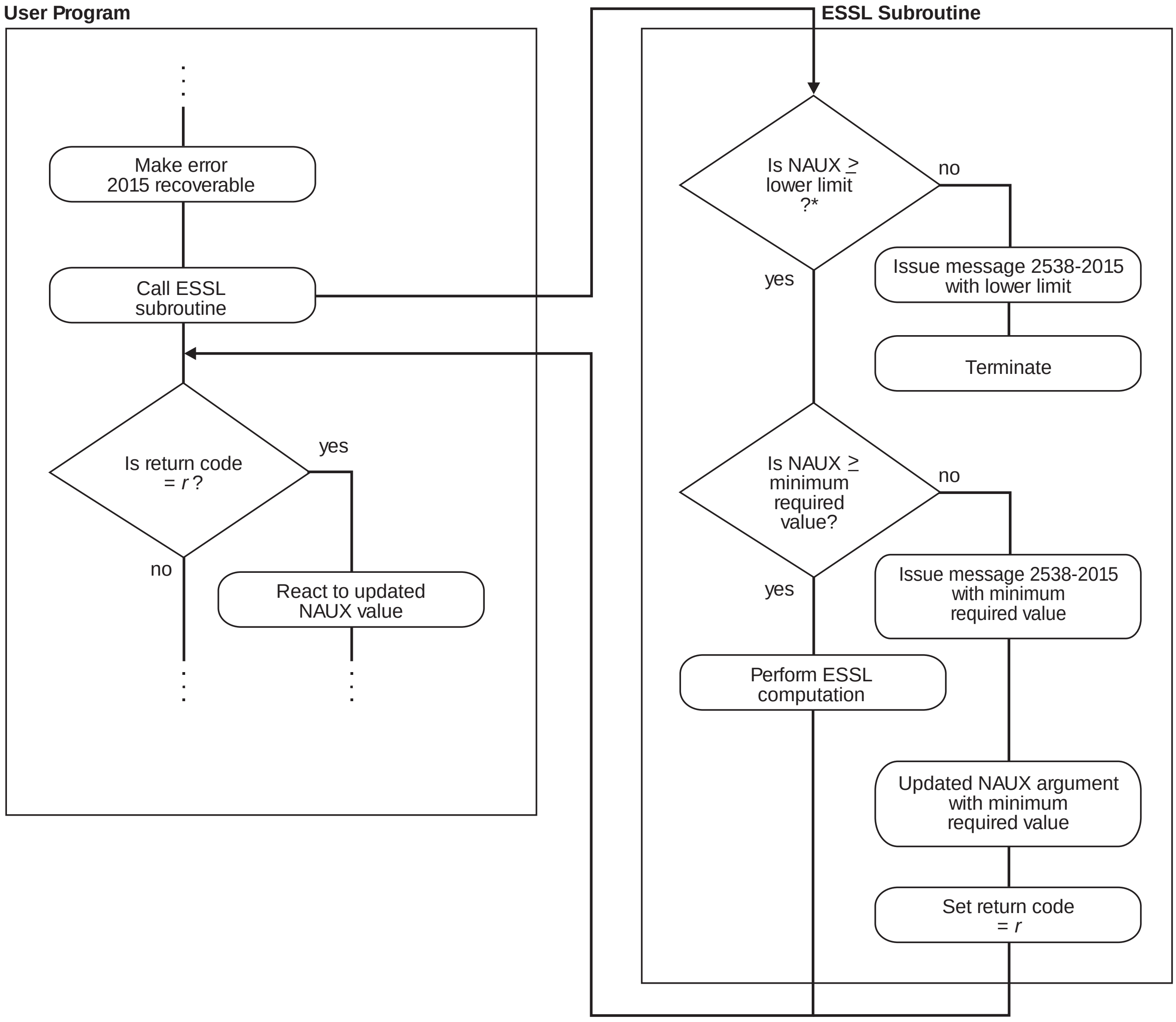
*Figure 1. How to Obtain an NAUX Value from an Error Message, but Terminate*

### The Second Way

In this case, you obtain the minimum required value of *naux* from the error message and from the updated *naux* argument returned to your program.

Use EINFO and ERRSET with an ESSL error exit routine, ENOTRM, to make error 2015 recoverable. This allows you to dynamically determine in your program the minimum sizes required for the auxiliary working storage areas, specified in the *naux* arguments. Run your program with the *naux* values smaller than required by the subroutine for the particular processor you are running on. As a general guideline, specify values smaller than those listed in the formulas. However, if a lower limit is specified in the syntax (only for several *naux1* arguments in the

Fourier transform, convolution, and correlation subroutines), you should not go below that limit. The ESSL error monitor returns the necessary sizes of the *aux* storage areas in the input-argument error message and a return code is passed back to your program, indicating that updated values are also returned in the *naux* arguments. You can then react to these updated values during run time in your program. ESSL does not perform any computation when this error occurs. For details on how to do this, see Chapter 4 on page 111. (If you accidentally specify a sufficient amount of storage for the ESSL subroutine to perform the computation, error handling does not issue an error message and processing continues normally.) Figure 2 illustrates what happens when error 2015 is recoverable.


User Program
Make error 2015 recoverable
Call ESSL subroutine
Is return code = *r* ?
yes
no
React to updated NAUX value
ESSL Subroutine
Is NAUX ≥ lower limit ?*
no
yes
Issue message 2538-2015 with lower limit
Terminate
Is NAUX ≥ minimum required value?
no
yes
Issue message 2538-2015 with minimum required value
Perform ESSL computation
Updated NAUX argument with minimum required value
Set return code = *r*

* This check applies only to several NAUX1 arguments in the Fourier transform, convolution, and correlation subroutines.

*Figure 2. How to Obtain an NAUX Value from an Error Message and in Your Program*

## Here Is an Example of What Happens When You Use These Two Techniques

The following example illustrates all the actions taken by the ESSL error-handling facility for each possible value of a recoverable input argument, *naux*. A key point here is that if you want to have the updated argument value returned to your program, you must make error 2015 recoverable and then specify an *naux* value greater than or equal to 20 and less than 300. For values out of that range, the error recovery facility is not in effect. (These values of *naux*, 20 and 300, are used only for the purposes of this example and do not relate to any of the ESSL subroutines.)

**NAUX** **Meaning of the NAUX Value**

20 Lower limit of *naux* required for using recoverable input-argument error-handling facilities in ESSL. (This applies only to several *naux1* arguments in the Fourier transform, convolution, and correlation subroutines. You can find the lower limit in the syntax description for the *naux1* argument. For a list of subroutines, see Table 22 on page 32.)

300 Minimum value of *naux*, required for successful running (on the processor the program is being run on).

Table 23 describes the actions taken by ESSL in every possible situation for the values given in this example.

*Table 23. Example of Input-Argument Error Recovery for Auxiliary Storage Sizes*

| NAUX Value | Action When 2015 Is an Unrecoverable Input-Argument Error | Action When 2015 Is a Recoverable Input-Argument Error |
|---|---|---|
| $naux < 20$ | An input-argument error message is issued. The value in the error message is the lower limit, 20. The application program stops. | An input-argument error message is issued. The value in the error message is the lower limit, 20. The application program stops. |
| $20 \le naux < 300$ | An input-argument error message is issued. The value in the error message is the minimum required value, 300. The application program stops. | ESSL returns the value of *naux* as 300 to the application program, and an input-argument error message is issued. The value in the error message is the minimum required value, 300. ESSL does no computation, and control is returned to the application program. |
| $naux \ge 300$ | Your application program runs successfully. | Your application program runs successfully. |

## Here Is How You Code It in Your Program

If you leave error 2015 unrecoverable, you **do not code anything** in your program. You just look at the error messages to get the sizes of auxiliary storage. On the other hand, if you want to make error 2015 recoverable to obtain the auxiliary storage sizes dynamically in your program, you need to **add some coding statements** to your program. For details on coding these statements in each programming language, see the following examples:

- For Fortran, see page 121
- For C, see page 139
- For C++, see page 155

You may want to provide a separate subroutine to calculate the auxiliary storage size whenever you need it. Figure 3 on page 38 shows how you might code a separate Fortran subroutine. Before calling SCFT in your program, call this subroutine, SCFT which calculates the minimum size and stores it in the *naux* arguments. Upon return, your program checks the return code. If it is nonzero, the *naux* arguments were updated, as planned. You should then make sure adequate storage is available and call SCFT. On the other hand, if the return code is zero, error handling was not invoked, the *naux* arguments were not updated, and the initialization step was performed for SCFT.

```
      SUBROUTINE SCFT
     * N, M, ISIGN, SCALE, AUX1, NAUX1,AUX2,NAUX2)
      REAL*4 X(0:*),Y(0:*),SCALE
      REAL*8 AUX1(7),AUX2(0:*)
      INTEGER*4 INIT,INC1X,INC2X,INC1Y,INC2Y,N,M,ISIGN,NAUX1,NAUX2
      EXTERNAL ENOTRM
      CHARACTER*8   S2015
          CALL EINFO(0)
          CALL ERRSAV(2015,S2015)
          CALL ERRSET(2015,0,-1,1,ENOTRM,0)
C     SETS NAUX1 AND NAUX2 TO THE MINIMUM VALUES REQUIRED TO USE
C     THE RECOVERABLE INPUT-ARGUMENT ERROR-HANDLING FACILITY
          NAUX1 = 7
          NAUX2 = 0
          CALL SCFT(INIT,X,INC1X,INC2X,Y,INC1Y,INC2Y,
     *              N,M,ISIGN,SCALE,AUX1,NAUX1,AUX2,NAUX2,*10)
          CALL ERRSTR(2015,S2015)
          RETURN
10        CONTINUE
          CALL ERRSTR(2015,S2015)
          RETURN 1
          END
```

*Figure 3. Fortran Subroutine to Calculate Auxiliary Storage Sizes*

## Providing a Correct Transform Length to ESSL

This section describes how to calculate the length of your transform by use of formulas or error-handling capabilities provided in ESSL.

## What ESSL Subroutines Require Transform Lengths?

For the ESSL subroutines listed in Table 24 on page 39, you need to provide one or more transform lengths for the computation of a Fourier transform. These transform lengths are supplied through one or more arguments, such as *n*, *n1*, *n2*, and *n3*, in the calling sequence for the ESSL subroutine. Only certain lengths of transforms are permitted in the computation.

*Table 24. ESSL Subroutines Requiring Transform Lengths*

| **Subroutine Names** |
|---|
| Fourier Transforms:<br>_CFT _RCFT _CRFT _COSF _SINF<br>SCOSFT _CFT2 _RCFT2 _CRFT2 _CFT3<br>_RCFT3 _CRFT3 SCFTP SCFT2P SCFT3P |

## Who Do You Want to Calculate the Length? You or ESSL?

You have a choice of two methods for determining an acceptable length for your transform to be processed by ESSL:

- Use the formula or large table in “Acceptable Lengths for the Transforms” on page 741 to determine an acceptable length. For details, see “How Do You Calculate the Length Using the Table or Formula?.”
- Use the ESSL error-handling facilities to return to you an acceptable length. For details, see “How Do You Get ESSL to Calculate the Length Using ESSL Error Handling?.”

## How Do You Calculate the Length Using the Table or Formula?

The lengths ESSL accepts for transforms in the Fourier transform subroutines are listed in “Acceptable Lengths for the Transforms” on page 741. You should use the table in that section to find the two values your length falls between. You then specify the **larger** length for your transform. If you find a perfect match, you can use that value without having to change it. The formula provided in that section expresses how to calculate the acceptable values listed in the table. If necessary, you can use the formula to dynamically check lengths in your program.

## How Do You Get ESSL to Calculate the Length Using ESSL Error Handling?

This section describes how to get ESSL to calculate transform lengths.

### Here Are the Two Ways You Can Do It

Ask yourself which of the following ways you prefer to obtain the information from ESSL:

- **By leaving error 2030 unrecoverable,** you can obtain an acceptable value for *n* from the input-argument error message, but your program terminates.
- **By making error 2030 recoverable,** you obtain an acceptable value for *n* from the input-argument error message and have the updated *n* argument returned to your program.

Because the Fourier transform subroutines allow only certain lengths for transforms, ESSL provides this error-handling capability to return acceptable lengths to your program. It returns them in the transform length arguments. The value ESSL returns is the **next larger acceptable length** for a transform, based on the length you specify in the *n* argument.

## The First Way

In this case, you obtain an acceptable value of *n* from the error message, but your program terminates.

Leave error 2030 as unrecoverable, without calls to EINFO and ERRSET. Run your program with a close approximation of the transform length you want to use. If this happens not to be an acceptable length, the ESSL error monitor returns an acceptable length of the transform in input-argument error message. This does, however, terminates your program when the error is encountered. (If you do happen to specify an acceptable length for the transform, error handling does not issue an error message and processing continues normally.) Figure 4 illustrates what happens when error 2030 is unrecoverable.

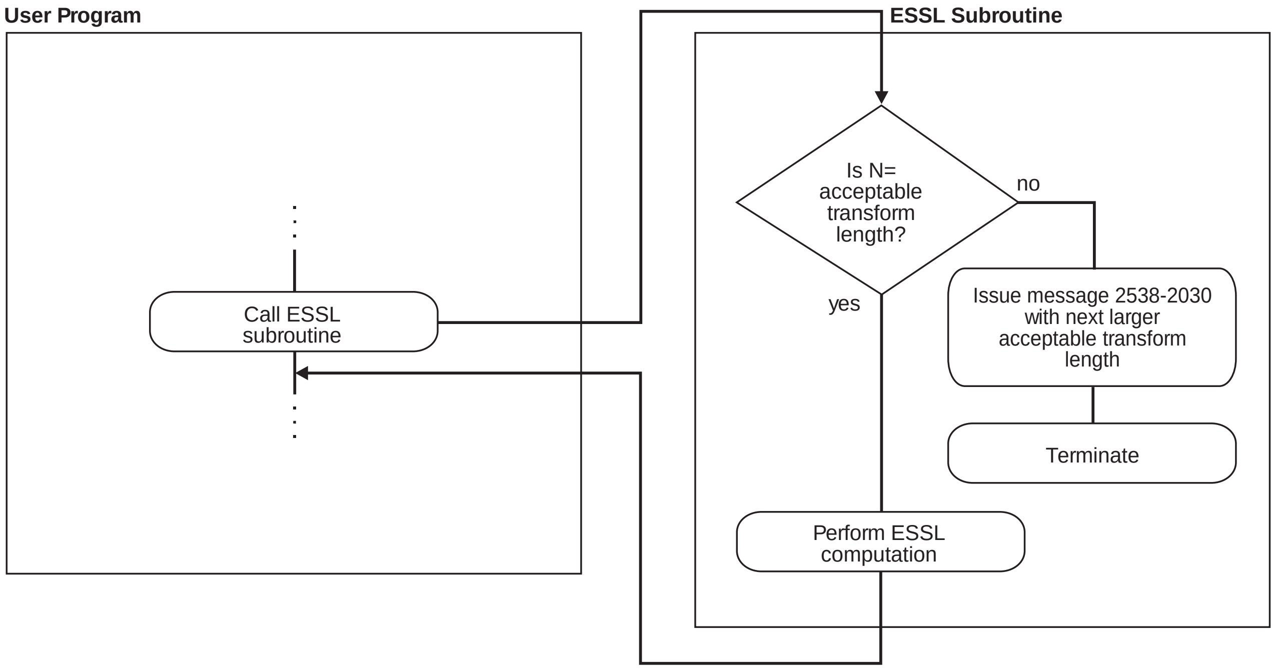

User Program
ESSL Subroutine
Is N= acceptable transform length?
no
yes
Call ESSL subroutine
Issue message 2538-2030 with next larger acceptable transform length
Terminate
Perform ESSL computation

*Figure 4. How to Obtain an N Value from an Error Message, but Terminate*

## The Second Way

In this case, you obtain an acceptable value of *n* from the error message and from the updated *n* argument returned to your program.

Use EINFO and ERRSET with an ESSL error exit routine, ENOTRM, to make error 2030 recoverable. This allows you to dynamically determine in your program an acceptable length for your transform, specified in the *n* argument(s). Run your program with a close approximation of the transform length you want to use. If this happens not to be an acceptable length, the ESSL error monitor returns an acceptable length of the transform in the input-argument error message and a return code is passed back to your program, indicating that updated values are also returned in the *n* argument(s). You can then react to these updated values during run time in your program. ESSL does not perform any computation when this error occurs. For details on how to do this, see Chapter 4 on page 111. (If you do happen to specify an acceptable length for the transform, error handling does not issue an error message and processing continues normally.) Figure 5 on page 41 illustrates what happens when error 2030 is recoverable.

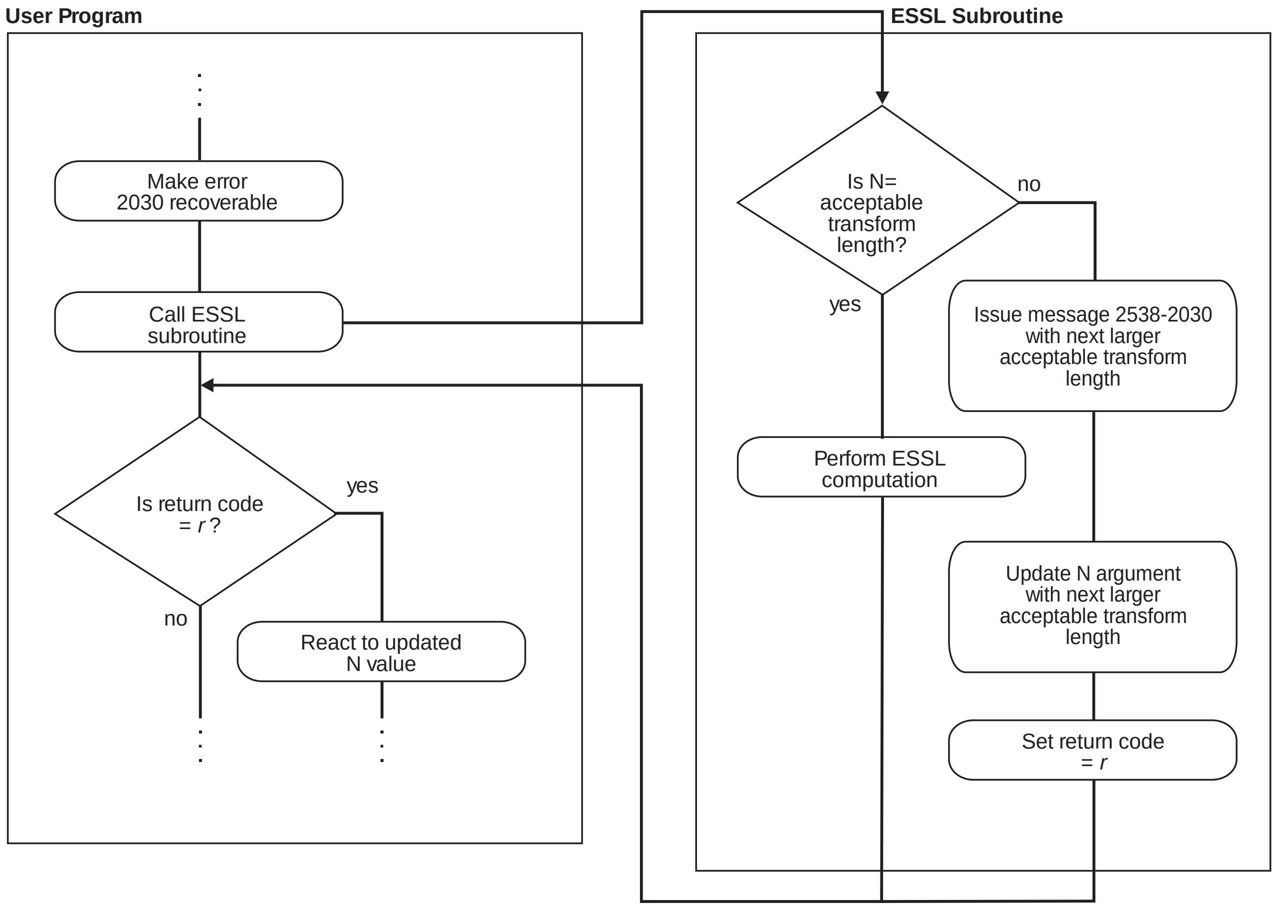

User Program
ESSL Subroutine
Make error 2030 recoverable
Call ESSL subroutine
Is return code = *r*?
yes
no
React to updated N value
Is N= acceptable transform length?
no
yes
Issue message 2538-2030 with next larger acceptable transform length
Perform ESSL computation
Update N argument with next larger acceptable transform length
Set return code = *r*

*Figure 5. How to Obtain an N Value from an Error Message and in Your Program*

## Here Is an Example of What Happens When You Use These Two Techniques

The following example illustrates all the actions taken by the ESSL error-handling facility for each possible value of a recoverable input argument, *n*. The values of *n* used in the example are as follows:

**N** **Meaning of the N Value**

7208960
An acceptable transform length, required for successful computing of a Fourier transform

7340032
The next larger acceptable transform length, required for successful computing of a Fourier transform

Table 25 on page 42 describes the actions taken by ESSL in every possible situation for the values given in this example.

*Table 25. Example of Input-Argument Error Recovery for Transform Lengths*

| N Value | Action When 2030 Is an Unrecoverable Input-Argument Error | Action When 2030 Is a Recoverable Input-Argument Error |
|---|---|---|
| $n = 7208960$<br>**–or–**<br>$n = 7340032$ | Your application program runs successfully. | Your application program runs successfully. |
| $7208960 < n < 7340032$ | An input-argument error message is issued. The value in the error message is 7340032. The application program stops. | ESSL returns the value of *n* as 7340032 to the application program, and an input-argument error message is issued. The value in the error message is 7340032. ESSL does no computation, and control is returned to the application program. |

## Here Is How You Code It in Your Program

If you leave error 2030 unrecoverable, you **do not code anything** in your program. You just look at the error messages to get the transform lengths. On the other hand, if you want to make error 2030 recoverable to obtain the transform lengths dynamically in your program, you need to **add some coding statements** to your program. For details on coding these statements in each programming language, see the following examples:

- For Fortran, see page 121
- For C, see page 139
- For C++, see page 155

You may want to provide a separate subroutine to calculate the transform length whenever you need it. Figure 6 on page 43 shows how you might code a separate Fortran subroutine. Before calling SCFT in your program, you call this subroutine, SCFT which calculates the correct length and stores it in *n*. Upon return, your program checks the return code. If it is nonzero, the *n* argument was updated, as planned. You then do any necessary data setup and call SCFT. On the other hand, if the return code is zero, error handling was not invoked, the *n* argument was not updated, and the initialization step was performed for SCFT.

```
      SUBROUTINE SCFT
     * N, M, ISIGN, SCALE, AUX1, NAUX1,AUX2,NAUX2)
      REAL*4 X(0:*),Y(0:*),SCALE
      REAL*8 AUX1(7),AUX2(0:*)
      INTEGER*4 INIT,INC1X,INC2X,INC1Y,INC2Y,N,M,ISIGN,NAUX1,NAUX2
      EXTERNAL ENOTRM
      CHARACTER*8   S2030
          CALL EINFO(0)
          CALL ERRSAV(2030,S2030)
          CALL ERRSET(2030,0,-1,1,ENOTRM,0)
          CALL SCFT(INIT,X,INC1X,INC2X,Y,INC1Y,INC2Y,
     *              N,M,ISIGN,SCALE,AUX1,NAUX1,AUX2,NAUX2,*10)
          CALL ERRSTR(2030,S2030)
          RETURN
10        CONTINUE
          CALL ERRSTR(2030,S2030)
          RETURN 1
          END
```

*Figure 6. Fortran Subroutine to Calculate Transform Length*

You might want to combine the request for auxiliary storage sizes along with your request for transform lengths. Figure 7 shows how you might code a separate Fortran subroutine combining both requests. It combines the functions performed by the subroutines in Figure 3 on page 38 and Figure 6.

```
      SUBROUTINE SCFT
     * N, M, ISIGN, SCALE, AUX1, NAUX1,AUX2,NAUX2)
      REAL*4 X(0:*),Y(0:*),SCALE
      REAL*8 AUX1(7),AUX2(0:*)
      INTEGER*4 INIT,INC1X,INC2X,INC1Y,INC2Y,N,M,ISIGN,NAUX1,NAUX2
      EXTERNAL ENOTRM
      CHARACTER*8   S2015,S2030
          CALL EINFO(0)
          CALL ERRSAV(2015,S2015)
          CALL ERRSAV(2030,S2030)
          CALL ERRSET(2015,0,-1,1,ENOTRM,0)
          CALL ERRSET(2030,0,-1,1,ENOTRM,0)
C     SETS NAUX1 AND NAUX2 TO THE MINIMUM VALUES REQUIRED TO USE
C     THE RECOVERABLE INPUT-ARGUMENT ERROR-HANDLING FACILITY
          NAUX1 = 7
          NAUX2 = 0
          CALL SCFT(INIT,X,INC1X,INC2X,Y,INC1Y,INC2Y,
     *              N,M,ISIGN,SCALE,AUX1,NAUX1,AUX2,NAUX2,*10)
          CALL ERRSTR(2015,S2015)
          CALL ERRSTR(2030,S2030)
          RETURN
10        CONTINUE
          CALL ERRSTR(2015,S2015)
          CALL ERRSTR(2030,S2030)
          RETURN 1
          END
```

*Figure 7. Fortran Subroutine to Calculate Auxiliary Storage Sizes and Transform Length*

## Getting the Best Accuracy

This section explains how accuracy of your results can be affected in various situations and what you can do to achieve the best possible accuracy.

### What Precisions Do ESSL Subroutines Operate On?

Both short- and long-precision real versions of the subroutines are provided in most areas of ESSL. In some areas, short- and long-precision complex versions are also provided, and, occasionally, an integer version is provided. The subroutine names are distinguished by a one- or two-letter prefix based on the following letters:

S for short-precision real
D for long-precision real
C for short-precision complex
Z for long-precision complex
I for integer

For a description of these data types, see “How Do You Set Up Your Scalar Data?” on page 28. The scalar data types and how you should code them for each programming language are listed under “Coding Your Scalar Data” in each language section in Chapter 4 on page 111.

### How does the Nature of the ESSL Computation Affect Accuracy?

In subroutines performing operations such as copy and swap, the accuracy of data is not affected. In subroutines performing computations involving mathematical operations on array data, the accuracy of the result may be affected by the following:

- The algorithm, which can vary depending on values or array sizes within the computation or the number of threads used.
- The matrix and vector sizes

For this reason, the ESSL subroutines do **not** have a closed formula for the error of computation. In other words, there is no formula with which you can calculate the error of computation in each subroutine.

Short-precision subroutines sometimes provide increased accuracy of results by accumulating intermediate results in long precision. This is also noted in the functional description for each subroutine. See the *RS/6000 POWERstation and POWERserver Hardware Technical Reference Information—General Architectures* manual for details.

For the RS/6000 POWER and POWER2, the short-precision, floating-point operands are stored by the hardware in the floating-point registers as long-precision values, and, as a result, all arithmetic operations are performed in long-precision. Where applicable, the ESSL subroutines use the Multiply-Add instructions, which combine a Multiply and Add operation without an intermediate rounding operation.

For the **ESSL POWER Library**, **ESSL Thread-Safe Library**, and **ESSL SMP Library**, results obtained by 32-bit environment and 64-bit environment applications using the same ESSL library are mathematically equivalent but may not be bit identical.

### What Data Type Standards Are Used by ESSL, and What Exceptions Should You Know About?

The data types operated on by the short-precision, long-precision, and integer versions of the subroutines are ANSI/IEEE 32-bit and 64-bit binary floating-point format, and 32-bit integer. See the *ANSI/IEEE Standard for Binary Floating-Point Arithmetic, ANSI/IEEE Standard 754-1985* for more detail.

There are ESSL-specific rules that apply to the results of computations using the ANSI/IEEE standards. When running your program, the result of a multiplication of NaN (“Not-a-Number”) by a scalar zero, under certain circumstances, may differ in the ESSL subroutines from the result you expect.

Usually, when NaN is multiplied by a scalar zero, the result is NaN; however, in some ESSL subroutines where scaling is performed, the result may be zero. For example, in computing $\alpha$***A***, where $\alpha$ is a scalar and ***A*** is a matrix, if $\alpha$ is zero and one (or more) of the elements of ***A*** is NaN, the scaled result, using that element, may be a zero, rather than NaN. To avoid problems, you should consider this when designing your program.

### How is Underflow Handled?

ESSL does not mask underflow. If your program incurs a number of unmasked underflows, its overall performance decreases. **For the RS/6000, floating-point exception trapping is disabled by default. Therefore, you do not have to mask underflow unless you have changed the default.**

### Where Can You Find More Information on Accuracy?

Information about accuracy can be found in the following places:

- Migration considerations concerning accuracy of results between releases, platforms, and so forth are described in Chapter 6 on page 169.
- Specific information on accuracy for each area of ESSL is given in “Performance and Accuracy Considerations” in each chapter introduction in Part 2.
- The functional description under “Function” for each subroutine explains what you need to know about the accuracy of the computation. Varying implementation techniques are sometimes used to improve performance. To let you know how accuracy is affected, the functional description may explain in general terms the different techniques used in the computation.

## Getting the Best Performance

This section describes how you can achieve the best possible performance from the ESSL subroutines.

### What General Coding Techniques Can You Use to Improve Performance?

There are many ways in which you can improve the performance of your program. Here are some of them:

- Use the basic linear algebra subprograms and matrix operations in the order of optimum performance: matrix-matrix computations, matrix-vector computations, and vector-scalar computations. When data is presented in matrices or vectors, rather than vectors or scalars, multiple operations can be performed by a single ESSL subroutine.

- Where possible, use subroutines that do multiple computations, such as SNDOT and SNAXPY, rather than individual computations, such as SDOT and SAXPY.
- Use a stride of 1 for the data in your computations. Not having vector elements consecutively accessed in storage can degrade your performance. The closer the vector elements are to each other in storage, the better your performance. For an explanation of stride, see "How Stride Is Used for Vectors" on page 58.
- Do **not** specify the size of the leading dimension of an array (*lda*) or stride of a vector (*inc*) equal to or near a multiple of:
  - 128 for a long-precision array
  - 256 for a short-precision array
- Do **not** specify the individual sizes of your one-dimensional arrays as multiples of 128. This is especially important when you are passing several one-dimensional arrays to an ESSL subroutine. (The multiplicity can cause a performance problem that otherwise might not occur.)
- For small problems, avoid using a large leading dimension (*lda*) for your matrix.
- In general, align your arrays on doubleword boundaries, regardless of the type of data; however, when running on a POWER2 processor, it is best to align your long-precision arrays on a quadword boundary. For information on how your programming language aligns data, see your programming language manuals.
- One subroutine may do scaling while another does not. If scaling is not necessary for your data, you get better performance by using the subroutine without scaling. SNORM2 and DNORM2 are examples of subroutines that do not do scaling, versus SNRM2 and DNRM2, which do scaling.
- Use the STRIDE subroutine to calculate the optimal stride values for your input or output data when using any of the Fourier transform subroutines, except _RCFT and _CRFT. Using these stride values for your data allows the Fourier transform subroutines to achieve maximum performance. You first obtain the optimal stride values from STRIDE, calling it once for each stride value desired. You then arrange your data using these stride values. After the data is set up, you call the Fourier transform subroutine. For details on the STRIDE subroutine and how to use it for each Fourier transform subroutine, see "STRIDE—Determine the Stride Value for Optimal Performance in Specified Fourier Transform Subroutines" on page 971. For additional information, see "Setting Up Your Data" on page 744.

## Where Can You Find More Information on Performance?

Information about performance can be found in the following places:

- Many of the techniques ESSL uses to achieve the best possible performance are described in the "High Performance of ESSL" on page 6.
- Migration considerations concerning performance are described in "Migrating ESSL Version 2 Programs to Version 3" on page 170.

- Specific information on performance for each area of ESSL is given in “Performance and Accuracy Considerations” in each chapter introduction in Part 2.
- Detailed performance information for selected subroutines can be found in reference [30], [41 ], [42 ] and on the IBM RS/6000 web site at:

  **http://www.rs6000.ibm.com/software/Apps/essl.html**

## Dealing with Errors when Using ESSL

At run time, you can encounter different types of errors or messages that are related to the use of the ESSL subroutines:

- Program exceptions
- ESSL input-argument errors
- ESSL computational errors
- ESSL resource errors
- ESSL attention messages

This section explains how to handle all these situations.

## What Can You Do about Program Exceptions?

The program exceptions you can encounter in ESSL are described in the RS/6000 architecture manuals. For details, see:

- The *ANSI/IEEE Standard for Binary Floating-Point Arithmetic, ANSI/IEEE Standard 754-1985*
- The *RS/6000 POWERstation and POWERserver Hardware Technical Reference Information—General Architectures* manual

## What Can You Do about ESSL Input-Argument Errors?

This section gives an overview on how you can handle input-argument errors.

### All Input-Argument Errors

ESSL checks the validity of most input arguments. If it finds that any are invalid, it issues the appropriate error messages. Also, except for the two recoverable errors described below, it terminates your program. You should use standard programming techniques to diagnose and fix unrecoverable input-argument errors, as described in Chapter 7 on page 173.

You can determine the input-argument errors that can occur in a subroutine by looking under “Error Conditions” in the subroutine description in Part 2 of this book. Error messages for all input-argument errors are listed in “Input-Argument Error Messages” on page 179.

### Recoverable Errors 2015 and 2030 Can Return Updated Values in the NAUX and N Arguments

For two input-argument errors, 2015 and 2030, in Fortran, C, C++, and PL/I programs, you have the option to continue running and have an updated value of the input argument returned to your program for subsequent use. These are called recoverable errors. This recoverable error-handling capability gives you flexibility in determining the correct values for the arguments. You can:

- Determine the correct size of an auxiliary work area by using error 2015. For help in deciding whether you want to use this capability and details on how to use it, see “Using Auxiliary Storage in ESSL” on page 32.
- Determine the correct length of a transform by using error 2030. For help in deciding whether you want to use this capability and details on how to use it, see “Providing a Correct Transform Length to ESSL” on page 38.

If you chose to leave errors 2015 and 2030 unrecoverable, you do not need to make any coding changes to your program. The input-argument error message is issued upon termination, containing the updated values you could have specified for the program to run successfully. You then make the necessary corrections in your program and rerun it.

If you choose to make errors 2015 and 2030 recoverable, you call the ERRSET subroutine to set up the ESSL error exit routine, ENOTRM, and then call the ESSL subroutine. When one or more of these errors occurs, the input-argument error message is issued with the updated values. In addition, the updated values are returned to your program in the input arguments named in the error message, along with a nonzero return code and processing continues. Return code values associated with these recoverable errors are described under “Error Conditions” for each ESSL subroutine in Part 2.

For details on how to code the necessary statements in your program to make 2015 and 2030 recoverable, see the following sections:

- “Input-Argument Errors in Fortran” on page 119
- “Input-Argument Errors in C” on page 136
- “Input-Argument Errors in C++” on page 152

## What Can You Do about ESSL Computational Errors?

This section gives an overview on how you can handle computational errors.

### All Computational Errors

ESSL computational errors are errors occurring in the computational data, such as in your vectors and matrices. You can determine the computational errors that can occur in a subroutine by looking under “Error Conditions” in the subroutine description in Part 2 of this book. These errors cause your program to terminate abnormally unless you take preventive action. A message is also provided in your output, containing information about the error. Messages are listed in “Computational Error Messages” on page 187.

When a computational error occurs, you should assume that the results are unpredictable. The result of the computation is valid only if no errors have occurred. In this case, a zero return code is returned.

Figure 8 on page 49 shows what happens when a computational error occurs.

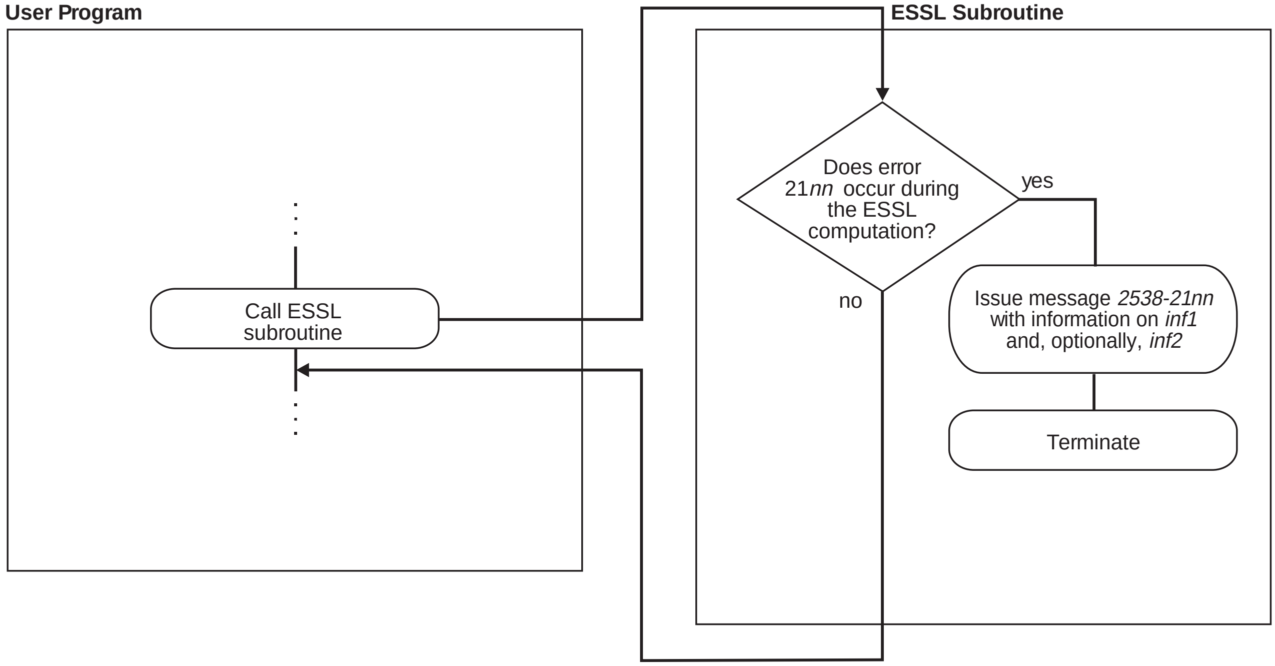

User Program
ESSL Subroutine
Call ESSL subroutine
Does error 21*nn* occur during the ESSL computation?
yes
no
Issue message *2538-21nn* with information on *inf1* and, optionally, *inf2*
Terminate

*Figure 8. How to Obtain Computational Error Information from an Error Message, but Terminate*

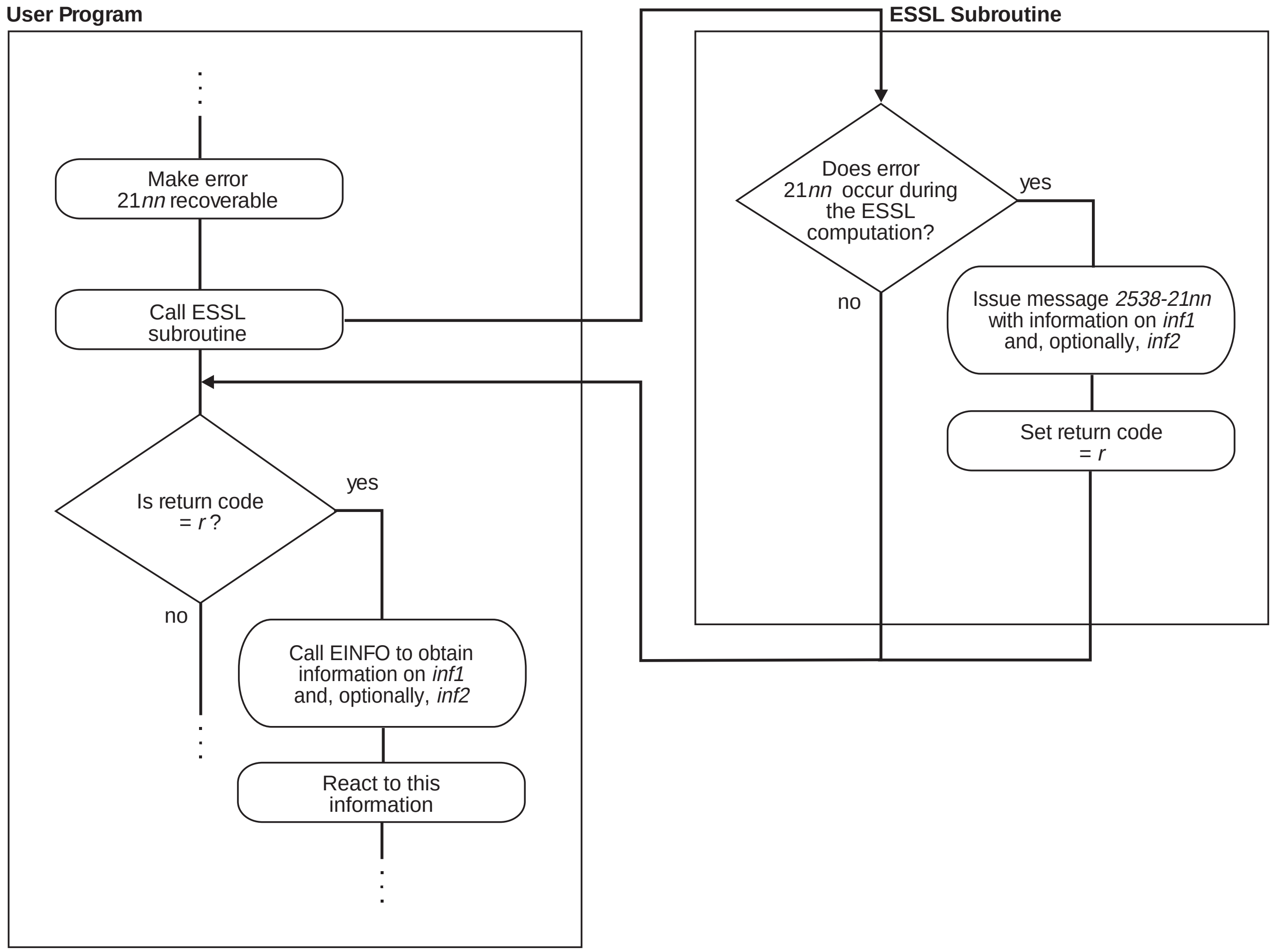
## Recoverable Computational Errors Can Return Values Through EINFO

In Fortran, C, C++, and PL/I programs, you have the capability to make certain computational errors recoverable and have information returned to your program about the errors. Recoverable computational errors are listed in Table 167 on page 962. First, you call EINFO in the beginning of your program to initialize the ESSL error option table. You then call ERRSET to reset the number of allowable errors for the computational error codes in which you are interested. When a computational error occurs, a nonzero return code is returned for each computational error. Return code values associated with these errors are described under “Error Conditions” in each subroutine description. Based on the return code, your program can branch to an appropriate statement to call the ESSL error information-handler subroutine, EINFO, to obtain specific information about the data involved in the error. This information is returned in the EINFO output arguments, *inf1* and, optionally, *inf2*. You can then check the information returned and continue processing, if you choose. The syntax for EINFO is described under “EINFO—ESSL Error Information-Handler Subroutine” on page 962. You also get a message in your output for each computational error encountered, containing information about the error. The EINFO subroutine provides the same information in the messages as it provides to your program.

For details on how to code the necessary statements in your program to obtain specific information on computational errors, see the following sections:

- “Computational Errors in Fortran” on page 122
- “Computational Errors in C” on page 140
- “Computational Errors in C++” on page 157

Figure 9 on page 50 shows what happens if you make a computational error recoverable.



User Program
Make error 21*nn* recoverable
Call ESSL subroutine
Is return code = *r*?
yes
no
Call EINFO to obtain information on *inf1* and, optionally, *inf2*
React to this information
ESSL Subroutine
Does error 21*nn* occur during the ESSL computation?
yes
no
Issue message *2538-21nn* with information on *inf1* and, optionally, *inf2*
Set return code = *r*

*Figure 9. How to Obtain Computational Error Information in an Error Message and in Your Program*

## What Can You Do about ESSL Resource Errors?

This section gives an overview on how you can handle resource errors.

### All Resource Errors

ESSL returns a resource error and terminates your program when an attempt to allocate work area fails. Some ESSL subroutines attempt to allocate work area for their internal use. Other ESSL subroutines attempt to dynamically allocate auxiliary storage when a user requests it through calling sequence arguments, such as *aux* and *naux*. For information on how you could reduce memory constraints on the system or increase the amount of memory available before rerunning the application program, see “ESSL Resource Error Messages” on page 176.

You can determine the resource errors that can occur in a subroutine by looking under “Error Conditions” in the subroutine description in Part 2 of this book. Error messages for all resource errors are listed in “Resource Error Messages” on page 190.

## What Can You Do about ESSL Attention Messages?

This section gives an overview on how you can handle attention messages.

### All Attention Messages

ESSL returns an attention message to describe a condition that occurred, however, ESSL is able to continue processing. For information on how you could reduce memory constraints on the system or increase the amount of memory available, see “ESSL Resource Error Messages” on page 176.

For example, an attention message may be issued when enough work area was available to continue processing, but was not the amount initially requested. An attention message would be issued to indicate that performance may be degraded.

For a list of subroutines that may generate an attention message, see Table 31 on page 177. For a list of attention messages, see “Informational and Attention Error Messages” on page 190.

## How Do You Control Error Handling by Setting Values in the ESSL Error Option Table?

This section explains all aspects of using the ESSL error option table.

### What Values Are Set in the ESSL Error Option Table?

The ESSL error option table contains information that tells ESSL what to do every time it encounters an ESSL-generated error. Table 26 shows the default values established in the table when ESSL is installed.

*Table 26 (Page 1 of 2). ESSL Error Option Table Default Values*

| Range of Error Messages (From–To) | Number of Allowable Errors (ALLOW) | Number of Messages Printed (PRINT) | Modifiable Table Entry (MODENT) |
|---|---|---|---|
| 2538–2000 | Unlimited | 255 | NO |
| 2538–2001 through 2538–2073 | Unlimited | 255 | YES |
| 2538–2074 | Unlimited | 5 | YES |
| 2538–2075 through 2538–2098 | Unlimited | 255 | YES |
| 2538–2099 | 1 | 255 | YES |
| 2538–2100 through 2538–2101 | 1 | 255 | YES |
| 2538–2102 | Unlimited | 255 | YES |
| 2538–2103 through 2538–2113 | 1 | 255 | YES |
| 2538–2114 | Unlimited | 255 | YES |
| 2538–2115 through 2538–2122 | 1 | 255 | YES |
| 2538–2123 through 2538–2124 | Unlimited | 255 | YES |
| 2538–2125 through 2538–2126 | 1 | 255 | YES |
| 2538–2127 | Unlimited | 255 | YES |
| 2538–2128 through 2538–2137 | 1 | 255 | YES |
| 2538–2138 through 2538–2143 | Unlimited | 255 | YES |
| 2538–2144 through 2538–2145 | 1 | 255 | YES |

*Table 26 (Page 2 of 2). ESSL Error Option Table Default Values*

| Range of Error Messages (From–To) | Number of Allowable Errors (ALLOW) | Number of Messages Printed (PRINT) | Modifiable Table Entry (MODENT) |
|---|---|---|---|
| 2538–2146 | Unlimited | 255 | YES |
| 2538–2147 through 2538–2198 | 1 | 255 | YES |
| 2538–2199 | 1 | 255 | YES |
| 2538–2400 through 2538–2499 | 1 | 255 | NO |
| 2538–2600 through 2538–2699 | Unlimited | 255 | NO |
| 2538–2700 through 2538–2799 | 1 | 255 | NO |

## How Can You Change the Values in the Error Option Table?

You can change any of the values in the ESSL error option table by calling the ERRSET subroutine in your program. This dynamically changes values at run time. You can also save and restore entries in the table by using the ERRSAV and ERRSTR subroutines, respectively. For a description of the ERRSET, ERRSAV, and ERRSTR subroutines see Chapter 17 on page 959.

## When Do You Change the Values in the Error Option Table?

Because you can change the information in the error option table, you can control what happens when any of the ESSL errors occur. There are a number of instances when you may want to do this:

***To Customize Your Error-Handling Environment:*** You may simply want to adjust the number of times an error is allowed to occur before your program terminates. You can use any of the capabilities available in ERRSET.

***To Obtain Auxiliary Storage Sizes and Transform Lengths:*** You may want to make ESSL input-argument error 2015 or 2030 recoverable, so ESSL returns updated auxiliary storage sizes or transform lengths, respectively, to your program. For a more detailed discussion, see “What Can You Do about ESSL Input-Argument Errors?” on page 47. For how to use ERRSET to do this, see the section for your programming language in Chapter 4 on page 111.

***To Get More Information About a Computational Error:*** You may want ESSL to return information about a computational error to your program. For a more detailed discussion, see “What Can You Do about ESSL Computational Errors?” on page 48. For how to do use ERRSET to do this, see the section for your programming language in Chapter 4 on page 111.

***To Allow Parts of Your Application to Have Unique Error-Handling Environments:*** If your program is part of a large application, you may want to dynamically save and restore entries in the error option table that have been altered by ERRSET. This ensures the integrity of the error option table when it is used by multiple programs within an application. For a more detailed discussion, see “ How Can You Control Error Handling in Large Applications by Saving and Restoring Entries in the Error Option Table?” on page 53 For how to use ERRSAV and ERRSTR, see the section for your programming language in Chapter 4 on page 111.

### How Can You Control Error Handling in Large Applications by Saving and Restoring Entries in the Error Option Table?

When your program is part of a larger application, you should consider that one of the following can occur:

- If you use ERRSET in your program to reset any of the values in the error option table for any of the ESSL input-argument errors or computational errors, some other program in the application may be adversely affected. It may be expecting its original values.
- If some other program in the application uses ERRSET to reset any of the values in the error option table for any of the ESSL input-argument errors or computational errors, your program may be adversely affected. You may need a certain value in the error option table, and the application may have reset that value.

These situations can be avoided if every program that uses ERRSET, in the large application, also uses the ERRSAV and ERRSTR facilities. For a particular error number, ERRSAV saves an entry from the error option table in an area accessible to your program. ERRSTR then stores the entry back into the error option table from the storage area. You code an ERRSAV and ERRSTR for each input-argument error number and computational error number for which you do an ERRSET to reset the values in the error option table. Call ERRSAV at the beginning of your program after you call EINFO, and then call ERRSTR at the end of your program after all ESSL computations are completed. This saves the original contents of the error option table while your program is running with different values, and then restores it to its original contents when your program is done. For details on how to code these statements in your program, see Chapter 4 on page 111.

## How does Error Handling Work in a Threaded Environment?

When your application program or Fortran first creates a thread, ESSL initializes the error option table information to the default settings shown in Table 26 on page 51. You can change the default settings for each thread you created by calling the appropriate error handling subroutines (ERRSET, ERRSAV, or ERRSTR) from each thread. An example of how to initialize the error option table and change the default settings on multiple threads is shown in “Example of Handling Errors in a Multithreaded Application Program” on page 127.

ESSL issues error messages as they occur in a threaded environment. Error messages issued from any of the existing threads are written to standard output in the order in which they occur.

When a terminating condition occurs on any of the existing threads (for example, the number of allowable errors was exceeded), ESSL terminates your application program. One set of summary information corresponding to the terminating thread is always printed. Summary information corresponding to other threads may also be printed.

## Where Can You Find More Information on Errors?

Information about errors and how to handle them can be found in the following places:

- How to code your program to use the ESSL error-handling facilities is described in Chapter 4 on page 111.
- All ESSL error messages are listed under “Messages” on page 178.
- The errors and return codes associated with each ESSL subroutine are listed under “Error Conditions” in each subroutine description in Part 2.
- Complete diagnostic procedures for all types of ESSL programming and documentation problems, along with how to collect information and report a problem, are provided in Chapter 7 on page 173.

# Chapter 3. Setting Up Your Data Structures

This chapter provides you with information that you need to set up your data structures, consisting of vectors, matrices, and sequences. These techniques apply to programs in all programming languages.

## Concepts

Vectors, matrices, and sequences are conceptual data structures contained in arrays. In many cases, ESSL uses stride or leading dimension to select the elements of the vector, matrix, or sequence from an array. In other cases, ESSL uses a specific mapping, or storage layout, that identifies the elements of the vector, matrix, or sequence in an array, sometimes requiring several arrays to help define the mapping. These elements selected from the array(s) make up the conceptual data structure.

When you call an ESSL subroutine, it assumes that the data structure is set up properly in the array(s) you pass to it. If it is not, your results are unpredictable. ESSL also uses these same storage layouts for data structures passed back to your program.

The use of the terms vector, matrix, and sequence in this book is consistent with standard mathematical definitions, and their representations are consistent with conventions used in mathematical texts. Special notations and conventions used in this book for describing vectors, matrices, and sequences are explained in “Special Notations and Conventions” on page xxv.

**Overlapping Data Structures:** Most of the subroutines do not allow vectors, matrices, or sequences to overlap. If this occurs, results are unpredictable. Where this applies, it is explained in Notes in each subroutine description. This means the elements of the data structure cannot reside in the same storage locations as any of the other data structures. It is possible, however, to have elements of different data structures in the same array, as long as the elements are interleaved through storage using strides greater than 1. For example, using vectors ***x*** and ***y*** with strides of 2, where ***x*** starts at `A(1)` and ***y*** starts at `A(2)`, the elements reside in array `A` in the order $x_1$, $y_1$, $x_2$, $y_2$, $x_3$, $y_3$, ... and so forth.

When you use this technique, you should be careful that you specify different starting locations for each data structure contained in the array.

## Vectors

A vector is a one-dimensional, ordered collection of numbers. It can be a column vector, which represents an *n* by 1 ordered collection, or a row vector, which represents a 1 by *n* ordered collection.

The column vector appears symbolically as follows:

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ . \\ x_n \end{bmatrix}$$

A row vector appears symbolically as follows:

$$\boldsymbol{x} = \begin{bmatrix} x_1 \; x_2 \; x_3 \; \ldots \; x_n \end{bmatrix}$$

Vectors can contain either real or complex numbers. When they contain real numbers, they are sometimes called real vectors. When they contain complex numbers, they are called complex vectors.

## Transpose of a Vector

The transpose of a vector changes a column vector to a row vector, or vice versa:

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ . \\ x_n \end{bmatrix} \qquad \boldsymbol{x}^{\mathrm{T}} = \begin{bmatrix} x_1 \;\; x_2 \;\; x_3 \ldots x_n \end{bmatrix} \qquad (\boldsymbol{x}^{\mathrm{T}})^{\mathrm{T}} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ . \\ x_n \end{bmatrix}$$

The ESSL subroutines use the vector as it is intended in the computation, as either a column vector or a row vector; therefore, no movement of data is necessary.

In the examples provided with the subroutine descriptions in Part 2 of this book, both types of vectors are represented in the same way, showing the elements of the array that make up the vector ***x***, as follows:

```
(1.0, 2.0, 3.0, 4.0, 5.0, 6.0)
```

## Conjugate Transpose of a Vector

The conjugate transpose of a vector ***x***, containing complex numbers, is denoted by $\boldsymbol{x}^{\mathrm{H}}$ and is expressed as follows:

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ . \\ x_n \end{bmatrix} \qquad \boldsymbol{x}^{\mathrm{H}} = \begin{bmatrix} \bar{x}_1 \;\; \bar{x}_2 \;\; \bar{x}_3 \ldots \bar{x}_n \end{bmatrix}$$

Just as for the transpose of a vector, no movement of data is necessary for the conjugate transpose of a vector.

## In Storage

A vector is usually stored within a one- or two-dimensional array. Its elements are stored sequentially in the array, but not necessarily contiguously.

The **location** of the vector in the array is specified by the argument for the vector in the ESSL calling sequence. It can be specified in a number of ways. For example, if A is an array of length 12, and you want to specify vector ***x*** as starting at the first element of array A, specify A as the argument, such as in:

```
X = SASUM (4,A,2)
```

where the number of elements to be summed in the vector is 4, the location of the vector is A, and the stride is 2.

If you want to specify vector ***x*** as starting at element 3 in array A, which is declared as A(1:12), specify:

```
X = SASUM (4,A(3),2)
```

If A is declared as A(-1:8), specify the following for element 3:

```
X = SASUM (4,A(1),2)
```

If A is a two-dimensional array and declared as A(1:4,1:10), and you want vector ***x*** to start at the second row and third column of A, specify the following:

```
X = SASUM (4,A(2,3),2)
```

The **stride** specified in the ESSL calling sequence is used to step through the array to select the vector elements. The direction in which the vector elements are selected from the array—that is, front to back or back to front—is indicated by the sign (+ or −) of the stride. The absolute value of the stride gives the spacing between each element selected from the array.

To calculate the total number of elements needed in an array for a vector, you can use the following formula, which takes into account the number of elements, *n*, in the array and the stride, *inc*, specified for the vector:

$$1+(n-1)|inc|$$

**An array can be much larger than the vector that it contains; that is, there can be many elements following the vector in the array, as well as elements preceding the vector.**

For a complete description of how vectors are stored within arrays, see “How Stride Is Used for Vectors” on page 58.

For a complex vector, a special storage arrangement is used to accommodate the two parts, *a* and *b*, of each complex number (*a*+*bi*) in the array. For each complex number, two sequential storage locations are required in the array. Therefore, exactly twice as much storage is required for complex vectors and matrices as for real vectors and matrices of the same precision. See “How Do You Set Up Your Scalar Data?” on page 28 for a description of real and complex numbers, and

"How Do You Set Up Your Arrays?" on page 29 for a description of how real and complex data is stored in arrays.

## How Stride Is Used for Vectors

The stride for a vector is an increment that is used to step through array storage to select the vector elements from an array. To define exactly which elements become the conceptual vector in the array, the following items are used together:

- The location of the vector within the array
- The stride for the vector
- The number of elements, *n*, to be processed

The stride can be positive, negative, or 0. For positive and negative strides, if you specify vector elements beyond the range of the array, your results are be unpredictable, and you may get program errors.

This section explains how each of the three types of stride is used to select the vector elements from the array.

### Positive Stride

When a positive stride is specified for a vector, the location specified by the argument for the vector is the location of the first element in the vector, element $x_1$. The vector is in forward order in the array: ($x_1$, $x_2$, ..., $x_n$). For example, if you specify `X(1)` for vector ***x***, where `X` is declared as `X(0:12)` and defined as:

```
X = (1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0, 11.0, 12.0, 13.0)
```

then processing begins at the second element in `X`, which is 2.0.

To find each successive element, the stride is added cumulatively to the starting point of vector ***x*** in the array. In this case, the starting point is `X(1)`. If the stride specified for vector ***x*** is 3 and the number of elements to be processed is 4, then the resulting elements selected from `X` for vector ***x*** are: `X(1),X(4),X(7),` and `X(10)`.

Vector ***x*** is then:

```
(2.0, 5.0, 8.0, 11.0)
```

As shown in this example, a vector does not have to extend to the end of the array. Elements are selected from the second to the eleventh element of the array, and the array elements after that are not used.

This element selection can be expressed in general terms. Using `BEGIN` as the starting point in an array `X` and *inc* as the stride, this results in the following elements being selected from the array:

X(BEGIN)
X(BEGIN+*inc*)
X(BEGIN+(2)*inc*)
X(BEGIN+(3)*inc*)
.
.
.
X(BEGIN+(*n*–1)*inc*)

The following general formula can be used to calculate each vector element position in a one-dimensional array:

$x_i = \mathtt{X}(\mathtt{BEGIN} + (i-1)(inc))$ for $i = 1, n$

When using an array with more than one dimension, you should understand how the array elements are stored to ensure that elements are selected properly. For a description of array storage, see “Setting Up Arrays in Fortran” on page 112. You should remember that the elements of an array are selected as they are arranged in storage, regardless of the number of dimensions defined in the array. Stride is used to step through array storage until *n* elements are selected. ESSL processing stops at that point. For example, given the following two-dimensional array, declared as A(1:7,1:4).

Matrix A is:

$$\begin{bmatrix} 1.0 & 8.0 & 15.0 & 22.0 \\ 2.0 & 9.0 & 16.0 & 23.0 \\ 3.0 & 10.0 & 17.0 & 24.0 \\ 4.0 & 11.0 & 18.0 & 25.0 \\ 5.0 & 12.0 & 19.0 & 26.0 \\ 6.0 & 13.0 & 20.0 & 27.0 \\ 7.0 & 14.0 & 21.0 & 28.0 \end{bmatrix}$$

with A(3,1) specified for vector ***x***, a stride of 2, and the number of elements to be processed as 12, the resulting vector ***x*** is:

(3.0, 5.0, 7.0, 9.0, 11.0, 13.0, 15.0, 17.0, 19.0, 21.0, 23.0, 25.0)

This is not a conventional use of arrays, and you should be very careful when using this technique.

## Zero Stride

When a zero stride is specified for a vector, the starting point for the vector is the only element used in the computation. The starting point for the vector is at the location specified by the argument for the vector, just as though you had specified a positive stride. For example, if you specify X for vector ***x***, where X is defined as:

X = (5.0, 4.0, 3.0, 2.0, 1.0)

and you specify the number of elements, *n*, to be processed as 6, then processing begins at the first element, which is 5.0. This element is used for each of the six elements in vector ***x***.

This makes the conceptual vector ***x*** appear as:

(5.0, 5.0, 5.0, 5.0, 5.0, 5.0)

The following general formula shows how to calculate each vector position in a one-dimensional array:

$x_i = \mathtt{X}(\mathtt{BEGIN})$ for $i = 1, n$

### Negative Stride

When a negative stride is specified for a vector, the location specified for the vector is actually the location of the last element in the vector. In other words, the vector is in reverse order in the array: ($x_n$, $x_{n-1}$, ..., $x_1$). You specify the end of the vector, ($x_n$). ESSL then calculates where the starting point ($x_1$) is by using the following arguments:

- The location of the vector in the array
- The stride for the vector, *inc*
- The number of elements, *n*, to be processed

If you specify vector ***x*** at location X(BEGIN) in array X with a negative stride of *inc* and *n* elements to be processed, then the following formula gives the starting point of vector ***x*** in the array:

X(BEGIN + (–*n*+1)(*inc*))

For example, if you specify X(2) for vector ***x***, where X is declared as X(1:9) and defined as:

```
X = (1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0)
```

and if you specify a stride of −2, and four elements to be processed, processing begins at the following element in X:

X(2+(–4+1)(–2)) = X(8)

where element X(8) is 8.0.

To find each of the *n* successive element positions in the array, you successively add the stride to the starting point *n*–1 times. Suppose the formula calculated a starting point of X(SP); the elements selected are:

X(SP)
X(SP+*inc*)
X(SP+(2)*inc*)
X(SP+(3)*inc*)
.
.
.
X(SP+(*n*–1)*inc*)

In the above example, the resulting elements selected from X for vector ***x*** are X(8), X(6), X(4), and X(2). This makes the resulting vector ***x*** appear as follows:

```
(8.0, 6.0, 4.0, 2.0)
```

The following general formula can be used to calculate each vector element position in a one-dimensional array:

$x_i$ = X(BEGIN + (–*n*+*i*)(*inc*)) for $i = 1, n$

## Sparse Vector

A sparse vector is a vector having a relatively small number of nonzero elements.

Consider the following as an example of a sparse vector ***x*** with *n* elements, where *n* is 11, and vector ***x*** is:

```
(0.0, 0.0, 1.0, 0.0, 2.0, 3.0, 0.0, 4.0, 0.0, 5.0, 0.0)
```

## In Storage

There are two storage modes that apply to sparse vectors: full-vector storage mode and compressed-vector storage mode. When a sparse vector is stored in **full-vector storage mode**, all its elements, including its zero elements, are stored in an array.

For example, sparse vector ***x*** is stored in full-vector storage mode in a one-dimensional array X, as follows:

```
X  = (0.0, 0.0, 1.0, 0.0, 2.0, 3.0, 0.0, 4.0, 0.0, 5.0, 0.0)
```

When a sparse vector is stored in **compressed-vector storage mode**, it is stored without its zero elements. It consists of two one-dimensional arrays, each with a length of *nz*, where *nz* is the number of nonzero elements in vector ***x***:

- The first array contains the nonzero elements of the sparse vector ***x***, stored contiguously within the array.

  **Note:** The ESSL subroutines do not check that all elements are nonzero. You do not get an error if any elements are zero.

- The second array contains a sequence of integers indicating the element positions (indices) of the nonzero elements of the sparse vector ***x*** stored in full-vector storage mode. This is referred to as the indices array.

For example, the sparse vector ***x*** shown above might have its five nonzero elements stored in ascending order in array X of length 5, as follows:

```
X = (1.0, 2.0, 3.0, 4.0, 5.0)
```

in which case, the array of indices, INDX, also of length 5, contains:

```
INDX = (3, 5, 6, 8, 10)
```

If the sparse vector ***x*** has its elements stored in random order in the array X as:

```
X  = (5.0, 3.0, 4.0, 1.0, 2.0)
```

then the array INDX contains:

```
INDX = (10, 6, 8, 3, 5)
```

In general terms, this storage technique can be expressed as follows:

For each $x_j \neq 0$, for $j = 1, n$
there exists $i$, where $1 \leq i \leq nz$,
such that X($i$) = $x_j$ and INDX($i$) = $j$.

where:

$x_1, ..., x_n$ are the $n$ elements of sparse vector ***x***, stored in full-vector storage mode.
X is the array containing the *nz* nonzero elements of sparse vector ***x***; that is, vector ***x*** is stored in compressed-vector storage mode.
INDX is the array containing the *nz* indices indicating the element positions.

To avoid an error when using the `INDX` array to access the elements in any other target vector, the length of the target vector must be greater than or equal to max(`INDX`(*i*)) for *i* = 1, *nz*.

## Matrices

A matrix, also referred to as a general matrix, is an *m* by *n* ordered collection of numbers. It is represented symbolically as:

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & . & . & . & a_{1n} \\ . & & & & . \\ . & & & & . \\ . & & & & . \\ a_{m1} & . & . & . & a_{mn} \end{bmatrix}$$

where the matrix is named ***A*** and has *m* rows and *n* columns. The elements of the matrix are $a_{ij}$, where *i* = 1, *m* and *j* = 1, *n*.

Matrices can contain either real or complex numbers. Those containing real numbers are called real matrices; those containing complex numbers are called complex matrices.

## Transpose of a Matrix

The transpose of a matrix ***A*** is a matrix formed from ***A*** by interchanging the rows and columns such that row *i* of matrix ***A*** becomes column *i* of the transposed matrix. The transpose of ***A*** is denoted by $\boldsymbol{A}^T$. Each element $a_{ij}$ in ***A*** becomes element $a_{ji}$ in $\boldsymbol{A}^T$. If ***A*** is an *m* by *n* matrix, then $\boldsymbol{A}^T$ is an *n* by *m* matrix. The following represents a matrix and its transpose:

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & . & . & . & a_{1n} \\ . & & & & . \\ . & & & & . \\ . & & & & . \\ a_{m1} & . & . & . & a_{mn} \end{bmatrix} \qquad \boldsymbol{A}^{\mathrm{T}} = \begin{bmatrix} a_{11} & . & . & . & a_{m1} \\ . & & & & . \\ . & & & & . \\ . & & & & . \\ a_{1n} & . & . & . & a_{mn} \end{bmatrix}$$

ESSL assumes that all matrices are stored in untransformed format, such as matrix ***A*** shown above. No movement of data is necessary in your application program when you are processing transposed matrices. The ESSL subroutines adjust their selection of elements from the matrix when an argument in the calling sequence indicates that the transposed matrix is to be used in the computation. Examples of this are the *transa* and *transb* arguments specified for SGEADD, matrix addition.

## Conjugate Transpose of a Matrix

The conjugate transpose of a matrix ***A***, containing complex numbers, is denoted by $\boldsymbol{A}^H$ and is expressed as follows:

$$A = \begin{bmatrix} a_{11} & . \ . \ . & a_{1n} \\ . & & . \\ . & & . \\ . & & . \\ a_{m1} & . \ . \ . & a_{mn} \end{bmatrix} \qquad A^{\mathrm{H}} = \begin{bmatrix} \bar{a}_{11} & . \ . \ . & \bar{a}_{m1} \\ . & & . \\ . & & . \\ . & & . \\ \bar{a}_{1n} & . \ . \ . & \bar{a}_{mn} \end{bmatrix}$$

Just as for the transpose of a matrix, the conjugate transpose of a matrix is stored in untransformed format. No movement of data is necessary in your program.

## In Storage

A matrix is usually stored in a two-dimensional array. Its elements are stored successively within the array. Each column of the matrix is stored successively in the array. The leading dimension argument is used to select the matrix elements from each successive column of the array. The starting point of the matrix in the array is specified as the argument for the matrix in the ESSL calling sequence. For example, if matrix ***A*** is contained in array `A` and starts at the first element in the first row and first column of `A`, you should specify `A` as the argument for matrix ***A***, such as in:

```
CALL  SGEMX (5,2,1.0,A,6,X,1,Y,1)
```

where, in the matrix-vector product, the number of rows in matrix ***A*** is 5, the number of columns in matrix ***A*** is 2, the scaling constant is 1.0, the location of the matrix is `A`, the leading dimension is 6, the vectors used in the matrix-vector product are `X` and `Y`, and their strides are 1.

If matrix ***A*** is contained in the array `BIG`, declared as `BIG(1:20,1:30)`, and starts at the second row and third column of `BIG`, you should specify `BIG(2,3)` as the argument for matrix ***A***, such as in:

```
CALL  SGEMX (5,2,1.0,BIG(2,3),6,X,1,Y,1)
```

See “How Leading Dimension Is Used for Matrices” for a complete description of how matrices are stored within arrays.

For a complex matrix, a special storage arrangement is used to accommodate the two parts, *a* and *b*, of each complex number (*a*+*bi*) in the array. For each complex number, two sequential storage locations are required in the array. Therefore, exactly twice as much storage is required for complex matrices as for real matrices of the same precision. See “How Do You Set Up Your Scalar Data?” on page 28 for a description of real and complex numbers, and “How Do You Set Up Your Arrays?” on page 29 for a description of how real and complex data is stored in arrays.

## How Leading Dimension Is Used for Matrices

The leading dimension for a two-dimensional array is an increment that is used to find the starting point for the matrix elements in each successive column of the array. To define exactly which elements become the conceptual matrix in the array, the following items are used together:

- The location of the matrix within the array
- The leading dimension
- The number of rows, *m*, to be processed in the array

- The number of columns, *n*, to be processed in the array

The leading dimension must always be positive. It must always be greater than or equal to *m*, the number of rows in the matrix to be processed. For an array, `A`, declared as `A(E1:E2,F1:F2)`, the leading dimension is equal to:

```
(E2–E1+1)
```

The starting point for selecting the matrix elements from the array is at the location specified by the argument for the matrix in the ESSL calling sequence. For example, if you specify `A(3,0)` for a 4 by 4 matrix ***A***, where `A` is declared as `A(1:7,0:4)`:

$$\begin{bmatrix} 1.0 & 8.0 & 15.0 & 22.0 & 29.0 \\ 2.0 & 9.0 & 16.0 & 23.0 & 30.0 \\ 3.0 & 10.0 & 17.0 & 24.0 & 31.0 \\ 4.0 & 11.0 & 18.0 & 25.0 & 32.0 \\ 5.0 & 12.0 & 19.0 & 26.0 & 33.0 \\ 6.0 & 13.0 & 20.0 & 27.0 & 34.0 \\ 7.0 & 14.0 & 21.0 & 28.0 & 35.0 \end{bmatrix}$$

then processing begins at the element at row 3 and column 0 in array `A`, which is 3.0.

The leading dimension is used to find the starting point for the matrix elements in each of the *n* successive columns in the array. ESSL subroutines assume that the arrays are stored in column-major order, as described under “How Do You Set Up Your Arrays?” on page 29, and they add the leading dimension (times the size of the element in bytes) to the starting point. They do this *n*–1 times. This finds the starting point in each of the *n* columns of the array.

In the above example, the leading dimension is:

```
E2–E1+1 = 7–1+1 = 7
```

If the number of columns, *n*, to be processed is 4, the starting points are: `A(3,0)`, `A(3,1)`, `A(3,2)`, and `A(3,3)`. These are elements 3.0, 10.0, 17.0, and 24.0 for $a_{11}$, $a_{12}$, $a_{13}$, and $a_{14}$, respectively.

In general terms, this results in the following starting positions of each column in the matrix being calculated as follows:

```
A(BEGINI, BEGINJ)
A(BEGINI, BEGINJ+1)
A(BEGINI, BEGINJ+2)
.
.
.
A(BEGINI, BEGINJ+n–1)
```

To find the elements in each column of the array, 1 is added successively to the starting point in the column until *m* elements are selected. This is why the leading dimension must be greater than or equal to *m*; otherwise, you go past the end of each dimension of the array. In the above example, if the number of elements, *m*, to be processed in each column is 4, the following elements are selected from array `A` for the first column of the matrix: `A(3,0)`, `A(4,0)`, `A(5,0)`, and `A(6,0)`.

These are elements 3.0, 4.0, 5.0, and 6.0, corresponding to the matrix elements $a_{11}$, $a_{21}$, $a_{31}$, and $a_{41}$, respectively.

Column element selection can also be expressed in general terms. Using `A(BEGINI,BEGINJ)` as the starting point in the array, this results in the following elements being selected from each column in the array:

```
A(BEGINI, BEGINJ)
A(BEGINI+1, BEGINJ)
A(BEGINI+2, BEGINJ)
.
.
.
A(BEGINI+m–1, BEGINJ)
```

Combining this with the technique already described for finding the starting point in each column of the array, the resulting matrix in the example is:

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & . \; . \; . & a_{14} \\ . & & . \\ . & & . \\ . & & . \\ a_{41} & . \; . \; . & a_{44} \end{bmatrix} = \begin{bmatrix} 3.0 & 10.0 & 17.0 & 24.0 \\ 4.0 & 11.0 & 18.0 & 25.0 \\ 5.0 & 12.0 & 19.0 & 26.0 \\ 6.0 & 13.0 & 20.0 & 27.0 \end{bmatrix}$$

As shown in this example, a matrix does not have to include all columns and rows of an array. The elements of matrix ***A*** are selected from rows 3 through 6 and columns 0 through 3 of the array. Rows 1, 2, and 7 and column 4 of the array are not used.

## Symmetric Matrix

The matrix ***A*** is symmetric if it has the property $\boldsymbol{A} = \boldsymbol{A}^T$, which means:

- It has the same number of rows as it has columns; that is, it has *n* rows and *n* columns.
- The value of every element $a_{ij}$ on one side of the main diagonal equals its mirror image $a_{ji}$ on the other side: $a_{ij} = a_{ji}$ for $1 \le i \le n$ and $1 \le j \le n$.

The following matrix illustrates a symmetric matrix of order *n*; that is, it has *n* rows and *n* columns. The subscripts on each side of the diagonal appear the same to show which elements are equal:

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{21} & a_{31} & . & . & . & a_{n1} \\ a_{21} & a_{22} & a_{32} & & & & . \\ a_{31} & a_{32} & a_{33} & & & & . \\ . & & & . & & & . \\ . & & & & . & & . \\ . & & & & & . & . \\ a_{n1} & . & . & . & . & . & a_{nn} \end{bmatrix}$$

### In Storage

The four storage modes used for storing symmetric matrices are described in the following sections:

- “Lower-Packed Storage Mode”
- “Upper-Packed Storage Mode” on page 67
- “Lower Storage Mode” on page 68
- “Upper Storage Mode” on page 69

The storage technique you should use depends on the ESSL subroutine you are using and is given under Notes in each subroutine description.

***Lower-Packed Storage Mode:*** When a symmetric matrix is stored in lower-packed storage mode, the lower triangular part of the symmetric matrix is stored, including the diagonal, in a one-dimensional array. The lower triangle is packed by columns. (This is equivalent to packing the upper triangle by rows.) The matrix is packed sequentially column by column in $n(n+1)/2$ elements of a one-dimensional array. To calculate the location of each element $a_{ij}$ of matrix ***A*** in an array, `AP`, using the lower triangular packed technique, use the following formula:

$$\mathtt{AP}(i + ((2n-j)(j-1)/2)) = a_{ij} \quad \text{where } i \geq j$$

This results in the following storage arrangement for the elements of a symmetric matrix ***A*** in an array `AP`:

| | |
|---|---|
| `AP(1)` | = $a_{11}$ (start the first column) |
| `AP(2)` | = $a_{21}$ |
| `AP(3)` | = $a_{31}$ |
| . | . |
| . | . |
| . | . |
| `AP`($n$) | = $a_{n1}$ |
| `AP`($n$+1) | = $a_{22}$ (start the second column) |
| `AP`($n$+2) | = $a_{32}$ |
| . | . |
| . | . |
| . | . |
| `AP`(2$n$–1) | = $a_{n2}$ |
| `AP`(2$n$) | = $a_{33}$ (start the third column and so forth) |
| `AP`(2$n$+1) | = $a_{43}$ |
| . | . |
| . | . |
| . | . |
| `AP`($n$($n$+1)/2) | = $a_{nn}$ |

Following is an example of a symmetric matrix that uses the element values to show the order in which the matrix elements are stored in the array.

Given the following matrix ***A***:

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 6 & 7 & 8 & 9 \\ 3 & 7 & 10 & 11 & 12 \\ 4 & 8 & 11 & 13 & 14 \\ 5 & 9 & 12 & 14 & 15 \end{bmatrix}$$

the array is:

```
AP = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)
```

**Note:** Additional work storage is required in the array for some ESSL subroutines; for example, in the simultaneous linear algebraic equation subroutines SPPF, DPPF, SPPS, and DPPS. See the description of those subroutines in Part 2 for details.

Following is an example of how to transform your symmetric matrix to lower-packed storage mode:

```
      K = 0
      DO 1 J=1,N
         DO 2 I=J,N
            K = K+1
            AP(K)=A(I,J)
  2      CONTINUE
  1   CONTINUE
```

***Upper-Packed Storage Mode:*** When a symmetric matrix is stored in upper-packed storage mode, the upper triangular part of the symmetric matrix is stored, including the diagonal, in a one-dimensional array. The upper triangle is packed by columns. (This is equivalent to packing the lower triangle by rows.) The matrix is packed sequentially column by column in $n(n+1)/2$ elements of a one-dimensional array. To calculate the location of each element $a_{ij}$ of matrix ***A*** in an array AP using the upper triangular packed technique, use the following formula:

$$\mathrm{AP}(i+(j(j-1)/2)) = a_{ij} \quad \text{where } j \geq i$$

This results in the following storage arrangement for the elements of a symmetric matrix ***A*** in an array AP:

| | |
|---|---|
| AP(1) | = $a_{11}$ (start the first column) |
| AP(2) | = $a_{12}$ (start the second column) |
| AP(3) | = $a_{22}$ |
| AP(4) | = $a_{13}$ (start the third column) |
| AP(5) | = $a_{23}$ |
| AP(6) | = $a_{33}$ |
| AP(7) | = $a_{14}$ (start the fourth column) |
| . | . |
| . | . |
| . | . |
| AP($j(j-1)/2+1$) | = $a_{1j}$ (start the $j$-th column) |
| AP($j(j-1)/2+2$) | = $a_{2j}$ |
| AP($j(j-1)/2+3$) | = $a_{3j}$ |
| . | . |
| . | . |
| . | . |
| AP($j(j-1)/2+j$) | = $a_{jj}$ (end of the $j$-th column) |

.                                .
.                                .
.                                .
AP($n$($n$+1)/2)                 = $a_{nn}$

Following is an example of a symmetric matrix that uses the element values to show the order in which the matrix elements are stored in the array. Given the following matrix ***A***:

$$\begin{bmatrix} 1 & 2 & 4 & 7 & 11 \\ 2 & 3 & 5 & 8 & 12 \\ 4 & 5 & 6 & 9 & 13 \\ 7 & 8 & 9 & 10 & 14 \\ 11 & 12 & 13 & 14 & 15 \end{bmatrix}$$

the array is:

```
AP = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)
```

Following is an example of how to transform your symmetric matrix to upper-packed storage mode:

```
      K = 0
      DO 1 J=1,N
         DO 2 I=1,J
            K = K+1
            AP(K)=A(I,J)
 2       CONTINUE
 1    CONTINUE
```

***Lower Storage Mode:*** When a symmetric matrix is stored in lower storage mode, the lower triangular part of the symmetric matrix is stored, including the diagonal, in a two-dimensional array. These elements are stored in the array in the same way they appear in the matrix. The upper part of the matrix is not required to be stored in the array.

Following is an example of a symmetric matrix ***A*** of order 5 and how it is stored in an array AL.

Given the following matrix ***A***:

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 6 & 7 & 8 & 9 \\ 3 & 7 & 10 & 11 & 12 \\ 4 & 8 & 11 & 13 & 14 \\ 5 & 9 & 12 & 14 & 15 \end{bmatrix}$$

the array is:

$$AL = \begin{bmatrix} 1 & * & * & * & * \\ 2 & 6 & * & * & * \\ 3 & 7 & 10 & * & * \\ 4 & 8 & 11 & 13 & * \\ 5 & 9 & 12 & 14 & 15 \end{bmatrix}$$

where "*" means you do not have to store a value in that position in the array. However, these storage positions are required.

***Upper Storage Mode:*** When a symmetric matrix is stored in upper storage mode, the upper triangular part of the symmetric matrix is stored, including the diagonal, in a two-dimensional array. These elements are stored in the array in the same way they appear in the matrix. The lower part of the matrix is not required to be stored in the array.

Following is an example of a symmetric matrix ***A*** of order 5 and how it is stored in an array AU.

Given the following matrix ***A***:

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 6 & 7 & 8 & 9 \\ 3 & 7 & 10 & 11 & 12 \\ 4 & 8 & 11 & 13 & 14 \\ 5 & 9 & 12 & 14 & 15 \end{bmatrix}$$

the array is:

$$AU = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ * & 6 & 7 & 8 & 9 \\ * & * & 10 & 11 & 12 \\ * & * & * & 13 & 14 \\ * & * & * & * & 15 \end{bmatrix}$$

where "*" means you do not have to store a value in that position in the array. However, these storage positions are required.

## Positive Definite or Negative Definite Symmetric Matrix

A real symmetric matrix ***A*** is positive definite if and only if $\boldsymbol{x}^{T}\boldsymbol{Ax}$ is positive for all nonzero vectors ***x***.

A real symmetric matrix ***A*** is negative definite if and only if $\boldsymbol{x}^{T}\boldsymbol{Ax}$ is negative for all nonzero vectors ***x***.

### In Storage

The positive definite or negative definite symmetric matrix is stored in the same way the symmetric matrix is stored. For a description of this storage technique, see "Symmetric Matrix" on page 65.

## Complex Hermitian Matrix

A complex matrix is Hermitian if it is equal to its conjugate transpose:

$$\boldsymbol{H} = \boldsymbol{H}^{\mathrm{H}}$$

### In Storage

The complex Hermitian matrix is stored using the same four techniques used for symmetric matrices:

- Lower-packed storage mode, as described in “Lower-Packed Storage Mode” on page 66. (The complex Hermitian matrix is not symmetric; therefore, lower-packed storage mode is not equivalent to packing the upper triangle by rows, as it is for a symmetric matrix.)
- Upper-packed storage mode, as described in “Upper-Packed Storage Mode” on page 67. (The complex Hermitian matrix is not symmetric; therefore, upper-packed storage mode is not equivalent to packing the lower triangle by rows, as it is for a symmetric matrix.)
- Lower storage mode, as described in “Lower Storage Mode” on page 68.
- Upper storage mode, as described in “Upper Storage Mode” on page 69.

Following is an example of a complex Hermitian matrix ***H*** of order 5.

Given the following matrix ***H***:

$$\begin{bmatrix}
(11,\ 0) & (21,\ -1) & (31,\ 1) & (41,\ -1) & (51,\ -1) \\
(21,\ 1) & (22,\ 0) & (32,\ -1) & (42,\ -1) & (52,\ 1) \\
(31,\ -1) & (32,\ 1) & (33,\ 0) & (43,\ -1) & (53,\ -1) \\
(41,\ 1) & (42,\ 1) & (43,\ 1) & (44,\ 0) & (54,\ -1) \\
(51,\ 1) & (52,\ -1) & (53,\ 1) & (54,\ 1) & (55,\ 0)
\end{bmatrix}$$

it is stored in a one-dimensional array, `HP`, in $n(n+1)/2 = 15$ elements as follows:

- In lower-packed storage mode:

```
HP = ((11, *), (21, 1), (31, -1), (41, 1), (51, 1),
      (22, *), (32, 1), (42, 1), (52, -1), (33, *),
      (43, 1), (53, 1), (44, *), (54, 1), (55, *))
```

- In upper-packed storage mode:

```
HP = ((11, *), (21, -1), (22, *), (31, 1), (32, -1),
      (33, *), (41, -1), (42, -1), 43, -1), (44, *),
      (51, -1), (52, 1), (53, -1), (54, -1), (55, *))
```

or it is stored in a two-dimensional array, `HP`, as follows:

- In lower storage mode:

$$\text{HP} = \begin{bmatrix}
(11,\ *) & * & * & * & * \\
(21,\ 1) & (22,\ *) & * & * & * \\
(31,\ -1) & (32,\ 1) & (33,\ *) & * & * \\
(41,\ 1) & (42,\ 1) & (43,\ 1) & (44,\ *) & * \\
(51,\ 1) & (52,\ -1) & (53,\ 1) & (54,\ 1) & (55,\ *)
\end{bmatrix}$$

- In upper storage mode

```
         ┌                                              ┐
         │ (11, *) (21, -1) (31,  1) (41, -1) (51, -1) │
         │    *    (22,  *) (32, -1) (42, -1) (52,  1) │
  HP =   │    *       *     (33,  *) (43, -1) (53, -1) │
         │    *       *        *     (44,  *) (54, -1) │
         │    *       *        *        *     (55,  *) │
         └                                              ┘
```

where “*” means you do not have to store a value in that position in the array. The imaginary parts of the diagonal elements of a complex Hermitian matrix are always 0, so you do not need to set these values. The ESSL subroutines always assume that the values in these positions are 0.

## Positive Definite or Negative Definite Complex Hermitian Matrix

A complex Hermitian matrix ***A*** is positive definite if and only if $\boldsymbol{x}^{\mathrm{H}}\boldsymbol{Ax}$ is positive for all nonzero vectors ***x***.

A complex Hermitian matrix ***A*** is negative definite if and only if $\boldsymbol{x}^{\mathrm{H}}\boldsymbol{Ax}$ is negative for all nonzero vectors ***x***.

### In Storage

The positive definite or negative definite complex Hermitian matrix is stored in the same way the complex Hermitian matrix is stored. For a description of this storage technique, see “Complex Hermitian Matrix” on page 70.

## Positive Definite or Negative Definite Symmetric Toeplitz Matrix

A positive definite or negative definite symmetric matrix ***A*** of order *n* is also a Toeplitz matrix if and only if:

$a_{ij} = a_{i-1,j-1}$     for $i = 2, n$ and $j = 2, n$

The elements on each diagonal of the Toeplitz matrix have a constant value. For the definition of a positive definite or negative definite symmetric matrix, see “Positive Definite or Negative Definite Symmetric Matrix” on page 69.

The following matrix illustrates a symmetric Toeplitz matrix of order *n*; that is, it has *n* rows and *n* columns:

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{21} & a_{31} & . & . & . & a_{n1} \\ a_{21} & a_{11} & a_{21} & . & & & . \\ a_{31} & a_{21} & a_{11} & & . & & . \\ . & . & & . & & . & . \\ . & & . & & . & & a_{31} \\ . & & & . & & . & a_{21} \\ a_{n1} & . & . & . & a_{31} & a_{21} & a_{11} \end{bmatrix}$$

A symmetric Toeplitz matrix of order *n* is represented by a vector ***x*** of length *n* containing the elements of the first column of the matrix (or the elements of the first row), such that $x_i = a_{i1}$ for $i = 1, n$.

The following vector represents the matrix ***A*** shown above:

$$\boldsymbol{x} = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ \cdot \\ \cdot \\ a_{n1} \end{bmatrix}$$

### In Storage

The elements of the vector ***x***, which represent a positive definite symmetric Toeplitz matrix, are stored sequentially in an array. This is called packed-symmetric-Toeplitz storage mode. Following is an example of a positive definite symmetric Toeplitz matrix ***A*** and how it is stored in an array X.

Given the following matrix ***A***:

```
┌                       ┐
| 99  12  13  14  15  16 |
| 12  99  12  13  14  15 |
| 13  12  99  12  13  14 |
| 14  13  12  99  12  13 |
| 15  14  13  12  99  12 |
| 16  15  14  13  12  99 |
└                       ┘
```

the array is:

```
X = (99, 12, 13, 14, 15, 16)
```

## Positive Definite or Negative Definite Complex Hermitian Toeplitz Matrix

A positive definite or negative definite complex Hermitian matrix ***A*** of order *n* is also a Toeplitz matrix if and only if:

$a_{ij} = a_{i-1,j-1}$ for $i = 2, n$ and $j = 2, n$

The real part of the diagonal elements of the Toeplitz matrix must have a constant value. The imaginary part of the diagonal elements must be zero.

For the definition of a positive definite of negative definite complex Hermitian matrix, see "Positive Definite or Negative Definite Complex Hermitian Matrix" on page 71.

The following matrix illustrates a complex Hermitian Toeplitz matrix of order *n*; that is, it has *n* rows and *n* columns:

$$
\boldsymbol{A} = \begin{bmatrix}
a_{11} & a_{12} & a_{13} & \cdot & \cdot & \cdot & a_{1n} \\
\bar{a}_{12} & a_{11} & a_{12} & \cdot & & & \cdot \\
\bar{a}_{13} & \bar{a}_{12} & a_{11} & & \cdot & & \cdot \\
\cdot & \cdot & & \cdot & & \cdot & \cdot \\
\cdot & & \cdot & & \cdot & & a_{13} \\
\cdot & & & \cdot & & \cdot & a_{12} \\
\bar{a}_{1n} & \cdot & \cdot & \cdot & \bar{a}_{13} & \bar{a}_{12} & a_{11}
\end{bmatrix}
$$

A complex Hermitian Toeplitz matrix of order *n* is represented by a vector ***x*** of length *n* containing the elements of the first row of the matrix.

The following vector represents the matrix ***A*** shown above.

$$
\boldsymbol{x} = \begin{bmatrix}
a_{11} \\ a_{12} \\ a_{13} \\ \cdot \\ \cdot \\ a_{1n}
\end{bmatrix}
$$

### In Storage

The elements of the vector ***x***, which represent a positive definite complex Hermitian Toeplitz matrix, are stored sequentially in an array. This is called packed-Hermitian-Toeplitz storage mode. Following is an example of a positive definite complex Hermitian Toeplitz matrix ***A*** and how it is stored in an array `X`.

Given the following matrix ***A***:

```
┌                                                            ┐
│  (10.0, 0.0)   (2.0, -3.0)  (-3.0, 1.0)   (1.0, 1.0)  │
│   (2.0, 3.0)   (10.0, 0.0)  (2.0, -3.0)  (-3.0, 1.0)  │
│ (-3.0, -1.0)    (2.0, 3.0)  (10.0, 0.0)  (2.0, -3.0)  │
│  (1.0, -1.0)  (-3.0, -1.0)   (2.0, 3.0)  (10.0, 0.0)  │
└                                                            ┘
```

the array is:

```
X = ((10.0, 0.0), (2.0, -3.0), (-3.0, 1.0), (1.0, 1.0))
```

## Triangular Matrix

There are two types of triangular matrices: upper triangular matrix and lower triangular matrix. Triangular matrices have the same number of rows as they have columns; that is, they have *n* rows and *n* columns.

A matrix ***U*** is an upper triangular matrix if its nonzero elements are found only in the upper triangle of the matrix, including the main diagonal; that is:

$u_{ij} = 0 \quad \text{if } i > j$

A matrix ***L*** is an lower triangular matrix if its nonzero elements are found only in the lower triangle of the matrix, including the main diagonal; that is:

$l_{ij} = 0 \quad \text{if } i < j$

The following matrices, ***U*** and ***L***, illustrate upper and lower triangular matrices of order *n*, respectively:

$$\boldsymbol{U} = \begin{bmatrix} u_{11} & u_{12} & u_{13} & . & . & . & u_{1n} \\ 0 & u_{22} & u_{23} & & & & . \\ 0 & 0 & u_{33} & & & & . \\ . & & & . & & & . \\ . & & & & . & & . \\ . & & & & & . & . \\ 0 & . & . & . & . & 0 & u_{nn} \end{bmatrix} \qquad \boldsymbol{L} = \begin{bmatrix} l_{11} & 0 & 0 & . & . & . & 0 \\ l_{21} & l_{22} & 0 & & & & . \\ l_{31} & l_{32} & l_{33} & & & & . \\ . & & & . & & & . \\ . & & & & . & & . \\ . & & & & & . & 0 \\ l_{n1} & . & . & . & . & . & l_{nn} \end{bmatrix}$$

A unit triangular matrix is a triangular matrix in which all the diagonal elements have a value of one; that is:

- For an upper triangular matrix, $u_{ij} = 1$ if $i = j$.
- For an lower triangular matrix, $l_{ij} = 1$ if $i = j$.

The following matrices, ***U*** and ***L***, illustrate upper and lower unit real triangular matrices of order *n*, respectively:

$$\boldsymbol{U} = \begin{bmatrix} 1 & u_{12} & u_{13} & . & . & . & u_{1n} \\ 0 & 1 & u_{23} & & & & . \\ 0 & 0 & 1 & & & & . \\ . & & & . & & & . \\ . & & & & . & & . \\ . & & & & & . & . \\ 0 & . & . & . & . & 0 & 1 \end{bmatrix} \qquad \boldsymbol{L} = \begin{bmatrix} 1 & 0 & 0 & . & . & . & 0 \\ l_{21} & 1 & 0 & & & & . \\ l_{31} & l_{32} & 1 & & & & . \\ . & & & . & & & . \\ . & & & & . & & . \\ . & & & & & . & 0 \\ l_{n1} & . & . & . & . & . & 1 \end{bmatrix}$$

## In Storage

The four storage modes used for storing triangular matrices are described in the following sections:

- “Upper-Triangular-Packed Storage Mode”
- “Lower-Triangular-Packed Storage Mode” on page 75
- “Upper-Triangular Storage Mode” on page 75
- “Lower-Triangular Storage Mode” on page 76

**It is important to note that because the diagonal elements of a unit triangular matrix are always one, you do not need to set these values in the array for these four storage modes. ESSL always assumes that the values in these positions are one.**

***Upper-Triangular-Packed Storage Mode:*** When an upper-triangular matrix is stored in upper-triangular-packed storage mode, the upper triangle of the matrix is stored, including the diagonal, in a one-dimensional array. The upper triangle is packed by columns. The elements are packed sequentially, column by column, in $n(n+1)/2$ elements of a one-dimensional array. To calculate the location of each element of the triangular matrix in the array, use the technique described in “Upper-Packed Storage Mode” on page 67.

Following is an example of an upper triangular matrix ***U*** of order 5 and how it is stored in array UP. It uses the element values to show the order in which the elements are stored in the one-dimensional array.

Given the following matrix ***U***:

$$\begin{bmatrix} 1 & 2 & 4 & 7 & 11 \\ 0 & 3 & 5 & 8 & 12 \\ 0 & 0 & 6 & 9 & 13 \\ 0 & 0 & 0 & 10 & 14 \\ 0 & 0 & 0 & 0 & 15 \end{bmatrix}$$

the array is:

```
UP = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)
```

***Lower-Triangular-Packed Storage Mode:*** When a lower-triangular matrix is stored in lower-triangular-packed storage mode, the lower triangle of the matrix is stored, including the diagonal, in a one-dimensional array. The lower triangle is packed by columns. The elements are packed sequentially, column by column, in $n(n+1)/2$ elements of a one-dimensional array. To calculate the location of each element of the triangular matrix in the array, use the technique described in “Lower-Packed Storage Mode” on page 66.

Following is an example of a lower triangular matrix ***L*** of order 5 and how it is stored in array LP. It uses the element values to show the order in which the elements are stored in the one-dimensional array.

Given the following matrix ***L***:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 6 & 0 & 0 & 0 \\ 3 & 7 & 10 & 0 & 0 \\ 4 & 8 & 11 & 13 & 0 \\ 5 & 9 & 12 & 14 & 15 \end{bmatrix}$$

the array is:

```
LP = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)
```

***Upper-Triangular Storage Mode:*** A triangular matrix is stored in upper-triangular storage mode in a two-dimensional array. Only the elements in the upper triangle of the matrix, including the diagonal, are stored in the upper triangle of the array.

Following is an example of an upper triangular matrix ***U*** of order 5 and how it is stored in array UTA.

Given the following matrix ***U***:

$$\begin{bmatrix} 11 & 12 & 13 & 14 & 15 \\ 0 & 22 & 23 & 24 & 25 \\ 0 & 0 & 33 & 34 & 35 \\ 0 & 0 & 0 & 44 & 45 \\ 0 & 0 & 0 & 0 & 55 \end{bmatrix}$$

the array is:

$$\text{UTA} = \begin{bmatrix} 11 & 12 & 13 & 14 & 15 \\ * & 22 & 23 & 24 & 25 \\ * & * & 33 & 34 & 35 \\ * & * & * & 44 & 45 \\ * & * & * & * & 55 \end{bmatrix}$$

where "*" means you do not have to store a value in that position in the array.

***Lower-Triangular Storage Mode:*** A triangular matrix is stored in lower-triangular storage mode in a two-dimensional array. Only the elements in the lower triangle of the matrix, including the diagonal, are stored in the lower triangle of the array.

Following is an example of a lower triangular matrix ***L*** of order 5 and how it is stored in array LTA.

Given the following matrix ***L***:

$$\begin{bmatrix} 11 & 0 & 0 & 0 & 0 \\ 21 & 22 & 0 & 0 & 0 \\ 31 & 32 & 33 & 0 & 0 \\ 41 & 42 & 43 & 44 & 0 \\ 51 & 52 & 53 & 54 & 55 \end{bmatrix}$$

the array is:

$$\text{LTA} = \begin{bmatrix} 11 & * & * & * & * \\ 21 & 22 & * & * & * \\ 31 & 32 & 33 & * & * \\ 41 & 42 & 43 & 44 & * \\ 51 & 52 & 53 & 54 & 55 \end{bmatrix}$$

where "*" means you do not have to store a value in that position in the array.

## General Band Matrix

A general band matrix has its nonzero elements arranged uniformly near the diagonal, such that:

$$a_{ij} = 0 \quad \text{if } (i-j) > ml \text{ or } (j-i) > mu$$

where *ml* and *mu* are the lower and upper band widths, respectively, and *ml*+*mu*+1 is the total band width.

The following matrix illustrates a **square** general band matrix of order *n*, where the band widths are *ml* = *q*–1 and *mu* = *p*–1:

$$\begin{array}{cc}
 & \left| \leftarrow mu \rightarrow \right| \\
A = \begin{array}{c} \\ - \\ \uparrow \\ ml \\ \downarrow \\ - \\ \\ \\ \\ \\ \end{array} &
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & . & . & a_{1p} & 0 & . & . & 0 \\
a_{21} & a_{22} & a_{23} & & & & . & 0 & & . \\
a_{31} & a_{32} & a_{33} & & & & & . & 0 & . \\
. & & & . & & & & & . & 0 \\
. & & & & . & & & & & . \\
a_{q1} & & & & & . & & & & . \\
0 & . & & & & & . & & & . \\
. & 0 & . & & & & & . & & . \\
. & & 0 & . & & & & & . & \\
0 & . & . & 0 & . & . & . & & & a_{nn}
\end{bmatrix}
\end{array}$$

Some special types of band matrices are:

- Tridiagonal matrix: *ml* = *mu* = 1
- 9-diagonal matrix: *ml* = *mu* = 4

The following two matrices illustrate *m* by *n* **rectangular** general band matrices, where the band widths are *ml* = *q*–1 and *mu* = *p*–1. For both matrices, the leading diagonal is $a_{11}$, $a_{22}$, $a_{33}$, ..., $a_{nn}$. Following is a general band matrix with *m* > *n*:

$$\begin{array}{cc}
 & \left| \leftarrow mu \rightarrow \right| \\
A = \begin{array}{c} \\ \\ \\ - \\ \uparrow \\ ml \\ \downarrow \\ - \\ \\ \\ \\ \\ \\ \end{array} &
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & . & . & a_{1p} & 0 & . & . & 0 \\
a_{21} & a_{22} & a_{23} & & & & . & 0 & & . \\
a_{31} & a_{32} & a_{33} & & & & & . & 0 & . \\
. & & & . & & & & & . & 0 \\
. & & & & . & & & & & . \\
a_{q1} & & & & & . & & & & . \\
0 & . & & & & & . & & & . \\
. & 0 & . & & & & & . & & . \\
. & & 0 & . & & & & & . & \\
. & & & 0 & . & & & & & a_{nn} \\
. & & & & 0 & . & & & & . \\
. & & & & & 0 & . & & & . \\
0 & . & . & . & . & . & 0 & . & . & a_{mn}
\end{bmatrix}
\end{array}$$

Following is a general band matrix with *m* < *n*:

$$\begin{array}{lcl} & & \quad\;\; \left|\leftarrow mu \rightarrow\right| \\ \boldsymbol{A} = & \begin{array}{c} \\ - \\ \uparrow \\ ml \\ \downarrow \\ - \\ \\ \\ \\ \\ \end{array} & \left[\begin{array}{ccccccccccccc} a_{11} & a_{12} & a_{13} & . & . & a_{1p} & 0 & . & . & . & . & . & 0 \\ a_{21} & a_{22} & a_{23} & & & & . & 0 & & & & & . \\ a_{31} & a_{32} & a_{33} & & & & & . & 0 & & & & . \\ . & & & . & & & & & . & 0 & & & . \\ . & & & & . & & & & & . & 0 & & . \\ a_{q1} & & & & & . & & & & & . & 0 & . \\ 0 & . & & & & & . & & & & & . & 0 \\ . & 0 & . & & & & & . & & & & & . \\ . & & 0 & . & & & & & . & & & & . \\ 0 & . & . & 0 & . & . & . & . & . & a_{nn} & . & . & a_{mn} \end{array}\right] \end{array}$$

## In Storage

The two storage modes used for storing general band matrices are described in the following sections:

- "General-Band Storage Mode"
- "BLAS-General-Band Storage Mode" on page 80

***General-Band Storage Mode:*** (This storage mode is used only for square matrices.) Only the band elements of a general band matrix are stored for general-band storage mode. Additional storage must also be provided for fill- in. General-band storage mode packs the matrix elements by columns into a two-dimensional array, such that each diagonal of the matrix appears as a row in the packed array.

For a matrix ***A*** of order *n* with band widths *ml* and *mu*, the array must have a leading dimension, *lda*, greater than or equal to 2*ml*+*mu*+16. The size of the second dimension must be (at least) *n*, the number of columns in the matrix.

Using array AGB, which is declared as AGB(2*ml*+*mu*+16, *n*), the columns of elements in matrix ***A*** are stored in each column in array AGB as follows, where $a_{11}$ is stored at AGB(*ml*+*mu*+1, 1):

$$
\begin{array}{c}
\\
\overline{\uparrow}\; ml \\
\downarrow \\
\uparrow \\
\\
mu \\
\\
\\
\downarrow \\
\overline{\underline{1}} \\
\uparrow \\
ml \\
\\
\downarrow \\
\uparrow \\
15 \\
\\
\downarrow
\end{array}
\quad
\texttt{AGB} = 
\begin{array}{c}
\\
\\
\\
\\
\\
\\
\\
\\
\\
\\
\\
\\
\\
\\
\\
\\
\\
\\
\end{array}
\begin{array}{c}
\leftarrow \qquad\qquad\qquad n \qquad\qquad\qquad \rightarrow \\
\left[\begin{array}{cccccccccccc}
* & . & . & . & & & & & & & & * \\
. & & & & & & & & & & & . \\
. & & & & & & & & & & & * \\
. & & & & & a_{1p} & a_{2,p+1} & . & . & . & . & a_{n-mu,n} \\
. & & & & . & . & . & & & & & . \\
. & & & . & & . & . & & & & & . \\
. & & a_{13} & & & . & . & & & & & . \\
* & a_{12} & . & & & & & & & & & \\
a_{11} & . & . & & & & & & & & & a_{nn} \\
. & . & . & & & & & & & & . & * \\
. & . & & & & & & & & . & & . \\
. & & & & & & & & & & & . \\
a_{q1} & a_{q+1,2} & . & & & a_{q+mu,p} & . & . & a_{n,n-ml} & & & . \\
* & & & & & & & & & & & \\
. & & & & & & & & & & & \\
. & & & & & & & & & & & \\
. & & & & & & & & & & & \\
* & . & . & . & & & & & & & & *
\end{array}\right]
\end{array}
$$

where “*” means you do not store an element in that position in the array.

In the ESSL subroutine computation, some of the positions in the array indicated by an “*” are used for fill- in. Other positions may not be accessed at all.

Following is an example of a band matrix ***A*** of order 9 and band widths of *ml* = 2 and *mu* = 3.

Given the following matrix ***A***:

$$
\begin{bmatrix}
11 & 12 & 13 & 14 & 0 & 0 & 0 & 0 & 0 \\
21 & 22 & 23 & 24 & 25 & 0 & 0 & 0 & 0 \\
31 & 32 & 33 & 34 & 35 & 36 & 0 & 0 & 0 \\
0 & 42 & 43 & 44 & 45 & 46 & 47 & 0 & 0 \\
0 & 0 & 53 & 54 & 55 & 56 & 57 & 58 & 0 \\
0 & 0 & 0 & 64 & 65 & 66 & 67 & 68 & 69 \\
0 & 0 & 0 & 0 & 75 & 76 & 77 & 78 & 79 \\
0 & 0 & 0 & 0 & 0 & 86 & 87 & 88 & 89 \\
0 & 0 & 0 & 0 & 0 & 0 & 97 & 98 & 99
\end{bmatrix}
$$

you store it in general-band storage mode in a 23 by 9 array `AGB` as follows, where $a_{11}$ is stored in `AGB(6,1)`:

$$
\mathrm{AGB} = \begin{bmatrix}
* & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * \\
* & * & * & 14 & 25 & 36 & 47 & 58 & 69 \\
* & * & 13 & 24 & 35 & 46 & 57 & 68 & 79 \\
* & 12 & 23 & 34 & 45 & 56 & 67 & 78 & 89 \\
11 & 22 & 33 & 44 & 55 & 66 & 77 & 88 & 99 \\
21 & 32 & 43 & 54 & 65 & 76 & 87 & 98 & * \\
31 & 42 & 53 & 64 & 75 & 86 & 97 & * & * \\
* & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & * \\
* & * & * & * & * & * & * & * & *
\end{bmatrix}
$$

Following is an example of how to transform your general band matrix, of order *n*, to general-band storage mode:

```
      MD=ML+MU+1
      DO 1 J=1,N
         DO 1 I=MAX(J-MU,1),MIN(J+ML,N)
            AGB(I-J+MD,J)=A(I,J)
1     CONTINUE
```

***BLAS-General-Band Storage Mode:*** (This storage mode is used for both square and rectangular matrices.) Only the band elements of a general band matrix are stored for BLAS-general-band storage mode. The storage mode packs the matrix elements by columns into a two-dimensional array, such that each diagonal of the matrix appears as a row in the packed array.

For an *m* by *n* matrix ***A*** with band widths *ml* and *mu*, the array AGB must have a leading dimension, *lda*, greater than or equal to *ml*+*mu*+1. The size of the second dimension must be (at least) *n*, the number of columns in the matrix.

Using the array AGB, which is declared as AGB(*ml*+*mu*+1, *n*), the columns of elements in matrix ***A*** are stored in each column in array AGB as follows, where $a_{11}$ is stored at AGB(*mu*+1, 1):

$$
\mathtt{AGB} = \begin{array}{c|cccccccccc|}
 & \multicolumn{10}{c}{|\leftarrow \qquad\qquad\qquad n \qquad\qquad\qquad \rightarrow|} \\
\overline{\uparrow} & * & \cdot & \cdot & \cdot & * & a_{1p} & a_{2,p+1} & \cdot & \cdot & \cdot \\
 & \cdot & & & & \cdot & \cdot & \cdot & & & \\
mu & \cdot & & & \cdot & & \cdot & \cdot & & & \\
 & \cdot & & a_{13} & & & \cdot & \cdot & & & \\
\downarrow & * & a_{12} & \cdot & & & & & & & \\
\overline{1} & a_{11} & \cdot & \cdot & & & & & & & \\
\overline{\uparrow} & \cdot & \cdot & \cdot & & & & & & & \\
ml & \cdot & \cdot & & & & & & & & \\
 & \cdot & & & & & & & & & \\
\underline{\downarrow} & a_{q1} & a_{q+1,2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{array}
$$

where “*” means you do not store an element in that position in the array. These positions are not accessed by ESSL. **Unused positions in the array always occur in the upper left triangle of the array, but may not occur in the lower right triangle of the array, as you can see from the examples given here.**

Following is an example where $m > n$, and general band matrix ***A*** is 9 by 8 with band widths of $ml = 2$ and $mu = 3$.

Given the following matrix ***A***:

$$
\begin{bmatrix}
11 & 12 & 13 & 14 & 0 & 0 & 0 & 0 \\
21 & 22 & 23 & 24 & 25 & 0 & 0 & 0 \\
31 & 32 & 33 & 34 & 35 & 36 & 0 & 0 \\
0 & 42 & 43 & 44 & 45 & 46 & 47 & 0 \\
0 & 0 & 53 & 54 & 55 & 56 & 57 & 58 \\
0 & 0 & 0 & 64 & 65 & 66 & 67 & 68 \\
0 & 0 & 0 & 0 & 75 & 76 & 77 & 78 \\
0 & 0 & 0 & 0 & 0 & 86 & 87 & 88 \\
0 & 0 & 0 & 0 & 0 & 0 & 97 & 98
\end{bmatrix}
$$

you store it in array `AGB`, declared as `AGB(6,8)`, as follows, where $a_{11}$ is stored in `AGB(4,1)`:

$$
\mathtt{AGB} = \begin{bmatrix}
* & * & * & 14 & 25 & 36 & 47 & 58 \\
* & * & 13 & 24 & 35 & 46 & 57 & 68 \\
* & 12 & 23 & 34 & 45 & 56 & 67 & 78 \\
11 & 22 & 33 & 44 & 55 & 66 & 77 & 88 \\
21 & 32 & 43 & 54 & 65 & 76 & 87 & 98 \\
31 & 42 & 53 & 64 & 75 & 86 & 97 & *
\end{bmatrix}
$$

Following is an example where $m < n$, and general band matrix ***A*** is 7 by 9 with band widths of $ml = 2$ and $mu = 3$.

Given the following matrix ***A***:

$$\begin{bmatrix} 11 & 12 & 13 & 14 & 0 & 0 & 0 & 0 & 0 \\ 21 & 22 & 23 & 24 & 25 & 0 & 0 & 0 & 0 \\ 31 & 32 & 33 & 34 & 35 & 36 & 0 & 0 & 0 \\ 0 & 42 & 43 & 44 & 45 & 46 & 47 & 0 & 0 \\ 0 & 0 & 53 & 54 & 55 & 56 & 57 & 58 & 0 \\ 0 & 0 & 0 & 64 & 65 & 66 & 67 & 68 & 69 \\ 0 & 0 & 0 & 0 & 75 & 76 & 77 & 78 & 79 \end{bmatrix}$$

you store it in array `AGB`, declared as `AGB(6,9)`, as follows, where $a_{11}$ is stored in `AGB(4,1)` and the leading diagonal does not fill up the whole row:

$$\mathtt{AGB} = \begin{bmatrix} * & * & * & 14 & 25 & 36 & 47 & 58 & 69 \\ * & * & 13 & 24 & 35 & 46 & 57 & 68 & 79 \\ * & 12 & 23 & 34 & 45 & 56 & 67 & 78 & * \\ 11 & 22 & 33 & 44 & 55 & 66 & 77 & * & * \\ 21 & 32 & 43 & 54 & 65 & 76 & * & * & * \\ 31 & 42 & 53 & 64 & 75 & * & * & * & * \end{bmatrix}$$

and where "*" means you do not store an element in that position in the array.

Following is an example of how to transform your general band matrix, for all values of *m* and *n*, to BLAS-general-band storage mode:

```
        DO 20 J=1,N
           K=MU+1-J
           DO 10 I=MAX(1,J-MU),MIN(M,J+ML)
              AGB(K+I,J)=A(I,J)
   10      CONTINUE
   20   CONTINUE
```

## Symmetric Band Matrix

A symmetric band matrix is a symmetric matrix whose nonzero elements are arranged uniformly near the diagonal, such that:

$$a_{ij} = 0 \quad \text{if } |i-j| > k$$

where *k* is the half band width.

The following matrix illustrates a symmetric band matrix of order *n*, where the half band width $k = q-1$:

$$\begin{array}{cccccccccccc}
 & & \multicolumn{6}{c}{|\leftarrow\ k\ \rightarrow|} & & & & \\
 & & a_{11} & a_{21} & a_{31} & . & . & a_{q1} & 0 & . & . & 0 \\
 & & a_{21} & a_{22} & a_{32} & & & & & 0 & & . \\
 & & a_{31} & a_{32} & a_{33} & & & & & & 0 & . \\
 & & . & & & . & & & & & & 0 \\
\boldsymbol{A} & = & . & & & & . & & & & & . \\
 & & a_{q1} & & & & & . & & & & . \\
 & & 0 & & & & & & . & & & . \\
 & & . & 0 & & & & & & . & & . \\
 & & . & & 0 & & & & & & . & \\
 & & 0 & . & . & 0 & . & . & . & & & a_{nn}
\end{array}$$

## In Storage

The two storage modes used for storing symmetric band matrices are described in the following sections:

- “Upper-Band-Packed Storage Mode”
- “Lower-Band-Packed Storage Mode” on page 84

***Upper-Band-Packed Storage Mode:*** Only the band elements of the upper triangular part of a symmetric band matrix, including the main diagonal, are stored for upper-band-packed storage mode.

For a matrix ***A*** of order *n* and a half band width of *k*, the array must have a leading dimension, *lda*, greater than or equal to *k*+1, and the size of the second dimension must be (at least) *n*.

Using array ASB, which is declared as ASB(*lda*,*n*), where *p* = *lda* = *k*+1, the elements of a symmetric band matrix are stored as follows:

$$\texttt{ASB} = \left[\begin{array}{cccccccccc}
* & . & . & . & & * & a_{1p} & a_{2,p+1} & . \ . \ . & a_{n-k,n} \\
. & & & & . & & & & & \\
. & & & . & & & & & & \\
. & & . & & & & & & & . \\
 & * & a_{13} & a_{24} & . & . & . & & & . \\
* & a_{12} & a_{23} & . & . & . & & & & . \\
a_{11} & a_{22} & . & . & . & & & & & a_{nn}
\end{array}\right]$$

where “*” means you do not store an element in that position in the array.

Following is an example of a symmetric band matrix ***A*** of order 6 and a half band width of 3.

Given the following matrix ***A***:

$$\begin{bmatrix} 11 & 12 & 13 & 14 & 0 & 0 \\ 12 & 22 & 23 & 24 & 25 & 0 \\ 13 & 23 & 33 & 34 & 35 & 36 \\ 14 & 24 & 34 & 44 & 45 & 46 \\ 0 & 25 & 35 & 45 & 55 & 56 \\ 0 & 0 & 36 & 46 & 56 & 66 \end{bmatrix}$$

you store it in upper-band-packed storage mode in array ASB, declared as ASB(4,6), as follows.

$$\text{ASB} = \begin{bmatrix} * & * & * & 14 & 25 & 36 \\ * & * & 13 & 24 & 35 & 46 \\ * & 12 & 23 & 34 & 45 & 56 \\ 11 & 22 & 33 & 44 & 55 & 66 \end{bmatrix}$$

Following is an example of how to transform your symmetric band matrix to upper-band-packed storage mode:

```
      DO 20 J=1,N
         M=K+1-J
         DO 10 I=MAX(1,J-K),J
            ASB(M+I,J)=A(I,J)
      10 CONTINUE
   20 CONTINUE
```

***Lower-Band-Packed Storage Mode:*** Only the band elements of the lower triangular part of a symmetric band matrix, including the main diagonal, are stored for lower-band-packed storage mode.

For a matrix ***A*** of order *n* and a half band width of *k*, the array must have a leading dimension, *lda*, greater than or equal to *k*+1, and the size of the second dimension must be (at least) *n*.

Using array ASB, which is declared as ASB(*lda*,*n*), where *q* = *lda* = *k*+1, the elements of a symmetric band matrix are stored as follows:

$$\text{ASB} = \begin{bmatrix} a_{11} & a_{22} & \cdots & & & a_{nn} \\ a_{21} & a_{32} & & & & * \\ a_{31} & a_{42} & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \\ a_{q1} & a_{q+1,2} & \cdots & a_{n,n-k} & * \cdots & * \end{bmatrix}$$

where “*” means you do not store an element in that position in the array.

Following is an example of a symmetric band matrix ***A*** of order 6 and a half band width of 2.

Given the following matrix ***A***:

$$\begin{bmatrix} 11 & 21 & 31 & 0 & 0 & 0 \\ 21 & 22 & 32 & 42 & 0 & 0 \\ 31 & 32 & 33 & 43 & 53 & 0 \\ 0 & 42 & 43 & 44 & 54 & 64 \\ 0 & 0 & 53 & 54 & 55 & 65 \\ 0 & 0 & 0 & 64 & 65 & 66 \end{bmatrix}$$

you store it in lower-band-packed storage mode in array `ASB`, declared as `ASB(3,6)`, as follows:

$$\text{ASB} = \begin{bmatrix} 11 & 22 & 33 & 44 & 55 & 66 \\ 21 & 32 & 43 & 54 & 65 & * \\ 31 & 42 & 53 & 64 & * & * \end{bmatrix}$$

Following is an example of how to transform your symmetric band matrix to lower-band-packed storage mode:

```
      DO 20 J=1,N
         DO 10 I=J,MIN(J+K,N)
            ASB(I-J+1,J)=A(I,J)
      10 CONTINUE
   20 CONTINUE
```

## Positive Definite Symmetric Band Matrix

A real symmetric band matrix $\boldsymbol{A}$ is positive definite if and only if $\boldsymbol{x}^T\boldsymbol{A}\boldsymbol{x}$ is positive for all nonzero vectors $\boldsymbol{x}$.

### In Storage

The positive definite symmetric band matrix is stored in the same way a symmetric band matrix is stored. For a description of this storage technique, see “Symmetric Band Matrix” on page 82.

## Complex Hermitian Band Matrix

A complex band matrix is Hermitian if it is equal to its conjugate transpose:

$$\boldsymbol{H} = \boldsymbol{H}^H$$

### In Storage

The complex Hermitian band matrix is stored using the same two techniques used for symmetric band matrices:

- Lower-band-packed storage mode, as described in “Lower-Band-Packed Storage Mode” on page 84
- Upper-band-packed storage mode, as described in “Upper-Band-Packed Storage Mode” on page 83

Following is an example of a complex Hermitian band matrix $\boldsymbol{H}$ of order 5, having a half band width of 2.

Given the following matrix $\boldsymbol{H}$:

```
┌                                                ┐
| (11,  0) (21, -1) (31,  1)  (0,  0)  (0,  0) |
| (21,  1) (22,  0) (32, -1) (42, -1)  (0,  0) |
| (31, -1) (32,  1) (33,  0) (43, -1) (53, -1) |
|  (0,  0) (42,  1) (43,  1) (44,  0) (54, -1) |
|  (0,  0)  (0,  0) (53,  1) (54,  1) (55,  0) |
└                                                ┘
```

you store it in a two-dimensional array HP, as follows:

- In lower-band-packed storage mode:

```
       ┌                                              ┐
       | (11,  *) (22, *) (33, *) (44,  *) (55, *) |
HP =   | (21,  1) (32, 1) (43, 1) (54,  1)    *    |
       | (31, -1) (42, 1) (53, 1)    *        *    |
       └                                              ┘
```

- In upper-band-packed storage mode:

```
       ┌                                                  ┐
       |    *        *     (31,  1) (42, -1) (53, -1) |
HP =   |    *     (21, -1) (32, -1) (43, -1) (54, -1) |
       | (11, *)  (22,  *) (33,  *) (44,  *) (55,  *) |
       └                                                  ┘
```

where "*" means you do not have to store a value in that position in the array. The imaginary parts of the diagonal elements of a complex Hermitian band matrix are always 0, so you do not need to set these values. The ESSL subroutines always assume that the values in these positions are 0.

## Triangular Band Matrix

There are two types of triangular band matrices: upper triangular band matrix and lower triangular band matrix. Triangular band matrices have the same number of rows as they have columns; that is, they have *n* rows and *n* columns. They have an upper or lower band width of *k*.

A band matrix ***U*** is an upper triangular band matrix if its nonzero elements are found only in the upper triangle of the matrix, including the main diagonal; that is:

$u_{ij} = 0 \quad \text{if } i > j$

Its band elements are arranged uniformly near the diagonal in the upper triangle of the matrix, such that:

$u_{ij} = 0 \quad \text{if } j-i > k$

The following matrix ***U*** illustrates an upper triangular band matrix of order *n* with an upper band width $k = q-1$:

$$\begin{array}{c} \left|\leftarrow k \rightarrow\right| \qquad\qquad\qquad\qquad \\ \boldsymbol{U} = \left[\begin{array}{cccccccccc} u_{11} & u_{21} & u_{31} & . & . & u_{q1} & 0 & . & . & 0 \\ 0 & u_{22} & u_{32} & & & & & 0 & & . \\ . & 0 & u_{33} & & & & & & 0 & . \\ . & & & . & & & & & & 0 \\ . & & & & . & & & & & . \\ . & & & & & . & & & & . \\ . & & & & & & . & & & . \\ & & & & & & & . & & \\ & & & & & & & & . & \\ 0 & . & . & . & & & & & 0 & u_{nn} \end{array}\right] \end{array}$$

A band matrix ***L*** is a lower triangular band matrix if its nonzero elements are found only in the lower triangle of the matrix, including the main diagonal; that is:

$l_{ij} = 0 \quad \text{if } i < j$

Its band elements are arranged uniformly near the diagonal in the lower triangle of the matrix such that:

$l_{ij} = 0 \quad \text{if } i-j > k$

The following matrix ***L*** illustrates an upper triangular band matrix of order *n* with a lower band width $k = q-1$:

$$\boldsymbol{L} = \begin{array}{c} \\ - \\ \uparrow \\ k \\ \downarrow \\ - \\ \\ \\ \\ \\ \end{array} \left[\begin{array}{cccccccccc} l_{11} & 0 & . & . & . & & & & & 0 \\ l_{21} & l_{22} & 0 & & & & & & & . \\ l_{31} & l_{32} & l_{33} & & & & & & & . \\ . & & & . & & & & & & . \\ . & & & & . & & & & & . \\ l_{q1} & & & & & . & & & & . \\ 0 & & & & & & . & & & \\ . & 0 & & & & & & . & & \\ . & & 0 & & & & & & . & 0 \\ 0 & . & . & 0 & . & . & . & & & l_{nn} \end{array}\right]$$

A triangular band matrix can also be a unit triangular band matrix if all the diagonal elements have a value of 1. For an illustration of a unit triangular matrix, see “Triangular Matrix” on page 73.

## In Storage

The two storage modes used for storing triangular band matrices are described in the following sections:

- “Upper-Triangular-Band-Packed Storage Mode” on page 88
- “Lower-Triangular-Band-Packed Storage Mode” on page 89

**It is important to note that because the diagonal elements of a unit triangular band matrix are always one, you do not need to set these values in the array for these two storage modes. ESSL always assumes that the values in these positions are one.**

***Upper-Triangular-Band-Packed Storage Mode:*** Only the band elements of the upper triangular part of an upper triangular band matrix, including the main diagonal, are stored for upper-triangular-band-packed storage mode.

For a matrix ***U*** of order *n* and an upper band width of *k*, the array must have a leading dimension, *lda*, greater than or equal to *k*+1, and the size of the second dimension must be (at least) *n*.

Using array `UTB`, which is declared as `UTB`(*lda*,*n*), where *p* = *lda* = *k*+1, the elements of an upper triangular band matrix are stored as follows:

$$\mathtt{UTB} = \begin{bmatrix}
* & \cdot & \cdot & \cdot & & * & u_{1p} & u_{2,p+1} & \cdot\ \cdot\ \cdot & u_{n-k,n} \\
\cdot & & & & \cdot & & & & & \\
\cdot & & & \cdot & & & & & & \\
\cdot & & \cdot & & & & & & & \cdot \\
 & * & u_{13} & u_{24} & \cdot & \cdot & \cdot & & & \cdot \\
* & u_{12} & u_{23} & \cdot & \cdot & \cdot & & & & \cdot \\
u_{11} & u_{22} & \cdot & \cdot & \cdot & & & & & u_{nn}
\end{bmatrix}$$

where "*" means you do not store an element in that position in the array.

Following is an example of an upper triangular band matrix ***U*** of order 6 and an upper band width of 3.

Given the following matrix ***U***:

$$\begin{bmatrix}
11 & 12 & 13 & 14 & 0 & 0 \\
0 & 22 & 23 & 24 & 25 & 0 \\
0 & 0 & 33 & 34 & 35 & 36 \\
0 & 0 & 0 & 44 & 45 & 46 \\
0 & 0 & 0 & 0 & 55 & 56 \\
0 & 0 & 0 & 0 & 0 & 66
\end{bmatrix}$$

you store it in upper-triangular-band-packed storage mode in array `UTB`, declared as `UTB(4,6)`, as follows:

$$\mathtt{UTB} = \begin{bmatrix}
* & * & * & 14 & 25 & 36 \\
* & * & 13 & 24 & 35 & 46 \\
* & 12 & 23 & 34 & 45 & 56 \\
11 & 22 & 33 & 44 & 55 & 66
\end{bmatrix}$$

Following is an example of how to transform your upper triangular band matrix to upper-triangular-band-packed storage mode:

```
      DO 20 J=1,N
         M=K+1-J
         DO 10 I=MAX(1,J-K),J
            UTB(M+I,J)=U(I,J)
      10 CONTINUE
   20 CONTINUE
```

***Lower-Triangular-Band-Packed Storage Mode:*** Only the band elements of the lower triangular part of a lower triangular band matrix, including the main diagonal, are stored for lower-triangular-band-packed storage mode.

**Note:** As an alternative to this storage mode, you can specify your arguments in your subroutine in a special way so that ESSL selects the matrix elements properly, and you can leave your matrix stored in full-matrix storage mode. For details, see the Notes in the subroutine description in Part 2 of this book.

For a matrix ***L*** of order *n* and a lower band width of *k*, the array must have a leading dimension, *lda*, greater than or equal to *k*+1, and the size of the second dimension must be (at least) *n*.

Using array LTB, which is declared as LTB(*lda*,*n*), where *q* = *lda* = *k*+1, the elements of a lower triangular band matrix are stored as follows:

$$\text{LTB} = \begin{bmatrix} l_{11} & l_{22} & \cdots & & & l_{nn} \\ l_{21} & l_{32} & & & & * \\ l_{31} & l_{42} & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & \\ l_{q1} & l_{q+1,2} & \cdots & l_{n,n-k} & * \; \cdots & * \end{bmatrix}$$

where “*” means you do not store an element in that position in the array.

Following is an example of a lower triangular band matrix ***L*** of order 6 and a lower band width of 2.

Given the following matrix ***L***:

$$\begin{bmatrix} 11 & 0 & 0 & 0 & 0 & 0 \\ 21 & 22 & 0 & 0 & 0 & 0 \\ 31 & 32 & 33 & 0 & 0 & 0 \\ 0 & 42 & 43 & 44 & 0 & 0 \\ 0 & 0 & 53 & 54 & 55 & 0 \\ 0 & 0 & 0 & 64 & 65 & 66 \end{bmatrix}$$

you store it in lower-triangular-band-packed storage mode in array LTB, declared as LTB(3,6), as follows:

```
              ┌                         ┐
              │ 11  22  33  44  55  66 │
      LTB =   │ 21  32  43  54  65   * │
              │ 31  42  53  64   *   * │
              └                         ┘
```

Following is an example of how to transform your lower triangular band matrix to lower-triangular-band-packed storage mode:

```
   DO 20 J=1,N
      M=1-J
      DO 10 I=J,MIN(N,J+K)
         LTB(M+I,J)=L(I,J)
   10 CONTINUE
20 CONTINUE
```

## General Tridiagonal Matrix

A general tridiagonal matrix is a matrix whose nonzero elements are found only on the diagonal, subdiagonal, and superdiagonal of the matrix; that is:

$a_{ij} = 0 \quad \text{if } |i–j| > 1$

The following matrix illustrates a general tridiagonal matrix of order *n*:

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & 0 & . & . & . & 0 \\ a_{21} & a_{22} & a_{23} & 0 & & & . \\ 0 & a_{32} & a_{33} & a_{34} & 0 & & . \\ . & 0 & a_{43} & a_{44} & . & . & . \\ . & & 0 & . & . & .\ . & \\ . & & & . & . & .\ . & . \\ & & & & . & .\ . & . \\ 0 & . & . & . & & .\ . & a_{nn} \end{bmatrix}$$

### In Storage

Only the diagonal, subdiagonal, and superdiagonal elements of the general tridiagonal matrix are stored. This is called tridiagonal storage mode. The elements of a general tridiagonal matrix, ***A***, of order *n* are stored in three one-dimensional arrays, C, D, and E, each of length *n*, where array C contains the subdiagonal elements, stored as follows:

C = $(*, a_{21}, a_{32}, a_{43}, ..., a_{n,n–1})$

and array D contains the main diagonal elements, stored as follows:

D = $(a_{11}, a_{22}, a_{33}, ..., a_{nn})$

and array E contains the superdiagonal elements, stored as follows:

E = $(a_{12}, a_{23}, a_{34}, ..., a_{n–1,n}, *)$

where "*" means you do not store an element in that position in the array.

Following is an example of a general tridiagonal matrix ***A*** of order 5:

$$\begin{bmatrix} 11 & 12 & 0 & 0 & 0 \\ 21 & 22 & 23 & 0 & 0 \\ 0 & 32 & 33 & 34 & 0 \\ 0 & 0 & 43 & 44 & 45 \\ 0 & 0 & 0 & 54 & 55 \end{bmatrix}$$

which you store in tridiagonal storage mode in arrays C, D, and E, each of length 5, as follows:

```
C = (*, 21, 32, 43, 54)

D = (11, 22, 33, 44, 55)

E = (12, 23, 34, 45, *)
```

**Note:** Some ESSL subroutines provide an option for specifying at least *n* additional locations at the end of each of the arrays C, D, and E. These additional locations are used for working storage by the ESSL subroutine. The reasons for choosing this option are explained in the subroutine descriptions.

## Symmetric Tridiagonal Matrix

A tridiagonal matrix ***A*** is also symmetric if and only if its nonzero elements are found only on the diagonal, subdiagonal, and superdiagonal of the matrix, and its subdiagonal elements and superdiagonal elements are equal; that is:

$(a_{ij} = 0 \quad \text{if } |i-j| > 1)$ and $(a_{ij} = a_{ji} \quad \text{if } |i-j| = 1)$

The following matrix illustrates a symmetric tridiagonal matrix of order *n*:

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{21} & 0 & . & . & . & & 0 \\ a_{21} & a_{22} & a_{32} & 0 & & & & . \\ 0 & a_{32} & a_{33} & a_{43} & 0 & & & . \\ . & 0 & a_{43} & a_{44} & . & . & & . \\ . & & 0 & . & . & . & . & \\ . & & & . & . & . & . & . \\ & & & & . & . & . & . \\ 0 & . & . & . & & . & . & a_{nn} \end{bmatrix}$$

### In Storage

Only the diagonal and subdiagonal elements of the positive definite symmetric tridiagonal matrix are stored. This is called symmetric-tridiagonal storage mode. The elements of a symmetric tridiagonal matrix ***A*** of order *n* are stored in two one-dimensional arrays C and D, each of length *n*, where array C contains the subdiagonal elements, stored as follows:

C = $(*, a_{21}, a_{32}, a_{43}, ..., a_{n,n-1})$

where “*” means you do not store an element in that position in the array. Then array D contains the main diagonal elements,stored as follows:

D = $(a_{11}, a_{22}, a_{33}, ..., a_{nn})$

Following is an example of a symmetric tridiagonal matrix ***A*** of order 5:

$$\begin{bmatrix} 10 & 1 & 0 & 0 & 0 \\ 1 & 20 & 2 & 0 & 0 \\ 0 & 2 & 30 & 3 & 0 \\ 0 & 0 & 3 & 40 & 4 \\ 0 & 0 & 0 & 4 & 50 \end{bmatrix}$$

which you store in symmetric-tridiagonal storage mode in arrays C and D, each of length 5, as follows:

$$C = (*, 1, 2, 3, 4)$$

$$D = (10, 20, 30, 40, 50)$$

**Note:** Some ESSL subroutines provide an option for specifying at least *n* additional locations at the end of each of the arrays C and D. These additional locations are used for working storage by the ESSL subroutine. The reasons for choosing this option are explained in the subroutine descriptions.

## Positive Definite Symmetric Tridiagonal Matrix

A real symmetric tridiagonal matrix ***A*** is positive definite if and only if $\boldsymbol{x}^{T}\boldsymbol{Ax}$ is positive for all nonzero vectors ***x***.

### In Storage

The positive definite symmetric tridiagonal matrix is stored in the same way the symmetric tridiagonal matrix is stored. For a description of this storage technique, see “Symmetric Tridiagonal Matrix” on page 91.

## Sparse Matrix

A sparse matrix is a matrix having a relatively small number of nonzero elements.

Consider the following as an example of a sparse matrix ***A***:

$$\begin{bmatrix} 11 & 0 & 13 & 0 & 0 & 0 \\ 21 & 22 & 0 & 24 & 0 & 0 \\ 0 & 32 & 33 & 0 & 35 & 0 \\ 0 & 0 & 43 & 44 & 0 & 46 \\ 51 & 0 & 0 & 54 & 55 & 0 \\ 61 & 62 & 0 & 0 & 65 & 66 \end{bmatrix}$$

### In Storage

A sparse matrix can be stored in full-matrix storage mode or a packed storage mode. When a sparse matrix is stored in **full-matrix storage mode**, all its elements, including its zero elements, are stored in an array.

The seven packed storage modes used for storing sparse matrices are described in the following sections:

- “Compressed-Matrix Storage Mode” on page 93
- “Compressed-Diagonal Storage Mode” on page 94

- “Storage-by-Indices” on page 97
- “Storage-by-Columns” on page 98
- “Storage-by-Rows” on page 99
- “Diagonal-Out Skyline Storage Mode” on page 101
- “Profile-In Skyline Storage Mode” on page 103

**Note:** When the elements of a sparse matrix are stored using any of these storage modes, the ESSL subroutines do not check that all elements are nonzero. You do not get an error if any elements are zero.

***Compressed-Matrix Storage Mode:*** The sparse matrix ***A***, stored in compressed-matrix storage mode, uses two two-dimensional arrays to define the sparse matrix storage, `AC` and `KA`. See reference [70 ]. Given the *m* by *n* sparse matrix ***A***, having a maximum of *nz* nonzero elements in each row:

- `AC` is defined as `AC`(*lda*,*nz*), where the leading dimension, *lda*, must be greater than or equal to *m*. Each row of array `AC` contains the nonzero elements of the corresponding row of matrix ***A***. For each row in matrix ***A*** containing less than *nz* nonzero elements, the corresponding row in array `AC` is padded with zeros. The elements in each row can be stored in any order.
- `KA` is an integer array defined as `KA`(*lda*,*nz*), where the leading dimension, *lda*, must be greater than or equal to *m*. It contains the column numbers of the matrix ***A*** elements that are stored in the corresponding positions in array `AC`. For each row in matrix ***A*** containing less than *nz* nonzero elements, the corresponding row in array `KA` is padded with any values from 1 to *n*. **Because this array is used by the ESSL subroutines to access other target vectors in the computation, you must adhere to these required values to avoid errors.**

**Unless all the rows of sparse matrix *A* contain approximately the same number of nonzero elements, this storage mode requires a large amount of storage. This diminishes the performance you can obtain by using this storage mode.**

Consider the following as an example of a 6 by 6 sparse matrix ***A*** with a maximum of four nonzero elements in each row. It shows how matrix ***A*** can be stored in arrays `AC` and `KA`.

Given the following matrix ***A***:

$$\begin{bmatrix} 11 & 0 & 13 & 0 & 0 & 0 \\ 21 & 22 & 0 & 24 & 0 & 0 \\ 0 & 32 & 33 & 0 & 35 & 0 \\ 0 & 0 & 43 & 44 & 0 & 46 \\ 51 & 0 & 0 & 54 & 55 & 0 \\ 61 & 62 & 0 & 0 & 65 & 66 \end{bmatrix}$$

the arrays are:

$$\text{AC} = \begin{bmatrix} 11 & 13 & 0 & 0 \\ 22 & 21 & 24 & 0 \\ 33 & 32 & 35 & 0 \\ 44 & 43 & 46 & 0 \\ 55 & 51 & 54 & 0 \\ 66 & 61 & 62 & 65 \end{bmatrix}$$

$$\text{KA} = \begin{bmatrix} 1 & 3 & * & * \\ 2 & 1 & 4 & * \\ 3 & 2 & 5 & * \\ 4 & 3 & 6 & * \\ 5 & 1 & 4 & * \\ 6 & 1 & 2 & 5 \end{bmatrix}$$

where “*” means you can store any value from 1 to 6 in that position in the array.

Symmetric sparse matrices use the same storage technique as nonsymmetric sparse matrices; that is, all nonzero elements of a symmetric matrix ***A*** must be stored in array AC, not just the elements of the upper triangle and diagonal of matrix ***A***.

In general terms, this storage technique can be expressed as follows:

> For each $a_{ij} \neq 0$, for $i = 1, m$ and $j = 1, n$
> there exists $k$, where $1 \leq k \leq nz$,
> such that AC($i$,$k$) = $a_{ij}$ and KA($i$,$k$) = $j$.
>
> For all other elements of AC and KA,
> AC($i$,$k$) = 0 and 1 ≤ KA($i$,$k$) ≤ $n$

where:

- $a_{ij}$ are the elements of the $m$ by $n$ matrix ***A*** that has a maximum of $nz$ nonzero elements in each row.
- Array AC is defined as AC($lda$,$nz$), where $lda \geq m$.
- Array KA is defined as KA($lda$,$nz$), where $lda \geq m$.

***Compressed-Diagonal Storage Mode:*** The storage mode used for square sparse matrices stored in compressed-diagonal storage mode has two variations, depending on whether the matrix is a general sparse matrix or a symmetric sparse matrix. This section explains both of these variations. This section begins, however, by explaining the conventions used for numbering the diagonals in the matrix, which apply to the storage descriptions.

Matrix ***A*** of order $n$ has $2n$–1 diagonals. Because $k = j–i$ is constant for the elements $a_{ij}$ along each diagonal, each diagonal can be assigned a diagonal number, $k$, having a value from $1–n$ to $n–1$. Then the diagonals can be referred to as $\boldsymbol{d}_k$, where $k = 1–n, n–1$.

The following matrix shows the starting position of each diagonal, $\boldsymbol{d}_k$:

$$A = \begin{array}{c|ccccccc|}
 & \boldsymbol{d}_0 & \boldsymbol{d}_1 & \boldsymbol{d}_2 & . & . & . & \boldsymbol{d}_{n-1} \\
 & a_{11} & a_{12} & a_{13} & . & . & . & a_{1n} \\
\boldsymbol{d}_{-1} & a_{21} & a_{22} & a_{23} & & & & . \\
\boldsymbol{d}_{-2} & a_{31} & a_{32} & a_{33} & & & & . \\
. & . & & & . & & & . \\
. & . & & & & . & & . \\
. & . & & & & & . & . \\
\boldsymbol{d}_{1-n} & a_{n1} & . & . & . & . & . & a_{nn}
\end{array}$$

For a **general** (square) sparse matrix ***A***, compressed-diagonal storage mode uses two arrays to define the sparse matrix storage, `AD` and `LA`. Using the above convention for numbering the diagonals, and given that sparse matrix ***A*** contains *nd* diagonals having nonzero elements, arrays `AD` and `LA` are set up as follows:

- `AD` is defined as `AD`(*lda*,*nd*), where the leading dimension, *lda*, must be greater than or equal to *n*. Each diagonal of matrix ***A*** that has at least one nonzero element is stored in a column of array `AD`. All of the elements of the diagonal, including its zero elements, are stored in *n* contiguous locations in the array, in the same order as they appear in the diagonal. Padding with zeros is required as follows to fill the *n* locations in each column of array `AD`:

  - Each superdiagonal ($k > 0$), which has $n–k$ elements, is padded with *k* trailing zeros.
  - The main diagonal ($k = 0$), which has *n* elements, does not require padding.
  - Each subdiagonal ($k < 0$), which has $n–|k|$ elements, is padded with $|k|$ leading zeros.

  The diagonals can be stored in any columns in array `AD`.

- `LA` is a one-dimensional integer array of length *nd*, containing the diagonal numbers *k* for the diagonals stored in each corresponding column in array `AD`.

**Because this storage mode requires entire diagonals to be stored, if the nonzero elements in matrix *A* are not concentrated along a few diagonals, this storage mode requires a large amount of storage. This diminishes the performance you obtain by using this storage mode.**

Consider the following as an example of how a 6 by 6 general sparse matrix ***A*** with 5 nonzero diagonals is stored in arrays `AD` and `LA`.

Given the following matrix ***A***:

$$\begin{bmatrix}
11 & 0 & 13 & 0 & 0 & 0 \\
21 & 22 & 0 & 24 & 0 & 0 \\
0 & 32 & 33 & 0 & 35 & 0 \\
0 & 0 & 43 & 44 & 0 & 46 \\
51 & 0 & 0 & 54 & 55 & 0 \\
61 & 62 & 0 & 0 & 65 & 66
\end{bmatrix}$$

the arrays are:

$$\texttt{AD} = \begin{bmatrix} 11 & 13 & 0 & 0 & 0 \\ 22 & 24 & 21 & 0 & 0 \\ 33 & 35 & 32 & 0 & 0 \\ 44 & 46 & 43 & 0 & 0 \\ 55 & 0 & 54 & 51 & 0 \\ 66 & 0 & 65 & 62 & 61 \end{bmatrix}$$

$$\texttt{LA} = (0, 2, -1, -4, -5)$$

For a **symmetric** sparse matrix, where each superdiagonal *k* is equal to subdiagonal −*k*, compressed-diagonal storage mode uses the same storage technique as for the general sparse matrix, except that only the nonzero main diagonal and one diagonal of each couple of nonzero diagonals, *k* and −*k*, are used in setting up arrays AD and LA. You can store either the upper or the lower diagonal of each couple.

Consider the following as an example of a symmetric sparse matrix of order 6 and how it is stored in arrays AD and LA, using only three nonzero diagonals in the matrix.

Given the following matrix ***A***:

$$\begin{bmatrix} 11 & 0 & 13 & 0 & 51 & 0 \\ 0 & 22 & 0 & 24 & 0 & 62 \\ 13 & 0 & 33 & 0 & 35 & 0 \\ 0 & 24 & 0 & 44 & 0 & 46 \\ 51 & 0 & 35 & 0 & 55 & 0 \\ 0 & 62 & 0 & 46 & 0 & 66 \end{bmatrix}$$

the arrays are:

$$\texttt{AD} = \begin{bmatrix} 11 & 13 & 0 \\ 22 & 24 & 0 \\ 33 & 35 & 0 \\ 44 & 46 & 0 \\ 55 & 0 & 51 \\ 66 & 0 & 62 \end{bmatrix}$$

$$\texttt{LA} = (0, 2, -4)$$

In general terms, this storage technique can be expressed as follows:

> For each $\boldsymbol{d}_k \neq (0, ..., 0)$, for $k = 1-n, n-1$
>     for **general** square sparse matrices, or
> for each unique $\boldsymbol{d}_k \neq (0, ..., 0)$, for $k = 1-n, n-1$
>     for **symmetric** sparse matrices,
> there exists *l*, where $1 \leq l \leq nd$,
> such that LA(*l*) = *k* and column *l* in array AD contains $\boldsymbol{dp}_k$.

where:

- Array AD is defined as AD(*lda*,*nd*), where $lda \geq n$, and where *nd* is the number of nonzero diagonals, $\boldsymbol{d}_k$ that are stored in array AD.
- Array LA has *nd* elements.
- *k* is the diagonal number of each diagonal, $\boldsymbol{d}_k$, where $k = i-j$.
- $\boldsymbol{dp}_k$ are the diagonals, $\boldsymbol{d}_k$, with padding, which are constructed from the sparse matrix ***A*** elements, $a_{ij}$, for $i, j = 1, n$ as follows:

  For superdiagonals ($k > 0$), $\boldsymbol{dp}_k$ has *k* trailing zeros: $\boldsymbol{dp}_k = (a_{1,k+1}, a_{2,k+2}, ..., a_{n-k,n}, 0_1, ..., 0_k)$
  For the main diagonal ($k = 0$), $\boldsymbol{dp}_0$ has no padding: $\boldsymbol{dp}_0 = (a_{11}, a_{22}, ..., a_{nn})$
  For subdiagonals ($k < 0$), $\boldsymbol{dp}_k$ has $|k|$ leading zeros: $\boldsymbol{dp}_k = (0_1, ..., 0_{|k|}, a_{|k|+1,1}, a_{|k|+2,2}, ..., a_{n,\, n-|k|})$

***Storage-by-Indices:*** For a sparse matrix ***A***, storage-by-indices uses three one-dimensional arrays to define the sparse matrix storage, AR, IA, and JA. Given the *m* by *n* sparse matrix ***A*** having *ne* nonzero elements, the arrays are set up as follows:

- AR of (at least) length *ne* contains the *ne* nonzero elements of the sparse matrix ***A***, stored contiguously in **any** order.
- IA, an integer array of (at least) length *ne* contains the corresponding row numbers of each nonzero element, $a_{ij}$, in matrix ***A***.
- JA, an integer array of (at least) length *ne* contains the corresponding column numbers of each nonzero element, $a_{ij}$, in matrix ***A***.

Consider the following as an example of a 6 by 6 sparse matrix ***A*** and how it can be stored in arrays AR, IA, and JA.:

Given the following matrix ***A***:

$$\begin{bmatrix} 11 & 0 & 13 & 0 & 0 & 0 \\ 21 & 22 & 0 & 24 & 0 & 0 \\ 0 & 32 & 33 & 0 & 35 & 0 \\ 0 & 0 & 43 & 44 & 0 & 46 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 61 & 62 & 0 & 0 & 65 & 66 \end{bmatrix}$$

the arrays are:

```
AR = (11, 22, 32, 33, 13, 21, 43, 24, 66, 46, 35, 62, 61, 65, 44)

IA = (1, 2, 3, 3, 1, 2, 4, 2, 6, 4, 3, 6, 6, 6, 4)

JA = (1, 2, 2, 3, 3, 1, 3, 4, 6, 6, 5, 2, 1, 5, 4)
```

In general terms, this storage technique can be expressed as follows:

For each $a_{ij} \neq 0$, for $i = 1, m$ and $j = 1, n$
there exists *k*, where $1 \leq k \leq ne$, such that:
  AR(*k*) = $a_{ij}$
  IA(*k*) = *i*
  JA(*k*) = *j*

where:

- $a_{ij}$ are the elements of the *m* by *n* sparse matrix ***A***.
- Arrays `AR`, `IA`, and `JA` each have *ne* elements.

***Storage-by-Columns:*** For a sparse matrix, ***A***, storage-by-columns uses three one-dimensional arrays to define the sparse matrix storage, `AR`, `IA`, and `JA`. Given the *m* by *n* sparse matrix ***A*** having *ne* nonzero elements, the arrays are set up as follows:

- `AR` of (at least) length *ne* contains the *ne* nonzero elements of the sparse matrix ***A***, stored contiguously. The columns of matrix ***A*** are stored consecutively from 1 to *n* in `AR`. The elements in each column of ***A*** are stored in any order in `AR`.
- `IA`, an integer array of (at least) length *ne* contains the corresponding row numbers of each nonzero element, $a_{ij}$, in matrix ***A***.
- `JA`, an integer array of (at least) length *n*+1 contains the relative starting position of each column of matrix ***A*** in array `AR`; that is, each element `JA`(*j*) of the column pointer array indicates where column *j* begins in array `AR`. If all elements in column *j* are zero, then `JA`(*j*) = `JA`(*j*+1). The last element, `JA`(*n*+1), indicates the position after the last element in array `AR`, which is *ne*+1.

Consider the following as an example of a 6 by 6 sparse matrix ***A*** and how it can be stored in arrays `AR`, `IA`, and `JA`.

Given the following matrix ***A***:

$$\begin{bmatrix} 11 & 0 & 13 & 0 & 0 & 0 \\ 21 & 22 & 0 & 24 & 0 & 0 \\ 0 & 32 & 33 & 0 & 0 & 0 \\ 0 & 0 & 43 & 44 & 0 & 46 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 61 & 62 & 0 & 0 & 0 & 66 \end{bmatrix}$$

the arrays are:

```
AR = (11, 61, 21, 62, 32, 22, 13, 33, 43, 44, 24, 46, 66)

IA = (1, 6, 2, 6, 3, 2, 1, 3, 4, 4, 2, 4, 6)

JA = (1, 4, 7, 10, 12, 12, 14)
```

In general terms, this storage technique can be expressed as follows:

```
For each $a_{ij} \neq 0$, for $i = 1, m$ and $j = 1, n$
there exists $k$, where $1 \leq k \leq ne$, such that
    AR($k$) = $a_{ij}$
    IA($k$) = $i$
And for $j = 1, n$,
    JA($j$) = $k$, where $a_{ij}$, in AR($k$), is the first element stored in AR for column $j$
    JA($j$) = JA($j$+1), where all $a_{ij} = 0$ in column $j$
    JA($n$+1) = $ne$+1
```

where:

- $a_{ij}$ are the elements of the *m* by *n* sparse matrix ***A***.

- Arrays AR and IA each have *ne* elements.
- Array JA has *n*+1 elements.

***Storage-by-Rows:*** The storage mode used for sparse matrices stored by rows has three variations, depending on whether the matrix is a general sparse matrix or a symmetric sparse matrix. This section explains these variations.

For a **general** sparse matrix ***A***, storage-by-rows uses three one-dimensional arrays to define the sparse matrix storage, AR, IA, and JA. Given the *m* by *n* sparse matrix ***A*** having *ne* nonzero elements, the arrays are set up as follows:

- AR of (at least) length *ne* contains the *ne* nonzero elements of the sparse matrix ***A***, stored contiguously. The rows of matrix ***A*** are stored consecutively from 1 to *m* in AR. The elements in each row of ***A*** are stored in any order in AR.
- IA, an integer array of (at least) length *m*+1 contains the relative starting position of each row of matrix ***A*** in array AR; that is, each element IA(*i*) of the row pointer array indicates where row *i* begins in array AR. If all elements in row *i* are zero, then IA(*i*) = IA(*i*+1). The last element, IA(*m*+1), indicates the position after the last element in array AR, which is *ne*+1.
- JA, an integer array of (at least) length *ne* contains the corresponding column numbers of each nonzero element, $a_{ij}$, in matrix ***A***.

Consider the following as an example of a 6 by 6 general sparse matrix ***A*** and how it can be stored in arrays AR, IA, and JA.

Given the following matrix ***A***:

$$\begin{bmatrix} 11 & 0 & 13 & 0 & 0 & 0 \\ 21 & 22 & 0 & 24 & 0 & 0 \\ 0 & 32 & 33 & 0 & 0 & 0 \\ 0 & 0 & 43 & 44 & 0 & 46 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 61 & 62 & 0 & 0 & 0 & 66 \end{bmatrix}$$

the arrays are:

```
AR = (11, 13, 24, 22, 21, 32, 33, 44, 43, 46, 61, 62, 66)

IA = (1, 3, 6, 8, 11, 11, 14)

JA = (1, 3, 4, 2, 1, 2, 3, 4, 3, 6, 1, 2, 6)
```

For a **symmetric** sparse matrix of order *m*, storage-by-rows uses the same storage technique as for the general sparse matrix, except that only the upper or lower triangle and diagonal elements are used in setting up arrays AR, IA, and JA.

Consider the following as an example of a symmetric sparse matrix ***A*** of order 6 and how it can be stored in arrays AR, IA, and JA using upper-storage-by-rows, which stores only the upper triangle and diagonal elements.

Given the following matrix A:

$$\begin{bmatrix} 11 & 0 & 13 & 0 & 0 & 0 \\ 0 & 22 & 23 & 24 & 0 & 0 \\ 13 & 23 & 33 & 0 & 35 & 0 \\ 0 & 24 & 0 & 44 & 0 & 46 \\ 0 & 0 & 35 & 0 & 55 & 0 \\ 0 & 0 & 0 & 46 & 0 & 0 \end{bmatrix}$$

the arrays are:

```
AR = (11, 13, 22, 24, 23, 33, 35, 46, 44, 55)

IA = (1, 3, 6, 8, 10, 11, 11)

JA = (1, 3, 2, 3, 4, 3, 5, 4, 6, 5)
```

Using the same symmetric matrix ***A***, consider the following as an example of how it can be stored in arrays `AR`, `IA`, and `JA` using lower-storage-by-rows, which stores only the lower triangle and diagonal elements:

```
AR = (11, 22, 23, 33, 13, 24, 44, 55, 35, 46)

IA = (1, 2, 3, 6, 8, 10, 11 )

JA = (1, 2, 2, 3, 1, 2, 4, 5, 3, 4)
```

In general terms, this storage technique can be expressed as follows:

```
For each $a_{ij} \neq 0$,
    for $i = 1, m$ and $j = 1, n$ for general sparse matrices
       or
    for $i = 1, m$ and $j = i, m$ for symmetric sparse matrices using the lower
    triangle
       or
    for $i = 1, m$ and $j = 1, i$ for symmetric sparse matrices using the upper
    triangle
there exists $k$, where $1 \leq k \leq ne$, such that
    AR($k$) = $a_{ij}$
    JA($k$) = $j$
And for $i = 1, m$,
    IA($i$) = $k$, where $a_{ij}$, in AR($k$), is the first element stored in AR for row $i$
    IA($i$) = IA($i$+1), where all $a_{ij} = 0$ in row $i$
    IA($m$+1) = $ne$+1
```

where:

- $a_{ij}$ are the elements of sparse matrix ***A***, which is either an *m* by *n* general sparse matrix or a symmetric sparse matrix of order *m* containing *ne* nonzero elements.
- Arrays `AR` and `JA` each have *ne* elements.
- Array `IA` has *m*+1 elements.

***Diagonal-Out Skyline Storage Mode:*** The diagonal-out skyline storage mode used for sparse matrices has two variations, depending on whether the matrix is a general sparse matrix or a symmetric sparse matrix. Both of these variations are explained here.

For a **general** sparse matrix ***A***, diagonal-out skyline storage mode uses four one-dimensional arrays to define the sparse matrix storage, AU, IDU, AL, and IDL. Given the sparse matrix ***A*** of order $n$, containing $nu+nl-n$ elements under the top and left profiles, the arrays are set up as follows:

- AU of (at least) length $nu$ contains the upper triangle of the sparse matrix ***A***, where the columns are stored consecutively from 1 to $n$ in AU in the following way. For each column, the elements starting at the diagonal element and ending at the topmost nonzero element in the column are stored contiguously in AU. The elements stored may include zero elements along with the nonzero elements. If all elements in the column to be stored are zero, the diagonal element, $a_{ii}$, having a value of zero, is stored in AU for that column. A total of $nu$ elements are stored for the upper triangle of ***A***.
- IDU, an integer array of (at least) length $n$+1 contains the relative position of each diagonal element of matrix ***A*** in array AU; that is, each element IDU($i$) of the diagonal pointer array indicates where diagonal element $a_{ii}$ is stored in array AU. One-origin is used, so the first element of IDU is always 1. The last element, IDU($n$+1), indicates the position after the last element in array AU, which is $nu$+1.
- AL of (at least) length $nl$ contains the lower triangle of the sparse matrix ***A***, where the rows are stored consecutively from 1 to $n$ in AL in the following way. For each row, the elements starting at the diagonal element and ending at the leftmost nonzero element in the row are stored contiguously in AL. The elements stored may include zero elements along with the nonzero elements. If all elements in the row to be stored are zero, the diagonal element, $a_{ii}$, having a value of zero, is stored in AL for that row. A total of $nl$ elements are stored for the lower triangle of ***A***. The values of the diagonal elements are meaningless, so you can store any values in those positions in AL.
- IDL, an integer array of (at least) length $n$+1 contains the relative position of each diagonal element of matrix ***A*** in array AL; that is, each element IDL($i$) of the diagonal pointer array indicates where diagonal element $a_{ii}$ is stored in array AL. One-origin is used, so the first element of IDL is always 1. The last element, IDL($n$+1), indicates the position after the last element in array AL, which is $nl$+1.

Consider the following as an example of a 6 by 6 general sparse matrix ***A*** and how it is stored in arrays AU, IDU, AL, and IDL.

Given the following matrix ***A***:

$$\begin{bmatrix} 0 & 12 & 13 & 0 & 0 & 0 \\ 21 & 22 & 0 & 24 & 0 & 0 \\ 31 & 0 & 33 & 34 & 0 & 36 \\ 41 & 42 & 43 & 44 & 45 & 0 \\ 0 & 0 & 0 & 54 & 55 & 56 \\ 0 & 0 & 63 & 0 & 65 & 66 \end{bmatrix}$$

the arrays are:

AU = (0, 22, 12, 33, 0, 13, 44, 34, 24, 55, 45, 66, 56, 0, 36)

IDU = (1, 2, 4, 7, 10, 12, 16) where $nu$=15

AL = (*, *, 21, *, 0, 31, *, 43, 42, 41, *, 54, *, 65, 0, 63)

IDL = (1, 2, 4, 7, 11, 13, 17) where $nl$=16

and where "*" means you do not have to store a value in that position in the array. However, these storage positions are required.

For a **symmetric** sparse matrix of order *n*, diagonal-out skyline storage mode uses the same storage technique as for the upper triangle and diagonal elements of the general sparse matrix; therefore, only the AU and IDU arrays are needed.

Consider the following as an example of a symmetric sparse matrix ***A*** of order 6 and how it is stored in arrays AU and IDU.

Given the following matrix ***A***:

$$\begin{bmatrix} 0 & 12 & 13 & 0 & 0 & 0 \\ 12 & 22 & 0 & 24 & 0 & 0 \\ 13 & 0 & 33 & 34 & 0 & 36 \\ 0 & 24 & 34 & 44 & 45 & 0 \\ 0 & 0 & 0 & 45 & 55 & 56 \\ 0 & 0 & 36 & 0 & 56 & 66 \end{bmatrix}$$

the arrays are:

AU = (0, 22, 12, 33, 0, 13, 44, 34, 24, 55, 45, 66, 56, 0, 36)

IDU = (1, 2, 4, 7, 10, 12, 16) where $nu$=15

In general terms, this storage technique can be expressed as follows:

```
For general sparse matrices and symmetric sparse matrices:
    For each $a_{ij}$ for $j = 1, n$ and $i = j, k$,
    where $a_{kj}$ is the topmost $a_{ij} \neq 0$ in each column $j$,
    there exists $m$, where $1 \leq m \leq nu$, such that
        AU($m+j-i$) = $a_{ij}$
        IDU($j$) = $m$ for each $a_{jj}$
        IDU($n$+1) = $nu$+1
Also, for general sparse matrices:
    For each $a_{ij}$ for $i = 1, n$ and $i = j, k$,
    where $a_{ik}$ is the leftmost $a_{ij} \neq 0$ in each row $i$,
    there exists $m$, where $1 \leq m \leq nl$, such that
        AL($m+i-j$) = $a_{ij}$
        IDL($i$) = $m$ for each $a_{ii}$
        IDL($n$+1) = $nl$+1
```

where:

- $a_{ij}$ are the elements of sparse matrix ***A***, of order *n*.

- Array AU has *nu* elements.
- Array AL has *nl* elements.
- Arrays IDU and IDL each have *n*+1 elements.

***Profile-In Skyline Storage Mode:*** The profile-in skyline storage mode used for sparse matrices has two variations, depending on whether the matrix is a general sparse matrix or a symmetric sparse matrix. Both of these variations are explained here.

For a **general** sparse matrix ***A***, profile-in skyline storage mode uses four one-dimensional arrays to define the sparse matrix storage, AU, IDU, AL, and IDL. Given the sparse matrix ***A*** of order *n*, containing *nu*+*nl*–*n* elements under the top and left profiles, the arrays are set up as follows:

- AU of (at least) length *nu* contains the upper triangle of the sparse matrix ***A***, where the columns are stored consecutively from 1 to *n* in AU in the following way. For each column, the elements starting at the topmost nonzero element in the column and ending at the diagonal element are stored contiguously in AU. The elements stored may include zero elements along with the nonzero elements. If all elements in the column to be stored are zero, the diagonal element, $a_{ii}$, having a value of zero, is stored in AU for that column. A total of *nu* elements are stored for the upper triangle of ***A***.
- IDU, an integer array of (at least) length *n*+1 contains the relative position of each diagonal element of matrix ***A*** in array AU; that is, each element IDU(*i*) of the diagonal pointer array indicates where diagonal element $a_{ii}$ is stored in array AU. One-origin is used, so the first element of IDU is always 1. The last element, IDU(*n*+1), indicates the position after the last element in array AU, which is *nu*+1.
- AL of (at least) length *nl* contains the lower triangle of the sparse matrix ***A***, where the rows are stored consecutively from 1 to *n* in AL in the following way. For each row, the elements starting at the leftmost nonzero element in the row and ending at the diagonal element are stored contiguously in AL. The elements stored may include zero elements along with the nonzero elements. If all elements in the row to be stored are zero, the diagonal element, $a_{ii}$, having a value of zero, is stored in AL for that row. A total of *nl* elements are stored for the lower triangle of ***A***. The values of the diagonal elements are meaningless, so you can store any values in those positions in AL.
- IDL, an integer array of (at least) length *n*+1 contains the relative position of each diagonal element of matrix ***A*** in array AL; that is, each element IDL(*i*) of the diagonal pointer array indicates where diagonal element $a_{ii}$ is stored in array AL. One-origin is used, so the first element of IDL is always 1. The last element, IDL(*n*+1), indicates the position after the last element in array AL, which is *nl*+1.

Consider the following as an example of a 6 by 6 general sparse matrix ***A*** and how it is stored in arrays AU, IDU, AL, and IDL.

Given the following matrix ***A***:

$$\begin{bmatrix} 0 & 12 & 13 & 0 & 0 & 0 \\ 21 & 22 & 0 & 24 & 0 & 0 \\ 31 & 0 & 33 & 34 & 0 & 36 \\ 41 & 42 & 43 & 44 & 45 & 0 \\ 0 & 0 & 0 & 54 & 55 & 56 \\ 0 & 0 & 63 & 0 & 65 & 66 \end{bmatrix}$$

the arrays are:

`AU = (0, 12, 22, 13, 0, 33, 24, 34, 44, 45, 55, 36, 0, 56, 66)`

`IDU = (1, 3, 6, 9, 11, 15, 16)` where *nu*=15

`AL = (*, 21, *, 31, 0, *, 41, 42, 43, *, 54, *, 63, 0, 65, *)`

`IDL = (1, 3, 6, 10, 12, 16, 17)` where *nl*=16

and where “*” means you do not have to store a value in that position in the array. However, these storage positions are required.

For a **symmetric** sparse matrix of order *n*, profile-in skyline storage mode uses the same storage technique as for the upper triangle and diagonal elements of the general sparse matrix; therefore, only the AU and IDU arrays are needed.

Consider the following as an example of a symmetric sparse matrix ***A*** of order 6 and how it is stored in arrays AU and IDU.

Given the following matrix ***A***:

$$\begin{bmatrix} 0 & 12 & 13 & 0 & 0 & 0 \\ 12 & 22 & 0 & 24 & 0 & 0 \\ 13 & 0 & 33 & 34 & 0 & 36 \\ 0 & 24 & 34 & 44 & 45 & 0 \\ 0 & 0 & 0 & 45 & 55 & 56 \\ 0 & 0 & 36 & 0 & 56 & 66 \end{bmatrix}$$

the arrays are:

`AU = (0, 12, 22, 13, 0, 33, 24, 34, 44, 45, 55, 36, 0, 56, 66)`

`IDU = (1, 3, 6, 9, 11, 15, 16)` where *nu*=15

In general terms, this storage technique can be expressed as follows:

```
For general sparse matrices and symmetric sparse matrices:
    For each $a_{ij}$ for $j = 1, n$ and $i = k, j$,
    where $a_{kj}$ is the topmost $a_{ij} \neq 0$ in each column $j$,
    there exists $m$, where $1 \leq m \leq nu$, such that
        AU($m-j+i$) = $a_{ij}$
        IDU($j$) = $m$ for each $a_{jj}$
        IDU($n$+1) = $nu$+1
Also, for general sparse matrices:
    For each $a_{ij}$ for $i = 1, n$ and $j = k, i$,
```

where $a_{ik}$ is the leftmost $a_{ij} \neq 0$ in each row $i$,
there exists $m$, where $1 \leq m \leq nl$, such that
AL($m$–$i$+$j$) = $a_{ij}$
IDL($i$) = $m$ for each $a_{ii}$
IDL($n$+1) = $nl$+1

where:

- $a_{ij}$ are the elements of sparse matrix ***A***, of order $n$.
- Array AU has *nu* elements.
- Array AL has *nl* elements.
- Arrays IDU and IDL each have $n$+1 elements.

# Sequences

A sequence is an ordered collection of numbers. It can be a one-, two-, or three-dimensional sequence. Sequences are used in the areas of sorting, searching, Fourier transforms, convolutions, and correlations.

## Real and Complex Elements in Storage

Sequences can contain either real or complex data. For sequences containing complex data, a special storage arrangement is used to accommodate the two parts, *a* and *b*, of each complex number, *a*+*bi*, in the array. For each complex number, two sequential storage locations are required in the array. Therefore, exactly twice as much storage is required for complex sequences as for real sequences of the same precision. See “How Do You Set Up Your Scalar Data?” on page 28 for a description of real and complex numbers, and “How Do You Set Up Your Arrays?” on page 29 for a description of how real and complex data is stored in arrays.

## One-Dimensional Sequences

A one-dimensional sequence appears symbolically as follows, where the subscripts indicate the element positions within the sequence:

$(x_1, x_2, x_3, \ldots x_n)$

### In Storage

A one-dimensional sequence is stored in an array using stride in the same way a vector uses stride. For details, see “How Stride Is Used for Vectors” on page 58.

## Two-Dimensional Sequences

A two-dimensional sequence appears symbolically as a series of columns of elements. (They are represented in the same way as a matrix without the square brackets.) The two subscripts indicate the element positions in the first and second dimensions, respectively:

$$\begin{array}{cccc}
a_{0,0} & a_{0,1} & \cdots & a_{0,n-1} \\
a_{1,0} & a_{1,1} & \cdots & a_{1,n-1} \\
\vdots & \vdots & & \vdots \\
a_{m-1,0} & a_{m-1,1} & \cdots & a_{m-1,n-1}
\end{array}$$

### In Storage

A two-dimensional sequence is stored in an array using the stride for the second dimension in the same way that a matrix uses leading dimension. It uses a stride of 1 for the first dimension. For details, see “How Leading Dimension Is Used for Matrices” on page 63. (In the area of Fourier transforms, a two-dimensional sequence may be stored in transposed form in an array. In this case, the stride for the second dimension is 1, and the stride for the first dimension is the leading dimension of the array.)

## Three-Dimensional Sequences

A three-dimensional sequence is represented as a series of blocks of elements. Each block is equivalent to a two-dimensional sequence. The number of blocks indicates the length of the third dimension. The three subscripts indicate the element positions in the first, second, and third dimensions, respectively:

**Plane 0:**

$$\begin{array}{ccc}
a_{0,0,0} & \cdot\ \cdot\ \cdot & a_{0,n-1,0} \\
a_{1,0,0} & \cdot\ \cdot\ \cdot & a_{1,n-1,0} \\
\cdot & & \cdot \\
\cdot & & \cdot \\
\cdot & & \cdot \\
a_{m-1,0,0} & \cdot\ \cdot\ \cdot & a_{m-1,n-1,0}
\end{array}$$

**Plane 1:**

$$\begin{array}{ccc}
a_{0,0,1} & \cdot\ \cdot\ \cdot & a_{0,n-1,1} \\
a_{1,0,1} & \cdot\ \cdot\ \cdot & a_{1,n-1,1} \\
\cdot & & \cdot \\
\cdot & & \cdot \\
\cdot & & \cdot \\
a_{m-1,0,1} & \cdot\ \cdot\ \cdot & a_{m-1,n-1,1}
\end{array}$$

$$\begin{array}{c}
\cdot \\
\cdot \\
\cdot
\end{array}$$

**Plane $(p-1)$:**

$$\begin{array}{ccc}
a_{0,0,p-1} & \cdot\ \cdot\ \cdot & a_{0,n-1,p-1} \\
a_{1,0,p-1} & \cdot\ \cdot\ \cdot & a_{1,n-1,p-1} \\
\cdot & & \cdot \\
\cdot & & \cdot \\
\cdot & & \cdot \\
a_{m-1,0,p-1} & \cdot\ \cdot\ \cdot & a_{m-1,n-1,p-1}
\end{array}$$

## In Storage

Each block of elements in a three-dimensional sequence is stored successively in an array. The stride for the third dimension is used to select the elements for each successive block of elements in the array. The starting point of the three-dimensional sequence is specified as the argument for the sequence in the ESSL calling statement. For example, if the three-dimensional sequence is contained in array `BIG`, declared as `BIG(1:20,1:30,1:10)`, and starts at the second element in the first dimension, the third element in the second dimension, and the first element in the third dimension of array `BIG`, you should specify `BIG(2,3,1)` as the argument for the sequence, such as in:

```
CALL  SCFT3 (BIG(2,3,1),20,600,Y,32,2056,16,20,10,1,1.0,AUX,30000)
```

See “How Stride Is Used for Three-Dimensional Sequences” on page 108 for a detailed description of how three-dimensional sequences are stored within arrays using strides.

## How Stride Is Used for Three-Dimensional Sequences

The elements of the three-dimensional sequence can be defined as $a_{ijk}$ for $i = 1, m$, $j = 1, n$, and $k = 1, p$. The first two subscripts, *i* and *j*, define the elements in the first two dimensions of the sequence, and the third subscript, *k*, defines the elements in the third dimension. Using this definition of three-dimensional sequences, this section explains how these elements are mapped into an array using the concepts of stride. (Remember that the elements $a_{ijk}$ are the elements of the conceptual data structure, the three-dimensional sequence to be processed by ESSL. The sequence does not have to include all the elements in the array. Strides are used by the ESSL subroutines to select the desired elements to be processed in the array.)

The sequence elements in the first two dimensions are mapped into an array in the same way a matrix or two-dimensional sequence is mapped into an array. It uses all the items listed in "How Leading Dimension Is Used for Matrices" on page 63, such as the starting point, the number of rows and columns, and the leading dimension. The stride for the first dimension, *inc1*, of a three-dimensional sequence is assumed to be 1, as for matrices. The stride for the second dimension, *inc2*, of a three-dimensional sequence is equivalent to the leading dimension for a matrix.

The stride for the third dimension, *inc3*, is used to define the array elements that make up the third dimension of the three-dimensional sequence. The stride for the third dimension is used as an increment to step through the array to find the starting point for each of the *p* successive blocks of elements in the array. The stride, *inc3*, must always be positive. It must always be greater than or equal to the number of elements to be processed in the first two dimensions; that is, *inc*3 ≥ (*inc*2)(*n*).

A three-dimensional sequence is usually stored in a one-, two-, or three-dimensional array; however, for the sake of this discussion, a three-dimensional array is used here. For an array, `A`, declared as `A(E1:E2,F1:F2,G1:G2)`, the strides in the first, second, and third dimensions are:

*inc1* = 1
*inc2* = (E2−E1+1)
*inc3* = (E2−E1+1)(F2−F1+1)

Given an array `A`, declared as `A(1:7,1:3,0:3)`, where the lengths of the first, second, and third dimensions are 7, 3, and 4, respectively, the resulting strides are *inc1* = 1, *inc2* = 7, and *inc3* = 21.

The starting point for a three-dimensional sequence in an array is at the location specified by the argument for the sequence in the ESSL calling statement. Using the array `A`, described above, if you specify `A(2,2,1)` for a three-dimensional sequence, where `A` is defined as follows, in four blocks, for planes 0 - 3, respectively:

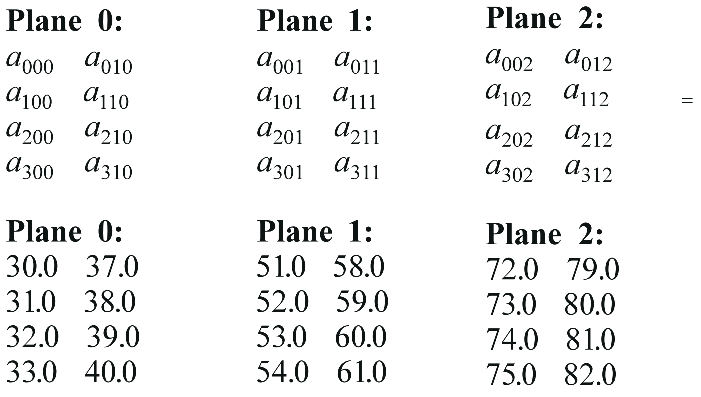
```
1.0  8.0 15.0    22.0 29.0 36.0    43.0 50.0 57.0    64.0 71.0 78.0
2.0  9.0 16.0    23.0 30.0 37.0    44.0 51.0 58.0    65.0 72.0 79.0
3.0 10.0 17.0    24.0 31.0 38.0    45.0 52.0 59.0    66.0 73.0 80.0
4.0 11.0 18.0    25.0 32.0 39.0    46.0 53.0 60.0    67.0 74.0 81.0
5.0 12.0 19.0    26.0 33.0 40.0    47.0 54.0 61.0    68.0 75.0 82.0
6.0 13.0 20.0    27.0 34.0 41.0    48.0 55.0 62.0    69.0 76.0 83.0
7.0 14.0 21.0    28.0 35.0 42.0    49.0 56.0 63.0    70.0 77.0 84.0
```

then processing begins in the second block of elements at row 2 and column 2 in array A, which is 30.0. The stride in the third dimension is then used to find the starting point for each of the next *p*–1 successive blocks of elements in the array. The stride, *inc3*, is added to the starting point *p*–1 times. In this example, the stride for the third dimension is 21, and the number of blocks of elements, *p*, to be processed is 3, so the starting points in array A are A(2,2,1), A(2,2,2), and A(2,2,3). These are elements 30.0, 51.0, and 72.0. These array elements then correspond to the sequence elements $a_{111}$, $a_{112}$, and $a_{113}$, respectively.

In general terms, this results in the following starting positions for the blocks of elements in the array:

```
A(BEGINI, BEGINJ, BEGINK)
A(BEGINI, BEGINJ, BEGINK+1)
A(BEGINI, BEGINJ, BEGINK+2)
.
.
A(BEGINI, BEGINJ, BEGINK+p–1)
```

Using *m* = 4, *n* = 2, and *p* = 3 to define the elements of the three-dimensional data structure in this example, the resulting three-dimensional sequence is defined as follows, in three blocks, for planes 0 - 2, respectively:



| Plane 0: | | Plane 1: | | Plane 2: | |
|---|---|---|---|---|---|
| $a_{000}$ | $a_{010}$ | $a_{001}$ | $a_{011}$ | $a_{002}$ | $a_{012}$ |
| $a_{100}$ | $a_{110}$ | $a_{101}$ | $a_{111}$ | $a_{102}$ | $a_{112}$ |
| $a_{200}$ | $a_{210}$ | $a_{201}$ | $a_{211}$ | $a_{202}$ | $a_{212}$ |
| $a_{300}$ | $a_{310}$ | $a_{301}$ | $a_{311}$ | $a_{302}$ | $a_{312}$ |

=

| Plane 0: | | Plane 1: | | Plane 2: | |
|---|---|---|---|---|---|
| 30.0 | 37.0 | 51.0 | 58.0 | 72.0 | 79.0 |
| 31.0 | 38.0 | 52.0 | 59.0 | 73.0 | 80.0 |
| 32.0 | 39.0 | 53.0 | 60.0 | 74.0 | 81.0 |
| 33.0 | 40.0 | 54.0 | 61.0 | 75.0 | 82.0 |

As shown in this example, the three-dimensional sequence does not have to include all the blocks of elements in the array. In this case, the three-dimensional sequence includes only the second through the fourth block of elements in the array. The first block is not used. Elements of an array are selected as they are arranged in storage, regardless of the number of dimensions defined in the array. Therefore, when using a one- or two-dimensional array to store your three-dimensional sequence, you should understand how your array elements are stored to ensure that elements are selected properly. See “Setting Up Arrays in Fortran” on page 112 for a description of array storage.

**Note:** Three-dimensional sequences are used by the three-dimensional Fourier transform subroutines. By specifying certain stride values for *inc2* and *inc3* and declaring your arrays to have certain number of dimensions, you achieve optimal performance in these subroutines. For details, see “Setting Up Your Data” on page 744 and the Notes section for each subroutine.

# Chapter 4. Coding Your Program

This chapter provides you with information you need to code your Fortran, C, C++, and PL/I programs using ESSL.

## Fortran Programs

This section describes how to code your Fortran program using any of the ESSL run-time libraries.

### Calling ESSL Subroutines and Functions in Fortran

In Fortran programs, most ESSL subroutines are invoked with the CALL statement:

```
CALL  subroutine-name  (argument-1, . . . , argument-n)
```

An example of a calling sequence for the SAXPY subroutine might be:

```
CALL SAXPY (5,A,X,J+INC,Y,1)
```

The remaining ESSL subroutines are invoked as functions by coding a function reference. You first declare the type of value returned by the function: short- or long-precision real, short- or long-precision complex, or integer. Then you code the function reference as part of an expression in a statement. An example of declaring and invoking the DASUM function might be:

```
DOUBLE PRECISION DASUM,SUM,X
      .
      .
      .
SUM = DASUM (N,X,INCX)
```

Values are returned differently for ESSL subroutines and functions. For subroutines, the results of the computation are returned in an argument specified in the calling sequence. In the CALL statement above, the result is returned in argument Y. For functions, the result is returned as the value of the function. In the assignment statement above, the result is assigned to SUM.

See the Fortran publications for details on how to code the CALL statement and a function reference.

### Setting Up a User-Supplied Subroutine for ESSL in Fortran

Some ESSL numerical quadrature subroutines call a user-supplied subroutine, *subf*, identified in the ESSL calling sequence. If your program that calls the numerical quadrature subroutines is coded in Fortran, there are some coding rules you must follow:

- You must declare *subf* as EXTERNAL in your program.
- You should code the *subf* subroutine to the specifications given in " Programming Considerations for the SUBF Subroutine" on page 922. For examples of coding a *subf* subroutine in Fortran, see the subroutine descriptions in that chapter.

# Setting Up Scalar Data in Fortran

Table 27 lists the scalar data types in Fortran that are used for ESSL. Only those types and lengths used by ESSL are listed.

*Table 27. Scalar Data Types in Fortran Programs*

| Terminology Used by ESSL | Fortran Equivalent |
|---|---|
| Character item[1]<br>'N', 'T', 'C' or 'n', 't', 'c' | CHARACTER*1<br>'N', 'T', 'C' |
| Logical item<br>.TRUE., .FALSE. | LOGICAL<br>.TRUE., .FALSE. |
| 32-bit environment integer<br>12345, –12345 | INTEGER or INTEGER*4<br>12345, –12345 |
| 64-bit environment integer[2]<br>12345, –12345 | INTEGER*8[3]<br>12345_8, –12345_8 |
| Short-precision real number[4]<br>12.345 | REAL or REAL*4<br>0.12345E2 |
| Long-precision real number[4]<br>12.345 | DOUBLE PRECISION or REAL*8<br>0.12345D2 |
| Short-precision complex number[4]<br>(123.45, –54321.0) | COMPLEX or COMPLEX*8<br>(123.45E0, –543.21E2) |
| Long-precision complex number[4]<br>(123.45, –54321.0) | COMPLEX*16<br>(123.45D0, –543.21D2) |

[1] ESSL accepts character data in either upper- or lowercase in its calling sequences.

[2] In accordance with the LP64 data model, all ESSL integer arguments remain 32-bits except for the **iusadr** argument for ERRSET.

[3] INTEGER may be used if you specify the compiler option **-qintsize=8**.

[4] Short- and long-precision numbers look the same in this book.

# Setting Up Arrays in Fortran

Arrays are declared in Fortran by specifying the array name, the number of dimensions, and the range of each dimension in a DIMENSION statement or an explicit data type statement, such as REAL, DOUBLE PRECISION, and so forth.

## Real and Complex Array Elements

Each array element can be either a real or complex data item of short or long precision. The type of the array determines the size of the element storage locations. Short-precision data requires 4 bytes, and long-precision data requires 8 bytes. Complex data requires two storage locations of either 4 or 8 bytes each, for short or long precision, respectively, to accommodate the two parts of the complex number: $c = a+bi$. Therefore, exactly twice as much storage is required for complex data as for real data of the same precision. See “How Do You Set Up Your Scalar Data?” on page 28 for a description of real and complex numbers.

Even though complex data items require two storage locations, the same number of elements exist in the array as for real data. A reference to an element—for example, C(3)—in an array containing complex data gives you the whole complex number; that is, it contains both *a* and *b*, where the complex number is expressed as follows:

C(I) ← ($a_i$, $b_i$) for a one-dimensional array
C(I,J) ← ($a_{ij}$, $b_{ij}$) for a two-dimensional array
C(I,J,K) ← ($a_{ijk}$, $b_{ijk}$) for a three-dimensional arra y

## One-Dimensional Array

For a one-dimensional array in Fortran 77, you can code:

```
DIMENSION  A(E1:E2)
```

where A is the name of the array, E1 is the lower bound, and E2 is the upper bound of the single dimension in the array. If the lower bound is not specified, such as in A(E2), the value is assumed to be 1. The upper bound is required.

A one-dimensional array is stored in ascending storage locations (relative to some base storage address) in the following order:

| **Relative Location** | **Array Element** |
|---|---|
| 1 | A(E1) |
| 2 | A(E1+1) |
| 3 | A(E1+2) |
| . | . |
| . | . |
| . | . |
| E2–E1+1 | A(E2) |

For example, the array A of length 4 specified in the DIMENSION statement as A(0:3) and containing the following elements:

A = (1, 2, 3, 4)

has its elements arranged in storage as follows:

| **Relative Location** | **Array Element Value** |
|---|---|
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |
| 4 | 4 |

## Two-Dimensional Array

For a two-dimensional array in Fortran 77, you can code:

```
DIMENSION  A(E1:E2,F1:F2)
```

where A is the name of the array. E1 and F1 are the lower bounds of the first and second dimensions, respectively, and E2 and F2 are the upper bounds of the first and second dimensions, respectively. If either of the lower bounds is not specified, such as in A(E2,F1:F2), the value is assumed to be 1. The upper bounds are always required for each dimension. For examples of Fortran 77 usage, see “SGEMV, DGEMV, CGEMV, ZGEMV, SGEMX, DGEMX, SGEMTX, and DGEMTX—Matrix-Vector Product for a General Matrix, Its Transpose, or Its Conjugate Transpose” on page 296.

The elements of a two-dimensional array are stored in column-major order; that is, they are stored in the following ascending storage locations (relative to some base storage address) with the value of the first (row) subscript expression increasing most rapidly and the value of the second (column) subscript expression increasing least rapidly. Following are the locations of the elements in the array:

| **Relative Location** | **Array Element** |
|---|---|
| 1 | A(E1,F1) (starting column 1) |
| 2 | A(E1+1,F1) |
| . | . |
| . | . |
| . | . |
| E2–E1+1 | A(E2,F1) |
| (E2–E1+1)+1 | A(E1,F1+1) (starting column 2) |
| (E2–E1+1)+2 | A(E1+1,F1+1) |
| . | . |
| . | . |
| . | . |
| (E2–E1+1)(2) | A(E2,F1+1) |
| (E2–E1+1)(2)+1 | A(E1,F1+2) (starting column 3) |
| (E2–E1+1)(2)+2 | A(E1+1,F1+2) |
| . | . |
| . | . |
| . | . |
| (E2–E1+1)(F2–F1) | A(E2,F2-1) |
| (E2–E1+1)(F2–F1)+1 | A(E1,F2) (starting column F2–F1+1) |
| (E2–E1+1)(F2–F1)+2 | A(E1+1,F2) |
| . | . |
| . | . |
| . | . |
| (E2–E1+1)(F2–F1+1) | A(E2,F2) |

For example, the 3 by 4 array A specified in the DIMENSION statement as A(2:4,1:4) and containing the following elements:

$$A = \begin{bmatrix} 11 & 12 & 13 & 14 \\ 21 & 22 & 23 & 24 \\ 31 & 32 & 33 & 34 \end{bmatrix}$$

has its elements arranged in storage as follows:

| **Relative Location** | **Array Element Value** |
|---|---|
| 1 | 11 (starting column 1) |
| 2 | 21 |
| 3 | 31 |
| 4 | 12 (starting column 2) |
| 5 | 22 |
| 6 | 32 |
| 7 | 13 (starting column 3) |
| 8 | 23 |
| 9 | 33 |
| 10 | 14 (starting column 4) |
| 11 | 24 |
| 12 | 34 |

Each element A(I,J) of the array A, declared A(1:*n*, 1:*m*), containing real or complex data, occupies the storage location whose address is given by the following formula:

address {A(I,J)} = address {A} + (I–1 + *n*(J–1))*f*

for:

I = 1, *n* and
J = 1, *m*

where:

*f* = 4 for short-precision real numbers
*f* = 8 for long-precision real numbers
*f* = 8 for short-precision complex numbers
*f* = 16 for long-precision complex numbers

## Three-Dimensional Array

For a three-dimensional array in Fortran 77, you can code:

```
DIMENSION  A(E1:E2,F1:F2,G1:G2)
```

where A is the name of the array. E1, F1, and G1 are the lower bounds of the first, second, and third dimensions, respectively, and E2, F2, and G2 are the upper bounds of the first, second, and third dimensions, respectively. If any of the lower bounds are not specified, such as in A(E1:E2,F1:F2,G2), the value is assumed to be 1. The upper bounds are always required for each dimension. For examples of Fortran 77 usage, see "SCFT3 and DCFT3—Complex Fourier Transform in Three Dimensions" on page 809.

The elements of a three-dimensional array can be thought of as a set of two-dimensional arrays, stored sequentially in ascending storage locations in the array. The elements in each two-dimensional array are stored as defined in the previous section. In the three-dimensional array, the value of the first (row) subscript expression increases most rapidly, the second (column) subscript expression increases less rapidly, and the third subscript expression (set of rows and columns) increases least rapidly. Following are the locations of the elements in the array:

| **Relative Location** | **Array Element** |
|---|---|
| 1 | A(E1,F1,G1) (starting the first set) |
| 2 | A(E1+1,F1,G1) |
| . | . |
| . | . |
| . | . |
| (E2–E1+1)(F2–F1+1) | A(E2,F2,G1) |
| (E2–E1+1)(F2–F1+1)+1 | A(E1,F1,G1+1) (starting the second set) |
| (E2–E1+1)(F2–F1+1)+2 | A(E1+1,F1,G1+1) |
| . | . |
| . | . |
| . | . |
| (E2–E1+1)(F2–F1+1)(2) | A(E2,F2,G1+1) |
| (E2–E1+1)(F2–F1+1)(2)+1 | A(E1,F1,G1+2) (starting the third set) |
| (E2–E1+1)(F2–F1+1)(2)+2 | A(E1+1,F1+2) |
| . | . |
| . | . |

| | |
|---|---|
| . | . |
| (E2–E1+1)(F2–F1+1)(G2–G1) | A(E2,F2,G2–1) |
| (E2–E1+1)(F2–F1+1)(G2–G1)+1 | A(E1,F1,G2) (starting the last set*) |
| (E2–E1+1)(F2–F1+1)(G2–G1)+2 | A(E1+1,F1,G2) |
| . | . |
| . | . |
| . | . |
| (E2–E1+1)(F2–F1+1)(G2–G1+1) | A(E2,F2,G2) |

* The last set is the G2–G1+1 set.

For example, the 3 by 2 by 4 array A specified in the DIMENSION statement as A(1:3,0:1,2:5) and containing the following sets of rows and columns of elements:

$$A = \begin{bmatrix} 111 & 121 \\ 211 & 221 \\ 311 & 321 \end{bmatrix} \begin{bmatrix} 112 & 122 \\ 212 & 222 \\ 312 & 322 \end{bmatrix} \begin{bmatrix} 113 & 123 \\ 213 & 223 \\ 313 & 323 \end{bmatrix} \begin{bmatrix} 114 & 124 \\ 214 & 224 \\ 314 & 324 \end{bmatrix}$$

has its elements arranged in storage as follows:

| **Relative Location** | **Array Element Value** |
|---|---|
| 1 | 111 (starting the first set) |
| 2 | 211 |
| 3 | 311 |
| 4 | 121 |
| 5 | 221 |
| 6 | 321 |
| 7 | 112 (starting the second set) |
| 8 | 212 |
| 9 | 312 |
| 10 | 122 |
| 11 | 222 |
| 12 | 322 |
| 13 | 113 (starting the third set) |
| 14 | 213 |
| 15 | 313 |
| 16 | 123 |
| 17 | 223 |
| 18 | 323 |
| 19 | 114 (starting the fourth set) |
| 20 | 214 |
| 21 | 314 |
| 22 | 124 |
| 23 | 224 |
| 24 | 324 |

Each element A(I,J,K) of the array A, declared A(1:*n*, 1:*m*, 1:*p*), containing real or complex data, occupies the storage location whose address is given by the following formula:

$$\text{address } \{A(I,J,K)\} = \text{address } \{A\} + (I-1 + n(J-1) + mn(K-1))f$$

for:

I = 1, *n*

J = 1, *m*
K = 1, *p*

where:

*f* = 4 for short-precision real numbers
*f* = 8 for long-precision real numbers
*f* = 8 for short-precision complex numbers
*f* = 16 for long-precision complex numbers

## Creating Multiple Threads and Calling ESSL from Your Fortran Program

The example shown below shows how to create up to a maximum of eight threads, where each thread calls the DURAND and DGEICD subroutines.

Be sure to compile this program with the `xlf_r` command and the `-qnosave` option.

```
       program matinv_example
       implicit none
!
!  program to invert m nxn random matrices
!

       real(8), allocatable :: A(:,:,:), det(:,:), rcond(:)
       real(8)              :: dummy_aux, seed=1998, sd
       integer              :: rc, i, m=8, n=500, iopt=3, naux=0
!
! allocate storage
!

       allocate(A(n,n,m),stat=rc)
       call error_exit(rc,"Allocation of matrix A")
       allocate(det(2,m),stat=rc)
       call error_exit(rc,"Allocation of det")
       allocate(rcond(m),stat=rc)
       call error_exit(rc,"Allocation of rcond")

!
! Calculate inverses in parallel
!
!SMP$  parallel do private(i,sd), schedule(static),
!SMP$&    share(n,a,iopt,rcond,det,dummy_aux,naux)
       do i=1,m

         sd = seed + 100*i
         call durand(sd,n*n,A(1,1,i))
         call dgeicd(A(1,1,i),n,n,iopt,rcond(i),det(1,i),
     &               dummy_aux,naux)
       enddo
```

```
      write(*,*)'Reciprocal condition numbers of the matrices are:'
      write(*,'(4E12.4)') rcond
!
!
!
      deallocate(A,stat=rc)

      call error_exit(rc,"Deallocation of matrix A")
      deallocate(det,stat=rc)
      call error_exit(rc,"Deallocation of det")
      deallocate(rcond,stat=rc)
      call error_exit(rc,"Deallocation of rcond")
      stop

      contains
        subroutine error_exit(error_code,string)
        character(*)    :: string
        integer         :: error_code
        if(error_code .eq. 0 ) return
        write(0,*)string,": failing return code was ",error_code
        stop 1
        end subroutine error_exit
      end
```

## Handling Errors in Your Fortran Program

ESSL provides you with flexibilities in handling both input-argument errors and computational errors:

- For input-argument errors 2015 and 2030, which are optionally-recoverable errors, ESSL allows you to obtain corrected input-argument values and react at run time.

  **Note:** In the case where error 2015 is unrecoverable, you have the option of dynamic allocation for most of the *aux* arguments. For details see the subroutine descriptions in Part 2 of this book.

- For computational errors, ESSL provides a return code and additional information to help you analyze the problem in your program and react at run time.

“Input-Argument Errors in Fortran” on page 119 and “Computational Errors in Fortran” on page 122 explain how to use these facilities by describing the additional statements you must code in your program.

For multithreaded application programs, if you want to initialize the error option table and change the default settings for input-argument and computational errors, you need to implement the steps shown in “Input-Argument Errors in Fortran” on page 119 and “Computational Errors in Fortran” on page 122 on each thread that calls ESSL. An example is shown in “Example of Handling Errors in a Multithreaded Application Program” on page 127.

## Input-Argument Errors in Fortran

To obtain corrected input-argument values in a Fortran program and to avert program termination for the optionally-recoverable input-argument errors 2015 and 2030, add the statements in the following steps your program. Steps 3 and 7 for ERRSAV and ERRSTR, respectively, are optional. Adding these steps makes the effect of the call to ERRSET temporary.

### *Step 1. Declare ENOTRM as External*

```
  EXTERNAL ENOTRM
```

This declares the ESSL error exit routine ENOTRM as an external reference in your program. This should be coded in the beginning of your program before any of the following statements.

### *Step 2. Call EINFO for Initialization*

```
  CALL EINFO (0)
```

This calls the EINFO subroutine with one argument of value 0 to initialize the ESSL error option table. It is required only if you call ERRSET in your program. It is coded only once in the beginning of your program before any calls to ERRSET. For a description of EINFO, see "EINFO—ESSL Error Information-Handler Subroutine" on page 962.

### *Step 3. Call ERRSAV*

```
  CALL  ERRSAV (ierno,tabent)
```

(This is an optional step.) This calls the ERRSAV subroutine, which stores the error option table entry for error number *ierno* in an 8-byte storage area, *tabent*, which is accessible to your program. ERRSAV must be called for each entry you want to save. This step is used, along with step 7, for ERRSTR. For information on whether you should use ERRSAV and ERRSTR, see " How Can You Control Error Handling in Large Applications by Saving and Restoring Entries in the Error Option Table?" on page 53. For an example, see "Example 3" on page 126, as the use is the same as for computational errors.

### *Step 4. Call ERRSET*

```
CALL  ERRSET (ierno,inoal,inomes,itrace,iusadr,irange)
```

This calls the ERRSET subroutine, which allows you to dynamically modify the action taken when an error occurs. For optionally-recoverable ESSL input-argument errors, you need to call ERRSET only if you want to avoid terminating your program and you want the input arguments associated with this error to be assigned correct values in your program when the error occurs. For one error (*ierno*) or a range of errors (*irange*), you can specify:

- How many times each error can occur before execution terminates (*inoal*)
- How many times each error message can be printed (*inomes*)
- The ESSL exit routine ENOTRM, to be invoked for the error indicated (*iusadr*)

ERRSET must be called for each error code you want to indicate as being recoverable. For ESSL, *ierno* should have a value of 2015 or 2030. If you want to eliminate error messages, you should indicate a negative number for *inomes*; otherwise, you should specify 0 for this argument. All the other ERRSET arguments should be specified as 0.

For a list of the default values set in the ESSL error option table, see Table 26 on page 51. For a description of the input-argument errors, see "Input-Argument Error Messages" on page 179. For a description of ERRSET, see Chapter 17 on page 959.

***Step 5. Call ESSL***

```
CALL  name  (arg-1,...,arg-n,*yyy,*zzz,...)
```

This calls the ESSL subroutine and specifies a branch on one or more return code values, where:

- *name* specifies the ESSL subroutine.
- *arg-1,..., arg-n* are the input and output arguments.
- *yyy*, *zzz*, and any other statement numbers preceded by an "*" are the Fortran statement numbers indicating where you want to branch when you get a nonzero return code. Each corresponds to a different ESSL value. Control goes to the corresponding statement number when a nonzero return code value is returned for the CALL statement. Return code values are described under "Error Conditions" in each ESSL subroutine description in Part 2 of this book.

***Step 6. Perform the Desired Action:*** These are the statements at statement number *yyy* or *zzz*, shown in the CALL statement in Step 5, and preceded by an "*." The statement to which control is passed corresponds to the return code value for the error.

These statements perform whatever action is desired when the recoverable error occurs. These statements may check the new values set in the input arguments to determine whether adequate program storage is available, and then decide whether to continue or terminate the program. Otherwise, these statements may check that the size of the working storage arrays or the length of the transform agrees with other data in the program. The program may also store this corrected input argument value for future reference.

***Step 7. Call ERRSTR***

```
CALL  ERRSTR (ierno,tabent)
```

(This is an optional step.) This calls the ERRSTR subroutine, which stores an entry in the error option table for error number *ierno* from an 8-byte storage area, *tabent*, which is accessible to your program. ERRSTR must be called for each entry you want to store. This step is used, along with step 3, for ERRSAV. For information on whether you should use ERRSAV and ERRSTR, see “ How Can You Control Error Handling in Large Applications by Saving and Restoring Entries in the Error Option Table?” on page 53. For an example, see “Example 3” on page 126, as the use is the same as for computational errors.

## Example

This example shows an error code 2015, which resets the size of the work area *aux*, specified in *naux*, if the value specified is too small. It also indicates that no error messages should be issued.

```
      .
      .
      .
C                   DECLARE ENOTRM AS EXTERNAL
      EXTERNAL ENOTRM
      .
      .
      .
C                   INITIALIZE THE ESSL ERROR
C                   OPTION TABLE
      CALL EINFO(0)
      .
      .
      .
C                   MAKE ERROR CODE 2015 A RECOVERABLE
C                   ERROR AND SUPPRESS PRINTING ALL
C                   ERROR MESSAGES FOR IT
      CALL ERRSET(2015,0,-1,0,ENOTRM,2015)
      .
      .
      .
C                   CALL ESSL ROUTINE SWLEV.
C                   IF THE NAUX INPUT
C                   ARGUMENT IS TOO SMALL, ERROR
C                   2015 OCCURS.  THE MINIMUM VALUE
C                   REQUIRED IS STORED IN THE NAUX
C                   INPUT ARGUMENT AND CONTROL GOES
C                   TO LABEL 400.
      CALL SWLEV(X,INCX,U,INCU,Y,INCY,N,AUX,NAUX,*400)
      .
      .
      .
```

```
C                    CHECK THE RESULTING INPUT ARGUMENT
C                    VALUE  IN NAUX AND TAKE THE
C                    DESIRED ACTION
400    .
       .
       .
```

## Computational Errors in Fortran

To obtain information about an ESSL computational error in a Fortran program, add the statements in the following steps to your program. Steps 2 and 7 for ERRSAV and ERRSTR, respectively, are optional. Adding these steps makes the effect of the call to ERRSET temporary. For a list of those computational errors that return information and to which these steps apply, see "EINFO—ESSL Error Information-Handler Subroutine" on page 962.

***Step 1. Call EINFO for Initialization***

```
  CALL EINFO (0)
```

This calls the EINFO subroutine with one argument of value 0 to initialize the ESSL error option table. It is required only if you call ERRSET in your program. It is coded only once in the beginning of your program before any calls to ERRSET. For a description of EINFO, see "EINFO—ESSL Error Information-Handler Subroutine" on page 962.

***Step 2. Call ERRSAV***

```
  CALL  ERRSAV (ierno,tabent)
```

(This is an optional step.) This calls the ERRSAV subroutine, which stores the error option table entry for error number *ierno* in an 8-byte storage area, *tabent*, which is accessible to your program. ERRSAV must be called for each entry you want to save. This step is used, along with step 7, for ERRSTR. For information on whether you should use ERRSAV and ERRSTR, see " How Can You Control Error Handling in Large Applications by Saving and Restoring Entries in the Error Option Table?" on page 53.

***Step 3. Call ERRSET***

```
CALL  ERRSET (ierno,inoal,inomes,itrace,iusadr,irange)
```

This calls the ERRSET subroutine, which allows you to dynamically modify the action taken when an error occurs. For ESSL computational errors, you need to call ERRSET only if you want to change the default values in the ESSL error option table. For one error (*ierno*) or a range of errors (*irange*), you can specify:

- How many times each error can occur before execution terminates (*inoal*)

- How many times each error message can be printed (*inomes*)

ERRSET must be called for each error code for which you want to change the default values. For ESSL, *ierno* should be set to one of the eligible values listed in Table 167 on page 962. To allow your program to continue after an error in the specified range occurs, *inoal* must be set to a value greater than 1. For ESSL, *iusadr* should be specified as either 0 or 1 in a 32-bit environment (0_8 or 1_8 in a 64-bit environment), so a user exit is not taken.

For a list of the default values set in the ESSL error option table, see Table 26 on page 51. For a description of the computational errors, see “Computational Error Messages” on page 187. For a description of ERRSET, see Chapter 17 on page 959.

***Step 4. Call ESSL***

```
CALL  name  (arg-1,...,arg-n,*yyy,*zzz,...)
```

This calls the ESSL subroutine and specifies a branch on one or more return code values, where:

- *name* specifies the ESSL subroutine.
- *arg-1*,..., *arg-n* are the input and output arguments.
- *yyy*, *zzz*, and any other statement numbers preceded by an “*” are the Fortran statement numbers indicating where you want to branch when you get a nonzero return code. Each corresponds to a different ESSL value. Control goes to the corresponding statement number when a nonzero return code value is returned for the CALL statement. Return code values are described under “Error Conditions” in each ESSL subroutine description in Part 2 of this book.

***Step 5. Call EINFO for Information***

```
nmbr  CALL EINFO (icode,inf1)
-or-
nmbr  CALL EINFO (icode,inf1,inf2)
```

This calls the EINFO subroutine, which returns information about certain computational errors, where:

- *nmbr* is the statement number *yyy*, *zzz*, or any of the other statement numbers preceded by an “*” in the CALL statement in Step 4, corresponding to the return code value for this error code.
- *icode* is the error code of interest.
- *inf1* and *inf2* are the integer variables used to receive the information, where *inf1* is assigned a value for all errors, and *inf2* is assigned a value for some errors. For a description of EINFO, see “EINFO—ESSL Error Information-Handler Subroutine” on page 962.

***Step 6. Check the Values in the Information Receivers:*** These statements check the values returned in the output argument information receivers, *inf1* and *inf2*, which contain the information about the computational error.

***Step 7. Call ERRSTR***

```
  CALL  ERRSTR (ierno,tabent)
```

(This is an optional step.) This calls the ERRSTR subroutine, which stores an entry in the error option table for error number *ierno* from an 8-byte storage area, *tabent*, which is accessible to your program. ERRSTR must be called for each entry you want to store. This step is used, along with step 2, for ERRSAV. For information on whether you should use ERRSAV and ERRSTR, see “ How Can You Control Error Handling in Large Applications by Saving and Restoring Entries in the Error Option Table?” on page 53.

## Example 1

This 32-bit environment example shows an error code 2104, which returns one piece of information: the index of the last diagonal with nonpositive value (I1).

```
       .
       .
       .
C                    INITIALIZE THE ESSL ERROR
C                    OPTION TABLE
       CALL EINFO(0)
       .
       .
       .
C                    ALLOW 100 ERRORS FOR CODE 2104
       CALL ERRSET(2104,100,0,0,0,2104)
       .
       .
       .
C                    CALL ESSL ROUTINE DPPF.
C                    IF THE INPUT MATRIX IS NOT
C                    POSITIVE DEFINITE, CONTROL GOES TO
C                    LABEL 400
       IOPT=0
       CALL DPPF(APP,N,IOPT,*400)
       .
       .
       .
```

```
C                    CALL THE INFORMATION-HANDLER
C                    ROUTINE FOR ERROR CODE 2104 TO
C                    RETURN ONE PIECE OF INFORMATION
C                    IN VARIABLE I1, THE INDEX OF THE
C                    LAST NONPOSITIVE DIAGONAL FOUND
C                    BY ROUTINE DPPF
400    CALL EINFO (2104,I1)
       .
       .
       .
```

### Example 2

This 32-bit environment example shows an error code 2103, which returns one piece of information: the index of the zero diagonal (I1) found by DGEF.

```
       .
       .
       .
C                    INITIALIZE THE ESSL ERROR
C                    OPTION TABLE
       CALL EINFO(0)
       .
       .
       .
C                    ALLOW 100 ERRORS FOR CODE 2103
       CALL ERRSET(2103,100,0,0,0,2103)
       .
       .
       .
C                    CALL ESSL SUBROUTINE DGEF.
C                    IF THE INPUT MATRIX IS
C                    SINGULAR, CONTROL GOES TO
C                    LABEL 400
       CALL DGEF(A,LDA,N,IPVT,*400)
       .
       .
       .
```

```
C                    CALL THE INFORMATION-HANDLER
C                    ROUTINE FOR ERROR CODE 2103 TO
C                    RETURN ONE PIECE OF INFORMATION
C                    IN VARIABLE I1, THE INDEX OF THE
C                    LAST ZERO DIAGONAL FOUND BY
C                    SUBROUTINE DGEF
400    CALL EINFO (2103,I1)
       .
       .
       .
```

### Example 3

This 32-bit environment example shows an error code 2101, which returns two pieces of information: the eigenvalue (I1) that failed to converge after the indicated (I2) number of iterations. It uses ERRSAV and ERRSTR to insulate the effects of the error handling for error 2101 by this program.

```
       .
       .
C                    DECLARE AN AREA TO SAVE THE
C                    ERROR OPTION TABLE INFORMATION
C                    FOR ERROR CODE 2101
       CHARACTER*8 SAV2101
       .
       .
C                    INITIALIZE THE ESSL ERROR
C                    OPTION TABLE
       CALL EINFO(0)
C                    SAVE THE EXISTING ERROR OPTION
C                    TABLE ENTRY FOR ERROR CODE 2101
       CALL ERRSAV(2101,SAV2101)
       .
       .
C                    ALLOW 255 ERRORS FOR CODE 2101
       CALL ERRSET(2101,255,0,0,0,2101)
       .
       .
C                    CALL ESSL SUBROUTINE DGEEV.
C                    IF THE EIGENVALUE FAILED TO
C                    CONVERGE, CONTROL GOES TO LABEL 400
       CALL DGEEV(IOPT,A,LDA,W,Z,LDZ,SELECT,N,AUX,NAUX,*400)
       .
       .
```

```
C                   CALL THE INFORMATION-HANDLER
C                   ROUTINE FOR ERROR CODE 2101 TO
C                   RETURN TWO PIECES OF INFORMATION.
C                   VARIABLE I1 CONTAINS THE EIGENVALUE
C                   THAT FAILED TO CONVERGE.  VARIABLE
C                   I2 CONTAINS THE NUMBER OF ITERATIONS.
400   CALL EINFO (2101,I1,I2)
      .
      .
C                   RESTORE THE PREVIOUS ERROR OPTION
C                   TABLE ENTRY FOR ERROR CODE 2101.
C                   ERROR PROCESSING RETURNS TO HOW IT
C                   WAS BEFORE IT WAS ALTERED BY THE ABOVE
C                   ERRSET STATEMENT.
      CALL ERRSTR(2101,SAV2101)
      .
      .
```

## Example of Handling Errors in a Multithreaded Application Program

This 32-bit environment example shows how to modify the MATINV_EXAMPLE program in “Creating Multiple Threads and Calling ESSL from Your Fortran Program” on page 117 with calls to the ESSL error handling subroutines. The ESSL error handling subroutines are called from each thread to: initialize the error option table, save the current error option table values for input-argument error 2015 and computational error 2105, change the default values for errors 2015 and 2105, and then restore the original default values for errors 2015 and 2105.

```
      program matinv_example
      implicit none
!
!  program to invert m nxn random matrices
!
      real(8), allocatable :: A(:,:,:), det(:,:), rcond(:)
      real(8)              :: dummy_aux, seed=1998, sd
      integer              :: rc, i, m=8, n=500, iopt=3, naux=0
      integer              :: inf1(8)
      character(8)         :: sav2015(8)
      character(8)         :: sav2105(8)
      integer              :: ENOTRM
```

```
!
       external ENOTRM
!
! allocate storage
!
       allocate(A(n,n,m),stat=rc)
       call error_exit(rc,"Allocation of matrix A")
       allocate(det(2,m),stat=rc)
       call error_exit(rc,"Allocation of det")
       allocate(rcond(m),stat=rc)
       call error_exit(rc,"Allocation of rcond")
!
! Calculate inverses in parallel
!
!SMP$  parallel do private(i,sd), schedule(static),
!SMP$&    share(n,m,a,iopt,rcond,det,dummy_aux,naux,sav2015,sav2105,inf1)
       do i=1,m
!
!       initialize error handling
         call einfo(0)
!
!       Save existing option table values for error 2015
         call errsav(2015,sav2015(i))
!
!       Set Error 2015 to be non-recoverable so dgeicd will dynamically
!       allocate the work area.
         call errset(2015,100,100,0,1,2015)
!
!       Save existing option table values for error 2105
         call errsav(2105,sav2105(i))
!
!       Set Error 2105 to be recoverable
         call errset(2105,100,100,0,ENOTRM,2105)
!
         sd = seed + 100*i
         call durand(sd,n*n,A(1,1,i))
         call dgeicd(A(1,1,i),n,n,iopt,rcond(i),det(1,i),
     &                dummy_aux,naux,*10,*20)
10       goto 30
!
```

```
!       Catch singular matrix returned by dgeicd.
20       CALL EINFO(2105,inf1(i))
         WRITE(*,*) 'ERROR: Zero pivot found at location ',inf1(i)
!
!       Restore the error option table entries
30       continue
         call errstr(2015,SAV2015(i))
         call errstr(2105,SAV2105(i))

       enddo

       write(*,*)'Reciprocal condition numbers of the matrices are:'
       write(*,'(4E12.4)') rcond
!
!
!
       deallocate(A,stat=rc)
       call error_exit(rc,"Deallocation of matrix A")
       deallocate(det,stat=rc)
       call error_exit(rc,"Deallocation of det")

       deallocate(rcond,stat=rc)
       call error_exit(rc,"Deallocation of rcond")
       stop
       contains
         subroutine error_exit(error_code,string)
         character(*)    :: string
         integer         :: error_code
         if(error_code .eq. 0 ) return
         write(0,*)string,": failing return code was ",error_code
         stop 1
         end subroutine error_exit
       end
```

# C Programs

This section describes how to code your C program.

## Calling ESSL Subroutines and Functions in C

This section shows how to call ESSL subroutines and functions from your C program.

### Before You Call ESSL

Before you can call the ESSL subroutines from your C program, you must have the appropriate ESSL header file installed on your system. The ESSL header file allows you to code your function calls as described in this section. It contains entries for all the ESSL subroutines. The ESSL header file is distributed with the ESSL package. The ESSL header file to be used with the C compiler is named `essl.h`. You should check with your system support group to verify that the appropriate ESSL header file is installed.

In the beginning of your program, before you call any of the ESSL subroutines, **you must code the following statement for the ESSL header file:**

```
#include <essl.h>
```

If you are planning to create your own threads for the ESSL Thread-Safety or SMP Library, you must include the `pthread.h` header file as the first include file in your C program. For an example, see “Creating Multiple Threads and Calling ESSL from Your C Program” on page 134.

### Coding the Calling Sequences

In C programs, the ESSL subroutines, not returning a function value, are invoked with the following type of statement:

```
subroutine-name (argument-1, . . . , argument-n);
```

An example of a calling sequence for SAXPY might be:

```
saxpy (5,a,x,incx,y,1);
```

The ESSL subroutines returning a function value are invoked with the following type of statement:

```
function-value-name=subroutine-name (argument-1, . . . , argument-n);
```

An example of invoking DASUM might be:

```
sum = dasum (n,x,incx);
```

See the C publications for details about how to code the function calls.

## Passing Arguments in C

This section describes how to pass arguments in your C program.

### About the Syntax Shown in This Book

The argument syntax shown in this book assumes that you have installed and are using the ESSL header file. For further details, see “Calling ESSL Subroutines and Functions in C” on page 129.

### No Optional Arguments

In the ESSL calling sequences for C, there are no optional arguments, as for some programming languages. You must code all the arguments listed in the syntax.

### Arguments That Must Be Passed by Value

All scalar arguments that are not modified must be passed by value in the ESSL calling sequence. (This refers to input-only scalar arguments, such as *incx*, *m*, and *lda*.)

### Arguments That Must Be Passed by Reference

Following are the instances in which you pass your arguments by reference (as a pointer) in the ESSL calling sequence:

***Arrays:*** Arguments that are arrays are passed by reference, as usual.

***Subroutine Names:*** Some ESSL subroutines call a user-supplied subroutine. The name is part of the ESSL calling sequence. It must be passed by reference.

***Output Scalar Arguments:*** When an output argument is a scalar data item, it must be passed by reference. This is true for all scalar data types: real, complex, and so forth. **When this occurs, it is listed in the notes of each subroutine description in Part 2 of this book.**

***Character Arguments:*** Character arguments must be passed as strings, by reference. You specify the character, in upper- or lowercase, in the ESSL calling sequence with double quotation marks around it, as in "t". Following is an example of how you can call SGEADD, specifying the *transa* and *transb* arguments as strings *n* and *t*, respectively:

```
sgeadd (a,5,"n",b,3,"t",c,4,4,3);
```

***Altered Arguments When Using Error Handling:*** If you use ESSL error handling in your C program, as described in “Handling Errors in Your C Program” on page 136, you must pass by reference all the arguments that can potentially be altered by ESSL error handling. This applies to all your ESSL call statements after the point where you code the #define statement, shown in step 1 in “Input-Argument Errors in C” on page 136 and in step 1 in “Computational Errors in C” on page 140. The two types of ESSL arguments are:

- *naux* arguments for auxiliary storage
- *n* arguments for transform lengths

## Setting Up a User-Supplied Subroutine for ESSL in C

Some ESSL numerical quadrature subroutines call a user-supplied subroutine, *subf*, identified in the ESSL calling sequence. If your program that calls the numerical quadrature subroutines is coded in C, there are some coding rules you must follow for the *subf* subroutine:

- You can code the *subf* subroutine using only C or Fortran.
- You must declare *subf* as an external subroutine in your application program.
- You should code the *subf* subroutine to the specifications given in “ Programming Considerations for the SUBF Subroutine” on page 922. For an example of coding a *subf* subroutine in C, see “Example 1” on page 933.

## Setting Up Scalar Data in C

Table 28 lists the scalar data types in C that are used for ESSL. Only those types and lengths used by ESSL are listed.

*Table 28 (Page 1 of 2). Scalar Data Types in C Programs*

| **Terminology Used by ESSL** | **C Equivalent** |
|---|---|
| Character item[1]<br>'N', 'T', 'C' or 'n', 't', 'c' | char *<br>“n”, “t”, “c” |
| Logical item<br>.TRUE., .FALSE. | Specify it as described in “Setting Up Complex and Logical Data Types in C” on page 132.[2] |

*Table 28 (Page 2 of 2). Scalar Data Types in C Programs*

| Terminology Used by ESSL | C Equivalent |
|---|---|
| 32-bit environment integer<br>12345, −12345 | signed int |
| 64-bit environment integer[3]<br>12345l, −12345l | long |
| Short-precision real number[4]<br>12.345 | float |
| Long-precision real number[4]<br>12.345 | double |
| Short-precision complex number[4]<br>(123.45, −54321.0) | Specify it as described in “Setting Up Complex and Logical Data Types in C” on page 132.[2] |
| Long-precision complex number[4]<br>(123.45, −54321.0) | Specify it as described in “Setting Up Complex and Logical Data Types in C” on page 132.[2] |

[1] ESSL accepts character data in either upper- or lowercase in its calling sequences.

[2] There are no equivalent data types for logical and complex data in C. These require special procedures. For details, see the referenced section.

[3] In accordance with the LP64 data model, all ESSL integer arguments remain 32-bits except for the **iusadr** argument for ERRSET.

[4] Short- and long-precision numbers look the same in this book.

## Setting Up Complex and Logical Data Types in C

Complex and logical data types are not part of the C language; however, some ESSL subroutines require arguments of these data types.

### Complex Data

ESSL provides identifiers, `cmplx` and `dcmplx`, for complex data types, defined in the ESSL header file, as well as two macro definitions, RE and IM, for handling the real and imaginary parts of complex numbers:

```
#ifndef _CMPLX
#ifndef _REIM
#define _REIM  1
#endif
typedef union  { struct { float __re, __im;}
                 __data;  double __align;} cmplx;
#endif
#ifndef _DCMPLX
#ifndef _REIM
#define _REIM  1
#endif
typedef union  { struct { double __re, __im;}
                 __data;  double __align;} dcmplx;
#endif
#ifdef _REIM
#define RE(x)  ((x).__data.__re)
#define IM(x)  ((x).__data.__im)
#endif
```

You must, therefore, code an include statement for the ESSL header file in the beginning of your program to use these definitions. For details, see “Calling ESSL Subroutines and Functions in C” on page 129.

Assuming you are using the ESSL header file, if you declare data items to be of type `cmplx` or `dcmplx`, you can pass them as short- and long-precision complex data to ESSL, respectively. You may want to write a CSET macro to initialize complex variables, using the RE and IM macros provided in the ESSL header file. Following is an example of how to use the CSET macro to initialize the complex variable `alpha`:

```
#include <essl.h>
#define CSET(x,a,b)  (RE(x)=a, IM(x)=b)
main()
{
cmplx alpha,t[3],s[5];
.
.
.
CSET (alpha,2.0,3.0);
caxpy (3,alpha,s,1,t,2);
.
.
.
}
```

If you choose to use your own definitions for complex data, instead of those provided in the ESSL header file, you can define _CMPLX and _DCMPLX in your program for short- and long-precision complex data, respectively, using the following #define statements. These statements are coded with your global declares in the front of your program and must be coded before the `#include` statement for the ESSL header file.

```
#define _CMPLX
#define _DCMPLX
```

If you prefer to define your complex data at compile time, you can use the job processing procedures described in “Compiling” on page 163.

### Logical Data

By coding the following simple macro definitions in your program, you can then use `TRUE` or `FALSE` in assigning values to or specifying any logical arguments passed to ESSL:

```
#define FALSE 0
#define TRUE 1
```

## Setting Up Arrays in C

C arrays are arranged in storage in row-major order. This means that the last subscript expression increases most rapidly, the next-to-the-last subscript expression increases less rapidly, and so forth, with the first subscript expression increasing least rapidly. ESSL subroutines require that arrays passed as arguments be in column-major order. This is the array storage convention used by Fortran, described in “Setting Up Arrays in Fortran” on page 112. To pass an array from your C program to ESSL, to have ESSL process the data correctly, and to get a result that is in the proper form for your C program, you can do any of the following:

- Build and process the matrix, logically transposed from the outset, and transpose the results as necessary.
- Before the ESSL call, transpose the input arrays. Then, following the ESSL call, transpose any arrays updated as output.
- If there are arguments in the ESSL calling sequence indicating whether the arrays are to be processed in normal or transposed form, such as the *transa* and *transb* arguments in the _GEMM subroutines, use these arguments in combination with the matrix equivalence rules to avoid having to transpose your data in separate operations. For further detail, see “SGEMMS, DGEMMS, CGEMMS, and ZGEMMS—Matrix Multiplication for General Matrices, Their Transposes, or Conjugate Transposes Using Winograd's Variation of Strassen's Algorithm” on page 405.

## Creating Multiple Threads and Calling ESSL from Your C Program

The example shown below shows how to create two threads, where each thread calls the ISAMAX subroutine. To use the AIX pthreads library, you must specify the `pthread.h` header file as the first include file in your program.

Be sure to compile this program with the `cc_r` command.

```
#include <pthread.h>
#include <stdio.h>
#include <unistd.h>
#include <essl.h>


/* Create structure for argument list  */
typedef struct {
  int        n;
  float     *x;
  int     incx;
} arg_list;
```

```
/* Define prototype for thread routine */
void *Thread(void *v);

int main()
{
 float  sx1[9] = { 1., 2., 7., -8., -5., -10., -9., 10., 6. };
 float  sx2[8] = { 1.,12., 7., -8., -5., -10., -9., 19.};
 pthread_t first_th;
 pthread_t second_th;
 int rc;
 arg_list a_l,b_l;


 /* Creating argument list for the first thread  */
 a_l.n = 9;
 a_l.incx = 1;
 a_l.x = sx1;

 /* Creating argument list for the second thread  */
 b_l.n = 8;
 b_l.incx = 1;
 b_l.x = sx2;


 /*  Creating first thread which calls the ESSL subroutine ISAMAX  */
 rc = pthread_create(&first_th, NULL, Thread, (void *) &a_l);
 if (rc) exit(-1);

 /*  Creating second thread which calls the ESSL subroutine ISAMAX */
 rc = pthread_create(&second_th, NULL, Thread, (void *) &b_l);
 if (rc) exit(-1);

 sleep(1);
 exit(0);
}


/* Thread routine which call ESSL routine ISAMAX */
void *Thread(void *v)
{
 arg_list *al;
 float *x;
 int n,incx;
 int i;

 al = (arg_list *)(v);
 x = al->x;
 n = al->n;
 incx = al->incx;
 /* Calling the ESSL subroutine ISAMAX  */

 i = isamax(n,x,incx);
 if ( i == 8)
   printf("max for sx2 should be 8 = %d\n",i);
 else
   printf("max for sx1 should be 6 = %d\n",i);
}
```

## Handling Errors in Your C Program

ESSL provides you with flexibilities in handling both input-argument errors and computational errors:

- For input-argument errors 2015 and 2030, which are optionally-recoverable errors, ESSL allows you to obtain corrected input-argument values and react at run time.

  **Note:** In the case where error 2015 is unrecoverable, you have the option of dynamic allocation for most of the *aux* arguments. For details see the subroutine descriptions in Part 2 of this book.

- For computational errors, ESSL provides a return code and additional information to help you analyze the problem in your program and react at run time.

"Input-Argument Errors in C" and "Computational Errors in C" on page 140 explain how to use these facilities by describing the additional statements you must code in your program.

For multithreaded application programs, if you want to initialize the error option table and change the default settings for input-argument and computational errors, you need to implement the steps shown in "Input-Argument Errors in C" and "Computational Errors in C" on page 140 on each thread that calls ESSL.

### Input-Argument Errors in C

To obtain corrected input-argument values in a C program and to avert program termination for the optionally-recoverable input-argument errors 2015 and 2030, add the statements in the following steps to your program. Steps 4 and 8 for ERRSAV and ERRSTR, respectively, are optional. Adding these steps makes the effect of the call to ERRSET temporary.

***Step 1.  Code the Global Statements for ESSL Error Handling***

```
/* Code two underscores */
/* before the letters ESVERR */
#define __ESVERR
#include <essl.h>
      extern int enotrm();
```

These statements are coded with your global declares in the front of your program. The `#define` must be coded before the `#include` statement for the ESSL header file. The extern statement declares the ESSL error exit routine ENOTRM as an external reference in your program. **After the point where you code these statements in your program, you must pass by reference all ESSL calling sequence arguments that can potentially be altered by ESSL error handling.** This applies to all your ESSL call statements. The two types of arguments are:

- *naux* arguments for auxiliary storage
- *n* arguments for transform lengths

***Step 2. Declare the Variables***

```
  int (*iusadr) ();
  int ierno,inoal,inomes,itrace,irange,irc,dummy;
  char storarea[8];
```

This declares a pointer, *iusadr*, to be used for the ESSL error exit routine ENOTRM. Also included are declares for the variables used by the ESSL and Fortran error-handling subroutines. Note that *storarea* must be 8 characters long. These should be coded in the beginning of your program before any of the following statements.

***Step 3. Do Initialization for ESSL***

```
iusadr = enotrm;
einfo (0,&dummy,&dummy);
```

The first statement sets the function pointer, *iusadr*, to ENOTRM, the ESSL error exit routine. The last statement calls the EINFO subroutine to initialize the ESSL error option table, where *dummy* is a declared integer and is a placeholder. For a description of EINFO, see "EINFO—ESSL Error Information-Handler Subroutine" on page 962. These statements should be coded only once in the beginning of your program before calls to ERRSET.

***Step 4. Call ERRSAV***

```
  errsav (&ierno,storarea);
```

(This is an optional step.) This calls the ERRSAV subroutine, which stores the error option table entry for error number *ierno* in an 8-byte storage area, *storarea*, which is accessible to your program. ERRSAV must be called for each entry you want to save. This step is used, along with step 8, for ERRSTR. For information on whether you should use ERRSAV and ERRSTR, see " How Can You Control Error Handling in Large Applications by Saving and Restoring Entries in the Error Option Table?" on page 53. For an example, see "Example 1" on page 143, as the use is the same as for computational errors.

***Step 5. Call ERRSET***

```
  errset (&ierno,&inoal,&inomes,&itrace,&iusadr,&irange);
```

This calls the ERRSET subroutine, which allows you to dynamically modify the action taken when an error occurs. For optionally-recoverable ESSL input-argument errors, you need to call ERRSET only if you want to avoid terminating your program and you want the input arguments associated with this error to be assigned correct values in your program when the error occurs. For one error (*ierno*) or a range of errors (*irange*), you can specify:

- How many times each error can occur before execution terminates (*inoal*)
- How many times each error message can be printed (*inomes*)
- The ESSL exit routine ENOTRM, to be invoked for the error indicated (*iusadr*)

ERRSET must be called for each error code you want to indicate as being recoverable. For ESSL, *ierno* should have a value of 2015 or 2030. If you want to eliminate error messages, you should indicate a negative number for *inomes*; otherwise, you should specify 0 for this argument. All the other ERRSET arguments should be specified as 0.

For a list of the default values set in the ESSL error option table, see Table 26 on page 51. For a description of the input-argument errors, see "Input-Argument Error Messages" on page 179. For a description of ERRSET, see Chapter 17 on page 959.

***Step 6. Call ESSL***

```
irc = name (arg1,...,argn);
if irc == rc1
        {
           .
           .
           .
              }
```

This calls the ESSL subroutine and specifies a branch on one or more return code values, where:

- *name* specifies the ESSL subroutine.
- *arg1*,...,*argn* are the input and output arguments. As explained in step 1, all arguments that can potentially be altered by error handling must be coded by reference.
- *irc* is the integer variable containing the return code resulting from the computation performed by the ESSL subroutine.
- *rc1*, *rc2*, and so forth are the possible return code values that can be passed back from the ESSL subroutine to C. The values can be 0, 1, 2, and so forth. Return code values are described under "Error Conditions" in each ESSL subroutine description in Part 2 of this book.

***Step 7. Perform the Desired Action:*** These are the statements following the test for each value of the return code, returned in *irc* in step 6. These statements perform whatever action is desired when the recoverable error occurs. These statements may check the new values set in the input arguments to determine whether adequate program storage is available, and then decide whether to continue or terminate the program. Otherwise, these statements may check that the size of the working storage arrays or the length of the transform agrees with other data in the program. The program may also store this corrected input argument value for future reference.

***Step 8. Call ERRSTR***

```
errstr (&ierno,storarea);
```

(This is an optional step.) This calls the ERRSTR subroutine, which stores an entry in the error option table for error number *ierno* from an 8-byte storage area, *storarea*, which is accessible to your program. ERRSTR must be called for each entry you want to store. This step is used, along with step 4, for ERRSAV. For information on whether you should use ERRSAV and ERRSTR, see " How Can You Control Error Handling in Large Applications by Saving and Restoring Entries in the Error Option Table?" on page 53. For an example, see "Example 1" on page 143, as the use is the same as for computational errors.

## Example 1

This example shows an error code 2015, which resets the size of the work area *aux*, specified in *naux*, if the value specified is too small. It also indicates that no error messages should be issued.

```
.
.
.
    /*GLOBAL STATEMENTS FOR ESSL ERROR HANDLING*/
#define __ESVERR
#include <essl.h>
extern int enotrm();

.
.
.
    /*DECLARE THE VARIABLES*/
main ()
{
int (*iusadr) ();
int ierno,inoal,inomes,itrace,irc,dummy;
int naux;

.
.
.
    /*INITIALIZE THE POINTER TO THE ENOTRM ROUTINE*/
iusadr = enotrm;
.
.
.
    /*INITIALIZE THE ESSL ERROR OPTION TABLE*/
einfo (0,&dummy,&dummy);
.
.
.
    /*MAKE ERROR CODE 2015 A RECOVERABLE ERROR AND
      SUPPRESS PRINTING ALL ERROR MESSAGES FOR IT*/
ierno = 2015;
inoal = 0;
inomes = –1;
itrace = 0;
irange = 2015;
errset (&ierno,&inoal,&inomes,&itrace,&iusadr,&irange);
.
.
.
    /*CALL ESSL SUBROUTINE SWLEV.  NAUX IS PASSED BY
      REFERENCE.  IF THE NAUX INPUT IS TOO SMALL,
      ERROR 2015 OCCURS.  THE MINIMUM VALUE REQUIRED
      IS STORED IN THE NAUX INPUT ARGUMENT, AND THE
      RETURN CODE OF 1 IS SET IN IRC.*/
irc = swlev (x,incx,u,incu,y,incy,n,aux,&naux);
if irc == 1
{
.    /*CHECK THE RESULTING INPUT ARGUMENT VALUE
.      IN NAUX AND TAKE THE DESIRED ACTION*/
.
}
.
.
.
}
```

## Computational Errors in C

To obtain information about an ESSL computational error in a C program, add the statements in the following steps to your program. Steps 4 and 9 for ERRSAV and ERRSTR, respectively, are optional. Adding these steps makes the effect of the call to ERRSET temporary. For a list of those computational errors that return information and to which these steps apply, see “EINFO—ESSL Error Information-Handler Subroutine” on page 962.

***Step 1.  Code the Global Statements for ESSL Error Handling***

```
/* Code two underscores */
/* before the letters ESVERR */
#define __ESVERR
#include <essl.h>
```

These statements are coded with your global declares in the front of your program. The `#define` must be coded before the `#include` statement for the ESSL header file. **After the point where you code these statements in your program, you must pass by reference all ESSL calling sequence arguments that can potentially be altered by ESSL error handling.** This applies to all your ESSL call statements. The two types of arguments are:

- *naux* arguments for auxiliary storage
- *n* arguments for transform lengths

### *Step 2. Declare the Variables*

```
  int ierno,inoal,inomes,itrace,iusadr,irange,irc;
  int inf1,inf2,dummy;
  char storarea[8];
```

These statements include declares for the variables used by the ESSL and Fortran error-handling subroutines. Note that *storarea* must be 8 characters long. These should be coded in the beginning of your program before any of the following statements.

### *Step 3. Do Initialization for ESSL*

```
einfo (0,&dummy,&dummy);
```

This statement calls the EINFO subroutine to initialize the ESSL error option table, where *dummy* is a declared integer and is a placeholder. For a description of EINFO, see “EINFO—ESSL Error Information-Handler Subroutine” on page 962. These statements should be coded only once in the beginning of your program before calls to ERRSET.

### *Step 4. Call ERRSAV*

```
  errsav (&ierno,storarea);
```

(This is an optional step.) This calls the ERRSAV subroutine, which stores the error option table entry for error number *ierno* in an 8-byte storage area, *storarea*, which is accessible to your program. ERRSAV must be called for each entry you want to save. This step is used, along with step 8, for ERRSTR. For information on whether you should use ERRSAV and ERRSTR, see “ How Can You Control Error Handling in Large Applications by Saving and Restoring Entries in the Error Option Table?” on page 53. For an example, see “Example 1” on page 143.

### *Step 5. Call ERRSET*

```
   errset (&ierno,&inoal,&inomes,&itrace,&iusadr,&irange);
```

This calls the ERRSET subroutine, which allows you to dynamically modify the action taken when an error occurs. For ESSL computational errors, you need to call ERRSET only if you want to change the default values in the ESSL error option table. For one error (*ierno*) or a range of errors (*irange*), you can specify:

- How many times each error can occur before execution terminates (*inoal*)
- How many times each error message can be printed (*inomes*)

ERRSET must be called for each error code for which you want to change the default values. For ESSL, *ierno* should be set to one of the eligible values listed in Table 167 on page 962. To allow your program to continue after an error in the specified range occurs, *inoal* must be set to a value greater than 1. For ESSL, *iusadr* should be specified as either 0 or 1 in a 32-bit environment (0l or 1l in a 64-bit environment), so a user exit is not taken.

For a list of the default values set in the ESSL error option table, see Table 26 on page 51. For a description of the computational errors, see “Computational Error Messages” on page 187. For a description of ERRSET, see Chapter 17 on page 959.

***Step 6. Call ESSL***

```
irc = name (arg1,...,argn);
if irc == rc1
        {
          .
          .
          .
             }
if irc == rc2
        {
          .
          .
          .
            }
```

This calls the ESSL subroutine and specifies a branch on one or more return code values, where:

- *name* specifies the ESSL subroutine.
- *arg1*,...,*argn* are the input and output arguments. As explained in step 1, all arguments that can potentially be altered by error handling must be coded by reference.
- *irc* is the integer variable containing the return code resulting from the computation performed by the ESSL subroutine.
- *rc1*, *rc2*, and so forth are the possible return code values that can be passed back from the ESSL subroutine to C. The values can be 0, 1, 2, and so forth. Return code values are described under “Error Conditions” in each ESSL subroutine description in Part 2 of this book.

The statements following each test of the return code can perform any desired action. This includes calling EINFO for more information about the error, as described in step 7.

***Step 7. Call EINFO for Information***

```
einfo (ierno,&inf1,&inf2);
```

This calls the EINFO subroutine, which returns information about certain computational errors, where:

- *ierno* is the error code of interest.
- *inf1* and *inf2* are the integer variables used to receive the information, where *inf1* is assigned a value for all errors, and *inf2* is assigned a value for some errors. You must specify both arguments, as there are no optional arguments for C. Both arguments must be passed by reference, because they are output scalar arguments. For a description of EINFO, see “EINFO—ESSL Error Information-Handler Subroutine” on page 962.

***Step 8. Check the Values in the Information Receivers:*** These statements check the values returned in the output argument information receivers, *inf1* and *inf2*, which contain the information about the computational error.

***Step 9. Call ERRSTR***

```
errstr (&ierno,storarea);
```

(This is an optional step.) This calls the ERRSTR subroutine, which stores an entry in the error option table for error number *ierno* from an 8-byte storage area, *storarea*, which is accessible to your program. ERRSTR must be called for each entry you want to store. This step is used, along with step 4, for ERRSAV. For information on whether you should use ERRSAV and ERRSTR, see “ How Can You Control Error Handling in Large Applications by Saving and Restoring Entries in the Error Option Table?” on page 53. For an example, see “Example 1.”

## Example 1

This 32-bit environment example shows an error code 2105, which returns one piece of information: the index of the pivot element (*i*) near zero, causing factorization to fail. It uses ERRSAV and ERRSTR to insulate the effects of the error handling for error 2105 by this program.

```
.
.
            /*GLOBAL STATEMENTS FOR ESSL ERROR HANDLING*/
#define __ESVERR
#include <essl.h>
.
.
            /*DECLARE THE VARIABLES*/
main ()
{
int ierno,inoal,inomes,itrace,iusadr,irange,irc;
int inf1,inf2,dummy;
char sav2105[8];
.
.
            /*INITIALIZE THE ESSL ERROR OPTION TABLE*/
einfo (0,&dummy,&dummy);
            /*SAVE THE EXISTING ERROR OPTION TABLE ENTRY
              FOR ERROR CODE 2105*/
ierno = 2105;
errsav (&ierno,sav2105);
.
.
            /*MAKE ERROR CODES 2101 THROUGH 2105 RECOVERABLE
              ERRORS AND SUPPRESS PRINTING ALL ERROR MESSAGES
              FOR THEM.  THIS SHOWS HOW YOU CODE THE
              ERRSET ARGUMENTS FOR A RANGE OF ERRORS. */
ierno = 2101;
inoal = 0;
inomes = 0; /*A DUMMY ARGUMENT*/
itrace = 0; /*A DUMMY ARGUMENT*/
iusadr = 0; /*A DUMMY ARGUMENT*/
irange = 2105
errset (&ierno,&inoal,&inomes,&itrace, &iusadr,&irange);

.
.
            /*CALL ESSL SUBROUTINE DGEICD. IF THE INPUT MATRIX
              IS SINGULAR OR NEARLY SINGULAR, ERROR 2105
              OCCURS.  A RETURN CODE OF 2 IS SET IN IRC.*/
irc = dgeicd (a,lda,n,iopt,&rcond,det,aux,&naux);
if irc == 2
```

```
       {
                /*CALL THE INFORMATION-HANDLER ROUTINE FOR ERROR
                  CODE 2105 TO RETURN ONE PIECE OF INFORMATION
                  IN VARIABLE INF1, THE INDEX OF THE PIVOT ELEMENT
                  NEAR ZERO, CAUSING FACTORIZATION TO FAIL.
                  INF2 IS NOT USED, BUT MUST BE SPECIFIED.
                  BOTH INF1 AND INF2 ARE PASSED BY REFERENCE,
                  BECAUSE THEY ARE OUTPUT SCALAR ARGUMENTS.*/
       ierno = 2105;
       einfo (ierno,&inf1,&inf2);
                /*CHECK THE VALUE IN VARIABLE INF1 AND TAKE THE
                  DESIRED ACTION*/
       .
       .
       }

    .
    .
                /*RESTORE THE PREVIOUS ERROR OPTION TABLE ENTRY
                  FOR ERROR CODE 2105.  ERROR PROCESSING
                  RETURNS TO HOW IT WAS BEFORE IT WAS ALTERED BY
                  THE ABOVE ERRSAV STATEMENT*/
    ierno = 2105;
    errstr (&ierno,sav2105);
    .
    .
    }
```

## C++ Programs

This section describes how to code your C++ program.

## Calling ESSL Subroutines and Functions in C++

This section shows how to call ESSL subroutines and functions from your C++ program.

### Before You Call ESSL

Before you can call the ESSL subroutines from your C++ program, you must have the appropriate ESSL header file installed on your system. The ESSL header file allows you to code your function calls as described in this section. It contains entries for all the ESSL subroutines. The ESSL header file is distributed with the ESSL package. The ESSL header file to be used with the C++ compiler is named `essl.h`.

In the beginning of your program, before you call any of the ESSL subroutines, **you must code the following statement for the ESSL header file:**

```
#include <essl.h>
```

If you are creating your own threads for the ESSL Thread-Safe or SMP Library, you must include the `pthread.h` header file in your C++ program. For an example, see “Creating Multiple Threads and Calling ESSL from Your C++ Program” on page 150.

### Coding the Calling Sequences

In C++ programs, the ESSL subroutines, not returning a function value, are invoked with the following type of statement:

```
    subroutine-name (argument-1, . . . , argument-n);
```

An example of a calling sequence for SAXPY might be:

```
saxpy (5,a,x,incx,y,1);
```

The ESSL subroutines returning a function value are invoked with the following type of statement:

```
function-value-name=subroutine-name (argument-1, . . . , argument-n);
```

An example of invoking DASUM might be:

```
sum = dasum (n,x,incx);
```

See the C++ publications for details about how to code the function calls.

## Passing Arguments in C++

This section describes how to pass arguments in your C++ program.

### About the Syntax Shown in This Book

The argument syntax shown in this book assumes that you have installed and are using the ESSL header file. For further details, see “Calling ESSL Subroutines and Functions in C++” on page 145.

### No Optional Arguments

In the ESSL calling sequences for C++, there are no optional arguments, as for some programming languages. You must code all the arguments listed in the syntax.

### Arguments That Must Be Passed by Value

All scalar arguments that are not modified must be passed by value in the ESSL calling sequence. (This refers to input-only scalar arguments, such as *incx*, *m*, and *lda*.)

### Arguments That Must Be Passed by Reference

Following are the instances in which you pass your arguments by reference (as a pointer) in the ESSL calling sequence:

***Arrays:*** Arguments that are arrays are passed by reference, as usual.

***Subroutine Names:*** Some ESSL subroutines call a user-supplied subroutine. The name is part of the ESSL calling sequence. It must be passed by reference.

***Character Arguments:*** Character arguments must be passed as strings, by reference. You specify the character, in upper- or lowercase, in the ESSL calling sequence with double quotation marks around it, as in "t". Following is an example of how you can call SGEADD, specifying the *transa* and *transb* arguments as strings *n* and *t*, respectively:

```
sgeadd (a,5,"n",b,3,"t",c,4,4,3);
```

## Setting Up a User-Supplied Subroutine for ESSL in C++

Some ESSL numerical quadrature subroutines call a user-supplied subroutine, *subf*, identified in the ESSL calling sequence. If your program that calls the numerical quadrature subroutines is coded in C++, there are some coding rules you must follow for the *subf* subroutine:

- You can code the *subf* subroutine using only C, C++, or Fortran.
- You must declare *subf* as an external subroutine in your application program.
- You should code the *subf* subroutine to the specifications given in “ Programming Considerations for the SUBF Subroutine” on page 922. For an example of coding a *subf* subroutine in C++, see “Example 1” on page 933.

## Setting Up Scalar Data in C++

Table 29 lists the scalar data types in C++ that are used for ESSL. Only those types and lengths used by ESSL are listed.

*Table 29 (Page 1 of 2). Scalar Data Types in C++ Programs*

| **Terminology Used by ESSL** | **C++ Equivalent** |
|---|---|
| Character item[1]<br>'N', 'T', 'C' or 'n', 't', 'c' | char *<br>“n”, “t”, “c” |
| Logical item<br>.TRUE., .FALSE. | Specify it as described in “Setting Up Short-Precision Complex Data Types and Logical Data Types in C++” on page 148.[2] |
| 32-bit environment integer<br>12345, −12345 | signed int |
| 64-bit environment integer[3]<br>12345l, −12345l | long |
| Short-precision real number[4]<br>12.345 | float |
| Long-precision real number[4]<br>12.345 | double |
| Short-precision complex number[4]<br>(123.45, −54321.0) | Specify it as described in “Setting Up Short-Precision Complex Data Types and Logical Data Types in C++” on page 148.[2] |
| Long-precision complex number[4]<br>(123.45, −54321.0) | complex[5] |

| Table 29 (Page 2 of 2). Scalar Data Types in C++ Programs | |
|---|---|
| **Terminology Used by ESSL** | **C++ Equivalent** |
| [1] ESSL accepts character data in either upper- or lowercase in its calling sequences.<br>[2] There are no equivalent data types for logical and short-precision complex data in C++. These require special procedures. For details, see the referenced section.<br>[3] In accordance with the LP64 data model, all ESSL integer arguments remain 32-bits except for the **iusadr** argument for ERRSET.<br>[4] Short- and long-precision numbers look the same in this book.<br>[5] This data type is defined in file <complex.h> for C++. | |

## Setting Up Short-Precision Complex Data Types and Logical Data Types in C++

Short-precision complex data types and logical data types are not part of the C++ language; however, some ESSL subroutines require arguments of these data types.

### Short-Precision Complex Data

ESSL provides an identifier, `cmplx`, for the short-precision complex data type, defined in the ESSL header file, as well as two member functions, `sreal` and `simag`, for handling the real and imaginary parts of short-precision complex numbers:

```
#ifndef _CMPLX
class cmplx
  {
  private:
    union { struct { float _re,_im; } _data; double _esvalign;};
  public:
    cmplx() { _data._re = 0.0; _data._im = 0.0; }
    cmplx(float r, float i = 0.0) { _data._re = r; _data._im = i; }
    cmplx(cmplx &c) { _data._re = c._data._re;
                       _data._im = c._data._im; }
    friend inline float sreal(const cmplx& a) { return a._data._re; }
    friend inline float simag(const cmplx& a) { return a._data._im; }
  };
#endif
```

You must, therefore, code an include statement for the ESSL header file in the beginning of your program to use these definitions. For details, see “Calling ESSL Subroutines and Functions in C++” on page 145.

Assuming you are using the ESSL header file, if you declare data items to be of type `cmplx` or `complex`, you can pass them as short- or long-precision complex data to ESSL, respectively. Following is an example of how you might code your program:

```
#include <complex.h>
#include <essl.h>
main()
{
cmplx alpha,t[3],s[5];
complex beta,td[3],sd[5];
.
.
.
alpha = cmplx(2.0,3.0);
caxpy (3,alpha,s,1,t,2);
.
.
.
beta = complex(2.0,3.0);
zaxpy (3,beta,sd,1,td,2);
.
.
.
}
```

If you choose to use your own definition for short-precision complex data, instead of that provided in the ESSL header file, you can define _CMPLX in your program, using the following `#define` statement. This statement is coded with your global declares in the front of your program and must be coded before the `#include` statement for the ESSL header file.

```
#define _CMPLX
```

If you prefer to define your short-precision complex data at compile time, you can use the job processing procedures described in “Compiling” on page 163.

### Logical Data

By coding the following simple macro definitions in your program, you can then use `TRUE` or `FALSE` in assigning values to or specifying any logical arguments passed to ESSL:

```
#define FALSE 0
#define TRUE 1
```

## Setting Up Arrays in C++

C++ arrays are arranged in storage in row-major order. This means that the last subscript expression increases most rapidly, the next-to-the-last subscript expression increases less rapidly, and so forth, with the first subscript expression increasing least rapidly. ESSL subroutines require that arrays passed as arguments be in column-major order. This is the array storage convention used by Fortran, described in “Setting Up Arrays in Fortran” on page 112. To pass an array from your C++ program to ESSL, to have ESSL process the data correctly, and to get a result that is in the proper form for your C++ program, you can do any of the following:

- Build and process the matrix, logically transposed from the outset, and transpose the results as necessary.
- Before the ESSL call, transpose the input arrays. Then, following the ESSL call, transpose any arrays updated as output.

- If there are arguments in the ESSL calling sequence indicating whether the arrays are to be processed in normal or transposed form, such as the *transa* and *transb* arguments in the _GEMM subroutines, use these arguments in combination with the matrix equivalence rules to avoid having to transpose your data in separate operations. For further detail, see “SGEMMS, DGEMMS, CGEMMS, and ZGEMMS—Matrix Multiplication for General Matrices, Their Transposes, or Conjugate Transposes Using Winograd's Variation of Strassen's Algorithm” on page 405.

## Creating Multiple Threads and Calling ESSL from Your C++ Program

The example shown below shows how to create two threads, where each thread calls the ISAMAX subroutine. To use the AIX pthreads library, you must remember to code the `pthread.h` header file in your C++ program.

Be sure to compile this program with the `xlC_r` command.

```
#include "essl.h"
#include <iostream.h>

/* Define prototype for thread routine */
void *Thread(void *v);


/* Define prototype for thread library routine, which is in C */
extern "C" {
#include <pthread.h>
#include <stdlib.h>
int pthread_create(pthread_t *tid, const pthread_attr_t *attr,
                   void *(*start_routine)(void *), void *arg);
}


extern "Fortran" int isamax(const int &,  float *, const int &);

/* Create structure for argument list  */
struct arg_list {
  int    n;
  float   *x;
  int   incx;
};
```

```
void main()
{
 float  sx1[9] = { 1., 2., 7., -8., -5., -10., -9., 10., 6. };
 float  sx2[8] = { 1.,12., 7., -8., -5., -10., -9., 19.};
 pthread_t first_th;
 pthread_t second_th;
 int rc;
 struct arg_list a_l,b_l;


 a_l.n = 9;
 a_l.incx = 1;
 a_l.x = sx1;

 b_l.n = 8;
 b_l.incx = 1;
 b_l.x = sx2;


 /* Creating argument list for first thread  */
 rc = pthread_create(&first_th, NULL, Thread, (void *) &a_l);
 if (rc) exit(-1);

 /* Creating argument list for second thread  */
 rc = pthread_create(&second_th, NULL, Thread, (void *) &b_l);
 if (rc) exit(-1);


 sleep(20);
 exit(0);
}


/* Thread routine which calls the ESSL subroutine ISAMAX */
void* Thread(void *v)
{
 struct arg_list *al;
 float *t;
 int n,incx;
 int i;


 al = (struct arg_list *)(v);
 t = al->x;
 n = al->n;
 incx = al->incx;
 /* Calling the ESSL subroutine ISAMAX  */

 i = isamax(n,t,incx);
 if ( i == 8)
   cout << "max for sx2 should be 8 = " << i << "\n";
 else
   cout << "max for sx1 should be 6 = " << i << "\n";
 return NULL;
}
```

## Handling Errors in Your C++ Program

ESSL provides you with flexibilities in handling both input-argument errors and computational errors:

- For input-argument errors 2015 and 2030, which are optionally-recoverable errors, ESSL allows you to obtain corrected input-argument values and react at run time.

  **Note:** In the case where error 2015 is unrecoverable, you have the option of dynamic allocation for most of the *aux* arguments. For details see the subroutine descriptions in Part 2 of this book.

- For computational errors, ESSL provides a return code and additional information to help you analyze the problem in your program and react at run time.

“Input-Argument Errors in C++” and “Computational Errors in C++” on page 157 explain how to use these facilities by describing the additional statements you must code in your program.

For multithreaded application programs, if you want to initialize the error option table and change the default settings for input-argument and computational errors, you need to implement the steps shown in “Input-Argument Errors in C++” and “Computational Errors in C++” on page 157 on each thread that calls ESSL.

### Input-Argument Errors in C++

To obtain corrected input-argument values in a C++ program and to avert program termination for the optionally-recoverable input-argument errors 2015 and 2030, add the statements in the following steps to your program. Steps 4 and 8 for ERRSAV and ERRSTR, respectively, are optional. Adding these steps makes the effect of the call to ERRSET temporary.

***Step 1.  Code the Global Statements for ESSL Error Handling***

```
/* Code one underscore */
/* before the letters ESVERR */
#define _ESVERR
#include <iostream.h>
#include <stdio.h>
#include <essl.h>
extern “Fortran” int enotrm(int &,int &);
```

These statements are coded with your global declares in the front of your program. The #define must be coded before the #include statements for the ESSL header file. The extern statement declares the ESSL error exit routine ENOTRM as an external reference in your program.

***Step 2. Declare the Variables***

```
int (*iusadr) (int &,int &);
int ierno,inoal,inomes,itrace,irange,irc,dummy;
char storarea[8];
```

This declares a pointer, *iusadr*, to be used for the ESSL error exit routine ENOTRM. Also included are declares for the variables used by the ESSL and Fortran error-handling subroutines. Note that *storarea* must be 8 characters long. These should be coded in the beginning of your program before any of the following statements.

***Step 3. Do Initialization for ESSL***

```
iusadr = enotrm;
dummy = 0;
einfo (0,dummy,dummy);
```

The first statement sets the function pointer, *iusadr*, to ENOTRM, the ESSL error exit routine. The last statement calls the EINFO subroutine to initialize the ESSL error option table, where *dummy* is a declared integer and is a placeholder. For a description of EINFO, see “EINFO—ESSL Error Information-Handler Subroutine” on page 962. These statements should be coded only once in the beginning of your program before calls to ERRSET.

***Step 4. Call ERRSAV***

```
errsav (ierno,storarea);
```

(This is an optional step.) This calls the ERRSAV subroutine, which stores the error option table entry for error number *ierno* in an 8-byte storage area, *storarea*, which is accessible to your program. ERRSAV must be called for each entry you want to save. This step is used, along with step 8, for ERRSTR. For information on whether you should use ERRSAV and ERRSTR, see “ How Can You Control Error Handling in Large Applications by Saving and Restoring Entries in the Error Option Table?” on page 53. For an example, see “Example” on page 159, as the use is the same as for computational errors.

***Step 5. Call ERRSET***

```
errset (ierno,inoal,inomes,itrace,iusadr,irange);
```

This calls the ERRSET subroutine, which allows you to dynamically modify the action taken when an error occurs. For optionally-recoverable ESSL input-argument errors, you need to call ERRSET only if you want to avoid terminating your program and you want the input arguments associated with this error to be assigned correct values in your program when the error occurs. For one error (*ierno*) or a range of errors (*irange*), you can specify:

- How many times each error can occur before execution terminates (*inoal*)
- How many times each error message can be printed (*inomes*)
- The ESSL exit routine ENOTRM, to be invoked for the error indicated (*iusadr*)

ERRSET must be called for each error code you want to indicate as being recoverable. For ESSL, *ierno* should have a value of 2015 or 2030. If you want to eliminate error messages, you should indicate a negative number for *inomes*; otherwise, you should specify 0 for this argument. All the other ERRSET arguments should be specified as 0.

For a list of the default values set in the ESSL error option table, see Table 26 on page 51. For a description of the input-argument errors, see “Input-Argument Error Messages” on page 179. For a description of ERRSET, see Chapter 17 on page 959.

***Step 6. Call ESSL***

```
irc = name (arg1,...,argn);
if irc == rc1
    {
      .
      .
      .
        }
if irc == rc2
    {
      .
      .
      .
        }
```

This calls the ESSL subroutine and specifies a branch on one or more return code values, where:

- *name* specifies the ESSL subroutine.
- *arg1*,...,*argn* are the input and output arguments.
- *irc* is the integer variable containing the return code resulting from the computation performed by the ESSL subroutine.
- *rc1*, *rc2*, and so forth are the possible return code values that can be passed back from the ESSL subroutine to C++. The values can be 0, 1, 2, and so forth. Return code values are described under “Error Conditions” in each ESSL subroutine description in Part 2 of this book.

***Step 7. Perform the Desired Action:*** These are the statements following the test for each value of the return code, returned in *irc* in step 6. These statements perform whatever action is desired when the recoverable error occurs. These statements may check the new values set in the input arguments to determine whether adequate program storage is available, and then decide whether to continue or terminate the program. Otherwise, these statements may check that the size of the working storage arrays or the length of the transform agrees with other data in the program. The program may also store this corrected input argument value for future reference.

***Step 8. Call ERRSTR***

```
errstr (ierno,storarea);
```

(This is an optional step.) This calls the ERRSTR subroutine, which stores an entry in the error option table for error number *ierno* from an 8-byte storage area, *storarea*, which is accessible to your program. ERRSTR must be called for each entry you want to store. This step is used, along with step 4, for ERRSAV. For information on whether you should use ERRSAV and ERRSTR, see " How Can You Control Error Handling in Large Applications by Saving and Restoring Entries in the Error Option Table?" on page 53. For an example, see "Example" on page 159, as the use is the same as for computational errors.

## Example

This example shows an error code 2015, which resets the size of the work area *aux*, specified in *naux*, if the value specified is too small. It also indicates that no error messages should be issued.

```
.
.
.
    /*GLOBAL STATEMENTS FOR ESSL ERROR HANDLING*/
#define _ESVERR
#include <essl.h>
#include <iostream.h>
#include <stdio.h>
extern "Fortran" int enotrm(int &,int &);

.
.
.
```

```
    /*DECLARE THE VARIABLES*/
main ()
{
int (*iusadr) (int &,int &);
int ierno,inoal,inomes,itrace,irc,dummy;
int naux;

.
.
.
    /*INITIALIZE THE POINTER TO THE ENOTRM ROUTINE*/
iusadr = enotrm;

.
.
.
    /*INITIALIZE THE ESSL ERROR OPTION TABLE*/
dummy = 0;
einfo (0,dummy,dummy);

.
.
.
    /*MAKE ERROR CODE 2015 A RECOVERABLE ERROR AND
      SUPPRESS PRINTING ALL ERROR MESSAGES FOR IT*/
ierno = 2015;
inoal = 0;
inomes = –1;
itrace = 0;
irange = 2015;
errset (ierno,inoal,inomes,itrace,iusadr,irange);

.
.
.
    /*CALL ESSL SUBROUTINE SWLEV.  NAUX IS PASSED BY
      REFERENCE.  IF THE NAUX INPUT IS TOO SMALL,
      ERROR 2015 OCCURS.  THE MINIMUM VALUE REQUIRED
      IS STORED IN THE NAUX INPUT ARGUMENT, AND THE
      RETURN CODE OF 1 IS SET IN IRC.*/
irc = swlev (x,incx,u,incu,y,incy,n,aux,naux);
if irc == 1

{
.    /*CHECK THE RESULTING INPUT ARGUMENT VALUE
.      IN NAUX AND TAKE THE DESIRED ACTION*/
.
}

.
.
.
}
```

## Computational Errors in C++

To obtain information about an ESSL computational error in a C++ program, add the statements in the following steps to your program. Steps 4 and 9 for ERRSAV and ERRSTR, respectively, are optional. Adding these steps makes the effect of the call to ERRSET temporary. For a list of those computational errors that return information and to which these steps apply, see “EINFO—ESSL Error Information-Handler Subroutine” on page 962.

### *Step 1. Code the Global Statements for ESSL Error Handling*

```
/* Code one underscore */
/* before the letters ESVERR */
#define _ESVERR
#include <iostream.h>
#include <stdio.h>
#include <essl.h>
```

These statements are coded with your global declares in the front of your program. The `#define` must be coded before the `#include` statement for the ESSL header file.

### *Step 2. Declare the Variables*

```
int ierno,inoal,inomes,itrace,iusadr,irange,irc;
int inf1,inf2,dummy;
char storarea[8];
```

These statements include declares for the variables used by the ESSL and Fortran error-handling subroutines. Note that *storarea* must be 8 characters long. These should be coded in the beginning of your program before any of the following statements.

### *Step 3. Do Initialization for ESSL*

```
dummy = 0;
einfo (0,dummy,dummy);
```

The last statement calls the EINFO subroutine to initialize the ESSL error option table, where *dummy* is a declared integer and is a placeholder. For a description of EINFO, see “EINFO—ESSL Error Information-Handler Subroutine” on page 962. These statements should be coded only once in the beginning of your program before calls to ERRSET.

### *Step 4. Call ERRSAV*

```
errsav (ierno,storarea);
```

(This is an optional step.) This calls the ERRSAV subroutine, which stores the error option table entry for error number *ierno* in an 8-byte storage area, *storarea*, which is accessible to your program. ERRSAV must be called for each entry you want to save. This step is used, along with step 8, for ERRSTR. For information on whether you should use ERRSAV and ERRSTR, see " How Can You Control Error Handling in Large Applications by Saving and Restoring Entries in the Error Option Table?" on page 53. For an example, see "Example" on page 159.

***Step 5. Call ERRSET***

```
errset (ierno,inoal,inomes,itrace,iusadr,irange);
```

This calls the ERRSET subroutine, which allows you to dynamically modify the action taken when an error occurs. For ESSL computational errors, you need to call ERRSET only if you want to change the default values in the ESSL error option table. For one error (*ierno*) or a range of errors (*irange*), you can specify:

- How many times each error can occur before execution terminates (*inoal*)
- How many times each error message can be printed (*inomes*)

ERRSET must be called for each error code for which you want to change the default values. For ESSL, *ierno* should be set to one of the eligible values listed in Table 167 on page 962. To allow your program to continue after an error in the specified range occurs, *inoal* must be set to a value greater than 1. For ESSL, *iusadr* should be specified as either 0 or 1 in a 32-bit environment (0l or 1l in a 64-bit environment), so a user exit is not taken.

For a list of the default values set in the ESSL error option table, see Table 26 on page 51. For a description of the computational errors, see "Computational Error Messages" on page 187. For a description of ERRSET, see Chapter 17 on page 959.

***Step 6. Call ESSL***

```
irc = name (arg1,...,argn);
if irc == rc1
    {
      .
      .
      .
        }
if irc == rc2
    {
      .
      .
      .
        }
```

This calls the ESSL subroutine and specifies a branch on one or more return code values, where:

- *name* specifies the ESSL subroutine.

- *arg1*,...,*argn* are the input and output arguments.
- *irc* is the integer variable containing the return code resulting from the computation performed by the ESSL subroutine.
- *rc1*, *rc2*, and so forth are the possible return code values that can be passed back from the ESSL subroutine to C++. The values can be 0, 1, 2, and so forth. Return code values are described under “Error Conditions” in each ESSL subroutine description in Part 2 of this book.

The statements following each test of the return code can perform any desired action. This includes calling EINFO for more information about the error, as described in step 7.

***Step 7. Call EINFO for Information***

```
einfo (ierno,inf1,inf2);
```

This calls the EINFO subroutine, which returns information about certain computational errors, where:

- *ierno* is the error code of interest.
- *inf1* and *inf2* are the integer variables used to receive the information, where *inf1* is assigned a value for all errors, and *inf2* is assigned a value for some errors. You must specify both arguments, as there are no optional arguments for C. For a description of EINFO, see “EINFO—ESSL Error Information-Handler Subroutine” on page 962.

***Step 8. Check the Values in the Information Receivers:*** These statements check the values returned in the output argument information receivers, *inf1* and *inf2*, which contain the information about the computational error.

***Step 9. Call ERRSTR***

```
errstr (ierno,storarea);
```

(This is an optional step.) This calls the ERRSTR subroutine, which stores an entry in the error option table for error number *ierno* from an 8-byte storage area, *storarea*, which is accessible to your program. ERRSTR must be called for each entry you want to store. This step is used, along with step 4, for ERRSAV. For information on whether you should use ERRSAV and ERRSTR, see “ How Can You Control Error Handling in Large Applications by Saving and Restoring Entries in the Error Option Table?” on page 53. For an example, see “Example.”

## Example

This 32-bit environment example shows an error code 2105, which returns one piece of information: the index of the pivot element (*i*) near zero, causing factorization to fail. It uses ERRSAV and ERRSTR to insulate the effects of the error handling for error 2105 by this program.

```
.
.
            /*GLOBAL STATEMENTS FOR ESSL ERROR HANDLING*/
#define _ESVERR
#include <essl.h>
#include <iostream.h>
#include <stdio.h>
.
.
            /*DECLARE THE VARIABLES*/
main ()
{
int ierno,inoal,inomes,itrace,iusadr,irange,irc;
int inf1,inf2,dummy;
char sav2105[8];
.
.
            /*INITIALIZE THE ESSL ERROR OPTION TABLE*/
dummy = 0;
einfo (0,dummy,dummy);
            /*SAVE THE EXISTING ERROR OPTION TABLE ENTRY
              FOR ERROR CODE 2105*/
ierno = 2105;
errsav (ierno,sav2105);
.
.
            /*MAKE ERROR CODES 2101 THROUGH 2105 RECOVERABLE
              ERRORS AND SUPPRESS PRINTING ALL ERROR MESSAGES
              FOR THEM.  THIS SHOWS HOW YOU CODE THE
              ERRSET ARGUMENTS FOR A RANGE OF ERRORS. */
ierno = 2101;
inoal = 0;
inomes = 0; /*A DUMMY ARGUMENT*/
itrace = 0; /*A DUMMY ARGUMENT*/
iusadr = 0; /*A DUMMY ARGUMENT*/
irange = 2105
errset (ierno,inoal,inomes,itrace, iusadr,irange);

.
.
            /*CALL ESSL SUBROUTINE DGEICD. IF THE INPUT MATRIX
              IS SINGULAR OR NEARLY SINGULAR, ERROR 2105
              OCCURS.  A RETURN CODE OF 2 IS SET IN IRC.*/
irc = dgeicd (a,lda,n,iopt,rcond,det,aux,naux);
if irc == 2
```

```
   {
            /*CALL THE INFORMATION-HANDLER ROUTINE FOR ERROR
              CODE 2105 TO RETURN ONE PIECE OF INFORMATION
              IN VARIABLE INF1, THE INDEX OF THE PIVOT ELEMENT
              NEAR ZERO, CAUSING FACTORIZATION TO FAIL.
              INF2 IS NOT USED, BUT MUST BE SPECIFIED.
              BOTH INF1 AND INF2 ARE PASSED BY REFERENCE,
              BECAUSE THEY ARE OUTPUT SCALAR ARGUMENTS.*/
   ierno = 2105;
   einfo (ierno,inf1,inf2);
            /*CHECK THE VALUE IN VARIABLE INF1 AND TAKE THE
              DESIRED ACTION*/
   .
   .
   }

.
.
            /*RESTORE THE PREVIOUS ERROR OPTION TABLE ENTRY
              FOR ERROR CODE 2105.  ERROR PROCESSING
              RETURNS TO HOW IT WAS BEFORE IT WAS ALTERED BY
              THE ABOVE ERRSAV STATEMENT*/
ierno = 2105;
errstr (ierno,sav2105);
.
.
}
```

## PL/I Programs

If you are using ESSL with PL/I Set for AIX, Version 1, see the PL/I publications for details on calling subroutines and functions.

# Chapter 5. Processing Your Program

This chapter provides information on how to process your program. It contains processing procedures that apply to Fortran, C, and C++ programs using ESSL. It describes **only the ESSL-specific changes** you need to make to your job setup procedures. For complete examples of job setup procedures, see your programming language's programming guide.

**Notes:**

1. For the ESSL SMP Library, you can use the XL Fortran XLSMPOPTS environment variable to specify options which affect SMP execution. For details, see the Fortran publications.
2. If you are using either the ESSL POWER2 Library or the ESSL Thread-Safe POWER2 Library, you must be running on a RS/6000 POWER2 processor.

## Compiling

This section describes how to compile your program.

## General Procedures

You can use any procedures you are currently using for compiling your program. ESSL requires no changes to the compile-time setup procedures for C or C++ programs, because the ESSL header file, `essl.h`, which is used for C and C++ programs, is installed in the `/usr/include` directory.

ESSL supports the XL Fortran compile-time option **-qextname**. For details, see the Fortran manuals.

You must use only the allowable compilers or assemblers listed in Table 2 on page 8.

## Using Your Own Complex Data Definitions in C Programs

If you want to specify your own definitions for short- and long-precision complex data, add `-D_CMPLX` and `-D_DCMPLX`, respectively, to your compile command, as shown here:

| ESSL Library Name | Command |
|---|---|
| SMP *–or–*<br>Thread-Safe *–or–*<br>Thread-Safe POWER2 | `cc_r -cO -D_CMPLX -D_DCMPLX` *xyz*`.c` |
| POWER2 *–or–*<br>POWER | `cc -cO -D_CMPLX -D_DCMPLX` *xyz*`.c` |

where *xyz*.c is the name of your C program. Otherwise, you automatically use the definitions of short- and long-precision complex data provided in the ESSL header file.

## Using Your Own Short Complex Data Definitions in C++ Programs

If you want to specify your own definition for short-precision complex data, add `-D_CMPLX` to your command, as shown here:

| ESSL Library Name | Command |
|---|---|
| SMP *–or–*<br>Thread-Safe *–or–*<br>Thread-Safe POWER2 | `xlC_r  -cO  -D_CMPLX  xyz.C -qnocinc=/usr/include/essl` |
| POWER2 *–or–*<br>POWER | `xlC  -cO  -D_CMPLX  xyz.C -qnocinc=/usr/include/essl` |

where *xyz*`.C` is the name of your C++ program. Otherwise, you automatically use the definition of short-precision complex data provided in the ESSL header file.

## Compiling and Linking

You can use any procedures you are currently using to link or run your program, as long as you make the necessary modifications for ESSL. This section describes these modifications. For details on the complete procedures, see your operating system and programming language manuals.

### 64-bit environment

The ESSL libraries for AIX 4.3.2 which support a 64-bit environment are:

- The **ESSL POWER Library**
- The **ESSL Thread-Safe Library**
- The **ESSL SMP Library**

64-bit environment applications can be created on any AIX 4.3.2 system, but can run only on 64-bit hardware.

If you are accessing ESSL from a 64-bit environment program, you must add the **-q64** compiler option. This compiler option is language independent.

| ESSL Library Name | Command |
|---|---|
| SMP | `xlf_r  -O  -qnosave -q64 xyz.f  -lesslsmp` |
| Thread-Safe | `cc_r  -O -q64 xyz.c  -lessl_r` |
| POWER | `xlC  -O  -q64 xyz.C  -lessl` |

where *xyz*`.f` is the name of your Fortran program, *xyz*`.c` is the name of your C program, and *xyz*`.C` is the name of your C++ program.

## Fortran Programs

If you are accessing ESSL from a Fortran program and want to compile and link in one step, you can use the following command:

| ESSL Library Name | Command |
|---|---|
| SMP | xlf_r -O -qnosave *xyz*.f -lesslsmp |
| Thread-Safe | xlf_r -O -qnosave *xyz*.f -lessl_r |
| Thread-Safe POWER2 | xlf_r -O -qnosave *xyz*.f -lesslp2_r |
| POWER2 | xlf -O *xyz*.f -lesslp2 |
| POWER | xlf -O *xyz*.f -lessl |

where *xyz*.f is the name of your Fortran program.

If you want to compile and link your Fortran program in separate steps, you can use the following commands:

| ESSL Library Name | Command |
|---|---|
| SMP | xlf_r -O -c -qnosave *xyz*.f<br>xlf_r *xyz*.o -lesslsmp |
| Thread-Safe | xlf_r -O -c -qnosave *xyz*.f<br>xlf_r *xyz*.o -lessl_r |
| Thread-Safe POWER2 | xlf_r -O -c -qnosave *xyz*.f<br>xlf_r *xyz*.o -lesslp2_r |
| POWER2 | xlf -O -c *xyz*.f<br>xlf *xyz*.o -lesslp2 |
| POWER | xlf -O -c *xyz*.f<br>xlf *xyz*.o -lessl |

where *xyz*.f is the name of your Fortran program, and *xyz*.0 is the name of your object file.

## C Programs

If you are accessing ESSL from a C program and want to compile and link in one step, you can use the following command:

| ESSL Library Name | Command |
|---|---|
| SMP | cc_r -O *xyz*.c -lesslsmp |
| Thread-Safe | cc_r -O *xyz*.c -lessl_r |
| Thread-Safe POWER2 | cc_r -O *xyz*.c -lesslp2_r |
| POWER2 | cc -O *xyz*.c -lesslp2 |
| POWER | cc -O *xyz*.c -lessl |

where *xyz*.c is the name of your C program.

If you want to compile and link your C program in separate steps, you can use the following commands:

| ESSL Library Name | Command |
|---|---|
| SMP | cc_r  -cO  *xyz*.c<br>cc_r  *xyz*.o  -lesslsmp |
| Thread-Safe | cc_r  -cO  *xyz*.c<br>cc_r  *xyz*.o  -lessl_r |
| Thread-Safe POWER2 | cc_r  -cO  *xyz*.c<br>cc_r  *xyz*.o  -lesslp2_r |
| POWER2 | cc  -cO  *xyz*.c<br>cc  *xyz*.o  -lesslp2 |
| POWER | cc  -cO  *xyz*.c<br>cc  *xyz*.o  -lessl |

where *xyz*.c is the name of your C program and *xyz*.o is the name of your object file.

In the above cases, you automatically use the definitions of short- and long-precision complex data provided in the ESSL header file. If you prefer to specify your own definitions for short- and long-precision complex data, add -D_CMPLX and -D_DCMPLX, respectively, to your commands, as shown here:

| ESSL Library Name | Command |
|---|---|
| SMP | cc_r  -O  -D_CMPLX  -D_DCMPLX  *xyz*.c  -lesslsmp |
| Thread-Safe | cc_r  -O  -D_CMPLX  -D_DCMPLX  *xyz*.c  -lessl_r |
| Thread-Safe POWER2 | cc_r  -O  -D_CMPLX  -D_DCMPLX  *xyz*.c  -lesslp2_r |
| POWER2 | cc  -O  -D_CMPLX  -D_DCMPLX  *xyz*.c  -lesslp2 |
| POWER | cc  -O  -D_CMPLX  -D_DCMPLX  *xyz*.c  -lessl |

## C++ Programs

If you are accessing ESSL from a C++ program and want to compile and link in one step, you can use the following command:

| ESSL Library Name | Command |
|---|---|
| SMP | xlC_r  -O  *xyz*.C  -lesslsmp -qnocinc=/usr/include/essl |
| Thread-Safe | xlC_r  -O  *xyz*.C  -lessl_r -qnocinc=/usr/include/essl |
| Thread-Safe POWER2 | xlC_r  -O  *xyz*.C  -lesslp2_r -qnocinc=/usr/include/essl |
| POWER2 | xlC  -O  *xyz*.C  -lesslp2 -qnocinc=/usr/include/essl |
| POWER | xlC  -O  *xyz*.C  -lessl -qnocinc=/usr/include/essl |

where *xyz*.C is the name of your C++ program.

If you want to compile and link your C++ program in separate steps, you can use the following commands:

| ESSL Library Name | Command |
|---|---|
| SMP | xlC_r  -cO  *xyz*.C -qnocinc=/usr/include/essl<br>xlC_r  *xyz*.o  -lesslsmp |

| ESSL Library Name | Command |
|---|---|
| Thread-Safe | xlC_r  -cO  *xyz*.C -qnocinc=/usr/include/essl<br>xlC_r  *xyz*.o  -lessl_r |
| Thread-Safe POWER2 | xlC_r  -cO  *xyz*.C -qnocinc=/usr/include/essl<br>xlC_r  *xyz*.o  -lesslp2_r |
| POWER2 | xlC  -cO  *xyz*.C -qnocinc=/usr/include/essl<br>xlC  *xyz*.o  -lesslp2 |
| POWER | xlC  -cO  *xyz*.C -qnocinc=/usr/include/essl<br>xlC  *xyz*.o  -lessl |

where *xyz*.C is the name of your C++ program, and *xyz*.o is the name of your object file.

In the above cases, you automatically use the definition of short-precision complex data provided in the ESSL header file. If you prefer to specify your own definition for short-precision complex data, add -D_CMPLX to your commands, as shown here:

| ESSL Library Name | Command |
|---|---|
| SMP | xlC_r  -O  -D_CMPLX  *xyz*.C  -lesslsmp -qnocinc=/usr/include/essl |
| Thread-Safe | xlC_r  -O  -D_CMPLX  *xyz*.C  -lessl_r -qnocinc=/usr/include/essl |
| Thread-Safe POWER2 | xlC_r  -O  -D_CMPLX  *xyz*.C  -lesslp2_r -qnocinc=/usr/include/essl |
| POWER2 | xlC  -O  -D_CMPLX  *xyz*.C  -lesslp2 -qnocinc=/usr/include/essl |
| POWER | xlC  -O  -D_CMPLX  *xyz*.C  -lessl -qnocinc=/usr/include/essl |

# Chapter 6. Migrating Your Programs

This chapter explains many aspects of migrating your application programs to use the ESSL subroutines. It covers:

- Migrating ESSL Version 3 programs to Version 3 Release 1.2
- Migrating ESSL Version 3 programs to Version 3 Release 1.1
- Migrating ESSL Version 2 programs to Version 3
- Planning for future migration
- Migrating between RS/6000 processors
- Migrating from other libraries to ESSL

## Migrating ESSL Version 3 Programs to Version 3 Release 1.2

This section describes all the aspects of migrating your ESSL Version 3 application programs to Version 3 Release 1.2.

**Note:** For a list of the new features added in ESSL Version 3 Release 1.2, see "What's New for ESSL for AIX" on page xxxiii.

### ESSL Subroutines

The calling sequences for the subroutines in ESSL Version 3 Release 1.1 and ESSL Version 3 Release 1.2 are identical. No changes to your application programs are required if you are migrating from ESSL Version 3 Release 1.1 to ESSL Version 3 Release 1.2.

## Migrating ESSL Version 3 Programs to Version 3 Release 1.1

This section describes all the aspects of migrating your ESSL Version 3 application programs to Version 3 Release 1.1.

**Note:** For a list of the new features added in ESSL Version 3 Release 1.1, see "Changes for ESSL Version 3 Release 1.1" on page xxxiii.

### ESSL Subroutines

The calling sequences for the subroutines in ESSL Version 3 and ESSL Version 3 Release 1.1 are identical.

Distinct libraries are provided for AIX 4.2.1 and AIX 4.3.2

- For AIX 4.2.1, the **ESSL Thread-Safe Library**, the **ESSL Thread-Safe POWER2 Library**, and the **ESSL SMP Library** were built using the pthreads draft 7 supplied on AIX 4.2.1. (This is the same as ESSL 3.1)
- For AIX 4.3.2, the **ESSL Thread-Safe Library**, the **ESSL Thread-Safe POWER2 Library**, and the **ESSL SMP Library** were built using the pthreads library that conforms to the IEEE POSIX 1003.1-1996 specification supplied on AIX 4.3.

Threaded applications built using ESSL 3.1 will continue to run with ESSL 3.1.1.

If you are migrating to a 64-bit environment you may need to make changes to your call to **ERRSET**. See "ERRSET—ESSL ERRSET Subroutine for ESSL" on page 966.

# Migrating ESSL Version 2 Programs to Version 3

This section describes all the aspects of migrating your ESSL Version 2 application programs to Version 3.

**Note:** For a list of the new features and subroutines added in ESSL Version 3, see “Changes for ESSL Version 3” on page xxxiii.

## ESSL Subroutines

The calling sequences for the subroutines in ESSL Version 2 and ESSL Version 3 are identical. This includes the new ESSL SMP and Thread-Safe Libraries that are included in the ESSL Version 3 product. You do not have to change your existing application programs that call ESSL subroutines when migrating to the ESSL Version 3 product. You must, however, re-link your application program. Therefore, you can simply re-link your existing programs to take advantage of the the increased performance of using the ESSL SMP Library on the SMP processors.

For the _GEF and _GEFCD subroutines, the first column of the matrix ***L*** with the corresponding $\boldsymbol{U}_{ii} = 0$ diagonal element is identified in a computational error message. Previously, the last column was identified. You do not have to make any modifications to your existing application programs that call these subroutines.

## ESSL Messages

The text of message format has changed. **ESV** has been removed from the message text. For details on the new format, see “Message Format” on page 178.

Some input-argument and computational error message numbers have been changed. The old message numbers can still be used when calling ERRSET, however, you should migrate to the new message numbers. The following describes which error messages have been modified:

| Old Error Message Number | New Error Message Number |
|---|---|
| 2074 | 2608 |
| 2123 | 2609 |
| 2128 | 2700 |

# Planning for Future Migration

With respect to planning for the future, if working storage does not need to persist after the subroutine call, you should use dynamic allocation. Otherwise, you should use the processor-independent formulas or simple formulas for calculating the values for the *naux* arguments in the ESSL calling sequences. Two things may occur that could cause the minimum values of *naux*, returned by ESSL error handling, to increase in the future:

- If changes are made to the ESSL subroutines to improve performance
- If changes are necessary to support future processors

The formulas allow you to specify your auxiliary storage large enough to accommodate any future improvements to ESSL and any future processors. If you do not provide, at least, these amounts of storage, your program may not run in the future.

You should use the following rule of thumb: To protect your application from having to be recoded in the future because of possible increased requirements for auxiliary storage, use dynamic allocation if possible. If the working storage must persists after the subroutine call, then you should provide as much storage as possible in your current application. In determining the right amount to specify, you should weigh your storage constraints against the inconvenience of making future changes, then specify what you think is best. If possible, you should provide this larger amount of storage to prevent future migration problems.

## Migrating between RS/6000 Processors

This section describes all the aspects of migrating your ESSL application programs (back and forth) between the RS/6000 PowerPC, POWER, POWER2, and POWER3 processors.

### Auxiliary Storage

The minimum amount of auxiliary storage returned by ESSL error handling may vary among the RS/6000 processors for the following subroutines: all the Fourier transform subroutines, SCONF, SCORF, and SACORF. Therefore, to guarantee that your application programs always migrate from any platform to any other platform, you should use the processor independent formulas to determine the amount of auxiliary storage to use.

### Bitwise-Identical Results

Because of hardware and ESSL design differences, the results you obtain when migrating from one ESSL Library to another may not be bitwise identical. The results, however, are mathematically equivalent.

## Migrating from Other Libraries to ESSL

This section describes some general aspects of moving from an IBM or non-IBM engineering and scientific library to ESSL.

### Migrating from ESSL/370

There is a high degree of compatibility between ESSL/370 and ESSL for AIX. However you may need to make some coding changes for certain subroutines. See the *Engineering and Scientific Subroutine Library Version 2.2 Guide and Reference* for details.

### Migrating from Another IBM Subroutine Library

If you are migrating from other IBM library products—such as Subroutine Library—Mathematics (SL MATH) or Scientific Subroutine Package (SSP), which have some functions similar to ESSL—the ESSL calling sequences differ from the calling sequences you are currently using. Your program must be modified to add the ESSL calling sequences and make the other ESSL-related coding changes.

If you are migrating from the Basic Linear Algebra Subroutine Library provided with the RS/6000 basic operating system, your calling sequences do not need to be changed.

## Migrating from LAPACK

ESSL contains a few subroutines that conform to the LAPACK interface (see Appendix B on page APB-1). If you are using these subroutines, no coding changes are needed to migrate to ESSL.

Additionally, you may be interested in using the Call Conversion Interface (CCI) that is available with LAPACK. The CCI substitutes a call to an ESSL subroutine in place of an LAPACK subroutine whenever an ESSL subroutine provides either functional or near-functional equivalence. Using the CCI allows LAPACK users to obtain the optimized performance of ESSL for an additional subset of LAPACK subroutines. For details, see reference [40].

## Migrating from a Non-IBM Subroutine Library

If you are using a non-IBM library, ESSL may provide subroutines corresponding to those you are currently using. You may choose to migrate your program to benefit from the increased performance offered by the ESSL subroutines. In this case, you may have to recode your program to use the ESSL calling sequences, because the names and arguments used by ESSL may be different from those used by the non-IBM library. On the other hand, if you are using any of the standard Level 1, 2, and 3 BLAS or LAPACK routines that correspond to ESSL subroutines, you do not need to recode the calling sequences. The ESSL calling sequences are the same as the public domain code.

# Chapter 7. Handling Problems

This chapter provides the following information for your use when dealing with errors:

- How to obtain IBM support.
- What to do about NLS (National Language Support) problems.
- A description of the different types of errors that can occur in ESSL. It explains what happens when an error occurs and, in some instances, how you can use error handling to obtain further information.
- All of the ESSL error messages are categorized into the different error types. There is also a description of the error message format.

## Where to Find More Information About Errors

Specific errors associated with each ESSL subroutine are listed under "Error Conditions" in each subroutine description in Part 3 of this book.

## Getting Help from IBM Support

Should you require help from IBM in resolving an ESSL problem, report it and provide the following information, if available and appropriate.

1. Your customer number
2. The ESSL program number:

   - 5765-C42

   This is important information that speeds up the correct routing of your call.
3. The version and release of the operating system that you are running on. To get this information on AIX, enter the following command:

   ```
   oslevel
   ```
4. The names and versions of key products being run. To get this information on AIX, enter the following command:

   ```
   lslpp  -h product
   ```

   where the appropriate values of *product* for AIX Version 4 are listed in Table 30.

*Table 30. Product File Set Names when Using AIX Version 4*

| Product File Sets | Descriptive Name |
|---|---|
| essl.* | ESSL |
| xlfrte | XL Fortran Run-Time Environment |
| xlhpf.rte | XL HPF Run-Time Environment |
| xlsmp.rte | SMP Run-Time Environment |
| xlfcmp | XL Fortran Compiler |
| vac.C | C for AIX Compiler |
| ibmcxx.cmp | IBM C, C++ Version 3.6 Compilers |

5. The message that is returned when an error is detected.
6. Any error message relating to core dumps.
7. The compiler listings, including compiler options in effect, and any run-time listings produced
8. Program changes made in comparison with a previous successful run
9. A small test case demonstrating the problem using the minimum number of statements and variables, including input data

Consult your IBM Service representative for more assistance.

## National Language Support

For National Language Support (NLS), all ESSL subroutines display messages located in externalized message catalogs. English versions of the message catalogs are shipped with the IBM Engineering and Scientific Subroutine Library for AIX product, but your site maybe using its own translated message catalogs. The AIX environment variable **NLSPATH** is used by the various ESSL subroutines to find the appropriate message catalog. **NLSPATH** specifies a list of directories to search for message catalogs. The directories are searched, in the order listed, to locate the message catalog. In resolving the path to the message catalog, **NLSPATH** is affected by the value of the environment variables **LC_MESSAGES** and **LANG**. If you get an error saying that a message catalog is not found and want the default message catalog, enter the following:

```
export NLSPATH = /usr/lib/nls/msg/%L/%N

export LANG = C
```

The ESSL message catalogs are in English, and are located in the following directories:

```
/usr/lib/nls/msg/C
/usr/lib/nls/msg/En_US
/usr/lib/nls/msg/en_US
```

If your site is using its own translations of the message catalogs, consult your system administrator for the appropriate value of **NLSPATH** or **LANG**. For additional information on NLS and message catalogs, see *IBM AIX Version 4 for RISC System/6000 General Programming Concepts*.

If ESSL cannot successfully find a message, ESSL returns message 2799, indicating which message could not be located. Message 2799 is described in “Miscellaneous Error Messages” on page 178.

## Dealing with Errors

At run time, you can encounter a number of different types of errors that are specifically related to the use of the ESSL subroutines:

- Program exceptions
- Input-argument errors (2001-2099) and (2801-2899)
- Computational errors (2100-2199)

- Resource errors (2401-2499)
- Informational and Attention messages (2600-2699)
- Miscellaneous errors (2700-2799)

## Program Exceptions

The program exceptions you can encounter in ESSL are described in the RS/6000 architecture manuals. For details, see:

- *ANSI/IEEE Standard for Binary Floating-Point Arithmetic, ANSI/IEEE Standard 754-1985*.
- *RS/6000 POWERstation and POWERserver Hardware Technical Reference Information—General Architectures*.

## ESSL Input-Argument Error Messages

If you receive an error message in the form 2538-20*nn*, you have an input-argument error in the calling sequence for an ESSL subroutine. Your program terminated at this point unless you did one of the following:

- Specified the ESSL user exit routine, ENOTRM, with ERRSET to determine the correct input argument values in your program for the optionally-recoverable ESSL errors 2015 or 2030. For details on how to do this, see Chapter 4 on page 111.
- Reset the number of allowable errors (2099) during ESSL installation or using ERRSET in your program. **This is not recommended for input-argument errors.**

**Note:** For many of the ESSL subroutines requiring auxiliary storage, you can avoid program termination due to error 2015 by allowing ESSL to dynamically allocate auxiliary storage for you. You do this by setting *naux* = 0 and making error 2015 unrecoverable. For details on which *aux* arguments allow dynamic allocation and how to specify them, see the subroutine descriptions in Part 2 of this book.

The name of the ESSL subroutine detecting the error is listed as part of the message. The argument number(s) involved in the error appears in the message text. See “Input-Argument Error Messages” on page 179 for a complete description of the information contained in each message and for an indication of which messages correspond to optionally-recoverable errors. Regardless of whether the name in the message is a user-callable ESSL subroutine or an internal ESSL routine, the message-text and its unique parts apply to the user-callable ESSL subroutine. Return code values are described under “Error Conditions” for each ESSL subroutine in Part 2 of this book.

You may get more than one error message, because most of the arguments are checked by ESSL for possible errors during each call to the subroutine. The ESSL subroutine returns as many messages as there are errors detected. As a result, fewer runs are necessary to diagnose your program.

Fix the error(s), recompile, relink, and rerun your program.

## ESSL Computational Error Messages

If you receive an error message in the form 2538-21*nn*, you have a computational error in the ESSL subroutine. A computational error is any error occurring in the ESSL subroutine while using the computational data (that is, scalar and array data). The name of the ESSL subroutine detecting the error is listed as part of the message. Regardless of whether the name in the message is a user-callable ESSL subroutine or an internal ESSL routine, the message-text and its unique parts apply to the user-callable ESSL subroutine. A nonzero return code is returned when the ESSL subroutine encounters a computational error. See “Computational Error Messages” on page 187 for a complete description of the information in each message. Return code values are described under “Error Conditions” for each ESSL subroutine in Part 2 of this book.

Your program terminates for some computational errors unless you have called ERRSET to reset the number of allowable errors for that particular error, and the number has not been exceeded. A message is issued for each computational error. You should use the message to determine where the error occurred in your program.

If you called ERRSET and you have not reached the limit of errors you had set, you can check the return code. If it is not 0, you should call the EINFO subroutine to obtain information about the data involved in the error. EINFO provides the same information provided in the messages; however, it is provided to your program so your program can check the information during run time. Depending on what you want to do, you may choose to continue processing or terminate your program after the error occurs. For information on how to make these changes in your program to reset the number of allowable errors, how to diagnose the error, and how to decide whether to continue or terminate your program, see Chapter 4 on page 111.

If you are unable to solve the problem, report it and provide the following information, if available and appropriate:

- The message number and the module that detected an error
- The system dump, system error code, and system log of this job
- The compiler listings, including compiler options in effect, and any run-time listings produced
- Program changes made in comparison with a previous successful run
- A small test case demonstrating the problem using the minimum number of statements and variables, including input data
- A brief description of the problem

## ESSL Resource Error Messages

If you receive a message in the form 2538-24*nn*, it means that ESSL issued a resource error message.

A resource error occurs when a buffer storage allocation request fails in a ESSL subroutine. In general, the ESSL subroutines allocate internal auxiliary storage dynamically as needed. Without sufficient storage, the subroutine cannot complete the computation.

When a buffer storage allocation request fails, a resource error message is issued, and the application program is terminated. You need to reduce the memory constraint on the system or increase the amount of memory available before rerunning the application program.

The following ways may reduce memory constraints:

- Investigate the load of your process and run in a more dedicated environment.
- Increase your processor's paging space.
- Select a machine with more memory.
- Consider specifying the `-bmaxdata` binder option when linking your program. For details see the Fortran publications.
- Check the setting of your user ID's user limit (`ulimit`). (see the *IBM AIX Version 4 Commands Reference*).

## ESSL Informational and Attention Messages

If you receive a message in the form 2538-26*nn*, it means that ESSL issued an informational or attention message.

### Informational Messages

When you receive an informational message, check your application to determine why the condition was detected.

### ESSL Attention Messages

An attention message is issued to describe a condition that occurred. ESSL is able to continue processing, but performance may be degraded.

One condition that may produce an attention message is when enough work area was available to continue processing, but was not the amount initially requested. ESSL does not terminate your application program, but performance may be degraded. If you want to reduce the memory constraint on the system or increase the amount of memory available to eliminate the attention message, see the suggestions in "ESSL Resource Error Messages" on page 176. For a list of subroutines that may generate this type of attention message, see Table 31.

*Table 31 (Page 1 of 2). ESSL Subroutines*

| **Subroutine Names** |
|---|
| Matrix-Vector Linear Algebra Subprograms:<br>_GEMV, _GER, _SPMV, _SYMV, _SPR, _SYR, _SPR2, _SYR2<br>_GERC, _GERU, _HPMV, _HEMV, _HPR, _HER, _HPR2, _HER2<br>_SBMV, _TBMV, _GBMV<br>_TPMV<br>_TRMV |
| Matrix Operations:<br>_GEMM, _GEMUL<br>_SYMM, _SYR2K, _TRMM<br>_HEMM, _HER2K |

*Table 31 (Page 2 of 2). ESSL Subroutines*

| **Subroutine Names** |
|---|
| Dense Linear Algebraic Equations:<br>_POF, _POICD, _PPF, _PPFCD, _PPICD<br>_GEICD, _TPI, _TRI<br>_TRSM, _TPSV<br>_TRSV |
| Banded Linear Algebraic Equations:<br>STBSV, DTBSV |
| Linear Least Squares:<br>_GESVS |
| Fourier Transforms:<br>_CFT, _CFT3<br>_RCFT, _RCFT3<br>_CRFT, _CRFT3 |

## Miscellaneous Error Messages

If you receive a message in the form 2538-27*nn*, it means that ESSL issued a miscellaneous error message.

A miscellaneous error is an error that does not fall under any other categories.

When ESSL detects a miscellaneous error, you receive an error message with information on how to proceed and your application program is terminated.

## Messages

This section explains the conventions used for the ESSL messages and lists all the ESSL messages. For a description of each of the four types of ESSL messages, see “Dealing with Errors” on page 174.

## Message Conventions

This section describes the message conventions for the ESSL product.

### About Upper- and Lowercase

The literals, such as, 'N', 'T', 'U', and so forth, appear in the messages in this book in uppercase; however, they may be specified in your ESSL calling sequence in either upper- or lowercase, for example, 'n', 't', and 'u'.

### Message Format

The ESSL messages are issued in your output in the following format:

*rtn-name* : 2538-*mmnn*
*message-text*

*Figure 10. Message Format*

The parts of the ESSL message are as follows:

*rtn-name* gives the name of the ESSL subroutine that encountered the error.

2538 is the ESSL component identification number.

*mm* indicates the type of ESSL error message:

20—Input-argument error message
21—Computational error message
24—Resource error message
26—Information and attention message
27—Miscellaneous error message

*nn* is the message identification number.

*message-text* describes the nature of the error. Where one of several possible message-texts can be issued for a particular ESSL error, they are listed in this book with an “or” between them. The possible unique parts are:

- The argument number of each argument involved in the error is included in the message description as (ARG NO. _ )
- Additional information about the error is included in the message. The placement of this information is shown in the messages as (_)

## Input-Argument Error Messages

```
RTN_NAME : 2538-2001
The number of elements (ARG NO. _) in a vector must be greater than
or equal to zero.

RTN_NAME : 2538-2002
The stride (ARG NO. _) for a vector must be nonzero.

RTN_NAME : 2538-2003
The number of rows (ARG NO. _) in a matrix must be greater than or equal
to zero.

RTN_NAME : 2538-2004
The number of columns (ARG NO. _) in a matrix must be greater than or
equal to zero.

RTN_NAME : 2538-2005
The size of the leading dimension (ARG NO. _) of an array must be greater
than zero.

RTN_NAME : 2538-2006
The number of rows (ARG NO. _) of a matrix must be less than or equal
to the size of the leading dimension (ARG NO. _) of its array.

RTN_NAME : 2538-2007
The degree of a polynomial (ARG NO. _) must be greater than or equal to zero.

RTN_NAME : 2538-2008
The number of elements (ARG NO. _) to be scanned must be greater than or
equal to 2.
```

```
RTN_NAME : 2538-2009
The number of elements (ARG NO. _) in a vector to be processed must be
greater than or equal to 3.

RTN_NAME : 2538-2010
The transform length (ARG NO. _) must be a power of 2.

RTN_NAME : 2538-2011
The number of points used in the interpolation (ARG NO. _) must be
greater than or equal to zero and less than or equal to the number of data
points (ARG NO. _).

RTN_NAME : 2538-2012
The transform length (ARG NO. _) must be less than or equal to (_).

RTN_NAME : 2538-2013
The transform length (ARG NO. _) must be greater than or equal to (_).

RTN_NAME : 2538-2014
The routine must be initialized with the present value of (ARG NO. _).

RTN_NAME : 2538-2015
The number of elements (ARG NO. _) in a work array must be greater than
or equal to (_).

RTN_NAME : 2538-2016
The form (ARG NO. _) of a matrix must be 'N' or 'T'.
   or
The form (ARG NO. _) of a matrix must be 'N', 'T', or 'C'.
   or
The form (ARG NO. _) of a matrix must be 'N' or 'C'.

RTN_NAME : 2538-2017
The dimension (ARG NO. _) of the matrices must be greater than or equal to zero.

RTN_NAME : 2538-2018
The matrix form is specified by (ARG NO. _); therefore, the leading
dimension (ARG NO. _) of its array must be greater than or equal to the
number of its rows (ARG NO. _).

RTN_NAME : 2538-2019
The number of sequences (ARG NO. _) must be greater than zero.

RTN_NAME : 2538-2020
(ARG NO. _) must be nonzero.

RTN_NAME : 2538-2021
The storage control switch (ARG NO. _) must be 1, 2, 3, or 4.

RTN_NAME : 2538-2022
(ARG NO. _) must be less than (_).

RTN_NAME : 2538-2023
The outer loop increment (ARG NO. _) must be greater than or equal to zero.

RTN_NAME : 2538-2024
The stride (ARG NO. _) for a vector must be greater than or equal to zero.
```

```
RTN_NAME : 2538-2025
The stride (ARG NO. _) for a vector must be greater than zero.

RTN_NAME : 2538-2026
The stride (ARG NO. _) for a vector must be greater than or equal to (_).

RTN_NAME : 2538-2027
The order (ARG NO. _) of a matrix must be greater than or equal to zero.

RTN_NAME : 2538-2028
The job option argument (ARG NO. _) must be 0, 1, or 2.
   or
The job option argument (ARG NO. _) must be 0, 1, 2, or 3.
   or
The job option argument (ARG NO. _) must be 0, 1, 2, 10, 11, or 12.
   or
The job option argument (ARG NO. _) must be 0, 1, 10, or 11.
   or
The job option argument (ARG NO. _) must be 0, 1, 20, or 21.
   or
The job option argument (ARG NO. _) must be 0, 1, 10, 11, 20, 21, 30,
or 31.
   or
The job option argument (ARG NO. _) must be 0, 1, 2, 3, or 4.

RTN_NAME : 2538-2029
The job option argument (ARG NO. _) must be 0 or 1.

RTN_NAME : 2538-2030
The transform length (ARG NO. _) is not an allowed value. The next higher
allowed value is (_).

RTN_NAME : 2538-2031
The resulting convolution length obtained from ARG NO. 10 = (_),
ARG NO.11 = (_), ARG NO.13 = (_), and ARG NO.14 = (_)
must be less than (_).

RTN_NAME : 2538-2032
The size of the leading dimension (ARG NO. _) of the matrix must be greater
than or equal to (_), the bandwidth constraint.

RTN_NAME : 2538-2033
The lower bandwidth (ARG NO. _) must be greater than or equal to zero.

RTN_NAME : 2538-2034
The upper bandwidth (ARG NO. _) must be greater than or equal to zero.

RTN_NAME : 2538-2035
The half-band bandwidth (ARG NO. _) must be greater than or equal to zero.

RTN_NAME : 2538-2036
The lower bandwidth (ARG NO. _) must be less than the order (ARG NO. _)
of the matrix.

RTN_NAME : 2538-2037
The upper bandwidth (ARG NO. _) must be less than the order (ARG NO. _)
of the matrix.
```

```
RTN_NAME : 2538-2038
The half-band bandwidth (ARG NO. _) must be less than the order (ARG NO. _)
of the matrix.

RTN_NAME : 2538-2039
(ARG NO. _) must be greater than zero.

RTN_NAME : 2538-2040
Insufficient storage allocated for positive definite solve.
(_) additional bytes required.

RTN_NAME : 2538-2041
The resulting correlation length obtained from ARG NO. 8  = (_) and
ARG NO. 10 = (_) must be less than (_).

RTN_NAME : 2538-2042
(ARG NO. _) must be greater than or equal to zero.

RTN_NAME : 2538-2043
(ARG NO. _) must be greater than (_).

RTN_NAME : 2538-2044
The number of initialized coefficients (ARG NO. _) cannot exceed the size
of the coefficient vector (ARG NO. _).

RTN_NAME : 2538-2045
The order specified (ARG NO. _) is not supported for this quadrature method.
The nearest supported order is (_).

RTN_NAME : 2538-2046
The scaling parameter (ARG NO. _) must be greater than zero for this
quadrature method.

RTN_NAME : 2538-2047
The scaling parameter (ARG NO. _) must be nonzero for this quadrature
method.

RTN_NAME : 2538-2048
The sum of (ARG NO. _) and (ARG NO. _) must be nonzero for this quadrature
method.

RTN_NAME : 2538-2049
The number of data points (ARG NO. _) must be greater than one in order
to perform numerical quadrature.

RTN_NAME : 2538-2050
The number of columns specified for the arrays to store the matrix in
compressed matrix mode (ARG NO. _) must be greater than or equal to (_).

RTN_NAME : 2538-2051
The number of columns (ARG NO. _) specified for the matrix used to store
the sparse matrix in compressed mode must be greater than zero.

RTN_NAME : 2538-2052
The total number of non-zero elements of the input sparse matrix stored by
rows, obtained from element (_) of the row pointers array (ARG NO. _),
must be greater than or equal to zero.
```

```
RTN_NAME : 2538-2053
The number of non-zero elements in row (_) obtained from the row pointer
array (ARG NO. _) is less than zero.

RTN_NAME : 2538-2054
The number of diagonals (ARG NO. _) specified for the matrix used to store the
sparse matrix in compressed diagonal mode must be greater than zero.

RTN_NAME : 2538-2055
Element (_) of the vector used to store the diagonal numbers (ARG NO. _)
is incompatible with the order of the sparse matrix (ARG NO. _).

RTN_NAME : 2538-2056
The matrix is singular because the number of non-zero entries (ARG NO. _)
is zero.

RTN_NAME : 2538-2057
Element (_) in the integer parameter vector (ARG NO. _) must be greater than
or equal to zero.

RTN_NAME : 2538-2058
Element (_) in the integer parameter vector (ARG NO. _) must be (_),(_), or (_).

RTN_NAME : 2538-2059
Element (_) in the real parameter vector (ARG NO. _) must be greater than zero.

RTN_NAME : 2538-2060
The size of the leading dimension (ARG NO. _) of an array must be greater
than or equal to the maximum of (ARG NO. _) and (ARG NO. _).

RTN_NAME : 2538-2061
Parameter (ARG NO. _), which specifies the number of columns of the input
sparse matrix (ARG NO. _ and ARG NO. _) must be greater than or equal to (_).

RTN_NAME : 2538-2062
The number of random numbers generated (ARG NO. _) must be even and
greater than or equal to zero.

RTN_NAME : 2538-2063
SIDE (ARG NO. _), which specifies whether the triangular input matrix
(ARG NO. _) appears on the left or right of the other input matrix, must
be 'L' or 'R'.

RTN_NAME : 2538-2064
UPLO (ARG NO. _), which specifies whether an input matrix (ARG NO. _)
is upper or lower triangular, must be 'U' or 'L'.

RTN_NAME : 2538-2065
DIAG (ARG NO. _), which specifies whether an input matrix (ARG NO. _)
is unit triangular, must be 'U' or 'N'.

RTN_NAME : 2538-2066
Given the value which has been assigned to SIDE (ARG NO. _), the leading
dimension (ARG NO. _) for the triangular input matrix must be greater
than or equal to (ARG NO. _).
```

```
RTN_NAME : 2538-2067
TRANSA (ARG NO. _) specifies whether an input matrix (ARG NO. _), its transpose,
or its conjugate transpose should be used. TRANSA must be 'N', 'T', or 'C'.

RTN_NAME : 2538-2068
The size of the leading dimension (ARG NO. _) of an array must be greater
than or equal to zero.

RTN_NAME : 2538-2069
The vector section size of the scalar library (ARG NO. _) must be 128 or 256.

RTN_NAME : 2538-2070
Element (_) in (ARG NO. _) must be 0 or 1.
   or
Element (_) in (ARG NO. _) must be greater than zero.
   or
Element (_) in (ARG NO. _) must be greater than or equal to zero.
   or
Element (_) in (ARG NO. _) must be greater than or equal to zero and
less than or equal to 1.
   or
Element (_) in (ARG NO. _) must be greater than the preceding element.
   or
Element (_) in (ARG NO. _) must be greater than or equal to 1 and
less than or equal to n.
   or
Element (_) in (ARG NO. _) must be -1 or 1.
   or
Element (_) in (ARG NO. _) must be nonzero.
   or
Element (_) in (ARG NO. _) must be 0, 1, 2, 10, or 11.
   or
Element (_) in (ARG NO. _) must be 0, 1, 2, 10, 11, 100, 102, or 110.
   or
Element (_) in (ARG NO. _) must be 0.
   or
Element (_) in (ARG NO. _) must be 1.
   or
Element (_) in (ARG NO. _) must be 0, 1, 2, 10, 11, 100, 101, 102,
110, or 111.
   or
Element (_) in (ARG NO. _) must be 1, 2, 3, or 4.
   or
Element (_) in (ARG NO. _) must be 1, 2, 3, 4, or 5.

RTN_NAME : 2538-2071
The number of eigenvalues (ARG NO. _) must be less than or equal to the
order of the matrix (ARG NO. _).

RTN_NAME : 2538-2072
The work area (ARG NO. _) does not contain a valid vector seed.  The
routine must be called with a nonzero value of ISEED (ARG NO. _).

RTN_NAME : 2538-2073
(ARG NO. _) must be a double precision whole number greater than or equal
to 1.0 and less than 2147483647.0.
```

```
RTN_NAME : 2538-2074
Performance can be improved by using a larger work array. For best performance,
specify the number of elements (ARG NO. _) in the work array to be greater
than or equal to (_).

RTN_NAME : 2538-2075
The data type parameter (ARG NO. _) must be 'S', 'D', 'C', or 'Z'.

RTN_NAME : 2538-2076
(ARG NO. _) must be greater than or equal to (_) and smaller than (_).

RTN_NAME : 2538-2077
The matrix is singular. Column (_) is empty in the matrix specified by
(ARG NO. _), (ARG NO. _), and (ARG NO. _).

RTN_NAME : 2538-2078
The matrix is singular. Row (_) is empty in the matrix specified by
(ARG NO. _), (ARG NO. _), and (ARG NO. _).

RTN_NAME : 2538-2079
The matrix, specified by (ARG NO. _), (ARG NO. _), and (ARG NO. _),
contains at least one duplicate column index in row (_).

RTN_NAME : 2538-2080
Element (_) in (ARG NO. _) must be greater than or equal to (_)
and less than or equal to (_).
   or
Element (_) in (ARG NO. _) must be greater than or equal to (_) and
less than or equal to (ARG NO. _).
   or
Element (_) in (ARG NO. _) must be greater than or equal to element (_)
and less than or equal to (_).
   or
Element (_) in (ARG NO. _) must be zero or must be greater than or
equal to (_).

RTN_NAME : 2538-2081
Element (_) in (ARG NO. _) must be less than or equal to (_).

RTN_NAME : 2538-2082
Element (_) in (ARG NO. _) may cause incorrect or misleading results.
A nonzero number with absolute value less than or equal to 1 is recommended.
   or
Element (_) in (ARG NO. _) may cause incorrect or misleading results.
A positive number less than or equal to 1 is recommended.

RTN_NAME : 2538-2083
The pivot tolerance (element (_) in (ARG NO. _)) may cause incorrect
or misleading results. A number greater than or equal to 0 and less than or
equal to 1 is recommended.

RTN_NAME : 2538-2084
The dimension (ARG NO. _) of the array (ARG NO. _) must be greater than or
equal to (_).

RTN_NAME : 2538-2085
The number of steps after which the generalized minimum residual method is
restarted, element (_) in (ARG NO. _), must be greater than 0.
```

```
RTN_NAME : 2538-2086
The acceleration parameter, element (_) in (ARG NO. _), must be greater than 0
when using the SSOR preconditioner.

RTN_NAME : 2538-2087
STOR (ARG NO. _), which specifies the storage variation used to represent
the input sparse matrix, must be 'G', 'L', or 'U'.

RTN_NAME : 2538-2088
INIT (ARG NO. _), which specifies the type of computation to be performed,
must be 'I', or 'S'.

RTN_NAME : 2538-2089
Element (_) in (ARG NO. _) must be greater than or equal to (_).
   or
Element (_) in (ARG NO. _) must be greater than or equal to element (_).

RTN_NAME : 2538-2090
For level (_), the number of grid points for dimension (_) must be
an odd number greater than 1.

RTN_NAME : 2538-2091
Since the mesh spacing (ARG NO. _) here is not constant, the second order
prolongation method must be used.  That is, element (_) of (ARG NO. _)
must be (_).

RTN_NAME : 2538-2092
The index into (ARG NO. _) is out of range.
This index is element (_,_) of (ARG NO. _).

RTN_NAME : 2538-2093
The index into (ARG NO. _) is out of range.
This index is element (_,_,_) of (ARG NO. _).

RTN_NAME : 2538-2094
For dimension (_) on level (_), the mesh spacing must be changed to
a positive value.

RTN_NAME : 2538-2095
Excess space in (ARG NO. _) has been decreased and may be inadequate.
To avoid this, specify the coarse level matrix as the final item in this
argument.

RTN_NAME : 2538-2096
For level (_), the matrix type, solver, and preconditioner are incompatible.

RTN_NAME : 2538-2097
The solver requested for level (_) requires a square matrix.
Elements (_,_,_) and (_,_,_) in (ARG NO. _) must be equal.

RTN_NAME : 2538-2098
Element (_,_) of (ARG NO. _) must be greater than or equal to (_).

RTN_NAME : 2538-2099
End of input argument error reporting. For more information, refer to
Engineering and Scientific Subroutine Library Guide and Reference (SA22-7272).
```

## Computational Error Messages

```
RTN_NAME : 2538-2100
The computed index of a vector is out of the range (_) to (_).

RTN_NAME : 2538-2101
Eigenvalue (_) failed to converge after (_) iterations.

RTN_NAME : 2538-2102
Eigenvector (_) failed to converge after (_) iterations.

RTN_NAME : 2538-2103
The matrix (ARG NO. _) is singular.
Zero diagonal element (_) has been detected.

RTN_NAME : 2538-2104
The matrix (ARG NO. _) is not positive definite. The last diagonal element
with nonpositive value is (_).

RTN_NAME : 2538-2105
Factorization failed due to near zero pivot number (_).

RTN_NAME : 2538-2106
Vector boundary misalignment detected in ESSL scalar library.

RTN_NAME : 2538-2107
Singular value (_) failed to converge after (_) iterations.

RTN_NAME : 2538-2108
The matrix specified by (ARG NO. _) and (ARG NO. _) is not definite
because the diagonal is not of constant sign.

RTN_NAME : 2538-2109
The matrix specified by (ARG NO. _) and (ARG NO. _) is not definite
and the iterative process is stopped at iteration number (_).

RTN_NAME : 2538-2110
The maximum allowed number of iterations, element number (_) of (ARG NO. _),
were performed but the iterative process did not converge to a solution
according to the stopping procedure.

RTN_NAME : 2538-2111
The factorization matrix (ARG NO. _) is not consistent with the sparse
matrix specified by (ARG NO. _) and (ARG NO. _).

RTN_NAME : 2538-2112
The incomplete factorization of the sparse matrix specified by
(ARG NO. _) and (ARG NO. _) is not stable.

RTN_NAME : 2538-2113
Unexpected nonzero vector mask detected in ESSL scalar routine.
Contact your IBM Service Representative.

RTN_NAME : 2538-2114
Eigenvalue (_) failed to converge after (_) iterations.
```

```
RTN_NAME : 2538-2115
The matrix (ARG NO. _) is not positive definite.
The leading minor of order (_) has a nonpositive determinant.

RTN_NAME : 2538-2116
The matrix specified by (ARG NO. _) and (ARG NO. _) is singular.

RTN_NAME : 2538-2117
The pivot element in column (_) is smaller than the first element in
(ARG NO. _).

RTN_NAME : 2538-2118
The pivot element in row (_) is smaller than the first element in
(ARG NO. _).

RTN_NAME : 2538-2119
The storage space, specified by (ARG NO. _), is insufficient.

RTN_NAME : 2538-2120
The matrix is singular. The last row processed in the matrix was row (_).

RTN_NAME : 2538-2121
The matrix is singular. the last column processed was column (_).

RTN_NAME : 2538-2122
The factorization failed. No pivot element was found in the active submatrix.

RTN_NAME : 2538-2123
Performance can be improved by specifying a larger value for (ARG NO. _).
(_) compressions were performed.

RTN_NAME : 2538-2124
The data contained in AUX1, (ARG NO. _), was computed for a different
algorithm.

RTN_NAME : 2538-2125
This subroutine initializes part of the ESSL run-time environment.  It may
be called only once, at the beginning of the run.  It must be called before
any ESSL computational routines are called.

RTN_NAME : 2538-2126
The pivot value at row (_) is not acceptable based on pivot criteria
((ARG NO. _) and (ARG NO. _)). No fixup was applicable to this pivot.
The matrix (ARG NO. _) may be singular or not definite.

RTN_NAME : 2538-2127
The pivot value at row (_) was replaced with element (_) in
(ARG NO. _).  The matrix (ARG NO. _) may be singular or not definite.

RTN_NAME : 2538-2128
Internal ESSL error. contact your IBM service representative.

RTN_NAME : 2538-2129
The matrix specified by (ARG NO. _), (ARG NO. _), and (ARG NO. _)
is not definite because the diagonal is not of constant sign or some diagonal
element is zero.
```

```
RTN_NAME : 2538-2130
The incomplete factorization of the sparse matrix specified by (ARG NO. _),
(ARG NO. _), and (ARG NO. _) is not stable.

RTN_NAME : 2538-2131
The matrix specified by (ARG NO. _), (ARG NO. _), and (ARG NO. _) is singular.

RTN_NAME : 2538-2132
Element (_) in (ARG NO. _) indicates that factorization was done on a previous
call.  The data passed is not the result of a prior valid factorization.

RTN_NAME : 2538-2133
An error occurred on level (_), in the user-supplied subroutine specified
by (ARG NO. _).

RTN_NAME : 2538-2134
The data contained in (ARG NO. _) is not consistent with the sparse matrix
specified by (ARG NO. _), (ARG NO. _), and (ARG NO. _).

RTN_NAME : 2538-2135
For level (_), loss of orthogonality occurred in a minimum residual solver
because the input matrix (element (_,_) of (ARG NO. _)) is inappropriate.
Choose one of the other non-symmetric solvers.

RTN_NAME : 2538-2136
For level (_), the main diagonal element for row (_) of a matrix is 0.

RTN_NAME : 2538-2137
This subroutine may be called only once in the beginning of the program.

RTN_NAME : 2538-2138
An error was detected while attempting to open the ESSLPARM file.  A default
ESSL processor family number of (_) will be assumed.  This may not be the
best choice for optimal performance.  Be sure that you have defined the
interface to ESSLPARM correctly for the run.  If you have, contact ESSL
system installation personnel.

RTN_NAME : 2538-2139
An error was detected while attempting to close the ESSLPARM file.  An ESSL
processor family number of (_) was assigned.

RTN_NAME : 2538-2140
An input/output error was detected while attempting to read line (_) of
the ESSLPARM file.  A default ESSL process or family number of (_) will
be assumed.  This may not be the best choice for optimal performance.
Be sure that you have defined the interface to ESSLPARM correctly for the run.
If you have, contact ESSL system installation personnel.

RTN_NAME : 2538-2141
The end of file marker on line (_) of the ESSLPARM file was reached
before a valid specification for the ESSL processor family number was read.
A default ESSL processor family number of (_) will be assumed.
This may not be the best choice for optimal performance.  Be sure that you
have defined the interface to ESSLPARM correctly for the run.  If you have,
contact ESSL system installation personnel.

RTN_NAME : 2538-2142
A syntax error was detected on line (_), column (_), of the ESSLPARM
```

```
file.  No valid specification for the ESSL processor family number was read.
A default ESSL processor family number of (_) will be assumed.
This may not be the best choice for optimal performance.  Contact ESSL system
installation personnel.

RTN_NAME : 2538-2143
The processor family number (_) on line (_) of the ESSLPARM file is
not in the allowable range.  Please specify a processor family number greater
than or equal to (_) and less than or equal to (_).  A default ESSL
processor family number of (_) will be assumed.  This may not be the best
choice for optimal performance.  Contact ESSL system installation personnel.

RTN_NAME : 2538-2144
This subroutine must be called before any computational ESSL routines are
called.

RTN_NAME : 2538-2145
The input matrix (ARG NO. _) is singular.  The first diagonal element found to
be exactly 0, was in column (_).

RTN_NAME : 2538-2146
The input matrix (ARG NO. _) is singular.  The first diagonal element found to
be exactly 0, was in column (_).

RTN_NAME : 2538-2199
End of computational error reporting. For more information, refer to
Engineering and Scientific Subroutine Library Guide and Reference (SA22-7272).
```

## Resource Error Messages

```
RTN_NAME : 2538-2400
An internal buffer allocation has failed due to insufficient memory.
```

## Informational and Attention Error Messages

```
RTN_NAME : 2538-2600
Performance may be degraded due to limited buffer space availability.

RTN_NAME : 2538-2601
Execution terminating due to error count for error number (_)
Message summary: Message number - Count

RTN_NAME : 2538-2602
User error corrective routine entered.
User corrective action taken.  Execution continuing.

RTN_NAME : 2538-2603
Standard corrective action taken.  Execution continuing.

RTN_NAME : 2538-2604
Execution terminating due to error count for error number _.

RTN_NAME : 2538-2605
Message summary:      _     - _

RTN_NAME : 2538-2606
Serial execution is taking place since the input array is equal
to the output array and either:
```

```
INC2X (ARG NO. _) is not equal to 2 times INC2Y (ARG NO. _) or
INC3X (ARG NO. _) is not equal to 2 times INC3Y (ARG NO. _).

RTN_NAME : 2538-2607
Serial execution is taking place since the input array is equal
to the output array and either:
INC2X (ARG NO. _) is not equal to INC2Y (ARG NO. _) or
INC3X (ARG NO. _) is not equal to INC3Y (ARG NO. _).

RTN_NAME : 2538-2608
Performance may be improved by using a larger work array. For best
performance, specify the number of elements (ARG NO. _) in the work array
to be greater than or equal to (_).

RTN_NAME : 2538-2609
Performance may be improved by specifying a larger value for (ARG NO. _).
(_) compressions were performed.
```

## Miscellaneous Error Messages

```
RTN_NAME : 2538-2700
Internal ESSL error number (_).
Contact your IBM service representative.

RTN_NAME : 2538-2703
Internal ESSL error: message number requested (_) is outside of the
valid range.  Contact your IBM service representative.
```

# Part 2. Reference Information

This part of the book is organized into ten areas, providing reference information for coding the ESSL calling sequences. It is organized as follows:

- Linear Algebra Subprograms
- Matrix Operations
- Linear Algebraic Equations
- Eigensystem Analysis
- Fourier Transforms, Convolutions and Correlations, and Related Computations
- Sorting and Searching
- Interpolation
- Numerical Quadrature
- Random Number Generation
- Utilities

# Chapter 8. Linear Algebra Subprograms

The linear algebra subprograms, provided in four areas, are described in this chapter.

## Overview of the Linear Algebra Subprograms

This section describes the subprograms in each of the four linear algebra subprogram areas:

- Vector-scalar linear algebra subprograms (Table 32 )
- Sparse vector-scalar linear algebra subprograms (Table 33 )
- Matrix-vector linear algebra subprograms (Table 34 )
- Sparse matrix-vector linear algebra subprograms (Table 35 )

**Notes:**

1. The term **subprograms** is used to be consistent with the Basic Linear Algebra Subprograms (BLAS), because many of these subprograms correspond to the BLAS.
2. Some of the linear algebra subprograms were designed in accordance with the Level 1 and Level 2 BLAS de facto standard. If these subprograms do not comply with the standard as approved, IBM will consider updating them to do so. If IBM updates these subprograms, the updates could require modifications of the calling application program.

## Vector-Scalar Linear Algebra Subprograms

The vector-scalar linear algebra subprograms include a subset of the standard set of Level 1 BLAS. For details on the BLAS, see reference [76]. The remainder of the vector-scalar linear algebra subprograms are commonly used computations provided for your applications. Both real and complex versions of the subprograms are provided.

*Table 32 (Page 1 of 2). List of Vector-Scalar Linear Algebra Subprograms*

| Descriptive Name | Short-Precision Subprogram | Long-Precision Subprogram | Page |
|---|---|---|---|
| Position of the First or Last Occurrence of the Vector Element Having the Largest Magnitude | ISAMAX†▪<br>ICAMAX†▪ | IDAMAX†▪<br>IZAMAX†▪ | 201 |
| Position of the First or Last Occurrence of the Vector Element Having Minimum Absolute Value | ISAMIN† | IDAMIN† | 204 |
| Position of the First or Last Occurrence of the Vector Element Having Maximum Value | ISMAX† | IDMAX† | 207 |
| Position of the First or Last Occurrence of the Vector Element Having Minimum Value | ISMIN† | IDMIN† | 210 |
| Sum of the Magnitudes of the Elements in a Vector | SASUM†▪<br>SCASUM†▪ | DASUM†▪<br>DZASUM†▪ | 213 |
| Multiply a Vector X by a Scalar, Add to a Vector Y, and Store in the Vector Y | SAXPY▪<br>CAXPY▪ | DAXPY▪<br>ZAXPY▪ | 216 |

*Table 32 (Page 2 of 2). List of Vector-Scalar Linear Algebra Subprograms*

| **Descriptive Name** | **Short-Precision Subprogram** | **Long-Precision Subprogram** | **Page** |
|---|---|---|---|
| Copy a Vector | SCOPY▪<br>CCOPY▪ | DCOPY▪<br>ZCOPY▪ | 219 |
| Dot Product of Two Vectors | SDOT†▪<br>CDOTU†▪<br>CDOTC†▪ | DDOT†▪<br>ZDOTU†▪<br>ZDOTC†▪ | 222 |
| Compute SAXPY or DAXPY N Times | SNAXPY | DNAXPY | 226 |
| Compute Special Dot Products N Times | SNDOT | DNDOT | 231 |
| Euclidean Length of a Vector with Scaling of Input to Avoid Destructive Underflow and Overflow | SNRM2†▪<br>SCNRM2†▪ | DNRM2†▪<br>DZNRM2†▪ | 236 |
| Euclidean Length of a Vector with No Scaling of Input | SNORM2†<br>CNORM2† | DNORM2†<br>ZNORM2† | 239 |
| Construct a Givens Plane Rotation | SROTG▪<br>CROTG▪ | DROTG▪<br>ZROTG▪ | 242 |
| Apply a Plane Rotation | SROT▪<br>CROT▪<br>CSROT▪ | DROT▪<br>ZROT▪<br>ZDROT▪ | 249 |
| Multiply a Vector X by a Scalar and Store in the Vector X | SSCAL▪<br>CSCAL▪<br>CSSCAL▪ | DSCAL▪<br>ZSCAL▪<br>ZDSCAL▪ | 253 |
| Interchange the Elements of Two Vectors | SSWAP▪<br>CSWAP▪ | DSWAP▪<br>ZSWAP▪ | 256 |
| Add a Vector X to a Vector Y and Store in a Vector Z | SVEA<br>CVEA | DVEA<br>ZVEA | 259 |
| Subtract a Vector Y from a Vector X and Store in a Vector Z | SVES<br>CVES | DVES<br>ZVES | 263 |
| Multiply a Vector X by a Vector Y and Store in a Vector Z | SVEM<br>CVEM | DVEM<br>ZVEM | 267 |
| Multiply a Vector X by a Scalar and Store in a Vector Y | SYAX<br>CYAX<br>CSYAX | DYAX<br>ZYAX<br>ZDYAX | 271 |
| Multiply a Vector X by a Scalar, Add to a Vector Y, and Store in a Vector Z | SZAXPY<br>CZAXPY | DZAXPY<br>ZZAXPY | 274 |

† This subprogram is invoked as a function in a Fortran program.

▪ Level 1 BLAS

## Sparse Vector-Scalar Linear Algebra Subprograms

The sparse vector-scalar linear algebra subprograms operate on sparse vectors using optimized storage techniques; that is, only the nonzero elements of the vector are stored. These subprograms provide similar functions to the vector-scalar subprograms. These subprograms represent a subset of the sparse extensions to the Level 1 BLAS described in reference [29]. Both real and complex versions of the subprograms are provided.

*Table 33. List of Sparse Vector-Scalar Linear Algebra Subprograms*

| Descriptive Name | Short-Precision Subprogram | Long-Precision Subprogram | Page |
|---|---|---|---|
| Scatter the Elements of a Sparse Vector X in Compressed-Vector Storage Mode into Specified Elements of a Sparse Vector Y in Full-Vector Storage Mode | SSCTR<br>CSCTR | DSCTR<br>ZSCTR | 279 |
| Gather Specified Elements of a Sparse Vector Y in Full-Vector Storage Mode into a Sparse Vector X in Compressed-Vector Storage Mode | SGTHR<br>CGTHR | DGTHR<br>ZGTHR | 282 |
| Gather Specified Elements of a Sparse Vector Y in Full-Vector Mode into a Sparse Vector X in Compressed-Vector Mode, and Zero the Same Specified Elements of Y | SGTHRZ<br>CGTHRZ | DGTHRZ<br>ZGTHRZ | 285 |
| Multiply a Sparse Vector X in Compressed-Vector Storage Mode by a Scalar, Add to a Sparse Vector Y in Full-Vector Storage Mode, and Store in the Vector Y | SAXPYI<br>CAXPYI | DAXPYI<br>ZAXPYI | 288 |
| Dot Product of a Sparse Vector X in Compressed-Vector Storage Mode and a Sparse Vector Y in Full-Vector Storage Mode | SDOTI†<br>CDOTCI†<br>CDOTUI† | DDOTI†<br>ZDOTCI†<br>ZDOTUI† | 291 |

† This subprogram is invoked as a function in a Fortran program.

## Matrix-Vector Linear Algebra Subprograms

The matrix-vector linear algebra subprograms operate on a higher-level data structure—matrix-vector rather than vector-scalar—using optimized algorithms to improve performance. These subprograms represent a subset of the Level 2 BLAS described in references [34] and [35 ]. Both real and complex versions of the subprograms are provided.

*Table 34 (Page 1 of 2). List of Matrix-Vector Linear Algebra Subprograms*

| Descriptive Name | Short-Precision Subprogram | Long-Precision Subprogram | Page |
|---|---|---|---|
| Matrix-Vector Product for a General Matrix, Its Transpose, or Its Conjugate Transpose | SGEMV◂<br>CGEMV◂<br>SGEMX§<br>SGEMTX§ | DGEMV◂<br>ZGEMV◂<br>DGEMX§<br>DGEMTX§ | 296 |
| Rank-One Update of a General Matrix | SGER◂<br>CGERU◂<br>CGERC◂ | DGER◂<br>ZGERU◂<br>ZGERC◂ | 307 |
| Matrix-Vector Product for a Real Symmetric or Complex Hermitian Matrix | SSPMV◂<br>CHPMV◂<br>SSYMV◂<br>CHEMV◂<br>SSLMX§ | DSPMV◂<br>ZHPMV◂<br>DSYMV◂<br>ZHEMV◂<br>DSLMX§ | 315 |
| Rank-One Update of a Real Symmetric or Complex Hermitian Matrix | SSPR◂<br>CHPR◂<br>SSYR◂<br>CHER◂<br>SSLR1§ | DSPR◂<br>ZHPR◂<br>DSYR◂<br>ZHER◂<br>DSLR1§ | 323 |

*Table 34 (Page 2 of 2). List of Matrix-Vector Linear Algebra Subprograms*

| Descriptive Name | Short-Precision Subprogram | Long-Precision Subprogram | Page |
|---|---|---|---|
| Rank-Two Update of a Real Symmetric or Complex Hermitian Matrix | SSPR2◂<br>CHPR2◂<br>SSYR2◂<br>CHER2◂<br>SSLR2§ | DSPR2◂<br>ZHPR2◂<br>DSYR2◂<br>ZHER2◂<br>DSLR2§ | 331 |
| Matrix-Vector Product for a General Band Matrix, Its Transpose, or Its Conjugate Transpose | SGBMV◂<br>CGBMV◂ | DGBMV◂<br>ZGBMV◂ | 340 |
| Matrix-Vector Product for a Real Symmetric or Complex Hermitian Band Matrix | SSBMV◂<br>CHBMV◂ | DSBMV◂<br>ZHBMV◂ | 347 |
| Matrix-Vector Product for a Triangular Matrix, Its Transpose, or Its Conjugate Transpose | STRMV◂<br>CTRMV◂<br>STPMV◂<br>CTPMV◂ | DTRMV◂<br>ZTRMV◂<br>DTPMV◂<br>ZTPMV◂ | 352 |
| Matrix-Vector Product for a Triangular Band Matrix, Its Transpose, or Its Conjugate Transpose | STBMV◂<br>CTBMV◂ | DTBMV◂<br>ZTBMV◂ | 358 |

◂ Level 2 BLAS

§ These subroutines are provided only for migration from earlier releases of ESSL and are not intended for use in new programs.

## Sparse Matrix-Vector Linear Algebra Subprograms

The sparse matrix-vector linear algebra subprograms operate on sparse matrices using optimized storage techniques; that is, only the nonzero elements of the vector are stored. These subprograms provide similar functions to the matrix-vector subprograms.

*Table 35. List of Sparse Matrix-Vector Linear Algebra Subprograms*

| Descriptive Name | Long- Precision Subprogram | Page |
|---|---|---|
| Matrix-Vector Product for a Sparse Matrix in Compressed-Matrix Storage Mode | DSMMX | 365 |
| Transpose a Sparse Matrix in Compressed-Matrix Storage Mode | DSMTM | 368 |
| Matrix-Vector Product for a Sparse Matrix or Its Transpose in Compressed-Diagonal Storage Mode | DSDMX | 372 |

## Use Considerations

If your program uses a sparse matrix stored by rows, as defined in “Storage-by-Rows” on page 99, you should first convert your sparse matrix to compressed-matrix storage mode by using the subroutine DSRSM on page 981. DSRSM converts a matrix to compressed-matrix storage mode. To convert your sparse matrix to compressed-diagonal storage mode, you need to perform this conversion in your application program before calling the ESSL subroutine.

## Performance and Accuracy Considerations

1. In ESSL, the SSCAL and DSCAL subroutines provide the fastest way to zero out contiguous (stride 1) arrays, by specifying *incx* = 1 and $\alpha = 0$.

2. Where possible, use the matrix-vector linear algebra subprograms, rather than the vector-scalar, to optimize performance. Because data is presented in matrices rather than vectors, multiple operations can be performed by a single ESSL subprogram.

3. Where possible, use subprograms that do multiple computations, such as SNDOT and SNAXPY, rather than individual computations, such as SDOT and SAXPY. You get better performance.

4. Many of the short-precision subprograms provide increased accuracy by accumulating results in long precision. This is noted in the functional description of each subprogram.

5. In some of the subprograms, because implementation techniques vary to optimize performance, accuracy of the results may vary for different array sizes. In the subprograms in which this occurs, a general description of the implementation techniques is given in the functional description for each subprogram.

6. To select the sparse matrix subroutine that gives you the best performance, you must consider the layout of the data in your matrix. From this, you can determine the most efficient storage mode for your sparse matrix. ESSL provides two versions of each of its sparse matrix-vector subroutines that you can use. One operates on sparse matrices stored in compressed-matrix storage mode, and the other operates on sparse matrices stored in compressed-diagonal storage mode. These two storage modes are described in “Sparse Matrix” on page 92.

   Compressed-matrix storage mode is generally applicable. It should be used when each row of the matrix contains approximately the same number of nonzero elements. However, if the matrix has a special form—that is, where the nonzero elements are concentrated along a few diagonals—compressed-diagonal storage mode gives improved performance.

7. There are some ESSL-specific rules that apply to the results of computations on the workstation processors using the ANSI/IEEE standards. For details, see “What Data Type Standards Are Used by ESSL, and What Exceptions Should You Know About?” on page 45.

## Vector-Scalar Subprograms

This section contains the vector-scalar subprogram descriptions.

## ISAMAX, IDAMAX, ICAMAX, and IZAMAX—Position of the First or Last Occurrence of the Vector Element Having the Largest Magnitude

ISAMAX and IDAMAX find the position *i* of the first or last occurrence of a vector element having the maximum absolute value. ICAMAX and IZAMAX find the position *i* of the first or last occurrence of a vector element having the largest sum of the absolute values of the real and imaginary parts of the vector elements.

You get the position of the first or last occurrence of an element by specifying positive or negative stride, respectively, for vector ***x***. Regardless of the stride, the position *i* is always relative to the location specified in the calling sequence for vector ***x*** (in argument *x*).

*Table 36. Data Types*

| ***x*** | **Subprogram** |
|---|---|
| Short-precision real | ISAMAX |
| Long-precision real | IDAMAX |
| Short-precision complex | ICAMAX |
| Long-precision complex | IZAMAX |

***Syntax***

| | |
|---|---|
| **Fortran** | ISAMAX \| IDAMAX \| ICAMAX \| IZAMAX (*n*, *x*, *incx*) |
| **C and C++** | isamax \| idamax \| icamax \| izamax (*n*, *x*, *incx*); |
| **PL/I** | ISAMAX \| IDAMAX \| ICAMAX \| IZAMAX (*n*, *x*, *incx*); |

*On Entry*

*n*
is the number of elements in vector ***x***. Specified as: a fullword integer; $n \geq 0$.

*x*
is the vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length $1+(n-1)|incx|$, containing numbers of the data type indicated in Table 36.

*incx*
is the stride for vector ***x***. Specified as: a fullword integer. It can have any value.

*On Return*

*Function value*

is the position *i* of the element in the array, where:

If $incx \geq 0$, *i* is the position of the first occurrence.

If $incx < 0$, *i* is the position of the last occurrence.

Returned as: a fullword integer; $0 \leq i \leq n$.

***Note:*** Declare the ISAMAX, IDAMAX, ICAMAX, and IZAMAX functions in your program as returning a fullword integer value.

***Function:*** ISAMAX and IDAMAX find the first element $x_k$, where $k$ is defined as the smallest index $k$, such that:

$|x_k| = \max\{|x_j| \text{ for } j = 1, n\}$

ICAMAX and IZAMAX find the first element $x_k$, where $k$ is defined as the smallest index $k$, such that:

$|a_k|+|b_k| = \max\{|a_j|+|b_j| \text{ for } j = 1, n\}$
where $x_k = (a_k, b_k)$

By specifying a positive or negative stride for vector ***x***, the first or last occurrence, respectively, is found in the array. The position $i$, returned as the value of the function, is always figured relative to the location specified in the calling sequence for vector ***x*** (in argument *x*). Therefore, depending on the stride specified for *incx*, *i* has the following values:

For *incx* ≥ 0, $i = k$
For *incx* < 0, $i = n-k+1$

See reference [76 ]. The result is returned as a function value. If $n$ is 0, then 0 is returned as the value of the function.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Example 1:*** This example shows a vector, ***x***, with a stride of 1.

*Function Reference and Input*

```
               N   X   INCX
               |   |    |
IMAX = ISAMAX( 9 , X ,  1   )

X        =  (1.0, 2.0, 7.0, -8.0, -5.0, -10.0, -9.0, 10.0, 6.0)
```

*Output*

```
IMAX     =  6
```

***Example 2:*** This example shows a vector, ***x***, with a stride greater than 1.

*Function Reference and Input*

```
               N   X   INCX
               |   |    |
IMAX = ISAMAX( 5 , X ,  2   )

X        =  (1.0, . , 7.0, . , -5.0, . , -9.0, . , 6.0)
```

*Output*

```
IMAX     =  4
```

***Example 3:*** This example shows a vector, ***x***, with a stride of 0.

*Function Reference and Input*

```
               N   X   INCX
               |   |    |
IMAX = ISAMAX( 9 , X ,  0   )

X         =  (1.0, . , . , . , . , . , . , . , . , .)
```

*Output*

```
IMAX      =  1
```

***Example 4:*** This example shows a vector, ***x***, with a negative stride. Processing begins at element `X(15)`, which is 2.0.

*Function Reference and Input*

```
               N   X   INCX
               |   |    |
IMAX = ISAMAX( 8 , X , -2   )

X         =  (3.0, . , 5.0, . , -8.0, . , 6.0, . , 8.0, . ,
              4.0, . , 8.0, . , 2.0)
```

*Output*

```
IMAX      =  7
```

***Example 5:*** This example shows a vector, ***x***, containing complex numbers and having a stride of 1.

*Function Reference and Input*

```
               N   X   INCX
               |   |    |
IMAX = ICAMAX( 5 , X ,  1   )

X         =  ((9.0 , 2.0) , (7.0 , -8.0) , (-5.0 , -10.0) , (-4.0 , 10.0),
              (6.0 , 3.0))
```

*Output*

```
IMAX      =  2
```

# ISAMIN and IDAMIN—Position of the First or Last Occurrence of the Vector Element Having Minimum Absolute Value

These subprograms find the position *i* of the first or last occurrence of a vector element having the minimum absolute value.

You get the position of the first or last occurrence of an element by specifying positive or negative stride, respectively, for vector ***x***. Regardless of the stride, the position *i* is always relative to the location specified in the calling sequence for vector ***x*** (in argument *x*).

*Table 37. Data Types*

| ***x*** | **Subprogram** |
|---|---|
| Short-precision real | ISAMIN |
| Long-precision real | IDAMIN |

### *Syntax*

| | |
|---|---|
| **Fortran** | ISAMIN \| IDAMIN (*n*, *x*, *incx*) |
| **C and C++** | isamin \| idamin (*n*, *x*, *incx*); |
| **PL/I** | ISAMIN \| IDAMIN (*n*, *x*, *incx*); |

*On Entry*

*n*
is the number of elements in vector ***x***. Specified as: a fullword integer; $n \geq 0$.

*x*
is the vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length $1+(n-1)|incx|$, containing numbers of the data type indicated in Table 37.

*incx*
is the stride for vector ***x***. Specified as: a fullword integer. It can have any value.

*On Return*

*Function value*

is the position *i* of the element in the array, where:

If $incx \geq 0$, *i* is the position of the first occurrence.

If $incx < 0$, *i* is the position of the last occurrence.

Returned as: a fullword integer; $0 \leq i \leq n$.

***Note:*** Declare the ISAMIN and IDAMIN functions in your program as returning a fullword integer value.

***Function:*** These subprograms find the first element $x_k$, where *k* is defined as the smallest index *k*, such that:

$|x_k| = \min\{|x_j| \text{ for } j = 1, n\}$

By specifying a positive or negative stride for vector ***x***, the first or last occurrence, respectively, is found in the array. The position *i*, returned as the value of the function, is always figured relative to the location specified in the calling sequence

for vector ***x*** (in argument *x*). Therefore, depending on the stride specified for *incx*, *i* has the following values:

For *incx* ≥ 0, *i* = *k*
For *incx* < 0, *i* = *n*−*k*+1

See reference [76 ]. The result is returned as a function value. If *n* is 0, then 0 is returned as the value of the function.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Example 1:*** This example shows a vector, ***x***, with a stride of 1.

*Function Reference and Input*

```
               N   X   INCX
               |   |    |
IMIN = ISAMIN( 6 , X ,  1  )

X        =  (3.0, 4.0, 1.0, 8.0, 1.0, 3.0)
```

*Output*

```
IMIN     =  3
```

***Example 2:*** This example shows a vector, ***x***, with a stride greater than 1.

*Function Reference and Input*

```
               N   X   INCX
               |   |    |
IMIN = ISAMIN( 4 , X ,  2  )

X        =  (-3.0, . , -9.0, . , -8.0, . , 3.0)
```

*Output*

```
IMIN     =  1
```

***Example 3:*** This example shows a vector, ***x***, with a positive stride and two elements with the minimum absolute value. The position of the first occurrence is returned.

*Function Reference and Input*

```
               N   X   INCX
               |   |    |
IMIN = ISAMIN( 4 , X ,  2  )

X        =  (2.0, . , -1.0, . , 4.0, . , 1.0)
```

*Output*

```
IMIN     =  2
```

***Example 4:*** This example shows a vector, ***x***, with a negative stride and two elements with the minimum absolute value. The position of the last occurrence is returned. Processing begins at element `X(7)`, which is 1.0.

*Function Reference and Input*

```
               N   X   INCX
               |   |    |
IMIN = ISAMIN( 4 , X , -2  )

X        =  (2.0, . , -1.0, . , 4.0, . , 1.0)
```

*Output*

```
IMIN     =  4
```

## ISMAX and IDMAX—Position of the First or Last Occurrence of the Vector Element Having the Maximum Value

These subprograms find the position *i* of the first or last occurrence of a vector element having the maximum value.

You get the position of the first or last occurrence of an element by specifying positive or negative stride, respectively, for vector ***x***. Regardless of the stride, the position *i* is always relative to the location specified in the calling sequence for vector ***x*** (in argument *x*).

*Table 38. Data Types*

| ***x*** | **Subprogram** |
|---|---|
| Short-precision real | ISMAX |
| Long-precision real | IDMAX |

### *Syntax*

| | |
|---|---|
| **Fortran** | ISMAX \| IDMAX (*n*, *x*, *incx*) |
| **C and C++** | ismax \| idmax (*n*, *x*, *incx*); |
| **PL/I** | ISMAX \| IDMAX (*n*, *x*, *incx*); |

*On Entry*

*n*

is the number of elements in vector ***x***. Specified as: a fullword integer; $n \geq 0$.

*x*

is the vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length $1+(n-1)|incx|$, containing numbers of the data type indicated in Table 38.

*incx*

is the stride for vector ***x***. Specified as: a fullword integer. It can have any value.

*On Return*

*Function value*

is the position *i* of the element in the array, where:

If $incx \geq 0$, *i* is the position of the first occurrence.

If $incx < 0$, *i* is the position of the last occurrence.

Returned as: a fullword integer; $0 \leq i \leq n$.

***Note:*** Declare the ISMAX and IDMAX functions in your program as returning a fullword integer value.

***Function:*** These subprograms find the first element $x_k$, where *k* is defined as the smallest index *k*, such that:

$x_k = \max\{x_j \text{ for } j = 1, n\}$

By specifying a positive or negative stride for vector ***x***, the first or last occurrence, respectively, is found in the array. The position *i*, returned as the value of the function, is always figured relative to the location specified in the calling sequence

for vector ***x*** (in argument *x*). Therefore, depending on the stride specified for *incx*, *i* has the following values:

For *incx* ≥ 0, *i* = *k*
For *incx* < 0, *i* = *n*−*k*+1

See reference [76 ]. The result is returned as a function value. If *n* is 0, then 0 is returned as the value of the function.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Example 1:*** This example shows a vector, ***x***, with a stride of 1.

*Function Reference and Input*

```
              N   X  INCX
              |   |   |
IMAX = ISMAX( 6 , X , 1  )

X        =  (3.0, 4.0, 1.0, 8.0, 1.0, 8.0)
```

*Output*

```
IMAX     =  4
```

***Example 2:*** This example shows a vector, ***x***, with a stride greater than 1.

*Function Reference and Input*

```
              N   X  INCX
              |   |   |
IMAX = ISMAX( 4 , X , 2  )

X        =  (-3.0, . , 9.0, . , -8.0, . , 3.0)
```

*Output*

```
IMAX     =  2
```

***Example 3:*** This example shows a vector, ***x***, with a positive stride and two elements with the maximum value. The position of the first occurrence is returned.

*Function Reference and Input*

```
              N   X  INCX
              |   |   |
IMAX = ISMAX( 4 , X , 2  )

X        =  (2.0, . , 4.0, . , 4.0, . , 1.0)
```

*Output*

```
IMAX     =  2
```

***Example 4:*** This example shows a vector, ***x***, with a negative stride and two elements with the maximum value. The position of the last occurrence is returned. Processing begins at element X(7), which is 1.0.

*Function Reference and Input*

```
              N   X   INCX
              |   |    |
IMAX = ISMAX( 4 , X , -2  )

X        =  (2.0, . , 4.0, . , 4.0, . , 1.0)
```

*Output*

```
IMAX     =  3
```

## ISMIN and IDMIN—Position of the First or Last Occurrence of the Vector Element Having Minimum Value

These subprograms find the position *i* of the first or last occurrence of a vector element having the minimum value.

You get the position of the first or last occurrence of an element by specifying positive or negative stride, respectively, for vector ***x***. Regardless of the stride, the position *i* is always relative to the location specified in the calling sequence for vector ***x*** (in argument *x*).

*Table 39. Data Types*

| ***x*** | **Subprogram** |
|---|---|
| Short-precision real | ISMIN |
| Long-precision real | IDMIN |

***Syntax***

| | |
|---|---|
| **Fortran** | ISMIN \| IDMIN (*n*, *x*, *incx*) |
| **C and C++** | ismin \| idmin (*n*, *x*, *incx*); |
| **PL/I** | ISMIN \| IDMIN (*n*, *x*, *incx*); |

*On Entry*

*n*

is the number of elements in vector ***x***. Specified as: a fullword integer; $n \geq 0$.

*x*

is the vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length $1+(n-1)|incx|$, containing numbers of the data type indicated in Table 39.

*incx*

is the stride for vector ***x***. Specified as: a fullword integer. It can have any value.

*On Return*

*Function value*

is the position *i* of the element in the array, where:

If $incx \geq 0$, *i* is the position of the first occurrence.

If $incx < 0$, *i* is the position of the last occurrence.

Returned as: a fullword integer; $0 \leq i \leq n$.

***Note:*** Declare the ISMIN and IDMIN functions in your program as returning a fullword integer value.

***Function:*** These subprograms find the first element $x_k$, where *k* is defined as the smallest index *k*, such that:

$x_k = \min\{x_j \text{ for } j = 1, n\}$

By specifying a positive or negative stride for vector ***x***, the first or last occurrence, respectively, is found in the array. The position *i*, returned as the value of the function, is always figured relative to the location specified in the calling sequence

for vector ***x*** (in argument *x*). Therefore, depending on the stride specified for *incx*, *i* has the following values:

For $incx \ge 0$, $i = k$
For $incx < 0$, $i = n-k+1$

See reference [76 ]. The result is returned as a function value. If *n* is 0, then 0 is returned as the value of the function.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Example 1:*** This example shows a vector, ***x***, with a stride of 1.

*Function Reference and Input*

```
               N   X  INCX
               |   |   |
IMIN = ISMIN( 6 , X , 1  )

X        =  (3.0, 4.0, 1.0, 8.0, 1.0, 3.0)
```

*Output*

```
IMIN     =  3
```

***Example 2:*** This example shows a vector, ***x***, with a stride greater than 1.

*Function Reference and Input*

```
               N   X  INCX
               |   |   |
IMIN = ISMIN( 4 , X , 2  )

X        =  (-3.0, . , -9.0, . , -8.0, . , 3.0)
```

*Output*

```
IMIN     =  2
```

***Example 3:*** This example shows a vector, ***x***, with a positive stride and two elements with the minimum value. The position of the first occurrence is returned. Processing begins at element `X(7)`, which is 1.0.

*Function Reference and Input*

```
               N   X  INCX
               |   |   |
IMIN = ISMIN( 4 , X , 2  )

X        =  (2.0, . , 1.0, . , 4.0, . , 1.0)
```

*Output*

```
IMIN     =  2
```

***Example 4:*** This example shows a vector, ***x***, with a negative stride and two elements with the minimum value. The position of the last occurrence is returned. Processing begins at element `X(7)`, which is 1.0.

*Function Reference and Input*

```
              N   X   INCX
              |   |    |
IMIN = ISMIN( 4 , X , -2  )

X        =  (2.0, . , 1.0, . , 4.0, . , 1.0)
```

*Output*

```
IMIN     =  4
```

## SASUM, DASUM, SCASUM, and DZASUM—Sum of the Magnitudes of the Elements in a Vector

SASUM and DASUM compute the sum of the absolute values of the elements in vector ***x***. SCASUM and DZASUM compute the sum of the absolute values of the real and imaginary parts of the elements in vector ***x***.

*Table 40. Data Types*

| ***x*** | **Result** | **Subprogram** |
|---|---|---|
| Short-precision real | Short-precision real | SASUM |
| Long-precision real | Long-precision real | DASUM |
| Short-precision complex | Short-precision real | SCASUM |
| Long-precision complex | Long-precision real | DZASUM |

***Syntax***

| | |
|---|---|
| **Fortran** | SASUM \| DASUM \| SCASUM \| DZASUM (*n*, *x*, *incx*) |
| **C and C++** | sasum \| dasum \| scasum \| dzasum (*n*, *x*, *incx*); |
| **PL/I** | SASUM \| DASUM \| SCASUM \| DZASUM (*n*, *x*, *incx*); |

*On Entry*

*n*
: is the number of elements in vector ***x***. Specified as: a fullword integer; $n \geq 0$.

*x*
: is the vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length 1+(*n*–1)|*incx*|, containing numbers of the data type indicated in Table 40.

*incx*
: is the stride for vector ***x***. Specified as: a fullword integer. It can have any value.

*On Return*

*Function value*
: is the result of the summation. Returned as: a number of the data type indicated in Table 40.

***Note:*** Declare this function in your program as returning a value of the type indicated in Table 40.

***Function:*** SASUM and DASUM compute the sum of the absolute values of the elements of ***x***, which is expressed as follows:

$$\sum_{i=1}^{n} |x_i| = |x_1| + |x_2| + \ldots + |x_n|$$

SCASUM and DZASUM compute the sum of the absolute values of the real and imaginary parts of the elements of ***x***, which is expressed as follows:

$$\sum_{i=1}^{n}(|a_i| + |b_i|) = (|a_1| + |b_1|) + (|a_2| + |b_2|) + \dots + (|a_n| + |b_n|)$$
$$\text{where } x_i = (a_i, b_i)$$

See reference [76 ]. The result is returned as a function value. If *n* is 0, then 0.0 is returned as the value of the function. For SASUM and SCASUM, intermediate results are accumulated in long precision.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Example 1:*** This example shows a vector, ***x***, with a stride of 1.

*Function Reference and Input*

```
              N   X  INCX
              |   |   |
SUMM = SASUM( 7 , X , 1  )

X        =  (1.0, -3.0, -6.0, 7.0, 5.0, 2.0, -4.0)
```

*Output*

```
SUMM     =  28.0
```

***Example 2:*** This example shows a vector, ***x***, with a stride greater than 1.

*Function Reference and Input*

```
              N   X  INCX
              |   |   |
SUMM = SASUM( 4 , X , 2  )

X        =  (1.0, . , -6.0, . , 5.0, . , -4.0)
```

*Output*

```
SUMM     =  16.0
```

***Example 3:*** This example shows a vector, ***x***, with negative stride. Processing begins at element `X(7)`, which is –4.0.

*Function Reference and Input*

```
              N   X   INCX
              |   |    |
SUMM = SASUM( 4 , X , -2  )

X        =  (1.0, . , -6.0, . , 5.0, . , -4.0)
```

*Output*

```
SUMM     =  16.0
```

***Example 4:*** This example shows a vector, ***x***, with a stride of 0. The result in SUMM is $nx_1$.

*Function Reference and Input*

```
              N   X  INCX
              |   |   |
SUMM = SASUM( 7 , X , 0  )

X        =  (-2.0, . , . , . , . , . , . , .)
```

*Output*

```
SUMM     =  14.0
```

***Example 5:*** This example shows a vector, ***x***, containing complex numbers and having a stride of 1.

*Function Reference and Input*

```
               N   X  INCX
               |   |   |
SUMM = SCASUM( 5 , X , 1  )

X        =  ((1.0, 2.0), (-3.0, 4.0), (5.0, -6.0 ), (-7.0, -8.0),
             (9.0, 10.0))
```

*Output*

```
SUMM     =  55.0
```

## SAXPY, DAXPY, CAXPY, and ZAXPY—Multiply a Vector X by a Scalar, Add to a Vector Y, and Store in the Vector Y

These subprograms perform the following computation, using the scalar $\alpha$ and vectors $\boldsymbol{x}$ and $\boldsymbol{y}$:

$$\boldsymbol{y} \leftarrow \boldsymbol{y}+\alpha\boldsymbol{x}$$

*Table 41. Data Types*

| *alpha*, ***x***, ***y*** | **Subprogram** |
|---|---|
| Short-precision real | SAXPY |
| Long-precision real | DAXPY |
| Short-precision complex | CAXPY |
| Long-precision complex | ZAXPY |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SAXPY \| DAXPY \| CAXPY \| ZAXPY (*n*, *alpha*, *x*, *incx*, *y*, *incy*) |
| **C and C++** | saxpy \| daxpy \| caxpy \| zaxpy (*n*, *alpha*, *x*, *incx*, *y*, *incy*); |
| **PL/I** | CALL SAXPY \| DAXPY \| CAXPY \| ZAXPY (*n*, *alpha*, *x*, *incx*, *y*, *incy*); |

*On Entry*

*n*
is the number of elements in vectors ***x*** and ***y***. Specified as: a fullword integer; $n \geq 0$.

*alpha*
is the scalar *alpha*. Specified as: a number of the data type indicated in Table 41.

*x*
is the vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length $1+(n-1)|incx|$, containing numbers of the data type indicated in Table 41.

*incx*
is the stride for vector ***x***. Specified as: a fullword integer. It can have any value.

*y*
is the vector ***y*** of length *n*. Specified as: a one-dimensional array of (at least) length $1+(n-1)|incy|$, containing numbers of the data type indicated in Table 41.

*incy*
is the stride for vector ***y***. Specified as: a fullword integer. It can have any value.

*On Return*

*y*
is the vector ***y***, containing the results of the computation $\boldsymbol{y}+\alpha\boldsymbol{x}$. Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 41.

### *Notes*

1. If you specify the same vector for ***x*** and ***y***, *incx* and *incy* must be equal; otherwise, results are unpredictable.

2. If you specify different vectors for ***x*** and ***y***, they must have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.

***Function:*** The computation is expressed as follows:

$$\begin{bmatrix} y_1 \\ . \\ . \\ . \\ y_n \end{bmatrix} \leftarrow \begin{bmatrix} y_1 \\ . \\ . \\ . \\ y_n \end{bmatrix} + \alpha \begin{bmatrix} x_1 \\ . \\ . \\ . \\ x_n \end{bmatrix}$$

See reference [76 ]. If *alpha* or *n* is zero, no computation is performed. For CAXPY, intermediate results are accumulated in long precision.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Example 1:*** This example shows vectors ***x*** and ***y*** with positive strides.

*Call Statement and Input*

```
            N  ALPHA  X  INCX  Y  INCY
            |    |    |   |    |   |
CALL SAXPY( 5 , 2.0 , X , 1  , Y , 2  )

X         =  (1.0, 2.0, 3.0, 4.0, 5.0)
Y         =  (1.0, . , 1.0, . , 1.0, . , 1.0, . , 1.0)
```

*Output*

```
Y         =  (3.0, . , 5.0, . , 7.0, . , 9.0, . , 11.0)
```

***Example 2:*** This example shows vectors ***x*** and ***y*** having strides of opposite signs. For ***y***, which has negative stride, processing begins at element `Y(5)`, which is 1.0.

*Call Statement and Input*

```
            N  ALPHA  X  INCX  Y   INCY
            |    |    |   |    |    |
CALL SAXPY( 5 , 2.0 , X , 1  , Y , -1  )

X         =  (1.0, 2.0, 3.0, 4.0, 5.0)
Y         =  (5.0, 4.0, 3.0, 2.0, 1.0)
```

*Output*

```
Y         =  (15.0, 12.0, 9.0, 6.0, 3.0)
```

***Example 3:*** This example shows a vector, ***x***, with 0 stride. Vector ***x*** is treated like a vector of length *n*, all of whose elements are the same as the single element in ***x***.

*Call Statement and Input*

```
            N  ALPHA  X  INCX  Y  INCY
            |    |    |   |    |   |
CALL SAXPY( 5 , 2.0 , X , 0  , Y , 1  )

X        =  (1.0)
Y        =  (5.0, 4.0, 3.0, 2.0, 1.0)
```

*Output*

```
Y        =  (7.0, 6.0, 5.0, 4.0, 3.0)
```

***Example 4:*** This example shows how SAXPY can be used to compute a scalar value. In this case, vectors ***x*** and ***y*** contain scalar values and the strides for both vectors are 0. The number of elements to be processed, *n*, is 1.

*Call Statement and Input*

```
            N  ALPHA  X  INCX  Y  INCY
            |    |    |   |    |   |
CALL SAXPY( 1 , 2.0 , X , 0  , Y , 0  )

X        =  (1.0)
Y        =  (5.0)
```

*Output*

```
Y        =  (7.0)
```

***Example 5:*** This example shows how to use CAXPY, where vectors ***x*** and ***y*** contain complex numbers. In this case, vectors ***x*** and ***y*** have positive strides.

*Call Statement and Input*

```
            N  ALPHA  X  INCX  Y  INCY
            |    |    |   |    |   |
CALL CAXPY( 3 ,ALPHA, X , 1  , Y , 2  )

ALPHA    =  (2.0, 3.0)
X        =  ((1.0, 2.0), (2.0, 0.0), (3.0, 5.0))
Y        =  ((1.0, 1.0 ), . , (0.0, 2.0), . , (5.0, 4.0))
```

*Output*

```
Y        =  ((-3.0, 8.0), . , (4.0, 8.0), . , (-4.0, 23.0))
```

## SCOPY, DCOPY, CCOPY, and ZCOPY—Copy a Vector

These subprograms copy vector ***x*** to another vector, ***y***:

$$\boldsymbol{y} \leftarrow \boldsymbol{x}$$

*Table 42. Data Types*

| ***x***, ***y*** | **Subprogram** |
|---|---|
| Short-precision real | SCOPY |
| Long-precision real | DCOPY |
| Short-precision complex | CCOPY |
| Long-precision complex | ZCOPY |

***Syntax***

| | |
|---|---|
| **Fortran** | CALL SCOPY \| DCOPY \| CCOPY \| ZCOPY (*n*, *x*, *incx*, *y*, *incy*) |
| **C and C++** | scopy \| dcopy \| ccopy \| zcopy (*n*, *x*, *incx*, *y*, *incy*); |
| **PL/I** | CALL SCOPY \| DCOPY \| CCOPY \| ZCOPY (*n*, *x*, *incx*, *y*, *incy*); |

*On Entry*

*n*
is the number of elements in vectors ***x*** and ***y***. Specified as: a fullword integer; $n \ge 0$.

*x*
is the vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length 1+(*n*–1)|*incx*|, containing numbers of the data type indicated in Table 42.

*incx*
is the stride for vector ***x***. Specified as: a fullword integer. It can have any value.

*y*
See “On Return.”

*incy*
is the stride for vector ***y.*** Specified as: a fullword integer. It can have any value.

*On Return*

*y*
is the vector ***y*** of length *n*. Returned as: a one-dimensional array of (at least) length 1+(*n*–1)|*incy*|, containing numbers of the data type indicated in Table 42.

***Notes***

1. If you specify the same vector for ***x*** and ***y***, *incx* and *incy* must be equal; otherwise, results are unpredictable.
2. If you specify different vectors for ***x*** and ***y***, they must have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.

***Function:*** The copy is expressed as follows:

$$\begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} \leftarrow \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix}$$

See reference [76 ]. If $n$ is 0, no copy is performed.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Example 1:*** This example shows input vector ***x*** and output vector ***y*** with positive strides.

*Call Statement and Input*

```
            N   X  INCX  Y  INCY
            |   |   |    |   |
CALL SCOPY( 5 , X , 1  , Y , 2  )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
```

*Output*

```
Y        =  (1.0, . , 2.0, . , 3.0, . , 4.0, . , 5.0)
```

***Example 2:*** This example shows how to obtain a reverse copy of the input vector ***x*** by specifying strides with the same absolute value, but with opposite signs, for input vector ***x*** and output vector ***y***. For ***y***, which has a negative stride, results are stored beginning at element `Y(5)`.

*Call Statement and Input*

```
            N   X  INCX  Y   INCY
            |   |   |    |    |
CALL SCOPY( 5 , X , 1  , Y , -1  )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
```

*Output*

```
Y        =  (5.0, 4.0, 3.0, 2.0, 1.0)
```

***Example 3:*** This example shows an input vector, ***x***, with 0 stride. Vector ***x*** is treated like a vector of length $n$, all of whose elements are the same as the single element in ***x***. This is a technique for replicating an element of a vector.

*Call Statement and Input*

```
            N   X  INCX  Y  INCY
            |   |   |    |   |
CALL SCOPY( 5 , X , 0  , Y , 1  )

X        =  (13.0)
```

*Output*

```
Y        =  (13.0, 13.0, 13.0, 13.0, 13.0)
```

***Example 4:*** This example shows input vector ***x*** and output vector ***y***, containing complex numbers and having positive strides.

*Call Statement and Input*

```
            N   X  INCX  Y  INCY
            |   |   |    |   |
CALL CCOPY( 4 , X , 1  , Y , 2  )

X        =  ((1.0, 1.0), (2.0, 2.0), (3.0, 3.0), (4.0, 4.0))
```

*Output*

```
Y        =  ((1.0, 1.0), . , (2.0, 2.0), . , (3.0, 3.0), . ,
             (4.0, 4.0))
```

# SDOT, DDOT, CDOTU, ZDOTU, CDOTC, and ZDOTC—Dot Product of Two Vectors

SDOT, DDOT, CDOTU, and ZDOTU compute the dot product of vectors ***x*** and ***y***:

$$x \bullet y$$

CDOTC and ZDOTC compute the dot product of the complex conjugate of vector ***x*** with vector ***y***:

$$\bar{x} \bullet y$$

*Table 43. Data Types*

| ***x***, ***y***, **Result** | **Subprogram** |
|---|---|
| Short-precision real | SDOT |
| Long-precision real | DDOT |
| Short-precision complex | CDOTU and CDOTC |
| Long-precision complex | ZDOTU and ZDOTC |

### *Syntax*

| | |
|---|---|
| **Fortran** | SDOT \| DDOT \| CDOTU \| ZDOTU \| CDOTC \| ZDOTC (*n*, *x*, *incx*, *y*, *incy*) |
| **C and C++** | sdot \| ddot \| cdotu \| zdotu \| cdotc \| zdotc (*n*, *x*, *incx*, *y*, *incy*); |
| **PL/I** | SDOT \| DDOT \| CDOTU \| ZDOTU \| CDOTC \| ZDOTC (*n*, *x*, *incx*, *y*, *incy*); |

*On Entry*

*n*
is the number of elements in vectors ***x*** and ***y***. Specified as: a fullword integer; $n \geq 0$.

*x*
is the vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length 1+(*n*–1)|*incx*|, containing numbers of the data type indicated in Table 43.

*incx*
is the stride for vector ***x***. Specified as: a fullword integer. It can have any value.

*y*
is the vector ***y*** of length *n*. Specified as: a one-dimensional array of (at least) length 1+(*n*–1)|*incy*|, containing numbers of the data type indicated in Table 43.

*incy*
is the stride for vector ***y***. Specified as: a fullword integer. It can have any value.

*On Return*

*Function value*
is the result of the dot product computation. Returned as: a number of the data type indicated in Table 43.

***Note:*** Declare this function in your program as returning a value of the data type indicated in Table 43.

***Function:*** SDOT, DDOT, CDOTU, and ZDOTU compute the dot product of the vectors ***x*** and ***y***, which is expressed as follows:

$$x \bullet y = x_1 y_1 + x_2 y_2 + \ldots + x_n y_n$$

CDOTC and ZDOTC compute the dot product of the complex conjugate of vector ***x*** with vector ***y***, which is expressed as follows:

$$\bar{x} \bullet y = \bar{x}_1 y_1 + \bar{x}_2 y_2 + \ldots + \bar{x}_n y_n$$

See reference [76 ]. The result is returned as a function value. If *n* is 0, then zero is returned as the value of the function.

For SDOT, CDOTU, and CDOTC, intermediate results are accumulated in long precision.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Example 1:*** This example shows how to compute the dot product of two vectors, ***x*** and ***y***, having strides of 1.

*Function Reference and Input*

```
             N   X  INCX  Y  INCY
             |   |   |    |   |
DOTT = SDOT( 5 , X , 1  , Y , 1  )

X        =  (1.0, 2.0, -3.0, 4.0, 5.0)
Y        =  (9.0, 8.0, 7.0, -6.0, 5.0)
```

*Output*

```
DOTT     =  (9.0 + 16.0 - 21.0 - 24.0 + 25.0) = 5.0
```

***Example 2:*** This example shows how to compute the dot product of a vector, ***x***, with a stride of 1, and a vector, ***y***, with a stride greater than 1.

*Function Reference and Input*

```
             N   X  INCX  Y  INCY
             |   |   |    |   |
DOTT = SDOT( 5 , X , 1  , Y , 2  )

X        =  (1.0, 2.0, -3.0, 4.0, 5.0)
Y        =  (9.0, . , 7.0, . , 5.0, . , -3.0, . , 1.0)
```

*Output*

```
DOTT     =  (9.0 + 14.0 - 15.0 - 12.0 + 5.0) = 1.0
```

***Example 3:*** This example shows how to compute the dot product of a vector, ***x***, with a negative stride, and a vector, ***y***, with a stride greater than 1. For ***x***, processing begins at element `X(5)`, which is 5.0.

*Function Reference and Input*

```
             N   X   INCX  Y  INCY
             |   |    |    |   |
DOTT = SDOT( 5 , X , -1  , Y , 2  )

X        =  (1.0, 2.0, -3.0, 4.0, 5.0)
Y        =  (9.0, . , 7.0, . , 5.0, . , -3.0, . , 1.0)
```

*Output*

```
DOTT     =  (45.0 + 28.0 - 15.0 - 6.0 + 1.0) = 53.0
```

***Example 4:*** This example shows how to compute the dot product of a vector, ***x***, with a stride of 0, and a vector, ***y***, with a stride of 1. The result in `DOTT` is $x_1(y_1+...+y_n)$.

*Function Reference and Input*

```
             N   X  INCX  Y  INCY
             |   |   |    |   |
DOTT = SDOT( 5 , X , 0  , Y , 1  )

X        =  (1.0, . , . , . , .)
Y        =  (9.0, 8.0, 7.0, -6.0, 5.0)
```

*Output*

```
DOTT     =  (1.0) × (9.0 + 8.0 + 7.0 - 6.0 + 5.0) = 23.0
```

***Example 5:*** This example shows how to compute the dot product of two vectors, ***x*** and ***y***, with strides of 0. The result in `DOTT` is $nx_1y_1$.

*Function Reference and Input*

```
             N   X  INCX  Y  INCY
             |   |   |    |   |
DOTT = SDOT( 5 , X , 0  , Y , 0  )

X        =  (1.0, . , . , . , .)
Y        =  (9.0, . , . , . , .)
```

*Output*

```
DOTT     =  (5) × (1.0) × (9.0) = 45.0
```

***Example 6:*** This example shows how to compute the dot product of two vectors, ***x*** and ***y***, containing complex numbers, where ***x*** has a stride of 1, and ***y*** has a stride greater than 1.

*Function Reference and Input*

```
              N   X  INCX  Y  INCY
              |   |   |    |   |
DOTT = CDOTU( 3 , X , 1  , Y , 2  )

X        =  ((1.0, 2.0), (3.0, -4.0), (-5.0, 6.0))
Y        =  ((10.0, 9.0), . , (-6.0, 5.0), . , (2.0, 1.0))
```

*Output*

```
DOTT     =  ((10.0 - 18.0 - 10.0) - (18.0 - 20.0 + 6.0),
             (9.0 + 15.0 - 5.0) + (20.0 + 24.0 + 12.0))
         =  (-22.0, 75.0)
```

***Example 7:*** This example shows how to compute the dot product of the conjugate of a vector, ***x***, with vector ***y***, both containing complex numbers, where ***x*** has a stride of 1, and ***y*** has a stride greater than 1.

*Function Reference and Input*

```
              N   X  INCX  Y  INCY
              |   |   |    |   |
DOTT = CDOTC( 3 , X , 1  , Y , 2  )

X        =  ((1.0, 2.0), (3.0, -4.0), (-5.0, 6.0))
Y        =  ((10.0, 9.0), . , (-6.0, 5.0), . , (2.0, 1.0))
```

*Output*

```
DOTT     =  ((10.0 - 18.0 - 10.0) + (18.0 - 20.0 + 6.0),
             (9.0  +  15.0  -  5.0)  -  (20.0  +  24.0  +  12.0))
         =  (-14.0, -37.0)
```

## SNAXPY and DNAXPY—Compute SAXPY or DAXPY N Times

These subprograms compute SAXPY or DAXPY, respectively, *n* times:

$$\boldsymbol{y}_i \leftarrow \boldsymbol{y}_i + \alpha_i \boldsymbol{x}_i \quad \text{for } i = 1, n$$

where each $\alpha_i$ is a scalar value, contained in the vector ***a***, and each $\boldsymbol{x}_i$ and $\boldsymbol{y}_i$ are vectors, contained in vectors (or matrices) ***x*** and ***y***, respectively. For an explanation of the SAXPY and DAXPY computations, see "SAXPY, DAXPY, CAXPY, and ZAXPY—Multiply a Vector X by a Scalar, Add to a Vector Y, and Store in the Vector Y" on page 216.

*Table 44. Data Types*

| ***a***, ***x***, ***y*** | **Subprogram** |
|---|---|
| Short-precision real | SNAXPY |
| Long-precision real | DNAXPY |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SNAXPY \| DNAXPY (*n*, *m*, *a*, *inca*, *x*, *incxi*, *incxo*, *y*, *incyi*, *incyo*) |
| **C and C++** | snaxpy \| dnaxpy (*n*, *m*, *a*, *inca*, *x*, *incxi*, *incxo*, *y*, *incyi*, *incyo*); |
| **PL/I** | CALL SNAXPY \| DNAXPY (*n*, *m*, *a*, *inca*, *x*, *incxi*, *incxo*, *y*, *incyi*, *incyo*); |

*On Entry*

*n*

is the number of SAXPY or DAXPY computations to be performed and the number of elements in vector ***a***. Specified as: a fullword integer; $n \geq 0$.

*m*

is the number of elements in vectors $\boldsymbol{x}_i$ and $\boldsymbol{y}_i$ for each SAXPY or DAXPY computation. Specified as: a fullword integer; $m \geq 0$.

*a*

is the vector ***a*** of length *n*, containing the scalar values $\alpha_i$, used in each computation of $\boldsymbol{y}_i + \alpha_i \boldsymbol{x}_i$. Specified as: a one-dimensional array of (at least) length $1+(n-1)|inca|$, containing numbers of the data type indicated in Table 44.

*inca*

is the stride for vector ***a***. Specified as: a fullword integer. It can have any value.

*x*

is the vector (or matrix) ***x***, containing the $\boldsymbol{x}_i$ vectors of length *m*, used in the *n* computations of $\boldsymbol{y}_i + \alpha_i \boldsymbol{x}_i$. Specified as: a one- or two-dimensional array of (at least) length $(1+(n-1)(incxo)) + (m-1)|incxi|$, containing numbers of the data type indicated in Table 44.

*incxi*

is the stride for ***x*** in the inner loop—that is, the stride identifying the elements in each vector $\boldsymbol{x}_i$. Specified as: a fullword integer. It can have any value.

*incxo*

is the stride for ***x*** in the outer loop—that is, the stride identifying each vector $\boldsymbol{x}_i$ in ***x***. Specified as: a fullword integer; $incxo \geq 0$.

*y*

is the vector (or matrix) ***y***, containing the $\boldsymbol{y}_i$ vectors of length *m*, used in the *n* computations of $\boldsymbol{y}_i + \alpha_i \boldsymbol{x}_i$. Specified as: a one- or two-dimensional array of (at least) length $(1+(n-1)(incyo)) + (m-1)|incyi|$, containing numbers of the data type indicated in Table 44.

*incyi*
is the stride for $\boldsymbol{y}$ in the inner loop—that is, the stride identifying the elements in each vector $\boldsymbol{y}_i$ in $\boldsymbol{y}$. Specified as: a fullword integer; $incyi > 0$ or $incyi < 0$.

*incyo*
is the stride for $\boldsymbol{y}$ in the outer loop—that is, the stride identifying each vector $\boldsymbol{y}_i$ in $\boldsymbol{y}$. Specified as: a fullword integer; $incyo \geq 0$.

*On Return*

*y*
is the vector (or matrix) $\boldsymbol{y}$, containing the $\boldsymbol{y}_i$ vectors of length *m*, which contain the results of the *n* SAXPY or DAXPY computations, $\boldsymbol{y}_i + \alpha_i\boldsymbol{x}_i$ for $i = 1, n$. Returned as: a one- or two-dimensional array, containing numbers of the data type indicated in Table 44 on page 226.

***Note:*** Vector $\boldsymbol{y}$ must have no common elements with vector $\boldsymbol{a}$ or vector $\boldsymbol{x}$; otherwise, results are unpredictable. See “Concepts” on page 55.

***Function:*** The SAXPY or DAXPY computations:

$$\boldsymbol{y} \leftarrow \boldsymbol{y} + \alpha\boldsymbol{x}$$

are performed *n* times. This is expressed as follows:

$$\boldsymbol{y}_i \leftarrow \boldsymbol{y}_i + \alpha_i\boldsymbol{x}_i \quad \text{for } i = 1, n$$

where each $\alpha_i$ is a scalar value, contained in the vector $\boldsymbol{a}$, and each $\boldsymbol{x}_i$ and $\boldsymbol{y}_i$ are vectors, contained in vectors (or matrices) $\boldsymbol{x}$ and $\boldsymbol{y}$, respectively.

Each computation of SAXPY or DAXPY on page 216 uses the length of the $\boldsymbol{x}_i$ and $\boldsymbol{y}_i$ vectors, *m*, for its input argument, *n*. It also uses the strides for the inner loop, *incxi* and *incyi*, for its parameters *incx* and *incy*, respectively. See “Function” on page 217 for a description of how the computation is done.

The outer loop of the SNAXPY or DNAXPY computation uses the strides of *inca*, *incxo*, and *incyo* to locate the elements in $\boldsymbol{a}$ and vectors in $\boldsymbol{x}$ and $\boldsymbol{y}$ for each *i*-th computation. These are:

For $i = 1, n$:

$\alpha_{((i-1)inca+1)}$ for $inca \geq 0$
$\alpha_{((i-n)inca+1)}$ for $inca < 0$
$x_{((i-1)incxo+1)}$
$y_{((i-1)incyo+1)}$

If *m* or *n* is 0, no computation is performed.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. $n < 0$
2. $m < 0$
3. $incxo < 0$
4. $incyi = 0$
5. $incyo < 0$

***Example 1:*** This example shows vectors, contained in matrices, with the stride of the inner loops *incxi* and *incyi* equal to 1.

*Call Statement and Input*

```
                N   M   A  INCA  X  INCXI INCXO  Y  INCYI INCYO
                |   |   |   |    |    |     |    |    |     |
CALL SNAXPY( 3 , 4 , A , 1  , X ,  1  , 10  , Y ,  1  ,  5  )

A        =  (3.0, 2.0, 4.0)

          ┌               ┐
          │ 1.0  4.0  3.0 │
          │ 2.0  3.0  4.0 │
          │ 3.0  2.0  2.0 │
          │ 4.0  1.0  1.0 │
X    =    │  .    .    .  │
          │  .    .    .  │
          │  .    .    .  │
          │  .    .    .  │
          │  .    .    .  │
          │  .    .    .  │
          └               ┘

          ┌               ┐
          │ 4.0  1.0  3.0 │
          │ 3.0  2.0  4.0 │
Y    =    │ 2.0  3.0  2.0 │
          │ 1.0  4.0  1.0 │
          │  .    .    .  │
          └               ┘
```

*Output*

```
          ┌                 ┐
          │  7.0  9.0  15.0 │
          │  9.0  8.0  20.0 │
Y    =    │ 11.0  7.0  10.0 │
          │ 13.0  6.0   5.0 │
          │   .    .     .  │
          └                 ┘
```

***Example 2:*** This example shows vectors, contained in matrices, with a stride of the inner loop *incxi* greater than 1.

*Call Statement and Input*

```
                N   M   A  INCA  X  INCXI INCXO  Y  INCYI INCYO
                |   |   |   |    |    |     |    |    |     |
CALL SNAXPY( 3 , 4 , A , 1  , X ,  2  , 10  , Y ,  1  ,  5  )

A        =  (3.0, 2.0, 4.0)
```

```
                ┌                ┐
                │ 1.0  4.0  3.0  │
                │  .    .    .   │
                │ 2.0  3.0  4.0  │
                │  .    .    .   │
X     =         │ 3.0  2.0  2.0  │
                │  .    .    .   │
                │ 4.0  1.0  1.0  │
                │  .    .    .   │
                │  .    .    .   │
                │  .    .    .   │
                └                ┘
```

```
                ┌                ┐
                │ 4.0  1.0  3.0  │
                │ 3.0  2.0  4.0  │
Y     =         │ 2.0  3.0  2.0  │
                │ 1.0  4.0  1.0  │
                │  .    .    .   │
                └                ┘
```

*Output*

```
Y          =(same as output Y in Example 1)
```

***Example 3:*** This example shows vectors, contained in matrices, with a negative stride, *incyi*, for the inner loop.

*Call Statement and Input*

```
             N   M   A  INCA  X  INCXI INCXO  Y  INCYI INCYO
             |   |   |   |    |    |     |    |    |     |
CALL SNAXPY( 3 , 4 , A , 1  , X ,  1  , 10  , Y , -1  ,  5  )

A        =  (3.0, 2.0, 4.0)
```

```
                ┌                ┐
                │ 1.0  4.0  3.0  │
                │ 2.0  3.0  4.0  │
                │ 3.0  2.0  2.0  │
                │ 4.0  1.0  1.0  │
X     =         │  .    .    .   │
                │  .    .    .   │
                │  .    .    .   │
                │  .    .    .   │
                │  .    .    .   │
                │  .    .    .   │
                └                ┘
```

```
                ┌                ┐
                │ 1.0  4.0  1.0  │
                │ 2.0  3.0  2.0  │
Y     =         │ 3.0  2.0  4.0  │
                │ 4.0  1.0  3.0  │
                │  .    .    .   │
                └                ┘
```

*Output*

```
        ┌                  ┐
        | 13.0  6.0   5.0  |
        | 11.0  7.0  10.0  |
Y    =  |  9.0  8.0  20.0  |
        |  7.0  9.0  15.0  |
        |   .    .     .   |
        └                  ┘
```

***Example 4:*** This example shows vectors, contained in matrices, with a negative stride, *inca*, for vector ***a***. For vector ***a***, processing begins at element `A(5)`, which is 3.0.

*Call Statement and Input*

```
             N   M   A   INCA  X  INCXI INCXO  Y  INCYI INCYO
             |   |   |    |    |    |     |    |    |     |
CALL SNAXPY( 3 , 4 , A , -2  , X ,  1   , 10  , Y ,  1  ,  5  )

A        =  (4.0, . , 2.0, . , 3.0)

        ┌               ┐
        | 1.0  4.0  3.0 |
        | 2.0  3.0  4.0 |
        | 3.0  2.0  2.0 |
        | 4.0  1.0  1.0 |
X    =  |  .    .    .  |
        |  .    .    .  |
        |  .    .    .  |
        |  .    .    .  |
        |  .    .    .  |
        |  .    .    .  |
        └               ┘

        ┌               ┐
        | 4.0  1.0  3.0 |
        | 3.0  2.0  4.0 |
Y    =  | 2.0  3.0  2.0 |
        | 1.0  4.0  1.0 |
        |  .    .    .  |
        └               ┘
```

*Output*

`Y` =(same as output Y in Example 1)

## SNDOT and DNDOT—Compute Special Dot Products N Times

These subprograms compute one of the following special dot products *n* times:

| | |
|---|---|
| $s_i \leftarrow \mathbf{x}_i \bullet \mathbf{y}_i$ | Store positive dot product |
| $s_i \leftarrow -\mathbf{x}_i \bullet \mathbf{y}_i$ | Store negative dot product |
| $s_i \leftarrow s_i + \mathbf{x}_i \bullet \mathbf{y}_i$ | Accumulate positive dot product |
| $s_i \leftarrow s_i - \mathbf{x}_i \bullet \mathbf{y}_i$ | Accumulate negative dot product |

for $i = 1, n$

where each $s_i$ is an element in vector $\mathbf{s}$, and each $\mathbf{x}_i$ and $\mathbf{y}_i$ are vectors contained in vectors (or matrices) $\mathbf{x}$ and $\mathbf{y}$, respectively.

*Table 45. Data Types*

| ***s***, ***x***, ***y*** | **Subprogram** |
|---|---|
| Short-precision real | SNDOT |
| Long-precision real | DNDOT |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SNDOT \| DNDOT (*n*, *m*, *s*, *incs*, *isw*, *x*, *incxi*, *incxo*, *y*, *incyi*, *incyo*) |
| **C and C++** | sndot \| dndot (*n*, *m*, *s*, *incs*, *isw*, *x*, *incxi*, *incxo*, *y*, *incyi*, *incyo*); |
| **PL/I** | CALL SNDOT \| DNDOT (*n*, *m*, *s*, *incs*, *isw*, *x*, *incxi*, *incxo*, *y*, *incyi*, *incyo*); |

*On Entry*

*n*

is the number of dot product computations to be performed and the number of elements in the vector $\mathbf{s}$. Specified as: a fullword integer; $n \geq 0$.

*m*

is the number of elements in vectors $\mathbf{x}_i$ and $\mathbf{y}_i$ for each dot product computation. Specified as: a fullword integer; $m \geq 0$.

*s*

is the vector $\mathbf{s}$, containing the *n* scalar values $s_i$, where: If $isw = 1$ or 2, $s_i$ is not used in the computation (no input value specified.)

If $isw = 3$ or 4, $s_i$ is used in the computation (input value specified.)

Specified as: a one-dimensional array of (at least) length $1+(n-1)|incs|$, containing numbers of the data type indicated in Table 45.

*incs*

is the stride for vector $\mathbf{s}$. Specified as: a fullword integer; $incs > 0$ or $incs < 0$.

*isw*

indicates the type of computation to perform, depending on the value specified:

If $isw = 1$, $s_i \leftarrow \mathbf{x}_i \bullet \mathbf{y}_i$

If $isw = 2$, $s_i \leftarrow -\mathbf{x}_i \bullet \mathbf{y}_i$

If $isw = 3$, $s_i \leftarrow s_i + \mathbf{x}_i \bullet \mathbf{y}_i$

If $isw = 4$, $s_i \leftarrow s_i - \mathbf{x}_i \bullet \mathbf{y}_i$

where $i = 1, n$

Specified as: a fullword integer. Its value must be 1, 2, 3, or 4.

*x*

is the vector (or matrix) $\mathbf{x}$, containing the $\mathbf{x}_i$ vectors of length *m*, used in the *n* dot product computations. Specified as: a one- or two-dimensional array of (at least) length $(1+(n-1)(incxo))+(m-1)|incxi|$, containing numbers of the data type indicated in Table 45.

*incxi*

is the stride for $\mathbf{x}$ in the inner loop—that is, the stride identifying the elements in each vector $\mathbf{x}_i$. Specified as: a fullword integer. It can have any value.

*incxo*

is the stride for $\mathbf{x}$ in the outer loop—that is, the stride identifying each vector $\mathbf{x}_i$ in $\mathbf{x}$. Specified as: a fullword integer; $incxo \geq 0$.

*y*

is the vector (or matrix) $\mathbf{y}$, containing the $\mathbf{y}_i$ vectors of length *m*, used in the *n* dot product computations. Specified as: a one- or two-dimensional array of (at least) length $(1+(n-1)(incyo)) + (m-1)|incyi|$, containing numbers of the data type indicated in Table 45 on page 231.

*incyi*

is the stride for $\mathbf{y}$ in the inner loop—that is, the stride identifying the elements in each vector $\mathbf{y}_i$. Specified as: a fullword integer. It can have any value.

*incyo*

is the stride for $\mathbf{y}$ in the outer loop—that is, the stride identifying each vector $\mathbf{y}_i$ in $\mathbf{y}$. Specified as: a fullword integer; $incyo \geq 0$.

*On Return*

*s*

is the vector $\mathbf{s}$ of length *n*, containing the results of the *n* dot product computations. The type of dot product computation depends of the value specified for *isw*.

If *isw* = 1, $s_i \leftarrow \mathbf{x}_i \bullet \mathbf{y}_i$

If *isw* = 2, $s_i \leftarrow -\mathbf{x}_i \bullet \mathbf{y}_i$

If *isw* = 3, $s_i \leftarrow s_i + \mathbf{x}_i \bullet \mathbf{y}_i$

If *isw* = 4, $s_i \leftarrow s_i - \mathbf{x}_i \bullet \mathbf{y}_i$
where $i = 1, n$

Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 45 on page 231.

***Function:*** The four possible computations that can be performed by these subprograms are:

| | |
|---|---|
| $s_i \leftarrow \mathbf{x}_i \bullet \mathbf{y}_i$ | Store positive dot product |
| $s_i \leftarrow -\mathbf{x}_i \bullet \mathbf{y}_i$ | Store negative dot product |
| $s_i \leftarrow s_i+\mathbf{x}_i \bullet \mathbf{y}_i$ | Accumulate positive dot product |
| $s_i \leftarrow s_i-\mathbf{x}_i \bullet \mathbf{y}_i$ | Accumulate negative dot product |

for $i = 1, n$

where each $s_i$ is a scalar element in the vector $\mathbf{s}$ of length *n*, and each of the *n* $\mathbf{x}_i$ and $\mathbf{y}_i$ vectors of length *m* are contained in vectors (or matrices) $\mathbf{x}$ and $\mathbf{y}$, respectively. Each computation uses the dot product, which is expressed:

$$\mathbf{x}_i \bullet \mathbf{y}_i = u_1v_1 + u_2v_2 + ... + u_mv_m$$

where $u_i$ and $v_i$ are elements of $\mathbf{x}_i$ and $\mathbf{y}_i$, respectively. To find the elements for the computation, it uses:

- The strides for the inner loops, *incxi* and *incyi*, to locate the elements in vectors $\mathbf{x}_i$ and $\mathbf{y}_i$, respectively.
- The strides for the outer loops, *incs, incxo*, and *incyo*, to locate the element $s_i$ in vector ***s*** and the vectors $\mathbf{x}_i$ and $\mathbf{y}_i$ in vectors (or matrices) ***x*** and ***y***, respectively.

If *m* or *n* is 0, no computation is performed. For SNDOT, intermediate results are accumulated in long precision.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. $n < 0$
2. $m < 0$
3. $incs = 0$
4. $isw < 1$ or $isw > 4$
5. $incxo < 0$
6. $incyo < 0$

***Example 1:*** This example shows a store positive dot product computation using vectors with positive strides.

*Call Statement and Input*

```
            N   M   S  INCS ISW  X  INCXI INCXO  Y  INCYI INCYO
            |   |   |   |    |   |    |     |    |    |     |
CALL SNDOT( 3 , 4 , S , 1  , 1 , X ,  1  ,  4  , Y ,  1  ,  4  )


          ┌                ┐
          | 1.0  2.0  3.0 |
X    =    | 2.0  3.0  4.0 |
          | 3.0  4.0  5.0 |
          | 4.0  5.0  6.0 |
          └                ┘


          ┌                ┐
          | 4.0  3.0  2.0 |
Y    =    | 3.0  2.0  1.0 |
          | 2.0  1.0  4.0 |
          | 1.0  4.0  3.0 |
          └                ┘
```

*Output*

```
S        =  (20.0, 36.0, 48.0)
```

***Example 2:*** This example shows a store negative dot product computation using vectors with positive and negative strides.

*Call Statement and Input*

```
            N   M   S   INCS ISW  X  INCXI INCXO  Y  INCYI INCYO
            |   |   |    |    |   |    |     |    |    |     |
CALL SNDOT( 3 , 4 , S , -1  , 2 , X ,  2   , 10  , Y , -1  ,  6  )

        ┌                ┐
        │ 1.0  2.0  3.0  │
        │  .    .    .   │
        │ 2.0  3.0  4.0  │
        │  .    .    .   │
X    =  │ 3.0  4.0  5.0  │
        │  .    .    .   │
        │ 4.0  5.0  6.0  │
        │  .    .    .   │
        │  .    .    .   │
        │  .    .    .   │
        └                ┘

        ┌                ┐
        │ 4.0  3.0  2.0  │
        │ 3.0  2.0  1.0  │
Y    =  │ 2.0  1.0  4.0  │
        │ 1.0  4.0  3.0  │
        │  .    .    .   │
        │  .    .    .   │
        └                ┘
```

*Output*

```
S        =  (-42.0, -34.0, -30.0)
```

***Example 3:*** This example shows an accumulative positive dot product using vectors with positive and negative strides.

*Call Statement and Input*

```
            N   M   S  INCS ISW  X  INCXI INCXO  Y  INCYI INCYO
            |   |   |   |    |   |    |     |    |    |     |
CALL SNDOT( 3 , 4 , S , 1  , 3 , X , -2   , 10  , Y ,  2  , 10  )

S        =  (2.0, 5.0, 8.0)

        ┌                ┐
        │ 1.0  2.0  3.0  │
        │  .    .    .   │
        │ 2.0  3.0  4.0  │
        │  .    .    .   │
X    =  │ 3.0  4.0  5.0  │
        │  .    .    .   │
        │ 4.0  5.0  6.0  │
        │  .    .    .   │
        │  .    .    .   │
        │  .    .    .   │
        └                ┘
```

```
          ┌                ┐
          │ 4.0  3.0  2.0  │
          │  .    .    .   │
          │ 3.0  2.0  1.0  │
          │  .    .    .   │
Y    =    │ 2.0  1.0  4.0  │
          │  .    .    .   │
          │ 1.0  4.0  3.0  │
          │  .    .    .   │
          │  .    .    .   │
          │  .    .    .   │
          └                ┘
```

*Output*

```
S        =  (32.0, 39.0, 50.0)
```

***Example 4:*** This example shows an accumulative negative dot product using vectors with positive and negative strides.

*Call Statement and Input*

```
             N   M   S   INCS ISW  X  INCXI INCXO  Y  INCYI INCYO
             |   |   |    |    |   |    |     |    |    |     |
CALL SNDOT( 3 , 4 , S , -1  , 4 , X ,  1  ,  6  , Y ,  2  , 10  )

S        =  (3.0, 6.0, 9.0)
          ┌                ┐
          │ 1.0  2.0  3.0  │
          │ 2.0  3.0  4.0  │
X    =    │ 3.0  4.0  5.0  │
          │ 4.0  5.0  6.0  │
          │  .    .    .   │
          │  .    .    .   │
          └                ┘

          ┌                ┐
          │ 4.0  3.0  2.0  │
          │  .    .    .   │
          │ 3.0  2.0  1.0  │
          │  .    .    .   │
Y    =    │ 2.0  1.0  4.0  │
          │  .    .    .   │
          │ 1.0  4.0  3.0  │
          │  .    .    .   │
          │  .    .    .   │
          │  .    .    .   │
          └                ┘
```

*Output*

```
S        =  (-45.0, -30.0, -11.0)
```

# SNRM2, DNRM2, SCNRM2, and DZNRM2—Euclidean Length of a Vector with Scaling of Input to Avoid Destructive Underflow and Overflow

These subprograms compute the Euclidean length ($l_2$ norm) of vector ***x***, with scaling of input to avoid destructive underflow and overflow.

*Table 46. Data Types*

| ***x*** | **Result** | **Subprogram** |
|---|---|---|
| Short-precision real | Short-precision real | SNRM2 |
| Long-precision real | Long-precision real | DNRM2 |
| Short-precision complex | Short-precision real | SCNRM2 |
| Long-precision complex | Long-precision real | DZNRM2 |

**Note:** If there is a possibility that your data will cause the computation to overflow or underflow, you should use these subroutines instead of SNORM2, DNORM2, CNORM2, and ZNORM2, because the intermediate computational results may exceed the maximum or minimum limits of the machine. “Notes” on page 239 explains how to estimate whether your data will cause an overflow or underflow.

***Syntax***

| | |
|---|---|
| **Fortran** | SNRM2 \| DNRM2 \| SCNRM2 \| DZNRM2 (*n*, *x*, *incx*) |
| **C and C++** | snrm2 \| dnrm2 \| scnrm2 \| dznrm2 (*n*, *x*, *incx*); |
| **PL/I** | SNRM2 \| DNRM2 \| SCNRM2 \| DZNRM2 (*n*, *x*, *incx*); |

*On Entry*

*n*
is the number of elements in vector ***x***. Specified as: a fullword integer; $n \geq 0$.

*x*
is the vector ***x*** of length *n*, whose Euclidean length is to be computed. Specified as: a one-dimensional array of (at least) length $1+(n-1)|incx|$, containing numbers of the data type indicated in Table 46.

*incx*
is the stride for vector ***x***. Specified as: a fullword integer. It can have any value.

*On Return*

*Function value*
is the Euclidean length ($l_2$ norm) of the vector ***x***. Returned as: a number of the data type indicated in Table 46.

***Note:*** Declare this function in your program as returning a value of the data type indicated in Table 46.

***Function:*** The Euclidean length ($l_2$ norm) of vector ***x*** is expressed as follows, with scaling of input to avoid destructive underflow and overflow:

$$\sqrt{|x_1|^2 + |x_2|^2 + \ldots + |x_n|^2}$$

See reference [76 ]. The result is returned as the function value. If *n* is 0, then 0.0 is returned as the value of the function.

For SNRM2 and SCNRM2, the sum of the squares of the absolute values of the elements is accumulated in long precision. The square root of this long-precision sum is then computed and, if necessary, is unscaled.

Although these subroutines eliminate destructive underflow, nondestructive underflows may occur if the input elements differ greatly in magnitude. This does not affect accuracy, but it degrades performance. The system default is to mask underflow, which improves the performance of these subroutines.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Important Information About the Following Examples:*** Workstations use workstation architecture precisions: ANSI/IEEE 32-bit and 64-bit binary floating-point format. The ranges are:

- For short-precision: $3.37\times10^{-38}$ to $3.37\times10^{38}$
- For long-precision: $1.67\times10^{-308}$ to $1.67\times10^{308}$

***Example 1:*** This example shows a vector, ***x***, whose elements must be scaled to prevent overflow.

*Function Reference and Input*

```
               N   X  INCX
               |   |   |
DNORM = DNRM2( 6 , X , 1  )

X      = (0.68056D+200, 0.25521D+200, 0.34028D+200,
          0.85071D+200, 0.25521D+200, 0.85071D+200)
```

*Output*

```
DNORM    =  0.1469D+201
```

***Example 2:*** This example shows a vector, ***x***, whose elements must be scaled to prevent destructive underflow.

*Function Reference and Input*

```
               N   X  INCX
               |   |   |
DNORM = DNRM2( 4 , X , 2  )

X     = (0.10795D-200, . , 0.10795D-200, . , 0.10795D-200,
         . , 0.10795D-200)
```

*Output*

```
DNORM    =  0.21590D-200
```

***Example 3:*** This example shows a vector, ***x***, with a stride of 0. The result in `SNORM` is:

$$\sqrt{nx_1^2}$$

*Function Reference and Input*

```
               N   X  INCX
               |   |   |
SNORM = SNRM2( 4 , X , 0  )

X        =  (4.0)
```

*Output*

```
SNORM    =  8.0
```

***Example 4:*** This example shows a vector, ***x***, containing complex numbers, and whose elements must be scaled to prevent overflow.

*Function Reference and Input*

```
                 N   X  INCX
                 |   |   |
DZNORM = DZNRM2( 3 , X , 1  )

X     = ((0.68056D+200, 0.25521D+200), (0.34028D+200, 0.85071D+200),
         (0.25521D+200, 0.85071D+200))
```

*Output*

```
DZNORM   =  0.1469D+201
```

***Example 5:*** This example shows a vector, ***x***, containing complex numbers, and whose elements must be scaled to prevent destructive underflow.

*Function Reference and Input*

```
                 N   X  INCX
                 |   |   |
DZNORM = DZNRM2( 2 , X , 2  )

X     = ((0.10795D-200, 0.10795D-200), . ,
         (0.10795D-200, 0.10795D-200))
```

*Output*

```
DZNORM   =  0.2159D-200
```

## SNORM2, DNORM2, CNORM2, and ZNORM2—Euclidean Length of a Vector with No Scaling of Input

These subprograms compute the euclidean length ($l_2$ norm) of vector ***x*** with no scaling of input.

*Table 47. Data Types*

| *x* | Result | Subprogram |
|---|---|---|
| Short-precision real | Short-precision real | SNORM2 |
| Long-precision real | Long-precision real | DNORM2 |
| Short-precision complex | Short-precision real | CNORM2 |
| Long-precision complex | Long-precision real | ZNORM2 |

***Syntax***

| | |
|---|---|
| **Fortran** | SNORM2 \| DNORM2 \| CNORM2 \| ZNORM2 (*n*, *x*, *incx*) |
| **C and C++** | snorm2 \| dnorm2 \| cnorm2 \| znorm2 (*n*, *x*, *incx*); |
| **PL/I** | SNORM2 \| DNORM2 \| CNORM2 \| ZNORM2 (*n*, *x*, *incx*); |

*On Entry*

*n*
is the number of elements in vector ***x***. Specified as: a fullword integer; $n \geq 0$.

*x*
is the vector ***x*** of length *n*, whose euclidean length is to be computed. Specified as: a one-dimensional array of (at least) length $1+(n-1)|incx|$, containing numbers of the data type indicated in Table 47.

*incx*
is the stride for vector ***x***. Specified as: a fullword integer. It can have any value.

*On Return*

*Function value*
is the euclidean length ($l_2$ norm) of the vector ***x***. Returned as: a number of the data type indicated in Table 47.

***Notes***

1. This subroutine does not underflow or overflow if the values of the elements in vector ***x*** conform to the following conditions. If these conditions are violated, overflow or destructive underflow may occur:
   - For short-precision numbers:

     Any valid short-precision number.
   - For long-precision numbers:

     $|x_i| = 0$ or $0.10010\text{E}{-145} < |x_i| < 0.13408\text{E}{+155}$ for $i = 1, n$
2. Declare this function in your program as returning a value of the data type indicated in Table 47.

***Function:*** The euclidean length ($l_2$ norm) of vector ***x*** is expressed as follows with no scaling of input:

$$\sqrt{|x_1|^2 + |x_2|^2 + \ldots + |x_n|^2}$$

See reference [76 ]. The result is returned as the function value. If *n* is 0, then 0.0 is returned as the value of the function.

For SNORM2 and CNORM2, the sum of the squares of the absolute values of the elements is accumulated in long-precision. The square root of this long-precision sum is then computed.

This subroutine should not be used if the values in vector ***x*** do not conform to the restriction given in “Notes” on page 239.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Example 1:*** This example shows a vector, ***x***, with a stride of 1.

*Function Reference and Input*

```
                N   X  INCX
                |   |   |
SNORM = SNORM2( 6 , X , 1  )

X        =  (3.0, 4.0, 1.0, 8.0, 1.0, 3.0)
```

*Output*

```
SNORM    =  10.0
```

***Example 2:*** This example shows a vector, ***x***, with a stride greater than 1.

*Function Reference and Input*

```
                N   X  INCX
                |   |   |
SNORM = SNORM2( 6 , X , 2  )

X        =  (3.0, . , 4.0, . , 1.0, . , 8.0, . , 1.0, . , 3.0)
```

*Output*

```
SNORM    =  10.0
```

***Example 3:*** This example shows a vector, ***x***, with a stride of 0. The result in `SNORM` is:

$$\sqrt{nx_1^2}$$

*Function Reference and Input*

```
                N   X  INCX
                |   |   |
SNORM = SNORM2( 4 , X , 0  )

X        =  (4.0)
```

*Output*

```
SNORM    =  8.0
```

***Example 4:*** This example shows a vector, ***x***, containing complex numbers and having a stride of 1.

*Function Reference and Input*

```
                N   X  INCX
                |   |   |
CNORM = CNORM2( 3 , X , 1  )

X        =  ((3.0, 4.0), (1.0, 8.0), (-1.0, 3.0))
```

*Output*

```
CNORM    =  10.0
```

# SROTG, DROTG, CROTG, and ZROTG—Construct a Givens Plane Rotation

SROTG and DROTG construct a real Givens plane rotation, and CROTG and ZROTG construct a complex Givens plane rotation. The computations use rotational elimination parameters *a* and *b*. Values are returned for *r*, as well as the cosine *c* and the sine *s* of the angle of rotation. SROTG and DROTG also return a value for *z*.

**Note:** Throughout this description, the symbols *r* and *z* are used to represent two of the output values returned for this computation. It is important to note that the values for *r* and *z* are actually returned in the input-output arguments *a* and *b*, respectively, overwriting the original values passed in *a* and *b*.

*Table 48. Data Types*

| *a*, *b*, *r*, *s* | *c* | *z* | **Subprogram** |
|---|---|---|---|
| Short-precision real | Short-precision real | Short-precision real | SROTG |
| Long-precision real | Long-precision real | Long-precision real | DROTG |
| Short-precision complex | Short-precision real | (No value returned) | CROTG |
| Long-precision complex | Long-precision real | (No value returned) | ZROTG |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SROTG \| DROTG \| CROTG \| ZROTG (*a*, *b*, *c*, *s*) |
| **C and C++** | srotg \| drotg \| crotg \| zrotg (*a*, *b*, *c*, *s*); |
| **PL/I** | CALL SROTG \| DROTG \| CROTG \| ZROTG (*a*, *b*, *c*, *s*); |

*On Entry*

*a*

is the rotational elimination parameter *a*. Specified as: a number of the data type indicated in Table 48.

*b*

is the rotational elimination parameter *b*. Specified as: a number of the data type indicated in Table 48.

*c*

See “On Return.”

*s*

See “On Return.”

*On Return*

*a*

is the value computed for *r*.

For SROTG and DROTG:

$$r = \sigma\sqrt{a^2 + b^2}$$

where:

$\sigma$ = SIGN(*a*) if |*a*| > |*b*|

$\sigma$ = SIGN($b$) if $|a| \le |b|$

For CROTG and ZROTG:

$$r = \psi \sqrt{|a|^2 + |b|^2} \qquad \text{if } |a| \ne 0$$

$$r = b \qquad \text{if } |a| = 0$$

where:
$\psi$ = $a/|a|$

Returned as: a number of the data type indicated in Table 48 on page 242.

*b*

is the value computed for *z*.

For SROTG and DROTG:
$z = s$ if $|a| > |b|$
$z = 1/c$ if $|a| \le |b|$ and $c \ne 0$ and $r \ne 0$
$z = 1$ if $|a| \le |b|$ and $c = 0$ and $r \ne 0$
$z = 0$ if $r = 0$

For CROTG and ZROTG: no value is returned, and the input value is not changed.

Returned as: a number of the data type indicated in Table 48 on page 242.

*c*

is the cosine *c* of the angle of (Givens) rotation. For SROTG and DROTG:
$c = a/r$ if $r \ne 0$
$c = 1$ if $r = 0$

For CROTG and ZROTG:

$$c = \frac{|a|}{\sqrt{|a|^2 + |b|^2}} \qquad \text{if } |a| \ne 0$$

$$c = 0 \qquad \text{if } |a| = 0$$

Returned as: a number of the data type indicated in Table 48 on page 242.

*s*

is the sine *s* of the angle of (Givens) rotation.

For SROTG and DROTG:
$s = b/r$ if $r \ne 0$
$s = 0$ if $r = 0$

For CROTG and ZROTG:

$$s = \frac{\psi \bar{b}}{\sqrt{|a|^2 + |b|^2}} \quad \text{if } |a| \neq 0$$

$$s = (1.0, 0.0) \quad \text{if } |a| = 0$$

where $\psi = a/|a|$

Returned as: a number of the data type indicated in Table 48 on page 242.

***Note:*** In your C program, arguments *a*, *b*, *c*, and *s* must be passed by reference.

***Function***

*SROTG and DROTG:* A real Givens plane rotation is constructed for values *a* and *b* by computing values for *r, c, s*, and *z*, where:

$$r = \sigma \sqrt{a^2 + b^2}$$

where:

$\sigma$ = SIGN($a$) if $|a| > |b|$
$\sigma$ = SIGN($b$) if $|a| \le |b|$

$c = a/r$ if $r \neq 0$

$c = 1$ if $r = 0$

$s = b/r$ if $r \neq 0$

$s = 0$ if $r = 0$

$z = s$ if $|a| > |b|$

$z = 1/c$ if $|a| \le |b|$ and $c \neq 0$ and $r \neq 0$

$z = 1$ if $|a| \le |b|$ and $c = 0$ and $r \neq 0$

$z = 0$ if $r = 0$

See reference [76 ].

Following are some important points about the computation:

1. The numbers for *c*, *s*, and *r* satisfy:

$$\begin{bmatrix} c & s \\ -s & c \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} r \\ 0 \end{bmatrix}$$

2. Where necessary, scaling is used to avoid overflow and destructive underflow in the computation of *r*, which is expressed as follows:

$$r = \sigma(|a|+|b|)\sqrt{\left(\frac{a}{|a|+|b|}\right)^2+\left(\frac{b}{|a|+|b|}\right)^2}$$

3. $\sigma$ is not essential to the computation of a Givens rotation matrix, but its use permits later stable reconstruction of *c* and *s* from just one stored number, *z*. See reference [88 ]. *c* and *s* are reconstructed from *z* as follows:

   $$\text{For } z = 1,\ c = 0 \text{ and } s = 1$$
   $$\text{For } |z| < 1,\ c = \sqrt{1-z^2} \text{ and } s = z$$
   $$\text{For } |z| > 1,\ c = 1/z \text{ and } s = \sqrt{1-c^2}$$

*CROTG and ZROTG:* A complex Givens plane rotation is constructed for values *a* and *b* by computing values for *r, c*, and *s*, where:

$$r = \psi\sqrt{|a|^2+|b|^2} \qquad \text{if } |a| \neq 0$$

$$r = b \qquad \text{if } |a| = 0$$

where:

$\psi$ = *a*/|*a*|

$$c = \frac{|a|}{\sqrt{|a|^2+|b|^2}} \qquad \text{if } |a| \neq 0$$

$$c = 0 \qquad \text{if } |a| = 0$$

$$s = \frac{\psi\,\overline{b}}{\sqrt{|a|^2+|b|^2}} \qquad \text{if } |a| \neq 0$$

$$s = (1.0, 0.0) \qquad \text{if } |a| = 0$$

See reference [76 ].

Following are some important points about the computation:

1. The numbers for *c, s*, and *r* satisfy:

$$\begin{bmatrix} c & s \\ -\bar{s} & c \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} r \\ 0 \end{bmatrix}$$

2. Where necessary, scaling is used to avoid overflow and destructive underflow in the computation of *r*, which is expressed as follows:

$$r = \psi \left( |a| + |b| \right) \sqrt{ \left| \frac{a}{|a| + |b|} \right|^2 + \left| \frac{b}{|a| + |b|} \right|^2 }$$

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* None

***Example 1:*** This example shows the construction of a real Givens plane rotation, where *r* is 0.

*Call Statement and Input*

```
             A     B    C   S
             |     |    |   |
CALL SROTG( 0.0 , 0.0 , C , S )
```

*Output*

```
A        =  0.0
B        =  0.0
C        =  1.0
S        =  0.0
```

***Example 2:*** This example shows the construction of a real Givens plane rotation, where *c* is 0.

*Call Statement and Input*

```
             A     B    C   S
             |     |    |   |
CALL SROTG( 0.0 , 2.0 , C , S )
```

*Output*

```
A        =  2.0
B        =  1.0
C        =  0.0
S        =  1.0
```

***Example 3:*** This example shows the construction of a real Givens plane rotation, where $|b| > |a|$.

*Call Statement and Input*

```
             A      B    C   S
             |      |    |   |
CALL SROTG( 6.0 , -8.0 , C , S )
```

*Output*

```
A        =  -10.0
                 _
B        =  -1.666
C        =  -0.6
S        =  0.8
```

***Example 4:*** This example shows the construction of a real Givens plane rotation, where |*a*| > |*b*|.

*Call Statement and Input*

```
             A     B    C   S
             |     |    |   |
CALL SROTG( 8.0 , 6.0 , C , S )
```

*Output*

```
A        =  10.0
B        =  0.6
C        =  0.8
S        =  0.6
```

***Example 5:*** This example shows the construction of a complex Givens plane rotation, where |*a*| = 0.

*Call Statement and Input*

```
            A   B   C   S
            |   |   |   |
CALL CROTG( A , B , C , S )

A        =  (0.0, 0.0)
B        =  (1.0, 0.0)
```

*Output*

```
A        =  (1.0, 0.0)
C        =  0.0
S        =  (1.0, 0.0)
```

***Example 6:*** This example shows the construction of a complex Givens plane rotation, where |*a*| ≠ 0.

*Call Statement and Input*

```
            A   B   C   S
            |   |   |   |
CALL CROTG( A , B , C , S )

A        =  (3.0, 4.0)
B        =  (4.0, 6.0)
```

*Output*

```
A        =  (5.26, 7.02)
C        =  0.57
S        =  (0.82, -0.05)
```

# SROT, DROT, CROT, ZROT, CSROT, and ZDROT—Apply a Plane Rotation

SROT and DROT apply a real plane rotation to real vectors; CROT and ZROT apply a complex plane rotation to complex vectors; and CSROT and ZDROT apply a real plane rotation to complex vectors. The plane rotation is applied to *n* points, where the points to be rotated are contained in vectors ***x*** and ***y***, and where the cosine and sine of the angle of rotation are *c* and *s*, respectively.

*Table 49. Data Types*

| ***x***, ***y*** | *c* | *s* | **Subprogram** |
|---|---|---|---|
| Short-precision real | Short-precision real | Short-precision real | SROT |
| Long-precision real | Long-precision real | Long-precision real | DROT |
| Short-precision complex | Short-precision real | Short-precision complex | CROT |
| Long-precision complex | Long-precision real | Long-precision complex | ZROT |
| Short-precision complex | Short-precision real | Short-precision real | CSROT |
| Long-precision complex | Long-precision real | Long-precision real | ZDROT |

***Syntax***

| | |
|---|---|
| **Fortran** | CALL SROT \| DROT \| CROT \| ZROT \| CSROT \| ZDROT (*n*, *x*, *incx*, *y*, *incy*, *c*, *s*) |
| **C and C++** | srot \| drot \| crot \| zrot \| csrot \| zdrot (*n*, *x*, *incx*, *y*, *incy*, *c*, *s*); |
| **PL/I** | CALL SROT \| DROT \| CROT \| ZROT \| CSROT \| ZDROT (*n*, *x*, *incx*, *y*, *incy*, *c*, *s*); |

*On Entry*

*n*
: is the number of points to be rotated—that is, the number of elements in vectors ***x*** and ***y***. Specified as: a fullword integer; $n \geq 0$.

*x*
: is the vector ***x*** of length *n*, containing the $x_i$ coordinates of the points to be rotated. Specified as: a one-dimensional array of (at least) length $1+(n-1)|incx|$, containing numbers of the data type indicated in Table 49.

*incx*
: is the stride for vector ***x***. Specified as: a fullword integer. It can have any value.

*y*
: is the vector ***y*** of length *n*, containing the $y_i$ coordinates of the points to be rotated. Specified as: a one-dimensional array of (at least) length $1+(n-1)|incy|$, containing numbers of the data type indicated in Table 49.

*incy*
: is the stride for vector ***y***. Specified as: a fullword integer. It can have any value.

*c*
: the cosine, *c*, of the angle of rotation. Specified as: a number of the data type indicated in Table 49.

*s*
: the sine, *s*, of the angle of rotation. Specified as: a number of the data type indicated in Table 49.

*On Return*

*x*

is the vector ***x*** of length *n*, containing the rotated $x_i$ coordinates, where:
$x_i \leftarrow cx_i+sy_i$ for $i$ = 1,

Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 49 on page 249.

*y*

is the vector ***y*** of length *n*, containing the rotated $y_i$ coordinates, where:

For SROT, DROT, CSROT, and ZDROT:
$y_i \leftarrow -sx_i+cy_i$ for $i$ = 1, *n*

For CROT and ZROT:

$$y_i \leftarrow -\bar{s}x_i + cy_i \quad \text{for } i = 1, n$$

Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 49 on page 249.

***Note:*** The vectors ***x*** and ***y*** must have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.

***Function:*** Applying a plane rotation to *n* points, where the points to be rotated are contained in vectors ***x*** and ***y***, is expressed as follows, where *c* and *s* are the cosine and sine of the angle of rotation, respectively. For SROT, DROT, CSROT, and ZDROT:

$$\begin{bmatrix} x_i \\ y_i \end{bmatrix} \leftarrow \begin{bmatrix} c & s \\ -s & c \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} \quad \text{for } i = 1, n$$

For CROT and ZROT:

$$\begin{bmatrix} x_i \\ y_i \end{bmatrix} \leftarrow \begin{bmatrix} c & s \\ -\bar{s} & c \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} \quad \text{for } i = 1, n$$

See references [55 ] and [76 ]. No computation is performed if *n* is 0 or if *c* is 1.0 and *s* is zero. For SROT, CROT, and CSROT, intermediate results are accumulated in long precision.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Example 1:*** This example shows how to apply a real plane rotation to real vectors ***x*** and ***y*** having positive strides.

*Call Statement and Input*

```
           N   X  INCX  Y  INCY   C    S
           |   |   |    |   |     |    |
CALL SROT( 5 , X , 1  , Y , 2  , 0.5 , S )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
Y        =  (-1.0, . , -2.0, . , -3.0, . , -4.0, . , -5.0)
```

$$S = \frac{\sqrt{3.0}}{2.0}$$

*Output*

```
X        =  (-0.366, -0.732, -1.098, -1.464, -1.830)
Y        =  (-1.366, -2.732, -4.098, -5.464, -6.830)
```

***Example 2:*** This example shows how to apply a real plane rotation to real vectors ***x*** and ***y*** having strides of opposite sign.

*Call Statement and Input*

```
           N   X  INCX  Y   INCY   C    S
           |   |   |    |    |     |    |
CALL SROT( 5 , X , 1  , Y , -1  , 0.5 , S )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
Y        =  (-5.0, -4.0, -3.0, -2.0, -1.0)
```

$$S = \frac{\sqrt{3.0}}{2.0}$$

*Output*

```
X        =(same as output X in Example 1)
Y        =  (-6.830, -5.464, -4.098, -2.732, -1.366)
```

***Example 3:*** This example shows how scalar values in vectors ***x*** and ***y*** can be processed by specifying 0 strides and the number of elements to be processed, *n*, equal to 1.

*Call Statement and Input*

```
           N   X  INCX  Y  INCY   C    S
           |   |   |    |   |     |    |
CALL SROT( 1 , X , 0  , Y , 0  , 0.5 , S )

X        =  (1.0)
Y        =  (-1.0)
```

$$S = \frac{\sqrt{3.0}}{2.0}$$

*Output*

```
X        =  (-0.366)
Y        =  (-1.366)
```

***Example 4:*** This example shows how to apply a complex plane rotation to complex vectors ***x*** and ***y*** having positive strides.

*Call Statement and Input*

```
           N   X  INCX  Y  INCY   C    S
           |   |   |    |   |     |    |
CALL CROT( 3 , X , 1  , Y , 2  , 0.5 , S )

X        =  ((1.0, 2.0), (2.0, 3.0), (3.0, 4.0))
Y        =  ((-1.0, 5.0), . , (-2.0, 4.0), . , (-3.0, 3.0))
S        =  (0.75, 0.50)
```

*Output*

```
X        =  ((-2.750, 4.250), (-2.500, 3.500), (-2.250, 2.750))
Y        =  ((-2.250, 1.500), . , (-4.000, 0.750), . ,
             (-5.750, 0.000))
```

***Example 5:*** This example shows how to apply a real plane rotation to complex vectors ***x*** and ***y*** having positive strides.

*Call Statement and Input*

```
            N   X  INCX  Y  INCY   C    S
            |   |   |    |   |     |    |
CALL CSROT( 3 , X , 1  , Y , 2  , 0.5 , S )

X        =  ((1.0, 2.0), (2.0, 3.0), (3.0, 4.0))
Y        =  ((-1.0, 5.0), . , (-2.0, 4.0), . , (-3.0, 3.0))
```

$$S = \frac{\sqrt{3.0}}{2.0}$$

*Output*

```
X        =  ((-0.366, 5.330), (-0.732, 4.964), (-1.098, 4.598))
Y        =  ((-1.366, 0.768), . , (-2.732, -0.598), . ,
             (-4.098, -1.964))
```

## SSCAL, DSCAL, CSCAL, ZSCAL, CSSCAL, and ZDSCAL—Multiply a Vector X by a Scalar and Store in the Vector X

These subprograms perform the following computation, using the scalar $\alpha$ and the vector $\boldsymbol{x}$:

$$\boldsymbol{x} \leftarrow \alpha\boldsymbol{x}$$

*Table 50. Data Types*

| $\alpha$ | ***x*** | **Subprogram** |
|---|---|---|
| Short-precision real | Short-precision real | SSCAL |
| Long-precision real | Long-precision real | DSCAL |
| Short-precision complex | Short-precision complex | CSCAL |
| Long-precision complex | Long-precision complex | ZSCAL |
| Short-precision real | Short-precision complex | CSSCAL |
| Long-precision real | Long-precision complex | ZDSCAL |

***Syntax***

| | |
|---|---|
| **Fortran** | CALL SSCAL \| DSCAL \| CSCAL \| ZSCAL \| CSSCAL \| ZDSCAL (*n*, *alpha*, *x*, *incx*) |
| **C and C++** | sscal \| dscal \| cscal \| zscal \| csscal \| zdscal (*n*, *alpha*, *x*, *incx*); |
| **PL/I** | CALL SSCAL \| DSCAL \| CSCAL \| ZSCAL \| CSSCAL \| ZDSCAL (*n*, *alpha*, *x*, *incx*); |

*On Entry*

*n*
is the number of elements in vector ***x***. Specified as: a fullword integer; $n \geq 0$.

*alpha*
is the scalar $\alpha$. Specified as: a number of the data type indicated in Table 50.

*x*
is the vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length $1+(n-1)|incx|$, containing numbers of the data type indicated in Table 50.

*incx*
is the stride for vector ***x***. Specified as: a fullword integer. It can have any value.

*On Return*

*x*
is the vector ***x*** of length *n*, containing the result of the computation $\alpha\boldsymbol{x}$. Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 50.

***Note:*** The fastest way in ESSL to zero out contiguous (stride 1) arrays is to call SSCAL or DSCAL, specifying *incx* = 1 and $\alpha = 0$.

***Function:*** The computation is expressed as follows:

$$\begin{bmatrix} x_1 \\ . \\ . \\ . \\ x_n \end{bmatrix} \leftarrow \alpha \begin{bmatrix} x_1 \\ . \\ . \\ . \\ x_n \end{bmatrix}$$

See reference [76 ]. If *n* is 0, no computation is performed. For CSCAL, intermediate results are accumulated in long precision.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Example 1:*** This example shows a vector, ***x***, with a stride of 1.

*Call Statement and Input*

```
            N  ALPHA  X  INCX
            |    |    |   |
CALL SSCAL( 5 , 2.0 , X , 1  )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
```

*Output*

```
X        =  (2.0, 4.0, 6.0, 8.0, 10.0)
```

***Example 2:*** This example shows vector, ***x***, with a stride greater than 1.

*Call Statement and Input*

```
            N  ALPHA  X  INCX
            |    |    |   |
CALL SSCAL( 5 , 2.0 , X , 2  )

X        =  (1.0, . , 2.0, . , 3.0, . , 4.0, . , 5.0)
```

*Output*

```
X        =  (2.0, . , 4.0, . , 6.0, . , 8.0, . , 10.0)
```

***Example 3:*** This example illustrates that when the strides for two similar computations (Example 1 and Example 3) have the same absolute value but have opposite signs, the output is the same. This example is the same as Example 1, except the stride for ***x*** is negative (–1). For performance reasons, it is better to specify the positive stride. For ***x***, processing begins at element `X(5)`, which is 5.0, and results are stored beginning at the same element.

*Call Statement and Input*

```
            N  ALPHA  X   INCX
            |    |    |    |
CALL SSCAL( 5 , 2.0 , X , -1  )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
```

*Output*

```
X        =  (2.0, 4.0, 6.0, 8.0, 10.0)
```

***Example 4:*** This example shows how SSCAL can be used to compute a scalar value. In this case, input vector ***x*** contains a scalar value, and the stride is 0. The number of elements to be processed, *n*, is 1.

*Call Statement and Input*

```
            N  ALPHA  X  INCX
            |    |    |   |
CALL SSCAL( 1 , 2.0 , X , 0  )

X        =  (1.0)
```

*Output*

```
X        =  (2.0)
```

***Example 5:*** This example shows a scalar, α, and a vector, ***x***, containing complex numbers, where vector ***x*** has a stride of 1.

*Call Statement and Input*

```
            N  ALPHA  X  INCX
            |    |    |   |
CALL CSCAL( 3 ,ALPHA, X , 1  )

ALPHA    =  (2.0, 3.0)
X        =  ((1.0, 2.0), (2.0, 0.0), (3.0, 5.0))
```

*Output*

```
X        =  ((-4.0, 7.0), (4.0, 6.0), (-9.0, 19.0))
```

***Example 6:*** This example shows a scalar, α, containing a real number, and a vector, ***x***, containing complex numbers, where vector ***x*** has a stride of 1.

*Call Statement and Input*

```
             N  ALPHA  X  INCX
             |    |    |   |
CALL CSSCAL( 3 , 2.0 , X , 1  )

X        =  ((1.0, 2.0), (2.0, 0.0), (3.0, 5.0))
```

*Output*

```
X        =  ((2.0, 4.0), (4.0, 0.0), (6.0, 10.0))
```

# SSWAP, DSWAP, CSWAP, and ZSWAP—Interchange the Elements of Two Vectors

These subprograms interchange the elements of vectors ***x*** and ***y***:

$\boldsymbol{y} \longleftrightarrow \boldsymbol{x}$

*Table 51. Data Types*

| ***x***, ***y*** | **Subprogram** |
|---|---|
| Short-precision real | SSWAP |
| Long-precision real | DSWAP |
| Short-precision complex | CSWAP |
| Long-precision complex | ZSWAP |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SSWAP \| DSWAP \| CSWAP \| ZSWAP (*n*, *x*, *incx*, *y*, *incy*) |
| **C and C++** | sswap \| dswap \| cswap \| zswap (*n*, *x*, *incx*, *y*, *incy*); |
| **PL/I** | CALL SSWAP \| DSWAP \| CSWAP \| ZSWAP (*n*, *x*, *incx*, *y*, *incy*); |

*On Entry*

*n*
is the number of elements in vectors ***x*** and ***y***. Specified as: a fullword integer; $n \geq 0$.

*x*
is the vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length 1+(*n*−1)|*incx*|, containing numbers of the data type indicated in Table 51.

*incx*
is the stride for vector ***x***. Specified as: a fullword integer. It can have any value.

*y*
is the vector ***y*** of length *n*. Specified as: a one-dimensional array of (at least) length 1+(*n*−1)|*incy*|, containing numbers of the data type indicated in Table 51.

*incy*
is the stride for vector ***y***. Specified as: a fullword integer. It can have any value.

*On Return*

*x*
is the vector ***x*** of length *n*, containing the elements that were swapped from vector ***y***. Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 51.

*y*
is the vector ***y*** of length *n*, containing the elements that were swapped from vector ***x***. Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 51.

### *Notes*

1. If you specify the same vector for ***x*** and ***y***, then *incx* and *incy* must be equal; otherwise, results are unpredictable.

2. If you specify different vectors for ***x*** and ***y***, they must have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.

***Function:*** The elements of vectors ***x*** and ***y*** are interchanged as follows:

$$\begin{bmatrix} y_1 \\ . \\ . \\ . \\ y_n \end{bmatrix} \leftarrow \rightarrow \begin{bmatrix} x_1 \\ . \\ . \\ . \\ x_n \end{bmatrix}$$

See reference [76 ]. If *n* is 0, no elements are interchanged.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Example 1:*** This example shows vectors ***x*** and ***y*** with positive strides.

*Call Statement and Input*

```
            N   X  INCX  Y  INCY
            |   |   |    |   |
CALL SSWAP( 5 , X , 1  , Y , 2  )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
Y        =  (-1.0, . , -2.0, . , -3.0, . , -4.0, . , -5.0)
```

*Output*

```
X        =  (-1.0, -2.0, -3.0, -4.0, -5.0)
Y        =  (1.0, . , 2.0, . , 3.0, . , 4.0, . , 5.0)
```

***Example 2:*** This example shows how to obtain output vectors ***x*** and ***y*** that are reverse copies of the input vectors ***y*** and ***x***. You must specify strides with the same absolute value, but with opposite signs. For ***y***, which has negative stride, processing begins at element `Y(5)`, which is –5.0, and the results of the swap are stored beginning at the same element.

*Call Statement and Input*

```
            N   X  INCX  Y   INCY
            |   |   |    |    |
CALL SSWAP( 5 , X , 1  , Y , -1  )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
Y        =  (-1.0, -2.0, -3.0, -4.0, -5.0)
```

*Output*

```
X        =  (-5.0, -4.0, -3.0, -2.0, -1.0)
Y        =  (5.0, 4.0, 3.0, 2.0, 1.0)
```

***Example 3:*** This example shows how SSWAP can be used to interchange scalar values in vectors ***x*** and ***y*** by specifying 0 strides and the number of elements to be processed as 1.

*Call Statement and Input*

```
            N   X  INCX  Y  INCY
            |   |   |    |   |
CALL SSWAP( 1 , X , 0  , Y , 0  )

X        =  (1.0)
Y        =  (-4.0)
```

*Output*

```
X        =  (-4.0)
Y        =  (1.0)
```

***Example 4:*** This example shows vectors ***x*** and ***y***, containing complex numbers and having positive strides.

*Call Statement and Input*

```
            N   X  INCX  Y  INCY
            |   |   |    |   |
CALL CSWAP( 4 , X , 1  , Y , 2  )

X        =  ((1.0, 6.0), (2.0, 7.0), (3.0, 8.0), (4.0, 9.0))
Y        =  ((-1.0, -1.0), . , (-2.0, -2.0), . , (-3.0, -3.0), . ,
             (-4.0, -4.0))
```

*Output*

```
X        =  ((-1.0, -1.0), (-2.0, -2.0), (-3.0, -3.0), (-4.0, -4.0))
Y        =  ((1.0, 6.0), . , (2.0, 7.0), . , (3.0, 8.0), . ,
             (4.0, 9.0))
```

# SVEA, DVEA, CVEA, and ZVEA—Add a Vector X to a Vector Y and Store in a Vector Z

These subprograms perform the following computation, using vectors ***x, y,*** and ***z***:

$$\boldsymbol{z} \leftarrow \boldsymbol{x}+\boldsymbol{y}$$

*Table 52. Data Types*

| ***x***, ***y***, ***z*** | **Subprogram** |
|---|---|
| Short-precision real | SVEA |
| Long-precision real | DVEA |
| Short-precision complex | CVEA |
| Long-precision complex | ZVEA |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SVEA \| DVEA \| CVEA \| ZVEA (*n*, *x*, *incx*, *y*, *incy*, *z*, *incz*) |
| **C and C++** | svea \| dvea \| cvea \| zvea (*n*, *x*, *incx*, *y*, *incy*, *z*, *incz*); |
| **PL/I** | CALL SVEA \| DVEA \| CVEA \| ZVEA (*n*, *x*, *incx*, *y*, *incy*, *z*, *incz*); |

*On Entry*

*n*
is the number of elements in vectors ***x, y,*** and ***z***. Specified as: a fullword integer; $n \geq 0$.

*x*
is the vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length 1+(*n*–1)|*incx*|, containing numbers of the data type indicated in Table 52.

*incx*
is the stride for vector ***x***. Specified as: a fullword integer. It can have any value.

*y*
is the vector ***y*** of length *n*. Specified as: a one-dimensional array of (at least) length 1+(*n*–1)|*incy*|, containing numbers of the data type indicated in Table 52.

*incy*
is the stride for vector ***y***. Specified as: a fullword integer. It can have any value.

*z*
See “On Return.”

*incz*
is the stride for vector ***z***. Specified as: a fullword integer. It can have any value.

*On Return*

*z*
is the vector ***z*** of length *n*, containing the result of the computation. Returned as: a one-dimensional array of (at least) length 1+(*n*–1)|*incz*|, containing numbers of the data type indicated in Table 52.

### *Notes*

1. If you specify the same vector for ***x*** and ***z***, then *incx* and *incz* must be equal; otherwise, results are unpredictable. The same is true for ***y*** and ***z***.

2. If you specify different vectors for ***x*** and ***z***, they must have no common elements; otherwise, results are unpredictable. The same is true for ***y*** and ***z***. See “Concepts” on page 55.

***Function:*** The computation is expressed as follows:

$$\begin{bmatrix} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_n \end{bmatrix} \leftarrow \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} + \begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix}$$

If *n* is 0, no computation is performed.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Example 1:*** This example shows vectors ***x***, ***y***, and ***z***, with positive strides.

*Call Statement and Input*

```
           N   X  INCX  Y  INCY  Z  INCZ
           |   |  |     |  |     |  |
CALL SVEA( 5 , X , 1  , Y , 2  , Z , 1  )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
Y        =  (1.0, . , 1.0, . , 1.0, . , 1.0, . , 1.0)
```

*Output*

```
Z        =  (2.0, 3.0, 4.0, 5.0, 6.0)
```

***Example 2:*** This example shows vectors ***x*** and ***y*** having strides of opposite sign, and an output vector ***z*** having a positive stride. For ***y***, which has negative stride, processing begins at element `Y(5)`, which is 1.0.

*Call Statement and Input*

```
           N   X  INCX  Y   INCY  Z  INCZ
           |   |  |     |   |     |  |
CALL SVEA( 5 , X , 1  , Y , -1  , Z , 2  )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
Y        =  (5.0, 4.0, 3.0, 2.0, 1.0)
```

*Output*

```
Z        =  (2.0, . , 4.0, . , 6.0, . , 8.0, . , 10.0)
```

***Example 3:*** This example shows a vector, ***x***, with 0 stride and a vector, ***z***, with negative stride. ***x*** is treated like a vector of length *n*, all of whose elements are the same as the single element in ***x***. For vector ***z***, results are stored beginning in element `Z(5)`.

*Call Statement and Input*

```
          N   X  INCX  Y  INCY  Z   INCZ
          |   |   |    |   |    |    |
CALL SVEA( 5 , X , 0  , Y , 1  , Z , -1  )

X        =  (1.0)
Y        =  (5.0, 4.0, 3.0, 2.0, 1.0)
```

*Output*

```
Z        =  (2.0, 3.0, 4.0, 5.0, 6.0)
```

***Example 4:*** This example shows a vector, ***y***, with 0 stride. ***y*** is treated like a vector of length *n*, all of whose elements are the same as the single element in ***y***.

*Call Statement and Input*

```
          N   X  INCX  Y  INCY  Z  INCZ
          |   |   |    |   |    |   |
CALL SVEA( 5 , X , 1  , Y , 0  , Z , 1  )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
Y        =  (5.0)
```

*Output*

```
Z        =  (6.0, 7.0, 8.0, 9.0, 10.0)
```

***Example 5:*** This example shows the output vector, ***z***, with 0 stride, where the vector ***x*** has positive stride, and the vector ***y*** has 0 stride. The number of elements to be processed, *n*, is greater than 1.

*Call Statement and Input*

```
          N   X  INCX  Y  INCY  Z  INCZ
          |   |   |    |   |    |   |
CALL SVEA( 5 , X , 1  , Y , 0  , Z , 0  )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
Y        =  (5.0)
```

*Output*

```
Z        =  (10.0)
```

***Example 6:*** This example shows the output vector ***z***, with 0 stride, where the vector ***x*** has 0 stride, and the vector ***y*** has negative stride. The number of elements to be processed, *n*, is greater than 1.

*Call Statement and Input*

```
          N   X  INCX  Y   INCY  Z  INCZ
          |   |   |    |    |    |   |
CALL SVEA( 5 , X , 0  , Y , -1  , Z , 0  )

X        =  (1.0)
Y        =  (5.0, 4.0, 3.0, 2.0, 1.0)
```

*Output*

```
Z        =  (6.0)
```

***Example 7:*** This example shows how SVEA can be used to compute a scalar value. In this case, vectors ***x*** and ***y*** contain scalar values. The strides of all vectors, ***x***, ***y***, and ***z***, are 0. The number of elements to be processed, *n*, is 1.

*Call Statement and Input*

```
           N   X  INCX  Y  INCY  Z  INCZ
           |   |  |     |  |     |  |
CALL SVEA( 1 , X , 0  , Y , 0  , Z , 0  )

X        =  (1.0)
Y        =  (5.0)
```

*Output*

```
Z        =  (6.0)
```

***Example 8:*** This example shows vectors ***x*** and ***y***, containing complex numbers and having positive strides.

*Call Statement and Input*

```
           N   X  INCX  Y  INCY  Z  INCZ
           |   |  |     |  |     |  |
CALL CVEA( 3 , X , 1  , Y , 2  , Z , 1  )

X        =  ((1.0, 2.0), (3.0, 4.0), (5.0, 6.0))
Y        =  ((7.0, 8.0), . , (9.0, 10.0), . , (11.0, 12.0))
```

*Output*

```
Z        =  ((8.0, 10.0), (12.0, 14.0), (16.0, 18.0))
```

# SVES, DVES, CVES, and ZVES—Subtract a Vector Y from a Vector X and Store in a Vector Z

These subprograms perform the following computation, using vectors ***x, y,*** and ***z***:

$$\boldsymbol{z} \leftarrow \boldsymbol{x} - \boldsymbol{y}$$

*Table 53. Data Types*

| ***x***, ***y***, ***z*** | **Subprogram** |
|---|---|
| Short-precision real | SVES |
| Long-precision real | DVES |
| Short-precision complex | CVES |
| Long-precision complex | ZVES |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SVES \| DVES \| CVES \| ZVES (*n*, *x*, *incx*, *y*, *incy*, *z*, *incz*) |
| **C and C++** | sves \| dves \| cves \| zves (*n*, *x*, *incx*, *y*, *incy*, *z*, *incz*); |
| **PL/I** | CALL SVES \| DVES \| CVES \| ZVES (*n*, *x*, *incx*, *y*, *incy*, *z*, *incz*); |

*On Entry*

*n*
is the number of elements in vectors ***x, y,*** and ***z***. Specified as: a fullword integer; $n \geq 0$.

*x*
is the vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length 1+(*n*–1)|*incx*|, containing numbers of the data type indicated in Table 53.

*incx*
is the stride for vector ***x***. Specified as: a fullword integer. It can have any value.

*y*
is the vector ***y*** of length *n*. Specified as: a one-dimensional array of (at least) length 1+(*n*–1)|*incy*|, containing numbers of the data type indicated in Table 53.

*incy*
is the stride for vector ***y***. Specified as: a fullword integer. It can have any value.

*z*
See On Return.

*incz*
is the stride for vector ***z***. Specified as: a fullword integer. It can have any value.

*On Return*

*z*
is the vector ***z*** of length *n*, containing the result of the computation. Returned as: a one-dimensional array of (at least) length 1+(*n*–1)|*incz*|, containing numbers of the data type indicated in Table 53.

### *Notes*

1. If you specify the same vector for ***x*** and ***z***, then *incx* and *incz* must be equal; otherwise, results are unpredictable. The same is true for ***y*** and ***z***.

2. If you specify different vectors for ***x*** and ***z***, they must have no common elements; otherwise, results are unpredictable. The same is true for ***y*** and ***z***. See “Concepts” on page 55.

***Function:*** The computation is expressed as follows:

$$\begin{bmatrix} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_n \end{bmatrix} \leftarrow \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} - \begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix}$$

If *n* is 0, no computation is performed.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Example 1:*** This example shows vectors ***x***, ***y***, and ***z***, with positive strides.

*Call Statement and Input*

```
          N   X  INCX  Y  INCY  Z  INCZ
          |   |  |     |  |     |  |
CALL SVES( 5 , X , 1  , Y , 2  , Z , 1  )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
Y        =  (1.0, . , 1.0, . , 1.0, . , 1.0, . , 1.0)
```

*Output*

```
Z        =  (0.0, 1.0, 2.0, 3.0, 4.0)
```

***Example 2:*** This example shows vectors ***x*** and ***y*** having strides of opposite sign, and an output vector ***z*** having a positive stride. For ***y***, which has negative stride, processing begins at element `Y(5)`, which is 1.0.

*Call Statement and Input*

```
          N   X  INCX  Y   INCY  Z  INCZ
          |   |  |     |   |     |  |
CALL SVES( 5 , X , 1  , Y , -1  , Z , 2  )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
Y        =  (5.0, 4.0, 3.0, 2.0, 1.0)
```

*Output*

```
Z        =  (0.0, . , 0.0, . , 0.0, . , 0.0, . , 0.0)
```

***Example 3:*** This example shows a vector, ***x***, with 0 stride, and a vector, ***z***, with negative stride. ***x*** is treated like a vector of length *n*, all of whose elements are the same as the single element in ***x***. For vector ***z***, results are stored beginning in element `Z(5)`.

*Call Statement and Input*

```
          N   X  INCX  Y  INCY  Z   INCZ
          |   |   |    |   |    |    |
CALL SVES( 5 , X , 0  , Y , 1  , Z , -1  )

X        =  (1.0)
Y        =  (5.0, 4.0, 3.0, 2.0, 1.0)
```

*Output*

```
Z        =  (0.0, -1.0, -2.0, -3.0, -4.0)
```

***Example 4:*** This example shows a vector, ***y***, with 0 stride. ***y*** is treated like a vector of length *n*, all of whose elements are the same as the single element in ***y***.

*Call Statement and Input*

```
          N   X  INCX  Y  INCY  Z  INCZ
          |   |   |    |   |    |   |
CALL SVES( 5 , X , 1  , Y , 0  , Z , 1  )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
Y        =  (5.0)
```

*Output*

```
Z        =  (-4.0, -3.0, -2.0, -1.0, 0.0)
```

***Example 5:*** This example shows the output vector ***z***, with 0 stride, where the vector ***x*** has positive stride, and the vector ***y*** has 0 stride. The number of elements to be processed, *n*, is greater than 1.

*Call Statement and Input*

```
          N   X  INCX  Y  INCY  Z  INCZ
          |   |   |    |   |    |   |
CALL SVES( 5 , X , 1  , Y , 0  , Z , 0  )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
Y        =  (5.0)
```

*Output*

```
Z        =  (0.0)
```

***Example 6:*** This example shows the output vector ***z***, with 0 stride, where the vector ***x*** has 0 stride, and the vector ***y*** has negative stride. The number of elements to be processed, *n*, is greater than 1.

*Call Statement and Input*

```
          N   X  INCX  Y   INCY  Z  INCZ
          |   |   |    |    |    |   |
CALL SVES( 5 , X , 0  , Y , -1  , Z , 0  )

X        =  (1.0)
Y        =  (5.0, 4.0, 3.0, 2.0, 1.0)
```

*Output*

```
Z        =  (-4.0)
```

***Example 7:*** This example shows how SVES can be used to compute a scalar value. In this case, vectors ***x*** and ***y*** contain scalar values. The strides of all vectors, ***x***, ***y***, and ***z***, are 0. The number of elements to be processed, *n*, is 1.

*Call Statement and Input*

```
           N   X  INCX  Y  INCY  Z  INCZ
           |   |   |    |   |    |   |
CALL SVES( 1 , X , 0  , Y , 0  , Z , 0  )

X        =  (1.0)
Y        =  (5.0)
```

*Output*

```
Z        =  (-4.0)
```

***Example 8:*** This example shows vectors ***x*** and ***y***, containing complex numbers and having positive strides.

*Call Statement and Input*

```
           N   X  INCX  Y  INCY  Z  INCZ
           |   |   |    |   |    |   |
CALL CVES( 3 , X , 1  , Y , 2  , Z , 1  )

X        =  ((1.0, 2.0), (3.0, 4.0), (5.0, 6.0))
Y        =  ((7.0, 8.0), . , (9.0, 10.0), . , (11.0, 12.0))
```

*Output*

```
Z        =  ((-6.0, -6.0), (-6.0, -6.0), (-6.0, -6.0))
```

# SVEM, DVEM, CVEM, and ZVEM—Multiply a Vector X by a Vector Y and Store in a Vector Z

These subprograms perform the following computation, using vectors ***x, y,*** and ***z***:

$$\boldsymbol{z} \leftarrow \boldsymbol{xy}$$

*Table 54. Data Types*

| ***x***, ***y***, ***z*** | **Subprogram** |
|---|---|
| Short-precision real | SVEM |
| Long-precision real | DVEM |
| Short-precision complex | CVEM |
| Long-precision complex | ZVEM |

### Syntax

| | |
|---|---|
| **Fortran** | CALL SVEM \| DVEM \| CVEM \| ZVEM (*n*, *x*, *incx*, *y*, *incy*, *z*, *incz*) |
| **C and C++** | svem \| dvem \| cvem \| zvem (*n*, *x*, *incx*, *y*, *incy*, *z*, *incz*); |
| **PL/I** | CALL SVEM \| DVEM \| CVEM \| ZVEM (*n*, *x*, *incx*, *y*, *incy*, *z*, *incz*); |

*On Entry*

*n*
is the number of elements in vectors ***x, y,*** and ***z***. Specified as: a fullword integer; $n \geq 0$.

*x*
is the vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length 1+(*n*−1)|*incx*|, containing numbers of the data type indicated in Table 54.

*incx*
is the stride for vector ***x***. Specified as: a fullword integer. It can have any value.

*y*
is the vector ***y*** of length *n*. Specified as: a one-dimensional array of (at least) length 1+(*n*−1)|*incy*|, containing numbers of the data type indicated in Table 54.

*incy*
is the stride for vector ***y***. Specified as: a fullword integer. It can have any value.

*z*
See “On Return.”

*incz*
is the stride for vector ***z***. Specified as: a fullword integer. It can have any value.

*On Return*

*z*
is the vector ***z*** of length *n*, containing the result of the computation. Returned as: a one-dimensional array of (at least) length 1+(*n*−1)|*incz*|, containing numbers of the data type indicated in Table 54.

### Notes

1. If you specify the same vector for ***x*** and ***z***, then *incx* and *incz* must be equal; otherwise, results are unpredictable. The same is true for ***y*** and ***z***.

2. If you specify different vectors for ***x*** and ***z***, they must have no common elements; otherwise, results are unpredictable. The same is true for ***y*** and ***z***. See “Concepts” on page 55.

***Function:*** The computation is expressed as follows:

$z_i \leftarrow x_i y_i$ for $i = 1, n$

If *n* is 0, no computation is performed. For CVEM, intermediate results are accumulated in long precision (short-precision Multiply followed by a long-precision Add), with the final result truncated to short precision.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Example 1:*** This example shows vectors ***x***, ***y***, and ***z***, with positive strides.

*Call Statement and Input*

```
           N   X  INCX  Y  INCY  Z  INCZ
           |   |   |    |   |    |   |
CALL SVEM( 5 , X , 1  , Y , 2  , Z , 1  )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
Y        =  (1.0, . , 1.0, . , 1.0, . , 1.0, . , 1.0)
```

*Output*

```
Z        =  (1.0, 2.0, 3.0, 4.0, 5.0)
```

***Example 2:*** This example shows vectors ***x*** and ***y*** having strides of opposite sign, and an output vector ***z*** having a positive stride. For ***y***, which has negative stride, processing begins at element `Y(5)`, which is 1.0.

*Call Statement and Input*

```
           N   X  INCX  Y   INCY  Z  INCZ
           |   |   |    |    |    |   |
CALL SVEM( 5 , X , 1  , Y , -1  , Z , 2  )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
Y        =  (5.0, 4.0, 3.0, 2.0, 1.0)
```

*Output*

```
Z        =  (1.0, . , 4.0, . , 9.0, . , 16.0, . , 25.0)
```

***Example 3:*** This example shows a vector, ***x***, with 0 stride, and a vector, ***z***, with negative stride. ***x*** is treated like a vector of length *n*, all of whose elements are the same as the single element in ***x***. For vector ***z***, results are stored beginning in element `Z(5)`.

*Call Statement and Input*

```
           N   X  INCX  Y  INCY  Z   INCZ
           |   |   |    |   |    |    |
CALL SVEM( 5 , X , 0  , Y , 1  , Z , -1  )

X        =  (1.0)
Y        =  (5.0, 4.0, 3.0, 2.0, 1.0)
```

*Output*

```
Z        =  (1.0, 2.0, 3.0, 4.0, 5.0)
```

***Example 4:*** This example shows a vector, ***y***, with 0 stride. ***y*** is treated like a vector of length *n*, all of whose elements are the same as the single element in ***y***.

*Call Statement and Input*

```
           N   X  INCX  Y  INCY  Z  INCZ
           |   |   |    |   |    |   |
CALL SVEM( 5 , X , 1  , Y , 0  , Z , 1  )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
Y        =  (5.0)
```

*Output*

```
Z        =  (5.0, 10.0, 15.0, 20.0, 25.0)
```

***Example 5:*** This example shows the output vector, ***z***, with 0 stride, where the vector ***x*** has positive stride, and the vector ***y*** has 0 stride. The number of elements to be processed, *n*, is greater than 1.

*Call Statement and Input*

```
           N   X  INCX  Y  INCY  Z  INCZ
           |   |   |    |   |    |   |
CALL SVEM( 5 , X , 1  , Y , 0  , Z , 0  )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
Y        =  (5.0)
```

*Output*

```
Z        =  (25.0)
```

***Example 6:*** This example shows the output vector ***z***, with 0 stride, where the vector ***x*** has 0 stride, and the vector ***y*** has negative stride. The number of elements to be processed, *n*, is greater than 1.

*Call Statement and Input*

```
           N   X  INCX  Y   INCY  Z  INCZ
           |   |   |    |    |    |   |
CALL SVEM( 5 , X , 0  , Y , -1  , Z , 0  )

X        =  (1.0)
Y        =  (5.0, 4.0, 3.0, 2.0, 1.0)
```

*Output*

```
Z        =  (5.0)
```

***Example 7:*** This example shows how SVEM can be used to compute a scalar value. In this case, vectors ***x*** and ***y*** contain scalar values. The strides of all vectors, ***x***, ***y***, and ***z***, are 0. The number of elements to be processed, *n*, is 1.

*Call Statement and Input*

```
           N   X  INCX  Y  INCY  Z  INCZ
           |   |   |    |   |    |   |
CALL SVEM( 1 , X , 0  , Y , 0  , Z , 0  )

X        =  (1.0)
Y        =  (5.0)
```

*Output*

```
Z        =  (5.0)
```

***Example 8:*** This example shows vectors ***x*** and ***y***, containing complex numbers and having positive strides.

*Call Statement and Input*

```
           N   X  INCX  Y  INCY  Z  INCZ
           |   |   |    |   |    |   |
CALL CVEM( 3 , X , 1  , Y , 2  , Z , 1  )

X        =  ((1.0, 2.0), (3.0, 4.0), (5.0, 6.0))
Y        =  ((7.0, 8.0), . , (9.0, 10.0), . , (11.0, 12.0))
```

*Output*

```
Z        =  ((-9.0, 22.0), (-13.0, 66.0), (-17.0, 126.0))
```

# SYAX, DYAX, CYAX, ZYAX, CSYAX, and ZDYAX—Multiply a Vector X by a Scalar and Store in a Vector Y

These subprograms perform the following computation, using the scalar $\alpha$ and vectors ***x*** and ***y***:

$$\boldsymbol{y} \leftarrow \alpha \boldsymbol{x}$$

*Table 55. Data Types*

| $\alpha$ | ***x***, ***y*** | **Subprogram** |
|---|---|---|
| Short-precision real | Short-precision real | SYAX |
| Long-precision real | Long-precision real | DYAX |
| Short-precision complex | Short-precision complex | CYAX |
| Long-precision complex | Long-precision complex | ZYAX |
| Short-precision real | Short-precision complex | CSYAX |
| Long-precision real | Long-precision complex | ZDYAX |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SYAX \| DYAX \| CYAX \| ZYAX \| CSYAX \| ZDYAX (*n*, *alpha*, *x*, *incx*, *y*, *incy*) |
| **C and C++** | syax \| dyax \| cyax \| zyax \| csyax \| zdyax (*n*, *alpha*, *x*, *incx*, *y*, *incy*); |
| **PL/I** | CALL SYAX \| DYAX \| CYAX \| ZYAX \| CSYAX \| ZDYAX (*n*, *alpha*, *x*, *incx*, *y*, *incy*); |

*On Entry*

*n*
is the number of elements in vector ***x*** and ***y***. Specified as: a fullword integer; $n \geq 0$.

*alpha*
is the scalar $\alpha$. Specified as: a number of the data type indicated in Table 55.

*x*
is the vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length 1+(*n*–1)|*incx*|, containing numbers of the data type indicated in Table 55.

*incx*
is the stride for vector ***x***. Specified as: a fullword integer. It can have any value.

*y*
See “On Return.”

*incy*
is the stride for vector ***y***. Specified as: a fullword integer. It can have any value.

*On Return*

*y*
is the vector ***y*** of length *n*, containing the result of the computation $\alpha$***x***. Returned as: a one-dimensional array of (at least) length 1+(*n*–1)|*incy*|, containing numbers of the data type indicated in Table 55.

### *Notes*

1. If you specify the same vector for ***x*** and ***y***, then *incx* and *incy* must be equal; otherwise, results are unpredictable.

2. If you specify different vectors for ***x*** and ***y***, they must have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.

***Function:*** The computation is expressed as follows:

$$\begin{bmatrix} y_1 \\ . \\ . \\ . \\ y_n \end{bmatrix} \leftarrow \alpha \begin{bmatrix} x_1 \\ . \\ . \\ . \\ x_n \end{bmatrix}$$

See reference [76 ]. If *n* is 0, no computation is performed. For CYAX, intermediate results are accumulated in long precision.

***Error Condition***

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Example 1:*** This example shows vectors ***x*** and ***y*** with positive strides.

*Call Statement and Input*

```
           N  ALPHA  X  INCX  Y  INCY
           |    |    |   |    |   |
CALL SYAX( 5 , 2.0 , X , 1  , Y , 2  )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
```

*Output*

```
Y        =  (2.0, . , 4.0, . , 6.0, . , 8.0, . , 10.0)
```

***Example 2:*** This example shows vectors ***x*** and ***y*** that have strides of opposite signs. For ***y***, which has negative stride, results are stored beginning in element `Y(5)`.

*Call Statement and Input*

```
           N  ALPHA  X  INCX  Y   INCY
           |    |    |   |    |    |
CALL SYAX( 5 , 2.0 , X , 1  , Y , -1  )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
```

*Output*

```
Y        =  (10.0, 8.0, 6.0, 4.0, 2.0)
```

***Example 3:*** This example shows a vector, ***x***, with 0 stride. ***x*** is treated like a vector of length *n*, all of whose elements are the same as the single element in ***x***.

*Call Statement and Input*

```
           N  ALPHA  X  INCX  Y  INCY
           |   |     |   |    |   |
CALL SYAX( 5 , 2.0 , X , 0  , Y , 1  )

X        =  (1.0)
```

*Output*

```
Y        =  (2.0, 2.0, 2.0, 2.0, 2.0)
```

***Example 4:*** This example shows how SYAX can be used to compute a scalar value. In this case both vectors ***x*** and ***y*** contain scalar values, and the strides for both vectors are 0. The number of elements to be processed, *n*, is 1.

*Call Statement and Input*

```
           N  ALPHA  X  INCX  Y  INCY
           |   |     |   |    |   |
CALL SYAX( 1 , 2.0 , X , 0  , Y , 0  )

X        =  (1.0)
```

*Output*

```
Y        =  (2.0)
```

***Example 5:*** This example shows a scalar, α, and vectors ***x*** and ***y***, containing complex numbers, where both vectors have a stride of 1.

*Call Statement and Input*

```
           N  ALPHA  X  INCX  Y  INCY
           |   |     |   |    |   |
CALL CYAX( 3 ,ALPHA, X , 1  , Y , 1  )

ALPHA    =  (2.0, 3.0)
X        =  ((1.0, 2.0), (2.0, 0.0), (3.0, 5.0))
```

*Output*

```
Y        =  ((-4.0, 7.0), (4.0, 6.0), (-9.0, 19.0))
```

***Example 6:*** This example shows a scalar, α, containing a real number, and vectors ***x*** and ***y***, containing complex numbers, where both vectors have a stride of 1.

*Call Statement and Input*

```
            N  ALPHA  X  INCX  Y  INCY
            |   |     |   |    |   |
CALL CSYAX( 3 , 2.0 , X , 1  , Y , 1  )

X        =  ((1.0, 2.0), (2.0, 0.0), (3.0, 5.0))
```

*Output*

```
Y        =  ((2.0, 4.0), (4.0, 0.0), (6.0, 10.0))
```

# SZAXPY, DZAXPY, CZAXPY, and ZZAXPY—Multiply a Vector X by a Scalar, Add to a Vector Y, and Store in a Vector Z

These subprograms perform the following computation, using the scalar $\alpha$ and vectors ***x***, ***y***, and ***z***:

$$\boldsymbol{z} \leftarrow \boldsymbol{y}+\alpha\boldsymbol{x}$$

*Table 56. Data Types*

| $\alpha$, ***x***, ***y***, ***z*** | **Subprogram** |
|---|---|
| Short-precision real | SZAXPY |
| Long-precision real | DZAXPY |
| Short-precision complex | CZAXPY |
| Long-precision complex | ZZAXPY |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SZAXPY \| DZAXPY \| CZAXPY \| ZZAXPY (*n*, *alpha*, *x*, *incx*, *y*, *incy*, *z*, *incz*) |
| **C and C++** | szaxpy \| dzaxpy \| czaxpy \| zzaxpy (*n*, *alpha*, *x*, *incx*, *y*, *incy*, *z*, *incz*); |
| **PL/I** | CALL SZAXPY \| DZAXPY \| CZAXPY \| ZZAXPY (*n*, *alpha*, *x*, *incx*, *y*, *incy*, *z*, *incz*); |

*On Entry*

*n*
is the number of elements in vectors ***x, y,*** and ***z***. Specified as: a fullword integer; $n \geq 0$.

*alpha*
is the scalar $\alpha$. Specified as: a number of the data type indicated in Table 56.

*x*
is the vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length $1+(n-1)|incx|$, containing numbers of the data type indicated in Table 56.

*incx*
is the stride for vector ***x***. Specified as: a fullword integer. It can have any value.

*y*
is the vector ***y*** of length *n*. Specified as: a one-dimensional array of (at least) length $1+(n-1)|incy|$, containing numbers of the data type indicated in Table 56.

*incy*
is the stride for vector ***y***. Specified as: a fullword integer. It can have any value.

*z*
See “On Return.”

*incz*
is the stride for vector ***z***. Specified as: a fullword integer. It can have any value.

*On Return*

*z*
is the vector ***z*** of length *n*, containing the result of the computation $\boldsymbol{y}+\alpha\boldsymbol{x}$ . Returned as: a one-dimensional array of (at least) length $1+(n-1)|incz|$, containing numbers of the data type indicated in Table 56.

***Notes***

1. If you specify the same vector for ***x*** and ***z***, then *incx* and *incz* must be equal; otherwise, results are unpredictable. The same is true for ***y*** and ***z***.
2. If you specify different vectors for ***x*** and ***z***, they must have no common elements; otherwise, results are unpredictable. The same is true for ***y*** and ***z***. See "Concepts" on page 55.

***Function:*** The computation is expressed as follows:

$$\begin{bmatrix} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_n \end{bmatrix} \leftarrow \begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} + \alpha \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix}$$

See reference [76 ]. If *n* is 0, no computation is performed. For CZAXPY, intermediate results are accumulated in long precision.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Example 1:*** This example shows vectors ***x*** and ***y*** with positive strides.

*Call Statement and Input*

```
             N  ALPHA  X  INCX  Y  INCY  Z  INCZ
             |    |    |   |    |   |    |   |
CALL SZAXPY( 5 , 2.0 , X , 1  , Y , 2  , Z , 1  )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
Y        =  (1.0, . , 1.0, . , 1.0, . , 1.0, . , 1.0)
```

*Output*

```
Z        =  (3.0, 5.0, 7.0, 9.0, 11.0)
```

***Example 2:*** This example shows vectors ***x*** and ***y*** having strides of opposite sign, and an output vector ***z*** having a positive stride. For ***y***, which has negative stride, processing begins at element `Y(5)`, which is 1.0.

*Call Statement and Input*

```
             N  ALPHA  X  INCX  Y   INCY  Z  INCZ
             |    |    |   |    |    |    |   |
CALL SZAXPY( 5 , 2.0 , X , 1  , Y , -1  , Z , 2  )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
Y        =  (5.0, 4.0, 3.0, 2.0, 1.0)
```

*Output*

```
Z        =  (3.0, . , 6.0, . , 9.0, . , 12.0, . , 15.0)
```

***Example 3:*** This example shows a vector, ***x***, with 0 stride, and a vector, ***z***, with negative stride. ***x*** is treated like a vector of length *n*, all of whose elements are the same as the single element in ***x***. For vector ***z***, results are stored beginning in element Z(5).

*Call Statement and Input*

```
             N  ALPHA  X  INCX  Y  INCY  Z   INCZ
             |    |    |   |    |   |    |    |
CALL SZAXPY( 5 , 2.0 , X , 0  , Y , 1  , Z , -1  )

X        =  (1.0)
Y        =  (5.0, 4.0, 3.0, 2.0, 1.0)
```

*Output*

```
Z        =  (3.0, 4.0, 5.0, 6.0, 7.0)
```

***Example 4:*** This example shows a vector, ***y***, with 0 stride. ***y*** is treated like a vector of length *n*, all of whose elements are the same as the single element in ***y***.

*Call Statement and Input*

```
             N  ALPHA  X  INCX  Y  INCY  Z  INCZ
             |    |    |   |    |   |    |   |
CALL SZAXPY( 5 , 2.0 , X , 1  , Y , 0  , Z , 1  )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
Y        =  (5.0)
```

*Output*

```
Z        =  (7.0, 9.0, 11.0, 13.0, 15.0)
```

***Example 5:*** This example shows how SZAXPY can be used to compute a scalar value. In this case, vectors ***x*** and ***y*** contain scalar values. The strides of all vectors, ***x***, ***y***, and ***z***, are 0. The number of elements to be processed, *n*, is 1.

*Call Statement and Input*

```
             N  ALPHA  X  INCX  Y  INCY  Z  INCZ
             |    |    |   |    |   |    |   |
CALL SZAXPY( 1 , 2.0 , X , 0  , Y , 0  , Z , 0  )

X        =  (1.0)
Y        =  (5.0)
```

*Output*

```
Z        =  (7.0)
```

***Example 6:*** This example shows vectors ***x*** and ***y***, containing complex numbers and having positive strides.

*Call Statement and Input*

```
               N  ALPHA  X  INCX  Y  INCY  Z  INCZ
               |    |    |   |    |   |    |   |
CALL CZAXPY( 3 ,ALPHA, X , 1  , Y , 2  , Z , 1  )

ALPHA    =  (2.0, 3.0)
X        =  ((1.0, 2.0), (2.0, 0.0), (3.0, 5.0))
Y        =  ((1.0, 1.0), . , (0.0, 2.0), . , (5.0, 4.0))
```

*Output*

```
Z        =  ((-3.0, 8.0), (4.0, 8.0), (-4.0, 23.0))
```

## Sparse Vector-Scalar Subprograms

This section contains the sparse vector-scalar subprogram descriptions.

# SSCTR, DSCTR, CSCTR, ZSCTR—Scatter the Elements of a Sparse Vector X in Compressed-Vector Storage Mode into Specified Elements of a Sparse Vector Y in Full-Vector Storage Mode

These subprograms scatter the elements of sparse vector ***x***, stored in compressed-vector storage mode, into specified elements of sparse vector ***y***, stored in full-vector storage mode.

*Table 57. Data Types*

| ***x***, ***y*** | **Subprogram** |
|---|---|
| Short-precision real | SSCTR |
| Long-precision real | DSCTR |
| Short-precision complex | CSCTR |
| Long-precision complex | ZSCTR |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SSCTR \| DSCTR \| CSCTR \| ZSCTR (*nz*, *x*, *indx*, *y*) |
| **C and C++** | ssctr \| dsctr \| csctr \| zsctr (*nz*, *x*, *indx*, *y*); |
| **PL/I** | CALL SSCTR \| DSCTR \| CSCTR \| ZSCTR (*nz*, *x*, *indx*, *y*); |

*On Entry*

*nz*

is the number of elements in sparse vector ***x***, stored in compressed-vector storage mode. Specified as: a fullword integer; $nz \geq 0$.

*x*

is the sparse vector ***x***, containing *nz* elements, stored in compressed-vector storage mode in an array, referred to as X. Specified as: a one-dimensional array of (at least) length *nz*, containing numbers of the data type indicated in Table 57.

*indx*

is the array, referred to as INDX, containing the *nz* indices that indicate the positions of the elements of the sparse vector ***x*** when in full-vector storage mode. They also indicate the positions in vector ***y*** into which the elements are copied.

Specified as: a one-dimensional array of (at least) length *nz*, containing fullword integers.

*y*

See "On Return."

*On Return*

*y*

is the sparse vector ***y***, stored in full-vector storage mode, of (at least) length max(INDX(*i*)) for *i* = 1, *nz*, into which *nz* elements of vector ***x*** are copied at positions indicated by the indices array INDX.

Returned as: a one-dimensional array of (at least) length max(INDX(*i*)) for *i* = 1, *nz*, containing numbers of the data type indicated in Table 57.

***Notes***

1. Each value specified in array INDX must be unique; otherwise, results are unpredictable.
2. Vectors ***x*** and ***y*** must have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.
3. For a description of how sparse vectors are stored, see “Sparse Vector” on page 60.

***Function:*** The copy is expressed as follows:

$y_{\text{INDX}(i)} \leftarrow x_i \quad \text{for } i = 1, nz$

where:

***x*** is a sparse vector, stored in compressed-vector storage mode.
INDX is the indices array for sparse vector ***x***.
***y*** is a sparse vector, stored in full-vector storage mode.

See reference [29 ]. If *nz* is 0, no copy is performed.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $nz < 0$

***Example 1:*** This example shows how to use SSCTR to copy a sparse vector ***x*** of length 5 into the following vector ***y***, where the elements of array INDX are in ascending order:

```
          Y = (6.0, 2.0, 4.0, 7.0, 6.0, 10.0, -2.0, 8.0, 9.0, 0.0 )
```

*Call Statement and Input*

```
            NZ  X   INDX   Y
            |   |    |     |
CALL SSCTR( 5 , X , INDX , Y )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
INDX     =  (1, 3, 4, 7, 10)
```

*Output*

```
Y        =  (1.0, 2.0, 2.0, 3.0, 6.0, 10.0, 4.0, 8.0, 9.0, 5.0)
```

***Example 2:*** This example shows how to use SSCTR to copy a sparse vector ***x*** of length 5 into the following vector ***y***, where the elements of array INDX are in random order:

```
          Y = (6.0, 2.0, 4.0, 7.0, 6.0, 10.0, -2.0, 8.0, 9.0, 0.0 )
```

*Call Statement and Input*

```
            NZ  X   INDX   Y
            |   |    |     |
CALL SSCTR( 5 , X , INDX , Y )

X         =  (1.0, 2.0, 3.0, 4.0, 5.0)
INDX      =  (4, 3, 1, 10, 7)
```

*Output*

```
Y         =  (3.0, 2.0, 2.0, 1.0, 6.0, 10.0, 5.0, 8.0, 9.0, 4.0)
```

***Example 3:*** This example shows how to use CSCTR to copy a sparse vector ***x*** of length 3 into the following vector ***y***, where the elements of array `INDX` are in random order:

```
         Y = ((6.0, 5.0), (-2.0, 3.0), (15.0, 4.0), (9.0, 0.0))
```

*Call Statement and Input*

```
            NZ  X   INDX   Y
            |   |    |     |
CALL CSCTR( 3 , X , INDX , Y )

X         =  ((1.0, 2.0), (3.0, 4.0), (5.0, 6.0))
INDX      =  (4, 1, 3)
```

*Output*

```
Y         =  ((3.0, 4.0), (-2.0, 3.0), (5.0, 6.0), (1.0, 2.0))
```

# SGTHR, DGTHR, CGTHR, and ZGTHR—Gather Specified Elements of a Sparse Vector Y in Full-Vector Storage Mode into a Sparse Vector X in Compressed-Vector Storage Mode

These subprograms gather specified elements of vector ***y***, stored in full-vector storage mode, into sparse vector ***x***, stored in compressed-vector storage mode.

*Table 58. Data Types*

| ***x***, ***y*** | **Subprogram** |
|---|---|
| Short-precision real | SGTHR |
| Long-precision real | DGTHR |
| Short-precision complex | CGTHR |
| Long-precision complex | ZGTHR |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SGTHR \| DGTHR \| CGTHR \| ZGTHR (*nz*, *y*, *x*, *indx*) |
| **C and C++** | sgthr \| dgthr \| cgthr \| zgthr (*nz*, *y*, *x*, *indx*); |
| **PL/I** | CALL SGTHR \| DGTHR \| CGTHR \| ZGTHR (*nz*, *y*, *x*, *indx*); |

*On Entry*

*nz*

is the number of elements in sparse vector ***x***, stored in compressed-vector storage mode. Specified as: a fullword integer; *nz* ≥ 0.

*y*

is the sparse vector ***y***, stored in full-vector storage mode, of (at least) length max(`INDX`(*i*)) for *i* = 1, *nz*, from which *nz* elements are copied from positions indicated by the indices array `INDX`.

Specified as: a one-dimensional array of (at least) length max(`INDX`(*i*)) for *i* = 1, *nz*, containing numbers of the data type indicated in Table 58.

*x*

See “On Return.”

*indx*

is the array, referred to as `INDX`, containing the *nz* indices that indicate the positions of the elements of the sparse vector ***x*** when in full-vector storage mode. They also indicate the positions in vector ***y*** from which elements are copied.

Specified as: a one-dimensional array of (at least) length *nz*, containing fullword integers.

*On Return*

*x*

is the sparse vector ***x***, containing *nz* elements, stored in compressed-vector storage mode in an array, referred to as `X`, into which are copied the elements of vector ***y*** from positions indicated by the indices array `INDX`.

Returned as: a one-dimensional array of (at least) length *nz*, containing numbers of the data type indicated in Table 58.

***Notes***

1. Vectors ***x*** and ***y*** must have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.
2. For a description of how sparse vectors are stored, see “Sparse Vector” on page 60.

***Function:*** The copy is expressed as follows:

$$x_i \leftarrow y_{\mathtt{INDX}(i)} \quad \text{for } i = 1, nz$$

where:

***x*** is a sparse vector, stored in compressed-vector storage mode.
INDX is the indices array for sparse vector ***x***.
***y*** is a sparse vector, stored in full-vector storage mode.

See reference [29 ]. If *nz* is 0, no copy is performed.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $nz < 0$

***Example 1:*** This example shows how to use SGTHR to copy specified elements of a vector ***y*** into a sparse vector ***x*** of length 5, where the elements of array INDX are in ascending order.

*Call Statement and Input*

```
            NZ  Y   X   INDX
            |   |   |    |
CALL SGTHR( 5 , Y , X , INDX )

Y        =  (6.0, 2.0, 4.0, 7.0, 6.0, 10.0, -2.0, 8.0, 9.0, 0.0)
INDX     =  (1, 3, 4, 7, 9)
```

*Output*

```
X        =  (6.0, 4.0, 7.0, -2.0, 9.0)
```

***Example 2:*** This example shows how to use SGTHR to copy specified elements of a vector ***y*** into a sparse vector ***x*** of length 5, where the elements of array INDX are in random order. (Note that the element 0.0 occurs in output vector ***x***. This does not produce an error.)

*Call Statement and Input*

```
            NZ  Y   X   INDX
            |   |   |    |
CALL SGTHR( 5 , Y , X , INDX )

Y        =  (6.0, 2.0, 4.0, 7.0, 6.0, 10.0, -2.0, 8.0, 9.0, 0.0)
INDX     =  (4, 3, 1, 10, 7)
```

*Output*

```
X        =  (7.0, 4.0, 6.0, 0.0, -2.0)
```

***Example 3:*** This example shows how to use CGTHR to copy specified elements of a vector, ***y***, into a sparse vector, ***x***, of length 3, where the elements of array `INDX` are in random order.

*Call Statement and Input*

```
            NZ  Y   X   INDX
            |   |   |    |
CALL CGTHR( 3 , Y , X , INDX )

Y        =  ((6.0, 5.0), (-2.0, 3.0), (15.0, 4.0), (9.0, 0.0))
INDX     =  (4, 1, 3)
```

*Output*

```
X        =  ((9.0, 0.0), (6.0, 5.0), (15.0, 4.0))
```

## SGTHRZ, DGTHRZ, CGTHRZ, and ZGTHRZ—Gather Specified Elements of a Sparse Vector Y in Full-Vector Mode into a Sparse Vector X in Compressed-Vector Mode, and Zero the Same Specified Elements of Y

These subprograms gather specified elements of sparse vector ***y***, stored in full-vector storage mode, into sparse vector ***x***, stored in compressed-vector storage mode, and zero the same specified elements of vector ***y***.

*Table 59. Data Types*

| ***x***, ***y*** | **Subprogram** |
|---|---|
| Short-precision real | SGTHRZ |
| Long-precision real | DGTHRZ |
| Short-precision complex | CGTHRZ |
| Long-precision complex | ZGTHRZ |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SGTHRZ \| DGTHRZ \| CGTHRZ \| ZGTHRZ (*nz*, *y*, *x*, *indx*) |
| **C and C++** | sgthrz \| dgthrz \| cgthrz \| zgthrz (*nz*, *y*, *x*, *indx*); |
| **PL/I** | CALL SGTHRZ \| DGTHRZ \| CGTHRZ \| ZGTHRZ (*nz*, *y*, *x*, *indx*); |

*On Entry*

*nz*

is the number of elements in sparse vector ***x***, stored in compressed-vector storage mode. Specified as: a fullword integer; *nz* ≥ 0.

*y*

is the sparse vector ***y***, stored in full-vector storage mode, of (at least) length max(INDX(*i*)) for *i* = 1, *nz*, from which *nz* elements are copied from positions indicated by the indices array INDX.

Specified as: a one-dimensional array of (at least) length max(INDX(*i*)) for *i* = 1, *nz*, containing numbers of the data type indicated in Table 59.

*x*

See “On Return.”

*indx*

is the array, referred to as INDX, containing the *nz* indices that indicate the positions of the elements of the sparse vector ***x*** when in full-vector storage mode. They also indicate the positions in vector ***y*** from which elements are copied then set to zero.

Specified as: a one-dimensional array of (at least) length *nz*, containing fullword integers.

*On Return*

*y*

is the sparse vector ***y***, stored in full-vector storage mode, of (at least) length max(INDX(*i*)) for *i* = 1, *nz*, whose elements are set to zero at positions indicated by the indices array INDX.

Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 59.

*x*

is the sparse vector ***x***, containing *nz* elements stored in compressed-vector storage mode in an array, referred to as X, into which are copied the elements of vector ***y*** from positions indicated by the indices array INDX.

Returned as: a one-dimensional array of (at least) length *nz*, containing numbers of the data type indicated in Table 59 on page 285.

***Notes***

1. Each value specified in array INDX must be unique; otherwise, results are unpredictable.
2. Vectors ***x*** and ***y*** must have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.
3. For a description of how sparse vectors are stored, see “Sparse Vector” on page 60.

***Function:*** The copy is expressed as follows:

$x_i \leftarrow y_{\mathrm{INDX}(i)}$
$y_{\mathrm{INDX}(i)} \leftarrow 0.0$ (for SGTHRZ and DGTHRZ)
$y_{\mathrm{INDX}(i)} \leftarrow (0.0,0.0)$ (for CGTHRZ and ZGTHRZ)
for $i = 1,nz$

where:

***x*** is a sparse vector, stored in compressed-vector storage mode.
INDX is the indices array for sparse vector ***x***.
***y*** is a sparse vector, stored in full-vector storage mode.

See reference [29 ]. If *nz* is 0, no computation is performed.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $nz < 0$

***Example 1:*** This example shows how to use SGTHRZ to copy specified elements of a vector ***y*** into a sparse vector ***x*** of length 5, where the elements of array INDX are in ascending order.

*Call Statement and Input*

```
              NZ  Y   X   INDX
              |   |   |    |
CALL SGTHRZ( 5 , Y , X , INDX )

Y        =  (6.0, 2.0, 4.0, 7.0, 6.0, 10.0, -2.0, 8.0, 9.0, 0.0)
INDX     =  (1, 3, 4, 7, 9)
```

*Output*

```
Y        =  (0.0, 2.0, 0.0, 0.0, 6.0, 10.0, 0.0, 8.0, 0.0, 0.0)
X        =  (6.0, 4.0, 7.0, -2.0, 9.0)
```

***Example 2:*** This example shows how to use SGTHRZ to copy specified elements of a vector ***y*** into a sparse vector ***x*** of length 5, where the elements of array `INDX` are in random order. (Note that the element 0.0 occurs in output vector ***x***. This does not produce an error.)

*Call Statement and Input*

```
             NZ  Y   X   INDX
             |   |   |    |
CALL SGTHRZ( 5 , Y , X , INDX )

Y        =  (6.0, 2.0, 4.0, 7.0, 6.0, 10.0, -2.0, 8.0, 9.0, 0.0)
INDX     =  (4, 3, 1, 10, 7)
```

*Output*

```
Y        =  (0.0, 2.0, 0.0, 0.0, 6.0, 10.0, 0.0, 8.0, 9.0, 0.0)
X        =  (7.0, 4.0, 6.0, 0.0, -2.0)
```

***Example 3:*** This example shows how to use CGTHRZ to copy specified elements of a vector ***y*** into a sparse vector ***x*** of length 3, where the elements of array `INDX` are in random order.

*Call Statement and Input*

```
             NZ  Y   X   INDX
             |   |   |    |
CALL CGTHRZ( 3 , Y , X , INDX )

Y        =  ((6.0, 5.0), (-2.0, 3.0), (15.0, 4.0), (9.0, 0.0))
INDX     =  (4, 1, 3)
```

*Output*

```
Y        =  ((0.0, 0.0), (-2.0, 3.0), (0.0, 0.0), (0.0, 0.0))
X        =  ((9.0, 0.0), (6.0, 5.0), (15.0, 4.0))
```

# SAXPYI, DAXPYI, CAXPYI, and ZAXPYI—Multiply a Sparse Vector X in Compressed-Vector Storage Mode by a Scalar, Add to a Sparse Vector Y in Full-Vector Storage Mode, and Store in the Vector Y

These subprograms multiply sparse vector ***x***, stored in compressed-vector storage mode, by scalar $\alpha$, add it to sparse vector ***y***, stored in full-vector storage mode, and store the result in vector ***y***.

*Table 60. Data Types*

| $\alpha$, ***x***, ***y*** | **Subprogram** |
|---|---|
| Short-precision real | SAXPYI |
| Long-precision real | DAXPYI |
| Short-precision complex | CAXPYI |
| Long-precision complex | ZAXPYI |

### *Syntax*

| **Fortran** | CALL SAXPYI \| DAXPYI \| CAXPYI \| ZAXPYI (*nz*, *alpha*, *x*, *indx*, *y*) |
|---|---|
| **C and C++** | saxpyi \| daxpyi \| caxpyi \| zaxpyi (*nz*, *alpha*, *x*, *indx*, *y*); |
| **PL/I** | CALL SAXPYI \| DAXPYI \| CAXPYI \| ZAXPYI (*nz*, *alpha*, *x*, *indx*, *y*); |

*On Entry*

*nz*

is the number of elements in sparse vector ***x***, stored in compressed-vector storage mode. Specified as: a fullword integer; $nz \geq 0$.

*alpha*

is the scalar $\alpha$. Specified as: a number of the data type indicated in Table 60.

*x*

is the sparse vector ***x***, containing *nz* elements, stored in compressed-vector storage mode in an array, referred to as `X`. Specified as: a one-dimensional array of (at least) length *nz*, containing numbers of the data type indicated in Table 60.

*indx*

is the array, referred to as `INDX`, containing the *nz* indices that indicate the positions of the elements of the sparse vector ***x*** when in full-vector storage mode. They also indicate the positions of the elements in vector ***y*** that are used in the computation.

Specified as: a one-dimensional array of (at least) length *nz*, containing fullword integers.

*y*

is the sparse vector ***y***, stored in full-vector storage mode, of (at least) length max(`INDX`(*i*)) for *i* = 1, *nz*. Specified as: a one-dimensional array of (at least) length max(`INDX`(*i*)) for *i* = 1, *nz*, containing numbers of the data type indicated in Table 60.

*On Return*

*y*

is the sparse vector ***y***, stored in full-vector storage mode, of (at least) length max(INDX(*i*)) for *i* = 1, *nz* containing the results of the computation, stored at positions indicated by the indices array INDX.

Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 60 on page 288.

***Notes***

1. Each value specified in array INDX must be unique; otherwise, results are unpredictable.
2. Vectors ***x*** and ***y*** must have no common elements; otherwise, results are unpredictable. See "Concepts" on page 55.
3. For a description of how sparse vectors are stored, see "Sparse Vector" on page 60.

***Function:*** The computation is expressed as follows:

$$y_{\mathtt{INDX}(i)} \leftarrow y_{\mathtt{INDX}(i)} + \alpha x_i \quad \text{for } i = 1, nz$$

where:

***x*** is a sparse vector, stored in compressed-vector storage mode.
INDX is the indices array for sparse vector ***x***.
***y*** is a sparse vector, stored in full-vector storage mode.

See reference [29 ]. If $\alpha$ or *nz* is zero, no computation is performed. For SAXPYI and CAXPYI, intermediate results are accumulated in long-precision.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $nz < 0$

***Example 1:*** This example shows how to use SAXPYI to perform a computation using a sparse vector ***x*** of length 5, where the elements of array INDX are in ascending order.

*Call Statement and Input*

```
             NZ ALPHA  X   INDX   Y
             |    |    |    |     |
CALL SAXPYI( 5 , 2.0 , X , INDX , Y )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
INDX     =  (1, 3, 4, 7, 10)
Y        =  (1.0, 5.0, 4.0, 3.0, 6.0, 10.0, -2.0, 8.0, 9.0, 0.0)
```

*Output*

```
Y        =  (3.0, 5.0, 8.0, 9.0, 6.0, 10.0, 6.0, 8.0, 9.0, 10.0)
```

***Example 2:*** This example shows how to use SAXPYI to perform a computation using a sparse vector ***x*** of length 5, where the elements of array INDX are in random order.

*Call Statement and Input*

```
             NZ ALPHA  X   INDX   Y
             |    |    |    |     |
CALL SAXPYI( 5 , 2.0 , X , INDX , Y )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
INDX     =  (4, 3, 1, 10, 7)
Y        =  (1.0, 5.0, 4.0, 3.0, 6.0, 10.0, -2.0, 8.0, 9.0, 0.0)
```

*Output*

```
Y        =  (7.0, 5.0, 8.0, 5.0, 6.0, 10.0, 8.0, 8.0, 9.0, 8.0)
```

***Example 3:*** This example shows how to use CAXPYI to perform a computation using a sparse vector ***x*** of length 3, where the elements of array `INDX` are in random order.

*Call Statement and Input*

```
             NZ  ALPHA   X   INDX   Y
             |     |     |    |     |
CALL CAXPYI( 3 , ALPHA , X , INDX , Y )

ALPHA    =  (2.0, 3.0)
X        =  ((1.0, 2.0), (3.0, 4.0), (5.0, 6.0))
INDX     =  (4, 1, 3)
Y        =  ((6.0, 5.0), (-2.0, 3.0), (15.0, 4.0), (9.0, 0.0))
```

*Output*

```
Y        =  ((0.0, 22.0), (-2.0, 3.0), (7.0, 31.0), (5.0, 7.0))
```

# SDOTI, DDOTI, CDOTUI, ZDOTUI, CDOTCI, and ZDOTCI—Dot Product of a Sparse Vector X in Compressed-Vector Storage Mode and a Sparse Vector Y in Full-Vector Storage Mode

SDOTI, DDOTI, CDOTUI, and ZDOTUI compute the dot product of sparse vector ***x***, stored in compressed-vector storage mode, and full vector ***y***, stored in full-vector storage mode.

CDOTCI and ZDOTCI compute the dot product of the complex conjugate of sparse vector ***x***, stored in compressed-vector storage mode, and full vector ***y***, stored in full-vector storage mode.

*Table 61. Data Types*

| ***x***, ***y***, **Result** | **Subprogram** |
|---|---|
| Short-precision real | SDOTI |
| Long-precision real | DDOTI |
| Short-precision complex | CDOTUI |
| Long-precision complex | ZDOTUI |
| Short-precision complex | CDOTCI |
| Long-precision complex | ZDOTCI |

### *Syntax*

| | |
|---|---|
| **Fortran** | SDOTI \| DDOTI \| CDOTUI \| ZDOTUI \| CDOTCI \| ZDOTCI (*nz*, *x*, *indx*, *y*) |
| **C and C++** | sdoti \| ddoti \| cdotui \| zdotui \| cdotci \| zdotci (*nz*, *x*, *indx*, *y*); |
| **PL/I** | SDOTI \| DDOTI \| CDOTUI \| ZDOTUI \| CDOTCI \| ZDOTCI (*nz*, *x*, *indx*, *y*); |

*On Entry*

*nz*

is the number of elements in sparse vector ***x***, stored in compressed-vector storage mode. Specified as: a fullword integer; $nz \geq 0$.

*x*

is the sparse vector ***x***, containing *nz* elements, stored in compressed-vector storage mode in an array, referred to as X. Specified as: a one-dimensional array of (at least) length *nz*, containing numbers of the data type indicated in Table 61.

*indx*

is the array, referred to as INDX, containing the *nz* indices that indicate the positions of the elements of the sparse vector ***x*** when in full-vector storage mode. They also indicate the positions of elements in vector ***y*** that are used in the computation.

Specified as: a one-dimensional array of (at least) length *nz*, containing fullword integers.

*y*

is the sparse vector ***y***, stored in full-vector storage mode, of (at least) length max(INDX(*i*)) for $i = 1, nz$. Specified as: a one-dimensional array of (at least) length max(INDX(*i*)) for $i = 1, nz$, containing numbers of the data type indicated in Table 61.

*On Return*

*Function value*

is the result of the dot product computation.

Returned as: a number of the data type indicated in Table 61 on page 291.

***Note***

1. Declare this function in your program as returning a value of the data type indicated in Table 61 on page 291.
2. For a description of how sparse vectors are stored, see “Sparse Vector” on page 60.

***Function:*** For SDOTI, DDOTI, CDOTUI, and ZDOTUI, the dot product computation is expressed as follows:

$$\sum_{i=1}^{nz} x_i y_{\mathrm{INDX}(i)} = x_1 y_{\mathrm{INDX}(1)} + x_2 y_{\mathrm{INDX}(2)} + \ldots + x_{nz} y_{\mathrm{INDX}(nz)}$$

For CDOTCI and ZDOTCI, the dot product computation is expressed as follows:

$$\sum_{i=1}^{nz} \bar{x}_i y_{\mathtt{INDX}(i)} = \bar{x}_1 y_{\mathtt{INDX}(1)} + \bar{x}_2 y_{\mathtt{INDX}(2)} + \ldots + \bar{x}_{nz} y_{\mathtt{INDX}(nz)}$$

where:

***x*** is a sparse vector, stored in compressed-vector storage mode.

$\bar{x}$ is the complex conjugate of a sparse vector, stored in compressed - vector storage mode.

`INDX` is the indices array for sparse vector ***x***.

***y*** is a sparse vector, stored in full-vector storage mode.

See reference [29 ]. The result is returned as the function value. If *nz* is 0, then zero is returned as the value of the function.

For SDOTI, CDOTUI, and CDOTCI, intermediate results are accumulated in long-precision.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $nz < 0$

***Example 1:*** This example shows how to use SDOTI to compute a dot product using a sparse vector ***x*** of length 5, where the elements of array INDX are in ascending order.

*Function Reference and Input*

```
              NZ  X   INDX   Y
              |   |    |     |
DOTT = SDOTI( 5 , X , INDX , Y )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
INDX     =  (1, 3, 4, 7, 10)
Y        =  (1.0, 5.0, 4.0, 3.0, 6.0, 10.0, -2.0, 8.0, 9.0, 0.0)
```

*Output*

```
DOTT     =  (1.0 + 8.0 + 9.0 -8.0 + 0.0) = 10.0
```

***Example 2:*** This example shows how to use SDOTI to compute a dot product using a sparse vector ***x*** of length 5, where the elements of array INDX are in random order.

*Function Reference and Input*

```
              NZ  X   INDX   Y
              |   |    |     |
DOTT = SDOTI( 5 , X , INDX , Y )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
INDX     =  (4, 3, 1, 10, 7)
Y        =  (1.0, 5.0, 4.0, 3.0, 6.0, 10.0, -2.0, 8.0, 9.0, 0.0)
```

*Output*

```
DOTT     =  (3.0 + 8.0 + 3.0 + 0.0 -10.0) = 4.0
```

***Example 3:*** This example shows how to use CDOTUI to compute a dot product using a sparse vector ***x*** of length 3, where the elements of array INDX are in ascending order.

*Function Reference and Input*

```
               NZ  X   INDX   Y
               |   |    |     |
DOTT = CDOTUI( 3 , X , INDX , Y )

X        =  ((1.0, 2.0), (3.0, 4.0), (5.0, 6.0))
INDX     =  (1, 3, 4)
Y        =  ((6.0, 5.0), (-2.0, 3.0), (15.0, 4.0), (9.0, 0.0))
```

*Output*

```
DOTT     =  (70.0, 143.0)
```

***Example 4:*** This example shows how to use CDOTCI to compute a dot product using the complex conjugate of a sparse vector ***x*** of length 3, where the elements of array INDX are in random order.

*Function Reference and Input*

```
               NZ  X   INDX   Y
               |   |    |     |
DOTT = CDOTCI( 3 , X , INDX , Y )

X        =  ((1.0, 2.0), (3.0, 4.0), (5.0, 6.0))
INDX     =  (4, 1, 3)
Y        =  ((6.0, 5.0), (-2.0, 3.0), (15.0, 4.0), (9.0, 0.0))
```

*Output*

```
DOTT     =  (146.0, -97.0)
```

## Matrix-Vector Subprograms

This section contains the matrix-vector subprogram descriptions.

# SGEMV, DGEMV, CGEMV, ZGEMV, SGEMX, DGEMX, SGEMTX, and DGEMTX—Matrix-Vector Product for a General Matrix, Its Transpose, or Its Conjugate Transpose

SGEMV and DGEMV compute the matrix-vector product for either a real general matrix or its transpose, using the scalars $\alpha$ and $\beta$, vectors ***x*** and ***y***, and matrix ***A*** or its transpose:

$$\boldsymbol{y} \leftarrow \beta\boldsymbol{y}+\alpha\boldsymbol{Ax}$$

$$\boldsymbol{y} \leftarrow \beta\boldsymbol{y}+\alpha\boldsymbol{A}^{\mathrm{T}}\boldsymbol{x}$$

CGEMV and ZGEMV compute the matrix-vector product for either a complex general matrix, its transpose, or its conjugate transpose, using the scalars $\alpha$ and $\beta$, vectors ***x*** and ***y***, and matrix ***A***, its transpose, or its conjugate transpose:

$$\boldsymbol{y} \leftarrow \beta\boldsymbol{y}+\alpha\boldsymbol{Ax}$$
$$\boldsymbol{y} \leftarrow \beta\boldsymbol{y}+\alpha\boldsymbol{A}^{\mathrm{T}}\boldsymbol{x}$$
$$\boldsymbol{y} \leftarrow \beta\boldsymbol{y}+\alpha\boldsymbol{A}^{\mathrm{H}}\boldsymbol{x}$$

SGEMX and DGEMX compute the matrix-vector product for a real general matrix, using the scalar $\alpha$, vectors ***x*** and ***y***, and matrix ***A***:

$$\boldsymbol{y} \leftarrow \boldsymbol{y}+\alpha\boldsymbol{Ax}$$

SGEMTX and DGEMTX compute the matrix-vector product for the transpose of a real general matrix, using the scalar $\alpha$, vectors ***x*** and ***y***, and the transpose of matrix ***A***:

$$\boldsymbol{y} \leftarrow \boldsymbol{y}+\alpha\boldsymbol{A}^{\mathrm{T}}\boldsymbol{x}$$

*Table 62. Data Types*

| $\alpha$, $\beta$, ***x***, ***y***, ***A*** | **Subprogram** |
|---|---|
| Short-precision real | SGEMV, SGEMX, and SGEMTX |
| Long-precision real | DGEMV, DGEMX, and DGEMTX |
| Short-precision complex | CGEMV |
| Long-precision complex | ZGEMV |

**Note:** SGEMV and DGEMV are Level 2 BLAS subroutines. It is suggested that these subroutines be used instead of SGEMX, DGEMX, SGEMTX, and DGEMTX, which are provided only for compatibility with earlier releases of ESSL.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SGEMV \| DGEMV \| CGEMV \| ZGEMV (*transa*, *m*, *n*, *alpha*, *a*, *lda*, *x*, *incx*, *beta*, *y*, *incy*)<br>CALL SGEMX \| DGEMX \| SGEMTX \| DGEMTX ( *m*, *n*, *alpha*, *a*, *lda*, *x*, *incx*, *y*, *incy*) |
| **C and C++** | sgemv \| dgemv \| cgemv \| zgemv (*transa*, *m*, *n*, *alpha*, *a*, *lda*, *x*, *incx*, *beta*, *y*, *incy*);<br>sgemx \| dgemx \| sgemtx \| dgemtx ( *m*, *n*, *alpha*, *a*, *lda*, *x*, *incx*, *y*, *incy*); |
| **PL/I** | CALL SGEMV \| DGEMV \| CGEMV \| ZGEMV (*transa*, *m*, *n*, *alpha*, *a*, *lda*, *x*, *incx*, *beta*, *y*, *incy*);<br>CALL SGEMX \| DGEMX \| SGEMTX \| DGEMTX ( *m*, *n*, *alpha*, *a*, *lda*, *x*, *incx*, *y*, *incy*); |

*On Entry*

*transa*

indicates the form of matrix ***A*** to use in the computation, where:

If *transa* = `'N'`, ***A*** is used in the computation.

If *transa* = `'T'`, $\boldsymbol{A}^T$ is used in the computation.

If *transa* = `'C'`, $\boldsymbol{A}^H$ is used in the computation.

Specified as: a single character. It must be `'N'`, `'T'`, or `'C'`.

*m*

is the number of rows in matrix ***A***, and:

For SGEMV, DGEMV, CGEMV, and ZGEMV:
If *transa* = `'N'`, it is the length of vector ***y***.
If *transa* = `'T'` or `'C'`, it is the length of vector ***x***.

For SGEMX and DGEMX, it is the length of vector ***y***.

For SGEMTX and DGEMTX, it is the length of vector ***x***.

Specified as: a fullword integer; $0 \le m \le lda$.

*n*

is the number of columns in matrix ***A***, and:

For SGEMV, DGEMV, CGEMV, and ZGEMV:
If *transa* = `'N'`, it is the length of vector ***x***.
If *transa* = `'T'` or `'C'`, it is the length of vector ***y***.

For SGEMX and DGEMX, it is the length of vector ***x***.

For SGEMTX and DGEMTX, it is the length of vector ***y***.

Specified as: a fullword integer; $n \ge 0$.

*alpha*

is the scaling constant $\alpha$. Specified as: a number of the data type indicated in Table 62 on page 296.

*a*

is the *m* by *n* matrix ***A***, where:

For SGEMV, DGEMV, CGEMV, and ZGEMV:
If *transa* = `'N'`, ***A*** is used in the computation.
If *transa* = `'T'`, $\boldsymbol{A}^T$ is used in the computation.
If *transa* = `'C'`, $\boldsymbol{A}^H$ is used in the computation.

For SGEMX and DGEMX, ***A*** is used in the computation.

For SGEMTX and DGEMTX, $\boldsymbol{A}^T$ is used in the computation.

**Note:** No data should be moved to form $\boldsymbol{A}^T$ or $\boldsymbol{A}^H$; that is, the matrix ***A*** should always be stored in its untransposed form.

Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 62 on page 296.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; $lda > 0$ and $lda \ge m$.

*x*

is the vector ***x***, where:

For SGEMV, DGEMV, CGEMV, and ZGEMV:
If *transa* = `'N'`, it has length *n*.
If *transa* = `'T'` or `'C'`, it has length *m*.

For SGEMX and DGEMX, it has length *n*.

For SGEMTX and DGEMTX, it has length *m*.

Specified as: a one-dimensional array, containing numbers of the data type indicated in Table 62 on page 296, where:

For SGEMV, DGEMV, CGEMV, and ZGEMV:
If *transa* = `'N'`, it must have at least 1+(*n*–1)|*incx*| elements.
If *transa* = `'T'` or `'C'`, it must have at least 1+(*m*–1)|*incx*| elements.

For SGEMX and DGEMX, it must have at least 1+(*n*–1)|*incx*| elements.

For SGEMTX and DGEMTX, it must have at least 1+(*m*–1)|*incx*| elements.

*incx*

is the stride for vector ***x***. Specified as: a fullword integer; It can have any value.

*beta*

is the scaling constant β. Specified as: a number of the data type indicated in Table 62 on page 296.

*y*

is the vector ***y***, where:

For SGEMV, DGEMV, CGEMV, and ZGEMV:
If *transa* = `'N'`, it has length *m*.
If *transa* = `'T'` or `'C'`, it has length *n*.

For SGEMX and DGEMX, it has length *m*.

For SGEMTX and DGEMTX, it has length *n*.

Specified as: a one-dimensional array, containing numbers of the data type indicated in Table 62 on page 296, where:

For SGEMV, DGEMV, CGEMV, and ZGEMV:
If *transa* = `'N'`, it must have at least 1+(*m*–1)|*incy*| elements.
If *transa* = `'T'` or `'C'`, it must have at least 1+(*n*–1)|*incy*| elements.

For SGEMX and DGEMX, it must have at least 1+(*m*–1)|*incy*| elements.

For SGEMTX and DGEMTX, it must have at least 1+(*n*–1)|*incy*| elements.

*incy*

is the stride for vector ***y***. Specified as: a fullword integer; *incy* > 0 or *incy* < 0.

*On Return*

*y*

is the vector ***y***, containing the result of the computation, where:

For SGEMV, DGEMV, CGEMV, and ZGEMV:
If *transa* = `'N'`, it has length *m*.
If *transa* = `'T'` or `'C'`, it has length *n*.

For SGEMX and DGEMX, it has length *m*.

For SGEMTX and DGEMTX, it has length *n*.

Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 62 on page 296.

***Notes***

1. For SGEMV and DGEMV, if you specify `'C'` for the *transa* argument, it is interpreted as though you specified `'T'`.
2. The SGEMV, DGEMV, CGEMV, and ZGEMV subroutines accept lowercase letters for the *transa* argument.
3. In the SGEMV, DGEMV, CGEMV, and ZGEMV subroutines, *incx* = 0 is valid; however, the Level 2 BLAS standard considers *incx* = 0 to be invalid. See references [34 ] and [35 ].
4. Vector ***y*** must have no common elements with matrix ***A*** or vector ***x***; otherwise, results are unpredictable. See "Concepts" on page 55.

***Function:*** The possible computations that can be performed by these subroutines are described in the following sections. Varying implementation techniques are used for this computation to improve performance. As a result, accuracy of the computational result may vary for different computations.

For SGEMV, CGEMV, SGEMX, and SGEMTX, intermediate results are accumulated in long precision. Occasionally, for performance reasons, these intermediate results are stored.

See references [34 ], [35 ], [38 ], [46 ], and [76]. No computation is performed if *m* or *n* is 0 or if α is zero and β is one.

*General Matrix:* For SGEMV, DGEMV, CGEMV, and ZGEMV, the matrix-vector product for a general matrix:

$$\boldsymbol{y} \leftarrow \beta\boldsymbol{y}+\alpha\boldsymbol{Ax}$$

is expressed as follows:

$$\begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_m \end{bmatrix} \leftarrow \beta \begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_m \end{bmatrix} + \alpha \begin{bmatrix} a_{11} & \ldots & a_{1n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{m1} & \ldots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix}$$

For SGEMX and DGEMX, the matrix-vector product for a real general matrix:

$$\boldsymbol{y} \leftarrow \boldsymbol{y}+\alpha\boldsymbol{Ax}$$

is expressed as follows:

$$\begin{bmatrix} y_1 \\ . \\ . \\ . \\ y_m \end{bmatrix} \leftarrow \begin{bmatrix} y_1 \\ . \\ . \\ . \\ y_m \end{bmatrix} + \alpha \begin{bmatrix} a_{11} & \dots & a_{1n} \\ . & & . \\ . & & . \\ . & & . \\ a_{m1} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ . \\ . \\ . \\ x_n \end{bmatrix}$$

In these expressions:

***y*** is a vector of length *m*.
$\alpha$ is a scalar.
$\beta$ is a scalar.
***A*** is an *m* by *n* matrix.
***x*** is a vector of length *n*.

*Transpose of a General Matrix:* For SGEMV, DGEMV, CGEMV and ZGEMV, the matrix-vector product for the transpose of a general matrix:

$\boldsymbol{y} \leftarrow \beta\boldsymbol{y}+\alpha\boldsymbol{A}^{\mathrm{T}}\boldsymbol{x}$

is expressed as follows:

$$\begin{bmatrix} y_1 \\ . \\ . \\ . \\ y_n \end{bmatrix} \leftarrow \beta \begin{bmatrix} y_1 \\ . \\ . \\ . \\ y_n \end{bmatrix} + \alpha \begin{bmatrix} a_{11} & \dots & a_{m1} \\ . & & . \\ . & & . \\ . & & . \\ a_{1n} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ . \\ . \\ . \\ x_m \end{bmatrix}$$

For SGEMTX and DGEMTX, the matrix-vector product for the transpose of a real general matrix:

$\boldsymbol{y} \leftarrow \boldsymbol{y}+\alpha\boldsymbol{A}^{\mathrm{T}}\boldsymbol{x}$

is expressed as follows:

$$\begin{bmatrix} y_1 \\ . \\ . \\ . \\ y_n \end{bmatrix} \leftarrow \begin{bmatrix} y_1 \\ . \\ . \\ . \\ y_n \end{bmatrix} + \alpha \begin{bmatrix} a_{11} & \dots & a_{m1} \\ . & & . \\ . & & . \\ . & & . \\ a_{1n} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ . \\ . \\ . \\ x_m \end{bmatrix}$$

In these expressions:

***y*** is a vector of length *n*.
$\alpha$ is a scalar.
$\beta$ is a scalar.
$\boldsymbol{A}^{\mathrm{T}}$ is the transpose of matrix ***A***, where ***A*** is an *m* by *n* matrix.
***x*** is a vector of length *m*.

*Conjugate Transpose of a General Matrix:* For CGEMV and ZGEMV, the matrix-vector product for the conjugate transpose of a general matrix:

$\boldsymbol{y} \leftarrow \beta\boldsymbol{y}+\alpha\boldsymbol{A}^{\mathrm{H}}\boldsymbol{x}$

is expressed as follows:

$$\begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} \leftarrow \beta \begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} + \alpha \begin{bmatrix} \bar{a}_{11} & \ldots & \bar{a}_{m1} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \bar{a}_{1n} & \ldots & \bar{a}_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_m \end{bmatrix}$$

where:

> ***y*** is a vector of length *n*.
> $\alpha$ is a scalar.
> $\beta$ is a scalar.
> ***A***$^H$ is the conjugate transpose of matrix ***A***, where ***A*** is an *m* by *n* matrix.
> ***x*** is a vector of length *m*.

***Error Conditions***

*Resource Errors:* Unable to allocate internal work area (for SGEMV, DGEMV, CGEMV, and ZGEMV).

*Computational Errors:* None

*Input-Argument Errors*

1. *transa* ≠ 'N', 'T', or 'C'
2. $m < 0$
3. $m > lda$
4. $n < 0$
5. $lda \le 0$
6. $incy = 0$

***Example 1:*** This example shows the computation for TRANSA equal to 'N', where the real general matrix ***A*** is used in the computation. Because *lda* is 10 and *n* is 3, array A must be declared as A(E1:E2,F1:F2), where E2-E1+1=10 and F2-F1+1 ≥ 3. In this example, array A is declared as A(1:10,0:2).

*Call Statement and Input*

```
           TRANSA M   N  ALPHA    A      LDA  X  INCX BETA   Y  INCY
             |    |   |    |      |       |   |   |    |     |   |
CALL SGEMV( 'N' , 4 , 3 , 1.0 , A(1,0) , 10 , X , 1  , 1.0 , Y , 2  )

    ┌               ┐
    | 1.0  2.0  3.0 |
    | 2.0  2.0  4.0 |
    | 3.0  2.0  2.0 |
    | 4.0  2.0  1.0 |
A = |  .    .    .  |
    |  .    .    .  |
    |  .    .    .  |
    |  .    .    .  |
    |  .    .    .  |
    |  .    .    .  |
    └               ┘
```

```
X        =  (3.0, 2.0, 1.0)
Y        =  (4.0, . , 5.0, . , 2.0, . , 3.0)
```

*Output*

```
Y        =  (14.0, . , 19.0, . , 17.0, . , 20.0)
```

***Example 2:*** This example shows the computation for `TRANSA` equal to `'T'`, where the transpose of the real general matrix ***A*** is used in the computation. Array `A` must follow the same rules as given in Example 1. In this example, array `A` is declared as `A(-1:8,1:3)`.

*Call Statement and Input*

```
           TRANSA M   N  ALPHA     A     LDA   X  INCX  BETA  Y  INCY
             |    |   |    |       |      |    |   |     |    |   |
CALL SGEMV( 'T' , 4 , 3 , 1.0 , A(-1,1) , 10 , X , 1  , 2.0 , Y , 2  )
```

```
A        =(same as input A in Example 1)
X        =  (3.0, 2.0, 1.0, 4.0)
Y        =  (1.0, . , 2.0, . , 3.0)
```

*Output*

```
Y        =  (28.0, . , 24.0, . , 29.0)
```

***Example 3:*** This example shows the computation for `TRANSA` equal to `'N'`, where the complex general matrix ***A*** is used in the computation.

*Call Statement and Input*

```
           TRANSA M   N   ALPHA   A   LDA  X  INCX  BETA   Y  INCY
             |    |   |     |     |    |   |   |     |     |   |
CALL CGEMV( 'N' , 5 , 3 , ALPHA , A , 10 , X , 1  , BETA , Y , 1  )
```

```
ALPHA    =  (1.0, 0.0)
```

```
    ┌                                    ┐
    │ (1.0, 2.0)  (3.0, 5.0)  (2.0, 0.0) │
    │ (2.0, 3.0)  (7.0, 9.0)  (4.0, 8.0) │
    │ (7.0, 4.0)  (1.0, 4.0)  (6.0, 0.0) │
    │ (8.0, 2.0)  (2.0, 5.0)  (8.0, 0.0) │
A = │ (9.0, 1.0)  (3.0, 6.0)  (1.0, 0.0) │
    │     .           .           .      │
    │     .           .           .      │
    │     .           .           .      │
    │     .           .           .      │
    │     .           .           .      │
    └                                    ┘
```

```
X        =  ((1.0, 2.0), (4.0, 0.0), (1.0, 1.0))
BETA     =  (1.0, 0.0)
Y        =  ((1.0, 2.0), (4.0, 0.0), (1.0, -1.0), (3.0, 4.0),
             (2.0, 0.0))
```

*Output*

```
Y        =  ((12.0, 28.0), (24.0, 55.0), (10.0, 39.0), (23.0, 50.0),
             (22.0, 44.0))
```

***Example 4:*** This example shows the computation for `TRANSA` equal to `'T'`, where the transpose of complex general matrix ***A*** is used in the computation. Because $\beta$ is zero, the result of the computation is $\alpha \boldsymbol{A}^{T} \boldsymbol{x}$

*Call Statement and Input*

```
           TRANSA M   N   ALPHA   A   LDA  X  INCX  BETA   Y  INCY
             |    |   |     |     |    |   |   |     |     |   |
CALL CGEMV( 'T' , 5 , 3 , ALPHA , A , 10 , X , 1  , BETA , Y , 1  )
```

```
ALPHA    =  (1.0, 0.0)
A        =(same as input A in Example 3)
X        =  ((1.0, 2.0), (4.0, 0.0), (1.0, 1.0), (3.0, 4.0),
             (2.0, 0.0))
BETA     =  (0.0, 0.0)
Y        =(not relevant)
```

*Output*

```
Y        =  ((42.0, 67.0), (10.0, 87.0), (50.0, 74.0))
```

***Example 5:*** This example shows the computation for `TRANSA` equal to `'C'`, where the conjugate transpose of the complex general matrix ***A*** is used in the computation.

*Call Statement and Input*

```
           TRANSA M   N   ALPHA   A   LDA  X  INCX  BETA   Y  INCY
             |    |   |     |     |    |   |   |     |     |   |
CALL CGEMV( 'C' , 5 , 3 , ALPHA , A , 10 , X , 1  , BETA , Y , 1  )
```

```
ALPHA    =  (-1.0, 0.0)
A        =(same as input A in Example 3)
X        =  ((1.0, 2.0), (4.0, 0.0), (1.0, 1.0), (3.0, 4.0),
             (2.0, 0.0))
BETA     =  (1.0, 0.0)
Y        =  ((1.0, 2.0), (4.0, 0.0), (1.0, -1.0))
```

*Output*

```
Y        =  ((-73.0, -13.0), (-74.0, 57.0), (-49.0, -11.0))
```

***Example 6:*** This example shows a matrix, ***A***, contained in a larger array, `A`. The strides of vectors ***x*** and ***y*** are positive. Because *lda* is 10 and *n* is 3, array `A` must be declared as `A(E1:E2,F1:F2)`, where `E2-E1+1=10` and `F2-F1+1 ≥` 3. For this example, array `A` is declared as `A(1:10,0:2)`.

*Call Statement and Input*

```
            M   N  ALPHA    A     LDA   X  INCX  Y  INCY
            |   |    |      |      |    |   |    |   |
CALL SGEMX( 4 , 3 , 1.0 , A(1,0) , 10 , X , 1  , Y , 2  )
```

```
     ┌                ┐
     | 1.0  2.0  3.0  |
     | 2.0  2.0  4.0  |
     | 3.0  2.0  2.0  |
     | 4.0  2.0  1.0  |
A =  |  .    .    .   |
     |  .    .    .   |
     |  .    .    .   |
     |  .    .    .   |
     |  .    .    .   |
     |  .    .    .   |
     └                ┘
```

```
X        =  (3.0, 2.0, 1.0)
Y        =  (4.0, . , 5.0, . , 2.0, . , 3.0)
```

*Output*

```
Y        =  (14.0, . , 19.0, . , 17.0, . , 20.0)
```

***Example 7:*** This example shows a matrix, ***A***, contained in a larger array, A. The strides of vectors ***x*** and ***y*** are of opposite sign. For ***y***, which has negative stride, processing begins at element Y(7), which is 4.0. Array A must follow the same rules as given in Example 6. For this example, array A is declared as A(-1:8,1:3).

*Call Statement and Input*

```
            M   N  ALPHA     A      LDA   X  INCX  Y   INCY
            |   |    |       |       |    |   |    |     |
CALL SGEMX( 4 , 3 , 1.0 , A(-1,1) , 10 , X , 1  , Y , -2  )
```

```
A        =(same as input A in Example 6)
X        =  (3.0, 2.0, 1.0)
Y        =  (3.0, . , 2.0, . , 5.0, . , 4.0)
```

*Output*

```
Y        =  (20.0, . , 17.0, . , 19.0, . , 14.0)
```

***Example 8:*** This example shows a matrix, ***A***, contained in a larger array, A, and the first element of the matrix is not the first element of the array. Array A must follow the same rules as given in Example 6. For this example, array A is declared as A(1:10,1:3).

*Call Statement and Input*

```
            M   N  ALPHA    A      LDA   X  INCX  Y  INCY
            |   |    |      |       |    |   |    |   |
CALL SGEMX( 4 , 3 , 1.0 , A(5,1) , 10 , X , 1  , Y , 1  )
```

```
    ┌              ┐
    | .    .    .  |
    | .    .    .  |
    | .    .    .  |
    | .    .    .  |
A = | 1.0  2.0  3.0 |
    | 2.0  2.0  4.0 |
    | 3.0  2.0  2.0 |
    | 4.0  2.0  1.0 |
    |  .    .    . |
    |  .    .    . |
    └              ┘

X        =  (3.0, 2.0, 1.0)
Y        =  (4.0, 5.0, 2.0, 3.0)
```

*Output*

```
Y        =  (14.0, 19.0, 17.0, 20.0)
```

***Example 9:*** This example shows a matrix, ***A***, and an array, `A`, having the same number of rows. For this case, *m* and *lda* are equal. Because *lda* is 4 and *n* is 3, array `A` must be declared as `A(E1:E2,F1:F2)`, where `E2-E1+1=4` and `F2-F1+1 ≥` 3. For this example, array `A` is declared as `A(1:4,0:2)`.

*Call Statement and Input*

```
           M   N  ALPHA    A     LDA  X  INCX  Y  INCY
           |   |    |      |      |   |   |    |   |
CALL SGEMX( 4 , 3 , 1.0 , A(1,0) , 4 , X , 1  , Y , 1  )

    ┌              ┐
    | 1.0  2.0  3.0 |
A = | 2.0  2.0  4.0 |
    | 3.0  2.0  2.0 |
    | 4.0  2.0  1.0 |
    └              ┘

X        =  (3.0, 2.0, 1.0)
Y        =  (4.0, 5.0, 2.0, 3.0)
```

*Output*

```
Y        =  (14.0, 19.0, 17.0, 20.0)
```

***Example 10:*** This example shows a matrix, ***A***, and an array, `A`, having the same number of rows. For this case, *m* and *lda* are equal. Because *lda* is 4 and *n* is 3, array `A` must be declared as `A(E1:E2,F1:F2)`, where `E2-E1+1=4` and `F2-F1+1 ≥` 3. For this example, array `A` is declared as `A(1:4,0:2)`.

*Call Statement and Input*

```
              M   N  ALPHA    A      LDA  X  INCX  Y  INCY
              |   |    |      |       |   |   |    |   |
CALL SGEMTX( 4 , 3 , 1.0 , A(1,0) , 4 , X , 1  , Y , 1  )
```

```
    ┌               ┐
    | 1.0  2.0  3.0 |
A = | 2.0  2.0  4.0 |
    | 3.0  2.0  2.0 |
    | 4.0  2.0  1.0 |
    └               ┘
```

```
X        =  (3.0, 2.0, 1.0, 4.0)
Y        =  (1.0, 2.0, 3.0)
```

*Output*

```
Y        =  (27.0, 22.0, 26.0)
```

***Example 11:*** This example shows a computation in which *alpha* is greater than 1. Array A must follow the same rules as given in Example 10. For this example, array A is declared as A(-1:2,1:3).

*Call Statement and Input*

```
              M   N  ALPHA     A      LDA  X  INCX  Y  INCY
              |   |    |       |       |   |   |    |   |
CALL SGEMTX( 4 , 3 , 2.0 , A(-1,1) , 4 , X , 1  , Y , 1  )
```

A =(same as input A in Example 10)

```
X        =  (3.0, 2.0, 1.0, 4.0)
Y        =  (1.0, 2.0, 3.0)
```

*Output*

```
Y        =  (53.0, 42.0, 49.0)
```

# SGER, DGER, CGERU, ZGERU, CGERC, and ZGERC—Rank-One Update of a General Matrix

SGER, DGER, CGERU, and ZGERU compute the rank-one update of a general matrix, using the scalar $\alpha$, matrix $\boldsymbol{A}$, vector $\boldsymbol{x}$, and the transpose of vector $\boldsymbol{y}$:

$$\boldsymbol{A} \leftarrow \boldsymbol{A}+\alpha \boldsymbol{x}\boldsymbol{y}^{T}$$

CGERC and ZGERC compute the rank-one update of a general matrix, using the scalar $\alpha$, matrix $\boldsymbol{A}$, vector $\boldsymbol{x}$, and the conjugate transpose of vector $\boldsymbol{y}$:

$$\boldsymbol{A} \leftarrow \boldsymbol{A}+\alpha \boldsymbol{x}\boldsymbol{y}^{H}$$

*Table 63. Data Types*

| $\alpha$, ***A***, ***x***, ***y*** | **Subprogram** |
|---|---|
| Short-precision real | SGER |
| Long-precision real | DGER |
| Short-precision complex | CGERU and CGERC |
| Long-precision complex | ZGERU and ZGERC |

**Note:** For compatibility with earlier releases of ESSL, you can use the names SGER1 and DGER1 for SGER and DGER, respectively.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SGER \| DGER \| CGERU \| ZGERU \| CGERC \| ZGERC (*m*, *n*, *alpha*, *x*, *incx*, *y*, *incy*, *a*, *lda*) |
| **C and C++** | sger \| dger \| cgeru \| zgeru \| cgerc \| zgerc (*m*, *n*, *alpha*, *x*, *incx*, *y*, *incy*, *a*, *lda*); |
| **PL/I** | CALL SGER \| DGER \| CGERU \| ZGERU \| CGERC \| ZGERC (*m*, *n*, *alpha*, *x*, *incx*, *y*, *incy*, *a*, *lda*); |

*On Entry*

*m*
is the number of rows in matrix ***A*** and the number of elements in vector ***x***. Specified as: a fullword integer; $0 \leq m \leq lda$.

*n*
is the number of columns in matrix ***A*** and the number of elements in vector ***y***. Specified as: a fullword integer; $n \geq 0$.

*alpha*
is the scaling constant $\alpha$. Specified as: a number of the data type indicated in Table 63.

*x*
is the vector ***x*** of length *m*. Specified as: a one-dimensional array of (at least) length $1+(m-1)|incx|$, containing numbers of the data type indicated in Table 63.

*incx*
is the stride for vector ***x***. Specified as: a fullword integer. It can have any value.

*y*
is the vector ***y*** of length *n*, whose transpose or conjugate transpose is used in the computation.

**Note:** No data should be moved to form $\boldsymbol{y}^{T}$ or $\boldsymbol{y}^{H}$; that is, the vector ***y*** should always be stored in its untransposed form.

Specified as: a one-dimensional array of (at least) length 1+(*n*−1)|*incy*|, containing numbers of the data type indicated in Table 63.

*incy*

is the stride for vector ***y***. Specified as: a fullword integer. It can have any value.

*a*

is the *m* by *n* matrix ***A***. Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 63 on page 307.

*lda*

is the size of the leading dimension of the array specified for *a*. Specified as: a fullword integer; *lda* > 0 and *lda* ≥ *m*.

*On Return*

*a*

is the *m* by *n* matrix ***A***, containing the result of the computation.

Returned as: a two-dimensional array, containing numbers of the data type indicated in Table 63 on page 307.

***Notes***

1. In these subroutines, *incx* = 0 and *incy* = 0 are valid; however, the Level 2 BLAS standard considers *incx* = 0 and *incy* = 0 to be invalid. See references [34] and [35 ].
2. Matrix ***A*** can have no common elements with vectors ***x*** and ***y***; otherwise, results are unpredictable. See “Concepts” on page 55.

***Function:*** SGER, DGER, CGERU, and ZGERU compute the rank-one update of a general matrix:

$$\boldsymbol{A} \leftarrow \boldsymbol{A} + \alpha \boldsymbol{x}\boldsymbol{y}^T$$

where:

***A*** is an *m* by *n* matrix.
$\alpha$ is a scalar.
***x*** is a vector of length *m*.
$\boldsymbol{y}^T$ is the transpose of vector ***y*** of length *n*.

It is expressed as follows:

$$\begin{bmatrix} a_{11} & \dots & a_{1n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{m1} & \dots & a_{mn} \end{bmatrix} \leftarrow \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{m1} & \dots & a_{mn} \end{bmatrix} + \alpha \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_m \end{bmatrix} \begin{bmatrix} y_1 & \dots & y_n \end{bmatrix}$$

It can also be expressed as:

$$\begin{bmatrix} a_{11} & \dots & a_{1n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{m1} & \dots & a_{mn} \end{bmatrix} \leftarrow \begin{bmatrix} a_{11} + \alpha x_1 y_1 & \dots & a_{1n} + \alpha x_1 y_n \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{m1} + \alpha x_m y_1 & \dots & a_{mn} + \alpha x_m y_n \end{bmatrix}$$

CGERC and ZGERC compute a slightly different rank-one update of a general matrix:

$$\boldsymbol{A} \leftarrow \boldsymbol{A} + \alpha \boldsymbol{x} \boldsymbol{y}^{\mathrm{H}}$$

where:

$\boldsymbol{A}$ is an *m* by *n* matrix.
$\alpha$ is a scalar.
$\boldsymbol{x}$ is a vector of length *m*.
$\boldsymbol{y}^{\mathrm{H}}$ is the conjugate transpose of vector $\boldsymbol{y}$ of length *n*.

It is expressed as follows:

$$\begin{bmatrix} a_{11} & \dots & a_{1n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{m1} & \dots & a_{mn} \end{bmatrix} \leftarrow \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{m1} & \dots & a_{mn} \end{bmatrix} + \alpha \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_m \end{bmatrix} \begin{bmatrix} \bar{y}_1 \; \dots \; \bar{y}_n \end{bmatrix}$$

It can also be expressed as:

$$\begin{bmatrix} a_{11} & \dots & a_{1n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{m1} & \dots & a_{mn} \end{bmatrix} \leftarrow \begin{bmatrix} a_{11} + \alpha x_1 \bar{y}_1 & \dots & a_{1n} + \alpha x_1 \bar{y}_n \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{m1} + \alpha x_m \bar{y}_1 & \dots & a_{mn} + \alpha x_m \bar{y}_n \end{bmatrix}$$

See references [34 ], [35 ], and [76 ]. No computation is performed if *m, n*, or $\alpha$ is zero. For CGERU and CGERC, intermediate results are accumulated in long precision. For SGER, intermediate results are accumulated in long precision on some platforms.

### Error Conditions

*Resource Errors:* Unable to allocate internal work area.

*Computational Errors:* None

*Input-Argument Errors*

1. $m < 0$
2. $n < 0$
3. $lda \le 0$
4. $m > lda$

***Example 1:*** This example shows a matrix, ***A***, contained in a larger array, `A`. The strides of vectors ***x*** and ***y*** are positive. Because *lda* is 10 and *n* is 3, array `A` must be declared as `A(E1:E2,F1:F2)`, where `E2-E1+1=10` and `F2-F1+1 ≥ 3`. For this example, array `A` is declared as `A(1:10,0:2)`.

*Call Statement and Input*

```
           M   N  ALPHA  X  INCX  Y  INCY    A     LDA
           |   |    |    |   |    |   |      |      |
CALL SGER( 4 , 3 , 1.0 , X , 1  , Y , 2  , A(1,0) , 10 )

X        =  (3.0, 2.0, 1.0, 4.0)
Y        =  (1.0, . , 2.0, . , 3.0)

    ┌               ┐
    │ 1.0  2.0  3.0 │
    │ 2.0  2.0  4.0 │
    │ 3.0  2.0  2.0 │
    │ 4.0  2.0  1.0 │
A = │  .    .    .  │
    │  .    .    .  │
    │  .    .    .  │
    │  .    .    .  │
    │  .    .    .  │
    │  .    .    .  │
    └               ┘
```

*Output*

```
    ┌                 ┐
    │ 4.0   8.0  12.0 │
    │ 4.0   6.0  10.0 │
    │ 4.0   4.0   5.0 │
    │ 8.0  10.0  13.0 │
A = │  .     .     .  │
    │  .     .     .  │
    │  .     .     .  │
    │  .     .     .  │
    │  .     .     .  │
    │  .     .     .  │
    └                 ┘
```

***Example 2:*** This example shows a matrix, ***A***, contained in a larger array, `A`. The strides of vectors ***x*** and ***y*** are of opposite sign. For ***y***, which has negative stride, processing begins at element `Y(5)`, which is 1.0. Array `A` must follow the same rules as given in Example 1. For this example, array `A` is declared as `A(-1:8,1:3)`.

*Call Statement and Input*

```
           M   N  ALPHA  X  INCX  Y   INCY     A      LDA
           |   |    |    |   |    |    |       |       |
CALL SGER( 4 , 3 , 1.0 , X , 1  , Y , -2  , A(-1,1) , 10 )

X        =  (3.0, 2.0, 1.0, 4.0)
Y        =  (3.0, . , 2.0, . , 1.0)
A        =(same as input A in Example 1)
```

*Output*

```
A          =(same as input A in Example 1)
```

***Example 3:*** This example shows a matrix, ***A***, contained in a larger array, `A`, and the first element of the matrix is not the first element of the array. Array `A` must follow the same rules as given in Example 1. For this example, array `A` is declared as `A(1:10,1:3)`.

*Call Statement and Input*

```
           M   N  ALPHA  X  INCX  Y  INCY    A     LDA
           |   |    |    |   |    |   |      |      |
CALL SGER( 4 , 3 , 1.0 , X , 3  , Y , 1  , A(4,1) , 10 )

X        =  (3.0, . , . , 2.0, . , . , 1.0, . , . , 4.0)
Y        =  (1.0, 2.0, 3.0)
```

```
    ┌               ┐
    | .    .    .   |
    | .    .    .   |
    | .    .    .   |
    | 1.0  2.0  3.0 |
A = | 2.0  2.0  4.0 |
    | 3.0  2.0  2.0 |
    | 4.0  2.0  1.0 |
    |  .    .    .  |
    |  .    .    .  |
    |  .    .    .  |
    └               ┘
```

*Output*

```
    ┌                 ┐
    | .     .     .   |
    | .     .     .   |
    | .     .     .   |
    | 4.0   8.0  12.0 |
A = | 4.0   6.0  10.0 |
    | 4.0   4.0   5.0 |
    | 8.0  10.0  13.0 |
    |  .     .     .  |
    |  .     .     .  |
    |  .     .     .  |
    └                 ┘
```

***Example 4:*** This example shows a matrix, ***A***, and array, `A`, having the same number of rows. For this case, *m* and *lda* are equal. Because *lda* is 4 and *n* is 3, array `A` must be declared as `A(E1:E2,F1:F2)`, where `E2-E1+1=4` and `F2-F1+1 ≥ 3`. For this example, array `A` is declared as `A(1:4,0:2)`.

*Call Statement and Input*

```
           M   N  ALPHA  X  INCX  Y  INCY    A     LDA
           |   |    |    |   |    |   |      |      |
CALL SGER( 4 , 3 , 1.0 , X , 1  , Y , 1  , A(1,0) , 4 )

X        =  (3.0, 2.0, 1.0, 4.0)
Y        =  (1.0, 2.0, 3.0)
```

```
    ┌               ┐
    | 1.0  2.0  3.0 |
A = | 2.0  2.0  4.0 |
    | 3.0  2.0  2.0 |
    | 4.0  2.0  1.0 |
    └               ┘
```

*Output*

```
    ┌                 ┐
    | 4.0   8.0  12.0 |
A = | 4.0   6.0  10.0 |
    | 4.0   4.0   5.0 |
    | 8.0  10.0  13.0 |
    └                 ┘
```

***Example 5:*** This example shows a computation in which scalar value for *alpha* is greater than 1. Array `A` must follow the same rules as given in Example 4. For this example, array `A` is declared as `A(-1:2,1:3)`.

*Call Statement and Input*

```
          M   N  ALPHA  X  INCX  Y  INCY     A      LDA
          |   |    |    |   |    |   |       |       |
CALL SGER( 4 , 3 , 2.0 , X , 1  , Y , 1  , A(-1,1) , 4 )

X        =  (3.0, 2.0, 1.0, 4.0)
Y        =  (1.0, 2.0, 3.0)
A        =(same as input A in Example 4)
```

*Output*

```
    ┌                  ┐
    |  7.0  14.0  21.0 |
A = |  6.0  10.0  16.0 |
    |  5.0   6.0   8.0 |
    | 12.0  18.0  25.0 |
    └                  ┘
```

***Example 6:*** This example shows a rank-one update in which all data items contain complex numbers, and the transpose $\boldsymbol{y}^T$ is used in the computation. Matrix ***A*** is contained in a larger array, `A`. The strides of vectors ***x*** and ***y*** are positive. The Fortran DIMENSION statement for array `A` must follow the same rules as given in Example 1. For this example, array `A` is declared as `A(1:10,0:2)`.

*Call Statement and Input*

```
           M   N   ALPHA   X  INCX  Y  INCY    A     LDA
           |   |     |     |   |    |   |      |      |
CALL CGERU( 5 , 3 , ALPHA , X , 1  , Y , 1  , A(1,0) , 10 )

ALPHA    =  (1.0, 0.0)
X        =  ((1.0, 2.0), (4.0, 0.0), (1.0, 1.0), (3.0, 4.0),
             (2.0, 0.0))
Y        =  ((1.0, 2.0), (4.0, 0.0), (1.0, -1.0))
```

```
    ┌                                        ┐
    │ (1.0, 2.0)  (3.0, 5.0)  (2.0, 0.0) │
    │ (2.0, 3.0)  (7.0, 9.0)  (4.0, 8.0) │
    │ (7.0, 4.0)  (1.0, 4.0)  (6.0, 0.0) │
    │ (8.0, 2.0)  (2.0, 5.0)  (8.0, 0.0) │
A = │ (9.0, 1.0)  (3.0, 6.0)  (1.0, 0.0) │
    │     .           .           .      │
    │     .           .           .      │
    │     .           .           .      │
    │     .           .           .      │
    │     .           .           .      │
    └                                        ┘
```

*Output*

```
    ┌                                                ┐
    │ (-2.0,   6.0)   (7.0,  13.0)   (5.0,   1.0) │
    │  (6.0,  11.0)  (23.0,   9.0)   (8.0,   4.0) │
    │  (6.0,   7.0)   (5.0,   8.0)   (8.0,   0.0) │
    │  (3.0,  12.0)  (14.0,  21.0)  (15.0,   1.0) │
A = │ (11.0,   5.0)  (11.0,   6.0)   (3.0,  -2.0) │
    │      .              .              .         │
    │      .              .              .         │
    │      .              .              .         │
    │      .              .              .         │
    │      .              .              .         │
    └                                                ┘
```

***Example 7:*** This example shows a rank-one update in which all data items contain complex numbers, and the conjugate transpose $\boldsymbol{y}^H$ is used in the computation. Matrix ***A*** is contained in a larger array, `A`. The strides of vectors ***x*** and ***y*** are positive. The Fortran DIMENSION statement for array `A` must follow the same rules as given in Example 1. For this example, array `A` is declared as `A(1:10,0:2)`.

*Call Statement and Input*

```
            M   N   ALPHA   X  INCX  Y  INCY    A     LDA
            |   |     |     |   |    |   |      |      |
CALL CGERC( 5 , 3 , ALPHA , X , 1  , Y , 1  , A(1,0) , 10 )

ALPHA    =  (1.0, 0.0)
X        =  ((1.0, 2.0), (4.0, 0.0), (1.0, 1.0), (3.0, 4.0),
             (2.0, 0.0))
Y        =  ((1.0, 2.0), (4.0, 0.0), (1.0, -1.0))
A        =(same as input A in Example 6 )
```

*Output*

$$A = \begin{bmatrix} (6.0, & 2.0) & (7.0, & 13.0) & (1.0, & 3.0) \\ (6.0, & -5.0) & (23.0, & 9.0) & (8.0, & 12.0) \\ (10.0, & 3.0) & (5.0, & 8.0) & (6.0, & 2.0) \\ (19.0, & 0.0) & (14.0, & 21.0) & (7.0, & 7.0) \\ (11.0, & -3.0) & (11.0, & 6.0) & (3.0, & 2.0) \\ . & & . & & . & \\ . & & . & & . & \\ . & & . & & . & \\ . & & . & & . & \\ . & & . & & . & \end{bmatrix}$$

# SSPMV, DSPMV, CHPMV, ZHPMV, SSYMV, DSYMV, CHEMV, ZHEMV, SSLMX, and DSLMX—Matrix-Vector Product for a Real Symmetric or Complex Hermitian Matrix

SSPMV, DSPMV, CHPMV, ZHPMV, SSYMV, DSYMV, CHEMV, and ZHEMV compute the matrix-vector product for either a real symmetric matrix or a complex Hermitian matrix, using the scalars $\alpha$ and $\beta$, matrix ***A***, and vectors ***x*** and ***y***:

$$\boldsymbol{y} \leftarrow \beta\boldsymbol{y}+\alpha\boldsymbol{Ax}$$

SSLMX and DSLMX compute the matrix-vector product for a real symmetric matrix, using the scalar $\alpha$, matrix ***A***, and vectors ***x*** and ***y***:

$$\boldsymbol{y} \leftarrow \boldsymbol{y}+\alpha\boldsymbol{Ax}$$

The following storage modes are used:

- For SSPMV, DSPMV, CHPMV, and ZHPMV, matrix ***A*** is stored in upper- or lower-packed storage mode.
- For SSYMV, DSYMV, CHEMV, and ZHEMV, matrix ***A*** is stored in upper or lower storage mode.
- For SSLMX and DSLMX, matrix ***A*** is stored in lower-packed storage mode.

*Table 64. Data Types*

| $\alpha$, $\beta$, ***A***, ***x***, ***y*** | **Subprogram** |
|---|---|
| Short-precision real | SSPMV, SSYMV, and SSLMX |
| Long-precision real | DSPMV, DSYMV, and DSLMX |
| Short-precision complex | CHPMV and CHEMV |
| Long-precision complex | ZHPMV and ZHEMV |

**Note:** SSPMV and DSPMV are Level 2 BLAS subroutines. You should use these subroutines instead of SSLMX and DSLMX, which are provided only for compatibility with earlier releases of ESSL.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SSPMV \| DSPMV \| CHPMV \| ZHPMV (*uplo*, *n*, *alpha*, *ap*, *x*, *incx*, *beta*, *y*, *incy*)<br>CALL SSYMV \| DSYMV \| CHEMV \| ZHEMV (*uplo*, *n*, *alpha*, *a*, *lda*, *x*, *incx*, *beta*, *y*, *incy*)<br>CALL SSLMX \| DSLMX (*n*, *alpha*, *ap*, *x*, *incx*, *y*, *incy*) |
| **C and C++** | sspmv \| dspmv \| chpmv \| zhpmv (*uplo*, *n*, *alpha*, *ap*, *x*, *incx*, *beta*, *y*, *incy*);<br>ssymv \| dsymv \| chemv \| zhemv (*uplo*, *n*, *alpha*, *a*, *lda*, *x*, *incx*, *beta*, *y*, *incy*);<br>sslmx \| dslmx (*n*, *alpha*, *ap*, *x*, *incx*, *y*, *incy*); |
| **PL/I** | CALL SSPMV \| DSPMV \| CHPMV \| ZHPMV (*uplo*, *n*, *alpha*, *ap*, *x*, *incx*, *beta*, *y*, *incy*);<br>CALL SSYMV \| DSYMV \| CHEMV \| ZHEMV (*uplo*, *n*, *alpha*, *a*, *lda*, *x*, *incx*, *beta*, *y*, *incy*);<br>CALL SSLMX \| DSLMX (*n*, *alpha*, *ap*, *x*, *incx*, *y*, *incy*); |

*On Entry*

*uplo*

indicates the storage mode used for matrix ***A***, where:

If *uplo* = 'U', ***A*** is stored in upper-packed or upper storage mode.

If *uplo* = 'L', ***A*** is stored in lower-packed or lower storage mode.

Specified as: a single character. It must be 'U' or 'L'.

*n*

is the number of elements in vectors ***x*** and ***y*** and the order of matrix ***A***. Specified as: a fullword integer; $n \geq 0$.

*alpha*

is the scaling constant α. Specified as: a number of the data type indicated in Table 64 on page 315.

*ap*

has the following meaning:

For SSPMV and DSPMV, *ap* is the real symmetric matrix ***A*** of order *n*, stored in upper- or lower-packed storage mode.

For CHPMV and ZHPMV, *ap* is the complex Hermitian matrix ***A*** of order *n*, stored in upper- or lower-packed storage mode.

For SSLMX and DSLMX, *ap* is the real symmetric matrix ***A*** of order *n*, stored in lower-packed storage mode.

Specified as: a one-dimensional array of (at least) length $n(n+1)/2$, containing numbers of the data type indicated in Table 64 on page 315.

*a*

has the following meaning:

For SSYMV and DSYMV, *a* is the real symmetric matrix ***A*** of order *n*, stored in upper or lower storage mode.

For CHEMV and ZHEMV, *a* is the complex Hermitian matrix ***A*** of order *n*, stored in upper or lower storage mode.

Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 64 on page 315.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; $lda > 0$ and $lda \geq n$.

*x*

is the vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length $1+(n-1)|incx|$, containing numbers of the data type indicated in Table 64 on page 315.

*incx*

is the stride for vector ***x***. Specified as: a fullword integer, where:

For SSPMV, DSPMV, CHPMV, ZHPMV, SSYMV, DSYMV, CHEMV, and ZHEMV, $incx < 0$ or $incx > 0$.

For SSLMX and DSLMX, *incx* can have any value.

*beta*

is the scaling constant β. Specified as: a number of the data type indicated in Table 64 on page 315.

*y*

is the vector ***y*** of length *n*. Specified as: a one-dimensional array of (at least) length $1+(n-1)|incy|$, containing numbers of the data type indicated in Table 64 on page 315.

*incy*

is the stride for vector ***y***. Specified as: a fullword integer; *incy* > 0 or *incy* < 0.

*On Return*

*y*

is the vector ***y*** of length *n*, containing the result of the computation. Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 64 on page 315.

***Notes***

1. All subroutines accept lowercase letters for the *uplo* argument.
2. The vector ***y*** must have no common elements with vector ***x*** or matrix ***A***; otherwise, results are unpredictable. See "Concepts" on page 55.
3. On input, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values.
4. For a description of how symmetric matrices are stored in upper- or lower-packed storage mode and upper or lower storage mode, see "Symmetric Matrix" on page 65. For a description of how complex Hermitian matrices are stored in upper- or lower-packed storage mode and upper or lower storage mode, see "Complex Hermitian Matrix" on page 70.

***Function:*** These subroutines perform the computations described in the two sections below. See references [34], [35 ], and [76 ]. For SSPMV, DSPMV, CHPMV, ZHPMV, SSYMV, DSYMV, CHEMV, and ZHEMV, if *n* is zero or if $\alpha$ is zero and $\beta$ is one, no computation is performed. For SSLMX and DSLMX, if *n* or $\alpha$ is zero, no computation is performed.

For SSLMX, SSPMV, SSYMV, CHPMV, and CHEMV, intermediate results are accumulated in long precision. However, several intermediate stores may occur for each element of the vector ***y***.

*For SSPMV, DSPMV, CHPMV, ZHPMV, SSYMV, DSYMV, CHEMV, and ZHEMV:* These subroutines compute the matrix-vector product for either a real symmetric matrix or a complex Hermitian matrix:

$$\boldsymbol{y} \leftarrow \beta\boldsymbol{y}+\alpha\boldsymbol{Ax}$$

where:

***y*** is a vector of length *n*.
$\alpha$ is a scalar.
$\beta$ is a scalar.
***A*** is a real symmetric or complex Hermitian matrix of order *n*.
***x*** is a vector of length *n*.

It is expressed as follows:

$$\begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} \leftarrow \beta \begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} + \alpha \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{n1} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix}$$

*For SSLMX and DSLMX:* These subroutines compute the matrix-vector product for a real symmetric matrix stored in lower-packed storage mode:

$$\boldsymbol{y} \leftarrow \boldsymbol{y} + \alpha \boldsymbol{A}\boldsymbol{x}$$

where:

***y*** is a vector of length *n*.
$\alpha$ is a scalar.
***A*** is a real symmetric matrix of order *n*.
***x*** is a vector of length *n*.

It is expressed as follows:

$$\begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} \leftarrow \begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} + \alpha \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{n1} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix}$$

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. *uplo* ≠ `'L'` or `'U'`
2. *n* < 0
3. *lda* < *n*
4. *lda* ≤ 0
5. *incx* = 0
6. *incy* = 0

***Example 1:*** This example shows vectors ***x*** and ***y*** with positive strides and a real symmetric matrix ***A*** of order 3, stored in lower-packed storage mode. Matrix ***A*** is:

$$\begin{bmatrix} 8.0 & 4.0 & 2.0 \\ 4.0 & 6.0 & 7.0 \\ 2.0 & 7.0 & 3.0 \end{bmatrix}$$

*Call Statement and Input*

```
            UPLO  N  ALPHA  AP   X  INCX BETA  Y  INCY
             |    |    |     |   |   |    |    |   |
CALL SSPMV( 'L' , 3 , 1.0 , AP , X , 1 , 1.0 , Y , 2 )

AP       =  (8.0, 4.0, 2.0, 6.0, 7.0, 3.0)
X        =  (3.0, 2.0, 1.0)
Y        =  (5.0, . , 3.0, . , 2.0)
```

*Output*

```
Y        =  (39.0, . , 34.0, . , 25.0)
```

***Example 2:*** This example shows vector ***x*** and ***y*** having strides of opposite signs. For ***x***, which has negative stride, processing begins at element `X(5)`, which is 1.0. The real symmetric matrix ***A*** of order 3 is stored in upper-packed storage mode. It uses the same input matrix ***A*** as in Example 1.

*Call Statement and Input*

```
            UPLO  N  ALPHA  AP   X  INCX  BETA  Y  INCY
             |    |    |     |   |    |    |    |   |
CALL SSPMV( 'U' , 3 , 1.0 , AP , X , -2 , 2.0 , Y , 1 )

AP       =  (8.0, 4.0, 6.0, 2.0, 7.0, 3.0)
X        =  (4.0, . , 2.0, . , 1.0)
Y        =  (6.0, 5.0, 4.0)
```

*Output*

```
Y        =  (36.0, 54.0, 36.0)
```

***Example 3:*** This example shows vector ***x*** and ***y*** with positive stride and a complex Hermitian matrix ***A*** of order 3, stored in lower-packed storage mode. Matrix ***A*** is:

```
            ┌                                      ┐
            │  (1.0, 0.0)  (3.0, 5.0)  (2.0, -3.0) │
            │ (3.0, -5.0)  (7.0, 0.0)  (4.0, -8.0) │
            │  (2.0, 3.0)  (4.0, 8.0)   (6.0, 0.0) │
            └                                      ┘
```

**Note:** On input, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values.

*Call Statement and Input*

```
            UPLO  N   ALPHA   AP   X  INCX BETA   Y  INCY
             |    |     |     |    |   |    |     |   |
CALL CHPMV( 'L' , 3 , ALPHA , AP , X , 1 , BETA , Y , 2 )

ALPHA    =  (1.0, 0.0)
AP       =  ((1.0, . ), (3.0, -5.0), (2.0, 3.0), (7.0, . ),
             (4.0, 8.0), (6.0, . ))
X        =  ((1.0, 2.0), (4.0, 0.0), (3.0, 4.0))
BETA     =  (1.0, 0.0)
Y        =  ((1.0, 0.0), . , (2.0, -1.0), . , (2.0, 1.0))
```

*Output*

```
Y        =  ((32.0, 21.0), . , (87.0, -8.0), . , (32.0, 64.0))
```

***Example 4:*** This example shows vector ***x*** and ***y*** having strides of opposite signs. For ***x***, which has negative stride, processing begins at element `X(5)`, which is (1.0, 2.0). The complex Hermitian matrix ***A*** of order 3 is stored in upper-packed storage mode. It uses the same input matrix ***A*** as in Example 3.

**Note:** On input, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values.

*Call Statement and Input*

```
            UPLO  N   ALPHA   AP   X  INCX  BETA   Y INCY
             |    |     |     |    |    |    |     |   |
CALL CHPMV( 'U' , 3 , ALPHA , AP , X , -2 , BETA , Y , 2 )

ALPHA    =  (1.0, 0.0)
AP       =  ((1.0, . ), (3.0, 5.0), (7.0, . ), (2.0, -3.0),
             (4.0, -8.0), (6.0, . ))
X        =  ((3.0, 4.0), . , (4.0, 0.0), . , (1.0, 2.0))
BETA     =  (0.0, 0.0)
Y        =(not relevant)
```

*Output*

```
Y        =  ((31.0, 21.0), . , (85.0, -7.0), . , (30.0, 63.0))
```

***Example 5:*** This example shows vectors ***x*** and ***y*** with positive strides and a real symmetric matrix ***A*** of order 3, stored in lower storage mode. It uses the same input matrix ***A*** as in Example 1.

*Call Statement and Input*

```
            UPLO  N  ALPHA  A  LDA  X  INCX BETA  Y INCY
             |    |    |    |   |   |   |    |    |   |
CALL SSYMV( 'L' , 3 , 1.0 , A , 3 , X , 1 , 1.0 , Y , 2 )


    ┌                ┐
    | 8.0   .    .   |
A = | 4.0  6.0   .   |
    | 2.0  7.0  3.0  |
    └                ┘


X        =  (3.0, 2.0, 1.0)
Y        =  (5.0, . , 3.0, . , 2.0)
```

*Output*

```
Y        =  (39.0, . , 34.0, . , 25.0)
```

***Example 6:*** This example shows vector ***x*** and ***y*** having strides of opposite signs. For ***x***, which has negative stride, processing begins at element `X(5)`, which is 1.0. The real symmetric matrix ***A*** of order 3 is stored in upper storage mode. It uses the same input matrix ***A*** as in Example 1.

*Call Statement and Input*

```
            UPLO  N  ALPHA  A  LDA  X  INCX  BETA  Y  INCY
             |    |    |    |   |   |    |     |    |   |
CALL SSYMV( 'U' , 3 , 1.0 , A , 4 , X , -2 , 2.0 , Y , 1 )
```

```
    ┌                ┐
    │ 8.0  4.0  2.0 │
A = │  .   6.0  7.0 │
    │  .    .   3.0 │
    │  .    .    .  │
    └                ┘
```

```
X        =  (4.0, . , 2.0, . , 1.0)
Y        =  (6.0, 5.0, 4.0)
```

*Output*

```
A        =  (36.0, 54.0, 36.0)
```

***Example 7:*** This example shows vector ***x*** and ***y*** with positive stride and a complex Hermitian matrix ***A*** of order 3, stored in lower storage mode. It uses the same input matrix ***A*** as in Example 3.

**Note:** On input, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values.

*Call Statement and Input*

```
            UPLO  N   ALPHA   A  LDA  X  INCX BETA   Y INCY
             |    |     |     |   |   |   |    |     |  |
CALL CHEMV( 'L' , 3 , ALPHA , A , 3 , X , 1 , BETA , Y , 2 )
```

```
ALPHA    =  (1.0, 0.0)
```

```
        ┌                                 ┐
        │ (1.0,  . )        .          .   │
A    =  │ (3.0, -5.0)  (7.0, .  )      .   │
        │ (2.0,  3.0)  (4.0, 8.0) (6.0,  . ) │
        └                                 ┘
```

```
X        =  ((1.0, 2.0), (4.0, 0.0), (3.0, 4.0))
BETA     =  (1.0, 0.0)
Y        =  ((1.0, 0.0), . , (2.0, -1.0), . , (2.0, 1.0))
```

*Output*

```
Y        =  ((32.0, 21.0), . , (87.0, -8.0), . , (32.0, 64.0))
```

***Example 8:*** This example shows vector ***x*** and ***y*** having strides of opposite signs. For ***x***, which has negative stride, processing begins at element `X(5)`, which is (1.0, 2.0). The complex Hermitian matrix ***A*** of order 3 is stored in upper storage mode. It uses the same input matrix ***A*** as in Example 3.

**Note:** On input, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values.

*Call Statement and Input*

```
            UPLO  N   ALPHA   A  LDA  X  INCX  BETA   Y INCY
             |    |     |     |   |   |    |     |    |   |
CALL CHEMV( 'U' , 3 , ALPHA , A , 3 , X , -2 , BETA , Y , 2 )

ALPHA    =  (1.0, 0.0)

        ┌                                      ┐
        | (1.0, . )  (3.0, 5.0)  (2.0, -3.0) |
A    =  |     .      (7.0,  . )  (4.0, -8.0) |
        |     .          .       (6.0,   . ) |
        └                                      ┘

X        =  ((3.0, 4.0), . , (4.0, 0.0), . , (1.0, 2.0))
BETA     =  (0.0, 0.0)
Y        =(not relevant)
```

*Output*

```
Y        =  ((31.0, 21.0), . , (85.0, -7.0), . , (30.0, 63.0))
```

***Example 9:*** This example shows vectors ***x*** and ***y*** with positive strides and a real symmetric matrix ***A*** of order 3. Matrix ***A*** is:

```
┌                ┐
| 8.0  4.0  2.0 |
| 4.0  6.0  7.0 |
| 2.0  7.0  3.0 |
└                ┘
```

*Call Statement and Input*

```
            N  ALPHA  AP   X  INCX  Y  INCY
            |    |    |    |   |    |   |
CALL SSLMX( 3 , 1.0 , AP , X , 1  , Y , 2  )

AP       =  (8.0, 4.0, 2.0, 6.0, 7.0, 3.0)
X        =  (3.0, 2.0, 1.0)
Y        =  (5.0, . , 3.0, . , 2.0)
```

*Output*

```
Y        =  (39.0, . , 34.0, . , 25.0)
```

# SSPR, DSPR, CHPR, ZHPR, SSYR, DSYR, CHER, ZHER, SSLR1, and DSLR1 —Rank-One Update of a Real Symmetric or Complex Hermitian Matrix

SSPR, DSPR, SSYR, DSYR, SSLR1, and DSLR1 compute the rank-one update of a real symmetric matrix, using the scalar $\alpha$, matrix $\boldsymbol{A}$, vector $\boldsymbol{x}$, and its transpose $\boldsymbol{x}^T$:

$$\boldsymbol{A} \leftarrow \boldsymbol{A}+\alpha\boldsymbol{x}\boldsymbol{x}^T$$

CHPR, ZHPR, CHER, and ZHER compute the rank-one update of a complex Hermitian matrix, using the scalar $\alpha$, matrix $\boldsymbol{A}$, vector $\boldsymbol{x}$, and its conjugate transpose $\boldsymbol{x}^H$:

$$\boldsymbol{A} \leftarrow \boldsymbol{A}+\alpha\boldsymbol{x}\boldsymbol{x}^H$$

The following storage modes are used:

- For SSPR, DSPR, CHPR, and ZHPR, matrix ***A*** is stored in upper- or lower-packed storage mode.
- For SSYR, DSYR, CHER, and ZHER, matrix ***A*** is stored in upper or lower storage mode.
- For SSLR1 and DSLR1, matrix ***A*** is stored in lower-packed storage mode.

*Table 65. Data Types*

| ***A***, ***x*** | $\alpha$ | **Subprogram** |
|---|---|---|
| Short-precision real | Short-precision real | SSPR, SSYR, and SSLR1 |
| Long-precision real | Long-precision real | DSPR, DSYR, and DSLR1 |
| Short-precision complex | Short-precision real | CHPR and CHER |
| Long-precision complex | Long-precision real | ZHPR and ZHER |

**Note:** SSPR and DSPR are Level 2 BLAS subroutines. You should use these subroutines instead of SSLR1 and DSLR1, which are only provided for compatibility with earlier releases of ESSL.

***Syntax***

| | |
|---|---|
| **Fortran** | CALL SSPR \| DSPR \| CHPR \| ZHPR (*uplo*, *n*, *alpha*, *x*, *incx*, *ap*)<br>CALL SSYR \| DSYR \| CHER \| ZHER (*uplo*, *n*, *alpha*, *x*, *incx*, *a*, *lda*)<br>CALL SSLR1 \| DSLR1 (*n*, *alpha*, *x*, *incx*, *ap*) |
| **C and C++** | sspr \| dspr \| chpr \| zhpr (*uplo*, *n*, *alpha*, *x*, *incx*, *ap*);<br>ssyr \| dsyr \| cher \| zher (*uplo*, *n*, *alpha*, *x*, *incx*, *a*, *lda*);<br>sslr1 \| dslr1 (*n*, *alpha*, *x*, *incx*, *ap*); |
| **PL/I** | CALL SSPR \| DSPR \| CHPR \| ZHPR (*uplo*, *n*, *alpha*, *x*, *incx*, *ap*);<br>CALL SSYR \| DSYR \| CHER \| ZHER (*uplo*, *n*, *alpha*, *x*, *incx*, *a*, *lda*);<br>CALL SSLR1 \| DSLR1 (*n*, *alpha*, *x*, *incx*, *ap*); |

*On Entry*

*uplo*
indicates the storage mode used for matrix ***A***, where:

If *uplo* = 'U', ***A*** is stored in upper-packed or upper storage mode.

If *uplo* = 'L', ***A*** is stored in lower-packed or lower storage mode.

Specified as: a single character. It must be 'U' or 'L'.

*n*
is the number of elements in vector ***x*** and the order of matrix ***A***. Specified as: a fullword integer; $n \ge 0$.

*alpha*
is the scaling constant $\alpha$. Specified as: a number of the data type indicated in Table 65 on page 323.

*x*
is the vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length $1+(n-1)|incx|$, containing numbers of the data type indicated in Table 65 on page 323.

*incx*
is the stride for vector ***x***. Specified as: a fullword integer, where:

For SSPR, DSPR, CHPR, ZHPR, SSYR, DSYR, CHER, and ZHER, $incx < 0$ or $incx > 0$.

For SSLR1 and DSLR1, *incx* can have any value.

*ap*
has the following meaning:

For SSPR and DSPR, *ap* is the real symmetric matrix ***A*** of order *n*, stored in upper- or lower-packed storage mode.

For CHPR and ZHPR, *ap* is the complex Hermitian matrix ***A*** of order *n*, stored in upper- or lower-packed storage mode.

For SSLR1 and DSLR1, *ap* is the real symmetric matrix ***A*** of order *n*, stored in lower-packed storage mode.

Specified as: a one-dimensional array of (at least) length $n(n+1)/2$, containing numbers of the data type indicated in Table 65 on page 323.

*a*
has the following meaning:

For SSYR and DSYR, *a* is the real symmetric matrix ***A*** of order *n*, stored in upper or lower storage mode.

For CHER and ZHER, *a* is the complex Hermitian matrix ***A*** of order *n*, stored in upper or lower storage mode.

Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 65 on page 323.

*lda*
is the leading dimension of the array specified for *a*. Specified as: a fullword integer; $lda > 0$ and $lda \ge n$.

*On Return*

*ap*
is the matrix ***A*** of order *n*, containing the results of the computation. Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 65 on page 323.

*a*

is the matrix ***A*** of order *n*, containing the results of the computation. Returned as: a two-dimensional array, containing numbers of the data type indicated in Table 65 on page 323.

***Notes***

1. All subroutines accept lowercase letters for the *uplo* argument.
2. The vector ***x*** must have no common elements with matrix ***A***; otherwise, results are unpredictable. See “Concepts” on page 55.
3. On input, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values. On output, if $\alpha \neq 0.0$, they are set to zero.
4. For a description of how symmetric matrices are stored in upper- or lower-packed storage mode and upper or lower storage mode, see “Symmetric Matrix” on page 65. For a description of how complex Hermitian matrices are stored in upper- or lower-packed storage mode and upper or lower storage mode, see “Complex Hermitian Matrix” on page 70.

***Function:*** These subroutines perform the computations described in the two sections below. See references [34], [35 ], and [76 ]. If *n* or $\alpha$ is 0, no computation is performed.

For CHPR and CHER, intermediate results are accumulated in long precision. For SSPR, SSYR, and SSLR1, intermediate results are accumulated in long precision on some platforms.

*For SSPR, DSPR, SSYR, DSYR, SSLR1, and DSLR1:* These subroutines compute the rank-one update of a real symmetric matrix:

$$\boldsymbol{A} \leftarrow \boldsymbol{A} + \alpha \boldsymbol{x}\boldsymbol{x}^{\mathrm{T}}$$

where:

***A*** is a real symmetric matrix of order *n*.
$\alpha$ is a scalar.
***x*** is a vector of length *n*.
$\boldsymbol{x}^{\mathrm{T}}$ is the transpose of vector ***x***.

It is expressed as follows:

$$\begin{bmatrix} a_{11} & \dots & a_{1n} \\ . & & . \\ . & & . \\ . & & . \\ a_{n1} & \dots & a_{nn} \end{bmatrix} \leftarrow \begin{bmatrix} a_{11} & \dots & a_{1n} \\ . & & . \\ . & & . \\ . & & . \\ a_{n1} & \dots & a_{nn} \end{bmatrix} + \alpha \begin{bmatrix} x_1 \\ . \\ . \\ . \\ x_n \end{bmatrix} \begin{bmatrix} x_1 & \dots & x_n \end{bmatrix}$$

*For CHPR, ZHPR, CHER, and ZHER:* These subroutines compute the rank-one update of a complex Hermitian matrix:

$$\boldsymbol{A} \leftarrow \boldsymbol{A} + \alpha \boldsymbol{x}\boldsymbol{x}^{\mathrm{H}}$$

where:

***A*** is a complex Hermitian matrix of order *n*.
$\alpha$ is a scalar.
***x*** is a vector of length *n*.
***x***<sup>H</sup> is the conjugate transpose of vector ***x***.

It is expressed as follows:

$$\begin{bmatrix} a_{11} \;\ldots\; a_{1n} \\ \cdot \qquad \cdot \\ \cdot \qquad \cdot \\ \cdot \qquad \cdot \\ a_{n1} \;\ldots\; a_{nn} \end{bmatrix} \leftarrow \begin{bmatrix} a_{11} \;\ldots\; a_{1n} \\ \cdot \qquad \cdot \\ \cdot \qquad \cdot \\ \cdot \qquad \cdot \\ a_{n1} \;\ldots\; a_{nn} \end{bmatrix} + \alpha \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} \begin{bmatrix} \bar{x}_1 \;\ldots\; \bar{x}_n \end{bmatrix}$$

***Error Condition***

*Computational Errors:* None

*Input-Argument Errors*

1. *uplo* ≠ `'L'` or `'U'`
2. *n* < 0
3. *incx* = 0
4. *lda* ≤ 0
5. *lda* < *n*

***Example 1:*** This example shows a vector ***x*** with a positive stride, and a real symmetric matrix ***A*** of order 3, stored in lower-packed storage mode. Matrix ***A*** is:

```
┌                 ┐
| 8.0  4.0  2.0 |
| 4.0  6.0  7.0 |
| 2.0  7.0  3.0 |
└                 ┘
```

*Call Statement and Input*

```
          UPLO   N  ALPHA  X  INCX  AP
            |    |    |    |   |    |
CALL SSPR( 'L' , 3 , 1.0 , X , 1  , AP )

X        =  (3.0, 2.0, 1.0)
AP       =  (8.0, 4.0, 2.0, 6.0, 7.0, 3.0)
```

*Output*

```
AP       =  (17.0, 10.0, 5.0, 10.0, 9.0, 4.0)
```

***Example 2:*** This example shows a vector ***x*** with a negative stride, and a real symmetric matrix ***A*** of order 3, stored in upper-packed storage mode. It uses the same input matrix ***A*** as in Example 1.

*Call Statement and Input*

```
          UPLO   N  ALPHA  X  INCX  AP
            |    |    |    |    |   |
CALL SSPR( 'U' , 3 , 1.0 , X , -2 , AP )

X        =  (1.0, . , 2.0, . , 3.0)
AP       =  (8.0, 4.0, 6.0, 2.0, 7.0, 3.0)
```

*Output*

```
AP       =  (17.0, 10.0, 10.0, 5.0, 9.0, 4.0)
```

***Example 3:*** This example shows a vector ***x*** with a positive stride, and a complex Hermitian matrix ***A*** of order 3, stored in lower-packed storage mode. Matrix ***A*** is:

```
┌                                          ┐
|  (1.0, 0.0)  (3.0, 5.0)  (2.0, -3.0) |
| (3.0, -5.0)  (7.0, 0.0)  (4.0, -8.0) |
|  (2.0, 3.0)  (4.0, 8.0)   (6.0, 0.0) |
└                                          ┘
```

**Note:** On input, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values. On output, if $\alpha \neq 0.0$, they are set to zero.

*Call Statement and Input*

```
           UPLO  N  ALPHA  X  INCX AP
            |    |    |    |   |   |
CALL CHPR( 'L' , 3 , 1.0 , X , 1 , AP )

X        =  ((1.0, 2.0), (4.0, 0.0), (3.0, 4.0))
AP       =  ((1.0, . ), (3.0, -5.0), (2.0, 3.0), (7.0, . ),
             (4.0, 8.0), (6.0, . ))
```

*Output*

```
AP       =  ((6.0, 0.0), (7.0, -13.0), (13.0, 1.0), (23.0, 0.0),
             (16.0, 24.0), (31.0, 0.0))
```

***Example 4:*** This example shows a vector ***x*** with a negative stride, and a complex Hermitian matrix ***A*** of order 3, stored in upper-packed storage mode. It uses the same input matrix ***A*** as in Example 3.

**Note:** On input, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values. On output, if $\alpha \neq 0.0$, they are set to zero.

*Call Statement and Input*

```
           UPLO  N  ALPHA  X  INCX  AP
            |    |    |    |    |   |
CALL CHPR( 'U' , 3 , 1.0 , X , -2 , AP )

X        =  ((3.0, 4.0), . , (4.0, 0.0), . , (1.0, 2.0))
AP       =  ((1.0, . ), (3.0, 5.0), (7.0, . ), (2.0, -3.0),
             (4.0, -8.0), (6.0, . ))
```

*Output*

```
AP       =  ((6.0, 0.0), (7.0, 13.0), (23.0, 0.0), (13.0, -1.0),
             (16.0, -24.0), (31.0, 0.0))
```

***Example 5:*** This example shows a vector ***x*** with a positive stride, and a real symmetric matrix ***A*** of order 3, stored in lower storage mode. It uses the same input matrix ***A*** as in Example 1.

*Call Statement and Input*

```
           UPLO  N  ALPHA  X  INCX A  LDA
            |    |    |    |   |   |   |
CALL SSYR( 'L' , 3 , 1.0 , X , 1 , A , 3 )

X        =  (3.0, 2.0, 1.0)

        ┌               ┐
        | 8.0   .    . |
A    =  | 4.0  6.0   . |
        | 2.0  7.0  3.0 |
        └               ┘
```

*Output*

```
        ┌                 ┐
        | 17.0    .    .  |
A    =  | 10.0  10.0   .  |
        |  5.0   9.0  4.0 |
        └                 ┘
```

***Example 6:*** This example shows a vector ***x*** with a negative stride, and a real symmetric matrix ***A*** of order 3, stored in upper storage mode. It uses the same input matrix ***A*** as in Example 1.

*Call Statement and Input*

```
           UPLO  N  ALPHA  X  INCX  A  LDA
            |    |    |    |    |   |   |
CALL SSYR( 'U' , 3 , 1.0 , X , -2 , A , 4 )

X        =  (1.0, . , 2.0, . , 3.0)

        ┌               ┐
        | 8.0  4.0  2.0 |
A    =  |  .   6.0  7.0 |
        |  .    .   3.0 |
        |  .    .    .  |
        └               ┘
```

*Output*

```
        ┌                 ┐
        | 17.0  10.0  5.0 |
A    =  |   .   10.0  9.0 |
        |   .     .   4.0 |
        |   .     .    .  |
        └                 ┘
```

***Example 7:*** This example shows a vector ***x*** with a positive stride, and a complex Hermitian matrix ***A*** of order 3, stored in lower storage mode. It uses the same input matrix ***A*** as in Example 3.

**Note:** On input, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values. On output, if $\alpha \neq 0.0$, they are set to zero.

*Call Statement and Input*

```
           UPLO  N  ALPHA  X  INCX A  LDA
            |    |    |    |   |   |   |
CALL CHER( 'L' , 3 , 1.0 , X , 1 , A , 3 )

X        =  ((1.0, 2.0), (4.0, 0.0), (3.0, 4.0))

        ┌                                     ┐
        │   (1.0, . )        .            .   │
A    =  │ (3.0, -5.0)   (7.0, . )         .   │
        │  (2.0, 3.0)  (4.0, 8.0)  (6.0, . )  │
        └                                     ┘
```

*Output*

```
        ┌                                          ┐
        │   (6.0, 0.0)        .            .       │
A    =  │ (7.0, -13.0)   (23.0, 0.0)        .       │
        │  (13.0, 1.0)  (16.0, 24.0)  (31.0, 0.0)  │
        └                                          ┘
```

***Example 8:*** This example shows a vector ***x*** with a negative stride, and a complex Hermitian matrix ***A*** of order 3, stored in upper storage mode. It uses the same input matrix ***A*** as in Example 3.

**Note:** On input, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values. On output, if $\alpha \neq 0.0$, they are set to zero.

*Call Statement and Input*

```
           UPLO  N  ALPHA  X  INCX  A  LDA
            |    |    |    |    |   |   |
CALL CHER( 'U' , 3 , 1.0 , X , -2 , A , 3 )

X        =  ((3.0, 4.0), . , (4.0, 0.0), . , (1.0, 2.0))

        ┌                                     ┐
        │ (1.0, . )  (3.0, 5.0)  (2.0, -3.0)  │
A    =  │     .      (7.0,  . )  (4.0, -8.0)  │
        │     .          .          (6.0, . )  │
        └                                     ┘
```

*Output*

```
        ┌                                         ┐
        │ (6.0, 0.0)  (7.0, 13.0)   (13.0, -1.0)  │
A    =  │     .       (23.0, 0.0)  (16.0, -24.0)  │
        │     .            .          (31.0, 0.0)  │
        └                                         ┘
```

***Example 9:*** This example shows a vector ***x*** with a positive stride, and a real symmetric matrix ***A*** of order 3, stored in lower-packed storage mode. It uses the same input matrix ***A*** as in Example 1.

*Call Statement and Input*

```
            N  ALPHA  X  INCX  AP
            |    |    |   |    |
CALL SSLR1( 3 , 1.0 , X , 1  , AP )

X        =  (3.0, 2.0, 1.0)
AP       =  (8.0, 4.0, 2.0, 6.0, 7.0, 3.0)
```

*Output*

```
AP       =  (17.0, 10.0, 5.0, 10.0, 9.0, 4.0)
```

# SSPR2, DSPR2, CHPR2, ZHPR2, SSYR2, DSYR2, CHER2, ZHER2, SSLR2, and DSLR2—Rank-Two Update of a Real Symmetric or Complex Hermitian Matrix

SSPR2, DSPR2, SSYR2, DSYR2, SSLR2, and DSLR2 compute the rank-two update of a real symmetric matrix, using the scalar $\alpha$, matrix ***A***, vectors ***x*** and ***y***, and their transposes $\boldsymbol{x}^T$ and $\boldsymbol{y}^T$:

$$\boldsymbol{A} \leftarrow \boldsymbol{A}+\alpha\boldsymbol{x}\boldsymbol{y}^T + \alpha\boldsymbol{y}\boldsymbol{x}^T$$

CHPR2, ZHPR2, CHER2, and ZHER2, compute the rank-two update of a complex Hermitian matrix, using the scalar $\alpha$, matrix ***A***, vectors ***x*** and ***y***, and their conjugate transposes $\boldsymbol{x}^H$ and $\boldsymbol{y}^H$:

$$\boldsymbol{A} \leftarrow \boldsymbol{A} + \alpha\boldsymbol{x}\boldsymbol{y}^H + \bar{\alpha}\boldsymbol{y}\boldsymbol{x}^H$$

The following storage modes are used:

- For SSPR2, DSPR2, CHPR2, and ZHPR2, matrix ***A*** is stored in upper- or lower-packed storage mode.
- For SSYR2, DSYR2, CHER2, and ZHER2, matrix ***A*** is stored in upper or lower storage mode.
- For SSLR2 and DSLR2, matrix ***A*** is stored in lower-packed storage mode.

*Table 66. Data Types*

| $\alpha$, ***A***, ***x***, ***y*** | **Subprogram** |
|---|---|
| Short-precision real | SSPR2, SSYR2, and SSLR2 |
| Long-precision real | DSPR2, DSYR2, and DSLR2 |
| Short-precision complex | CHPR2 and CHER2 |
| Long-precision complex | ZHPR2 and ZHER2 |

**Note:** SSPR2 and DSPR2 are Level 2 BLAS subroutines. You should use these subroutines instead of SSLR2 and DSLR2, which are only provided for compatibility with earlier releases of ESSL.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SSPR2 \| DSPR2 \| CHPR2 \| ZHPR2 (*uplo*, *n*, *alpha*, *x*, *incx*, *y*, *incy*, *ap*)<br>CALL SSYR2 \| DSYR2 \| CHER2 \| ZHER2 (*uplo*, *n*, *alpha*, *x*, *incx*, *y*, *incy*, *a*, *lda*)<br>CALL SSLR2 \| DSLR2 (*n*, *alpha*, *x*, *incx*, *y*, *incy*, *ap*) |
| **C and C++** | sspr2 \| dspr2 \| chpr2 \| zhpr2 (*uplo*, *n*, *alpha*, *x*, *incx*, *y*, *incy*, *ap*);<br>ssyr2 \| dsyr2 \| cher2 \| zher2 (*uplo*, *n*, *alpha*, *x*, *incx*, *y*, *incy*, *a*, *lda*);<br>sslr2 \| dslr2 (*n*, *alpha*, *x*, *incx*, *y*, *incy*, *ap*); |
| **PL/I** | CALL SSPR2 \| DSPR2 \| CHPR2 \| ZHPR2 (*uplo*, *n*, *alpha*, *x*, *incx*, *y*, *incy*, *ap*);<br>CALL SSYR2 \| DSYR2 \| CHER2 \| ZHER2 (*uplo*, *n*, *alpha*, *x*, *incx*, *y*, *incy*, *a lda*);<br>CALL SSLR2 \| DSLR2 (*n*, *alpha*, *x*, *incx*, *y*, *incy*, *ap*); |

*On Entry*

*uplo*

indicates the storage mode used for matrix ***A***, where:

If *uplo* = `'U'`, ***A*** is stored in upper-packed or upper storage mode.

If *uplo* = `'L'`, ***A*** is stored in lower-packed or lower storage mode.

Specified as: a single character. It must be ‘U&csq or `'L'`.

*n*

is the number of elements in vectors ***x*** and ***y*** and the order of matrix ***A***. Specified as: a fullword integer; $n \geq 0$.

*alpha*

is the scaling constant α. Specified as: a number of the data type indicated in Table 66 on page 331.

*x*

is the vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length 1+(*n*−1)|*incx*|, containing numbers of the data type indicated in Table 66 on page 331.

*incx*

is the stride for vector ***x***.

Specified as: a fullword integer, where:

For SSPR2, DSPR2, CHPR2, ZHPR2, SSYR2, DSYR2, CHER2, and ZHER2, $incx < 0$ or $incx > 0$.

For SSLR2 and DSLR2, *incx* can have any value.

*y*

is the vector ***y*** of length *n*. Specified as: a one-dimensional array of (at least) length 1+(*n*−1)|*incy*|, containing numbers of the data type indicated in Table 66 on page 331.

*incy*

is the stride for vector ***y***. Specified as: a fullword integer, where:

For SSPR2, DSPR2, CHPR2, ZHPR2, SSYR2, DSYR2, CHER2, and ZHER2, $incy < 0$ or $incy > 0$.

For SSLR2 and DSLR2, *incy* can have any value.

*ap*

has the following meaning:

For SSPR2 and DSPR2, *ap* is the real symmetric matrix ***A*** of order *n*, stored in upper- or lower-packed storage mode.

For CHPR2 and ZHPR2, *ap* is the complex Hermitian matrix ***A*** of order *n*, stored in upper- or lower-packed storage mode.

For SSLR2 and DSLR2, *ap* is the real symmetric matrix ***A*** of order *n*, stored in lower-packed storage mode.

Specified as: a one-dimensional array of (at least) length *n*(*n*+1)/2, containing numbers of the data type indicated in Table 66 on page 331.

*a*

has the following meaning:

For SSYR2 and DSYR2, *a* is the real symmetric matrix ***A*** of order *n*, stored in upper or lower storage mode.

For CHER2 and ZHER2, *a* is the complex Hermitian matrix ***A*** of order *n*, stored in upper or lower storage mode.

Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 66 on page 331.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; *lda* > 0 and *lda* ≥ *n*.

*On Return*

*ap*

is the matrix ***A*** of order *n*, containing the results of the computation. Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 66 on page 331.

*a*

is the matrix ***A*** of order *n*, containing the results of the computation. Returned as: a two-dimensional array, containing numbers of the data type indicated in Table 66 on page 331.

***Notes***

1. All subroutines accept lowercase letters for the *uplo* argument.
2. The vectors ***x*** and ***y*** must have no common elements with matrix ***A***; otherwise, results are unpredictable. See "Concepts" on page 55.
3. On input, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values. On output, if $\alpha \neq$ zero, the imaginary parts of the diagonal elements are set to zero.
4. For a description of how symmetric matrices are stored in upper- or lower-packed storage mode and upper or lower storage mode, see "Symmetric Matrix" on page 65. For a description of how complex Hermitian matrices are stored in upper- or lower-packed storage mode and upper or lower storage mode, see "Complex Hermitian Matrix" on page 70.

***Function:*** These subroutines perform the computation described in the two sections below. See references [34], [35 ], and [76 ]. If *n* or $\alpha$ is zero, no computation is performed.

For SSPR2, SSYR2, SSLR2, CHPR2, and CHER2, intermediate results are accumulated in long precision.

*SSPR2, DSPR2, SSYR2, DSYR2, SSLR2, and DSLR2:* These subroutines compute the rank-two update of a real symmetric matrix:

$$\boldsymbol{A} \leftarrow \boldsymbol{A} + \alpha \boldsymbol{x}\boldsymbol{y}^{\mathrm{T}} + \alpha \boldsymbol{y}\boldsymbol{x}^{\mathrm{T}}$$

where:

***A*** is a real symmetric matrix of order *n*.
$\alpha$ is a scalar.
***x*** is a vector of length *n*.
$\boldsymbol{x}^{\mathrm{T}}$ is the transpose of vector ***x***.
***y*** is a vector of length *n*.
$\boldsymbol{y}^{\mathrm{T}}$ is the transpose of vector ***y***.

It is expressed as follows:

$$\begin{bmatrix} a_{11} & \dots & a_{1n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{n1} & \dots & a_{nn} \end{bmatrix} \leftarrow \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{n1} & \dots & a_{nn} \end{bmatrix} + \alpha \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} \begin{bmatrix} y_1 & \dots & y_n \end{bmatrix}$$

$$+ \alpha \begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} \begin{bmatrix} x_1 & \dots & x_n \end{bmatrix}$$

*CHPR2, ZHPR2, CHER2, and ZHER2:* These subroutines compute the rank-two update of a complex Hermitian matrix:

$$\boldsymbol{A} \leftarrow \boldsymbol{A} + \alpha \boldsymbol{x}\boldsymbol{y}^{\mathrm{H}} + \bar{\alpha}\boldsymbol{y}\boldsymbol{x}^{\mathrm{H}}$$

where:

$\boldsymbol{A}$ is a complex Hermitian matrix of order *n*.
$\alpha$ is a scalar.
$\boldsymbol{x}$ is a vector of length *n*.
$\boldsymbol{x}^{\mathrm{H}}$ is the conjugate transpose of vector $\boldsymbol{x}$.
$\boldsymbol{y}$ is a vector of length *n*.
$\boldsymbol{y}^{\mathrm{H}}$ is the conjugate transpose of vector $\boldsymbol{y}$.

It is expressed as follows:

$$\begin{bmatrix} a_{11} & \dots & a_{1n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{n1} & \dots & a_{nn} \end{bmatrix} \leftarrow \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{n1} & \dots & a_{nn} \end{bmatrix} + \alpha \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} \begin{bmatrix} \bar{y}_1 & \dots & \bar{y}_n \end{bmatrix}$$

$$+ \bar{\alpha} \begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} \begin{bmatrix} \bar{x}_1 & \dots & \bar{x}_n \end{bmatrix}$$

### Error Condition

*Computational Errors:* None

*Input-Argument Errors*

1. *uplo* ≠ `'L'` or `'U'`
2. $n < 0$
3. $incx = 0$

4. *incy* = 0
5. *lda* ≤ 0
6. *lda* < *n*

***Example 1:*** This example shows vectors ***x*** and ***y*** with positive strides and a real symmetric matrix ***A*** of order 3, stored in lower-packed storage mode. Matrix ***A*** is:

```
┌               ┐
| 8.0  4.0  2.0 |
| 4.0  6.0  7.0 |
| 2.0  7.0  3.0 |
└               ┘
```

*Call Statement and Input*

```
            UPLO  N  ALPHA  X  INCX  Y  INCY AP
             |    |    |    |   |    |   |   |
CALL SSPR2( 'L' , 3 , 1.0 , X , 1 ,  Y , 2 , AP )

X        =  (3.0, 2.0, 1.0)
Y        =  (5.0, . , 3.0, . , 2.0)
AP       =  (8.0, 4.0, 2.0, 6.0, 7.0, 3.0)
```

*Output*

```
AP       =  (38.0, 23.0, 13.0, 18.0, 14.0, 7.0)
```

***Example 2:*** This example shows vector ***x*** and ***y*** having strides of opposite signs. For ***x***, which has negative stride, processing begins at element `X(5)`, which is 3.0. The real symmetric matrix ***A*** of order 3 is stored in upper-packed storage mode. It uses the same input matrix ***A*** as in Example 1.

*Call Statement and Input*

```
            UPLO  N  ALPHA  X   INCX Y  INCY AP
             |    |    |    |    |   |   |   |
CALL SSPR2( 'U' , 3 , 1.0 , X , -2 , Y , 2 , AP )

X        =  (1.0, . , 2.0, . , 3.0)
Y        =  (5.0, . , 3.0, . , 2.0)
AP       =  (8.0, 4.0, 6.0, 2.0, 7.0, 3.0)
```

*Output*

```
AP       =  (38.0, 23.0, 18.0, 13.0, 14.0, 7.0)
```

***Example 3:*** This example shows vector ***x*** and ***y*** with positive stride and a complex Hermitian matrix ***A*** of order 3, stored in lower-packed storage mode. Matrix ***A*** is:

```
┌                                        ┐
|  (1.0, 0.0)  (3.0, 5.0)  (2.0, -3.0) |
| (3.0, -5.0)  (7.0, 0.0)  (4.0, -8.0) |
|  (2.0, 3.0)  (4.0, 8.0)   (6.0, 0.0) |
└                                        ┘
```

**Note:** On input, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values. On output, if $\alpha \neq$ zero, the imaginary parts of the diagonal elements are set to zero.

*Call Statement and Input*

```
            UPLO  N   ALPHA   X  INCX Y  INCY AP
             |    |     |     |   |   |   |   |
CALL CHPR2( 'L' , 3 , ALPHA , X , 1 , Y , 2 , AP )

ALPHA    =  (1.0, 0.0)
X        =  ((1.0, 2.0), (4.0, 0.0), (3.0, 4.0))
Y        =  ((1.0, 0.0), . , (2.0, -1.0), . , (2.0, 1.0))
AP       =  ((1.0, . ), (3.0, -5.0), (2.0, 3.0), (7.0, . ),
             (4.0, 8.0), (6.0, . ))
```

*Output*

```
AP       =  ((3.0, 0.0), (7.0, -10.0), (9.0, 4.0), (23.0, 0.0),
             (14.0, 23.0), (26.0, 0.0))
```

***Example 4:*** This example shows vector ***x*** and ***y*** having strides of opposite signs. For ***x***, which has negative stride, processing begins at element X(5), which is (1.0,2.0). The complex Hermitian matrix ***A*** of order 3 is stored in upper-packed storage mode. It uses the same input matrix ***A*** as in Example 3.

**Note:** On input, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values. On output, if $\alpha \neq$ zero, the imaginary parts of the diagonal elements are set to zero.

*Call Statement and Input*

```
            UPLO  N   ALPHA   X  INCX  Y  INCY AP
             |    |     |     |    |   |   |   |
CALL CHPR2( 'U' , 3 , ALPHA , X , -2 , Y , 2 , AP )

ALPHA    =  (1.0, 0.0)
X        =  ((3.0, 4.0), . , (4.0, 0.0), . , (1.0, 2.0))
Y        =  ((1.0, 0.0), . , (2.0, -1.0), . , (2.0, 1.0))
AP       =  ((1.0, . ), (3.0, 5.0), (7.0, . ), (2.0, -3.0),
             (4.0, -8.0), (6.0, . ))
```

*Output*

```
AP       =  ((3.0, 0.0), (7.0, 10.0), (23.0, 0.0), (9.0, -4.0),
             (14.0, -23.0), (26.0, 0.0))
```

***Example 5:*** This example shows vectors ***x*** and ***y*** with positive strides, and a real symmetric matrix ***A*** of order 3, stored in lower storage mode. It uses the same input matrix ***A*** as in Example 1.

*Call Statement and Input*

```
            UPLO  N  ALPHA  X  INCX Y  INCY A  LDA
             |    |    |    |   |   |   |   |   |
CALL SSYR2( 'L' , 3 , 1.0 , X , 1 , Y , 2 , A , 3 )

X        =  (3.0, 2.0, 1.0)
Y        =  (5.0, . , 3.0, . , 2.0)
```

```
         ┌                ┐
         │ 8.0   .     .  │
A    =   │ 4.0  6.0    .  │
         │ 2.0  7.0  3.0  │
         └                ┘
```

*Output*

```
         ┌                  ┐
         │ 38.0    .     .  │
A    =   │ 23.0  18.0    .  │
         │ 13.0  14.0  7.0  │
         └                  ┘
```

***Example 6:*** This example shows vector ***x*** and ***y*** having strides of opposite signs. For ***x***, which has negative stride, processing begins at element `X(5)`, which is 3.0. The real symmetric matrix ***A*** of order 3 is stored in upper storage mode. It uses the same input matrix ***A*** as in Example 1.

*Call Statement and Input*

```
            UPLO  N  ALPHA  X  INCX  Y  INCY A  LDA
             |    |    |    |    |   |   |   |   |
CALL SSYR2( 'U' , 3 , 1.0 , X , -2 , Y , 2 , A , 4 )

X        =  (1.0, . , 2.0, . , 3.0)
Y        =  (5.0, . , 3.0, . , 2.0)
```

```
         ┌                ┐
         │ 8.0  4.0  2.0  │
A    =   │  .   6.0  7.0  │
         │  .    .   3.0  │
         │  .    .    .   │
         └                ┘
```

*Output*

```
         ┌                   ┐
         │ 38.0  23.0  13.0  │
A    =   │   .   18.0  14.0  │
         │   .     .    7.0  │
         │   .     .     .   │
         └                   ┘
```

***Example 7:*** This example shows vector ***x*** and ***y*** with positive stride, and a complex Hermitian matrix ***A*** of order 3, stored in lower storage mode. It uses the same input matrix ***A*** as in Example 3.

**Note:** On input, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values. On output, if $\alpha \neq$ zero, the imaginary parts of the diagonal elements are set to zero.

*Call Statement and Input*

```
            UPLO  N   ALPHA   X  INCX Y  INCY A  LDA
             |    |     |     |   |   |   |   |   |
CALL CHER2( 'L' , 3 , ALPHA , X , 1 , Y , 2 , A , 3 )

ALPHA    =  (1.0, 0.0)
X        =  ((1.0, 2.0), (4.0, 0.0), (3.0, 4.0))
Y        =  ((1.0, 0.0), . , (2.0, -1.0), . , (2.0, 1.0))

          ┌                                  ┐
          │   (1.0, . )       .          .   │
A    =    │ (3.0, -5.0)   (7.0, . )      .   │
          │  (2.0, 3.0)  (4.0, 8.0)  (6.0, . )│
          └                                  ┘
```

*Output*

```
          ┌                                          ┐
          │   (3.0, 0.0)       .             .       │
A    =    │ (7.0, -10.0)  (23.0, 0.0 )       .       │
          │   (9.0, 4.0)  (14.0, 23.0)  (26.0, 0.0 ) │
          └                                          ┘
```

***Example 8:*** This example shows vector ***x*** and ***y*** having strides of opposite signs. For ***x***, which has negative stride, processing begins at element `X(5)`, which is (1.0, 2.0). The complex Hermitian matrix ***A*** of order 3 is stored in upper storage mode. It uses the same input matrix ***A*** as in Example 3.

**Note:** On input, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values. On output, if α ≠ zero, the imaginary parts of the diagonal elements are set to zero.

*Call Statement and Input*

```
            UPLO  N   ALPHA   X  INCX  Y  INCY A  LDA
             |    |     |     |    |   |   |   |   |
CALL CHER2( 'U' , 3 , ALPHA , X , -2 , Y , 2 , A , 3 )

ALPHA    =  (1.0, 0.0)
X        =  ((3.0, 4.0), . , (4.0, 0.0), . , (1.0, 2.0))
Y        =  ((1.0, 0.0), . , (2.0, -1.0), . , (2.0, 1.0))

          ┌                                  ┐
          │ (1.0, . ) (3.0, 5.0)   (2.0, -3.0) │
A    =    │    .        (7.0, . )   (4.0, -8.0) │
          │    .           .          (6.0, . ) │
          └                                  ┘
```

*Output*

```
          ┌                                        ┐
          │ (3.0, 0.0)  (7.0, 10.0)    (9.0, -4.0) │
A    =    │    .       (23.0, 0.0)  (14.0, -23.0) │
          │    .            .          (26.0, 0.0) │
          └                                        ┘
```

***Example 9:*** This example shows vectors ***x*** and ***y*** with positive strides and a real symmetric matrix ***A*** of order 3, stored in lower-packed storage mode. It uses the same input matrix ***A*** as in Example 1.

*Call Statement and Input*

```
            N  ALPHA  X  INCX  Y  INCY  AP
            |    |    |   |    |   |    |
CALL SSLR2( 3 , 1.0 , X , 1  , Y , 2  , AP )

X        =  (3.0, 2.0, 1.0)
Y        =  (5.0, . , 3.0, . , 2.0)
AP       =  (8.0, 4.0, 2.0, 6.0, 7.0, 3.0)
```

*Output*

```
AP       =  (38.0, 23.0, 13.0, 18.0, 14.0, 7.0)
```

# SGBMV, DGBMV, CGBMV, and ZGBMV—Matrix-Vector Product for a General Band Matrix, Its Transpose, or Its Conjugate Transpose

SGBMV and DGBMV compute the matrix-vector product for either a real general band matrix or its transpose, where the general band matrix is stored in BLAS-general-band storage mode. It uses the scalars $\alpha$ and $\beta$, vectors ***x*** and ***y***, and general band matrix ***A*** or its transpose:

$$\boldsymbol{y} \leftarrow \beta\boldsymbol{y}+\alpha\boldsymbol{Ax}$$

$$\boldsymbol{y} \leftarrow \beta\boldsymbol{y}+\alpha\boldsymbol{A}^{\mathrm{T}}\boldsymbol{x}$$

CGBMV and ZGBMV compute the matrix-vector product for either a complex general band matrix, its transpose, or its conjugate transpose, where the general band matrix is stored in BLAS-general-band storage mode. It uses the scalars $\alpha$ and $\beta$, vectors ***x*** and ***y***, and general band matrix ***A***, its transpose, or its conjugate transpose:

$$\boldsymbol{y} \leftarrow \beta\boldsymbol{y}+\alpha\boldsymbol{Ax}$$
$$\boldsymbol{y} \leftarrow \beta\boldsymbol{y}+\alpha\boldsymbol{A}^{\mathrm{T}}\boldsymbol{x}$$
$$\boldsymbol{y} \leftarrow \beta\boldsymbol{y}+\alpha\boldsymbol{A}^{\mathrm{H}}\boldsymbol{x}$$

*Table 67. Data Types*

| $\alpha$, $\beta$, ***x***, ***y***, ***A*** | **Subprogram** |
|---|---|
| Short-precision real | SGBMV |
| Long-precision real | DGBMV |
| Short-precision complex | CGBMV |
| Long-precision complex | ZGBMV |

### *Syntax*

| **Fortran** | CALL SGBMV \| DGBMV \| CGBMV \| ZGBMV (*transa*, *m*, *n*, *ml*, *mu*, *alpha*, *a*, *lda*, *x*, *incx*, *beta*, *y*, *incy*) |
|---|---|
| **C and C++** | sgbmv \| dgbmv \| cgbmv \| zgbmv (*transa*, *m*, *n*, *ml*, *mu*, *alpha*, *a*, *lda*, *x*, *incx*, *beta*, *y*, *incy*); |
| **PL/I** | CALL SGBMV \| DGBMV \| CGBMV \| ZGBMV (*transa*, *m*, *n*, *ml*, *mu*, *alpha*, *a*, *lda*, *x*, *incx*, *beta*, *y*, *incy*); |

*On Entry*

*transa*

indicates the form of matrix ***A*** to use in the computation, where:

If *transa* = `'N'`, ***A*** is used in the computation.

If *transa* = `'T'`, $\boldsymbol{A}^{\mathrm{T}}$ is used in the computation.

If *transa* = `'C'`, $\boldsymbol{A}^{\mathrm{H}}$ is used in the computation.

Specified as: a single character. It must be `'N'`, `'T'`, or `'C'`.

*m*

is the number of rows in matrix ***A***, and:

If *transa* = `'N'`, it is the length of vector ***y***.

If *transa* = `'T'` or `'C'`, it is the length of vector ***x***.

Specified as: a fullword integer; $m \geq 0$.

*n*

is the number of columns in matrix ***A***, and:

If *transa* = `'N'`, it is the length of vector ***x***.

If *transa* = `'T'` or `'C'`, it is the length of vector ***y***.

Specified as: a fullword integer; $n \geq 0$.

*ml*

is the lower band width *ml* of the matrix ***A***. Specified as: a fullword integer; $ml \geq 0$.

*mu*

is the upper band width *mu* of the matrix ***A***. Specified as: a fullword integer; $mu \geq 0$.

*alpha*

is the scaling constant $\alpha$. Specified as: a number of the data type indicated in Table 67 on page 340.

*a*

is the *m* by *n* general band matrix ***A***, stored in BLAS-general-band storage mode. It has an upper band width *mu* and a lower band width *ml*. Also:

If *transa* = `'N'`, ***A*** is used in the computation.

If *transa* = `'T'`, $\boldsymbol{A}^T$ is used in the computation.

If *transa* = `'C'`, $\boldsymbol{A}^H$ is used in the computation.

**Note:** No data should be moved to form $\boldsymbol{A}^T$ or $\boldsymbol{A}^H$; that is, the matrix ***A*** should always be stored in its untransposed form in BLAS-general-band storage mode.

Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 67 on page 340, where $lda \geq ml+mu+1$.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; $lda > 0$ and $lda \geq ml+mu+1$.

*x*

is the vector ***x***, where:

If *transa* = `'N'`, it has length *n*.

If *transa* = `'T'` or `'C'`, it has length *m*.

Specified as: a one-dimensional array, containing numbers of the data type indicated in Table 67 on page 340, where:

If *transa* = `'N'`, it must have at least $1+(n-1)|incx|$ elements.

If *transa* = `'T'` or `'C'`, it must have at least $1+(m-1)|incx|$ elements.

*incx*

is the stride for vector ***x***. Specified as: a fullword integer; $incx > 0$ or $incx < 0$.

*beta*

is the scaling constant $\beta$. Specified as: a number of the data type indicated in Table 67 on page 340.

*y*

is the vector ***y***, where:

If *transa* = `'N'`, it has length *m*.

If *transa* = `'T'` or `'C'`, it has length *n*.

Specified as: a one-dimensional array, containing numbers of the data type indicated in Table 67 on page 340, where:

If *transa* = `'N'`, it must have at least 1+(*m*–1)|*incy*| elements.

If *transa* = `'T'` or `'C'`, it must have at least 1+(*n*–1)|*incy*| elements.

*incy*

is the stride for vector ***y***. Specified as: a fullword integer; *incy* > 0 or *incy* < 0.

*On Return*

*y*

is the vector ***y***, containing the result of the computation, where:

If *transa* = `'N'`, it has length *m*.

If *transa* = `'T'` or `'C'`, it has length *n*.

Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 67 on page 340.

***Notes***

1. For SGBMV and DGBMV, if you specify `'C'` for the *transa* argument, it is interpreted as though you specified `'T'`.
2. All subroutines accept lowercase letters for the *transa* argument.
3. Vector ***y*** must have no common elements with matrix ***A*** or vector ***x***; otherwise, results are unpredictable. See “Concepts” on page 55.
4. To achieve optimal performance, use *lda* = *mu*+*ml*+1.
5. For general band matrices, if you specify *ml* ≥ *m* or *mu* ≥ *n*, ESSL assumes, **only for purposes of the computation**, that the lower band width is *m*–1 or the upper band width is *n*–1, respectively. However, ESSL uses the original values for *ml* and *mu* **for the purposes of finding the locations** of element $a_{11}$ and all other elements in the array specified for ***A***, as described in “General Band Matrix” on page 76. For an illustration of this technique, see “Example 4” on page 345.
6. For a description of how a general band matrix is stored in BLAS-general-band storage mode in an array, see “General Band Matrix” on page 76.

***Function:*** The possible computations that can be performed by these subroutines are described in the following sections. Varying implementation techniques are used for this computation to improve performance. As a result, accuracy of the computational result may vary for different computations.

In all the computations, general band matrix ***A*** is stored in its untransposed form in an array, using BLAS-general-band storage mode.

For SGBMV and CGBMV, intermediate results are accumulated in long precision. Occasionally, for performance reasons, these intermediate results are truncated to short precision and stored.

See references [34 ], [35 ], [38 ], [46 ], and [76]. No computation is performed if *m* or *n* is 0 or if $\alpha$ is zero and $\beta$ is one.

*General Band Matrix:* For SGBMV, DGBMV, CGBMV, and ZGBMV, the matrix-vector product for a general band matrix is expressed as follows:

$$\boldsymbol{y} \leftarrow \beta\boldsymbol{y}+\alpha\boldsymbol{A}\boldsymbol{x}$$

where:

$\boldsymbol{x}$ is a vector of length *n*.
$\boldsymbol{y}$ is a vector of length *m*.
$\alpha$ is a scalar.
$\beta$ is a scalar.
$\boldsymbol{A}$ is an *m* by *n* general band matrix, having a lower band width of *ml* and an upper band width of *mu*.

*Transpose of a General Band Matrix:* For SGBMV, DGBMV, CGBMV, and ZGBMV, the matrix-vector product for the transpose of a general band matrix is expressed as:

$$\boldsymbol{y} \leftarrow \beta\boldsymbol{y}+\alpha\boldsymbol{A}^{\mathrm{T}}\boldsymbol{x}$$

where:

$\boldsymbol{x}$ is a vector of length *m*.
$\boldsymbol{y}$ is a vector of length *n*.
$\alpha$ is a scalar.
$\beta$ is a scalar.
$\boldsymbol{A}^{\mathrm{T}}$ is the transpose of an *m* by *n* general band matrix $\boldsymbol{A}$, having a lower band width of *ml* and an upper band width of *mu*.

*Conjugate Transpose of a General Band Matrix:* For CGBMV and ZGBMV, the matrix-vector product for the conjugate transpose of a general band matrix is expressed as follows:

$$\boldsymbol{y} \leftarrow \beta\boldsymbol{y}+\alpha\boldsymbol{A}^{\mathrm{H}}\boldsymbol{x}$$

where:

$\boldsymbol{x}$ is a vector of length *m*.
$\boldsymbol{y}$ is a vector of length *n*.
$\alpha$ is a scalar.
$\beta$ is a scalar.
$\boldsymbol{A}^{\mathrm{H}}$ is the conjugate transpose of an *m* by *n* general band matrix $\boldsymbol{A}$ of order *n*, having a lower band width of *ml* and an upper band width of *mu*.

### *Error Conditions*

*Computational Errors:* None

*Input-Argument Errors*

1. *transa* ≠ `'N'`, `'T'`, or `'C'`
2. $m < 0$
3. $n < 0$
4. $ml < 0$
5. $mu < 0$
6. $lda \le 0$
7. $lda < ml+mu+1$
8. $incx = 0$

9. *incy* = 0

***Example 1:*** This example shows how to use SGBMV to perform the computation $\boldsymbol{y} \leftarrow \beta\boldsymbol{y}+\alpha\boldsymbol{A}\boldsymbol{x}$, where TRANSA is equal to 'N', and the following real general band matrix ***A*** is used in the computation. Matrix ***A*** is:

```
┌                     ┐
│ 1.0  1.0  1.0  0.0 │
│ 2.0  2.0  2.0  2.0 │
│ 3.0  3.0  3.0  3.0 │
│ 4.0  4.0  4.0  4.0 │
│ 0.0  5.0  5.0  5.0 │
└                     ┘
```

*Call Statement and Input*

```
           TRANSA M   N   ML  MU  ALPHA  A  LDA  X  INCX  BETA   Y  INCY
             |    |   |   |   |     |    |   |   |   |      |    |   |
CALL SGBMV( 'N' , 5 , 4 , 3 , 2 ,  2.0 , A , 8 , X , 1  , 10.0 , Y , 2  )
```

```
        ┌                     ┐
        │  .    .   1.0  2.0 │
        │  .   1.0  2.0  3.0 │
        │ 1.0  2.0  3.0  4.0 │
A    =  │ 2.0  3.0  4.0  5.0 │
        │ 3.0  4.0  5.0   .  │
        │ 4.0  5.0   .    .  │
        │  .    .    .    .  │
        │  .    .    .    .  │
        └                     ┘

X        =  (1.0, 2.0, 3.0, 4.0)
Y        =  (1.0, . , 2.0, . , 3.0, . , 4.0, . , 5.0, . )
```

*Output*

```
Y        =  (22.0, . , 60.0, . , 90.0, . , 120.0, . , 140.0, . )
```

***Example 2:*** This example shows how to use SGBMV to perform the computation $\boldsymbol{y} \leftarrow \beta\boldsymbol{y}+\alpha\boldsymbol{A}^{T}\boldsymbol{x}$, where TRANSA is equal to 'T', and the transpose of a real general band matrix ***A*** is used in the computation. It uses the same input as Example 1.

*Call Statement and Input*

```
           TRANSA M   N   ML  MU  ALPHA  A  LDA  X  INCX  BETA   Y  INCY
             |    |   |   |   |     |    |   |   |   |      |    |   |
CALL SGBMV( 'T' , 5 , 4 , 3 , 2 ,  2.0 , A , 8 , X , 1  , 10.0 , Y , 2  )
```

*Output*

```
Y        =  (70.0, . , 130.0, . , 140.0, . , 148.0, . )
```

***Example 3:*** This example shows how to use CGBMV to perform the computation $\boldsymbol{y} \leftarrow \beta\boldsymbol{y}+\alpha\boldsymbol{A}^{H}\boldsymbol{x}$, where TRANSA is equal to 'C', and the complex conjugate of the following general band matrix ***A*** is used in the computation. Matrix ***A*** is:

```
┌                                                ┐
│ (1.0, 1.0)  (1.0, 1.0)  (1.0, 1.0)  (0.0, 0.0) │
│ (2.0, 2.0)  (2.0, 2.0)  (2.0, 2.0)  (2.0, 2.0) │
│ (3.0, 3.0)  (3.0, 3.0)  (3.0, 3.0)  (3.0, 3.0) │
│ (4.0, 4.0)  (4.0, 4.0)  (4.0, 4.0)  (4.0, 4.0) │
│ (0.0, 0.0)  (5.0, 5.0)  (5.0, 5.0)  (0.0, 0.0) │
└                                                ┘
```

*Call Statement and Input*

```
           TRANSA M   N   ML  MU  ALPHA   A  LDA  X  INCX  BETA   Y  INCY
             |    |   |   |   |     |     |   |   |   |     |     |   |
CALL CGBMV( 'C' , 5 , 4 , 3 , 2 , ALPHA , A , 8 , X , 1  , BETA , Y , 2  )
```

```
          ┌                                                ┐
          │     .           .       (1.0, 1.0)  (2.0, 2.0) │
          │     .       (1.0, 1.0)  (2.0, 2.0)  (3.0, 3.0) │
          │ (1.0, 1.0)  (2.0, 2.0)  (3.0, 3.0)  (4.0, 4.0) │
A     =   │ (2.0, 2.0)  (3.0, 3.0)  (4.0, 4.0)  (5.0, 5.0) │
          │ (3.0, 3.0)  (4.0, 4.0)  (5.0, 5.0)      .      │
          │ (4.0, 4.0)  (5.0, 5.0)      .           .      │
          │     .           .           .           .      │
          │     .           .           .           .      │
          └                                                ┘
```

```
X         =  ((1.0, 2.0), (2.0, 3.0), (3.0, 4.0), (4.0, 5.0),
              (5.0, 6.0))
ALPHA     =  (1.0, 1.0)
BETA      =  (10.0, 0.0)
Y         =  ((1.0, 2.0), . , (2.0, 3.0), . , (3.0, 4.0), . ,
              (4.0, 5.0), . )
```

*Output*

```
Y         =  ((70.0, 100.0), . , (130.0, 170.0), . ,
              (140.0, 180.0), . , (148.0, 186.0), . )
```

***Example 4:*** This example shows how to use SGBMV to perform the computation ***y*** ← β***y***+α***Ax***, where *ml* ≥ *m* and *mu* ≥ *n*, TRANSA is equal to 'N', and the following real general band matrix ***A*** is used in the computation. Matrix ***A*** is:

```
┌                          ┐
│ 1.0  1.0  1.0  1.0  1.0 │
│ 2.0  2.0  2.0  2.0  2.0 │
│ 3.0  3.0  3.0  3.0  3.0 │
│ 4.0  4.0  4.0  4.0  4.0 │
└                          ┘
```

*Call Statement and Input*

```
           TRANSA M   N   ML  MU  ALPHA  A  LDA   X  INCX  BETA   Y  INCY
             |    |   |   |   |     |    |   |    |   |     |     |   |
CALL SGBMV( 'N' , 4 , 5 , 6 , 5 ,  2.0 , A , 12 , X , 1  , 10.0 , Y , 2  )
```

```
        ┌                         ┐
        |  .    .    .    .    .  |
        |  .    .    .    .   1.0 |
        |  .    .    .   1.0  2.0 |
        |  .    .   1.0  2.0  3.0 |
        |  .   1.0  2.0  3.0  4.0 |
A    =  | 1.0  2.0  3.0  4.0   .  |
        | 2.0  3.0  4.0   .    .  |
        | 3.0  4.0   .    .    .  |
        | 4.0   .    .    .    .  |
        |  .    .    .    .    .  |
        |  .    .    .    .    .  |
        |  .    .    .    .    .  |
        └                         ┘

X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
Y        =  (1.0, . , 2.0, . , 3.0, . , 4.0, . )
```

*Output*

```
Y        =  (40.0, . , 80.0, . , 120.0, . , 160.0, . )
```

## SSBMV, DSBMV, CHBMV, and ZHBMV—Matrix-Vector Product for a Real Symmetric or Complex Hermitian Band Matrix

SSBMV and DSBMV compute the matrix-vector product for a real symmetric band matrix. CHBMV and ZHBMV compute the matrix-vector product for a complex Hermitian band matrix. The band matrix ***A*** is stored in either upper- or lower-band-packed storage mode. It uses the scalars α and β, vectors ***x*** and ***y***, and band matrix ***A***:

$$\boldsymbol{y} \leftarrow \beta\boldsymbol{y}+\alpha\boldsymbol{Ax}$$
$$\boldsymbol{y} \leftarrow \beta\boldsymbol{y}+\alpha\boldsymbol{Ax}$$

*Table 68. Data Types*

| α, β, ***x***, ***y***, ***A*** | **Subprogram** |
|---|---|
| Short-precision real | SSBMV |
| Long-precision real | DSBMV |
| Short-precision complex | CHBMV |
| Long-precision complex | ZHBMV |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SSBMV \| DSBMV \| CHBMV \| ZHBMV (*uplo*, *n*, *k*, *alpha*, *a*, *lda*, *x*, *incx*, *beta*, *y*, *incy*) |
| **C and C++** | ssbmv \| dsbmv \| chbmv \| zhbmv (*uplo*, *n*, *k*, *alpha*, *a*, *lda*, *x*, *incx*, *beta*, *y*, *incy*); |
| **PL/I** | CALL SSBMV \| DSBMV \| CHBMV \| ZHBMV (*uplo*, *n*, *k*, *alpha*, *a*, *lda*, *x*, *incx*, *beta*, *y*, *incy*); |

*On Entry*

*uplo*

indicates the storage mode used for matrix ***A***, where either the upper or lower triangle can be stored:

If *uplo* = `'U'`, ***A*** is stored in upper-band-packed storage mode.

If *uplo* = `'L'`, ***A*** is stored in lower-band-packed storage mode.

Specified as: a single character. It must be `'U'` or `'L'`.

*n*

is the order of matrix ***A*** and the number of elements in vectors ***x*** and ***y***. Specified as: a fullword integer; $n \geq 0$.

*k*

is the half band width *k* of the matrix ***A***. Specified as: a fullword integer; $k \geq 0$.

*alpha*

is the scaling constant α. Specified as: a number of the data type indicated in Table 68.

*a*

is the real symmetric or complex Hermitian band matrix ***A*** of order *n*, having a half band width of *k*, where:

If *uplo* = `'U'`, ***A*** is stored in upper-band-packed storage mode.

If *uplo* = `'L'`, ***A*** is stored in lower-band-packed storage mode.

Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 68, where $lda \geq k+1$.

*lda*
is the leading dimension of the array specified for *a*. Specified as: a fullword integer; *lda* > 0 and *lda* ≥ *k*+1.

*x*
is the vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length 1+(*n*−1)|*incx*|, containing numbers of the data type indicated in Table 68 on page 347.

*incx*
is the stride for vector ***x***. Specified as: a fullword integer; *incx* > 0 or *incx* < 0.

*beta*
is the scaling constant β. Specified as: a number of the data type indicated in Table 68 on page 347.

*y*
is the vector ***y*** of length *n*. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 68 on page 347.

*incy*
is the stride for vector ***y***. Specified as: a fullword integer; *incy* > 0 or *incy* < 0.

*On Return*

*y*
is the vector ***y*** of length *n*, containing the result of the computation. Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 68 on page 347.

***Notes***

1. All subroutines accept lowercase letters for the *uplo* argument.

2. Vector ***y*** must have no common elements with matrix ***A*** or vector ***x***; otherwise, results are unpredictable. See “Concepts” on page 55.

3. To achieve optimal performance in these subroutines, use *lda* = *k*+1.

4. The imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values.

5. For real symmetric and complex Hermitian band matrices, if you specify *k* ≥ *n*, ESSL assumes, **only for purposes of the computation**, that the half band width of matrix ***A*** is *n*−1; that is, it processes matrix ***A***, of order *n*, as though it is a (nonbanded) real symmetric or complex Hermitian matrix. However, ESSL uses the original value for *k* **for the purposes of finding the locations** of element $a_{11}$ and all other elements in the array specified for ***A***, as described in the storage modes referenced in the next note. For an illustration of this technique, see “Example 3” on page 350.

6. For a description of how a real symmetric band matrix is stored, see “Upper-Band-Packed Storage Mode” on page 83 or “Lower-Band-Packed Storage Mode” on page 84. For a description of how a complex Hermitian band matrix is stored, see “Complex Hermitian Matrix” on page 70.

***Function:*** These subroutines perform the following matrix-vector product, using a real symmetric or complex Hermitian band matrix ***A***, stored in either upper- or lower-band-packed storage mode:

$$\boldsymbol{y} \leftarrow \beta\boldsymbol{y}+\alpha\boldsymbol{A}\boldsymbol{x}$$

where:

> ***x*** and ***y*** are vectors of length *n*.
> α and β are scalars.
> ***A*** is an real symmetric or complex Hermitian band matrix of order *n*, having a half band width of *k*.

For SSBMV and CHBMV, intermediate results are accumulated in long precision. Occasionally, for performance reasons, these intermediate results are truncated to short precision and stored.

See references [34 ], [38 ], [46 ], and [76 ]. No computation is performed if *n* is 0 or if α is zero and β is one.

### *Error Conditions*

*Computational Errors:* None

*Input-Argument Errors*

1. *uplo* ≠ `'U'` or `'L'`
2. $n < 0$
3. $k < 0$
4. $lda \le 0$
5. $lda < k+1$
6. $incx = 0$
7. $incy = 0$

***Example 1:*** This example shows how to use SSBMV to perform the matrix-vector product, where the real symmetric band matrix ***A*** of order 7 and half band width of 3 is stored in upper-band-packed storage mode. Matrix ***A*** is:

```
┌                                    ┐
| 1.0  1.0  1.0  1.0  0.0  0.0  0.0 |
| 1.0  2.0  2.0  2.0  2.0  0.0  0.0 |
| 1.0  2.0  3.0  3.0  3.0  3.0  0.0 |
| 1.0  2.0  3.0  4.0  4.0  4.0  4.0 |
| 0.0  2.0  3.0  4.0  5.0  5.0  5.0 |
| 0.0  0.0  3.0  4.0  5.0  6.0  6.0 |
| 0.0  0.0  0.0  4.0  5.0  6.0  7.0 |
└                                    ┘
```

*Call Statement and Input*

```
            UPLO  N   K  ALPHA  A  LDA  X  INCX  BETA   Y  INCY
             |    |   |    |    |   |   |   |     |     |   |
CALL SSBMV( 'U' , 7 , 3 , 2.0 , A , 5 , X , 1  , 10.0 , Y , 2  )
```

```
          ┌                                    ┐
          |  .    .    .   1.0  2.0  3.0  4.0 |
          |  .    .   1.0  2.0  3.0  4.0  5.0 |
A    =    |  .   1.0  2.0  3.0  4.0  5.0  6.0 |
          | 1.0  2.0  3.0  4.0  5.0  6.0  7.0 |
          |  .    .    .    .    .    .    .  |
          └                                    ┘
```

```
X         =  (1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0)
Y         =  (1.0, . , 2.0, . , 3.0, . , 4.0, . , 5.0, . , 6.0, . , 7.0)
```

*Output*

```
Y        =  (30.0, . , 78.0, . , 148.0, . , 244.0, . , 288.0, . ,
             316.0, . , 322.0)
```

***Example 2:*** This example shows how to use CHBMV to perform the matrix-vector product, where the complex Hermitian band matrix ***A*** of order 7 and half band width of 3 is stored in lower-band-packed storage mode. Matrix ***A*** is:

```
┌                                                                                                     ┐
│  (1.0, 0.0)  (1.0, 1.0)  (1.0, 1.0)  (1.0, 1.0)  (0.0, 0.0)  (0.0, 0.0)  (0.0, 0.0) │
│ (1.0, -1.0)  (2.0, 0.0)  (2.0, 2.0)  (2.0, 2.0)  (2.0, 2.0)  (0.0, 0.0)  (0.0, 0.0) │
│ (1.0, -1.0) (2.0, -2.0)  (3.0, 0.0)  (3.0, 3.0)  (3.0, 3.0)  (3.0, 3.0)  (0.0, 0.0) │
│ (1.0, -1.0) (2.0, -2.0) (3.0, -3.0)  (4.0, 0.0)  (4.0, 4.0)  (4.0, 4.0)  (4.0, 4.0) │
│  (0.0, 0.0) (2.0, -2.0) (3.0, -3.0) (4.0, -4.0)  (5.0, 0.0)  (5.0, 5.0)  (5.0, 5.0) │
│  (0.0, 0.0)  (0.0, 0.0) (3.0, -3.0) (4.0, -4.0) (5.0, -5.0)  (6.0, 0.0)  (6.0, 6.0) │
│  (0.0, 0.0)  (0.0, 0.0)  (0.0, 0.0) (4.0, -4.0) (5.0, -5.0) (6.0, -6.0)  (7.0, 0.0) │
└                                                                                                     ┘
```

**Note:** The imaginary parts of the diagonal elements of a complex Hermitian matrix are assumed to be zero, so you do not need to set these values.

*Call Statement and Input*

```
            UPLO  N   K   ALPHA   A  LDA  X  INCX  BETA   Y  INCY
             |    |   |     |     |   |   |   |     |     |   |
CALL CHBMV( 'L' , 7 , 3 , ALPHA , A , 5 , X , 1  , BETA , Y , 2  )

ALPHA    =  (2.0, 0.0)
BETA     =  (10.0, 0.0)
```

```
          ┌                                                                             ┐
          │  (1.0, . )  (2.0, . )  (3.0, . )  (4.0, . )  (5.0, . )  (6.0, . ) (7.0, . ) │
          │ (1.0, 1.0) (2.0, 2.0) (3.0, 3.0) (4.0, 4.0) (5.0, 5.0) (6.0, 6.0)     .     │
A     =   │ (1.0, 1.0) (2.0, 2.0) (3.0, 3.0) (4.0, 4.0) (5.0, 5.0)    .           .     │
          │ (1.0, 1.0) (2.0, 2.0) (3.0, 3.0) (4.0, 4.0)    .          .           .     │
          │     .          .          .          .         .          .           .     │
          └                                                                             ┘
```

```
X        =  ((1.0, 1.0), (2.0, 2.0), (3.0, 3.0), (4.0, 4.0),
             (5.0, 5.0), (6.0, 6.0), (7.0, 7.0))
Y        =  ((1.0, 1.0), . , (2.0, 2.0), . , (3.0, 3.0), . ,
             (4.0, 4.0), . , (5.0, 5.0), . , (6.0, 6.0), . ,
             (7.0, 7.0))
```

*Output*

```
Y        =  ((48.0, 12.0), . , (124.0, 32.0), . , (228.0, 68.0), . ,
             (360.0, 128.0), . , (360.0, 216.0), . ,
             (300.0, 332.0), . , (168.0, 476.0))
```

***Example 3:*** This example shows how to use SSBMV to perform the matrix-vector product, where *n* ≥ *k*. Matrix ***A*** is a real 5 by 5 symmetric band matrix with a half band width of 5, stored in upper-band-packed storage mode. Matrix ***A*** is:

```
                       ┌                          ┐
                       │ 1.0  1.0  1.0  1.0  1.0 │
                       │ 1.0  2.0  2.0  2.0  2.0 │
                       │ 1.0  2.0  3.0  3.0  3.0 │
                       │ 1.0  2.0  3.0  4.0  4.0 │
                       │ 1.0  2.0  3.0  4.0  5.0 │
                       └                          ┘
```

*Call Statement and Input*

```
            UPLO  N   K  ALPHA  A  LDA  X  INCX  BETA   Y  INCY
             |    |   |    |    |   |   |   |      |    |   |
CALL SSBMV( 'U' , 5 , 5 , 2.0 , A , 7 , X , 1  , 10.0 , Y , 2  )
```

```
        ┌                          ┐
        │  .    .    .    .    .  │
        │  .    .    .    .   1.0 │
        │  .    .    .   1.0  2.0 │
A     = │  .    .   1.0  2.0  3.0 │
        │  .   1.0  2.0  3.0  4.0 │
        │ 1.0  2.0  3.0  4.0  5.0 │
        │  .    .    .    .    .  │
        └                          ┘
```

```
X        =  (1.0, 2.0, 3.0, 4.0, 5.0)
Y        =  (1.0, . , 2.0, . , 3.0, . , 4.0, . , 5.0, . )
```

*Output*

```
Y        =  (40.0, . , 78.0, . , 112.0, . , 140.0, . , 160.0, . )
```

# STRMV, DTRMV, CTRMV, ZTRMV, STPMV, DTPMV, CTPMV, and ZTPMV—Matrix-Vector Product for a Triangular Matrix, Its Transpose, or Its Conjugate Transpose

STRMV, DTRMV, STPMV, and DTPMV compute one of the following matrix-vector products, using the vector ***x*** and triangular matrix ***A*** or its transpose:

$$\boldsymbol{x} \leftarrow \boldsymbol{A}\boldsymbol{x}$$
$$\boldsymbol{x} \leftarrow \boldsymbol{A}^{\mathrm{T}}\boldsymbol{x}$$

CTRMV, ZTRMV, CTPMV, and ZTPMV compute one of the following matrix-vector products, using the vector ***x*** and triangular matrix ***A***, its transpose, or its conjugate transpose:

$$\boldsymbol{x} \leftarrow \boldsymbol{A}\boldsymbol{x}$$
$$\boldsymbol{x} \leftarrow \boldsymbol{A}^{\mathrm{T}}\boldsymbol{x}$$
$$\boldsymbol{x} \leftarrow \boldsymbol{A}^{\mathrm{H}}\boldsymbol{x}$$

Matrix ***A*** can be either upper or lower triangular, where:

- For the _TRMV subroutines, it is stored in upper- or lower-triangular storage mode, respectively.
- For the _TPMV subroutines, it is stored in upper- or lower-triangular-packed storage mode, respectively.

*Table 69. Data Types*

| ***A***, ***x*** | **Subprogram** |
|---|---|
| Short-precision real | STRMV and STPMV |
| Long-precision real | DTRMV and DTPMV |
| Short-precision complex | CTRMV and CTPMV |
| Long-precision complex | ZTRMV and ZTPMV |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL STRMV \| DTRMV \| CTRMV \| ZTRMV (*uplo*, *transa*, *diag*, *n*, *a*, *lda*, *x*, *incx*)<br>CALL STPMV \| DTPMV \| CTPMV \| ZTPMV (*uplo*, *transa*, *diag*, *n*, *ap*, *x*, *incx*) |
| **C and C++** | strmv \| dtrmv \| ctrmv \| ztrmv (*uplo*, *transa*, *diag*, *n*, *a*, *lda*, *x*, *incx*);<br>stpmv \| dtpmv \| ctpmv \| ztpmv (*uplo*, *transa*, *diag*, *n*, *ap*, *x*, *incx*); |
| **PL/I** | CALL STRMV \| DTRMV \| CTRMV \| ZTRMV (*uplo*, *transa*, *diag*, *n*, *a*, *lda*, *x*, *incx*);<br>CALL STPMV \| DTPMV \| CTPMV \| ZTPMV (*uplo*, *transa*, *diag*, *n*, *ap*, *x*, *incx*); |

*On Entry*

*uplo*

indicates whether matrix ***A*** is an upper or lower triangular matrix, where:

If *uplo* = `'U'`, ***A*** is an upper triangular matrix.

If *uplo* = `'L'`, ***A*** is a lower triangular matrix.

Specified as: a single character. It must be `'U'` or `'L'`.

*transa*
indicates the form of matrix ***A*** to use in the computation, where:

If *transa* = `'N'`, ***A*** is used in the computation.

If *transa* = `'T'`, $\boldsymbol{A}^T$ is used in the computation.

If *transa* = `'C'`, $\boldsymbol{A}^H$ is used in the computation.

Specified as: a single character. It must be `'N'`, `'T'`, or `'C'`.

*diag*
indicates the characteristics of the diagonal of matrix ***A***, where:

If *diag* = `'U'`, ***A*** is a unit triangular matrix.

If *diag* = `'N'`, ***A*** is not a unit triangular matrix.

Specified as: a single character. It must be `'U'` or `'N'`.

*n*
is the order of triangular matrix ***A***. Specified as: a fullword integer; $0 \leq n \leq lda$.

*a*
is the upper or lower triangular matrix ***A*** of order *n*, stored in upper- or lower-triangular storage mode, respectively.

**Note:** No data should be moved to form $\boldsymbol{A}^T$ or $\boldsymbol{A}^H$; that is, the matrix ***A*** should always be stored in its untransposed form.

Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 69 on page 352.

*lda*
is the leading dimension of the array specified for *a*. Specified as: a fullword integer; $lda > 0$ and $lda \geq n$.

*ap*
is the upper or lower triangular matrix ***A*** of order *n*, stored in upper- or lower-triangular-packed storage mode, respectively. Specified as: a one-dimensional array of (at least) length $n(n+1)/2$, containing numbers of the data type indicated in Table 69 on page 352.

*x*
is the vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length $1+(n-1)|incx|$, containing numbers of the data type indicated in Table 69 on page 352.

*incx*
is the stride for vector ***x***. Specified as: a fullword integer; $incx > 0$ or $incx < 0$.

*On Return*

*x*
is the vector ***x*** of length *n*, containing the results of the computation. Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 69 on page 352.

***Notes***

1. These subroutines accept lowercase letters for the *uplo*, *transa*, and *diag* arguments.
2. For STRMV, DTRMV, STPMV, and DTPMV if you specify `'C'` for the *transa* argument, it is interpreted as though you specified `'T'`.
3. Matrix ***A*** and vector ***x*** must have no common elements; otherwise, results are unpredictable.

4. ESSL assumes certain values in your array for parts of a triangular matrix. As a result, you do not have to set these values. For unit triangular matrices, the elements of the diagonal are assumed to be 1.0 for real matrices and (1.0, 0.0) for complex matrices. When using upper- or lower-triangular storage, the unreferenced elements in the lower and upper triangular part, respectively, are assumed to be zero.
5. For a description of triangular matrices and how they are stored in upper- and lower-triangular storage mode and in upper- and lower-triangular-packed storage mode, see "Triangular Matrix" on page 73.

***Function:*** These subroutines can perform the following matrix-vector product computations, using the triangular matrix ***A***, its transpose, or its conjugate transpose, where ***A*** can be either upper or lower triangular:

$\boldsymbol{x} \leftarrow \boldsymbol{A}\boldsymbol{x}$
$\boldsymbol{x} \leftarrow \boldsymbol{A}^{T}\boldsymbol{x}$
$\boldsymbol{x} \leftarrow \boldsymbol{A}^{H}\boldsymbol{x}$ (for CTRMV, ZTRMV, CTPMV, and ZTPMV only)

where:

***x*** is a vector of length *n*.
***A*** is an upper or lower triangular matrix of order *n*. For _TRMV, it is stored in upper- or lower-triangular storage mode, respectively. For _TPMV, it is stored in upper- or lower-triangular-packed storage mode, respectively.

See references [32 ] and [38 ]. If *n* is 0, no computation is performed.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. $uplo \neq$ 'L' or 'U'
2. $transa \neq$ 'T', 'N', or 'C'
3. $diag \neq$ 'N' or 'U'
4. $n < 0$
5. $lda \leq 0$
6. $lda < n$
7. $incx = 0$

***Example 1:*** This example shows the computation $\boldsymbol{x} \leftarrow \boldsymbol{A}\boldsymbol{x}$. Matrix ***A*** is a real 4 by 4 lower triangular matrix that is unit triangular, stored in lower-triangular storage mode. Vector ***x*** is a vector of length 4. Matrix ***A*** is:

$$\begin{bmatrix} 1.0 & . & . & . \\ 1.0 & 1.0 & . & . \\ 2.0 & 3.0 & 1.0 & . \\ 3.0 & 4.0 & 3.0 & 1.0 \end{bmatrix}$$

**Note:** Because matrix ***A*** is unit triangular, the diagonal elements are not referenced. ESSL assumes a value of 1.0 for the diagonal elements.

*Call Statement and Input*

```
            UPLO TRANSA DIAG  N   A  LDA  X  INCX
             |     |     |    |   |   |   |   |
CALL STRMV( 'L' , 'N' , 'U' , 4 , A , 4 , X , 1  )
```

```
        ┌                   ┐
        │  .    .    .    . │
A    =  │ 1.0   .    .    . │
        │ 2.0  3.0   .    . │
        │ 3.0  4.0  3.0   . │
        └                   ┘
```

```
X        =  (1.0, 2.0, 3.0, 4.0)
```

*Output*

```
X        =  (1.0, 3.0, 11.0, 24.0)
```

***Example 2:*** This example shows the computation $\boldsymbol{x} \leftarrow \boldsymbol{A}^{T}\boldsymbol{x}$. Matrix ***A*** is a real 4 by 4 upper triangular matrix that is unit triangular, stored in upper-triangular storage mode. Vector ***x*** is a vector of length 4. Matrix ***A*** is:

```
┌                   ┐
│ 1.0  2.0  3.0  2.0│
│  .   1.0  2.0  5.0│
│  .    .   1.0  3.0│
│  .    .    .   1.0│
└                   ┘
```

**Note:** Because matrix ***A*** is unit triangular, the diagonal elements are not referenced. ESSL assumes a value of 1.0 for the diagonal elements.

*Call Statement and Input*

```
            UPLO TRANSA DIAG  N   A  LDA  X  INCX
             |     |     |    |   |   |   |   |
CALL STRMV( 'U' , 'T' , 'U' , 4 , A , 4 , X , 1  )
```

```
        ┌                    ┐
        │  .   2.0  3.0  2.0 │
A    =  │  .    .   2.0  5.0 │
        │  .    .    .   3.0 │
        │  .    .    .    .  │
        └                    ┘
```

```
X        =  (5.0, 4.0, 3.0, 2.0)
```

*Output*

```
X        =  (5.0, 14.0, 26.0, 41.0)
```

***Example 3:*** This example shows the computation $\boldsymbol{x} \leftarrow \boldsymbol{A}^{H}\boldsymbol{x}$. Matrix ***A*** is a complex 4 by 4 upper triangular matrix that is unit triangular, stored in upper-triangular storage mode. Vector ***x*** is a vector of length 4. Matrix ***A*** is:

```
┌                                                ┐
│ (1.0, 0.0)  (2.0, 2.0)  (3.0, 3.0)  (2.0, 2.0) │
│     .       (1.0, 0.0)  (2.0, 2.0)  (5.0, 5.0) │
│     .           .       (1.0, 0.0)  (3.0, 3.0) │
│     .           .           .       (1.0, 0.0) │
└                                                ┘
```

**Note:** Because matrix ***A*** is unit triangular, the diagonal elements are not referenced. ESSL assumes a value of (1.0, 0.0) for the diagonal elements.

*Call Statement and Input*

```
            UPLO TRANSA DIAG  N   A  LDA  X  INCX
             |     |     |    |   |   |   |   |
CALL CTRMV( 'U' , 'C' , 'U' , 4 , A , 4 , X , 1  )
```

```
        ┌                                        ┐
        │ .  (2.0, 2.0)  (3.0, 3.0)  (2.0, 2.0) │
A    =  │ .      .       (2.0, 2.0)  (5.0, 5.0) │
        │ .      .           .       (3.0, 3.0) │
        │ .      .           .           .      │
        └                                        ┘
```

```
X        =  ((5.0, 5.0), (4.0, 4.0), (3.0, 3.0), (2.0, 2.0))
```

*Output*

```
X        =  ((5.0, 5.0), (24.0, 4.0), (49.0, 3.0), (80.0, 2.0))
```

***Example 4:*** This example shows the computation ***x*** ← ***Ax***. Matrix ***A*** is a real 4 by 4 lower triangular matrix that is unit triangular, stored in lower-triangular-packed storage mode. Vector ***x*** is a vector of length 4. Matrix ***A*** is:

```
┌                     ┐
│ 1.0   .    .    .   │
│ 1.0  1.0   .    .   │
│ 2.0  3.0  1.0   .   │
│ 3.0  4.0  3.0  1.0  │
└                     ┘
```

**Note:** Because matrix ***A*** is unit triangular, the diagonal elements are not referenced. ESSL assumes a value of 1.0 for the diagonal elements.

*Call Statement and Input*

```
            UPLO TRANSA DIAG  N   AP   X  INCX
             |     |     |    |   |    |   |
CALL STPMV( 'L' , 'N' , 'U' , 4 , AP , X , 1  )

AP       =  ( . , 1.0, 2.0, 3.0, . , 3.0, 4.0, . , 3.0, . )
X        =  (1.0, 2.0, 3.0, 4.0)
```

*Output*

```
X        =  (1.0, 3.0, 11.0, 24.0)
```

***Example 5:*** This example shows the computation ***x*** ← ***A***$^{T}$***x***. Matrix ***A*** is a real 4 by 4 upper triangular matrix that is not unit triangular, stored in upper-triangular-packed storage mode. Vector ***x*** is a vector of length 4. Matrix ***A*** is:

```
┌                     ┐
│ 1.0  2.0  3.0  2.0  │
│  .   2.0  2.0  5.0  │
│  .    .   3.0  3.0  │
│  .    .    .   1.0  │
└                     ┘
```

*Call Statement and Input*

```
            UPLO TRANSA DIAG  N   AP   X  INCX
             |     |     |    |   |    |   |
CALL STPMV( 'U' , 'T' , 'N' , 4 , AP , X , 1  )

AP       =  (1.0, 2.0, 2.0, 3.0, 2.0, 3.0, 2.0, 5.0, 3.0, 1.0)
X        =  (5.0, 4.0, 3.0, 2.0)
```

*Output*

```
X        =  (5.0, 18.0, 32.0, 41.0)
```

***Example 6:*** This example shows the computation $\boldsymbol{x} \leftarrow \boldsymbol{A}^{H}\boldsymbol{x}$. Matrix ***A*** is a complex 4 by 4 upper triangular matrix that is unit triangular, stored in upper-triangular-packed storage mode. Vector ***x*** is a vector of length 4. Matrix ***A*** is:

```
┌                                             ┐
| (1.0, 0.0) (2.0, 2.0) (3.0, 3.0) (2.0, 2.0) |
|     .      (1.0, 0.0) (2.0, 2.0) (5.0, 5.0) |
|     .          .      (1.0, 0.0) (3.0, 3.0) |
|     .          .          .      (1.0, 0.0) |
└                                             ┘
```

**Note:** Because matrix ***A*** is unit triangular, the diagonal elements are not referenced. ESSL assumes a value of (1.0, 0.0) for the diagonal elements.

*Call Statement and Input*

```
            UPLO TRANSA DIAG  N   AP   X  INCX
             |     |     |    |   |    |   |
CALL CTPMV( 'U' , 'C' , 'U' , 4 , AP , X , 1  )

AP       =  ( . , (2.0, 2.0), . , (3.0, 3.0), (2.0, 2.0), . ,
             (2.0, 2.0), (5.0, 5.0), (3.0, 3.0), . )
X        =  ((5.0, 5.0), (4.0, 4.0), (3.0, 3.0), (2.0, 2.0))
```

*Output*

```
X        =  ((5.0, 5.0), (24.0, 4.0), (49.0, 3.0), (80.0, 2.0))
```

# STBMV, DTBMV, CTBMV, and ZTBMV—Matrix-Vector Product for a Triangular Band Matrix, Its Transpose, or Its Conjugate Transpose

STBMV and DTBMV compute one of the following matrix-vector products, using the vector ***x*** and triangular band matrix ***A*** or its transpose:

$\boldsymbol{x} \leftarrow \boldsymbol{Ax}$
$\boldsymbol{x} \leftarrow \boldsymbol{A}^{\mathrm{T}}\boldsymbol{x}$

CTBMV and ZTBMV compute one of the following matrix-vector products, using the vector ***x*** and triangular band matrix ***A***, its transpose, or its conjugate transpose:

$\boldsymbol{x} \leftarrow \boldsymbol{Ax}$
$\boldsymbol{x} \leftarrow \boldsymbol{A}^{\mathrm{T}}\boldsymbol{x}$
$\boldsymbol{x} \leftarrow \boldsymbol{A}^{\mathrm{H}}\boldsymbol{x}$

Matrix ***A*** can be either upper or lower triangular and is stored in upper- or lower-triangular-band-packed storage mode, respectively.

*Table 70. Data Types*

| ***A***, ***x*** | **Subprogram** |
|---|---|
| Short-precision real | STBMV |
| Long-precision real | DTBMV |
| Short-precision complex | CTBMV |
| Long-precision complex | ZTBMV |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL STBMV \| DTBMV \| CTBMV \| ZTBMV (*uplo*, *transa*, *diag*, *n*, *k*, *a*, *lda*, *x*, *incx*) |
| **C and C++** | stbmv \| dtbmv \| ctbmv \| ztbmv (*uplo*, *transa*, *diag*, *n*, *k*, *a*, *lda*, *x*, *incx*); |
| **PL/I** | CALL STBMV \| DTBMV \| CTBMV \| ZTBMV (*uplo*, *transa*, *diag*, *n*, *k*, *a*, *lda*, *x*, *incx*); |

*On Entry*

*uplo*
indicates whether matrix ***A*** is an upper or lower triangular band matrix, where:

If *uplo* = `'U'`, ***A*** is an upper triangular matrix.

If *uplo* = `'L'`, ***A*** is a lower triangular matrix.

Specified as: a single character. It must be `'U'` or `'L'`.

*transa*
indicates the form of matrix ***A*** to use in the computation, where:

If *transa* = `'N'`, ***A*** is used in the computation.

If *transa* = `'T'`, $\boldsymbol{A}^{\mathrm{T}}$ is used in the computation.

If *transa* = `'C'`, $\boldsymbol{A}^{\mathrm{H}}$ is used in the computation.

Specified as: a single character. It must be `'N'`, `'T'`, or `'C'`.

*diag*
indicates the characteristics of the diagonal of matrix ***A***, where:

If *diag* = `'U'`, ***A*** is a unit triangular matrix.

If *diag* = 'N', ***A*** is not a unit triangular matrix.

Specified as: a single character. It must be 'U' or 'N'.

*n*

is the order of triangular band matrix ***A***. Specified as: a fullword integer; $n \geq 0$.

*k*

is the upper or lower band width *k* of the matrix ***A***. Specified as: a fullword integer; $k \geq 0$.

*a*

is the upper or lower triangular band matrix ***A*** of order *n*, stored in upper- or lower-triangular-band-packed storage mode, respectively.

**Note:** No data should be moved to form $\boldsymbol{A}^T$ or $\boldsymbol{A}^H$; that is, the matrix ***A*** should always be stored in its untransposed form.

Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 70 on page 358.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; $lda > 0$ and $lda \geq k+1$.

*x*

is the vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length $1+(n-1)|incx|$, containing numbers of the data type indicated in Table 70 on page 358.

*incx*

is the stride for vector ***x***. Specified as: a fullword integer; $incx > 0$ or $incx < 0$.

*On Return*

*x*

is the vector ***x*** of length *n*, containing the results of the computation. Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 70 on page 358.

***Notes***

1. These subroutines accept lowercase letters for the *uplo*, *transa*, and *diag* arguments.
2. For STBMV and DTBMV, if you specify 'C' for the *transa* argument, it is interpreted as though you specified 'T'.
3. Matrix ***A*** and vector ***x*** must have no common elements; otherwise, results are unpredictable.
4. To achieve optimal performance in these subroutines, use $lda = k+1$.
5. For unit triangular matrices, the elements of the diagonal are assumed to be 1.0 for real matrices and (1.0, 0.0) for complex matrices. As a result, you do not have to set these values.
6. For both upper and lower triangular band matrices, if you specify $k \geq n$, ESSL assumes, **only for purposes of the computation**, that the upper or lower band width of matrix ***A*** is $n-1$; that is, it processes matrix ***A***, of order *n*, as though it is a (nonbanded) triangular matrix. However, ESSL uses the original value for *k* **for the purposes of finding the locations** of element $a_{11}$ and all other elements in the array specified for ***A***, as described in “Triangular Band Matrix” on page 86. For an illustration of this technique, see “Example 4” on page 362.

7. For a description of triangular band matrices and how they are stored in upper- and lower-triangular-band-packed storage mode, see “Triangular Band Matrix” on page 86.
8. If you are using a lower triangular band matrix, you may want to use this alternate approach instead of using lower-triangular-band-packed storage mode. Leave matrix ***A*** in full-matrix storage mode when you pass it to ESSL and specify the *lda* argument to be *lda*+1, which is the leading dimension of matrix ***A*** plus 1. ESSL then processes the matrix elements in the same way as though you had set them up in lower-triangular-band-packed storage mode.

***Function:*** These subroutines can perform the following matrix-vector product computations, using the triangular band matrix ***A***, its transpose, or its conjugate transpose, where ***A*** can be either upper or lower triangular:

$\boldsymbol{x} \leftarrow \boldsymbol{A}\boldsymbol{x}$
$\boldsymbol{x} \leftarrow \boldsymbol{A}^{T}\boldsymbol{x}$
$\boldsymbol{x} \leftarrow \boldsymbol{A}^{H}\boldsymbol{x}$ (for CTBMV and ZTBMV only)

where:

***x*** is a vector of length *n*.
***A*** is an upper or lower triangular band matrix of order *n*, stored in upper- or lower-triangular-band-packed storage mode, respectively.

See references [34 ], [46 ], and [38 ]. If *n* is 0, no computation is performed.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. *uplo* ≠ 'L' or 'U'
2. *transa* ≠ 'T', 'N', or 'C'
3. *diag* ≠ 'N' or 'U'
4. $n < 0$
5. $k < 0$
6. $lda \le 0$
7. $lda < k+1$
8. $incx = 0$

***Example 1:*** This example shows the computation $\boldsymbol{x} \leftarrow \boldsymbol{A}\boldsymbol{x}$. Matrix ***A*** is a real 7 by 7 upper triangular band matrix with a half band width of 3 that is not unit triangular, stored in upper-triangular-band-packed storage mode. Vector ***x*** is a vector of length 7. Matrix ***A*** is:

$$\begin{bmatrix}
1.0 & 1.0 & 1.0 & 1.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 2.0 & 2.0 & 2.0 & 2.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 3.0 & 3.0 & 3.0 & 3.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 4.0 & 4.0 & 4.0 & 4.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 5.0 & 5.0 & 5.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 6.0 & 6.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 7.0
\end{bmatrix}$$

*Call Statement and Input*

```
            UPLO TRANSA DIAG  N   K   A  LDA  X  INCX
             |    |     |    |   |   |   |   |   |
CALL STBMV( 'U' , 'N' , 'N' , 7 , 3 , A , 5 , X , 1  )
```

```
        ┌                                   ┐
        │  .    .    .   1.0  2.0  3.0  4.0 │
        │  .    .   1.0  2.0  3.0  4.0  5.0 │
A    =  │  .   1.0  2.0  3.0  4.0  5.0  6.0 │
        │ 1.0  2.0  3.0  4.0  5.0  6.0  7.0 │
        │  .    .    .    .    .    .    .  │
        └                                   ┘
```

```
X        =  (1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0)
```

*Output*

```
X        =  (10.0, 28.0, 54.0, 88.0, 90.0, 78.0, 49.0)
```

***Example 2:*** This example shows the computation $\boldsymbol{x} \leftarrow \boldsymbol{A}^{T}\boldsymbol{x}$. Matrix ***A*** is a real 7 by 7 lower triangular band matrix with a half band width of 3 that is not unit triangular, stored in lower-triangular-band-packed storage mode. Vector ***x*** is a vector of length 7. Matrix ***A*** is:

```
┌                                   ┐
│ 1.0  0.0  0.0  0.0  0.0  0.0  0.0 │
│ 1.0  2.0  0.0  0.0  0.0  0.0  0.0 │
│ 1.0  2.0  3.0  0.0  0.0  0.0  0.0 │
│ 1.0  2.0  3.0  4.0  0.0  0.0  0.0 │
│ 0.0  2.0  3.0  4.0  5.0  0.0  0.0 │
│ 0.0  0.0  3.0  4.0  5.0  6.0  0.0 │
│ 0.0  0.0  0.0  4.0  5.0  6.0  7.0 │
└                                   ┘
```

*Call Statement and Input*

```
            UPLO TRANSA DIAG  N   K   A  LDA  X  INCX
             |    |     |    |   |   |   |   |   |
CALL STBMV( 'L' , 'T' , 'N' , 7 , 3 , A , 5 , X , 1  )
```

```
        ┌                                   ┐
        │ 1.0  2.0  3.0  4.0  5.0  6.0  7.0 │
        │ 1.0  2.0  3.0  4.0  5.0  6.0   .  │
A    =  │ 1.0  2.0  3.0  4.0  5.0   .    .  │
        │ 1.0  2.0  3.0  4.0   .    .    .  │
        │  .    .    .    .    .    .    .  │
        └                                   ┘
```

```
X        =  (1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0)
```

*Output*

```
X        =  (10.0, 28.0, 54.0, 88.0, 90.0, 78.0, 49.0)
```

***Example 3:*** This example shows the computation $\boldsymbol{x} \leftarrow \boldsymbol{A}^{H}\boldsymbol{x}$. Matrix ***A*** is a complex 7 by 7 upper triangular band matrix with a half band width of 3 that is not unit triangular, stored in upper-triangular-band-packed storage mode. Vector ***x*** is a vector of length 7. Matrix ***A*** is:

```
┌                                                                           ┐
| (1.0, 1.0) (1.0, 1.0) (1.0, 1.0) (1.0, 1.0) (0.0, 0.0) (0.0, 0.0) (0.0, 0.0) |
| (0.0, 0.0) (2.0, 2.0) (2.0, 2.0) (2.0, 2.0) (2.0, 2.0) (0.0, 0.0) (0.0, 0.0) |
| (0.0, 0.0) (0.0, 0.0) (3.0, 3.0) (3.0, 3.0) (3.0, 3.0) (3.0, 3.0) (0.0, 0.0) |
| (0.0, 0.0) (0.0, 0.0) (0.0, 0.0) (4.0, 4.0) (4.0, 4.0) (4.0, 4.0) (4.0, 4.0) |
| (0.0, 0.0) (0.0, 0.0) (0.0, 0.0) (0.0, 0.0) (5.0, 5.0) (5.0, 5.0) (5.0, 5.0) |
| (0.0, 0.0) (0.0, 0.0) (0.0, 0.0) (0.0, 0.0) (0.0, 0.0) (6.0, 6.0) (6.0, 6.0) |
| (0.0, 0.0) (0.0, 0.0) (0.0, 0.0) (0.0, 0.0) (0.0, 0.0) (0.0, 0.0) (7.0, 7.0) |
└                                                                           ┘
```

*Call Statement and Input*

```
            UPLO TRANSA DIAG  N   K   A  LDA  X  INCX
             |     |     |    |   |   |   |   |   |
CALL CTBMV( 'U' , 'C' , 'N' , 7 , 3 , A , 5 , X , 1  )
```

```
     ┌                                                                                  ┐
     |     .           .           .       (1.0, 1.0)  (2.0, 2.0)  (3.0, 3.0)  (4.0, 4.0) |
     |     .           .       (1.0, 1.0)  (2.0, 2.0)  (3.0, 3.0)  (4.0, 4.0)  (5.0, 5.0) |
A  = |     .       (1.0, 1.0)  (2.0, 2.0)  (3.0, 3.0)  (4.0, 4.0)  (5.0, 5.0)  (6.0, 6.0) |
     | (1.0, 1.0)  (2.0, 2.0)  (3.0, 3.0)  (4.0, 4.0)  (5.0, 5.0)  (6.0, 6.0)  (7.0, 7.0) |
     |     .           .           .           .           .           .           .     |
     └                                                                                  ┘
```

```
X        =  ((1.0, 2.0), (2.0, 4.0), (3.0, 6.0), (4.0, 8.0),
             (5.0, 10.0), (6.0, 12.0), (7.0, 14.0))
```

*Output*

```
X        =  ((1.0, 2.0), (7.0, 9.0), (24.0, 23.0), (58.0, 46.0),
             (112.0, 79.0), (186.0, 122.0), (280.0, 175.0))
```

***Example 4:*** This example shows the computation $\boldsymbol{x} \leftarrow \boldsymbol{A}^{T}\boldsymbol{x}$, where $k > n$. Matrix ***A*** is a real 4 by 4 upper triangular band matrix with a half band width of 5 that is not unit triangular, stored in upper-triangular-band-packed storage mode. Vector ***x*** is a vector of length 4. Matrix ***A*** is:

```
┌                    ┐
| 1.0  1.0  1.0  1.0 |
|  .   2.0  2.0  2.0 |
|  .    .   3.0  3.0 |
|  .    .    .   4.0 |
└                    ┘
```

*Call Statement and Input*

```
               UPLO TRANSA DIAG  N   K   A  LDA  X  INCX
                 |     |     |   |   |   |   |   |   |
CALL STBMV( 'U' , 'T' , 'N' , 4 , 5 , A , 6 , X , 1  )

          ┌                      ┐
          |  .    .    .    .    |
A    =    |  .    .    .    .    |
          |  .    .    .   1.0   |
          |  .    .   1.0  2.0   |
          |  .   1.0  2.0  3.0   |
          | 1.0  2.0  3.0  4.0   |
          └                      ┘

X         =  (1.0, 2.0, 3.0, 4.0)
```

*Output*

```
X         =  (1.0, 5.0, 14.0, 30.0)
```

## Sparse Matrix-Vector Subprograms

This section contains the sparse matrix-vector subprogram descriptions.

# DSMMX—Matrix-Vector Product for a Sparse Matrix in Compressed-Matrix Storage Mode

This subprogram computes the matrix-vector product for sparse matrix ***A***, stored in compressed-matrix storage mode, using the matrix and vectors ***x*** and ***y***:

$$\boldsymbol{y} \leftarrow \boldsymbol{Ax}$$

where ***A***, ***x***, and ***y*** contain long-precision real numbers. You can use DSMTM to transpose matrix ***A*** before calling this subroutine. The resulting computation performed by this subroutine is then $\boldsymbol{y} \leftarrow \boldsymbol{A}^{\mathrm{T}}\boldsymbol{x}$.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL DSMMX (*m*, *nz*, *ac*, *ka*, *lda*, *x*, *y*) |
| **C and C++** | dsmmx (*m*, *nz*, *ac*, *ka*, *lda*, *x*, *y*); |
| **PL/I** | CALL DSMMX (*m*, *nz*, *ac*, *ka*, *lda*, *x*, *y*); |

*On Entry*

*m*
is the number of rows in sparse matrix ***A*** and the number of elements in vector ***y***. Specified as: a fullword integer; $m \geq 0$.

*nz*
is the maximum number of nonzero elements in each row of sparse matrix ***A***. Specified as: a fullword integer; $nz \geq 0$.

*ac*
is the *m* by *n* sparse matrix ***A***, stored in compressed-matrix storage mode in an array, referred to as AC. Specified as: an *lda* by (at least) *nz* array, containing long-precision real numbers.

*ka*
is the array, referred to as KA, containing the column numbers of the matrix ***A*** elements stored in the corresponding positions in array AC. Specified as: an *lda* by (at least) *nz* array, containing fullword integers, where 1 ≤ (elements of KA) ≤ *n*.

*lda*
is the size of the leading dimension of the arrays specified for *ac* and *ka*. Specified as: a fullword integer; $lda > 0$ and $lda \geq m$.

*x*
is the vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length *n*, containing long-precision real numbers.

*y*
See "On Return."

*On Return*

*y*
is the vector ***y*** of length *m*, containing the result of the computation. Returned as: a one-dimensional array of (at least) length *m*, containing long-precision real numbers.

### *Notes*

1. Matrix ***A*** must have no common elements with vectors ***x*** and ***y***; otherwise, results are unpredictable.

2. For the KA array, where there are no corresponding nonzero elements in AC, you must still fill in a number between 1 and *n*. See the “Example” on page 366.
3. For a description of how sparse matrices are stored in compressed-matrix storage mode, see “Compressed-Matrix Storage Mode” on page 93.
4. If your sparse matrix is stored by rows, as defined in “Storage-by-Rows” on page 99, you should first use the DSRSM utility subroutine, described in “DSRSM—Convert a Sparse Matrix from Storage-by-Rows to Compressed-Matrix Storage Mode” on page 981, to convert your sparse matrix to compressed-matrix storage mode.

***Function:*** The matrix-vector product is computed for a sparse matrix, stored in compressed matrix mode:

$$\boldsymbol{y} \leftarrow \boldsymbol{Ax}$$

where:

***A*** is an *m* by *n* sparse matrix, stored in compressed-matrix storage mode in arrays AC and KA.
***x*** is a vector of length *n*.
***y*** is a vector of length *m*.

It is expressed as follows:

$$\begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_m \end{bmatrix} \leftarrow \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{m1} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix}$$

See reference [70 ]. If *m* is 0, no computation is performed; if *nz* is 0, output vector ***y*** is set to zero, because matrix ***A*** contains all zeros.

If your program uses a sparse matrix stored by rows and you want to use this subroutine, you should first convert your sparse matrix to compressed-matrix storage mode by using the DSRSM utility subroutine described in “DSRSM—Convert a Sparse Matrix from Storage-by-Rows to Compressed-Matrix Storage Mode” on page 981.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. $m < 0$
2. $lda \leq 0$
3. $m > lda$
4. $nz < 0$

***Example:*** This example shows the matrix-vector product computed for the following sparse matrix ***A***, which is stored in compressed-matrix storage mode in arrays AC and KA. Matrix ***A*** is:

```
┌                               ┐
│ 4.0  0.0  7.0  0.0  0.0  0.0 │
│ 3.0  4.0  0.0  2.0  0.0  0.0 │
│ 0.0  2.0  4.0  0.0  4.0  0.0 │
│ 0.0  0.0  7.0  4.0  0.0  1.0 │
│ 1.0  0.0  0.0  3.0  4.0  0.0 │
│ 1.0  1.0  0.0  0.0  3.0  4.0 │
└                               ┘
```

*Call Statement and Input*

```
            M   NZ  AC   KA  LDA  X   Y
            |   |   |    |    |   |   |
CALL DSMMX( 6 , 4 , AC , KA , 6 , X , Y )

        ┌                     ┐
        │ 4.0  7.0  0.0  0.0 │
        │ 4.0  3.0  2.0  0.0 │
AC   =  │ 4.0  2.0  4.0  0.0 │
        │ 4.0  7.0  1.0  0.0 │
        │ 4.0  1.0  3.0  0.0 │
        │ 4.0  1.0  1.0  3.0 │
        └                     ┘

        ┌             ┐
        │ 1  3  1  1 │
        │ 2  1  4  1 │
KA   =  │ 3  2  5  1 │
        │ 4  3  6  1 │
        │ 5  1  4  1 │
        │ 6  1  2  5 │
        └             ┘
X        =  (1.0, 2.0, 3.0, 4.0, 5.0, 6.0)
```

*Output*

```
Y        =  (25.0, 19.0, 36.0, 43.0, 33.0, 42.0)
```

# DSMTM—Transpose a Sparse Matrix in Compressed-Matrix Storage Mode

This subprogram transposes sparse matrix ***A***, stored in compressed-matrix storage mode, where ***A*** contains long-precision real numbers.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL DSMTM (*m*, *nz*, *ac*, *ka*, *lda*, *n*, *nt*, *at*, *kt*, *ldt*, *aux*, *naux*) |
| **C and C++** | dsmtm (*m*, *nz*, *ac*, *ka*, *lda*, *n*, *nt*, *at*, *kt*, *ldt*, *aux*, *naux*); |
| **PL/I** | CALL DSMTM (*m*, *nz*, *ac*, *ka*, *lda*, *n*, *nt*, *at*, *kt*, *ldt*, *aux*, *naux*); |

*On Entry*

*m*
is the number of rows in sparse matrix ***A***. Specified as: a fullword integer; $m \geq 0$.

*nz*
is the maximum number of nonzero elements in each row of sparse matrix ***A***. Specified as: a fullword integer; $nz \geq 0$.

*ac*
is the *m* by *n* sparse matrix ***A***, stored in compressed-matrix storage mode in an array, referred to as AC. Specified as: an *lda* by (at least) *nz* array, containing long-precision real numbers.

*ka*
is the array, referred to as KA, containing the column numbers of the matrix ***A*** elements stored in the corresponding positions in array AC. Specified as: an *lda* by (at least) *nz* array, containing fullword integers, where 1 ≤ (elements of KA) ≤ *n*.

*lda*
is the size of the leading dimension of the arrays specified for *ac* and *ka*. Specified as: a fullword integer; $lda > 0$ and $lda \geq m$.

*n*
is the number of columns in sparse matrix ***A***. Specified as: a fullword integer; $0 \leq n \leq ldt$ and $n \geq$ (maximum column index in KA).

*nt*
is the number of columns in output arrays AT and KT that are available for use. Specified as: a fullword integer; $nt > 0$.

*at*
See “On Return” on page 369.

*kt*
See “On Return” on page 369.

*ldt*
is the size of the leading dimension of the arrays specified for *at* and *kt*. Specified as: a fullword integer; $ldt > 0$ and $ldt \geq n$.

*aux*
has the following meaning:

If $naux = 0$ and error 2015 is unrecoverable, *aux* is ignored.

Otherwise, it is a storage work area used by this subroutine. Its size is specified by *naux*.

Specified as: an area of storage, containing long-precision real numbers. They can have any value.

*naux*

is the size of the work area specified by *aux*—that is, the number of elements in *aux*. Specified as: a fullword integer, where:

If *naux* = 0 and error 2015 is unrecoverable, DSMTM dynamically allocates the work area used by this subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, *naux* ≥ *n*.

*On Return*

*n*

is the number of rows in the transposed matrix $\boldsymbol{A}^T$. Returned as: a fullword integer; *n* = (maximum column index in KA).

*nt*

is the maximum number of nonzero elements, *nt*, in each row of the transposed matrix $\boldsymbol{A}^T$. Returned as: a fullword integer; *nt* ≤ *m*.

*at*

is the *n* by (at least) *m* sparse matrix transpose $\boldsymbol{A}^T$, stored in compressed-matrix storage mode in an array, referred to as AT. Returned as: an *ldt* by (at least) *nt* array, containing long-precision real numbers.

*kt*

is the array, referred to as KT, containing the column numbers of the transposed matrix $\boldsymbol{A}^T$ elements, stored in the corresponding positions in array AT. Returned as: an *ldt* by (at least) *nt* array, containing fullword integers, where 1 ≤ (elements of KT) ≤ *m*.

***Notes***

1. In your C program, arguments *n* and *nt* must be passed by reference.
2. The value specified for input argument *nt* should be greater than or equal to the number of nonzero elements you estimate to be in each row of the transposed sparse matrix $\boldsymbol{A}^T$. The output value is less than or equal to the input value you specify.
3. For the KA array, where there are no corresponding nonzero elements in AC, you must still fill in a number between 1 and *n*. See the “Example” on page 370.
4. For a description of how sparse matrices are stored in compressed-matrix storage mode, see “Compressed-Matrix Storage Mode” on page 93.
5. If your sparse matrix is stored by rows, as defined in “Storage-by-Rows” on page 99, you should first use the DSRSM utility subroutine, described in “DSRSM—Convert a Sparse Matrix from Storage-by-Rows to Compressed-Matrix Storage Mode” on page 981, to convert your sparse matrix to compressed-matrix storage mode.
6. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see “Using Auxiliary Storage in ESSL” on page 32.

***Function:*** A sparse matrix $\boldsymbol{A}$, stored in arrays AC and KA in compressed-matrix storage mode, is transposed, forming $\boldsymbol{A}^T$, and is stored in arrays AT and KT in compressed-matrix storage mode. See reference [70 ]. This subroutine is provided for when you want to do a matrix-vector product using a transposed matrix, $\boldsymbol{A}^T$. First, you transpose a matrix, $\boldsymbol{A}$, using this subroutine, then you call DSMMX with the transposed matrix $\boldsymbol{A}^T$. This results in the following computation being performed: $\boldsymbol{y} \leftarrow \boldsymbol{A}^T\boldsymbol{x}$.

If your program uses a sparse matrix stored by rows and you want to use this subroutine, you should first convert your sparse matrix to compressed-matrix storage mode by using the DSRSM utility subroutine described in “DSRSM—Convert a Sparse Matrix from Storage-by-Rows to Compressed-Matrix Storage Mode” on page 981.

***Error Conditions***

*Resource Errors:* Error 2015 is unrecoverable, *naux* = 0, and unable to allocate work area.

*Computational Errors:* None

*Input-Argument Errors*

1. *m*, *n* < 0
2. *lda*, *ldt* < 1
3. *lda* < *m*
4. *ldt* < *n*
5. *nz* < 0
6. *n* is less than the maximum column index in KA.
7. *nt* or *ldt* are too small.
8. When the following two errors occur, arrays AT, KT, and AUX are overwritten:
   *naux* < *n*
   *nt* ≤ 0
9. Error 2015 is recoverable or *naux*≠0, and *naux* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example:*** This example shows how to transpose the following 5 by 4 sparse matrix ***A***, which is stored in compressed-matrix storage mode in arrays AC and KA. Matrix ***A*** is:

$$\begin{bmatrix} 11.0 & 0.0 & 0.0 & 0.0 \\ 21.0 & 0.0 & 23.0 & 0.0 \\ 0.0 & 0.0 & 33.0 & 34.0 \\ 0.0 & 42.0 & 0.0 & 44.0 \\ 51.0 & 0.0 & 53.0 & 0.0 \end{bmatrix}$$

The resulting 4 by 5 matrix transpose $\boldsymbol{A}^T$, stored in compressed-matrix storage mode in arrays AT and KT, is as follows. Matrix $\boldsymbol{A}^T$ is:

$$\begin{bmatrix} 11.0 & 21.0 & 0.0 & 0.0 & 51.0 \\ 0.0 & 0.0 & 0.0 & 42.0 & 0.0 \\ 0.0 & 23.0 & 33.0 & 0.0 & 53.0 \\ 0.0 & 0.0 & 34.0 & 44.0 & 0.0 \end{bmatrix}$$

As shown here, the value of N is larger than the actual number of columns in the matrix ***A***. On output, the exact number of rows in the transposed matrix is returned in the output argument N.

On output, row 6 of AT and KT is is not accessed or modified by the subroutine. Column 4 and row 5 are accessed and modified. They are of no use in further computations and will not be used, because NT = 3 and M = 4.

*Call Statement and Input*

```
            M   NZ  AC   KA  LDA  N   NT  AT   KT   LDT  AUX  NAUX
            |   |   |    |   |    |   |   |    |    |    |    |
CALL DSMTM( 5 , 2 , AC , KA , 5 , 5 , 4 , AT , KT ,  6 , AUX , 5  )
```

```
        ┌            ┐
        │ 11.0   0.0 │
        │ 21.0  23.0 │
AC   =  │ 33.0  34.0 │
        │ 42.0  44.0 │
        │ 51.0  53.0 │
        └            ┘
```

```
        ┌      ┐
        │ 1  1 │
        │ 1  3 │
KA   =  │ 3  4 │
        │ 2  4 │
        │ 1  3 │
        └      ┘
```

*Output*

```
N        =  4
NT       =  3
```

```
        ┌                       ┐
        │ 11.0  21.0  51.0  0.0 │
        │ 42.0   0.0   0.0  0.0 │
AT   =  │ 33.0  23.0  53.0  0.0 │
        │ 34.0  44.0   0.0  0.0 │
        │  0.0   0.0   0.0  0.0 │
        │   .     .     .    .  │
        └                       ┘
```

```
        ┌            ┐
        │ 1  2  5  1 │
        │ 4  1  1  1 │
KT   =  │ 3  2  5  1 │
        │ 3  4  1  1 │
        │ 1  1  1  1 │
        │ .  .  .  . │
        └            ┘
```

## DSDMX—Matrix-Vector Product for a Sparse Matrix or Its Transpose in Compressed-Diagonal Storage Mode

This subprogram computes the matrix-vector product for square sparse matrix ***A***, stored in compressed-diagonal storage mode, using either the matrix or its transpose, and vectors ***x*** and ***y***:

$$\boldsymbol{y} \leftarrow \boldsymbol{A}\boldsymbol{x}$$
$$\boldsymbol{y} \leftarrow \boldsymbol{A}^{T}\boldsymbol{x}$$

where ***A***, ***x***, and ***y*** contain long-precision real numbers.

***Syntax***

| | |
|---|---|
| **Fortran** | CALL DSDMX (*iopt*, *n*, *nd*, *ad*, *lda*, *trans*, *la*, *x*, *y*) |
| **C and C++** | dsdmx (*iopt*, *n*, *nd*, *ad*, *lda*, *trans*, *la*, *x*, *y*); |
| **PL/I** | CALL DSDMX (*iopt*, *n*, *nd*, *ad*, *lda*, *trans*, *la*, *x*, *y*); |

*On Entry*

*iopt*

indicates the storage variation used for sparse matrix ***A***, stored in compressed-diagonal storage mode, where:

If *iopt* = 0, matrix ***A*** is a general sparse matrix, where all the nonzero diagonals in matrix ***A*** are used to set up the storage arrays.

If *iopt* = 1, matrix ***A*** is a symmetric sparse matrix, where only the nonzero main diagonal and one of each of the unique nonzero diagonals are used to set up the storage arrays.

Specified as: a fullword integer; *iopt* = 0 or 1.

*n*

is the order of sparse matrix ***A*** and the number of elements in vectors ***x*** and ***y***. Specified as: a fullword integer; *n* ≥ 0.

*nd*

is the number of diagonals stored in the columns of array AD, as well as the number of columns in AD and the number of elements in array LA. Specified as: a fullword integer; *nd* ≥ 0.

*ad*

is the sparse matrix ***A*** of order *n*, stored in compressed diagonal storage in an array, referred to as AD. The *iopt* argument indicates the storage variation used for storing matrix ***A***. The *trans* argument indicates the following:

If *trans* = 'N', ***A*** is used in the computation.

If *trans* = 'T', $\boldsymbol{A}^{T}$ is used in the computation.

**Note:** No data should be moved to form $\boldsymbol{A}^{T}$; that is, the matrix ***A*** should always be stored in its untransposed form.

Specified as: an *lda* by (at least) *nd* array, containing long-precision real numbers; *lda* ≥ *n*.

*lda*

is the size of the leading dimension of the array specified for *ad*. Specified as: a fullword integer; *lda* > 0 and *lda* ≥ *n*.

*trans*

indicates the form of matrix ***A*** to use in the computation, where:

If *trans* = `'N'`, ***A*** is used in the computation.

If *trans* = `'T'`, $\boldsymbol{A}^T$ is used in the computation.

Specified as: a single character; *trans* = `'N'` or `'T'`.

*la*

is the array, referred to as `LA`, containing the diagonal numbers *k* for the diagonals stored in each corresponding column in array `AD`. (For an explanation of how diagonal numbers are assigned, see “Compressed-Diagonal Storage Mode” on page 94.)

Specified as: a one-dimensional array of (at least) length *nd*, containing fullword integers; $1-n \le$ `LA`$(i) \le n-1$.

*x*

is the vector ***x*** of length *n*. Specified as: a one-dimensional array, containing long-precision real numbers.

*y*

See “On Return.”

*On Return*

*y*

is the vector ***y*** of length *n*, containing the result of the computation. Returned as: a one-dimensional array, containing long-precision real numbers.

***Notes***

1. All subroutines accept lowercase letters for the *trans* argument.
2. Matrix ***A*** must have no common elements with vectors ***x*** and ***y***; otherwise, results are unpredictable.
3. For a description of how sparse matrices are stored in compressed-diagonal storage mode, see “Compressed-Diagonal Storage Mode” on page 94.

***Function:*** The matrix-vector product of a square sparse matrix or its transpose, is computed for a matrix stored in compressed-diagonal storage mode:

$$\boldsymbol{y} \leftarrow \boldsymbol{Ax}$$
$$\boldsymbol{y} \leftarrow \boldsymbol{A}^T\boldsymbol{x}$$

where:

***A*** is a sparse matrix of order *n*, stored in compressed-diagonal storage mode in `AD` and `LA`, using the storage variation for either general or symmetric sparse matrices, as indicated by the *iopt* argument.
***x*** and ***y*** are vectors of length *n*.

It is expressed as follows for $\boldsymbol{y} \leftarrow \boldsymbol{Ax}$:

$$\begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} \leftarrow \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{n1} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix}$$

It is expressed as follows for $\boldsymbol{y} \leftarrow \boldsymbol{A}^{T}\boldsymbol{x}$:

$$\begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} \leftarrow \begin{bmatrix} a_{11} & \dots & a_{n1} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{1n} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix}$$

If *n* is 0, no computation is performed; if *nd* is 0, output vector ***y*** is set to zero, because matrix ***A*** contains all zeros.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. *iopt* ≠ 0 or 1
2. *n* < 0
3. *lda* ≤ 0
4. *n* > *lda*
5. *trans* ≠ 'N' or 'T'
6. *nd* < 0
7. LA(*j*) ≤ −*n* or LA(*j*) ≥ *n*, for any *j* = 1, *n*

***Example 1:*** This example shows the matrix-vector product using *trans* = 'N', which is computed for the following sparse matrix ***A*** of order 6. The matrix is stored in compressed-matrix storage mode in arrays AD and LA using the storage variation for general sparse matrices, storing all nonzero diagonals. Matrix ***A*** is:

```
┌                               ┐
│ 4.0  0.0  7.0  0.0  0.0  0.0 │
│ 3.0  4.0  0.0  2.0  0.0  0.0 │
│ 0.0  2.0  4.0  0.0  4.0  0.0 │
│ 0.0  0.0  7.0  4.0  0.0  1.0 │
│ 1.0  0.0  0.0  3.0  4.0  0.0 │
│ 1.0  1.0  0.0  0.0  3.0  4.0 │
└                               ┘
```

*Call Statement and Input*

```
           IOPT  N   ND  AD  LDA TRANS  LA   X   Y
            |    |   |   |    |    |    |    |   |
CALL DSDMX( 0  , 6 , 5 , AD , 6 , 'N' , LA , X , Y )

         ┌                          ┐
         │ 4.0  0.0  0.0  0.0  7.0 │
         │ 4.0  0.0  0.0  3.0  2.0 │
AD   =   │ 4.0  0.0  0.0  2.0  4.0 │
         │ 4.0  0.0  0.0  7.0  1.0 │
         │ 4.0  0.0  1.0  3.0  0.0 │
         │ 4.0  1.0  1.0  3.0  0.0 │
         └                          ┘

LA       =  (0, -5, -4, -1, 2)
X        =  (1.0, 2.0, 3.0, 4.0, 5.0, 6.0)
```

*Output*

```
Y        =  (25.0, 19.0, 36.0, 43.0, 33.0, 42.0)
```

***Example 2:*** This example shows the matrix-vector product using *trans* = 'N', which is computed for the following sparse matrix ***A*** of order 6. The matrix is stored in compressed-matrix storage mode in arrays AD and LA using the storage variation for symmetric sparse matrices, storing the nonzero main diagonal and one of each of the unique nonzero diagonals. Matrix ***A*** is:

```
                  ┌                                     ┐
                  | 11.0   0.0  13.0   0.0  15.0   0.0 |
                  |  0.0  22.0   0.0  24.0   0.0  26.0 |
                  | 13.0   0.0  33.0   0.0  35.0   0.0 |
                  |  0.0  24.0   0.0  44.0   0.0  46.0 |
                  | 15.0   0.0  35.0   0.0  55.0   0.0 |
                  |  0.0  26.0   0.0  46.0   0.0  66.0 |
                  └                                     ┘
```

*Call Statement and Input*

```
           IOPT  N   ND  AD  LDA TRANS  LA   X   Y
            |    |   |   |    |    |    |    |   |
CALL DSDMX( 1  , 6 , 3 , AD , 6 , 'N' , LA , X , Y )

          ┌                  ┐
          | 11.0  13.0   0.0 |
          | 22.0  24.0   0.0 |
AD   =    | 33.0  35.0   0.0 |
          | 44.0  46.0   0.0 |
          | 55.0   0.0  15.0 |
          | 66.0   0.0  26.0 |
          └                  ┘

LA       =  (0, 2, -4)
X        =  (1.0, 2.0, 3.0, 4.0, 5.0, 6.0)
```

*Output*

```
Y        =  (125.0, 296.0, 287.0, 500.0, 395.0, 632.0)
```

***Example 3:*** This example is the same as Example 1 except that it shows the matrix-vector product for the transpose of a matrix, using *trans* = 'T'. It is computed using the transpose of the following sparse matrix ***A*** of order 6, which is stored in compressed-matrix storage mode in arrays AD and LA, using the storage variation for general sparse matrices, storing all nonzero diagonals. It uses the same matrix ***A*** as in Example 1.

*Call Statement and Input*

```
           IOPT  N   ND  AD  LDA TRANS  LA   X   Y
            |    |   |   |    |    |    |    |   |
CALL DSDMX( 0  , 6 , 5 , AD , 6 , 'T' , LA , X , Y )
```

AD =(same as input AD in Example 1)
LA =(same as input LA in Example 1)
X =(same as input X in Example 1)

*Output*

```
Y        =  (21.0, 20.0, 47.0, 35.0, 50.0, 28.0)
```

# Chapter 9. Matrix Operations

The matrix operation subroutines are described in this chapter.

## Overview of the Matrix Operation Subroutines

Some of the matrix operation subroutines were designed in accordance with the Level 3 BLAS de facto standard. If these subroutines do not comply with the standard as approved, IBM will consider updating them to do so. If IBM updates these subroutines, the updates could require modifications of the calling application program. For details on the Level 3 BLAS, see reference [32]. The matrix operation subroutines also include the commonly used matrix operations: addition, subtraction, multiplication, and transposition (Table 71).

*Table 71. List of Matrix Operation Subroutines*

| **Descriptive Name** | **Short-Precision Subroutine** | **Long-Precision Subroutine** | **Page** |
|---|---|---|---|
| Matrix Addition for General Matrices or Their Transposes | SGEADD<br>CGEADD | DGEADD<br>ZGEADD | 381 |
| Matrix Subtraction for General Matrices or Their Transposes | SGESUB<br>CGESUB | DGESUB<br>ZGESUB | 388 |
| Matrix Multiplication for General Matrices, Their Transposes, or Conjugate Transposes | SGEMUL<br>CGEMUL | DGEMUL<br>ZGEMUL<br>DGEMLP§ | 395 |
| Matrix Multiplication for General Matrices, Their Transposes, or Conjugate Transposes Using Winograd's Variation of Strassen's Algorithm | SGEMMS<br>CGEMMS | DGEMMS<br>ZGEMMS | 405 |
| Combined Matrix Multiplication and Addition for General Matrices, Their Transposes, or Conjugate Transposes | SGEMM♦<br>CGEMM♦ | DGEMM♦<br>ZGEMM♦ | 411 |
| Matrix-Matrix Product Where One Matrix is Real or Complex Symmetric or Complex Hermitian | SSYMM♦<br>CSYMM♦<br>CHEMM♦ | DSYMM♦<br>ZSYMM♦<br>ZHEMM♦ | 420 |
| Triangular Matrix-Matrix Product | STRMM♦<br>CTRMM♦ | DTRMM♦<br>ZTRMM♦ | 428 |
| Rank-K Update of a Real or Complex Symmetric or a Complex Hermitian Matrix | SSYRK♦<br>CSYRK♦<br>CHERK♦ | DSYRK♦<br>ZSYRK♦<br>ZHERK♦ | 435 |
| Rank-2K Update of a Real or Complex Symmetric or a Complex Hermitian Matrix | SSYR2K♦<br>CSYR2K♦<br>CHER2K♦ | DSYR2K♦<br>ZSYR2K♦<br>ZHER2K♦ | 442 |
| General Matrix Transpose (In-Place) | SGETMI<br>CGETMI | DGETMI<br>ZGETMI | 450 |
| General Matrix Transpose (Out-of-Place) | SGETMO<br>CGETMO | DGETMO<br>ZGETMO | 453 |

♦ Level 3 BLAS

§ This subroutine is provided only for migration from earlier releases of ESSL and is not intended for use in new programs. Documentation for this subroutine is no longer provided.

# Use Considerations

This section describes some key points about using the matrix operations subroutines.

## Specifying Normal, Transposed, or Conjugate Transposed Input Matrices

On each invocation, the matrix operation subroutines can perform one of several possible computations, using different forms of the input matrices ***A*** and ***B***. For the real and complex versions of the subroutines, there are four and nine combinations, respectively, depending on the characters specified for the *transa* and *transb* arguments:

| | |
|---|---|
| `'N'` | Normal form |
| `'T'` | Transposed form subroutines) |
| `'C'` | Conjugate transposed form |

The four and nine possible combinations are defined as follows:

| Real Combinations | Complex Combinations |
|---|---|
| $\boldsymbol{AB}$ | $\boldsymbol{AB}$ |
| $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}$ | $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}$ |
| | $\boldsymbol{A}^{\mathrm{H}}\boldsymbol{B}$ |
| $\boldsymbol{AB}^{\mathrm{T}}$ | $\boldsymbol{AB}^{\mathrm{T}}$ |
| $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}$ | $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}$ |
| | $\boldsymbol{A}^{\mathrm{H}}\boldsymbol{B}^{\mathrm{T}}$ |
| | $\boldsymbol{AB}^{\mathrm{H}}$ |
| | $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{H}}$ |
| | $\boldsymbol{A}^{\mathrm{H}}\boldsymbol{B}^{\mathrm{H}}$ |

## Transposing or Conjugate Transposing:

This section describes some key points about using transposed and conjugate transposed matrices.

### On Input

In every case, the input arrays for the matrix, its transpose, or its conjugate transpose should be stored in the original untransposed form. You then specify the desired form of the matrix to be used in the computation in the *transa* or *transb* arguments. For a description of matrix transpose and matrix conjugate transpose, see “Matrices” on page 62.

### On Output

If you want to compute the transpose or the conjugate transpose of a matrix operation—that is, the output stored in matrix ***C***—you should use the matrix identities described in Special Usage for each subroutine description. Examples are provided in the subroutine descriptions to show the use of these matrix identities. This accomplishes the transpose or conjugate transpose as part of the multiply operation.

# Performance and Accuracy Considerations

This section describes some key points about performance and accuracy in the matrix operations subroutines.

## In General

1. The matrix operation subroutines use algorithms that are tuned specifically to the workstation processors they run on. The techniques involve using any one of several computational methods, based on certain operation counts and sizes of data.
2. The short-precision multiplication subroutines provide increased accuracy by partially accumulating results in long precision.
3. Strassen's method is not stable for certain row or column scalings of the input matrices ***A*** and ***B***. Therefore, for matrices ***A*** and ***B*** with divergent exponent values, Strassen's method may give inaccurate results. For these cases, you should use the _GEMUL or _GEMM subroutines.
4. There are ESSL-specific rules that apply to the results of computations on the workstation processors using the ANSI/IEEE standards. For details, see “What Data Type Standards Are Used by ESSL, and What Exceptions Should You Know About?” on page 45.

## For Large Matrices

If you are using large square matrices in your matrix multiplication operations, you get better performance by using SGEMMS, DGEMMS, CGEMMS, and ZGEMMS. These subroutines use Winograd's variation of Strassen's algorithm for both real and complex matrices.

## For Combined Operations

If you want to perform a combined matrix multiplication and addition with scaling, SGEMM, DGEMM, CGEMM, and ZGEMM provide better performance than if you perform the parts of the computation separately in your program. See references [32] and [35 ].

## Matrix Operation Subroutines

This section contains the matrix operation subroutine descriptions.

# SGEADD, DGEADD, CGEADD, and ZGEADD—Matrix Addition for General Matrices or Their Transposes

These subroutines can perform any one of the following matrix additions, using matrices ***A*** and ***B*** or their transposes, and matrix ***C***:

$\boldsymbol{C} \leftarrow \boldsymbol{A}+\boldsymbol{B}$
$\boldsymbol{C} \leftarrow \boldsymbol{A}^{T}+\boldsymbol{B}$
$\boldsymbol{C} \leftarrow \boldsymbol{A}+\boldsymbol{B}^{T}$
$\boldsymbol{C} \leftarrow \boldsymbol{A}^{T}+\boldsymbol{B}^{T}$

*Table 72. Data Types*

| ***A***, ***B***, ***C*** | **Subroutine** |
|---|---|
| Short-precision real | SGEADD |
| Long-precision real | DGEADD |
| Short-precision complex | CGEADD |
| Long-precision complex | ZGEADD |

***Syntax***

| | |
|---|---|
| **Fortran** | CALL SGEADD \| DGEADD \| CGEADD \| ZGEADD (*a*, *lda*, *transa*, *b*, *ldb*, *transb*, *c*, *ldc*, *m*, *n*) |
| **C and C++** | sgeadd \| dgeadd \| cgeadd \| zgeadd (*a*, *lda*, *transa*, *b*, *ldb*, *transb*, *c*, *ldc*, *m*, *n*); |
| **PL/I** | CALL SGEADD \| DGEADD \| CGEADD \| ZGEADD (*a*, *lda*, *transa*, *b*, *ldb*, *transb*, *c*, *ldc*, *m*, *n*); |

*On Entry*

*a*

is the matrix ***A***, where:

If *transa* = 'N', ***A*** is used in the computation, and ***A*** has *m* rows and *n* columns.

If *transa* = 'T', $\boldsymbol{A}^{T}$ is used in the computation, and ***A*** has *n* rows and *m* columns.

**Note:** No data should be moved to form $\boldsymbol{A}^{T}$; that is, the matrix ***A*** should always be stored in its untransposed form.

Specified as: a two-dimensional array, containing numbers of the data type indicated in Table 72, where:

If *transa* = 'N', its size must be *lda* by (at least) *n*.

If *transa* = 'T', its size must be *lda* by (at least) *m*.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; $lda > 0$ and:

If *transa* = 'N', $lda \geq m$.

If *transa* = 'T', $lda \geq n$.

*transa*

indicates the form of matrix ***A*** to use in the computation, where:

If *transa* = 'N', ***A*** is used in the computation.

If *transa* = 'T', $\boldsymbol{A}^{T}$ is used in the computation.

Specified as: a single character; *transa* = `'N'` or `'T'`.

*b*

is the matrix ***B***, where:

If *transb* = `'N'`, ***B*** is used in the computation, and ***B*** has *m* rows and *n* columns.

If *transb* = `'T'`, $\boldsymbol{B}^T$ is used in the computation, and ***B*** has *n* rows and *m* columns.

**Note:** No data should be moved to form $\boldsymbol{B}^T$; that is, the matrix ***B*** should always be stored in its untransposed form.

Specified as: a two-dimensional array, containing numbers of the data type indicated in Table 72 on page 381, where:

If *transb* = `'N'`, its size must be *ldb* by (at least) *n*.

If *transb* = `'T'`, its size must be *ldb* by (at least) *m*.

*ldb*

is the leading dimension of the array specified for *b*. Specified as: a fullword integer; $ldb > 0$ and:

If *transb* = `'N'`, $ldb \geq m$.

If *transb* = `'T'`, $ldb \geq n$.

*transb*

indicates the form of matrix ***B*** to use in the computation, where:

If *transb* = `'N'`, ***B*** is used in the computation.

If *transb* = `'T'`, $\boldsymbol{B}^T$ is used in the computation.

Specified as: a single character; *transb* = `'N'` or `'T'`.

*c*

See “On Return.”

*ldc*

is the leading dimension of the array specified for *c*. Specified as: a fullword integer; $ldc > 0$ and $ldc \geq m$.

*m*

is the number of rows in matrix ***C***. Specified as: a fullword integer; $0 \leq m \leq ldc$.

*n*

is the number of columns in matrix ***C***. Specified as: a fullword integer; $0 \leq n$.

*On Return*

*c*

is the *m* by *n* matrix ***C***, containing the results of the computation. Returned as: an *ldc* by (at least) *n* array, containing numbers of the data type indicated in Table 72 on page 381.

### *Notes*

1. All subroutines accept lowercase letters for the *transa* and *transb* arguments.
2. Matrix ***C*** must have no common elements with matrices ***A*** or ***B***. However, ***C*** may (exactly) coincide with ***A*** if *transa* = `'N'`, and ***C*** may (exactly) coincide with ***B*** if *transb* = `'N'`. Otherwise, results are unpredictable. See “Concepts” on page 55.

***Function:*** The matrix sum is expressed as follows, where $a_{ij}$, $b_{ij}$, and $c_{ij}$ are elements of matrices ***A, B,*** and ***C***, respectively:

$c_{ij} = a_{ij}+b_{ij}$ for $\boldsymbol{C} \leftarrow \boldsymbol{A}+\boldsymbol{B}$
$c_{ij} = a_{ij}+b_{ji}$ for $\boldsymbol{C} \leftarrow \boldsymbol{A}+\boldsymbol{B}^{\mathrm{T}}$
$c_{ij} = a_{ji}+b_{ij}$ for $\boldsymbol{C} \leftarrow \boldsymbol{A}^{\mathrm{T}}+\boldsymbol{B}$
$c_{ij} = a_{ji}+b_{ji}$ for $\boldsymbol{C} \leftarrow \boldsymbol{A}^{\mathrm{T}}+\boldsymbol{B}^{\mathrm{T}}$
for $i = 1, m$ and $j = 1, n$

If *m* or *n* is 0, no computation is performed.

***Special Usage:*** You can compute the transpose $\boldsymbol{C}^{\mathrm{T}}$ of each of the four computations listed under “ Function” by using the following matrix identities:

$(\boldsymbol{A}+\boldsymbol{B})^{\mathrm{T}} = \boldsymbol{A}^{\mathrm{T}}+\boldsymbol{B}^{\mathrm{T}}$
$(\boldsymbol{A}+\boldsymbol{B}^{\mathrm{T}})^{\mathrm{T}} = \boldsymbol{A}^{\mathrm{T}}+\boldsymbol{B}$
$(\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{B})^{\mathrm{T}} = \boldsymbol{A}+\boldsymbol{B}^{\mathrm{T}}$
$(\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{B}^{\mathrm{T}})^{\mathrm{T}} = \boldsymbol{A}+\boldsymbol{B}$

Be careful that your output array receiving $\boldsymbol{C}^{\mathrm{T}}$ has dimensions large enough to hold the transposed matrix. See “Example 4” on page 385.

### ***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. *lda*, *ldb*, *ldc* $\leq 0$
2. *m*, *n* $< 0$
3. $m > ldc$
4. *transa*, *transb* ≠ `'N'` or `'T'`
5. *transa* = `'N'` and $m > lda$
6. *transa* = `'T'` and $n > lda$
7. *transb* = `'N'` and $m > ldb$
8. *transb* = `'T'` and $n > ldb$

***Example 1:*** This example shows the computation $\boldsymbol{C} \leftarrow \boldsymbol{A}+\boldsymbol{B}$, where ***A*** and ***C*** are contained in larger arrays `A` and `C`, respectively, and ***B*** is the same size as array `B`, in which it is contained.

*Call Statement and Input*

```
             A  LDA TRANSA  B  LDB TRANSB  C  LDC  M   N
             |   |    |     |   |    |     |   |   |   |
CALL SGEADD( A , 6 , 'N'  , B , 4 , 'N'  , C , 5 , 4 , 3 )
```

```
          ┌                              ┐
          | 110000.0  120000.0  130000.0 |
          | 210000.0  220000.0  230000.0 |
A     =   | 310000.0  320000.0  330000.0 |
          | 410000.0  420000.0  430000.0 |
          |       .         .         .  |
          |       .         .         .  |
          └                              ┘
```

```
        ┌                ┐
        │ 11.0  12.0  13.0 │
B    =  │ 21.0  22.0  23.0 │
        │ 31.0  32.0  33.0 │
        │ 41.0  42.0  43.0 │
        └                ┘
```

*Output*

```
        ┌                            ┐
        │ 110011.0  120012.0  130013.0 │
        │ 210021.0  220022.0  230023.0 │
C    =  │ 310031.0  320032.0  330033.0 │
        │ 410041.0  420042.0  430043.0 │
        │        .         .         . │
        └                            ┘
```

***Example 2:*** This example shows the computation ***C*** ← ***A***$^T$+***B***, where ***A, B,*** and ***C*** are the same size as arrays `A`, `B`, and `C`, in which they are contained.

*Call Statement and Input*

```
             A  LDA TRANSA  B  LDB TRANSB  C  LDC  M   N
             |   |    |     |   |    |     |   |   |   |
CALL SGEADD( A , 3 , 'T'  , B , 4 , 'N'  , C , 4 , 4 , 3 )
```

```
        ┌                                      ┐
        │ 110000.0  120000.0  130000.0  140000.0 │
A    =  │ 210000.0  220000.0  230000.0  240000.0 │
        │ 310000.0  320000.0  330000.0  340000.0 │
        └                                      ┘
```

```
        ┌                ┐
        │ 11.0  12.0  13.0 │
B    =  │ 21.0  22.0  23.0 │
        │ 31.0  32.0  33.0 │
        │ 41.0  42.0  43.0 │
        └                ┘
```

*Output*

```
        ┌                            ┐
        │ 110011.0  210012.0  310013.0 │
C    =  │ 120021.0  220022.0  320023.0 │
        │ 130031.0  230032.0  330033.0 │
        │ 140041.0  240042.0  340043.0 │
        └                            ┘
```

***Example 3:*** This example shows computation ***C*** ← ***A***+***B***$^T$, where ***A*** is contained in a larger array `A`, and ***B*** and ***C*** are the same size as arrays `B` and `C`, in which they are contained.

*Call Statement and Input*

```
             A  LDA TRANSA  B  LDB TRANSB  C  LDC  M   N
             |   |    |     |   |    |     |   |   |   |
CALL SGEADD( A , 5 , 'N'  , B , 3 , 'T'  , C , 4 , 4 , 3 )
```

```
        ┌                              ┐
        | 110000.0  120000.0  130000.0 |
        | 210000.0  220000.0  230000.0 |
A     = | 310000.0  320000.0  330000.0 |
        | 410000.0  420000.0  430000.0 |
        |        .         .         . |
        └                              ┘
```

```
        ┌                        ┐
        | 11.0  12.0  13.0  14.0 |
B     = | 21.0  22.0  23.0  24.0 |
        | 31.0  32.0  33.0  34.0 |
        └                        ┘
```

*Output*

```
        ┌                              ┐
        | 110011.0  120021.0  130031.0 |
C     = | 210012.0  220022.0  230032.0 |
        | 310013.0  320023.0  330033.0 |
        | 410014.0  420024.0  430034.0 |
        └                              ┘
```

***Example 4:*** This example shows how to produce the transpose of the result of the computation performed in “Example 3” on page 384, $\boldsymbol{C} \leftarrow \boldsymbol{A}+\boldsymbol{B}^T$, which uses the calling sequence:

```
CALL SGEADD( A , 5 , 'N' , B , 3 , 'T' , C , 4 , 4 , 3 )
```

You instead code a calling sequence for $\boldsymbol{C}^T \leftarrow \boldsymbol{A}^T+\boldsymbol{B}$, as shown below, where the resulting matrix $\boldsymbol{C}^T$ in the output array `CT` is the transpose of the matrix in the output array `C` in Example 3. Note that the array `CT` has dimensions large enough to receive the transposed matrix. For a description of all the matrix identities, see “Special Usage” on page 383.

*Call Statement and Input*

```
             A  LDA TRANSA  B  LDB TRANSB  C   LDC  M   N
             |   |    |     |   |    |     |    |   |   |
CALL SGEADD( A , 5 , 'T'  , B , 3 , 'N'  , CT , 4 , 3 , 4 )
```

```
        ┌                              ┐
        | 110000.0  120000.0  130000.0 |
        | 210000.0  220000.0  230000.0 |
A     = | 310000.0  320000.0  330000.0 |
        | 410000.0  420000.0  430000.0 |
        |        .         .         . |
        └                              ┘
```

```
        ┌                        ┐
        | 11.0  12.0  13.0  14.0 |
B     = | 21.0  22.0  23.0  24.0 |
        | 31.0  32.0  33.0  34.0 |
        └                        ┘
```

*Output*

```
        ┌                                          ┐
        │ 110011.0  210012.0  310013.0  410014.0 │
CT   =  │ 120021.0  220022.0  320023.0  420024.0 │
        │ 130031.0  230032.0  330033.0  430034.0 │
        │       .         .         .         .  │
        └                                          ┘
```

***Example 5:*** This example shows the computation $\boldsymbol{C} \leftarrow \boldsymbol{A}^T+\boldsymbol{B}^T$, where ***A, B,*** and ***C*** are the same size as the arrays `A`, `B`, and `C`, in which they are contained.

*Call Statement and Input*

```
             A  LDA TRANSA  B  LDB TRANSB  C  LDC  M   N
             |   |   |      |   |   |      |   |   |   |
CALL SGEADD( A , 3 , 'T'  , B , 3 , 'T'  , C , 4 , 4 , 3 )
```

```
        ┌                                          ┐
        │ 110000.0  120000.0  130000.0  140000.0 │
A    =  │ 210000.0  220000.0  230000.0  240000.0 │
        │ 310000.0  320000.0  330000.0  340000.0 │
        └                                          ┘
```

```
        ┌                          ┐
        │ 11.0  12.0  13.0  14.0 │
B    =  │ 21.0  22.0  23.0  24.0 │
        │ 31.0  32.0  33.0  34.0 │
        └                          ┘
```

*Output*

```
        ┌                                ┐
        │ 110011.0  210021.0  310031.0 │
C    =  │ 120012.0  220022.0  320032.0 │
        │ 130013.0  230023.0  330033.0 │
        │ 140014.0  240024.0  340034.0 │
        └                                ┘
```

***Example 6:*** This example shows the computation $\boldsymbol{C} \leftarrow \boldsymbol{A}+\boldsymbol{B}$, where ***A***, ***B***, and ***C*** are contained in larger arrays `A`, `B`, and `C`, respectively, and the arrays contain complex data.

*Call Statement and Input*

```
             A  LDA TRANSA  B  LDB TRANSB  C  LDC  M   N
             |   |   |      |   |   |      |   |   |   |
CALL CGEADD( A , 6 , 'N'  , B , 5 , 'N'  , C , 5 , 4 , 3 )
```

```
        ┌                                        ┐
        │ (1.0, 5.0)  (9.0, 2.0)  (1.0, 9.0) │
        │ (2.0, 4.0)  (8.0, 3.0)  (1.0, 8.0) │
A    =  │ (3.0, 3.0)  (7.0, 5.0)  (1.0, 7.0) │
        │ (6.0, 6.0)  (3.0, 6.0)  (1.0, 4.0) │
        │     .           .           .      │
        │     .           .           .      │
        └                                        ┘
```

```
               ┌                                    ┐
               | (1.0, 8.0)  (2.0, 7.0)  (3.0, 2.0) |
               | (4.0, 4.0)  (6.0, 8.0)  (6.0, 3.0) |
B     =        | (6.0, 2.0)  (4.0, 5.0)  (4.0, 5.0) |
               | (7.0, 2.0)  (6.0, 4.0)  (1.0, 6.0) |
               |     .           .           .      |
               └                                    ┘
```

*Output*

```
               ┌                                         ┐
               |  (2.0, 13.0)  (11.0,  9.0)  (4.0, 11.0) |
               |  (6.0,  8.0)  (14.0, 11.0)  (7.0, 11.0) |
C     =        |  (9.0,  5.0)  (11.0, 10.0)  (5.0, 12.0) |
               | (13.0,  8.0)   (9.0, 10.0)  (2.0, 10.0) |
               |      .              .             .     |
               └                                         ┘
```

# SGESUB, DGESUB, CGESUB, and ZGESUB—Matrix Subtraction for General Matrices or Their Transposes

These subroutines can perform any one of the following matrix subtractions, using matrices ***A*** and ***B*** or their transposes, and matrix ***C***:

$\boldsymbol{C} \leftarrow \boldsymbol{A}-\boldsymbol{B}$
$\boldsymbol{C} \leftarrow \boldsymbol{A}^{T}-\boldsymbol{B}$
$\boldsymbol{C} \leftarrow \boldsymbol{A}-\boldsymbol{B}^{T}$
$\boldsymbol{C} \leftarrow \boldsymbol{A}^{T}-\boldsymbol{B}^{T}$

*Table 73. Data Types*

| ***A***, ***B***, ***C*** | **Subroutine** |
|---|---|
| Short-precision real | SGESUB |
| Long-precision real | DGESUB |
| Short-precision complex | CGESUB |
| Long-precision complex | ZGESUB |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SGESUB \| DGESUB \| CGESUB \| ZGESUB (*a*, *lda*, *transa*, *b*, *ldb*, *transb*, *c*, *ldc*, *m*, *n*) |
| **C and C++** | sgesub \| dgesub \| cgesub \| zgesub (*a*, *lda*, *transa*, *b*, *ldb*, *transb*, *c*, *ldc*, *m*, *n*); |
| **PL/I** | CALL SGESUB \| DGESUB \| CGESUB \| ZGESUB (*a*, *lda*, *transa*, *b*, *ldb*, *transb*, *c*, *ldc*, *m*, *n*); |

*On Entry*

*a*

is the matrix ***A***, where:

If *transa* = `'N'`, ***A*** is used in the computation, and ***A*** has *m* rows and *n* columns.

If *transa* = `'T'`, $\boldsymbol{A}^{T}$ is used in the computation, and ***A*** has *n* rows and *m* columns.

**Note:** No data should be moved to form $\boldsymbol{A}^{T}$; that is, the matrix ***A*** should always be stored in its untransposed form.

Specified as: a two-dimensional array, containing numbers of the data type indicated in Table 73, where:

If *transa* = `'N'`, its size must be *lda* by (at least) *n*.

If *transa* = `'T'`, its size must be *lda* by (at least) *m*.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; $lda > 0$ and:

If *transa* = `'N'`, $lda \geq m$.

If *transa* = `'T'`, $lda \geq n$.

*transa*

indicates the form of matrix ***A*** to use in the computation, where:

If *transa* = `'N'`, ***A*** is used in the computation.

If *transa* = `'T'`, $\boldsymbol{A}^{T}$ is used in the computation.

Specified as: a single character; *transa* = `'N'` or `'T'`.

*b*

is the matrix ***B***, where:

If *transb* = `'N'`, ***B*** is used in the computation, and ***B*** has *m* rows and *n* columns.

If *transb* = `'T'`, $\boldsymbol{B}^T$ is used in the computation, and ***B*** has *n* rows and *m* columns.

**Note:** No data should be moved to form $\boldsymbol{B}^T$; that is, the matrix ***B*** should always be stored in its untransposed form.

Specified as: a two-dimensional array, containing numbers of the data type indicated in Table 72 on page 381, where:

If *transb* = `'N'`, its size must be *ldb* by (at least) *n*.

If *transb* = `'T'`, its size must be *ldb* by (at least) *m*.

*ldb*

is the leading dimension of the array specified for *b*. Specified as: a fullword integer; $ldb > 0$ and:

If *transb* = `'N'`, $ldb \ge m$.

If *transb* = `'T'`, $ldb \ge n$.

*transb*

indicates the form of matrix ***B*** to use in the computation, where:

If *transb* = `'N'`, ***B*** is used in the computation.

If *transb* = `'T'`, $\boldsymbol{B}^T$ is used in the computation.

Specified as: a single character; *transb* = `'N'` or `'T'`.

*c*

See “On Return.”

*ldc*

is the leading dimension of the array specified for *c*. Specified as: a fullword integer; $ldc > 0$ and $ldc \ge m$.

*m*

is the number of rows in matrix ***C***. Specified as: a fullword integer; $0 \le m \le ldc$.

*n*

is the number of columns in matrix ***C***. Specified as: a fullword integer; $0 \le n$.

*On Return*

*c*

is the *m* by *n* matrix ***C***, containing the results of the computation. Returned as: an *ldc* by (at least) *n* array, containing numbers of the data type indicated in Table 73 on page 388.

***Notes***

1. All subroutines accept lowercase letters for the *transa* and *transb* arguments.
2. Matrix ***C*** must have no common elements with matrices ***A*** or ***B***. However, ***C*** may (exactly) coincide with ***A*** if *transa* = `'N'`, and ***C*** may (exactly) coincide with ***B*** if *transb* = `'N'`. Otherwise, results are unpredictable. See “Concepts” on page 55.

***Function:*** The matrix subtraction is expressed as follows, where $a_{ij}$, $b_{ij}$, and $c_{ij}$ are elements of matrices ***A, B,*** and ***C***, respectively:

$c_{ij} = a_{ij}-b_{ij}$ for $\boldsymbol{C} \leftarrow \boldsymbol{A}-\boldsymbol{B}$
$c_{ij} = a_{ij}-b_{ji}$ for $\boldsymbol{C} \leftarrow \boldsymbol{A}-\boldsymbol{B}^T$
$c_{ij} = a_{ji}-b_{ij}$ for $\boldsymbol{C} \leftarrow \boldsymbol{A}^T-\boldsymbol{B}$
$c_{ij} = a_{ji}-b_{ji}$ for $\boldsymbol{C} \leftarrow \boldsymbol{A}^T-\boldsymbol{B}^T$
for $i = 1, m$ and $j = 1, n$

If *m* or *n* is 0, no computation is performed.

***Special Usage:*** You can compute the transpose $\boldsymbol{C}^T$ of each of the four computations listed under “Function” by using the following matrix identities:

$(\boldsymbol{A}-\boldsymbol{B})^T = \boldsymbol{A}^T-\boldsymbol{B}^T$
$(\boldsymbol{A}-\boldsymbol{B}^T)^T = \boldsymbol{A}^T-\boldsymbol{B}$
$(\boldsymbol{A}^T-\boldsymbol{B})^T = \boldsymbol{A}-\boldsymbol{B}^T$
$(\boldsymbol{A}^T-\boldsymbol{B}^T)^T = \boldsymbol{A}-\boldsymbol{B}$

Be careful that your output array receiving $\boldsymbol{C}^T$ has dimensions large enough to hold the transposed matrix. See “Example 5” on page 392.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. *lda*, *ldb*, *ldc* ≤ 0
2. *m*, *n* < 0
3. *m* > *ldc*
4. *transa*, *transb* ≠ 'N' or 'T'
5. *transa* = 'N' and *m* > *lda*
6. *transa* = 'T' and *n* > *lda*
7. *transb* = 'N' and *m* > *ldb*
8. *transb* = 'T' and *n* > *ldb*

***Example 1:*** This example shows the computation ***C*** ← ***A***–***B***, where ***A*** and ***C*** are contained in larger arrays A and C, respectively, and ***B*** is the same size as array B, in which it is contained.

*Call Statement and Input*

```
             A  LDA TRANSA  B  LDB TRANSB  C  LDC  M   N
             |   |    |     |   |    |     |   |   |   |
CALL SGESUB( A , 6 , 'N'  , B , 4 , 'N'  , C , 5 , 4 , 3 )

        ┌                                ┐
        | 110000.0  120000.0  130000.0 |
        | 210000.0  220000.0  230000.0 |
A    =  | 310000.0  320000.0  330000.0 |
        | 410000.0  420000.0  430000.0 |
        |      .         .         .   |
        |      .         .         .   |
        └                                ┘
```

```
          ┌                     ┐
          │ -11.0  -12.0  -13.0 │
B    =    │ -21.0  -22.0  -23.0 │
          │ -31.0  -32.0  -33.0 │
          │ -41.0  -42.0  -43.0 │
          └                     ┘
```

*Output*

```
          ┌                              ┐
          │ 110011.0  120012.0  130013.0 │
          │ 210021.0  220022.0  230023.0 │
C    =    │ 310031.0  320032.0  330033.0 │
          │ 410041.0  420042.0  430043.0 │
          │       .         .         .  │
          └                              ┘
```

***Example 2:*** This example shows the computation $\boldsymbol{C} \leftarrow \boldsymbol{A}^{T} - \boldsymbol{B}$, where ***A, B,*** and ***C*** are the same size as arrays `A`, `B`, and `C`, in which they are contained.

*Call Statement and Input*

```
             A  LDA TRANSA  B  LDB TRANSB  C  LDC  M   N
             |   |    |     |   |    |     |   |   |   |
CALL SGESUB( A , 3 , 'T'  , B , 4 , 'N'  , C , 4 , 4 , 3 )
```

```
          ┌                                        ┐
          │ 110000.0  120000.0  130000.0  140000.0 │
A    =    │ 210000.0  220000.0  230000.0  240000.0 │
          │ 310000.0  320000.0  330000.0  340000.0 │
          └                                        ┘
```

```
          ┌                     ┐
          │ -11.0  -12.0  -13.0 │
B    =    │ -21.0  -22.0  -23.0 │
          │ -31.0  -32.0  -33.0 │
          │ -41.0  -42.0  -43.0 │
          └                     ┘
```

*Output*

```
          ┌                              ┐
          │ 110011.0  210012.0  310013.0 │
C    =    │ 120021.0  220022.0  320023.0 │
          │ 130031.0  230032.0  330033.0 │
          │ 140041.0  240042.0  340043.0 │
          └                              ┘
```

***Example 3:*** This example shows computation $\boldsymbol{C} \leftarrow \boldsymbol{A} - \boldsymbol{B}^{T}$, where ***A*** is contained in a larger array `A`, and ***B*** and ***C*** are the same size as arrays `B` and `C`, in which they are contained.

*Call Statement and Input*

```
             A  LDA TRANSA  B  LDB TRANSB  C  LDC  M   N
             |   |    |     |   |    |     |   |   |   |
CALL SGESUB( A , 5 , 'N'  , B , 3 , 'T'  , C , 4 , 4 , 3 )
```

```
          ┌                              ┐
          │ 110000.0  120000.0  130000.0 │
          │ 210000.0  220000.0  230000.0 │
A    =    │ 310000.0  320000.0  330000.0 │
          │ 410000.0  420000.0  430000.0 │
          │       .         .         .  │
          └                              ┘
```

```
          ┌                          ┐
          │ -11.0  -12.0  -13.0  -14.0 │
B    =    │ -21.0  -22.0  -23.0  -24.0 │
          │ -31.0  -32.0  -33.0  -34.0 │
          └                          ┘
```

*Output*

```
          ┌                              ┐
          │ 110011.0  120021.0  130031.0 │
C    =    │ 210012.0  220022.0  230032.0 │
          │ 310013.0  320023.0  330033.0 │
          │ 410014.0  420024.0  430034.0 │
          └                              ┘
```

***Example 4:*** This example shows the computation $\boldsymbol{C} \leftarrow \boldsymbol{A}^T - \boldsymbol{B}^T$, where ***A, B,*** and ***C*** are the same size as the arrays A, B, and C, in which they are contained.

*Call Statement and Input*

```
             A  LDA TRANSA  B  LDB TRANSB  C  LDC  M   N
             |   |    |     |   |    |     |   |   |   |
CALL SGESUB( A , 3 , 'T'  , B , 3 , 'T'  , C , 4 , 4 , 3 )
```

```
          ┌                                        ┐
          │ 110000.0  120000.0  130000.0  140000.0 │
A    =    │ 210000.0  220000.0  230000.0  240000.0 │
          │ 310000.0  320000.0  330000.0  340000.0 │
          └                                        ┘
```

```
          ┌                          ┐
          │ -11.0  -12.0  -13.0  -14.0 │
B    =    │ -21.0  -22.0  -23.0  -24.0 │
          │ -31.0  -32.0  -33.0  -34.0 │
          └                          ┘
```

*Output*

```
          ┌                              ┐
          │ 110011.0  210021.0  310031.0 │
C    =    │ 120012.0  220022.0  320032.0 │
          │ 130013.0  230023.0  330033.0 │
          │ 140014.0  240024.0  340034.0 │
          └                              ┘
```

***Example 5:*** This example shows how to produce the transpose of the result of the computation performed in “Example 4,” $\boldsymbol{C} \leftarrow \boldsymbol{A}^T - \boldsymbol{B}^T$, which uses the calling sequence:

```
CALL SGESUB( A , 3 , 'T' , B , 3 , 'T' , C , 4 , 4 , 3 )
```

You instead code a calling sequence for $\boldsymbol{C}^T \leftarrow \boldsymbol{A} - \boldsymbol{B}$, as shown below, where the resulting matrix $\boldsymbol{C}^T$ in the output array CT is the transpose of the matrix in the output array C in Example 4. Note that the array CT has dimensions large enough to receive the transposed matrix. For a description of all the matrix identities, see “Special Usage” on page 390.

*Call Statement and Input*

```
            A  LDA TRANSA  B  LDB TRANSB  C    LDC  M   N
            |   |    |     |   |    |     |     |   |   |
CALL SGESUB( A , 3 , 'N'  , B , 3 , 'N'  , CT , 3 , 3 , 4 )
```

```
        ┌                                        ┐
        │ 110000.0  120000.0  130000.0  140000.0 │
A    =  │ 210000.0  220000.0  230000.0  240000.0 │
        │ 310000.0  320000.0  330000.0  340000.0 │
        └                                        ┘
```

```
        ┌                            ┐
        │ -11.0  -12.0  -13.0  -14.0 │
B    =  │ -21.0  -22.0  -23.0  -24.0 │
        │ -31.0  -32.0  -33.0  -34.0 │
        └                            ┘
```

*Output*

```
        ┌                                        ┐
        │ 110011.0  120012.0  130013.0  140014.0 │
CT   =  │ 210021.0  220022.0  230023.0  240024.0 │
        │ 310031.0  320032.0  330033.0  340034.0 │
        └                                        ┘
```

***Example 6:*** This example shows the computation ***C*** ← ***A***–***B***, where ***A***, ***B***, and ***C*** are contained in larger arrays A, B, and C, respectively, and the arrays contain complex data.

*Call Statement and Input*

```
            A  LDA TRANSA  B  LDB TRANSB  C  LDC  M   N
            |   |    |     |   |    |     |   |   |   |
CALL CGESUB( A , 6 , 'N'  , B , 5 , 'N'  , C , 5 , 4 , 3 )
```

```
        ┌                                    ┐
        │ (1.0, 5.0)  (9.0, 2.0)  (1.0, 9.0) │
        │ (2.0, 4.0)  (8.0, 3.0)  (1.0, 8.0) │
A    =  │ (3.0, 3.0)  (7.0, 5.0)  (1.0, 7.0) │
        │ (6.0, 6.0)  (3.0, 6.0)  (1.0, 4.0) │
        │     .           .           .      │
        │     .           .           .      │
        └                                    ┘
```

```
        ┌                                    ┐
        │ (1.0, 8.0)  (2.0, 7.0)  (3.0, 2.0) │
        │ (4.0, 4.0)  (6.0, 8.0)  (6.0, 3.0) │
B    =  │ (6.0, 2.0)  (4.0, 5.0)  (4.0, 5.0) │
        │ (7.0, 2.0)  (6.0, 4.0)  (1.0, 6.0) │
        │     .           .           .      │
        └                                    ┘
```

*Output*

```
                ┌                                            ┐
                |  (0.0, -3.0)   (7.0, -5.0)  (-2.0,  7.0) |
                | (-2.0,  0.0)   (2.0, -5.0)  (-5.0,  5.0) |
C     =         | (-3.0,  1.0)   (3.0,  0.0)  (-3.0,  2.0) |
                | (-1.0,  4.0)  (-3.0,  2.0)   (0.0, -2.0) |
                |       .             .              .     |
                └                                            ┘
```

# SGEMUL, DGEMUL, CGEMUL, and ZGEMUL—Matrix Multiplication for General Matrices, Their Transposes, or Conjugate Transposes

SGEMUL and DGEMUL can perform any one of the following matrix multiplications, using matrices ***A*** and ***B*** or their transposes, and matrix ***C***:

| | |
|---|---|
| $\boldsymbol{C} \leftarrow \boldsymbol{AB}$ | $\boldsymbol{C} \leftarrow \boldsymbol{AB}^{\mathrm{T}}$ |
| $\boldsymbol{C} \leftarrow \boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}$ | $\boldsymbol{C} \leftarrow \boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}$ |

CGEMUL and ZGEMUL can perform any one of the following matrix multiplications, using matrices ***A*** and ***B***, their transposes or their conjugate transposes, and matrix ***C***:

| | | |
|---|---|---|
| $\boldsymbol{C} \leftarrow \boldsymbol{AB}$ | $\boldsymbol{C} \leftarrow \boldsymbol{AB}^{\mathrm{T}}$ | $\boldsymbol{C} \leftarrow \boldsymbol{AB}^{\mathrm{H}}$ |
| $\boldsymbol{C} \leftarrow \boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}$ | $\boldsymbol{C} \leftarrow \boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}$ | $\boldsymbol{C} \leftarrow \boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{H}}$ |
| $\boldsymbol{C} \leftarrow \boldsymbol{A}^{\mathrm{H}}\boldsymbol{B}$ | $\boldsymbol{C} \leftarrow \boldsymbol{A}^{\mathrm{H}}\boldsymbol{B}^{\mathrm{T}}$ | $\boldsymbol{C} \leftarrow \boldsymbol{A}^{\mathrm{H}}\boldsymbol{B}^{\mathrm{H}}$ |

*Table 74. Data Types*

| ***A***, ***B***, ***C*** | **Subroutine** |
|---|---|
| Short-precision real | SGEMUL |
| Long-precision real | DGEMUL |
| Short-precision complex | CGEMUL |
| Long-precision complex | ZGEMUL |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SGEMUL \| DGEMUL \| CGEMUL \| ZGEMUL (*a*, *lda*, *transa*, *b*, *ldb*, *transb*, *c*, *ldc*, *l*, *m*, *n*) |
| **C and C++** | sgemul \| dgemul \| cgemul \| zgemul (*a*, *lda*, *transa*, *b*, *ldb*, *transb*, *c*, *ldc*, *l*, *m*, *n*); |
| **PL/I** | CALL SGEMUL \| DGEMUL \| CGEMUL \| ZGEMUL (*a*, *lda*, *transa*, *b*, *ldb*, *transb*, *c*, *ldc*, *l*, *m*, *n*); |
| **APL2** | SGEMUL \| DGEMUL \| CGEMUL \| ZGEMUL *a lda transa b ldb transb* `'c'` *ldc l m n* |

*On Entry*

*a*

is the matrix ***A***, where:

If *transa* = `'N'`, ***A*** is used in the computation, and ***A*** has *l* rows and *m* columns.

If *transa* = `'T'`, $\boldsymbol{A}^{\mathrm{T}}$ is used in the computation, and ***A*** has *m* rows and *l* columns.

If *transa* = `'C'`, $\boldsymbol{A}^{\mathrm{H}}$ is used in the computation, and ***A*** has *m* rows and *l* columns.

**Note:** No data should be moved to form $\boldsymbol{A}^{\mathrm{T}}$ or $\boldsymbol{A}^{\mathrm{H}}$; that is, the matrix ***A*** should always be stored in its untransposed form.

Specified as: a two-dimensional array, containing numbers of the data type indicated in Table 74, where:

If *transa* = `'N'`, its size must be *lda* by (at least) *m*.

If *transa* = `'T'` or `'C'`, its size must be *lda* by (at least) *l*.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; $lda > 0$ and:

If *transa* = 'N', $lda \geq l$.

If *transa* = 'T' or 'C', $lda \geq m$.

*transa*

indicates the form of matrix $\boldsymbol{A}$ to use in the computation, where:

If *transa* = 'N', $\boldsymbol{A}$ is used in the computation.

If *transa* = 'T', $\boldsymbol{A}^T$ is used in the computation.

If *transa* = 'C', $\boldsymbol{A}^H$ is used in the computation.

Specified as: a single character; *transa* = 'N' or 'T' for SGEMUL and DGEMUL; *transa* = 'N', 'T', or 'C' for CGEMUL and ZGEMUL.

*b*

is the matrix $\boldsymbol{B}$, where:

If *transb* = 'N', $\boldsymbol{B}$ is used in the computation, and $\boldsymbol{B}$ has $m$ rows and $n$ columns.

If *transb* = 'T', $\boldsymbol{B}^T$ is used in the computation, and $\boldsymbol{B}$ has $n$ rows and $m$ columns.

If *transb* = 'C', $\boldsymbol{B}^H$ is used in the computation, and $\boldsymbol{B}$ has $n$ rows and $m$ columns.

**Note:** No data should be moved to form $\boldsymbol{B}^T$ or $\boldsymbol{B}^H$; that is, the matrix $\boldsymbol{B}$ should always be stored in its untransposed form.

Specified as: a two-dimensional array, containing numbers of the data type indicated in Table 74 on page 395, where:

If *transb* = 'N', its size must be *ldb* by (at least) *n*.

If *transb* = 'T' or 'C', its size must be *ldb* by (at least) *m*.

*ldb*

is the leading dimension of the array specified for *b*. Specified as: a fullword integer; $ldb > 0$ and:

If *transb* = 'N', $ldb \geq m$.

If *transb* = 'T' or 'C', $ldb \geq n$.

*transb*

indicates the form of matrix $\boldsymbol{B}$ to use in the computation, where:

If *transb* = 'N', $\boldsymbol{B}$ is used in the computation.

If *transb* = 'T', $\boldsymbol{B}^T$ is used in the computation.

If *transb* = 'C', $\boldsymbol{B}^H$ is used in the computation.

Specified as: a single character; *transb* = 'N' or 'T' for SGEMUL and DGEMUL; *transb* = 'N', 'T', or 'C' for CGEMUL and ZGEMUL.

*c*

See “On Return” on page 397.

*ldc*

is the leading dimension of the array specified for *c*. Specified as: a fullword integer; $ldc > 0$ and $ldc \geq l$.

*l*

is the number of rows in matrix $\boldsymbol{C}$. Specified as: a fullword integer; $0 \leq l \leq ldc$.

*m*

has the following meaning, where:

If *transa* = `'N'`, it is the number of columns in matrix ***A***.

If *transa* = `'T'` or `'C'`, it is the number of rows in matrix ***A***.

In addition:

If *transb* = `'N'`, it is the number of rows in matrix ***B***.

If *transb* = `'T'` or `'C'`, it is the number of columns in matrix ***B***.

Specified as: a fullword integer; $m \geq 0$.

*n*

is the number of columns in matrix ***C***. Specified as: a fullword integer; $n \geq 0$.

*On Return*

*c*

is the *l* by *n* matrix ***C***, containing the results of the computation. Returned as: an *ldc* by (at least) *n* numbers of the data type indicated in Table 74 on page 395.

### Notes

1. All subroutines accept lowercase letters for the *transa* and *transb* arguments.
2. Matrix ***C*** must have no common elements with matrices ***A*** or ***B***; otherwise, results are unpredictable. See “Concepts” on page 55.

***Function:*** The matrix multiplication is expressed as follows, where $a_{ik}$, $b_{kj}$, and $c_{ij}$ are elements of matrices ***A, B,*** and ***C***, respectively:

$$c_{ij} = \sum_{k=1}^{m} a_{ik}\, b_{kj} \quad \text{for } \boldsymbol{C} \leftarrow \boldsymbol{A}\ \boldsymbol{B}$$

$$c_{ij} = \sum_{k=1}^{m} a_{ki}\, b_{kj} \quad \text{for } \boldsymbol{C} \leftarrow \boldsymbol{A}^{\mathrm{T}}\ \boldsymbol{B}$$

$$c_{ij} = \sum_{k=1}^{m} \bar{a}_{ki}\, b_{kj} \quad \text{for } \boldsymbol{C} \leftarrow \boldsymbol{A}^{\mathrm{H}}\ \boldsymbol{B}$$

$$c_{ij} = \sum_{k=1}^{m} a_{ik}\, b_{jk} \quad \text{for } \boldsymbol{C} \leftarrow \boldsymbol{A}\ \boldsymbol{B}^{\mathrm{T}}$$

$$c_{ij} = \sum_{k=1}^{m} a_{ki}\, b_{jk} \quad \text{for } \boldsymbol{C} \leftarrow \boldsymbol{A}^{\mathrm{T}}\ \boldsymbol{B}^{\mathrm{T}}$$

$$c_{ij} = \sum_{k=1}^{m} \bar{a}_{ki}\, b_{jk} \quad \text{for } \boldsymbol{C} \leftarrow \boldsymbol{A}^{\mathrm{H}}\ \boldsymbol{B}^{\mathrm{T}}$$

$$c_{ij} = \sum_{k=1}^{m} a_{ik}\, \bar{b}_{jk} \quad \text{for } \boldsymbol{C} \leftarrow \boldsymbol{A}\ \boldsymbol{B}^{\mathrm{H}}$$

$$c_{ij} = \sum_{k=1}^{m} a_{ki}\, \bar{b}_{jk} \quad \text{for } \boldsymbol{C} \leftarrow \boldsymbol{A}^{\mathrm{T}}\ \boldsymbol{B}^{\mathrm{H}}$$

$$c_{ij} = \sum_{k=1}^{m} \bar{a}_{ki}\, \bar{b}_{jk} \quad \text{for } \boldsymbol{C} \leftarrow \boldsymbol{A}^{\mathrm{H}}\ \boldsymbol{B}^{\mathrm{H}}$$

$$\text{for} \quad i = 1,\ l \ \text{ and } \ j = 1, n$$

See reference [38 ]. If *l* or *n* is 0, no computation is performed. If *l* and *n* are greater than 0, and *m* is 0, an *l* by *n* matrix of zeros is returned.

***Special Usage***

*Equivalence Rules:* By using the following equivalence rules, you can compute the transpose $\boldsymbol{C}^{\mathrm{T}}$ or the conjugate transpose $\boldsymbol{C}^{\mathrm{H}}$ of some of the computations performed by these subroutines:

| Transpose | Conjugate Transpose |
|---|---|
| $(\boldsymbol{AB})^{\mathrm{T}} = \boldsymbol{B}^{\mathrm{T}}\boldsymbol{A}^{\mathrm{T}}$ | $(\boldsymbol{AB})^{\mathrm{H}} = \boldsymbol{B}^{\mathrm{H}}\boldsymbol{A}^{\mathrm{H}}$ |
| $(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{B})^{\mathrm{T}} = \boldsymbol{B}^{\mathrm{T}}\boldsymbol{A}$ | $(\boldsymbol{A}^{\mathrm{H}}\boldsymbol{B})^{\mathrm{H}} = \boldsymbol{B}^{\mathrm{H}}\boldsymbol{A}$ |
| $(\boldsymbol{AB}^{\mathrm{T}})^{\mathrm{T}} = \boldsymbol{BA}^{\mathrm{T}}$ | $(\boldsymbol{AB}^{\mathrm{H}})^{\mathrm{H}} = \boldsymbol{BA}^{\mathrm{H}}$ |
| $(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}})^{\mathrm{T}} = \boldsymbol{BA}$ | $(\boldsymbol{A}^{\mathrm{H}}\boldsymbol{B}^{\mathrm{H}})^{\mathrm{H}} = \boldsymbol{BA}$ |

When coding the calling sequences for these cases, be careful to code your matrix arguments and dimension arguments in the order indicated by the rule. Also, be careful that your output array, receiving $\boldsymbol{C}^{\mathrm{T}}$ or $\boldsymbol{C}^{\mathrm{H}}$, has dimensions large enough to hold the resulting transposed or conjugate transposed matrix. See “Example 2” on page 400 and “Example 4” on page 401.

***Error Conditions***

*Resource Errors:* Unable to allocate internal work area (CGEMUL and ZGEMUL only).

*Computational Errors:* None

*Input-Argument Errors*

1. *lda*, *ldb*, *ldc* ≤ 0
2. *l*, *m*, *n* < 0
3. *l* > *ldc*
4. *transa*, *transb* ≠ `'N'` or `'T'` for SGEMUL and DGEMUL
5. *transa*, *transb* ≠ `'N'`, `'T'`, or `'C'` for CGEMUL and ZGEMUL
6. *transa* = `'N'` and *l* > *lda*
7. *transa* = `'T'` or `'C'` and *m* > *lda*
8. *transb* = `'N'` and *m* > *ldb*
9. *transb* = `'T'` or `'C'` and *n* > *ldb*

***Example 1:*** This example shows the computation ***C*** ← ***AB***, where ***A, B,*** and ***C*** are contained in larger arrays `A`, `B`, and `C`, respectively.

*Call Statement and Input*

```
             A  LDA TRANSA  B  LDB TRANSB  C  LDC  L   M   N
             |   |    |     |   |    |     |   |   |   |   |
CALL SGEMUL( A , 8 , 'N'  , B , 6 , 'N'  , C , 7 , 6 , 5 , 4 )
```

```
        ┌                              ┐
        |  1.0   2.0  -1.0  -1.0   4.0 |
        |  2.0   0.0   1.0   1.0  -1.0 |
        |  1.0  -1.0  -1.0   1.0   2.0 |
A    =  | -3.0   2.0   2.0   2.0   0.0 |
        |  4.0   0.0  -2.0   1.0  -1.0 |
        | -1.0  -1.0   1.0  -3.0   2.0 |
        |   .     .     .     .     .  |
        |   .     .     .     .     .  |
        └                              ┘
```

```
        ┌                        ┐
        |  1.0  -1.0   0.0   2.0 |
        |  2.0   2.0  -1.0  -2.0 |
B    =  |  1.0   0.0  -1.0   1.0 |
        | -3.0  -1.0   1.0  -1.0 |
        |  4.0   2.0  -1.0   1.0 |
        |   .     .     .     .  |
        └                        ┘
```

*Output*

```
        ┌                         ┐
        | 23.0  12.0  -6.0    2.0 |
        | -4.0  -5.0   1.0    3.0 |
        |  3.0   0.0   1.0    4.0 |
C    =  | -3.0   5.0  -2.0  -10.0 |
        | -5.0  -7.0   4.0    4.0 |
        | 15.0   6.0  -5.0    6.0 |
        |   .     .     .      .  |
        └                         ┘
```

***Example 2:*** This example shows how to produce the transpose of the result of the computation performed in “Example 1,” $\boldsymbol{C} \leftarrow \boldsymbol{AB}$, which uses the calling sequence:

```
CALL  SGEMUL (A,8,'N',B,6,'N',C,7,6,5,4)
```

You instead code a calling sequence for $\boldsymbol{C}^T \leftarrow \boldsymbol{B}^T\boldsymbol{A}^T$, as shown below, where the resulting matrix $\boldsymbol{C}^T$ in the output array `CT` is the transpose of the matrix in the output array `C` in Example 1. Note that the array `CT` has dimensions large enough to receive the transposed matrix. For a description of all the matrix identities, see “Special Usage” on page 398.

*Call Statement and Input*

```
             A  LDA TRANSA  B  LDB TRANSB  C   LDC  L   M   N
             |   |    |     |   |    |     |    |   |   |   |
CALL SGEMUL( B , 6 , 'T'  , A , 8 , 'T'  , CT , 5 , 4 , 5 , 6 )
```

```
        ┌                      ┐
        │  1.0  -1.0   0.0   2.0 │
        │  2.0   2.0  -1.0  -2.0 │
B    =  │  1.0   0.0  -1.0   1.0 │
        │ -3.0  -1.0   1.0  -1.0 │
        │  4.0   2.0  -1.0   1.0 │
        │   .     .     .     .  │
        └                      ┘
```

```
        ┌                            ┐
        │  1.0   2.0  -1.0  -1.0   4.0 │
        │  2.0   0.0   1.0   1.0  -1.0 │
        │  1.0  -1.0  -1.0   1.0   2.0 │
A    =  │ -3.0   2.0   2.0   2.0   0.0 │
        │  4.0   0.0  -2.0   1.0  -1.0 │
        │ -1.0  -1.0   1.0  -3.0   2.0 │
        │   .     .     .     .     .  │
        │   .     .     .     .     .  │
        └                            ┘
```

*Output*

```
        ┌                                    ┐
        │ 23.0  -4.0  3.0   -3.0  -5.0  15.0 │
        │ 12.0  -5.0  0.0    5.0  -7.0   6.0 │
CT   =  │ -6.0   1.0  1.0   -2.0   4.0  -5.0 │
        │  2.0   3.0  4.0  -10.0   4.0   6.0 │
        │   .     .    .      .     .     .  │
        └                                    ┘
```

***Example 3:*** This example shows the computation $\boldsymbol{C} \leftarrow \boldsymbol{A}^T\boldsymbol{B}$, where ***A*** and ***C*** are contained in larger arrays `A` and `C`, respectively, and ***B*** is the same size as the

*Call Statement and Input*

```
             A  LDA TRANSA  B  LDB TRANSB  C  LDC  L   M   N
             |   |    |     |   |    |     |   |   |   |   |
CALL SGEMUL( A , 4 , 'T'  , B , 3 , 'N'  , C , 5 , 3 , 3 , 6 )
```

```
              ┌                    ┐
              | 1.0  -3.0   2.0 |
A      =      | 2.0   4.0   0.0 |
              | 1.0  -1.0  -1.0 |
              |  .     .     .  |
              └                    ┘
```

```
              ┌                                       ┐
              | 1.0  -3.0   2.0   2.0  -1.0   2.0 |
B      =      | 2.0   4.0   0.0   0.0   1.0  -2.0 |
              | 1.0  -1.0  -1.0  -1.0  -1.0   1.0 |
              └                                       ┘
```

*Output*

```
              ┌                                        ┐
              | 6.0   4.0   1.0   1.0   0.0   -1.0 |
              | 4.0  26.0  -5.0  -5.0   8.0  -15.0 |
C      =      | 1.0  -5.0   5.0   5.0  -1.0    3.0 |
              |  .     .     .     .     .      .  |
              |  .     .     .     .     .      .  |
              └                                        ┘
```

***Example 4:*** This example shows how to produce the transpose of the result of the computation performed in “Example 3” on page 400, $\boldsymbol{C} \leftarrow \boldsymbol{A}^{T}\boldsymbol{B}$, which uses the calling sequence:

```
CALL  SGEMUL (A,4,'T',B,3,'N',C,5,3,3,6)
```

You instead code the calling sequence for $\boldsymbol{C}^{T} \leftarrow \boldsymbol{B}^{T}\boldsymbol{A}$, as shown below, where the resulting matrix $\boldsymbol{C}^{T}$ in the output array CT is the transpose of the matrix in the output array C in Example 3. Note that the array CT has dimensions large enough to receive the transposed matrix. For a description of all the matrix identities, see “Special Usage” on page 398.

*Call Statement and Input*

```
             A  LDA TRANSA  B  LDB TRANSB  C    LDC  L   M   N
             |   |    |     |   |    |     |     |   |   |   |
CALL SGEMUL( B , 3 , 'T'  , A , 4 , 'N'  , CT , 8 , 6 , 3 , 3 )
```

```
              ┌                                       ┐
              | 1.0  -3.0   2.0   2.0  -1.0   2.0 |
B      =      | 2.0   4.0   0.0   0.0   1.0  -2.0 |
              | 1.0  -1.0  -1.0  -1.0  -1.0   1.0 |
              └                                       ┘
```

```
              ┌                    ┐
              | 1.0  -3.0   2.0 |
A      =      | 2.0   4.0   0.0 |
              | 1.0  -1.0  -1.0 |
              |  .     .     .  |
              └                    ┘
```

*Output*

```
         ┌                    ┐
         │  6.0    4.0   1.0  │
         │  4.0   26.0  -5.0  │
         │  1.0   -5.0   5.0  │
CT    =  │  1.0   -5.0   5.0  │
         │  0.0    8.0  -1.0  │
         │ -1.0  -15.0   3.0  │
         │   .      .     .   │
         │   .      .     .   │
         └                    ┘
```

***Example 5:*** This example shows the computation $\boldsymbol{C} \leftarrow \boldsymbol{A}\boldsymbol{B}^{T}$, where ***A*** and ***C*** are contained in larger arrays `A` and `C`, respectively, and ***B*** is the same size as the array `B` in which it is contained.

*Call Statement and Input*

```
             A  LDA TRANSA  B  LDB TRANSB  C  LDC  L   M   N
             |   |    |     |   |    |     |   |   |   |   |
CALL SGEMUL( A , 4 , 'N'  , B , 3 , 'T'  , C , 5 , 3 , 2 , 3 )
```

```
         ┌           ┐
         │ 1.0  -3.0 │
A     =  │ 2.0   4.0 │
         │ 1.0  -1.0 │
         │  .     .  │
         └           ┘
```

```
         ┌           ┐
         │ 1.0  -3.0 │
B     =  │ 2.0   4.0 │
         │ 1.0  -1.0 │
         └           ┘
```

*Output*

```
         ┌                    ┐
         │  10.0  -10.0   4.0 │
         │ -10.0   20.0  -2.0 │
C     =  │   4.0   -2.0   2.0 │
         │    .      .     .  │
         │    .      .     .  │
         └                    ┘
```

***Example 6:*** This example shows the computation $\boldsymbol{C} \leftarrow \boldsymbol{A}^{T}\boldsymbol{B}^{T}$, where ***A, B,*** and ***C*** are the same size as the arrays `A`, `B`, and `C` in which they are contained. (Based on the dimensions of the matrices, ***A*** is actually a column vector, and ***C*** is actually a row vector.)

*Call Statement and Input*

```
             A  LDA TRANSA  B  LDB TRANSB  C  LDC  L   M   N
             |   |    |     |   |    |     |   |   |   |   |
CALL SGEMUL( A , 3 , 'T'  , B , 3 , 'T'  , C , 1 , 1 , 3 , 3 )
```

```
         ┌     ┐
         │ 1.0 │
A     =  │ 2.0 │
         │ 1.0 │
         └     ┘
         ┌                 ┐
         │ 1.0  -3.0   2.0 │
B    =   │ 2.0   4.0   0.0 │
         │ 1.0  -1.0  -1.0 │
         └                 ┘
```

*Output*

```
         ┌                  ┐
B    =   │ -3.0  10.0  -2.0 │
         └                  ┘
```

***Example 7:*** This example shows the computation $\boldsymbol{C} \leftarrow \boldsymbol{A}^{T}\boldsymbol{B}$ using complex data, where ***A***, ***B***, and ***C*** are contained in larger arrays `A`, `B`, and `C`, respectively.

*Call Statement and Input*

```
             A  LDA TRANSA  B  LDB TRANSB  C  LDC  L   M   N
             |   |    |     |   |    |     |   |   |   |   |
CALL CGEMUL( A , 6 , 'T'  , B , 7 , 'N'  , C , 3 , 2 , 3 , 3 )
```

```
         ┌                        ┐
         │ (1.0, 2.0)  (3.0, 4.0) │
         │ (4.0, 6.0)  (7.0, 1.0) │
A    =   │ (6.0, 3.0)  (2.0, 5.0) │
         │     .           .      │
         │     .           .      │
         │     .           .      │
         └                        ┘
```

```
         ┌                                    ┐
         │ (1.0, 9.0)  (2.0, 6.0)  (5.0, 6.0) │
         │ (2.0, 5.0)  (6.0, 2.0)  (6.0, 4.0) │
         │ (2.0, 6.0)  (5.0, 4.0)  (2.0, 6.0) │
B    =   │     .           .           .      │
         │     .           .           .      │
         │     .           .           .      │
         │     .           .           .      │
         └                                    ┘
```

*Output*

```
         ┌                                               ┐
         │ (-45.0, 85.0)  (20.0, 93.0)  (-13.0, 110.0) │
C    =   │ (-50.0, 90.0)  (12.0, 79.0)    (3.0,  94.0) │
         │       .              .              .        │
         └                                               ┘
```

***Example 8:*** This example shows the computation $\boldsymbol{C} \leftarrow \boldsymbol{A}\boldsymbol{B}^{H}$ using complex data, where ***A*** and ***C*** are contained in larger arrays `A` and `C`, respectively, and ***B*** is the same size as the array `B` in which it is contained.

*Call Statement and Input*

```
             A  LDA TRANSA  B  LDB TRANSB  C  LDC  L   M   N
             |   |    |     |   |    |     |   |   |   |   |
CALL CGEMUL( A , 4 , 'N'  , B , 3 , 'C'  , C , 4 , 3 , 2 , 3 )
```

```
             ┌                       ┐
             | (1.0, 2.0)  (-3.0, 2.0) |
A     =      | (2.0, 6.0)   (4.0, 5.0) |
             | (1.0, 2.0)  (-1.0, 8.0) |
             |     .            .      |
             └                        ┘
```

```
             ┌                        ┐
             | (1.0, 3.0)  (-3.0, 2.0) |
B     =      | (2.0, 5.0)   (4.0, 6.0) |
             | (1.0, 1.0)  (-1.0, 9.0) |
             └                        ┘
```

*Output*

```
             ┌                                             ┐
             | (20.0,  -1.0)  (12.0, 25.0)  (24.0,  26.0) |
C     =      | (18.0, -23.0)  (80.0, -2.0)  (49.0, -37.0) |
             | (26.0, -23.0)  (56.0, 37.0)  (76.0,   2.0) |
             |       .              .              .       |
             └                                             ┘
```

# SGEMMS, DGEMMS, CGEMMS, and ZGEMMS—Matrix Multiplication for General Matrices, Their Transposes, or Conjugate Transposes Using Winograd's Variation of Strassen's Algorithm

These subroutines use Winograd's variation of the Strassen's algorithm to perform the matrix multiplication for both real and complex matrices. SGEMMS and DGEMMS can perform any one of the following matrix multiplications, using matrices ***A*** and ***B*** or their transposes, and matrix ***C***:

$$\boldsymbol{C} \leftarrow \boldsymbol{AB} \qquad \boldsymbol{C} \leftarrow \boldsymbol{AB}^{\mathrm{T}}$$
$$\boldsymbol{C} \leftarrow \boldsymbol{A}^{\mathrm{T}}\boldsymbol{B} \qquad \boldsymbol{C} \leftarrow \boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}$$

CGEMMS and ZGEMMS can perform any one of the following matrix multiplications, using matrices ***A*** and ***B***, their transposes or their conjugate transposes, and matrix ***C***:

$$\boldsymbol{C} \leftarrow \boldsymbol{AB} \qquad \boldsymbol{C} \leftarrow \boldsymbol{AB}^{\mathrm{T}} \qquad \boldsymbol{C} \leftarrow \boldsymbol{AB}^{\mathrm{H}}$$
$$\boldsymbol{C} \leftarrow \boldsymbol{A}^{\mathrm{T}}\boldsymbol{B} \qquad \boldsymbol{C} \leftarrow \boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}} \qquad \boldsymbol{C} \leftarrow \boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{H}}$$
$$\boldsymbol{C} \leftarrow \boldsymbol{A}^{\mathrm{H}}\boldsymbol{B} \qquad \boldsymbol{C} \leftarrow \boldsymbol{A}^{\mathrm{H}}\boldsymbol{B}^{\mathrm{T}} \qquad \boldsymbol{C} \leftarrow \boldsymbol{A}^{\mathrm{H}}\boldsymbol{B}^{\mathrm{H}}$$

*Table 75. Data Types*

| ***A***, ***B***, ***C*** | *aux* | **Subroutine** |
|---|---|---|
| Short-precision real | Short-precision real | SGEMMS |
| Long-precision real | Long-precision real | DGEMMS |
| Short-precision complex | Short-precision real | CGEMMS |
| Long-precision complex | Long-precision real | ZGEMMS |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SGEMMS \| DGEMMS \| CGEMMS \| ZGEMMS (*a*, *lda*, *transa*, *b*, *ldb*, *transb*, *c*, *ldc*, *l*, *m*, *n*, *aux*, *naux*) |
| **C and C++** | sgemms \| dgemms \| cgemms \| zgemms (*a*, *lda*, *transa*, *b*, *ldb*, *transb*, *c*, *ldc*, *l*, *m*, *n*, *aux*, *naux*); |
| **PL/I** | CALL SGEMMS \| DGEMMS \| CGEMMS \| ZGEMMS (*a*, *lda*, *transa*, *b*, *ldb*, *transb*, *c*, *ldc*, *l*, *m*, *n*, *aux*, *naux*); |

*On Entry*

*a*

is the matrix ***A***, where:

If *transa* = `'N'`, ***A*** is used in the computation, and ***A*** has *l* rows and *m* columns.

If *transa* = `'T'`, $\boldsymbol{A}^{\mathrm{T}}$ is used in the computation, and ***A*** has *m* rows and *l* columns.

If *transa* = `'C'`, $\boldsymbol{A}^{\mathrm{H}}$ is used in the computation, and ***A*** has *m* rows and *l* columns.

**Note:** No data should be moved to form $\boldsymbol{A}^{\mathrm{T}}$ or $\boldsymbol{A}^{\mathrm{H}}$; that is, the matrix ***A*** should always be stored in its untransposed form.

Specified as: a two-dimensional array, containing numbers of the data type indicated in Table 75, where:

If *transa* = `'N'`, its size must be *lda* by (at least) *m*.

If *transa* = `'T'` or `'C'`, its size must be *lda* by (at least) *l*.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; *lda* > 0 and:

If *transa* = `'N'`, *lda* ≥ *l*.

If *transa* = `'T'` or `'C'`, *lda* ≥ *m*.

*transa*

indicates the form of matrix ***A*** to use in the computation, where:

If *transa* = `'N'`, ***A*** is used in the computation.

If *transa* = `'T'`, $\boldsymbol{A}^T$ is used in the computation.

If *transa* = `'C'`, $\boldsymbol{A}^H$ is used in the computation.

Specified as: a single character; *transa* = `'N'` or `'T'` for SGEMMS and DGEMMS; *transa* = `'N'`, `'T'`, or `'C'` for CGEMMS and ZGEMMS.

*b*

is the matrix ***B***, where:

If *transb* = `'N'`, ***B*** is used in the computation, and ***B*** has *m* rows and *n* columns.

If *transb* = `'T'`, $\boldsymbol{B}^T$ is used in the computation, and ***B*** has *n* rows and *m* columns.

If *transb* = `'C'`, $\boldsymbol{B}^H$ is used in the computation, and ***B*** has *n* rows and *m* columns.

**Note:** No data should be moved to form $\boldsymbol{B}^T$ or $\boldsymbol{B}^H$; that is, the matrix ***B*** should always be stored in its untransposed form.

Specified as: a two-dimensional array, containing numbers of the data type indicated in Table 75 on page 405, where:

If *transb* = `'N'`, its size must be *ldb* by (at least) *n*.

If *transb* = `'T'` or `'C'`, its size must be *ldb* by (at least) *m*.

*ldb*

is the leading dimension of the array specified for *b*. Specified as: a fullword integer; *ldb* > 0 and:

If *transb* = `'N'`, *ldb* ≥ *m*.

If *transb* = `'T'` or `'C'`, *ldb* ≥ *n*.

*transb*

indicates the form of matrix ***B*** to use in the computation, where:

If *transb* = `'N'`, ***B*** is used in the computation.

If *transb* = `'T'`, $\boldsymbol{B}^T$ is used in the computation.

If *transb* = `'C'`, $\boldsymbol{B}^H$ is used in the computation.

Specified as: a single character; *transb* = `'N'` or `'T'` for SGEMMS and DGEMMS; *transb* = `'N'`, `'T'`, or `'C'` for CGEMMS and ZGEMMS.

*c*

See “On Return” on page 407.

*ldc*

is the leading dimension of the array specified for *c*. Specified as: a fullword integer; *ldc* > 0 and *ldc* ≥ *l*.

*l*

is the number of rows in matrix ***C***. Specified as: a fullword integer; $0 \le l \le ldc$.

*m*

has the following meaning, where:

If *transa* = `'N'`, it is the number of columns in matrix ***A***.

If *transa* = `'T'` or `'C'`, it is the number of rows in matrix ***A***.

In addition:

If *transb* = `'N'`, it is the number of rows in matrix ***B***.

If *transb* = `'T'` or `'C'`, it is the number of columns in matrix ***B***.

Specified as: a fullword integer; $m \ge 0$.

*n*

is the number of columns in matrix ***C***. Specified as: a fullword integer; $n \ge 0$.

*aux*

has the following meaning:

If *naux* = 0 and error 2015 is unrecoverable, *aux* is ignored.

Otherwise, is the storage work area used by this subroutine. Its size is specified by *naux*.

Specified as: an area of storage containing numbers of the data type indicated in Table 75 on page 405.

*naux*

is the size of the work area specified by *aux*—that is, the number of elements in *aux*.

Specified as: a fullword integer, where:

If *naux* = 0 and error 2015 is unrecoverable, SGEMMS, DGEMMS, CGEMMS, and ZGEMMS dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise,

When this subroutine uses Strassen's algorithm:

- For SGEMMS and DGEMMS:

  Use $naux = \max[(n)(l), 0.7m(l+n)]$.

- For CGEMMS and ZGEMMS:

  Use $naux = \max[(n)(l), 0.7m(l+n)]+nb1+nb2$, where:
  If $l \ge n$, then $nb1 \ge (l)(n+20)$ and $nb2 \ge \max[(n)(l), (m)(n+20)]$.
  If $l < n$, then $nb1 \ge (m)(n+20)$ and $nb2 \ge \max[(n)(l), (l)(m+20)]$.

When this subroutine uses the direct method (_GEMUL), use $naux \ge 0$.

**Notes:**

1. In most cases, these formulas provide an overestimate.
2. For an explanation of when this subroutine uses the direct method versus Strassen's algorithm, see “Notes” on page 408.

*On Return*

*c*

is the *l* by *n* matrix ***C***, containing the results of the computation. Returned as: an *ldc* by (at least) *n* array, containing numbers of the data type indicated in Table 75 on page 405.

***Notes***

1. There are two instances when these subroutines use the direct method (_GEMUL), rather than using Strassen's algorithm:
   - When either or both of the input matrices are small
   - For CGEMMS and ZGEMMS, when input matrices ***A*** and ***B*** overlap

   In these instances when the direct method is used, the subroutine does not use auxiliary storage, and you can specify *naux* = 0.
2. For CGEMMS and ZGEMMS, one of the input matrices, ***A*** or ***B***, is rearranged during the computation and restored to its original form on return. Keep this in mind when diagnosing an abnormal termination.
3. All subroutines accept lowercase letters for the *transa* and *transb* arguments.
4. Matrix ***C*** must have no common elements with matrices ***A*** or ***B***; otherwise, results are unpredictable. See “Concepts” on page 55.
5. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see “Using Auxiliary Storage in ESSL” on page 32.

***Function:*** The matrix multiplications performed by these subroutines are functionally equivalent to those performed by SGEMUL, DGEMUL, CGEMUL, and ZGEMUL. For details on the computations performed, see “Function” on page 397.

SGEMMS, DGEMMS, CGEMMS, and ZGEMMS use Winograd's variation of the Strassen's algorithm with minor changes for tuning purposes. (See pages 45 and 46 in reference [11].) The subroutines compute matrix multiplication for both real and complex matrices of large sizes. Complex matrix multiplication uses a special technique, using three real matrix multiplications and five real matrix additions. Each of these three resulting matrix multiplications then uses Strassen's algorithm.

*Strassen's Algorithm:* The steps of Strassen's algorithm can be repeated up to four times by these subroutines, with each step reducing the dimensions of the matrix by a factor of two. The number of steps used by this subroutine depends on the size of the input matrices. Each step reduces the number of operations by about 10% from the normal matrix multiplication. On the other hand, if the matrix is small, a normal matrix multiplication is performed without using the Strassen's algorithm, and no improvement is gained. For details about small matrices, see “Notes.”

*Complex Matrix Multiplication:* The complex multiplication is performed by forming the real and imaginary parts of the input matrices. These subroutines uses three real matrix multiplications and five real matrix additions, instead of the normal four real matrix multiplications and two real matrix additions. Using only three real matrix multiplications allows the subroutine to achieve up to a 25% reduction in matrix operations, which can result in a significant savings in computing time for large matrices.

*Accuracy Considerations:* Strassen's method is not stable for certain row or column scalings of the input matrices ***A*** and ***B***. Therefore, for matrices ***A*** and ***B*** with divergent exponent values Strassen's method may give inaccurate results. For these cases, you should use the _GEMUL or _GEMM subroutines.

***Special Usage:*** The equivalence rules, defined for matrix multiplication of ***A*** and ***B*** in “Special Usage” on page 398, also apply to these subroutines. You should use the equivalence rules when you want to transpose or conjugate transpose the result of the multiplication computation. When coding the calling sequences for these cases, be careful to code your matrix arguments and dimension arguments in the order indicated by the rule. Also, be careful that your output array, receiving $\boldsymbol{C}^T$ or $\boldsymbol{C}^H$, has dimensions large enough to hold the resulting transposed or conjugate transposed matrix. See “Example 2” on page 400 and “Example 4” on page 401.

***Error Conditions***

*Resource Errors:* Error 2015 is unrecoverable, *naux* = 0, and unable to allocate work area.

*Computational Errors:* None

*Input-Argument Errors*

1. *lda*, *ldb*, *ldc* ≤ 0
2. *l*, *m*, *n* < 0
3. *l* > *ldc*
4. *transa*, *transb* ≠ `'N'` or `'T'` for SGEMMS and DGEMMS
5. *transa*, *transb* ≠ `'N'`, `'T'`, or `'C'` for CGEMMS and ZGEMMS
6. *transa* = `'N'` and *l* > *lda*
7. *transa* = `'T'` or `'C'` and *m* > *lda*
8. *transb* = `'N'` and *m* > *ldb*
9. *transb* = `'T'` or `'C'` and *n* > *ldb*
10. Error 2015 is recoverable or *naux*≠0, and *naux* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example 1:*** This example shows the computation ***C*** ← ***AB***, where ***A***, ***B***, and ***C*** are contained in larger arrays `A`, `B`, and `C`, respectively. It shows how to code the calling sequence for SGEMMS, but does not use the Strassen algorithm for doing the computation. The calling sequence is shown below. The input and output, other than auxiliary storage, is the same as in “Example 1” on page 399 for SGEMUL.

*Call Statement and Input*

```
             A  LDA TRANSA  B  LDB TRANSB  C  LDC  L   M   N   AUX  NAUX
             |   |   |      |   |   |      |   |   |   |   |    |    |
CALL SGEMMS( A , 8 , 'N'  , B , 6 , 'N'  , C , 7 , 6 , 5 , 4 , AUX , 0  )
```

***Example 2:*** This example shows the computation ***C*** ← $\boldsymbol{AB}^H$, where ***A*** and ***C*** are contained in larger arrays `A` and `C`, respectively, and ***B*** is the same size as the array `B` in which it is contained. The arrays contain complex data. This example shows how to code the calling sequence for CGEMMS, but does not use the Strassen algorithm for doing the computation. The calling sequence is shown below. The input and output, other than auxiliary storage, is the same as in “Example 8” on page 403 for CGEMUL.

*Call Statement and Input*

```
                A  LDA TRANSA  B  LDB TRANSB  C  LDC  L   M   N   AUX  NAUX
                |   |    |     |   |    |     |   |   |   |   |    |    |
CALL CGEMMS( A , 4 , 'N'  , B , 3 , 'C'  , C , 4 , 3 , 2 , 3 , AUX , 0 )
```

# SGEMM, DGEMM, CGEMM, and ZGEMM—Combined Matrix Multiplication and Addition for General Matrices, Their Transposes, or Conjugate Transposes

SGEMM and DGEMM can perform any one of the following combined matrix computations, using scalars $\alpha$ and $\beta$, matrices ***A*** and ***B*** or their transposes, and matrix ***C***:

| | |
|---|---|
| $\boldsymbol{C} \leftarrow \alpha \boldsymbol{AB}+\beta \boldsymbol{C}$ | $\boldsymbol{C} \leftarrow \alpha \boldsymbol{AB}^{\mathrm{T}}+\beta \boldsymbol{C}$ |
| $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}+\beta \boldsymbol{C}$ | $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}+\beta \boldsymbol{C}$ |

CGEMM and ZGEMM can perform any one of the following combined matrix computations, using scalars $\alpha$ and $\beta$, matrices ***A*** and ***B***, their transposes or their conjugate transposes, and matrix ***C***:

| | | |
|---|---|---|
| $\boldsymbol{C} \leftarrow \alpha \boldsymbol{AB}+\beta \boldsymbol{C}$ | $\boldsymbol{C} \leftarrow \alpha \boldsymbol{AB}^{\mathrm{T}}+\beta \boldsymbol{C}$ | $\boldsymbol{C} \leftarrow \alpha \boldsymbol{AB}^{\mathrm{H}}+\beta \boldsymbol{C}$ |
| $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}+\beta \boldsymbol{C}$ | $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}+\beta \boldsymbol{C}$ | $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{H}}+\beta \boldsymbol{C}$ |
| $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{\mathrm{H}}\boldsymbol{B}+\beta \boldsymbol{C}$ | $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{\mathrm{H}}\boldsymbol{B}^{\mathrm{T}}+\beta \boldsymbol{C}$ | $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{\mathrm{H}}\boldsymbol{B}^{\mathrm{H}}+\beta \boldsymbol{C}$ |

*Table 76. Data Types*

| ***A***, ***B***, ***C***, $\alpha$, $\beta$ | **Subroutine** |
|---|---|
| Short-precision real | SGEMM |
| Long-precision real | DGEMM |
| Short-precision complex | CGEMM |
| Long-precision complex | ZGEMM |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SGEMM \| DGEMM \| CGEMM \| ZGEMM (*transa*, *transb*, *l*, *n*, *m*, *alpha*, *a*, *lda*, *b*, *ldb*, *beta*, *c*, *ldc*) |
| **C and C++** | sgemm \| dgemm \| cgemm \| zgemm (*transa*, *transb*, *l*, *n*, *m*, *alpha*, *a*, *lda*, *b*, *ldb*, *beta*, *c*, *ldc*); |
| **PL/I** | CALL SGEMM \| DGEMM \| CGEMM \| ZGEMM (*transa*, *transb*, *l*, *n*, *m*, *alpha*, *a*, *lda*, *b*, *ldb*, *beta*, *c*, *ldc*); |

*On Entry*

*transa*

indicates the form of matrix ***A*** to use in the computation, where:

If *transa* = `'N'`, ***A*** is used in the computation.

If *transa* = `'T'`, $\boldsymbol{A}^{\mathrm{T}}$ is used in the computation.

If *transa* = `'C'`, $\boldsymbol{A}^{\mathrm{H}}$ is used in the computation.

Specified as: a single character; *transa* = `'N'`, `'T'`, or `'C'`.

*transb*

indicates the form of matrix ***B*** to use in the computation, where:

If *transb* = `'N'`, ***B*** is used in the computation.

If *transb* = `'T'`, $\boldsymbol{B}^{\mathrm{T}}$ is used in the computation.

If *transb* = `'C'`, $\boldsymbol{B}^{\mathrm{H}}$ is used in the computation.

Specified as: a single character; *transb* = `'N'`, `'T'`, or `'C'`.

*l*

is the number of rows in matrix ***C***. Specified as: a fullword integer; 0 ≤ *l* ≤ *ldc*.

*n*

is the number of columns in matrix ***C***. Specified as: a fullword integer; *n* ≥ 0.

*m*

has the following meaning, where:

If *transa* = `'N'`, it is the number of columns in matrix ***A***.

If *transa* = `'T'` or `'C'`, it is the number of rows in matrix ***A***.

In addition:

If *transb* = `'N'`, it is the number of rows in matrix ***B***.

If *transb* = `'T'` or `'C'`, it is the number of columns in matrix ***B***.

Specified as: a fullword integer; *m* ≥ 0.

*alpha*

is the scalar $\alpha$. Specified as: a number of the data type indicated in Table 76 on page 411.

*a*

is the matrix ***A***, where:

If *transa* = `'N'`, ***A*** is used in the computation, and ***A*** has *l* rows and *m* columns.

If *transa* = `'T'`, $\boldsymbol{A}^T$ is used in the computation, and ***A*** has *m* rows and *l* columns.

If *transa* = `'C'`, $\boldsymbol{A}^H$ is used in the computation, and ***A*** has *m* rows and *l* columns.

**Note:** No data should be moved to form $\boldsymbol{A}^T$ or $\boldsymbol{A}^H$; that is, the matrix ***A*** should always be stored in its untransposed form.

Specified as: a two-dimensional array, containing numbers of the data type indicated in Table 76 on page 411, where:

If *transa* = `'N'`, its size must be *lda* by (at least) *m*.

If *transa* = `'T'` or `'C'`, its size must be *lda* by (at least) *l*.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; *lda* > 0 and:

If *transa* = `'N'`, *lda* ≥ *l*.

If *transa* = `'T'` or `'C'`, *lda* ≥ *m*.

*b*

is the matrix ***B***, where:

If *transb* = `'N'`, ***B*** is used in the computation, and ***B*** has *m* rows and *n* columns.

If *transb* = `'T'`, $\boldsymbol{B}^T$ is used in the computation, and ***B*** has *n* rows and *m* columns.

If *transb* = `'C'`, $\boldsymbol{B}^H$ is used in the computation, and ***B*** has *n* rows and *m* columns.

**Note:** No data should be moved to form $\boldsymbol{B}^T$ or $\boldsymbol{B}^H$; that is, the matrix ***B*** should always be stored in its untransposed form.

Specified as: a two-dimensional array, containing numbers of the data type indicated in Table 76 on page 411, where:

If *transb* = 'N', its size must be *ldb* by (at least) *n*.

If *transb* = 'T' or 'C', its size must be *ldb* by (at least) *m*.

*ldb*

is the leading dimension of the array specified for *b*. Specified as: a fullword integer; *ldb* > 0 and:

If *transb* = 'N', *ldb* ≥ *m*.

If *transb* = 'T' or 'C', *ldb* ≥ *n*.

*beta*

is the scalar β. Specified as: a number of the data type indicated in Table 76 on page 411.

*c*

is the *l* by *n* matrix ***C***. Specified as: a two-dimensional array, containing numbers of the data type indicated in Table 76 on page 411.

*ldc*

is the leading dimension of the array specified for *c*. Specified as: a fullword integer; *ldc* > 0 and *ldc* ≥ *l*.

*On Return*

*c*

is the *l* by *n* matrix ***C***, containing the results of the computation. Returned as: an *ldc* by (at least) *n* array, containing numbers of the data type indicated in Table 76 on page 411.

***Notes***

1. All subroutines accept lowercase letters for the *transa* and *transb* arguments.
2. For SGEMM and DGEMM, if you specify 'C' for the *transa* or *transb* argument, it is interpreted as though you specified 'T'.
3. Matrix ***C*** must have no common elements with matrices ***A*** or ***B***; otherwise, results are unpredictable. See “Concepts” on page 55.

***Function:*** The combined matrix addition and multiplication is expressed as follows, where $a_{ik}$, $b_{kj}$, and $c_{ij}$ are elements of matrices ***A, B,*** and ***C***, respectively:

$$c_{ij} = \left( \alpha \sum_{k=1}^{m} a_{ik} \, b_{kj} \right) + \beta c_{ij} \quad \text{for } \boldsymbol{C} \leftarrow \alpha \boldsymbol{A} \, \boldsymbol{B} + \beta \boldsymbol{C}$$

$$c_{ij} = \left( \alpha \sum_{k=1}^{m} a_{ki} \, b_{kj} \right) + \beta c_{ij} \quad \text{for } \boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{\mathrm{T}} \, \boldsymbol{B} + \beta \boldsymbol{C}$$

$$c_{ij} = \left( \alpha \sum_{k=1}^{m} \bar{a}_{ki} \, b_{kj} \right) + \beta c_{ij} \quad \text{for } \boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{\mathrm{H}} \, \boldsymbol{B} + \beta \boldsymbol{C}$$

$$c_{ij} = \left( \alpha \sum_{k=1}^{m} a_{ik} \, b_{jk} \right) + \beta c_{ij} \quad \text{for } \boldsymbol{C} \leftarrow \alpha \boldsymbol{A} \, \boldsymbol{B}^{\mathrm{T}} + \beta \boldsymbol{C}$$

$$c_{ij} = \left( \alpha \sum_{k=1}^{m} a_{ki} \, b_{jk} \right) + \beta c_{ij} \quad \text{for } \boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{\mathrm{T}} \, \boldsymbol{B}^{\mathrm{T}} + \beta \boldsymbol{C}$$

$$c_{ij} = \left( \alpha \sum_{k=1}^{m} \bar{a}_{ki} \, b_{jk} \right) + \beta c_{ij} \quad \text{for } \boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{\mathrm{H}} \, \boldsymbol{B}^{\mathrm{T}} + \beta \boldsymbol{C}$$

$$c_{ij} = \left( \alpha \sum_{k=1}^{m} a_{ik} \, \bar{b}_{jk} \right) + \beta c_{ij} \quad \text{for } \boldsymbol{C} \leftarrow \alpha \boldsymbol{A} \, \boldsymbol{B}^{\mathrm{H}} + \beta \boldsymbol{C}$$

$$c_{ij} = \left( \alpha \sum_{k=1}^{m} a_{ki} \, \bar{b}_{jk} \right) + \beta c_{ij} \quad \text{for } \boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{\mathrm{T}} \, \boldsymbol{B}^{\mathrm{H}} + \beta \boldsymbol{C}$$

$$c_{ij} = \left( \alpha \sum_{k=1}^{m} \bar{a}_{ki} \, \bar{b}_{jk} \right) + \beta c_{ij} \quad \text{for } \boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{\mathrm{H}} \, \boldsymbol{B}^{\mathrm{H}} + \beta \boldsymbol{C}$$

$$\text{for} \quad i = 1,\ l \ \text{ and } \ j = 1, n$$

See references [32 ] and [38 ]. In the following three cases, no computation is performed:

- *l* is 0.
- *n* is 0.
- $\beta$ is 1 and $\alpha$ is 0.

Assuming the above conditions do not exist, if $\beta \neq 1$ and *m* is 0, then $\beta$***C*** is returned.

***Special Usage***

*Equivalence Rules:* The equivalence rules, defined for matrix multiplication of ***A*** and ***B*** in “Special Usage” on page 398, also apply to the matrix multiplication part of the computation performed by this subroutine. You should use the equivalent rules when you want to transpose or conjugate transpose the multiplication part of the computation. When coding the calling sequences for these cases, be careful to code your matrix arguments and dimension arguments in the order indicated by the rule. Also, be careful that your input and output array C has dimensions large enough to hold the resulting matrix. See “Example 4” on page 417.

### Error Conditions

*Resource Errors:* Unable to allocate internal work area (CGEMM and ZGEMM only).

*Computational Errors:* None

*Input-Argument Errors*

1. *lda*, *ldb*, *ldc* ≤ 0
2. *l*, *m*, *n* < 0
3. *l* > *ldc*
4. *transa*, *transb* ≠ 'N', 'T', or 'C'
5. *transa* = 'N' and *l* > *lda*
6. *transa* = 'T' or 'C' and *m* > *lda*
7. *transb* = 'N' and *m* > *ldb*
8. *transb* = 'T' or 'C' and *n* > *ldb*

***Example 1:*** This example shows the computation $\boldsymbol{C} \leftarrow \alpha \boldsymbol{AB}+\beta \boldsymbol{C}$, where ***A***, ***B***, and ***C*** are contained in larger arrays A, B, and C, respectively.

*Call Statement and Input*

```
                      TRANSA TRANSB  L   N   M  ALPHA  A  LDA  B  LDB  BETA  C  LDC
                        |      |     |   |   |    |    |   |   |   |     |   |   |
CALL SGEMM( 'N'  , 'N'  , 6 , 4 , 5 , 1.0 , A , 8 , B , 6 , 2.0 , C , 7 )
```

```
        ┌                         ┐
        |  1.0   2.0  -1.0  -1.0   4.0 |
        |  2.0   0.0   1.0   1.0  -1.0 |
        |  1.0  -1.0  -1.0   1.0   2.0 |
A    =  | -3.0   2.0   2.0   2.0   0.0 |
        |  4.0   0.0  -2.0   1.0  -1.0 |
        | -1.0  -1.0   1.0  -3.0   2.0 |
        |   .     .     .     .     .  |
        |   .     .     .     .     .  |
        └                         ┘
```

```
        ┌                    ┐
        |  1.0  -1.0   0.0   2.0 |
        |  2.0   2.0  -1.0  -2.0 |
B    =  |  1.0   0.0  -1.0   1.0 |
        | -3.0  -1.0   1.0  -1.0 |
        |  4.0   2.0  -1.0   1.0 |
        |   .     .     .     .  |
        └                    ┘
```

```
        ┌                 ┐
        | 0.5  0.5  0.5  0.5 |
        | 0.5  0.5  0.5  0.5 |
        | 0.5  0.5  0.5  0.5 |
C    =  | 0.5  0.5  0.5  0.5 |
        | 0.5  0.5  0.5  0.5 |
        | 0.5  0.5  0.5  0.5 |
        |  .    .    .    .  |
        └                 ┘
```

*Output*

```
        ┌                          ┐
        │ 24.0  13.0  -5.0   3.0 │
        │ -3.0  -4.0   2.0   4.0 │
        │  4.0   1.0   2.0   5.0 │
C    =  │ -2.0   6.0  -1.0  -9.0 │
        │ -4.0  -6.0   5.0   5.0 │
        │ 16.0   7.0  -4.0   7.0 │
        │   .     .     .     .  │
        └                          ┘
```

***Example 2:*** This example shows the computation ***C*** ← α***AB***$^T$+β***C***, where ***A*** and ***C*** are contained in larger arrays `A` and `C`, respectively, and ***B*** is the same size as array `B` in which it is contained.

*Call Statement and Input*

```
          TRANSA TRANSB  L   N   M  ALPHA  A  LDA  B  LDB  BETA  C  LDC
            |      |     |   |   |    |    |   |   |   |    |    |   |
CALL SGEMM( 'N'  , 'T'  , 3 , 3 , 2 , 1.0 , A , 4 , B , 3 , 2.0 , C , 5 )
```

```
        ┌            ┐
        │ 1.0  -3.0 │
A    =  │ 2.0   4.0 │
        │ 1.0  -1.0 │
        │  .     .  │
        └            ┘
```

```
        ┌            ┐
        │ 1.0  -3.0 │
B    =  │ 2.0   4.0 │
        │ 1.0  -1.0 │
        └            ┘
```

```
        ┌                ┐
        │ 0.5  0.5  0.5 │
        │ 0.5  0.5  0.5 │
C    =  │ 0.5  0.5  0.5 │
        │  .    .    .  │
        │  .    .    .  │
        └                ┘
```

*Output*

```
        ┌                    ┐
        │ 11.0  -9.0   5.0 │
        │ -9.0  21.0  -1.0 │
C    =  │  5.0  -1.0   3.0 │
        │   .     .     .  │
        │   .     .     .  │
        └                    ┘
```

***Example 3:*** This example shows the computation ***C*** ← α***AB***+β***C*** using complex data, where ***A***, ***B***, and ***C*** are contained in larger arrays, `A`, `B`, and `C`, respectively.

*Call Statement and Input*

```
          TRANSA TRANSB  L   N   M   ALPHA   A  LDA  B  LDB  BETA   C  LDC
            |      |     |   |   |     |     |   |   |   |    |     |   |
CALL CGEMM( 'N'  , 'N'  , 6 , 2 , 3 , ALPHA , A , 8 , B , 4 , BETA , C , 8 )
```

```
ALPHA    =  (1.0, 0.0)
BETA     =  (2.0, 0.0)

            ┌                                         ┐
            | (1.0, 5.0)  (9.0, 2.0)  (1.0, 9.0) |
            | (2.0, 4.0)  (8.0, 3.0)  (1.0, 8.0) |
            | (3.0, 3.0)  (7.0, 5.0)  (1.0, 7.0) |
A     =     | (4.0, 2.0)  (4.0, 7.0)  (1.0, 5.0) |
            | (5.0, 1.0)  (5.0, 1.0)  (1.0, 6.0) |
            | (6.0, 6.0)  (3.0, 6.0)  (1.0, 4.0) |
            |     .           .           .      |
            |     .           .           .      |
            └                                         ┘

            ┌                         ┐
            | (1.0, 8.0)  (2.0, 7.0) |
B     =     | (4.0, 4.0)  (6.0, 8.0) |
            | (6.0, 2.0)  (4.0, 5.0) |
            |     .           .      |
            └                         ┘

            ┌                         ┐
            | (0.5, 0.0)  (0.5, 0.0) |
            | (0.5, 0.0)  (0.5, 0.0) |
            | (0.5, 0.0)  (0.5, 0.0) |
C     =     | (0.5, 0.0)  (0.5, 0.0) |
            | (0.5, 0.0)  (0.5, 0.0) |
            | (0.5, 0.0)  (0.5, 0.0) |
            |     .           .      |
            |     .           .      |
            └                         ┘
```

*Output*

```
            ┌                                     ┐
            | (-22.0, 113.0)  (-35.0, 142.0) |
            | (-19.0, 114.0)  (-35.0, 141.0) |
            | (-20.0, 119.0)  (-43.0, 146.0) |
C     =     | (-27.0, 110.0)  (-58.0, 131.0) |
            |   (8.0, 103.0)    (0.0, 112.0) |
            | (-55.0, 116.0)  (-75.0, 135.0) |
            |       .               .        |
            |       .               .        |
            └                                     ┘
```

***Example 4:*** This example shows how to obtain the conjugate transpose of ***AB***$^{\mathrm{H}}$.

$$\left(\boldsymbol{AB}^{\mathrm{H}}\right)^{\mathrm{H}} = \overline{\overline{\boldsymbol{B}}\boldsymbol{A}^{\mathrm{T}}} = \boldsymbol{BA}^{\mathrm{H}}$$

This shows the conjugate transpose of the computation performed in “Example 8” on page 403 for CGEMUL, which uses the following calling sequence:

```
CALL CGEMUL( A , 4 , 'N' , B , 3 , 'C' , C , 4 , 3 , 2 , 3 )
```

You instead code the calling sequence for ***C*** ← $\beta$***C***+$\alpha$***BA***$^{\mathrm{H}}$, where $\beta = 0$, $\alpha = 1$, and the array ***C*** has the correct dimensions to receive the transposed matrix.

Because β is zero, β***C*** = 0. For a description of all the matrix identities, see “Special Usage” on page 398.

*Call Statement and Input*

```
           TRANSA TRANSB  L   N   M   ALPHA   A  LDA  B  LDB  BETA   C  LDC
             |      |     |   |   |     |     |   |   |   |    |     |   |
CALL CGEMM( 'N'  , 'C'  , 3 , 3 , 2 , ALPHA , B , 3 , A , 3 , BETA , C , 4 )
```

```
ALPHA    =  (1.0, 0.0)
BETA     =  (0.0, 0.0)
```

```
        ┌                        ┐
        | (1.0, 3.0)  (-3.0, 2.0) |
B    =  | (2.0, 5.0)   (4.0, 6.0) |
        | (1.0, 1.0)  (-1.0, 9.0) |
        └                        ┘
```

```
        ┌                        ┐
        | (1.0, 2.0)  (-3.0, 2.0) |
A    =  | (2.0, 6.0)   (4.0, 5.0) |
        | (1.0, 2.0)  (-1.0, 8.0) |
        |     .            .      |
        └                        ┘
```

```
C        =(not relevant)
```

*Output*

```
        ┌                                                  ┐
        | (20.0,   1.0)  (18.0, 23.0)  (26.0,  23.0) |
C    =  | (12.0, -25.0)  (80.0,  2.0)  (56.0, -37.0) |
        | (24.0, -26.0)  (49.0, 37.0)  (76.0,  -2.0) |
        |      .              .              .       |
        └                                                  ┘
```

***Example 5:*** This example shows the computation ***C*** ← $\alpha$***A***$^{T}$***B***$^{H}$+β***C*** using complex data, where ***A, B***, and ***C*** are the same size as the arrays `A`, `B`, and `C`, in which they are contained. Because β is zero, β***C*** = 0. (Based on the dimensions of the matrices, ***A*** is actually a column vector, and ***C*** is actually a row vector.)

*Call Statement and Input*

```
           TRANSA TRANSB  L   N   M   ALPHA   A  LDA  B  LDB  BETA   C  LDC
             |      |     |   |   |     |     |   |   |   |    |     |   |
CALL CGEMM( 'T'  , 'C'  , 1 , 3 , 3 , ALPHA , A , 3 , B , 3 , BETA , C , 1 )
```

```
ALPHA    =  (1.0, 1.0)
BETA     =  (0.0, 0.0)
```

```
        ┌            ┐
        | (1.0,  2.0) |
A    =  | (2.0,  5.0) |
        | (1.0,  6.0) |
        └            ┘
```

```
        ┌                                      ┐
        | (1.0, 6.0)  (-3.0, 4.0)   (2.0, 6.0) |
B    =  | (2.0, 3.0)   (4.0, 6.0)   (0.0, 3.0) |
        | (1.0, 3.0)  (-1.0, 6.0)  (-1.0, 9.0) |
        └                                      ┘
```

C =(not relevant)

*Output*

$$A = \begin{bmatrix} (86.0, 44.0) & (58.0, 70.0) & (121.0, 55.0) \end{bmatrix}$$

# SSYMM, DSYMM, CSYMM, ZSYMM, CHEMM, and ZHEMM—Matrix-Matrix Product Where One Matrix is Real or Complex Symmetric or Complex Hermitian

These subroutines compute one of the following matrix-matrix products, using the scalars $\alpha$ and $\beta$ and matrices ***A***, ***B***, and ***C***:

1. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}\boldsymbol{B}+\beta \boldsymbol{C}$
2. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{B}\boldsymbol{A}+\beta \boldsymbol{C}$

where matrix ***A*** is stored in either upper or lower storage mode, and:

- For SSYMM and DSYMM, matrix ***A*** is real symmetric.
- For CSYMM and ZSYMM, matrix ***A*** is complex symmetric.
- For CHEMM and ZHEMM, matrix ***A*** is complex Hermitian.

*Table 77. Data Types*

| $\alpha$, ***A***, ***B***, $\beta$, ***C*** | **Subprogram** |
|---|---|
| Short-precision real | SSYMM |
| Long-precision real | DSYMM |
| Short-precision complex | CSYMM and CHEMM |
| Long-precision complex | ZSYMM and ZHEMM |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SSYMM \| DSYMM \| CSYMM \| ZSYMM \| CHEMM \| ZHEMM (*side*, *uplo*, *m*, *n*, *alpha*, *a*, *lda*, *b*, *ldb*, *beta*, *c*, *ldc*) |
| **C and C++** | ssymm \| dsymm \| csymm \| zsymm \| chemm \| zhemm (*side*, *uplo*, *m*, *n*, *alpha*, *a*, *lda*, *b*, *ldb*, *beta*, *c*, *ldc*); |
| **PL/I** | CALL SSYMM \| DSYMM \| CSYMM \| ZSYMM \| CHEMM \| ZHEMM (*side*, *uplo*, *m*, *n*, *alpha*, *a*, *lda*, *b*, *ldb*, *beta*, *c*, *ldc*); |

*On Entry*

*side*

indicates whether matrix ***A*** is located to the left or right of rectangular matrix ***B*** in the equation used for this computation, where:

If *side* = `'L'`, ***A*** is to the left of ***B***, resulting in equation 1.

If *side* = `'R'`, ***A*** is to the right of ***B***, resulting in equation 2.

Specified as: a single character. It must be `'L'` or `'R'`.

*uplo*

indicates the storage mode used for matrix ***A***, where:

If *uplo* = `'U'`, ***A*** is stored in upper storage mode.

If *uplo* = `'L'`, ***A*** is stored in lower storage mode.

Specified as: a single character. It must be `'U'` or `'L'`.

*m*

is the number of rows in rectangular matrices ***B*** and ***C***, and:

If *side* = `'L'`, *m* is the order of matrix ***A*.**

Specified as: a fullword integer; $0 \le m \le ldb$, $m \le ldc$, and:

If *side* = `'L'`, $m \le lda$.

*n*

is the number of columns in rectangular matrices ***B*** and ***C***, and:

If *side* = `'R'`, *n* is the order of matrix ***A***.

Specified as: a fullword integer; $n \ge 0$ and:

If *side* = `'R'`, $n \le lda$.

*alpha*

is the scalar α. Specified as: a number of the data type indicated in Table 77 on page 420.

*a*

is the real symmetric, complex symmetric, or complex Hermitian matrix ***A***, where:

If *side* = `'L'`, ***A*** is order *m*.

If *side* = `'R'`, ***A*** is order *n*.

and where it is stored as follows:

If *uplo* = `'U'`, ***A*** is stored in upper storage mode.

If *uplo* = `'L'`, ***A*** is stored in lower storage mode.

Specified as: a two-dimensional array, containing numbers of the data type indicated in Table 77 on page 420, where:

If *side* = `'L'`, its size must be *lda* by (at least) *m*.

If *side* = `'R'`, it size must be *lda* by (at least) *n*.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; $lda > 0$ and:

If *side* = `'L'`, $lda \ge m$.

If *side* = `'R'`, $lda \ge n$.

*b*

is the *m* by *n* rectangular matrix ***B***. Specified as: an *ldb* by (at least) *n* array, containing numbers of the data type indicated in Table 77 on page 420.

*ldb*

is the leading dimension of the array specified for *b*. Specified as: a fullword integer; $ldb > 0$ and $ldb \ge m$.

*beta*

is the scalar β. Specified as: a number of the data type indicated in Table 77 on page 420.

*c*

is the *m* by *n* rectangular matrix ***C***. Specified as: an *ldc* by (at least) *n* array, containing numbers of the data type indicated in Table 77 on page 420.

*ldc*

is the leading dimension of the array specified for *c*. Specified as: a fullword integer; $ldc > 0$ and $ldc \ge m$.

*On Return*

*c*

is the *m* by *n* matrix ***C***, containing the results of the computation.

Returned as: an *ldc* by (at least) *n* array, containing numbers of the data type indicated in Table 77 on page 420.

***Notes***

1. These subroutines accept lowercase letters for the *side* and *uplo* arguments.
2. Matrices ***A***, ***B***, and ***C*** must have no common elements; otherwise, results are unpredictable.
3. If matrix ***A*** is upper triangular (*uplo* = `'U'`), these subroutines use only the data in the upper triangular portion of the array. If matrix ***A*** is lower triangular, (*uplo* = `'L'`), these subroutines use only the data in the lower triangular portion of the array. In each case, the other portion of the array is altered during the computation, but restored before exit.
4. The imaginary parts of the diagonal elements of a complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values.
5. For a description of how symmetric matrices are stored in upper and lower storage mode, see "Symmetric Matrix" on page 65. For a description of how complex Hermitian matrices are stored in upper and lower storage mode, see "Complex Hermitian Matrix" on page 70.

***Function:*** These subroutines can perform the following matrix-matrix product computations using matrix ***A***, which is real symmetric for SSYMM and DSYMM, complex symmetric for CSYMM and ZSYMM, and complex Hermitian for CHEMM and ZHEMM:

1. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{AB} + \beta \boldsymbol{C}$
2. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{BA} + \beta \boldsymbol{C}$

   where:

   > $\alpha$ and $\beta$ are scalars.
   > ***A*** is a matrix of the type indicated above, stored in upper or lower storage mode. It is order *m* for equation 1 and order *n* for equation 2.
   > ***B*** and ***C*** are *m* by *n* rectangular matrices.

See references [32 ] and [38 ]. In the following two cases, no computation is performed:

- *n* or *m* is 0.
- $\beta$ is one and $\alpha$ is zero.

***Error Conditions***

*Resource Errors:* Unable to allocate internal work area.

*Computational Errors:* None

*Input-Argument Errors*

1. $m < 0$
2. $m > ldb$
3. $m > ldc$
4. $n < 0$
5. $lda, ldb, ldc \leq 0$
6. *side* ≠ `'L'` or `'R'`
7. *uplo* ≠ `'L'` or `'U'`
8. *side* = `'L'` and $m > lda$
9. *side* = `'R'` and $n > lda$

***Example 1:*** This example shows the computation ***C*** ← α***AB***+β***C***, where ***A*** is a real symmetric matrix of order 5, stored in upper storage mode, and ***B*** and ***C*** are 5 by 4 rectangular matrices.

*Call Statement and Input*

```
            SIDE  UPLO  M   N   ALPHA   A  LDA  B  LDB  BETA  C  LDC
             |     |    |   |     |     |   |   |   |    |    |   |
CALL SSYMM( 'L' , 'U' , 5 , 4 ,  2.0  , A , 8 , B , 6 , 1.0 , C , 5 )
```

```
        ┌                            ┐
        │ 1.0  2.0  -1.0  -1.0   4.0 │
        │  .   0.0   1.0   1.0  -1.0 │
        │  .    .   -1.0   1.0   2.0 │
A    =  │  .    .     .    2.0   0.0 │
        │  .    .     .     .   -1.0 │
        │  .    .     .     .     .  │
        │  .    .     .     .     .  │
        │  .    .     .     .     .  │
        └                            ┘
```

```
        ┌                        ┐
        │  1.0  -1.0   0.0   2.0 │
        │  2.0   2.0  -1.0  -2.0 │
B    =  │  1.0   0.0  -1.0   1.0 │
        │ -3.0  -1.0   1.0  -1.0 │
        │  4.0   2.0  -1.0   1.0 │
        │   .     .     .     .  │
        └                        ┘
```

```
        ┌                        ┐
        │ 23.0  12.0  -6.0   2.0 │
        │ -4.0  -5.0   1.0   3.0 │
C    =  │  5.0   6.0  -1.0  -4.0 │
        │ -4.0   1.0   0.0  -5.0 │
        │  8.0  -4.0  -2.0  13.0 │
        └                        ┘
```

*Output*

```
        ┌                            ┐
        │  69.0   36.0  -18.0    6.0 │
        │ -12.0  -15.0    3.0    9.0 │
C    =  │  15.0   18.0   -3.0  -12.0 │
        │ -12.0    3.0    0.0  -15.0 │
        │   8.0  -20.0   -2.0   35.0 │
        └                            ┘
```

***Example 2:*** This example shows the computation ***C*** ← α***AB***+β***C***, where ***A*** is a real symmetric matrix of order 3, stored in lower storage mode, and ***B*** and ***C*** are 3 by 6 rectangular matrices.

*Call Statement and Input*

```
            SIDE  UPLO  M   N   ALPHA   A  LDA  B  LDB  BETA  C  LDC
             |     |    |   |     |     |   |   |   |    |    |   |
CALL SSYMM( 'L' , 'L' , 3 , 6 ,  2.0  , A , 4 , B , 3 , 2.0 , C , 5 )
```

```
          ┌              ┐
          | 1.0    .    . |
A    =    | 2.0   4.0   . |
          | 1.0 -1.0 -1.0 |
          |  .    .    .  |
          └              ┘
```

```
          ┌                                ┐
          | 1.0 -3.0  2.0  2.0 -1.0   2.0 |
B    =    | 2.0  4.0  0.0  0.0  1.0  -2.0 |
          | 1.0 -1.0 -1.0 -1.0 -1.0   1.0 |
          └                                ┘
```

```
          ┌                               ┐
          |  6.0   4.0  1.0  1.0   0.0  -1.0 |
          |  9.0  11.0  5.0  5.0   3.0  -5.0 |
C    =    | -2.0  -6.0  3.0  3.0  -1.0  32.0 |
          |   .     .    .    .     .     .  |
          |   .     .    .    .     .     .  |
          └                               ┘
```

*Output*

```
          ┌                                     ┐
          | 24.0   16.0   4.0   4.0   0.0   -4.0 |
          | 36.0   44.0  20.0  20.0  12.0  -20.0 |
C    =    | -8.0  -24.0  12.0  12.0  -4.0   12.0 |
          |   .      .     .     .     .      .  |
          |   .      .     .     .     .      .  |
          └                                     ┘
```

***Example 3:*** This example shows the computation $\boldsymbol{C} \leftarrow \alpha \boldsymbol{BA}+\beta \boldsymbol{C}$, where ***A*** is a real symmetric matrix of order 3, stored in upper storage mode, and ***B*** and ***C*** are 2 by 3 rectangular matrices.

*Call Statement and Input*

```
            SIDE  UPLO  M   N   ALPHA   A  LDA  B  LDB  BETA  C  LDC
             |     |    |   |     |     |   |   |   |    |    |   |
CALL SSYMM( 'R' , 'U' , 2 , 3 ,  2.0  , A , 4 , B , 3 , 1.0 , C , 5 )
```

```
          ┌              ┐
          | 1.0 -3.0  1.0 |
A    =    |  .   4.0 -1.0 |
          |  .    .   2.0 |
          |  .    .    .  |
          └              ┘
```

```
          ┌              ┐
          | 1.0 -3.0  3.0 |
B    =    | 2.0  4.0 -1.0 |
          |  .    .    .  |
          └              ┘
```

```
          ┌                   ┐
          |  13.0  -18.0  10.0 |
          | -11.0   11.0  -4.0 |
C    =    |   .      .      .  |
          |   .      .      .  |
          |   .      .      .  |
          └                   ┘
```

*Output*

```
         ┌                    ┐
         │  39.0  -54.0   30.0 │
         │ -33.0   33.0  -12.0 │
C    =   │   .      .      .   │
         │   .      .      .   │
         │   .      .      .   │
         └                    ┘
```

***Example 4:*** This example shows the computation ***C*** ← α***BA***+β***C***, where ***A*** is a real symmetric matrix of order 3, stored in lower storage mode, and ***B*** and ***C*** are 3 by 3 square matrices.

*Call Statement and Input*

```
            SIDE  UPLO  M   N   ALPHA   A  LDA  B  LDB  BETA  C  LDC
             |     |    |   |     |     |   |   |   |    |    |   |
CALL SSYMM( 'R' , 'L' , 3 , 3 , -1.0  , A , 3 , B , 3 , 1.0 , C , 3 )
```

```
         ┌                ┐
         │ 1.0    .    .  │
A    =   │ 2.0  10.0   .  │
         │ 1.0  11.0  4.0 │
         └                ┘
```

```
         ┌                 ┐
         │ 1.0  -3.0   2.0 │
B    =   │ 2.0   4.0   0.0 │
         │ 1.0  -1.0  -1.0 │
         └                 ┘
```

```
         ┌                  ┐
         │  1.0   5.0  -9.0 │
C    =   │ -3.0  10.0  -2.0 │
         │ -2.0   8.0   0.0 │
         └                  ┘
```

*Output*

```
         ┌                    ┐
         │   4.0   11.0   15.0 │
C    =   │ -13.0  -34.0  -48.0 │
         │   0.0   27.0   14.0 │
         └                    ┘
```

***Example 5:*** This example shows the computation ***C*** ← α***BA***+β***C***, where ***A*** is a complex symmetric matrix of order 3, stored in upper storage mode, and ***B*** and ***C*** are 2 by 3 rectangular matrices.

*Call Statement and Input*

```
            SIDE  UPLO  M   N   ALPHA   A  LDA  B  LDB  BETA   C  LDC
             |     |    |   |     |     |   |   |   |    |     |   |
CALL CSYMM( 'R' , 'U' , 2 , 3 , ALPHA , A , 4 , B , 3 , BETA , C , 5 )
```

```
ALPHA    =  (2.0, 3.0)

BETA     =  (1.0, 6.0)
```

```
          ┌                                      ┐
          │ (1.0, 5.0)  (-3.0, 2.0)   (1.0, 6.0) │
A    =    │     .        (4.0, 5.0)  (-1.0, 4.0) │
          │     .            .        (2.0, 5.0) │
          │     .            .            .      │
          └                                      ┘
```

```
          ┌                                      ┐
          │ (1.0, 1.0)  (-3.0, 2.0)   (3.0, 3.0) │
B    =    │ (2.0, 6.0)   (4.0, 5.0)  (-1.0, 4.0) │
          │     .            .            .      │
          └                                      ┘
```

```
          ┌                                          ┐
          │  (13.0, 6.0)  (-18.0, 6.0)  (10.0, 7.0)  │
          │ (-11.0, 8.0)   (11.0, 1.0)  (-4.0, 2.0)  │
C    =    │       .             .            .       │
          │       .             .            .       │
          │       .             .            .       │
          └                                          ┘
```

*Output*

```
          ┌                                                          ┐
          │  (-96.0,   72.0)  (-141.0, -226.0)  (-112.0,   38.0)  │
          │ (-230.0, -269.0)  (-133.0,  -23.0)  (-272.0, -198.0)  │
C    =    │        .                  .                 .          │
          │        .                  .                 .          │
          │        .                  .                 .          │
          └                                                          ┘
```

***Example 6:*** This example shows the computation ***C*** ← α***BA***+β***C***, where ***A*** is a complex Hermitian matrix of order 3, stored in lower storage mode, and ***B*** and ***C*** are 3 by 3 square matrices.

**Note:** The imaginary parts of the diagonal elements of a complex Hermitian matrix are assumed to be zero, so you do not have to set these values.

*Call Statement and Input*

```
            SIDE  UPLO  M   N   ALPHA   A  LDA  B  LDB  BETA   C  LDC
             |     |    |   |     |     |   |   |   |     |    |   |
CALL CHEMM( 'R' , 'L' , 2 , 3 , ALPHA , A , 4 , B , 3 , BETA , C , 5 )
```

```
ALPHA    =  (2.0, 3.0)
```

```
BETA     =  (1.0, 6.0)
```

```
          ┌                                       ┐
          │  (1.0,  . )       .           .       │
A    =    │  (3.0, 2.0)  (4.0,  . )       .       │
          │ (-1.0, 6.0)  (1.0, 4.0)  (2.0,  . )   │
          │      .            .           .       │
          └                                       ┘
```

```
          ┌                                      ┐
          │ (1.0, 1.0)  (-3.0, 2.0)   (3.0, 3.0) │
B    =    │ (2.0, 6.0)   (4.0, 5.0)  (-1.0, 4.0) │
          │     .            .            .      │
          └                                      ┘
```

```
                ┌                                         ┐
                |  (13.0, 6.0)  (-18.0, 6.0)  (10.0, 7.0) |
                | (-11.0, 8.0)   (11.0, 1.0)  (-4.0, 2.0) |
C     =         |       .             .            .      |
                |       .             .            .      |
                |       .             .            .      |
                └                                         ┘
```

*Output*

```
                ┌                                                     ┐
                | (-137.0,  17.0)  (-158.0, -102.0)  (-39.0, 141.0) |
                | (-154.0, -77.0)   (-63.0,  186.0)  (159.0, 104.0) |
C     =         |        .                 .                .       |
                |        .                 .                .       |
                |        .                 .                .       |
                └                                                     ┘
```

# STRMM, DTRMM, CTRMM, and ZTRMM—Triangular Matrix-Matrix Product

STRMM and DTRMM compute one of the following matrix-matrix products, using the scalar $\alpha$, rectangular matrix $\boldsymbol{B}$, and triangular matrix $\boldsymbol{A}$ or its transpose:

1. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{AB}$
2. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}$
3. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{BA}$
4. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{BA}^{\mathrm{T}}$

CTRMM and ZTRMM compute one of the following matrix-matrix products, using the scalar $\alpha$, rectangular matrix $\boldsymbol{B}$, and triangular matrix $\boldsymbol{A}$, its transpose, or its conjugate transpose:

1. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{AB}$
2. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}$
3. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{BA}$
4. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{BA}^{\mathrm{T}}$
5. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{A}^{\mathrm{H}}\boldsymbol{B}$
6. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{BA}^{\mathrm{H}}$

*Table 78. Data Types*

| ***A***, ***B***, $\alpha$ | **Subroutine** |
|---|---|
| Short-precision real | STRMM |
| Long-precision real | DTRMM |
| Short-precision complex | CTRMM |
| Long-precision complex | ZTRMM |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL STRMM \| DTRMM \| CTRMM \| ZTRMM (*side*, *uplo*, *transa*, *diag*, *m*, *n*, *alpha*, *a*, *lda*, *b*, *ldb*) |
| **C and C++** | strmm \| dtrmm \| ctrmm \| ztrmm (*side*, *uplo*, *transa*, *diag*, *m*, *n*, *alpha*, *a*, *lda*, *b*, *ldb*); |
| **PL/I** | CALL STRMM \| DTRMM \| CTRMM \| ZTRMM (*side*, *uplo*, *transa*, *diag*, *m*, *n*, *alpha*, *a*, *lda*, *b*, *ldb*); |

*On Entry*

*side*

indicates whether the triangular matrix ***A*** is located to the left or right of rectangular matrix ***B*** in the equation used for this computation, where:

If *side* = `'L'`, ***A*** is to the left of ***B*** in the equation, resulting in either equation 1, 2, or 5.

If *side* = `'R'`, ***A*** is to the right of ***B*** in the equation, resulting in either equation 3, 4, or 6.

Specified as: a single character. It must be `'L'` or `'R'`.

*uplo*

indicates whether matrix ***A*** is an upper or lower triangular matrix, where:

If *uplo* = `'U'`, ***A*** is an upper triangular matrix.

If *uplo* = `'L'`, ***A*** is a lower triangular matrix.

Specified as: a single character. It must be `'U'` or `'L'`.

*transa*

indicates the form of matrix ***A*** to use in the computation, where:

If *transa* = `'N'`, ***A*** is used in the computation, resulting in either equation 1 or 3.

If *transa* = `'T'`, $\boldsymbol{A}^T$ is used in the computation, resulting in either equation 2 or 4.

If *transa* = `'C'`, $\boldsymbol{A}^H$ is used in the computation, resulting in either equation 5 or 6.

Specified as: a single character. It must be `'N'`, `'T'`, or `'C'`.

*diag*

indicates the characteristics of the diagonal of matrix ***A***, where:

If *diag* = `'U'`, ***A*** is a unit triangular matrix.

If *diag* = `'N'`, ***A*** is not a unit triangular matrix.

Specified as: a single character. It must be `'U'` or `'N'`.

*m*

is the number of rows in rectangular matrix ***B***, and:

If *side* = `'L'`, *m* is the order of triangular matrix ***A***.

Specified as: a fullword integer, where:

If *side* = `'L'`, $0 \le m \le lda$ and $m \le ldb$.

If *side* = `'R'`, $0 \le m \le ldb$.

*n*

is the number of columns in rectangular matrix ***B***, and:

If *side* = `'R'`, *n* is the order of triangular matrix ***A***.

Specified as: a fullword integer; $n \ge 0$ and:

If *side* = `'R'`, $n \le lda$.

*alpha*

is the scalar $\alpha$. Specified as: a number of the data type indicated in Table 78 on page 428.

*a*

is the triangular matrix ***A***, of which only the upper or lower triangular portion is used, where:

If *side* = `'L'`, ***A*** is order *m*.

If *side* = `'R'`, ***A*** is order *n*.

**Note:** No data should be moved to form $\boldsymbol{A}^T$ or $\boldsymbol{A}^H$; that is, the matrix ***A*** should always be stored in its untransposed form.

Specified as: a two-dimensional array, containing numbers of the data type indicated in Table 78 on page 428, where:

If *side* = `'L'`, its size must be *lda* by (at least) *m*.

If *side* = `'R'`, it size must be *lda* by (at least) *n*.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; $lda > 0$ and:

If *side* = `'L'`, $lda \ge m$.

If *side* = `'R'`, $lda \ge n$.

*b*

is the *m* by *n* rectangular matrix ***B***. Specified as: an *ldb* by (at least) *n* array, containing numbers of the data type indicated in Table 78 on page 428.

*ldb*
is the leading dimension of the array specified for *b*. Specified as: a fullword integer; *ldb* > 0 and *ldb* ≥ *m*.

*On Return*

*b*
is the *m* by *n* matrix ***B***, containing the results of the computation. Returned as: an *ldb* by (at least) *n* array, containing numbers of the data type indicated in Table 78 on page 428.

***Notes***

1. These subroutines accept lowercase letters for the *side*, *uplo*, *transa*, and *diag* arguments.
2. For STRMM and DTRMM, if you specify `'C'` for the *transa* argument, it is interpreted as though you specified `'T'`.
3. Matrices ***A*** and ***B*** must have no common elements; otherwise, results are unpredictable.
4. ESSL assumes certain values in your array for parts of a triangular matrix. As a result, you do not have to set these values. For unit triangular matrices, the elements of the diagonal are assumed to be 1.0 for real matrices and (1.0, 0.0) for complex matrices. When using upper- or lower-triangular storage, the unreferenced elements in the lower and upper triangular part, respectively, are assumed to be zero.
5. For a description of triangular matrices and how they are stored, see "Triangular Matrix" on page 73.

***Function:*** These subroutines can perform the following matrix-matrix product computations, using the triangular matrix ***A***, its transpose, or its conjugate transpose, where ***A*** can be either upper- or lower-triangular:

1. $\boldsymbol{B} \leftarrow \alpha \boldsymbol{AB}$
2. $\boldsymbol{B} \leftarrow \alpha \boldsymbol{A}^{T}\boldsymbol{B}$
3. $\boldsymbol{B} \leftarrow \alpha \boldsymbol{A}^{H}\boldsymbol{B}$ (for CTRMM and ZTRMM only)

   where:

   $\alpha$ is a scalar.
   ***A*** is a triangular matrix of order *m*.
   ***B*** is an *m* by *n* rectangular matrix.

4. $\boldsymbol{B} \leftarrow \alpha \boldsymbol{BA}$
5. $\boldsymbol{B} \leftarrow \alpha \boldsymbol{BA}^{T}$
6. $\boldsymbol{B} \leftarrow \alpha \boldsymbol{BA}^{H}$ (for CTRMM and ZTRMM only)

   where:

   $\alpha$ is a scalar.
   ***A*** is a triangular matrix of order *n*.
   ***B*** is an *m* by *n* rectangular matrix.

See references [32 ] and [38 ]. If *n* or *m* is 0, no computation is performed.

***Error Conditions***

*Resource Errors:* Unable to allocate internal work area.

*Computational Errors:* None

*Input-Argument Errors*

1. $m < 0$
2. $n < 0$
3. *lda*, *ldb* ≤ 0
4. *side* ≠ 'L' or 'R'
5. *uplo* ≠ 'L' or 'U'
6. *transa* ≠ 'T', 'N', or 'C'
7. *diag* ≠ 'N' or 'U'
8. *side* = 'L' and *m* > *lda*
9. *m* > *ldb*
10. *side* = 'R' and *n* > *lda*

***Example 1:*** This example shows the computation $\boldsymbol{B} \leftarrow \alpha\boldsymbol{AB}$, where ***A*** is a 5 by 5 upper triangular matrix that is not unit triangular, and ***B*** is a 5 by 3 rectangular matrix.

*Call Statement and Input*

```
               SIDE  UPLO TRANSA  DIAG  M    N   ALPHA   A  LDA  B  LDB
                |     |     |      |    |    |     |     |   |   |   |
CALL STRMM( 'L' , 'U' , 'N'  , 'N' , 5 , 3 ,  1.0  , A , 7 , B , 6 )
```

```
          ┌                              ┐
          | 3.0  -1.0   2.0   2.0   1.0 |
          |  .   -2.0   4.0  -1.0   3.0 |
          |  .     .   -3.0   0.0   2.0 |
A     =   |  .     .     .    4.0  -2.0 |
          |  .     .     .     .    1.0 |
          |  .     .     .     .     .  |
          |  .     .     .     .     .  |
          └                              ┘
```

```
          ┌                ┐
          |  2.0  3.0   1.0 |
          |  5.0  5.0   4.0 |
B     =   |  0.0  1.0   2.0 |
          |  3.0  1.0  -3.0 |
          | -1.0  2.0   1.0 |
          |   .    .     .  |
          └                ┘
```

*Output*

```
          ┌                     ┐
          |   6.0  10.0   -2.0 |
          | -16.0  -1.0    6.0 |
B     =   |  -2.0   1.0   -4.0 |
          |  14.0   0.0  -14.0 |
          |  -1.0   2.0    1.0 |
          |    .     .      .  |
          └                     ┘
```

***Example 2:*** This example shows the computation $\boldsymbol{B} \leftarrow \alpha \boldsymbol{A}^T\boldsymbol{B}$, where ***A*** is a 5 by 5 upper triangular matrix that is not unit triangular, and ***B*** is a 5 by 4 rectangular matrix.

*Call Statement and Input*

```
            SIDE  UPLO TRANSA  DIAG  M   N    ALPHA  A  LDA  B  LDB
             |     |     |      |    |   |     |     |   |   |   |
CALL STRMM( 'L' , 'U' , 'T'  , 'N' , 5 , 4 ,  1.0  , A , 7 , B , 6 )
```

```
        ┌                              ┐
        | -1.0  -4.0  -2.0   2.0   3.0 |
        |   .   -2.0   2.0   2.0   2.0 |
        |   .     .   -3.0  -1.0   4.0 |
A    =  |   .     .     .    1.0   0.0 |
        |   .     .     .     .   -2.0 |
        |   .     .     .     .     .  |
        |   .     .     .     .     .  |
        └                              ┘
```

```
        ┌                        ┐
        |  1.0   2.0   3.0   4.0 |
        |  3.0   3.0  -1.0   2.0 |
B    =  | -2.0  -1.0   0.0   1.0 |
        |  4.0   4.0  -3.0  -3.0 |
        |  2.0   2.0   2.0   2.0 |
        |   .     .     .     .  |
        └                        ┘
```

*Output*

```
        ┌                          ┐
        | -1.0  -2.0   -3.0   -4.0 |
        |  2.0  -2.0  -14.0  -12.0 |
B    =  | 10.0   5.0   -8.0   -7.0 |
        | 14.0  15.0    1.0    8.0 |
        | -3.0   4.0    3.0   16.0 |
        |   .     .      .      .  |
        └                          ┘
```

***Example 3:*** This example shows the computation $\boldsymbol{B} \leftarrow \alpha \boldsymbol{B}\boldsymbol{A}$, where ***A*** is a 5 by 5 lower triangular matrix that is not unit triangular, and ***B*** is a 3 by 5 rectangular matrix.

*Call Statement and Input*

```
            SIDE  UPLO TRANSA  DIAG  M   N    ALPHA  A  LDA  B  LDB
             |     |     |      |    |   |     |     |   |   |   |
CALL STRMM( 'R' , 'L' , 'N'  , 'N' , 3 , 5 ,  1.0  , A , 7 , B , 4 )
```

```
        ┌                            ┐
        | 2.0   .     .     .     .  |
        | 2.0  3.0    .     .     .  |
        | 2.0  1.0   1.0    .     .  |
A    =  | 0.0  3.0   0.0  -2.0    .  |
        | 2.0  4.0  -1.0   2.0  -1.0 |
        |  .    .     .     .     .  |
        |  .    .     .     .     .  |
        └                            ┘
        ┌                              ┐
        │  3.0   4.0  -1.0  -1.0  -1.0 │
B     = │  2.0   1.0  -1.0   0.0   3.0 │
        │ -2.0  -1.0  -3.0   0.0   2.0 │
        │   .     .     .     .     .  │
        └                              ┘
```

*Output*

```
        ┌                            ┐
        │ 10.0   4.0   0.0  0.0   1.0 │
B     = │ 10.0  14.0  -4.0  6.0  -3.0 │
        │ -8.0   2.0  -5.0  4.0  -2.0 │
        │   .     .     .    .     .  │
        └                            ┘
```

***Example 4:*** This example shows the computation ***B*** ← α***BA***, where ***A*** is a 6 by 6 upper triangular matrix that is unit triangular, and ***B*** is a 1 by 6 rectangular matrix.

**Note:** Because matrix ***A*** is unit triangular, the diagonal elements are not referenced. ESSL assumes a value of 1.0 for the diagonal elements.

*Call Statement and Input*

```
            SIDE  UPLO TRANSA  DIAG  M   N    ALPHA  A  LDA  B  LDB
             |     |     |      |    |   |     |     |   |   |   |
CALL STRMM( 'R' , 'U' , 'N'  , 'U' , 1 , 6 ,  1.0  , A , 7 , B , 2 )
```

```
        ┌                                 ┐
        │ .  2.0  -3.0  1.0   2.0   4.0 │
        │ .   .    0.0  1.0   1.0  -2.0 │
        │ .   .     .   4.0  -1.0   1.0 │
A     = │ .   .     .    .    0.0  -1.0 │
        │ .   .     .    .     .    2.0 │
        │ .   .     .    .     .     .  │
        │ .   .     .    .     .     .  │
        └                                 ┘
```

```
        ┌                               ┐
B     = │ 1.0  2.0  1.0  3.0  -1.0  -2.0 │
        │  .    .    .    .     .     .  │
        └                               ┘
```

*Output*

```
        ┌                                ┐
B     = │ 1.0  4.0  -2.0  10.0  2.0  -6.0 │
        │  .    .     .     .    .     .  │
        └                                ┘
```

***Example 5:*** This example shows the computation ***B*** ← α***A***$^H$***B***, where ***A*** is a 5 by 5 upper triangular matrix that is not unit triangular, and ***B*** is a 5 by 1 rectangular matrix.

*Call Statement and Input*

```
            SIDE  UPLO TRANSA  DIAG  M   N    ALPHA   A  LDA  B  LDB
             |     |     |      |    |   |      |     |   |   |   |
CALL CTRMM( 'L' , 'U' , 'C'  , 'N' , 5 , 1 ,  ALPHA , A , 6 , B , 6 )

ALPHA    =  (1.0, 0.0)
```

```
          ┌                                                                     ┐
          | (-4.0, 1.0)  (4.0, -3.0)  (-1.0,  3.0)  (0.0,  0.0)  (-1.0,  0.0) |
          |      .      (-2.0,  0.0)  (-3.0, -1.0) (-2.0, -1.0)   (4.0,  3.0) |
A     =   |      .            .       (-5.0,  3.0) (-3.0, -3.0)  (-5.0, -5.0) |
          |      .            .             .       (4.0, -4.0)   (2.0,  0.0) |
          |      .            .             .            .        (2.0, -1.0) |
          |      .            .             .            .             .      |
          └                                                                     ┘
```

```
          ┌              ┐
          |  (3.0, 4.0)  |
          | (-4.0, 2.0)  |
B     =   | (-5.0, 0.0)  |
          |  (1.0, 3.0)  |
          |  (3.0, 1.0)  |
          |      .       |
          └              ┘
```

*Output*

```
          ┌                ┐
          | (-8.0, -19.0)  |
          |  (8.0,  21.0)  |
B     =   | (44.0,  -8.0)  |
          | (13.0,  -7.0)  |
          | (19.0,   2.0)  |
          |      .         |
          └                ┘
```

## SSYRK, DSYRK, CSYRK, ZSYRK, CHERK, and ZHERK—Rank-K Update of a Real or Complex Symmetric or a Complex Hermitian Matrix

These subroutines compute one of the following rank-k updates, where matrix ***C*** is stored in either upper or lower storage mode. SSYRK, DSYRK, CSYRK, and ZSYRK use the scalars $\alpha$ and $\beta$, real or complex matrix ***A*** or its transpose, and real or complex symmetric matrix ***C*** to compute:

1. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}\boldsymbol{A}^{T}+\beta \boldsymbol{C}$
2. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{T}\boldsymbol{A}+\beta \boldsymbol{C}$

CHERK and ZHERK use the scalars $\alpha$ and $\beta$, complex matrix ***A*** or its complex conjugate transpose, and complex Hermitian matrix ***C*** to compute:

3. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}\boldsymbol{A}^{H}+\beta \boldsymbol{C}$
4. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{H}\boldsymbol{A}+\beta \boldsymbol{C}$

*Table 79. Data Types*

| ***A***, ***C*** | $\alpha$, $\beta$ | **Subprogram** |
|---|---|---|
| Short-precision real | Short-precision real | SSYRK |
| Long-precision real | Long-precision real | DSYRK |
| Short-precision complex | Short-precision complex | CSYRK |
| Long-precision complex | Long-precision complex | ZSYRK |
| Short-precision complex | Short-precision real | CHERK |
| Long-precision complex | Long-precision real | ZHERK |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SSYRK \| DSYRK \| CSYRK \| ZSYRK \| CHERK \| ZHERK (*uplo*, *trans*, *n*, *k*, *alpha*, *a*, *lda*, *beta*, *c*, *ldc*) |
| **C and C++** | ssyrk \| dsyrk \| csyrk \| zsyrk \| cherk \| zherk (*uplo*, *trans*, *n*, *k*, *alpha*, *a*, *lda*, *beta*, *c*, *ldc*); |
| **PL/I** | CALL SSYRK \| DSYRK \| CSYRK \| ZSYRK \| CHERK \| ZHERK (*uplo*, *trans*, *n*, *k*, *alpha*, *a*, *lda*, *beta*, *c*, *ldc*); |

*On Entry*

*uplo*

indicates the storage mode used for matrix ***C***, where:

If *uplo* = `'U'`, ***C*** is stored in upper storage mode.

If *uplo* = `'L'`, ***C*** is stored in lower storage mode.

Specified as: a single character. It must be `'U'` or `'L'`.

*trans*

indicates the form of matrix ***A*** to use in the computation, where:

If *trans* = `'N'`, ***A*** is used, resulting in equation 1 or 3.

If *trans* = `'T'`, $\boldsymbol{A}^{T}$ is used, resulting in equation 2.

If *trans* = `'C'`, $\boldsymbol{A}^{H}$ is used, resulting in equation 4.

Specified as: a single character, where:

For SSYRK and DSYRK, it must be `'N'`, `'T'`, or `'C'`.

For CSYRK and ZSYRK, it must be 'N' or 'T'.

For CHERK and ZHERK, it must be 'N' or 'C'.

*n*

is the order of matrix ***C***. Specified as: a fullword integer; $0 \leq n \leq ldc$ and:

If *trans* = 'N', then $n \leq lda$.

*k*

has the following meaning, where:

If *trans* = 'N', it is the number of columns in matrix ***A***.

If *trans* = 'T' or 'C', it is the number of rows in matrix ***A***.

Specified as: a fullword integer; $k \geq 0$ and:

If *trans* = 'T' or 'C', then $k \leq lda$.

*alpha*

is the scalar $\alpha$. Specified as: a number of the data type indicated in Table 79 on page 435.

*a*

is the rectangular matrix ***A***, where:

If *trans* = 'N', ***A*** is *n* by *k*.

If *trans* = 'T' or 'C', ***A*** is *k* by *n*.

**Note:** No data should be moved to form $\boldsymbol{A}^T$ or $\boldsymbol{A}^H$; that is, the matrix ***A*** should always be stored in its untransposed form.

Specified as: a two-dimensional array, containing numbers of the data type indicated in Table 79 on page 435, where:

If *trans* = 'N', its size must be *lda* by (at least) *k*.

If *trans* = 'T' or 'C', its size must be *lda* by (at least) *n*.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; $lda > 0$ and:

If *trans* = 'N', $lda \geq n$.

If *trans* = 'T' or 'C', $lda \geq k$.

*beta*

is the scalar $\beta$. Specified as: a number of the data type indicated in Table 79 on page 435.

*c*

is matrix ***C*** of order *n*, which is real symmetric, complex symmetric, or complex Hermitian, where:

If *uplo* = 'U', ***C*** is stored in upper storage mode.

If *uplo* = 'L', ***C*** is stored in lower storage mode.

Specified as: an *ldc* by (at least) *n* array, containing numbers of the data type indicated in Table 79 on page 435.

*ldc*

is the leading dimension of the array specified for *c*. Specified as: a fullword integer; $ldc > 0$ and $ldc \geq n$.

*On Return*

*c*

is matrix ***C*** of order *n*, which is real symmetric, complex symmetric, or complex Hermitian, containing the results of the computation, where:

If *uplo* = `'U'`, ***C*** is stored in upper storage mode.

If *uplo* = `'L'`, ***C*** is stored in lower storage mode.

Returned as: an *ldc* by (at least) *n* array, containing numbers of the data type indicated in Table 79 on page 435.

***Notes***

1. These subroutines accept lowercase letters for the *uplo* and *trans* arguments.
2. For SSYRK and DSYRK, if you specify `'C'` for the *trans* argument, it is interpreted as though you specified `'T'`.
3. Matrices ***A*** and ***C*** must have no common elements; otherwise, results are unpredictable.
4. The imaginary parts of the diagonal elements of a complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values. On output, they are set to zero, except when $\beta$ is one and $\alpha$ or *k* is zero, in which case no computation is performed.
5. For a description of how symmetric matrices are stored in upper and lower storage mode, see “Symmetric Matrix” on page 65. For a description of how complex Hermitian matrices are stored in upper and lower storage mode, see “Complex Hermitian Matrix” on page 70.

***Function:*** These subroutines can perform the following rank-k updates. For SSYRK and DSYRK, matrix ***C*** is real symmetric. For CSYRK and ZSYRK, matrix ***C*** is complex symmetric. They perform:

1. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}\boldsymbol{A}^{\mathrm{T}}+\beta \boldsymbol{C}$
2. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}+\beta \boldsymbol{C}$

For CHERK and ZHERK, matrix ***C*** is complex Hermitian. They perform:

3. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}\boldsymbol{A}^{\mathrm{H}}+\beta \boldsymbol{C}$
4. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{\mathrm{H}}\boldsymbol{A}+\beta \boldsymbol{C}$

where:

$\alpha$ and $\beta$ are scalars.
***A*** is a rectangular matrix, which is *n* by *k* for equations 1 and 3, and is *k* by *n* for equations 2 and 4.
***C*** is a matrix of order *n* of the type indicated above, stored in upper or lower storage mode.

See references [32 ] and [38 ]. In the following two cases, no computation is performed:

- *n* is 0.
- $\beta$ is one, and $\alpha$ is zero or *k* is zero.

Assuming the above conditions do not exist, if $\beta$ is not one, and $\alpha$ is zero or *k* is zero, then $\beta$***C*** is returned.

***Error Conditions***

*Resource Errors:* Unable to allocate internal work area.

*Computational Errors:* None

*Input-Argument Errors*

1. *lda*, *ldc* ≤ 0
2. *ldc* < *n*
3. *k*, *n* < 0
4. *uplo* ≠ 'U' or 'L'
5. *trans* ≠ 'N', 'T', or 'C' for SSYRK and DSYRK
6. *trans* ≠ 'N' or 'T' for CSYRK and ZSYRK
7. *trans* ≠ 'N' or 'C' for CHERK and ZHERK
8. *trans* = 'N' and *lda* < *n*
9. *trans* = 'T' or 'C' and *lda* < *k*

***Example 1:*** This example shows the computation $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}\boldsymbol{A}^{T}+\beta\boldsymbol{C}$, where ***A*** is an 8 by 2 real rectangular matrix, and ***C*** is a real symmetric matrix of order 8, stored in upper storage mode.

*Call Statement and Input*

```
            UPLO TRANS   N   K   ALPHA   A  LDA  BETA  C   LDC
             |    |      |   |     |     |   |    |    |    |
CALL SSYRK( 'U' , 'N' ,  8 , 2 ,  1.0  , A , 9 , 1.0 , C , 10 )
```

```
        ┌           ┐
        | 0.0   8.0 |
        | 1.0   9.0 |
        | 2.0  10.0 |
        | 3.0  11.0 |
A    =  | 4.0  12.0 |
        | 5.0  13.0 |
        | 6.0  14.0 |
        | 7.0  15.0 |
        |  .     .  |
        └           ┘
```

```
        ┌                                                 ┐
        | 0.0  1.0  3.0  6.0  10.0  15.0  21.0  28.0 |
        |  .   2.0  4.0  7.0  11.0  16.0  22.0  29.0 |
        |  .    .   5.0  8.0  12.0  17.0  23.0  30.0 |
        |  .    .    .   9.0  13.0  18.0  24.0  31.0 |
C    =  |  .    .    .    .   14.0  19.0  25.0  32.0 |
        |  .    .    .    .     .   20.0  26.0  33.0 |
        |  .    .    .    .     .     .   27.0  34.0 |
        |  .    .    .    .     .     .     .   35.0 |
        |  .    .    .    .     .     .     .     .  |
        |  .    .    .    .     .     .     .     .  |
        └                                                 ┘
```

*Output*

```
          ┌                                                       ┐
          │ 64.0  73.0   83.0   94.0  106.0  119.0  133.0  148.0 │
          │   .   84.0   96.0  109.0  123.0  138.0  154.0  171.0 │
          │   .     .   109.0  124.0  140.0  157.0  175.0  194.0 │
          │   .     .      .   139.0  157.0  176.0  196.0  217.0 │
C    =    │   .     .      .      .   174.0  195.0  217.0  240.0 │
          │   .     .      .      .      .   214.0  238.0  263.0 │
          │   .     .      .      .      .      .   259.0  286.0 │
          │   .     .      .      .      .      .      .   309.0 │
          │   .     .      .      .      .      .      .      .  │
          │   .     .      .      .      .      .      .      .  │
          └                                                       ┘
```

***Example 2:*** This example shows the computation $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{T}\boldsymbol{A}+\beta \boldsymbol{C}$, where ***A*** is a 3 by 8 real rectangular matrix, and ***C*** is a real symmetric matrix of order 8, stored in lower storage mode.

*Call Statement and Input*

```
            UPLO TRANS   N   K   ALPHA   A  LDA  BETA  C  LDC
              |     |    |   |     |     |   |    |    |   |
CALL SSYRK( 'L' , 'T' ,  8 , 3 ,  1.0  , A , 4 , 1.0 , C , 8 )
```

```
          ┌                                            ┐
          │ 0.0  3.0  6.0   9.0  12.0  15.0  18.0  21.0 │
A    =    │ 1.0  4.0  7.0  10.0  13.0  16.0  19.0  22.0 │
          │ 2.0  5.0  8.0  11.0  14.0  17.0  20.0  23.0 │
          │  .    .    .     .     .     .     .     .  │
          └                                            ┘
```

```
          ┌                                              ┐
          │ 0.0    .     .     .     .     .     .     .  │
          │ 1.0   8.0    .     .     .     .     .     .  │
          │ 2.0   9.0  15.0    .     .     .     .     .  │
C    =    │ 3.0  10.0  16.0  21.0    .     .     .     .  │
          │ 4.0  11.0  17.0  22.0  26.0    .     .     .  │
          │ 5.0  12.0  18.0  23.0  27.0  30.0    .     .  │
          │ 6.0  13.0  19.0  24.0  28.0  31.0  33.0    .  │
          │ 7.0  14.0  20.0  25.0  29.0  32.0  34.0  35.0 │
          └                                              ┘
```

*Output*

```
          ┌                                                           ┐
          │  5.0     .      .      .      .       .       .       .  │
          │ 15.0   58.0     .      .      .       .       .       .  │
          │ 25.0   95.0  164.0     .      .       .       .       .  │
C    =    │ 35.0  132.0  228.0  323.0     .       .       .       .  │
          │ 45.0  169.0  292.0  414.0  535.0      .       .       .  │
          │ 55.0  206.0  356.0  505.0  653.0   800.0      .       .  │
          │ 65.0  243.0  420.0  596.0  771.0   945.0  1118.0      .  │
          │ 75.0  280.0  484.0  687.0  889.0  1090.0  1290.0  1489.0 │
          └                                                           ┘
```

***Example 3:*** This example shows the computation $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}\boldsymbol{A}^{T}+\beta \boldsymbol{C}$, where ***A*** is a 3 by 5 complex rectangular matrix, and ***C*** is a complex symmetric matrix of order 3, stored in upper storage mode.

*Call Statement and Input*

```
            UPLO TRANS   N   K   ALPHA   A  LDA  BETA   C  LDC
              |    |     |   |     |     |   |    |     |   |
CALL CSYRK( 'U' , 'N' ,  3 , 5 , ALPHA , A , 3 , BETA , C , 4 )

ALPHA    =  (1.0, 1.0)
BETA     =  (1.0, 1.0)

        ┌                                                           ┐
        | (2.0, 0.0) (3.0, 2.0) (4.0, 1.0) (1.0, 7.0) (0.0, 0.0) |
A    =  | (3.0, 3.0) (8.0, 0.0) (2.0, 5.0) (2.0, 4.0) (1.0, 2.0) |
        | (1.0, 3.0) (2.0, 1.0) (6.0, 0.0) (3.0, 2.0) (2.0, 2.0) |
        └                                                           ┘

        ┌                                      ┐
        | (2.0, 1.0) (1.0, 9.0) (4.0, 5.0) |
C    =  |     .      (3.0, 1.0) (6.0, 7.0) |
        |     .          .      (8.0, 1.0) |
        |     .          .          .      |
        └                                      ┘
```

*Output*

```
        ┌                                                ┐
        | (-57.0, 13.0) (-63.0, 79.0) (-24.0,  70.0) |
C    =  |       .       (-28.0, 90.0) (-55.0, 103.0) |
        |       .             .        (13.0,  75.0) |
        |       .             .              .       |
        └                                                ┘
```

***Example 4:*** This example shows the computation $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{H}\boldsymbol{A}+\beta \boldsymbol{C}$, where ***A*** is a 5 by 3 complex rectangular matrix, and ***C*** is a complex Hermitian matrix of order 3, stored in lower storage mode.

**Note:** The imaginary parts of the diagonal elements of a complex Hermitian matrix are assumed to be zero, so you do not have to set these values. On output, they are set to zero.

*Call Statement and Input*

```
            UPLO TRANS   N   K   ALPHA   A  LDA  BETA   C  LDC
              |    |     |   |     |     |   |    |     |   |
CALL CHERK( 'L' , 'C' ,  3 , 5 ,  1.0  , A , 5 , 1.0  , C , 4 )

        ┌                                      ┐
        | (2.0, 0.0) (3.0, 2.0) (4.0, 1.0) |
        | (3.0, 3.0) (8.0, 0.0) (2.0, 5.0) |
A    =  | (1.0, 3.0) (2.0, 1.0) (6.0, 0.0) |
        | (3.0, 3.0) (8.0, 0.0) (2.0, 5.0) |
        | (1.0, 9.0) (3.0, 0.0) (6.0, 7.0) |
        └                                      ┘

        ┌                                          ┐
        | (6.0,  . )       .           .       |
C    =  | (3.0, 4.0)  (10.0,  . )      .       |
        | (9.0, 1.0)  (12.0, 2.0)  (3.0, . ) |
        |     .            .           .       |
        └                                          ┘
```

*Output*

```
            ┌                                                 ┐
            │ (138.0,  0.0)        .                .         │
C     =     │  (65.0, 80.0)  (165.0,   0.0)         .         │
            │ (134.0, 46.0)   (88.0, -88.0)  (199.0, 0.0)     │
            │       .              .                .         │
            └                                                 ┘
```

# SSYR2K, DSYR2K, CSYR2K, ZSYR2K, CHER2K, and ZHER2K—Rank-2K Update of a Real or Complex Symmetric or a Complex Hermitian Matrix

These subroutines compute one of the following rank-2k updates, where matrix ***C*** is stored in upper or lower storage mode. SSYR2K, DSYR2K, CSYR2K, and ZSYR2K use the scalars $\alpha$ and $\beta$, real or complex matrices ***A*** and ***B*** or their transposes, and real or complex symmetric matrix ***C*** to compute:

1. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}\boldsymbol{B}^{\mathrm{T}}+\alpha \boldsymbol{B}\boldsymbol{A}^{\mathrm{T}}+\beta \boldsymbol{C}$
2. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}+\alpha \boldsymbol{B}^{\mathrm{T}}\boldsymbol{A}+\beta \boldsymbol{C}$

CHER2K and ZHER2K use the scalars $\alpha$ and $\beta$, complex matrices ***A*** and ***B*** or their complex conjugate transposes, and complex Hermitian matrix ***C*** to compute:

3. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}\boldsymbol{B}^{\mathrm{H}} + \bar{\alpha} \boldsymbol{B}\boldsymbol{A}^{\mathrm{H}} + \beta \boldsymbol{C}$

4. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{\mathrm{H}}\boldsymbol{B} + \bar{\alpha} \boldsymbol{B}^{\mathrm{H}}\boldsymbol{A} + \beta \boldsymbol{C}$

*Table 80. Data Types*

| ***A***, ***B***, ***C***, $\alpha$ | $\beta$ | **Subprogram** |
|---|---|---|
| Short-precision real | Short-precision real | SSYR2K |
| Long-precision real | Long-precision real | DSYR2K |
| Short-precision complex | Short-precision complex | CSYR2K |
| Long-precision complex | Long-precision complex | ZSYR2K |
| Short-precision complex | Short-precision real | CHER2K |
| Long-precision complex | Long-precision real | ZHER2K |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SSYR2K \| DSYR2K \| CSYR2K \| ZSYR2K \| CHER2K \| ZHER2K (*uplo*, *trans*, *n*, *k*, *alpha*, *a*, *lda*, *b*, *ldb*, *beta*, *c*, *ldc*) |
| **C and C++** | ssyr2k \| dsyr2k \| csyr2k \| zsyr2k \| cher2k \| zher2k (*uplo*, *trans*, *n*, *k*, *alpha*, *a*, *lda*, *b*, *ldb*, *beta*, *c*, *ldc*); |
| **PL/I** | CALL SSYR2K \| DSYR2K \| CSYR2K \| ZSYR2K \| CHER2K \| ZHER2K (*uplo*, *trans*, *n*, *k*, *alpha*, *a*, *lda*, *b*, *ldb*, *beta*, *c*, *ldc*); |

*On Entry*

*uplo*

indicates the storage mode used for matrix ***C***, where:

If *uplo* = `'U'`, ***C*** is stored in upper storage mode.

If *uplo* = `'L'`, ***C*** is stored in lower storage mode.

Specified as: a single character. It must be `'U'` or `'L'`.

*trans*

indicates the form of matrices ***A*** and ***B*** to use in the computation, where:

If *trans* = `'N'`, ***A*** and ***B*** are used, resulting in equation 1 or 3.

If *trans* = 'T', $\boldsymbol{A}^T$ and $\boldsymbol{B}^T$ are used, resulting in equation 2.

If *trans* = 'C', $\boldsymbol{A}^H$ and $\boldsymbol{B}^H$ are used, resulting in equation 4.

Specified as: a single character, where:

For SSYR2K and DSYR2K, it must be 'N', 'T', or 'C'.

For CSYR2K and ZSYR2K, it must be 'N' or 'T'.

For CHER2K and ZHER2K, it must be 'N' or 'C'.

*n*

is the order of matrix ***C***. Specified as: a fullword integer; $0 \le n \le ldc$ and:

If *trans* = 'N', then $n \le lda$ and $n \le ldb$.

*k*

has the following meaning, where:

If *trans* = 'N', it is the number of columns in matrices ***A*** and ***B***.

If *trans* = 'T' or 'C', it is the number of rows in matrices ***A*** and ***B***.

Specified as: a fullword integer; $k \ge 0$ and:

If *trans* = 'T' or 'C', then $k \le lda$ and $k \le ldb$.

*alpha*

is the scalar $\alpha$. Specified as: a number of the data type indicated in Table 80 on page 442.

*a*

is the rectangular matrix ***A***, where:

If *trans* = 'N', ***A*** is *n* by *k*.

If *trans* = 'T' or 'C', ***A*** is *k* by *n*.

**Note:** No data should be moved to form $\boldsymbol{A}^T$ or $\boldsymbol{A}^H$; that is, the matrix ***A*** should always be stored in its untransposed form.

Specified as: a two-dimensional array, containing numbers of the data type indicated in Table 80 on page 442, where:

If *trans* = 'N', its size must be *lda* by (at least) *k*.

If *trans* = 'T' or 'C', its size must be *lda* by (at least) *n*.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; $lda > 0$ and:

If *trans* = 'N', $lda \ge n$.

If *trans* = 'T' or 'C', $lda \ge k$.

*b*

is the rectangular matrix ***B***, where:

If *trans* = 'N', ***B*** is *n* by *k*.

If *trans* = 'T' or 'C', ***B*** is *k* by *n*.

**Note:** No data should be moved to form $\boldsymbol{B}^T$ or $\boldsymbol{B}^H$; that is, the matrix ***B*** should always be stored in its untransposed form.

Specified as: a two-dimensional array, containing numbers of the data type indicated in Table 80 on page 442, where:

If *trans* = 'N', its size must be *ldb* by (at least) *k*.

If *trans* = 'T' or 'C', its size must be *ldb* by (at least) *n*.

*ldb*

is the leading dimension of the array specified for *b*. Specified as: a fullword integer; *ldb* > 0 and:

If *trans* = 'N', *ldb* ≥ *n*.

If *trans* = 'T' or 'C', *ldb* ≥ *k*.

*beta*

is the scalar β. Specified as: a number of the data type indicated in Table 80 on page 442.

*c*

is matrix ***C*** of order *n*, which is real symmetric, complex symmetric, or complex Hermitian, where:

If *uplo* = 'U', ***C*** is stored in upper storage mode.

If *uplo* = 'L', ***C*** is stored in lower storage mode.

Specified as: an *ldc* by (at least) *n* array, containing numbers of the data type indicated in Table 80 on page 442.

*ldc*

is the leading dimension of the array specified for *c*. Specified as: a fullword integer; *ldc* > 0 and *ldc* ≥ *n*.

*On Return*

*c*

is matrix ***C*** of order *n*, which is real symmetric, complex symmetric, or complex Hermitian, containing the results of the computation, where:

If *uplo* = 'U', ***C*** is stored in upper storage mode.

If *uplo* = 'L', ***C*** is stored in lower storage mode.

Returned as: an *ldc* by (at least) *n* array, containing numbers of the data type indicated in Table 80 on page 442.

***Notes***

1. These subroutines accept lowercase letters for the *uplo* and *trans* arguments.
2. For SSYR2K and DSYR2K, if you specify 'C' for the *trans* argument, it is interpreted as though you specified 'T'.
3. Matrices ***A*** and ***B*** must have no common elements with matrix ***C***; otherwise, results are unpredictable.
4. The imaginary parts of the diagonal elements of a complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values. On output, they are set to zero, except when β is one and α or *k* is zero, in which case no computation is performed.
5. For a description of how symmetric matrices are stored in upper and lower storage mode, see “Symmetric Matrix” on page 65. For a description of how complex Hermitian matrices are stored in upper and lower storage mode, see “Complex Hermitian Matrix” on page 70.

***Function:*** These subroutines can perform the following rank-2k updates. For SSYR2K and DSYR2K, matrix ***C*** is real symmetric. For CSYR2K and ZSYR2K, matrix ***C*** is complex symmetric. They perform:

1. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}\boldsymbol{B}^{\mathrm{T}} + \alpha \boldsymbol{B}\boldsymbol{A}^{\mathrm{T}} + \beta \boldsymbol{C}$
2. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{\mathrm{T}}\boldsymbol{B} + \alpha \boldsymbol{B}^{\mathrm{T}}\boldsymbol{A} + \beta \boldsymbol{C}$

For CHER2K and ZHER2K, matrix ***C*** is complex Hermitian. They perform:

3. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}\boldsymbol{B}^{\mathrm{H}} + \overline{\alpha} \boldsymbol{B}\boldsymbol{A}^{\mathrm{H}} + \beta \boldsymbol{C}$
4. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{\mathrm{H}}\boldsymbol{B} + \overline{\alpha} \boldsymbol{B}^{\mathrm{H}}\boldsymbol{A} + \beta \boldsymbol{C}$

where:

$\alpha$ and $\beta$ are scalars.
***A*** and ***B*** are rectangular matrices, which are *n* by *k* for equations 1 and 3, and are *k* by *n* for equations 2 and 4.
***C*** is a matrix of order *n* of the type indicated above, stored in upper or lower storage mode.

See references [32 ], [38 ], and [69 ]. In the following two cases, no computation is performed:

- *n* is 0.
- $\beta$ is one, and $\alpha$ is zero or *k* is zero.

Assuming the above conditions do not exist, if $\beta$ is not one, and $\alpha$ is zero or *k* is zero, then $\beta$***C*** is returned.

***Error Conditions***

*Resource Errors:* Unable to allocate internal work area.

*Computational Errors:* None

*Input-Argument Errors*

1. *lda*, *ldb*, *ldc* ≤ 0
2. *ldc* < *n*
3. *k*, *n* < 0
4. *uplo* ≠ 'U' or 'L'
5. *trans* ≠ 'N', 'T', or 'C' for SSYR2K and DSYR2K
6. *trans* ≠ 'N' or 'T' for CSYR2K and ZSYR2K
7. *trans* ≠ 'N' or 'C' for CHER2K and ZHER2K
8. *trans* = 'N' and *lda* < *n*
9. *trans* = 'T' or 'C' and *lda* < *k*
10. *trans* = 'N' and *ldb* < *n*
11. *trans* = 'T' or 'C' and *ldb* < *k*

***Example 1:*** This example shows the computation $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}\boldsymbol{B}^{\mathrm{T}}+\alpha \boldsymbol{B}\boldsymbol{A}^{\mathrm{T}}+\beta \boldsymbol{C}$, where ***A*** and ***B*** are 8 by 2 real rectangular matrices, and ***C*** is a real symmetric matrix of order 8, stored in upper storage mode.

*Call Statement and Input*

```
               UPLO TRANS   N   K   ALPHA   A  LDA  B  LDB  BETA  C   LDC
                |     |     |   |     |     |   |   |   |    |    |    |
CALL SSYR2K( 'U' , 'N' ,  8 , 2 ,  1.0  , A , 9 , B , 8 , 1.0 , C , 10 )
```

```
                ┌            ┐
                │ 0.0   8.0  │
                │ 1.0   9.0  │
                │ 2.0  10.0  │
                │ 3.0  11.0  │
A     =         │ 4.0  12.0  │
                │ 5.0  13.0  │
                │ 6.0  14.0  │
                │ 7.0  15.0  │
                │  .     .   │
                └            ┘
```

```
                ┌            ┐
                │ 15.0  7.0  │
                │ 14.0  6.0  │
                │ 13.0  5.0  │
B     =         │ 12.0  4.0  │
                │ 11.0  3.0  │
                │ 10.0  2.0  │
                │  9.0  1.0  │
                │  8.0  0.0  │
                └            ┘
```

```
                ┌                                                  ┐
                │ 0.0  1.0  3.0  6.0  10.0  15.0  21.0  28.0 │
                │  .   2.0  4.0  7.0  11.0  16.0  22.0  29.0 │
                │  .    .   5.0  8.0  12.0  17.0  23.0  30.0 │
                │  .    .    .   9.0  13.0  18.0  24.0  31.0 │
C     =         │  .    .    .    .   14.0  19.0  25.0  32.0 │
                │  .    .    .    .     .   20.0  26.0  33.0 │
                │  .    .    .    .     .     .   27.0  34.0 │
                │  .    .    .    .     .     .     .   35.0 │
                │  .    .    .    .     .     .     .     .  │
                │  .    .    .    .     .     .     .     .  │
                └                                                  ┘
```

*Output*

```
                ┌                                                          ┐
                │ 112.0  127.0  143.0  160.0  178.0  197.0  217.0  238.0 │
                │   .    138.0  150.0  163.0  177.0  192.0  208.0  225.0 │
                │   .      .    157.0  166.0  176.0  187.0  199.0  212.0 │
                │   .      .      .    169.0  175.0  182.0  190.0  199.0 │
C     =         │   .      .      .      .    174.0  177.0  181.0  186.0 │
                │   .      .      .      .      .    172.0  172.0  173.0 │
                │   .      .      .      .      .      .    163.0  160.0 │
                │   .      .      .      .      .      .      .    147.0 │
                │   .      .      .      .      .      .      .      .   │
                │   .      .      .      .      .      .      .      .   │
                └                                                          ┘
```

***Example 2:*** This example shows the computation $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{T}\boldsymbol{B}+\alpha \boldsymbol{B}^{T}\boldsymbol{A}+\beta \boldsymbol{C}$, where ***A*** and ***B*** are 3 by 8 real rectangular matrices, and ***C*** is a real symmetric matrix of order 8, stored in lower storage mode.

*Call Statement and Input*

```
             UPLO TRANS   N   K   ALPHA   A  LDA  B  LDB  BETA  C  LDC
              |    |      |   |     |     |   |   |   |    |    |   |
CALL SSYR2K( 'L' , 'T' ,  8 , 3 ,  1.0  , A , 4 , B , 5 , 1.0 , C , 8 )
```

```
                ┌                                              ┐
                │ 0.0  3.0  6.0   9.0  12.0  15.0  18.0  21.0 │
A     =         │ 1.0  4.0  7.0  10.0  13.0  16.0  19.0  22.0 │
                │ 2.0  5.0  8.0  11.0  14.0  17.0  20.0  23.0 │
                │  .    .    .     .     .     .     .     .  │
                └                                              ┘

                ┌                                          ┐
                │ 1.0  2.0  3.0  4.0  5.0  6.0  7.0   8.0 │
                │ 2.0  3.0  4.0  5.0  6.0  7.0  8.0   9.0 │
B     =         │ 3.0  4.0  5.0  6.0  7.0  8.0  9.0  10.0 │
                │  .    .    .    .    .    .    .     .  │
                │  .    .    .    .    .    .    .     .  │
                └                                          ┘

                ┌                                                ┐
                │ 0.0    .     .     .     .     .     .     .  │
                │ 1.0   8.0    .     .     .     .     .     .  │
                │ 2.0   9.0  15.0    .     .     .     .     .  │
C     =         │ 3.0  10.0  16.0  21.0    .     .     .     .  │
                │ 4.0  11.0  17.0  22.0  26.0    .     .     .  │
                │ 5.0  12.0  18.0  23.0  27.0  30.0    .     .  │
                │ 6.0  13.0  19.0  24.0  28.0  31.0  33.0    .  │
                │ 7.0  14.0  20.0  25.0  29.0  32.0  34.0  35.0 │
                └                                                ┘
```

*Output*

```
                ┌                                                    ┐
                │  16.0    .     .     .     .     .      .      .  │
                │  38.0  84.0    .     .     .     .      .      .  │
                │  60.0 124.0 187.0    .     .     .      .      .  │
C     =         │  82.0 164.0 245.0 325.0    .     .      .      .  │
                │ 104.0 204.0 303.0 401.0 498.0    .      .      .  │
                │ 126.0 244.0 361.0 477.0 592.0 706.0     .      .  │
                │ 148.0 284.0 419.0 553.0 686.0 818.0  949.0     .  │
                │ 170.0 324.0 477.0 629.0 780.0 930.0 1079.0 1227.0 │
                └                                                    ┘
```

***Example 3:*** This example shows the computation $\boldsymbol{C} \leftarrow \alpha \boldsymbol{AB}^T+\alpha \boldsymbol{BA}^T+\beta \boldsymbol{C}$, where ***A*** and ***B*** are 3 by 5 complex rectangular matrices, and ***C*** is a complex symmetric matrix of order 3, stored in lower storage mode.

*Call Statement and Input*

```
             UPLO TRANS   N   K   ALPHA   A  LDA  B  LDB  BETA   C  LDC
              |     |     |   |     |     |   |   |   |    |     |   |
CALL CSYR2K( 'L' , 'N' ,  3 , 5 , ALPHA , A , 3 , B , 3 , BETA , C , 4 )

ALPHA    =  (1.0, 1.0)

BETA     =  (1.0, 1.0)

                ┌                                                      ┐
                │ (2.0, 5.0) (3.0, 2.0) (4.0, 1.0) (1.0, 7.0) (0.0, 0.0) │
A     =         │ (3.0, 3.0) (8.0, 5.0) (2.0, 5.0) (2.0, 4.0) (1.0, 2.0) │
                │ (1.0, 3.0) (2.0, 1.0) (6.0, 5.0) (3.0, 2.0) (2.0, 2.0) │
                └                                                      ┘
```

```
          ┌                                                        ┐
          | (1.0, 5.0) (6.0, 2.0) (3.0, 1.0) (2.0, 0.0) (1.0, 0.0) |
B    =    | (2.0, 4.0) (7.0, 5.0) (2.0, 5.0) (2.0, 4.0) (0.0, 0.0) |
          | (3.0, 5.0) (8.0, 1.0) (1.0, 5.0) (1.0, 0.0) (1.0, 1.0) |
          └                                                        ┘
```

```
          ┌                                  ┐
          | (2.0, 3.0)     .          .      |
C    =    | (1.0, 9.0) (3.0, 3.0)     .      |
          | (4.0, 5.0) (6.0, 7.0) (8.0, 3.0) |
          |     .          .          .      |
          └                                  ┘
```

*Output*

```
          ┌                                                     ┐
          | (-101.0, 121.0)        .                .           |
C    =    | (-182.0, 192.0) (-274.0, 248.0)         .           |
          |  (-98.0, 146.0) (-163.0, 205.0) (-151.0, 115.0)     |
          |        .               .                .           |
          └                                                     ┘
```

***Example 4:*** This example shows the computation:

$$\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{\mathrm{H}}\boldsymbol{B} + \overline{\alpha}\boldsymbol{B}^{\mathrm{H}}\boldsymbol{A} + \beta\boldsymbol{C}$$

where ***A*** and ***B*** are 5 by 3 complex rectangular matrices, and ***C*** is a complex Hermitian matrix of order 3, stored in upper storage mode.

**Note:** The imaginary parts of the diagonal elements of a complex Hermitian matrix are assumed to be zero, so you do not have to set these values. On output, they are set to zero.

*Call Statement and Input*

```
             UPLO TRANS   N   K   ALPHA   A  LDA  B  LDB  BETA   C  LDC
              |     |     |   |     |     |   |   |   |     |    |   |
CALL CHER2K( 'U' , 'C' ,  3 , 5 , ALPHA , A , 5 , B , 5 , 1.0  , C , 4 )

ALPHA    =  (1.0, 1.0)
```

```
          ┌                                  ┐
          | (2.0, 0.0) (3.0, 2.0) (4.0, 1.0) |
          | (3.0, 3.0) (8.0, 0.0) (2.0, 5.0) |
A    =    | (1.0, 3.0) (2.0, 1.0) (6.0, 0.0) |
          | (3.0, 3.0) (8.0, 0.0) (2.0, 5.0) |
          | (1.0, 9.0) (3.0, 0.0) (6.0, 7.0) |
          └                                  ┘
```

```
          ┌                                  ┐
          | (4.0, 5.0) (6.0, 7.0) (8.0, 0.0) |
          | (1.0, 9.0) (3.0, 0.0) (6.0, 7.0) |
B    =    | (3.0, 3.0) (8.0, 0.0) (2.0, 5.0) |
          | (1.0, 3.0) (2.0, 1.0) (6.0, 0.0) |
          | (2.0, 0.0) (3.0, 2.0) (4.0, 1.0) |
          └                                  ┘
```

```
        ┌                                    ┐
        | (6.0,  . )  (3.0, 4.0)  (9.0, 1.0) |
C    =  |     .      (10.0,  . ) (12.0, 2.0) |
        |     .           .       (3.0,  . ) |
        |     .           .           .      |
        └                                    ┘
```

*Output*

```
        ┌                                             ┐
        | (102.0, 0.0)  (56.0, -143.0) (244.0, -96.0) |
C    =  |       .      (174.0,    0.0) (238.0,  78.0) |
        |       .            .         (363.0,   0.0) |
        |       .            .               .        |
        └                                             ┘
```

# SGETMI, DGETMI, CGETMI, and ZGETMI—General Matrix Transpose (In-Place)

These subroutines transpose an *n* by *n* matrix ***A*** in place—that is, in matrix ***A***:

$$\boldsymbol{A} \leftarrow \boldsymbol{A}^T$$

*Table 81. Data Types*

| ***A*** | **Subroutine** |
|---|---|
| Short-precision real | SGETMI |
| Long-precision real | DGETMI |
| Short-precision complex | CGETMI |
| Long-precision complex | ZGETMI |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SGETMI \| DGETMI \| CGETMI \| ZGETMI (*a*, *lda*, *n*) |
| **C and C++** | sgetmi \| dgetmi \| cgetmi \| zgetmi (*a*, *lda*, *n*); |
| **PL/I** | CALL SGETMI \| DGETMI \| CGETMI \| ZGETMI (*a*, *lda*, *n*); |

*On Entry*

*a*
is the matrix ***A*** having *n* rows and *n* columns. Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 81.

*lda*
is the leading dimension of the array specified for *a*. Specified as: a fullword integer; $lda > 0$ and $lda \geq n$.

*n*
is the number of rows and columns in matrix ***A***. Specified as: a fullword integer; $n \geq 0$.

*On Return*

*a*
is the *n* by *n* matrix $\boldsymbol{A}^T$, containing the results of the matrix transpose operation Returned as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 81.

### *Notes*

1. To achieve optimal performance in these subroutines, specify an even value for *lda*. An odd value may degrade performance.
2. To achieve optimal performance in CGETMI, align the array specified for *a* on a doubleword boundary.

***Function:*** Matrix ***A*** is transposed in place; that is, the *n* rows and *n* columns in matrix ***A*** are exchanged. For matrix ***A*** with elements $a_{ij}$, where $i, j = 1, n$, the in-place transpose is expressed as $a_{ji} = a_{ij}$ for $i, j = 1, n$.

For the following input matrix ***A***:

$$A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ . & & . \\ . & & . \\ . & & . \\ a_{n1} & \dots & a_{nn} \end{bmatrix}$$

the in-place matrix transpose operation $\boldsymbol{A} \leftarrow \boldsymbol{A}^T$ is expressed as:

$$\begin{bmatrix} a_{11} & \dots & a_{1n} \\ . & & . \\ . & & . \\ . & & . \\ a_{n1} & \dots & a_{nn} \end{bmatrix} \leftarrow \begin{bmatrix} a_{11} & \dots & a_{n1} \\ . & & . \\ . & & . \\ . & & . \\ a_{1n} & \dots & a_{nn} \end{bmatrix}$$

If $n$ is 0, no computation is performed.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. $n < 0$ or $n > lda$
2. $lda \le 0$

***Example:*** This example shows an in-place matrix transpose of matrix ***A*** having 5 rows and 5 columns.

*Call Statement and Input*

```
                A     LDA   N
                |      |    |
CALL SGETMI( A(2,3) , 10 , 5 )
```

```
        ┌                                       ┐
        | .   .   .     .     .     .     .   |
        | .   .  1.0   6.0  11.0  16.0  21.0  |
        | .   .  2.0   7.0  12.0  17.0  22.0  |
        | .   .  3.0   8.0  13.0  18.0  23.0  |
A     = | .   .  4.0   9.0  14.0  19.0  24.0  |
        | .   .  5.0  10.0  15.0  20.0  25.0  |
        | .   .   .     .     .     .     .   |
        | .   .   .     .     .     .     .   |
        | .   .   .     .     .     .     .   |
        | .   .   .     .     .     .     .   |
        └                                       ┘
```

*Output*

```
                 ┌                                       ┐
                 | .   .    .     .     .     .     .    |
                 | .   .   1.0   2.0   3.0   4.0   5.0   |
                 | .   .   6.0   7.0   8.0   9.0  10.0   |
                 | .   .  11.0  12.0  13.0  14.0  15.0   |
A     =          | .   .  16.0  17.0  18.0  19.0  20.0   |
                 | .   .  21.0  22.0  23.0  24.0  25.0   |
                 | .   .    .     .     .     .     .    |
                 | .   .    .     .     .     .     .    |
                 | .   .    .     .     .     .     .    |
                 | .   .    .     .     .     .     .    |
                 └                                       ┘
```

# SGETMO, DGETMO, CGETMO, and ZGETMO—General Matrix Transpose (Out-of-Place)

These subroutines transpose an *m* by *n* matrix ***A*** out of place, returning the result in matrix ***B***:

$$\boldsymbol{B} \leftarrow \boldsymbol{A}^T$$

*Table 82. Data Types*

| ***A***, ***B*** | **Subroutine** |
|---|---|
| Short-precision real | SGETMO |
| Long-precision real | DGETMO |
| Short-precision complex | CGETMO |
| Long-precision complex | ZGETMO |

***Syntax***

| | |
|---|---|
| **Fortran** | CALL SGETMO \| DGETMO \| CGETMO \| ZGETMO (*a*, *lda*, *m*, *n*, *b*, *ldb*) |
| **C and C++** | sgetmo \| dgetmo \| cgetmo \| zgetmo (*a*, *lda*, *m*, *n*, *b*, *ldb*); |
| **PL/I** | CALL SGETMO \| DGETMO \| CGETMO \| ZGETMO (*a*, *lda*, *m*, *n*, *b*, *ldb*); |

*On Entry*

*a*
is the matrix ***A*** having *m* rows and *n* columns. Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 82.

*lda*
is the leading dimension of the array specified for *a*. Specified as: a fullword integer; $lda > 0$ and $lda \ge m$.

*m*
is the number of rows in matrix ***A*** and the number of columns in matrix ***B***. Specified as: a fullword integer; $m \ge 0$.

*n*
is the number of columns in matrix ***A*** and the number of rows in matrix ***B***. Specified as: a fullword integer; $n \ge 0$.

*b*
See "On Return."

*ldb*
is the leading dimension of the array specified for *b*. Specified as: a fullword integer; $ldb > 0$ and $ldb \ge n$.

*On Return*

*b*
is the matrix ***B*** having *n* rows and *m* columns, containing the results of the matrix transpose operation, $\boldsymbol{A}^T$. Returned as: an *ldb* by (at least) *m* array, containing numbers of the data type indicated in Table 82.

***Notes***

1. The matrix ***B*** must have no common elements with matrix ***A***; otherwise, results are unpredictable. See "Concepts" on page 55.

2. To achieve optimal performance in CGETMO, align the arrays specified for *a* and *b* on doubleword boundaries.

***Function:*** Matrix ***A*** is transposed out of place; that is, the *m* rows and *n* columns in matrix ***A*** are stored in *n* rows and *m* columns of matrix ***B***. For matrix ***A*** with elements $a_{ij}$, where $i = 1, m$ and $j = 1, n$, the out-of-place transpose is expressed as $b_{ji} = a_{ij}$ for $i = 1, m$ and $j = 1, n$.

For the following input matrix ***A***:

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{m1} & \dots & a_{mn} \end{bmatrix}$$

the out-of-place matrix transpose operation $\boldsymbol{B} \leftarrow \boldsymbol{A}^{\mathrm{T}}$ is expressed as:

$$\begin{bmatrix} b_{11} & \dots & b_{1m} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ b_{n1} & \dots & b_{nm} \end{bmatrix} \leftarrow \begin{bmatrix} a_{11} & \dots & a_{m1} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{1n} & \dots & a_{mn} \end{bmatrix}$$

If *m* or *n* is 0, no computation is performed.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. $m < 0$ or $m > lda$
2. $n < 0$ or $n > ldb$
3. $lda \le 0$
4. $ldb \le 0$

***Example 1:*** This example shows an out-of-place matrix transpose of matrix ***A***, having 5 rows and 4 columns, with the result going into matrix ***B***.

*Call Statement and Input*

```
               A      LDA   M   N      B      LDB
               |       |    |   |      |       |
CALL SGETMO( A(2,3) , 10 , 5 , 4 , B(2,2) , 6 )
```

```
          ┌                                 ┐
          │ .  .   .     .     .     .   .  │
          │ .  .  1.0   6.0  11.0  16.0  .  │
          │ .  .  2.0   7.0  12.0  17.0  .  │
          │ .  .  3.0   8.0  13.0  18.0  .  │
A     =   │ .  .  4.0   9.0  14.0  19.0  .  │
          │ .  .  5.0  10.0  15.0  20.0  .  │
          │ .  .   .     .     .     .   .  │
          │ .  .   .     .     .     .   .  │
          │ .  .   .     .     .     .   .  │
          │ .  .   .     .     .     .   .  │
          └                                 ┘
```

*Output*

```
          ┌                                       ┐
          │ .    .     .     .     .     .     .  │
          │ .   1.0   2.0   3.0   4.0   5.0    .  │
B     =   │ .   6.0   7.0   8.0   9.0  10.0    .  │
          │ .  11.0  12.0  13.0  14.0  15.0    .  │
          │ .  16.0  17.0  18.0  19.0  20.0    .  │
          │ .    .     .     .     .     .     .  │
          └                                       ┘
```

***Example 2:*** This example uses the same input matrix ***A*** as in Example 1 to show that transposes can be achieved in the same array as long as the input and output data do not overlap. On output, the input data is not overwritten in the array.

*Call Statement and Input*

```
               A     LDA   M   N     B     LDB
               |      |    |   |     |      |
CALL SGETMO( A(2,3) , 10 , 5 , 4 , A(7,1) , 10 )
```

```
          ┌                                            ┐
          │   .     .     .     .     .     .     .   │
          │   .     .    1.0   6.0  11.0  16.0    .   │
          │   .     .    2.0   7.0  12.0  17.0    .   │
          │   .     .    3.0   8.0  13.0  18.0    .   │
A     =   │   .     .    4.0   9.0  14.0  19.0    .   │
          │   .     .    5.0  10.0  15.0  20.0    .   │
          │  1.0   2.0   3.0   4.0   5.0    .     .   │
          │  6.0   7.0   8.0   9.0  10.0    .     .   │
          │ 11.0  12.0  13.0  14.0  15.0    .     .   │
          │ 16.0  17.0  18.0  19.0  20.0    .     .   │
          └                                            ┘
```

# Chapter 10. Linear Algebraic Equations

The linear algebraic equation subroutines, provided in four areas, are described in this chapter.

## Overview of the Linear Algebraic Equation Subroutines

This section describes the subroutines in each of the four linear algebraic equation areas:

- Dense linear algebraic equations (Table 83 )
- Banded linear algebraic equations (Table 84 )
- Sparse linear algebraic equations (Table 85 )
- Linear least squares (Table 86 )

**Note:** Some of the linear algebraic equations were designed in accordance with the Level 2 BLAS, Level 3 BLAS, and LAPACK de facto standard. If these subprograms do not comply with the standard as approved, IBM will consider updating them to do so. If IBM updates these subprograms, the updates could require modifications of the calling application program. For details on the Level 2 and 3 BLAS, see references [32] and [34 ]. For details on the LAPACK routines, see reference [8].

## Dense Linear Algebraic Equation Subroutines

The dense linear algebraic equation subroutines provide solutions to linear systems of equations for both real and complex general matrices and their transposes, positive definite real symmetric and complex Hermitian matrices, and triangular matrices. Some of these subroutines correspond to the Level 2 BLAS, Level 3 BLAS, and LAPACK routines described in references [32], [34 ], and [8 ].

*Table 83 (Page 1 of 2). List of Dense Linear Algebraic Equation Subroutines*

| Descriptive Name | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|
| General Matrix Factorization | SGEF<br>CGEF<br>SGETRF[Δ]<br>CGETRF[Δ] | DGEF<br>ZGEF<br>DGETRF[Δ]<br>ZGETRF[Δ]<br>DGEFP[§] | 466<br><br>479 |
| General Matrix, Its Transpose, or Its Conjugate Transpose Solve | SGES<br>CGES | DGES<br>ZGES | 469 |
| General Matrix, Its Transpose, or Its Conjugate Transpose Multiple Right-Hand Side Solve | SGESM<br>CGESM<br>SGETRS[Δ]<br>CGETRS[Δ] | DGESM<br>ZGESM<br>DGETRS[Δ]<br>ZGETRS[Δ] | 473<br><br>483 |
| General Matrix Factorization, Condition Number Reciprocal, and Determinant | SGEFCD | DGEFCD | 488 |
| Positive Definite Real Symmetric or Complex Hermitian Matrix Factorization | SPPF<br>SPOF<br>CPOF | DPPF<br>DPOF<br>ZPOF<br>DPPFP[§] | 492 |

*Table 83 (Page 2 of 2). List of Dense Linear Algebraic Equation Subroutines*

| Descriptive Name | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|
| Positive Definite Real Symmetric Matrix Solve | SPPS | DPPS | 500 |
| Positive Definite Real Symmetric or Complex Hermitian Matrix Multiple Right-Hand Side Solve | SPOSM<br>CPOSM | DPOSM<br>ZPOSM | 503 |
| Positive Definite Real Symmetric Matrix Factorization, Condition Number Reciprocal, and Determinant | SPPFCD<br>SPOFCD | DPPFCD<br>DPOFCD | 508 |
| General Matrix Inverse, Condition Number Reciprocal, and Determinant | SGEICD | DGEICD | 515 |
| Positive Definite Real Symmetric Matrix Inverse, Condition Number Reciprocal, and Determinant | SPPICD<br>SPOICD | DPPICD<br>DPOICD | 520 |
| Solution of a Triangular System of Equations with a Single Right-Hand Side | STRSV◂<br>CTRSV◂<br>STPSV◂<br>CTPSV◂ | DTRSV◂<br>ZTRSV◂<br>DTPSV◂<br>ZTPSV◂ | 527 |
| Solution of Triangular Systems of Equations with Multiple Right-Hand Sides | STRSM♦<br>CTRSM♦ | DTRSM♦<br>ZTRSM♦ | 533 |
| Triangular Matrix Inverse | STRI<br>STPI | DTRI<br>DTPI | 541 |

◂ Level 2 BLAS

♦ Level 3 BLAS

Δ LAPACK

§ This subroutine is provided only for migration from earlier releases of ESSL and is not intended for use is new programs. Documentation for this subroutine is no longer provided.

## Banded Linear Algebraic Equation Subroutines

The banded linear algebraic equation subroutines provide solutions to linear systems of equations for real general band matrices, real positive definite symmetric band matrices, real or complex general tridiagonal matrices, real positive definite symmetric tridiagonal matrices, and real or complex triangular band matrices.

*Table 84 (Page 1 of 2). List of Banded Linear Algebraic Equation Subroutines*

| Descriptive Name | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|
| General Band Matrix Factorization | SGBF | DGBF | 547 |
| General Band Matrix Solve | SGBS | DGBS | 551 |
| Positive Definite Symmetric Band Matrix Factorization | SPBF<br>SPBCHF | DPBF<br>DPBCHF | 554 |
| Positive Definite Symmetric Band Matrix Solve | SPBS<br>SPBCHS | DPBS<br>DPBCHS | 558 |
| General Tridiagonal Matrix Factorization | SGTF | DGTF | 561 |
| General Tridiagonal Matrix Solve | SGTS | DGTS | 564 |

*Table 84 (Page 2 of 2). List of Banded Linear Algebraic Equation Subroutines*

| Descriptive Name | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|
| General Tridiagonal Matrix Combined Factorization and Solve with No Pivoting | SGTNP<br>CGTNP | DGTNP<br>ZGTNP | 566 |
| General Tridiagonal Matrix Factorization with No Pivoting | SGTNPF<br>CGTNPF | DGTNPF<br>ZGTNPF | 569 |
| General Tridiagonal Matrix Solve with No Pivoting | SGTNPS<br>CGTNPS | DGTNPS<br>ZGTNPS | 572 |
| Positive Definite Symmetric Tridiagonal Matrix Factorization | SPTF | DPTF | 575 |
| Positive Definite Symmetric Tridiagonal Matrix Solve | SPTS | DPTS | 577 |
| Triangular Band Equation Solve | STBSV◂<br>CTBSV◂ | DTBSV◂<br>ZTBSV◂ | 579 |

◂ Level 2 BLAS

## Sparse Linear Algebraic Equation Subroutines

The sparse linear algebraic equation subroutines provide direct and iterative solutions to linear systems of equations both for general sparse matrices and their transposes and for sparse symmetric matrices.

*Table 85. List of Sparse Linear Algebraic Equation Subroutines*

| Descriptive Name | Long- Precision Subroutine | Page |
|---|---|---|
| General Sparse Matrix Factorization Using Storage by Indices, Rows, or Columns | DGSF | 586 |
| General Sparse Matrix or Its Transpose Solve Using Storage by Indices, Rows, or Columns | DGSS | 592 |
| General Sparse Matrix or Its Transpose Factorization, Determinant, and Solve Using Skyline Storage Mode | DGKFS | 596 |
| Symmetric Sparse Matrix Factorization, Determinant, and Solve Using Skyline Storage Mode | DSKFS | 614 |
| Iterative Linear System Solver for a General or Symmetric Sparse Matrix Stored by Rows | DSRIS | 633 |
| Sparse Positive Definite or Negative Definite Symmetric Matrix Iterative Solve Using Compressed-Matrix Storage Mode | DSMCG§ | 644 |
| Sparse Positive Definite or Negative Definite Symmetric Matrix Iterative Solve Using Compressed-Diagonal Storage Mode | DSDCG | 652 |
| General Sparse Matrix Iterative Solve Using Compressed-Matrix Storage Mode | DSMGCG§ | 660 |
| General Sparse Matrix Iterative Solve Using Compressed-Diagonal Storage Mode | DSDGCG | 667 |

§ These subroutines are provided only for migration from earlier releases of ESSL and are not intended for use in new programs. Use DSRIS instead.

## Linear Least Squares Subroutines

The linear least squares subroutines provide least squares solutions to linear systems of equations for real general matrices. Two methods are provided: one that uses the singular value decomposition and another that uses a QR decomposition with column pivoting.

*Table 86. List of Linear Least Squares Subroutines*

| Descriptive Name | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|
| Singular Value Decomposition for a General Matrix | SGESVF | DGESVF | 675 |
| Linear Least Squares Solution for a General Matrix Using the Singular Value Decomposition | SGESVS | DGESVS | 683 |
| Linear Least Squares Solution for a General Matrix Using a QR Decomposition with Column Pivoting | SGELLS | DGELLS | 688 |

## Dense and Banded Linear Algebraic Equation Considerations

This section provides some key points about using the dense and banded linear algebraic equation subroutines.

### Use Considerations

1. To solve a system of equations, you need to use both the factorization and solve subroutines for the type of matrix you have. Each factorization subroutine should be followed in your program by the corresponding solve subroutine. The output from the factorization subroutine should be used as input to the solve subroutine.
2. To solve a system of equations with one or more right-hand sides, follow the call to the factorization subroutine with one or more calls to a solve subroutine or one call to a multiple solve subroutine.
3. The ESSL naming conventions for the dense and banded linear algebraic equation subroutines are similar to those used in the LAPACK documentation. (LAPACK, as well as its documentation, is available from the sources listed in reference [8].) The following ESSL subroutines correspond to the LAPACK subroutines: SGETRF, DGETRF, CGETRF, ZGETRF, SGETRS, DGETRS, CGETRS, and ZGETRS.

### Performance and Accuracy Considerations

1. Except in a few instances, the _GTNP subroutines provide better performance than the _GTNPF and _GTNPS subroutines. For details, see the subroutine descriptions.
2. The general subroutines (dense and banded) use partial pivoting for accuracy and fast performance.
3. The short-precision subroutines provide increased accuracy by accumulating intermediate results in long precision. Occasionally, for performance reasons, these intermediate results are stored.

4. There are ESSL-specific rules that apply to the results of computations on the workstation processors using the ANSI/IEEE standards. For details, see "What Data Type Standards Are Used by ESSL, and What Exceptions Should You Know About?" on page 45.

## Sparse Matrix Direct Solver Considerations

This section provides some key points about using the sparse matrix direct solver subroutines.

### Use Considerations

1. To solve a sparse system of equations by a direct method, you must use both the factorization and solve subroutines. The factorization subroutine should be followed in your program by the corresponding solve subroutine; that is, the output from the factorization subroutine should be used as input to the solve subroutine.

2. To solve a system of equations with one or more right-hand sides, follow the call to the factorization subroutine with one or more calls to the solve subroutine.

3. The amount of storage required for the arrays depends on the sparsity pattern of the matrix. The requirement that $lna > 2nz$ on entry to DGSF does not guarantee a successful run of the program. Some programs may be terminated because of the large number of fill-ins generated upon factorization. Fill-ins generated in a program depend on the structure of each matrix. If a large number of fill-ins is anticipated when factoring a matrix, the value of *lna* should be large enough to accommodate your problem.

### Performance and Accuracy Considerations

1. To make the subroutine more efficient, an input matrix comprised of all nonzero elements is preferable. See the syntax description of each subroutine for details.

2. DGSF optionally checks the validity of the indices and pointers of the input matrix. Use of this option is suggested; however, it may affect performance. For details, see the syntax description for DGSF.

3. In DGSS, if there are multiple sparse right-hand sides to be solved, you should take advantage of the sparsity by selecting a proper value for *jopt* (such as $jopt = 10$ or 11). If there is only one right-hand side to be solved, it is suggested that you do not exploit the sparsity.

4. In DGSF, the value you enter for the lower bound of all elements in the matrix (`RPARM(1)`) affects the accuracy of the result. Specifying a larger number allows you to gain some performance; however, you may lose some accuracy in the solution.

5. In DGSF, the threshold pivot tolerance (`RPARM(2)`) is used to select pivots. A value that is close to 0.0 approaches no pivoting. A value close to 1.0 approaches partial pivoting. A value of 0.1 is considered to be a good compromise between numerical stability and sparsity.

6. If the ESSL subroutine performs storage compressions, you receive an attention message. When this occurs, the performance of this subroutine is

affected. You can improve the performance by increasing the value specified for *lna*.

7. There are ESSL-specific rules that apply to the results of computations on the workstation processors using the ANSI/IEEE standards. For details, see “What Data Type Standards Are Used by ESSL, and What Exceptions Should You Know About?” on page 45.

## Sparse Matrix Skyline Solver Considerations

This section provides some key points about using the sparse matrix skyline solver subroutines.

## Use Considerations

1. To solve a system of equations with one or more right-hand sides, where the matrix is stored in skyline storage mode, you can use either of the following methods. The factored output matrix is the same for both of these methods.

   - Call the skyline subroutine with the combined factor-and-solve option.
   - Call the skyline subroutine with the factor-only option, followed in your program by a call to the same subroutine with the solve-only option. The factored output matrix resulting from the factorization should be used as input to the same subroutine to do the solve. You can solve for the right-hand sides in a single call or in individual calls.

   You also have the option of doing a partial factorization, where the subroutine assumes that the initial part of the input matrix is already factored. It then factors the remaining rows and columns. If you want, you can factor a very large matrix progressively by using this option.

2. Forward elimination can be done with or without scaling the right-hand side by the diagonal matrix elements. To perform the computation without scaling, call DGKFS with the normal solve-only option, and define the upper triangular skyline matrix (AU) as a diagonal. To perform the computation with scaling, call DGKFS with the transpose solve-only, option and define the lower triangular skyline matrix (AL) as a diagonal.

3. Back substitution can be done with or without scaling the right-hand side by the diagonal matrix elements. To perform the computation without scaling, call DGKFS with the transpose solve-only option, and define the upper triangular skyline matrix (AU) as a diagonal. To perform the computation with scaling, call DGKFS with the normal solve-only option, and define the lower triangular skyline matrix (AL) as a diagonal.

## Performance and Accuracy Considerations

1. For optimal performance, use diagonal-out skyline storage mode for both your input and output matrices. If you specify profile-in skyline storage mode for your input matrix, and either you do not plan to use the factored output or you plan to do a solve only, it is more efficient to specify diagonal-out skyline storage mode for your output matrix. These rules apply to all the computations.

2. In some cases, elapsed time may be reduced significantly by using the combined factor-and-solve option to solve for all right-hand sides at once, in conjunction with the factorization, rather than doing the factorization and solve separately.

3. If you do a solve only, and you solve for more than one right-hand side, it is most efficient to call the skyline subroutine once with all right-hand sides, rather than once for each right-hand side.

4. The skyline subroutines allow some control over processing of the pivot (diagonal) elements of the matrix during the factorization phase. Pivot processing is controlled by `IPARM(10)` through `IPARM(15)` and `RPARM(10)` through `RPARM(15)`. If a pivot occurs within a range that is designated to be fixed (`IPARM(0) = 1`, `IPARM(10) = 1`, and the appropriate element `IPARM(11)` through `IPARM(15) = 1`), it is replaced with the corresponding element of `RPARM(11)` through `RPARM(15)`. Should this pivot fix-up occur, you receive an attention message. This message indicates that the matrix being factored may be unstable (singular or not definite). The results produced in this situation may be inaccurate, and you should review them carefully.

## Sparse Matrix Iterative Solver Considerations

This section provides some key points about using the sparse matrix iterative solver subroutines.

### Use Considerations

If you need to solve linear systems with different right-hand sides but with the same matrix using the preconditioned algorithms, you can reuse the incomplete factorization computed during the first call to the subroutine.

### Performance and Accuracy Considerations

1. The DSMCG and DSMGCG subroutines are provided for migration purposes from earlier releases of ESSL. You get better performance and a wider choice of algorithms if you use the DSRIS subroutine.

2. To select the sparse matrix subroutine that provides the best performance, you must consider the sparsity pattern of the matrix. From this, you can determine the most efficient storage mode for your sparse matrix. ESSL provides a number of versions of the sparse matrix iterative solve subroutines. They operate on sparse matrices stored in row-wise, diagonal, and compressed-matrix storage modes. These storage modes are described in “Sparse Matrix” on page 92.

   Storage-by-rows is generally applicable. You should use this storage mode unless your matrices are already set up in one of the other storage modes. If, however, your matrix has a regular sparsity pattern—that is, where the nonzero elements are concentrated along a few diagonals—you may want to use compressed-diagonal storage mode. This can save some storage space. Compressed-matrix storage mode is provided for migration purposes from earlier releases of ESSL and is not intended for use. (You get better performance and a wider choice of algorithms if you use the DSRIS subroutine, which uses storage-by-rows.)

3. The performance achieved in the sparse matrix iterative solver subroutines depends on the value specified for the relative accuracy $\varepsilon$. For details, see Notes for each subroutine.

4. You can select the iterative algorithm you want to use to solve your linear system. The methods include conjugate gradient (CG), conjugate gradient squared (CGS), generalized minimum residual (GMRES), more smoothly

converging variant of the CGS method (Bi-CGSTAB), or transpose-free quasi-minimal residual method (TFQMR).

5. For a general sparse or positive definite symmetric matrix, the iterative algorithm may fail to converge for one of the following reasons:

   - The value of $\varepsilon$ is too small, asking for too much precision.
   - The maximum number of iterations is too small, allowing too few iterations for the algorithm to converge.
   - The matrix is not positive real; that is, the symmetric part, $(\boldsymbol{A}+\boldsymbol{A}^T)/2$, is not positive definite.
   - The matrix is ill-conditioned, which may cause overflows during the computation.

6. These algorithms have a tendency to generate underflows that may hurt overall performance. The system default is to mask underflow, which improves the performance of these subroutines.

## Linear Least Squares Considerations

This section provides some key points about using the linear least squares subroutines.

## Use Considerations

If you want to use a singular value decomposition method to compute the minimal norm linear least squares solution of $\boldsymbol{AX} \cong \boldsymbol{B}$, calls to SGESVF or DGESVF should be followed by calls to SGESVS or DGESVS, respectively.

## Performance and Accuracy Considerations

1. Least squares solutions obtained by using a singular value decomposition require more storage and run time than those obtained using a QR decomposition with column pivoting. The singular value decomposition method, however, is a more reliable way to handle rank deficiency.

2. The short-precision subroutines provide increased accuracy by accumulating intermediate results in long precision. Occasionally, for performance reasons, these intermediate results are stored.

3. The accuracy of the resulting singular values and singular vectors varies between the short- and long-precision versions of each subroutine. The degree of difference depends on the size and conditioning of the matrix computation.

4. There are ESSL-specific rules that apply to the results of computations on the workstation processors using the ANSI/IEEE standards. For details, see “What Data Type Standards Are Used by ESSL, and What Exceptions Should You Know About?” on page 45.

## Dense Linear Algebraic Equation Subroutines

This section contains the dense linear algebraic equation subroutine descriptions.

# SGEF, DGEF, CGEF, and ZGEF—General Matrix Factorization

This subroutine factors a square general matrix ***A*** using Gaussian elimination with partial pivoting. To solve the system of equations with one or more right-hand sides, follow the call to these subroutines with one or more calls to SGES/SGESM, DGES/DGESM, CGES/CGESM, or ZGES/ZGESM, respectively. To compute the inverse of matrix ***A***, follow the call to these subroutines with a call to SGEICD or DGEICD, respectively.

*Table 87. Data Types*

| ***A*** | **Subroutine** |
|---|---|
| Short-precision real | SGEF |
| Long-precision real | DGEF |
| Short-precision complex | CGEF |
| Long-precision complex | ZGEF |

**Note:** The output from these factorization subroutines should be used only as input to the following subroutines for performing a solve or inverse: SGES/SGESM/SGEICD, DGES/DGESM/DGEICD, CGES/CGESM, and ZGES/ZGESM, respectively.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SGEF \| DGEF \| CGEF \| ZGEF (*a*, *lda*, *n*, *ipvt*) |
| **C and C++** | sgef \| dgef \| cgef \| zgef (*a*, *lda*, *n*, *ipvt*); |
| **PL/I** | CALL SGEF \| DGEF \| CGEF \| ZGEF (*a*, *lda*, *n*, *ipvt*); |

*On Entry*

*a*
is the *n* by *n* general matrix ***A*** to be factored. Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 87.

*lda*
is the leading dimension of the array specified for *a*. Specified as: a fullword integer; $lda > 0$ and $lda \geq n$.

*n*
is the order of matrix ***A***. Specified as: a fullword integer; $0 \leq n \leq lda$.

*ipvt*
See "On Return."

*On Return*

*a*
is the *n* by *n* transformed matrix ***A***, containing the results of the factorization. See "Function" on page 467. Returned as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 87.

*ipvt*
is the integer vector ***ipvt*** of length *n*, containing the pivot indices. Returned as: a one-dimensional array of (at least) length *n*, containing fullword integers.

***Notes***

1. Calling SGEFCD or DGEFCD with *iopt* = 0 is equivalent to calling SGEF or DGEF.
2. On both input and output, matrix ***A*** conforms to LAPACK format.

***Function:*** The matrix ***A*** is factored using Gaussian elimination with partial pivoting (***ipvt***) to compute the ***LU*** factorization of ***A***, where (***A*** = ***PLU***) :

***L*** is a unit lower triangular matrix.
***U*** is an upper triangular matrix.
***P*** is the permutation matrix.

On output, the transformed matrix ***A*** contains ***U*** in the upper triangle and ***L*** in the strict lower triangle where ***ipvt*** contains the pivots representing permutation ***P***, such that ***A*** = ***PLU***.

If *n* is 0, no computation is performed. See references [36 ] and [38 ].

***Error Conditions***

*Resource Errors:* Unable to allocate internal work area.

*Computational Errors:* Matrix ***A*** is singular.

- One or more columns of ***L*** and the corresponding diagonal of ***U*** contain all zeros (all columns of ***L*** are checked). The first column, *i*, of ***L*** with a corresponding ***U*** = 0 diagonal element is identified in the computational error message.
- The return code is set to 1.
- *i* can be determined at run time by use of the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error code 2103 in the ESSL error option table; otherwise, the default value causes your program to terminate when this error occurs. For details, see “What Can You Do about ESSL Computational Errors?” on page 48.

*Input-Argument Errors*

1. $lda \le 0$
2. $n < 0$
3. $n > lda$

***Example 1:*** This example shows a factorization of a real general matrix ***A*** of order 9.

*Call Statement and Input*

```
           A  LDA  N   IPVT
           |   |   |    |
CALL SGEF( A , 9 , 9 , IPVT )
```

```
          ┌                                                ┐
          │ 1.0  1.0  1.0  1.0  0.0  0.0   0.0   0.0   0.0 │
          │ 1.0  1.0  1.0  1.0  1.0  0.0   0.0   0.0   0.0 │
          │ 4.0  1.0  1.0  1.0  1.0  1.0   0.0   0.0   0.0 │
          │ 0.0  5.0  1.0  1.0  1.0  1.0   1.0   0.0   0.0 │
A    =    │ 0.0  0.0  6.0  1.0  1.0  1.0   1.0   1.0   0.0 │
          │ 0.0  0.0  0.0  7.0  1.0  1.0   1.0   1.0   1.0 │
          │ 0.0  0.0  0.0  0.0  8.0  1.0   1.0   1.0   1.0 │
          │ 0.0  0.0  0.0  0.0  0.0  9.0   1.0   1.0   1.0 │
          │ 0.0  0.0  0.0  0.0  0.0  0.0  10.0  11.0  12.0 │
          └                                                ┘
```

*Output*

```
        ┌                                                                                 ┐
        │ 4.0000  1.0000  1.0000  1.0000   1.0000   1.0000   0.0000   0.0000   0.0000 │
        │ 0.0000  5.0000  1.0000  1.0000   1.0000   1.0000   1.0000   0.0000   0.0000 │
        │ 0.0000  0.0000  6.0000  1.0000   1.0000   1.0000   1.0000   1.0000   0.0000 │
        │ 0.0000  0.0000  0.0000  7.0000   1.0000   1.0000   1.0000   1.0000   1.0000 │
A    =  │ 0.0000  0.0000  0.0000  0.0000   8.0000   1.0000   1.0000   1.0000   1.0000 │
        │ 0.0000  0.0000  0.0000  0.0000   0.0000   9.0000   1.0000   1.0000   1.0000 │
        │ 0.0000  0.0000  0.0000  0.0000   0.0000   0.0000  10.0000  11.0000  12.0000 │
        │ 0.2500  0.1500  0.1000  0.0714   0.0536  -0.0694  -0.0306   0.1806   0.3111 │
        │ 0.2500  0.1500  0.1000  0.0714  -0.0714  -0.0556  -0.0194   0.9385  -0.0031 │
        └                                                                                 ┘
```

```
IPVT     =  (3, 4, 5, 6, 7, 8, 9, 8, 9)
```

***Example 2:*** This example shows a factorization of a complex general matrix ***A*** of order 4.

*Call Statement and Input*

```
          A  LDA  N   IPVT
          |   |   |    |
CALL CGEF( A , 4 , 4 , IPVT )
```

```
          ┌                                            ┐
          │ (1.0, 2.0) (1.0, 7.0) (2.0, 4.0) (3.0, 1.0) │
A    =    │ (2.0, 0.0) (1.0, 3.0) (4.0, 4.0) (2.0, 3.0) │
          │ (2.0, 1.0) (5.0, 0.0) (3.0, 6.0) (0.0, 0.0) │
          │ (8.0, 5.0) (1.0, 9.0) (6.0, 6.0) (8.0, 1.0) │
          └                                            ┘
```

*Output*

```
        ┌                                                                        ┐
        │  (8.0000, 5.0000)   (1.0000, 9.0000)  (6.0000, 6.0000)  (8.0000, 1.0000) │
A    =  │  (0.2022, 0.1236)   (1.9101, 5.0562)  (1.5281, 2.0449) (1.5056, -0.1910) │
        │ (0.2360, -0.0225) (-0.0654, -0.9269) (-0.3462, 6.2692) (-1.6346, 1.3269) │
        │ (0.1798, -0.1124)   (0.2462, 0.1308) (0.4412, -0.3655)  (0.2900, 2.3864) │
        └                                                                        ┘
```

```
IPVT     =  (4, 4, 3, 4)
```

# SGES, DGES, CGES, and ZGES—General Matrix, Its Transpose, or Its Conjugate Transpose Solve

These subroutines solve the system $\boldsymbol{Ax} = \boldsymbol{b}$ for $\boldsymbol{x}$, where $\boldsymbol{A}$ is a general matrix and $\boldsymbol{x}$ and $\boldsymbol{b}$ are vectors. Using the *iopt* argument, they can also solve the real system $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{x} = \boldsymbol{b}$ or the complex system $\boldsymbol{A}^{\mathrm{H}}\boldsymbol{x} = \boldsymbol{b}$ for $\boldsymbol{x}$. These subroutines use the results of the factorization of matrix $\boldsymbol{A}$, produced by a preceding call to SGEF/SGEFCD, DGEF/DGEFP/DGEFCD, CGEF, or ZGEF, respectively.

*Table 88. Data Types*

| ***A***, ***b***, ***x*** | **Subroutine** |
|---|---|
| Short-precision real | SGES |
| Long-precision real | DGES |
| Short-precision complex | CGES |
| Long-precision complex | ZGES |

**Note:** The input to these solve subroutines must be the output from the factorization subroutines SGEF/SGEFCD, DGEF/DGEFP/DGEFCD, CGEF, and ZGEF, respectively.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SGES \| DGES \| CGES \| ZGES (*a*, *lda*, *n*, *ipvt*, *bx*, *iopt*) |
| **C and C++** | sges \| dges \| cges \| zges (*a*, *lda*, *n*, *ipvt*, *bx*, *iopt*); |
| **PL/I** | CALL SGES \| DGES \| CGES \| ZGES (*a*, *lda*, *n*, *ipvt*, *bx*, *iopt*); |

*On Entry*

*a*

is the factorization of matrix ***A***, produced by a preceding call to SGEF/SGEFCD, DGEF/DGEFP/DGEFCD, CGEF, or ZGEF, respectively. Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 88.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; $lda > 0$ and $lda \geq n$.

*n*

is the order of matrix ***A***. Specified as: a fullword integer; $0 \leq n \leq lda$.

*ipvt*

is the integer vector ***ipvt*** of length *n*, containing the pivot indices produced by a preceding call to SGEF/SGEFCD, DGEF/DGEFP/DGEFCD, CGEF, or ZGEF, respectively. Specified as: a one-dimensional array of (at least) length *n*, containing fullword integers.

*bx*

is the vector ***b*** of length *n*, containing the right-hand side of the system. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 88.

*iopt*

determines the type of computation to be performed, where:

If $iopt = 0$, ***A*** is used in the computation.

If *iopt* = 1, $\boldsymbol{A}^T$ is used in SGES and DGES. $\boldsymbol{A}^H$ is used in CGES and ZGES.

**Note:** No data should be moved to form $\boldsymbol{A}^T$ or $\boldsymbol{A}^H$; that is, the matrix ***A*** should always be stored in its untransposed form.

Specified as: a fullword integer; *iopt* = 0 or 1.

*On Return*

*bx*

is the solution vector ***x*** of length *n*, containing the results of the computation. Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 88 on page 469.

### *Notes*

1. The scalar data specified for input arguments *lda* and *n* for these subroutines must be the same as the corresponding input arguments specified for SGEF/SGEFCD, DGEF/DGEFP/DGEFCD, CGEF, and ZGEF, respectively.
2. The array data specified for input arguments *a* and *ipvt* for these subroutines must be the same as the corresponding output arguments for SGEF/SGEFCD, DGEF/DGEFP/DGEFCD, CGEF, and ZGEF, respectively.
3. The vectors and matrices used in this computation must have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.

***Function:*** The system $\boldsymbol{Ax} = \boldsymbol{b}$ is solved for ***x***, where ***A*** is a general matrix and ***x*** and ***b*** are vectors. Using the *iopt* argument, this subroutine can also solve the real system $\boldsymbol{A}^T\boldsymbol{x} = \boldsymbol{b}$ or the complex system $\boldsymbol{A}^H\boldsymbol{x} = \boldsymbol{b}$ for ***x***. These subroutines use the results of the factorization of matrix ***A***, produced by a preceding call to SGEF/SGEFCD, DGEF/DGEFP/DGEFCD, CGEF, or ZGEF, respectively. For a description of how ***A*** is factored, see “SGEF, DGEF, CGEF, and ZGEF—General Matrix Factorization” on page 466.

If *n* is 0, no computation is performed. See references [36 ] and [38 ].

### *Error Conditions*

*Computational Errors:* None

**Note:** If the factorization performed by SGEF, DGEF, CGEF, ZGEF, SGEFCD, DGEFCD, or DGEFP failed because a pivot element is zero, the results returned by this subroutine are unpredictable, and there may be a divide-by-zero program exception message.

*Input-Argument Errors*

1. $lda \le 0$
2. $n < 0$
3. $n > lda$
4. $iopt \ne 0$ or 1

### *Example 1*

*Part 1:* This part of the example shows how to solve the system $\boldsymbol{Ax} = \boldsymbol{b}$, where matrix ***A*** is the same matrix factored in the “Example 1” on page 467 for SGEF and DGEF.

*Call Statement and Input*

```
          A  LDA  N   IPVT   BX  IOPT
          |   |   |    |     |    |
CALL SGES( A , 9 , 9 , IPVT , BX , 0  )

IPVT     =  (3, 4, 5, 6, 7, 8, 9, 8, 9)
BX       =  (4.0, 5.0, 9.0, 10.0, 11.0, 12.0, 12.0, 12.0, 33.0)
A        = (same as output A in “Example 1” on page 467)
```

*Output*

```
BX       =  (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0)
```

*Part 2:* This part of the example shows how to solve the system $\boldsymbol{A}^{T}\boldsymbol{x} = \boldsymbol{b}$, where matrix ***A*** is the input matrix factored in “Example 1” on page 467 for SGEF and DGEF. Most of the input is the same in Part 2 as in Part 1.

*Call Statement and Input*

```
          A  LDA  N   IPVT   BX  IOPT
          |   |   |    |     |    |
CALL SGES( A , 9 , 9 , IPVT , BX , 1  )

IPVT     =  (3, 4, 5, 6, 7, 8, 9, 8, 9)
BX       =  (6.0, 8.0, 10.0, 12.0, 13.0, 14.0, 15.0, 15.0, 15.0)
A        = (same as output A in “Example 1” on page 467)
```

*Output*

```
BX       =  (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0)
```

***Example 2***

*Part 1:* This part of the example shows how to solve the system $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$, where matrix ***A*** is the same matrix factored in the “Example 2” on page 468 for CGEF and ZGEF.

*Call Statement and Input*

```
          A  LDA  N   IPVT   BX  IOPT
          |   |   |    |     |    |
CALL CGES( A , 4 , 4 , IPVT , BX , 0  )

IPVT     =  (4, 4, 3, 4)
BX       =  ((-10.0, 85.0), (-6.0, 61.0), (10.0, 38.0),
             (58.0, 168.0))
A        = (same as output A in “Example 1” on page 467)
```

*Output*

```
BX       =  ((9.0, 0.0), (5.0, 1.0), (1.0, 6.0), (3.0, 4.0))
```

*Part 2:* This part of the example shows how to solve the system $\boldsymbol{A}^{H}\boldsymbol{x} = \boldsymbol{b}$, where matrix ***A*** is the input matrix factored in “Example 2” on page 468 for CGEF and ZGEF. Most of the input is the same in Part 2 as in Part 1.

*Call Statement and Input*

```
           A  LDA  N   IPVT   BX  IOPT
           |   |   |    |     |    |
CALL CGES( A , 4 , 4 , IPVT , BX , 1  )

IPVT     =  (4, 4, 3, 4)
BX       =  ((71.0, 12.0), (61.0, -70.0), (123.0, -34.0),
             (68.0, 7.0))
```

A = (same as output A in “Example 1” on page 467)

*Output*

```
BX       =  ((9.0, 0.0), (5.0, 1.0), (1.0, 6.0), (3.0, 4.0))
```

## SGESM, DGESM, CGESM, and ZGESM—General Matrix, Its Transpose, or Its Conjugate Transpose Multiple Right-Hand Side Solve

These subroutines solve the following systems of equations for multiple right-hand sides, where ***A***, ***X***, and ***B*** are general matrices. SGESM and DGESM solve one of the following:

1. $AX = B$
2. $A^TX = B$

CGESM and ZGESM solve one of the following:

1. $AX = B$
2. $A^TX = B$
3. $A^HX = B$

These subroutines use the results of the factorization of matrix ***A***, produced by a preceding call to SGEF/SGEFCD, DGEF/DGEFP/DGEFCD, CGEF, or ZGEF, respectively.

*Table 89. Data Types*

| ***A***, ***B***, ***X*** | **Subroutine** |
|---|---|
| Short-precision real | SGESM |
| Long-precision real | DGESM |
| Short-precision complex | CGESM |
| Long-precision complex | ZGESM |

**Note:** The input to these solve subroutines must be the output from the factorization subroutines SGEF/SGEFCD, DGEF/DGEFP/DGEFCD, CGEF, and ZGEF, respectively.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SGESM \| DGESM \| CGESM \| ZGESM (*trans*, *a*, *lda*, *n*, *ipvt*, *b*, *ldb*, *nrhs*) |
| **C and C++** | sgesm \| dgesm \| cgesm \| zgesm (*trans*, *a*, *lda*, *n*, *ipvt*, *b*, *ldb*, *nrhs*); |
| **PL/I** | CALL SGESM \| DGESM \| CGESM \| ZGESM (*trans*, *a*, *lda*, *n*, *ipvt*, *b*, *ldb*, *nrhs*); |

*On Entry*

*trans*

indicates the form of matrix ***A*** to use in the computation, where:

If *transa* = 'N', ***A*** is used in the computation, resulting in equation 1.

If *transa* = 'T', $A^T$ is used in the computation, resulting in equation 2.

If *transa* = 'C', $A^H$ is used in the computation, resulting in equation 3.

Specified as: a single character. It must be 'N', 'T', or 'C'.

*a*

is the factorization of matrix ***A***, produced by a preceding call to SGEF/SGEFCD, DGEF/DGEFP/DGEFCD, CGEF, or ZGEF, respectively. Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 89.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; *lda* > 0 and *lda* ≥ *n*.

*n*

is the order of matrix ***A***. Specified as: a fullword integer; 0 ≤ *n* ≤ *lda*.

*ipvt*

is the integer vector ***ipvt*** of length *n*, containing the pivot indices produced by a preceding call to SGEF/SGEFCD, DGEF/DGEFP/DGEFCD, CGEF, or ZGEF, respectively. Specified as: a one-dimensional array of (at least) length *n*, containing fullword integers.

*b*

is the matrix ***B***, containing the *nrhs* right-hand sides of the system. The right-hand sides, each of length *n*, reside in the columns of matrix ***B***. Specified as: an *ldb* by (at least) *nrhs* array, containing numbers of the data type indicated in Table 89 on page 473.

*ldb*

is the leading dimension of the array specified for *b*. Specified as: a fullword integer; *ldb* > 0 and *ldb* ≥ *n*.

*nrhs*

is the number of right-hand sides in the system to be solved. Specified as: a fullword integer; *nrhs* ≥ 0.

*On Return*

*b*

is the matrix ***B***, containing the *nrhs* solutions to the system in the columns of ***B***. Specified as: an *ldb* by (at least) *nrhs* array, containing numbers of the data type indicated in Table 89 on page 473.

***Notes***

1. For SGESM and DGESM, if you specify `'C'` for the *trans* argument, it is interpreted as though you specified `'T'`.
2. The scalar data specified for input arguments *lda* and *n* for these subroutines must be the same as the corresponding input arguments specified for SGEF/SGEFCD, DGEF/DGEFP/DGEFCD, CGEF, and ZGEF, respectively.
3. The array data specified for input arguments *a* and *ipvt* for these subroutines must be the same as the corresponding output arguments for SGEF/SGEFCD, DGEF/DGEFP/DGEFCD, CGEF, and ZGEF, respectively.
4. The vectors and matrices used in this computation must have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.

***Function:*** One of the following systems of equations is solved for multiple right-hand sides:

1. $\boldsymbol{AX} = \boldsymbol{B}$
2. $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{X} = \boldsymbol{B}$
3. $\boldsymbol{A}^{\mathrm{H}}\boldsymbol{X} = \boldsymbol{B}$ (only for CGESM and ZGESM)

where ***A***, ***B***, and ***X*** are general matrices. These subroutines use the results of the factorization of matrix ***A***, produced by a preceding call to SGEF/SGEFCD, DGEF/DGEFP/DGEFCD, CGEF, or ZGEF, respectively. For a description of how ***A*** is factored, see “SGEF, DGEF, CGEF, and ZGEF—General Matrix Factorization” on page 466.

If *n* or *nrhs* is 0, no computation is performed. See references [36] and [38 ].

### *Error Conditions*

*Computational Errors:* None

**Note:** If the factorization performed by SGEF, DGEF, CGEF, ZGEF, SGEFCD, DGEFCD, or DGEFP failed because a pivot element is zero, the results returned by this subroutine are unpredictable, and there may be a divide-by-zero program exception message.

*Input-Argument Errors*

1. *trans* ≠ 'N', 'T', or 'C'
2. *lda*, *ldb* ≤ 0
3. *n* < 0
4. *n* > *lda*, *ldb*
5. *nrhs* < 0

### *Example 1*

*Part 1:* This part of the example shows how to solve the system ***AX*** = ***B*** for two right-hand sides, where matrix ***A*** is the same matrix factored in the “Example 1” on page 467 for SGEF and DGEF.

*Call Statement and Input*

```
           TRANS  A  LDA  N   IPVT   B  LDB  NRHS
             |    |   |   |    |     |   |    |
CALL SGESM( 'N' , A , 9 , 9 , IPVT , B , 9 ,  2  )

IPVT     =  (3, 4, 5, 6, 7, 8, 9, 8, 9)
A        =(same as output A in “Example 1” on page 467)
```

```
        ┌             ┐
        |  4.0   10.0 |
        |  5.0   15.0 |
        |  9.0   24.0 |
        | 10.0   35.0 |
B    =  | 11.0   48.0 |
        | 12.0   63.0 |
        | 12.0   70.0 |
        | 12.0   78.0 |
        | 33.0  266.0 |
        └             ┘
```

*Output*

```
        ┌          ┐
        | 1.0  1.0 |
        | 1.0  2.0 |
        | 1.0  3.0 |
        | 1.0  4.0 |
B    =  | 1.0  5.0 |
        | 1.0  6.0 |
        | 1.0  7.0 |
        | 1.0  8.0 |
        | 1.0  9.0 |
        └          ┘
```

*Part 2:* This part of the example shows how to solve the system $\mathbf{A}^{T}\mathbf{X} = \mathbf{B}$ for two right-hand sides, where matrix ***A*** is the input matrix factored in “Example 1” on page 467 for SGEF and DGEF.

*Call Statement and Input*

```
           TRANS  A  LDA  N   IPVT   B  LDB  NRHS
             |    |   |   |    |     |   |    |
CALL SGESM( 'T' , A , 9 , 9 , IPVT , B , 9 ,  2  )

IPVT     =  (3, 4, 5, 6, 7, 8, 9, 8, 9)
A        =(same as output A in “Example 1” on page 467)
```

```
        ┌             ┐
        |  6.0   15.0 |
        |  8.0   26.0 |
        | 10.0   40.0 |
        | 12.0   57.0 |
B    =  | 13.0   76.0 |
        | 14.0   97.0 |
        | 15.0  120.0 |
        | 15.0  125.0 |
        | 15.0  129.0 |
        └             ┘
```

*Output*

```
        ┌          ┐
        | 1.0  1.0 |
        | 1.0  2.0 |
        | 1.0  3.0 |
        | 1.0  4.0 |
B    =  | 1.0  5.0 |
        | 1.0  6.0 |
        | 1.0  7.0 |
        | 1.0  8.0 |
        | 1.0  9.0 |
        └          ┘
```

***Example 2***

*Part 1:* This part of the example shows how to solve the system $\mathbf{A}\mathbf{X} = \mathbf{B}$ for two right-hand sides, where matrix ***A*** is the same matrix factored in the “Example 2” on page 468 for CGEF and ZGEF.

*Call Statement and Input*

```
           TRANS  A  LDA  N   IPVT   B  LDB  NRHS
             |    |   |   |    |     |   |    |
CALL CGESM( 'N' , A , 4 , 4 , IPVT , B , 4 ,  2  )

IPVT     =  (4, 4, 3, 4)
A        =(same as output A in “Example 2” on page 468)
```

```
        ┌                                ┐
        | (-10.0, 85.0)  (-11.0, 53.0) |
B    =  |  (-6.0, 61.0)   (-6.0, 54.0) |
        |  (10.0, 38.0)    (2.0, 40.0) |
        | (58.0, 168.0)  (15.0, 105.0) |
        └                                ┘
```

*Output*

```
        ┌                          ┐
        | (9.0, 0.0)  (1.0, 1.0) |
B    =  | (5.0, 1.0)  (2.0, 2.0) |
        | (1.0, 6.0)  (3.0, 3.0) |
        | (3.0, 4.0)  (4.0, 4.0) |
        └                          ┘
```

*Part 2:* This part of the example shows how to solve the system $\boldsymbol{A}^{T}\boldsymbol{X} = \boldsymbol{B}$ for two right-hand sides, where matrix ***A*** is the input matrix factored in “Example 2” on page 468 for CGEF and ZGEF.

*Call Statement and Input*

```
           TRANS  A  LDA  N   IPVT   B  LDB  NRHS
             |    |   |   |    |     |   |    |
CALL CGESM( 'T' , A , 4 , 4 , IPVT , B , 4 ,  2  )

IPVT     =  (4, 4, 3, 4)
A        =(same as output A in “Example 2” on page 468)
```

```
        ┌                                 ┐
        |   (71.0, 12.0)   (18.0, 68.0) |
B    =  |  (61.0, -70.0)  (-27.0, 71.0) |
        | (123.0, -34.0)  (-11.0, 97.0) |
        |    (68.0, 7.0)   (28.0, 50.0) |
        └                                 ┘
```

*Output*

```
        ┌                          ┐
        | (9.0, 0.0)  (1.0, 1.0) |
B    =  | (5.0, 1.0)  (2.0, 2.0) |
        | (1.0, 6.0)  (3.0, 3.0) |
        | (3.0, 4.0)  (4.0, 4.0) |
        └                          ┘
```

*Part 3:* This part of the example shows how to solve the system $\boldsymbol{A}^{H}\boldsymbol{X} = \boldsymbol{B}$ for two right-hand sides, where matrix ***A*** is the input matrix factored in “Example 2” on page 468 for CGEF and ZGEF.

*Call Statement and Input*

```
           TRANS  A  LDA  N   IPVT   B  LDB  NRHS
             |    |   |   |    |     |   |    |
CALL CGESM( 'C' , A , 4 , 4 , IPVT , B , 4 ,  2  )

IPVT     =  (4, 4, 3, 4)
A        =(same as output A in “Example 2” on page 468)
```

```
                ┌                                  ┐
                |   (58.0, -3.0)    (45.0, 20.0)   |
B     =         |  (68.0, -31.0)   (83.0, -20.0)   |
                |  (89.0, -22.0)     (98.0, 1.0)   |
                |   (53.0, 15.0)    (45.0, 25.0)   |
                └                                  ┘
```

*Output*

```
                ┌                          ┐
                | (1.0, 4.0)  (4.0, 5.0)   |
B     =         | (2.0, 3.0)  (3.0, 4.0)   |
                | (3.0, 2.0)  (2.0, 3.0)   |
                | (4.0, 1.0)  (1.0, 2.0)   |
                └                          ┘
```

# SGETRF, DGETRF, CGETRF and ZGETRF—General Matrix Factorization

These subroutines factor general matrix ***A*** using Gaussian elimination with partial pivoting. To solve the system of equations with one or more right-hand sides, follow the call to these subroutines with one or more calls to SGETRS, DGETRS CGETRS, or ZGETRS, respectively. To compute the inverse of matrix ***A***, follow the call to these subroutines with a call to SGEICD or DGEICD, respectively.

*Table 90. Data Types*

| ***A*** | **Subroutine** |
|---|---|
| Short-precision real | SGETRF |
| Long-precision real | DGETRF |
| Short-precision complex | CGETRF |
| Long-precision complex | ZGETRF |

**Note:** The output from these factorization subroutines should be used only as input to the following subroutines for performing a solve or inverse: SGETRS, DGETRS, CGETRS, ZGETRS, SGEICD or DGEICD respectively.

***Syntax***

| | |
|---|---|
| **Fortran** | CALL SGETRF \| DGETRF \| CGETRF \| ZGETRF (*m*, *n*, *a*, *lda*, *ipvt*, *info*) |
| **C and C++** | sgetrf \| dgetrf \| cgetrf \| zgetrf (*m*, *n*, *a*, *lda*, *ipvt*, *info*); |
| **PL/I** | CALL SGETRF \| DGETRF \| CGETRF \| ZGETRF (*m*, *n*, *a*, *lda*, *ipvt*, *info*); |

*On Entry*

*m*
the number of rows in general matrix ***A*** used in the computation. Specified as: a fullword integer; $0 \leq m \leq lda$.

*n*
the number of columns in general matrix ***A*** used in the computation. Specified as: a fullword integer; $n \geq 0$.

*a*
is the *m* by *n* general matrix ***A*** to be factored. Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 90.

*lda*
is the leading dimension of matrix ***A***. Specified as: a fullword integer; $lda > 0$ and $lda \geq m$.

*ipvt*
See “On Return.”

*info*
See “On Return.”

*On Return*

*a*
is the *m* by *n* transformed matrix ***A***, containing the results of the factorization. See “Function” on page 480. Returned as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 90.

*ipvt*
is the integer vector ***ipvt*** of length min($m$,$n$), containing the pivot indices. Returned as: a one-dimensional array of (at least) length min($m$,$n$), containing fullword integers,where $1 \le ipvt(i) \le m$.

*info*
has the following meaning:

If $info = 0$, the factorization of general matrix ***A*** completed successfully.

If $info > 0$, *info* is set equal to the first $i$, where $\boldsymbol{U}_{ii}$ is singular and its inverse could not be computed.

Specified as: a fullword integer; $info \ge 0$.

### *Notes*

1. In your C program, argument *info* must be passed by reference.
2. The matrix ***A*** and vector ***ipvt*** must have no common elements; otherwise results are unpredictable.
3. The way these subroutines handle singularity differs from LAPACK. These subroutines us the *info* argument to provide information about the singularity of ***A***, like LAPACK, but also provide an error message.
4. On both input and output, matrix ***A*** conforms to LAPACK format.

***Function:*** The matrix ***A*** is factored using Gaussian elimination with partial pivoting (***ipvt***) to compute the ***LU*** factorization of ***A***, where (***A=PLU***):

***L*** is a unit lower triangular matrix.
***U*** is an upper triangular matrix.
***P*** is the permutation matrix.

On output, the transformed matrix ***A*** contains ***U*** in the upper triangle (if $m \ge n$) or upper trapezoid (if $m < n$) and ***L*** in the strict lower triangle (if $m \le n$) or lower trapezoid (if $m > n$). ***ipvt*** contains the pivots representing permutation ***P***, such that $\boldsymbol{A} = \boldsymbol{PLU}$.

If $m$ or $n$ is 0, no computation is performed and the subroutine returns after doing some parameter checking. See references [36] and [60 ].

### *Error Conditions*

*Resource Errors:* Unable to allocate internal work area.

*Computational Errors:* Matrix ***A*** is singular.

- The first column, $i$, of ***L*** with a corresponding $\boldsymbol{U}_{ii} = 0$ diagonal element is identified in the computational error message.
- The computational error message may occur multiple times with processing continuing after each error, because the default for the number of allowable errors for error code 2146 is set to be unlimited in the ESSL error option table.

*Input-Argument Errors*

1. $m < 0$
2. $n < 0$
3. $m > lda$
4. $lda \le 0$

***Example 1:*** This example shows a factorization of a real general matrix ***A*** of order 9.

*Call Statement and Input*

```
             M   N   A  LDA  IPVT  INFO
             |   |   |   |     |     |
CALL DGETRF( 9 , 9 , A,  9 , IPVT, INFO )
```

```
        ┌                                             ┐
        | 1.0  1.2  1.4  1.6  1.8  2.0  2.2  2.4  2.6 |
        | 1.2  1.0  1.2  1.4  1.6  1.8  2.0  2.2  2.4 |
        | 1.4  1.2  1.0  1.2  1.4  1.6  1.8  2.0  2.2 |
        | 1.6  1.4  1.2  1.0  1.2  1.4  1.6  1.8  2.0 |
A    =  | 1.8  1.6  1.4  1.2  1.0  1.2  1.4  1.6  1.8 |
        | 2.0  1.8  1.6  1.4  1.2  1.0  1.2  1.4  1.6 |
        | 2.2  2.0  1.8  1.6  1.4  1.2  1.0  1.2  1.4 |
        | 2.4  2.2  2.0  1.8  1.6  1.4  1.2  1.0  1.2 |
        | 2.6  2.4  2.2  2.0  1.8  1.6  1.4  1.2  1.0 |
        └                                             ┘
```

*Output*

```
        ┌                                              ┐
        | 2.6   2.4  2.2  2.0  1.8  1.6  1.4  1.2  1.0 |
        | 0.4   0.3  0.6  0.8  1.1  1.4  1.7  1.9  2.2 |
        | 0.5  -0.4  0.4  0.8  1.2  1.6  2.0  2.4  2.8 |
        | 0.5  -0.3  0.0  0.4  0.8  1.2  1.6  2.0  2.4 |
A    =  | 0.6  -0.3  0.0  0.0  0.4  0.8  1.2  1.6  2.0 |
        | 0.7  -0.2  0.0  0.0  0.0  0.4  0.8  1.2  1.6 |
        | 0.8  -0.2  0.0  0.0  0.0  0.0  0.4  0.8  1.2 |
        | 0.8  -0.1  0.0  0.0  0.0  0.0  0.0  0.4  0.8 |
        | 0.9  -0.1  0.0  0.0  0.0  0.0  0.0  0.0  0.4 |
        └                                              ┘

IPVT     =  (9, 9, 9, 9, 9, 9, 9, 9, 9)
INFO     =  0
```

***Example 2:*** This example shows a factorization of a complex general matrix ***A*** of order 9.

*Call Statement and Input*

```
             M   N   A  LDA  IPVT  INFO
             |   |   |   |     |     |
CALL ZGETRF( 9 , 9 , A,  9 , IPVT, INFO )
```

```
        ┌                                                                                                          ┐
        | (2.0, 1.0) (2.4,-1.0) (2.8,-1.0) (3.2,-1.0)  (3.6,-1.0) (4.0,-1.0) (4.4,-1.0) (4.8,-1.0) (5.2,-1.0) |
        | (2.4, 1.0) (2.0, 1.0) (2.4,-1.0) (2.8,-1.0)  (3.2,-1.0) (3.6,-1.0) (4.0,-1.0) (4.4,-1.0) (4.8,-1.0) |
        | (2.8, 1.0) (2.4, 1.0) (2.0, 1.0) (2.4,-1.0)  (2.8,-1.0) (3.2,-1.0) (3.6,-1.0) (4.0,-1.0) (4.4,-1.0) |
        | (3.2, 1.0) (2.8, 1.0) (2.4, 1.0) (2.0, 1.0)  (2.4,-1.0) (2.8,-1.0) (3.2,-1.0) (3.6,-1.0) (4.0,-1.0) |
A    =  | (3.6, 1.0) (3.2, 1.0) (2.8, 1.0) (2.4, 1.0)  (2.0, 1.0) (2.4,-1.0) (2.8,-1.0) (3.2,-1.0) (3.6,-1.0) |
        | (4.0, 1.0) (3.6, 1.0) (3.2, 1.0) (2.8, 1.0)  (2.4, 1.0) (2.0, 1.0) (2.4,-1.0) (2.8,-1.0) (3.2,-1.0) |
        | (4.4, 1.0) (4.0, 1.0) (3.6, 1.0) (3.2, 1.0)  (2.8, 1.0) (2.4, 1.0) (2.0, 1.0) (2.4,-1.0) (2.8,-1.0) |
        | (4.8, 1.0) (4.4, 1.0) (4.0, 1.0) (3.6, 1.0)  (3.2, 1.0) (2.8, 1.0) (2.4, 1.0) (2.0, 1.0) (2.4,-1.0) |
        | (5.2, 1.0) (4.8, 1.0) (4.4, 1.0) (4.0, 1.0)  (3.6, 1.0) (3.2, 1.0) (2.8, 1.0) (2.4, 1.0) (2.0, 1.0) |
        └                                                                                                          ┘
```

*Output*

```
        ┌                                                                                                        ┐
        │ (5.2, 1.0) (4.8, 1.0) (4.4, 1.0)   (4.0, 1.0)   (3.6, 1.0)  (3.2, 1.0) (2.8, 1.0) (2.4, 1.0) (2.0, 1.0) │
        │ (0.4, 0.1) (0.6,-2.0) (1.1,-1.9)   (1.7,-1.9)   (2.3,-1.8)  (2.8,-1.8) (3.4,-1.7) (3.9,-1.7) (4.5,-1.6) │
        │ (0.5, 0.1) (0.0,-0.1) (0.6,-1.9)   (1.2,-1.8)   (1.8,-1.7)  (2.5,-1.6) (3.1,-1.5) (3.7,-1.4) (4.3,-1.3) │
        │ (0.6, 0.1) (0.0,-0.1) (-0.1,-0.1)  (0.7,-1.9)   (1.3,-1.7)  (2.0,-1.6) (2.7,-1.5) (3.4,-1.4) (4.0,-1.2) │
A    =  │ (0.6, 0.1) (0.0,-0.1) (-0.1,-0.1) (-0.1, 0.0)   (0.7,-1.9)  (1.5,-1.7) (2.2,-1.6) (2.9,-1.5) (3.7,-1.3) │
        │ (0.7, 0.1) (0.0,-0.1)  (0.0, 0.0) (-0.1, 0.0)  (-0.1, 0.0)  (0.8,-1.9) (1.6,-1.8) (2.4,-1.6) (3.2,-1.5) │
        │ (0.8, 0.0) (0.0, 0.0)  (0.0, 0.0)  (0.0, 0.0)   (0.0, 0.0)  (0.0, 0.0) (0.8,-1.9) (1.7,-1.8) (2.5,-1.8) │
        │ (0.9, 0.0) (0.0, 0.0)  (0.0, 0.0)  (0.0, 0.0)   (0.0, 0.0)  (0.0, 0.0) (0.0, 0.0) (0.8,-2.0) (1.7,-1.9) │
        │ (0.9, 0.0) (0.0, 0.0)  (0.0, 0.0)  (0.0, 0.0)   (0.0, 0.0)  (0.0, 0.0) (0.0, 0.0) (0.0, 0.0) (0.8,-2.0) │
        └                                                                                                        ┘

                       IPVT      =  (9, 9, 9, 9, 9, 9, 9, 9, 9)
                       INFO      =  0
```

## SGETRS, DGETRS, CGETRS, and ZGETRS—General Matrix Multiple Right-Hand Side Solve

SGETRS and DGETRS solve one of the following systems of equations for multiple right-hand sides:

1. $\boldsymbol{AX} = \boldsymbol{B}$
2. $\boldsymbol{A}^{T}\boldsymbol{X} = \boldsymbol{B}$

CGETRS and ZGETRS solve one of the following systems of equations for multiple right-hand sides:

1. $\boldsymbol{AX} = \boldsymbol{B}$
2. $\boldsymbol{A}^{T}\boldsymbol{X} = \boldsymbol{B}$
3. $\boldsymbol{A}^{H}\boldsymbol{X} = \boldsymbol{B}$

In the formulas above:

***A*** represents the general matrix ***A*** containing the ***LU*** factorization.
***B*** represents the general matrix ***B*** containing the right-hand sides in its columns.
***X*** represents the general matrix ***B*** containing the solution vectors in its columns.

These subroutines use the results of the factorization of matrix ***A***, produced by a preceding call to SGETRF, DGETRF, CGETRF, or ZGETRF, respectively.

*Table 91. Data Types*

| ***A***, ***B*** | **Subroutine** |
|---|---|
| Short-precision real | SGETRS |
| Long-precision real | DGETRS |
| Short-precision complex | CGETRS |
| Long-precision complex | ZGETRS |

**Note:** The input to these solve subroutines must be the output from the factorization subroutines SGETRF, DGETRF, CGETRF and ZGETRF, respectively.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SGETRS \| DGETRS \| CGETRS \| ZGETRS (*transa*, *n*, *nrhs*, *a*, *lda*, *ipvt*, *bx*, *ldb*, *info*) |
| **C and C++** | sgetrs \| dgetrs \| cgetrs \| zgetrs (*transa*, *n*, *nrhs*, *a*, *lda*, *ipvt*, *bx*, *ldb*, *info*); |
| **PL/I** | CALL SGETRS \| DGETRS \| CGETRS \| ZGETRS (*transa*, *n*, *nrhs*, *a*, *lda*, *ipvt*, *bx*, *ldb*, *info*); |

*On Entry*

*transa*
indicates the form of matrix ***A*** to use in the computation, where:

If *transa* = `'N'`, ***A*** is used in the computation, resulting in solution 1.

If *transa* = `'T'`, $\boldsymbol{A}^{T}$ is used in the computation, resulting in solution 2.

If *transa* = `'C'`, $\boldsymbol{A}^{H}$ is used in the computation, resulting in solution 3.

Specified as: a single character; *transa* = 'N', 'T', or 'C'.

*n*

is the order of factored matrix ***A*** and the number of rows in matrix ***B***. Specified as: a fullword integer; $n \geq 0$.

*nrhs*

the number of right-hand sides—that is, the number of columns in matrix ***B*** used in the computation. Specified as: a fullword integer; $nrhs \geq 0$.

*a*

is the factorization of matrix ***A***, produced by a preceding call to SGETRF, DGETRF, CGETRF, or ZGETRF, respectively. Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 91 on page 483.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; $lda > 0$ and $lda \geq n$.

*ipvt*

is the integer vector ***ipvt*** of length *n*, containing the pivot indices produced by a preceding call to SGETRF, DGETRF, CGETRF, or ZGETRF, respectively. Specified as: a one-dimensional array of (at least) length *n*, containing fullword integers, where $1 \leq ipvt(i) \leq n$.

*bx*

is the general matrix ***B*** containing the right-hand side of the system. Specified as: an *ldb* by (at least) *nrhs* array, containing numbers of the data type indicated in Table 91 on page 483.

*ldb*

is the leading dimension of the array specified for *b*. Specified as: a fullword integer; $ldb > 0$ and $ldb \geq n$.

*info*

See “On Return.”

*On Return*

*bx*

is the solution ***X*** containing the results of the computation. Returned as: an *ldb* by (at least) *nrhs* array, containing numbers of the data type indicated in Table 91 on page 483.

*info*

*info* has the following meaning:

If *info* = 0, the solve of general matrix ***A*** completed successfully.

***Notes***

1. In your C program, argument *info* must be passed by reference.
2. These subroutines accept lower case letters for the *transa* argument.
3. For SGETRS and DGETRS, if you specify 'C' for the *transa* argument, it is interpreted as though you specified 'T'.
4. The scalar data specified for input argument *n* must be the same for both _GETRF and _GETRS. In addition, the scalar data specified for input argument *m* in _GETRF **must be the same** as input argument *n* in both _GETRF and _GETRS.

   If, however, you do **not** plan to call _GETRS after calling _GETRF, then input arguments *m* and *n* in _GETRF do not need to be equal.

5. The array data specified for input arguments *a* and *ipvt* for these subroutines must be the same as the corresponding output arguments for SGETRF, DGETRF, CGETRF, and ZGETRF, respectively.
6. The matrices and vector used in this computation must have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.
7. On both input and output, matrices ***A*** and ***B*** conform to LAPACK format.

***Function:*** One of the following systems of equations is solved for multiple right-hand sides:

1. $\boldsymbol{AX} = \boldsymbol{B}$
2. $\boldsymbol{A}^T\boldsymbol{X} = \boldsymbol{B}$
3. $\boldsymbol{A}^H\boldsymbol{X} = \boldsymbol{B}$ (only for CGETRS and ZGETRS)

where ***A***, ***B***, and ***X*** are general matrices. These subroutines uses the results of the factorization of matrix ***A***, produced by a preceding call to SGETRF, DGETRF, CGETRF or ZGETRF, respectively. For details on the factorization, see “SGETRF, DGETRF, CGETRF and ZGETRF—General Matrix Factorization” on page 479.

If $n = 0$ or $nrhs = 0$, no computation is performed and the subroutine returns after doing some parameter checking. See references [36] and [60 ].

### *Error Conditions*

*Computational Errors:* None

**Note:** If the factorization performed by SGETRF, DGETRF, CGETRF or ZGETRF failed because a pivot element is zero, the results returned by this subroutine are unpredictable, and there may be a divide-by-zero program exception message.

*Input-Argument Errors*

1. *transa* ≠ 'N', 'T', or 'C'
2. $n < 0$
3. $nrhs < 0$
4. $n > lda$
5. $n > ldb$
6. $lda \le 0$
7. $ldb \le 0$

***Example 1:*** This example shows how to solve the system $\boldsymbol{AX} = \boldsymbol{B}$, where matrix ***A*** is the same matrix factored in the “Example 1” on page 481 for DGETRF.

*Call Statement and Input*

```
            TRANSA  N  NRHS  A  LDA  IPIV  BX LDB  INFO
              |     |    |   |   |     |   |   |     |
CALL DGETRS('N' ,  9 ,  5 , A , 9 , IPIV, B , 9 , INFO)

IPVT     =  (9, 9, 9, 9, 9, 9, 9, 9, 9)
```

A = (same as output A in “Example 1” on page 481)

```
              ┌                                     ┐
              │ 93.0  186.0  279.0  372.0  465.0 │
              │ 84.4  168.8  253.2  337.6  422.0 │
              │ 76.6  153.2  229.8  306.4  383.0 │
              │ 70.0  140.0  210.0  280.0  350.0 │
B     =       │ 65.0  130.0  195.0  260.0  325.0 │
              │ 62.0  124.0  186.0  248.0  310.0 │
              │ 61.4  122.8  184.2  245.6  307.0 │
              │ 63.6  127.2  190.8  254.4  318.0 │
              │ 69.0  138.0  207.0  276.0  345.0 │
              └                                     ┘
```

*Output*

```
              ┌                              ┐
              │ 1.0   2.0   3.0   4.0   5.0 │
              │ 2.0   4.0   6.0   8.0  10.0 │
              │ 3.0   6.0   9.0  12.0  15.0 │
              │ 4.0   8.0  12.0  16.0  20.0 │
B     =       │ 5.0  10.0  15.0  20.0  25.0 │
              │ 6.0  12.0  18.0  24.0  30.0 │
              │ 7.0  14.0  21.0  28.0  35.0 │
              │ 8.0  16.0  24.0  32.0  40.0 │
              │ 9.0  18.0  27.0  36.0  45.0 │
              └                              ┘

INFO      =  0
```

***Example 2:*** This example shows how to solve the system ***AX*** = ***b***, where matrix ***A*** is the same matrix factored in the “Example 2” on page 481 for ZGETRF.

*Call Statement and Input*

```
           TRANS   N  NRHS  A  LDA  IPIV  B  LDB  INFO
             |     |    |   |   |     |   |   |     |
CALL ZGETRS('N' ,  9 ,  5 , A , 9 , IPIV, B , 9 , INFO)

IPVT      =  (9, 9, 9, 9, 9, 9, 9, 9, 9)
A         = (same as output A in “Example 2” on page 481)
```

```
          ┌                                                                                  ┐
          │ (193.0,-10.6)  (200.0, 21.8)  (207.0, 54.2)  (214.0, 86.6)  (221.0,119.0) │
          │ (173.8, -9.4)  (178.8, 20.2)  (183.8, 49.8)  (188.8, 79.4)  (193.8,109.0) │
          │ (156.2, -5.4)  (159.2, 22.2)  (162.2, 49.8)  (165.2, 77.4)  (168.2,105.0) │
          │ (141.0,  1.4)  (142.0, 27.8)  (143.0, 54.2)  (144.0, 80.6)  (145.0,107.0) │
B     =   │ (129.0, 11.0)  (128.0, 37.0)  (127.0, 63.0)  (126.0, 89.0)  (125.0,115.0) │
          │ (121.0, 23.4)  (118.0, 49.8)  (115.0, 76.2)  (112.0,102.6)  (109.0,129.0) │
          │ (117.8, 38.6)  (112.8, 66.2)  (107.8, 93.8)  (102.8,121.4)   (97.8,149.0) │
          │ (120.2, 56.6)  (113.2, 86.2)  (106.2,115.8)   (99.2,145.4)   (92.2,175.0) │
          │ (129.0, 77.4)  (120.0,109.8)  (111.0,142.2). (102.0,174.6)   (93.0,207.0) │
          └                                                                                  ┘
```

*Output*

```
          ┌                                                         ┐
          | (1.0,1.0)  (1.0,2.0)  (1.0,3.0) (1.0,4.0) (1.0,5.0) |
          | (2.0,1.0)  (2.0,2.0)  (2.0,3.0) (2.0,4.0) (2.0,5.0) |
          | (3.0,1.0)  (3.0,2.0)  (3.0,3.0) (3.0,4.0) (3.0,5.0) |
          | (4.0,1.0)  (4.0,2.0)  (4.0,3.0) (4.0,4.0) (4.0,5.0) |
B       = | (5.0,1.0)  (5.0,2.0)  (5.0,3.0) (5.0,4.0) (5.0,5.0) |
          | (6.0,1.0)  (6.0,2.0)  (6.0,3.0) (6.0,4.0) (6.0,5.0) |
          | (7.0,1.0)  (7.0,2.0)  (7.0,3.0) (7.0,4.0) (7.0,5.0) |
          | (8.0,1.0)  (8.0,2.0)  (8.0,3.0) (8.0,4.0) (8.0,5.0) |
          | (9.0,1.0)  (9.0,2.0)  (9.0,3.0) (9.0,4.0) (9.0,5.0) |
          └                                                         ┘

INFO      =  0
```

# SGEFCD and DGEFCD—General Matrix Factorization, Condition Number Reciprocal, and Determinant

These subroutines factor general matrix ***A*** using Gaussian elimination. An estimate of the reciprocal of the condition number and the determinant of matrix ***A*** can also be computed. To solve a system of equations with one or more right-hand sides, follow the call to these subroutines with one or more calls to SGES/SGESM or DGES/DGESM, respectively. To compute the inverse of matrix ***A***, follow the call to these subroutines with a call to SGEICD and DGEICD, respectively.

*Table 92. Data Types*

| ***A***, *aux*, *rcond*, ***det*** | **Subroutine** |
|---|---|
| Short-precision real | SGEFCD |
| Long-precision real | DGEFCD |

**Note:** The output from these factorization subroutines should be used only as input to the following subroutines for performing a solve or inverse: SGES/SGESM/SGEICD and DGES/DGESM/DGEICD, respectively.

***Syntax***

| | |
|---|---|
| **Fortran** | CALL SGEFCD \| DGEFCD (*a*, *lda*, *n*, *ipvt*, *iopt*, *rcond*, *det*, *aux*, *naux*) |
| **C and C++** | sgefcd \| dgefcd (*a*, *lda*, *n*, *ipvt*, *iopt*, *rcond*, *det*, *aux*, *naux*); |
| **PL/I** | CALL SGEFCD \| DGEFCD (*a*, *lda*, *n*, *ipvt*, *iopt*, *rcond*, *det*, *aux*, *naux*); |

*On Entry*

*a*
is a general matrix ***A*** of order *n*, whose factorization, reciprocal of condition number, and determinant are computed. Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 92.

*lda*
is the leading dimension of the array specified for *a*. Specified as: a fullword integer; $lda > 0$ and $lda \geq n$.

*n*
is the order of matrix ***A***. Specified as: a fullword integer; $0 \leq n \leq lda$.

*ipvt*
See "On Return" on page 489.

*iopt*
indicates the type of computation to be performed, where:

If $iopt = 0$, the matrix is factored.

If $iopt = 1$, the matrix is factored, and the reciprocal of the condition number is computed.

If $iopt = 2$, the matrix is factored, and the determinant is computed.

If $iopt = 3$, the matrix is factored, and the reciprocal of the condition number and the determinant are computed.

Specified as: a fullword integer; $iopt = 0$, 1, 2, or 3.

*rcond*
See "On Return" on page 489.

*det*

See “On Return” on page 489.

*aux*

has the following meaning:

If *naux = 0* and error 2015 is unrecoverable, *aux* is ignored.

Otherwise, it is is a storage work area used by this subroutine. Its size is specified by *naux*.

Specified as: an area of storage, containing numbers of the data type indicated in Table 92 on page 488.

*naux*

is the size of the work area specified by *aux*—that is, the number of elements in *aux*. Specified as: a fullword integer, where:

If *naux* = 0 and error 2015 is unrecoverable, SGEFCD and DGEFCD dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, *naux* ≥ *n*.

*On Return*

*a*

is the transformed matrix ***A*** of order *n*, containing the results of the factorization. See “Function” on page 490. Returned as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 92 on page 488.

*ipvt*

is the integer vector ***ipvt*** of length *n*, containing the pivot indices. Returned as: a one-dimensional array of (at least) length *n*, containing fullword integers.

*rcond*

is an estimate of the reciprocal of the condition number, *rcond*, of matrix ***A***. Returned as: a number of the data type indicated in Table 92 on page 488; *rcond* ≥ 0.

*det*

is the vector ***det***, containing the two components, $det_1$ and $det_2$, of the determinant of matrix ***A***. The determinant is:

$$det_1\left(10^{det_2}\right)$$

where $1 \le det_1 < 10$. Returned as: an array of length 2, containing numbers of the data type indicated in Table 92 on page 488.

***Notes***

1. In your C program, argument *rcond* must be passed by reference.
2. When *iopt* = 0, these subroutines provide the same function as a call to SGEF or DGEF, respectively.
3. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see “Using Auxiliary Storage in ESSL” on page 32.
4. On both input and output, matrix **A** conforms to LAPACK format.

***Function:*** Matrix ***A*** is factored using Gaussian elimination with partial pivoting (**ipvt**) to compute the ***LU*** factorization of ***A***, where (***A=PLU***):

***L*** is a unit lower triangular matrix.
***U*** is an upper triangular matrix.
***P*** is the permutation matrix.

On output, the transformed matrix ***A*** contains ***U*** in the upper triangle and ***L*** in the strict lower triangle where ***ipvt*** contains the pivots representing permutation ***P***, such that ***A*** = ***PLU***.

An estimate of the reciprocal of the condition number, *rcond*, and the determinant, ***det***, can also be computed by this subroutine. The estimate of the condition number uses an enhanced version of the algorithm described in references [66] and [67 ].

If *n* is 0, no computation is performed. See reference [36 ].

These subroutines call SGEF and DGEF, respectively, to perform the factorization. ***ipvt*** is an output vector of SGEF and DGEF. It is returned for use by SGES/SGESM and DGES/DGESM, the solve subroutines.

### *Error Conditions*

*Resource Errors:* Error 2015 is unrecoverable, *naux* = 0, and unable to allocate work area.

*Computational Errors:* Matrix ***A*** is singular.

- If your program is not terminated by SGEF and DGEF, then SGEFCD and DGEFCD, respectively, return 0 for *rcond* and *det*.
- One or more columns of ***L*** and the corresponding diagonal of ***U*** contain all zeros (all columns of ***L*** are checked). The first column, *i*, of ***L*** with a corresponding ***U*** = 0 diagonal element is identified in the computational error message, issued by SGEF or DGEF, respectively.
- *i* can be determined at run time by using the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error code 2103 in the ESSL error option table; otherwise, the default value causes your program to be terminated by SGEF or DGEF, respectively, when this error occurs. If your program is not terminated by SGEF or DGEF, respectively, the return code is set to 2. For details, see “What Can You Do about ESSL Computational Errors?” on page 48.

*Input-Argument Errors*

1. $lda \leq 0$
2. $n < 0$
3. $n > lda$
4. $iopt \neq$ 0, 1, 2, or 3
5. Error 2015 is recoverable or $naux \neq 0$, and *naux* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example:*** This example shows a factorization of matrix ***A*** of order 9. The input is the same as used in SGEF and DGEF. See “Example 1” on page 467. The reciprocal of the condition number and the determinant of matrix ***A*** are also computed. The values used to estimate the reciprocal of the condition number in this example are obtained with the following values:

$\|\boldsymbol{A}\|_1$ = max(6.0, 8.0, 10.0, 12.0, 13.0, 14.0, 15.0, 15.0, 15.0) = 15.0

Estimate of $\|\boldsymbol{A}^{-1}\|_1$ = 1091.87

This estimate is equal to the actual *rcond* of 5.436($10^{-5}$), which is computed by SGEICD and DGEICD. (See “Example 1” on page 518.) On output, the value in ***det***, |***A***|, is equal to 336.

*Call Statement and Input*

```
             A  LDA  N   IPVT  IOPT  RCOND   DET   AUX  NAUX
             |   |   |    |     |     |       |     |    |
CALL DGEFCD( A , 9 , 9 , IPVT , 3  , RCOND , DET , AUX , 9  )
```

A         =(same as input A in “Example 1” on page 467)

*Output*

```
A         =(same as output A in “Example 1” on page 467)
IPVT      =  (3, 4, 5, 6, 7, 8, 9, 8, 9)
RCOND     =  0.00005436
DET       =  (3.36, 2.00)
```

## SPPF, DPPF, SPOF, DPOF, CPOF, and ZPOF—Positive Definite Real Symmetric or Complex Hermitian Matrix Factorization

The SPPF and DPPF subroutines factor positive definite symmetric matrix ***A***, stored in lower-packed storage mode, using Gaussian elimination ($\boldsymbol{LDL}^T$) or the Cholesky factorization method. To solve a system of equations with one or more right-hand sides, follow the call to these subroutines with one or more calls to SPPS or DPPS, respectively. To find the inverse of matrix ***A***, follow the call to these subroutines, performing Cholesky factorization, with a call to SPPICD or DPPICD, respectively.

The SPOF, DPOF, CPOF, and ZPOF subroutines factor matrix ***A*** stored in upper or lower storage mode, where:

- For SPOF and DPOF, ***A*** is a positive definite symmetric matrix.
- For CPOF and ZPOF, ***A*** is a positive definite complex Hermitian matrix.

Matrix ***A*** is factored using Cholesky factorization, ($\boldsymbol{LL}^T$ or $\boldsymbol{U}^T\boldsymbol{U}$ for SPOF and DPOF and $\boldsymbol{LL}^H$ or $\boldsymbol{U}^H\boldsymbol{U}$ for CPOF and ZPOF). To solve the system of equations with one or more right-hand sides, follow the call to these subroutines with a call to SPOSM, DPOSM, CPOSM, or ZPOSM. To find the inverse of matrix ***A***, follow the call to SPOF or DPOF with a call to SPOICD or DPOICD.

*Table 93. Data Types*

| ***A*** | **Subroutine** |
|---|---|
| Short-precision real | SPPF and SPOF |
| Long-precision real | DPPF and DPOF |
| Short-precision complex | CPOF |
| Long-precision complex | ZPOF |

**Note:** The output from SPPF and DPPF should be used only as input to the following subroutines for performing a solve or inverse: SPPS/SPPICD and DPPS/DPPICD, respectively. The output from SPOF, DPOF, CPOF, and ZPOF should be used only as input to the following subroutines for performing a solve or inverse: SPOSM/SPOICD, DPOSM/DPOICD, CPOSM, and ZPOSM, respectively.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SPPF \| DPPF (*ap*, *n*, *iopt*)<br>CALL SPOF \| DPOF \| CPOF \| ZPOF (*uplo*, *a*, *lda*, *n*) |
| **C and C++** | sppf \| dppf (*ap*, *n*, *iopt*);<br>spof \| dpof \| cpof \| zpof (*uplo*, *a*, *lda*, *n*); |
| **PL/I** | CALL SPPF \| DPPF (*ap*, *n*, *iopt*);<br>CALL SPOF \| DPOF \| CPOF \| ZPOF (*uplo*, *a*, *lda*, *n*); |

*On Entry*

*uplo*

indicates whether matrix ***A*** is stored in upper or lower storage mode, where:

If *uplo* = `'U'`, ***A*** is stored in upper storage mode.

If *uplo* = 'L', ***A*** is stored in lower storage mode.

Specified as: a single character. It must be 'U' or 'L'.

*ap*

is array, referred to as AP, in which matrix ***A***, to be factored, is stored in lower-packed storage mode.

Specified as: a one-dimensional array, containing numbers of the data type indicated in Table 93 on page 492. See Notes.

If *iopt* = 0, the array must have at least *n*(*n*+1)/2+*n* elements.

If *iopt* = 1, the array must have at least *n*(*n*+1)/2 elements.

*a*

is the positive definite matrix ***A***, to be factored.

Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 93 on page 492.

*lda*

is the leading dimension of the array specified for *a*.

Specified as: a fullword integer; *lda* > 0 and *lda* ≥ *n*.

*n*

is the order *n* of matrix ***A***.

Specified as: a fullword integer; *n* ≥ 0.

*iopt*

determines the type of computation to be performed, where:

If *iopt* = 0, the matrix is factored using the $\boldsymbol{LDL}^{T}$ method.

If *iopt* = 1, the matrix is factored using Cholesky factorization.

Specified as: a fullword integer; *iopt* = 0 or 1.

*On Return*

*ap*

is the transformed matrix ***A*** of order *n*, containing the results of the factorization. See “Notes” and “Function” on page 494.

Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 93 on page 492.

If *iopt* = 0, the array contains *n*(*n*+1)/2+*n* elements.

If *iopt* = 1, the array contains *n*(*n*+1)/2 elements.

*a*

is the transformed matrix ***A*** of order *n*, containing the results of the factorization. See “Function” on page 494.

Returned as: a two-dimensional array, containing numbers of the data type indicated in Table 93 on page 492.

***Notes***

1. All subroutines accept lowercase letters for the *uplo* argument.
2. In the input and output arrays specified for *ap*, the first *n*(*n*+1)/2 elements are matrix elements. The additional *n* locations, required in the array when *iopt* = 0, are used for working storage by this subroutine and should not be altered between calls to the factorization and solve subroutines.

3. On input, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values. On output, they are set to zero.
4. The serial and SMP performance of SPPF and DPPF has been improved in many cases by utilizing new algorithms based on recursive packed storage format. As a result, on output, the array specified for AP may be stored in this new format rather than the conventional lower packed format. (See references [50], [63 ], and [64 ]).

   The array specified for AP should not be altered between calls to the factorization and solve subroutines; otherwise unpredictable results may occur.
5. For a description of the storage modes used for the matrices, see:
   - For positive definite symmetric matrices, see “Positive Definite or Negative Definite Symmetric Matrix” on page 69.
   - For positive definite complex Hermitian matrices, see “Positive Definite or Negative Definite Complex Hermitian Matrix” on page 71.

***Function:*** The functions for these subroutines are described in the sections below.

*For SPPF and DPPF:* If *iopt* = 0, the positive definite symmetric matrix ***A***, stored in lower-packed storage mode, is factored using Gaussian elimination, where ***A*** is expressed as:

$\boldsymbol{A} = \boldsymbol{LDL}^T$

where:

***L*** is a unit lower triangular matrix.
$\boldsymbol{L}^T$ is the transpose of matrix ***L***.
***D*** is a diagonal matrix.

If *iopt* = 1, the positive definite symmetric matrix ***A*** is factored using Cholesky factorization, where ***A*** is expressed as:

$\boldsymbol{A} = \boldsymbol{LL}^T$

where ***L*** is a lower triangular matrix.

If *n* is 0, no computation is performed. See references [36 ] and [38 ].

*For SPOF, DPOF, CPOF, and ZPOF:* The positive definite matrix ***A***, stored in upper or lower storage mode, is factored using Cholesky factorization, where ***A*** is expressed as:

$\boldsymbol{A} = \boldsymbol{LL}^T$ or $\boldsymbol{A} = \boldsymbol{U}^T\boldsymbol{U}$ for SPOF and DPOF
$\boldsymbol{A} = \boldsymbol{LL}^H$ or $\boldsymbol{A} = \boldsymbol{U}^H\boldsymbol{U}$ for CPOF and ZPOF

where:

***L*** is a lower triangular matrix.
$\boldsymbol{L}^T$ is the transpose of matrix ***L***.
$\boldsymbol{L}^H$ is the conjugate transpose of matrix ***L***.
***U*** is an upper triangular matrix.
$\boldsymbol{U}^T$ is the transpose of matrix ***U***.
$\boldsymbol{U}^H$ is the conjugate transpose of matrix ***U***.

If *n* is 0, no computation is performed. See references [8 ], [67 ], and [36 ].

### *Error Conditions*

*Resource Errors:* Unable to allocate internal work area.

*Computational Errors*

1. Matrix ***A*** is not positive definite (for SPPF and DPPF when $iopt = 0$).
   - Processing continues to the end of the matrix.
   - One or more elements of ***D*** contain values less than or equal to 0; all elements of ***D*** are checked. The index *i* of the **last** nonpositive element encountered is identified in the computational error message.
   - The return code is set to 1.
   - *i* can be determined at run time by use of the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error code 2104 in the ESSL error option table; otherwise, the default value causes your program to terminate when this error occurs. For details, see "What Can You Do about ESSL Computational Errors?" on page 48.
2. Matrix ***A*** is not positive definite (for SPPF and DPPF when $iopt = 1$ and for SPOF, DPOF, CPOF, and ZPOF).
   - Processing stops at the first occurrence of a nonpositive definite diagonal element.
   - The order *i* of the **first** minor encountered having a nonpositive determinant is identified in the computational error message.
   - The return code is set to 1.
   - *i* can be determined at run time by use of the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error code 2115 in the ESSL error option table; otherwise, the default value causes your program to terminate when this error occurs. For details, see "What Can You Do about ESSL Computational Errors?" on page 48.

*Input-Argument Errors*

1. $n < 0$
2. $iopt \neq 0$ or 1
3. $uplo \neq$ `'U'` or `'L'`
4. $lda \leq 0$
5. $n > lda$

***Example 1:*** This example shows a factorization of positive definite symmetric matrix ***A*** of order 9, stored in lower-packed storage mode, where on input matrix ***A*** is:

$$\begin{bmatrix}
1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\
1.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 \\
1.0 & 2.0 & 3.0 & 3.0 & 3.0 & 3.0 & 3.0 & 3.0 & 3.0 \\
1.0 & 2.0 & 3.0 & 4.0 & 4.0 & 4.0 & 4.0 & 4.0 & 4.0 \\
1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 5.0 & 5.0 & 5.0 & 5.0 \\
1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 & 6.0 & 6.0 & 6.0 \\
1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 & 7.0 & 7.0 & 7.0 \\
1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 & 7.0 & 8.0 & 8.0 \\
1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 & 7.0 & 8.0 & 9.0
\end{bmatrix}$$

On output, all elements of this matrix ***A*** are 1.0.

**Note:** The AP arrays are formatted in a triangular arrangement for readability; however, they are stored in lower-packed storage mode.

*Call Statement and Input*

```
          AP  N  IOPT
          |   |   |
CALL SPPF( AP, 9,  0 )
```

```
AP = (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0,
      2.0, 2.0, 2.0, 2.0, 2.0, 2.0, 2.0, 2.0,
      3.0, 3.0, 3.0, 3.0, 3.0, 3.0, 3.0,
      4.0, 4.0, 4.0, 4.0, 4.0, 4.0,
      5.0, 5.0, 5.0, 5.0, 5.0,
      6.0, 6.0, 6.0, 6.0,
      7.0, 7.0, 7.0,
      8.0, 8.0,
      9.0,
       . ,  . ,  . ,  . ,  . ,  . ,  . ,  . ,  .  )
```

*Output*

```
AP = (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0,
      1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0,
      1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0,
      1.0, 1.0, 1.0, 1.0, 1.0, 1.0,
      1.0, 1.0, 1.0, 1.0, 1.0,
      1.0, 1.0, 1.0, 1.0,
      1.0, 1.0, 1.0,
      1.0, 1.0,
      1.0,
      1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0)
```

***Example 2:*** This example shows a factorization of the same positive definite symmetric matrix ***A*** of order 9 used in Example 1, stored in lower-packed storage mode.

**Note:** The AP arrays are formatted in a triangular arrangement for readability; however, they are stored in lower-packed storage mode.

*Call Statement and Input*

```
          AP  N  IOPT
          |   |   |
CALL SPPF( AP, 9,  1 )
```

```
AP = (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0,
      2.0, 2.0, 2.0, 2.0, 2.0, 2.0, 2.0, 2.0,
      3.0, 3.0, 3.0, 3.0, 3.0, 3.0, 3.0,
      4.0, 4.0, 4.0, 4.0, 4.0, 4.0,
      5.0, 5.0, 5.0, 5.0, 5.0,
      6.0, 6.0, 6.0, 6.0,
      7.0, 7.0, 7.0,
      8.0, 8.0,
      9.0)
```

*Output*

```
AP = (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0,
      1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0,
      1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0,
      1.0, 1.0, 1.0, 1.0, 1.0, 1.0,
      1.0, 1.0, 1.0, 1.0, 1.0,
      1.0, 1.0, 1.0, 1.0,
      1.0, 1.0, 1.0,
      1.0, 1.0,
      1.0)
```

***Example 3:*** This example shows a factorization of the same positive definite symmetric matrix ***A*** of order 9 used in Example 1, but stored in lower storage mode.

*Call Statement and Input*

```
           UPLO  A  LDA  N
            |    |   |   |
CALL SPOF( 'L' , A , 9 , 9 )
```

```
        ┌                                            ┐
        | 1.0   .    .    .    .    .    .    .    . |
        | 1.0  2.0   .    .    .    .    .    .    . |
        | 1.0  2.0  3.0   .    .    .    .    .    . |
        | 1.0  2.0  3.0  4.0   .    .    .    .    . |
A    =  | 1.0  2.0  3.0  4.0  5.0   .    .    .    . |
        | 1.0  2.0  3.0  4.0  5.0  6.0   .    .    . |
        | 1.0  2.0  3.0  4.0  5.0  6.0  7.0   .    . |
        | 1.0  2.0  3.0  4.0  5.0  6.0  7.0  8.0   . |
        | 1.0  2.0  3.0  4.0  5.0  6.0  7.0  8.0  9.0 |
        └                                            ┘
```

*Output*

```
        ┌                                            ┐
        | 1.0   .    .    .    .    .    .    .    . |
        | 1.0  1.0   .    .    .    .    .    .    . |
        | 1.0  1.0  1.0   .    .    .    .    .    . |
        | 1.0  1.0  1.0  1.0   .    .    .    .    . |
A    =  | 1.0  1.0  1.0  1.0  1.0   .    .    .    . |
        | 1.0  1.0  1.0  1.0  1.0  1.0   .    .    . |
        | 1.0  1.0  1.0  1.0  1.0  1.0  1.0   .    . |
        | 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0   . |
        | 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0 |
        └                                            ┘
```

***Example 4:*** This example shows a factorization of the same positive definite symmetric matrix ***A*** of order 9 used in Example 1, but stored in upper storage mode.

*Call Statement and Input*

```
           UPLO  A  LDA  N
            |    |   |   |
CALL SPOF( 'U' , A , 9 , 9 )
```

```
        ┌                                             ┐
        │ 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0 │
        │  .   2.0  2.0  2.0  2.0  2.0  2.0  2.0  2.0 │
        │  .    .   3.0  3.0  3.0  3.0  3.0  3.0  3.0 │
        │  .    .    .   4.0  4.0  4.0  4.0  4.0  4.0 │
A    =  │  .    .    .    .   5.0  5.0  5.0  5.0  5.0 │
        │  .    .    .    .    .   6.0  6.0  6.0  6.0 │
        │  .    .    .    .    .    .   7.0  7.0  7.0 │
        │  .    .    .    .    .    .    .   8.0  8.0 │
        │  .    .    .    .    .    .    .    .   9.0 │
        └                                             ┘
```

*Output*

```
        ┌                                             ┐
        │ 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0 │
        │  .   1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0 │
        │  .    .   1.0  1.0  1.0  1.0  1.0  1.0  1.0 │
        │  .    .    .   1.0  1.0  1.0  1.0  1.0  1.0 │
A    =  │  .    .    .    .   1.0  1.0  1.0  1.0  1.0 │
        │  .    .    .    .    .   1.0  1.0  1.0  1.0 │
        │  .    .    .    .    .    .   1.0  1.0  1.0 │
        │  .    .    .    .    .    .    .   1.0  1.0 │
        │  .    .    .    .    .    .    .    .   1.0 │
        └                                             ┘
```

***Example 5:*** This example shows a factorization of positive definite complex Hermitian matrix ***A*** of order 3, stored in lower storage mode, where on input matrix ***A*** is:

```
              ┌                                           ┐
              │  (25.0, 0.0)  (-5.0, -5.0)  (10.0, 5.0)  │
              │  (-5.0, 5.0)   (51.0, 0.0)  (4.0, -6.0)  │
              │ (10.0, -5.0)    (4.0, 6.0)  (71.0, 0.0)  │
              └                                           ┘
```

**Note:** On input, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values. On output, they are set to zero.

*Call Statement and Input*

```
          UPLO  A  LDA  N
           |    |   |   |
CALL CPOF( 'L' , A , 3 , 3 )
```

```
        ┌                                        ┐
        │   (25.0, . )       .           .       │
A    =  │  (-5.0, 5.0) (51.0,  . )       .       │
        │ (10.0, -5.0)  (4.0, 6.0) (71.0, . )    │
        └                                        ┘
```

*Output*

```
        ┌                                       ┐
        │  (5.0, 0.0)       .           .       │
A    =  │ (-1.0, 1.0)  (7.0, 0.0)       .       │
        │ (2.0, -1.0)  (1.0, 1.0)  (8.0, 0.0)   │
        └                                       ┘
```

***Example 6:*** This example shows a factorization of positive definite complex Hermitian matrix ***A*** of order 3, stored in upper storage mode, where on input matrix ***A*** is:

```
┌                                    ┐
|  (9.0, 0.0)  (3.0, 3.0) (3.0, -3.0) |
| (3.0, -3.0) (18.0, 0.0) (8.0, -6.0) |
|  (3.0, 3.0)  (8.0, 6.0) (43.0, 0.0) |
└                                    ┘
```

**Note:** On input, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values. On output, they are set to zero.

*Call Statement and Input*

```
           UPLO  A  LDA  N
            |    |   |   |
CALL CPOF( 'U' , A , 3 , 3 )
```

```
        ┌                                    ┐
        | (9.0,  . )   (3.0,3.0)   (3.0,-3.0) |
A    =  |     .        (18.0, . )   (8.0,-6.0) |
        |     .             .      (43.0,  . ) |
        └                                    ┘
```

*Output*

```
        ┌                                        ┐
        | (3.0, 0.0)    (1.0, 1.0)  (1.0, -1.0) |
A    =  |     .         (4.0, 0.0)  (2.0, -1.0) |
        |     .             .        (6.0, 0.0) |
        └                                        ┘
```

## SPPS and DPPS—Positive Definite Real Symmetric Matrix Solve

These subroutines solve the system $\boldsymbol{Ax} = \boldsymbol{b}$ for $\boldsymbol{x}$, where $\boldsymbol{A}$ is a positive definite symmetric matrix, and $\boldsymbol{x}$ and $\boldsymbol{b}$ are vectors. The subroutines use the results of the factorization of matrix $\boldsymbol{A}$, produced by a preceding call to SPPF/SPPFCD or DPPF/DPPFP/DPPFCD, respectively.

*Table 94. Data Types*

| ***A***, ***b***, ***x*** | **Subroutine** |
|---|---|
| Short-precision real | SPPS |
| Long-precision real | DPPS |

**Note:** The input to these solve subroutines must be the output from the factorization subroutines SPPF/SPPFCD and DPPF/DPPFP/DPPFCD, respectively.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SPPS \| DPPS (*ap*, *n*, *bx*, *iopt*) |
| **C and C++** | spps \| dpps (*ap*, *n*, *bx*, *iopt*); |
| **PL/I** | CALL SPPS \| DPPS (*ap*, *n*, *bx*, *iopt*); |

*On Entry*

*ap*

is the factorization of matrix $\boldsymbol{A}$, produced by a preceding call to SPPF/SPPFCD or DPPF/DPPFP/DPPFCD, respectively. Specified as: a one-dimensional array, containing numbers of the data type indicated in Table 94, where:

If $iopt = 0$, the array must contain $n(n+1)/2+n$ elements.

If $iopt = 1$, the array must contain $n(n+1)/2$ elements.

*n*

is the order of matrix $\boldsymbol{A}$ used in the factorization, and the lengths of vectors $\boldsymbol{b}$ and $\boldsymbol{x}$. Specified as: a fullword integer; $n \geq 0$.

*bx*

is the vector $\boldsymbol{b}$ of length $n$, containing the right-hand side of the system. Specified as: a one-dimensional array of (at least) length $n$, containing numbers of the data type indicated in Table 94.

*iopt*

indicates the type of factorization that was performed on matrix $\boldsymbol{A}$, where:

If $iopt = 0$, the matrix was factored using the $\boldsymbol{LDL}^{\mathrm{T}}$ method.

If $iopt = 1$, the matrix was factored using Cholesky factorization.

Specified as: a fullword integer; $iopt = 0$ or 1.

*On Return*

*bx*

is the solution vector $\boldsymbol{x}$ of length $n$, containing the results of the computation. Specified as: a one-dimensional array, containing numbers of the data type indicated in Table 94.

***Notes***

1. The array data specified for input argument *ap* for these subroutines must be the same as the corresponding output argument for SPPF/SPPFCD and DPPF/DPPFP/DPPFCD, respectively.
2. The scalar data specified for input argument *n* for these subroutines must be the same as that specified for SPPF/SPPFCD and DPPF/DPPFP/DPPFCD, respectively.
3. When you call these subroutines after calling SPPF or DPPF, the value of input argument *iopt* must be the same as that specified for SPPF and DPPF.
4. When you call these subroutines after calling SPPFCD or DPPFCD, the value of input argument *iopt* must be 0.
5. When you call these subroutines after calling DPPFP, the value of input argument *iopt* must be 1.
6. In the input array specified for *ap*, the first $n(n+1)/2$ elements are matrix elements. The additional *n* locations, required in the array when $iopt = 0$, are used for working storage by this subroutine and should not be altered between calls to the factorization and solve subroutines.
7. The vectors and matrices used in this computation must have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.
8. For a description of how a positive definite symmetric matrix is stored in lower-packed storage mode in an array, see “Symmetric Matrix” on page 65.

***Function:*** The system $\boldsymbol{Ax} = \boldsymbol{b}$ is solved for $\boldsymbol{x}$, where $\boldsymbol{A}$ is a positive definite symmetric matrix, stored in lower-packed storage mode in array AP, and $\boldsymbol{x}$ and $\boldsymbol{b}$ are vectors. These subroutines use the results of the factorization of matrix $\boldsymbol{A}$, produced by a preceding call to SPPF/SPPFCD or DPPF/DPPFP/DPPFCD, respectively.

If *n* is 0, no computation is performed. See references [36 ] and [38 ].

***Error Conditions***

*Computational Errors:* None

**Note:** If a call to SPPF, DPPF, SPPFCD, DPPFCD, or DPPFP resulted in a nonpositive definite matrix, error 2104 or 2115, SPPS or DPPS results may be unpredictable or numerically unstable.

*Input-Argument Errors*

1. $n < 0$
2. $iopt \neq 0$ or 1

***Example 1:*** This example shows how to solve the system $\boldsymbol{Ax} = \boldsymbol{b}$, where matrix $\boldsymbol{A}$ is the same matrix factored in the “Example 1” on page 495 for SPPF and DPPF.

*Call Statement and Input*

```
            AP   N   BX   IOPT
            |    |   |     |
CALL SPPS ( AP , 9 , BX ,  0 )
```

```
AP       =(same as output AP in “Example 1” on page 495 for SPPF and DPPF)
BX       =  (9.0, 17.0, 24.0, 30.0, 35.0, 39.0, 42.0, 44.0, 45.0)
```

*Output*

```
BX       =  (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0)
```

***Example 2:*** This example shows how to solve the same system as in Example 1, where matrix ***A*** is the same matrix factored in the “Example 2” on page 496 for SPPF and DPPF.

*Call Statement and Input*

```
           AP   N   BX  IOPT
           |    |   |    |
CALL SPPS( AP , 9 , BX , 1 )
```

```
AP       =(same as output AP in “Example 2” on page 496 for SPPF and DPPF)
BX       =  (9.0, 17.0, 24.0, 30.0, 35.0, 39.0, 42.0, 44.0, 45.0)
```

*Output*

```
BX       =  (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0)
```

# SPOSM, DPOSM, CPOSM, and ZPOSM—Positive Definite Real Symmetric or Complex Hermitian Matrix Multiple Right-Hand Side Solve

These subroutines solve the system $\boldsymbol{AX} = \boldsymbol{B}$ for $\boldsymbol{X}$, using multiple right-hand sides, where $\boldsymbol{X}$ and $\boldsymbol{B}$ are general matrices and:

- For SPOSM and DPOSM, $\boldsymbol{A}$ is a positive definite symmetric matrix.
- For CPOSM and ZPOSM, $\boldsymbol{A}$ is a positive definite complex Hermitian matrix.

These subroutines use the results of the factorization of matrix $\boldsymbol{A}$, produced by a preceding call to SPOF/SPOFCD, DPOF/DPOFCD, CPOF, or ZPOF, respectively.

*Table 95. Data Types*

| ***A***, ***B***, ***X*** | **Subroutine** |
|---|---|
| Short-precision real | SPOSM |
| Long-precision real | DPOSM |
| Short-precision complex | CPOSM |
| Long-precision complex | ZPOSM |

**Note:** The input to these solve subroutines must be the output from the factorization subroutines SPOF/SPOFCD, DPOF/DPOFCD, CPOF, and ZPOF, respectively.

***Syntax***

| | |
|---|---|
| **Fortran** | CALL SPOSM \| DPOSM \| CPOSM \| ZPOSM (*uplo*, *a*, *lda*, *n*, *b*, *ldb*, *nrhs*) |
| **C and C++** | sposm \| dposm \| cposm \| zposm (*uplo*, *a*, *lda*, *n*, *b*, *ldb*, *nrhs*); |
| **PL/I** | CALL SPOSM \| DPOSM \| CPOSM \| ZPOSM (*uplo*, *a*, *lda*, *n*, *b*, *ldb*, *nrhs*); |

*On Entry*

*uplo*

indicates whether the original matrix $\boldsymbol{A}$ is stored in upper or lower storage mode, where:

If *uplo* = `'U'`, $\boldsymbol{A}$ is stored in upper storage mode.

If *uplo* = `'L'`, $\boldsymbol{A}$ is stored in lower storage mode.

Specified as: a single character. It must be `'U'` or `'L'`.

*a*

is the factorization of positive definite matrix $\boldsymbol{A}$, produced by a preceding call to SPOF/SPOFCD, DPOF/DPOFCD, CPOF, or ZPOF, respectively. Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 95.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; $lda > 0$ and $lda \geq n$.

*n*

is the order of matrix $\boldsymbol{A}$. Specified as: a fullword integer; $0 \leq n \leq lda$.

*b*

is the matrix ***B***, containing the *nrhs* right-hand sides of the system. The right-hand sides, each of length *n*, reside in the columns of matrix ***B***. Specified as: an *ldb* by (at least) *nrhs* array, containing numbers of the data type indicated in Table 95.

*ldb*

is the leading dimension of the array specified for *b*. Specified as: a fullword integer; $ldb > 0$ and $ldb \geq n$.

*nrhs*

is the number of right-hand sides in the system to be solved. Specified as: a fullword integer; $nrhs \geq 0$.

*On Return*

*b*

is the matrix ***B***, containing the *nrhs* solutions to the system in the columns of ***B***. Specified as: an *ldb* by (at least) *nrhs* array, containing numbers of the data type indicated in Table 95 on page 503.

***Notes***

1. All subroutines accept lowercase letters for the *uplo* argument.
2. The scalar data specified for input arguments *uplo*, *lda*, and *n* for these subroutines must be the same as the corresponding input arguments specified for SPOF/SPOFCD, DPOF/DPOFCD, CPOF, and ZPOF, respectively.
3. The array data specified for input argument *a* for these subroutines must be the same as the corresponding output arguments for SPOF/SPOFCD, DPOF/DPOFCD, CPOF, and ZPOF, respectively.
4. The vectors and matrices used in this computation must have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.
5. For a description of how the matrices are stored:
   - For positive definite symmetric matrices, see “Positive Definite or Negative Definite Symmetric Matrix” on page 69.
   - For positive definite complex Hermitian matrices, see “Positive Definite or Negative Definite Complex Hermitian Matrix” on page 71.

***Function:*** The system ***AX*** = ***B*** is solved for ***X***, using multiple right-hand sides, where ***X*** and ***B*** are general matrices, and ***A*** is a positive definite symmetric matrix for SPOSM and DPOSM and a positive definite complex Hermitian matrix for CPOSM and ZPOSM. These subroutines use the results of the factorization of matrix ***A***, produced by a preceding call to SPOF/SPOFCD, DPOF/DPOFCD, CPOF, or ZPOF, respectively. For a description of how ***A*** is factored, see “SPPF, DPPF, SPOF, DPOF, CPOF, and ZPOF—Positive Definite Real Symmetric or Complex Hermitian Matrix Factorization” on page 492.

If *n* or *nrhs* is 0, no computation is performed. See references [8] and [36 ].

***Error Conditions***

*Computational Errors:* None

**Note:** If the factorization performed by SPOF, DPOF, CPOF, ZPOF, SPOFCD, or DPOFCD failed because matrix ***A*** was not positive definite, the results returned by this subroutine are unpredictable, and there may be a divide-by-zero program exception message.

*Input-Argument Errors*

1. *uplo* ≠ `'U'` or `'L'`
2. *lda*, *ldb* ≤ 0
3. *n* < 0
4. *n* > *lda*
5. *n* > *ldb*
6. *nrhs* < 0

***Example 1:*** This example shows how to solve the system ***AX*** = ***B*** for two right-hand sides, where matrix ***A*** is the same matrix factored in the “Example 3” on page 497 for SPOF.

*Call Statement and Input*

```
              UPLO  A  LDA  N   B  LDB  NRHS
               |    |   |   |   |   |    |
CALL SPOSM( 'L' , A , 9 , 9 , B , 9 ,  2  )
```

A =(same as output A in “Example 3” on page 497)

```
          ┌             ┐
          |  9.0   45.0 |
          | 17.0   89.0 |
          | 24.0  131.0 |
          | 30.0  170.0 |
B    =    | 35.0  205.0 |
          | 39.0  235.0 |
          | 42.0  259.0 |
          | 44.0  276.0 |
          | 45.0  285.0 |
          └             ┘
```

*Output*

```
          ┌          ┐
          | 1.0  1.0 |
          | 1.0  2.0 |
          | 1.0  3.0 |
          | 1.0  4.0 |
B    =    | 1.0  5.0 |
          | 1.0  6.0 |
          | 1.0  7.0 |
          | 1.0  8.0 |
          | 1.0  9.0 |
          └          ┘
```

***Example 2:*** This example shows how to solve the system ***A***$^T$***X*** = ***B*** for two right-hand sides, where matrix ***A*** is the input matrix factored in “Example 4” on page 497 for SPOF.

*Call Statement and Input*

```
              UPLO  A  LDA  N   B  LDB  NRHS
               |    |   |   |   |   |    |
CALL SPOSM( 'U' , A , 9 , 9 , B , 9 ,  2  )
```

`A` =(same as output `A` in “Example 4” on page 497)

```
                ┌              ┐
                │  9.0   45.0 │
                │ 17.0   89.0 │
                │ 24.0  131.0 │
                │ 30.0  170.0 │
B     =         │ 35.0  205.0 │
                │ 39.0  235.0 │
                │ 42.0  259.0 │
                │ 44.0  276.0 │
                │ 45.0  285.0 │
                └              ┘
```

*Output*

```
                ┌          ┐
                │ 1.0  1.0 │
                │ 1.0  2.0 │
                │ 1.0  3.0 │
                │ 1.0  4.0 │
B     =         │ 1.0  5.0 │
                │ 1.0  6.0 │
                │ 1.0  7.0 │
                │ 1.0  8.0 │
                │ 1.0  9.0 │
                └          ┘
```

***Example 3:*** This example shows how to solve the system ***AX*** = ***B*** for two right-hand sides, where matrix ***A*** is the same matrix factored in the “Example 5” on page 498 for CPOF.

*Call Statement and Input*

```
            UPLO  A  LDA  N   B  LDB  NRHS
             |    |   |   |   |   |    |
CALL CPOSM( 'L' , A , 3 , 3 , B , 3 ,  2  )
```

`A` =(same as output `A` in “Example 5” on page 498)

```
        ┌                                   ┐
        │  (60.0, -55.0)    (70.0, 10.0) │
B     = │   (34.0, 58.0)  (-51.0, 110.0) │
        │ (13.0, -152.0)    (75.0, 63.0) │
        └                                   ┘
```

*Output*

```
        ┌                              ┐
        │ (2.0, -1.0)   (2.0, 0.0) │
B     = │  (1.0, 1.0)  (-1.0, 2.0) │
        │ (0.0, -2.0)   (1.0, 1.0) │
        └                              ┘
```

***Example 4:*** This example shows how to solve the system ***AX*** = ***B*** for two right-hand sides, where matrix ***A*** is the input matrix factored in “Example 6” on page 499 for CPOF.

*Call Statement and Input*

```
            UPLO  A  LDA  N   B  LDB  NRHS
             |    |   |   |   |   |    |
CALL CPOSM( 'U' , A , 3 , 3 , B , 3 ,  2  )
```

A =(same as output A in “Example 6” on page 499)

```
        ┌                               ┐
        | (33.0, -18.0)   (15.0, -3.0) |
B    =  | (45.0, -45.0)    (8.0, -2.0) |
        |  (152.0, 1.0)  (43.0, -29.0) |
        └                               ┘
```

*Output*

```
        ┌                           ┐
        | (2.0, -1.0)   (2.0, 0.0) |
B    =  | (1.0, -1.0)   (0.0, 1.0) |
        |  (3.0, 0.0)  (1.0, -1.0) |
        └                           ┘
```

# SPPFCD, DPPFCD, SPOFCD, and DPOFCD—Positive Definite Real Symmetric Matrix Factorization, Condition Number Reciprocal, and Determinant

The SPPFCD and DPPFCD subroutines factor positive definite symmetric matrix ***A***, stored in lower-packed storage mode, using Gaussian elimination (***LDL***$^T$). The reciprocal of the condition number and the determinant of matrix ***A*** can also be computed. To solve the system of equations with one or more right-hand sides, follow the call to these subroutines with one or more calls to SPPS or DPPS, respectively.

The SPOFCD and DPOFCD subroutines factor positive definite symmetric matrix ***A***, stored in upper or lower storage mode, using Cholesky factorization (***LL***$^T$ or ***U***$^T$***U***). The reciprocal of the condition number and the determinant of matrix ***A*** can also be computed. To solve the system of equations with one or more right-hand sides, follow the call to these subroutines with a call to SPOSM or DPOSM, respectively. To find the inverse of matrix ***A***, follow the call to these subroutines with a call to SPOICD or DPOICD, respectively.

*Table 96. Data Types*

| ***A***, *aux*, *rcond*, ***det*** | **Subroutine** |
|---|---|
| Short-precision real | SPPFCD and SPOFCD |
| Long-precision real | DPPFCD and DPOFCD |

**Note:** The output factorization from SPPFCD and DPPFCD should be used only as input to the solve subroutines SPPS and DPPS, respectively. The output from SPOFCD and DPOFCD should be used only as input to the following subroutines for performing a solve or inverse: SPOSM/SPOICD and DPOSM/DPOICD, respectively.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SPPFCD \| DPPFCD (*ap*, *n*, *iopt*, *rcond*, *det*, *aux*, *naux*)<br>CALL SPOFCD \| DPOFCD (*uplo*, *a*, *lda*, *n*, *iopt*, *rcond*, *det*, *aux*, *naux*) |
| **C and C++** | sppfcd \| dppfcd (*ap*, *n*, *iopt*, *rcond*, *det*, *aux*, *naux*);<br>spofcd \| dpofcd (*uplo*, *a*, *lda*, *n*, *iopt*, *rcond*, *det*, *aux*, *naux*); |
| **PL/I** | CALL SPPFCD \| DPPFCD (*ap*, *n*, *iopt*, *rcond*, *det*, *aux*, *naux*);<br>CALL SPOFCD \| DPOFCD (*uplo*, *a*, *lda*, *n*, *iopt*, *rcond*, *det*, *aux*, *naux*); |

*On Entry*

*uplo*

indicates whether matrix ***A*** is stored in upper or lower storage mode, where:

If *uplo* = `'U'`, ***A*** is stored in upper storage mode.

If *uplo* = `'L'`, ***A*** is stored in lower storage mode.

Specified as: a single character. It must be `'U'` or `'L'`.

*ap*

is the array, referred to as AP, in which the matrix ***A***, to be factored, is stored in lower-packed storage mode. Specified as: a one-dimensional array of (at least) length $n(n+1)/2+n$, containing numbers of the data type indicated in Table 96.

*a*

is the positive definite symmetric matrix ***A***, to be factored. Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 96 on page 508.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; $lda > 0$ and $lda \geq n$.

*n*

is the order *n* of matrix ***A***. Specified as: a fullword integer, where:

For SPPFCD and DPPFCD, $n \geq 0$.

For SPOFCD and DPOFCD, $0 \leq n \leq lda$.

*iopt*

indicates the type of computation to be performed, where:

If $iopt = 0$, the matrix is factored.

If $iopt = 1$, the matrix is factored, and the reciprocal of the condition number is computed.

If $iopt = 2$, the matrix is factored, and the determinant is computed.

If $iopt = 3$, the matrix is factored and the reciprocal of the condition number and the determinant are computed.

Specified as: a fullword integer; $iopt = 0$, 1, 2, or 3.

*rcond*

See “On Return.”

*det*

See “On Return.”

*aux*

has the following meaning:

If $naux = 0$ and error 2015 is unrecoverable, *aux* is ignored.

Otherwise, is the storage work area used by these subroutines. Its size is specified by *naux*. Specified as: an area of storage, containing numbers of the data type indicated in Table 96 on page 508.

*naux*

is the size of the work area specified by *aux*—that is, the number of elements in *aux*. Specified as: a fullword integer, where:

If $naux = 0$ and error 2015 is unrecoverable, SPPFCD, DPPFCD, SPOFCD, and DPOFCD dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, $naux \geq n$.

*On Return*

*ap*

is the transformed matrix ***A*** of order *n*, containing the results of the factorization. See “Function” on page 510. Returned as: a one-dimensional array of (at least) length $n(n+1)/2+n$, containing numbers of the data type indicated in Table 96 on page 508.

*a*

is the transformed matrix ***A*** of order *n*, containing the results of the factorization. See “Function” on page 510. Returned as: a two-dimensional array, containing numbers of the data type indicated in Table 96 on page 508.

*rcond*

is the estimate of the reciprocal of the condition number, *rcond*, of matrix ***A***. Returned as: a number of the data type indicated in Table 96 on page 508; *rcond* ≥ 0.

*det*

is the vector ***det***, containing the two components $det_1$ and $det_2$ of the determinant of matrix ***A***. The determinant is:

$$det_1\left(10^{det_2}\right)$$

where $1 \le det_1 < 10$. Returned as: an array of length 2, containing numbers of the data type indicated in Table 96 on page 508.

***Notes***

1. All subroutines accept lowercase letters for the *uplo* argument.
2. In your C program, argument *rcond* must be passed by reference.
3. When *iopt* = 0, SPPFCD and DPPFCD provide the same function as a call to SPPF or DPPF, respectively. When *iopt* = 0, SPOFCD and DPOFCD provide the same function as a call to SPOF or DPOF, respectively.
4. The serial and SMP performance of SPPFCD and DPPFCD has been improved in many cases by utilizing new algorithms based on recursive packed storage format. As a result, on output, the array specified for AP may be stored in this new format rather than the conventional lower packed format. (See references [50], [63 ], and [64 ]).

   The array specified for AP should not be altered between calls to the factorization and solve subroutines; otherwise unpredictable results may occur.
5. See “Notes” on page 501 for information on specifying a value for *iopt* in the SPPS and DPPS subroutines after calling SPPFCD and DPPFCD, respectively.
6. In the input and output arrays specified for *ap*, the first *n*(*n*+1)/2 elements are matrix elements. The additional *n* locations in the array are used for working storage by this subroutine and should not be altered between calls to the factorization and solve subroutines.
7. For a description of how a positive definite symmetric matrix is stored in lower-packed storage mode in an array, see “Symmetric Matrix” on page 65. For a description of how a positive definite symmetric matrix is stored in upper or lower storage mode, see “Positive Definite or Negative Definite Symmetric Matrix” on page 69.
8. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see “Using Auxiliary Storage in ESSL” on page 32.

***Function:*** The functions for these subroutines are described in the sections below.

*For SPPFCD and DPPFCD:* The positive definite symmetric matrix ***A***, stored in lower-packed storage mode, is factored using Gaussian elimination, where ***A*** is expressed as:

$\boldsymbol{A} = \boldsymbol{LDL}^{\mathrm{T}}$

where:

***L*** is a unit lower triangular matrix.
$\boldsymbol{L}^{\mathrm{T}}$ is the transpose of matrix ***L***.
***D*** is a diagonal matrix.

An estimate of the reciprocal of the condition number, *rcond*, and the determinant, ***det***, can also be computed by this subroutine. The estimate of the condition number uses an enhanced version of the algorithm described in references [66] and [67 ].

If *n* is 0, no computation is performed. See references [36 ] and [38 ].

These subroutines call SPPF and DPPF, respectively, to perform the factorization using Gaussian elimination ($\boldsymbol{LDL}^{\mathrm{T}}$). If you want to use the Cholesky factorization method, you must call SPPF and DPPF directly.

*For SPOFCD and DPOFCD:* The positive definite symmetric matrix ***A***, stored in upper or lower storage mode, is factored using Cholesky factorization, where ***A*** is expressed as:

$\boldsymbol{A} = \boldsymbol{LL}^{\mathrm{T}}$ or $\boldsymbol{A} = \boldsymbol{U}^{\mathrm{T}}\boldsymbol{U}$

where:

***L*** is a lower triangular matrix.
$\boldsymbol{L}^{\mathrm{T}}$ is the transpose of matrix ***L***.
***U*** is an upper triangular matrix.
$\boldsymbol{U}^{\mathrm{T}}$ is the transpose of matrix ***U***.

If specified, the estimate of the reciprocal of the condition number and the determinant can also be computed. The estimate of the condition number uses an enhanced version of the algorithm described in references [66] and [67 ].

If *n* is 0, no computation is performed. See references [8 ] and [36 ].

### Error Conditions

*Resource Errors:* Error 2015 is unrecoverable, *naux* = 0, and unable to allocate work area.

*Computational Errors*

1. Matrix ***A*** is not positive definite (for SPPFCD and DPPFCD).
   - If matrix ***A*** is singular (at least one of the diagonal elements are 0), then *rcond* and *det*, if you requested them, are set to 0.
   - If matrix ***A*** is nonsingular and nonpositive definite (none of the diagonal elements are 0 and at least one diagonal element is negative), then *rcond* and ***det***, if you requested them, are computed.
   - One or more elements of ***D*** contain values less than or equal to 0; all elements of ***D*** are checked. The index *i* of the last nonpositive element

encountered is identified in the computational error message, issued by SPPF or DPPF, respectively.

- *i* can be determined at run time by using the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error code 2104 in the ESSL error option table; otherwise, the default value causes your program to be terminated by SPPF or DPPF, respectively, when this error occurs. If your program is not terminated by SPPF or DPPF, respectively, the return code is set to 2. For details, see “What Can You Do about ESSL Computational Errors?” on page 48.

2. Matrix ***A*** is not positive definite (for SPOFCD and DPOFCD).
   - If matrix ***A*** is singular (at least one of the diagonal elements are 0), then *rcond* and *det*, if you requested them, are set to 0.
   - If matrix ***A*** is nonsingular and nonpositive definite (none of the diagonal elements are 0 and at least one diagonal element is negative), then *rcond* and ***det***, if you requested them, are computed.
   - Processing stops at the first occurrence of a nonpositive definite diagonal element.
   - The order *i* of the **first** minor encountered having a nonpositive determinant is identified in the computational error message.
   - *i* can be determined at run time by using the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error code 2115 in the ESSL error option table; otherwise, the default value causes your program to be terminated by SPPF or DPPF, respectively, when this error occurs. If your program is not terminated by SPPF or DPPF, respectively, the return code is set to 2. For details, see “What Can You Do about ESSL Computational Errors?” on page 48.

*Input-Argument Errors*

1. $uplo \neq$ `'U'` or `'L'`
2. $lda \leq 0$
3. $lda < n$
4. $n < 0$
5. $iopt \neq 0, 1, 2,$ or 3
6. Error 2015 is recoverable or *naux*≠0, and *naux* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example 1:*** This example computes the factorization, reciprocal of the condition number, and determinant of matrix ***A***. The input is the same as used in “Example 1” on page 495 for SPPF.

The values used to estimate the reciprocal of the condition number are obtained with the following values:

$\|\boldsymbol{A}\|_1 = \max(9.0, 17.0, 24.0, 30.0, 35.0, 39.0, 42.0, 44.0, 45.0) = 45.0$
Estimate of $\|\boldsymbol{A}\|$ = `4.0`

On output, the value in ***det***, |***A***|, is equal to 1.

*Call Statement and Input*

```
                AP   N  IOPT  RCOND   DET   AUX  NAUX
                |    |  |     |       |     |    |
CALL DPPFCD( AP , 9 , 3  , RCOND , DET , AUX , 9  )
```

AP =(same as input AP in “Example 1” on page 495)

*Output*

AP =(same as output AP in “Example 1” on page 495)
RCOND = 0.0055555
DET = (1.0, 0.0)

***Example 2:*** This example computes the factorization, reciprocal of the condition number, and determinant of matrix ***A***. The input is the same as used in “Example 3” on page 497 for SPOF.

The values used to estimate the reciprocal of the condition number are obtained with the following values:

$\|\boldsymbol{A}\|_1$ = max(9.0, 17.0, 24.0, 30.0, 35.0, 39.0, 42.0, 44.0, 45.0) = 45.0
Estimate of $\|\boldsymbol{A}\|$ = 4.0

On output, the value in ***det***, |***A***|, is equal to 1.

*Call Statement and Input*

```
                UPLO A  LDA  N IOPT  RCOND   DET   AUX  NAUX
                 |   |   |   |  |     |       |     |    |
CALL SPOFCD( 'L', A , 9 , 9 , 3 , RCOND , DET , AUX , 9  )
```

A =(same as input A in “Example 3” on page 497)

*Output*

A =(same as output A in “Example 3” on page 497)
RCOND = 0.0055555
DET = (1.0, 0.0)

***Example 3:*** This example computes the factorization, reciprocal of the condition number, and determinant of matrix ***A***. The input is the same as used in “Example 4” on page 497 for SPOF.

The values used to estimate the reciprocal of the condition number are obtained with the following values:

$\|\boldsymbol{A}\|_1$ = max(9.0, 17.0, 24.0, 30.0, 35.0, 39.0, 42.0, 44.0, 45.0) = 45.0
Estimate of $\|\boldsymbol{A}\|$ = 4.0

On output, the value in ***det***, |***A***|, is equal to 1.

*Call Statement and Input*

```
                UPLO A  LDA  N IOPT  RCOND   DET   AUX  NAUX
                 |   |   |   |  |     |       |     |    |
CALL SPOFCD( 'U', A , 9 , 9 , 3 , RCOND , DET , AUX , 9  )
```

A =(same as input A in “Example 4” on page 497)

*Output*

```
A        =(same as output A in “Example 4” on page 497)
RCOND    =  0.0055555
DET      =  (1.0, 0.0)
```

## SGEICD and DGEICD—General Matrix Inverse, Condition Number Reciprocal, and Determinant

These subroutines find the inverse, the reciprocal of the condition number, and the determinant of matrix ***A***.

*Table 97. Data Types*

| ***A***, *aux*, *rcond*, ***det*** | **Subroutine** |
|---|---|
| Short-precision real | SGEICD |
| Long-precision real | DGEICD |

**Note:** If you call these subroutines with *iopt* = 0, 1, 2, or 3 the input must be the output from the factorization subroutines SGEF/SGEFCD/SGETRF or DGEF/DGEFCD/DGEFP/DGETRF, respectively.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SGEICD \| DGEICD (*a*, *lda*, *n*, *iopt*, *rcond*, *det*, *aux*, *naux*) |
| **C and C++** | sgeicd \| dgeicd (*a*, *lda*, *n*, *iopt*, *rcond*, *det*, *aux*, *naux*); |
| **PL/I** | CALL SGEICD \| DGEICD (*a*, *lda*, *n*, *iopt*, *rcond*, *det*, *aux*, *naux*); |

*On Entry*

*a*

has the following meaning, where:

If *iopt* = 0, 1, 2, or 3, it is matrix ***A*** of order *n*, whose inverse, reciprocal of condition number, and determinant are computed.

If *iopt* = 4, it is the transformed matrix ***A*** of order *n*, resulting from the factorization performed in a previous call to SGEF/SGEFCD or DGEF/DGEFCD/DGEFP, respectively, whose inverse is computed.

Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 97.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; *lda* > 0 and *lda* ≥ *n*.

*n*

is the order of matrix ***A***. Specified as: a fullword integer; 0 ≤ *n* ≤ *lda*.

*iopt*

indicates the type of computation to be performed, where:

If *iopt* = 0, the inverse is computed for matrix ***A***.

If *iopt* = 1, the inverse and the reciprocal of the condition number are computed for matrix ***A***.

If *iopt* = 2, the inverse and the determinant are computed for matrix ***A***.

If *iopt* = 3, the inverse, the reciprocal of the condition number, and the determinant are computed for matrix ***A***.

If *iopt* = 4, the inverse is computed using the factored matrix ***A***.

Specified as: a fullword integer; *iopt* = 0, 1, 2, 3, 4.

*rcond*

See “On Return” on page 516.

*det*

See “On Return.”

*aux*

has the following meaning, and its size is specified by *naux*:

If *iopt* = 0, 1, 2, or 3, then if *naux* = 0 and error 2015 is unrecoverable, *aux* is ignored. Otherwise, it is the storage work area used by this subroutine.

If *iopt* = 4, *aux* has the following meaning:

- For SGEICD, the first *n* locations in *aux* must contain the ***ipvt*** integer vector of length *n*, resulting from a previous call to SGEF, SGETRF, or SGEFCD.
- For DGEICD, the first ceiling(*n*/2) locations in *aux* must contain the ***ipvt*** integer vector of length *n*, resulting from a previous call to DGEF, DGETRF, DGEFCD, or DGEFP.

Specified as: an area of storage, containing numbers of the data type indicated in Table 97 on page 515.

*naux*

is the size of the work area specified by *aux*—that is, the number of elements in *aux*. Specified as: a fullword integer, where:

If *iopt* ≠ 4, then if *naux* = 0 and error 2015 is unrecoverable, SGEICD and DGEICD dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise *naux* must have the following value:

For the RS/6000 POWER, POWER3, or PowerPC processors, *naux* ≥ 100*n*.

For the RS/6000 POWER2 processors, *naux* ≥ 200*n*.

**Note:** *naux* values specified for releases prior to ESSL Version 2 Release 2 will still work, but you may not achieve optimal performance.

*On Return*

*a*

is the resulting inverse of matrix ***A*** of order *n*. Returned as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 97 on page 515.

*rcond*

is the reciprocal of the condition number, *rcond*, of matrix ***A***. Returned as: a real number of the data type indicated in Table 97 on page 515; *rcond* ≥ 0.

*det*

is the vector ***det***, containing the two components $det_1$ and $det_2$ of the determinant of matrix ***A***. The determinant is:

$$det_1\left(10^{det_2}\right)$$

where $1 \leq det_1 < 10$. Returned as: an array of length 2, containing numbers of the data type indicated in Table 97 on page 515.

***Notes***

1. In your C program, argument *rcond* must be passed by reference.
2. If *iopt* = 4, the following input arguments for SGEICD and DGEICD must be set to the same values in the previous call to SGEF/SGEFCD or DGEF/DGEFCD/DGEFP, respectively:

| **For _GEF_** | **For _GEICD** |
|---|---|
| Input arguments *n* and *lda* | Input arguments *n* and *lda* |
| Output arguments *a* and *ipvt* | Input arguments *a* and *aux* |

3. You have the option of having the value for *naux* dynamically returned to your program. For details, see “Using Auxiliary Storage in ESSL” on page 32.

***Function:*** The inverse, the reciprocal of the condition number, and the determinant of a general square matrix ***A*** are computed using partial pivoting to preserve accuracy, where:

- $\boldsymbol{A}^{-1}$ is the inverse of matrix ***A***, where $\boldsymbol{A}\boldsymbol{A}^{-1} = \boldsymbol{A}^{-1}\boldsymbol{A} = \boldsymbol{I}$, and ***I*** is the identity matrix.
- $1/(\|\boldsymbol{A}\|_1)(\|\boldsymbol{A}^{-1}\|_1)$ is the reciprocal of the condition number, where $\|\boldsymbol{A}\|_1$ is the one-norm of matrix ***A***.
- |***A***| is the determinant of matrix ***A***, where |***A***| is expressed as:

$$det_1\left(10^{det_2}\right)$$

The *iopt* argument is used to determine the combination of output items produced by this subroutine: the inverse, the reciprocal of the condition number, and the determinant.

If *n* is 0, no computation is performed. See references [36 ], [38 ], and [44 ].

***Error Conditions***

*Resource Errors:* If *iopt* = 0, 1, 2, or 3, then error 2015 is unrecoverable, *naux* = 0, and unable to allocate work area.

*Computational Errors:* Matrix ***A*** is singular or nearly singular.

- The index *i* of the first pivot element having a value equal to 0, is identified in the computational error message.
- These subroutines return 0 for *rcond* and ***det***, if you requested them.
- The return code is set to 2.
- *i* can be determined at run time by use of the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error code 2105 in the ESSL error option table; otherwise, the default value causes your program to terminate when this error occurs. For details, see “What Can You Do about ESSL Computational Errors?” on page 48.

*Input-Argument Errors*

1. *lda* ≤ 0
2. *n* < 0
3. *n* > *lda*
4. *iopt* ≠ 0, 1, 2, 3, or 4
5. Error 2015 is recoverable or *naux*≠0, and *naux* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example 1:*** This example computes the inverse, the reciprocal of the condition number, and the determinant of matrix ***A***. The values used to compute the reciprocal of the condition number in this example are obtained with the following values:

$\|\boldsymbol{A}\|_1$ = max(6.0, 8.0, 10.0, 12.0, 13.0, 14.0, 15.0, 15.0, 15.0) = 15.0
$\|\boldsymbol{A}^{-1}\|_1$ = 1226.33

On output, the value in ***det***, |***A***|, is equal to 336.

*Call Statement and Input*

```
             A  LDA  N  IOPT  RCOND   DET   AUX   NAUX
             |   |   |   |       |     |     |     |
CALL DGEICD( A , 9 , 9 , 3  , RCOND , DET , AUX , 293 )
```

```
        ┌                                                ┐
        │ 1.0  1.0  1.0  1.0  0.0  0.0   0.0   0.0   0.0 │
        │ 1.0  1.0  1.0  1.0  1.0  0.0   0.0   0.0   0.0 │
        │ 4.0  1.0  1.0  1.0  1.0  1.0   0.0   0.0   0.0 │
        │ 0.0  5.0  1.0  1.0  1.0  1.0   1.0   0.0   0.0 │
A     = │ 0.0  0.0  6.0  1.0  1.0  1.0   1.0   1.0   0.0 │
        │ 0.0  0.0  0.0  7.0  1.0  1.0   1.0   1.0   1.0 │
        │ 0.0  0.0  0.0  0.0  8.0  1.0   1.0   1.0   1.0 │
        │ 0.0  0.0  0.0  0.0  0.0  9.0   1.0   1.0   1.0 │
        │ 0.0  0.0  0.0  0.0  0.0  0.0  10.0  11.0  12.0 │
        └                                                ┘
```

*Output*

```
        ┌                                                                        ┐
        │    0.333   -0.667   0.333  0.000  0.000  0.000   0.042 -0.042  0.000 │
        │   56.833  -52.167  -1.167 -0.500 -0.500 -0.357   6.836 -0.479 -0.500 │
        │  -55.167   51.833   0.833  0.500  0.500  0.214  -6.735  0.521  0.500 │
        │   -1.000    1.000   0.000  0.000  0.000  0.143  -0.143  0.000  0.000 │
A     = │   -1.000    1.000   0.000  0.000  0.000  0.000   0.000  0.000  0.000 │
        │   -1.000    1.000   0.000  0.000  0.000  0.000  -0.125  0.125  0.000 │
        │ -226.000  206.000   5.000  3.000  2.000  1.429 -27.179  1.750  2.000 │
        │  560.000 -520.000 -10.000 -6.000 -4.000 -2.857  67.857 -5.000 -5.000 │
        │ -325.000  305.000   5.000  3.000  2.000  1.429 -39.554  3.125  3.000 │
        └                                                                        ┘
```

```
RCOND    =  0.00005436
DET      =  (3.36, 2.00)
```

***Example 2:*** This example computes the inverse of matrix ***A***, where *iopt* = 4 and matrix ***A*** is the transformed matrix factored in “Example 1” on page 467 by SGEF. The input contents of `AUX`, shown here, is the same as the output contents of `IPVT` in that example.

*Call Statement and Input*

```
             A  LDA  N  IOPT  RCOND   DET   AUX   NAUX
             |   |   |   |      |      |     |     |
CALL SGEICD( A , 9 , 9 , 4  , RCOND , DET , AUX , 300 )

A        =(same as output A in “Example 1” on page 467)
AUX      =  (3, 4, 5, 6, 7, 8, 9, 8, 9)
```

*Output*

```
        ┌                                                                   ┐
        │    0.333   -0.667   0.333  0.000  0.000  0.000   0.042 -0.042  0.000 │
        │   56.833  -52.167  -1.167 -0.500 -0.500 -0.357   6.836 -0.479 -0.500 │
        │  -55.167   51.833   0.833  0.500  0.500  0.214  -6.735  0.521  0.500 │
        │   -1.000    1.000   0.000  0.000  0.000  0.143  -0.143  0.000  0.000 │
A    =  │   -1.000    1.000   0.000  0.000  0.000  0.000   0.000  0.000  0.000 │
        │   -1.000    1.000   0.000  0.000  0.000  0.000  -0.125  0.125  0.000 │
        │ -226.000  206.000   5.000  3.000  2.000  1.429 -27.179  1.750  2.000 │
        │  560.000 -520.000 -10.000 -6.000 -4.000 -2.857  67.857 -5.000 -5.000 │
        │ -325.000  305.000   5.000  3.000  2.000  1.429 -39.554  3.125  3.000 │
        └                                                                   ┘
```

# SPPICD, DPPICD, SPOICD, and DPOICD—Positive Definite Real Symmetric Matrix Inverse, Condition Number Reciprocal, and Determinant

These subroutines find the inverse, the reciprocal of the condition number, and the determinant of positive definite symmetric matrix ***A*** using Cholesky factorization, where:

- For SPPICD and DPPICD, ***A*** is stored in lower-packed storage mode.
- For SPOICD and DPOICD, ***A*** is stored in upper or lower storage mode.

*Table 98. Data Types*

| ***A***, *aux*, *rcond*, ***det*** | **Subroutine** |
|---|---|
| Short-precision real | SPPICD and SPOICD |
| Long-precision real | DPPICD and DPOICD |

**Note:** If you call these subroutines with *iopt* = 4, the input must be the output from the factorization subroutines SPPF, DPPF, SPOF/SPOFCD, or DPOF/DPOFCD, respectively, where Cholesky factorization was performed.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SPPICD \| DPPICD (*ap*, *n*, *iopt*, *rcond*, *det*, *aux*, *naux*)<br>CALL SPOICD \| DPOICD (*uplo*, *a*, *lda*, *n*, *iopt*, *rcond*, *det*, *aux*, *naux*) |
| **C and C++** | sppicd \| dppicd (*ap*, *n*, *iopt*, *rcond*, *det*, *aux*, *naux*);<br>spoicd \| dpoicd (*uplo*, *a*, *lda*, *n*, *iopt*, *rcond*, *det*, *aux*, *naux*); |
| **PL/I** | CALL SPPICD \| DPPICD (*ap*, *n*, *iopt*, *rcond*, *det*, *aux*, *naux*);<br>CALL SPOICD \| DPOICD (*uplo*, *a*, *lda*, *n*, *iopt*, *rcond*, *det*, *aux*, *naux*); |

*On Entry*

*uplo*

indicates whether matrix ***A*** is stored in upper or lower storage mode, where:

If *uplo* = `'U'`, ***A*** is stored in upper storage mode.

If *uplo* = `'L'`, ***A*** is stored in lower storage mode.

Specified as: a single character. It must be `'U'` or `'L'`.

*ap*

is the array, referred to as `AP`, where:

If *iopt* = 0, 1, 2, or 3, then `AP` contains the positive definite real symmetric matrix ***A***, whose inverse, condition number reciprocal, and determinant are computed, where matrix ***A*** is stored in lower-packed storage mode.

If *iopt* = 4, then `AP` contains the transformed matrix ***A*** of order *n*, resulting from the Cholesky factorization performed in a previous call to SPPF or DPPF, respectively, whose inverse is computed.

Specified as: a one-dimensional array of (at least) length *n*(*n*+1)/2, containing numbers of the data type indicated in Table 98.

*a*

has the following meaning, where:

If *iopt* = 0, 1, 2, or 3, it is the positive definite real symmetric matrix ***A***, whose inverse, condition number reciprocal, and determinant are computed, where matrix ***A*** is stored in upper or lower storage mode.

If *iopt* = 4, it is the transformed matrix ***A*** of order *n*, containing results of the factorization from a previous call to SPOF/SPOFCD or DPOF/DPOFCD, respectively, whose inverse is computed.

Specified as: an *n* by (at least) *n* array, containing numbers of the data type indicated in Table 98 on page 520.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; *lda* > 0 and *lda* ≥ *n*.

*n*

is the order *n* of matrix ***A***. Specified as: a fullword integer; *n* ≥ 0.

*iopt*

indicates the type of computation to be performed, where:

If *iopt* = 0, the inverse is computed for matrix ***A***.

If *iopt* = 1, the inverse and the reciprocal of the condition number are computed for matrix ***A***.

If *iopt* = 2, the inverse and the determinant are computed for matrix ***A***.

If *iopt* = 3, the inverse, the reciprocal of the condition number, and the determinant are computed for matrix ***A***.

If *iopt* = 4, the inverse is computed for the (Cholesky) factored matrix ***A***.

Specified as: a fullword integer; *iopt* = 0, 1, 2, 3, or 4.

*rcond*

See “On Return.”

*det*

See “On Return.”

*aux*

has the following meaning:

If *naux* = 0 and error 2015 is unrecoverable, *aux* is ignored.

Otherwise, it is the storage work area used by this subroutine. Its size is specified by *naux*. Specified as: an area of storage, containing numbers of the data type indicated in Table 98 on page 520.

*naux*

is the size of the work area specified by *aux*—that is, the number of elements in *aux*. Specified as: a fullword integer, where:

If *naux* = 0 and error 2015 is unrecoverable, SPPICD, DPPICD, SPOICD, AND DPOICD dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, *naux* ≥ *n*.

*On Return*

*ap*

is the resulting array, referred to as AP, containing the inverse of the matrix in lower-packed storage mode. Returned as: a one-dimensional array of (at least) length $n(n+1)/2$, containing numbers of the data type indicated in Table 98 on page 520.

*a*

is the transformed matrix ***A*** of order *n*, containing the inverse of the matrix in upper or lower storage mode. Returned as: a two-dimensional array, containing numbers of the data type indicated in Table 98 on page 520.

*rcond*

is the reciprocal of the condition number, *rcond*, of matrix ***A***. Returned as: a real number of the data type indicated in Table 98 on page 520; $rcond \geq 0$.

*det*

is the vector ***det***, containing the two components $det_1$ and $det_2$ of the determinant of matrix ***A***. The determinant is:

$$det_1\left(10^{det_2}\right)$$

where $1 \leq det_1 < 10$. Returned as: an array of length 2, containing numbers of the data type indicated in Table 98 on page 520.

***Notes***

1. For these subroutines, when you specify $iopt = 4$, you must do the following:
   - For SPPICD and DPPICD, use Cholesky factorization in the previous call to SPPF and DPPF, respectively.
   - For SPOICD and DPOICD, specify the same storage mode for matrix ***A*** that was specified in the previous call to SPOF/SPOFCD and DPOF/DPOFCD, respectively.
   - The scalar data specified for input arguments *uplo*, *lda*, and *n* for these subroutines must be the same as the corresponding input arguments specified for SPOF/SPOFCD and DPOF/DPOFCD, respectively.
2. All subroutines accept lowercase letters for the *uplo* argument.
3. In your C program, argument *rcond* must be passed by reference.
4. The serial and SMP performance of SPPICD and DPPICD have been improved in many cases by utilizing new algorithms based on recursive packed storage format. (See references [50], [63 ], and [64 ]).
5. For a description of how a positive definite symmetric matrix is stored in lower-packed storage mode in an array, see “Symmetric Matrix” on page 65. For a description of how a positive definite symmetric matrix is stored in upper or lower storage mode, see “Positive Definite or Negative Definite Symmetric Matrix” on page 69.
6. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see “Using Auxiliary Storage in ESSL” on page 32.

***Function:*** These subroutines find the inverse, the reciprocal of the condition number, and the determinant of positive definite symmetric matrix ***A*** using Cholesky factorization, where:

- $\boldsymbol{A}^{-1}$ is the inverse of matrix ***A***, where $\boldsymbol{AA}^{-1} = \boldsymbol{A}^{-1}\boldsymbol{A} = \boldsymbol{I}$, and ***I*** is the identity matrix.

- $1/(\|\boldsymbol{A}\|_1)(\|\boldsymbol{A}^{-1}\|_1)$ is the reciprocal of the condition number, where $\|\boldsymbol{A}\|_1$ is the one-norm of matrix ***A***.
- |***A***| is the determinant of matrix ***A***, where |***A***| is expressed as:

$$det_1\left(10^{det_2}\right)$$

The *iopt* argument is used to determine the combination of output items produced by this subroutine: the inverse, the reciprocal of the condition number, and the determinant.

If *n* is 0, no computation is performed. See references [36 ], [38 ], and [44 ].

### ***Error Conditions***

*Resource Errors*

- Error 2015 is unrecoverable, *naux* = 0, and unable to allocate work area.
- Unable to allocate internal work area.

*Computational Errors:* Matrix ***A*** is not positive definite.

- These subroutines do not perform the inverse, determinant, and reciprocal of the condition number computations.
- For *iopt* = 1, 2, or 3, the leading minor of order *i* has a nonpositive determinant. The order *i* is identified in the computational error message, issued by SPPF, DPPF, SPOF, or DPOF, respectively.

  For *iopt* = 4 for SPPICD and DPPICD, if the Cholesky factorization performed by SPPF or DPPF, respectively, failed due to a nonpositive definite matrix ***A***, the results are unpredictable, and a computational error message may be issued.

  For *iopt* = 4 for SPOICD and DPOICD, if the factorization performed by SPOF/SPOFCD or DPOF/DPOFCD, respectively, failed due to a nonpositive definite matrix ***A***, the results are unpredictable, and a computational error message may be issued.
- *i* can be determined at run time by using the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error code 2115 in the ESSL error option table; otherwise, the default value causes your program to be terminated by SPPF, DPPF, SPOF, or DPOF, respectively, when this error occurs. If your program is not terminated by SPPF, DPPF, SPOF, or DPOF, respectively, the return code is set to 2. For details, see “What Can You Do about ESSL Computational Errors?” on page 48.

*Input-Argument Errors*

1. *uplo* ≠ `'U'` or `'L'`
2. *n* < 0
3. *lda* ≤ 0
4. *lda* < *n*
5. *iopt* ≠ 0, 1, 2, 3, or 4
6. Error 2015 is recoverable or *naux*≠0, and *naux* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example 1:*** This example uses SPPICD to compute the inverse, reciprocal of the condition number, and determinant of matrix ***A***. Where ***A*** is:

$$\begin{bmatrix}
1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\
1.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 \\
1.0 & 2.0 & 3.0 & 3.0 & 3.0 & 3.0 & 3.0 & 3.0 & 3.0 \\
1.0 & 2.0 & 3.0 & 4.0 & 4.0 & 4.0 & 4.0 & 4.0 & 4.0 \\
1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 5.0 & 5.0 & 5.0 & 5.0 \\
1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 & 6.0 & 6.0 & 6.0 \\
1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 & 7.0 & 7.0 & 7.0 \\
1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 & 7.0 & 8.0 & 8.0 \\
1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 & 7.0 & 8.0 & 9.0
\end{bmatrix}$$

The values used to compute the reciprocal of the condition number in this example are obtained with the following values:

$\|\boldsymbol{A}\|_1$ = max(9.0, 17.0, 24.0, 30.0, 35.0, 39.0, 42.0, 44.0, 45.0) = 45.0
$\|\boldsymbol{A}^{-1}\|_1$ = 4.0

On output, the value in ***det***, |***A***|, is equal to 1, and RCOND = 1/180.

**Note:** The AP arrays are formatted in a triangular arrangement for readability; however, they are stored in lower-packed storage mode.

*Call Statement and Input*

```
             AP   N  IOPT RCOND   DET   AUX  NAUX
             |    |   |     |      |     |    |
CALL SPPICD( AP , 9 , 3 , RCOND , DET , AUX , 9  )

AP   =   (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0,
          2.0, 2.0, 2.0, 2.0, 2.0, 2.0, 2.0, 2.0,
          3.0, 3.0, 3.0, 3.0, 3.0, 3.0, 3.0,
          4.0, 4.0, 4.0, 4.0, 4.0, 4.0,
          5.0, 5.0, 5.0, 5.0, 5.0,
          6.0, 6.0, 6.0, 6.0,
          7.0, 7.0, 7.0,
          8.0, 8.0,
          9.0)
```

*Output*

```
AP   =   (2.0, -1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,
          2.0, -1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,
          2.0, -1.0, 0.0, 0.0, 0.0, 0.0, 0.0,
          2.0, -1.0, 0.0, 0.0, 0.0, 0.0,
          2.0, -1.0, 0.0, 0.0, 0.0,
          2.0, -1.0, 0.0, 0.0,
          2.0, -1.0, 0.0,
          2.0, -1.0,
          1.0)

RCOND    =  0.005556
DET      =  (1.0, 0.0)
```

***Example 2:*** This example uses SPPICD to compute the inverse of matrix ***A***, where *iopt* = 4, and matrix ***A*** is the transformed matrix factored in “Example 1” on page 495 by SPPF.

**Note:** The AP arrays are formatted in a triangular arrangement for readability; however, they are stored in lower-packed storage mode.

*Call Statement and Input*

```
            AP   N  IOPT RCOND   DET   AUX NAUX
            |    |   |     |      |     |    |
CALL SPPICD(AP , 9 , 4 , RCOND , DET , AUX , 9)
```

AP =(same as output AP in “Example 2” on page 496 for SPPF)

*Output*

```
AP   =   (2.0, -1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,
          2.0, -1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,
          2.0, -1.0, 0.0, 0.0, 0.0, 0.0, 0.0,
          2.0, -1.0, 0.0, 0.0, 0.0, 0.0,
          2.0, -1.0, 0.0, 0.0, 0.0,
          2.0, -1.0, 0.0, 0.0,
          2.0, -1.0, 0.0,
          2.0, -1.0,
          1.0)
```

***Example 3:*** This example uses SPOICD to compute the inverse, reciprocal of the condition number, and determinant of the same matrix ***A*** used in Example 1; however, matrix ***A*** is stored in upper storage mode in this example.

The values used to compute the reciprocal of the condition number in this example are obtained with the following values:

$\|\boldsymbol{A}\|_1$ = max(9.0, 17.0, 24.0, 30.0, 35.0, 39.0, 42.0, 44.0, 45.0) = 45.0
$\|\boldsymbol{A}^{-1}\|_1$ = 4.0

On output, the value in ***det***, |***A***|, is equal to 1, and RCOND = 1/180.

*Call Statement and Input*

```
             UPLO   A  LDA   N  IOPT RCOND   DET   AUX  NAUX
              |     |   |    |   |     |      |     |    |
CALL SPOICD( 'U' , A , 9 ,  9 , 3 , RCOND , DET , AUX , 9  )
```

$$A = \begin{bmatrix} 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ . & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 \\ . & . & 3.0 & 3.0 & 3.0 & 3.0 & 3.0 & 3.0 & 3.0 \\ . & . & . & 4.0 & 4.0 & 4.0 & 4.0 & 4.0 & 4.0 \\ . & . & . & . & 5.0 & 5.0 & 5.0 & 5.0 & 5.0 \\ . & . & . & . & . & 6.0 & 6.0 & 6.0 & 6.0 \\ . & . & . & . & . & . & 7.0 & 7.0 & 7.0 \\ . & . & . & . & . & . & . & 8.0 & 8.0 \\ . & . & . & . & . & . & . & . & 9.0 \end{bmatrix}$$

*Output*

```
          ┌                                                     ┐
          │ 2.0  -1.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0 │
          │  .    2.0  -1.0   0.0   0.0   0.0   0.0   0.0   0.0 │
          │  .     .    2.0  -1.0   0.0   0.0   0.0   0.0   0.0 │
          │  .     .     .    2.0  -1.0   0.0   0.0   0.0   0.0 │
A     =   │  .     .     .     .    2.0  -1.0   0.0   0.0   0.0 │
          │  .     .     .     .     .    2.0  -1.0   0.0   0.0 │
          │  .     .     .     .     .     .    2.0  -1.0   0.0 │
          │  .     .     .     .     .     .     .    2.0  -1.0 │
          │  .     .     .     .     .     .     .     .    1.0 │
          └                                                     ┘

RCOND    =  0.005555556
DET      =  (1.0, 0.0)
```

***Example 4:*** This example uses SPOICD to compute the inverse of matrix ***A***, where *iopt* = 4, and matrix ***A*** is the transformed matrix factored in "Example 1" on page 495 by SPOF.

*Call Statement and Input*

```
             UPLO  A  LDA   N  IOPT RCOND   DET   AUX  NAUX
              |    |   |    |   |     |      |     |    |
CALL SPOICD( 'U' , A , 9 ,  9 , 4 , RCOND , DET , AUX , 9  )
```

`A` =(same as output `A` in "Example 4" on page 497 for SPOF)

*Output*

```
          ┌                                                     ┐
          │ 2.0  -1.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0 │
          │  .    2.0  -1.0   0.0   0.0   0.0   0.0   0.0   0.0 │
          │  .     .    2.0  -1.0   0.0   0.0   0.0   0.0   0.0 │
          │  .     .     .    2.0  -1.0   0.0   0.0   0.0   0.0 │
A     =   │  .     .     .     .    2.0  -1.0   0.0   0.0   0.0 │
          │  .     .     .     .     .    2.0  -1.0   0.0   0.0 │
          │  .     .     .     .     .     .    2.0  -1.0   0.0 │
          │  .     .     .     .     .     .     .    2.0  -1.0 │
          │  .     .     .     .     .     .     .     .    1.0 │
          └                                                     ┘
```

# STRSV, DTRSV, CTRSV, ZTRSV, STPSV, DTPSV, CTPSV, and ZTPSV—Solution of a Triangular System of Equations with a Single Right-Hand Side

STRSV, DTRSV, STPSV, and DTPSV perform one of the following solves for a triangular system of equations with a single right-hand side, using the vector ***x*** and triangular matrix ***A*** or its transpose:

| Solution | Equation |
|---|---|
| 1. $\boldsymbol{x} \leftarrow \boldsymbol{A}^{-1}\boldsymbol{x}$ | $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$ |
| 2. $\boldsymbol{x} \leftarrow \boldsymbol{A}^{-\mathrm{T}}\boldsymbol{x}$ | $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{x} = \boldsymbol{b}$ |

CTRSV, ZTRSV, CTPSV, and ZTPSV perform one of the following solves for a triangular system of equations with a single right-hand side, using the vector ***x*** and and triangular matrix ***A***, its transpose, or its conjugate transpose:

| Solution | Equation |
|---|---|
| 1. $\boldsymbol{x} \leftarrow \boldsymbol{A}^{-1}\boldsymbol{x}$ | $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$ |
| 2. $\boldsymbol{x} \leftarrow \boldsymbol{A}^{-\mathrm{T}}\boldsymbol{x}$ | $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{x} = \boldsymbol{b}$ |
| 3. $\boldsymbol{x} \leftarrow \boldsymbol{A}^{-\mathrm{H}}\boldsymbol{x}$ | $\boldsymbol{A}^{\mathrm{H}}\boldsymbol{x} = \boldsymbol{b}$ |

Matrix ***A*** can be either upper or lower triangular, where:

- For the _TRSV subroutines, it is stored in upper- or lower-triangular storage mode, respectively.
- For the _TPSV subroutines, it is stored in upper- or lower-triangular-packed storage mode, respectively.

**Note:** The term ***b*** used in the systems of equations listed above represents the right-hand side of the system. It is important to note that in these subroutines the right-hand side of the equation is actually provided in the input-output argument *x*.

*Table 99. Data Types*

| ***A***, ***x*** | Subroutine |
|---|---|
| Short-precision real | STRSV and STPSV |
| Long-precision real | DTRSV and DTPSV |
| Short-precision complex | CTRSV and CTPSV |
| Long-precision complex | ZTRSV and ZTPSV |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL STRSV \| DTRSV \| CTRSV \| ZTRSV (*uplo*, *transa*, *diag*, *n*, *a*, *lda*, *x*, *incx*)<br>CALL STPSV \| DTPSV \| CTPSV \| ZTPSV (*uplo*, *transa*, *diag*, *n*, *ap*, *x*, *incx*) |
| **C and C++** | strsv \| dtrsv \| ctrsv \| ztrsv (*uplo*, *transa*, *diag*, *n*, *a*, *lda*, *x*, *incx*);<br>stpsv \| dtpsv \| ctpsv \| ztpsv (*uplo*, *transa*, *diag*, *n*, *ap*, *x*, *incx*); |
| **PL/I** | CALL STRSV \| DTRSV \| CTRSV \| ZTRSV (*uplo*, *transa*, *diag*, *n*, *a*, *lda*, *x*, *incx*);<br>CALL STPSV \| DTPSV \| CTPSV \| ZTPSV (*uplo*, *transa*, *diag*, *n*, *ap*, *x*, *incx*); |

*On Entry*

*uplo*

indicates whether matrix ***A*** is an upper or lower triangular matrix, where:

If *uplo* = `'U'`, ***A*** is an upper triangular matrix.

If *uplo* = `'L'`, ***A*** is a lower triangular matrix.

Specified as: a single character. It must be `'U'` or `'L'`.

*transa*

indicates the form of matrix ***A*** used in the system of equations, where:

If *transa* = `'N'`, ***A*** is used, resulting in solution 1.

If *transa* = `'T'`, $\boldsymbol{A}^{T}$ is used, resulting in solution 2.

If *transa* = `'C'`, $\boldsymbol{A}^{H}$ is used, resulting in solution 3.

Specified as: a single character. It must be `'N'`, `'T'`, or `'C'`.

*diag*

indicates the characteristics of the diagonal of matrix ***A***, where:

If *diag* = `'U'`, ***A*** is a unit triangular matrix.

If *diag* = `'N'`, ***A*** is not a unit triangular matrix.

Specified as: a single character. It must be `'U'` or `'N'`.

*n*

is the order of triangular matrix ***A***. Specified as: a fullword integer; $n \geq 0$ and $n \leq lda$.

*a*

is the upper or lower triangular matrix ***A*** of order *n*, stored in upper- or lower-triangular storage mode, respectively. Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 99 on page 527.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; $lda > 0$ and $lda \geq n$.

*ap*

is the upper or lower triangular matrix ***A*** of order *n*, stored in upper- or lower-triangular-packed storage mode, respectively. Specified as: a one-dimensional array of (at least) length $n(n+1)/2$, containing numbers of the data type indicated in Table 99 on page 527.

*x*

is the vector ***x*** of length *n*, containing the right-hand side of the triangular system to be solved. Specified as: a one-dimensional array of (at least) length $1+(n-1)|incx|$, containing numbers of the data type indicated in Table 99 on page 527.

*incx*

is the stride for vector ***x***. Specified as: a fullword integer; $incx > 0$ or $incx < 0$.

*On Return*

*x*

is the solution vector ***x*** of length *n*, containing the results of the computation. Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 99 on page 527.

***Notes***

1. These subroutines accept lowercase letters for the *uplo*, *transa*, and *diag* arguments.

2. For STRSV, DTRSV, STPSV, and DTPSV, if you specify `'C'` for the *transa* argument, it is interpreted as though you specified `'T'`.
3. Matrix ***A*** and vector ***x*** must have no common elements; otherwise, results are unpredictable.
4. ESSL assumes certain values in your array for parts of a triangular matrix. As a result, you do not have to set these values. For unit diagonal matrices, the elements of the diagonal are assumed to be 1.0 for real matrices and (1.0, 0.0) for complex matrices. When using upper- or lower-triangular storage, the unreferenced elements in the lower and upper triangular part, respectively, are assumed to be zero.
5. For a description of triangular matrices and how they are stored in upper- and lower-triangular storage mode and in upper- and lower-triangular-packed storage mode, see “Triangular Matrix” on page 73.

***Function:*** These subroutines solve a triangular system of equations with a single right-hand side. The solution ***x*** may be any of the following, where triangular matrix ***A***, its transpose, or its conjugate transpose is used, and where ***A*** can be either upper- or lower-triangular:

1. $\mathbf{x} \leftarrow \mathbf{A}^{-1}\mathbf{x}$
2. $\mathbf{x} \leftarrow \mathbf{A}^{-T}\mathbf{x}$
3. $\mathbf{x} \leftarrow \mathbf{A}^{-H}\mathbf{x}$ (only for CTRSV, ZTRSV, CTPSV, and ZTPSV)

where:

***x*** is a vector of length *n*.
***A*** is an upper or lower triangular matrix of order *n*. For _TRSV, it is stored in upper- or lower-triangular storage mode, respectively. For _TPSV, it is stored in upper- or lower-triangular-packed storage mode, respectively.

If *n* is 0, no computation is performed. See references [32 ], [36 ], and [38 ].

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. *uplo* ≠ `'L'` or `'U'`
2. *transa* ≠ `'T'`, `'N'`, or `'C'`
3. *diag* ≠ `'N'` or `'U'`
4. $n < 0$
5. $lda \le 0$
6. $lda < n$
7. $incx = 0$

***Example 1:*** This example shows the solution $\mathbf{x} \leftarrow \mathbf{A}^{-1}\mathbf{x}$. Matrix ***A*** is a real 4 by 4 lower unit triangular matrix, stored in lower-triangular storage mode. Vector ***x*** is a vector of length 4.

**Note:** Because matrix ***A*** is unit triangular, the diagonal elements are not referenced. ESSL assumes a value of 1.0 for the diagonal elements.

*Call Statement and Input*

```
            UPLO TRANSA DIAG  N   A  LDA  X  INCX
             |     |     |    |   |   |   |   |
CALL STRSV( 'L' , 'N' , 'U' , 4 , A , 4 , X , 1  )
```

```
        ┌                  ┐
        |  .    .    .   . |
        | 1.0   .    .   . |
A    =  | 2.0  3.0   .   . |
        | 3.0  4.0  3.0  . |
        └                  ┘
```

```
X        =  (1.0, 3.0, 11.0, 24.0)
```

*Output*

```
X        =  (1.0, 2.0, 3.0, 4.0)
```

***Example 2:*** This example shows the solution ***x*** ← $\boldsymbol{A}^{-T}\boldsymbol{x}$. Matrix ***A*** is a real 4 by 4 upper nonunit triangular matrix, stored in upper-triangular storage mode. Vector ***x*** is a vector of length 4.

*Call Statement and Input*

```
            UPLO TRANSA DIAG  N   A  LDA  X  INCX
             |     |     |    |   |   |   |   |
CALL STRSV( 'U' , 'T' , 'N' , 4 , A , 4 , X , 1  )
```

```
        ┌                    ┐
        | 1.0  2.0  3.0  2.0 |
A    =  |  .   2.0  2.0  5.0 |
        |  .    .   3.0  3.0 |
        |  .    .    .   1.0 |
        └                    ┘
```

```
X        =  (5.0, 18.0, 32.0, 41.0)
```

*Output*

```
X        =  (5.0, 4.0, 3.0, 2.0)
```

***Example 3:*** This example shows the solution ***x*** ← $\boldsymbol{A}^{-H}\boldsymbol{x}$. Matrix ***A*** is a complex 4 by 4 upper unit triangular matrix, stored in upper-triangular storage mode. Vector ***x*** is a vector of length 4.

**Note:** Because matrix ***A*** is unit triangular, the diagonal elements are not referenced. ESSL assumes a value of (1.0, 0.0) for the diagonal elements.

*Call Statement and Input*

```
                UPLO TRANSA DIAG  N   A  LDA  X  INCX
                 |    |     |    |   |   |   |   |
CALL CTRSV( 'U' , 'C' , 'U' , 4 , A , 4 , X , 1  )

        ┌                                         ┐
        | . (2.0, 2.0) (3.0,  3.0) (2.0, 2.0) |
A    =  | .     .      (2.0,  2.0) (5.0, 5.0) |
        | .     .         .        (3.0, 3.0) |
        | .     .         .            .      |
        └                                         ┘

X         =  ((5.0, 5.0), (24.0, 4.0), (49.0, 3.0), (80.0, 2.0))
```

*Output*

```
X         =  ((5.0, 5.0), (4.0, 4.0), (3.0, 3.0), (2.0, 2.0))
```

***Example 4:*** This example shows the solution $\boldsymbol{x} \leftarrow \boldsymbol{A}^{-1}\boldsymbol{x}$. Matrix ***A*** is a real 4 by 4 lower unit triangular matrix, stored in lower-triangular-packed storage mode. Vector ***x*** is a vector of length 4. Matrix ***A*** is:

```
┌                     ┐
| 1.0   .    .    .   |
| 1.0  1.0   .    .   |
| 2.0  3.0  1.0   .   |
| 3.0  4.0  3.0  1.0  |
└                     ┘
```

**Note:** Because matrix ***A*** is unit triangular, the diagonal elements are not referenced. ESSL assumes a value of 1.0 for the diagonal elements.

*Call Statement and Input*

```
                UPLO TRANSA DIAG  N   AP   X  INCX
                 |    |     |    |   |    |   |
CALL STPSV( 'L' , 'N' , 'U' , 4 , AP , X , 1  )

AP        =  ( . , 1.0, 2.0, 3.0, . , 3.0, 4.0, . , 3.0, . )
X         =  (1.0, 3.0, 11.0, 24.0)
```

*Output*

```
X         =  (1.0, 2.0, 3.0, 4.0)
```

***Example 5:*** This example shows the solution $\boldsymbol{x} \leftarrow \boldsymbol{A}^{-T}\boldsymbol{x}$. Matrix ***A*** is a real 4 by 4 upper nonunit triangular matrix, stored in upper-triangular-packed storage mode. Vector ***x*** is a vector of length 4. Matrix ***A*** is:

```
┌                     ┐
| 1.0  2.0  3.0  2.0  |
|  .   2.0  2.0  5.0  |
|  .    .   3.0  3.0  |
|  .    .    .   1.0  |
└                     ┘
```

*Call Statement and Input*

```
            UPLO TRANSA DIAG  N   AP   X  INCX
             |     |     |    |   |    |   |
CALL STPSV( 'U' , 'T' , 'N' , 4 , AP , X , 1  )

AP       =  (1.0, 2.0, 2.0, 3.0, 2.0, 3.0, 2.0, 5.0, 3.0, 1.0)
X        =  (5.0, 18.0, 32.0, 41.0)
```

*Output*

```
X        =  (5.0, 4.0, 3.0, 2.0)
```

***Example 6:*** This example shows the solution $\boldsymbol{x} \leftarrow \boldsymbol{A}^{-H}\boldsymbol{x}$. Matrix ***A*** is a complex 4 by 4 upper unit triangular matrix, stored in upper-triangular-packed storage mode. Vector ***x*** is a vector of length 4. Matrix ***A*** is:

$$\begin{bmatrix} (1.0, 0.0) & (2.0, 2.0) & (3.0, 3.0) & (2.0, 2.0) \\ . & (1.0, 0.0) & (2.0, 2.0) & (5.0, 5.0) \\ . & . & (1.0, 0.0) & (3.0, 3.0) \\ . & . & . & (1.0, 0.0) \end{bmatrix}$$

**Note:** Because matrix ***A*** is unit triangular, the diagonal elements are not referenced. ESSL assumes a value of (1.0, 0.0) for the diagonal elements.

*Call Statement and Input*

```
            UPLO TRANSA DIAG  N   AP   X  INCX
             |     |     |    |   |    |   |
CALL CTPSV( 'U' , 'C' , 'U' , 4 , AP , X , 1  )

AP       =  ( . , (2.0, 2.0), . , (3.0, 3.0), (2.0, 2.0), . ,
             (2.0, 2.0), (5.0, 5.0), (3.0, 3.0), . )
X        =  ((5.0, 5.0), (24.0, 4.0), (49.0, 3.0), (80.0, 2.0))
```

*Output*

```
X        =  ((5.0, 5.0), (4.0, 4.0), (3.0, 3.0), (2.0, 2.0))
```

# STRSM, DTRSM, CTRSM, and ZTRSM—Solution of Triangular Systems of Equations with Multiple Right-Hand Sides

STRSM and DTRSM perform one of the following solves for a triangular system of equations with multiple right-hand sides, using scalar $\alpha$, rectangular matrix ***B***, and triangular matrix ***A*** or its transpose:

| Solution | Equation |
|---|---|
| 1. $\boldsymbol{B} \leftarrow \alpha(\boldsymbol{A}^{-1})\boldsymbol{B}$ | $\boldsymbol{AX} = \alpha\boldsymbol{B}$ |
| 2. $\boldsymbol{B} \leftarrow \alpha(\boldsymbol{A}^{-T})\boldsymbol{B}$ | $\boldsymbol{A}^{T}\boldsymbol{X} = \alpha\boldsymbol{B}$ |
| 3. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{B}(\boldsymbol{A}^{-1})$ | $\boldsymbol{XA} = \alpha\boldsymbol{B}$ |
| 4. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{B}(\boldsymbol{A}^{-T})$ | $\boldsymbol{XA}^{T} = \alpha\boldsymbol{B}$ |

CTRSM and ZTRSM perform one of the following solves for a triangular system of equations with multiple right-hand sides, using scalar $\alpha$, rectangular matrix ***B***, and triangular matrix ***A***, its transpose, or its conjugate transpose:

| Solution | Equation |
|---|---|
| 1. $\boldsymbol{B} \leftarrow \alpha(\boldsymbol{A}^{-1})\boldsymbol{B}$ | $\boldsymbol{AX} = \alpha\boldsymbol{B}$ |
| 2. $\boldsymbol{B} \leftarrow \alpha(\boldsymbol{A}^{-T})\boldsymbol{B}$ | $\boldsymbol{A}^{T}\boldsymbol{X} = \alpha\boldsymbol{B}$ |
| 3. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{B}(\boldsymbol{A}^{-1})$ | $\boldsymbol{XA} = \alpha\boldsymbol{B}$ |
| 4. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{B}(\boldsymbol{A}^{-T})$ | $\boldsymbol{XA}^{T} = \alpha\boldsymbol{B}$ |
| 5. $\boldsymbol{B} \leftarrow \alpha(\boldsymbol{A}^{-H})\boldsymbol{B}$ | $\boldsymbol{A}^{H}\boldsymbol{X} = \alpha\boldsymbol{B}$ |
| 6. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{B}(\boldsymbol{A}^{-H})$ | $\boldsymbol{XA}^{H} = \alpha\boldsymbol{B}$ |

**Note:** The term ***X*** used in the systems of equations listed above represents the output solution matrix. It is important to note that in these subroutines the solution matrix is actually returned in the input-output argument *b*.

*Table 100. Data Types*

| ***A***, ***B***, $\alpha$ | **Subroutine** |
|---|---|
| Short-precision real | STRSM |
| Long-precision real | DTRSM |
| Short-precision complex | CTRSM |
| Long-precision complex | ZTRSM |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL STRSM \| DTRSM \| CTRSM \| ZTRSM (*side*, *uplo*, *transa*, *diag*, *m*, *n*, *alpha*, *a*, *lda*, *b*, *ldb*) |
| **C and C++** | strsm \| dtrsm \| ctrsm \| ztrsm (*side*, *uplo*, *transa*, *diag*, *m*, *n*, *alpha*, *a*, *lda*, *b*, *ldb*); |
| **PL/I** | CALL STRSM \| DTRSM \| CTRSM \| ZTRSM (*side*, *uplo*, *transa*, *diag*, *m*, *n*, *alpha*, *a*, *lda*, *b*, *ldb*); |

*On Entry*

*side*

indicates whether the triangular matrix ***A*** is located to the left or right of rectangular matrix ***B*** in the system of equations, where:

If *side* = `'L'`, ***A*** is to the left of ***B***, resulting in solution 1, 2, or 5.

If *side* = `'R'`, ***A*** is to the right of ***B***, resulting in solution 3, 4, or 6.

Specified as: a single character. It must be `'L'` or `'R'`.

*uplo*

indicates whether matrix ***A*** is an upper or lower triangular matrix, where:

If *uplo* = 'U', ***A*** is an upper triangular matrix.

If *uplo* = 'L', ***A*** is a lower triangular matrix.

Specified as: a single character. It must be 'U' or 'L'.

*transa*

indicates the form of matrix ***A*** used in the system of equations, where:

If *transa* = 'N', ***A*** is used, resulting in solution 1 or 3.

If *transa* = 'T', $\boldsymbol{A}^T$ is used, resulting in solution 2 or 4.

If *transa* = 'C', $\boldsymbol{A}^H$ is used, resulting in solution 5 or 6.

Specified as: a single character. It must be 'N', 'T', or 'C'.

*diag*

indicates the characteristics of the diagonal of matrix ***A***, where:

If *diag* = 'U', ***A*** is a unit triangular matrix.

If *diag* = 'N', ***A*** is not a unit triangular matrix.

Specified as: a single character. It must be 'U' or 'N'.

*m*

is the number of rows in rectangular matrix ***B***, and:

If *side* = 'L', *m* is the order of triangular matrix ***A***.

Specified as: a fullword integer, where:

If *side* = 'L', $0 \le m \le lda$ and $m \le ldb$.

If *side* = 'R', $0 \le m \le ldb$.

*n*

is the number of columns in rectangular matrix ***B***, and:

If *side* = 'R', *n* is the order of triangular matrix ***A***.

Specified as: a fullword integer; $n \ge 0$, and:

If *side* = 'R', $n \le lda$.

*alpha*

is the scalar $\alpha$. Specified as: a number of the data type indicated in Table 100 on page 533.

*a*

is the triangular matrix ***A***, of which only the upper or lower triangular portion is used, where:

If *side* = 'L', ***A*** is order *m*.

If *side* = 'R', ***A*** is order *n*.

Specified as: a two-dimensional array, containing numbers of the data type indicated in Table 100 on page 533, where:

If *side* = 'L', its size must be *lda* by (at least) *m*.

If *side* = 'R', it size must be *lda* by (at least) *n*.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; $lda > 0$, and:

If *side* = 'L', $lda \ge m$.

If *side* = `'R'`, *lda* ≥ *n*.

*b*

is the *m* by *n* rectangular matrix ***B***, which contains the right-hand sides of the triangular system to be solved. Specified as: an *ldb* by (at least) *n* array, containing numbers of the data type indicated in Table 100 on page 533.

*ldb*

is the leading dimension of the array specified for *b*. Specified as: a fullword integer; *ldb* > 0 and *ldb* ≥ *m*.

*On Return*

*b*

is the *m* by *n* matrix ***B***, containing the results of the computation.

Returned as: an *ldb* by (at least) *n* array, containing numbers of the data type indicated in Table 100 on page 533.

***Notes***

1. These subroutines accept lowercase letters for the *transa*, *side*, *diag*, and *uplo* arguments.
2. For STRSM and DTRSM, if you specify `'C'` for the *transa* argument, it is interpreted as though you specified `'T'`.
3. Matrices ***A*** and ***B*** must have no common elements or results are unpredictable.
4. If matrix ***A*** is upper triangular (*uplo* = `'U'`), these subroutines refer to only the upper triangular portion of the matrix. If matrix ***A*** is lower triangular, (*uplo* = `'L'`), these subroutines refer to only the lower triangular portion of the matrix. The unreferenced elements are assumed to be zero.
5. The elements of the diagonal of a unit triangular matrix are always one, so you do not need to set these values. The ESSL subroutines always assume that the values in these positions are 1.0 for STRSM and DTRSM and (1.0, 0.0) for CTRSM and ZTRSM.
6. For a description of triangular matrices and how they are stored, see “Triangular Matrix” on page 73.

***Function:*** These subroutines solve a triangular system of equations with multiple right-hand sides. The solution ***B*** may be any of the following, where ***A*** is a triangular matrix and ***B*** is a rectangular matrix:

1. $\boldsymbol{B} \leftarrow \alpha(\boldsymbol{A}^{-1})\boldsymbol{B}$
2. $\boldsymbol{B} \leftarrow \alpha(\boldsymbol{A}^{-T})\boldsymbol{B}$
3. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{B}(\boldsymbol{A}^{-1})$
4. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{B}(\boldsymbol{A}^{-T})$
5. $\boldsymbol{B} \leftarrow \alpha(\boldsymbol{A}^{-H})\boldsymbol{B}$ (only for CTRSM and ZTRSM)
6. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{B}(\boldsymbol{A}^{-H})$ (only for CTRSM and ZTRSM)

where:

$\alpha$ is a scalar.
***B*** is an *m* by *n* rectangular matrix.
***A*** is an upper or lower triangular matrix, where:
If *side* = `'L'`, it has order *m*, and equation 1, 2, or 5 is performed.
If *side* = `'R'`, it has order *n*, and equation 3, 4, or 6 is performed.

If *n* or *m* is 0, no computation is performed. See references [32] and [36 ].

***Error Conditions***

*Resource Errors:* Unable to allocate internal work area.

*Computational Errors:* None

**Note:** If the triangular matrix ***A*** is singular, the results returned by this subroutine are unpredictable, and there may be a divide-by-zero program exception message.

*Input-Argument Errors*

1. $m < 0$
2. $n < 0$
3. *lda*, *ldb* $\leq 0$
4. *side* ≠ `'L'` or `'R'`
5. *uplo* ≠ `'L'` or `'U'`
6. *transa* ≠ `'T'`, `'N'`, or `'C'`
7. *diag* ≠ `'N'` or `'U'`
8. *side* = `'L'` and $m > lda$
9. $m > ldb$
10. *side* = `'R'` and $n > lda$

***Example 1:*** This example shows the solution $\boldsymbol{B} \leftarrow \alpha(\boldsymbol{A}^{-1})\boldsymbol{B}$, where ***A*** is a real 5 by 5 upper triangular matrix that is not unit triangular, and ***B*** is a real 5 by 3 rectangular matrix.

*Call Statement and Input*

```
            SIDE  UPLO TRANSA  DIAG  M   N   ALPHA   A  LDA  B  LDB
             |     |     |      |    |   |     |     |   |   |   |
CALL STRSM( 'L' , 'U' , 'N'  , 'N' , 5 , 3 ,  1.0  , A , 7 , B , 6 )
```

```
        ┌                             ┐
        │ 3.0  -1.0   2.0   2.0   1.0 │
        │  .   -2.0   4.0  -1.0   3.0 │
        │  .     .   -3.0   0.0   2.0 │
A    =  │  .     .     .    4.0  -2.0 │
        │  .     .     .     .    1.0 │
        │  .     .     .     .     .  │
        │  .     .     .     .     .  │
        └                             ┘
```

```
        ┌                     ┐
        │   6.0  10.0   -2.0 │
        │ -16.0  -1.0    6.0 │
B    =  │  -2.0   1.0   -4.0 │
        │  14.0   0.0  -14.0 │
        │  -1.0   2.0    1.0 │
        │    .     .      .  │
        └                     ┘
```

*Output*

```
        ┌                 ┐
        │  2.0  3.0   1.0 │
        │  5.0  5.0   4.0 │
B    =  │  0.0  1.0   2.0 │
        │  3.0  1.0  -3.0 │
        │ -1.0  2.0   1.0 │
        │   .    .     .  │
        └                 ┘
```

***Example 2:*** This example shows the solution $\boldsymbol{B} \leftarrow \alpha(\boldsymbol{A}^{-T})\boldsymbol{B}$, where ***A*** is a real 5 by 5 upper triangular matrix that is not unit triangular, and ***B*** is a real 5 by 4 rectangular matrix.

*Call Statement and Input*

```
            SIDE  UPLO TRANSA  DIAG  M   N    ALPHA  A  LDA  B  LDB
             |     |     |      |    |   |     |     |   |   |   |
CALL STRSM( 'L' , 'U' , 'T'  , 'N' , 5 , 4 ,  1.0  , A , 7 , B , 6 )
```

```
        ┌                              ┐
        │ -1.0  -4.0  -2.0   2.0   3.0 │
        │   .   -2.0   2.0   2.0   2.0 │
        │   .     .   -3.0  -1.0   4.0 │
A    =  │   .     .     .    1.0   0.0 │
        │   .     .     .     .   -2.0 │
        │   .     .     .     .     .  │
        │   .     .     .     .     .  │
        └                              ┘
```

```
        ┌                          ┐
        │ -1.0  -2.0   -3.0   -4.0 │
        │  2.0  -2.0  -14.0  -12.0 │
B    =  │ 10.0   5.0   -8.0   -7.0 │
        │ 14.0  15.0    1.0    8.0 │
        │ -3.0   4.0    3.0   16.0 │
        │   .     .      .      .  │
        └                          ┘
```

*Output*

```
        ┌                        ┐
        │  1.0   2.0   3.0   4.0 │
        │  3.0   3.0  -1.0   2.0 │
B    =  │ -2.0  -1.0   0.0   1.0 │
        │  4.0   4.0  -3.0  -3.0 │
        │  2.0   2.0   2.0   2.0 │
        │   .     .     .     .  │
        └                        ┘
```

***Example 3:*** This example shows the solution $\boldsymbol{B} \leftarrow \alpha\boldsymbol{B}(\boldsymbol{A}^{-1})$, where ***A*** is a real 5 by 5 lower triangular matrix that is not unit triangular, and ***B*** is a real 3 by 5 rectangular matrix.

*Call Statement and Input*

```
            SIDE  UPLO TRANSA  DIAG  M   N    ALPHA  A  LDA  B  LDB
             |     |     |      |    |   |     |     |   |   |   |
CALL STRSM( 'R' , 'L' , 'N'  , 'N' , 3 , 5 ,  1.0  , A , 7 , B , 4 )
```

```
        ┌                            ┐
        | 2.0   .     .     .     .  |
        | 2.0  3.0    .     .     .  |
        | 2.0  1.0   1.0    .     .  |
A    =  | 0.0  3.0   0.0  -2.0    .  |
        | 2.0  4.0  -1.0   2.0  -1.0 |
        |  .    .     .     .     .  |
        |  .    .     .     .     .  |
        └                            ┘
```

```
        ┌                             ┐
        | 10.0   4.0   0.0  0.0   1.0 |
B    =  | 10.0  14.0  -4.0  6.0  -3.0 |
        | -8.0   2.0  -5.0  4.0  -2.0 |
        |   .     .     .    .     .  |
        └                             ┘
```

*Output*

```
        ┌                              ┐
        |  3.0   4.0  -1.0  -1.0  -1.0 |
B    =  |  2.0   1.0  -1.0   0.0   3.0 |
        | -2.0  -1.0  -3.0   0.0   2.0 |
        |   .     .     .     .     .  |
        └                              ┘
```

***Example 4:*** This example shows the solution $\boldsymbol{B} \leftarrow \alpha\boldsymbol{B}(\boldsymbol{A}^{-1})$, where ***A*** is a real 6 by 6 upper triangular matrix that is unit triangular, and ***B*** is a real 1 by 6 rectangular matrix.

**Note:** Because matrix ***A*** is unit triangular, the diagonal elements are not referenced. ESSL assumes a value of 1.0 for the diagonal element.

*Call Statement and Input*

```
            SIDE  UPLO TRANSA  DIAG  M   N    ALPHA  A  LDA  B  LDB
             |     |     |      |    |   |     |     |   |   |   |
CALL STRSM( 'R' , 'U' , 'N'  , 'U' , 1 , 6 ,  1.0  , A , 7 , B , 2 )
```

```
        ┌                               ┐
        | .  2.0  -3.0  1.0   2.0   4.0 |
        | .   .    0.0  1.0   1.0  -2.0 |
        | .   .     .   4.0  -1.0   1.0 |
A    =  | .   .     .    .    0.0  -1.0 |
        | .   .     .    .     .    2.0 |
        | .   .     .    .     .     .  |
        | .   .     .    .     .     .  |
        └                               ┘
```

```
        ┌                                 ┐
B    =  | 1.0  4.0  -2.0  10.0  2.0  -6.0 |
        |  .    .     .     .    .     .  |
        └                                 ┘
```

*Output*

```
        ┌                                ┐
B    =  | 1.0  2.0  1.0  3.0  -1.0  -2.0 |
        |  .    .    .    .     .     .  |
        └                                ┘
```

***Example 5:*** This example shows the solution $\boldsymbol{B} \leftarrow \alpha\boldsymbol{B}(\boldsymbol{A}^{-1})$, where ***A*** is a complex 5 by 5 lower triangular matrix that is not unit triangular, and ***B*** is a complex 3 by 5 rectangular matrix.

*Call Statement and Input*

```
             SIDE  UPLO TRANSA  DIAG  M   N    ALPHA   A  LDA  B  LDB
              |     |     |      |    |   |      |     |   |   |   |
CALL CTRSM( 'R' , 'L' , 'N'  , 'N' , 3 , 5 ,  ALPHA , A , 7 , B , 4 )

ALPHA    =  (1.0, 0.0)

          ┌                                                                ┐
          | (2.0, -3.0)     .           .           .           .          |
          | (2.0, -4.0) (3.0, -1.0)     .           .           .          |
          | (2.0,  2.0) (1.0,  2.0)  (1.0,  1.0)    .           .          |
A     =   | (0.0,  0.0) (3.0, -1.0)  (0.0, -1.0) (-2.0,  1.0)    .          |
          | (2.0,  2.0) (4.0,  0.0) (-1.0,  2.0)  (2.0, -4.0) (-1.0, -4.0) |
          |     .           .           .           .           .          |
          |     .           .           .           .           .          |
          └                                                                ┘

          ┌                                                                            ┐
          | (22.0, -41.0)  (7.0, -26.0)  (9.0, 0.0)  (-15.0, -3.0)  (-15.0, 8.0) |
B     =   | (29.0, -18.0) (24.0, -10.0)  (9.0, 6.0) (-12.0, -24.0) (-19.0, -8.0) |
          |  (-15.0, 2.0) (-3.0, -21.0) (-2.0, 4.0)  (-4.0, -12.0) (-10.0, -6.0) |
          |       .             .             .            .             .          |
          └                                                                            ┘
```

*Output*

```
          ┌                                                                    ┐
          |  (3.0, 0.0)   (4.0, 0.0) (-1.0, -2.0) (-1.0, -1.0) (-1.0, -4.0) |
B     =   | (2.0, -1.0)   (1.0, 2.0) (-1.0, -3.0)   (0.0, 2.0)  (3.0, -4.0) |
          | (-2.0, 1.0) (-1.0, -3.0)  (-3.0, 1.0)   (0.0, 0.0)  (2.0, -2.0) |
          |     .            .             .            .            .      |
          └                                                                    ┘
```

***Example 6:*** This example shows the solution $\boldsymbol{B} \leftarrow \alpha(\boldsymbol{A}^{-H})\boldsymbol{B}$, where ***A*** is a complex 5 by 5 upper triangular matrix that is not unit triangular, and ***B*** is a complex 5 by 1 rectangular matrix.

*Call Statement and Input*

```
             SIDE  UPLO TRANSA  DIAG  M   N    ALPHA   A  LDA  B  LDB
              |     |     |      |    |   |      |     |   |   |   |
CALL CTRSM( 'L' , 'U' , 'C'  , 'N' , 5 , 1 ,  ALPHA , A , 6 , B , 6 )

ALPHA    =  (1.0, 0.0)

          ┌                                                                  ┐
          | (-4.0, 1.0) (4.0, -3.0)  (-1.0, 3.0)   (0.0, 0.0)  (-1.0, 0.0) |
          |      .      (-2.0, 0.0) (-3.0, -1.0) (-2.0, -1.0)   (4.0, 3.0) |
A     =   |      .            .      (-5.0, 3.0) (-3.0, -3.0) (-5.0, -5.0) |
          |      .            .            .      (4.0, -4.0)   (2.0, 0.0) |
          |      .            .            .           .       (2.0, -1.0) |
          |      .            .            .           .            .      |
          └                                                                  ┘
          ┌                 ┐
          | (-8.0, -19.0)   |
          |  (8.0,  21.0)   |
B     =   | (44.0,  -8.0)   |
          | (13.0,  -7.0)   |
          | (19.0,   2.0)   |
          |       .         |
          └                 ┘
```

*Output*

```
          ┌              ┐
          |  (3.0, 4.0)  |
          | (-4.0, 2.0)  |
B     =   | (-5.0, 0.0)  |
          |  (1.0, 3.0)  |
          |  (3.0, 1.0)  |
          |      .       |
          └              ┘
```

## STRI, DTRI, STPI, and DTPI—Triangular Matrix Inverse

These subroutines find the inverse of triangular matrix ***A***:

$$\boldsymbol{A} \leftarrow \boldsymbol{A}^{-1}$$

Matrix ***A*** can be either upper or lower triangular, where:

- For the _TRI subroutines, it is stored in upper- or lower-triangular storage mode, respectively.
- For the _TPI subroutines, it is stored in upper- or lower-triangular-packed storage mode, respectively.

*Table 101. Data Types*

| ***A*** | **Subroutine** |
|---|---|
| Short-precision real | STRI and STPI |
| Long-precision real | DTRI and DTPI |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL STRI \| DTRI (*uplo*, *diag*, *a*, *lda*, *n*)<br>CALL STPI \| DTPI (*uplo*, *diag*, *ap*, *n*) |
| **C and C++** | stri \| dtri (*uplo*, *diag*, *a*, *lda*, *n*);<br>stpi \| dtpi (*uplo*, *diag*, *ap*, *n*); |
| **PL/I** | CALL STRI \| DTRI (*uplo*, *diag*, *a*, *lda*, *n*);<br>CALL STPI \| DTPI (*uplo*, *diag*, *ap*, *n*); |

*On Entry*

*uplo*

indicates whether matrix ***A*** is an upper or lower triangular matrix, where:

If *uplo* = 'U', ***A*** is an upper triangular matrix.

If *uplo* = 'L', ***A*** is a lower triangular matrix.

Specified as: a single character. It must be 'U' or 'L'.

*diag*

indicates the characteristics of the diagonal of matrix ***A***, where:

If *diag* = 'U', ***A*** is a unit triangular matrix.

If *diag* = 'N', ***A*** is not a unit triangular matrix.

Specified as: a single character. It must be 'U' or 'N'.

*a*

is the upper or lower triangular matrix ***A*** of order *n*, stored in upper- or lower-triangular storage mode, respectively. Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 101.

*lda*

is the leading dimension of the arrays specified for *a*. Specified as: a fullword integer; *lda* > 0 and *lda* ≥ *n*.

*ap*

is the upper or lower triangular matrix ***A*** of order *n*, stored in upper- or lower-triangular-packed storage mode, respectively. Specified as: a one-dimensional array of (at least) length *n*(*n*+1)/2, containing numbers of the data type indicated in Table 101.

*n*

is the order of matrix ***A***. Specified as: a fullword integer; $n \geq 0$, where:

*On Return*

*a*

is the inverse of the upper or lower triangular matrix ***A*** of order *n*, stored in upper- or lower-triangular storage mode, respectively. Returned as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 101 on page 541.

*ap*

is the inverse of the upper or lower triangular matrix ***A*** of order *n*, stored in upper- or lower-triangular-packed storage mode, respectively. Returned as: a one-dimensional array of (at least) length *n*(*n*+1)/2, containing numbers of the data type indicated in Table 101 on page 541.

***Notes***

1. These subroutines accept lowercase letters for the *uplo* and *diag* arguments.
2. If matrix ***A*** is upper triangular (*uplo* = `'U'`), these subroutines refer to only the upper triangular portion of the matrix. If matrix ***A*** is lower triangular, (*uplo* = `'L'`), these subroutines refer to only the lower triangular portion of the matrix. The unreferenced elements are assumed to be zero.
3. The elements of the diagonal of a unit triangular matrix are always one, so you do not need to set these values.
4. For a description of triangular matrices and how they are stored in upper- and lower-triangular storage mode and in upper- and lower-triangular-packed storage mode, see “Triangular Matrix” on page 73.

***Function:*** These subroutines find the inverse of triangular matrix ***A***, where ***A*** is either upper or lower triangular:

$$\boldsymbol{A} \leftarrow \boldsymbol{A}^{-1}$$

where:

***A*** is the triangular matrix of order *n*.
$\boldsymbol{A}^{-1}$ the inverse of the triangular matrix of order *n*.

If *n* is 0, no computation is performed. See references [8 ] and [36 ].

***Error Conditions***

*Resource Errors:* Unable to allocate internal work area.

*Computational Errors:* Matrix ***A*** is singular.

- One or more of the diagonal elements of matrix ***A*** are zero. The first column, *i*, of matrix ***A***, in which a zero diagonal element is found, is identified in the computational error message.
- The return code is set to 1.

- *i* can be determined at run time by use of the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error code 2145 in the ESSL error option table; otherwise, the default value causes your program to terminate when this error occurs. For details, see “What Can You Do about ESSL Computational Errors?” on page 48.

*Input-Argument Errors*

1. *uplo* ≠ `'U'` or `'L'`
2. *diag* ≠ `'U'` or `'N'`
3. *n* < 0
4. *lda* ≤ 0
5. *lda* < *n*

***Example 1:*** This example shows how the inverse of matrix ***A*** is computed, where ***A*** is a 5 by 5 upper triangular matrix that is not unit triangular and is stored in upper-triangular storage mode. Matrix ***A*** is:

```
┌                                ┐
│ 1.00  3.00  4.00   5.00   6.00 │
│ 0.00  2.00  8.00   9.00   1.00 │
│ 0.00  0.00  4.00   8.00   4.00 │
│ 0.00  0.00  0.00  -2.00   6.00 │
│ 0.00  0.00  0.00   0.00  -1.00 │
└                                ┘
```

and where the following inverse matrix is computed. Matrix ***A***$^{-1}$ is:

```
┌                                    ┐
│ 1.00  -1.50   2.00   3.75   35.00 │
│ 0.00   0.50  -1.00  -1.75  -14.00 │
│ 0.00   0.00   0.25   1.00    7.00 │
│ 0.00   0.00   0.00  -0.50   -3.00 │
│ 0.00   0.00   0.00   0.00   -1.00 │
└                                    ┘
```

*Call Statement and Input*

```
           UPLO  DIAG  A  LDA  N
            |     |    |   |   |
CALL STRI( 'U' , 'N' , A , 5 , 5)
```

```
        ┌                                ┐
        │ 1.00  3.00  4.00   5.00   6.00 │
        │  .    2.00  8.00   9.00   1.00 │
A    =  │  .     .    4.00   8.00   4.00 │
        │  .     .     .    -2.00   6.00 │
        │  .     .     .      .    -1.00 │
        └                                ┘
```

*Output*

```
        ┌                                    ┐
        │ 1.00  -1.50   2.00   3.75   35.00 │
        │  .     0.50  -1.00  -1.75  -14.00 │
A    =  │  .      .     0.25   1.00    7.00 │
        │  .      .      .    -0.50   -3.00 │
        │  .      .      .      .     -1.00 │
        └                                    ┘
```

***Example 2:*** This example shows how the inverse of matrix ***A*** is computed, where ***A*** is a 5 by 5 lower triangular matrix that is unit triangular and is stored in lower-triangular storage mode. Matrix ***A*** is:

```
┌                         ┐
│ 1.0  0.0  0.0  0.0  0.0 │
│ 3.0  1.0  0.0  0.0  0.0 │
│ 4.0  8.0  1.0  0.0  0.0 │
│ 5.0  9.0  8.0  1.0  0.0 │
│ 6.0  1.0  4.0  6.0  1.0 │
└                         ┘
```

and where the following inverse matrix is computed. Matrix ***A***$^{-1}$ is:

```
┌                                   ┐
│    1.0     0.0   0.0   0.0  0.0   │
│   -3.0     1.0   0.0   0.0  0.0   │
│   20.0    -8.0   1.0   0.0  0.0   │
│ -138.0    55.0  -8.0   1.0  0.0   │
│  745.0  -299.0  44.0  -6.0  1.0   │
└                                   ┘
```

**Note:** Because matrix ***A*** is unit triangular, the diagonal elements are not referenced. ESSL assumes a value of 1.0 for the diagonal elements.

*Call Statement and Input*

```
           UPLO  DIAG  A  LDA  N
            |     |    |   |   |
CALL STRI( 'L' , 'U' , A , 5 , 5)

         ┌                       ┐
         │  .    .    .    .   . │
         │ 3.0   .    .    .   . │
A    =   │ 4.0  8.0   .    .   . │
         │ 5.0  9.0  8.0   .   . │
         │ 6.0  1.0  4.0  6.0  . │
         └                       ┘
```

*Output*

```
         ┌                                      ┐
         │     .        .     .     .    .      │
         │   -3.0       .     .     .    .      │
A    =   │   20.0     -8.0    .     .    .      │
         │ -138.0     55.0  -8.0    .    .      │
         │  745.0   -299.0  44.0  -6.0   .      │
         └                                      ┘
```

***Example 3:*** This example shows how the inverse of matrix ***A*** is computed, where ***A*** is the same matrix shown in Example 1 and is stored in upper-triangular-packed storage mode. The inverse matrix computed here is the same as the inverse matrix shown in Example 1 and is stored in upper-triangular-packed storage mode.

*Call Statement and Input*

```
           UPLO  DIAG  AP   N
            |     |    |    |
CALL STPI( 'U' , 'N' , AP , 5)
```

```
AP       =  (1.00, 3.00, 2.00, 4.00, 8.00, 4.00, 5.00, 9.00, 8.00,
             -2.00, 6.00, 1.00, 4.00, 6.00, -1.00)
```

*Output*

```
AP       =  (1.00, -1.50, 0.50, 2.00, -1.00, 0.25, 3.75, -1.75, 1.00,
             -0.50, 35.00, -14.00, 7.00, -3.00, -1.00)
```

***Example 4:*** This example shows how the inverse of matrix ***A*** is computed, where ***A*** is the same matrix shown in Example 2 and is stored in lower-triangular-packed storage mode. The inverse matrix computed here is the same as the inverse matrix shown in Example 2 and is stored in lower-triangular-packed storage mode.

**Note:** Because matrix ***A*** is unit triangular, the diagonal elements are not referenced. ESSL assumes a value of 1.0 for the diagonal elements.

*Call Statement and Input*

```
           UPLO  DIAG  AP   N
            |     |    |    |
CALL STPI( 'L' , 'U' , AP , 5)
```

```
AP       =  ( . , 3.0, 4.0, 5.0, 6.0, . , 8.0, 9.0, 1.0, . , 8.0, 4.0,
             . , 6.0, . )
```

*Output*

```
AP       =  ( . , -3.0, 20.0, -138.0, 745.0, . , -8.0, 55.0, -299.0,
             . , -8.0, 44.0, . , -6.0, . )
```

# Banded Linear Algebraic Equation Subroutines

This section contains the banded linear algebraic equation subroutine descriptions.

## SGBF and DGBF—General Band Matrix Factorization

These subroutines factor general band matrix ***A***, stored in general-band storage mode, using Gaussian elimination. To solve the system of equations with one or more right-hand sides, follow the call to these subroutines with one or more calls to SGBS or DGBS, respectively.

*Table 102. Data Types*

| ***A*** | **Subroutine** |
|---|---|
| Short-precision real | SGBF |
| Long-precision real | DGBF |

**Note:** The output from these factorization subroutines should be used only as input to the solve subroutines SGBS and DGBS, respectively.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SGBF \| DGBF (*agb*, *lda*, *n*, *ml*, *mu*, *ipvt*) |
| **C and C++** | sgbf \| dgbf (*agb*, *lda*, *n*, *ml*, *mu*, *ipvt*); |
| **PL/I** | CALL SGBF \| DGBF (*agb*, *lda*, *n*, *ml*, *mu*, *ipvt*); |

*On Entry*

*agb*
is the general band matrix ***A*** of order *n*, stored in general-band storage mode, to be factored. It has an upper band width *mu* and a lower band width *ml*. Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 102, where $lda \geq 2ml+mu+16$.

*lda*
is the leading dimension of the array specified for *agb*. Specified as: a fullword integer; $lda > 0$ and $lda \geq 2ml+mu+16$.

*n*
is the order of the matrix ***A***. Specified as: a fullword integer; $n > ml$ and $n > mu$.

*ml*
is the lower band width *ml* of the matrix ***A***. Specified as: a fullword integer; $0 \leq ml < n$.

*mu*
is the upper band width *mu* of the matrix ***A***. Specified as: a fullword integer; $0 \leq mu < n$.

*ipvt*
See “On Return.”

*On Return*

*agb*
is the transformed matrix ***A*** of order *n*, containing the results of the factorization. See “Function” on page 548. Returned as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 102.

*ipvt*
is the integer vector ***ipvt*** of length *n*, containing the pivot information necessary to construct matrix ***L*** from the information contained in the output array *agb*.

Returned as: a one-dimensional array of (at least) length *n*, containing fullword integers.

***Notes***

1. ***ipvt*** is not a permutation vector in the strict sense. It is used to record column interchanges in ***L*** due to partial pivoting and to improve performance.
2. The entire *lda* by *n* array specified for *agb* must remain unchanged between calls to the factorization and solve subroutines.
3. This subroutine can be used for tridiagonal matrices ($ml = mu = 1$); however, the tridiagonal subroutines SGTF/DGTF and SGTS/DGTS are faster.
4. For a description of how a general band matrix is stored in general-band storage mode in an array, see “General Band Matrix” on page 76.

***Function:*** The general band matrix ***A***, stored in general-band storage mode, is factored using Gaussian elimination with partial pivoting to compute the ***LU*** factorization of ***A***, where:

***ipvt*** is a vector containing the pivoting information.
***L*** is a unit lower triangular band matrix.
***U*** is an upper triangular band matrix.

The transformed matrix ***A*** contains ***U*** in packed format, along with the multipliers necessary to construct, with the help of ***ipvt***, a matrix ***L***, such that ***A*** = ***LU***. This factorization can then be used by SGBS or DGBS, respectively, to solve the system of equations. See reference [38].

***Error Conditions***

*Resource Errors:* Unable to allocate internal work area.

*Computational Errors:* Matrix ***A*** is singular.

- One or more columns of ***L*** and the corresponding diagonal of ***U*** contain all zeros (all columns of ***L*** are checked). The last column, *i*, of ***L*** with a corresponding ***U*** = 0 diagonal element is identified in the computational error message.
- The return code is set to 1.
- *i* can be determined at run time by use of the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error code 2103 in the ESSL error option table; otherwise, the default value causes your program to terminate when this error occurs. For details, see “What Can You Do about ESSL Computational Errors?” on page 48.

*Input-Argument Errors*

1. $lda \le 0$
2. $ml < 0$
3. $ml \ge n$
4. $mu < 0$
5. $mu \ge n$
6. $lda < 2ml+mu+16$

***Example:*** This example shows a factorization of a general band matrix ***A*** of order 9, with a lower band width of 2 and an upper band width of 3. On input matrix ***A*** is:

```
┌                                               ┐
| 1.0  1.0  1.0  1.0  0.0  0.0   0.0   0.0   0.0 |
| 1.0  1.0  1.0  1.0  1.0  0.0   0.0   0.0   0.0 |
| 4.0  1.0  1.0  1.0  1.0  1.0   0.0   0.0   0.0 |
| 0.0  5.0  1.0  1.0  1.0  1.0   1.0   0.0   0.0 |
| 0.0  0.0  6.0  1.0  1.0  1.0   1.0   1.0   0.0 |
| 0.0  0.0  0.0  7.0  1.0  1.0   1.0   1.0   1.0 |
| 0.0  0.0  0.0  0.0  8.0  1.0   1.0   1.0   1.0 |
| 0.0  0.0  0.0  0.0  0.0  9.0   1.0   1.0   1.0 |
| 0.0  0.0  0.0  0.0  0.0  0.0  10.0  11.0  12.0 |
└                                               ┘
```

Matrix ***A*** is stored in general-band storage mode in the two-dimensional array AGB of size LDA by N, where LDA = 2*ml*+*mu*+16 = 23. The array AGB is declared as AGB(1:23,1:9).

**Note:** Matrix ***A*** is the same matrix used in the examples in subroutines SGEF and DGEF (see “Example 1” on page 467) and SGEFCD and DGEFCD (see “Example” on page 490).

*Call Statement and Input*

```
           AGB  LDA   N   ML  MU  IPVT )
            |    |    |   |   |    |
CALL SGBF( AGB , 23 , 9 , 2 , 3 , IPVT )
```

```
          ┌                                                                          ┐
          | 0.0000  0.0000  0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000 |
          | 0.0000  0.0000  0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000 |
          | 0.0000  0.0000  0.0000  1.0000  1.0000  1.0000   1.0000   1.0000   1.0000 |
          | 0.0000  0.0000  1.0000  1.0000  1.0000  1.0000   1.0000   1.0000   1.0000 |
          | 0.0000  1.0000  1.0000  1.0000  1.0000  1.0000   1.0000   1.0000   1.0000 |
          | 1.0000  1.0000  1.0000  1.0000  1.0000  1.0000   1.0000   1.0000  12.0000 |
          | 1.0000  1.0000  1.0000  1.0000  1.0000  1.0000   1.0000  11.0000   0.0000 |
          | 4.0000  5.0000  6.0000  7.0000  8.0000  9.0000  10.0000   0.0000   0.0000 |
          | 0.0000  0.0000  0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000 |
          | 0.0000  0.0000  0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000 |
          | 0.0000  0.0000  0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000 |
AGB   =   | 0.0000  0.0000  0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000 |
          | 0.0000  0.0000  0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000 |
          | 0.0000  0.0000  0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000 |
          | 0.0000  0.0000  0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000 |
          | 0.0000  0.0000  0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000 |
          | 0.0000  0.0000  0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000 |
          | 0.0000  0.0000  0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000 |
          | 0.0000  0.0000  0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000 |
          | 0.0000  0.0000  0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000 |
          | 0.0000  0.0000  0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000 |
          | 0.0000  0.0000  0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000 |
          | 0.0000  0.0000  0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000 |
          └                                                                          ┘
```

*Output*

```
            ┌                                                                      ┐
            | 0.0000  0.0000  0.0000  0.0000   0.0000   1.0000   1.0000   1.0000   1.0000 |
            | 0.0000  0.0000  0.0000  0.0000   1.0000   1.0000   1.0000   1.0000   1.0000 |
            | 0.0000  0.0000  0.0000  1.0000   1.0000   1.0000   1.0000   1.0000   1.0000 |
            | 0.0000  0.0000  1.0000  1.0000   1.0000   1.0000   1.0000   1.0000  12.0000 |
            | 0.0000  1.0000  1.0000  1.0000   1.0000   1.0000   1.0000  11.0000   0.3111 |
            | 0.2500  0.2000  0.1600  0.1400   0.1250   0.1100   0.1000   5.5380  -325.00 |
            | 0.0000  0.1500  0.0000  0.0714   0.0000  -0.0556  -0.0306   0.9385   0.0000 |
            | 0.2500  0.1500  0.1000  0.0714  -0.0714  -0.0694  -0.0194   0.0000   0.0000 |
            | 0.2500  0.0000  0.1000  0.0000   0.0536   0.0000   0.0000   0.0000   0.0000 |
            | 0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000 |
            | 0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000 |
AGB   =     | 0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000 |
            | 0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000 |
            | 0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000 |
            | 0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000 |
            | 0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000 |
            | 0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000 |
            | 0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000 |
            | 0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000 |
            | 0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000 |
            | 0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000 |
            | 0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000 |
            | 0.0000  0.0000  0.0000  0.0000   0.0000   0.0000   0.0000   0.0000   0.0000 |
            └                                                                      ┘


                       IPVT     =  (2, -65534, -131070, -196606, -262142, -327678, -327678,
                                    -327680, -327680)
```

# SGBS and DGBS—General Band Matrix Solve

These subroutines solve the system $\boldsymbol{Ax} = \boldsymbol{b}$ for $\boldsymbol{x}$, where $\boldsymbol{A}$ is a general band matrix, and $\boldsymbol{x}$ and $\boldsymbol{b}$ are vectors. They use the results of the factorization of matrix $\boldsymbol{A}$, produced by a preceding call to SGBF or DGBF, respectively.

*Table 103. Data Types*

| ***A***, ***b***, ***x*** | **Subroutine** |
|---|---|
| Short-precision real | SGBS |
| Long-precision real | DGBS |

**Note:** The input to these solve subroutines must be the output from the factorization subroutines SGBF and DGBF, respectively.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SGBS \| DGBS (*agb*, *lda*, *n*, *ml*, *mu*, *ipvt*, *bx*) |
| **C and C++** | sgbs \| dgbs (*agb*, *lda*, *n*, *ml*, *mu*, *ipvt*, *bx*); |
| **PL/I** | CALL SGBS \| DGBS (*agb*, *lda*, *n*, *ml*, *mu*, *ipvt*, *bx*); |

*On Entry*

*agb*
is the factorization of general band matrix $\boldsymbol{A}$, produced by a preceding call to SGBF or DGBF. Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 103, where $lda \geq 2ml+mu+16$.

*lda*
is the leading dimension of the array specified for *agb*. Specified as: a fullword integer; $lda > 0$ and $lda \geq 2ml+mu+16$.

*n*
is the order of the matrix $\boldsymbol{A}$. Specified as: a fullword integer; $n > ml$ and $n > mu$.

*ml*
is the lower band width *ml* of the matrix $\boldsymbol{A}$. Specified as: a fullword integer; $0 \leq ml < n$.

*mu*
is the upper band width *mu* of the matrix $\boldsymbol{A}$. Specified as: a fullword integer; $0 \leq mu < n$.

*ipvt*
is the integer vector ***ipvt*** of length *n*, produced by a preceding call to SGBF or DGBF. It contains the pivot information necessary to construct matrix $\boldsymbol{L}$ from the information contained in the array specified for *agb*.

Specified as: a one-dimensional array of (at least) length *n*, containing fullword integers.

*bx*
is the vector $\boldsymbol{b}$ of length *n*, containing the right-hand side of the system. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 103.

*On Return*

*bx*

is the solution vector ***x*** of length *n*, containing the results of the computation. Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 103.

***Notes***

1. The scalar data specified for input arguments *lda*, *n*, *ml*, and *mu* for these subroutines must be the same as that specified for SGBF and DGBF, respectively.
2. The array data specified for input arguments *agb* and *ipvt* for these subroutines must be the same as the corresponding output arguments for SGBF and DGBF, respectively.
3. The entire *lda* by *n* array specified for *agb* must remain unchanged between calls to the factorization and solve subroutines.
4. The vectors and matrices used in this computation must have no common elements; otherwise, results are unpredictable. See "Concepts" on page 55.
5. This subroutine can be used for tridiagonal matrices ($ml = mu = 1$); however, the tridiagonal subroutines, SGTF/DGTF and SGTS/DGTS, are faster.
6. For a description of how a general band matrix is stored in general-band storage mode in an array, see "General Band Matrix" on page 76.

***Function:*** The real system ***Ax*** = ***b*** is solved for ***x***, where ***A*** is a real general band matrix, stored in general-band storage mode, and ***x*** and ***b*** are vectors. These subroutines use the results of the factorization of matrix ***A***, produced by a preceding call to SGBF or DGBF, respectively. The transformed matrix ***A***, used by this computation, consists of the upper triangular matrix ***U*** and the multipliers necessary to construct ***L*** using ***ipvt***, as defined in "Function" on page 548. See reference [38 ].

***Error Conditions***

*Computational Errors*

**Note:** If the factorization performed by SGBF or DGBF failed due to a singular matrix argument, the results returned by this subroutine are unpredictable, and there may be a divide-by-zero program exception message.

*Input-Argument Errors*

1. $lda \le 0$
2. $ml < 0$
3. $ml \ge n$
4. $mu < 0$
5. $mu \ge n$
6. $lda < 2ml+mu+16$

***Example:*** This example shows how to solve the system ***Ax*** = ***b***, where general band matrix ***A*** is the same matrix factored in "Example" on page 549 for SGBF and DGBF. The input for `AGB` and `IPVT` in this example is the same as the output for that example.

*Call Statement and Input*

```
           AGB  LDA   N   ML  MU  IPVT   BX
            |    |    |   |   |    |     |
CALL SGBS( AGB , 23 , 9 , 2 , 3 , IPVT , BX )
```

```
IPVT     =  (2, -65534, -131070, -196606, -262142, -327678, -327678,
             -327680, -327680)
BX       =  (4.0000, 5.0000, 9.0000, 10.0000, 11.0000, 12.0000,
             12.0000, 12.0000, 33.0000)
```

AGB = (same as output AGB in “Example” on page 549)

*Output*

```
BX       =  (1.0000, 1.0000, 1.0000, 1.0000, 1.0000, 1.0000, 1.0000,
             0.9999, 1.0001)
```

# SPBF, DPBF, SPBCHF, and DPBCHF—Positive Definite Symmetric Band Matrix Factorization

These subroutines factor positive definite symmetric band matrix ***A***, stored in lower-band-packed storage mode, using:

- Gaussian elimination for SPBF and DPBF
- Cholesky factorization for SPBCHF and DPBCHF

To solve the system of equations with one or more right-hand sides, follow the call to these subroutines with one or more calls to SPBS, DPBS, SPBCHS, or DPBCHS, respectively.

*Table 104. Data Types*

| ***A*** | **Subroutine** |
|---|---|
| Short-precision real | SPBF and SPBCHF |
| Long-precision real | DPBF and DPBCHF |

**Notes:**

1. The output from these factorization subroutines should be used only as input to the solve subroutines SPBS, DPBS, SPBCHS, and DPBCHS, respectively.
2. For optimal performance:
   - For wide band widths, use _PBCHF.
   - For narrow band widths, use either _PBF or _PBCHF.
   - For very narrow band widths:
     - Use either SPBF or SPBCHF.
     - Use DPBF.

***Syntax***

| | |
|---|---|
| **Fortran** | CALL SPBF \| DPBF \| SPBCHF \| DPBCHF (*apb*, *lda*, *n*, *m*) |
| **C and C++** | spbf \| dpbf \| spbchf \| dpbchf (*apb*, *lda*, *n*, *m*); |
| **PL/I** | CALL SPBF \| DPBF \| SPBCHF \| DPBCHF (*apb*, *lda*, *n*, *m*); |

*On Entry*

*apb*
is the positive definite symmetric band matrix ***A*** of order *n*, stored in lower-band-packed storage mode, to be factored. It has a half band width of *m*. Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 104. See “Notes” on page 555.

*lda*
is the leading dimension of the array specified for *apb*. Specified as: a fullword integer; $lda > 0$ and $lda > m$.

*n*
is the order *n* of matrix ***A***. Specified as: a fullword integer; $n > m$.

*m*
is the half band width of the matrix ***A***. Specified as: a fullword integer; $0 \le m < n$.

*On Return*

*apb*

is the transformed matrix ***A*** of order *n*, containing the results of the factorization. See "Function" on page 555. Returned as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 104. For further details, see "Notes" on page 555.

### *Notes*

1. These subroutines can be used for tridiagonal matrices ($m = 1$); however, the tridiagonal subroutines, SPTF/DPTF and SPTS/DPTS, are faster.
2. For SPBF and DPBF when $m > 0$, location APB(2,*n*) is sometimes set to 0.
3. For a description of how a positive definite symmetric band matrix is stored in lower-band-packed storage mode in an array, see "Positive Definite Symmetric Band Matrix" on page 85.

***Function:*** The positive definite symmetric band matrix ***A***, stored in lower-band-packed storage mode, is factored using Gaussian elimination in SPBF and DPBF and Cholesky factorization in SPBCHF and DPBCHF. The transformed matrix ***A*** contains the results of the factorization in packed format. This factorization can then be used by SPBS, DPBS, SPBCHS, and DPBCHS, respectively, to solve the system of equations.

For performance reasons, divides are done in a way that reduces the effective exponent range for which DPBF works properly, when processing narrow band widths; therefore, you may want to scale your problem.

### *Error Conditions*

*Resource Errors:* Unable to allocate internal work area.

*Computational Errors*

1. Matrix ***A*** is not positive definite (for SPBF and DPBF).
   - One or more elements of ***D*** contain values less than or equal to 0; all elements of ***D*** are checked. The index *i* of the last nonpositive element encountered is identified in the computational error message.
   - The return code is set to 1.
   - *i* can be determined at run time by use of the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error code 2104 in the ESSL error option table; otherwise, the default value causes your program to terminate when this error occurs. For details, see Chapter 4 on page 111.
2. Matrix ***A*** is not positive definite (for SPBCHF and DPBCHF).
   - The leading minor of order *i* has a nonpositive determinant. The order *i* is identified in the computational error message.
   - The return code is set to 1.
   - *i* can be determined at run time by using the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error code 2115 in the ESSL error option table; otherwise, the default value causes your program to be terminate when this error occurs. For details, see Chapter 4 on page 111.

*Input-Argument Errors*

1. *lda* ≤ 0
2. *m* < 0
3. *m* ≥ *n*
4. *m* ≥ *lda*

***Example 1:*** This example shows a factorization of a real positive definite symmetric band matrix ***A*** of order 9, using Gaussian elimination, where on input, matrix ***A*** is:

```
┌                                             ┐
| 1.0  1.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0 |
| 1.0  2.0  2.0  1.0  0.0  0.0  0.0  0.0  0.0 |
| 1.0  2.0  3.0  2.0  1.0  0.0  0.0  0.0  0.0 |
| 0.0  1.0  2.0  3.0  2.0  1.0  0.0  0.0  0.0 |
| 0.0  0.0  1.0  2.0  3.0  2.0  1.0  0.0  0.0 |
| 0.0  0.0  0.0  1.0  2.0  3.0  2.0  1.0  0.0 |
| 0.0  0.0  0.0  0.0  1.0  2.0  3.0  2.0  1.0 |
| 0.0  0.0  0.0  0.0  0.0  1.0  2.0  3.0  2.0 |
| 0.0  0.0  0.0  0.0  0.0  0.0  1.0  2.0  3.0 |
└                                             ┘
```

and on output, matrix ***A*** is:

```
┌                                             ┐
| 1.0  1.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0 |
| 1.0  1.0  1.0  1.0  0.0  0.0  0.0  0.0  0.0 |
| 1.0  1.0  1.0  1.0  1.0  0.0  0.0  0.0  0.0 |
| 0.0  1.0  1.0  1.0  1.0  1.0  0.0  0.0  0.0 |
| 0.0  0.0  1.0  1.0  1.0  1.0  1.0  0.0  0.0 |
| 0.0  0.0  0.0  1.0  1.0  1.0  1.0  1.0  0.0 |
| 0.0  0.0  0.0  0.0  1.0  1.0  1.0  1.0  1.0 |
| 0.0  0.0  0.0  0.0  0.0  1.0  1.0  1.0  1.0 |
| 0.0  0.0  0.0  0.0  0.0  0.0  1.0  1.0  1.0 |
└                                             ┘
```

where array location `APB(2,9)` is set to 0.0.

*Call Statement and Input*

```
           APB  LDA  N   M
            |    |   |   |
CALL SPBF( APB , 3 , 9 , 2 )
```

```
         ┌                                             ┐
         | 1.0  2.0  3.0  3.0  3.0  3.0  3.0  3.0  3.0 |
APB  =   | 1.0  2.0  2.0  2.0  2.0  2.0  2.0  2.0   .  |
         | 1.0  1.0  1.0  1.0  1.0  1.0  1.0   .    .  |
         └                                             ┘
```

*Output*

```
         ┌                                             ┐
         | 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0 |
APB  =   | 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  0.0 |
         | 1.0  1.0  1.0  1.0  1.0  1.0  1.0   .    .  |
         └                                             ┘
```

***Example 2:*** This example shows a Cholesky factorization of the same matrix used in Example 1.

*Call Statement and Input*

```
             APB  LDA  N   M
              |    |   |   |
CALL SPBCHF( APB , 3 , 9 , 2 )
```

```
APB      = (same as input APB in Example 1)
```

*Output*

```
        ┌                                             ┐
        | 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0 |
APB  =  | 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0   .  |
        | 1.0  1.0  1.0  1.0  1.0  1.0  1.0   .    .  |
        └                                             ┘
```

# SPBS, DPBS, SPBCHS, and DPBCHS—Positive Definite Symmetric Band Matrix Solve

These subroutines solve the system ***Ax*** = ***b*** for ***x***, where ***A*** is a positive definite symmetric band matrix, and ***x*** and ***b*** are vectors. They use the results of the factorization of matrix ***A***, produced by a preceding call to SPBF, DPBF, SPBCHF, and DPBCHF, respectively, where:

- Gaussian elimination was used by SPBF and DPBF.
- Cholesky factorization was used by SPBCHF and DPBCHF.

*Table 105. Data Types*

| ***A***, ***b***, ***x*** | **Subroutine** |
|---|---|
| Short-precision real | SPBS and SPBCHS |
| Long-precision real | DPBS and DPBCHS |

**Notes:**

1. The input to these solve subroutines must be the output from the factorization subroutines SPBF, DPBF, SPBCHF, and DPBCHF, respectively.
2. For performance tradeoffs, see "SPBF, DPBF, SPBCHF, and DPBCHF—Positive Definite Symmetric Band Matrix Factorization" on page 554.

***Syntax***

| | |
|---|---|
| **Fortran** | CALL SPBS \| DPBS \| SPBCHS \| DPBCHS (*apb*, *lda*, *n*, *m*, *bx*) |
| **C and C++** | spbs \| dpbs \| spbchs \| dpbchs (*apb*, *lda*, *n*, *m*, *bx*); |
| **PL/I** | CALL SPBS \| DPBS \| SPBCHS \| DPBCHS (*apb*, *lda*, *n*, *m*, *bx*); |

*On Entry*

*apb*
is the factorization of matrix ***A***, produced by a preceding call to SPBF or DPBF. Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 105. See "Notes" on page 559.

*lda*
is the leading dimension of the array specified for *apb*. Specified as: a fullword integer; $lda > 0$ and $lda > m$.

*n*
is the order *n* of matrix ***A***. Specified as: a fullword integer; $n > m$.

*m*
is the half band width of the matrix ***A***. Specified as: a fullword integer; $0 \le m < n$.

*bx*
is the vector ***b*** of length *n*, containing the right-hand side of the system. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 105.

*On Return*

*bx*

is the solution vector ***x*** of length *n*, containing the results of the computation. Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 105.

***Notes***

1. The scalar data specified for input arguments *lda*, *n*, and *m* for these subroutines must be the same as that specified for SPBF, DPBF, SPBCHF, and DPBCHF, respectively.
2. The array data specified for input argument *apb* for these subroutines must be the same as the corresponding output argument for SPBF, DPBF, SPBCHF, and DPBCHF, respectively.
3. These subroutines can be used for tridiagonal matrices ($m = 1$); however, the tridiagonal subroutines, SPTF/DPTF and SPTS/DPTS, are faster.
4. The vectors and matrices used in this computation must have no common elements; otherwise, results are unpredictable. See "Concepts" on page 55.
5. For a description of how a positive definite symmetric band matrix is stored in lower-band-packed storage mode in an array, see "Positive Definite Symmetric Band Matrix" on page 85.

***Function:*** The system $\boldsymbol{Ax} = \boldsymbol{b}$ is solved for ***x***, where ***A*** is a positive definite symmetric band matrix, stored in lower-band-packed storage mode, and ***x*** and ***b*** are vectors. These subroutines use the results of the factorization of matrix ***A***, produced by a preceding call to SPBF, DPBF, SPBCHF, or DPBCHF, respectively.

***Error Conditions***

*Computational Errors:* None

**Note:** If the factorization subroutine resulted in a nonpositive definite matrix, error 2104 for SPBF and DPBF or error 2115 for SPBCHF and DPBCHF, results of these subroutines may be unpredictable.

*Input-Argument Errors*

1. $lda \le 0$
2. $m < 0$
3. $m \ge n$
4. $m \ge lda$

***Example 1:*** This example shows how to solve the system $\boldsymbol{Ax} = \boldsymbol{b}$, where matrix ***A*** is the same matrix factored in the "Example 1" on page 556 for SPBF and DPBF, using Gaussian elimination.

*Call Statement and Input*

```
           APB  LDA  N   M   BX
            |    |   |   |   |
CALL SPBS( APB , 3 , 9 , 2 , BX )
```

```
APB       =  (same as output APB in "Example 1" on page 556)
BX        =  (3.0, 6.0, 9.0, 9.0, 9.0, 9.0, 9.0, 8.0, 6.0)
```

*Output*

```
BX       =  (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0)
```

This example shows how to solve the system ***Ax*** = ***b***, where matrix ***A*** is the same matrix factored in the “Example 2” on page 557 for SPBCHF and DPBCHF, using Cholesky factorization.

*Call Statement and Input*

```
             APB  LDA  N   M   BX
              |    |   |   |   |
CALL SPBCHS( APB , 3 , 9 , 2 , BX )
```

APB = (same as output APB in “Example 2” on page 557)

```
BX       =  (3.0, 6.0, 9.0, 9.0, 9.0, 9.0, 9.0, 8.0, 6.0)
```

*Output*

```
BX       =  (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0)
```

## SGTF and DGTF—General Tridiagonal Matrix Factorization

These subroutines compute the standard Gaussian factorization with partial pivoting for tridiagonal matrix ***A***, stored in tridiagonal storage mode. To solve a tridiagonal system with one or more right-hand sides, follow the call to these subroutines with one or more calls to SGTS or DGTS, respectively.

*Table 106. Data Types*

| ***c, d, e, f*** | **Subroutine** |
|---|---|
| Short-precision real | SGTF |
| Long-precision real | DGTF |

**Note:** The output from these factorization subroutines should be used only as input to the solve subroutines SGTS and DGTS, respectively.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SGTF \| DGTF (*n*, *c*, *d*, *e*, *f*, *ipvt*) |
| **C and C++** | sgtf \| dgtf (*n*, *c*, *d*, *e*, *f*, *ipvt*); |
| **PL/I** | CALL SGTF \| DGTF (*n*, *c*, *d*, *e*, *f*, *ipvt*); |

*On Entry*

*n*
is the order *n* of tridiagonal matrix ***A***. Specified as: a fullword integer; $n \ge 0$.

*c*
is the vector ***c***, containing the lower subdiagonal of matrix ***A*** in positions 2 through *n* in an array, referred to as `C`. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 106.

*d*
is the vector ***d***, containing the main diagonal of matrix ***A***, in positions 1 through *n* in an array, referred to as `D`. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 106.

*e*
is the vector ***e***, containing the upper subdiagonal of matrix ***A***, in positions 1 through *n*–1 in an array, referred to as `E`. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 106.

*f*
See “On Return.”

*ipvt*
See “On Return.”

*On Return*

*c*
is the vector ***c***, containing part of the factorization of matrix ***A*** in positions 1 through *n* in an array, referred to as `C`. Returned as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 106.

*d*

is the vector ***d***, containing part of the factorization of matrix ***A*** in an array, referred to as D. Returned as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 106.

*e*

is the vector ***e***, containing part of the factorization of the matrix ***A*** in positions 1 through *n* in an array, referred to as E. Returned as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 106 on page 561.

*f*

is the vector ***f***, containing part of the factorization of matrix ***A*** in the first *n* positions in an array, referred to as F. Returned as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 106 on page 561.

*ipvt*

is the integer vector ***ipvt*** of length *n*, containing the pivot information. Returned as: a one-dimensional array of (at least) length *n*, containing fullword integers.

***Notes***

1. For a description of how tridiagonal matrices are stored, see “General Tridiagonal Matrix” on page 90.
2. ***ipvt*** is not a permutation vector in the strict sense. It is used to record column interchanges in the tridiagonal matrix due to partial pivoting.
3. The factorization matrix ***A*** is stored in nonstandard format.

***Function:*** The standard Gaussian elimination with partial pivoting of tridiagonal matrix ***A*** is computed. The factorization is returned by overwriting input arrays C, D, and E, and by writing into output array F, along with pivot information in vector ***ipvt***. This factorization can then be used by SGTS or DGTS, respectively, to solve tridiagonal systems of linear equations. See references [43], [52 ], [53 ], and [87]. If *n* is 0, no computation is performed.

***Error Conditions***

*Computational Errors:* Matrix ***A*** is singular or nearly singular.

- A pivot element has a value that cannot be reciprocated or is equal to 0. The index *i* of the element is identified in the computational error message.
- The return code is set to 1.
- *i* can be determined at run time by use of the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error code 2105 in the ESSL error option table; otherwise, the default value causes your program to terminate when this error occurs. For details, see “What Can You Do about ESSL Computational Errors?” on page 48.

*Input-Argument Errors:* $n < 0$

***Example:*** This example shows how to factor the following tridiagonal matrix ***A*** of order 4:

$$\begin{bmatrix} 2.0 & 2.0 & 0.0 & 0.0 \\ 1.0 & 3.0 & 2.0 & 0.0 \\ 0.0 & 1.0 & 3.0 & 2.0 \\ 0.0 & 0.0 & 1.0 & 3.0 \end{bmatrix}$$

*Call Statement and Input*

```
           N   C   D   E   F   IPVT
           |   |   |   |   |    |
CALL DGTF( 4 , C , D , E , F , IPVT )

C        =  ( . , 1.0, 1.0, 1.0)
D        =  (2.0, 3.0, 3.0, 3.0)
E        =  (2.0, 2.0, 2.0, . )
```

*Output*

```
C        =  ( . , -0.5, -0.5, -0.5)
D        =  (-0.5, -0.5, -0.5, -0.5)
E        =  (2.0, 2.0, 2.0, . )
IPVT     =  (X'00', X'00', X'00', X'00')
```

*Notes*

1. `F` is stored in an internal format and is passed unchanged to the solve subroutine.
2. A “`.`” means you do not have to store a value in that position in the array. However, these storage positions are required and may be overwritten during the computation.

# SGTS and DGTS—General Tridiagonal Matrix Solve

These subroutines solve a tridiagonal system of linear equations using the factorization of tridiagonal matrix *A*, stored in tridiagonal storage mode, produced by SGTF or DGTF, respectively.

*Table 107. Data Types*

| ***c***, ***d***, ***e***, ***f***, ***b***, ***x*** | **Subroutine** |
|---|---|
| Short-precision real | SGTS |
| Long-precision real | DGTS |

**Note:** The input to these solve subroutines must be the output from the factorization subroutines SGTF and DGTF, respectively.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SGTS \| DGTS (*n*, *c*, *d*, *e*, *f*, *ipvt*, *bx*) |
| **C and C++** | sgts \| dgts (*n*, *c*, *d*, *e*, *f*, *ipvt*, *bx*); |
| **PL/I** | CALL SGTS \| DGTS (*n*, *c*, *d*, *e*, *f*, *ipvt*, *bx*); |

*On Entry*

*n*

is the order *n* of tridiagonal matrix ***A***. Specified as: a fullword integer; $n \geq 0$.

*c*

is the vector ***c***, containing part of the factorization of matrix ***A*** from SGTF or DGTF, respectively, in an array, referred to as `C`. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 107.

*d*

is the vector ***d***, containing part of the factorization of matrix ***A*** from SGTF or DGTF, respectively, in an array, referred to as `D`. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 107.

*e*

is the vector ***e***, containing part of the factorization of matrix ***A*** from SGTF or DGTF, respectively, in an array, referred to as `E`. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 107.

*f*

is the vector ***f***, containing part of the factorization of matrix ***A*** from SGTF or DGTF, respectively, in an array, referred to as `F`. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 107.

*ipvt*

is the integer vector ***ipvt*** of length *n*, containing the pivot information, produced by a preceding call to SGTF and DGTF, respectively. Specified as: a one-dimensional array of (at least) length *n*, containing fullword integers.

*bx*

is the vector ***b*** of length *n*, containing the right-hand side of the system in the first *n* positions in an array, referred to as `BX`. Specified as: a one-dimensional array of (at least) length $n$+1, containing numbers of the data type indicated in Table 107. For details on specifying the length, see “Notes” on page 565.

*On Return*

*bx*

is the solution vector ***x*** (at least) of length $n$, containing the solution of the tridiagonal system in the first $n$ positions in an array, referred to as BX. Returned as: a one-dimensional array, of (at least) length ($n$+1), containing numbers of the data type indicated in Table 107 on page 564. For details about the length, see “Notes.”

***Notes***

1. For a description of how tridiagonal matrices are stored, see “General Tridiagonal Matrix” on page 90.
2. Array BX can have a length of $n$ if memory location BX($n$+1) is addressable—that is, not in read-protected storage. If it is in read-protected storage, array BX must have a length of $n$+1. In both cases, the vector ***b*** (on input) and vector ***x*** (on output) reside in positions 1 through $n$ in array BX. Array location BX($n$+1) is not altered by these subroutines.

***Function:*** Given the factorization produced by SGTF or DGTF, respectively, these subroutines use the standard forward elimination and back substitution to solve the tridiagonal system ***Ax*** = ***b***, where ***A*** is a general tridiagonal matrix. See references [43 ], [52 ], [53 ], and [87].

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Example:*** This example solves the tridiagonal system ***Ax*** = ***b,*** where matrix ***A*** is the same matrix factored in “Example” on page 562 for SGTF and DGTF, and where:

```
        b = (4.0, 6.0, 6.0, 4.0)
        x = (1.0, 1.0, 1.0, 1.0)
```

*Call Statement and Input*

```
           N   C   D   E   F   IPVT   BX
           |   |   |   |   |    |     |
CALL DGTS( 4 , C , D , E , F , IPVT , BX )
```

```
C        =  (same as output C in “Example” on page 562)
D        =  (same as output D in “Example” on page 562)
E        =  (same as output E in “Example” on page 562)
F        =  (same as output F in “Example” on page 562)
IPVT     =  (same as output IPVT in “Example” on page 562)
BX       =    (4.0, 6.0, 6.0, 4.0, . )
```

*Output*

```
BX       =  (1.0, 1.0, 1.0, 1.0, . )
```

# SGTNP, DGTNP, CGTNP, and ZGTNP—General Tridiagonal Matrix Combined Factorization and Solve with No Pivoting

These subroutines solve the tridiagonal system ***Ax*** = ***b*** using Gaussian elimination, where tridiagonal matrix ***A*** is stored in tridiagonal storage mode.

*Table 108. Data Types*

| ***c***, ***d***, ***e***, ***b***, ***x*** | **Subroutine** |
|---|---|
| Short-precision real | SGTNP |
| Long-precision real | DGTNP |
| Short-precision complex | CGTNP |
| Long-precision complex | ZGTNP |

**Note:** In general, these subroutines provide better performance than the _GTNPF and _GTNPS subroutines; however, in the following instances, you get better performance by using _GTNPF and _GTNPS:

- For small *n*
- When performing a single factorization followed by multiple solves

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SGTNP \| DGTNP \| CGTNP \| ZGTNP (*n*, *c*, *d*, *e*, *bx*) |
| **C and C++** | sgtnp \| dgtnp \| cgtnp \| zgtnp (*n*, *c*, *d*, *e*, *bx*); |
| **PL/I** | CALL SGTNP \| DGTNP \| CGTNP \| ZGTNP (*n*, *c*, *d*, *e*, *bx*); |

*On Entry*

*n*

is the order *n* of tridiagonal matrix ***A***. Specified as: a fullword integer; *n* ≥ 0.

*c*

is the vector ***c***, containing the lower subdiagonal of matrix ***A*** in positions 2 through *n* in an array, referred to as `C`. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 108. On output, `C` is overwritten; that is, the original input is not preserved.

*d*

is the vector ***d***, containing the main diagonal of matrix ***A*** in positions 1 through *n* in an array, referred to as `D`. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 108. On output, `D` is overwritten; that is, the original input is not preserved.

*e*

is the vector ***e***, containing the upper subdiagonal of matrix ***A*** in positions 1 through *n*–1 in an array, referred to as `E`. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 108. On output, `E` is overwritten; that is, the original input is not preserved.

*bx*

is the vector ***b***, containing the right-hand side of the system in the first *n* positions in an array, referred to as `BX`. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 108.

*On Return*

*bx*
is the solution vector ***x*** of length *n*, containing the solution of the tridiagonal system in the first *n* positions in an array, referred to as BX. Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 108 on page 566.

***Note:*** For a description of how tridiagonal matrices are stored, see “General Tridiagonal Matrix” on page 90.

***Function:*** The solution of the tridiagonal system ***Ax*** = ***b*** is computed by Gaussian elimination.

No pivoting is done. Therefore, these subroutines should not be used when pivoting is necessary to maintain the numerical accuracy of the solution. Overflow may occur if small main diagonal elements are generated. Underflow or accuracy loss may occur if large main diagonal elements are generated.

For performance reasons, complex divides are done without scaling. Computing the inverse in this way restricts the range of numbers for which the ZGTNP subroutine works properly.

For performance reasons, divides are done in a way that reduces the effective exponent range for which DGTNP and ZGTNP work properly; therefore, you may want to scale your problem, such that the diagonal elements are close to 1.0 for DGTNP and (1.0, 0.0) for ZGTNP.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Example 1:*** This example shows a factorization of the real tridiagonal matrix ***A,*** of order 4:

$$\begin{bmatrix} 7.0 & 4.0 & 0.0 & 0.0 \\ 1.0 & 8.0 & 5.0 & 0.0 \\ 0.0 & 2.0 & 9.0 & 6.0 \\ 0.0 & 0.0 & 3.0 & 10.0 \end{bmatrix}$$

It then finds the solution of the tridiagonal system ***Ax*** = ***b***, where ***b*** is:

```
(11.0, 14.0, 17.0, 13.0)
```

and ***x*** is:

```
(1.0, 1.0, 1.0, 1.0)
```

On output, arrays C, D, and E are overwritten.

*Call Statement and Input*

```
            N   C   D   E   BX
            |   |   |   |   |
CALL DGTNP( 4 , C , D , E , BX )

C        =  ( . , 1.0, 2.0, 3.0)
D        =  (7.0, 8.0, 9.0, 10.0)
E        =  (4.0, 5.0, 6.0, . )
BX       =  (11.0, 14.0, 17.0, 13.0)
```

*Output*

```
BX       =  (1.0, 1.0, 1.0, 1.0)
```

***Example 2:*** This example shows a factorization of the complex tridiagonal matrix ***A,*** of order 4:

```
┌                                                     ┐
| (7.0,  7.0)  (4.0,  4.0)  (0.0,  0.0)   (0.0,  0.0)  |
| (1.0,  1.0)  (8.0,  8.0)  (5.0,  5.0)   (0.0,  0.0)  |
| (0.0,  0.0)  (2.0,  2.0)  (9.0,  9.0)   (6.0,  6.0)  |
| (0.0,  0.0)  (0.0,  0.0)  (3.0,  3.0)  (10.0, 10.0)  |
└                                                     ┘
```

It then finds the solution of the tridiagonal system ***Ax*** = ***b***, where ***b*** is:

```
((-11.0,19.0), (-14.0,50.0), (-17.0,93.0), (-13.0,85.0))
```

and ***x*** is:

```
((1.0,-1.0), (2.0,-2.0), (3.0,-3.0), (4.0,-4.0))
```

On output, arrays `C`, `D`, and `E` are overwritten.

*Call Statement and Input*

```
            N   C   D   E   BX
            |   |   |   |   |
CALL ZGTNP( 4 , C , D , E , BX )

C        =  ( . , (1.0, 1.0), (2.0, 2.0), (3.0, 3.0))
D        =  ((7.0, 7.0), (8.0, 8.0), (9.0, 9.0), (10.0, 10.0))
E        =  ((4.0, 4.0), (5.0, 5.0), (6.0, 6.0), . )
BX       =  ((-11.0, 19.0), (-14.0, 50.0), (-17.0, 93.0), (-13.0, 85.0))
```

*Output*

```
BX       =  ((0.0, 1.0), (1.0, 2.0), (2.0, 3.0), (3.0, 4.0))
```

# SGTNPF, DGTNPF, CGTNPF, and ZGTNPF—General Tridiagonal Matrix Factorization with No Pivoting

These subroutines factor tridiagonal matrix ***A***, stored in tridiagonal storage mode, using Gaussian elimination. To solve a tridiagonal system of linear equations with one or more right-hand sides, follow the call to these subroutines with one or more calls to SGTNPS, DGTNPS, CGTNPS, or ZGTNPS, respectively.

*Table 109. Data Types*

| ***c***, ***d***, ***e*** | **Subroutine** |
|---|---|
| Short-precision real | SGTNPF |
| Long-precision real | DGTNPF |
| Short-precision complex | CGTNPF |
| Long-precision complex | ZGTNPF |

**Notes:**

1. The output from these factorization subroutines should be used only as input to the solve subroutines SGTNPS, DGTNPS, CGTNPS, and ZGTNPS, respectively.
2. In general, the _GTNP subroutines provide better performance than the _GTNPF and _GTNPS subroutines; however, in the following instances, you get better performance by using _GTNPF and _GTNPS:
   - For small *n*
   - When performing a single factorization followed by multiple solves

***Syntax***

| | |
|---|---|
| **Fortran** | CALL SGTNPF \| DGTNPF \| CGTNPF \| ZGTNPF (*n*, *c*, *d*, *e*, *iopt*) |
| **C and C++** | sgtnpf \| dgtnpf \| cgtnpf \| zgtnpf (*n*, *c*, *d*, *e*, *iopt*); |
| **PL/I** | CALL SGTNPF \| DGTNPF \| CGTNPF \| ZGTNPF (*n*, *c*, *d*, *e*, *iopt*); |

*On Entry*

*n*

is the order *n* of tridiagonal matrix ***A***. Specified as: a fullword integer; $n \geq 0$.

*c*

is the vector ***c***, containing the lower subdiagonal of matrix ***A*** in positions 2 through *n* in an array, referred to as `C`. Specified as: a one-dimensional array, of (at least) length `n`, containing numbers of the data type indicated in Table 109.

*d*

is the vector ***d***, containing the main diagonal of matrix ***A*** in positions 1 through *n* in an array, referred to as `D`. Specified as: a one-dimensional array, of (at least) length *n*, containing numbers of the data type indicated in Table 109.

*e*

is the vector ***e***, containing the upper subdiagonal of matrix ***A*** in positions 1 through *n*–1 in an array, referred to as `E`. Specified as: a one-dimensional array, of (at least) length *n*, containing numbers of the data type indicated in Table 109.

*iopt*

indicates the type of computation to be performed, where:

If *iopt* = 0 or 1, Gaussian elimination is used to factor the matrix.

Specified as: a fullword integer; *iopt* = 0 or 1.

*On Return*

*c*

is the vector ***c***, containing part of the factorization of matrix ***A*** in positions 1 through *n* in an array, referred to as `C`. Returned as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 109 on page 569.

*d*

is the vector ***d***, containing part of the factorization of matrix ***A*** in an array, referred to as `D`. Returned as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 109 on page 569.

*e*

is the vector ***e***, containing part of the factorization of matrix ***A*** in positions 1 through *n* in an array, referred to as `E`. Returned as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 109 on page 569. It has the same length as `E` on entry.

***Note:*** For a description of how tridiagonal matrices are stored, see "General Tridiagonal Matrix" on page 90.

***Function:*** The factorization of a diagonally-dominant tridiagonal matrix ***A*** is computed using Gaussian elimination, This factorization can then be used by SGTNPS, DGTNPS, CGTNPS, or ZGTNPS respectively, to solve the tridiagonal systems of linear equations. See reference [74].

No pivoting is done by these subroutines. Therefore, these subroutines should not be used when pivoting is necessary to maintain the numerical accuracy of the solution. Overflow may occur if small main diagonal elements are generated. Underflow or accuracy loss may occur if large main diagonal elements are generated.

For performance reasons, complex divides are done without scaling. Computing the inverse in this way restricts the range of numbers for which ZGTNPF works properly.

For performance reasons, divides are done in a way that reduces the effective exponent range for which DGTNPF and ZGTNPF work properly; therefore, you may want to scale your problem, such that the diagonal elements are close to 1.0 for DGTNPF and (1.0, 0.0) for ZGTNPF.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. $n < 0$
2. $iopt \neq 0$ or 1

***Example 1:*** This example shows a factorization of the tridiagonal matrix ***A,*** of order 4:

$$\begin{bmatrix} 1.0 & 1.0 & 0.0 & 0.0 \\ 1.0 & 2.0 & 1.0 & 0.0 \\ 0.0 & 1.0 & 3.0 & 1.0 \\ 0.0 & 0.0 & 1.0 & 1.0 \end{bmatrix}$$

*Call Statement and Input*

```
             N   C   D   E   IOPT
             |   |   |   |    |
CALL DGTNPF( 4 , C , D , E ,  0   )

C        =  ( . , 1.0, 1.0, 1.0)
D        =  (1.0, 2.0, 3.0, 1.0)
E        =  (1.0, 1.0, 1.0, .  )
```

*Output*

```
C        =  ( . , -1.0, -1.0, 1.0)
D        =  (-1.0, -1.0, -1.0, -1.0)
E        =  (1.0, 1.0, -1.0, . )
```

***Example 2:*** This example shows a factorization of the tridiagonal matrix ***A,*** of order 4:

$$\begin{bmatrix} (7.0, 7.0) & (4.0, 4.0) & (0.0, 0.0) & (0.0, 0.0) \\ (1.0, 1.0) & (8.0, 8.0) & (5.0, 5.0) & (0.0, 0.0) \\ (0.0, 0.0) & (2.0, 2.0) & (9.0, 9.0) & (6.0, 6.0) \\ (0.0, 0.0) & (0.0, 0.0) & (3.0, 3.0) & (10.0, 10.0) \end{bmatrix}$$

*Call Statement and Input*

```
             N   C   D   E   IOPT
             |   |   |   |    |
CALL ZGTNPF( 4 , C , D , E ,  0   )

C        =  ( . , (1.0, 1.0), (2.0, 2.0), (3.0, 3.0))
D        =  ((7.0, 7.0), (8.0, 8.0), (9.0, 9.0), (10.0, 10.0))
E        =  ((4.0, 4.0), (5.0, 5.0), (6.0, 6.0), . )
```

*Output*

```
C        =  ( . , (-0.142, 0.0), (-0.269, 0.0), (3.0, 3.0))
D        =  ((-0.0714, 0.0714), (-0.0673, 0.0673), (-0.0854, 0.0854),
             (-0.05, 0.05))
E        =  ((4.0, 4.0), (5.0, 5.0), (-0.6, 0.0), . )
```

*Notes*

1. A "**.**" means you do not have to store a value in that position in the array. However, these storage positions are required and may be overwritten during the computation.

## SGTNPS, DGTNPS, CGTNPS, and ZGTNPS—General Tridiagonal Matrix Solve with No Pivoting

These subroutines solve a tridiagonal system of equations using the factorization of matrix ***A***, stored in tridiagonal storage mode, produced by SGTNPF, DGTNPF, CGTNPF, or ZGTNPF, respectively.

*Table 110. Data Types*

| ***c***, ***d***, ***e***, ***b***, ***x*** | **Subroutine** |
|---|---|
| Short-precision real | SGTNPS |
| Long-precision real | DGTNPS |
| Short-precision complex | CGTNPS |
| Long-precision complex | ZGTNPS |

**Note:** The input to these solve subroutines must be the output from the factorization subroutines SGTNPF, DGTNPF, CGTNPF, and ZGTNPF, respectively.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SGTNPS \| DGTNPS \| CGTNPS \| ZGTNPS (*n*, *c*, *d*, *e*, *bx*) |
| **C and C++** | sgtnps \| dgtnps \| cgtnps \| zgtnps (*n*, *c*, *d*, *e*, *bx*); |
| **PL/I** | CALL SGTNPS \| DGTNPS \| CGTNPS \| ZGTNPS (*n*, *c*, *d*, *e*, *bx*); |

*On Entry*

*n*

is the order *n* of tridiagonal matrix ***A***. Specified as: a fullword integer; $n \geq 0$.

*c*

is the vector ***c***, containing part of the factorization of matrix ***A*** from SGTNPF, DGTNPF, CGTNPF, and ZGTNPF, respectively, in an array, referred to as `C`. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 110.

*d*

is the vector ***d***, containing part of the factorization of matrix ***A*** from SGTNPF, DGTNPF, CGTNPF, and ZGTNPF, respectively, in an array, referred to as `D`. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 110.

*e*

is the vector ***e***, containing part of the factorization of matrix ***A*** from SGTNPF, DGTNPF, CGTNPF, and ZGTNPF, respectively, in an array, referred to as `E`. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 110.

*bx*

is the vector ***b***, containing the right-hand side of the system in the first *n* positions in an array, referred to as `BX`. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 110.

*On Return*

*bx*

is the solution vector ***x*** of length *n*, containing the solution of the tridiagonal system in the first *n* positions in an array, referred to as BX. Returned as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 110.

***Note:*** For a description of how tridiagonal matrices are stored, see "General Tridiagonal Matrix" on page 90.

***Function:*** The solution of tridiagonal system $\boldsymbol{Ax} = \boldsymbol{b}$ is computed using the factorization produced by SGTNPF, DGTNPF, CGTNPF, or ZGTNPF, respectively. The factorization is based on Gaussian elimination. See reference [74].

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Example 1:*** This example finds the solution of tridiagonal system $\boldsymbol{Ax} = \boldsymbol{b}$, where matrix ***A*** is the same matrix factored in "Example 1" on page 571 for SGTNPF and DGTNPF. ***b*** is:

```
(2.0, 4.0, 5.0, 2.0)
```

and ***x*** is:

```
(1.0, 1.0, 1.0, 1.0)
```

*Call Statement and Input*

```
              N   C   D   E   BX
              |   |   |   |   |
CALL DGTNPS( 4 , C , D , E , BX )
```

```
C          =  (same as output C in "Example 1" on page 571)
D          =  (same as output D in "Example 1" on page 571)
E          =  (same as output E in "Example 1" on page 571)
BX         =    (2.0, 4.0, 5.0, 2.0)
```

*Output*

```
BX         =  (1.0, 1.0, 1.0, 1.0)
```

***Example 2:*** This example finds the solution of tridiagonal system $\boldsymbol{Ax} = \boldsymbol{b}$, where matrix ***A*** is the same matrix factored in "Example 2" on page 571 for CGTNPF and ZGTNPF. ***b*** is:

```
((-11.0,19.0), (-14.0,50.0), (-17.0,93.0), (-13.0,85.0))
```

and ***x*** is:

```
((0.0,1.0), (1.0,2.0), (2.0,3.0), (3.0,4.0))
```

*Call Statement and Input*

```
              N   C   D   E   BX
              |   |   |   |   |
CALL ZGTNPS( 4 , C , D , E , BX )
```

C = (same as output C in "Example 2" on page 571)

D = (same as output D in "Example 2" on page 571)

E = (same as output E in "Example 2" on page 571)

BX = ((-11.0, 19.0), (-14.0, 50.0), (-17.0, 93.0), (-13.0, 8))

*Output*

BX = ((0.0, 1.0), (1.0, 2.0), (2.0, 3.0), (3.0, 4.0))

# SPTF and DPTF—Positive Definite Symmetric Tridiagonal Matrix Factorization

These subroutines factor symmetric tridiagonal matrix *A*, stored in symmetric-tridiagonal storage mode, using Gaussian elimination. To solve a tridiagonal system of linear equations with one or more right-hand sides, follow the call to these subroutines with one or more calls to SPTS or DPTS, respectively.

*Table 111. Data Types*

| ***c***, ***d*** | **Subroutine** |
|---|---|
| Short-precision real | SPTF |
| Long-precision real | DPTF |

**Note:** The output from these factorization subroutines should be used only as input to the solve subroutines SPTS and DPTS, respectively.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SPTF \| DPTF (*n*, *c*, *d*, *iopt*) |
| **C and C++** | sptf \| dptf (*n*, *c*, *d*, *iopt*); |
| **PL/I** | CALL SPTF \| DPTF (*n*, *c*, *d*, *iopt*); |

*On Entry*

*n*
is the order *n* of tridiagonal matrix *A*. Specified as: a fullword integer; $n \geq 0$.

*c*
is the vector ***c***, containing the off-diagonal of matrix ***A*** in positions 2 through *n* in an array, referred to as `C`. Specified as: a one-dimensional array, of (at least) length *n*, containing numbers of the data type indicated in Table 111.

*d*
is the vector ***d***, containing the main diagonal of matrix ***A*** in positions 1 through *n* in an array referred to as `D`. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 111.

*iopt*
indicates the type of computation to be performed, where:

If *iopt* = 0 or 1, Gaussian elimination is used to factor the matrix.

Specified as: a fullword integer; *iopt* = 0 or 1.

*On Return*

*c*
is the vector ***c***, containing part of the factorization of matrix ***A*** in an array, referred to as `C`. Returned as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 111.

*d*
is the vector ***d***, containing part of the factorization of matrix ***A*** in positions 1 through *n* in an array, referred to as `D`. Returned as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 111. It has the same length as `D` on entry.

***Note:*** For a description of how positive definite symmetric tridiagonal matrices are stored, see “Positive Definite Symmetric Tridiagonal Matrix” on page 92.

***Function:*** The factorization of positive definite symmetric tridiagonal matrix ***A*** is computed using Gaussian elimination. This factorization can then be used by SPTS or DPTS, respectively, to solve the tridiagonal systems of linear equations. See reference [74].

No pivoting is done. Therefore, these subroutines should not be used when pivoting is necessary to maintain the numerical accuracy of the solution. Overflow may occur if small pivots are generated.

For performance reasons, divides are done in a way that reduces the effective exponent range for which DPTF works properly; therefore, you may want to scale your problem, such that the diagonal elements are close to 1.0 for DPTF.

### *Error Conditions*

*Computational Errors:* None

**Note:** There is no test for positive definiteness in these subroutines.

*Input-Argument Errors*

1. $n < 0$
2. $iopt \neq 0$ or 1

***Example:*** This example shows a factorization of the tridiagonal matrix ***A,*** of order 4:

$$\begin{bmatrix} 1.0 & 1.0 & 0.0 & 0.0 \\ 1.0 & 2.0 & 1.0 & 0.0 \\ 0.0 & 1.0 & 3.0 & 1.0 \\ 0.0 & 0.0 & 1.0 & 1.0 \end{bmatrix}$$

*Call Statement and Input*

```
           N   C   D  IOPT
           |   |   |   |
CALL DPTF( 4 , C , D , 0  )

C       =  ( . , 1.0, 1.0, 1.0)
D       =  (1.0, 2.0, 3.0, 1.0)
```

*Output*

```
C       =  ( . , -1.0, -1.0, -1.0)
D       =  (-1.0, -1.0, -1.0, -1.0)
```

*Notes*

1. A “**.**” means you do not have to store a value in that position in the array. However, these storage positions are required and may be overwritten during the computation.

## SPTS and DPTS—Positive Definite Symmetric Tridiagonal Matrix Solve

These subroutines solve a positive definite symmetric tridiagonal system of equations using the factorization of matrix ***A***, stored in symmetric-tridiagonal storage mode, produced by SPTF and DPTF, respectively.

*Table 112. Data Types*

| ***c***, ***d***, ***b***, ***x*** | **Subroutine** |
|---|---|
| Short-precision real | SPTS |
| Long-precision real | DPTS |

**Note:** The input to these solve subroutines must be the output from the factorization subroutines SPTF and DPTF, respectively.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SPTS \| DPTS (*n*, *c*, *d*, *bx*) |
| **C and C++** | spts \| dpts (*n*, *c*, *d*, *bx*); |
| **PL/I** | CALL SPTS \| DPTS (*n*, *c*, *d*, *bx*); |

*On Entry*

*n*
is the order *n* of tridiagonal matrix ***A***. Specified as: a fullword integer; $n \geq 0$.

*c*
is the vector ***c***, containing part of the factorization of matrix ***A*** from SPTF or DPTF, respectively, in an array, referred to as `C`. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 112.

*d*
is the vector ***d***, containing part of the factorization of matrix ***A*** from SPTF or DPTF, respectively, in an array, referred to as `D`. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 112.

*bx*
is the vector ***b***, containing the right-hand side of the system in the first *n* positions in an array, referred to as `BX`. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 112.

*On Return*

*bx*
is the solution vector ***x*** of length *n*, containing the solution of the tridiagonal system in the first *n* positions in an array, referred to as `BX`. Returned as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 112.

***Note:*** For a description of how tridiagonal matrices are stored, see “Positive Definite or Negative Definite Symmetric Matrix” on page 69.

***Function:*** The solution of positive definite symmetric tridiagonal system $\boldsymbol{Ax} = \boldsymbol{b}$ is computed using the factorization produced by SPTF or DPTF, respectively. The factorization is based on Gaussian elimination. See reference [74].

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Example:*** This example finds the solution of tridiagonal system $\boldsymbol{Ax} = \boldsymbol{b}$, where matrix $\boldsymbol{A}$ is the same matrix factored in "Example" on page 576 for SPTF and DPTF. $\boldsymbol{b}$ is:

```
(2.0, 4.0, 5.0, 2.0)
```

and $\boldsymbol{x}$ is:

```
(1.0, 1.0, 1.0, 1.0)
```

*Call Statement and Input*

```
           N   C   D   BX
           |   |   |   |
CALL DPTS( 4 , C , D , BX )

C         =  ( . , -1.0, -1.0, -1.0)
D         =  (-1.0, -1.0, -1.0, -1.0)
BX        =  (2.0, 4.0, 5.0, 2.0)
```

*Output*

```
BX        =  (1.0, 1.0, 1.0, 1.0)
```

# STBSV, DTBSV, CTBSV, and ZTBSV—Triangular Band Equation Solve

STBSV and DTBSV solve one of the following triangular banded systems of equations with a single right-hand side, using the vector ***x*** and triangular band matrix ***A*** or its transpose:

| Solution | Equation |
|---|---|
| 1. $\boldsymbol{x} \leftarrow \boldsymbol{A}^{-1}\boldsymbol{x}$ | $\boldsymbol{Ax} = \boldsymbol{b}$ |
| 2. $\boldsymbol{x} \leftarrow \boldsymbol{A}^{-\mathrm{T}}\boldsymbol{x}$ | $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{x} = \boldsymbol{b}$ |

CTBSV and ZTBSV solve one of the following triangular banded systems of equations with a single right-hand side, using the vector ***x*** and triangular band matrix ***A***, its transpose, or its conjugate transpose:

| Solution | Equation |
|---|---|
| 1. $\boldsymbol{x} \leftarrow \boldsymbol{A}^{-1}\boldsymbol{x}$ | $\boldsymbol{Ax} = \boldsymbol{b}$ |
| 2. $\boldsymbol{x} \leftarrow \boldsymbol{A}^{-\mathrm{T}}\boldsymbol{x}$ | $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{x} = \boldsymbol{b}$ |
| 3. $\boldsymbol{x} \leftarrow \boldsymbol{A}^{-\mathrm{H}}\boldsymbol{x}$ | $\boldsymbol{A}^{\mathrm{H}}\boldsymbol{x} = \boldsymbol{b}$ |

Matrix ***A*** can be either upper or lower triangular and is stored in upper- or lower-triangular-band-packed storage mode, respectively.

*Table 113. Data Types*

| ***A***, ***x*** | **Subprogram** |
|---|---|
| Short-precision real | STBSV |
| Long-precision real | DTBSV |
| Short-precision complex | CTBSV |
| Long-precision complex | ZTBSV |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL STBSV \| DTBSV \| CTBSV \| ZTBSV (*uplo*, *trans*, *diag*, *n*, *k*, *a*, *lda*, *x*, *incx*) |
| **C and C++** | stbsv \| dtbsv \| ctbsv \| ztbsv (*uplo*, *trans*, *diag*, *n*, *k*, *a*, *lda*, *x*, *incx*); |
| **PL/I** | CALL STBSV \| DTBSV \| CTBSV \| ZTBSV (*uplo*, *trans*, *diag*, *n*, *k*, *a*, *lda*, *x*, *incx*); |

*On Entry*

*uplo*

indicates whether matrix ***A*** is an upper or lower triangular band matrix, where:

If *uplo* = `'U'`, ***A*** is an upper triangular matrix.

If *uplo* = `'L'`, ***A*** is a lower triangular matrix.

Specified as: a single character. It must be `'U'` or `'L'`.

*trans*

indicates the form of matrix ***A*** used in the system of equations, where:

If *trans* = `'N'`, ***A*** is used, resulting in solution 1.

If *trans* = `'T'`, $\boldsymbol{A}^{\mathrm{T}}$ is used, resulting in solution 2.

If *trans* = `'C'`, $\boldsymbol{A}^{\mathrm{H}}$ is used, resulting in solution 3.

Specified as: a single character. It must be `'N'`, `'T'`, or `'C'`.

*diag*

indicates the characteristics of the diagonal of matrix ***A***, where:

If *diag* = 'U', ***A*** is a unit triangular matrix.

If *diag* = 'N', ***A*** is not a unit triangular matrix.

Specified as: a single character. It must be 'U' or 'N'.

*n*

is the order of triangular band matrix ***A***. Specified as: a fullword integer; $n \ge 0$.

*k*

is the upper or lower band width *k* of the matrix ***A***. Specified as: a fullword integer; $k \ge 0$.

*a*

is the upper or lower triangular band matrix ***A*** of order *n*, stored in upper- or lower-triangular-band-packed storage mode, respectively. Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 113 on page 579.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; $lda > 0$ and $lda \ge k+1$.

*x*

is the vector ***x*** of length *n*, containing the right-hand side of the triangular system to be solved. Specified as: a one-dimensional array of (at least) length $1+(n-1)|incx|$, containing numbers of the data type indicated in Table 113 on page 579.

*incx*

is the stride for vector ***x***. Specified as: a fullword integer; $incx > 0$ or $incx < 0$.

*On Return*

*x*

is the solution vector ***x*** of length *n*, containing the results of the computation. Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 113 on page 579.

***Notes***

1. These subroutines accept lowercase letters for the *uplo*, *trans*, and *diag* arguments.
2. For STBSV and DTBSV, if you specify 'C' for the *trans* argument, it is interpreted as though you specified 'T'.
3. Matrix ***A*** and vector ***x*** must have no common elements; otherwise, results are unpredictable.
4. For unit triangular matrices, the elements of the diagonal are assumed to be 1.0 for real matrices and (1.0, 0.0) for complex matrices, and you do not need to set these values in the array.
5. For both upper and lower triangular band matrices, if you specify $k \ge n$, ESSL assumes, **for purposes of the computation only**, that the upper or lower band width of matrix ***A*** is $n-1$; that is, it processes matrix ***A*** of order *n*, as though it is a (nonbanded) triangular matrix. However, ESSL uses the original value for *k* **for the purposes of finding the locations** of element $a_{11}$ and all other elements in the array specified for ***A***, as described in “Triangular Band Matrix” on page 86. For an illustration of this technique, see “Example 3” on page 583.

6. For a description of triangular band matrices and how they are stored in upper- and lower-triangular-band-packed storage mode, see “Triangular Band Matrix” on page 86.
7. If you are using a lower triangular band matrix, it may save your program some time if you use this alternate approach instead of using lower-triangular-band-packed storage mode. Leave matrix ***A*** in full-matrix storage mode when you pass it to ESSL and specify the *lda* argument to be *lda*+1, which is the leading dimension of matrix ***A*** plus 1. ESSL then processes the matrix elements in the same way as though you had set them up in lower-triangular-band-packed storage mode.

***Function:*** These subroutines solve a triangular banded system of equations with a single right-hand side. The solution, ***x***, may be any of the following, where triangular band matrix ***A***, its transpose, or its conjugate transpose is used, and where ***A*** can be either upper- or lower-triangular:

1. $\mathbf{x} \leftarrow \mathbf{A}^{-1}\mathbf{x}$
2. $\mathbf{x} \leftarrow \mathbf{A}^{-\mathrm{T}}\mathbf{x}$
3. $\mathbf{x} \leftarrow \mathbf{A}^{-\mathrm{H}}\mathbf{x}$ (for CTBSV and ZTBSV only)

where:

***x*** is a vector of length *n*.
***A*** is an upper or lower triangular band matrix of order *n*, stored in upper- or lower-triangular-band-packed storage mode, respectively.

See references [34 ], [46 ], and [38 ]. If *n* is 0, no computation is performed.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. $n < 0$
2. $k < 0$
3. $lda \leq 0$
4. $lda < k+1$
5. $incx = 0$
6. *uplo* ≠ `'L'` or `'U'`
7. *trans* ≠ `'T'`, `'N'`, or `'C'`
8. *diag* ≠ `'N'` or `'U'`

***Example 1:*** This example shows the solution $\mathbf{x} \leftarrow \mathbf{A}^{-1}\mathbf{x}$. Matrix ***A*** is a real 9 by 9 upper triangular band matrix with an upper band width of 2 that is not unit triangular, stored in upper-triangular-band-packed storage mode. Vector ***x*** is a vector of length 9, where matrix ***A*** is:

```
┌                                             ┐
| 1.0  1.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0 |
| 0.0  4.0  2.0  3.0  0.0  0.0  0.0  0.0  0.0 |
| 0.0  0.0  4.0  1.0  1.0  0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  4.0  2.0  2.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0  3.0  1.0  1.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0  0.0  3.0  2.0  2.0  0.0 |
| 0.0  0.0  0.0  0.0  0.0  0.0  3.0  1.0  0.0 |
| 0.0  0.0  0.0  0.0  0.0  0.0  0.0  2.0  2.0 |
| 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0 |
└                                             ┘
```

*Call Statement and Input*

```
            UPLO TRANS  DIAG  N   K   A  LDA  X  INCX
             |    |      |    |   |   |   |   |   |
CALL STBSV( 'U' , 'N' , 'N' , 9 , 2 , A , 3 , X , 1  )
```

```
        ┌                                             ┐
        |  .    .   1.0  3.0  1.0  2.0  1.0  2.0  0.0 |
A    =  |  .   1.0  2.0  1.0  2.0  1.0  2.0  1.0  2.0 |
        | 1.0  4.0  4.0  4.0  3.0  3.0  3.0  2.0  1.0 |
        └                                             ┘
```

```
X        =  (2.0, 7.0, 1.0, 8.0, 2.0, 8.0, 1.0, 8.0, 3.0)
```

*Output*

```
X        =  (1.0, 1.0, 0.0, 1.0, 0.0, 2.0, 0.0, 1.0, 3.0)
```

***Example 2:*** This example shows the solution $\boldsymbol{x} \leftarrow \boldsymbol{A}^{-T}\boldsymbol{x}$, solving the same system as in Example 1. Matrix ***A*** is a real 9 by 9 lower triangular band matrix with a lower band width of 2 that is not unit triangular, stored in lower-triangular-band-packed storage mode. Vector ***x*** is a vector of length 9 where matrix ***A*** is:

```
┌                                             ┐
| 1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |
| 1.0  4.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |
| 1.0  2.0  4.0  0.0  0.0  0.0  0.0  0.0  0.0 |
| 0.0  3.0  1.0  4.0  0.0  0.0  0.0  0.0  0.0 |
| 0.0  0.0  1.0  2.0  3.0  0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  2.0  1.0  3.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0  1.0  2.0  3.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0  0.0  2.0  1.0  2.0  0.0 |
| 0.0  0.0  0.0  0.0  0.0  0.0  0.0  2.0  1.0 |
└                                             ┘
```

*Call Statement and Input*

```
            UPLO TRANS  DIAG  N   K   A  LDA  X  INCX
             |    |      |    |   |   |   |   |   |
CALL STBSV( 'L' , 'T' , 'N' , 9 , 2 , A , 3 , X , 1  )
```

```
        ┌                                             ┐
        | 1.0  4.0  4.0  4.0  3.0  3.0  3.0  2.0  1.0 |
A    =  | 1.0  2.0  1.0  2.0  1.0  2.0  1.0  2.0   .  |
        | 1.0  3.0  1.0  2.0  1.0  2.0  0.0   .    .  |
        └                                             ┘
```

X = (same as input X in Example 1)

*Output*

X = (same as output X in Example 1)

***Example 3:*** This example shows the solution $\boldsymbol{x} \leftarrow \boldsymbol{A}^{-T}\boldsymbol{x}$, where $k > n$. Matrix $\boldsymbol{A}$ is a real 4 by 4 upper triangular band matrix with an upper band width of 3, even though $k$ is specified as 5. It is not unit triangular and is stored in upper-triangular-band-packed storage mode. Vector $\boldsymbol{x}$ is a vector of length 4 where matrix $\boldsymbol{A}$ is:

$$\begin{bmatrix} 1.0 & 2.0 & 3.0 & 2.0 \\ 0.0 & 2.0 & 2.0 & 5.0 \\ 0.0 & 0.0 & 3.0 & 3.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

*Call Statement and Input*

```
              UPLO TRANS  DIAG  N   K   A  LDA  X  INCX
               |     |     |    |   |   |   |   |   |
CALL STBSV( 'U' , 'T' , 'N' , 4 , 5 , A , 6 , X , 1  )
```

$$A = \begin{bmatrix} . & . & . & . \\ . & . & . & . \\ . & . & . & 2.0 \\ . & . & 3.0 & 5.0 \\ . & 2.0 & 2.0 & 3.0 \\ 1.0 & 2.0 & 3.0 & 1.0 \end{bmatrix}$$

```
X        =  (5.0, 18.0, 32.0, 41.0)
```

*Output*

```
X        =  (5.0, 4.0, 3.0, 2.0)
```

***Example 4:*** This example shows the solution $\boldsymbol{x} \leftarrow \boldsymbol{A}^{-T}\boldsymbol{x}$. Matrix $\boldsymbol{A}$ is a complex 7 by 7 lower triangular band matrix with a lower band width of 3 that is not unit triangular, stored in lower-triangular-band-packed storage mode. Vector $\boldsymbol{x}$ is a vector of length 7. Matrix $\boldsymbol{A}$ is:

$$\begin{bmatrix} (1.0, 0.0) & (0.0, 0.0) & (0.0, 0.0) & (0.0, 0.0) & (0.0, 0.0) & (0.0, 0.0) & (0.0, 0.0) \\ (1.0, 2.0) & (2.0, 1.0) & (0.0, 0.0) & (0.0, 0.0) & (0.0, 0.0) & (0.0, 0.0) & (0.0, 0.0) \\ (1.0, 3.0) & (2.0, 2.0) & (3.0, 1.0) & (0.0, 0.0) & (0.0, 0.0) & (0.0, 0.0) & (0.0, 0.0) \\ (1.0, 4.0) & (2.0, 3.0) & (3.0, 3.0) & (4.0, 1.0) & (0.0, 0.0) & (0.0, 0.0) & (0.0, 0.0) \\ (0.0, 0.0) & (2.0, 4.0) & (3.0, 3.0) & (4.0, 2.0) & (2.0, 1.0) & (0.0, 0.0) & (0.0, 0.0) \\ (0.0, 0.0) & (0.0, 0.0) & (3.0, 3.0) & (4.0, 3.0) & (5.0, 1.0) & (3.0, 1.0) & (0.0, 0.0) \\ (0.0, 0.0) & (0.0, 0.0) & (0.0, 0.0) & (4.0, 4.0) & (5.0, 2.0) & (6.0, 1.0) & (2.0, 1.0) \end{bmatrix}$$

*Call Statement and Input*

```
              UPLO TRANS  DIAG  N   K   A  LDA  X  INCX
               |     |     |    |   |   |   |   |   |
CALL CTBSV( 'L' , 'T' , 'N' , 7 , 3 , A , 4 , X , 1  )
```

```
               ┌                                                                               ┐
               │ (1.0, 0.0) (2.0, 1.0) (3.0, 1.0) (4.0, 1.0) (2.0, 1.0) (3.0, 1.0) (2.0, 1.0) │
A    =         │ (1.0, 2.0) (2.0, 2.0) (3.0, 3.0) (4.0, 2.0) (5.0, 1.0) (6.0, 1.0)     .      │
               │ (1.0, 3.0) (2.0, 3.0) (3.0, 3.0) (4.0, 3.0) (5.0, 2.0)     .          .      │
               │ (1.0, 4.0) (2.0, 4.0) (3.0, 3.0) (4.0, 4.0)     .          .          .      │
               └                                                                               ┘
```

```
X         =  ((2.0, 2.0), (7.0, 1.0), (1.0, 1.0), (8.0, 1.0),
              (2.0, 0.0), (8.0, 1.0), (1.0, 2.0))
```

*Output*

```
X         =  ((-12.048, -13.136), (6.304, -1.472), (-1.880, 1.040),
              (2.600, -1.800), (-2.160, 1.880), (0.800, -1.400),
              (0.800, 0.600))
```

## Sparse Linear Algebraic Equation Subroutines

This section contains the sparse linear algebraic equation subroutine descriptions.

# DGSF—General Sparse Matrix Factorization Using Storage by Indices, Rows, or Columns

This subroutine factors sparse matrix ***A*** by Gaussian elimination, using a modified Markowitz count with threshold pivoting. The sparse matrix can be stored by indices, rows, or columns. To solve the system of equations, follow the call to this subroutine with a call to DGSS.

***Syntax***

| | |
|---|---|
| **Fortran** | CALL DGSF (*iopt*, *n*, *nz*, *a*, *ia*, *ja*, *lna*, *iparm*, *rparm*, *oparm*, *aux*, *naux*) |
| **C and C++** | dgsf (*iopt*, *n*, *nz*, *a*, *ia*, *ja*, *lna*, *iparm*, *rparm*, *oparm*, *aux*, *naux*); |
| **PL/I** | CALL DGSF (*iopt*, *n*, *nz*, *a*, *ia*, *ja*, *lna*, *iparm*, *rparm*, *oparm*, *aux*, *naux*); |

*On Entry*

*iopt*

indicates the storage technique used for sparse matrix ***A***, where:

If *iopt* = 0, it is stored by indices.

If *iopt* = 1, it is stored by rows.

If *iopt* = 2, it is stored by columns.

Specified as: a fullword integer; *iopt* = 0, 1, or 2.

*n*

is the order *n* of sparse matrix ***A***. Specified as: a fullword integer; *n* ≥ 0.

*nz*

is the number of elements in sparse matrix ***A***, stored in an array, referred to as A. Specified as: a fullword integer; *nz* > 0.

*a*

is the sparse matrix ***A***, to be factored, stored in an array, referred to as A. Specified as: an array of length *lna*, containing long-precision real numbers.

*ia*

is the array, referred to as IA, where:

If *iopt* = 0, it contains the row numbers that correspond to the elements in array A.

If *iopt* = 1, it contains the row pointers.

If *iopt* = 2, it contains the row numbers that correspond to the elements in array A.

Specified as: an array of length *lna*, containing fullword integers; IA(*i*) ≥ 1. See "Sparse Matrix" on page 92 for more information on storage techniques.

*ja*

is the array, referred to as JA, where:

If *iopt* = 0, it contains the column numbers that correspond to the elements in array A.

If *iopt* = 1, it contains the column numbers that correspond to the elements in array A.

If *iopt* = 2, it contains the column pointers.

Specified as: an array of length *lna*, containing fullword integers; JA(*i*) ≥ 1. See "Sparse Matrix" on page 92 for more information on storage techniques.

*lna*

is the length of the arrays specified for *a*, *ia*, and *ja*. Specified as: a fullword integer; *lna* > 2*nz*. If you do not specify a sufficient amount, it results in an error. See "Error Conditions" on page 589.

The size of *lna* depends on the structure of the input matrix. The requirement that *lna* > 2*nz* does not guarantee a successful run of the program. If the input matrix is expected to have many fill-ins, *lna* should be set larger. Larger *lna* may result in a performance improvement.

For details on how *lna* relates to storage compressions, see "Performance and Accuracy Considerations" on page 461.

*iparm*

is an array of parameters, IPARM(*i*), where:

- IPARM(1) determines whether the default values for *iparm* and *rparm* are used by this subroutine.

  If IPARM(1) = 0, the following default values are used:

  IPARM(2) = 10
  IPARM(3) = 1
  IPARM(4) = 0
  RPARM(1) = $10^{-12}$
  RPARM(2) = 0.1

  If IPARM(1) = 1, the default values are not used.

- IPARM(2) determines the number of minimal Markowitz counts that are examined to determine a pivot. (See reference [98].)

- IPARM(3) has the following meaning, where:

  If IPARM(3) = 0, this subroutine checks the values in arrays IA and JA.

  If IPARM(3) = 1, this subroutine assumes that the input values are correct in arrays IA and JA.

- IPARM(4) has the following meaning, where:

  If IPARM(4) = 0, this computation is not performed.

  If IPARM(4) = 1, this subroutine computes:

  The absolute value of the smallest pivot element
  The absolute value of the largest element in ***U***.

  These values are stored in OPARM(2) and OPARM(3), respectively.

- IPARM(5) is reserved.

Specified as: an array of (at least) length 5, containing fullword integers, where the *iparm* values must be:
IPARM(1) = 0 or 1
IPARM(2) ≥ 1
IPARM(3) = 0 or 1
IPARM(4) = 0 or 1

*rparm*

is an array of parameters, RPARM(*i*), where:

- RPARM(1) contains the lower bound of the absolute value of all elements in the matrix. If a pivot element is less than this number, the matrix is reported as singular. Any computed element whose absolute value is less than this number is set to 0.

- RPARM(2) is the threshold pivot tolerance used to control the choice of pivots.
- RPARM(3) is reserved.
- RPARM(4) is reserved.
- RPARM(5) is reserved.

Specified as: a one-dimensional array of (at least) length 5, containing long-precision real numbers, where the *rparm* values must be:
RPARM(1) ≥ 0.0
0.0 ≤ RPARM(2) ≤ 1.0

For additional information about *rparm*, see “Performance and Accuracy Considerations” on page 461.

*oparm*
See “On Return.”

*aux*
is the storage work area used by this subroutine. Its size is specified by *naux*. Specified as: an area of storage, containing long-precision real numbers.

*naux*
is the size of the work area specified by *aux*—that is, the number of elements in *aux*. Specified as: a fullword integer; *naux* ≥ 10*n*+100.

*On Return*

*a*
is the transformed array, referred to as A, containing the factored matrix ***A***, required as input to DGSS. Returned as: a one-dimensional array of length *lna*, containing long-precision real numbers.

*ia*
is the transformed array, referred to as IA, required as input to DGSS. Returned as: a one-dimensional array of length *lna*, containing fullword integers.

*ja*
is the transformed array, referred to as JA, required as input to DGSS. Returned as: a one-dimensional array of length *lna*, containing fullword integers.

*oparm*
is an array of parameters, OPARM(*i*), where:

- OPARM(1) is the amount of fill-ins for the sparse processing portion of the algorithm.
- OPARM(2) contains the absolute value of the smallest pivot element of the matrix. This value is computed and set only if IPARM(4) = 1.
- OPARM(3) contains the absolute value of the largest element encountered in ***U*** after the factorization. This value is computed and set only if IPARM(4) = 1.
- OPARM(4) is reserved.
- OPARM(5) is reserved.

Returned as: a one-dimensional array of length 5, containing long-precision real numbers.

*aux*

is the storage work area used by this subroutine. It contains the information required as input for DGSS. Specified as: an area of storage, containing long-precision real numbers.

***Notes***

1. For a description of the three storage techniques used by this subroutine for sparse matrices, see "Sparse Matrix" on page 92.
2. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see "Using Auxiliary Storage in ESSL" on page 32.

***Function:*** The matrix ***A*** is factored by Gaussian elimination, using a modified Markowitz count with threshold pivoting to compute the sparse LU factorization of ***A***:

$$\boldsymbol{LU} = \boldsymbol{PAQ}$$

where:

***A*** is a general sparse matrix of order *n*, stored by indices, columns, or rows in arrays A, IA, and JA.
***L*** is a unit lower triangular matrix.
***U*** is an upper triangular matrix.
***P*** is a permutation matrix.
***Q*** is a permutation matrix.

To solve the system of equations, follow the call to this subroutine with a call to DGSS. If *n* is 0, no computation is performed. See references [10], [47 ], and [90 ].

***Error Conditions***

*Computational Errors*

1. If this subroutine has to perform storage compressions, an attention message is issued. When this occurs, the performance of this subroutine is affected. The performance can be improved by increasing the value specified for *lna*.
2. The following errors with their corresponding return codes can occur in this subroutine. Where a value of *i* is indicated, it can be determined at run time by use of the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for that particular error code in the ESSL error option table; otherwise, the default value causes your program to terminate when the error occurs. For details, see "What Can You Do about ESSL Computational Errors?" on page 48.
   - For error 2117, return code 2 indicates that the pivot element in a column, *i*, is smaller than the value specified in RPARM(1).
   - For error 2118, return code 3 indicates that pivot element in a row, *i*, is smaller than the value specified in RPARM(1).
   - For error 2120, return code 4 indicates that a row, *i*, is found empty on factorization. The matrix is singular.
   - For error 2121, return code 5 indicates that a column is found empty on factorization. The matrix is singular.

- For error 2119, return code 6 indicates that the storage space indicated by *lna* is insufficient.
- For error 2122, return code 7 indicates that no pivot element was found in the active submatrix.

*Input-Argument Errors*

1. $iopt \neq 0, 1$, or 2
2. $n < 0$
3. $nz \leq 0$
4. $lna \leq 2nz$
5. `IPARM(1)` ≠ 0 or 1
6. `IPARM(2)` ≤ 0
7. `IPARM(3)` ≠ 0 or 1
8. `IPARM(4)` ≠ 0 or 1
9. `RPARM(1)` < 0.0
10. `RPARM(2)` < 0.0 or `RPARM(2)` > 1.0
11. $iopt = 1$ and $ia(i) \geq ia\ (i+1)$, $i = 1, n$
12. $iopt = 2$ and $ja(i) \geq ja(i+1)$, $i = 1, n$
13. $iopt = 0$ or 1 and $ja(i) < 1$ or $ja(i) > n$, $i = 1, nz$
14. $iopt = 0$ or 1 and $ia(i) < 1$ or $ia(i) > n$, $i = 1, nz$
15. There are duplicate indices in a row or column of the input matrix.
16. The matrix is singular if a row or column of the input matrix is empty.
17. *naux* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example:*** This example factors 5 by 5 sparse matrix ***A***, which is stored by indices in arrays `A`, `IA`, and `JA`. The three storage techniques are shown in this example, and the output is the same regardless of the storage technique used. The matrix is factored using Gaussian elimination with threshold pivoting. Matrix ***A*** is:

$$\begin{bmatrix} 2.0 & 0.0 & 4.0 & 0.0 & 0.0 \\ 1.0 & 1.0 & 0.0 & 0.0 & 3.0 \\ 0.0 & 0.0 & 3.0 & 4.0 & 0.0 \\ 2.0 & 2.0 & 0.0 & 1.0 & 5.0 \\ 0.0 & 0.0 & 1.0 & 1.0 & 0.0 \end{bmatrix}$$

**Note:** In this example, only nonzero elements are used as input to the matrix.

*Call Statement and Input (Storage-By-Indices)*

```
          IOPT  N  NZ  A  IA  JA  LNA  IPARM  RPARM  OPARM  AUX  NAUX
           |    |   |  |   |   |   |     |      |      |     |    |
CALL DGSF( 0  , 5, 13, A, IA, JA, 27 , IPARM, RPARM, OPARM, AUX, 150 )

A       =  (2.0, 1.0, 1.0, 3.0, 4.0, 1.0, 5.0, 2.0, 2.0, 1.0, 1.0,
            4.0, 3.0, . , . , . , . , . , . , . , . , . , . , . , . ,
            . , . )
IA      =  (1, 2, 2, 3, 3, 4, 4, 4, 4, 5, 5, 1, 2, . , . , . , . ,
            . , . , . , . , . , . , . , . , . , . , . , . , . , . )
JA      =  (1, 1, 2, 3, 4, 4, 5, 1, 2, 3, 4, 3, 5, . , . , . , . ,
            . , . , . , . , . , . , . , . , . , . , . , . , . , . )
IPARM   =  (1, 3, 1, 1)
RPARM   =  (1.D-12, 0.1D0)
```

*Call Statement and Input (Storage-By-Rows)*

```
          IOPT  N  NZ  A  IA  JA  LNA  IPARM  RPARM  OPARM  AUX  NAUX
           |    |   |  |   |   |   |     |      |      |     |    |
CALL DGSF( 1  , 5, 13, A, IA, JA, 27 , IPARM, RPARM, OPARM, AUX, 150 )
```

```
A        =  (2.0, 4.0, 1.0, 1.0, 3.0, 3.0, 4.0, 2.0, 2.0, 1.0, 5.0,
             1.0, 1.0, . , . , . , . , . , . , . , . , . , . , . , . , . ,
             . , . )
IA       =  (1, 3, 6, 8, 12, 14, . , . , . , . , . , . , . , . , . ,
             . , . , . , . , . )
JA       =  (1, 3, 1, 2, 5, 3, 4, 1, 2, 4, 5, 3, 4, . , . , . , . ,
             . , . , . , . , . , . , . , . , . , . )
IPARM    =  (1, 3, 1, 1)
RPARM    =  (1.D-12, 0.1D0)
```

### *Call Statement and Input (Storage-By-Columns)*

```
          IOPT  N  NZ  A  IA  JA  LNA  IPARM  RPARM  OPARM  AUX  NAUX
           |    |   |  |   |   |   |     |      |      |     |    |
CALL DGSF( 2  , 5, 13, A, IA, JA, 27 , IPARM, RPARM, OPARM, AUX, 150 )
```

```
A        =  (2.0, 1.0, 2.0, 1.0, 2.0, 4.0, 3.0, 1.0, 4.0, 1.0, 1.0,
             3.0, 5.0, . , . , . , . , . , . , . , . , . , . , . , . , . ,
             . , . )
IA       =  (1, 2, 4, 2, 4, 1, 3, 5, 3, 4, 5, 2, 4, . , . , . , . ,
             . , . , . , . , . , . , . , . , . , . )
JA       =  (1, 4, 6, 9, 12, 14, . , . , . , . , . , . , . , . , . ,
             . , . , . , . , . )
IPARM    =  (1, 3, 0, 1)
RPARM    =  (1.D-12, 0.1D0)
```

### *Output*

```
A        =  (0.5, . , 0.3, 1.0, . , 1.0, . , 3.0, . , . , . , 1.0,
             1.0, . , . , . , . , . , . , . , -1.7, -0.5, -1.0, -1.0,
             4.0, -3.0, -4.0)
IA       =  (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, . , . , . , . ,
             . , . , . , 2, 1, 1, 3, 3, 5, 5)
JA       =  (1, 0, 5, 2, 0, 4, 0, 2, 0, 0, 0, 3, 4, . , . , . , . ,
             . , . , . , 4, 2, 4, 4, 1, 3, 1)
OPARM    =  (1.000000, 0.333333, 3.000000)
```

## DGSS—General Sparse Matrix or Its Transpose Solve Using Storage by Indices, Rows, or Columns

This subroutine solves either of the following systems:

$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$
$\boldsymbol{A}^{T}\boldsymbol{x} = \boldsymbol{b}$

where ***A*** is a sparse matrix, $\boldsymbol{A}^{T}$ is the transpose of sparse matrix ***A***, and ***x*** and ***b*** are vectors. DGSS uses the results of the factorization of matrix ***A***, produced by a preceding call to DGSF.

**Note:** The input to this solve subroutine must be the output from the factorization subroutine, DGSF.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL DGSS (*jopt*, *n*, *a*, *ia*, *ja*, *lna*, *bx*, *aux*, *naux*) |
| **C and C++** | dgss (*jopt*, *n*, *a*, *ia*, *ja*, *lna*, *bx*, *aux*, *naux*); |
| **PL/I** | CALL DGSS (*jopt*, *n*, *a*, *ia*, *ja*, *lna*, *bx*, *aux*, *naux*); |

*On Entry*

*jopt*

indicates the type of computation to be performed, where:

If *jopt* = 0, $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$ is solved, where the right-hand side is not sparse.

If *jopt* = 1, $\boldsymbol{A}^{T}\boldsymbol{x} = \boldsymbol{b}$ is solved, where the right-hand side is not sparse.

If *jopt* = 10, $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$ is solved, where the right-hand side is sparse.

If *jopt* = 11, $\boldsymbol{A}^{T}\boldsymbol{x} = \boldsymbol{b}$ is solved, where the right-hand side is sparse.

Specified as: a fullword integer; *jopt* = 0, 1, 10, or 11.

*n*

is the order *n* of sparse matrix ***A***. Specified as: a fullword integer; $n \ge 0$.

*a*

is the factorization of sparse matrix ***A***, stored in array `A`, produced by a preceding call to DGSF. Specified as: an array of length *lna*, containing long-precision real numbers.

*ia*

is the array, referred to as `IA`, produced by a preceding call to DGSF. Specified as: an array of length *lna*, containing fullword integers.

*ja*

is the array, referred to as `JA`, produced by a preceding call to DGSF. Specified as: an array of length *lna*, containing fullword integers.

*lna*

is the length of the arrays `A`, `IA`, and `JA`. In DGSS, *lna* must be identical to the value specified in DGSF; otherwise, results are unpredictable. Specified as: a fullword integer; $lna > 0$.

*bx*

is the vector ***b*** of length *n*, containing the right-hand side of the system. Specified as: a one-dimensional array of (at least) length *n*, containing long-precision real numbers.

*aux*

is the storage work area passed to this subroutine by a preceding call to DGSF. Its size is specified by *naux*. Specified as: an area of storage, containing long-precision real numbers.

*naux*

is the size of the work area specified by *aux*—that is, the number of elements in *aux*. Specified as: a fullword integer; *naux* $\geq$ 10*n*+100.

*On Return*

*ia*

is the transformed array, referred to as IA, which can be used as input in subsequent calls to this subroutine. This may result in a performance increase. Specified as: an array of length *lna*, containing fullword integers.

*bx*

is the solution vector ***x*** of length *n*, containing the results of the computation. Specified as: a one-dimensional array, containing long-precision real numbers.

***Notes***

1. The input arguments *n*, *lna*, and *naux*, must be the same as those specified for DGSF. Whereas, the input arguments *a, ia, ja*, and *aux* must be those produced on output by DGSF. Otherwise, results are unpredictable.
2. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see “Using Auxiliary Storage in ESSL” on page 32.

***Function:*** The system $\boldsymbol{Ax} = \boldsymbol{b}$ is solved for ***x***, where ***A*** is a sparse matrix and ***x*** and ***b*** are vectors. Depending on the value specified for the *jopt* argument, DGSS can also solve the system $\boldsymbol{A}^T\boldsymbol{x} = \boldsymbol{b}$, where $\boldsymbol{A}^T$ is the transpose of sparse matrix ***A***.

If the value specified for the *jopt* argument is 0 or 10, the following equation is solved:

$$\boldsymbol{Ax} = \boldsymbol{b}$$

If the value specified for the *jopt* argument is 1 or 11, the following equation is solved:

$$\boldsymbol{A}^T\boldsymbol{x} = \boldsymbol{b}$$

DGSS uses the results of the factorization of matrix ***A***, produced by a preceding call to DGSF. The transformed matrix ***A*** consists of the upper triangular matrix ***U*** and the lower triangular matrix ***L***.

See references [10 ], [47 ], and [90 ].

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. *jopt* $\neq$ 0, 1, 10, or 11
2. $n < 0$
3. $lna \leq 0$

4. *naux* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example 1:*** This example shows how to solve the system ***Ax*** = ***b***, where matrix ***A*** is a 5 by 5 sparse matrix. The right-hand side is not sparse.

**Note:** The input for this subroutine is the same as the output from DGSF, except for BX.

Matrix ***A*** is:

$$\begin{bmatrix} 2.0 & 0.0 & 4.0 & 0.0 & 0.0 \\ 1.0 & 1.0 & 0.0 & 0.0 & 3.0 \\ 0.0 & 0.0 & 3.0 & 4.0 & 0.0 \\ 2.0 & 2.0 & 0.0 & 1.0 & 5.0 \\ 0.0 & 0.0 & 1.0 & 1.0 & 0.0 \end{bmatrix}$$

*Call Statement and Input*

```
          JOPT  N   A   IA   JA   LNA   BX   AUX   NAUX
           |    |   |    |    |    |     |    |     |
CALL DGSS( 0  , 5 , A , IA , JA , 27  , BX , AUX , 150 )
```

```
A        =  (0.5, . , 0.3, 1.0, . , 1.0, . , 3.0, . , . , . , . , 1.0,
             1.0, . , . , . , . , . , . , . , . , . , -1.7, -0.5, -1.0, -1.0,
             4.0, -3.0, -4.0)
IA       =  (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, . , . , . , . , . , . ,
             . , . , . , . , 2, 1, 1, 3, 3, 5, 5)
JA       =  (1, 0, 5, 2, 0, 4, 0, 2, 0, 0, 0, 3, 4, . , . , . , . , . , . ,
             . , . , . , . , 4, 2, 4, 4, 1, 3, 1)
BX       =  (1.0, 1.0, 1.0, 1.0, 1.0)
```

*Output*

```
IA       =  (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, . , . , . , . , . , . ,
             . , . , . , . , 2, 1, 1, 3, 3, 5, 5)
BX       =  (-5.500000, 9.500000, 3.000000, -2.000000, -1.000000)
```

***Example 2:*** This example shows how to solve the system $\boldsymbol{A}^{T}\boldsymbol{x}$ = ***b***, using the same matrix ***A*** used in Example 1. The input is also the same as in Example 1, except for the *jopt* argument. The right-hand side is not sparse.

*Call Statement and Input*

```
          JOPT  N   A   IA   JA   LNA   BX   AUX   NAUX
           |    |   |    |    |    |     |    |     |
CALL DGSS( 1  , 5 , A , IA , JA , 27  , BX , AUX , 150 )

BX       =  (1.0, 1.0, 1.0, 1.0, 1.0)
```

*Output*

```
IA       =  (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, . , . , . , . , . , . ,
             . , . , . , . , 2, 1, 1, 3, 3, 5, 5)
BX       =  (0.000000, -3.000000, -2.000000, 2.000000, 7.000000)
```

***Example 3:*** This example shows how to solve the system ***Ax*** = ***b***, using the same matrix ***A*** as in Examples 1 and 2. The input is also the same as in Examples 1 and 2, except for the *jopt* and *bx* arguments. The right-hand side is sparse.

*Call Statement and Input*

```
          JOPT  N   A   IA   JA   LNA   BX   AUX   NAUX
           |    |   |    |    |    |     |    |     |
CALL DGSS( 10 , 5 , A , IA , JA , 27  , BX , AUX , 150 )

BX       =  (0.0, 0.0, 0.0, 1.0, 0.0)
```

*Output*

```
IA       =  (1, 4, 2, 5, 3, 5, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
             0, 2, 1, 1, 3, 3, 5, 5)
BX       =  (0.000000, 3.000000, 0.000000, 0.000000, -1.000000)
```

***Example 4:*** This example shows how to solve the system $\boldsymbol{A}^{T}\boldsymbol{x} = \boldsymbol{b}$, using the same matrix ***A*** as in Examples 1, 2, and 3. The input is also the same as in Examples 1, 2, and 3, except for the *jopt* argument. The right-hand side is sparse.

*Call Statement and Input*

```
          JOPT  N   A   IA   JA   LNA   BX   AUX   NAUX
           |    |   |    |    |    |     |    |     |
CALL DGSS( 11 , 5 , A , IA , JA , 27  , BX , AUX , 150 )

BX       =  (0.0, 0.0, 0.0, 1.0, 0.0)
```

*Output*

```
IA       =  (1, 4, 2, 5, 3, 5, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
             0, 2, 1, 1, 3, 3, 5, 5)
BX       =   (0.000000, 0.000000, 1.000000, 0.000000, -3.000000 )
```

# DGKFS—General Sparse Matrix or Its Transpose Factorization, Determinant, and Solve Using Skyline Storage Mode

This subroutine can perform either or both of the following functions for general sparse matrix ***A***, stored in skyline storage mode, and for vectors ***x*** and ***b***:

- Factor ***A*** and, optionally, compute the determinant of ***A***.
- Solve the system $\boldsymbol{Ax} = \boldsymbol{b}$ or $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{x} = \boldsymbol{b}$ using the results of the factorization of matrix ***A***, produced on this call or a preceding call to this subroutine.

You also have the choice of using profile-in or diagonal-out skyline storage mode for ***A*** on input or output.

**Note:** The input to the solve performed by this subroutine must be the output from the factorization performed by this subroutine.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL DGKFS (*n*, *au*, *nu*, *idu*, *al*, *nl*, *idl*, *iparm*, *rparm*, *aux*, *naux*, *bx*, *ldbx*, *mbx*) |
| **C and C++** | dgkfs (*n*, *au*, *nu*, *idu*, *al*, *nl*, *idl*, *iparm*, *rparm*, *aux*, *naux*, *bx*, *ldbx*, *mbx*); |
| **PL/I** | CALL DGKFS (*n*, *au*, *nu*, *idu*, *al*, *nl*, *idl*, *iparm*, *rparm*, *aux*, *naux*, *bx*, *ldbx*, *mbx*); |

*On Entry*

*n*
is the order of general sparse matrix ***A***. Specified as: a fullword integer; $n \geq 0$.

*au*
is the array, referred to as AU, containing one of three forms of the upper triangular part of general sparse matrix ***A***, depending on the type of computation performed, where:

- If you are doing a **factor and solve** or a **factor only**, and if IPARM(3) = 0, then AU contains the unfactored upper triangle of general sparse matrix ***A***.
- If you are doing a **factor only**, and if IPARM(3) > 0, then AU contains the partially factored upper triangle of general sparse matrix ***A***. The first IPARM(3) columns in the upper triangle of ***A*** are already factored. The remaining columns are factored in this computation.
- If you are doing a **solve only**, then AU contains the factored upper triangle of general sparse matrix ***A***, produced by a preceding call to this subroutine.

In each case:

If IPARM(4) = 0, diagonal-out skyline storage mode is used for ***A***.

If IPARM(4) = 1, profile-in skyline storage mode is used for ***A***.

Specified as: a one-dimensional array of (at least) length *nu*, containing long-precision real numbers.

*nu*
is the length of array AU. Specified as: a fullword integer; $nu \geq 0$ and $nu \geq$ (IDU($n$+1)−1).

*idu*
is the array, referred to as IDU, containing the relative positions of the diagonal elements of matrix ***A*** (in one of its three forms) in array AU. Specified as: a one-dimensional array of (at least) length *n*+1, containing fullword integers.

*al*

is the array, referred to as AL, containing one of three forms of the lower triangular part of general sparse matrix ***A***, depending on the type of computation performed, where:

- If you are doing a **factor and solve** or a **factor only**, and if IPARM(3) = 0, then AL contains the unfactored lower triangle of general sparse matrix ***A***.
- If you are doing a **factor only**, and if IPARM(3) > 0, then AL contains the partially factored lower triangle of general sparse matrix ***A***. The first IPARM(3) rows in the lower triangle of ***A*** are already factored. The remaining rows are factored in this computation.
- If you are doing a **solve only**, then AL contains the factored lower triangle of general sparse matrix ***A***, produced by a preceding call to this subroutine.

**Note:** In all these cases, entries in AL for diagonal elements of ***A*** are not assumed to have meaningful values.

In each case:

If IPARM(4) = 0, diagonal-out skyline storage mode is used for ***A***.

If IPARM(4) = 1, profile-in skyline storage mode is used for ***A***.

Specified as: a one-dimensional array of (at least) length *nl*, containing long-precision real numbers.

*nl*

is the length of array AL. Specified as: a fullword integer; *nl* ≥ 0 and *nl* ≥ (IDL(*n*+1)–1).

*idl*

is the array, referred to as IDL, containing the relative positions of the diagonal elements of matrix ***A*** (in one of its three forms) in array AL. Specified as: a one-dimensional array of (at least) length *n*+1, containing fullword integers.

*iparm*

is an array of parameters, IPARM(*i*), where:

- IPARM(1) indicates whether certain default values for *iparm* and *rparm* are used by this subroutine, where:

  If IPARM(1) = 0, the following default values are used. For restrictions, see “Notes” on page 603.

  ```
  IPARM(2) = 0
  IPARM(3) = 0
  IPARM(4) = 0
  IPARM(5) = 0
  IPARM(10) = 0
  IPARM(11) = –1
  IPARM(12) = –1
  IPARM(13) = –1
  IPARM(14) = –1
  IPARM(15) = 0
  ```
  RPARM(10) = $10^{-12}$

  If IPARM(1) = 1, the default values are not used.

- IPARM(2) indicates the type of computation performed by this subroutine. The following table gives the IPARM(2) values for each variation:

| **Type of Computation** | ***Ax* = *b*** | ***Ax* = *b* and Determinant(*A*)** | ***A*ᵀ*x* = *b*** | ***A*ᵀ*x* = *b* and Determinant(*A*)** |
|---|---|---|---|---|
| **Factor and Solve** | 0 | 10 | 100 | 110 |
| **Factor Only** | 1 | 11 | N/A | N/A |
| **Solve Only** | 2 | N/A | 102 | N/A |

- `IPARM(3)` indicates whether a full or partial factorization is performed on matrix ***A***, where:

  If `IPARM(3)` = 0, and:

  > If you are doing a **factor and solve** or a **factor only**, then a full factorization is performed for matrix ***A*** on rows and columns 1 through *n*.
  > If you are doing a **solve only**, this argument has no effect on the computation, but must be set to 0.

  If `IPARM(3)` > 0, and you are doing a **factor only**, then a partial factorization is performed on matrix ***A***. Rows 1 through `IPARM(3)` of columns 1 through `IPARM(3)` in matrix ***A*** must be in factored form from a preceding call to this subroutine. The factorization is performed on rows `IPARM(3)+1` through *n* and columns `IPARM(3)+1` through *n*. For an illustration, see “Notes” on page 603.

- `IPARM(4)` indicates the input storage mode used for matrix ***A***. This determines the arrangement of data in arrays `AU`, `IDU`, `AL`, and `IDL` on input, where:

  If `IPARM(4)` = 0, diagonal-out skyline storage mode is used.

  If `IPARM(4)` = 1, profile-in skyline storage mode is used.

- `IPARM(5)` indicates the output storage mode used for matrix ***A***. This determines the arrangement of data in arrays `AU`, `IDU`, `AL`, and `IDL` on output, where:

  If `IPARM(5)` = 0, diagonal-out skyline storage mode is used.

  If `IPARM(5)` = 1, profile-in skyline storage mode is used.

- `IPARM(6)` through `IPARM(9)` are reserved.

- `IPARM(10)` has the following meaning, where:

  If you are doing a **factor and solve** or a **factor only**, then `IPARM(10)` indicates whether certain default values for *iparm* and *rparm* are used by this subroutine, where:

  > If `IPARM(10)` = 0, the following default values are used. For restrictions, see “Notes” on page 603.
  > `IPARM(11)` = –1
  > `IPARM(12)` = –1
  > `IPARM(13)` = –1
  > `IPARM(14)` = –1
  > `IPARM(15)` = 0
  > `RPARM(10)` = $10^{-12}$
  > If `IPARM(10)` = 1, the default values are not used.

If you are doing a **solve only**, this argument is not used.

- IPARM(11) through IPARM(15) have the following meaning, where:

  If you are doing a **factor and solve** or a **factor only**, then IPARM(11) through IPARM(15) control the type of processing to apply to pivot elements occurring in regions 1 through 5, respectively. The pivot elements are $u_{kk}$ for $k$ = 1, $n$ when doing a full factorization, and they are $k$ = IPARM(3)+1, $n$ when doing a partial factorization. The region in which a pivot element falls depends on the sign and magnitude of the pivot element. The regions are determined by RPARM(10). For a description of the regions and associated pivot values, see "Notes" on page 603. For each region *i* for *i* = 1,5, where the pivot occurs in region *i*, the processing applied to the pivot element is determined by IPARM(10+*i*), where:

  If IPARM(10+*i*) = −1, the pivot element is trapped and computational error 2126 is generated. See "Error Conditions" on page 605.
  If IPARM(10+*i*) = 0, for *i* = 1, 2, 4, and 5, processing continues normally.

  **Note:** A value of 0 is not permitted for region 3, because if processing continues, a divide-by-zero exception occurs.

  If IPARM(10+*i*) = 1, the pivot element is replaced with the value in RPARM(10+*i*), and processing continues normally.

  If you are doing a **solve only**, these arguments are not used.

- IPARM(16) through IPARM(25), see "On Return" on page 601.

Specified as: a one-dimensional array of (at least) length 25, containing fullword integers, where:
IPARM(1) = 0 or 1
IPARM(2) = 0, 1, 2, 10, 11, 100, 102, or 110
If IPARM(2) = 0, 2, 10, 100, 102, or 110, then IPARM(3) = 0
If IPARM(2) = 1 or 11, then 0 ≤ IPARM(3) ≤ $n$
IPARM(4), IPARM(5) = 0 or 1
If IPARM(2) = 0, 1, 10, 11, 100, or 110, then:
IPARM(10) = 0 or 1
IPARM(11), IPARM(12) = −1, 0, or 1
IPARM(13) = −1 or 1
IPARM(14), IPARM(15) = −1, 0, or 1

*rparm*

is an array of parameters, RPARM(*i*), where:

- RPARM(1) through RPARM(9) are reserved.

- RPARM(10) has the following meaning, where:

  If you are doing a **factor and solve** or a **factor only**, RPARM(10) is the tolerance value for small pivots. This sets the bounds for the pivot regions, where pivots are processed according to the options you specify for the five regions in IPARM(11) through IPARM(15), respectively. The suggested value is $10^{-15}$ ≤ IPARM(10) ≤ 1.

  If you are doing a **solve only**, this argument is not used.

- RPARM(11) through RPARM(15) have the following meaning, where:

If you are doing a **factor and solve** or a **factor only**, RPARM(11) through RPARM(15) are the fix-up values to use for the pivots in regions 1 through 5, respectively. For each RPARM(10+*i*) for *i* = 1,5, where the pivot occurs in region *i*:

If IPARM(10+*i*) = 1, the pivot is replaced with RPARM(10+*i*), where |RPARM(10+*i*)| should be a sufficiently large nonzero value to avoid overflow when calculating the reciprocal of the pivot. The suggested value is $10^{-15}$ ≤ |RPARM(10+*i*)| ≤ 1.
If IPARM(10+*i*) ≠ 1, RPARM(10+*i*) is not used.

If you are doing a **solve only**, these arguments are not used.

- RPARM(16) through RPARM(25), see "On Return" on page 601.

Specified as: a one-dimensional array of (at least) length 25, containing long-precision real numbers, where if IPARM(2) = 0, 1, 10, 11, 100, or 110, then:
RPARM(10) ≥ 0.0
RPARM(11) through RPARM(15) ≠ 0.0

*aux*
has the following meaning:

If *naux* = 0 and error 2015 is unrecoverable, *aux* is ignored.

Otherwise, it is the storage work area used by this subroutine. Its size is specified by *naux*.

Specified as: an area of storage, containing long-precision real numbers.

*naux*
is the size of the work area specified by *aux*—that is, the number of elements in *aux*. Specified as: a fullword integer, where:

If *naux* = 0 and error 2015 is unrecoverable, DGKFS dynamically allocates the work area used by this subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise,

If you are doing a **factor only**, use *naux* ≥ 5*n*.

If you are doing a **factor and solve** or a **solve only**, use *naux* ≥ 5*n*+4*mbx*.

*bx*
has the following meaning, where:

If you are doing a **factor and solve** or a **solve only**, *bx* is the array, containing the *mbx* right-hand side vectors ***b*** of the system $\boldsymbol{Ax} = \boldsymbol{b}$ or $\boldsymbol{A}^T\boldsymbol{x} = \boldsymbol{b}$. Each vector ***b*** is length *n* and is stored in the corresponding column of the array.

If you are doing a **factor only**, this argument is not used in the computation.

Specified as: an *ldbx* by (at least) *mbx* array, containing long-precision real numbers.

*ldbx*
has the following meaning, where:

If you are doing a **factor and solve** or a **solve only**, *ldbx* is the leading dimension of the array specified for *bx*.

If you are doing a **factor only**, this argument is not used in the computation.

Specified as: a fullword integer; *ldbx* $\geq$ *n* and:

If *mbx* $\neq$ 0, then *ldbx* $>$ 0.

If *mbx* = 0, then *ldbx* $\geq$ 0.

*mbx*

has the following meaning, where:

If you are doing a **factor and solve** or a **solve only**, *mbx* is the number of right-hand side vectors, ***b***, in the array specified for *bx*.

If you are doing a **factor only**, this argument is not used in the computation.

Specified as: a fullword integer; *mbx* $\geq$ 0.

*On Return*

*au*

is the array, referred to as AU, containing the upper triangular part of the ***LU*** factored form of general sparse matrix ***A***, where:

If IPARM(5) = 0, diagonal-out skyline storage mode is used for ***A***.

If IPARM(5) = 1, profile-in skyline storage mode is used for ***A***.

(If *mbx* = 0 and you are doing a solve only, then *au* is unchanged on output.) Returned as: a one-dimensional array of (at least) length *nu*, containing long-precision real numbers.

*idu*

is the array, referred to as IDU, containing the relative positions of the diagonal elements of the factored output matrix ***A*** in array AU. (If *mbx* = 0 and you are doing a solve only, then *idu* is unchanged on output.) Returned as: a one-dimensional array of (at least) length *n*+1, containing fullword integers.

*al*

is the array, referred to as AL, containing the lower triangular part of the ***LU*** factored form of general sparse matrix ***A***, where:

If IPARM(5) = 0, diagonal-out skyline storage mode is used for ***A***.

If IPARM(5) = 1, profile-in skyline storage mode is used for ***A***.

**Note:** You should assume that entries in AL for diagonal elements of ***A*** do not have meaningful values.

(If *mbx* = 0 and you are doing a solve only, then *al* is unchanged on output.) Returned as: a one-dimensional array of (at least) length *nl*, containing long-precision real numbers.

*idl*

is the array, referred to as IDL, containing the relative positions of the diagonal elements of the factored output matrix ***A*** in array AL. (If *mbx* = 0 and you are doing a solve only, then *idl* is unchanged on output.) Returned as: a one-dimensional array of (at least) length *n*+1, containing fullword integers.

*iparm*

is an array of parameters, IPARM(*i*), where:

- IPARM(1) through IPARM(15) are unchanged.
- IPARM(16) has the following meaning, where:

  If you are doing a **factor and solve** or a **factor only**, and:

    If IPARM(16) = –1, your factorization did not complete successfully, resulting in computational error 2126.

If IPARM(16) > 0, it is the row number $k$, in which the maximum absolute value of the ratio $a_{kk}/u_{kk}$ occurred, where:

If IPARM(3) = 0, $k$ can be any of the rows, 1 through $n$, in the full factorization.

If IPARM(3) > 0, $k$ can be any of the rows, IPARM(3)+1 through $n$, in the partial factorization.

If you are doing a **solve only**, this argument is not used in the computation and is unchanged.

- IPARM(17) through IPARM(20) are reserved.
- IPARM(21) through IPARM(25) have the following meaning, where:

  If you are doing a **factor and solve** or a **factor only**, IPARM(21) through IPARM(25) have the following meanings for each region $i$ for $i$ = 1,5, respectively:

  If IPARM(20+$i$) = −1, your factorization did not complete successfully, resulting in computational error 2126.

  If IPARM(20+$i$) ≥ 0, it is the number of pivots in region $i$ for the columns that were factored in matrix ***A***, where:

  If IPARM(3) = 0, columns 1 through $n$ were factored in the full factorization.

  If IPARM(3) > 0, columns IPARM(3)+1 through $n$ were factored in the partial factorization.

  If you are doing a **solve only**, these arguments are not used in the computation and are unchanged.

Returned as: a one-dimensional array of (at least) length 25, containing fullword integers.

*rparm*

is an array of parameters, RPARM($i$), where:

- RPARM(1) through RPARM(15) are unchanged.
- RPARM(16) has the following meaning, where:

  If you are doing a **factor and solve** or a **factor only**, and:

  If RPARM(16) = 0.0, your factorization did not complete successfully, resulting in computational error 2126.

  If |RPARM(16)| > 0.0, it is the ratio for row $k$, $a_{kk}/u_{kk}$, having the maximum absolute value. Row $k$ is indicated in IPARM(16), and:

  If IPARM(3) = 0, the ratio corresponds to one of the rows, 1 through $n$, in the full factorization.

  If IPARM(3) > 0, the ratio corresponds to one of the rows, IPARM(3)+1 through $n$, in the partial factorization.

  If you are doing a **solve only**, this argument is not used in the computation and is unchanged.

- RPARM(17) and RPARM(18) have the following meaning, where:

  If you are **computing the determinant** of matrix ***A***, then RPARM(17) is the mantissa, *detbas*, and RPARM(18) is the power of 10, *detpwr*, used to express the value of the determinant: *detbas*($10^{detpwr}$), where $1 \leq detbas < 10$. Also:

If IPARM(3) = 0, the determinant is computed for columns 1 through *n* in the full factorization.
If IPARM(3) > 0, the determinant is computed for columns IPARM(3)+1 through *n* in the partial factorization.

If you are **not computing the determinant** of matrix ***A***, these arguments are not used in the computation and are unchanged.

- RPARM(19) through RPARM(25) are reserved.

Returned as: a one-dimensional array of (at least) length 25, containing long-precision real numbers.

*bx*

has the following meaning, where:

If you are doing a **factor and solve** or a **solve only**, *bx* is the array, containing the *mbx* solution vectors ***x*** of the system ***Ax*** = ***b*** or $\boldsymbol{A}^{T}\boldsymbol{x}$ = ***b***. Each vector ***x*** is length *n* and is stored in the corresponding column of the array. (If *mbx* = 0, then *bx* is unchanged on output.)

If you are doing a **factor only**, this argument is not used in the computation and is unchanged.

Returned as: an *ldbx* by (at least) *mbx* array, containing long-precision real numbers.

### Notes

1. If you set either IPARM(1) = 0 or IPARM(10) = 0, indicating you want to use the default values for IPARM(11) through IPARM(15) and RPARM(10), then:
   - Matrix ***A*** must be positive definite.
   - No pivots are fixed, using RPARM(11) through RPARM(15) values.
   - No small pivots are tolerated; that is, the value should be |*pivot*| > RPARM(10).
2. Many of the input and output parameters for *iparm* and *rparm* are defined for the five pivot regions handled by this subroutine. The limits of the regions are based on RPARM(10), as shown in Figure 11. The pivot values in each region are:

   Region 1: *pivot* < −RPARM(10)
   Region 2: −RPARM(10) ≤ *pivot* < 0
   Region 3: *pivot* = 0
   Region 4: 0 < *pivot* ≤ RPARM(10)
   Region 5: *pivot* > RPARM(10)

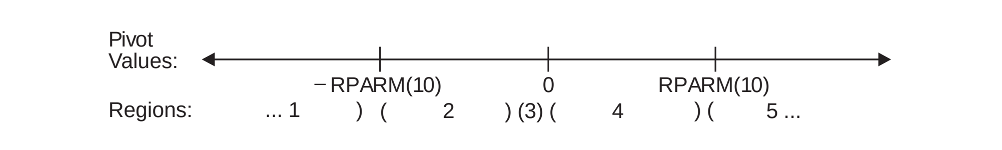


*Figure 11. Five Pivot Regions*

3. The IPARM(4) and IPARM(5) arguments allow you to specify the same or different skyline storage modes for your input and output arrays for matrix ***A***. This allows you to change storage modes as needed. However, if you are concerned with performance, you should use diagonal-out skyline storage mode for both input and output, if possible, because there is less overhead.

   For a description of how sparse matrices are stored in skyline storage mode, see “Profile-In Skyline Storage Mode” on page 103 and “Diagonal-Out Skyline Storage Mode” on page 101.

4. Following is an illustration of the portion of matrix ***A*** factored in the partial factorization when IPARM(3) > 0. In this case, the subroutine assumes that rows and columns 1 through IPARM(3) are already factored and that rows and columns IPARM(3)+1 through *n* are to be factored in this computation.

$$\begin{array}{cc}
 & \left|\leftarrow \text{factored} \rightarrow\right|\leftarrow \text{to be factored} \rightarrow\left|\right. \\
\begin{array}{c}\bar{\uparrow}\\ \\ \text{factored}\\ \\ \underline{\downarrow}\\ \bar{\uparrow}\\ \text{to be}\\ \text{factored}\\ \\ \underline{\downarrow}\end{array} & \left[\begin{array}{cccccc}
a_{11} & \cdots & a_{1j} & a_{1,j+1} & \cdots & a_{1n}\\
\vdots & \ddots & \vdots & & & \vdots\\
a_{j1} & \cdots & a_{jj} & & & \vdots\\
a_{j+1,1} & & & & & \vdots\\
\vdots & & & & \ddots & \vdots\\
a_{n1} & \cdots & \cdots & \cdots & \cdots & a_{nn}
\end{array}\right]
\end{array} \qquad \text{where } j = \mathrm{IPARM}(3)$$

   You use the partial factorization function when, for design or storage reasons, you must factor the matrix ***A*** in stages. When doing a partial factorization, you must use the same skyline storage mode for all parts of the matrix as it is progressively factored.

5. Your various arrays must have no common elements; otherwise, results are unpredictable.

6. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see “Using Auxiliary Storage in ESSL” on page 32.

***Function:*** This subroutine can factor, compute the determinant of, and solve general sparse matrix ***A***, stored in skyline storage mode. For all computations, input matrix ***A*** can be stored in either diagonal-out or profile-in skyline storage mode. Output matrix ***A*** can also be stored in either of these modes and can be different from the mode used for input.

Matrix ***A*** is factored into the following form using specified pivot processing:

***A*** = ***LU***

where:

***U*** is an upper triangular matrix.
***L*** is a lower triangular matrix.

The transformed matrix ***A***, factored into its ***LU*** form, is stored in packed format in arrays AU and AL. The inverse of the diagonal of matrix ***U*** is stored in the corresponding elements of array AU. The off-diagonal elements of the upper triangular matrix ***U*** are stored in the corresponding off-diagonal elements of array AU. The off-diagonal elements of the lower triangular matrix ***L*** are stored in the corresponding off-diagonal elements of array AL. (The diagonal elements stored in array AL do not have meaningful values.)

The partial factorization of matrix ***A***, which you can do when you specify the factor-only option, assumes that the first IPARM(3) rows and columns are already factored in the input matrix. It factors the remaining *n*−IPARM(3) rows and columns in matrix ***A***. (See "Notes" on page 603 for an illustration.) It updates only the elements in arrays AU and AL corresponding to the part of matrix ***A*** that is factored.

The determinant can be computed with any of the factorization computations. With a full factorization, you get the determinant for the whole matrix. With a partial factorization, you get the determinant for only that part of the matrix factored in this computation.

The system $\boldsymbol{Ax} = \boldsymbol{b}$ or $\boldsymbol{A}^T\boldsymbol{x} = \boldsymbol{b}$, having multiple right-hand sides, is solved for ***x***, using the transformed matrix ***A*** produced by this call or a subsequent call to this subroutine.

See references [9 ], [12 ], [25 ], [47 ], and [68]. If *n* is 0, no computation is performed. If *mbx* is 0, no solve is performed.

### *Error Conditions*

*Resource Errors*

- Error 2015 is unrecoverable, *naux* = 0, and unable to allocate work area.
- Unable to allocate internal work area.

*Computational Errors*

1. If a pivot occurs in region *i* for *i* = 1,5 and IPARM(10+*i*) = 1, the pivot value is replaced with RPARM(10+*i*), an attention message is issued, and processing continues.
2. Unacceptable pivot values occurred in the factorization of matrix ***A***.
    - One or more diagonal elements of ***U*** contains unacceptable pivots and no valid fixup is applicable. The row number *i* of the first unacceptable pivot element is identified in the computational error message.
    - The return code is set to 2.
    - *i* can be determined at run time by use of the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error code 2126 in the ESSL error option table; otherwise, the default value causes your program to terminate when this error occurs. For details, see "What Can You Do about ESSL Computational Errors?" on page 48.

*Input-Argument Errors*

1. $n < 0$
2. $nu < 0$
3. IDU(*n*+1) > *nu*+1
4. IDU(*i*+1) ≤ IDU(*i*) for *i* = 1, *n*

5. IDU(*i*+1) > IDU(*i*)+*i* and IPARM(4) = 0 for *i* = 1, *n*
6. IDU(*i*) > IDU(*i*–1)+*i* and IPARM(4) = 1 for *i* = 2, *n*
7. *nl* < 0
8. IDL(*n*+1) > *nl*+1
9. IDL(*i*+1) ≤ IDL(*i*) for *i* = 1, *n*
10. IDL(*i*+1) > IDL(*i*)+*i* and IPARM(4) = 0 for *i* = 1, *n*
11. IDL(*i*) > IDL(*i*–1)+*i* and IPARM(4) = 1 for *i* = 2, *n*
12. IPARM(1) ≠ 0 or 1
13. IPARM(2) ≠ 0, 1, 2, 10, 11, 100, 102, or 110
14. IPARM(3) < 0
15. IPARM(3) > *n*
16. IPARM(3) > 0 and IPARM(2) ≠ 1 or 11
17. IPARM(4), IPARM(5) ≠ 0 or 1
18. IPARM(2) = 0, 1, 10, 11, 100, or 110 and:
    - IPARM(10) ≠ 0 or 1
    - IPARM(11), IPARM(12) ≠ –1, 0, or 1
    - IPARM(13) ≠ –1 or 1
    - IPARM(14), IPARM(15) ≠ –1, 0, or 1
    - RPARM(10) < 0.0
    - RPARM(10+*i*) = 0.0 and IPARM(10+*i*) = 1 for *i* = 1,5
19. IPARM(2) = 0, 2, 10, 100, 102, or 110 and:
    - *ldbx* ≤ 0 and *mbx* ≠ 0 and *n* ≠ 0
    - *ldbx* < 0 and *mbx* = 0
    - *ldbx* < *n* and *mbx* ≠ 0
    - *mbx* < 0
20. Error 2015 is recoverable or *naux*≠0, and *naux* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example 1:*** This example shows how to factor a 9 by 9 general sparse matrix ***A*** and solve the system ***Ax*** = ***b*** with three right-hand sides. The default values are used for IPARM and RPARM. Input matrix ***A***, shown here, is stored in diagonal-out skyline storage mode. Matrix ***A*** is:

$$\begin{bmatrix}
2.0 & 2.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
2.0 & 4.0 & 4.0 & 2.0 & 2.0 & 0.0 & 0.0 & 0.0 & 2.0 \\
2.0 & 4.0 & 6.0 & 4.0 & 4.0 & 0.0 & 2.0 & 0.0 & 4.0 \\
2.0 & 4.0 & 6.0 & 6.0 & 6.0 & 2.0 & 4.0 & 0.0 & 6.0 \\
0.0 & 0.0 & 0.0 & 2.0 & 4.0 & 4.0 & 4.0 & 2.0 & 4.0 \\
0.0 & 2.0 & 4.0 & 6.0 & 8.0 & 6.0 & 8.0 & 4.0 & 10.0 \\
0.0 & 0.0 & 0.0 & 2.0 & 4.0 & 6.0 & 8.0 & 6.0 & 8.0 \\
0.0 & 0.0 & 0.0 & 2.0 & 4.0 & 6.0 & 8.0 & 8.0 & 10.0 \\
2.0 & 4.0 & 6.0 & 6.0 & 8.0 & 6.0 & 10.0 & 8.0 & 16.0
\end{bmatrix}$$

Output matrix ***A***, shown here, is in ***LU*** factored form with $\boldsymbol{U}^{-1}$ on the diagonal, and is stored in diagonal-out skyline storage mode. Matrix ***B*** is:

```
┌                                             ┐
│ 0.5  2.0  2.0  0.0  0.0  0.0  0.0  0.0  0.0 │
│ 1.0  0.5  2.0  2.0  2.0  0.0  0.0  0.0  2.0 │
│ 1.0  1.0  0.5  2.0  2.0  0.0  2.0  0.0  2.0 │
│ 1.0  1.0  1.0  0.5  2.0  2.0  2.0  0.0  2.0 │
│ 0.0  0.0  0.0  1.0  0.5  2.0  2.0  2.0  2.0 │
│ 0.0  1.0  1.0  1.0  1.0  0.5  2.0  2.0  2.0 │
│ 0.0  0.0  0.0  1.0  1.0  1.0  0.5  2.0  2.0 │
│ 0.0  0.0  0.0  1.0  1.0  1.0  1.0  0.5  2.0 │
│ 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  0.5 │
└                                             ┘
```

*Call Statement and Input*

```
            N   AU  NU  IDU  AL  NL  IDL  IPARM  RPARM  AUX  NAUX  BX  LDBX MBX
            |   |   |   |    |   |   |     |      |     |    |     |    |    |
CALL DGKFS( 9 , AU, 33, IDU, AL, 35, IDL, IPARM, RPARM, AUX,  57 , BX , 12 , 3 )
```

```
AU       =  (2.0, 4.0, 2.0, 6.0, 4.0, 2.0, 6.0, 4.0, 2.0, 4.0, 6.0,
             4.0, 2.0, 6.0, 4.0, 2.0, 8.0, 8.0, 4.0, 4.0, 2.0, 8.0,
             6.0, 4.0, 2.0, 16.0, 10.0, 8.0, 10.0, 4.0, 6.0, 4.0, 2.0)
IDU      =  (1, 2, 4, 7, 10, 14, 17, 22, 26, 34)
AL       =  (0.0, 0.0, 2.0, 0.0, 4.0, 2.0, 0.0, 6.0, 4.0, 2.0, 0.0,
             2.0, 0.0, 8.0, 6.0, 4.0, 2.0, 0.0, 6.0, 4.0, 2.0, 0.0,
             8.0, 6.0, 4.0, 2.0, 0.0, 8.0, 10.0, 6.0, 8.0, 6.0, 6.0,
             4.0, 2.0)
IDL      =  (1, 2, 4, 7, 11, 13, 18, 22, 27, 36)
IPARM    =  (0, . , . , . , . , . , . , . , . , . , . , . , . , . , . ,
             . , . , . , . , . , . , . , . , . , . , . , . , . , . )

RPARM    =(not relevant)

              ┌                        ┐
              │  6.00   12.00   18.00  │
              │ 16.00   32.00   48.00  │
              │ 26.00   52.00   78.00  │
              │ 36.00   72.00  108.00  │
              │ 20.00   40.00   60.00  │
BX    =       │ 48.00   96.00  144.00  │
              │ 34.00   68.00  102.00  │
              │ 38.00   76.00  114.00  │
              │ 66.00  132.00  198.00  │
              │   .       .       .    │
              │   .       .       .    │
              │   .       .       .    │
              └                        ┘
```

*Output*

```
AU       =  (0.5, 0.5, 2.0, 0.5, 2.0, 2.0, 0.5, 2.0, 2.0, 0.5, 2.0,
             2.0, 2.0, 0.5, 2.0, 2.0, 0.5, 2.0, 2.0, 2.0, 2.0, 0.5,
             2.0, 2.0, 2.0, 0.5, 2.0, 2.0, 2.0, 2.0, 2.0, 2.0, 2.0)
IDU      =(same as input)
AL       =  (0.0, 0.0, 1.0, 0.0, 1.0, 1.0, 0.0, 1.0, 1.0, 1.0, 0.0,
             1.0, 0.0, 1.0, 1.0, 1.0, 1.0, 0.0, 1.0, 1.0, 1.0, 0.0,
             1.0, 1.0, 1.0, 1.0, 0.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0,
             1.0, 1.0)
```

```
IDL      =(same as input)
IPARM    =  (0, . , . , . , . , . , . , . , . , . , . , . , . , . , . ,
              . , 9, . , . , . , . , . , . , 0, 0, 0, 0, 9)
RPARM    =  ( . , . , . , . , . , . , . , . , . , . , . , . , . , . , . ,
              . , 8.0, . , . , . , . , . , . , . , . , . , . , . , . )

          ┌                   ┐
          | 1.00  2.00  3.00  |
          | 1.00  2.00  3.00  |
          | 1.00  2.00  3.00  |
          | 1.00  2.00  3.00  |
          | 1.00  2.00  3.00  |
BX    =   | 1.00  2.00  3.00  |
          | 1.00  2.00  3.00  |
          | 1.00  2.00  3.00  |
          | 1.00  2.00  3.00  |
          |  .     .     .    |
          |  .     .     .    |
          |  .     .     .    |
          └                   ┘
```

***Example 2:*** This example shows how to factor the 9 by 9 general sparse matrix ***A*** from Example 1, solve the system $\boldsymbol{A}^{T}\boldsymbol{x} = \boldsymbol{b}$ with three right-hand sides, and compute the determinant of ***A***. The default values for pivot processing are used for IPARM. Input matrix ***A*** is stored in profile-in skyline storage mode. Output matrix ***A*** is in ***LU*** factored form with $\boldsymbol{U}^{-1}$ on the diagonal, and is stored in diagonal-out skyline storage mode. It is the same as output matrix ***A*** in Example 1.

*Call Statement and Input*

```
             N  AU  NU  IDU  AL  NL  IDL  IPARM  RPARM  AUX  NAUX  BX  LDBX MBX
             |  |   |    |   |   |    |     |      |     |    |    |    &darrow    |
CALL DGKFS( 9, AU, 33, IDU, AL, 35, IDL, IPARM, RPARM, AUX,  57 , BX , 12 , 3 )

AU       =  (2.0, 2.0, 4.0, 2.0, 4.0, 6.0, 2.0, 4.0, 6.0, 2.0, 4.0,
             6.0, 4.0, 2.0, 4.0, 6.0, 2.0, 4.0, 4.0, 8.0, 8.0, 2.0,
             4.0, 6.0, 8.0, 2.0, 4.0, 6.0, 4.0, 10.0, 8.0, 10.0, 16.0)
IDU      =  (1, 3, 6, 9, 13, 16, 21, 25, 33, 34)
AL       =  (0.0, 2.0, 0.0, 2.0, 4.0, 0.0, 2.0, 4.0, 6.0, 0.0, 2.0,
             0.0, 2.0, 4.0, 6.0, 8.0, 0.0, 2.0, 4.0, 6.0, 0.0, 2.0,
             4.0, 6.0, 8.0, 0.0, 2.0, 4.0, 6.0, 6.0, 8.0, 6.0, 10.0,
             8.0, 0.0)
IDL      =  (1, 3, 6, 10, 12, 17, 21, 26, 35, 36)
IPARM    =  (1, 110, 0, 1, 0, . , . , . , . , . , . , 0, . , . , . , . , . , . ,
              . , . , . , . , . , . , . , . , . , . , . , . , . , . , . , . )

RPARM    =(not relevant)
```

```
          ┌                        ┐
          | 10.00   20.00   30.00 |
          | 20.00   40.00   60.00 |
          | 28.00   56.00   84.00 |
          | 30.00   60.00   90.00 |
          | 40.00   80.00  120.00 |
BX    =   | 30.00   60.00   90.00 |
          | 44.00   88.00  132.00 |
          | 28.00   56.00   84.00 |
          | 60.00  120.00  180.00 |
          |   .       .       .   |
          |   .       .       .   |
          |   .       .       .   |
          └                        ┘
```

*Output*

```
AU         =(same as output AU in Example 1)
IDU        =(same as output IDU in Example 1)
AL         =(same as output AL in Example 1)
IDL        =(same as output IDL in Example 1)
IPARM      =  (1, 110, 0, 1, 0, . , . , . , . , 0, . , . , . , . , . ,
                9, . , . , . , . , 0, 0, 0, 0, 9)
RPARM      =  ( . , . , . , . , . , . , . , . , . , . , . , . , . , . ,
                . , 8.0, 5.12, 2.0, . , . , . , . , . , . , . )
BX         =(same as output BX in Example 1)
```

***Example 3:*** This example shows how to factor a 9 by 9 negative-definite general sparse matrix ***A***, solve the system ***Ax*** = ***b*** with three right-hand sides, and compute the determinant of ***A***. (Default values for pivot processing are not used for IPARM because ***A*** is negative-definite.) Input matrix ***A***, shown here, is stored in diagonal-out skyline storage mode:

```
┌                                                      ┐
| -2.0  -2.0  -2.0   0.0   0.0   0.0    0.0   0.0   0.0 |
| -2.0  -4.0  -4.0  -2.0  -2.0   0.0    0.0   0.0  -2.0 |
| -2.0  -4.0  -6.0  -4.0  -4.0   0.0   -2.0   0.0  -4.0 |
| -2.0  -4.0  -6.0  -6.0  -6.0  -2.0   -4.0   0.0  -6.0 |
|  0.0   0.0   0.0  -2.0  -4.0  -4.0   -4.0  -2.0  -4.0 |
|  0.0  -2.0  -4.0  -6.0  -8.0  -6.0   -8.0  -4.0 -10.0 |
|  0.0   0.0   0.0  -2.0  -4.0  -6.0   -8.0  -6.0  -8.0 |
|  0.0   0.0   0.0  -2.0  -4.0  -6.0   -8.0  -8.0 -10.0 |
| -2.0  -4.0  -6.0  -6.0  -8.0  -6.0  -10.0  -8.0 -16.0 |
└                                                      ┘
```

Output matrix ***A***, shown here, is in ***LU*** factored form with $\boldsymbol{U}^{-1}$ on the diagonal, and is stored in diagonal-out skyline storage mode. Matrix ***A*** is:

```
┌                                                     ┐
│ -0.5  -2.0  -2.0   0.0   0.0   0.0   0.0   0.0   0.0 │
│  1.0  -0.5  -2.0  -2.0  -2.0   0.0   0.0   0.0  -2.0 │
│  1.0   1.0  -0.5  -2.0  -2.0   0.0  -2.0   0.0  -2.0 │
│  1.0   1.0   1.0  -0.5  -2.0  -2.0  -2.0   0.0  -2.0 │
│  0.0   0.0   0.0   1.0  -0.5  -2.0  -2.0  -2.0  -2.0 │
│  0.0   1.0   1.0   1.0   1.0  -0.5  -2.0  -2.0  -2.0 │
│  0.0   0.0   0.0   1.0   1.0   1.0  -0.5  -2.0  -2.0 │
│  0.0   0.0   0.0   1.0   1.0   1.0   1.0  -0.5  -2.0 │
│  1.0   1.0   1.0   1.0   1.0   1.0   1.0   1.0  -0.5 │
└                                                     ┘
```

*Call Statement and Input*

```
          N  AU  NU  IDU  AL  NL  IDL  IPARM  RPARM  AUX  NAUX  BX  LDBX MBX
          |  |   |    |   |   |    |     |      |     |    |    |    |    |
CALL DGKFS( 9, AU, 33, IDU, AL, 35, IDL, IPARM, RPARM, AUX,  57 , BX , 12 , 3 )
```

```
AU       =  (-2.0, -4.0, -2.0, -6.0, -4.0, -2.0, -6.0, -4.0, -2.0,
             -4.0, -6.0, -4.0, -2.0, -6.0, -4.0, -2.0, -8.0, -8.0,
             -4.0, -4.0, -2.0, -8.0, -6.0, -4.0, -2.0, -16.0, -10.0,
             -8.0, -10.0, -4.0, -6.0, -4.0, -2.0)
IDU      =  (1, 2, 4, 7, 10, 14, 17, 22, 26, 34)
AL       =  (0.0, 0.0, -2.0, 0.0, -4.0, -2.0, 0.0, -6.0, -4.0, -2.0,
             0.0, -2.0, 0.0, -8.0, -6.0, -4.0, -2.0, 0.0, -6.0, -4.0,
             -2.0, 0.0, -8.0, -6.0, -4.0, -2.0, 0.0, -8.0, -10.0,
             -6.0, -8.0, -6.0, -6.0, -4.0, -2.0)
IDL      =  (1, 2, 4, 7, 11, 13, 18, 22, 27, 36)
IPARM    =  (1, 10, 0, 0, 0, . , . , . , . , . , 1, 0, -1, -1, -1, -1, . ,
             . , . , . , . , . , . , . , . , . )

RPARM    =  ( . , . , . , . , . , . , . , . , . , $10^{-15}$, . , . , . ,
              . , . , . , . , . , . , . , . , . , . , . , . , . , . )
BX       =(same as input BX in Example 1)
```

*Output*

```
AU       =  (-0.5, -0.5, -2.0, -0.5, -2.0, -2.0, -0.5, -2.0, -2.0,
             -0.5, -2.0, -2.0, -2.0, -0.5, -2.0, -2.0, -0.5, -2.0,
             -2.0, -2.0, -2.0, -0.5, -2.0, -2.0, -2.0, -0.5, -2.0,
             -2.0, -2.0, -2.0, -2.0, -2.0, -2.0)
IDU      =(same as input)
AL       =  (0.0, 0.0, 1.0, 0.0, 1.0, 1.0, 0.0, 1.0, 1.0, 1.0, 0.0,
             1.0, 0.0, 1.0, 1.0, 1.0, 1.0, 0.0, 1.0, 1.0, 1.0, 0.0,
             1.0, 1.0, 1.0, 1.0, 0.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0,
             1.0, 1.0)
IDL      =(same as input)
IPARM    =  (1, 10, 0, 0, 0, . , . , . , . , . , 1, 0, -1, -1, -1, -1, 9,
             . , . , . , . , 9, 0, 0, 0, 0)
RPARM    =  ( . , . , . , . , . , . , . , . , . , $10^{-15}$, . , . , . ,
             . , . , . , 8.0, -5.12, 2.0, . , . , . , . , . , . , . )
```

$$
BX = \begin{bmatrix}
-1.00 & -2.00 & -3.00 \\
-1.00 & -2.00 & -3.00 \\
-1.00 & -2.00 & -3.00 \\
-1.00 & -2.00 & -3.00 \\
-1.00 & -2.00 & -3.00 \\
-1.00 & -2.00 & -3.00 \\
-1.00 & -2.00 & -3.00 \\
-1.00 & -2.00 & -3.00 \\
-1.00 & -2.00 & -3.00 \\
. & . & . \\
. & . & . \\
. & . & .
\end{bmatrix}
$$

***Example 4:*** This example shows how to factor the first six rows and columns, referred to as matrix ***A1***, of the 9 by 9 general sparse matrix ***A*** from Example 1 and compute the determinant of ***A1***. Input matrix ***A1***, shown here, is stored in diagonal-out skyline storage mode. Input matrix ***A1*** is:

$$
\begin{bmatrix}
2.0 & 2.0 & 2.0 & 0.0 & 0.0 & 0.0 \\
2.0 & 4.0 & 4.0 & 2.0 & 2.0 & 0.0 \\
2.0 & 4.0 & 6.0 & 4.0 & 4.0 & 0.0 \\
2.0 & 4.0 & 6.0 & 6.0 & 6.0 & 2.0 \\
0.0 & 0.0 & 0.0 & 2.0 & 4.0 & 4.0 \\
0.0 & 2.0 & 4.0 & 6.0 & 8.0 & 6.0
\end{bmatrix}
$$

Output matrix ***A1***, shown here, is in ***LU*** factored form with $\boldsymbol{U}^{-1}$ on the diagonal, and is stored in diagonal-out skyline storage mode. Output matrix ***A1*** is:

$$
\begin{bmatrix}
0.5 & 2.0 & 2.0 & 0.0 & 0.0 & 0.0 \\
1.0 & 0.5 & 2.0 & 2.0 & 2.0 & 0.0 \\
1.0 & 1.0 & 0.5 & 2.0 & 2.0 & 0.0 \\
1.0 & 1.0 & 1.0 & 0.5 & 2.0 & 2.0 \\
0.0 & 0.0 & 0.0 & 1.0 & 0.5 & 2.0 \\
0.0 & 1.0 & 1.0 & 1.0 & 1.0 & 0.5
\end{bmatrix}
$$

*Call Statement and Input*

```
           N  AU  NU  IDU  AL  NL  IDL  IPARM  RPARM  AUX  NAUX  BX   LDBX   MBX
           |  |   |    |   |   |    |     |      |     |    |    |     |      |
CALL DGKFS( 6, AU, 33, IDU, AL, 35, IDL, IPARM, RPARM, AUX,  45 , BX , LDBX , MBX )
```

```
AU       =(same as input AU in Example 1)
IDU      =  (1, 2, 4, 7, 10, 14, 17)
AL       =(same as input AL in Example 1)
IDL      =  (1, 2, 4, 7, 11, 13, 18)
IPARM    =  (1, 11, 0, 0, 0, . , . , . , . , . , . , 0, . , . , . , . , . , . , . ,
              . , . , . , . , . , . , . , . , . , . , . , . , . , . , . , . , . )
RPARM    =(not relevant)
BX       =(not relevant)
LDBX     =(not relevant)
MBX      =(not relevant)
```

*Output*

```
AU       =  (0.5, 0.5, 2.0, 0.5, 2.0, 2.0, 0.5, 2.0, 2.0, 0.5, 2.0,
             2.0, 2.0, 0.5, 2.0, 2.0, 8.0, 8.0, 4.0, 4.0, 2.0, 8.0,
             6.0, 4.0, 2.0, 16.0, 10.0, 8.0, 10.0, 4.0, 6.0, 4.0, 2.0)
IDU      =(same as input)
AL       =  (0.0, 0.0, 1.0, 0.0, 1.0, 1.0, 0.0, 1.0, 1.0, 1.0, 0.0,
             1.0, 0.0, 1.0, 1.0, 1.0, 1.0, 0.0, 6.0, 4.0, 2.0, 0.0,
             8.0, 6.0, 4.0, 2.0, 0.0, 8.0, 10.0, 6.0, 8.0, 6.0, 6.0,
             4.0, 2.0)
IDL      =(same as input)
IPARM    =  (1, 11, 0, 0, 0, . , . , . , . , 0, . , . , . , . , . , 3,
             . , . , . , . , 0, 0, 0, 0, 6)
RPARM    =  ( . , . , . , . , . , . , . , . , . , . , . , . , . , . ,
             . , 3.0, 6.4, 1.0, . , . , . , . , . , . , . )
BX       =(same as input)
LDBX     =(same as input)
MBX      =(same as input)
```

***Example 5:*** This example shows how to do a partial factorization of the 9 by 9 general sparse matrix ***A*** from Example 1, where the first six rows and columns were factored in Example 4. It factors the remaining three rows and columns and computes the determinant of that part of the matrix. The input matrix, referred to as ***A2***, shown here, is made up of the output factored matrix ***A1*** plus the three remaining unfactored rows and columns of matrix ***A***. Matrix ***A2*** is:

$$\begin{bmatrix}
0.5 & 2.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
1.0 & 0.5 & 2.0 & 2.0 & 2.0 & 0.0 & 0.0 & 0.0 & 2.0 \\
1.0 & 1.0 & 0.5 & 2.0 & 2.0 & 0.0 & 2.0 & 0.0 & 4.0 \\
1.0 & 1.0 & 1.0 & 0.5 & 2.0 & 2.0 & 4.0 & 0.0 & 6.0 \\
0.0 & 0.0 & 0.0 & 1.0 & 0.5 & 2.0 & 4.0 & 2.0 & 4.0 \\
0.0 & 1.0 & 1.0 & 1.0 & 1.0 & 0.5 & 8.0 & 4.0 & 10.0 \\
0.0 & 0.0 & 0.0 & 2.0 & 4.0 & 6.0 & 8.0 & 6.0 & 8.0 \\
0.0 & 0.0 & 0.0 & 2.0 & 4.0 & 6.0 & 8.0 & 8.0 & 10.0 \\
2.0 & 4.0 & 6.0 & 6.0 & 8.0 & 6.0 & 10.0 & 8.0 & 16.0
\end{bmatrix}$$

Both parts of input matrix ***A2*** are stored in diagonal-out skyline storage mode.

Output matrix ***A2*** is the same as output matrix ***A*** in Example 1 and is stored in diagonal-out skyline storage mode.

*Call Statement and Input*

```
            N  AU  NU  IDU  AL  NL  IDL  IPARM  RPARM  AUX  NAUX  BX   LDBX   MBX
            |  |   |   |    |   |   |    |      |      |    |     |    |      |
CALL DGKFS( 9, AU, 33, IDU, AL, 35, IDL, IPARM, RPARM, AUX,  45 , BX , LDBX , MBX )

AU       =(same as output AU in Example 4)
IDU      =(same as input IDU in Example 1)
AL       =(same as output AL in Example 4)
IDL      =(same as input IDL in Example 1)
IPARM    =  (1, 11, 6, 0, 0, . , . , . , . , 0, . , . , . , . , . ,
             . , . , . , . , . , . , . , . , . , . )
RPARM    =(not relevant)
```

```
BX        =(not relevant)
LDBX      =(not relevant)
MBX       =(not relevant)
```

*Output*

```
AU        =(same as output AU in Example 1)
IDU       =(same as output IDU in Example 1)
AL        =(same as output AL in Example 1)
IDL       =(same as output IDL in Example 1)
IPARM     =  (1, 11, 6, 0, 0, . , . , . , . , 0, . , . , . , . , . , 9,
              . , . , . , . , 0, 0, 0, 0, 3)
RPARM     =  ( . , . , . , . , . , . , . , . , . , . , . , . , . , . ,
              . , 8.0, 8.0, 0.0, . , . , . , . , . , . , . )
BX        =(same as input)
LDBX      =(same as input)
MBX       =(same as input)
```

***Example 6:*** This example shows how to solve the system ***Ax*** = ***b*** with one right-hand side for a general sparse matrix ***A***. Input matrix ***A***, used here, is the same as factored output matrix ***A*** from Example 1, stored in profile-in skyline storage mode. Here, output matrix ***A*** is unchanged on output and is stored in profile-in skyline storage mode.

*Call Statement and Input*

```
               N  AU  NU  IDU  AL  NL  IDL  IPARM  RPARM  AUX NAUX  BX  LDBX MBX
               |  |   |    |   |   |    |     |      |     |   |    |    |    |
CALL DGKFS( 9, AU, 33, IDU, AL, 35, IDL, IPARM, RPARM, AUX, 49 , BX , 9 ,  1 )
```

```
AU        =  (0.5, 2.0, 0.5, 2.0, 2.0, 0.5, 2.0, 2.0, 0.5, 2.0, 2.0,
              2.0, 0.5, 2.0, 2.0, 0.5, 2.0, 2.0, 2.0, 2.0, 0.5, 2.0,
              2.0, 2.0, 0.5, 2.0, 2.0, 2.0, 2.0, 2.0, 2.0, 2.0, 0.5)
IDU       =  (1, 3, 6, 9, 13, 16, 21, 25, 33, 34)
AL        =  (0.0, 1.0, 0.0, 1.0, 1.0, 0.0, 1.0, 1.0, 1.0, 0.0, 1.0,
              0.0, 1.0, 1.0, 1.0, 1.0, 0.0, 1.0, 1.0, 1.0, 0.0, 1.0,
              1.0, 1.0, 1.0, 0.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0,
              1.0, 0.0)
IDL       =  (1, 3, 6, 10, 12, 17, 21, 26, 35, 36)
IPARM     =  (1, 2, 0, 1, 1, . , . , . , . , . , . , . , . , . , . ,
              . , . , . , . , . , . , . , . , . , . )
RPARM     =(not relevant)
BX        =  (12.0, 58.0, 114.0, 176.0, 132.0, 294.0, 240.0, 274.0,
              406.0)
```

*Output*

```
AU        =(same as input)
IDU       =(same as input)
AL        =(same as input)
IDL       =(same as input)
IPARM     =(same as input)
RPARM     =(not relevant)
BX        =  (1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0)
```

# DSKFS—Symmetric Sparse Matrix Factorization, Determinant, and Solve Using Skyline Storage Mode

This subroutine can perform either or both of the following functions for symmetric sparse matrix ***A***, stored in skyline storage mode, and for vectors ***x*** and ***b***:

- Factor ***A*** and, optionally, compute the determinant of ***A***.
- Solve the system ***Ax*** = ***b*** using the results of the factorization of matrix ***A***, produced on this call or a preceding call to this subroutine.

You have the choice of using either Gaussian elimination or Cholesky decomposition. You also have the choice of using profile-in or diagonal-out skyline storage mode for ***A*** on input or output.

**Note:** The input to the solve performed by this subroutine must be the output from the factorization performed by this subroutine.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL DSKFS (*n*, *a*, *na*, *idiag*, *iparm*, *rparm*, *aux*, *naux*, *bx*, *ldbx*, *mbx*) |
| **C and C++** | dskfs (*n*, *a*, *na*, *idiag*, *iparm*, *rparm*, *aux*, *naux*, *bx*, *ldbx*, *mbx*); |
| **PL/I** | CALL DSKFS (*n*, *a*, *na*, *idiag*, *iparm*, *rparm*, *aux*, *naux*, *bx*, *ldbx*, *mbx*); |

*On Entry*

*n*

is the order of symmetric sparse matrix ***A***. Specified as: a fullword integer; $n \geq 0$.

*a*

is the array, referred to as A, containing one of three forms of the upper triangular part of symmetric sparse matrix ***A***, depending on the type of computation performed, where:

- If you are doing a **factor and solve** or a **factor only**, and if IPARM(3) = 0, then A contains the unfactored upper triangle of symmetric sparse matrix ***A***.
- If you are doing a **factor only**, and if IPARM(3) > 0, then A contains the partially factored upper triangle of symmetric sparse matrix ***A***. The first IPARM(3) columns in the upper triangle of ***A*** are already factored. The remaining columns are factored in this computation.
- If you are doing a **solve only**, then A contains the factored upper triangle of sparse matrix ***A***, produced by a preceding call to this subroutine.

In each case:

If IPARM(4) = 0, diagonal-out skyline storage mode is used for ***A***.

If IPARM(4) = 1, profile-in skyline storage mode is used for ***A***.

Specified as: a one-dimensional array of (at least) length *na*, containing long-precision real numbers.

*na*

is the length of array A. Specified as: a fullword integer; $na \geq 0$ and $na \geq$ (IDIAG($n$+1)−1).

*idiag*

is the array, referred to as IDIAG, containing the relative positions of the diagonal elements of matrix ***A*** (in one of its three forms) in array A. Specified as: a one-dimensional array of (at least) length *n*+1, containing fullword integers.

*iparm*

is an array of parameters, IPARM(*i*), where:

- IPARM(1) indicates whether certain default values for *iparm* and *rparm* are used by this subroutine, where:

  If IPARM(1) = 0, the following default values are used. For restrictions, see “Notes” on page 621.

  IPARM(2) = 0
  IPARM(3) = 0
  IPARM(4) = 0
  IPARM(5) = 0
  IPARM(10) = 0
  IPARM(11) = –1
  IPARM(12) = –1
  IPARM(13) = –1
  IPARM(14) = –1
  IPARM(15) = 0
  RPARM(10) = $10^{-12}$

  If IPARM(1) = 1, the default values are not used.

- IPARM(2) indicates the type of computation performed by this subroutine. The following table gives the IPARM(2) values for each variation:

| **Type of Computation** | **Gaussian Elimination *Ax = b*** | **Gaussian Elimination *Ax = b* and Determinant(*A*)** | **Cholesky Decomposition *Ax = b*** | **Cholesky Decomposition *Ax = b* and Determinant(*A*)** |
|---|---|---|---|---|
| **Factor and Solve** | 0 | 10 | 100 | 110 |
| **Factor Only** | 1 | 11 | 101 | 111 |
| **Solve Only** | 2 | N/A | 102 | N/A |

- IPARM(3) indicates whether a full or partial factorization is performed on matrix ***A***, where:

  If IPARM(3) = 0, and:

  If you are doing a **factor and solve** or a **factor only**, then a full factorization is performed for matrix ***A*** on rows and columns 1 through *n*.
  If you are doing a **solve only**, this argument has no effect on the computation, but must be set to 0.

  If IPARM(3) > 0, and you are doing a **factor only**, then a partial factorization is performed on matrix ***A***. Rows 1 through IPARM(3) of columns 1 through IPARM(3) in matrix ***A*** must be in factored form from a preceding call to this subroutine. The factorization is performed on rows

IPARM(3)+1 through *n* and columns IPARM(3)+1 through *n*. For an illustration, see “Notes” on page 621.

- IPARM(4) indicates the input storage mode used for matrix ***A***. This determines the arrangement of data in arrays A and IDIAG on input, where:

  If IPARM(4) = 0, diagonal-out skyline storage mode is used.

  If IPARM(4) = 1, profile-in skyline storage mode is used.

- IPARM(5) indicates the output storage mode used for matrix ***A***. This determines the arrangement of data in arrays A and IDAIG on output, where:

  If IPARM(5) = 0, diagonal-out skyline storage mode is used.

  If IPARM(5) = 1, profile-in skyline storage mode is used.

- IPARM(6) through IPARM(9) are reserved.

- IPARM(10) has the following meaning, where:

  If you are doing a **factor and solve** or a **factor only**, then IPARM(10) indicates whether certain default values for *iparm* and *rparm* are used by this subroutine, where:

  If IPARM(10) = 0, the following default values are used. For restrictions, see “Notes” on page 621.
  IPARM(11) = –1
  IPARM(12) = –1
  IPARM(13) = –1
  IPARM(14) = –1
  IPARM(15) = 0
  RPARM(10) = $10^{-12}$
  If IPARM(10) = 1, the default values are not used.

  If you are doing a **solve only**, this argument is not used.

- IPARM(11) through IPARM(15) have the following meaning, where:

  If you are doing a **factor and solve** or a **factor only**, then IPARM(11) through IPARM(15) control the type of processing to apply to pivot elements occurring in regions 1 through 5, respectively. The pivot elements are $d_{kk}$ for Gaussian elimination and $r_{kk}$ for Cholesky decomposition for $k = 1, n$ when doing a full factorization, and they are $k$ = IPARM(3)+1, $n$ when doing a partial factorization. The region in which a pivot element falls depends on the sign and magnitude of the pivot element. The regions are determined by RPARM(10). For a description of the regions and associated pivot values, see “Notes” on page 621. For each region $i$ for $i = 1,5$, where the pivot occurs in region $i$, the processing applied to the pivot element is determined by IPARM(10+*i*), where:

  If IPARM(10+*i*) = –1, the pivot element is trapped and computational error 2126 is generated. See “Error Conditions” on page 623.
  If IPARM(10+*i*) = 0, processing continues normally.

  **Note:** A value of 0 is not permitted for region 3, because if processing continues, a divide-by-zero exception occurs. In addition, if you are doing a Cholesky decomposition, a value of 0 is not permitted in regions 1 and 2, because a square root exception occurs.

If IPARM(10+*i*) = 1, the pivot element is replaced with the value in RPARM(10+*i*), and processing continues normally.

If you are doing a **solve only**, these arguments are not used.

- IPARM(16) through IPARM(25), see "On Return" on page 619.

Specified as: a one-dimensional array of (at least) length 25, containing fullword integers, where:

IPARM(1) = 0 or 1
IPARM(2) = 0, 1, 2, 10, 11, 100, 101, 102, 110, or 111
If IPARM(2) = 0, 2, 10, 100, 102, or 110, then IPARM(3) = 0
If IPARM(2) = 1, 11, 101, or 111, then 0 ≤ IPARM(3) ≤ *n*
IPARM(4), IPARM(5) = 0 or 1
If IPARM(2) = 0, 1, 10, or 11, then:
IPARM(10) = 0 or 1
IPARM(11), IPARM(12) = –1, 0, or 1
IPARM(13) = –1 or 1
IPARM(14), IPARM(15) = –1, 0, or 1

If IPARM(2) = 100, 101, 110, or 111, then:
IPARM(10) = 0 or 1
IPARM(11), IPARM(12), IPARM(13) = –1 or 1
IPARM(14), IPARM(15) = –1, 0, or 1

*rparm*

is an array of parameters, RPARM(*i*), where:

- RPARM(1) through RPARM(9) are reserved.
- RPARM(10) has the following meaning, where:

  If you are doing a **factor and solve** or a **factor only**, RPARM(10) is the tolerance value for small pivots. This sets the bounds for the pivot regions, where pivots are processed according to the options you specify for the five regions in IPARM(11) through IPARM(15), respectively. The suggested value is $10^{-15}$ ≤ IPARM(10) ≤ 1.

  If you are doing a **solve only**, this argument is not used.

- RPARM(11) through RPARM(15) have the following meaning, where:

  If you are doing a **factor and solve** or a **factor only**, RPARM(11) through RPARM(15) are the fix-up values to use for the pivots in regions 1 through 5, respectively. For each RPARM(10+*i*) for *i* = 1,5, where the pivot occurs in region *i*:

  If IPARM(10+*i*) = 1, the pivot is replaced with RPARM(10+*i*), where |RPARM(10+*i*)| should be a sufficiently large nonzero value to avoid overflow when calculating the reciprocal of the pivot. For Gaussian elimination, the suggested value is $10^{-15}$ ≤ |RPARM(10+*i*)| ≤ 1. For Cholesky decomposition, the value must be RPARM(10+*i*) > 0.
  If IPARM(10+*i*) ≠ 1, RPARM(10+*i*) is not used.

  If you are doing a **solve only**, these arguments are not used.

- RPARM(16) through RPARM(25), see "On Return" on page 619.

Specified as: a one-dimensional array of (at least) length 25, containing long-precision real numbers, where if IPARM(2) = 0, 1, 10, 11, 100, 101, 110, or 111, then:

RPARM(10) ≥ 0.0
If IPARM(2) = 0, 1, 10, or 11, then RPARM(11) through RPARM(15) ≠ 0.0
If IPARM(2) = 100, 101, 110, or 111, then RPARM(11) through RPARM(15) > 0.0

*aux*

has the following meaning:

If *naux* = 0 and error 2015 is unrecoverable, *aux* is ignored.

Otherwise, it is the storage work area used by this subroutine. Its size is specified by *naux*.

Specified as: an area of storage, containing long-precision real numbers.

*naux*

is the size of the work area specified by *aux*—that is, the number of elements in *aux*. Specified as: a fullword integer, where:

If *naux* = 0 and error 2015 is unrecoverable, DSKFS dynamically allocates the work area used by this subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, If you are doing a **factor only**, you can use *naux* ≥ *n*; however, for optimal performance, use *naux* ≥ 3*n*.

If you are doing a **factor and solve** or a **solve only**, use *naux* ≥ 3*n*+4*mbx*.

For further details on error handling and the special factor-only case, see “Notes” on page 621.

*bx*

has the following meaning, where:

If you are doing a **factor and solve** or a **solve only**, *bx* is the array, containing the *mbx* right-hand side vectors ***b*** of the system ***Ax*** = ***b***. Each vector ***b*** is length *n* and is stored in the corresponding column of the array.

If you are doing a **factor only**, this argument is not used in the computation.

Specified as: an *ldbx* by (at least) *mbx* array, containing long-precision real numbers.

*ldbx*

has the following meaning, where:

If you are doing a **factor and solve** or a **solve only**, *ldbx* is the leading dimension of the array specified for *bx*.

If you are doing a **factor only**, this argument is not used in the computation.

Specified as: a fullword integer; *ldbx* ≥ *n* and:

If *mbx* ≠ 0, then *ldbx* > 0.

If *mbx* = 0, then *ldbx* ≥ 0.

*mbx*

has the following meaning, where:

If you are doing a **factor and solve** or a **solve only**, *mbx* is the number of right-hand side vectors, ***b***, in the array specified for *bx*.

If you are doing a **factor only**, this argument is not used in the computation.

Specified as: a fullword integer; *mbx* ≥ 0.

*On Return*

*a*

is the array, referred to as A, containing the upper triangular part of symmetric sparse matrix ***A*** in ***LDL***$^T$ or ***R***$^T$***R*** factored form, where:

If IPARM(5) = 0, diagonal-out skyline storage mode is used for ***A***.

If IPARM(5) = 1, profile-in skyline storage mode is used for ***A***.

(If *mbx* = 0 and you are doing a solve only, then *a* is unchanged on output.) Returned as: a one-dimensional array of (at least) length *na*, containing long-precision real numbers.

*idiag*

is the array, referred to as IDIAG, containing the relative positions of the diagonal elements of the factored output matrix ***A*** in array A. (If *mbx* = 0 and you are doing a solve only, then *idiag* is unchanged on output.)

Returned as: a one-dimensional array of (at least) length *n*+1, containing fullword integers.

*iparm*

is an array of parameters, IPARM(*i*), where:

- IPARM(1) through IPARM(15) are unchanged.
- IPARM(16) has the following meaning, where:

  If you are doing a **factor and solve** or a **factor only**, and:

  If IPARM(16) = –1, your factorization did not complete successfully, resulting in computational error 2126.
  If IPARM(16) > 0, it is the row number *k*, in which the maximum absolute value of the ratio $a_{kk}/d_{kk}$ for Gaussian elimination and $a_{kk}/r_{kk}$ for Cholesky decomposition occurred, where:
  If IPARM(3) = 0, *k* can be any of the rows, 1 through *n*, in the full factorization.
  If IPARM(3) > 0, *k* can be any of the rows, IPARM(3)+1 through *n*, in the partial factorization.

  If you are doing a **solve only**, this argument is not used in the computation and is unchanged.

- IPARM(17) through IPARM(20) are reserved.
- IPARM(21) through IPARM(25) have the following meaning, where:

  If you are doing a **factor and solve** or a **factor only**, IPARM(21) through IPARM(25) have the following meanings for each region *i* for *i* = 1,5, respectively:

  If IPARM(20+*i*) = –1, your factorization did not complete successfully, resulting in computational error 2126.
  If IPARM(20+*i*) ≥ 0, it is the number of pivots in region *i* for the columns that were factored in matrix ***A***, where:
  If IPARM(3) = 0, columns 1 through *n* were factored in the full factorization.

If IPARM(3) > 0, columns IPARM(3)+1 through *n* were factored in the partial factorization.

If you are doing a **solve only**, these arguments are not used in the computation and are unchanged.

Returned as: a one-dimensional array of (at least) length 25, containing fullword integers.

*rparm*

is an array of parameters, RPARM(*i*), where:

- RPARM(1) through RPARM(15) are unchanged.
- RPARM(16) has the following meaning, where:

  If you are doing a **factor and solve** or a **factor only**, and:

  If RPARM(16) = 0.0, your factorization did not complete successfully, resulting in computational error 2126.
  If |RPARM(16)| > 0.0, it is the ratio for row *k*, $a_{kk}/d_{kk}$ for Gaussian elimination and $a_{kk}/r_{kk}$ for Cholesky decomposition, having the maximum absolute value. Row *k* is indicated in IPARM(16), and:
  If IPARM(3) = 0, the ratio corresponds to one of the rows, 1 through *n*, in the full factorization.
  If IPARM(3) > 0, the ratio corresponds to one of the rows, IPARM(3)+1 through *n*, in the partial factorization.

  If you are doing a **solve only**, this argument is not used in the computation and is unchanged.

- RPARM(17) and RPARM(18) have the following meaning, where:

  If you are **computing the determinant** of matrix ***A***, then RPARM(17) is the mantissa, *detbas*, and RPARM(18) is the power of 10, *detpwr*, used to express the value of the determinant: $detbas(10^{detpwr})$, where $1 \leq detbas < 10$. Also:

  If IPARM(3) = 0, the determinant is computed for columns 1 through *n* in the full factorization.
  If IPARM(3) > 0, the determinant is computed for columns IPARM(3)+1 through *n* in the partial factorization.

  If you are **not computing the determinant** of matrix ***A***, these arguments are not used in the computation and are unchanged.

- RPARM(19) through RPARM(25) are reserved.

Returned as: a one-dimensional array of (at least) length 25, containing long-precision real numbers.

*bx*

has the following meaning, where:

If you are doing a **factor and solve** or a **solve only**, *bx* is the array, containing the *mbx* solution vectors ***x*** of the system ***Ax*** = ***b***. Each vector ***x*** is length *n* and is stored in the corresponding column of the array. (If *mbx* = 0, then *bx* is unchanged on output.)

If you are doing a **factor only**, this argument is not used in the computation and is unchanged.

Returned as: an *ldbx* by (at least) *mbx* array, containing long-precision real numbers.

***Notes***

1. When doing a **solve only**, you should specify the same factorization method in IPARM(2), Gaussian elimination or Cholesky decomposition, that you specified for your factorization on a previous call to this subroutine.
2. If you set either IPARM(1) = 0 or IPARM(10) = 0, indicating you want to use the default values for IPARM(11) through IPARM(15) and RPARM(10), then:
   - Matrix ***A*** must be positive definite.
   - No pivots are fixed, using RPARM(11) through RPARM(15) values.
   - No small pivots are tolerated; that is, the value should be |*pivot*| > RPARM(10).
3. Many of the input and output parameters for *iparm* and *rparm* are defined for the five pivot regions handled by this subroutine. The limits of the regions are based on RPARM(10), as shown in Figure 12. The pivot values in each region are:

   Region 1: *pivot* < −RPARM(10)
   Region 2: −RPARM(10) ≤ *pivot* < 0
   Region 3: *pivot* = 0
   Region 4: 0 < *pivot* ≤ RPARM(10)
   Region 5: *pivot* > RPARM(10)

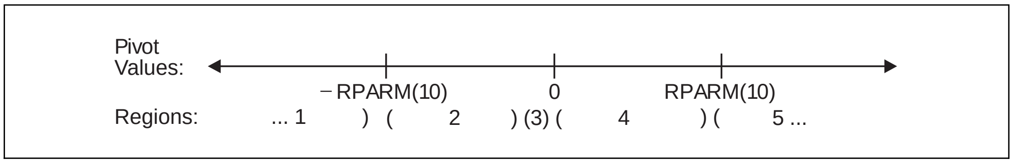


*Figure 12. Five Pivot Regions*

4. The IPARM(4) and IPARM(5) arguments allow you to specify the same or different skyline storage modes for your input and output arrays for matrix ***A***. This allows you to change storage modes as needed. However, if you are concerned with performance, you should use diagonal-out skyline storage mode for both input and output, if possible, because there is less overhead.

   For a description of how sparse matrices are stored in skyline storage mode, see “Profile-In Skyline Storage Mode” on page 103 and “Diagonal-Out Skyline Storage Mode” on page 101. Those descriptions use different array and variable names from the ones used here. To relate the two sets, use the following table:

| **Name Here** | **Name in the Storage Description** |
|---|---|
| A | AU |
| *na* | *nu* |
| IDIAG | IDU |

5. Following is an illustration of the portion of matrix ***A*** factored in the partial factorization when `IPARM(3) > 0`. In this case, the subroutine assumes that rows and columns 1 through `IPARM(3)` are already factored and that rows and columns `IPARM(3)+1` through *n* are to be factored in this computation.

$$\begin{array}{c|cccccccccc|}
 & \multicolumn{5}{c}{|\leftarrow \text{factored} \rightarrow|} & \multicolumn{5}{c}{\leftarrow \text{to be factored} \rightarrow|} \\
\overline{\uparrow} & a_{11} & \cdot & \cdot & \cdot & a_{1j} & a_{1,j+1} & \cdot & \cdot & \cdot & a_{1n} \\
\text{factored} & \cdot & \cdot & & & \cdot & & & & & \cdot \\
 & \cdot & & \cdot & & \cdot & & & & & \cdot \\
\underline{\downarrow} & \cdot & & & \cdot & \cdot & & & & & \cdot \\
\overline{\uparrow} & a_{1j} & \cdot & \cdot & \cdot & a_{jj} & & & & & \cdot \\
 & a_{1,j+1} & & & & & & & & & \cdot \\
\text{to be} & \cdot & & & & & & \cdot & & & \cdot \\
\text{factored} & \cdot & & & & & & & \cdot & & \cdot \\
 & \cdot & & & & & & & & \cdot & \cdot \\
\underline{\downarrow} & a_{1n} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{nn}
\end{array}
\qquad \text{where } j = \text{IPARM}(3)$$

   You use the partial factorization function when, for design or storage reasons, you must factor the matrix ***A*** in stages. When doing a partial factorization, you must use the same skyline storage mode for all parts of the matrix as it is progressively factored.

6. Your various arrays must have no common elements; otherwise, results are unpredictable.

7. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see "Using Auxiliary Storage in ESSL" on page 32.

***Function:*** This subroutine can factor, compute the determinant of, and solve symmetric sparse matrix ***A***, stored in skyline storage mode. It can use either Gaussian elimination or Cholesky decomposition. For all computations, input matrix ***A*** can be stored in either diagonal-out or profile-in skyline storage mode. Output matrix ***A*** can also be stored in either of these modes and can be different from the mode used for input.

For Gaussian elimination, matrix ***A*** is factored into the following form using specified pivot processing:

$$\boldsymbol{A} = \boldsymbol{LDL}^{\mathrm{T}}$$

where:

***D*** is a diagonal matrix.
***L*** is a lower triangular matrix.

The transformed matrix ***A***, factored into its ***LDL***$^{\mathrm{T}}$ form, is stored in packed format in array `A`, such that the inverse of the diagonal matrix ***D*** is stored in the corresponding elements of array `A`. The off-diagonal elements of the unit upper triangular matrix ***L***$^{\mathrm{T}}$ are stored in the corresponding off-diagonal elements of array `A`.

For Cholesky decomposition, matrix $\boldsymbol{A}$ is factored into the following form using specified pivot processing:

$$\boldsymbol{A} = \boldsymbol{R}^{T}\boldsymbol{R}$$

where $\boldsymbol{R}$ is an upper triangular matrix

The transformed matrix $\boldsymbol{A}$, factored into its $\boldsymbol{R}^{T}\boldsymbol{R}$ form, is stored in packed format in array A, such that the inverse of the diagonal elements of the upper triangular matrix $\boldsymbol{R}$ is stored in the corresponding elements of array A. The off-diagonal elements of matrix $\boldsymbol{R}$ are stored in the corresponding off-diagonal elements of array A.

The partial factorization of matrix $\boldsymbol{A}$, which you can do when you specify the factor-only option, assumes that the first IPARM(3) rows and columns are already factored in the input matrix. It factors the remaining *n*–IPARM(3) rows and columns in matrix $\boldsymbol{A}$. (See “Notes” on page 621 for an illustration.) It updates only the elements in array A corresponding to the part of matrix $\boldsymbol{A}$ that is factored.

The determinant can be computed with any of the factorization computations. With a full factorization, you get the determinant for the whole matrix. With a partial factorization, you get the determinant for only that part of the matrix factored in this computation.

The system $\boldsymbol{Ax} = \boldsymbol{b}$, having multiple right-hand sides, is solved for $\boldsymbol{x}$ using the transformed matrix $\boldsymbol{A}$ produced by this call or a subsequent call to this subroutine.

See references [9 ], [12 ], [25 ], [47 ], [68]. If *n* is 0, no computation is performed. If *mbx* is 0, no solve is performed.

### *Error Conditions*

*Resource Errors*

- Error 2015 is unrecoverable, *naux* = 0, and unable to allocate work area.
- Unable to allocate internal work area.

*Computational Errors*

1. If a pivot occurs in region *i* for *i* = 1,5 and IPARM(10+*i*) = 1, the pivot value is replaced with RPARM(10+*i*), an attention message is issued, and processing continues.
2. Unacceptable pivot values occurred in the factorization of matrix $\boldsymbol{A}$.
   - One or more diagonal elements of $\boldsymbol{D}$ or $\boldsymbol{R}$ contains unacceptable pivots and no valid fixup is applicable. The row number *i* of the first unacceptable pivot element is identified in the computational error message.
   - The return code is set to 2.
   - *i* can be determined at run time by use of the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error code 2126 in the ESSL error option table; otherwise, the default value causes your program to terminate when this error occurs. For details, see “What Can You Do about ESSL Computational Errors?” on page 48.

*Input-Argument Errors*

1. $n < 0$
2. $na < 0$
3. IDIAG($n$+1) > $na$+1
4. IDIAG($i$+1) ≤ IDIAG($i$) for $i = 1, n$
5. IDIAG($i$+1) > IDIAG($i$)+$i$ and IPARM(4) = 0 for $i = 1, n$
6. IDIAG($i$) > IDIAG($i$–1)+$i$ and IPARM(4) = 1 for $i = 2, n$
7. IPARM(1) ≠ 0 or 1
8. IPARM(2) ≠ 0, 1, 2, 10, 11, 100, 101, 102, 110, or 111
9. IPARM(3) < 0
10. IPARM(3) > $n$
11. IPARM(3) > 0 and IPARM(2) ≠ 1, 11, 101, or 111
12. IPARM(4), IPARM(5) ≠ 0 or 1
13. IPARM(2) = 0, 1, 10, or 11 and:
    IPARM(10) ≠ 0 or 1
    IPARM(11), IPARM(12) ≠ –1, 0, or 1
    IPARM(13) ≠ –1 or 1
    IPARM(14), IPARM(15) ≠ –1, 0, or 1
    RPARM(10) < 0.0
    RPARM(10+$i$) = 0.0 and IPARM(10+$i$) = 1 for $i$ = 1,5
14. IPARM(2) = 100, 101, 110, or 111 and:
    IPARM(10) ≠ 0 or 1
    IPARM(11), IPARM(12), IPARM(13) ≠ –1 or 1
    IPARM(14), IPARM(15) ≠ –1, 0, or 1
    RPARM(10) < 0.0
    RPARM(10+$i$) ≤ 0.0 and IPARM(10+$i$) = 1 for $i$ = 1,5
15. IPARM(2) = 0, 2, 10, 100, 102, or 110 and:
    $ldbx \le 0$ and $mbx \ne 0$ and $n \ne 0$
    $ldbx < 0$ and $mbx = 0$
    $ldbx < n$ and $mbx \ne 0$
    $mbx < 0$
16. Error 2015 is recoverable or $naux$≠0, and $naux$ is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example 1:*** This example shows how to factor a 9 by 9 symmetric sparse matrix ***A*** and solve the system ***Ax*** = ***b*** with three right-hand sides. It uses Gaussian elimination. The default values are used for IPARM and RPARM. Input matrix ***A***, shown here, is stored in diagonal-out skyline storage mode. Matrix ***A*** is:

$$\begin{bmatrix}
1.0 & 1.0 & 1.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
1.0 & 2.0 & 2.0 & 2.0 & 1.0 & 1.0 & 0.0 & 1.0 & 0.0 \\
1.0 & 2.0 & 3.0 & 3.0 & 2.0 & 2.0 & 0.0 & 2.0 & 0.0 \\
1.0 & 2.0 & 3.0 & 4.0 & 3.0 & 3.0 & 0.0 & 3.0 & 0.0 \\
0.0 & 1.0 & 2.0 & 3.0 & 4.0 & 4.0 & 1.0 & 4.0 & 0.0 \\
0.0 & 1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 2.0 & 5.0 & 1.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 2.0 & 3.0 & 3.0 & 2.0 \\
0.0 & 1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 3.0 & 7.0 & 3.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 2.0 & 3.0 & 4.0
\end{bmatrix}$$

Output matrix ***A***, shown here, is in ***LDL***$^{T}$ factored form with ***D***$^{-1}$ on the diagonal, and is stored in diagonal-out skyline storage mode. Matrix ***A*** is:

```
┌                                             ┐
│ 1.0  1.0  1.0  1.0  0.0  0.0  0.0  0.0  0.0 │
│ 1.0  1.0  1.0  1.0  1.0  1.0  0.0  1.0  0.0 │
│ 1.0  1.0  1.0  1.0  1.0  1.0  0.0  1.0  0.0 │
│ 1.0  1.0  1.0  1.0  1.0  1.0  0.0  1.0  0.0 │
│ 0.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  0.0 │
│ 0.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0 │
│ 0.0  0.0  0.0  0.0  1.0  1.0  1.0  1.0  1.0 │
│ 0.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0 │
│ 0.0  0.0  0.0  0.0  0.0  1.0  1.0  1.0  1.0 │
└                                             ┘
```

*Call Statement and Input*

```
           N  A  NA  IDIAG  IPARM  RPARM  AUX NAUX  BX  LDBX MBX
           |  |  |     |      |      |     |   |    |    |   |
CALL DSKFS( 9, A, 33, IDIAG, IPARM, RPARM, AUX, 39 , BX , 12 , 3 )
```

```
A        =  (1.0, 2.0, 1.0, 3.0, 2.0, 1.0, 4.0, 3.0, 2.0, 1.0, 4.0,
             3.0, 2.0, 1.0, 5.0, 4.0, 3.0, 2.0, 1.0, 3.0, 2.0, 1.0,
             7.0, 3.0, 5.0, 4.0, 3.0, 2.0, 1.0, 4.0, 3.0, 2.0, 1.0)
IDIAG    =  (1, 2, 4, 7, 11, 15, 20, 23, 30, 34)
IPARM    =  (0, . , . , . , . , . , . , . , . , . , . , . , . , . ,
             . , . , . , . , . , . , . , . , . , . , . , . )

RPARM    =(not relevant)
```

```
          ┌                    ┐
          │  4.00   8.00  12.00 │
          │ 10.00  20.00  30.00 │
          │ 15.00  30.00  45.00 │
          │ 19.00  38.00  57.00 │
          │ 19.00  38.00  57.00 │
BX    =   │ 23.00  46.00  69.00 │
          │ 11.00  22.00  33.00 │
          │ 28.00  56.00  84.00 │
          │ 10.00  20.00  30.00 │
          │   .      .      .   │
          │   .      .      .   │
          │   .      .      .   │
          └                    ┘
```

*Output*

```
A        =  (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0,
             1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0,
             1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0)
IDIAG    =(same as input)
IPARM    =  (0, . , . , . , . , . , . , . , . , . , . , . , . , . ,
             . , 8, . , . , . , . , . , 0, 0, 0, 0, 9)
RPARM    =  ( . , . , . , . , . , . , . , . , . , . , . , . , . , . ,
             . , 7.0, . , . , . , . , . , . , . , . , . )
```

```
         ┌                ┐
         │ 1.00  2.00  3.00 │
         │ 1.00  2.00  3.00 │
         │ 1.00  2.00  3.00 │
         │ 1.00  2.00  3.00 │
         │ 1.00  2.00  3.00 │
BX   =   │ 1.00  2.00  3.00 │
         │ 1.00  2.00  3.00 │
         │ 1.00  2.00  3.00 │
         │ 1.00  2.00  3.00 │
         │  .     .     .   │
         │  .     .     .   │
         │  .     .     .   │
         └                ┘
```

***Example 2:*** This example shows how to factor the 9 by 9 symmetric sparse matrix ***A*** from Example 1, solve the system ***Ax*** = ***b*** with three right-hand sides, and compute the determinant of ***A***. It uses Gaussian elimination. The default values for pivot processing are used for IPARM. Input matrix ***A*** is stored in profile-in skyline storage mode. Output matrix ***A*** is in ***LDL***$^{T}$ factored form with ***D***$^{-1}$ on the diagonal, and is stored in diagonal-out skyline storage mode. It is the same as output matrix ***A*** in Example 1.

*Call Statement and Input*

```
           N  A  NA  IDIAG  IPARM  RPARM  AUX NAUX  BX  LDBX MBX
           |  |  |     |      |      |     |   |     |    |   |
CALL DSKFS( 9, A, 33, IDIAG, IPARM, RPARM, AUX, 39 , BX , 12 , 3 )
```

```
A        =  (1.0, 1.0, 2.0, 1.0, 2.0, 3.0, 1.0, 2.0, 3.0, 4.0, 1.0,
             2.0, 3.0, 4.0, 1.0, 2.0, 3.0, 4.0, 5.0, 1.0, 2.0, 3.0,
             1.0, 2.0, 3.0, 4.0, 5.0, 3.0, 7.0, 1.0, 2.0, 3.0, 4.0)
IDIAG    =  (1, 3, 6, 10, 14, 19, 22, 29, 33, 34)
IPARM    =  (1, 10, 0, 1, 0, . , . , . , . , . , 0, . , . , . , . , . ,
             . , . , . , . , . , . , . , . , . , . , . )
```

RPARM =(not relevant)

```
         ┌                  ┐
         │  4.00   8.00  12.00 │
         │ 10.00  20.00  30.00 │
         │ 15.00  30.00  45.00 │
         │ 19.00  38.00  57.00 │
         │ 19.00  38.00  57.00 │
BX   =   │ 23.00  46.00  69.00 │
         │ 11.00  22.00  33.00 │
         │ 28.00  56.00  84.00 │
         │ 10.00  20.00  30.00 │
         │   .      .      .   │
         │   .      .      .   │
         │   .      .      .   │
         └                  ┘
```

*Output*

A =(same as output A in Example 1)

IDIAG =(same as input IDIAG in Example 1)

```
IPARM    =  (1, 10, 0, 1, 0, . , . , . , . , . , 0, . , . , . , . , . , 8,
                . , . , . , . , 0, 0, 0, 0, 9)
RPARM      =   ( . , . , . , . , . , . , . , . , . , . , . , . , . , . ,
                . , 7.0, 1.0, 0.0, . , . , . , . , . , . , . )
BX         =(same as output BX in Example 1)
```

***Example 3:*** This example shows how to factor a 9 by 9 negative-definite symmetric sparse matrix ***A***, solve the system ***Ax*** = ***b*** with three right-hand sides, and compute the determinant of ***A***. It uses Gaussian elimination. (Default values for pivot processing are not used for IPARM because ***A*** is negative-definite.) Input matrix ***A***, shown here, is stored in diagonal-out skyline storage mode. Matrix ***A*** is:

```
┌                                                        ┐
| -1.0  -1.0  -1.0  -1.0   0.0   0.0   0.0   0.0   0.0 |
| -1.0  -2.0  -2.0  -2.0  -1.0  -1.0   0.0  -1.0   0.0 |
| -1.0  -2.0  -3.0  -3.0  -2.0  -2.0   0.0  -2.0   0.0 |
| -1.0  -2.0  -3.0  -4.0  -3.0  -3.0   0.0  -3.0   0.0 |
|  0.0  -1.0  -2.0  -3.0  -4.0  -4.0  -1.0  -4.0   0.0 |
|  0.0  -1.0  -2.0  -3.0  -4.0  -5.0  -2.0  -5.0  -1.0 |
|  0.0   0.0   0.0   0.0  -1.0  -2.0  -3.0  -3.0  -2.0 |
|  0.0  -1.0  -2.0  -3.0  -4.0  -5.0  -3.0  -7.0  -3.0 |
|  0.0   0.0   0.0   0.0   0.0  -1.0  -2.0  -3.0  -4.0 |
└                                                        ┘
```

Output matrix ***A***, shown here, is in ***LDL***$^{T}$ factored form with ***D***$^{-1}$ on the diagonal, and is stored in diagonal-out skyline storage mode. Matrix ***A*** is:

```
┌                                                        ┐
| -1.0   1.0   1.0   1.0   0.0   0.0   0.0   0.0   0.0 |
|  1.0  -1.0   1.0   1.0   1.0   1.0   0.0   1.0   0.0 |
|  1.0   1.0  -1.0   1.0   1.0   1.0   0.0   1.0   0.0 |
|  1.0   1.0   1.0  -1.0   1.0   1.0   0.0   1.0   0.0 |
|  0.0   1.0   1.0   1.0  -1.0   1.0   1.0   1.0   0.0 |
|  0.0   1.0   1.0   1.0   1.0  -1.0   1.0   1.0   1.0 |
|  0.0   0.0   0.0   0.0   1.0   1.0  -1.0   1.0   1.0 |
|  0.0   1.0   1.0   1.0   1.0   1.0   1.0  -1.0   1.0 |
|  0.0   0.0   0.0   0.0   0.0   1.0   1.0   1.0  -1.0 |
└                                                        ┘
```

*Call Statement and Input*

```
          N  A  NA  IDIAG  IPARM  RPARM  AUX  NAUX  BX  LDBX MBX
          |  |  |     |      |      |     |    |    |    |   |
CALL DSKFS(9, A, 33, IDIAG, IPARM, RPARM, AUX,  39 , BX , 12 , 3 )

A          =  (-1.0, -2.0, -1.0, -3.0, -2.0, -1.0, -4.0, -3.0, -2.0,
                -1.0, -4.0, -3.0, -2.0, -1.0, -5.0, -4.0, -3.0, -2.0,
                -1.0, -3.0, -2.0, -1.0, -7.0, -3.0, -5.0, -4.0, -3.0,
                -2.0, -1.0, -4.0, -3.0, -2.0, -1.0)
IDIAG      =  (1, 2, 4, 7, 11, 15, 20, 23, 30, 34)
IPARM      =  (1, 10, 0, 0, 0, . , . , . , . , 1, 0, -1, -1, -1, -1, . ,
                . , . , . , . , . , . , . , . , . )

RPARM      =  ( . , . , . , . , . , . , . , . , . , $10^{-15}$, . , . ,
                 . , . , . , . , . , . , . , . , . , . , . , . , . )
BX         =(same as input BX in Example 1)
```

*Output:*

```
A        =  (-1.0, -1.0, 1.0, -1.0, 1.0, 1.0, -1.0, 1.0, 1.0, 1.0,
             -1.0, 1.0, 1.0, 1.0, -1.0, 1.0, 1.0, 1.0, 1.0, -1.0, 1.0,
             1.0, -1.0 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, -1.0, 1.0, 1.0,
             1.0)
IDIAG    =(same as input)
IPARM    =  (1, 10, 0, 0, 0, . , . , . , . , . , 1, 0, -1, -1, -1, -1, 8,
             . , . , . , . , . , 9, 0, 0, 0, 0)
RPARM    =  ( . , . , . , . , . , . , . , . , . , . , . ,10$^{-15}$, . , . , . ,
              . , . , . , 7.0, -1.0, 0.0, . , . , . , . , . , . , . )
```

$$
BX = \begin{bmatrix}
-1.00 & -2.00 & -3.00 \\
-1.00 & -2.00 & -3.00 \\
-1.00 & -2.00 & -3.00 \\
-1.00 & -2.00 & -3.00 \\
-1.00 & -2.00 & -3.00 \\
-1.00 & -2.00 & -3.00 \\
-1.00 & -2.00 & -3.00 \\
-1.00 & -2.00 & -3.00 \\
-1.00 & -2.00 & -3.00 \\
. & . & . \\
. & . & . \\
. & . & .
\end{bmatrix}
$$

***Example 4:*** This example shows how to factor the first six rows and columns, referred to as matrix ***A1***, of the 9 by 9 symmetric sparse matrix ***A*** from Example 1 and compute the determinant of ***A1***. It uses Gaussian elimination. Input matrix ***A1***, shown here, is stored in diagonal-out skyline storage mode. Input matrix ***A1*** is:

$$
\begin{bmatrix}
1.0 & 1.0 & 1.0 & 1.0 & 0.0 & 0.0 \\
1.0 & 2.0 & 2.0 & 2.0 & 1.0 & 1.0 \\
1.0 & 2.0 & 3.0 & 3.0 & 2.0 & 2.0 \\
1.0 & 2.0 & 3.0 & 4.0 & 3.0 & 3.0 \\
0.0 & 1.0 & 2.0 & 3.0 & 4.0 & 4.0 \\
0.0 & 1.0 & 2.0 & 3.0 & 4.0 & 5.0
\end{bmatrix}
$$

Output matrix ***A1***, shown here, is in ***LDL***$^{T}$ factored form with ***D***$^{-1}$ on the diagonal, and is stored in diagonal-out skyline storage mode. Output matrix ***A1*** is:

$$
\begin{bmatrix}
1.0 & 1.0 & 1.0 & 1.0 & 0.0 & 0.0 \\
1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\
1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\
1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\
0.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\
0.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0
\end{bmatrix}
$$

*Call Statement and Input*

```
             N   A   NA   IDIAG   IPARM   RPARM   AUX  NAUX  BX   LDBX   MBX
             |   |   |     |       |       |       |    |    |     |      |
CALL DSKFS (6 , A , 33 , IDIAG , IPARM , RPARM , AUX , 27 , BX , LDBX , MBX )
```

```
A        =(same as input A in Example 1)
IDIAG    =  (1, 2, 4, 7, 11, 15, 20)
IPARM    =  (1, 11, 0, 0, 0, . , . , . , . , 0, . , . , . , . , . ,
             . , . , . , . , . , . , . , . , . , . )
RPARM    =(not relevant)
BX       =(not relevant)
LDBX     =(not relevant)
MBX      =(not relevant)
```

*Output*

```
A        =  (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0,
             1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 3.0, 2.0, 1.0,
             7.0, 3.0, 5.0, 4.0, 3.0, 2.0, 1.0, 4.0, 3.0, 2.0, 1.0)
IDIAG    =(same as input)
IPARM    =  (1, 11, 0, 0, 0, . , . , . , . , 0, . , . , . , . , . , 6,
             . , . , . , . , 0, 0, 0, 0, 6)
RPARM    =  ( . , . , . , . , . , . , . , . , . , . , . , . , . , . ,
             . , 5.0, 1.0, 0.0, . , . , . , . , . , . , . )
BX       =(same as input)
LDBX     =(same as input)
MBX      =(same as input)
```

***Example 5:*** This example shows how to do a partial factorization of the 9 by 9 symmetric sparse matrix ***A*** from Example 1, where the first six rows and columns were factored in Example 4. It factors the remaining three rows and columns and computes the determinant of that part of the matrix. It uses Gaussian elimination. The input matrix, referred to as ***A2***, shown here, is made up of the output factored matrix ***A1*** plus the three remaining unfactored rows and columns of matrix ***A*** Matrix ***A2*** is:

```
┌                                             ┐
| 1.0  1.0  1.0  1.0  0.0  0.0  0.0  0.0  0.0 |
| 1.0  1.0  1.0  1.0  1.0  1.0  0.0  1.0  0.0 |
| 1.0  1.0  1.0  1.0  1.0  1.0  0.0  2.0  0.0 |
| 1.0  1.0  1.0  1.0  1.0  1.0  0.0  3.0  0.0 |
| 0.0  1.0  1.0  1.0  1.0  1.0  1.0  4.0  0.0 |
| 0.0  1.0  1.0  1.0  1.0  1.0  2.0  5.0  1.0 |
| 0.0  0.0  0.0  0.0  1.0  2.0  3.0  3.0  2.0 |
| 0.0  1.0  2.0  3.0  4.0  5.0  3.0  7.0  3.0 |
| 0.0  0.0  0.0  0.0  0.0  1.0  2.0  3.0  4.0 |
└                                             ┘
```

Both parts of input matrix ***A2*** are stored in diagonal-out skyline storage mode.

Output matrix ***A2*** is the same as output matrix ***A*** in Example 1 and is stored in diagonal-out skyline storage mode.

*Call Statement and Input*

```
             N   A   NA   IDIAG   IPARM   RPARM   AUX   NAUX  BX   LDBX   MBX
             |   |   |     |       |       |       |     |    |     |      |
CALL DSKFS (9 , A , 33 , IDIAG , IPARM , RPARM , AUX ,  27 , BX , LDBX , MBX )
```

```
A        =(same as output A in Example 4)
IDIAG    =(same as input IDIAG in Example 1)
```

```
IPARM    =  (1, 11, 6, 0, 0, . , . , . , . , . , 0, . , . , . , . , . ,
             . , . , . , . , . , . , . , . , . , . )
RPARM    =(not relevant)
BX       =(not relevant)
LDBX     =(not relevant)
MBX      =(not relevant)
```

*Output*

```
A        =(same as output A in Example 1)

IDIAG    =(same as output IDIAG in Example 1)
IPARM    =  (1, 11, 6, 0, 0, . , . , . , . , . , 0, . , . , . , . , . , 8,
             . , . , . , . , 0, 0, 0, 0, 3)
RPARM    =  ( . , . , . , . , . , . , . , . , . , . , . , . , . , . , . ,
             . , 7.0, 1.0, 0.0, . , . , . , . , . , . , . )
BX       =(same as input)
LDBX     =(same as input)
MBX      =(same as input)
```

***Example 6:*** This example shows how to solve the system ***Ax*** = ***b*** with one right-hand side for a symmetric sparse matrix ***A***. Input matrix ***A***, used here, is the same as factored output matrix ***A*** from Example 1, stored in profile-in skyline storage mode. It specifies Gaussian elimination, as used in Example 1. Here, output matrix ***A*** is unchanged on output and is stored in profile-in skyline storage mode.

*Call Statement and Input*

```
            N  A  NA  IDIAG  IPARM  RPARM  AUX NAUX  BX  LDBX MBX
            |  |  |     |      |      |     |   |    |    |    |
CALL DSKFS (9, A, 33, IDIAG, IPARM, RPARM, AUX, 31 , BX , 9 ,  1 )

A        =  (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0,
             1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0,
             1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0)
IDIAG    =  (1, 3, 6, 10, 14, 19, 22, 29, 33, 34)
IPARM    =  (1, 2, 0, 1, 1, . , . , . , . , . , . , . , . , . , . , . ,
             . , . , . , . , . , . , . , . , . , . )
RPARM    =(not relevant)
BX       =  (10.0, 38.0, 64.0, 87.0, 103.0, 133.0, 80.0, 174.0, 80.0)
```

*Output*

```
A        =(same as input)
IDIAG    =(same as input)
IPARM    =(same as input)
APARM    =(same as input)
BX       =  (1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0)
```

***Example 7:*** This example shows how to factor a 9 by 9 symmetric sparse matrix ***A*** and solve the system ***Ax*** = ***b*** with four right-hand sides. It uses Cholesky decomposition. Input matrix ***A***, shown here, is stored in profile-in skyline storage mode Matrix ***A*** is:

```
┌                                                ┐
│ 1.0  1.0   1.0   0.0   1.0   0.0   0.0   0.0   1.0 │
│ 1.0  5.0   3.0   0.0   3.0   0.0   0.0   0.0   3.0 │
│ 1.0  3.0  11.0   3.0   5.0   3.0   3.0   0.0   5.0 │
│ 0.0  0.0   3.0  17.0   5.0   5.0   5.0   0.0   5.0 │
│ 1.0  3.0   5.0   5.0  29.0   7.0   7.0   0.0   9.0 │
│ 0.0  0.0   3.0   5.0   7.0  39.0   9.0   6.0   9.0 │
│ 0.0  0.0   3.0   5.0   7.0   9.0  53.0   8.0  11.0 │
│ 0.0  0.0   0.0   0.0   0.0   6.0   8.0  66.0  10.0 │
│ 1.0  3.0   5.0   5.0   9.0   9.0  11.0  10.0  89.0 │
└                                                ┘
```

Output matrix ***A***, shown here, is in ***R***$^T$***R*** factored form with the inverse of the diagonal of ***R*** on the diagonal, and is stored in profile-in skyline storage mode. Matrix ***A*** is:

```
┌                                                ┐
│ 1.0  1.0   1.0  0.0  1.0   0.0   0.0   0.0   1.0 │
│ 1.0   .5   1.0  0.0  1.0   0.0   0.0   0.0   1.0 │
│ 1.0  1.0  .333  1.0  1.0   1.0   1.0   0.0   1.0 │
│ 0.0  0.0   1.0  .25  1.0   1.0   1.0   0.0   1.0 │
│ 1.0  1.0   1.0  1.0   .2   1.0   1.0   0.0   1.0 │
│ 0.0  0.0   1.0  1.0  1.0  .167   1.0   1.0   1.0 │
│ 0.0  0.0   1.0  1.0  1.0   1.0  .143   1.0   1.0 │
│ 0.0  0.0   0.0  0.0  0.0   1.0   1.0  .125   1.0 │
│ 1.0  1.0   1.0  1.0  1.0   1.0   1.0   1.0  .111 │
└                                                ┘
```

*Call Statement and Input*

```
            N  A  NA  IDIAG  IPARM  RPARM  AUX NAUX  BX  LDBX MBX
            |  |  |     |      |      |     |   |    |    |    |
CALL DSKFS( 9, A, 34, IDIAG, IPARM, RPARM, AUX, 43 , BX , 10 , 4 )

A        =  (1.0, 1.0, 5.0, 1.0, 3.0, 11.0, 3.0, 17.0, 1.0, 3.0, 5.0,
             5.0, 29.0, 3.0, 5.0, 7.0, 39.0, 3.0, 5.0, 7.0, 9.0, 53.0,
             6.0, 8.0, 66.0, 1.0, 3.0, 5.0, 5.0, 9.0, 9.0, 11.0, 10.0,
             89.0)
IDIAG    =  (1, 3, 6, 8, 13, 17, 22, 25, 34, 35)
IPARM    =  (1, 110, 0, 1, 1, . , . , . , . , . , . , 0, . , . , . , . , . , . , . ,
             . , . , . , . , . , . , . , . , . , . , . , . , . , . , . , . , . , . , . )

RPARM    =(not relevant)

            ┌                                ┐
            │   5.00   10.00   15.00   20.00 │
            │  15.00   30.00   45.00   60.00 │
            │  34.00   68.00  102.00  136.00 │
            │  40.00   80.00  120.00  160.00 │
BX    =     │  66.00  132.00  198.00  264.00 │
            │  78.00  156.00  234.00  312.00 │
            │  96.00  192.00  288.00  384.00 │
            │  90.00  180.00  270.00  360.00 │
            │ 142.00  284.00  426.00  568.00 │
            │    .       .       .       .   │
            └                                ┘
```

*Output*

```
A        =  (1.0, 1.0, .5, 1.0, 1.0, .333, 1.0, .25, 1.0, 1.0, 1.0,
             1.0, .2, 1.0, 1.0, 1.0, .167, 1.0, 1.0, 1.0, 1.0, .143,
             1.0, 1.0, .125, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0,
             .111)
IDIAG    =(same as input)
IPARM    =  (1, 110, 0, 1, 1, . , . , . , . , . , 0, . , . , . , . , . ,
             9, . , . , . , . , 0, 0, 0, 0, 9)
RPARM    =  ( . , . , . , . , . , . , . , . , . , . , . , . , . , . ,
             . , 9.89, 1.32, 11.0, . , . , . , . , . , . , . )

          ┌                        ┐
          | 1.00  2.00  3.00  4.00 |
          | 1.00  2.00  3.00  4.00 |
          | 1.00  2.00  3.00  4.00 |
          | 1.00  2.00  3.00  4.00 |
BX   =    | 1.00  2.00  3.00  4.00 |
          | 1.00  2.00  3.00  4.00 |
          | 1.00  2.00  3.00  4.00 |
          | 1.00  2.00  3.00  4.00 |
          | 1.00  2.00  3.00  4.00 |
          |  .     .     .     .   |
          └                        ┘
```

# DSRIS—Iterative Linear System Solver for a General or Symmetric Sparse Matrix Stored by Rows

This subroutine solves a general or symmetric sparse linear system of equations, using an iterative algorithm, with or without preconditioning. The methods include conjugate gradient (CG), conjugate gradient squared (CGS), generalized minimum residual (GMRES), more smoothly converging variant of the CGS method (Bi-CGSTAB), or transpose-free quasi-minimal residual method (TFQMR). The preconditioners include an incomplete LU factorization, an incomplete Cholesky factorization (for positive definite symmetric matrices), diagonal scaling, or symmetric successive over-relaxation (SSOR) with two possible choices for the diagonal matrix: one uses the absolute values sum of the input matrix, and the other uses the diagonal obtained from the LU factorization. The sparse matrix is stored using storage-by-rows for general matrices and upper- or lower-storage-by-rows for symmetric matrices. Matrix ***A*** and vectors ***x*** and ***b*** are used:

$$\boldsymbol{Ax} = \boldsymbol{b}$$

where ***A***, ***x***, and ***b*** contain long-precision real numbers.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL DSRIS (*stor*, *init*, *n*, *ar*, *ja*, *ia*, *b*, *x*, *iparm*, *rparm*, *aux1*, *naux1*, *aux2*, *naux2*) |
| **C and C++** | dsris (*stor*, *init*, *n*, *ar*, *ja*, *ia*, *b*, *x*, *iparm*, *rparm*, *aux1*, *naux1*, *aux2*, *naux2*); |
| **PL/I** | CALL DSRIS (*stor*, *init*, *n*, *ar*, *ja*, *ia*, *b*, *x*, *iparm*, *rparm*, *aux1*, *naux1*, *aux2*, *naux2*); |

*On Entry*

*stor*

indicates the form of sparse matrix ***A*** and the storage mode used, where:

If *stor* = 'G', ***A*** is a general sparse matrix, stored using storage-by-rows.

If *stor* = 'U', ***A*** is a symmetric sparse matrix, stored using upper-storage-by-rows.

If *stor* = 'L', ***A*** is a symmetric sparse matrix, stored using lower-storage-by-rows.

Specified as: a single character. It must be 'G', 'U', or 'L'.

*init*

indicates the type of computation to be performed, where:

If *init* = 'I', the preconditioning matrix is computed, the internal representation of the sparse matrix is generated, and the iteration procedure is performed. The coefficient matrix and preconditioner in internal format are saved in *aux1*.

If *init* = 'S', the iteration procedure is performed using the coefficient matrix and the preconditioner in internal format, stored in *aux1*, created in a preceding call to this subroutine with *init* = 'I'. You use this option to solve the same matrix for different right-hand sides, ***b***, optimizing your performance. As long as you do not change the coefficient matrix and preconditioner in *aux1*, any number of calls can be made with *init* = 'S'.

Specified as: a single character. It must be 'I' or 'S'.

*n*

is the order of the linear system ***Ax*** = ***b*** and the number of rows and columns in sparse matrix ***A***. Specified as: a fullword integer; $n \ge 0$.

*ar*

is the sparse matrix ***A*** of order *n*, stored by rows in an array, referred to as AR. The *stor* argument indicates the storage variation used for storing matrix ***A***. Specified as: a one-dimensional array, containing long-precision real numbers. The number of elements in this array can be determined by subtracting 1 from the value in IA(*n*+1).

*ja*

is the array, referred to as JA, containing the column numbers of each nonzero element in sparse matrix **A**. Specified as: a one-dimensional array, containing fullword integers; 1 ≤ (JA elements) ≤ *n*. The number of elements in this array can be determined by subtracting 1 from the value in IA(*n*+1).

*ia*

is the row pointer array, referred to as IA, containing the starting positions of each row of matrix ***A*** in array AR and one position past the end of array AR. Specified as: a one-dimensional array of (at least) length *n*+1, containing fullword integers; IA(*i*+1) ≥ IA(*i*) for *i* = 1, *n*+1.

*b*

is the vector ***b*** of length *n*, containing the right-hand side of the matrix problem. Specified as: a one-dimensional array of (at least) length *n*, containing long-precision real numbers.

*x*

is the vector ***x*** of length *n*, containing your initial guess of the solution of the linear system. Specified as: a one-dimensional array of (at least) length *n*, containing long-precision real numbers. The elements can have any value, and if no guess is available, the value can be zero.

*iparm*

is an array of parameters, IPARM(*i*), where:

- IPARM(1) controls the number of iterations.

  If IPARM(1) > 0, IPARM(1) is the maximum number of iterations allowed.

  If IPARM(1) = 0, the following default values are used:

  IPARM(1) = 300
  IPARM(2) = 4
  IPARM(4) = 4
  IPARM(5) = 1
  RPARM(1) = $10^{-6}$
  RPARM(2) = 1

- IPARM(2) is the flag used to select the iterative procedure used in this subroutine.

  If IPARM(2) = 1, the conjugate gradient (CG) method is used. Note that this algorithm should only be used with positive definite symmetric matrices.

  If IPARM(2) = 2, the conjugate gradient squared (CGS) method is used.

  If IPARM(2) = 3, the generalized minimum residual (GMRES) method, restarted after *k* steps, is used.

  If IPARM(2) = 4, the more smoothly converging variant of the CGS method (Bi-CGSTAB) is used.

If IPARM(2) = 5, the transpose-free quasi-minimal residual method (TFQMR) is used.

- IPARM(3) has the following meaning, where:

  If IPARM(2) ≠ 3, then IPARM(3) is not used.

  If IPARM(2) = 3, then IPARM(3) = *k*, where *k* is the number of steps after which the generalized minimum residual method is restarted. A value for *k* in the range of 5 to 10 is suitable for most problems.

- IPARM(4) is the flag that determines the type of preconditioning.

  If IPARM(4) = 1, the system is not preconditioned.

  If IPARM(4) = 2, the system is preconditioned by a diagonal matrix.

  If IPARM(4) = 3, the system is preconditioned by SSOR splitting with the diagonal given by the absolute values sum of the input matrix.

  If IPARM(4) = 4, the system is preconditioned by an incomplete LU factorization.

  If IPARM(4) = 5, the system is preconditioned by SSOR splitting with the diagonal given by the incomplete LU factorization.

  **Note:** The multithreaded version of DSRIS only runs on multiple threads when IPARM(4) = 1 or 2.

- IPARM(5) is the flag used to select the stopping criterion used in the computation, where the following items are used in the definitions of the stopping criteria below:

  – $\varepsilon$ is the desired relative accuracy and is stored in RPARM(1).

  – $x_j$ is the solution found at the *j*-th iteration.

  – $r_j$ and $r_0$ are the preconditioned residuals obtained at iterations *j* and 0, respectively. (The residual at iteration *j* is given by $\boldsymbol{b}-\boldsymbol{A}x_{j}$.)

  If IPARM(5) = 1, the iterative method is stopped when:

  $\|r_j\|_2 \,/\, \|x_j\|_2 < \varepsilon$

  **Note:** IPARM(5) = 1 is the default value assumed by ESSL if you do not specify one of the values described here; therefore, if you do not update your program to set an IPARM(5) value, you, by default, use the above stopping criterion.

  If IPARM(5) = 2, the iterative method is stopped when:

  $\|r_j\|_2 \,/\, \|r_0\|_2 < \varepsilon$

  If IPARM(5) = 3, the iterative method is stopped when:

  $\|x_j - x_{j-1}\|_2 \,/\, \|x_j\|_2 < \varepsilon$

  **Note:** Stopping criterion 3 performs poorly with the TFQMR method; therefore, if you specify TFQMR (IPARM(2) = 5), you should not specify stopping criterion 3.

- IPARM(6), see “On Return” on page 637.

Specified as: an array of (at least) length 6, containing fullword integers, where:
IPARM(1) ≥ 0

IPARM(2) = 1, 2, 3, 4, or 5
If IPARM(2) = 3, then IPARM(3) > 0
IPARM(4) = 1, 2, 3, 4, or 5
IPARM(5) = 1, 2, or 3 (Other values default to stopping criterion 1.)

*rparm*

is an array of parameters, RPARM(*i*), where:

RPARM(1) is the relative accuracy $\epsilon$ used in the stopping criterion. See "Notes" on page 638.

RPARM(2), see "On Return" on page 637.

RPARM(3) has the following meaning, where:

- If IPARM(4) ≠ 3, then RPARM(3) is not used.
- If IPARM(4) = 3, then RPARM(3) is the acceleration parameter used in SSOR. (A value in the range 0.5 to 2.0 is suitable for most problems.)

Specified as: a one-dimensional array of (at least) length 3, containing long-precision real numbers, where:
RPARM(1) ≥ 0
If IPARM(4) = 3, RPARM(3) > 0

*aux1*

is working storage for this subroutine, where:

If *init* = 'I', the working storage is computed. It can contain any values.

If *init* = 'S', the working storage is used in solving the linear system. It contains the coefficient matrix and preconditioner in internal format, computed in an earlier call to this subroutine.

Specified as: an area of storage, containing *naux1* long-precision real numbers.

*naux1*

is the number of doublewords in the working storage specified in *aux1*.

Specified as: a fullword integer, where:

In these formulas *nw* has the following value:
If *stor* = 'G', then *nw* = IA(*n*+1)−1+*n*.
If *stor* = 'U' or 'L', then *nw* = 2(IA(*n*+1)−1).

If IPARM(4) = 1, use *naux1* = (3/2)*nw*+(7/2)*n*+40.
If IPARM(4) = 2, use *naux1* = (3/2)*nw*+(9/2)*n*+40.
If IPARM(4) = 3, 4, or 5, then:
If IPARM(2) ≠ 1, use *naux1* = 3*nw*+10*n*+60.
If IPARM(2) = 1, use *naux1* = 3*nw*+(21/2)*n*+60.

**Note:** If you receive an attention message, you have not specified sufficient auxiliary storage to achieve optimal performance, but it is enough to perform the computation. To obtain optimal performance, you need to use the amount given by the attention message.

*aux2*

has the following meaning:

If *naux2* = 0 and error 2015 is unrecoverable, *aux2* is ignored.

Otherwise, it is working storage used by this subroutine that is available for use by the calling program between calls to this subroutine.

Specified as: an area of storage, containing *naux2* long-precision real numbers.

*naux2*

is the number of doublewords in the working storage specified in *aux2*. Specified as: a fullword integer, where:

If *naux2* = 0 and error 2015 is unrecoverable, DSRIS dynamically allocates the work area used by this subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise,

If IPARM(2) = 1, use *naux*2 ≥ 4*n*.
If IPARM(2) = 2, use *naux*2 ≥ 7*n*.
If IPARM(2) = 3, use *naux*2 ≥ (*k*+2)*n*+*k*(*k*+4)+1, where *k* = IPARM(3).
If IPARM(2) = 4, use *naux*2 ≥ 7*n*.
If IPARM(2) = 5, use *naux*2 ≥ 9*n*.

*On Return*

*ar*

is the sparse matrix ***A*** of order *n*, stored by rows in an array, referred to as AR. The *stor* argument indicates the storage variation used for storing matrix ***A***. The order of the elements in each row of ***A*** in AR may be changed on output.

Returned as: a one-dimensional array, containing long-precision real numbers. The number of elements in this array can be determined by subtracting 1 from the value in IA(*n*+1).

*ja*

is the array, referred to as JA, containing the column numbers of each nonzero element in sparse matrix **A**. These elements correspond to the arrangement of the contents of AR on output.

Returned as: a one-dimensional array, containing fullword integers; 1 ≤ (JA elements) ≤ *n*. The number of elements in this array can be determined by subtracting 1 from the value in IA(*n*+1).

*x*

is the vector ***x*** of length *n*, containing the solution of the system ***Ax*** = ***b***. Returned as: a one-dimensional array of (at least) length *n*, containing long-precision real numbers.

*iparm*

is an array of parameters, IPARM(*i*), where:

IPARM(1) through IPARM(5) are unchanged.

IPARM(6) contains the number of iterations performed by this subroutine.

Returned as: a one-dimensional array of length 6, containing fullword integers.

*rparm*

is an array of parameters, RPARM(*i*), where:

RPARM(1) is unchanged.

RPARM(2) contains the estimate of the error of the solution. If the process converged, RPARM(2) ≤ $\epsilon$.

RPARM(3) is unchanged.

Returned as: a one-dimensional array of length 3, containing long-precision real numbers.

*aux1*

is working storage for this subroutine, containing the coefficient matrix and preconditioner in internal format, ready to be passed in a subsequent invocation of this subroutine. Returned as: an area of storage, containing *naux1* long-precision real numbers.

***Notes***

1. If you want to solve the same sparse linear system of equations multiple times using a different algorithm with the same preconditioner and using a different right-hand side each time, you get the best performance by using the following technique. Call DSRIS the first time with *init* = 'I'. This solves the system, and then stores the coefficient matrix and preconditioner in internal format in *aux1*. On the subsequent invocations of DSRIS with different right-hand sides, specify *init* = 'S'. This indicates to DSRIS to use the contents of *aux1*, saving the time to convert your coefficient matrix and preconditioner to internal format. If you use this technique, you should not modify the contents of *aux1* between calls to DSRIS.

   In some cases, you can specify a different algorithm in IPARM(2) when making calls with *init* = 'S'. (See "Example 2" on page 641.) However, DSRIS sometimes needs different information in *aux1* for different algorithms. When this occurs, DSRIS issues an attention message, continues processing the computation, and then resets the contents of *aux1*. Your performance is not improved in this case, which is functionally equivalent to calling DSRIS with *init* = 'I'.

2. If you use the CG method with *init* = 'I', you must use the CG method when you specify *init* = 'S'. However, if you use a different method with *init* = 'I', you can use any other method, except CG, when you specify *init* = 'S'.

3. These subroutines accept lowercase letters for the *stor* and *init* arguments.

4. Matrix ***A***, vector ***x***, and vector ***b*** must have no common elements; otherwise, results are unpredictable.

5. In this subroutine, a value of RPARM(1) = 0 is permitted to force the solver to evaluate exactly IPARM(1) iterations. The algorithm computes a sequence of approximate solution vectors ***x*** that converge to the solution. The iterative procedure is stopped when the selected stopping criterion is satisfied or when more than the maximum number of iterations (in IPARM(1)) is reached.

   For the stopping criteria specified in IPARM(5), the relative accuracy $\varepsilon$ (in RPARM(1)) must be specified reasonably ($10^{-4}$ to $10^{-8}$). If you specify a larger $\varepsilon$, the algorithm takes fewer iterations to converge to a solution. If you specify a smaller $\varepsilon$, the algorithm requires more iterations and computer time, but converges to a more precise solution. If the value you specify is unreasonably small, the algorithm may fail to converge within the number of iterations it is allowed to perform.

6. For a description of how sparse matrices are stored by rows, see "Storage-by-Rows" on page 99.

7. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see "Using Auxiliary Storage in ESSL" on page 32.

***Function:*** The linear system:

***Ax*** = ***b***

is solved using one of the following methods: conjugate gradient (CG), conjugate gradient squared (CGS), generalized minimum residual (GMRES), more smoothly converging variant of the CGS method (Bi-CGSTAB), or transpose-free quasi-minimal residual method (TFQMR), where:

> ***A*** is a sparse matrix of order *n*. The matrix is stored in arrays `AR`, `IA`, and `JA`. If it is general, it is stored by rows. If it is symmetric, it can be stored using upper- or lower-storage-by-rows.
> ***x*** is a vector of length *n*.
> ***b*** is a vector of length *n*.

One of the following preconditioners is used:

- an incomplete LU factorization
- an incomplete Cholesky factorization (for positive definite symmetric matrices)
- diagonal scaling
- symmetric successive over-relaxation (SSOR) with two possible choices for the diagonal matrix:
  - the absolute values sum of the input matrix
  - the diagonal obtained from the LU factorization

See references [36 ], [54 ], [79 ], [83 ], [86], and [92 ].

When you call this subroutine to solve a system for the first time, you specify *init* = `'I'`. After that, you can solve the same system any number of times by calling this subroutine each time with *init* = `'S'`. These subsequent calls use the coefficient matrix and preconditioner, stored in internal format in *aux1*. You optimize performance by doing this, because certain portions of the computation have already been performed.

### *Error Conditions*

*Resource Errors:* Error 2015 is unrecoverable, *naux2* = 0, and unable to allocate work area.

*Computational Errors:* The following errors, with their corresponding return codes, can occur in this subroutine. For details on error handling, see “What Can You Do about ESSL Computational Errors?” on page 48.

- For error 2110, return code 1 indicates that the subroutine exceeded `IPARM(1)` iterations without converging. Vector ***x*** contains the approximate solution computed at the last iteration.
- For error 2130, return code 2 indicates that the incomplete LU factorization of ***A*** could not be completed, because one pivot was 0.
- For error 2124, the subroutine has been called with *init* = `'S'`, but the data contained in *aux1* was computed for a different algorithm. An attention message is issued. Processing continues, and the contents of *aux1* are reset correctly.
- For error 2134, return code 3 indicates that the data contained in *aux1* is not consistent with the input sparse matrix. The subroutine has been called with *init* = `'S'`, and *aux1* contains an incomplete factorization and internal data storage for the input matrix ***A*** that was computed by a previous call to the subroutine when *init* = `'I'`. This error indicates that *aux1* has been modified since the last call to the subroutine, or that the input matrix is not the same as

the one that was factored. If the default action has been overridden, the subroutine can be called again with the same parameters, with the exception of IPARM(4) = 1 or 4.

- For error 2131, return code 4 indicates that the matrix is singular, because all elements in one row of the matrix contain zero.
- For error 2129, return code 5 indicates that the matrix is not positive definite.
- For error 2128, return code 8 indicates an internal ESSL error. Please contact your IBM Representative.

*Input-Argument Errors*

1. $n < 0$
2. *stor* ≠ 'G', 'U', or 'L'
3. *init* ≠ 'I' or 'S'
4. IA($n$+1) < 1
5. IA($i$+1)−IA($i$) < 0, for any $i = 1, n$
6. IPARM(1) < 0
7. IPARM(2) ≠ 1, 2, 3, 4, or 5
8. IPARM(3) ≤ 0 and IPARM(2) = 3
9. IPARM(4) ≠ 1, 2, 3, 4, or 5
10. RPARM(1) < 0
11. RPARM(3) ≤ 0 and IPARM(4) = 3
12. *naux1* is too small—that is, less than the minimum required value. Return code 6 is returned if error 2015 is recoverable.
13. Error 2015 is recoverable or *naux2*≠0, and *naux2* is too small—that is, less than the minimum required value. Return code 7 is returned for *naux2* if error 2015 is recoverable.

***Example 1:*** This example finds the solution of the linear system ***Ax*** = ***b*** for the sparse matrix ***A***, which is stored by rows in arrays AR, IA, and JA. The system is solved using the Bi-CGSTAB algorithm. The iteration is stopped when the norm of the residual is less than the given threshold specified in RPARM(1). The algorithm is allowed to perform 20 iterations. The process converges after 9 iterations. Matrix ***A*** is:

$$\begin{bmatrix}
2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 2.0 & -1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 1.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
1.0 & 0.0 & 0.0 & 2.0 & -1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 1.0 & 2.0 & -1.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 2.0 & -1.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 2.0 & -1.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 2.0 & -1.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 2.0
\end{bmatrix}$$

*Call Statement and Input*

```
             STOR INIT   N   AR   JA   IA   B   X   IPARM   RPARM   AUX1   NAUX1   AUX2   NAUX2
              |    |     |   |    |    |    |   |     |       |       |      |       |      |
CALL DSRIS( 'G' , 'I' , 9 , AR , JA , IA , B , X , IPARM , RPARM , AUX1 ,   98   , AUX2 ,   63 )
```

```
AR       =  (2.0, 2.0, -1.0, 1.0, 2.0, 1.0, 2.0, -1.0, 1.0, 2.0, -1.0,
             1.0, 2.0, -1.0, 1.0, 2.0, -1.0, 1.0, 2.0, -1.0, 1.0, 2.0)
JA       =  (1, 2, 3, 2, 3, 1, 4, 5, 4, 5, 6, 5, 6, 7, 6, 7, 8, 7, 8,
             9, 8, 9)
IA       =  (1, 2, 4, 6, 9, 12, 15, 18, 21, 23)
B        =  (2.0, 1.0, 3.0, 2.0, 2.0, 2.0, 2.0, 2.0, 3.0)
X        =  (0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0)
IPARM(1) =  20
IPARM(2) =  4
IPARM(3) =  0
IPARM(4) =  1
IPARM(5) =  10
RPARM(1) =  1.D-7
RPARM(3) =  1.0
```

*Output*

```
X        =  (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0)
IPARM(6) =  9
RPARM(2) =  0.29D-16
```

***Example 2:*** This example finds the solution of the linear system ***Ax*** = ***b*** for the same sparse matrix ***A*** used in Example 1. It also uses the same right-hand side in ***b*** and the same initial guesses in ***x***. However, the system is solved using a different algorithm, conjugate gradient squared (CGS). Because INIT is 'S', the best performance is achieved. The iteration is stopped when the norm of the residual is less than the given threshold specified in RPARM(1). The algorithm is allowed to perform 20 iterations. The process converges after 9 iterations.

*Call Statement and Input*

```
           STOR INIT   N   AR   JA   IA   B   X   IPARM   RPARM   AUX1   NAUX1   AUX2   NAUX2
            |    |     |   |    |    |    |   |     |       |       |      |       |      |
CALL DSRIS( 'G' , 'S' , 9 , AR , JA , IA , B , X , IPARM , RPARM , AUX1 ,   98  , AUX2 ,   63 )
```

```
AR       =(same as input AR in Example 1)
JA       =(same as input JA in Example 1)
IA       =(same as input IA in Example 1)
B        =(same as input B in Example 1)
X        =(same as input X in Example 1)
IPARM(1) =  20
IPARM(2) =  2
IPARM(3) =  0
IPARM(4) =  1
IPARM(5) =  10
RPARM(1) =  1.D-7
RPARM(3) =  1.0
```

*Output*

```
X        =  (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0)
IPARM(6) =  9
RPARM(2) =  0.42D-19
```

***Example 3:*** This example finds the solution of the linear system ***Ax*** = ***b*** for the sparse matrix ***A***, which is stored by rows in arrays AR, IA, and JA. The system is solved using the two-term conjugate gradient method (CG), preconditioned by incomplete LU factorization. The iteration is stopped when the norm of the residual is less than the given threshold specified in RPARM(1). The algorithm is allowed to perform 20 iterations. The process converges after 1 iteration. Matrix ***A*** is:

```
┌                                                       ┐
│  2.0   0.0  -1.0   0.0   0.0   0.0   0.0   0.0   0.0 │
│  0.0   2.0   0.0  -1.0   0.0   0.0   0.0   0.0   0.0 │
│ -1.0   0.0   2.0   0.0  -1.0   0.0   0.0   0.0   0.0 │
│  0.0  -1.0   0.0   2.0   0.0  -1.0   0.0   0.0   0.0 │
│  0.0   0.0  -1.0   0.0   2.0   0.0  -1.0   0.0   0.0 │
│  0.0   0.0   0.0  -1.0   0.0   2.0   0.0  -1.0   0.0 │
│  0.0   0.0   0.0   0.0  -1.0   0.0   2.0   0.0  -1.0 │
│  0.0   0.0   0.0   0.0   0.0  -1.0   0.0   2.0   0.0 │
│  0.0   0.0   0.0   0.0   0.0   0.0  -1.0   0.0   2.0 │
└                                                       ┘
```

*Call Statement Input*

```
           STOR INIT   N   AR   JA   IA   B   X   IPARM   RPARM   AUX1   NAUX1   AUX2   NAUX2
            |     |    |   |    |    |    |   |     |       |      |       |      |       |
CALL DSRIS( 'G' , 'I' , 9 , AR , JA , IA , B , X , IPARM , RPARM , AUX1 ,  223  , AUX2 ,   36 )
```

```
AR       =  (2.0, -1.0, 2.0, -1.0, -1.0, 2.0, -1.0, -1.0, 2.0, -1.0,
             -1.0, 2.0, -1.0, -1.0, 2.0, -1.0, -1.0, 2.0, -1.0, -1.0,
             2.0, -1.0, 2.0)
JA       =  (1, 3, 2, 4, 1, 3, 5, 2, 4, 6, 3, 5, 7, 4, 6, 8, 5, 7, 9,
             6, 8, 7, 9)
IA       =  (1, 3, 5, 8, 11, 14, 17, 20, 22, 24)
B        =  (1.0, 1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 1.0, 1.0)
X        =  (0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0)
IPARM(1) =  20
IPARM(2) =  1
IPARM(3) =  0
IPARM(4) =  4
IPARM(5) =  1
RPARM(1) =  1.D-7
RPARM(3) =  1.0
```

*Output*

```
X        =  (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0)
IPARM(6) =  1
RPARM(2) =  0.16D-15
```

***Example 4:*** This example finds the solution of the linear system ***Ax*** = ***b*** for the same sparse matrix ***A*** used in Example 3. However, matrix ***A*** is stored using upper-storage-by-rows in arrays AR, IA, and JA. The system is solved using the generalized minimum residual (GMRES), restarted after 5 steps and preconditioned with SSOR splitting. The iteration is stopped when the norm of the residual is less than the given threshold specified in RPARM(1). The algorithm is allowed to perform 20 iterations. The process converges after 12 iterations.

*Call Statement Input*

```
           STOR INIT   N   AR   JA   IA   B   X   IPARM   RPARM   AUX1   NAUX1   AUX2   NAUX2
            |     |    |   |    |    |    |   |     |       |      |       |      |       |
CALL DSRIS( 'U' , 'I' , 9 , AR , JA , IA , B , X , IPARM , RPARM , AUX1 ,  219  , AUX2 ,  109 )
```

```
AR       =  (2.0, -1.0, 2.0, -1.0, 2.0, -1.0, 2.0, -1.0, 2.0, -1.0,
             2.0, -1.0, 2.0, -1.0, 2.0, 2.0)
JA       =  (1, 3, 2, 4, 3, 5, 4, 6, 5, 7, 6, 8, 7, 9, 8, 9)
IA       =  (1, 3, 5, 7, 9, 11, 13, 15, 16, 17)
B        =  (1.0, 1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 1.0, 1.0)
X        =  (0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0)
IPARM(1) =  20
IPARM(2) =  3
IPARM(3) =  5
IPARM(4) =  3
IPARM(5) =  1
RPARM(1) =  1.D-7
RPARM(3) =  2.0
```

*Output*

```
X        =  (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0)
IPARM(6) =  12
RPARM(2) =  0.33D-7
```

# DSMCG—Sparse Positive Definite or Negative Definite Symmetric Matrix Iterative Solve Using Compressed-Matrix Storage Mode

This subroutine solves a symmetric, positive definite or negative definite linear system, using the conjugate gradient method, with or without preconditioning by an incomplete Cholesky factorization, for a sparse matrix stored in compressed-matrix storage mode. Matrix ***A*** and vectors ***x*** and ***b*** are used:

$$\boldsymbol{Ax} = \boldsymbol{b}$$

where ***A***, ***x***, and ***b*** contain long-precision real numbers.

**Notes:**

1. These subroutines are provided only for migration purposes. You get better performance and a wider choice of algorithms if you use the DSRIS subroutine.
2. If your sparse matrix is stored by rows, as defined in “Storage-by-Rows” on page 99, you should first use the utility subroutine DSRSM to convert your sparse matrix to compressed-matrix storage mode. See “DSRSM—Convert a Sparse Matrix from Storage-by-Rows to Compressed-Matrix Storage Mode” on page 981

***Syntax***

| | |
|---|---|
| **Fortran** | CALL DSMCG (*m*, *nz*, *ac*, *ka*, *lda*, *b*, *x*, *iparm*, *rparm*, *aux1*, *naux1*, *aux2*, *naux2*) |
| **C and C++** | dsmcg (*m*, *nz*, *ac*, *ka*, *lda*, *b*, *x*, *iparm*, *rparm*, *aux1*, *naux1*, *aux2*, *naux2*); |
| **PL/I** | CALL DSMCG (*m*, *nz*, *ac*, *ka*, *lda*, *b*, *x*, *iparm*, *rparm*, *aux1*, *naux1*, *aux2*, *naux2*); |

*On Entry*

*m*
is the order of the linear system ***Ax*** = ***b*** and the number of rows in sparse matrix ***A***. Specified as: a fullword integer; *m* ≥ 0.

*nz*
is the maximum number of nonzero elements in each row of sparse matrix ***A***. Specified as: a fullword integer; *nz* ≥ 0.

*ac*
is the array, referred to as AC, containing the values of the nonzero elements of the sparse matrix, stored in compressed-matrix storage mode. Specified as: an *lda* by (at least) *nz* array, containing long-precision real numbers.

*ka*
is the array, referred to as KA, containing the column numbers of the matrix ***A*** elements stored in the corresponding positions in array AC. Specified as: an *lda* by (at least) *nz* array, containing fullword integers, where 1 ≤ (elements of KA) ≤ *m*.

*lda*
is the leading dimension of the arrays specified for *ac* and *ka*. Specified as: a fullword integer; *lda* > 0 and *lda* ≥ *m*.

*b*
is the vector ***b*** of length *m*, containing the right-hand side of the matrix problem. Specified as: a one-dimensional array of (at least) length *m*, containing long-precision real numbers.

*x*
is the vector ***x*** of length *m*, containing your initial guess of the solution of the linear system. Specified as: a one-dimensional array of (at least) length *m*,

containing long-precision real numbers. The elements can have any value, and if no guess is available, the value can be zero.

*iparm*

is an array of parameters, IPARM(*i*), where:

- IPARM(1) controls the number of iterations.

  If IPARM(1) > 0, IPARM(1) is the maximum number of iterations allowed.

  If IPARM(1) = 0, the following default values are used:

  IPARM(1) = 300
  IPARM(2) = 1
  IPARM(3) = 0
  RPARM(1) = $10^{-6}$

- IPARM(2) is the flag used to select the stopping criterion.

  If IPARM(2) = 0, the conjugate gradient iterative procedure is stopped when:

  $$\|\boldsymbol{r}\|_2 / \|\boldsymbol{x}\|_2 < \varepsilon$$

  where $\boldsymbol{r} = \boldsymbol{b} - \boldsymbol{Ax}$ is the residual, and $\varepsilon$ is the desired relative accuracy. $\varepsilon$ is stored in RPARM(1).

  If IPARM(2) = 1, the conjugate gradient iterative procedure is stopped when:

  $$\|\boldsymbol{r}\|_2 / \lambda\|\boldsymbol{x}\|_2 < \varepsilon$$

  where $\lambda$ is an estimate to the minimum eigenvalue of the iteration matrix. $\lambda$ is computed adaptively by this program and, on output, is stored in RPARM(2).

  If IPARM(2) = 2, the conjugate gradient iterative procedure is stopped when:

  $$\|\boldsymbol{r}\|_2 / \lambda\|\boldsymbol{x}\|_2 < \varepsilon$$

  where $\lambda$ is a predetermined estimate to the minimum eigenvalue of the iteration matrix. This eigenvalue estimate, on input, is stored in RPARM(2) and may be obtained by an earlier call to this subroutine with the same matrix.

- IPARM(3) is the flag that determines whether the system is to be solved using the conjugate gradient method, preconditioned by an incomplete Cholesky factorization with no fill-in.

  If IPARM(3) = 0, the system is not preconditioned.

  If IPARM(3) = 10, the system is preconditioned by an incomplete Cholesky factorization.

  If IPARM(3) = –10, the system is preconditioned by an incomplete Cholesky factorization, where the factorization matrix was computed in an earlier call to this subroutine and is stored in *aux2*.

- IPARM(4), see “On Return” on page 646.

Specified as: an array of (at least) length 4, containing fullword integers, where:

IPARM(1) ≥ 0
IPARM(2) = 0, 1, or 2
IPARM(3) = 0, 10, or –10

*rparm*

is an array of parameters, RPARM(*i*), where ε is stored in RPARM(1), and λ is stored in RPARM(2).

RPARM(1) > 0, is the relative accuracy ϵ used in the stopping criterion.

RPARM(2) > 0, is the estimate of the smallest eigenvalue, λ, of the iteration matrix. It is only used when IPARM(2) = 2.

RPARM(3), see “On Return.”

Specified as: a one-dimensional array of (at least) length 3, containing long-precision real numbers.

*aux1*

has the following meaning:

If *naux1* = 0 and error 2015 is unrecoverable, *aux1* is ignored.

Otherwise, it is a storage work area used by this subroutine, which is available for use by the calling program between calls to this subroutine. Its size is specified by *naux1*.

Specified as: an area of storage, containing long-precision real numbers.

*naux1*

is the size of the work area specified by *aux1*—that is, the number of elements in *aux1*. Specified as: a fullword integer, where:

If *naux1* = 0 and error 2015 is unrecoverable, DSMCG dynamically allocates the work area used by this subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, *naux1* must have at least the following value, where:

If IPARM(2) = 0 or 2, use *naux1* ≥ 3*m*.

If IPARM(2) = 1 and IPARM(1) ≠ 0, use *naux1* ≥ 3*m*+2(IPARM(1)).

If IPARM(2) = 1 and IPARM(1) = 0, use *naux1* ≥ 3*m*+600.

*aux2*

is a storage work area used by this subroutine. If IPARM(3) = –10, *aux2* must contain the incomplete Cholesky factorization of matrix ***A***, computed in an earlier call to DSMCG. The size of *aux2* is specified by *naux2*. Specified as: an area of storage, containing long-precision real numbers.

*naux2*

is the size of the work area specified by *aux2*—that is, the number of elements in *aux2*. Specified as: a fullword integer. When IPARM(3) = 10 or –10, *naux2* must have at least the following value: *naux2* ≥ *m*(*nz*–1)1.5+2(*m*+6).

*On Return*

*x*

is the vector ***x*** of length *m*, containing the solution of the system ***Ax*** = ***b***. Returned as: a one-dimensional array of (at least) length *m*, containing long-precision real numbers.

*iparm*

is an array of parameters, IPARM(*i*), where:

IPARM(1) is unchanged.

IPARM(2) is unchanged.

IPARM(3) is unchanged.

IPARM(4) contains the number of iterations performed by this subroutine.

Returned as: a one-dimensional array of length 4, containing fullword integers.

*rparm*

is an array of parameters, RPARM(*i*), where:

RPARM(1) is unchanged.

RPARM(2) is unchanged if IPARM(2) = 0 or 2. If IPARM(2) = 1, RPARM(2) contains λ, an estimate of the smallest eigenvalue of the iteration matrix.

RPARM(3) contains the estimate of the error of the solution. If the process converged, RPARM(3) $\leq \epsilon$.

Returned as: a one-dimensional array of length 3, containing long-precision real numbers; $\lambda > 0$.

*aux2*

is the storage work area used by this subroutine.

If IPARM(3) = 10, *aux2* contains the incomplete Cholesky factorization of matrix ***A***.

If IPARM(3) = –10, *aux2* is unchanged.

See “Notes” for additional information on *aux2*. Returned as: an area of storage, containing long-precision real numbers.

***Notes***

1. When IPARM(3) = –10, this subroutine uses the incomplete Cholesky factorization in *aux2*, computed in an earlier call to this subroutine. When IPARM(3) = 10, this subroutine computes the incomplete Cholesky factorization and stores it in *aux2*.
2. If you solve the same sparse linear system of equations several times with different right-hand sides using the preconditioned algorithm, specify IPARM(3) = 10 on the first invocation. The incomplete factorization is stored in *aux2*. You may save computing time on subsequent calls by setting IPARM(3) = –10. In this way, the algorithm reutilizes the incomplete factorization that was computed the first time. Therefore, you should not modify the contents of *aux2* between calls.
3. Matrix ***A*** must have no common elements with vectors ***x*** and ***b***; otherwise, results are unpredictable.
4. In the iterative solvers for sparse matrices, the relative accuracy ε (RPARM(1)) must be specified “reasonably” ($10^{-4}$ to $10^{-8}$). The algorithm computes a sequence of approximate solution vectors ***x*** that converge to the solution. The iterative procedure is stopped when the norm of the residual is sufficiently small—that is, when:

   $$\|\boldsymbol{b}-\boldsymbol{A}\boldsymbol{x}\|_2 \,/\, \lambda\|\boldsymbol{x}\|_2 < \varepsilon$$

   where λ is an estimate of the minimum eigenvalue of the iteration matrix, which is either estimated adaptively or given by the user. As a result, if you specify a larger ε, the algorithm takes fewer iterations to converge to a solution. If you specify a smaller ε, the algorithm requires more iterations and computer time, but converges to a more precise solution. If the value you specify is unreasonably small, the algorithm may fail to converge within the number of iterations it is allowed to perform.

5. For a description of how sparse matrices are stored in compressed-matrix storage mode, see “Compressed-Matrix Storage Mode” on page 93.
6. On output, array AC and vector ***b*** are not bitwise identical to what they were on input, because the matrix ***A*** and the right-hand side are scaled before starting the iterative process and are unscaled before returning control to the user. In addition, arrays AC and KA may be rearranged on output, but still contain a mathematically equivalent mapping of the elements in matrix ***A***.
7. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see “Using Auxiliary Storage in ESSL” on page 32.

***Function:*** The sparse positive definite or negative definite linear system:

$$\boldsymbol{Ax} = \boldsymbol{b}$$

is solved, where:

***A*** is a symmetric, positive definite or negative definite sparse matrix of order *m*, stored in compressed-matrix storage mode in AC and KA.
***x*** is a vector of length *m*.
***b*** is a vector of length *m*.

The system is solved using the two-term conjugate gradient method, with or without preconditioning by an incomplete Cholesky factorization. In both cases, the matrix is scaled by the square root of the diagonal.

See references [60 ] and [65 ]. [36 ].

If your program uses a sparse matrix stored by rows and you want to use this subroutine, first convert your sparse matrix to compressed-matrix storage mode by using the subroutine DSRSM described on page 981.

### *Error Conditions*

*Resource Errors:* Error 2015 is unrecoverable, *naux1* = 0, and unable to allocate work area.

*Computational Errors:* The following errors, with their corresponding return codes, can occur in this subroutine. Where a value of *i* is indicated, it can be determined at run time by use of the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for that particular error code in the ESSL error option table; otherwise, the default value causes your program to terminate when the error occurs. For details, see “What Can You Do about ESSL Computational Errors?” on page 48.

- For error 2110, return code 1 indicates that the subroutine exceeded IPARM(1) iterations without converging. Vector ***x*** contains the approximate solution computed at the last iteration.
- For error 2111, return code 2 indicates that *aux2* contains an incorrect factorization. The subroutine has been called with IPARM(3) = –10, and *aux2* contains an incomplete factorization of the input matrix ***A*** that was computed by a previous call to the subroutine when IPARM(3) = 10. This error indicates that *aux2* has been modified since the last call to the subroutine, or that the input

matrix is not the same as the one that was factored. If the default action has been overridden, the subroutine can be called again with the same parameters, with the exception of `IPARM(3) = 0` or 10.

- For error 2109, return code 3 indicates that the inner product (***y***,***Ay***) is negative in the iterative procedure after iteration *i*. This should not occur, because the input matrix is assumed to be positive or negative definite. Vector ***x*** contains the results of the last iteration. The value *i* is identified in the computational error message.
- For error 2108, return code 4 indicates that the matrix is not positive definite. `AC` is partially modified and does not represent the same matrix as on entry.

*Input-Argument Errors*

1. $m < 0$
2. $lda < 1$
3. $lda < m$
4. $nz < 0$
5. $nz = 0$ and $m > 0$
6. `IPARM(1) < 0`
7. `IPARM(2)` ≠ 0, 1, or 2
8. `IPARM(3)` ≠ 0, 10, or –10
9. `RPARM(1) < 0`
10. `RPARM(2) < 0`
11. Error 2015 is recoverable or *naux1*≠0, and *naux1* is too small—that is, less than the minimum required value. Return code 5 is returned if error 2015 is recoverable.
12. *naux2* is too small—that is, less than the minimum required value. Return code 5 is returned if error 2015 is recoverable.

***Example 1:*** This example finds the solution of the linear system ***Ax*** = ***b*** for the sparse matrix ***A***, which is stored in compressed-matrix storage mode in arrays `AC` and `KA`. The system is solved using the conjugate gradient method. Matrix ***A*** is:

$$\begin{bmatrix}
2.0 & 0.0 & 0.0 & -1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 2.0 & -1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & -1.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
-1.0 & 0.0 & 0.0 & 2.0 & -1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & -1.0 & 2.0 & -1.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & -1.0 & 2.0 & -1.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -1.0 & 2.0 & -1.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -1.0 & 2.0 & -1.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -1.0 & 2.0
\end{bmatrix}$$

**Note:** For input matrix `KA`, ( . ) indicates any value between 1 and 9.

*Call Statement and Input*

```
            M   NZ  AC   KA LDA  B   X  IPARM  RPARM  AUX1 NAUX1  AUX2 NAUX2
            |   |   |    |   |   |   |    |      |      |     |     |     |
CALL DSMCG( 9 , 3 , AC, KA, 9 , B , X, IPARM, RPARM, AUX1, 27  , AUX2,  0  )

IPARM(1) =  20
IPARM(2) =  0
IPARM(3) =  0
RPARM(1) =  1.D-7
```

```
                 ┌                   ┐
                 |  2.0  -1.0   0.0  |
                 |  2.0  -1.0   0.0  |
                 | -1.0   2.0   0.0  |
                 | -1.0   2.0  -1.0  |
AC    =          | -1.0   2.0  -1.0  |
                 | -1.0   2.0  -1.0  |
                 | -1.0   2.0  -1.0  |
                 | -1.0   2.0  -1.0  |
                 | -1.0   2.0   0.0  |
                 └                   ┘
```

```
                 ┌          ┐
                 | 1  4  .  |
                 | 2  3  .  |
                 | 2  3  .  |
                 | 1  4  5  |
KA    =          | 4  5  6  |
                 | 5  6  7  |
                 | 6  7  8  |
                 | 7  8  9  |
                 | 8  9  .  |
                 └          ┘
```

```
B         =  (1.0, 1.0, 1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 1.0)
X         =  (0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0)
```

*Output*

```
X         =  (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0)
IPARM(4)  =  5
RPARM(2)  =  0
RPARM(3)  =  0.351D-15
```

***Example 2:*** This example finds the solution of the linear system ***Ax*** = ***b*** for the same sparse matrix ***A*** as in Example 1, which is stored in compressed-matrix storage mode in arrays AC and KA. The system is solved using the conjugate gradient method, preconditioned with an incomplete Cholesky factorization. The smallest eigenvalue of the iteration matrix is computed and used in stopping the computation.

**Note:** For input matrix KA, ( . ) indicates any value between 1 and 9.

*Call Statement and Input*

```
            M   NZ  AC   KA LDA  B   X  IPARM  RPARM  AUX1 NAUX1  AUX2 NAUX2
            |   |   |    |   |   |   |    |      |     |     |     |     |
CALL DSMCG( 9 , 3 , AC, KA, 9 , B , X, IPARM, RPARM, AUX1, 67  , AUX2, 74  )

IPARM(1)  =  20
IPARM(2)  =  1
IPARM(3)  =  10
RPARM(1)  =  1.D-7
AC        =(same as input AC in Example 1)
KA        =(same as input KA in Example 1)
B         =  (1.0, 1.0, 1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 1.0)
X         =  (0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0)
```

*Output*

```
X        =  (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0)
IPARM(4) =  1
RPARM(2) =  1
RPARM(3) =  0.100D-15
```

# DSDCG—Sparse Positive Definite or Negative Definite Symmetric Matrix Iterative Solve Using Compressed-Diagonal Storage Mode

This subroutine solves a symmetric, positive definite or negative definite linear system, using the two-term conjugate gradient method, with or without preconditioning by an incomplete Cholesky factorization, for a sparse matrix stored in compressed-diagonal storage mode. Matrix ***A*** and vectors ***x*** and ***b*** are used:

$\boldsymbol{Ax} = \boldsymbol{b}$

where ***A***, ***x***, and ***b*** contain long-precision real numbers.

***Syntax***

| | |
|---|---|
| **Fortran** | CALL DSDCG (*iopt*, *m*, *nd*, *ad*, *lda*, *la*, *b*, *x*, *iparm*, *rparm*, *aux1*, *naux1*, *aux2*, *naux2*) |
| **C and C++** | dsdcg (*iopt*, *m*, *nd*, *ad*, *lda*, *la*, *b*, *x*, *iparm*, *rparm*, *aux1*, *naux1*, *aux2*, *naux2*); |
| **PL/I** | CALL DSDCG (*iopt*, *m*, *nd*, *ad*, *lda*, *la*, *b*, *x*, *iparm*, *rparm*, *aux1*, *naux1*, *aux2*, *naux2*); |

*On Entry*

*iopt*

indicates the type of storage used, where:

If *iopt* = 0, all the nonzero diagonals of the sparse matrix are stored in compressed-diagonal storage mode.

If *iopt* = 1, the sparse matrix, stored in compressed-diagonal storage mode, is symmetric. Only the main diagonal and one of each pair of identical diagonals are stored in array AD.

Specified as: a fullword integer; *iopt* = 0 or 1.

*m*

is the order of the linear system ***Ax*** = ***b*** and the number of rows in sparse matrix ***A***. Specified as: a fullword integer; $m \geq 0$.

*nd*

is the number of nonzero diagonals stored in the columns of array AD, the number of columns in the array AD, and the number of elements in array LA. Specified as: a fullword integer; it must have the following value, where:

If $m > 0$, then $nd > 0$.

If $m = 0$, then $nd \geq 0$.

*ad*

is the array, referred to as AD, containing the values of the nonzero elements of the sparse matrix stored in compressed-diagonal storage mode. If *iopt* = 1, the main diagonal and one of each pair of identical diagonals is stored in this array.

Specified as: an *lda* by (at least) *nd* array, containing long-precision real numbers.

*lda*

is the leading dimension of the array specified for *ad*. Specified as: a fullword integer; $lda > 0$ and $lda \geq m$.

*la*

is the array, referred to as LA, containing the diagonal numbers *k* for the diagonals stored in each corresponding column in array AD. For an explanation of how diagonal numbers are assigned, see “Compressed-Diagonal Storage Mode” on page 94.

Specified as: a one-dimensional array of (at least) length *nd*, containing fullword integers, where $1-m \leq$ (elements of LA) $\leq m-1$.

*b*

is the vector ***b*** of length *m*, containing the right-hand side of the matrix problem. Specified as: a one-dimensional array of (at least) length *m*, containing long-precision real numbers.

*x*

is the vector ***x*** of length *m*, containing your initial guess of the solution of the linear system. Specified as: a one-dimensional array of (at least) length *m*, containing long-precision real numbers. The elements can have any value, and if no guess is available, the value can be zero.

*iparm*

is an array of parameters, IPARM(*i*), where:

- IPARM(1) controls the number of iterations.

  If IPARM(1) > 0, IPARM(1) is the maximum number of iterations allowed.

  If IPARM(1) = 0, the following default values are used:

  IPARM(1) = 300
  IPARM(2) = 1
  IPARM(3) = 0
  RPARM(1) = $10^{-6}$

- IPARM(2) is the flag used to select the stopping criterion.

  If IPARM(2) = 0, the conjugate gradient iterative procedure is stopped when:

  $$\|\boldsymbol{r}\|_2 / \|\boldsymbol{x}\|_2 < \varepsilon$$

  where $\boldsymbol{r} = \boldsymbol{b}-\boldsymbol{Ax}$ is the residual and $\varepsilon$ is the desired relative accuracy. $\varepsilon$ is stored in RPARM(1).

  If IPARM(2) = 1, the conjugate gradient iterative procedure is stopped when:

  $$\|\boldsymbol{r}\|_2 / \lambda\|\boldsymbol{x}\|_2 < \varepsilon$$

  where $\lambda$ is an estimate to the minimum eigenvalue of the iteration matrix. $\lambda$ is computed adaptively by this program and, on output, is stored in RPARM(2).

  If IPARM(2) = 2, the conjugate gradient iterative procedure is stopped when:

  $$\|\boldsymbol{r}\|_2 / \lambda\|\boldsymbol{x}\|_2 < \varepsilon$$

  where $\lambda$ is a predetermined estimate to the minimum eigenvalue of the iteration matrix. This eigenvalue estimate, on input, is stored in RPARM(2) and may be obtained by an earlier call to this subroutine with the same matrix.

- IPARM(3) is the flag that determines whether the system is to be solved using the conjugate gradient method, preconditioned by an incomplete Cholesky factorization with no fill-in.

  If IPARM(3) = 0, the system is not preconditioned.

If IPARM(3) = 10, the system is preconditioned by an incomplete Cholesky factorization.

If IPARM(3) = –10, the system is preconditioned by an incomplete Cholesky factorization, where the factorization matrix was computed in an earlier call to this subroutine and is stored in *aux2*.

- IPARM(4), see “On Return” on page 655.

Specified as: an array of (at least) length 4, containing fullword integers, where:
IPARM(1) = 0
IPARM(2) = 0, 1, or 2
IPARM(3) = 0, 10, or –10

*rparm*

is an array of parameters, RPARM(*i*), where ε is stored in RPARM(1), and λ is stored in RPARM(2).

RPARM(1) > 0, is the relative accuracy ϵ used in the stopping criterion.

RPARM(2) > 0, is the estimate of the smallest eigenvalue, λ, of the iteration matrix. It is only used when IPARM(2) = 2.

RPARM(3), see “On Return” on page 655.

Specified as: a one-dimensional array of (at least) length 3, containing long-precision real numbers.

*aux1*

has the following meaning:

If *naux1* = 0 and error 2015 is unrecoverable, *aux1* is ignored.

Otherwise, it is a storage work area used by this subroutine, which is available for use by the calling program between calls to this subroutine. Its size is specified by *naux1*.

Specified as: an area of storage, containing long-precision real numbers.

*naux1*

is the size of the work area specified by *aux1*—that is, the number of elements in *aux1*.

Specified as: a fullword integer, where:

If *naux* = 0 and error 2015 is unrecoverable, DSDCG dynamically allocates the work area used by this subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, it must have at least the following value, where:

If IPARM(2) = 0 or 2, use *naux1* ≥ 3*m*.

If IPARM(2) = 1 and IPARM(1) ≠ 0, use *naux1* ≥ 3*m*+2(IPARM(1)).

If IPARM(2) = 1 and IPARM(1) = 0, use *naux1* ≥ 3*m*+600.

*aux2*

is the storage work area used by this subroutine. If IPARM(3) = –10, *aux2* must contain the incomplete Cholesky factorization of matrix ***A***, computed in an earlier call to DSDCG. Its size is specified by *naux2*. Specified as: an area of storage, containing long-precision real numbers.

*naux2*

is the size of the work area specified by *aux2*—that is, the number of elements in *aux2*. Specified as: a fullword integer. When IPARM(3) = 10 or –10, *naux2* must have at least the following value, where:

If *iopt* = 0, use *naux2* ≥ *m*(1.5*nd*+2)1.5+2(*m*+6).

If *iopt* = 1, use *naux2* ≥ *m*(3*nd*+2)+8.

*On Return*

*x*

is the vector ***x*** of length *m*, containing the solution of the system ***Ax*** = ***b***. Returned as: a one-dimensional array, containing long-precision real numbers.

*iparm*

As an array of parameters, IPARM(*i*), where:

IPARM(1) is unchanged.

IPARM(2) is unchanged.

IPARM(3) is unchanged.

IPARM(4) contains the number of iterations performed by this subroutine.

Returned as: a one-dimensional array of length 4, containing fullword integers.

*rparm*

is an array of parameters, RPARM(*i*), where:

RPARM(1) is unchanged.

RPARM(2) is unchanged if IPARM(2) = 0 or 2. If IPARM(2) = 1, RPARM(2) contains λ, an estimate of the smallest eigenvalue of the iteration matrix.

RPARM(3) contains the estimate of the error of the solution. If the process converged, RPARM(3) ≤ $\epsilon$.

Returned as: a one-dimensional array of length 3, containing long-precision real numbers; λ > 0.

*aux2*

is the storage work area used by this subroutine.

If IPARM(3) = 10, *aux2* contains the incomplete Cholesky factorization of matrix ***A***.

If IPARM(3) = –10, *aux2* is unchanged.

See “Notes” for additional information on *aux2*. Returned as: an area of storage, containing long-precision real numbers.

### Notes

1. When IPARM(3) = –10, this subroutine uses the incomplete Cholesky factorization in *aux2*, computed in an earlier call to this subroutine. When IPARM(3) = 10, this subroutine computes the incomplete Cholesky factorization and stores it in *aux2*.
2. If you solve the same sparse linear system of equations several times with different right-hand sides using the preconditioned algorithm, specify IPARM(3) = 10 on the first invocation. The incomplete factorization is stored in *aux2*. You may save computing time on subsequent calls by setting IPARM(3) = –10. In this way, the algorithm reutilizes the incomplete factorization that was computed the first time. Therefore, you should not modify the contents of *aux2* between calls.
3. Matrix ***A*** must have no common elements with vectors ***x*** and ***b***; otherwise, results are unpredictable.

4. In the iterative solvers for sparse matrices, the relative accuracy ε (RPARM(1)) must be specified "reasonably" ($10^{-4}$ to $10^{-8}$). The algorithm computes a sequence of approximate solution vectors ***x*** that converge to the solution. The iterative procedure is stopped when the norm of the residual is sufficiently small—that is, when:

   $$\|\boldsymbol{b}-\boldsymbol{Ax}\|_2 \,/\, \lambda\|\boldsymbol{x}\|_2 < \varepsilon$$

   where λ is an estimate of the minimum eigenvalue of the iteration matrix, which is either estimated adaptively or given by the user. As a result, if you specify a larger ε, the algorithm takes fewer iterations to converge to a solution. If you specify a smaller ε, the algorithm requires more iterations and computer time, but converges to a more precise solution. If the value you specify is unreasonably small, the algorithm may fail to converge within the number of iterations it is allowed to perform.

5. For a description of how sparse matrices are stored in compressed-matrix storage mode, see "Compressed-Matrix Storage Mode" on page 93.

6. On output, array AD and vector ***b*** are not bitwise identical to what they were on input, because the matrix ***A*** and the right-hand side are scaled before starting the iterative process and are unscaled before returning control to the user. In addition, arrays AD and LA may be rearranged on output, but still contain a mathematically equivalent mapping of the elements in matrix ***A***.

7. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see "Using Auxiliary Storage in ESSL" on page 32.

***Function:*** The sparse positive definite or negative definite linear system:

$$\boldsymbol{Ax} = \boldsymbol{b}$$

is solved, where:

> ***A*** is a symmetric, positive definite or negative definite sparse matrix of order *m*, stored in compressed-diagonal storage mode in arrays AD and LA.
> ***x*** is a vector of length *m*.
> ***b*** is a vector of length *m*.

The system is solved using the two-term conjugate gradient method, with or without preconditioning by an incomplete Cholesky factorization. In both cases, the matrix is scaled by the square root of the diagonal.

See references [60 ] and [65 ]. [36 ].

***Error Conditions***

*Resource Errors:* Error 2015 is unrecoverable, *naux1* = 0, and unable to allocate work area.

*Computational Errors:* The following errors, with their corresponding return codes, can occur in this subroutine. Where a value of *i* is indicated, it can be determined at run time by use of the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for that particular error code in the ESSL error option table; otherwise, the default value causes your program to terminate when the error occurs. For details, see “What Can You Do about ESSL Computational Errors?” on page 48.

- For error 2110, return code 1 indicates that the subroutine exceeded IPARM(1) iterations without converging. Vector ***x*** contains the approximate solution computed at the last iteration.
- For error 2111, return code 2 indicates that *aux2* contains an incorrect factorization. The subroutine has been called with IPARM(3) = –10, and *aux2* contains an incomplete factorization of the input matrix ***A*** that was computed by a previous call to the subroutine when IPARM(3) = 10. This error indicates that *aux2* has been modified since the last call to the subroutine, or that the input matrix is not the same as the one that was factored. If the default action has been overridden, the subroutine can be called again with the same parameters, with the exception of IPARM(3) = 0 or 10.
- For error 2109, return code 3 indicates that the inner product (***y***,***Ay***) is negative in the iterative procedure after iteration *i*. This should not occur, because the input matrix is assumed to be positive or negative definite. Vector ***x*** contains the results of the last iteration. The value *i* is identified in the computational error message.
- For error 2108, return code 4 indicates that the matrix is not positive definite. AC is partially modified and does not represent the same matrix as on entry.

*Input-Argument Errors*

1. $iopt \neq$ 0 or 1
2. $m < 0$
3. $lda < 1$
4. $lda < m$
5. $nd < 0$
6. $nd = 0$ and $m > 0$
7. $|\lambda(i)| > m{-}1$ for $i = 1, nd$
8. IPARM(1) < 0
9. IPARM(2) ≠ 0, 1, or 2
10. IPARM(3) ≠ 0, 10, or –10
11. RPARM(1) < 0
12. RPARM(2) < 0
13. Error 2015 is recoverable or $naux1 \neq 0$, and *naux1* is too small—that is, less than the minimum required value. Return code 5 is returned if error 2015 is recoverable.
14. *naux2* is too small—that is, less than the minimum required value. Return code 5 is returned if error 2015 is recoverable.

***Example 1:*** This example finds the solution of the linear system ***Ax*** = ***b*** for sparse matrix ***A***, which is stored in compressed-diagonal storage mode in arrays AD and LA. The system is solved using the two-term conjugate gradient method. In this example, IOPT = 0.. Matrix ***A*** is:

```
┌                                                 ┐
│  2.0   0.0  -1.0   0.0   0.0   0.0   0.0   0.0   0.0 │
│  0.0   2.0   0.0  -1.0   0.0   0.0   0.0   0.0   0.0 │
│ -1.0   0.0   2.0   0.0  -1.0   0.0   0.0   0.0   0.0 │
│  0.0  -1.0   0.0   2.0   0.0  -1.0   0.0   0.0   0.0 │
│  0.0   0.0  -1.0   0.0   2.0   0.0  -1.0   0.0   0.0 │
│  0.0   0.0   0.0  -1.0   0.0   2.0   0.0  -1.0   0.0 │
│  0.0   0.0   0.0   0.0  -1.0   0.0   2.0   0.0  -1.0 │
│  0.0   0.0   0.0   0.0   0.0  -1.0   0.0   2.0   0.0 │
│  0.0   0.0   0.0   0.0   0.0   0.0  -1.0   0.0   2.0 │
└                                                 ┘
```

*Call Statement and Input*

```
             IOPT  M   ND  AD  LDA  LA   B   X   IPARM   RPARM   AUX1  NAUX1  AUX2  NAUX2
              |    |   |   |    |   |    |   |     |       |       |     |      |     |
CALL DSDCG( 0   , 9 , 3 , AD , 9 , LA , B , X , IPARM , RPARM , AUX1 , 283 , AUX2 ,  0  )
```

```
IPARM(1) =  20
IPARM(2) =  0
IPARM(3) =  0
RPARM(1) =  1.D-7
```

```
          ┌                ┐
          │ 2.0   0.0  -1.0 │
          │ 2.0   0.0  -1.0 │
          │ 2.0  -1.0  -1.0 │
          │ 2.0  -1.0  -1.0 │
AD    =   │ 2.0  -1.0  -1.0 │
          │ 2.0  -1.0  -1.0 │
          │ 2.0  -1.0  -1.0 │
          │ 2.0  -1.0   0.0 │
          │ 2.0  -1.0   0.0 │
          └                ┘
```

```
LA       =  (0, -2, 2)
B        =  (1.0, 1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 1.0, 1.0)
X        =  (0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0)
```

*Output*

```
X        =  (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0)
IPARM(4) =  5
RPARM(2) =  0
RPARM(3) =  0.46D-16
```

***Example 2:*** This example finds the solution of the linear system ***Ax*** = ***b*** for the same sparse matrix ***A*** as in Example 1, which is stored in compressed-diagonal storage mode in arrays AD and LA. The system is solved using the two-term conjugate gradient method. In this example, IOPT = 1, indicating that the matrix is symmetric, and only the main diagonal and one of each pair of identical diagonals are stored in array AD.

*Call Statement and Input*

```
             IOPT  M   ND  AD  LDA  LA   B   X   IPARM   RPARM   AUX1  NAUX1  AUX2  NAUX2
              |    |   |   |    |   |    |   |     |       |       |     |      |     |
CALL DSDCG( 1   , 9 , 2 , AD , 9 , LA , B , X , IPARM , RPARM , AUX1 , 283 , AUX2 , 80  )
```

```
IPARM(1) =  20
IPARM(2) =  0
IPARM(3) =  10
RPARM(1) =  1.D-7

        ┌            ┐
        | 2.0   0.0  |
        | 2.0   0.0  |
        | 2.0  -1.0  |
        | 2.0  -1.0  |
AD   =  | 2.0  -1.0  |
        | 2.0  -1.0  |
        | 2.0  -1.0  |
        | 2.0  -1.0  |
        | 2.0  -1.0  |
        └            ┘

LA       =  (0, -2)
B        =  (1.0, 1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 1.0, 1.0)
X        =  (0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0)
```

*Output*

```
X        =  (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0)
IPARM(4) =  1
RPARM(2) =  0
RPARM(3) =  0.89D-16
```

# DSMGCG—General Sparse Matrix Iterative Solve Using Compressed-Matrix Storage Mode

This subroutine solves a general sparse linear system of equations using an iterative algorithm, conjugate gradient squared or generalized minimum residual, with or without preconditioning by an incomplete LU factorization. The subroutine is suitable for positive real matrices—that is, when the symmetric part of the matrix, $(\boldsymbol{A}+\boldsymbol{A}^T)/2$, is positive definite. The sparse matrix is stored in compressed-matrix storage mode. Matrix ***A*** and vectors ***x*** and ***b*** are used:

$$\boldsymbol{Ax} = \boldsymbol{b}$$

where ***A***, ***x***, and ***b*** contain long-precision real numbers.

**Notes:**

1. These subroutines are provided only for migration purposes. You get better performance and a wider choice of algorithms if you use the DSRIS subroutine.
2. If your sparse matrix is stored by rows, as defined in “Storage-by-Rows” on page 99, you should first use the utility subroutine DSRSM to convert your sparse matrix to compressed-matrix storage mode. See “DSRSM—Convert a Sparse Matrix from Storage-by-Rows to Compressed-Matrix Storage Mode” on page 981.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL DSMGCG (*m*, *nz*, *ac*, *ka*, *lda*, *b*, *x*, *iparm*, *rparm*, *aux1*, *naux1*, *aux2*, *naux2*) |
| **C and C++** | dsmgcg (*m*, *nz*, *ac*, *ka*, *lda*, *b*, *x*, *iparm*, *rparm*, *aux1*, *naux1*, *aux2*, *naux2*); |
| **PL/I** | CALL DSMGCG (*m*, *nz*, *ac*, *ka*, *lda*, *b*, *x*, *iparm*, *rparm*, *aux1*, *naux1*, *aux2*, *naux2*); |

*On Entry*

*m*
is the order of the linear system ***Ax*** = ***b*** and the number of rows in sparse matrix ***A***. Specified as: a fullword integer; $m \ge 0$.

*nz*
is the maximum number of nonzero elements in each row of sparse matrix ***A***. Specified as: a fullword integer; $nz \ge 0$.

*ac*
is the array, referred to as AC, containing the values of the nonzero elements of the sparse matrix, stored in compressed-matrix storage mode. Specified as: an *lda* by (at least) *nz* array, containing long-precision real numbers.

*ka*
is the array, referred to as KA, containing the column numbers of the matrix ***A*** elements stored in the corresponding positions in array AC. Specified as: an *lda* by (at least) *nz* array, containing fullword integers, where 1 ≤ (elements of KA) ≤ *m*.

*lda*
is the leading dimension of the arrays specified for *ac* and *ka*. Specified as: a fullword integer; $lda > 0$ and $lda \ge m$.

*b*
is the vector ***b*** of length *m*, containing the right-hand side of the matrix problem. Specified as: a one-dimensional array of (at least) length *m*, containing long-precision real numbers.

*x*

is the vector ***x*** of length *m*, containing your initial guess of the solution of the linear system. Specified as: a one-dimensional array of (at least) length *m*, containing long-precision real numbers. The elements can have any value, and if no guess is available, the value can be zero.

*iparm*

is an array of parameters, IPARM(*i*), where:

- IPARM(1) controls the number of iterations.

  If IPARM(1) > 0, IPARM(1) is the maximum number of iterations allowed.

  If IPARM(1) = 0, the following default values are used:

  IPARM(1) = 300
  IPARM(2) = 0
  IPARM(3) = 10
  RPARM(1) = $10^{-6}$

- IPARM(2) is the flag used to select the iterative procedure used in this subroutine.

  If IPARM(2) = 0, the conjugate gradient squared method is used.

  If IPARM(2) = *k*, the generalized minimum residual method, restarted after *k* steps, is used. Note that the size of the work area *aux1* becomes larger as *k* increases. A value for *k* in the range of 5 to 10 is suitable for most problems.

- IPARM(3) is the flag that determines whether the system is to be preconditioned by an incomplete LU factorization with no fill-in.

  If IPARM(3) = 0, the system is not preconditioned.

  If IPARM(3) = 10, the system is preconditioned by an incomplete LU factorization.

  If IPARM(3) = –10, the system is preconditioned by an incomplete LU factorization, where the factorization matrix was computed in an earlier call to this subroutine and is stored in *aux2*.

- IPARM(4), see “On Return” on page 662.

Specified as: an array of (at least) length 4, containing fullword integers, where:
IPARM(1) ≥ 0
IPARM(2) ≥ 0
IPARM(3) = 0, 10, or –10

*rparm*

is an array of parameters, RPARM(*i*), where:

RPARM(1) > 0, is the relative accuracy $\epsilon$ used in the stopping criterion. The iterative procedure is stopped when:

$$\|\boldsymbol{b}-\boldsymbol{Ax}\|_2 \;/\; \|\boldsymbol{x}\|_2 < \varepsilon$$

RPARM(2) is reserved.

RPARM(3), see “On Return” on page 662.

Specified as: a one-dimensional array of (at least) length 3, containing long-precision real numbers.

*aux1*

has the following meaning:

If *naux1* = 0 and error 2015 is unrecoverable, *aux1* is ignored.

Otherwise, it is a storage work area used by this subroutine, which is available for use by the calling program between calls to this subroutine. Its size is specified by *naux1*.

Specified as: an area of storage, containing long-precision real numbers.

*naux1*

is the size of the work area specified by *aux1*—that is, the number of elements in *aux1*. Specified as: a fullword integer, where:

If *naux1* = 0 and error 2015 is unrecoverable, DSMGCG dynamically allocates the work area used by this subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, it must have at least the following value, where:

If `IPARM(2)` = 0, use $naux1 \geq 7m$.

If `IPARM(2)` > 0, use $naux1 \geq (k+2)m+k(k+4)+1$, where $k$ = `IPARM(2)`.

*aux2*

is the storage work area used by this subroutine. If `IPARM(3)` = –10, *aux2* must contain the incomplete LU factorization of matrix ***A***, computed in an earlier call to DSMGCG. The size of *aux2* is specified by *naux2*.

Specified as: an area of storage, containing long-precision real numbers.

*naux2*

is the size of the work area specified by *aux2*—that is, the number of elements in *aux2*. Specified as: a fullword integer. When `IPARM(3)` = 10, *naux2* must have at least the following value: $naux2 \geq 3+2m+1.5nz(m)$.

*On Return*

*x*

is the vector ***x*** of length *m*, containing the solution of the system ***Ax*** = ***b***. Returned as: a one-dimensional array of (at least) length *m*, containing long-precision real numbers.

*iparm*

is an array of parameters, `IPARM(`*i*`)`, where:

`IPARM(1)` is unchanged.

`IPARM(2)` is unchanged.

`IPARM(3)` is unchanged.

`IPARM(4)` contains the number of iterations performed by this subroutine.

Returned as: a one-dimensional array of length 4, containing fullword integers.

*rparm*

is an array of parameters, `RPARM(`*i*`)`, where:

`RPARM(1)` is unchanged.

`RPARM(2)` is reserved.

`RPARM(3)` contains the estimate of the error of the solution. If the process converged, `RPARM(3)` ≤ `RPARM(1)`

Returned as: a one-dimensional array of length 3, containing long-precision real numbers.

*aux2*

is the storage work area used by this subroutine.

If IPARM(3) = 10, *aux2* contains the incomplete LU factorization of matrix ***A***.

If IPARM(3) = –10, *aux2* is unchanged.

See “Notes” for additional information on *aux2*. Returned as: an area of storage, containing long-precision real numbers.

***Notes***

1. When IPARM(3) = –10, this subroutine uses the incomplete LU factorization in *aux2*, computed in an earlier call to this subroutine. When IPARM(3) = 10, this subroutine computes the incomplete LU factorization and stores it in *aux2*.
2. If you solve the same sparse linear system of equations several times with different right-hand sides using the preconditioned algorithm, specify IPARM(2) = 10 on the first invocation. The incomplete factorization is stored in *aux2*. You may save computing time on subsequent calls by setting IPARM(3) equal to –10. In this way, the algorithm reutilizes the incomplete factorization that was computed the first time. Therefore, you should not modify the contents of *aux2* between calls.
3. Matrix ***A*** must have no common elements with vectors ***x*** and ***b***; otherwise, results are unpredictable.
4. In the iterative solvers for sparse matrices, the relative accuracy ε (RPARM(1)) must be specified “reasonably” ($10^{-4}$ to $10^{-8}$). The algorithm computes a sequence of approximate solution vectors ***x*** that converge to the solution. The iterative procedure is stopped when the norm of the residual is sufficiently small—that is, when:

   $$\|\boldsymbol{b}-\boldsymbol{Ax}\|_2 \,/\, \|\boldsymbol{x}\|_2 < \varepsilon$$

   As a result, if you specify a larger ε, the algorithm takes fewer iterations to converge to a solution. If you specify a smaller ε, the algorithm requires more iterations and computer time, but converges to a more precise solution. If the value you specify is unreasonably small, the algorithm may fail to converge within the number of iterations it is allowed to perform.
5. For a description of how sparse matrices are stored in compressed-matrix storage mode, see “Compressed-Matrix Storage Mode” on page 93.
6. On output, array AC is not bitwise identical to what it was on input because the matrix ***A*** is scaled before starting the iterative process and is unscaled before returning control to the user.
7. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see “Using Auxiliary Storage in ESSL” on page 32.

***Function:*** The linear system:

$$\boldsymbol{Ax} = \boldsymbol{b}$$

is solved using either the conjugate gradient squared method or the generalized minimum residual method, with or without preconditioning by an incomplete LU factorization, where:

> ***A*** is a sparse matrix of order *m*, stored in compressed-matrix storage mode in arrays AC and KA.
> ***x*** is a vector of length *m*.
> ***b*** is a vector of length *m*.

See references [83 ] and [85 ]. [36 ].

If your program uses a sparse matrix stored by rows and you want to use this subroutine, first convert your sparse matrix to compressed-matrix storage mode by using the subroutine DSRSM described on page 981.

### *Error Conditions*

*Resource Errors:* Error 2015 is unrecoverable, *naux1* = 0, and unable to allocate work area.

*Computational Errors:* The following errors, with their corresponding return codes, can occur in this subroutine. For details on error handling, see “What Can You Do about ESSL Computational Errors?” on page 48.

- For error 2110, return code 1 indicates that the subroutine exceeded IPARM(1) iterations without converging. Vector ***x*** contains the approximate solution computed at the last iteration.
- For error 2111, return code 2 indicates that *aux2* contains an incorrect factorization. The subroutine has been called with IPARM(3) = –10, and *aux2* contains an incomplete factorization of the input matrix ***A*** that was computed by a previous call to the subroutine when IPARM(3) = 10. This error indicates that *aux2* has been modified since the last call to the subroutine, or that the input matrix is not the same as the one that was factored. If the default action has been overridden, the subroutine can be called again with the same parameters, with the exception of IPARM(3) = 0 or 10.
- For error 2112, return code 3 indicates that the incomplete LU factorization of ***A*** could not be completed, because one pivot was 0.
- For error 2116, return code 4 indicates that the matrix is singular, because all elements in one row of the matrix contain 0. Array AC is partially modified and does not represent the same matrix as on entry.

*Input-Argument Errors*

1. $m < 0$
2. $lda < 1$
3. $lda < m$
4. $nz < 0$
5. $nz = 0$ and $m > 0$
6. IPARM(1) < 0
7. IPARM(2) < 0
8. IPARM(3) ≠ 0, 10, or –10
9. RPARM(1) < 0
10. RPARM(2) < 0
11. Error 2015 is recoverable or *naux1*≠0, and *naux1* is too small—that is, less than the minimum required value. Return code 5 is returned if error 2015 is recoverable.

12. *naux2* is too small—that is, less than the minimum required value. Return code 5 is returned if error 2015 is recoverable.

***Example 1:*** This example finds the solution of the linear system ***Ax*** = ***b*** for the sparse matrix ***A***, which is stored in compressed-matrix storage mode in arrays AC and KA. The system is solved using the conjugate gradient squared method. Matrix ***A*** is:

```
┌                                                    ┐
│ 2.0  0.0   0.0  0.0   0.0   0.0   0.0   0.0   0.0 │
│ 0.0  2.0  -1.0  0.0   0.0   0.0   0.0   0.0   0.0 │
│ 0.0  1.0   2.0  0.0   0.0   0.0   0.0   0.0   0.0 │
│ 1.0  0.0   0.0  2.0  -1.0   0.0   0.0   0.0   0.0 │
│ 0.0  0.0   0.0  1.0   2.0  -1.0   0.0   0.0   0.0 │
│ 0.0  0.0   0.0  0.0   1.0   2.0  -1.0   0.0   0.0 │
│ 0.0  0.0   0.0  0.0   0.0   1.0   2.0  -1.0   0.0 │
│ 0.0  0.0   0.0  0.0   0.0   0.0   1.0   2.0  -1.0 │
│ 0.0  0.0   0.0  0.0   0.0   0.0   0.0   1.0   2.0 │
└                                                    ┘
```

**Note:** For input matrix KA, ( . ) indicates any value between 1 and 9.

*Call Statement and Input*

```
              M   NZ   AC   KA   LDA   B   X   IPARM   RPARM   AUX1   NAUX1   AUX2   NAUX2
              |   |    |    |     |    |   |     |       |       |      |       |      |
CALL DSMGCG( 9 ,  3 , AC , KA ,  9   , B , X , IPARM , RPARM , AUX1 ,  63   , AUX2 ,   0 )
```

```
IPARM(1) =  20
IPARM(2) =  0
IPARM(3) =  0
RPARM(1) =  1.D-7

            ┌                 ┐
            │ 2.0   0.0   0.0 │
            │ 2.0  -1.0   0.0 │
            │ 1.0   2.0   0.0 │
            │ 1.0   2.0  -1.0 │
AC    =     │ 1.0   2.0  -1.0 │
            │ 1.0   2.0  -1.0 │
            │ 1.0   2.0  -1.0 │
            │ 1.0   2.0  -1.0 │
            │ 1.0   2.0   0.0 │
            └                 ┘


            ┌         ┐
            │ 1  .  . │
            │ 2  3  . │
            │ 2  3  . │
            │ 1  4  5 │
KA    =     │ 4  5  6 │
            │ 5  6  7 │
            │ 6  7  8 │
            │ 7  8  9 │
            │ 8  9  . │
            └         ┘

B           =  (2.0, 1.0, 3.0, 2.0, 2.0, 2.0, 2.0, 2.0, 3.0)
X           =  (0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0)
```

*Output*

```
X        =  (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0)
IPARM(4) =  9
RPARM(3) =  0.150D-19
```

***Example 2:*** This example finds the solution of the linear system ***Ax*** = ***b*** for the same sparse matrix ***A*** as in Example 1, which is stored in compressed-matrix storage mode in arrays AC and KA. The system is solved using the generalized minimum residual method, restarted after 5 steps and preconditioned with an incomplete LU factorization. Most of the input is the same as in Example 1.

**Note:** For input matrix KA, ( . ) indicates any value between 1 and 9.

*Call Statement and Input*

```
             M   NZ   AC   KA   LDA   B   X   IPARM   RPARM   AUX1   NAUX1   AUX2   NAUX2
             |   |    |    |    |     |   |     |       |      |       |      |       |
CALL DSMGCG( 9 ,  3 , AC , KA ,  9  , B , X , IPARM , RPARM , AUX1 ,  109  , AUX2 ,  46 )
```

```
IPARM(1) =  20
IPARM(2) =  5
IPARM(3) =  10
RPARM(1) =  1.D-7
AC       =(same as input AC in Example 1)
KA       =(same as input KA in Example 1)
B        =  (2.0, 1.0, 3.0, 2.0, 2.0, 2.0, 2.0, 2.0, 3.0)
X        =  (0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0)
```

*Output*

```
X        =  (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0)
IPARM(4) =  2
RPARM(3) =  0.290D-15
```

# DSDGCG—General Sparse Matrix Iterative Solve Using Compressed-Diagonal Storage Mode

This subroutine solves a general sparse linear system of equations using an iterative algorithm, conjugate gradient squared or generalized minimum residual, with or without preconditioning by an incomplete LU factorization. The subroutine is suitable for positive real matrices—that is, when the symmetric part of the matrix, $(\boldsymbol{A}+\boldsymbol{A}^T)/2$, is positive definite. The sparse matrix is stored in compressed-diagonal storage mode. Matrix ***A*** and vectors ***x*** and ***b*** are used:

$$\boldsymbol{Ax} = \boldsymbol{b}$$

where ***A***, ***x***, and ***b*** contain long-precision real numbers.

***Syntax***

| | |
|---|---|
| **Fortran** | CALL DSDGCG (*m*, *nd*, *ad*, *lda*, *la*, *b*, *x*, *iparm*, *rparm*, *aux1*, *naux1*, *aux2*, *naux2*) |
| **C and C++** | dsdgcg (*m*, *nd*, *ad*, *lda*, *la*, *b*, *x*, *iparm*, *rparm*, *aux1*, *naux1*, *aux2*, *naux2*); |
| **PL/I** | CALL DSDGCG (*m*, *nd*, *ad*, *lda*, *la*, *b*, *x*, *iparm*, *rparm*, *aux1*, *naux1*, *aux2*, *naux2*); |

*On Entry*

*m*

is the order of the linear system $\boldsymbol{Ax} = \boldsymbol{b}$ and the number of rows in sparse matrix ***A***. Specified as: a fullword integer; $m \ge 0$.

*nd*

is the number of nonzero diagonals stored in the columns of array AD, the number of columns in array AD, and the number of elements in array LA. Specified as: a fullword integer; it must have the following value, where:

If $m > 0$, then $nd > 0$.

If $m = 0$, then $nd \ge 0$.

*ad*

is the array, referred to as AD, containing the values of the nonzero elements of the sparse matrix, stored in compressed-matrix storage mode. Specified as: an *lda* by (at least) *nd* array, containing long-precision real numbers.

*lda*

is the leading dimension of the arrays specified for *ad*. Specified as: a fullword integer; $lda > 0$ and $lda \ge m$.

*la*

is the array, referred to as LA, containing the diagonal numbers *k* for the diagonals stored in each corresponding column in array AD. For an explanation of how diagonal numbers are stored, see "Compressed-Diagonal Storage Mode" on page 94.

Specified as: a one-dimensional array of (at least) length *nd*, containing fullword integers, where $1-m \le$ (elements of LA) $\le (m-1)$.

*b*

is the vector ***b*** of length *m*, containing the right-hand side of the matrix problem. Specified as: a one-dimensional array of (at least) length *m*, containing long-precision real numbers.

*x*

is the vector ***x*** of length *m*, containing your initial guess of the solution of the linear system. Specified as: a one-dimensional array of (at least) length *m*, containing long-precision real numbers. The elements can have any value, and if no guess is available, the value can be zero.

*iparm*

is an array of parameters, IPARM(*i*), where:

- IPARM(1) controls the number of iterations.

  If IPARM(1) > 0, IPARM(1) is the maximum number of iterations allowed.

  If IPARM(1) = 0, the following default values are used:

  IPARM(1) = 300
  IPARM(2) = 0
  IPARM(3) = 10
  RPARM(1) = $10^{-6}$

- IPARM(2) is the flag used to select the iterative procedure used in this subroutine.

  If IPARM(2) = 0, the conjugate gradient squared method is used.

  If IPARM(2) = *k*, the generalized minimum residual method, restarted after *k* steps, is used. Note that the size of the work area *aux1* becomes larger as *k* increases. A value for *k* in the range of 5 to 10 is suitable for most problems.

- IPARM(3) is the flag that determines whether the system is to be preconditioned by an incomplete LU factorization with no fill-in.

  If IPARM(3) = 0, the system is not preconditioned.

  If IPARM(3) = 10, the system is preconditioned by an incomplete LU factorization.

  If IPARM(3) = –10, the system is preconditioned by an incomplete LU factorization, where the factorization matrix was computed in an earlier call to this subroutine and is stored in *aux2*.

- IPARM(4), see “On Return” on page 669.

Specified as: an array of (at least) length 4, containing fullword integers, where:
IPARM(1) ≥ 0
IPARM(2) ≥ 0
IPARM(3) = 0, 10, or –10

*rparm*

is an array of parameters, RPARM(*i*), where:

If RPARM(1) > 0, is the relative accuracy $\epsilon$ used in the stopping criterion. The iterative procedure is stopped when:

$$\|\boldsymbol{b}-\boldsymbol{Ax}\|_2 / \|\boldsymbol{x}\|_2 < \varepsilon$$

RPARM(2) is reserved.

RPARM(3), see “On Return” on page 669.

Specified as: a one-dimensional array of (at least) length 3, containing long-precision real numbers.

*aux1*

has the following meaning:

If *naux1* = 0 and error 2015 is unrecoverable, *aux1* is ignored.

Otherwise, it is a storage work area used by this subroutine, which is available for use by the calling program between calls to this subroutine. Its size is specified by *naux1*.

Specified as: an area of storage, containing long-precision real numbers.

*naux1*

is the size of the work area specified by *aux1*—that is, the number of elements in *aux1*. Specified as: a fullword integer, where:

If *naux1* = 0 and error 2015 is unrecoverable, DSDGCG dynamically allocates the work area used by this subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, *naux1* > 0 and must have at least the following value, where:

If IPARM(2) = 0, use *naux1* ≥ 7*m*.

If IPARM(2) > 0, use *naux1* ≥ (*k*+2)*m*+*k*(*k*+4)+1, where *k* = PARM(2).

*aux2*

is a storage work area used by this subroutine. If IPARM(3) = –10, *aux2* must contain the incomplete LU factorization of matrix ***A***, computed in an earlier call to DSDGCG. The size of *aux2* is specified by *naux2*.

Specified as: an area of storage, containing long-precision real numbers.

*naux2*

is the size of the work area specified by *aux2*—that is, the number of elements in *aux2*. Specified as: a fullword integer. When IPARM(3) = 10 or –10, *naux2* must have at least the following value: *naux2* ≥ 3+2*m*+1.5*nd*(*m*).

*On Return*

*x*

is the vector ***x*** of length *m*, containing the solution of the system ***Ax*** = ***b***. Returned as: a one-dimensional array of (at least) length *m*, containing long-precision real numbers.

*iparm*

is an array of parameters, IPARM(*i*), where:

IPARM(1) is unchanged.

IPARM(2) is unchanged.

IPARM(3) is unchanged.

IPARM(4) contains the number of iterations performed by this subroutine.

Returned as: a one-dimensional array of length 4, containing fullword integers.

*rparm*

is an array of parameters, RPARM(*i*), where:

RPARM(1) is unchanged.

RPARM(2) is reserved.

RPARM(3) contains the estimate of the error of the solution. If the process converged, RPARM(3) ≤ RPARM(1).

Returned as: a one-dimensional array of length 3, containing long-precision real numbers.

*aux2*

is the storage work area used by this subroutine.

If IPARM(3) = 10, *aux2* contains the incomplete LU factorization of matrix ***A***.

If IPARM(3) = –10, *aux2* is unchanged.

See “Notes” for additional information on *aux2*. Returned as: an area of storage, containing long-precision real numbers.

***Notes***

1. When IPARM(3) = –10, this subroutine uses the incomplete LU factorization in *aux2*, computed in an earlier call to this subroutine. When IPARM(3) = 10, this subroutine computes the incomplete LU factorization and stores it in *aux2*.
2. If you solve the same sparse linear system of equations several times with different right-hand sides, using the preconditioned algorithm, specify IPARM(3) = 10 on the first invocation. The incomplete factorization is stored in *aux2*. You may save computing time on subsequent calls by setting IPARM(3) = –10. In this way, the algorithm reutilizes the incomplete factorization that was computed the first time. Therefore, you should not modify the contents of *aux2* between calls.
3. Matrix ***A*** must have no common elements with vectors ***x*** and ***b***; otherwise, results are unpredictable.
4. In the iterative solvers for sparse matrices, the relative accuracy ε (RPARM(1)) must be specified “reasonably” ($10^{-4}$ to $10^{-8}$). The algorithm computes a sequence of approximate solution vectors ***x*** that converge to the solution. The iterative procedure is stopped when the norm of the residual is sufficiently small—that is, when:

   $$\|\boldsymbol{b}-\boldsymbol{Ax}\|_2 \,/\, \|\boldsymbol{x}\|_2 < \varepsilon$$

   As a result, if you specify a larger ε, the algorithm takes fewer iterations to converge to a solution. If you specify a smaller ε, the algorithm requires more iterations and computer time, but converges to a more precise solution. If the value you specify is unreasonably small, the algorithm may fail to converge within the number of iterations it is allowed to perform.
5. For a description of how sparse matrices are stored in compressed-diagonal storage mode, see “Compressed-Diagonal Storage Mode” on page 94.
6. On output, array AD is not bitwise identical to what it was on input, because matrix ***A*** is scaled before starting the iterative process and is unscaled before returning control to the user.
7. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see “Using Auxiliary Storage in ESSL” on page 32.

***Function:*** The linear system:

$$\boldsymbol{Ax} = \boldsymbol{b}$$

is solved using either the conjugate gradient squared method or the generalized minimum residual method, with or without preconditioning by an incomplete LU factorization, where:

***A*** is a sparse matrix of order *m*, stored in compressed-diagonal storage mode in arrays AD and LA.
***x*** is a vector of length *m*.
***b*** is a vector of length *m*.

See references [83 ] and [85 ]. [36 ].

### *Error Conditions*

*Resource Errors:* Error 2015 is unrecoverable, *naux1* = 0, and unable to allocate work area.

*Computational Errors:* The following errors, with their corresponding return codes, can occur in this subroutine. For details on error handling, see "What Can You Do about ESSL Computational Errors?" on page 48.

- For error 2110, return code 1 indicates that the subroutine exceeded IPARM(1) iterations without converging. Vector ***x*** contains the approximate solution computed at the last iteration.
- For error 2111, return code 2 indicates that *aux2* contains an incorrect factorization. The subroutine has been called with IPARM(3) = –10, and *aux2* contains an incomplete factorization of the input matrix ***A*** that was computed by a previous call to the subroutine when IPARM(3) = 10. This error indicates that *aux2* has been modified since the last call to the subroutine, or that the input matrix is not the same as the one that was factored. If the default action has been overridden, the subroutine can be called again with the same parameters, with the exception of IPARM(3) = 0 or 10.
- For error 2112, return code 3 indicates that the incomplete LU factorization of ***A*** could not be completed, because one pivot was 0.
- For error 2116, return code 4 indicates that the matrix is singular, because all elements in one row of the matrix contain 0. Array AC is partially modified and does not represent the same matrix as on entry.

*Input-Argument Errors*

1. *m* < 0
2. *lda* < 1
3. *lda* < *m*
4. *nd* < 0
5. *nd* = 0 and *m* > 0
6. IPARM(1) < 0
7. IPARM(2) < 0
8. IPARM(3) ≠ 0, 10, or –10
9. RPARM(1) < 1.D0
10. Error 2015 is recoverable or *naux1*≠0, and *naux1* is too small—that is, less than the minimum required value. Return code 5 is returned if error 2015 is recoverable.
11. *naux2* is too small—that is, less than the minimum required value. Return code 5 is returned if error 2015 is recoverable.

***Example 1:*** This example finds the solution of the linear system ***Ax*** = ***b*** for the sparse matrix ***A***, which is stored in compressed-diagonal storage mode in arrays AD and LA. The system is solved using the conjugate gradient squared method. Matrix ***A*** is:

```
                        ┌                                                  ┐
                        | 2.0  0.0  -1.0   0.0   0.0   0.0   0.0   0.0   0.0 |
                        | 0.0  2.0   0.0  -1.0   0.0   0.0   0.0   0.0   0.0 |
                        | 0.0  0.0   2.0   0.0  -1.0   0.0   0.0   0.0   0.0 |
                        | 0.0  0.0   0.0   2.0   0.0  -1.0   0.0   0.0   0.0 |
                        | 1.0  0.0   0.0   0.0   2.0   0.0  -1.0   0.0   0.0 |
                        | 0.0  1.0   0.0   0.0   0.0   2.0   0.0  -1.0   0.0 |
                        | 0.0  0.0   1.0   0.0   0.0   0.0   2.0   0.0  -1.0 |
                        | 0.0  0.0   0.0   1.0   0.0   0.0   0.0   2.0   0.0 |
                        | 0.0  0.0   0.0   0.0   1.0   0.0   0.0   0.0   2.0 |
                        └                                                  ┘
```

*Call Statement and Input*

```
         M   ND   AD   LDA   LA   B   X   IPARM   RPARM   AUX1   NAUX1   AUX2   NAUX2
         |   |    |    |     |    |   |     |       |       |      |       |      |
DSDGCG( 9 ,  3 , AD ,  9  , LA , B , X , IPARM , RPARM , AUX1 ,  63   , AUX2 ,   0 )
```

```
IPARM(1) =  20
IPARM(2) =  0
IPARM(3) =  0
RPARM(1) =  1.D-7
```

```
            ┌                ┐
            | 2.0  -1.0  0.0 |
            | 2.0  -1.0  0.0 |
            | 2.0  -1.0  0.0 |
            | 2.0  -1.0  0.0 |
AD   =      | 2.0  -1.0  1.0 |
            | 2.0  -1.0  1.0 |
            | 2.0  -1.0  1.0 |
            | 2.0   0.0  1.0 |
            | 2.0   0.0  1.0 |
            └                ┘
```

```
LA       =  (0, 2, -4)
B        =  (1, 1, 1, 1, 2, 2, 2, 3, 3)
X        =  (0, 0, 0, 0, 0, 0, 0, 0, 0)
```

*Output*

```
X        =  (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0)
IPARM(4) =  8
RPARM(3) =  0.308D-17
```

***Example 2:*** This example finds the solution of the linear system ***Ax*** = ***b*** for the same sparse matrix ***A*** as in Example 1, which is stored in compressed-diagonal storage mode in arrays AD and LA. The system is solved using the generalized minimum residual method, restarted after 5 steps and preconditioned with an incomplete LU factorization. Most of the input is the same as in Example 1.

*Call Statement and Input*

```
              M   ND   AD   LDA   LA   B   X   IPARM   RPARM   AUX1   NAUX1   AUX2   NAUX2
              |   |    |    |     |    |   |     |       |       |      |       |      |
CALL DSDGCG( 9 ,  3 , AD ,  9  , LA , B , X , IPARM , RPARM , AUX1 ,  109  , AUX2 ,  46 )
```

```
IPARM(1) =  20
IPARM(2) =  5
IPARM(3) =  10
RPARM(1) =  1.D-7
AD       =(same as input AD in Example 1)
LA       =(same as input LA in Example 1)
B        =  (1, 1, 1, 1, 2, 2, 2, 3, 3)
X        =  (0, 0, 0, 0, 0, 0, 0, 0, 0)
```

*Output*

```
X        =  (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0)
IPARM(4) =  6
RPARM(3) =  0.250D-15
```

# Linear Least Squares Subroutines

This section contains the linear least squares subroutine descriptions.

# SGESVF and DGESVF—Singular Value Decomposition for a General Matrix

These subroutines compute the singular value decomposition of general matrix ***A*** in preparation for solving linear least squares problems. To compute the minimal norm linear least squares solution of ***AX*** ≅ ***B***, follow the call to these subroutines with a call to SGESVS or DGESVS, respectively.

| *Table 114. Data Types* | |
|---|---|
| ***A***, ***B***, ***s***, *aux* | **Subroutine** |
| Short-precision real | SGESVF |
| Long-precision real | DGESVF |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SGESVF \| DGESVF (*iopt*, *a*, *lda*, *b*, *ldb*, *nb*, *s*, *m*, *n*, *aux*, *naux*) |
| **C and C++** | sgesvf \| dgesvf (*iopt*, *a*, *lda*, *b*, *ldb*, *nb*, *s*, *m*, *n*, *aux*, *naux*); |
| **PL/I** | CALL SGESVF \| DGESVF (*iopt*, *a*, *lda*, *b*, *ldb*, *nb*, *s*, *m*, *n*, *aux*, *naux*); |

*On Entry*

*iopt*

indicates the type of computation to be performed, where:

If *iopt* = 0 or 10, singular values are computed.

If *iopt* = 1 or 11, singular values and ***V*** are computed.

If *iopt* = 2 or 12, singular values, ***V***, and $\boldsymbol{U}^T\boldsymbol{B}$ are computed.

Specified as: a fullword integer; *iopt* = 0, 1, 2, 10, 11, or 12.

If *iopt* < 10, singular values are unordered.

If *iopt* ≥ 10, singular values are sorted in descending order and, if applicable, the columns of ***V*** and the rows of $\boldsymbol{U}^T\boldsymbol{B}$ are swapped to correspond to the sorted singular values.

*a*

is the *m* by *n* general matrix ***A***, whose singular value decomposition is to be computed. Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 114.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; *lda* > 0 and *lda* ≥ max(*m*, *n*).

*b*

has the following meaning, where:

If *iopt* = 0, 1, 10, or 11, this argument is not used in the computation.

If *iopt* = 2 or 12, it is the *m* by *nb* matrix ***B***.

Specified as: an *ldb* by (at least) *nb* array, containing numbers of the data type indicated in Table 114.

If this subroutine is followed by a call to SGESVS or DGESVS, ***B*** should contain the right-hand side of the linear least squares problem, ***AX*** ≅ ***B***. (The *nb* column vectors of ***B*** contain right-hand sides for *nb* distinct linear least squares problems.) However, if the matrix $\boldsymbol{U}^T$ is desired on output, ***B*** should be equal to the identity matrix of order *m*.

*ldb*

has the following meaning, where:

If *iopt* = 0, 1, 10, or 11, this argument is not used in the computation.

If *iopt* = 2 or 12, it is the leading dimension of the array specified for *b*.

Specified as: a fullword integer. It must have the following values, where:

If *iopt* = 0, 1, 10, or 11, *ldb* > 0.

If *iopt* = 2 or 12, *ldb* > 0 and *ldb* ≥ max(*m*, *n*).

*nb*

has the following meaning, where:

If *iopt* = 0, 1, 10, or 11, this argument is not used in the computation.

If *iopt* = 2 or 12, it is the number of columns in matrix ***B***.

Specified as: a fullword integer; if *iopt* = 2 or 12, *nb* > 0.

*s*

See "On Return."

*m*

is the number of rows in matrices ***A*** and ***B***. Specified as: a fullword integer; *m* ≥ 0.

*n*

is the number of columns in matrix ***A*** and the number of elements in vector ***s***. Specified as: a fullword integer; *n* ≥ 0.

*aux*

has the following meaning:

If *naux* = 0 and error 2015 is unrecoverable, *aux* is ignored.

Otherwise, it is the storage work area used by this subroutine. Its size is specified by *naux*.

Specified as: an area of storage, containing numbers of the data type indicated in Table 114 on page 675.

*naux*

is the size of the work area specified by *aux*—that is, the number of elements in *aux*. Specified as: a fullword integer, where:

If *naux* = 0 and error 2015 is unrecoverable, SGESVF and DGESVF dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, It must have the following value, where:

If *iopt* = 0 or 10, *naux* ≥ *n*+max(*m*, *n*).

If *iopt* = 1 or 11, *naux* ≥ 2*n*+max(*m*, *n*).

If *iopt* = 2 or 12, *naux* ≥ 2*n*+max(*m*, *n*, *nb*).

*On Return*

*a*

has the following meaning, where:

If *iopt* = 0, or 10, ***A*** is overwritten; that is, the original input is not preserved.

If *iopt* = 1, 2, 11, or 12, ***A*** contains the real orthogonal matrix ***V***, of order *n*, in its first *n* rows and *n* columns. If *iopt* = 11 or 12, the columns of ***V*** are swapped to correspond to the sorted singular values. If *m* > *n*, rows *n*+1, *n*+2, ..., *m* of array A are overwritten; that is, the original input is not preserved.

Returned as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 114 on page 675.

*b*

has the following meaning, where:

If *iopt* = 0, 1, 10, or 11, ***B*** is not used in the computation.

If *iopt* = 2 or 12, ***B*** is overwritten by the *n* by *nb* matrix $\boldsymbol{U}^T\boldsymbol{B}$.

If *iopt* = 12, the rows of $\boldsymbol{U}^T\boldsymbol{B}$ are swapped to correspond to the sorted singular values. If *m* > *n*, rows *n*+1, *n*+2, ..., *m* of array B are overwritten; that is, the original input is not preserved.

Returned as: an *ldb* by (at least) *nb* array, containing numbers of the data type indicated in Table 114 on page 675.

*s*

is a the vector ***s*** of length *n*, containing the singular values of matrix ***A***. Returned as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 114 on page 675; $s_i \ge 0$, where:

If *iopt* < 10, the singular values are unordered in ***s***.

If *iopt* ≥ 10, the singular values are sorted in descending order in ***s***; that is, $s_1 \ge s_2 \ge ... \ge s_n \ge 0$. If applicable, the columns of ***V*** and the rows of $\boldsymbol{U}^T\boldsymbol{B}$ are swapped to correspond to the sorted singular values.

***Notes***

1. The following items must have no common elements; otherwise, results are unpredictable: matrices ***A*** and ***B***, vector ***s***, and the data area specified for *aux*.
2. When you specify *iopt* = 0, 1, 10, or 11, you must also specify:
   - A dummy argument for *b*
   - A positive value for *ldb*

   See “Example” on page 562.
3. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see “Using Auxiliary Storage in ESSL” on page 32.

***Function:*** The singular value decomposition of a real general matrix is computed as follows:

$$\boldsymbol{A} = \boldsymbol{U}\Sigma\boldsymbol{V}^T$$

where:

$\boldsymbol{U}^T\boldsymbol{U} = \boldsymbol{V}^T\boldsymbol{V} = \boldsymbol{V}\boldsymbol{V}^T = \boldsymbol{I}$
***A*** is an *m* by *n* real general matrix.
***V*** is a real general orthogonal matrix of order *n*. On output, ***V*** overwrites the first *n* rows and *n* columns of ***A***.
$\boldsymbol{U}^T\boldsymbol{B}$ is an *n* by *nb* real general matrix. On output, $\boldsymbol{U}^T\boldsymbol{B}$ overwrites the first *n* rows and *nb* columns of ***B***.
$\Sigma$ is an *n* by *n* real diagonal matrix. The diagonal elements of $\Sigma$ are the singular values of ***A***, returned in the vector ***s***.

If *m* or *n* is equal to 0, no computation is performed.

One of the following algorithms is used:

1. Golub-Reinsch Algorithm (See pages 134 to 151 in reference [96 ].)
   a. Reduce the real general matrix ***A*** to bidiagonal form using Householder transformations.
   b. Iteratively reduce the bidiagonal form to diagonal form using a variant of the QR algorithm.
2. Chan Algorithm (See reference [13 ].)
   a. Compute the QR decomposition of matrix ***A*** using Householder transformations; that is, ***A*** = ***QR***.
   b. Apply the Golub-Reinsch Algorithm to the matrix ***R***.

   If ***R*** = ***XWY***$^{\mathrm{T}}$ is the singular value decomposition of ***R***, the singular value decomposition of matrix ***A*** is given by:

$$\boldsymbol{A} = \boldsymbol{Q}\begin{bmatrix}\boldsymbol{X}\\0\end{bmatrix}\boldsymbol{WY}^{\mathrm{T}}$$

where:

$$\boldsymbol{U} = \boldsymbol{Q}\begin{bmatrix}\boldsymbol{X}\\0\end{bmatrix}$$

$$\Sigma = \boldsymbol{W}$$

$$\boldsymbol{V} = \boldsymbol{Y}$$

Also, see references [13 ], [56 ], [75 ], and pages 134 to 151 in reference [96]. These algorithms have a tendency to generate underflows that may hurt overall performance. The system default is to mask underflow, which improves the performance of these subroutines.

### Error Conditions

*Resource Errors:* Error 2015 is unrecoverable, *naux* = 0, and unable to allocate work area.

*Computational Errors:* Singular value (*i*) failed to converge after (*x*) iterations.

- The singular values ($s_j$, $j$ = $n$, $n$–1, ..., $i$+1) are correct. If *iopt* < 10, they are unordered. Otherwise, they are ordered.
- *a* has been modified.
- If *iopt* = 2 or 12, then *b* has been modified.
- The return code is set to 1.
- *i* and *x* can be determined at run time by use of the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error code 2107 in the ESSL error option table;

otherwise, the default value causes your program to terminate when this error occurs. See “What Can You Do about ESSL Computational Errors?” on page 48.

*Input-Argument Errors*

1. *iopt* ≠ 0, 1, 2, 10, 11, or 12
2. *lda* ≤ 0
3. max(*m*, *n*) > *lda*
4. *ldb* ≤ 0 and *iopt* = 2, 12
5. max(*m*, *n*) > *ldb* and *iopt* = 2, 12
6. *nb* ≤ 0 and *iopt* = 2, 12
7. *m* < 0
8. *n* < 0
9. Error 2015 is recoverable or *naux*≠0, and *naux* is too small—that is, less than the minimum required value. Return code 2 is returned if error 2015 is recoverable.

***Example 1:*** This example shows how to find only the singular values, ***s***, of a real long-precision general matrix ***A***, where:

- M is greater than N.
- NAUX is greater than or equal to N+max(M, N) = 7.
- LDB has been set to 1 to avoid a Fortran error message.
- DUMMY is a placeholder for argument *b*, which is not used in the computation.
- The singular values are returned in S.
- On output, matrix ***A*** is overwritten; that is, the original input is not preserved.

*Call Statement and Input*

```
            IOPT  A  LDA    B    LDB  NB  S   M   N   AUX  NAUX
             |    |   |     |     |   |   |   |   |    |    |
CALL DGESVF( 0  , A , 4 , DUMMY , 1 , 0 , S , 4 , 3 , AUX , 7  )


        ┌                  ┐
        |  1.0   2.0   3.0 |
A    =  |  4.0   5.0   6.0 |
        |  7.0   8.0   9.0 |
        | 10.0  11.0  12.0 |
        └                  ┘
```

*Output*

```
S       =   (25.462, 1.291, 0.000)
```

***Example 2:*** This example computes the singular values, ***s***, of a real long-precision general matrix ***A*** and the matrix ***V***, where:

- M is equal to N.
- NAUX is greater than or equal to 2N+max(M, N) = 9.
- LDB has been set to 1 to avoid a Fortran error message.
- DUMMY is a placeholder for argument *b*, which is not used in the computation.
- The singular values are returned in S.
- The matrix ***V*** is returned in A.

*Call Statement and Input*

```
            IOPT  A  LDA    B    LDB  NB  S   M   N   AUX  NAUX
              |   |   |     |     |   |   |   |   |    |    |
CALL DGESVF( 1  , A , 3 , DUMMY , 1 , 0 , S , 3 , 3 , AUX , 9  )
```

```
        ┌                ┐
        │  2.0  1.0  1.0 │
A    =  │  4.0  1.0  0.0 │
        │ -2.0  2.0  1.0 │
        └                ┘
```

*Output*

```
        ┌                        ┐
        │ -0.994   0.105  -0.041 │
A    =  │ -0.112  -0.870   0.480 │
        │ -0.015  -0.482  -0.876 │
        └                        ┘
```

```
S        =    (4.922, 2.724, 0.597)
```

***Example 3:*** This example computes the singular values, ***s***, and computes matrices ***V*** and ***U***$^{T}$***B*** in preparation for solving the underdetermined system ***AX*** ≅ ***B***, where:

- `M` is less than `N`.
- `NAUX` is greater than or equal to 2`N`+max(`M`, `N`, `NB`) = 9.
- The singular values are returned in `S`.
- The matrix ***V*** is returned in `A`.
- The matrix ***U***$^{T}$***B*** is returned in `B`.

*Call Statement and Input*

```
            IOPT  A  LDA  B  LDB  NB  S   M   N   AUX  NAUX
              |   |   |   |   |   |   |   |   |    |    |
CALL DGESVF( 2  , A , 3 , B , 3 , 1 , S , 2 , 3 , AUX , 9  )
```

```
        ┌               ┐
        │ 1.0  2.0  2.0 │
A    =  │ 2.0  4.0  5.0 │
        │  .    .    .  │
        └               ┘
```

```
        ┌     ┐
        │ 1.0 │
B    =  │ 4.0 │
        │  .  │
        └     ┘
```

*Output*

```
                ┌                            ┐
                │ -0.304  -0.894   0.328 │
A      =        │ -0.608   0.447   0.656 │
                │ -0.733   0.000  -0.680 │
                └                            ┘
```

```
                ┌          ┐
                │ -4.061 │
B      =        │  0.000 │
                │ -0.714 │
                └          ┘
```

```
S          =    (7.342, 0.000, 0.305)
```

***Example 4:*** This example computes the singular values, ***s***, and matrices ***V*** and ***U***$^T$***B*** in preparation for solving the overdetermined system ***AX*** ≅ ***B***, where:

- M is greater than N.
- NAUX is greater than or equal to 2N+max(M, N, NB) = 7.
- The singular values are returned in S.
- The matrix ***V*** is returned in A.
- The matrix ***U***$^T$***B*** is returned in B.

*Call Statement and Input*

```
            IOPT   A  LDA  B  LDB  NB  S   M   N   AUX  NAUX
             |     |   |   |   |   |   |   |   |    |    |
CALL DGESVF( 2   , A , 3 , B , 3 , 2 , S , 3 , 2 , AUX , 7   )
```

```
                ┌            ┐
                │ 1.0  4.0 │
A      =        │ 2.0  5.0 │
                │ 3.0  6.0 │
                └            ┘
```

```
                ┌              ┐
                │ 7.0  10.0 │
B      =        │ 8.0  11.0 │
                │ 9.0  12.0 │
                └              ┘
```

*Output*

```
                ┌                   ┐
                │  0.922  -0.386 │
A      =        │ -0.386  -0.922 │
                │    .       .    │
                └                   ┘
```

```
                ┌                       ┐
                │  -1.310   -2.321 │
B      =        │ -13.867  -18.963 │
                │     .        .    │
                └                       ┘
```

```
X          =    (0.773, 9.508)
```

***Example 5:*** This example computes the singular values, ***s***, and matrices ***V*** and $\boldsymbol{U^TB}$ in preparation for solving the overdetermined system $\boldsymbol{AX} \cong \boldsymbol{B}$. The singular values are sorted in descending order, and the columns of ***V*** and the rows of $\boldsymbol{U^TB}$ are swapped to correspond to the sorted singular values.

- M is greater than N.
- NAUX is greater than or equal to 2N+max(M, N, NB) = 7.
- The singular values are returned in S.
- The matrix ***V*** is returned in A.
- The matrix $\boldsymbol{U^TB}$ is returned in B.

*Call Statement and Input*

```
            IOPT  A  LDA  B  LDB  NB  S   M   N   AUX  NAUX
             |    |   |   |   |   |   |   |   |    |    |
CALL DGESVF( 12 , A , 3 , B , 3 , 2 , S , 3 , 2 , AUX , 7  )

        ┌          ┐
        | 1.0  4.0 |
A    =  | 2.0  5.0 |
        | 3.0  6.0 |
        └          ┘

        ┌           ┐
        | 7.0  10.0 |
B    =  | 8.0  11.0 |
        | 9.0  12.0 |
        └           ┘
```

*Output*

```
        ┌                ┐
        | -0.386   0.922 |
A    =  | -0.922  -0.386 |
        |   .       .    |
        └                ┘

        ┌                  ┐
        | -13.867  -18.963 |
B    =  |  -1.310   -2.321 |
        |    .        .    |
        └                  ┘

S        =   (9.508, 0.773)
```

## SGESVS and DGESVS—Linear Least Squares Solution for a General Matrix Using the Singular Value Decomposition

These subroutines compute the minimal norm linear least squares solution of $\boldsymbol{AX} \cong \boldsymbol{B}$, where $\boldsymbol{A}$ is a general matrix, using the singular value decomposition computed by SGESVF or DGESVF.

*Table 115. Data Types*

| ***V***, ***UB***, ***X***, ***s***, $\tau$ | **Subroutine** |
|---|---|
| Short-precision real | SGESVS |
| Long-precision real | DGESVS |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SGESVS \| DGESVS (*v*, *ldv*, *ub*, *ldub*, *nb*, *s*, *x*, *ldx*, *m*, *n*, *tau*) |
| **C and C++** | sgesvs \| dgesvs (*v*, *ldv*, *ub*, *ldub*, *nb*, *s*, *x*, *ldx*, *m*, *n*, *tau*); |
| **PL/I** | CALL SGESVS \| DGESVS (*v*, *ldv*, *ub*, *ldub*, *nb*, *s*, *x*, *ldx*, *m*, *n*, *tau*); |

*On Entry*

*v*

is the orthogonal matrix $\boldsymbol{V}$ of order *n* in the singular value decomposition of matrix $\boldsymbol{A}$. It is produced by a preceding call to SGESVF or DGESVF, where it corresponds to output argument *a*.

Specified as: an *ldv* by (at least) *n* array, containing numbers of the data type indicated in Table 115.

*ldv*

is the leading dimension of the array specified for *v*. Specified as: a fullword integer; $ldv > 0$ and $ldv \ge n$.

*ub*

is an *n* by *nb* matrix, containing $\boldsymbol{U}^T\boldsymbol{B}$. It is produced by a preceding call to SGESVF or DGESVF, where it corresponds to output argument *b*. On output, $\boldsymbol{U}^T\boldsymbol{B}$ is overwritten; that is, the original input is not preserved.

Specified as: an *ldub* by (at least) *nb* array, containing numbers of the data type indicated in Table 115.

*ldub*

is the leading dimension of the array specified for *ub*. Specified as: a fullword integer; $ldub > 0$ and $ldub \ge n$.

*nb*

is the number of columns in matrices $\boldsymbol{X}$ and $\boldsymbol{U}^T\boldsymbol{B}$. Specified as: a fullword integer; $nb > 0$.

*s*

is the vector $\boldsymbol{s}$ of length *n*, containing the singular values of matrix $\boldsymbol{A}$. It is produced by a preceding call to SGESVF or DGESVF, where it corresponds to output argument *s*.

Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 115; $s_i \ge 0$.

*x*

See "On Return" on page 684.

*ldx*
is the leading dimension of the array specified for *x*. Specified as: a fullword integer; *ldx* > 0 and *ldx* ≥ *n*.

*m*
is the number of rows in matrix ***A***. Specified as: a fullword integer; *m* ≥ 0.

*n*
is the number of columns in matrix ***A***, the order of matrix ***V***, the number of elements in vector ***s***, the number of rows in matrix ***UB***, and the number of rows in matrix ***X***. Specified as: a fullword integer; *n* ≥ 0.

*tau*
is the error tolerance $\tau$. Any singular values in vector ***s*** that are less than $\tau$ are treated as zeros when computing matrix ***X***. Specified as: a number of the data type indicated in Table 115 on page 683; $\tau \geq 0$. For more information on the values for $\tau$, see “Notes.”

*On Return*

*x*
is an *n* by *nb* matrix, containing the minimal norm linear least solutions of ***AX*** ≅ ***B***. The *nb* column vectors of ***X*** contain minimal norm solution vectors for *nb* distinct linear least squares problems.

Returned as: an *ldx* by (at least) *nb* array, containing numbers of the data type indicated in Table 115 on page 683.

***Notes***

1. ***V***, ***X***, ***s***, and $\boldsymbol{U}^{T}\boldsymbol{B}$ can have no common elements; otherwise the results are unpredictable.

2. In problems involving experimental data, $\tau$ should reflect the absolute accuracy of the matrix elements:

   $$\tau \geq \max(|\Delta_{ij}|)$$

   where $\Delta_{ij}$ are the errors in $a_{ij}$. In problems where the matrix elements are known exactly or are only affected by roundoff errors:

   $$\left[\tau \geq \varepsilon\left(\sqrt{mn}\right)\right] \max\left(s_j\right) \qquad \text{for } \; j = (1, \ldots, n)$$

   where:

   $\varepsilon$ is equal to 0.11920E−06 for SGESVS and 0.22204D−15 for DGESVS.
   ***s*** is a vector containing the singular values of matrix ***A***.

   For more information, see references [13 ], [56 ], [75 ], and pages 134 to 151 in reference [96].

***Function:*** The minimal norm linear least squares solution of ***AX*** ≅ ***B***, where ***A*** is a real general matrix, is computed using the singular value decomposition, produced by a preceding call to SGESVF or DGESVF. From SGESVF or DGESVF, the singular value decomposition of ***A*** is given by the following:

$$\boldsymbol{A} = \boldsymbol{U}\Sigma\boldsymbol{V}^{T}$$

The linear least squares of solution ***X***, for ***AX*** ≅ ***B***, is given by the following formula:

$$\boldsymbol{X} = \boldsymbol{V}\Sigma^{+}\boldsymbol{U}^{T}\boldsymbol{B}$$

where:

$\Sigma +$ is the diagonal matrix with elements $\sigma_j^{+}$, where:

$$\sigma_j^{+} = 1.0/\sigma_j \quad \text{if } \sigma_j \geq \tau \text{ and } \sigma_j \neq 0$$
$$\sigma_j^{+} = 0 \quad \text{for all other cases}$$

If *m* or *n* is equal to 0, no computation is performed. See references [13], [56 ], [75 ], and pages 134 to 151 in reference [96]. These algorithms have a tendency to generate underflows that may hurt overall performance. The system default is to mask underflow, which improves the performance of these subroutines.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. $ldv \leq 0$
2. $n > ldv$
3. $ldub \leq 0$
4. $n > ldub$
5. $ldx \leq 0$
6. $n > ldx$
7. $nb \leq 0$
8. $m < 0$
9. $n < 0$
10. $\tau < 0$

***Example 1:*** This example finds the linear least squares solution for the underdetermined system ***AX*** ≅ ***B***, using the singular value decomposition computed by DGESVF. Matrix ***A*** is:

```
┌               ┐
| 1.0  2.0  2.0 |
| 2.0  4.0  5.0 |
└               ┘
```

and matrix ***B*** is:

```
┌     ┐
| 1.0 |
| 4.0 |
└     ┘
```

On output, matrix ***U***$^{T}$***B*** is overwritten.

**Note:** This example corresponds to Example 3 of DGESVF on page 680.

*Call Statement and Input*

```
             V  LDV  UB  LDUB  NB  S   X  LDX  M   N   TAU
             |   |   |    |     |  |   |   |   |   |    |
CALL DGESVS( V , 3 , UB , 3  , 1 , S , X , 3 , 2 , 3 , TAU )

         ┌                         ┐
         │ -0.304  -0.894   0.328 │
V    =   │ -0.608   0.447   0.656 │
         │ -0.733   0.000  -0.680 │
         └                         ┘


         ┌        ┐
         │ -4.061 │
UB   =   │  0.000 │
         │ -0.714 │
         └        ┘


S        =  (7.342, 0.000, 0.305)
TAU      =  0.3993D-14
```

*Output*

```
         ┌        ┐
         │ -0.600 │
X    =   │ -1.200 │
         │  2.000 │
         └        ┘
```

***Example 2:*** This example finds the linear least squares solution for the overdetermined system $\boldsymbol{AX} \cong \boldsymbol{B}$, using the singular value decomposition computed by DGESVF. Matrix $\boldsymbol{A}$ is:

```
┌          ┐
│ 1.0  4.0 │
│ 2.0  5.0 │
│ 3.0  6.0 │
└          ┘
```

and where $\boldsymbol{B}$ is:

```
┌           ┐
│ 7.0  10.0 │
│ 8.0  11.0 │
│ 9.0  12.0 │
└           ┘
```

On output, matrix $\boldsymbol{U^TB}$ is overwritten.

**Note:** This example corresponds to Example 4 of DGESVF on page 681.

*Call Statement*

```
             V  LDV  UB  LDUB  NB  S   X  LDX  M   N   TAU
             |   |   |    |     |  |   |   |   |   |    |
CALL DGESVS( V , 3 , UB , 3  , 2 , S , X , 2 , 3 , 2 , TAU )
```

*Input*

```
               ┌                  ┐
               │  0.922   -0.386  │
V    =         │ -0.386   -0.922  │
               │    .        .    │
               └                  ┘
```

```
               ┌                    ┐
               │  -1.310    -2.321  │
UB   =         │ -13.867   -18.963  │
               │    .         .     │
               └                    ┘
```

```
S          =  (0.773, 9.508)
TAU        =  0.5171D-14
```

*Output*

```
               ┌                  ┐
X    =         │ -1.000   -2.000  │
               │  2.000    3.000  │
               └                  ┘
```

# SGELLS and DGELLS—Linear Least Squares Solution for a General Matrix Using a QR Decomposition with Column Pivoting

These subroutines compute the minimal norm linear least squares solution of $\boldsymbol{AX} \cong \boldsymbol{B}$, using a QR decomposition with column pivoting.

*Table 116. Data Types*

| ***A, B, X, rn,*** $\tau$, *aux* | **Subroutine** |
|---|---|
| Short-precision real | SGELLS |
| Long-precision real | DGELLS |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SGELLS \| DGELLS (*iopt*, *a*, *lda*, *b*, *ldb*, *x*, *ldx*, *rn*, *tau*, *m*, *n*, *nb*, *k*, *aux*, *naux*) |
| **C and C++** | sgells \| dgells (*iopt*, *a*, *lda*, *b*, *ldb*, *x*, *ldx*, *rn*, *tau*, *m*, *n*, *nb*, *k*, *aux*, *naux*); |
| **PL/I** | CALL SGELLS \| DGELLS (*iopt*, *a*, *lda*, *b*, *ldb*, *x*, *ldx*, *rn*, *tau*, *m*, *n*, *nb*, *k*, *aux*, *naux*); |

*On Entry*

*iopt*
indicates the type of computation to be performed, where:

If *iopt* = 0, ***X*** is computed.

If *iopt* = 1, ***X*** and the Euclidean Norm of the residual vectors are computed.

Specified as: a fullword integer; *iopt* = 0 or 1.

*a*
is the *m* by *n* coefficient matrix ***A***. On output, ***A*** is overwritten; that is, the original input is not preserved. Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 116.

*lda*
is the leading dimension of the array specified for *a*. Specified as: a fullword integer; *lda* > 0 and *lda* ≥ *m*.

*b*
is the *m* by *nb* matrix ***B***, containing the right-hand sides of the linear systems. The *nb* column vectors of ***B*** contain right-hand sides for *nb* distinct linear least squares problems. On output, ***B*** is overwritten; that is, the original input is not preserved.

Specified as: an *ldb* by (at least) *nb* array, containing numbers of the data type indicated in Table 116.

*ldb*
is the leading dimension of the array specified for *b*. Specified as: a fullword integer; *ldb* > 0 and *ldb* ≥ *m*.

*x*
See “On Return” on page 689.

*ldx*
is the leading dimension of the array specified for *x*. Specified as: a fullword integer; *ldx* > 0 and *ldx* ≥ *n*.

*rn*
See “On Return” on page 689.

*tau*

is the tolerance $\tau$, used to determine the subset of the columns of ***A*** used in the solution. Specified as: a number of the data type indicated in Table 116; $\tau \geq 0$. For more information on how to select a value for $\tau$, see "Notes" on page 690.

*m*

is the number of rows in matrices ***A*** and ***B***. Specified as: a fullword integer; $m \geq 0$.

*n*

is the number of columns in matrix ***A*** and the number of rows in matrix ***X***. Specified as: a fullword integer; $n \geq 0$.

*nb*

is the number of columns in matrices ***B*** and ***X*** and the number of elements in vector ***rn***. Specified as: a fullword integer; $nb > 0$.

*k*

See "On Return."

*aux*

has the following meaning:

If $naux = 0$ and error 2015 is unrecoverable, *aux* is ignored.

Otherwise, it is the storage work area used by this subroutine. Its size is specified by *naux*.

Specified as: an area of storage, containing numbers of the data type indicated in Table 116 on page 688. On output, the contents of *aux* are overwritten.

*naux*

is the size of the work area specified by *aux*—that is, the number of elements in *aux*. Specified as: a fullword integer, where:

If $naux = 0$ and error 2015 is unrecoverable, SGELLS and DGELLS dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, It must have the following values:

$naux \geq 3n+\max(n, nb)$ for SGELLS

$naux \geq [\text{ceiling}(2.5n)+\max(n, nb)]$ for DGELLS

*On Return*

*x*

is the solution matrix ***X***, with *n* rows and *nb* columns, where:

If $k \neq 0$, the *nb* column vectors of ***X*** contain minimal norm least squares solutions for *nb* distinct linear least squares problems. The elements in each solution vector correspond to the original columns of ***A***.

If $k = 0$, the *nb* column vectors of ***X*** are set to 0.

Returned as: an *ldx* by (at least) *nb* array, containing numbers of the data type indicated in Table 116 on page 688.

*rn*

is the vector ***rn*** of length *nb*, where:

If $iopt = 0$ or $k = 0$, ***rn*** is not used in the computation.

If $iopt = 1$, $rn_i$ is the Euclidean Norm of the residual vector for the linear least squares problem defined by the *i*-th column vector of ***B***.

Returned as: a one-dimensional array of (at least) *nb*, containing numbers of the data type indicated in Table 116 on page 688.

*k*

is the number of columns of matrix ***A*** used in the solution. Returned as: a fullword integer; $k$ = (the number of diagonal elements of matrix ***R*** exceeding $\tau$ in magnitude).

***Notes***

1. In your C program, argument *k* must be passed by reference.
2. If $ldb \geq \max(m, n)$, matrix ***X*** and matrix ***B*** can be the same; otherwise, matrix ***X*** and matrix ***B*** can have no common elements, or the results are unpredictable.
3. The following items must have no common elements; otherwise, results are unpredictable:
   - Matrices ***A*** and ***X***, vector ***rn***, and the data area specified for *aux*
   - Matrices ***A*** and ***B***, vector ***rn***, and the data area specified for *aux*.
4. If the relative uncertainty in the matrix ***B*** is $\rho$, then:

   $$\tau \geq \rho\|\boldsymbol{A}\|_F$$

   See references [44 ], [60 ], and [75 ] for additional guidance on determining suitable values for $\tau$.
5. When you specify $iopt = 0$, you must also specify a dummy argument for *rn*. For more details, see “Example 1” on page 691.
6. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see “Using Auxiliary Storage in ESSL” on page 32.

***Function:*** The minimal norm linear least squares solution of $\boldsymbol{AX} \cong \boldsymbol{B}$ is computed using a QR decomposition with column pivoting, where:

***A*** is an *m* by *n* real general matrix.
***B*** is an *m* by *nb* real general matrix.
***X*** is an *n* by *nb* real general matrix.

Optionally, the Euclidean Norms of the residual vectors can be computed. Following are the steps involved in finding the minimal norm linear least squares solution of $\boldsymbol{AX} \cong \boldsymbol{B}$. ***A*** is decomposed, using Householder transformations and column pivoting, into the following form:

$$\boldsymbol{AP} = \boldsymbol{QR}$$

where:

***P*** is a permutation matrix.
***Q*** is an orthogonal matrix.
***R*** is an upper triangular matrix.

*k* is the first index, where:

$$|r_{k+1,k+1}| \leq \tau$$

If $k = n$, the minimal norm linear least squares solution is obtained by solving $\boldsymbol{RX} = \boldsymbol{Q}^T\boldsymbol{B}$ and reordering ***X*** to correspond to the original columns of ***A.***

If $k < n$, $\boldsymbol{R}$ has the following form:

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{R}_{11} & \boldsymbol{R}_{12} \\ 0 & 0 \end{bmatrix}$$

To find the minimal norm linear least squares solution, it is necessary to zero the submatrix $\boldsymbol{R}_{12}$ using Householder transformations. See references [44], [60 ], and [75 ]. If *m* or *n* is equal to 0, no computation is performed. These algorithms have a tendency to generate underflows that may hurt overall performance. The system default is to mask underflow, which improves the performance of these subroutines.

***Error Conditions***

*Resource Errors:* Error 2015 is unrecoverable, *naux* = 0, and unable to allocate work area.

*Computational Errors:* None

*Input-Argument Errors*

1. *iopt* ≠ 0 or 1
2. *lda* ≤ 0
3. *m* > *lda*
4. *ldb* ≤ 0
5. *m* > *ldb*
6. *ldx* ≤ 0
7. *n* > *ldx*
8. *m* < 0
9. *n* < 0
10. *nb* ≤ 0
11. $\tau < 0$
12. Error 2015 is recoverable or *naux*≠0, and *naux* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example 1:*** This example solves the underdetermined system $\boldsymbol{AX} \cong \boldsymbol{B}$. On output, ***A*** and ***B*** are overwritten. `DUMMY` is used as a placeholder for argument *rn*, which is not used in the computation.

*Call Statement and Input*

```
            IOPT  A  LDA  B  LDB  X  LDX   RN     TAU   M   N   NB  K   AUX  NAUX
              |   |   |   |   |   |   |     |      |    |   |   |   |    |    |
CALL DGELLS( 0  , A , 2 , B , 2 , X , 3 , DUMMY , TAU , 2 , 3 , 1 , K , AUX , 11 )
```

```
                       ┌               ┐
A    =                 │ 1.0  2.0  2.0 │
                       │ 2.0  4.0  5.0 │
                       └               ┘


                       ┌     ┐
B    =                 │ 1.0 │
                       │ 4.0 │
                       └     ┘

TAU          =   0.0
```

*Output*

```
                       ┌        ┐
                       │ -0.600 │
X    =                 │ -1.200 │
                       │  2.000 │
                       └        ┘

K            =   2
```

***Example 2:*** This example solves the overdetermined system ***AX*** ≅ ***B***. On output, ***A*** and ***B*** are overwritten. DUMMY is used as a placeholder for argument *rn*, which is not used in the computation.

*Call Statement and Input*

```
           IOPT  A  LDA  B  LDB  X  LDX   RN     TAU   M   N   NB  K   AUX  NAUX
             |   |   |   |   |   |   |     |      |    |   |   |   |    |    |
CALL DGELLS( 0  , A , 3 , B , 3 , X , 2 , DUMMY , TAU , 3 , 2 , 2 , K , AUX , 7  )
```

```
                       ┌          ┐
                       │ 1.0  4.0 │
A    =                 │ 2.0  5.0 │
                       │ 3.0  6.0 │
                       └          ┘

                       ┌           ┐
                       │ 7.0  10.0 │
B    =                 │ 8.0  11.0 │
                       │ 9.0  12.0 │
                       └           ┘

TAU          =   0.0
```

*Output*

```
                       ┌                ┐
X    =                 │ -1.000  -2.000 │
                       │  2.000   3.000 │
                       └                ┘

K            =   2
```

***Example 3:*** This example solves the overdetermined system ***AX*** ≅ ***B*** and computes the Euclidean Norms of the residual vectors. On output, ***A*** and ***B*** are overwritten.

*Call Statement and Input*

```
                IOPT  A  LDA  B  LDB  X  LDX  RN   TAU   M   N   NB  K   AUX  NAUX
                 |    |   |   |   |   |   |   |     |    |   |   |   |    |    |
CALL DGELLS( 1  , A , 3 , B , 3 , X , 2 , RN , TAU , 3 , 2 , 2 , K , AUX , 7  )
```

```
                    ┌             ┐
                    | 1.1  -4.3 |
A       =   | 2.0  -5.0 |
                    | 3.0  -6.0 |
                    └             ┘
```

```
                    ┌               ┐
                    | -7.0  10.0 |
B       =   | -8.0  11.0 |
                    | -9.0  12.0 |
                    └               ┘

TAU        =    0.0
```

*Output*

```
                    ┌                   ┐
X       =   | 0.543  -1.360 |
                    | 1.785  -2.699 |
                    └                   ┘

                    ┌        ┐
RN      =   | 0.196 |
                    | 0.275 |
                    └        ┘

K          =    2
```

# Chapter 11. Eigensystem Analysis

The eigensystem analysis subroutines are described in this chapter.

## Overview of the Eigensystem Analysis Subroutines

The eigensystem analysis subroutines provide solutions to the algebraic eigensystem analysis problem ***Az*** = *w**z*** and the generalized eigensystem analysis problem ***Az*** = *w**Bz*** ( Table 117). Many of the eigensystem analysis subroutines use the algorithms presented in *Linear Algebra* by Wilkinson and Reinsch [96] or use adaptations of EISPACK routines, as described in the *EISPACK Guide Lecture Notes in Computer Science* in reference [84] or in the *EISPACK Guide Extension Lecture Notes in Computer Science* in reference [56 ]. (EISPACK is available from the sources listed in reference [49].)

*Table 117. List of Eigensystem Analysis Subroutines*

| **Descriptive Name** | **Short-Precision Subroutine** | **Long-Precision Subroutine** | **Page** |
|---|---|---|---|
| Eigenvalues and, Optionally, All or Selected Eigenvectors of a General Matrix | SGEEV<br>CGEEV | DGEEV<br>ZGEEV | 698 |
| Eigenvalues and, Optionally, the Eigenvectors of a Real Symmetric Matrix or a Complex Hermitian Matrix | SSPEV<br>CHPEV | DSPEV<br>ZHPEV | 709 |
| Extreme Eigenvalues and, Optionally, the Eigenvectors of a Real Symmetric Matrix or a Complex Hermitian Matrix | SSPSV<br>CHPSV | DSPSV<br>ZHPSV | 718 |
| Eigenvalues and, Optionally, the Eigenvectors of a Generalized Real Eigensystem, Az=wBz, where A and B Are Real General Matrices | SGEGV | DGEGV | 726 |
| Eigenvalues and, Optionally, the Eigenvectors of a Generalized Real Symmetric Eigensystem, Az=wBz, where A Is Real Symmetric and B Is Real Symmetric Positive Definite | SSYGV | DSYGV | 732 |

## Performance and Accuracy Considerations

1. The accuracy of the resulting eigenvalues and eigenvectors varies between the short- and long-precision versions of each subroutine. The degree of difference depends on the size and conditioning of the matrix computation. Some of the subroutines have examples illustrating this difference.

2. The short precision subroutines provide increased accuracy by accumulating intermediate results in long precision. Occasionally, for performance reasons, these intermediate results are stored.

3. If you want to compute 10% or fewer eigenvalues only, or you want to compute 30% or fewer eigenvalues and eigenvectors, you get better performance if you use _SPSV and _HPSV instead of _SPEV and _HPEV, respectively. For all other uses, you should use _SPEV and _HPEV.

4. There are some ESSL-specific rules that apply to the results of computations on the workstation processors using the ANSI/IEEE standards. For details, see

“What Data Type Standards Are Used by ESSL, and What Exceptions Should You Know About?” on page 45.

# Eigensystem Analysis Subroutines

This section contains the eigensystem analysis subroutine

## SGEEV, DGEEV, CGEEV, and ZGEEV—Eigenvalues and, Optionally, All or Selected Eigenvectors of a General Matrix

SGEEV and DGEEV compute the eigenvalues and, optionally, all or selected eigenvectors of real general matrix ***A***. CGEEV and ZGEEV compute the eigenvalues and, optionally, all or selected eigenvectors of complex general matrix ***A***. Eigenvalues are returned in complex vector ***w***, and eigenvectors are returned in complex matrix ***Z***:

$\boldsymbol{Az} = w\boldsymbol{z}$

*Table 118. Data Types*

| ***A*** | ***w*, *Z*** | *aux* | **Subroutine** |
|---|---|---|---|
| Short-precision real | Short-precision complex | Short-precision real | SGEEV |
| Long-precision real | Long-precision complex | Long-precision real | DGEEV |
| Short-precision complex | Short-precision complex | Short-precision real | CGEEV |
| Long-precision complex | Long-precision complex | Long-precision real | ZGEEV |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SGEEV \| DGEEV \| CGEEV \| ZGEEV (*iopt*, *a*, *lda*, *w*, *z*, *ldz*, *select*, *n*, *aux*, *naux*) |
| **C and C++** | sgeev \| dgeev \| cgeev \| zgeev (*iopt*, *a*, *lda*, *w*, *z*, *ldz*, *select*, *n*, *aux*, *naux*); |
| **PL/I** | CALL SGEEV \| DGEEV \| CGEEV \| ZGEEV (*iopt*, *a*, *lda*, *w*, *z*, *ldz*, *select*, *n*, *aux*, *naux*); |

*On Entry*

*iopt*

indicates the type of computation to be performed, where:

If *iopt* = 0, eigenvalues only are computed.

If *iopt* = 1, eigenvalues and eigenvectors are computed.

If *iopt* = 2, eigenvalues and eigenvectors corresponding to selected eigenvalues are computed.

Specified as: a fullword integer; *iopt* = 0, 1, or 2.

*a*

is the real or complex general matrix ***A*** of order *n*, whose eigenvalues and, optionally, eigenvectors are computed. Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 118. On output, A is overwritten; that is, the original input is not preserved.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; *lda* > 0 and *lda* ≥ *n*.

*w*

See “On Return” on page 699.

*z*

See “On Return” on page 699.

*ldz*

has the following meaning, where:

If *iopt* = 0, it is not used in the computation.

If *iopt* = 1 or 2, it is the leading dimension of the output array specified for *z*.

Specified as: a fullword integer. It must have the following values, where:

If *iopt* = 0, $ldz > 0$.

If *iopt* = 1 or 2, $ldz > 0$ and $ldz \geq n$.

*select*

has the following meaning, where:

If *iopt* = 0 or 1, it is not used in the computation.

If *iopt* = 2, it is the logical vector ***select*** of length *n* whose true elements indicate those eigenvalues whose corresponding eigenvectors are to be computed.

Specified as: a one-dimensional array of (at least) length *n*, containing logical data items. Element values can be true (.TRUE.) or false (.FALSE.).

*n*

is the order of matrix ***A***. Specified as: a fullword integer; $n \geq 0$.

*aux*

has the following meaning:

If *naux* = 0 and error 2015 is unrecoverable, *aux* is ignored.

Otherwise, it is a storage work area used by this subroutine. Its size is specified by *naux*.

Specified as: an area of storage, containing numbers of the data type indicated in Table 118 on page 698. On output, the contents are overwritten.

*naux*

is the size of the work area specified by *aux*—that is, the number of elements in *aux*. Specified as: a fullword integer, where:

If *naux* = 0 and error 2015 is unrecoverable, SGEEV, DGEEV, CGEEV, and ZGEEV dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, it must have the following value, where:

For SGEEV and DGEEV:
If *iopt* = 0, $naux \geq n$.
If *iopt* = 1, $naux \geq 2n$.
If *iopt* = 2,

$naux \geq n^2+4n$.

For CGEEV and ZGEEV:
If *iopt* = 0, $naux \geq 2n$.
If *iopt* = 1, $naux \geq 3n$.
If *iopt* = 2, $naux \geq 2n^2+5n$.

*On Return*

*w*

is the vector ***w*** of length *n*, containing the eigenvalues of ***A***. The eigenvalues are unordered. For SGEEV and DGEEV, complex conjugate pairs appear consecutively with the eigenvalue having the positive imaginary part first.

Returned as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 118 on page 698.

*z*

has the following meaning, where:

If *iopt* = 0, it is not used in the computation.

If *iopt* = 1, it is the matrix ***Z*** of order *n*, containing the eigenvectors of matrix ***A***. The eigenvector in column *i* of matrix ***Z*** corresponds to the eigenvalue $w_i$.

If *iopt* = 2, it is the *n* by *m* matrix ***Z***, containing the *m* eigenvectors of matrix ***A***, which correspond to the *m* selected eigenvalues in $w_i$,

where *m* is the number of true elements in the logical vector ***select***. The eigenvector in column *i* of matrix ***Z*** corresponds to the *i*-th selected eigenvalue. Any eigenvector that does not converge is set to 0.

Returned as: a two-dimensional array, containing numbers of the data type indicated in Table 118 on page 698, where:

If *iopt* = 1, the array must be *ldz* by (at least) *n*.

If *iopt* = 2, the array must be *ldz* by (at least) *m*.

***Notes***

1. When you specify *iopt* = 0, you must also specify:
   - A positive value for *ldz*
   - A dummy argument for *z* (see “Example 1” on page 703)
   - A dummy argument for *select*.
2. When you specify *iopt* = 1, you must also specify a dummy argument for *select*.
3. The following items must have no common elements: matrix ***A***, matrix ***Z***, vector ***w***, vector ***select***, and the data area specified for *aux*; otherwise, results are unpredictable. See “Concepts” on page 55.
4. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see “Using Auxiliary Storage in ESSL” on page 32.

***Function:*** The next two sections describe the methods used to compute the eigenvectors and eigenvalues for either a real general matrix or a complex general matrix. For more information on these methods, see references [39], [43 ], [45 ], [61 ], [84], [94 ], and [96 ]. If *n* is 0, no computation is performed. The results of the computations using short- and long-precision data can vary in accuracy. See the examples for an illustration of the difference in the results.

These algorithms have a tendency to generate underflows that may hurt overall performance. The system default is to mask underflow, which improves the performance of these subroutines.

*Real General Matrix:* The eigenvalues and, optionally, all or selected eigenvectors of a real general matrix ***A*** are computed as follows.

- For *iopt* = 0, the eigenvalues are computed as follows:
  1. Balance the real general matrix ***A***.
  2. Reduce the balanced matrix to a real upper Hessenberg matrix using orthogonal similarity transformations.
  3. Compute the eigenvalues of the real upper Hessenberg matrix using the QR algorithm.

4. The eigenvalues are returned in vector ***w***.

- For *iopt* = 1, the eigenvalues and eigenvectors are computed as follows:

  1. Balance the real general matrix ***A***.
  2. Reduce the balanced matrix to a real upper Hessenberg matrix using orthogonal similarity transformations.
  3. Accumulate the orthogonal similarity transformations used in the reduction of the real balanced matrix to upper Hessenberg form.
  4. Compute the eigenvalues of the real upper Hessenberg matrix and the eigenvectors of the corresponding real balanced matrix using the QR algorithm.
  5. Back transform the eigenvectors of the balanced matrix to the eigenvectors of the original matrix.
  6. The eigenvalues are returned in vector ***w***, and the eigenvectors are returned in matrix ***Z***.

- For *iopt* = 2, the eigenvalues and selected eigenvectors are computed as follows:

  1. Balance the real general matrix ***A***.
  2. Reduce the balanced matrix to a real upper Hessenberg matrix using orthogonal similarity transformations.
  3. Compute the eigenvalues of the real upper Hessenberg matrix using the QR algorithm.
  4. Compute the eigenvectors of the real upper Hessenberg matrix corresponding to selected eigenvalues, indicated in the logical vector ***select***, using inverse iteration.
  5. Back transform the eigenvectors of the real upper Hessenberg matrix to the eigenvectors of the balanced matrix.
  6. Back transform the eigenvectors of the balanced matrix to the eigenvectors of the original matrix.
  7. The eigenvalues are returned in vector ***w***, and the eigenvectors are returned in matrix ***Z***.

*Complex General Matrix:* The eigenvalues and, optionally, all or selected eigenvectors of a complex general matrix ***A*** are computed as follows.

- For *iopt* = 0, the eigenvalues are computed as follows:

  1. Balance the complex general matrix ***A***.
  2. Reduce the complex balanced matrix to a complex upper Hessenberg matrix using unitary similarity transformations.
  3. Compute the eigenvalues of the complex upper Hessenberg matrix using a QR algorithm with implicit shift.
  4. The eigenvalues are returned in vector ***w***.

- For *iopt* = 1, the eigenvalues and eigenvectors are computed as follows:

  1. Balance the complex general matrix ***A***.
  2. Reduce the complex balanced matrix to a complex upper Hessenberg matrix using unitary similarity transformations.
  3. Accumulate the unitary similarity transformations used in the reduction of the complex balanced matrix to complex upper Hessenberg form.
  4. Compute the eigenvalues of the complex upper Hessenberg matrix and the eigenvectors of the complex balanced matrix using a QR algorithm with implicit shift.

5. Back transform the eigenvectors of the complex balanced matrix to the eigenvectors of the original matrix.
6. The eigenvalues are returned in vector ***w***, and the eigenvectors are returned in matrix ***Z***.

- For *iopt* = 2, the eigenvalues and selected eigenvectors are computed as follows:

  1. Balance the complex general matrix ***A***.
  2. Reduce the complex balanced matrix to a complex upper Hessenberg matrix using unitary similarity transformations.
  3. Compute the eigenvalues of the complex upper Hessenberg matrix using a QR algorithm with implicit shift.
  4. Compute the eigenvectors of the complex upper Hessenberg matrix corresponding to selected eigenvalues, indicated in the logical vector ***select***, using inverse iteration.
  5. Back transform the eigenvectors of the complex upper Hessenberg matrix to the eigenvectors of the complex balanced matrix.
  6. Back transform the eigenvectors of the complex balanced matrix to the eigenvectors of the original matrix.
  7. The eigenvalues are returned in vector ***w***, and the eigenvectors are returned in matrix ***Z***.

### Error Conditions

*Resource Errors:* Error 2015 is unrecoverable, *naux* = 0, and unable to allocate work area.

*Computational Errors*

1. Eigenvalue (*i*) failed to converge after (*xxx*) iterations.
   - The eigenvalues ($w_j$, $j = n, n–1, ..., i+1$) are correct.
   - If *iopt* = 1, then *z* is modified, but no eigenvectors are correct.
   - *a* is modified.
   - The return code is set to 1.
   - *i* and *xxx* can be determined at run time by use of the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error code 2101 in the ESSL error option table; otherwise, the default value causes your program to terminate when this error occurs. See “What Can You Do about ESSL Computational Errors?” on page 48.
2. Eigenvector (*yyy*) failed to converge after (*zzz*) iterations. (The computational error message may occur multiple times with processing continuing after each error, because the number of allowable errors for error code 2102 is set to be unlimited in the ESSL error option table.)
   - All eigenvalues are correct.
   - The eigenvector that failed to converge is set to 0.
   - Any selected eigenvectors for which this message has not been issued are correct.
   - *a* is modified.
   - The return code is set to 2.
   - *yyy* and *zzz* for the last eigenvector that failed to converge can be determined at run time by use of the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error 2199 in the ESSL error option table. See “What Can You Do about ESSL Computational Errors?” on page 48.

*Input-Argument Errors*

1. *iopt* ≠ 0, 1, or 2
2. *n* < 0
3. *lda* ≤ 0
4. *n* > *lda*
5. *ldz* ≤ 0 and *iopt* ≠ 0
6. *n* > *ldz* and *iopt* ≠ 0
7. Error 2015 is recoverable or *naux*≠0, and *naux* is too small—that is, less than the minimum required value. Return code 3 is returned if error 2015 is recoverable.

***Example 1:*** This example shows how to find the eigenvalues only of a real short-precision general matrix ***A*** of order 4, where:

- `NAUX` is equal to `N`.
- `AUX` contains `N` elements.
- `LDZ` is set to avoid an error condition.
- `DUMMY1` and `DUMMY2` are used as placeholders for arguments *z* and *select*, which are not used in the computation.
- On output, `A` has been overwritten.

**Note:** This matrix is used in Example 5.5 in referenced text [61 ].

*Call Statement and Input*

```
             IOPT  A  LDA  W     Z     LDZ  SELECT   N   AUX  NAUX
              |    |   |   |     |      |     |      |    |    |
CALL SGEEV( 0  , A , 4 , W , DUMMY1 , 1 , DUMMY2 , 4 , AUX , 4  )
```

```
        ┌                      ┐
        │ -2.0  2.0  2.0  2.0  │
A    =  │ -3.0  3.0  2.0  2.0  │
        │ -2.0  0.0  4.0  2.0  │
        │ -1.0  0.0  0.0  5.0  │
        └                      ┘
```

*Output*

```
        ┌                        ┐
        │ (0.999999,  0.000000)  │
W    =  │ (2.000001,  0.000000)  │
        │ (2.999996,  0.000000)  │
        │ (3.999999,  0.000000)  │
        └                        ┘
```

***Example 2:*** This example shows how to find the eigenvalues and eigenvectors of a real short-precision general matrix ***A*** of order 3, where:

- `NAUX` is equal to 2`N`.
- `AUX` contains 2`N` elements.
- `DUMMY` is used as a placeholder for argument *select*, which is not used in the computation.
- On output, `A` has been overwritten.

**Note:** This matrix is used in Example 5.1 in referenced text [61 ].

*Call Statement and Input*

```
           IOPT  A  LDA  W   Z  LDZ  SELECT  N   AUX  NAUX
            |    |   |   |   |   |     |     |    |    |
CALL SGEEV( 1  , A , 3 , W , Z , 3 , DUMMY , 3 , AUX , 6  )
```

```
        ┌                     ┐
        │  33.0   16.0   72.0 │
A    =  │ -24.0  -10.0  -57.0 │
        │  -8.0   -4.0  -17.0 │
        └                     ┘
```

*Output*

```
        ┌                       ┐
        │ (3.000022, 0.000000)  │
W    =  │ (1.000019, 0.000000)  │
        │ (1.999961, 0.000000)  │
        └                       ┘
```

```
        ┌                                                                         ┐
        │  (2.498781, 0.000000)   (76.837608,  0.000000)   (79.999451,  0.000000) │
Z    =  │ (-1.874081, 0.000000)  (-61.470169,  0.000000)  (-64.999649,  0.000000) │
        │ (-0.624695, 0.000000)  (-20.489990,  0.000000)  (-19.999886,  0.000000) │
        └                                                                         ┘
```

***Example 3:*** This example shows how to find the eigenvalues and eigenvectors of a real short-precision general matrix ***A*** of order 3, where:

- NAUX is equal to 2N.
- AUX contains 2N elements.
- DUMMY is used as a placeholder for argument *select*, which is not used in the computation.
- On output, A has been overwritten.

**Note:** This matrix is used in Example 5.4 in referenced text [61 ].

*Call Statement and Input*

```
           IOPT  A  LDA  W   Z  LDZ  SELECT  N   AUX  NAUX
            |    |   |   |   |   |     |     |    |    |
CALL SGEEV( 1  , A , 3 , W , Z , 3 , DUMMY , 3 , AUX , 6  )
```

```
        ┌                  ┐
        │  8.0  -1.0  -5.0 │
A    =  │ -4.0   4.0  -2.0 │
        │ 18.0  -5.0  -7.0 │
        └                  ┘
```

*Output*

```
        ┌                       ┐
        │ (1.999999,  3.999998) │
W    =  │ (1.999999, -3.999998) │
        │ (0.999997,  0.000000) │
        └                       ┘
```

```
        ┌                                                                   ┐
        │  (0.044710, 0.410578)   (0.044710, -0.410578)  (1.732048, 0.000000) │
Z    =  │ (-0.365868, 0.455287)  (-0.365868, -0.455287)  (3.464096, 0.000000) │
        │  (0.455287, 0.365868)   (0.455287, -0.365868)  (1.732049, 0.000000) │
        └                                                                   ┘
```

***Example 4:*** This example shows how to find the eigenvalues and selected eigenvectors of a real short-precision general matrix ***A*** of order 4, where:

- NAUX is equal to $N^2+4N$.
- AUX contains NAUX elements.
- The first, third, and fourth eigenvectors are selected and appear in the first, second, and third columns of matrix Z, respectively.
- On output, A has been overwritten.

**Note:** This matrix is used in Example 5.5 in referenced text [61 ].

*Call Statement and Input*

```
           IOPT  A  LDA  W   Z  LDZ  SELECT   N   AUX  NAUX
            |    |   |   |   |   |     |      |    |    |
CALL SGEEV( 2  , A , 4 , W , Z , 4 , SELECT , 4 , AUX , 32 )
```

```
        ┌                     ┐
        │ -2.0  2.0  2.0  2.0 │
A    =  │ -3.0  3.0  2.0  2.0 │
        │ -2.0  0.0  4.0  2.0 │
        │ -1.0  0.0  0.0  5.0 │
        └                     ┘


           ┌         ┐
           │ .TRUE.  │
SELECT =   │ .FALSE. │
           │ .TRUE.  │
           │ .TRUE.  │
           └         ┘
```

*Output*

```
        ┌                       ┐
        │ (0.999999,  0.000000) │
W    =  │ (2.000001,  0.000000) │
        │ (2.999996,  0.000000) │
        │ (3.999999,  0.000000) │
        └                       ┘
```

```
        ┌                                                                     ┐
        │ (1.000000, 0.000000)  (-0.674014, 0.000000)  (-0.474306, 0.000000) │
Z    =  │ (0.750000, 0.000000)  (-0.674014, 0.000000)  (-0.474306, 0.000000) │
        │ (0.500000, 0.000000)  (-0.674013, 0.000000)  (-0.474306, 0.000000) │
        │ (0.250000, 0.000000)  (-0.337006, 0.000000)  (-0.474305, 0.000000) │
        └                                                                     ┘
```

***Example 5:*** This example shows how to find the eigenvalues and selected eigenvectors of a real short-precision general matrix ***A*** of order 3, where:

- NAUX is equal to $N^2+4N$.
- AUX contains NAUX elements.
- The first and second eigenvectors are selected and appear in the first and second columns of matrix Z, respectively.
- On output, A has been overwritten.

**Note:** This matrix is used in Example 5.4 in referenced text [61 ].

*Call Statement and Input*

```
           IOPT  A  LDA  W   Z  LDZ  SELECT   N   AUX  NAUX
            |    |   |   |   |   |     |      |    |    |
CALL SGEEV( 2  , A , 3 , W , Z , 3 , SELECT , 3 , AUX , 21 )
```

```
        ┌                  ┐
        │  8.0  -1.0  -5.0 │
A     = │ -4.0   4.0  -2.0 │
        │ 18.0  -5.0  -7.0 │
        └                  ┘
```

```
          ┌         ┐
          │ .TRUE.  │
SELECT =  │ .TRUE.  │
          │ .FALSE. │
          └         ┘
```

*Output*

```
        ┌                       ┐
        │ (1.999999,  3.999998) │
W     = │ (1.999999, -3.999998) │
        │ (0.999997,  0.000000) │
        └                       ┘
```

```
        ┌                                              ┐
        │ (0.500000,  0.000000)  (0.500000,  0.000000) │
Z     = │ (0.500000,  0.500000)  (0.500000, -0.500000) │
        │ (0.500000, -0.500000)  (0.500000,  0.500000) │
        └                                              ┘
```

***Example 6:*** This example shows how the results of Example 5 would differ if matrix ***A*** was a real long-precision general matrix. On output, A has been overwritten.

*Call Statement and Input*

```
           IOPT  A  LDA  W   Z  LDZ  SELECT   N   AUX  NAUX
            |    |   |   |   |   |     |      |    |    |
CALL DGEEV( 2  , A , 3 , W , Z , 3 , SELECT , 3 , AUX , 21 )
```

*Output*

```
        ┌                       ┐
        │ (2.000000,  4.000000) │
W     = │ (2.000000, -4.000000) │
        │ (1.000000,  0.000000) │
        └                       ┘
```

```
        ┌                                              ┐
        │ (0.500000,  0.000000)  (0.500000,  0.000000) │
Z     = │ (0.500000,  0.500000)  (0.500000, -0.500000) │
        │ (0.500000, -0.500000)  (0.500000,  0.500000) │
        └                                              ┘
```

***Example 7:*** This example shows how to find the eigenvalues only of a complex long-precision general matrix ***A*** of order 3, where:

- NAUX is equal to 2N.
- AUX contains 2N elements.
- LDZ is set to avoid an error condition.

- DUMMY1 and DUMMY2 are used as placeholders for arguments *z* and *select*, which are not used in the computation.
- On output, A has been overwritten.

**Note:** This matrix is used in Example 6.4 in referenced text [61 ].

*Call Statement and Input*

```
           IOPT  A  LDA  W     Z     LDZ  SELECT   N   AUX  NAUX
            |    |   |   |     |      |     |      |    |    |
CALL ZGEEV( 0   , A , 3 , W , DUMMY1 , 1 , DUMMY2 , 3 , AUX , 6  )
```

```
        ┌                                          ┐
        |  (1.0,  2.0)   (3.0,  4.0)  (21.0, 22.0) |
A    =  | (43.0, 44.0)  (13.0, 14.0)  (15.0, 16.0) |
        |  (5.0,  6.0)   (7.0,  8.0)  (25.0, 26.0) |
        └                                          ┘
```

*Output*

```
        ┌                           ┐
        | (39.776655, 42.995668) |
W    =  | (-7.477530,  6.880321) |
        |  (6.700876, -7.875989) |
        └                           ┘
```

***Example 8:*** This example shows how to find the eigenvalues and eigenvectors of a complex long-precision general matrix ***A*** of order 4, where:

- NAUX is equal to 3N.
- AUX contains 3N elements.
- DUMMY is used as a placeholder for argument *select*, which is not used in the computation.
- On output, A has been overwritten.

**Note:** This matrix is used in Example 6.5 in referenced text [61 ].

*Call Statement and Input*

```
           IOPT  A  LDA  W   Z  LDZ  SELECT  N   AUX  NAUX
            |    |   |   |   |   |     |     |    |    |
CALL ZGEEV( 1   , A , 4 , W , Z , 4 , DUMMY , 4 , AUX , 12 )
```

```
        ┌                                                       ┐
        | (5.0, 9.0)  (5.0,  5.0)  (-6.0, -6.0)  (-7.0, -7.0) |
A    =  | (3.0, 3.0)  (6.0, 10.0)  (-5.0, -5.0)  (-6.0, -6.0) |
        | (2.0, 2.0)  (3.0,  3.0)  (-1.0,  3.0)  (-5.0, -5.0) |
        | (1.0, 1.0)  (2.0,  2.0)  (-3.0, -3.0)   (0.0,  4.0) |
        └                                                       ┘
```

*Output*

```
        ┌                        ┐
        | (4.000000, 8.000000) |
W    =  | (2.000000, 6.000000) |
        | (3.000000, 7.000000) |
        | (1.000000, 5.000000) |
        └                        ┘
        ┌                                                                                        ┐
        │ (0.625817, 0.229776)  (0.333009, -0.729358)  (-1.535458, 1.519551)  (0.000000, 3.464102) │
Z    =  │ (0.625817, 0.229776)  (0.666017, -1.458715)  (-1.535458, 1.519551)  (0.000000, 1.732051) │
        │ (0.625817, 0.229776)  (0.333009, -0.729358)  ( 0.000000, 0.000000)  (0.000000, 1.732051) │
        │ (0.000000, 0.000000)  (0.333009, -0.729358)  (-1.535458, 1.519551)  (0.000000, 1.732051) │
        └                                                                                        ┘
```

***Example 9:*** This example shows how to find the eigenvalues and selected eigenvectors of a complex long-precision general matrix ***A*** of order 4, where:

- NAUX is equal to $2N^2+5N$.
- AUX contains NAUX elements.
- The first, third, and fourth eigenvectors are selected and appear in the first, second, and third columns of matrix Z, respectively.
- On output, A has been overwritten.

**Note:** This matrix is used in Example 6.5 in referenced text [61 ].

*Call Statement and Input*

```
          IOPT  A  LDA  W   Z  LDZ  SELECT   N   AUX  NAUX
           |    |   |   |   |   |     |      |    |    |
CALL ZGEEV( 2  , A , 4 , W , Z , 4 , SELECT , 4 , AUX , 52 )
```

```
        ┌                                              ┐
        │ (5.0, 9.0)  (5.0,  5.0)  (-6.0, -6.0)  (-7.0, -7.0) │
A    =  │ (3.0, 3.0)  (6.0, 10.0)  (-5.0, -5.0)  (-6.0, -6.0) │
        │ (2.0, 2.0)  (3.0,  3.0)  (-1.0,  3.0)  (-5.0, -5.0) │
        │ (1.0, 1.0)  (2.0,  2.0)  (-3.0, -3.0)   (0.0,  4.0) │
        └                                              ┘
```

```
         ┌         ┐
         │ .TRUE.  │
SELECT = │ .FALSE. │
         │ .TRUE.  │
         │ .TRUE.  │
         └         ┘
```

*Output*

```
        ┌                      ┐
        │ (4.000000, 8.000000) │
W    =  │ (2.000000, 6.000000) │
        │ (3.000000, 7.000000) │
        │ (1.000000, 5.000000) │
        └                      ┘
```

```
        ┌                                                                    ┐
        │ (-0.748331, 0.000000)  (-0.935414, 0.000000)  (-1.247219, 0.000000) │
Z    =  │ (-0.748331, 0.000000)  (-0.935414, 0.000000)  (-0.623610, 0.000000) │
        │ (-0.748331, 0.000000)   (0.000000, 0.000000)  (-0.623610, 0.000000) │
        │  (0.000000, 0.000000)  (-0.935414, 0.000000)  (-0.623610, 0.000000) │
        └                                                                    ┘
```

# SSPEV, DSPEV, CHPEV, and ZHPEV—Eigenvalues and, Optionally, the Eigenvectors of a Real Symmetric Matrix or a Complex Hermitian Matrix

SSPEV and DSPEV compute the eigenvalues and, optionally, the eigenvectors of real symmetric matrix ***A***, stored in lower- or upper-packed storage mode. CHPEV and ZHPEV compute the eigenvalues and, optionally, the eigenvectors of complex Hermitian matrix ***A***, stored in lower- or upper-packed storage mode. Eigenvalues are returned in vector ***w***, and eigenvectors are returned in matrix ***Z***:

$$\mathbf{A}\mathbf{z} = w\mathbf{z}$$

where $\mathbf{A} = \mathbf{A}^T$ or $\mathbf{A} = \mathbf{A}^H$.

*Table 119. Data Types*

| ***A***, ***z*** | ***w***, *aux* | **Subroutine** |
|---|---|---|
| Short-precision real | Short-precision real | SSPEV |
| Long-precision real | Long-precision real | DSPEV |
| Short-precision complex | Short-precision real | CHPEV |
| Long-precision complex | Long-precision real | ZHPEV |

**Note:** For compatibility with earlier releases of ESSL, you can use the names SSLEV, DSLEV, CHLEV, and ZHLEV for SSPEV, DSPEV, CHPEV, and ZHPEV, respectively.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SSPEV \| DSPEV \| CHPEV \| ZHPEV (*iopt*, *ap*, *w*, *z*, *ldz*, *n*, *aux*, *naux*) |
| **C and C++** | sspev \| dspev \| chpev \| zhpev (*iopt*, *ap*, *w*, *z*, *ldz*, *n*, *aux*, *naux*); |
| **PL/I** | CALL SSPEV \| DSPEV \| CHPEV \| ZHPEV (*iopt*, *ap*, *w*, *z*, *ldz*, *n*, *aux*, *naux*); |

*On Entry*

*iopt*

indicates the type of computation to be performed, where:

If *iopt* = 0 or 20, eigenvalues only are computed.

If *iopt* = 1 or 21, eigenvalues and eigenvectors are computed.

Specified as: a fullword integer; *iopt* = 0, 1, 20, or 21.

*ap*

is the real symmetric or complex Hermitian matrix ***A*** of order *n*, whose eigenvalues and, optionally, eigenvectors are computed. It is stored in an array, referred to as AP, where:

If *iopt* = 0 or 1, it is stored in lower-packed storage mode.

If *iopt* = 20 or 21, it is stored in upper-packed storage mode.

Specified as: a one-dimensional array of (at least) length *n*(*n*+1)/2, containing numbers of the data type indicated in Table 119. On output, for SSPEV and DSPEV if *iopt* = 0 or 20, and for CHPEV and ZHPEV, AP is overwritten; that is, the original input is not preserved.

*w*

See “On Return” on page 710.

*z*

See “On Return.”

*ldz*

has the following meaning, where:

If *iopt* = 0 or 20, it is not used in the computation.

If *iopt* = 1 or 21, it is the leading dimension of the output array specified for *z*.

Specified as: a fullword integer. It must have the following value, where:

If *iopt* = 0 or 20, *ldz* > 0.

If *iopt* = 1 or 21, *ldz* > 0 and *ldz* ≥ *n*.

*n*

is the order of matrix ***A***. Specified as: a fullword integer; *n* ≥ 0.

*aux*

has the following meaning:

If *naux* = 0 and error 2015 is unrecoverable, *aux* is ignored.

Otherwise, it is a storage work area used by this subroutine. Its size is specified by *naux*.

Specified as: an area of storage, containing numbers of the data type indicated in Table 119 on page 709. On output, the contents are overwritten.

*naux*

is the size of the work area specified by *aux*—that is, the number of elements in *aux*. Specified as: a fullword integer, where:

If *naux* = 0 and error 2015 is unrecoverable, SSPEV, DSPEV, CHPEV, and ZHPEV dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, It must have the following value, where:

For SSPEV and DSPEV:
If *iopt* = 0 or 20, *naux* ≥ *n*.
If *iopt* = 1 or 21, *naux* ≥ 2*n*.

For CHPEV and ZHPEV:
If *iopt* = 0 or 20, *naux* ≥ 3*n*.
If *iopt* = 1 or 21, *naux* ≥ 4*n*.

*On Return*

*w*

is the vector ***w*** of length *n*, containing the eigenvalues of ***A*** in ascending order. Returned as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 119 on page 709.

*z*

has the following meaning, where:

If *iopt* = 0 or 20, it is not used in the computation.

If *iopt* = 1 or 21, it is the matrix ***Z*** of order *n*, containing the orthonormal eigenvectors of matrix ***A***. The eigenvector in column *i* of matrix ***Z*** corresponds to the eigenvalue $w_i$.

Returned as: an *ldz* by (at least) *n* array, containing numbers of the data type indicated in Table 119 on page 709.

***Notes***

1. When you specify *iopt* = 0 or 20, you must specify:
   - A positive value for *ldz*
   - A dummy argument for *z* (see “Example 1” on page 713)
2. The following items must have no common elements: matrix ***A***, matrix ***Z***, vector ***w***, and the data area specified for *aux*; otherwise, results are unpredictable. See “Concepts” on page 55.
3. On input, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values.
4. For a description of how real symmetric matrices are stored in lower- or upper-packed storage mode, see “Lower-Packed Storage Mode” on page 66 or “Upper-Packed Storage Mode” on page 67, respectively.

   For a description of how complex Hermitian matrices are stored in lower- or upper-packed storage mode, see “Complex Hermitian Matrix” on page 70.
5. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see “Using Auxiliary Storage in ESSL” on page 32.

***Function:*** The next two sections describe the methods used to compute the eigenvalues and, optionally, the eigenvectors for either a real symmetric matrix or a complex Hermitian matrix. For more information on these methods, see references [39], [43 ], [61 ], [84 ], [94], and [96 ]. If *n* is 0, no computation is performed. The results of the computations using short- and long-precision data can vary in accuracy. See “Example 3” on page 714 and “Example 4” on page 715 for an illustration of the difference in results. Eigenvalues computed using equivalent *iopt* values are mathematically equivalent, but are not guaranteed to be bitwise identical. For example, the results computed using *iopt* = 0 and *iopt* = 20 are mathematically equivalent, but are not necessarily bitwise identical.

These algorithms have a tendency to generate underflows that may hurt overall performance. The system default is to mask underflow, which improves the performance of these subroutines.

*Real Symmetric Matrix:* The eigenvalues and, optionally, the eigenvectors of a real symmetric matrix ***A*** are computed as follows:

- For *iopt* = 0 or 20, the eigenvalues are computed as follows:
  1. Reduce the real symmetric matrix ***A*** to a real symmetric tridiagonal matrix using orthogonal similarity transformations.
  2. Compute the eigenvalues of the real symmetric tridiagonal matrix using the implicit QL algorithm.
  3. The eigenvalues are ordered and returned in vector ***w***.
- For *iopt* = 1 or 21, the eigenvalues and eigenvectors are computed as follows:
  1. Reduce the real symmetric matrix ***A*** to a real symmetric tridiagonal matrix using and accumulating orthogonal similarity transformations.

2. Compute the eigenvalues of the real symmetric tridiagonal matrix and the eigenvectors of the real symmetric matrix using the implicit QL algorithm.
3. The eigenvalues are ordered and returned in vector ***w***, and the corresponding eigenvectors are returned in matrix ***Z***.

*Complex Hermitian Matrix:* The eigenvalues and, optionally, the eigenvectors of a complex Hermitian matrix ***A*** are computed as follows:

- For *iopt* = 0 or 20, the eigenvalues are computed as follows:

  1. Reduce the complex Hermitian matrix ***A*** to a real symmetric tridiagonal matrix using unitary similarity transformations.
  2. Compute the eigenvalues of the real symmetric tridiagonal matrix using the implicit QL algorithm.
  3. The eigenvalues are ordered and returned in vector ***w***.

- For *iopt* = 1 or 21, the eigenvalues and eigenvectors are computed as follows:

  1. Reduce the complex Hermitian matrix ***A*** to a real symmetric tridiagonal matrix using unitary similarity transformations.
  2. Compute the eigenvalues and eigenvectors of the real symmetric tridiagonal matrix using the implicit QL algorithm.
  3. Back transform the eigenvectors of the real symmetric tridiagonal matrix to those of the original complex Hermitian matrix.
  4. The eigenvalues are ordered and returned in vector ***w***, and the corresponding eigenvectors are returned in matrix ***Z***.

### *Error Conditions*

*Resource Errors:* Error 2015 is unrecoverable, *naux* = 0, and unable to allocate work area.

*Computational Errors:* Eigenvalue (*i*) failed to converge after (*xxx*) iterations:

- The eigenvalues ($w_j$, *j* = 1, 2, ..., *i*–1) are correct, but are unordered.
- If *iopt* = 1 or 21, then *z* is modified, but no eigenvectors are correct.
- If *iopt* = 0 or 20 for SSPEV and DSPEV, *ap* is modified.
- For CHPEV and ZHPEV, *ap* is modified.
- The return code is set to 1.
- *i* and *xxx* can be determined at run time by use of the ESSL error-handling-facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error code 2101 in the ESSL error option table; otherwise, the default value causes your program to terminate when this error occurs. See “What Can You Do about ESSL Computational Errors?” on page 48.

*Input-Argument Errors*

1. *iopt* ≠ 0, 1, 20, or 21
2. *n* < 0
3. *ldz* ≤ 0 and *iopt* = 1 or 21
4. *n* > *ldz* and *iopt* = 1 or 21
5. Error 2015 is recoverable or *naux*≠0, and *naux* is too small—that is, less than the minimum required value. Return code 2 is returned if error 2015 is recoverable.

***Example 1:*** This example shows how to find the eigenvalues only of a real short-precision symmetric matrix ***A*** of order 3, stored in lower-packed storage mode Matrix ***A*** is:

$$\begin{bmatrix} 1.0 & -1.0 & 0.0 \\ -1.0 & 2.0 & -1.0 \\ 0.0 & -1.0 & 1.0 \end{bmatrix}$$

where:

- `NAUX` is equal to `N`.
- `AUX` contains `N` elements.
- `LDZ` is set to 1 to avoid an error condition.
- `DUMMY` is used as a placeholder for argument *z*, which is not used in the computation.
- On output, `AP` has been overwritten.

*Call Statement and Input*

```
           IOPT  AP   W     Z    LDZ  N   AUZ  NAUX
            |    |    |     |     |   |    |    |
CALL SSPEV( 0  , AP , W , DUMMY , 1 , 3 , AUX , 3  )

AP       =  (1.0, -1.0, 0.0, 2.0, -1.0, 1.0)
```

*Output*

```
        ┌          ┐
        | 0.000000 |
W    =  | 1.000000 |
        | 3.000000 |
        └          ┘
```

***Example 2:*** This example shows how to find the eigenvalues and eigenvectors of a real short-precision symmetric matrix ***A*** of order 4, stored in upper-packed storage mode. Matrix ***A*** is:

$$\begin{bmatrix} 5.0 & 4.0 & 1.0 & 1.0 \\ 4.0 & 5.0 & 1.0 & 1.0 \\ 1.0 & 1.0 & 4.0 & 2.0 \\ 1.0 & 1.0 & 2.0 & 4.0 \end{bmatrix}$$

where:

- `NAUX` is equal to 2`N`.
- `AUX` contains 2`N` elements.

**Note:** This matrix is used in Example 4.1 in referenced text [61 ].

*Call Statement and Input*

```
           IOPT  AP   W   Z  LDZ  N   AUZ  NAUX
            |    |    |   |   |   |    |    |
CALL SSPEV( 21 , AP , W , Z , 4 , 4 , AUX , 8  )

AP       =  (5.0, 4.0, 5.0, 1.0, 1.0, 4.0, 1.0, 1.0, 2.0, 4.0)
```

*Output*

```
         ┌          ┐
         │ 1.000000 │
W    =   │ 2.000000 │
         │ 5.000000 │
         │ 9.999999 │
         └          ┘
```

```
         ┌                                            ┐
         │  0.707107   0.000000   0.316227  0.632455 │
Z    =   │ -0.707107   0.000000   0.316228  0.632455 │
         │  0.000000  -0.707106  -0.632455  0.316227 │
         │  0.000000   0.707107  -0.632455  0.316228 │
         └                                            ┘
```

***Example 3:*** This example shows how to find the eigenvalues and eigenvectors of a real short-precision symmetric matrix ***A***, stored in lower-packed storage mode, which has an eigenvalue of multiplicity 2. Matrix ***A*** is:

```
┌                     ┐
│ 6.0  4.0  4.0  1.0 │
│ 4.0  6.0  1.0  4.0 │
│ 4.0  1.0  6.0  4.0 │
│ 1.0  4.0  4.0  6.0 │
└                     ┘
```

where:

- `NAUX` is equal to 2`N`.
- `AUX` contains 2`N` elements.

**Note:** This matrix is used in Example 4.2 in referenced text [61 ].

*Call Statement and Input*

```
           IOPT  AP   W   Z  LDZ  N   AUZ  NAUX
            |    |    |   |   |   |    |    |
CALL SSPEV( 1  , AP , W , Z , 7 , 4 , AUX , 8  )

AP       =  (6.0, 4.0, 4.0, 1.0, 6.0, 1.0, 4.0, 6.0, 4.0, 6.0)
```

*Output*

```
         ┌           ┐
         │ -1.000000 │
W    =   │  4.999999 │
         │  5.000000 │
         │ 15.000000 │
         └           ┘
```

```
         ┌                                            ┐
         │ -0.500000   0.000000   0.707107  0.500000 │
         │  0.500000   0.707107   0.000000  0.500000 │
         │  0.500000  -0.707107   0.000000  0.500000 │
Z    =   │ -0.500000   0.000000  -0.707107  0.500000 │
         │    .          .          .          .     │
         │    .          .          .          .     │
         │    .          .          .          .     │
         └                                            ┘
```

***Example 4:*** This example shows how the results of Example 3 differ if matrix ***A*** is a real long-precision symmetric matrix.

*Call Statement and Input*

```
           IOPT  AP   W   Z  LDZ  N   AUZ  NAUX
            |    |    |   |   |   |    |    |
CALL DSPEV( 1  , AP , W , Z , 7 , 4 , AUX , 8  )
```

*Output*

```
        ┌          ┐
        | -1.000000 |
W    =  |  5.000000 |
        |  5.000000 |
        | 15.000000 |
        └          ┘

        ┌                                            ┐
        | -0.500000  -0.216773  -0.673060  0.500000 |
        |  0.500000   0.673060  -0.216773  0.500000 |
        |  0.500000  -0.673060   0.216773  0.500000 |
Z    =  | -0.500000   0.216773   0.673060  0.500000 |
        |   .          .          .          .      |
        |   .          .          .          .      |
        |   .          .          .          .      |
        └                                            ┘
```

***Example 5:*** This example shows how to find the eigenvalues and eigenvectors of a complex Hermitian matrix ***A*** of order 2, stored in lower-packed storage mode. Matrix ***A*** is:

```
┌                         ┐
| (1.0, 0.0)  (0.0, -1.0) |
| (0.0, 1.0)  (1.0,  0.0) |
└                         ┘
```

where:

- `NAUX` is equal to 4`N`.
- `AUX` contains 4`N` elements.
- On output, `AP` has been overwritten.

**Note:** This matrix is used in Example 6.1 in referenced text [61 ].

*Call Statement and Input*

```
           IOPT  AP   W   Z  LDZ  N   AUZ  NAUX
            |    |    |   |   |   |    |    |
CALL ZHPEV( 1  , AP , W , Z , 2 , 2 , AUX , 8  )

AP       =  ((1.0, . ), (0.0, 1.0), (1.0, . ))
```

*Output*

```
        ┌          ┐
W    =  | 0.000000 |
        | 2.000000 |
        └          ┘
        ┌                                               ┐
Z    =  │  (0.000000, -0.707107)  (0.000000, -0.707107) │
        │ (-0.707107,  0.000000)  (0.707107,  0.000000) │
        └                                               ┘
```

***Example 6:*** This example shows how to find the eigenvalues only of a complex Hermitian matrix ***A*** of order 4, stored in upper-packed storage mode. Matrix ***A*** is:

```
        ┌                                                    ┐
        │ (3.0,  0.0)  (1.0, 0.0)  (0.0,  0.0)  (0.0, 2.0) │
        │ (1.0,  0.0)  (3.0, 0.0)  (0.0, -2.0)  (0.0, 0.0) │
        │ (0.0,  0.0)  (0.0, 2.0)  (1.0,  0.0)  (1.0, 0.0) │
        │ (0.0, -2.0)  (0.0, 0.0)  (1.0,  0.0)  (1.0, 0.0) │
        └                                                    ┘
```

where:

- NAUX is equal to 3N.
- AUX contains 3N elements.
- LDZ is set to 1 to avoid an error condition.
- DUMMY is used as a placeholder for argument *z*, which is not used in the computation.
- On output, AP has been overwritten.

**Note:** This matrix is used in Example 6.6 in referenced text [61 ].

*Call Statement and Input*

```
           IOPT  AP   W     Z    LDZ  N   AUZ  NAUX
            |    |    |     |     |   |    |    |
CALL ZHPEV( 20 , AP , W , DUMMY , 1 , 4 , AUX , 12 )

AP      =  ((3.0, . ), (1.0, 0.0), (3.0, . ), (0.0, 0.0),
             (0.0, -2.0), (1.0, . ), (0.0, 2.0), (0.0, 0.0),
             (1.0, 0.0), (1.0, . ))
```

*Output*

```
        ┌           ┐
        │ -0.828427 │
W    =  │  0.000000 │
        │  4.000000 │
        │  4.828427 │
        └           ┘
```

***Example 7:*** This example shows how to find the eigenvalues and eigenvectors of a complex Hermitian matrix ***A*** of order 2, stored in lower-packed storage mode. Matrix ***A*** is:

```
        ┌                         ┐
        │ (1.0, 0.0)  (1.0, -1.0) │
        │ (1.0, 1.0)  (1.0,  0.0) │
        └                         ┘
```

where:

- NAUX is equal to 4N.
- AUX contains 4N elements.
- On output, AP has been overwritten.

**Note:** This matrix is used in Example 6.2 in referenced text [61 ].

*Call Statement and Input*

```
           IOPT  AP   W   Z  LDZ  N   AUZ  NAUX
            |    |    |   |   |   |    |    |
CALL ZHPEV( 1  , AP , W , Z , 2 , 2 , AUX , 8  )

AP       =  ((1.0, . ), (1.0, 1.0), (1.0, . ))
```

*Output*

```
        ┌           ┐
W    =  │ -0.414214 │
        │  2.414214 │
        └           ┘


        ┌                                                ┐
Z    =  │  (0.500000, -0.500000)  (0.500000, -0.500000) │
        │ (-0.707107,  0.000000)  (0.707107,  0.000000) │
        └                                                ┘
```

# SSPSV, DSPSV, CHPSV, and ZHPSV—Extreme Eigenvalues and, Optionally, the Eigenvectors of a Real Symmetric Matrix or a Complex Hermitian Matrix

SSPSV and DSPSV compute the extreme eigenvalues and, optionally, the eigenvectors of real symmetric matrix ***A***, stored in lower- or upper-packed storage mode. CHPSV and ZHPSV compute the extreme eigenvalues and, optionally, the eigenvectors of complex Hermitian matrix ***A***, stored in lower- or upper-packed storage mode. The extreme eigenvalues are returned in vector ***w***, and the corresponding eigenvectors are returned in matrix ***Z***:

$$\boldsymbol{Az} = w\boldsymbol{z}$$

where $\boldsymbol{A} = \boldsymbol{A}^T$ or $\boldsymbol{A} = \boldsymbol{A}^H$.

*Table 120. Data Types*

| ***A***, ***z*** | ***w***, *aux* | **Subroutine** |
|---|---|---|
| Short-precision real | Short-precision real | SSPSV |
| Long-precision real | Long-precision real | DSPSV |
| Short-precision complex | Short-precision real | CHPSV |
| Long-precision complex | Long-precision real | ZHPSV |

**Note:** If you want to compute 10% or fewer eigenvalues only, or you want to compute 30% or fewer eigenvalues and eigenvectors, you get better performance if you use _SPSV and _HPSV instead of _SPEV and _HPEV, respectively. For all other uses, you should use _SPEV and _HPEV.

***Syntax***

| | |
|---|---|
| **Fortran** | CALL SSPSV \| DSPSV \| CHPSV \| ZHPSV (*iopt*, *ap*, *w*, *z*, *ldz*, *n*, *m*, *aux*, *naux*) |
| **C and C++** | sspsv \| dspsv \| chpsv \| zhpsv (*iopt*, *ap*, *w*, *z*, *ldz*, *n*, *m*, *aux*, *naux*); |
| **PL/I** | CALL SSPSV \| DSPSV \| CHPSV \| ZHPSV (*iopt*, *ap*, *w*, *z*, *ldz*, *n*, *m*, *aux*, *naux*); |

*On Entry*

*iopt*

indicates the type of computation to be performed, where:

If *iopt* = 0 or 20, the *m* smallest eigenvalues only are computed.

If *iopt* = 1 or 21, the *m* smallest eigenvalues and the eigenvectors are computed.

If *iopt* = 10 or 30, the *m* largest eigenvalues only are computed.

If *iopt* = 11 or 31, the *m* largest eigenvalues and the eigenvectors are computed.

Specified as: a fullword integer; *iopt* = 0, 1, 10, 11, 20, 21, 30, or 31.

*ap*

is the real symmetric or complex Hermitian matrix ***A*** of order *n*, whose *m* smallest or largest eigenvalues and, optionally, the corresponding eigenvectors are computed. It is stored in an array, referred to as `AP`, where:

If *iopt* = 0, 1, 10, or 11, it is stored in lower-packed storage mode.

If *iopt* = 20, 21, 30, or 31, it is stored in upper-packed storage mode.

Specified as: a one-dimensional array of (at least) length $n(n+1)/2$, containing numbers of the data type indicated in Table 120 on page 718. On output, AP is overwritten; that is, the original input is not preserved.

*w*

See “On Return.”

*z*

See “On Return.”

*ldz*

has the following meaning, where:

If *iopt* = 0, 10, 20, or 30, it is not used in the computation.

If *iopt* = 1, 11, 21, or 31, it is the leading dimension of the output array specified for *z*.

Specified as: a fullword integer. It must have the following value, where:

If *iopt* = 0, 10, 20, or 30, $ldz > 0$.

If *iopt* = 1, 11, 21, or 31, $ldz > 0$ and $ldz \geq n$.

*n*

is the order of matrix ***A***. Specified as: a fullword integer; $n \geq 0$.

*m*

is the number of eigenvalues and, optionally, eigenvectors to be computed. Specified as: a fullword integer; $0 \leq m \leq n$.

*aux*

has the following meaning:

If *naux* = 0 and error 2015 is unrecoverable, *aux* is ignored.

Otherwise, it is a storage work area used by this subroutine. Its size is specified by *naux*.

Specified as: an area of storage, containing numbers of the data type indicated in Table 120 on page 718. On output, the contents are overwritten.

*naux*

is the size of the work area specified by *aux*—that is, the number of elements in *aux*. Specified as: a fullword integer, where:

If *naux* = 0 and error 2015 is unrecoverable, SSPSV, DSPSV, CHPSV, and ZHPSV dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, It must have the following value, where:

For SSPSV and DSPSV:
If *iopt* = 0, 10, 20, or 30, $naux \geq 3n$.
If *iopt* = 1, 11, 21, or 31, $naux \geq 9n$.

For CHPSV and ZHPSV:
If *iopt* = 0, 10, 20, or 30, $naux \geq 5n$.
If *iopt* = 1, 11, 21, or 31, $naux \geq 11n$.

*On Return*

*w*

is the vector ***w*** of length *n*, containing in the first *m* positions of either the *m* smallest eigenvalues of ***A*** in ascending order or the *m* largest eigenvalues of ***A*** in descending order.

Returned as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 120 on page 718.

*z*

has the following meaning, where:

If *iopt* = 0, 10, 20, or 30, it is not used in the computation.

If *iopt* = 1, 11, 21, or 31, it is the *n* by *m* matrix ***Z***, containing *m* orthonormal eigenvectors of matrix ***A***. The eigenvector in column *i* of matrix ***Z*** corresponds to the eigenvalue $w_i$.

Returned as: an *ldz* by (at least) *m* array, containing numbers of the data type indicated in Table 120 on page 718.

### *Notes*

1. When you specify *iopt* = 0, 10, 20, or 30, you must specify:
   - A positive value for *ldz*
   - A dummy argument for *z* (see “Example 4” on page 724)
2. The following items must have no common elements: matrix ***A***, matrix ***Z***, vector ***w***, and the data area specified for *aux*; otherwise, results are unpredictable. See “Concepts” on page 55.
3. On input, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values.
4. For a description of how real symmetric matrices are stored in lower- or upper-packed storage mode, see “Lower-Packed Storage Mode” on page 66 or “Upper-Packed Storage Mode” on page 67, respectively.

   For a description of how complex Hermitian matrices are stored in lower- or upper-packed storage mode, see “Complex Hermitian Matrix” on page 70.
5. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see “Using Auxiliary Storage in ESSL” on page 32.

***Function:*** The methods used to compute the extreme eigenvalues and, optionally, the eigenvectors for either a real symmetric matrix or a complex Hermitian matrix are described in the steps below. For more information on these methods, see references [39], [43 ], [61 ], [84 ], [94], and [96 ]. If *n* or *m* is 0, no computation is performed. The results of the computations using short- and long-precision data can vary in accuracy. Eigenvalues computed using equivalent *iopt* values are mathematically equivalent, but are not guaranteed to be bitwise identical. For example, the results computed using *iopt* = 0 and *iopt* = 20 are mathematically equivalent, but are not necessarily bitwise identical.

These algorithms have a tendency to generate underflows that may hurt overall performance. The system default is to mask underflow, which improves the performance of these subroutines.

The extreme eigenvalues and, optionally, the eigenvectors of a real symmetric matrix ***A*** or complex Hermitian matrix ***A*** are computed as follows:

- For *iopt* = 0, 10, 20, or 30, the eigenvalues are computed as follows:
  1. Reduce the real symmetric matrix ***A*** (for SSPSV and DSPSV) or complex Hermitian matrix ***A*** (for CHPSV and ZHPSV) to a real symmetric tridiagonal matrix using orthogonal similarity transformations (for SSPSV and DSPSV) or unitary similarity transforms (for CHPSV and ZHPSV).
  2. Compute the *m* smallest eigenvalues or *m* largest eigenvalues of the real symmetric tridiagonal matrix using a rational variant of the QR method with Newton corrections.
  3. The eigenvalues are returned in vector ***w*** in the first *m* positions, where the *m* smallest are placed in ascending order, or the *m* largest are placed in descending order.
- For *iopt* = 1, 11, 21, or 31, the eigenvalues and eigenvectors are computed as follows:
  1. Reduce the real symmetric matrix ***A*** (for SSPSV and DSPSV) or complex Hermitian matrix ***A*** (for CHPSV and ZHPSV) to a real symmetric tridiagonal matrix using orthogonal similarity transformations (for SSPSV and DSPSV) or unitary similarity transforms (for CHPSV and ZHPSV).
  2. Compute the *m* smallest eigenvalues or *m* largest eigenvalues of the real symmetric tridiagonal matrix using a rational variant of the QR method with Newton corrections.
  3. Compute the corresponding eigenvectors of the real symmetric tridiagonal matrix using inverse iteration.
  4. Back transform the eigenvectors of the real symmetric tridiagonal matrix to those of the original matrix.
  5. The eigenvalues are returned in vector ***w*** in the first *m* positions, where the *m* smallest are placed in ascending order, or the *m* largest are placed in descending order. The corresponding eigenvectors are returned in matrix ***Z***.

### *Error Conditions*

*Resource Errors:* Error 2015 is unrecoverable, *naux* = 0, and unable to allocate work area.

*Computational Errors*

1. Eigenvalue (*i*) failed to converge after (*xxx*) iterations. (The computational error message may occur multiple times with processing continuing after each error, because the number of allowable errors for error code 2114 is set to be unlimited in the ESSL error option table.)
   - The eigenvalue, $w_i$, is the best estimate obtained. Any eigenvalues for which this message has not been issued are correct.
   - *ap* is modified.
   - The return code is set to 1.
   - *i* and *xxx* can be determined at run time by use of the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error 2199 in the ESSL error option table. See “What Can You Do about ESSL Computational Errors?” on page 48.
2. Eigenvector (*i*) failed to converge after (*xxx*) iterations. (The computational error message may occur multiple times with processing continuing after each error, because the number of allowable errors for error code 2102 is set to be unlimited in the ESSL error option table.)

- All eigenvalues are correct.
- The eigenvector that failed to converge is set to zero; however, any selected eigenvectors for which this message is not issued are correct.
- *ap* is modified.
- The return code is set to 2.
- *i* and *xxx* can be determined at run time by use of the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error 2199 in the ESSL error option table. See “What Can You Do about ESSL Computational Errors?” on page 48.

*Input-Argument Errors*

1. *iopt* ≠ 0, 1, 10, 11, 20, 21, 30, or 31
2. $n < 0$
3. $m < 0$
4. $m > n$
5. *ldz* ≤ 0 and *iopt* = 1, 11, 21, or 31
6. *n* > *ldz* and *iopt* = 1, 11, 21, or 31
7. Error 2015 is recoverable or *naux*≠0, and *naux* is too small—that is, less than the minimum required value. Return code 3 is returned if error 2015 is recoverable.

***Example 1:*** This example shows how to find the two smallest eigenvalues and corresponding eigenvectors of a real long-precision symmetric matrix ***A*** of order 4, stored in upper-packed storage mode. Matrix ***A*** is:

$$\begin{bmatrix} 5.0 & 4.0 & 1.0 & 1.0 \\ 4.0 & 5.0 & 1.0 & 1.0 \\ 1.0 & 1.0 & 4.0 & 2.0 \\ 1.0 & 1.0 & 2.0 & 4.0 \end{bmatrix}$$

where:

- NAUX is equal to 9N.
- AUX contains 9N elements.
- On output, AP has been overwritten.

**Note:** This matrix is used in Example 4.1 in referenced text [61 ].

*Call Statement and Input*

```
          IOPT  AP   W   Z  LDZ  N   M   AUX  NAUX
           |    |    |   |   |   |   |    |    |
CALL DSPSV( 21 , AP , W , Z , 4 , 4 , 2 , AUX , 36 )

AP      =  (5.0, 4.0, 5.0, 1.0, 1.0, 4.0, 1.0, 1.0, 2.0, 4.0)
```

*Output*

```
        ┌          ┐
        | 1.000000 |
W    =  | 2.000000 |
        |   .      |
        |   .      |
        └          ┘
```

```
         ┌                      ┐
         │ -0.707107   0.000000 │
Z    =   │  0.707107   0.000000 │
         │  0.000000  -0.707107 │
         │  0.000000   0.707107 │
         └                      ┘
```

***Example 2:*** This example shows how to find the three largest eigenvalues and corresponding eigenvectors of a real long-precision symmetric matrix ***A*** of order 4, stored in lower-packed storage mode, having an eigenvalue of multiplicity two. Matrix ***A*** is:

```
                         ┌                     ┐
                         │ 6.0  4.0  4.0  1.0  │
                         │ 4.0  6.0  1.0  4.0  │
                         │ 4.0  1.0  6.0  4.0  │
                         │ 1.0  4.0  4.0  6.0  │
                         └                     ┘
```

where:

- NAUX is equal to 9N.
- AUX contains 9N elements.
- On output, AP has been overwritten.

**Note:** This matrix is used in Example 4.2 in referenced text [61 ].

*Call Statement and Input*

```
           IOPT  AP   W   Z  LDZ  N   M   AUX  NAUX
            |    |    |   |   |   |   |    |    |
CALL DSPSV( 11 , AP , W , Z , 8 , 4 , 3 , AUX , 36 )

AP       =  (6.0, 4.0, 4.0, 1.0, 6.0, 1.0, 4.0, 6.0, 4.0, 6.0)
```

*Output*

```
         ┌           ┐
         │ 15.000000 │
W    =   │  5.000000 │
         │  5.000000 │
         │   .       │
         └           ┘

         ┌                                  ┐
         │  0.500000   0.707107   0.000000  │
         │  0.500000   0.000000  -0.707107  │
         │  0.500000   0.000000   0.707107  │
Z    =   │  0.500000  -0.707107   0.000000  │
         │   .          .          .        │
         │   .          .          .        │
         │   .          .          .        │
         │   .          .          .        │
         └                                  ┘
```

***Example 3:*** This example shows how to find the largest eigenvalue and the corresponding eigenvector of a complex Hermitian matrix ***A*** of order 2, stored in lower-packed storage mode. Matrix ***A*** is:

```
                        ┌                          ┐
                        │ (1.0, 0.0)  (0.0, -1.0) │
                        │ (0.0, 1.0)  (1.0,  0.0) │
                        └                          ┘
```

where:

- NAUX is equal to 11N.
- AUX contains 11N elements.
- On output, AP has been overwritten.

**Note:** This matrix is used in Example 6.1 in referenced text [61 ].

*Call Statement and Input*

```
           IOPT  AP   W   Z  LDZ  N   M   AUX  NAUX
            |    |    |   |   |   |   |    |    |
CALL ZHPSV( 11 , AP , W , Z , 2 , 2 , 1 , AUX , 22 )

AP       =  ((1.0, . ), (0.0, 1.0), (1.0, . ))
```

*Output*

```
         ┌          ┐
W    =   │ 2.000000 │
         │  .       │
         └          ┘


         ┌                       ┐
Z    =   │ (0.000000, -0.707107) │
         │ (0.707107,  0.000000) │
         └                       ┘
```

***Example 4:*** This example shows how to find the two smallest eigenvalues only of a complex Hermitian matrix ***A*** of order 4, stored in upper-packed storage mode. Matrix ***A*** is:

```
         ┌                                                  ┐
         │ (3.0,  0.0)  (1.0, 0.0)  (0.0,  0.0)  (0.0, 2.0) │
         │ (1.0,  0.0)  (3.0, 0.0)  (0.0, -2.0)  (0.0, 0.0) │
         │ (0.0,  0.0)  (0.0, 2.0)  (1.0,  0.0)  (1.0, 0.0) │
         │ (0.0, -2.0)  (0.0, 0.0)  (1.0,  0.0)  (1.0, 0.0) │
         └                                                  ┘
```

where:

- NAUX is equal to 5N.
- AUX contains 5N elements.
- LDZ is set to 1 to avoid an error condition.
- DUMMY is used as a placeholder for argument *z*, which is not used in the computation.
- On output, AP has been overwritten.

**Note:** This matrix is used in Example 6.6 in referenced text [61 ].

*Call Statement and Input*

```
           IOPT  AP   W     Z    LDZ  N   M   AUX  NAUX
            |    |    |     |     |   |   |    |    |
CALL ZHPSV( 20 , AP , W , DUMMY , 1 , 4 , 2 , AUX , 20 )

AP       =  ((3.0, . ), (1.0, 0.0), (3.0, . ), (0.0, 0.0),
             (0.0, -2.0), (1.0, . ), (0.0, 2.0), (0.0, 0.0),
             (1.0, 0.0), (1.0, . ))
```

*Output*

```
        ┌           ┐
        | -0.828427 |
W    =  |  0.000000 |
        |   .       |
        |   .       |
        └           ┘
```

# SGEGV and DGEGV—Eigenvalues and, Optionally, the Eigenvectors of a Generalized Real Eigensystem, Az=wBz, where A and B Are Real General Matrices

These subroutines compute the eigenvalues and, optionally, the eigenvectors of a generalized real eigensystem, where ***A*** and ***B*** are real general matrices. Eigenvalues ***w*** are based on the two parts returned in vectors α and β, such that $w_i = \alpha_i/\beta_i$ for $\beta_i \neq 0$, and $w_i = \infty$ for $\beta_i = 0$. Eigenvectors are returned in matrix ***Z***:

$$\boldsymbol{Az} = w\boldsymbol{Bz}$$

| Table 121. Data Types | | |
|---|---|---|
| ***A***, ***B***, β, *aux* | α, ***Z*** | **Subroutine** |
| Short-precision real | Short-precision complex | SGEGV |
| Long-precision real | Long-precision complex | DGEGV |

### Syntax

| | |
|---|---|
| **Fortran** | CALL SGEGV \| DGEGV (*iopt*, *a*, *lda*, *b*, *ldb*, *alpha*, *beta*, *z*, *ldz*, *n*, *aux*, *naux*) |
| **C and C++** | sgegv \| dgegv (*iopt*, *a*, *lda*, *b*, *ldb*, *alpha*, *beta*, *z*, *ldz*, *n*, *aux*, *naux*); |
| **PL/I** | CALL SGEGV \| DGEGV (*iopt*, *a*, *lda*, *b*, *ldb*, *alpha*, *beta*, *z*, *ldz*, *n*, *aux*, *naux*); |

*On Entry*

*iopt*
indicates the type of computation to be performed, where:

If *iopt* = 0, eigenvalues only are computed.

If *iopt* = 1, eigenvalues and eigenvectors are computed.

Specified as: a fullword integer; *iopt* = 0 or 1.

*a*
is the real general matrix ***A*** of order *n*. Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 121. On output, A is overwritten; that is, the original input is not preserved.

*lda*
is the leading dimension of the array specified for *a*. Specified as: a fullword integer; *lda* > 0 and *lda* ≥ *n*.

*b*
is the real general matrix ***B*** of order *n*. Specified as: an *ldb* by (at least) *n* array, containing numbers of the data type indicated in Table 121. On output, B is overwritten; that is, the original input is not preserved.

*ldb*
is the leading dimension of the array specified for *b*. Specified as: a fullword integer; *ldb* > 0 and *ldb* ≥ *n*.

*alpha*
See "On Return" on page 727.

*beta*
See "On Return" on page 727.

*z*
See "On Return" on page 727.

*ldz*

has the following meaning, where:

If *iopt* = 0, it is not used in the computation.

If *iopt* = 1, it is the leading dimension of the output array specified for *z*.

Specified as: a fullword integer. It must have the following value, where:

If *iopt* = 0, *ldz* > 0.

If *iopt* = 1, *ldz* > 0 and *ldz* ≥ *n*.

*n*

is the order of matrices ***A*** and ***B***. Specified as: a fullword integer; *n* ≥ 0.

*aux*

has the following meaning:

If *naux* = 0 and error 2015 is unrecoverable, *aux* is ignored.

Otherwise, it is a storage work area used by this subroutine. Its size is specified by *naux*.

Specified as: an area of storage, containing numbers of the data type indicated in Table 121 on page 726. On output, the contents are overwritten.

*naux*

is the size of the work area specified by *aux*—that is, the number of elements in *aux*. Specified as: a fullword integer, where:

If *naux* = 0 and error 2015 is unrecoverable, SGEGV and DGEGV dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, *naux* ≥ 3*n*.

*On Return*

*alpha*

is the vector $\alpha$ of length *n*, containing the numerators of the eigenvalues of the generalized real eigensystem ***Az*** = *w**Bz***. Returned as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 121 on page 726.

*beta*

is the vector $\beta$ of length *n*, containing the denominators of the eigenvalues of the generalized real eigensystem ***Az*** = *w**Bz***. Returned as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 121 on page 726.

*z*

has the following meaning, where:

If *iopt* = 0, it is not used in the computation.

If *iopt* = 1, it is the matrix ***Z*** of order *n*, containing the eigenvectors of the generalized real eigensystem, ***Az*** = *w**Bz***. The eigenvector in column *i* of matrix ***Z*** corresponds to the eigenvalue $w_i$, computed using the $\alpha_i$ and $\beta_i$ values. Each eigenvector is normalized so that the modulus of its largest element is 1.

Returned as: an *ldz* by (at least) *n* array, containing numbers of the data type indicated in Table 121 on page 726.

***Notes***

1. When you specify *iopt* = 0, you must specify:
   - A positive value for *ldz*
   - A dummy argument for *z* (see “Example 1” on page 729)
2. Matrices ***A***, ***B***, and ***Z***, vectors $\alpha$ and $\beta$, and the work area specified for *aux* must have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.
3. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see “Using Auxiliary Storage in ESSL” on page 32.

***Function:*** The following steps describe the methods used to compute the eigenvalues and, optionally, the eigenvectors of a generalized real eigensystem, ***Az*** = *w**Bz***, where ***A*** and ***B*** are real general matrices. The methods are based upon Moler and Stewart's QZ algorithm. You must calculate the resulting eigenvalues ***w*** based on the two parts returned in vectors $\alpha$ and $\beta$. Each eigenvalue is calculated as follows: $w_i = \alpha_i/\beta_i$ for $\beta_i \neq 0$ and $w_i = \infty$ for $\beta_i = 0$. Eigenvalues are unordered, except that complex conjugate pairs appear consecutively with the eigenvalue having the positive imaginary part first.

- For *iopt* = 0, the eigenvalues are computed as follows:
  1. Simultaneously reduce ***A*** to upper Hessenberg form and ***B*** to upper triangular form using orthogonal transformations.
  2. Reduce ***A*** from upper Hessenberg form to quasi-upper triangular form while maintaining the upper triangular form of ***B*** using orthogonal transformations.
  3. Compute the eigenvalues of the generalized real eigensystem with ***A*** in quasi-upper triangular form and ***B*** in upper triangular form using orthogonal transformations.
  4. The numerators and denominators of the eigenvalues are returned in vectors $\alpha$ and $\beta$, respectively.
- For *iopt* = 1, the eigenvalues and eigenvectors are computed as follows:
  1. Simultaneously reduce ***A*** to upper Hessenberg form and ***B*** to upper triangular form using and accumulating orthogonal transformations.
  2. Reduce ***A*** from upper Hessenberg form to quasi-upper triangular form while maintaining the upper triangular form of ***B*** using and accumulating orthogonal transformations.
  3. Compute the eigenvalues of the generalized real eigensystem with ***A*** in quasi-upper triangular form and ***B*** in upper triangular form using and accumulating orthogonal transformations.
  4. Compute the eigenvectors of the generalized real eigensystem with ***A*** in quasi-upper triangular form and ***B*** in upper triangular form using back substitution.
  5. The numerators and denominators of the eigenvalues are returned in vectors $\alpha$ and $\beta$, respectively, and the eigenvectors are returned in matrix ***Z***.

For more information on these methods, see references [39 ], [43 ], [56 ], [80 ], [61], [60 ], [84 ], [94], and [96 ]. If *n* is 0, no computation is performed. The results of the computations using short- and long-precision data can vary in accuracy.

These algorithms have a tendency to generate underflows that may hurt overall performance. The system default is to mask underflow, which improves the performance of these subroutines.

***Error Conditions***

*Resource Errors:* Error 2015 is unrecoverable, *naux* = 0, and unable to allocate work area.

*Computational Errors:* Eigenvalue (*i*) failed to converge after (*xxx*) iterations:

- The eigenvalues ($w_j$, $j = i+1, i+2, ..., n$) are correct.
- If *iopt* = 1, then ***Z*** is modified, but no eigenvectors are correct.
- ***A*** and ***B*** have been modified.
- The return code is set to 1.
- *i* and *xxx* can be determined at run time by use of the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error code 2101 in the ESSL error option table; otherwise, the default value causes your program to terminate when this error occurs. See “What Can You Do about ESSL Computational Errors?” on page 48.

*Input-Argument Errors*

1. *iopt* ≠ 0 or 1
2. $n < 0$
3. *lda* ≤ 0
4. *n* > *lda*
5. *ldb* ≤ 0
6. *n* > *ldb*
7. *ldz* ≤ 0 and *iopt* = 1
8. *n* > *ldz* and *iopt* = 1
9. Error 2015 is recoverable or *naux*≠0, and *naux* is too small—that is, less than the minimum required value. Return code 2 is returned if error 2015 is recoverable.

***Example 1:*** This example shows how to find the eigenvalues only of a real generalized eigensystem problem, ***AZ*** = ***wBZ***, where:

- NAUX is equal to 3N.
- AUX contains 3N elements.
- LDZ is set to 1 to avoid an error condition.
- DUMMY is used as a placeholder for argument *z*, which is not used in the computation.
- On output, matrices A and B are overwritten.

**Note:** These matrices are from page 257 in referenced text [60 ].

*Call Statement and Input*

```
              IOPT  A  LDA  B  LDB  ALPHA   BETA     Z    LDZ  N   AUX  NAUX
               |    |   |   |   |     |       |      |     |   |    |    |
CALL DGEGV( 0   , A , 3 , B , 3 , ALPHA , BETA , DUMMY , 1 , 3 , AUX , 9  )
```

```
             ┌                 ┐
             | 10.0  1.0   2.0 |
A     =      |  1.0  3.0  -1.0 |
             |  1.0  1.0   2.0 |
             └                 ┘
```

```
             ┌               ┐
             | 1.0  2.0  3.0 |
B     =      | 4.0  5.0  6.0 |
             | 7.0  8.0  9.0 |
             └               ┘
```

*Output*

```
        ┌                        ┐
        |  (4.778424, 0.000000)  |
ALPHA = | (-4.760580, 0.000000)  |
        |  (2.769466, 0.000000)  |
        └                        ┘
```

```
        ┌           ┐
        |  0.000000 |
BETA  = |  0.934851 |
        | 15.446215 |
        └           ┘
```

***Example 2:*** This example shows how to find the eigenvalues and eigenvectors of a real generalized eigensystem problem, ***AZ*** = ***wBZ***, where:

- NAUX is equal to 3N.
- AUX contains 3N elements.
- On output, matrices A and B are overwritten.

**Note:** These matrices are from page 263 in referenced text [60 ].

*Call Statement and Input*

```
           IOPT  A  LDA  B  LDB  ALPHA   BETA   Z  LDZ  N   AUX  NAUX
            |    |   |   |   |     |       |    |   |   |    |    |
CALL DGEGV( 1  , A , 5 , B , 5 , ALPHA , BETA , Z , 5 , 5 , AUX , 15 )
```

```
             ┌                          ┐
             | 2.0  3.0  4.0  5.0   6.0 |
             | 4.0  4.0  5.0  6.0   7.0 |
A     =      | 0.0  3.0  6.0  7.0   8.0 |
             | 0.0  0.0  2.0  8.0   9.0 |
             | 0.0  0.0  0.0  1.0  10.0 |
             └                          ┘
```

```
             ┌                              ┐
             | 1.0  -1.0  -1.0  -1.0  -1.0 |
             | 0.0   1.0  -1.0  -1.0  -1.0 |
B     =      | 0.0   0.0   1.0  -1.0  -1.0 |
             | 0.0   0.0   0.0   1.0  -1.0 |
             | 0.0   0.0   0.0   0.0   1.0 |
             └                              ┘
```

*Output*

```
         ┌                       ┐
         │  (7.950050, 0.000000) │
         │ (-0.277338, 0.000000) │
ALPHA =  │  (2.149669, 0.000000) │
         │  (6.720718, 0.000000) │
         │ (10.987556, 0.000000) │
         └                       ┘

         ┌          ┐
         │ 0.374183 │
         │ 1.480299 │
BETA  =  │ 1.636872 │
         │ 1.213574 │
         │ 0.908837 │
         └          ┘

        ┌
        │ (1.000000, 0.000000)  (-0.483408, 0.000000)   (0.540696, 0.000000)
        │ (0.565497, 0.000000)   (1.000000, 0.000000)   (0.684441, 0.000000)
Z     = │ (0.180429, 0.000000)  (-0.661372, 0.000000)  (-1.000000, 0.000000)
        │ (0.034182, 0.000000)   (0.180646, 0.000000)   (0.363671, 0.000000)
        │ (0.003039, 0.000000)  (-0.017732, 0.000000)  (-0.041865, 0.000000)
        └

                                                                      ┐
                          (1.000000, 0.000000)  (-1.000000, 0.000000) │
                          (0.722065, 0.000000)  (-0.610415, 0.000000) │
                         (-0.089003, 0.000000)  (-0.116987, 0.000000) │
                         (-0.223599, 0.000000)   (0.038979, 0.000000) │
                          (0.050111, 0.000000)   (0.018653, 0.000000) │
                                                                      ┘
```

## SSYGV and DSYGV—Eigenvalues and, Optionally, the Eigenvectors of a Generalized Real Symmetric Eigensystem, Az=wBz, where A Is Real Symmetric and B Is Real Symmetric Positive Definite

These subroutines compute the eigenvalues and, optionally, the eigenvectors of a generalized real symmetric eigensystem, where ***A*** is a real symmetric matrix, and ***B*** is a real positive definite symmetric matrix. Both ***A*** and ***B*** are stored in lower storage mode in two-dimensional arrays. Eigenvalues are returned in vector ***w***, and eigenvectors are returned in matrix ***Z***:

$$\boldsymbol{Az} = w\boldsymbol{Bz}$$

where $\boldsymbol{A} = \boldsymbol{A}^T$, $\boldsymbol{B} = \boldsymbol{B}^T$, and $\boldsymbol{x}^T\boldsymbol{Bx} > 0$.

*Table 122. Data Types*

| ***A***, ***B***, ***w***, ***Z***, *aux* | **Subroutine** |
|---|---|
| Short-precision real | SSYGV |
| Long-precision real | DSYGV |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SSYGV \| DSYGV (*iopt*, *a*, *lda*, *b*, *ldb*, *w*, *z*, *ldz*, *n*, *aux*, *naux*) |
| **C and C++** | ssygv \| dsygv (*iopt*, *a*, *lda*, *b*, *ldb*, *w*, *z*, *ldz*, *n*, *aux*, *naux*); |
| **PL/I** | CALL SSYGV \| DSYGV (*iopt*, *a*, *lda*, *b*, *ldb*, *w*, *z*, *ldz*, *n*, *aux*, *naux*); |

*On Entry*

*iopt*

indicates the type of computation to be performed, where:

If *iopt* = 0, eigenvalues only are computed.

If *iopt* = 1, eigenvalues and eigenvectors are computed.

Specified as: a fullword integer; *iopt* = 0 or 1.

*a*

is the real symmetric matrix ***A*** of order *n*. It is stored in lower storage mode. Specified as: an *lda* by (at least) *n* array, containing numbers of the data type indicated in Table 122. On output, the data in the lower triangle of ***A*** is overwritten; that is, the original input is not preserved.

*lda*

is the leading dimension of the array specified for *a*. Specified as: a fullword integer; *lda* > 0 and *lda* ≥ *n*.

*b*

is the real positive definite symmetric matrix ***B*** of order *n*. It is stored in lower storage mode. Specified as: an *ldb* by (at least) *n* array, containing numbers of the data type indicated in Table 122. On output, the data in the lower triangle of ***B*** is overwritten; that is, the original input is not preserved.

*ldb*

is the leading dimension of the array specified for *b*. Specified as: a fullword integer; *ldb* > 0 and *ldb* ≥ *n*.

*w*

See “On Return” on page 733.

*z*

See “On Return” on page 733.

*ldz*

has the following meaning, where:

If *iopt* = 0, it is not used in the computation.

If *iopt* = 1, it is the leading dimension of the output array specified for *z*.

Specified as: a fullword integer. It must have the following value, where:

If *iopt* = 0, *ldz* > 0.

If *iopt* = 1, *ldz* > 0 and *ldz* ≥ *n*.

*n*

is the order of matrices ***A*** and ***B***. Specified as: a fullword integer; *n* ≥ 0.

*aux*

has the following meaning:

If *naux* = 0 and error 2015 is unrecoverable, *aux* is ignored.

Otherwise, it is a storage work area used by this subroutine. Its size is specified by *naux*.

Specified as: an area of storage, containing numbers of the data type indicated in Table 122 on page 732. On output, the contents are overwritten.

*naux*

is the size of the work area specified by *aux*—that is, the number of elements in *aux*. Specified as: a fullword integer, where:

If *naux* = 0 and error 2015 is unrecoverable, SSYGV and DSYGV dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, It must have the following value, where:

If *iopt* = 0, *naux* ≥ *n*.

If *iopt* = 1, *naux* ≥ 2*n*.

*On Return*

*w*

is the vector ***w*** of length *n*, containing the eigenvalues of the generalized real symmetric eigensystem ***Az*** = *w**Bz*** in ascending order. Returned as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 122 on page 732.

*z*

has the following meaning, where:

If *iopt* = 0, it is not used in the computation.

If *iopt* = 1, it is the matrix ***Z*** of order *n*, containing the eigenvectors of the generalized real symmetric eigensystem, ***Az*** = *w**Bz***. The eigenvectors are normalized so that $\boldsymbol{Z}^{T}\boldsymbol{BZ} = I$. The eigenvector in column *i* of matrix ***Z*** corresponds to the eigenvalue $w_i$.

Returned as: an *ldz* by (at least) *n* array, containing numbers of the data type indicated in Table 122 on page 732.

***Notes***

1. When you specify *iopt* = 0, you must specify:
   - A positive value for *ldz*
   - A dummy argument for *z* (see “Example 1” on page 735)
2. Matrices ***A*** and ***Z*** may coincide. Matrices ***A*** and ***B***, vector ***w***, and the data area specified for *aux* must have no common elements; otherwise, results are unpredictable. Matrices ***Z*** and ***B***, vector ***w***, and the data area specified for *aux* must also have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.
3. For a description of how real symmetric matrices are stored in lower storage mode, see “Lower Storage Mode” on page 68.
4. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see “Using Auxiliary Storage in ESSL” on page 32.

***Function:*** The following steps describe the methods used to compute the eigenvalues and, optionally, the eigenvectors of a generalized real symmetric eigensystem, $\boldsymbol{Az} = w\boldsymbol{Bz}$, where ***A*** is a real symmetric matrix, and ***B*** is a real positive definite symmetric matrix. Both ***A*** and ***B*** are stored in lower storage mode in two-dimensional arrays.

1. Compute the Cholesky Decomposition of ***B***:

   $\boldsymbol{B} = \boldsymbol{LL}^{\mathrm{T}}$

   For a description of methods used in this computation, see “SPPF, DPPF, SPOF, DPOF, CPOF, and ZPOF—Positive Definite Real Symmetric or Complex Hermitian Matrix Factorization” on page 492.
2. Compute ***C***:

   $\boldsymbol{C} = \boldsymbol{L}^{-1}\boldsymbol{AL}^{-\mathrm{T}}$

   In this computation, ***C*** overwrites ***A***.
3. Solve the real symmetric eigensystems analysis problem, computing the eigenvalues ***w*** and, optionally, the eigenvectors ***Y***:

   $\boldsymbol{CY} = \boldsymbol{wY}$

   where:

   $\boldsymbol{Y} = \boldsymbol{L}^{\mathrm{T}}\boldsymbol{Z}$

   For a description of the methods used for this computation, see “Real Symmetric Matrix” on page 711. In this computation, ***Y*** overwrites ***Z***.
4. If eigenvectors are requested (with *iopt* = 1), transform the eigenvectors ***Y*** into the eigenvectors ***Z*** of the original system, $\boldsymbol{Az} = w\boldsymbol{Bz}$, by solving $\boldsymbol{L}^{\mathrm{T}}\boldsymbol{Z} = \boldsymbol{Y}$ for ***Z***:

   $\boldsymbol{Z} = \boldsymbol{L}^{-\mathrm{T}}\boldsymbol{Y}$

For more information on these methods, see references [39 ], [43 ], [56 ], [61 ], [60], [84 ], [94 ], and [96]. If *n* is 0, no computation is performed. The results of the computations using short- and long-precision data can vary in accuracy.

These algorithms have a tendency to generate underflows that may hurt overall performance. The system default is to mask underflow, which improves the performance of these subroutines.

### *Error Conditions*

*Resource Errors:* Error 2015 is unrecoverable, *naux* = 0, and unable to allocate work area.

*Computational Errors*

1. The ***B*** matrix is not positive definite. The leading minor of order *i* has a nonpositive determinant.
   - ***B*** is modified, but no eigenvalues or eigenvectors are correct.
   - The return code is set to 1.
   - *i* can be determined at run time by use of the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error code 2115 in the ESSL error option table; otherwise, the default value causes your program to terminate when this error occurs. See "What Can You Do about ESSL Computational Errors?" on page 48.
2. Eigenvalue (*i*) failed to converge after (*xxx*) iterations:
   - The eigenvalues ($w_j$, $j = 1, 2, ..., i–1$) are correct, but are unordered.
   - If *iopt* = 1, then *z* is modified, but no eigenvectors are correct.
   - ***A*** and ***B*** have been modified.
   - The return code is set to 2.
   - *i* and *xxx* can be determined at run time by use of the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error code 2101 in the ESSL error option table; otherwise, the default value causes your program to terminate when this error occurs. See "What Can You Do about ESSL Computational Errors?" on page 48.

*Input-Argument Errors*

1. *iopt* ≠ 0 or 1
2. *n* < 0
3. *lda* ≤ 0
4. *n* > *lda*
5. *ldb* ≤ 0
6. *n* > *ldb*
7. *ldz* ≤ 0 and *iopt* = 1
8. *n* > *ldz* and *iopt* = 1
9. Error 2015 is recoverable or *naux*≠0, and *naux* is too small—that is, less than the minimum required value. Return code 3 is returned if error 2015 is recoverable.

***Example 1:*** This example shows how to find the eigenvalues only of a real symmetric generalized eigensystem problem, ***AZ*** = ***wBZ***, where:

- `NAUX` is equal to `N`.
- `AUX` contains `N` elements.
- `LDZ` is set to 1 to avoid an error condition.
- `DUMMY` is used as a placeholder for argument *z*, which is not used in the computation.
- On output, the lower triangle of `A` and `B` is overwritten.

**Note:** These matrices are used in Example 8.6.2 in referenced text [60 ].

*Call Statement and Input*

```
           IOPT  A  LDA  B  LDB  W     Z    LDZ  N   AUX  NAUX
            |    |   |   |   |   |     |     |   |    |    |
CALL DSYGV( 0  , A , 2 , B , 2 , W , DUMMY , 1 , 2 , AUX , 2  )
```

```
        ┌             ┐
A    =  │ 229.0     .  │
        │ 163.0  116.0 │
        └             ┘
```

```
        ┌           ┐
B    =  │ 81.0    .  │
        │ 59.0  43.0 │
        └           ┘
```

*Output*

```
        ┌           ┐
W    =  │ -0.500000 │
        │  5.000000 │
        └           ┘
```

***Example 2:*** This example shows how to find the eigenvalues and eigenvectors of a real symmetric generalized eigensystem problem, ***AZ*** = ***wBZ***, where:

- NAUX is equal to 2N.
- AUX contains 2N elements.
- On output, the lower triangle of A and B is overwritten.

**Note:** These matrices are from page 67 in referenced text [56 ].

*Call Statement and Input*

```
           IOPT  A  LDA  B  LDB  W   Z  LDZ  N   AUX  NAUX
            |    |   |   |   |   |   |   |   |    |    |
CALL DSYGV( 1  , A , 3 , B , 3 , W , Z , 3 , N , AUX , 6  )
```

```
        ┌                ┐
        │ -1.0    .    .  │
A    =  │  1.0   1.0   .  │
        │ -1.0  -1.0  1.0 │
        └                ┘
```

```
        ┌              ┐
        │ 2.0   .    .  │
B    =  │ 1.0  2.0   .  │
        │ 0.0  1.0  2.0 │
        └              ┘
```

*Output*

```
        ┌           ┐
        │ -1.500000 │
W    =  │  0.000000 │
        │  2.000000 │
        └           ┘
```

```
            ┌                                  ┐
            │  0.866025   0.000000   0.000000  │
Z     =     │ -0.577350  -0.408248  -0.707107  │
            │  0.288675  -0.408248   0.707107  │
            └                                  ┘
```

# Chapter 12. Fourier Transforms, Convolutions and Correlations, and Related Computations

The signal processing subroutines, provided in three areas, are described in this chapter.

## Overview of the Signal Processing Subroutines

This section describes the subroutines in each of the three signal processing areas:

- Fourier transform subroutines (Table 123 )
- Convolution and correlation subroutines (Table 124 )
- Related-computation subroutines (Table 125 )

## Fourier Transforms Subroutines

The Fourier transform subroutines perform mixed-radix transforms in one, two, and three dimensions.

*Table 123. List of Fourier Transform Subroutines*

| **Descriptive Name** | **Short-Precision Subroutine** | **Long-Precision Subroutine** | **Page** |
|---|---|---|---|
| Complex Fourier Transform | SCFT<br>SCFTP§ | DCFT | 749 |
| Real-to-Complex Fourier Transform | SRCFT | DRCFT | 757 |
| Complex-to-Real Fourier Transform | SCRFT | DCRFT | 765 |
| Cosine Transform | SCOSF<br>SCOSFT§ | DCOSF | 773 |
| Sine Transform | SSINF | DSINF | 780 |
| Complex Fourier Transform in Two Dimensions | SCFT2<br>SCFT2P§ | DCFT2 | 787 |
| Real-to-Complex Fourier Transform in Two Dimensions | SRCFT2 | DRCFT2 | 794 |
| Complex-to-Real Fourier Transform in Two Dimensions | SCRFT2 | DCRFT2 | 801 |
| Complex Fourier Transform in Three Dimensions | SCFT3<br>SCFT3P§ | DCFT3 | 809 |
| Real-to-Complex Fourier Transform in Three Dimensions | SRCFT3 | DRCFT3 | 815 |
| Complex-to-Real Fourier Transform in Three Dimensions | SCRFT3 | DCRFT3 | 821 |
| § This subroutine is provided only for migration from earlier releases of ESSL and is not intended for use in new programs. Documentation for this subroutine is no longer provided. | | | |

## Convolution and Correlation Subroutines

The convolution and correlation subroutines provide the choice of using Fourier methods or direct methods. The Fourier-method subroutines contain a high-performance mixed-radix capability. There are also several direct-method subroutines that provide decimated output.

*Table 124. List of Convolution and Correlation Subroutines*

| **Descriptive Name** | **Short-Precision Subroutine** | **Long-Precision Subroutine** | **Page** |
|---|---|---|---|
| Convolution or Correlation of One Sequence with One or More Sequences | SCON§<br>SCOR§ | | 828 |
| Convolution or Correlation of One Sequence with Another Sequence Using a Direct Method | SCOND<br>SCORD | | 834 |
| Convolution or Correlation of One Sequence with One or More Sequences Using the Mixed-Radix Fourier Method | SCONF<br>SCORF | | 840 |
| Convolution or Correlation with Decimated Output Using a Direct Method | SDCON<br>SDCOR | DDCON<br>DDCOR | 849 |
| Autocorrelation of One or More Sequences | SACOR§ | | 853 |
| Autocorrelation of One or More Sequences Using the Mixed-Radix Fourier Method | SACORF | | 857 |
| § These subroutines are provided only for migration from earlier releases of ESSL and are not intended for use in new programs. | | | |

## Related-Computation Subroutines

The related-computation subroutines consist of a group of computations that can be used in general signal processing applications. They are similar to those provided on the IBM 3838 Array Processor; however, the ESSL subroutines generally solve a wider range of problems.

*Table 125. List of Related-Computation Subroutines*

| **Descriptive Name** | **Short-Precision Subroutine** | **Long-Precision Subroutine** | **Page** |
|---|---|---|---|
| Polynomial Evaluation | SPOLY | DPOLY | 863 |
| I-th Zero Crossing | SIZC | DIZC | 866 |
| Time-Varying Recursive Filter | STREC | DTREC | 869 |
| Quadratic Interpolation | SQINT | DQINT | 872 |
| Wiener-Levinson Filter Coefficients | SWLEV | DWLEV | 876 |

# Fourier Transforms, Convolutions, and Correlations Considerations

This section describes some global information applying to the Fourier transform, convolution, and correlation subroutines.

## Use Considerations

This section provides some key points about using the Fourier transform, convolution, and correlation subroutines.

### Understanding the Terminology and Conventions Used for Your Array Data

These subroutines use the term “sequences,” rather than vectors and matrices, to describe the data that is stored in the arrays. The conventions used for representing sequences are defined in “Sequences” on page xxvi.

Some of the sequences used in these computations use a zero origin rather than a one-origin. For example, $x_j$ can be expressed with $j = 0, 1, ..., n-1$ rather than $j = 1, 2, ..., n$. When using the formulas provided in this book to calculate array sizes or offsets into arrays, you need to be careful that you substitute the correct values. For example, the number of $x_j$ elements in the sequence is $n$, not $n-1$.

### Concerns about Lengths of Transforms

The length of the transform you can use in your program depends on the limits of the addressability of your processor.

### Determining an Acceptable Length of a Transform

To determine acceptable lengths of the transforms in the Fourier transform subroutines, you have several choices. First, you can use the formula or table of values in “Acceptable Lengths for the Transforms” to choose a value. Second, ESSL's input-argument error recovery provides a means of determining an acceptable length of the transform. It uses the optionally-recoverable error 2030. For details, see “Providing a Correct Transform Length to ESSL” on page 38.

### Acceptable Lengths for the Transforms

Use the following formula to determine acceptable transform lengths:

$$n = (2^h)\,(3^i)\,(5^j)\,(7^k)\,(11^m) \qquad \text{for } n \leq 37748736$$

where:

$h = 1, 2, ..., 25$
$i = 0, 1, 2$
$j, k, m = 0, 1$

Figure 13 on page 742 lists all the acceptable values for transform lengths in the Fourier transform subroutines.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 20 | 22 | 24 | 28 | 30 | 32 | 36 | 40 | 42 |
| 44 | 48 | 56 | 60 | 64 | 66 | 70 | 72 | 80 |
| 84 | 88 | 90 | 96 | 110 | 112 | 120 | 126 | 128 |
| 132 | 140 | 144 | 154 | 160 | 168 | 176 | 180 | 192 |
| 198 | 210 | 220 | 224 | 240 | 252 | 256 | 264 | 280 |
| 288 | 308 | 320 | 330 | 336 | 352 | 360 | 384 | 396 |
| 420 | 440 | 448 | 462 | 480 | 504 | 512 | 528 | 560 |
| 576 | 616 | 630 | 640 | 660 | 672 | 704 | 720 | 768 |
| 770 | 792 | 840 | 880 | 896 | 924 | 960 | 990 | 1008 |
| 1024 | 1056 | 1120 | 1152 | 1232 | 1260 | 1280 | 1320 | 1344 |
| 1386 | 1408 | 1440 | 1536 | 1540 | 1584 | 1680 | 1760 | 1792 |
| 1848 | 1920 | 1980 | 2016 | 2048 | 2112 | 2240 | 2304 | 2310 |
| 2464 | 2520 | 2560 | 2640 | 2688 | 2772 | 2816 | 2880 | 3072 |
| 3080 | 3168 | 3360 | 3520 | 3584 | 3696 | 3840 | 3960 | 4032 |
| 4096 | 4224 | 4480 | 4608 | 4620 | 4928 | 5040 | 5120 | 5280 |
| 5376 | 5544 | 5632 | 5760 | 6144 | 6160 | 6336 | 6720 | 6930 |
| 7040 | 7168 | 7392 | 7680 | 7920 | 8064 | 8192 | 8448 | 8960 |
| 9216 | 9240 | 9856 | 10080 | 10240 | 10560 | 10752 | 11088 | 11264 |
| 11520 | 12288 | 12320 | 12672 | 13440 | 13860 | 14080 | 14336 | 14784 |
| 15360 | 15840 | 16128 | 16384 | 16896 | 17920 | 18432 | 18480 | 19712 |
| 20160 | 20480 | 21120 | 21504 | 22176 | 22528 | 23040 | 24576 | 24640 |
| 25344 | 26880 | 27720 | 28160 | 28672 | 29568 | 30720 | 31680 | 32256 |
| 32768 | 33792 | 35840 | 36864 | 36960 | 39424 | 40320 | 40960 | 42240 |
| 43008 | 44352 | 45056 | 46080 | 49152 | 49280 | 50688 | 53760 | 55440 |
| 56320 | 57344 | 59136 | 61440 | 63360 | 64512 | 65536 | 67584 | 71680 |
| 73728 | 73920 | 78848 | 80640 | 81920 | 84480 | 86016 | 88704 | 90112 |
| 92160 | 98304 | 98560 | 101376 | 107520 | 110880 | 112640 | 114688 | 118272 |
| 122880 | 126720 | 129024 | 131072 | 135168 | 143360 | 147456 | 147840 | 157696 |
| 161280 | 163840 | 168960 | 172032 | 177408 | 180224 | 184320 | 196608 | 197120 |
| 202752 | 215040 | 221760 | 225280 | 229376 | 236544 | 245760 | 253440 | 258048 |
| 262144 | 270336 | 286720 | 294912 | 295680 | 315392 | 322560 | 327680 | 337920 |
| 344064 | 354816 | 360448 | 368640 | 393216 | 394240 | 405504 | 430080 | 443520 |
| 450560 | 458752 | 473088 | 491520 | 506880 | 516096 | 524288 | 540672 | 573440 |
| 589824 | 591360 | 630784 | 645120 | 655360 | 675840 | 688128 | 709632 | 720896 |
| 737280 | 786432 | 788480 | 811008 | 860160 | 887040 | 901120 | 917504 | 946176 |
| 983040 | 1013760 | 1032192 | 1048576 | 1081344 | 1146880 | 1179648 | 1182720 | 1261568 |
| 1290240 | 1310720 | 1351680 | 1376256 | 1419264 | 1441792 | 1474560 | 1572864 | 1576960 |
| 1622016 | 1720320 | 1774080 | 1802240 | 1835008 | 1892352 | 1966080 | 2027520 | 2064384 |
| 2097152 | 2162688 | 2293760 | 2359296 | 2365440 | 2523136 | 2580480 | 2621440 | 2703360 |
| 2752512 | 2838528 | 2883584 | 2949120 | 3145728 | 3153920 | 3244032 | 3440640 | 3548160 |
| 3604480 | 3670016 | 3784704 | 3932160 | 4055040 | 4128768 | 4194304 | 4325376 | 4587520 |
| 4718592 | 4730880 | 5046272 | 5160960 | 5242880 | 5406720 | 5505024 | 5677056 | 5767168 |
| 5898240 | 6291456 | 6307840 | 6488064 | 6881280 | 7096320 | 7208960 | 7340032 | 7569408 |
| 7864320 | 8110080 | 8257536 | 8388608 | 8650752 | 9175040 | 9437184 | 9461760 | 10092544 |
| 10321920 | 10485760 | 10813440 | 11010048 | 11354112 | 11534336 | 11796480 | 12582912 | 12615680 |
| 12976128 | 13762560 | 14192640 | 14417920 | 14680064 | 15138816 | 15728640 | 16220160 | 16515072 |
| 16777216 | 17301504 | 18350080 | 18874368 | 18923520 | 20185088 | 20643840 | 20971520 | 21626880 |
| 22020096 | 22708224 | 23068672 | 23592960 | 25165824 | 25231360 | 25952256 | 27525120 | 28385280 |
| 28835840 | 29360128 | 30277632 | 31457280 | 32440320 | 33030144 | 33554432 | 34603008 | 36700160 |
| 37748736 | | | | | | | | |

*Figure 13. Table of Acceptable Lengths for the Transforms*

### Understanding Auxiliary Working Storage Requirements

Auxiliary working storage is required by the Fourier transform subroutines and by the SCONF, SCORF, and SACORF subroutines. This storage is provided through the calling sequence arguments *aux*, *aux1*, and *aux2*. The sizes of these storage areas are specified by the calling sequence arguments *naux*, *naux1*, and *naux2*, respectively.

***AUX1:*** The *aux1* array is used for storing tables and other parameters when you call a Fourier transform, convolution, or correlation subroutine for initialization with *init* = 1. The initialized *aux1* array is then used on succeeding calls with *init* = 0, when the computation is actually done. You should not use this array between the initialization and the computation.

***AUX and AUX2:*** The *aux* and *aux2* arrays are used for temporary storage during the running of the subroutine and are available for use by your program between calls to the subroutine.

***AUX3:*** The *aux3* argument is provided for migration purposes only and is ignored.

## Initializing Auxiliary Working Storage

In many of those subroutines requiring *aux1* auxiliary working storage, two invocations of the subroutines are necessary. The first invocation initializes the working storage in *aux1* for the subroutine, and the second performs the computations. (For an explanation of auxiliary working storage, see Understanding Auxiliary Working Storage Requirements.) As a result, the working storage in *aux1* should not be used by the calling program between the two calls to the subroutine. However, it can be reused after intervening calls to the subroutine with different arguments.

If you plan to repeat a computation many times using the same set of arguments, you only need to do one initialization of the *aux1* array; that is, the initialized *aux1* array can be saved and reused as many times as needed for the computation.

If you plan to perform different computations, with different sets of arguments (except for input argument *x*), you need to do an initialization for each different computation; that is, you initialize the various *aux1* arrays for use with the different computations, saving and reusing them until they are not needed any more.

## Determining the Amount of Auxiliary Working Storage That You Need

To determine the size of auxiliary storage, you have several choices. First, you can use the formulas provided in each subroutine description. Second, ESSL's input-argument error recovery provides a means of determining the minimum size you need for auxiliary storage. It uses the optionally-recoverable error 2015. For details, see “Using Auxiliary Storage in ESSL” on page 32. Third, you can have ESSL dynamically allocate *aux* and *aux2*. For details, see “Dynamic Allocation of Auxiliary Storage” on page 33.

# Performance and Accuracy Considerations

The following sections explain the performance and accuracy considerations for the Fourier transforms, convolution, and correlation subroutines. For further details about performance and accuracy, see Chapter 2 on page 25.

# When Running on the Workstation Processors

There are ESSL-specific rules that apply to the results of computations on the workstation processors using the ANSI/IEEE standards. For details, see “What Data Type Standards Are Used by ESSL, and What Exceptions Should You Know About?” on page 45.

# Defining Arrays

The stride arguments, *inc1h, inc1x, inc1y, inc2x, inc2y*, *inc3x*, and *inc3y*, provide great flexibility in defining the input and output data arrays. The arrangement of data in storage, however, can have an effect upon cache performance. By using strides, you can have data scattered in storage. Best performance is obtained with data closely spaced in storage and with elements of the sequence in contiguous locations. The optimum values for *inc1h, inc1x*, and *inc1y* are 1.

In writing the calling program, you may find it convenient to declare `X` or `Y` as a two-dimensional array. For example, you can declare `X` in a DIMENSION statement as `X(INC2X,M)`.

# Fourier Transform Considerations

This section describes some ways to optimize performance in the Fourier transform subroutines.

## Setting Up Your Data

Many of the Fourier transform, convolution, and correlation subroutines provide the facility for processing many sequences in one call. For short sequences, for example 1024 elements or less, this facility should be used as much as possible. This provides improved performance compared to processing only one sequence at a time.

If possible, you should use the same array for input and output. In addition, the requirements for the strides of the input and output arrays are explained in the Notes for each subroutine.

For improved performance, small values of *inc1x* and *inc1y* should be used, where applicable, preferably *inc1x* = 1 and *inc1y* = 1. A stride of 1 means the sequence elements are stored contiguously. Also, if possible, the sequences should be stored close to each other. For all the Fourier transform subroutines except _RCFT and _CRFT, you should use the STRIDE subroutine to determine the optimal stride(s) for your input or output data. Complete instructions on how to use STRIDE for each of these subroutines is included in “STRIDE—Determine the Stride Value for Optimal Performance in Specified Fourier Transform Subroutines” on page 971.

To obtain the best performance in the three-dimensional Fourier transform subroutines, you should use strides, *inc2* and *inc3*, provided by the STRIDE subroutine and declare your three-dimensional data structure as a one-dimensional array. The three-dimensional Fourier transform subroutines assume that *inc1* for the array is 1. Therefore, each element $x_{ijk}$ for $i = 0, 1, ..., n1-1$, $j = 0, 1, ..., n2-1$, and $k = 0, 1, ..., n3-1$ of the three-dimensional data structure of dimensions *n1* by *n2* by *n3* is stored in a one-dimensional array X(0:L) at location X(*l*), where $l = i+inc2(j)+inc3(k)$. The minimum required value of L is calculated by inserting the maximum values for *i*, *j*, and *k* in the above equation, giving $L = (n1-1)+inc2(n2-1)+inc3(n3-1)$. The minimum total size of array X is L+1. To ensure that this mapping is unique so no two elements $x_{ijk}$ occupy the same array element, X(*l*), the subroutines have the following restriction: $inc2 \geq n1$ and $inc3 \geq (inc2)(n2)$. This arrangement of array data in storage leaves some blank space between successive planes of the array X. By determining the best size for this space, specifying an optimum *inc3* stride, the third dimension of the array does not create conflicts in the 3090 storage hierarchy.

If the *inc3* stride value returned by the STRIDE subroutine turns out to be a multiple of *inc2*, the array X can be declared as a three-dimensional array as X(*inc2*,*inc3*/*inc2*,*n3*); otherwise, it can be declared as either a one-dimensional array, X(0:L), as described above, or a two-dimensional array X(0:*inc3*−1,0:*n3*−1), where $x_{ijk}$ is stored in X(*l*,*k*) where $l = i+(inc2)(j)$.

### Using the Scale Argument

If you must multiply either the input or the output sequences by a common factor, you can avoid the multiplication by letting the *scale* argument contain the factor. The subroutines multiply the sine and cosine values by the scale factor during the initialization. Thus, scaling takes no time after the initialization of the Fourier transform calculations.

## How the Fourier Transform Subroutines Achieve High Performance

There are two levels of optimization for the fast Fourier transforms (FFTs) in the ESSL library. For sequences with a large power of 2 length, we provide efficient radix-2 and radix-8 transform implementations where cache use is optimized. The cache optimization includes ordering of operations to maximize stride-1 data access and prefetching cache lines.

Similar optimization techniques are used for sequence lengths which are not a power of 2 and mixed-radix FFT's are performed. Many short sequence FFT's have sequence size specific optimizations. Some of these optimizations were originally developed for a vector machine and have been adapted for cache based RISC machines (see references [1], [5 ], and [7 ])

The other optimization in the FFT routine is to treat multiple sequences as efficiently as possible. Techniques here include blocking sequences to fit into available CPU cache and transposing sequences to ensure stride-1 access. Whenever possible, the highest performance can be obtained when multiple sequences are transformed in a single call.

## Convolution and Correlation Considerations

This section describes some ways to optimize performance in the convolution and correlation subroutines.

## Performance Tradeoffs between Subroutines

The subroutines SCON, SCOR, SACOR, SCOND, SCORD, SDCON, SDCOR, DDCON, and DDCOR compute convolutions, correlations, and autocorrelations using essentially the same methods. They make a decision, based on estimated timings, to use one of two methods:

- A direct method that is most efficient when one or both of the input sequences are short
- A direct method that is most efficient when the output sequence is short

Using this approach has the following advantages:

- In most cases, improved performance can be achieved for direct methods because:
  - No initialization is required.
  - No working storage or padding of sequences is necessary.
- In some cases, greater accuracy may be available.
- Negative strides can be used.

In general, using SCONF, SCORF, and SACORF provides the best performance, because the mixed-radix Fourier transform subroutines are used. However, if you can determine from your arguments that a direct method is preferred, you should use SCOND and SCORD instead. These give you better performance for the direct methods, and also give you additional capabilities.

In cases where there is doubt as to the best choice of a subroutine, perform timing experiments.

## Special Uses of SCORD

The subroutine SCORD can perform the functions of SCON and SACOR; that is, it can compute convolutions and autocorrelations. To compute a convolution, you must specify a negative stride for ***h*** (see Example 4 in SCORD). To compute the autocorrelation, you must specify the two input sequences to be the same (see Example 5 in SCORD).

## Special Uses of _DCON and _DCOR

The _DCON and _DCOR subroutines compute convolutions and correlations, respectively, by the direct method with decimated output. Setting the decimation interval *id* = 1 in SDCON and SDCOR provides the same function as SCOND and SCORD, respectively. Doing the same in DDCON and DDCOR provides long-precision versions of SCOND and SCORD, respectively, which are not otherwise available.

## Accuracy When Direct Methods Are Used

The direct methods used by the convolution and correlation subroutines use vector operations to accumulate sums of products. The products are computed and accumulated in long precision. As a result, higher accuracy can be obtained in the final results for some types of data. For example, if input data consists only of integers, and if no intermediate and final numbers become too large (larger than $2^{24}-1$ for short-precision computations and larger than $2^{56}-1$ for long-precision computations), the results are exact.

### Accuracy When Fourier Methods Are Used

The Fourier methods used by the convolution and correlation subroutines compute Fourier transforms of input data that is multiplied element-by-element in short-precision arithmetic. The inverse Fourier transform is then computed. There are internally generated rounding errors in the Fourier transforms. It has been shown in references [ 93] and [ 82] that, in the case of white noise data, the relative root mean square (RMS) error of the Fourier transform is proportional to $log_2 n$ with a very small proportionality factor. In general, with random, evenly distributed data, this is better than the RMS error of the direct method. However, one must keep in mind the fact that, while the Fourier method may yield a smaller root mean square error, there can be points with large relative errors. Thus, it can happen that some points, usually at the ends of the output sequence, can be obtained with greater relative accuracy with direct methods.

### Convolutions and Correlations by Fourier Methods

The convolution and correlation subroutines that use the Fourier methods determine a sequence length *n*, whose Fourier transform is computed using ESSL subroutines. In the simple case where *iy0* = 0 for convolution or *iy0* = −*nh*+1 for correlation, *n* is chosen as a value greater than or equal to the following, which is also acceptable to the Fourier tranform subroutines:

*nt* = min(*nh*+*nx*−1, *ny*) for convolution and correlation
*nt* = min(*nx*+*nx*−1, *ny*) for autocorrelation

which is also acceptable to the Fourier subroutines.

## Related Computation Considerations

This section describes some key points about using the related-computation subroutines.

## Accuracy Considerations

- Many of the subroutines performing short-precision computations provide increased accuracy by accumulating results in long precision. This is noted in the functional description for each subroutine.
- There are ESSL-specific rules that apply to the results of computations on the workstation processors using the ANSI/IEEE standards. For details, see “What Data Type Standards Are Used by ESSL, and What Exceptions Should You Know About?” on page 45.

# Fourier Transform Subroutines

This section contains the Fourier transform subroutine descriptions.

## SCFT and DCFT—Complex Fourier Transform

These subroutines compute a set of *m* complex discrete *n*-point Fourier transforms of complex data.

*Table 126. Data Types*

| X, Y | *scale* | **Subroutine** |
|---|---|---|
| Short-precision complex | Short-precision real | SCFT |
| Long-precision complex | Long-precision real | DCFT |

**Note:** Two invocations of this subroutine are necessary: one to prepare the working storage for the subroutine, and the other to perform the computations.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SCFT \| DCFT (*init*, *x*, *inc1x*, *inc2x*, *y*, *inc1y*, *inc2y*, *n*, *m*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*) |
| **C and C++** | scft \| dcft (*init*, *x*, *inc1x*, *inc2x*, *y*, *inc1y*, *inc2y*, *n*, *m*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*); |
| **PL/I** | CALL SCFT \| DCFT (*init*, *x*, *inc1x*, *inc2x*, *y*, *inc1y*, *inc2y*, *n*, *m*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*); |

*On Entry*

*init*

is a flag, where:

If *init* ≠ 0, trigonometric functions and other parameters, depending on arguments other than *x*, are computed and saved in *aux1*. The contents of *x* and *y* are not used or changed.

If *init* = 0, the discrete Fourier transforms of the given sequences are computed. The only arguments that may change after initialization are *x*, *y*, and *aux2*. All scalar arguments must be the same as when the subroutine was called for initialization with *init* ≠ 0.

Specified as: a fullword integer. It can have any value.

*x*

is the array X, consisting of *m* sequences of length *n*. Specified as: an array of (at least) length 1+(*n*–1)*inc1x*+(*m*–1)*inc2x*, containing numbers of the data type indicated in Table 126.

*inc1x*

is the stride between the elements within each sequence in array X. Specified as: a fullword integer; *inc1x* > 0.

*inc2x*

is the stride between the first elements of the sequences in array X. (If *m* = 1, this argument is ignored.) Specified as: a fullword integer; *inc2x* > 0.

*y*

See “On Return” on page 751.

*inc1y*

is the stride between the elements within each sequence in array Y. Specified as: a fullword integer; *inc1y* > 0.

*inc2y*

is the stride between the first elements of each sequence in array `Y`. (If $m = 1$, this argument is ignored.) Specified as: a fullword integer; $inc2y > 0$.

*n*

is the length of each sequence to be transformed. Specified as: a fullword integer; $n \leq 37748736$ and must be one of the values listed in "Acceptable Lengths for the Transforms" on page 741. For all other values specified less than 37748736, you have the option of having the next larger acceptable value returned in this argument, as well as in the optionally-recoverable error 2030. For details, see "Providing a Correct Transform Length to ESSL" on page 38.

*m*

is the number of sequences to be transformed. Specified as: a fullword integer; $m > 0$.

*isign*

controls the direction of the transform, determining the sign *Isign* of the exponent of $W_n$, where:

If $isign$ = positive value, $Isign = +$ (transforming time to frequency).

If $isign$ = negative value, $Isign = -$ (transforming frequency to time).

Specified as: a fullword integer; $isign > 0$ or $isign < 0$.

*scale*

is the scaling constant *scale*. See "Function" on page 752 for its usage. Specified as: a number of the data type indicated in Table 126 on page 749, where $scale > 0.0$ or $scale < 0.0$

*aux1*

is the working storage for this subroutine, where:

If $init \neq 0$, the working storage is computed.

If $init = 0$, the working storage is used in the computation of the Fourier transforms.

Specified as: an area of storage, containing *naux1* long-precision real numbers.

*naux1*

is the number of doublewords in the working storage specified in *aux1*. Specified as: a fullword integer; $naux1 > 7$ and $naux1 \geq$ (minimum value required for successful processing). To determine a sufficient value, use the processor-independent formulas. For values between 7 and the minimum value, you have the option of having the minimum value returned in this argument. For details, see "Using Auxiliary Storage in ESSL" on page 32.

*aux2*

has the following meaning:

If $naux2 = 0$ and error 2015 is unrecoverable, *aux2* is ignored.

Otherwise, it is the working storage used by this subroutine, which is available for use by the calling program between calls to this subroutine.

Specified as: an area of storage, containing *naux2* long-precision real numbers. On output, the contents are overwritten.

*naux2*

is the number of doublewords in the working storage specified in *aux2*. Specified as: a fullword integer, where:

If $naux2 = 0$ and error 2015 is unrecoverable, SCFT and DCFT dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, *naux2* ≥ (minimum value required for successful processing). To determine a sufficient value, use the processor-independent formulas. For all other values specified less than the minimum value, you have the option of having the minimum value returned in this argument. For details, see “Using Auxiliary Storage in ESSL” on page 32.

*On Return*

*y*

has the following meaning, where:

If *init* ≠ 0, this argument is not used, and its contents remain unchanged.

If *init* = 0, this is array `Y`, consisting of the results of the *m* discrete Fourier transforms, each of length *n*.

Returned as: an array of (at least) length 1+(*n*–1)*inc1y*+(*m*–1)*inc2y*, containing numbers of the data type indicated in Table 126 on page 749. This array must be aligned on a doubleword boundary.

*aux1*

is the working storage for this subroutine, where:

If *init* ≠ 0, it contains information ready to be passed in a subsequent invocation of this subroutine.

If *init* = 0, its contents are unchanged.

Returned as: the contents are not relevant.

### ***Notes***

1. *aux1* should **not** be used by the calling program between calls to this subroutine with *init* ≠ 0 and *init* = 0. However, it can be reused after intervening calls to this subroutine with different arguments.
2. When using the ESSL SMP library, for optimal performance, the number of threads specified should be the same for *init* ≠ 0 and *init* = 0.
3. For optimal performance, the preferred value for *inc1x* and *inc1y* is 1. This implies that the sequences are stored with stride 1. The preferred value for *inc2x* and *inc2y* is *n*. This implies that sequences are stored one after another without any gap.

   It is possible to specify sequences in the transposed form—that is, as rows of a two-dimensional array. In this case, *inc2x* (or *inc2y*) = 1 and *inc1x* (or *inc1y*) is equal to the leading dimension of the array. One can specify either input, output, or both in the transposed form by specifying appropriate values for the stride parameters. For selecting optimal values of *inc1x* and *inc1y* for _CFT, you should use “STRIDE—Determine the Stride Value for Optimal Performance in Specified Fourier Transform Subroutines” on page 971. Example 1 in the STRIDE subroutine description explains how it is used for _CFT.

   If you specify the same array for `X` and `Y`, then *inc1x* and *inc1y* must be equal, and *inc2x* and *inc2y* must be equal. In this case, output overwrites input. If *m* = 1, the *inc2x* and *inc2y* values are not used by the subroutine. If you specify different arrays for `X` and `Y`, they must have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.

### ***Processor-Independent Formulas for SCFT for NAUX1 and NAUX2***

*NAUX1 Formulas:*

If $n \leq 8192$, use *naux1* = 20000.
If $n > 8192$, use *naux1* = 20000+1.14*n*.

*NAUX2 Formulas:*

If $n \leq 8192$, use *naux2* = 20000.
If $n > 8192$, use *naux2* = 20000+1.14*n*.
For the transposed case, where *inc2x* = 1 or *inc2y* = 1, and where $n \geq 252$, add the following to the above storage requirements:
(*n*+256)(min(64, *m*)).

***Processor-Independent Formulas for DCFT for NAUX1 and NAUX2***

*NAUX1 Formulas:*

If $n \leq 2048$, use *naux1* = 20000.
If $n > 2048$, use *naux1* = 20000+2.28*n*.

*NAUX2 Formulas:*

If $n \leq 2048$, use *naux2* = 20000.
If $n > 2048$, use *naux2* = 20000+2.28*n*.
For the transposed case, where *inc2x* = 1 or *inc2y* = 1, and where $n \geq 252$, add the following to the above storage requirements:
(2*n*+256)(min(64, *m*)).

***Function:*** The set of *m* complex discrete n-point Fourier transforms of complex data in array `X`, with results going into array `Y`, is expressed as follows:

$$y_{ki} = scale \sum_{j=0}^{n-1} x_{ji} W_n^{(Isign)jk}$$

for:

$k = 0, 1, ..., n-1$
$i = 1, 2, ..., m$

where:

$$W_n = e^{-2\pi\left(\sqrt{-1}\right)/n}$$

and where:

$x_{ji}$ are elements of the sequences in array `X`.
$y_{ki}$ are elements of the sequences in array `Y`.
*Isign* is + or − (determined by argument *isign*).
*scale* is a scalar value.

For *scale* = 1.0 and *isign* being positive, you obtain the discrete Fourier transform, a function of frequency. The inverse Fourier transform is obtained with *scale* = 1.0/*n* and *isign* being negative. See references [1 ], [3 ], [4 ], [19 ], and [20].

Two invocations of this subroutine are necessary:

1. With *init* ≠ 0, the subroutine tests and initializes arguments of the program, setting up the *aux1* working storage.
2. With *init* = 0, the subroutine checks that the initialization arguments in the *aux1* working storage correspond to the present arguments, and if so, performs the calculation of the Fourier transforms.

### *Error Conditions*

*Resource Errors:* Error 2015 is unrecoverable, *naux2* = 0, and unable to allocate work area.

*Computational Errors:* None

*Input-Argument Errors*

1. $n > 37748736$
2. *inc1x*, *inc2x*, *inc1y*, or *inc2y* ≤ 0
3. $m \le 0$
4. *isign* = 0
5. *scale* = 0.0
6. The subroutine has not been initialized with the present arguments.
7. The length of the transform in *n* is not an allowable value. Return code 1 is returned if error 2030 is recoverable.
8. *naux1* ≤ 7
9. *naux1* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.
10. Error 2015 is recoverable or *naux2*≠0, and *naux2* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example 1:*** This example shows an input array X with a set of four short-precision complex sequences:

$$e^{2\pi\left(\sqrt{-1}\right)jk/n}$$

for $j = 0, 1, ..., n–1$ with $n = 8$, and the single frequencies $k = 0, 1, 2,$ and 3. The arrays are declared as follows:

```
COMPLEX*8  X(0:1023),Y(0:1023)
REAL*8     AUX1(1693),AUX2(4096)
```

First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation.

*Call Statement and Input*

```
           INIT  X INC1X INC2X Y INC1Y INC2Y N   M ISIGN SCALE  AUX1   NAUX1  AUX2   NAUX2
            |    |  |     |    |  |     |    |   |  |     |     |       |      |       |
CALL SCFT(INIT, X , 1  ,  8 , Y , 1  ,  8 , 8 , 4 , 1 , SCALE, AUX1 , 1693 , AUX2 , 4096)
```

```
INIT      =  1(for initialization)
INIT      =  0(for computation)
SCALE     =  1.0
```

X contains the following four sequences:

```
(1.0000, 0.0000)   (1.0000,  0.0000)   (1.0000,  0.0000)   (1.0000,  0.0000)
(1.0000, 0.0000)   (0.7071,  0.7071)   (0.0000,  1.0000)  (-0.7071,  0.7071)
(1.0000, 0.0000)   (0.0000,  1.0000)  (-1.0000,  0.0000)   (0.0000, -1.0000)
(1.0000, 0.0000)  (-0.7071,  0.7071)   (0.0000, -1.0000)   (0.7071,  0.7071)
(1.0000, 0.0000)  (-1.0000,  0.0000)   (1.0000,  0.0000)  (-1.0000,  0.0000)
(1.0000, 0.0000)  (-0.7071, -0.7071)   (0.0000,  1.0000)   (0.7071, -0.7071)
(1.0000, 0.0000)   (0.0000, -1.0000)  (-1.0000,  0.0000)   (0.0000,  1.0000)
(1.0000, 0.0000)   (0.7071, -0.7071)   (0.0000, -1.0000)  (-0.7071, -0.7071)
```

*Output:* Y contains the following four sequences:

```
(8.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)
(0.0000, 0.0000)  (8.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)
(0.0000, 0.0000)  (0.0000, 0.0000)  (8.0000, 0.0000)  (0.0000, 0.0000)
(0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)  (8.0000, 0.0000)
(0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)
(0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)
(0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)
(0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)
```

***Example 2:*** This example shows an input array X with a set of four input spike sequences equal to the output of Example 1. This shows how you can compute the inverse of the transform in Example 1 by using a negative *isign*, giving as output the four sequences listed in the input for Example 1. First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation.

*Call Statement and Input*

```
          INIT  X INC1X INC2X Y INC1Y INC2Y N   M  ISIGN SCALE   AUX1   NAUX1  AUX2   NAUX2
           |    |  |     |    |  |     |    |   |    |     |      |       |     |       |
CALL SCFT(INIT, X , 1  ,  8 , Y , 1  ,  8 , 8 , 4 , -1 , SCALE , AUX1 , 1693 , AUX2 , 4096)
```

```
INIT      =  1(for initialization)
INIT      =  0(for computation)
SCALE     =  0.125
X         =(same as output Y in Example 1)
```

*Output*

```
Y         =(same as input X in Example 1)
```

***Example 3:*** This example shows an input array X with a set of four short-precision complex sequences

$$e^{2\pi\left(\sqrt{-1}\right)jk/n}$$

for $j = 0, 1, ..., n{-}1$ with $n = 12$, and the single frequencies $k = 0, 1, 2$, and 3. Also, $inc1x = inc1y = m$ and $inc2x = inc2y = 1$ to show how the input and output arrays can be stored in the transposed form. The arrays are declared as follows:

```
COMPLEX*8  X (4,0:11),Y(4,0:11)
REAL*8     AUX1(10000),AUX2(10000)
```

First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation.

*Call Statement and Input*

```
          INIT  X INC1X INC2X Y INC1Y INC2Y  N   M ISIGN SCALE  AUX1   NAUX1   AUX2   NAUX2
           |    |  |     |    |  |     |     |   |  |     |     |       |       |       |
CALL SCFT(INIT, X , 4  ,  1 , Y , 4  ,  1 , 12 , 4 , 1 , SCALE, AUX1 , 10000 , AUX2 , 10000)
```

```
INIT     =  1(for initialization)
INIT     =  0(for computation)
SCALE    =  1.0
```

X contains the following four sequences:

```
(1.0000, 0.0000)   (1.0000, 0.0000)   (1.0000, 0.0000)   (1.0000, 0.0000)
(1.0000, 0.0000)   (0.8660, 0.5000)   (0.5000, 0.8660)   (0.0000, 1.0000)
(1.0000, 0.0000)   (0.5000, 0.8660)  (-0.5000, 0.8660)  (-1.0000, 0.0000)
(1.0000, 0.0000)   (0.0000, 1.0000)  (-1.0000, 0.0000)  (0.0000, -1.0000)
(1.0000, 0.0000)  (-0.5000, 0.8660) (-0.5000, -0.8660)   (1.0000, 0.0000)
(1.0000, 0.0000)  (-0.8660, 0.5000)  (0.5000, -0.8660)   (0.0000, 1.0000)
(1.0000, 0.0000)  (-1.0000, 0.0000)   (1.0000, 0.0000)  (-1.0000, 0.0000)
(1.0000, 0.0000) (-0.8660, -0.5000)   (0.5000, 0.8660)  (0.0000, -1.0000)
(1.0000, 0.0000) (-0.5000, -0.8660)  (-0.5000, 0.8660)   (1.0000, 0.0000)
(1.0000, 0.0000)  (0.0000, -1.0000)  (-1.0000, 0.0000)   (0.0000, 1.0000)
(1.0000, 0.0000)  (0.5000, -0.8660) (-0.5000, -0.8660) (-1.0000,  0.0000)
(1.0000, 0.0000)  (0.8660, -0.5000)  (0.5000, -0.8660)  (0.0000, -1.0000)
```

*Output:* Y contains the following four sequences:

```
(12.0000, 0.0000)   (0.0000, 0.0000)   (0.0000, 0.0000)   (0.0000, 0.0000)
 (0.0000, 0.0000)  (12.0000, 0.0000)   (0.0000, 0.0000)   (0.0000, 0.0000)
 (0.0000, 0.0000)   (0.0000, 0.0000)  (12.0000, 0.0000)   (0.0000, 0.0000)
 (0.0000, 0.0000)   (0.0000, 0.0000)   (0.0000, 0.0000)  (12.0000, 0.0000)
 (0.0000, 0.0000)   (0.0000, 0.0000)   (0.0000, 0.0000)   (0.0000, 0.0000)
 (0.0000, 0.0000)   (0.0000, 0.0000)   (0.0000, 0.0000)   (0.0000, 0.0000)
 (0.0000, 0.0000)   (0.0000, 0.0000)   (0.0000, 0.0000)   (0.0000, 0.0000)
 (0.0000, 0.0000)   (0.0000, 0.0000)   (0.0000, 0.0000)   (0.0000, 0.0000)
 (0.0000, 0.0000)   (0.0000, 0.0000)   (0.0000, 0.0000)   (0.0000, 0.0000)
 (0.0000, 0.0000)   (0.0000, 0.0000)   (0.0000, 0.0000)   (0.0000, 0.0000)
 (0.0000, 0.0000)   (0.0000, 0.0000)   (0.0000, 0.0000)   (0.0000, 0.0000)
 (0.0000, 0.0000)   (0.0000, 0.0000)   (0.0000, 0.0000)   (0.0000, 0.0000)
```

***Example 4:*** This example shows an input array X with a set of four input spike sequences exactly equal to the output of Example 3. This shows how you can compute the inverse of the transform in Example 3 by using a negative *isign*, giving as output the four sequences listed in the input for Example 3. First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation.

*Call Statement and Input*

```
         INIT  X INC1X INC2X Y INC1Y INC2Y  N   M  ISIGN SCALE   AUX1  NAUX1  AUX2  NAUX2
          |    |  |      |   |  |      |    |   |    |     |       |     |      |     |
CALL SCFT(INIT, X , 4  ,  1 , Y , 4  ,  1 , 12 , 4 , -1 , SCALE , AUX1, 10000, AUX2, 10000)
```

INIT = 1(for initialization)
INIT = 0(for computation)
SCALE = 1.0/12.0
X =(same as output Y in Example 3)

*Output*

Y =(same as input X in Example 3)

***Example 5:*** This example shows how to compute a transform of a single long-precision complex sequence. It uses *isign* = 1 and *scale* = 1.0. The arrays are declared as follows:

```
COMPLEX*16  X(0:7),Y(0:7)
REAL*8      AUX1(26),AUX2(12)
```

The input in X is an impulse at zero, and the output in Y is constant for all frequencies. First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation.

*Call Statement and Input*

```
         INIT  X INC1X INC2X Y INC1Y INC2Y N   M ISIGN SCALE   AUX1  NAUX1  AUX2  NAUX2
          |    |  |      |   |  |      |   |   |  |     |       |     |      |     |
CALL DCFT(INIT, X , 1  ,  0 , Y , 1  ,  0 , 8 , 1 , 1 , SCALE , AUX1 , 26  , AUX2 , 12)
```

INIT = 1(for initialization)
INIT = 0(for computation)
SCALE = 1.0

X contains the following sequence:

```
(1.0000, 0.0000)
(0.0000, 0.0000)
(0.0000, 0.0000)
(0.0000, 0.0000)
(0.0000, 0.0000)
(0.0000, 0.0000)
(0.0000, 0.0000)
(0.0000, 0.0000)
```

*Output:*

```
(1.0000, 0.0000)
(1.0000, 0.0000)
(1.0000, 0.0000)
(1.0000, 0.0000)
(1.0000, 0.0000)
(1.0000, 0.0000)
(1.0000, 0.0000)
(1.0000, 0.0000)
```

## SRCFT and DRCFT—Real-to-Complex Fourier Transform

These subroutines compute a set of *m* complex discrete *n*-point Fourier transforms of real data.

*Table 127. Data Types*

| X, *scale* | Y | **Subroutine** |
|---|---|---|
| Short-precision real | Short-precision complex | SRCFT |
| Long-precision real | Long-precision complex | DRCFT |

**Note:** Two invocations of this subroutine are necessary: one to prepare the working storage for the subroutine, and the other to perform the computations.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SRCFT (*init*, *x*, *inc2x*, *y*, *inc2y*, *n*, *m*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*, *aux3*, *naux3*)<br>CALL DRCFT (*init*, *x*, *inc2x*, *y*, *inc2y*, *n*, *m*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*) |
| **C and C++** | srcft (*init*, *x*, *inc2x*, *y*, *inc2y*, *n*, *m*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*, *aux3*, *naux3*);<br>drcft (*init*, *x*, *inc2x*, *y*, *inc2y*, *n*, *m*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*); |
| **PL/I** | CALL SRCFT (*init*, *x*, *inc2x*, *y*, *inc2y*, *n*, *m*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*, *aux3*, *naux3*);<br>CALL DRCFT (*init*, *x*, *inc2x*, *y*, *inc2y*, *n*, *m*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*); |

*On Entry*

*init*

is a flag, where:

If *init* ≠ 0, trigonometric functions and other parameters, depending on arguments other than *x*, are computed and saved in *aux1*. The contents of *x* and *y* are not used or changed.

If *init* = 0, the discrete Fourier transforms of the given sequences are computed. The only arguments that may change after initialization are *x*, *y*, and *aux2*. All scalar arguments must be the same as when the subroutine was called for initialization with *init* ≠ 0.

Specified as: a fullword integer. It can have any value.

*x*

is the array X, consisting of *m* sequences of length *n*, which are to be transformed. The sequences are assumed to be stored with stride 1. Specified as: an array of (at least) length *n*+(*m*–1)*inc2x*, containing numbers of the data type indicated in Table 127. See “Notes” on page 759 for more details. (It can be declared as X(*inc2x*,*m*).)

*inc2x*

is the stride between the first elements of the sequences in array X. (If *m* = 1, this argument is ignored.) Specified as: a fullword integer; *inc2x* ≥ *n*.

*y*

See “On Return” on page 759.

*inc2y*

is the stride between the first elements of the sequences in array Y. (If *m* = 1, this argument is ignored.) Specified as: a fullword integer; *inc2y* ≥ (*n*/2)+1.

*n*

is the length of each sequence to be transformed. Specified as: a fullword integer; $n \le 37748736$ and must be one of the values listed in "Acceptable Lengths for the Transforms" on page 741. For all other values specified less than 37748736, you have the option of having the next larger acceptable value returned in this argument. For details, see "Providing a Correct Transform Length to ESSL" on page 38.

*m*

is the number of sequences to be transformed. Specified as: a fullword integer; $m > 0$.

*isign*

controls the direction of the transform, determining the sign *Isign* of the exponent of $W_n$, where:

If *isign* = positive value, *Isign* = + (transforming time to frequency).

If *isign* = negative value, *Isign* = − (transforming frequency to time).

Specified as: a fullword integer; $isign > 0$ or $isign < 0$.

*scale*

is the scaling constant *scale*. See "Function" on page 760 for its usage. Specified as: a number of the data type indicated in Table 127 on page 757, where $scale > 0.0$ or $scale < 0.0$.

*aux1*

is the working storage for this subroutine, where:

If $init \ne 0$, the working storage is computed.

If $init = 0$, the working storage is used in the computation of the Fourier transforms.

Specified as: an area of storage, containing *naux1* long-precision real numbers.

*naux1*

is the number of doublewords in the working storage specified in *aux1*. Specified as: a fullword integer; $naux1 > 14$ and $naux1 \ge$ (minimum value required for successful processing). To determine a sufficient value, use the processor-independent formulas. For values between 14 and the minimum value, you have the option of having the minimum value returned in this argument. For details, see "Using Auxiliary Storage in ESSL" on page 32.

*aux2*

has the following meaning:

If $naux2 = 0$ and error 2015 is unrecoverable, *aux2* is ignored.

Otherwise, it is the working storage used by this subroutine, which is available for use by the calling program between calls to this subroutine.

Specified as: an area of storage, containing *naux2* long-precision real numbers. On output, the contents are overwritten.

*naux2*

is the number of doublewords in the working storage specified in *aux2*. Specified as: a fullword integer, where:

If $naux2 = 0$ and error 2015 is unrecoverable, SRCFT and DRCFT dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, $naux2 \ge$ (minimum value required for successful processing). To determine a sufficient value, use the processor-independent formulas. For all other values specified less than the minimum value, you have the option of having the minimum value returned in this argument. For details, see “Using Auxiliary Storage in ESSL” on page 32.

*aux3*

this argument is provided for migration purposes only and is ignored.

Specified as: an area of storage, containing *naux3* long-precision real numbers.

*naux3*

this argument is provided for migration purposes only and is ignored.

Specified as: a fullword integer.

*On Return*

*y*

has the following meaning, where:

If *init* ≠ 0, this argument is not used, and its contents remain unchanged.

If *init* = 0, this is array Y, consisting of the results of the *m* complex discrete Fourier transforms, each of length *n*. The sequences are stored with the stride 1. Due to complex conjugate symmetry, only the first (n/2) + 1 elements of each sequence are given in the output—that is, $y_{ki}$, $k = 0, 1, ..., n/2$, $i = 1, 2, ..., m$.

Returned as: an array of (at least) length $n/2+1+(m-1)inc2y$, containing numbers of the data type indicated in Table 127 on page 757. This array must be aligned on a doubleword boundary. (It can be declared as Y(*inc2y*,*m*).)

*aux1*

is the working storage for this subroutine, where:

If *init* ≠ 0, it contains information ready to be passed in a subsequent invocation of this subroutine.

If *init* = 0, its contents are unchanged.

Returned as: the contents are not relevant.

***Notes***

1. *aux1* should **not** be used by the calling program between calls to this subroutine with *init* ≠ 0 and *init* = 0. However, it can be reused after intervening calls to this subroutine with different arguments.
2. When using the ESSL SMP library, for optimal performance, the number of threads specified should be the same for *init* ≠ 0 and *init* = 0.
3. In these subroutines, the elements in each sequence in ***x*** and ***y*** are assumed to be stored in contiguous storage locations, using a stride of 1; therefore, *inc1x* and *inc1y* values are not a part of the argument list. For optimal performance, the *inc2x* and *inc2y* values should be close to their respective minimum values, which are given below:

   min(*inc2x*) = *n*
   min(*inc2y*) = *n*/2+1

   If you specify the same array for X and Y, then *inc2x* must equal 2(*inc2y*). In this case, output overwrites input. If *m* = 1, the *inc2x* and *inc2y* values are not used by the subroutine. If you specify different arrays for X and Y, they must have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.

4. Be sure to align array X on a doubleword boundary, and specify an even number for *inc2x*, if possible.

***Processor-Independent Formulas for SRCFT for NAUX1 and NAUX2***

*NAUX1 Formulas*

If $n \le 16384$, use $naux1 = 25000$.
If $n > 16384$, use $naux1 = 20000+0.82n$.

*NAUX2 Formulas*

If $n \le 16384$, use $naux2 = 20000$.
If $n > 16384$, use $naux2 = 20000+0.57n$.

***Processor-Independent Formulas for DRCFT for NAUX1 and NAUX2***

*NAUX1 Formulas*

If $n \le 4096$, use $naux1 = 22000$.
If $n > 4096$, use $naux1 = 20000+1.64n$.

*NAUX2 Formulas*

If $n \le 4096$, use $naux2 = 20000$.
If $n > 4096$, use $naux2 = 20000+1.14n$.

***Function:*** The set of *m* complex conjugate even discrete n-point Fourier transforms of real data in array X, with results going into array Y, is expressed as follows:

$$y_{ki} = scale \sum_{j=0}^{n-1} x_{ji} W_n^{(Isign)jk}$$

for:

$k = 0, 1, ..., n-1$
$i = 1, 2, ..., m$

where:

$$W_n = e^{-2\pi\left(\sqrt{-1}\right)/n}$$

and where:

$x_{ji}$ are elements of the sequences in array X.
$y_{ki}$ are elements of the sequences in array Y.
*Isign* is + or – (determined by argument *isign*).
*scale* is a scalar value.

The output in array Y is complex. For $scale = 1.0$ and *isign* being positive, you obtain the discrete Fourier transform, a function of frequency. The inverse Fourier transform is obtained with $scale = 1.0/n$ and *isign* being negative. See references [1 ], [4 ], [19 ], and [20].

Two invocations of this subroutine are necessary:

1. With *init* ≠ 0, the subroutine tests and initializes arguments of the program, setting up the *aux1* working storage.
2. With *init* = 0, the subroutine checks that the initialization arguments in the *aux1* working storage correspond to the present arguments, and if so, performs the calculation of the Fourier transforms.

***Error Conditions***

*Resource Errors:* Error 2015 is unrecoverable, *naux2* = 0, and unable to allocate work area.

*Computational Errors:* None

*Input-Argument Errors*

1. *n* > 37748736
2. *m* ≤ 0
3. *inc2x* < *n*
4. *inc2y* < *n*/2+1
5. *isign* = 0
6. *scale* = 0.0
7. The subroutine has not been initialized with the present arguments.
8. The length of the transform in *n* is not an allowable value. Return code 1 is returned if error 2030 is recoverable.
9. *naux1* ≤ 14
10. *naux1* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.
11. Error 2015 is recoverable or *naux2*≠0, and *naux2* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example 1:*** This example shows an input array `X` with a set of *m* cosine sequences cos(2π*jk*/*n*), *j* = 0, 1, ..., 15 with the single frequencies *k* = 0, 1, 2, 3. The Fourier transform of the cosine sequence with frequency *k* = 0 or n/2 has 1.0 in the 0 or n/2 position, respectively, and zeros elsewhere. For all other *k*, the Fourier transform has 0.5 in the *k* position and zeros elsewhere. The arrays are declared as follows:

```
REAL*4     X(0:65535)
COMPLEX*8  Y(0:32768)
REAL*8     AUX1(41928), AUX2(35344), AUX3(1)
```

First, initialize `AUX1` using the calling sequence shown below with `INIT` ≠ 0. Then use the same calling sequence with `INIT` = 0 to do the calculation.

*Call Statement and Input*

```
           INIT  X  INC2X Y INC2Y  N   M ISIGN SCALE  AUX1   NAUX1   AUX2   NAUX2   AUX3  NAUX3
            |    |    |   |   |    |   |   |     |     |       |      |       |      |      |
CALL SRCFT(INIT, X , 16 , Y , 9 , 16 , 4 , 1 , SCALE, AUX1 , 41928 , AUX2 , 35344 , AUX3 ,  0 )
```

`INIT` = `1`(for initialization)
`INIT` = `0`(for computation)
`SCALE` = `1.0/16`

X contains the following four sequences:

```
1.0000   1.0000   1.0000   1.0000
1.0000   0.9239   0.7071   0.3827
1.0000   0.7071   0.0000  -0.7071
1.0000   0.3827  -0.7071  -0.9239
1.0000   0.0000  -1.0000   0.0000
1.0000  -0.3827  -0.7071   0.9239
1.0000  -0.7071   0.0000   0.7071
1.0000  -0.9239   0.7071  -0.3827
1.0000  -1.0000   1.0000  -1.0000
1.0000  -0.9239   0.7071  -0.3827
1.0000  -0.7071   0.0000   0.7071
1.0000  -0.3827  -0.7071   0.9239
1.0000   0.0000  -1.0000   0.0000
1.0000   0.3827  -0.7071  -0.9239
1.0000   0.7071   0.0000  -0.7071
1.0000   0.9239   0.7071   0.3827
```

*Output:* Y contains the following four sequences:

```
(1.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)
(0.0000, 0.0000)  (0.5000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)
(0.0000, 0.0000)  (0.0000, 0.0000)  (0.5000, 0.0000)  (0.0000, 0.0000)
(0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)  (0.5000, 0.0000)
(0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)
(0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)
(0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)
(0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)
(0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)
```

***Example 2:*** This example shows another transform computation with different data using the same initialized array AUX1 as in Example 1. The input is also a set of four cosine sequences cos($2\pi jk/n$), $j = 0, 1, ..., 15$ with the single frequencies $k = 8, 9, 10, 11$, thus including the middle frequency $k = 8$. The middle frequency has the value 1.0. For other frequencies, the transform has zeros, except for frequencies $k$ and $n-k$. Only the values for $j = n-k$ are given in the output.

*Call Statement and Input*

```
          INIT X  INC2X Y INC2Y  N   M ISIGN SCALE  AUX1   NAUX1   AUX2   NAUX2   AUX3  NAUX3
           |   |    |   |   |    |   |   |     |     |       |       |       |       |
CALL SRCFT( 0 , X , 16 , Y , 9 , 16 , 4 , 1 , SCALE, AUX1 , 41928 , AUX2 , 35344 , AUX3 ,  0  )
```

```
SCALE    =  1.0/16
```

X contains the following four sequences:

```
 1.0000   1.0000   1.0000   1.0000
-1.0000  -0.9239  -0.7071  -0.3827
 1.0000   0.7071   0.0000  -0.7071
-1.0000  -0.3827   0.7071   0.9239
 1.0000   0.0000  -1.0000   0.0000
-1.0000   0.3827   0.7071  -0.9239
 1.0000  -0.7071   0.0000   0.7071
-1.0000   0.9239  -0.7071   0.3827
 1.0000  -1.0000   1.0000  -1.0000
-1.0000   0.9239  -0.7071   0.3827
 1.0000  -0.7071   0.0000   0.7071
-1.0000   0.3827   0.7071  -0.9239
 1.0000   0.0000  -1.0000   0.0000
-1.0000  -0.3827   0.7071   0.9239
 1.0000   0.7071   0.0000  -0.7071
-1.0000  -0.9239  -0.7071  -0.3827
```

*Output:* Y contains the following four sequences:

```
(0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)
(0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)
(0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)
(0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)
(0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)
(0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)  (0.5000, 0.0000)
(0.0000, 0.0000)  (0.0000, 0.0000)  (0.5000, 0.0000)  (0.0000, 0.0000)
(0.0000, 0.0000)  (0.5000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)
(1.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)  (0.0000, 0.0000)
```

***Example 3:*** This example uses the mixed-radix capability. The arrays are declared as follows:

```
REAL*8     X(0:11)
COMPLEX*16 Y(0:6)
REAL*8     AUX1(50),AUX2(50)
```

Arrays X and Y are made equivalent by the following statement, making them occupy the same storage:

```
EQUIVALENCE (X,Y)
```

First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation.

*Call Statement and Input*

```
              INIT  X INC2X Y INC2Y  N   M ISIGN SCALE   AUX1  NAUX1 AUX2  NAUX2
               |    |   |   |   |    |   |   |     |      |      |    |      |
CALL DRCFT(INIT, X , 0 , Y , 0 , 12 , 1 , 1 , SCALE , AUX1 , 50 , AUX2 , 50)
```

```
INIT     =  1(for initialization)
INIT     =  0(for computation)
SCALE    =  1.0
X        =  (1.0000 , 1.0000 , 1.0000 , 1.0000 , 1.0000 , 1.0000 ,
             1.0000 , 1.0000 , 1.0000 , 1.0000 , 1.0000 , 1.0000)
```

*Output:* Y contains the following sequence:

```
(12.0000 , 0.0000)
 (0.0000 , 0.0000)
 (0.0000 , 0.0000)
 (0.0000 , 0.0000)
 (0.0000 , 0.0000)
 (0.0000 , 0.0000)
 (0.0000 , 0.0000)
```

# SCRFT and DCRFT—Complex-to-Real Fourier Transform

These subroutines compute a set of *m* real discrete *n*-point Fourier transforms of complex conjugate even data.

*Table 128. Data Types*

| X | Y, *scale* | **Subroutine** |
|---|---|---|
| Short-precision complex | Short-precision real | SCRFT |
| Long-precision complex | Long-precision real | DCRFT |

**Note:** Two invocations of this subroutine are necessary: one to prepare the working storage for the subroutine, and the other to perform the computations.

***Syntax***

| | |
|---|---|
| **Fortran** | CALL SCRFT (*init*, *x*, *inc2x*, *y*, *inc2y*, *n*, *m*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*, *aux3*, *naux3*)<br>CALL DCRFT (*init*, *x*, *inc2x*, *y*, *inc2y*, *n*, *m*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*) |
| **C and C++** | scrft (*init*, *x*, *inc2x*, *y*, *inc2y*, *n*, *m*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*, *aux3*, *naux3*);<br>dcrft (*init*, *x*, *inc2x*, *y*, *inc2y*, *n*, *m*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*); |
| **PL/I** | CALL SCRFT (*init*, *x*, *inc2x*, *y*, *inc2y*, *n*, *m*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*, *aux3*, *naux3*);<br>CALL DCRFT (*init*, *x*, *inc2x*, *y*, *inc2y*, *n*, *m*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*); |

*On Entry*

*init*

is a flag, where:

If *init* ≠ 0, trigonometric functions and other parameters, depending on arguments other than *x*, are computed and saved in *aux1*. The contents of *x* and *y* are not used or changed.

If *init* = 0, the discrete Fourier transforms of the given sequences are computed. The only arguments that may change after initialization are *x*, *y*, and *aux2*. All scalar arguments must be the same as when the subroutine was called for initialization with *init* ≠ 0.

Specified as: a fullword integer. It can have any value.

*x*

is the array X, consisting of *m* sequences. Due to complex conjugate symmetry, the input consists of only the first (*n*/2)+1 elements of each sequence; that is, $x_{ji}$, *j* = 0, 1, ..., *n*/2, *i* = 1, 2, ..., *m*. The sequences are assumed to be stored with stride 1.

Specified as: an array of (at least) length *n*/2+1+(*m*–1)*inc2x*, containing numbers of the data type indicated in Table 128. This array must be aligned on a doubleword boundary. (It can be declared as X(*inc2x*,*m*).)

*inc2x*

is the stride between the first elements of the sequences in array X. (If *m* = 1, this argument is ignored.) Specified as: a fullword integer; *inc2x* ≥ (*n*/2)+1.

*y*

See "On Return" on page 767.

*inc2y*

is the stride between the first elements of the sequences in array `Y`. (If $m = 1$, this argument is ignored.) Specified as: a fullword integer; $inc2y \geq n$.

*n*

is the length of each sequence to be transformed. Specified as: a fullword integer; $n \leq 37748736$ and must be one of the values listed in “Acceptable Lengths for the Transforms” on page 741. For all other values specified less than 37748736, you have the option of having the next larger acceptable value returned in this argument. For details, see “Providing a Correct Transform Length to ESSL” on page 38.

*m*

is the number of sequences to be transformed. Specified as: a fullword integer; $m > 0$.

*isign*

controls the direction of the transform, determining the sign *Isign* of the exponent of $W_n$, where:

If *isign* = positive value, *Isign* = + (transforming time to frequency).

If *isign* = negative value, *Isign* = − (transforming frequency to time).

Specified as: a fullword integer; $isign > 0$ or $isign < 0$.

*scale*

is the scaling constant *scale*. See “Function” on page 768 for its usage. Specified as: a number of the data type indicated in Table 128 on page 765, where $scale > 0.0$ or $scale < 0.0$.

*aux1*

is the working storage for this subroutine, where:

If $init \neq 0$, the working storage is computed.

If $init = 0$, the working storage is used in the computation of the Fourier transforms.

Specified as: an area of storage, containing *naux1* long-precision real numbers.

*naux1*

is the number of doublewords in the working storage specified in *aux1*. Specified as: a fullword integer; $naux1 > 13$ and $naux1 \geq$ (minimum value required for successful processing). To determine a sufficient value, use the processor-independent formulas. For values between 13 and the minimum value, you have the option of having the minimum value returned in this argument. For details, see “Using Auxiliary Storage in ESSL” on page 32.

*aux2*

has the following meaning:

If $naux2 = 0$ and error 2015 is unrecoverable, *aux2* is ignored.

Otherwise, it is the working storage used by this subroutine that is available for use by the calling program between calls to this subroutine.

Specified as: an area of storage, containing *naux2* long-precision real numbers. On output, the contents are overwritten.

*naux2*

is the number of doublewords in the working storage specified in *aux2*. Specified as: a fullword integer, where:

If $naux2 = 0$ and error 2015 is unrecoverable, SCRFT and DCRFT dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, *naux2* ≥ (minimum value required for successful processing). To determine a sufficient value, use the processor-independent formulas. For all other values specified less than the minimum value, you have the option of having the minimum value returned in this argument. For details, see "Using Auxiliary Storage in ESSL" on page 32.

*aux3*

this argument is provided for migration purposes only and is ignored.

Specified as: an area of storage, containing *naux3* long-precision real numbers.

*naux3*

this argument is provided for migration purposes only and is ignored.

Specified as: a fullword integer.

*On Return*

*y*

has the following meaning, where:

If *init* ≠ 0, this argument is not used, and its contents remain unchanged.

If *init* = 0, this is array Y, consisting of the results of the *m* discrete Fourier transforms of the complex conjugate even data, each of length *n*. The sequences are stored with stride 1.

Returned as: an array of (at least) length *n*+(*m*–1)*inc2y*, containing numbers of the data type indicated in Table 128 on page 765. See "Notes" for more details. (It can be declared as Y(*inc2y*,*m*).)

*aux1*

is the working storage for this subroutine, where:

If *init* ≠ 0, it contains information ready to be passed in a subsequent invocation of this subroutine.

If *init* = 0, its contents are unchanged.

Returned as: the contents are not relevant.

### *Notes*

1. *aux1* should **not** be used by the calling program between calls to this subroutine with *init* ≠ 0 and *init* = 0. However, it can be reused after intervening calls to this subroutine with different arguments.
2. When using the ESSL SMP library, for optimal performance, the number of threads specified should be the same for *init* ≠ 0 and *init* = 0.
3. The elements in each sequence in ***x*** and ***y*** are assumed to be stored in contiguous storage locations—that is, with a stride of 1. Therefore, *inc1x* and *inc1y* values are not a part of the argument list. For optimal performance, the *inc2x* and *inc2y* values should be close to their respective minimum values, which are given below:

   min(*inc2y*) = *n*
   min(*inc2x*) = *n*/2+1

   If you specify the same array for X and Y, then *inc2y* must equal 2(*inc2x*). In this case, output overwrites input. If *m* = 1, the *inc2x* and *inc2y* values are not used by the subroutine. If you specify different arrays for X and Y, they must have no common elements; otherwise, results are unpredictable. See "Concepts" on page 55.

4. Be sure to align array `Y` on a doubleword boundary, and specify an even number for *inc2y*, if possible.

***Processor-Independent Formulas for SCRFT for NAUX1 and NAUX2***

*NAUX1 Formulas*

If $n \le 16384$, use $naux1 = 25000$.
If $n > 16384$, use $naux1 = 20000+0.82n$.

*NAUX2 Formulas*

If $n \le 16384$, use $naux2 = 20000$.
If $n > 16384$, use $naux2 = 20000+0.57n$.

***Processor-Independent Formulas for DCRFT for NAUX1 and NAUX2***

*NAUX1 Formulas*

If $n \le 4096$, use $naux1 = 22000$.
If $n > 4096$, use $naux1 = 20000+1.64n$.

*NAUX2 Formulas*

If $n \le 4096$, use $naux2 = 20000$.
If $n > 4096$, use $naux2 = 20000+1.14n$.

***Function:*** The set of *m* real discrete *n*-point Fourier transforms of complex conjugate even data in array `X`, with results going into array `Y`, is expressed as follows:

$$y_{ki} = scale \sum_{j=0}^{n-1} x_{ji} W_n^{(Isign)jk}$$

for:

$k = 0, 1, ..., n–1$
$i = 1, 2, ..., m$

where:

$$W_n = e^{-2\pi\left(\sqrt{-1}\right)/n}$$

and where:

$x_{ji}$ are elements of the sequences in array `X`.
$y_{ki}$ are elements of the sequences in array `Y`.
*Isign* is + or − (determined by argument *isign*).
*scale* is a scalar value.

Because of the symmetry, `Y` has real data. For $scale = 1.0$ and *isign* being positive, you obtain the discrete Fourier transform, a function of frequency. The inverse Fourier transform is obtained with *scale* = 1.0/*n* and *isign* being negative. See references [1 ], [4 ], [19 ], and [20].

Two invocations of this subroutine are necessary:

1. With *init* ≠ 0, the subroutine tests and initializes arguments of the program, setting up the *aux1* working storage.
2. With *init* = 0, the subroutine checks that the initialization arguments in the *aux1* working storage correspond to the present arguments, and if so, performs the calculation of the Fourier transforms.

***Error Conditions***

*Resource Errors:* Error 2015 is unrecoverable, *naux2* = 0, and unable to allocate work area.

*Computational Errors:* None

*Input-Argument Errors*

1. *n* > 37748736
2. *m* ≤ 0
3. *inc2x* < *n*/2+1
4. *inc2y* < *n*
5. *scale* = 0.0
6. *isign* = 0
7. The subroutine has not been initialized with the present arguments.
8. The length of the transform in *n* is not an allowable value. Return code 1 is returned if error 2030 is recoverable.
9. *naux1* ≤ 13
10. *naux1* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.
11. Error 2015 is recoverable or *naux2*≠0, and *naux2* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example 1:*** This example uses the mixed-radix capability and shows how to compute a single transform. The arrays are declared as follows:

```
COMPLEX*8  X(0:6)
REAL*8     AUX1(50), AUX2(50), AUX3(1)
REAL*4     Y(0:11)
```

First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation.

**Note:** X shows the *n*/2+1 = 7 elements used in the computation.

*Call Statement and Input*

```
           INIT  X INC2X Y INC2Y  N   M ISIGN SCALE  AUX1  NAUX1 AUX2  NAUX2  AUX3  NAUX3
            |    |  |    |  |     |   |  |     |     |      |     |     |      |     |
CALL SCRFT(INIT, X , 0 , Y , 0 , 12 , 1 , 1 , SCALE, AUX1 , 50 , AUX2 , 50 ,  AUX3 ,  0  )
```

```
INIT     =  1(for initialization)
INIT     =  0(for computation)
SCALE    =  1.0
```

X contains the following sequence:

```
(1.0,  0.0)
(0.0,  0.0)
(0.0,  0.0)
(0.0,  0.0)
(0.0,  0.0)
(0.0,  0.0)
(0.0,  0.0)
```

*Output*

```
Y        =  (1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0)
```

***Example 2:*** This example shows another transform computation with different data using the same initialized array AUX1 as in Example 1.

*Call Statement and Input*

```
          INIT  X INC2X Y INC2Y  N   M ISIGN SCALE  AUX1 NAUX1  AUX2 NAUX2  AUX3  NAUX3
            |   |   |   |   |    |   |   |     |     |     |     |     |     |      |
CALL SCRFT( 0 , X , 0 , Y , 0 , 12 , 1 , 1 , SCALE, AUX1 , 50 , AUX2 , 50 , AUX3 ,  0 )
```

```
SCALE    =  1.0
```

X contains the following sequence:

```
(1.0, 0.0)
(1.0, 0.0)
(1.0, 0.0)
(1.0, 0.0)
(1.0, 0.0)
(1.0, 0.0)
(1.0, 0.0)
```

*Output*

```
Y        =  (12.0 , 0.0 , 0.0 , 0.0 , 0.0 , 0.0 , 0.0 , 0.0 ,
             0.0 , 0.0 , 0.0 , 0.0)
```

***Example 3:*** This example shows how to compute many transforms simultaneously. The arrays are declared as follows:

```
COMPLEX*8  X(0:8,2)
REAL*8     AUX1(50), AUX2(16), AUX3(1)
REAL*4     Y(0:15,2)
```

First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation.

*Call Statement and Input*

```
           INIT  X INC2X Y INC2Y   N   M ISIGN SCALE  AUX1  NAUX1 AUX2  NAUX2 AUX3  NAUX3
            |    |   |   |   |     |   |   |     |     |      |    |      |    |      |
CALL SCRFT(INIT, X , 9 , Y , 16 , 16 , 2 , 1 , SCALE, AUX1 , 50 , AUX2 , 16 , AUX3 ,  0 )
```

```
INIT     =  1(for initialization)
INIT     =  0(for computation)
SCALE    =  1.0
```

X contains the following two sequences:

```
(1.0, 0.0)  (0.0, 0.0)
(1.0, 0.0)  (0.0, 0.0)
(1.0, 0.0)  (0.0, 0.0)
(1.0, 0.0)  (0.0, 0.0)
(1.0, 0.0)  (0.0, 0.0)
(1.0, 0.0)  (0.0, 0.0)
(1.0, 0.0)  (0.0, 0.0)
(1.0, 0.0)  (0.0, 0.0)
(1.0, 0.0)  (1.0, 0.0)
```

*Output:* Y contains the following two sequences:

```
16.0   1.0
 0.0  -1.0
 0.0   1.0
 0.0  -1.0
 0.0   1.0
 0.0  -1.0
 0.0   1.0
 0.0  -1.0
 0.0   1.0
 0.0  -1.0
 0.0   1.0
 0.0  -1.0
 0.0   1.0
 0.0  -1.0
 0.0   1.0
 0.0  -1.0
```

***Example 4:*** This example shows the same array being used for input and output. The arrays are declared as follows:

```
COMPLEX*16 X(0:8,2)
REAL*8     AUX1(50), AUX2(16)
REAL*8     Y(0:17,2)
```

Arrays X and Y are made equivalent by the following statement, making them occupy the same storage:

```
EQUIVALENCE (X,Y)
```

This requires INC2Y = 2(INC2X). First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation.

*Call Statement and Input*

```
           INIT  X INC2X Y  INC2Y  N   M  ISIGN SCALE  AUX1  NAUX1 AUX2  NAUX2
            |    |   |   |    |    |   |    |     |     |     |     |     |
CALL DCRFT(INIT, X , 9 , Y , 18 , 16 , 2 , -1 , SCALE, AUX1 , 50 , AUX2 , 16)
```

```
INIT     =  1(for initialization)
INIT     =  0(for computation)
SCALE    =  0.0625
```

X contains the following two sequences:

```
 (1.0,  0.0)   (1.0,  0.0)
 (0.0,  1.0)   (0.0, -1.0)
(-1.0,  0.0)  (-1.0,  0.0)
 (0.0, -1.0)   (0.0,  1.0)
 (1.0,  0.0)   (1.0,  0.0)
 (0.0,  1.0)   (0.0, -1.0)
(-1.0,  0.0)  (-1.0,  0.0)
 (0.0, -1.0)   (0.0,  1.0)
 (1.0,  0.0)   (1.0,0.0)
```

*Output:* Y contains the following two sequences:

```
0.0  0.0
0.0  0.0
0.0  0.0
0.0  0.0
0.0  1.0
0.0  0.0
0.0  0.0
0.0  0.0
0.0  0.0
0.0  0.0
0.0  0.0
0.0  0.0
1.0  0.0
0.0  0.0
0.0  0.0
0.0  0.0
```

# SCOSF and DCOSF—Cosine Transform

These subroutines compute a set of *m* real even discrete *n*-point Fourier transforms of cosine sequences of real even data.

Table 129. Data Types

| X, Y, *scale* | **Subroutine** |
|---|---|
| Short-precision real | SCOSF |
| Long-precision real | DCOSF |

**Note:** Two invocations of this subroutine are necessary: one to prepare the working storage for the subroutine, and the other to perform the computations.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SCOSF \| DCOSF (*init*, *x*, *inc1x*, *inc2x*, *y*, *inc1y*, *inc2y*, *n*, *m*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*) |
| **C and C++** | scosf \| dcosf (*init*, *x*, *inc1x*, *inc2x*, *y*, *inc1y*, *inc2y*, *n*, *m*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*); |
| **PL/I** | CALL SCOSF \| DCOSF (*init*, *x*, *inc1x*, *inc2x*, *y*, *inc1y*, *inc2y*, *n*, *m*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*); |

*On Entry*

*init*

is a flag, where:

If *init* ≠ 0, trigonometric functions and other parameters, depending on arguments other than *x*, are computed and saved in *aux1*. The contents of *x* and *y* are not used or changed.

If *init* = 0, the discrete Fourier transforms of the given sequences are computed. The only arguments that may change after initialization are *x*, *y*, and *aux2*. All scalar arguments must be the same as when the subroutine was called for initialization with *init* ≠ 0.

Specified as: a fullword integer. It can have any value.

*x*

is the array X, consisting of *m* sequences of length *n*/2+1. Specified as: an array of (at least) length 1+(*n*/2)*inc1x*+(*m*–1)*inc2x*, containing numbers of the data type indicated in Table 129.

*inc1x*

is the stride between the elements within each sequence in array X. Specified as: a fullword integer; *inc1x* > 0.

*inc2x*

is the stride between the first elements of the sequences in array X. (If *m* = 1, this argument is ignored.) Specified as: a fullword integer; *inc2x* > 0.

*y*

See “On Return” on page 775.

*inc1y*

is the stride between the elements within each sequence in array Y. Specified as: a fullword integer; *inc1y* > 0.

*inc2y*

is the stride between the first elements of the sequences in array Y. (If $m = 1$, this argument is ignored.) Specified as: a fullword integer; $inc2y > 0$.

*n*

is the transform length. However, due to symmetry, only the first $n/2+1$ values are given in the input and output. Specified as: a fullword integer; $n \leq 37748736$ and must be one of the values listed in “Acceptable Lengths for the Transforms” on page 741. For all other values specified less than 37748736, you have the option of having the next larger acceptable value returned in this argument. For details, see “Providing a Correct Transform Length to ESSL” on page 38.

*m*

is the number of sequences to be transformed. Specified as: a fullword integer; $m > 0$.

*scale*

is the scaling constant *scale*. See “Function” on page 776 for its usage. Specified as: a number of the data type indicated in Table 129 on page 773, where $scale > 0.0$ or $scale < 0.0$.

*aux1*

is the working storage for this subroutine, where:

If $init \neq 0$, the working storage is computed.

If $init = 0$, the working storage is used in the computation of the Fourier transforms.

Specified as: an area of storage, containing *naux1* long-precision real numbers.

*naux1*

is the number of doublewords in the working storage specified in *aux1*. Specified as: a fullword integer; $naux1 \geq$ (minimum value required for successful processing). To determine a sufficient value, use the processor-independent formulas. For all other values specified less than the minimum value, you have the option of having the minimum value returned in this argument. For details, see “Using Auxiliary Storage in ESSL” on page 32.

*aux2*

has the following meaning:

If $naux2 = 0$ and error 2015 is unrecoverable, *aux2* is ignored.

Otherwise, it is the working storage used by this subroutine, which is available for use by the calling program between calls to this subroutine.

Specified as: an area of storage, containing *naux2* long-precision real numbers. On output, the contents are overwritten.

*naux2*

is the number of doublewords in the working storage specified in *aux2*. Specified as: a fullword integer, where:

If $naux2 = 0$ and error 2015 is unrecoverable, SCOSF and DCOSF dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, $naux2 \geq$ (minimum value required for successful processing). To determine a sufficient value, use the processor-independent formulas. For all other values specified less than the minimum value, you have the option of having the minimum value returned in this argument. For details, see “Using Auxiliary Storage in ESSL” on page 32.

*On Return*

*y*

has the following meaning, where:

If *init* ≠ 0, this argument is not used, and its contents remain unchanged.

If *init* = 0, this is array Y, consisting of the results of the *m* discrete Fourier transforms, where each Fourier transform is real and of length *n*. However, due to symmetry, only the first *n*/2+1 values are given in the output—that is, $y_{ki}$, *k* = 0, 1, ..., *n*/2 for each *i* = 1, 2, ..., *m*.

Returned as: an array of (at least) length 1+(*n*/2)*inc1y*+(*m*−1)*inc2y*, containing numbers of the data type indicated in Table 129 on page 773.

*aux1*

is the working storage for this subroutine, where:

If *init* ≠ 0, it contains information ready to be passed in a subsequent invocation of this subroutine.

If *init* = 0, its contents are unchanged.

Returned as: the contents are not relevant.

### *Notes*

1. *aux1* should **not** be used by the calling program between calls to this subroutine with *init* ≠ 0 and *init* = 0. However, it can be reused after intervening calls to this subroutine with different arguments.
2. When using the ESSL SMP library, for optimal performance, the number of threads specified should be the same for *init* ≠ 0 and *init* = 0.
3. For optimal performance, the preferred value for *inc1x* and *inc1y* is 1. This implies that the sequences are stored with stride 1. In addition, *inc2x* and *inc2y* should be close to *n*/2+1.

   It is possible to specify sequences in the transposed form—that is, as rows of a two-dimensional array. In this case, *inc2x* (or *inc2y*) = 1 and *inc1x* (or *inc1y*) is equal to the leading dimension of the array. One can specify either input, output, or both in the transposed form by specifying appropriate values for the stride parameters. For selecting optimal values of *inc1x* and *inc1y* for _COSF, you should use "STRIDE—Determine the Stride Value for Optimal Performance in Specified Fourier Transform Subroutines" on page 971. Example 2 in the STRIDE subroutine description explains how it is used for _COSF.

   If you specify the same array for X and Y, then *inc1x* and *inc1y* must be equal, and *inc2x* and *inc2y* must be equal. In this case, output overwrites input. If *m* = 1, the *inc2x* and *inc2y* values are not used by the subroutine. If you specify different arrays for X and Y, they must have no common elements; otherwise, results are unpredictable. See "Concepts" on page 55.

### *Processor-Independent Formulas for SCOSF for NAUX1 and NAUX2*

*NAUX1 Formulas:*

If *n* ≤ 16384, use *naux1* = 40000.
If *n* > 16384, use *naux1* = 20000+.30*n*.

*NAUX2 Formulas:*

If $n \leq 16384$, use *naux2* = 25000.
If $n > 16384$, use *naux2* = 20000+.32*n*.
For the transposed case, where *inc2x* = 1 or *inc2y* = 1, and where $n \geq 252$, add the following to the above storage requirements:
(*n*/4+257)(min(128, *m*)).

### *Processor-Independent Formulas for DCOSF for NAUX1 and NAUX2*

*NAUX1 Formulas:*

If $n \leq 16384$, use *naux1* = 35000.
If $n > 16384$, use *naux1* = 20000+.60*n*.

*NAUX2 Formulas:*

If $n \leq 16384$, use *naux2* = 20000.
If $n > 16384$, use *naux2* = 20000+.64*n*.
For the transposed case, where *inc2x* = 1 or *inc2y* = 1, and where $n \geq 252$, add the following to the above storage requirements:
(*n*/2+257)(min(128, *m*)).

***Function:*** The set of *m* real even discrete *n*-point Fourier transforms of the cosine sequences of real data in array X, with results going into array Y, is expressed as follows:

$$y_{ki} = scale \left( .5x_{0,i} + .5(-1)^k x_{n/2,i} + \sum_{j=1}^{n/2-1} x_{ji} \; \cos\left(jk(2\pi / n)\right) \right)$$

for:

$k = 0, 1, ..., n/2$
$i = 1, 2, ..., m$

where:

$x_{ji}$ are elements of the sequences in array X, where each sequence contains the $n/2+1$ real nonredundant data $x_{ji}$, $j = 0, 1, ..., n/2$.
$y_{ki}$ are elements of the sequences in array Y, where each sequence contains the $n/2+1$ real nonredundant data $y_{ki}$, $k = 0, 1, ..., n/2$.
*scale* is a scalar value.

You can obtain the inverse cosine transform by specifying *scale* = 4.0/*n*. Thus, if an X input is used with *scale* = 1.0, and its output is used as input on a subsequent call with *scale* = 4.0/*n*, the original X is obtained. See references [1 ], [4 ], [19 ], and [20 ].

Two invocations of this subroutine are necessary:

1. With *init* ≠ 0, the subroutine tests and initializes arguments of the program, setting up the *aux1* working storage.
2. With *init* = 0, the subroutine checks that the initialization arguments in the *aux1* working storage correspond to the present arguments, and if so, performs the calculation of the Fourier transforms.

These subroutines use a Fourier transform method with a mixed-radix capability. This provides maximum performance for your application.

### *Error Conditions*

*Resource Errors:* Error 2015 is unrecoverable, *naux2* = 0, and unable to allocate work area.

*Computational Errors:* None

*Input-Argument Errors*

1. *n* > 37748736
2. *inc1x* or *inc1y* ≤ 0
3. *inc2x* or *inc2y* ≤ 0
4. *m* ≤ 0
5. *scale* = 0.0
6. The subroutine has not been initialized with the present arguments.
7. The length of the transform in *n* is not an allowable value. Return code 1 is returned if error 2030 is recoverable.
8. *naux1* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.
9. Error 2015 is recoverable or *naux2*≠0, and *naux2* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example 1:*** This example shows an input array X with a set of *m* cosine sequences of length *n*/2+1, cos(*jk*(2π/*n*)), *j* = 0, 1, ..., *n*/2, with the single frequencies *k* = 0, 1, 2, 3. The Fourier transform of the cosine sequence with frequency *k* = 0 or *n*/2 has *n*/2 in the 0-th or *n*/2-th position, respectively, and zeros elsewhere. For all other *k*, the Fourier transform has *n*/4 in position *k* and zeros elsewhere. The arrays are declared as follows:

```
REAL*4     X(0:71),Y(0:71)
REAL*8     AUX1(414),AUX2(8960)
```

First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation.

*Call Statement and Input*

```
           INIT  X INC1X INC2X Y INC1Y INC2Y  N   M   SCALE  AUX1  NAUX1  AUX2  NAUX2
            |    |  |     |    |  |     |     |   |    |      |     |      |     |
CALL SCOSF(INIT, X , 1  , 18 , Y , 1  , 18 , 32 , 4 , SCALE, AUX1 , 414 , AUX2 , 8960)
```

```
INIT     =  1(for initialization)
INIT     =  0(for computation)
SCALE    =  1.0
```

X contains the following four sequences:

```
1.0000   1.0000   1.0000   1.0000
1.0000   0.9808   0.9239   0.8315
1.0000   0.9239   0.7071   0.3827
1.0000   0.8315   0.3827  -0.1951
1.0000   0.7071   0.0000  -0.7071
1.0000   0.5556  -0.3827  -0.9808
1.0000   0.3827  -0.7071  -0.9239
1.0000   0.1951  -0.9239  -0.5556
1.0000   0.0000  -1.0000   0.0000
1.0000  -0.1951  -0.9239   0.5556
1.0000  -0.3827  -0.7071   0.9239
1.0000  -0.5556  -0.3827   0.9808
1.0000  -0.7071   0.0000   0.7071
1.0000  -0.8315   0.3827   0.1951
1.0000  -0.9239   0.7071  -0.3827
1.0000  -0.9808   0.9239  -0.8315
1.0000  -1.0000   1.0000  -1.0000
  .        .        .        .
```

*Output:* Y contains the following four sequences:

```
16.0000  0.0000  0.0000  0.0000
 0.0000  8.0000  0.0000  0.0000
 0.0000  0.0000  8.0000  0.0000
 0.0000  0.0000  0.0000  8.0000
 0.0000  0.0000  0.0000  0.0000
 0.0000  0.0000  0.0000  0.0000
 0.0000  0.0000  0.0000  0.0000
 0.0000  0.0000  0.0000  0.0000
 0.0000  0.0000  0.0000  0.0000
 0.0000  0.0000  0.0000  0.0000
 0.0000  0.0000  0.0000  0.0000
 0.0000  0.0000  0.0000  0.0000
 0.0000  0.0000  0.0000  0.0000
 0.0000  0.0000  0.0000  0.0000
 0.0000  0.0000  0.0000  0.0000
 0.0000  0.0000  0.0000  0.0000
 0.0000  0.0000  0.0000  0.0000
   .       .       .       .
```

***Example 2:*** This example shows an input array X with a set of four input spike sequences equal to the output of Example 1. This shows how you can compute the inverse of the transform in Example 1 by using *scale* = 4.0/*n*, giving as output the four sequences listed in the input for Example 1. First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation.

*Call Statement and Input*

```
           INIT  X INC1X INC2X Y INC1Y INC2Y  N   M   SCALE  AUX1  NAUX1  AUX2  NAUX2
            |    |  |     |    |  |     |     |   |    |      |      |     |      |
CALL SCOSF(INIT, X , 1  , 18 , Y , 1  , 18 , 32 , 4 , SCALE, AUX1 , 414 , AUX2 , 8960)
```

INIT = 1(for initialization)
INIT = 0(for computation)
SCALE = 4.0/32
X =(same sequences as in output Y in Example 1)

*Output*

Y =(same sequences as in output X in Example 1)

***Example 3:*** This example shows another computation using the same arguments initialized in Example 1 and using different input sequence data. The data for this example has frequencies *k* = 14, 15, 16, 17. Because only the sequence data has changed, initialization does not have to be done again.

*Call Statement and Input*

```
           INIT  X INC1X INC2X Y INC1Y INC2Y N   M   SCALE  AUX1  NAUX1  AUX2  NAUX2
            |    |  |     |    |  |     |    |   |     |      |      |     |     |
CALL SCOSF( 0 , X , 1  , 18 , Y , 1  , 18 , 32 , 4 , SCALE, AUX1 , 414 , AUX2 , 8960)
```

```
SCALE    =  1.0
```

X contains the following four sequences:

```
 1.0000   1.0000   1.0000   1.0000
-0.9239  -0.9808  -1.0000  -0.9808
 0.7071   0.9239   1.0000   0.9239
-0.3827  -0.8315  -1.0000  -0.8315
 0.0000   0.7071   1.0000   0.7071
 0.3827  -0.5556  -1.0000  -0.5556
-0.7071   0.3827   1.0000   0.3827
 0.9239  -0.1951  -1.0000  -0.1951
-1.0000   0.0000   1.0000   0.0000
 0.9239   0.1951  -1.0000   0.1951
-0.7071  -0.3827   1.0000  -0.3827
 0.3827   0.5556  -1.0000   0.5556
 0.0000  -0.7071   1.0000  -0.7071
-0.3827   0.8315  -1.0000   0.8315
 0.7071  -0.9239   1.0000  -0.9239
-0.9239   0.9808  -1.0000   0.9808
 1.0000  -1.0000   1.0000  -1.0000
   .        .        .        .
```

*Output:* Y contains the following four sequences:

```
0.0000  0.0000   0.0000  0.0000
0.0000  0.0000   0.0000  0.0000
0.0000  0.0000   0.0000  0.0000
0.0000  0.0000   0.0000  0.0000
0.0000  0.0000   0.0000  0.0000
0.0000  0.0000   0.0000  0.0000
0.0000  0.0000   0.0000  0.0000
0.0000  0.0000   0.0000  0.0000
0.0000  0.0000   0.0000  0.0000
0.0000  0.0000   0.0000  0.0000
0.0000  0.0000   0.0000  0.0000
0.0000  0.0000   0.0000  0.0000
0.0000  0.0000   0.0000  0.0000
0.0000  0.0000   0.0000  0.0000
8.0000  0.0000   0.0000  0.0000
0.0000  8.0000   0.0000  8.0000
0.0000  0.0000  16.0000  0.0000
  .       .        .       .
```

## SSINF and DSINF—Sine Transform

These subroutines compute a set of *m* real even discrete *n*-point Fourier transforms of sine sequences of real even data.

*Table 130. Data Types*

| X, Y, *scale* | **Subroutine** |
|---|---|
| Short-precision real | SSINF |
| Long-precision real | DSINF |

**Note:** Two invocations of this subroutine are necessary: one to prepare the working storage for the subroutine, and the other to perform the computations.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SSINF \| DSINF (*init*, *x*, *inc1x*, *inc2x*, *y*, *inc1y*, *inc2y*, *n*, *m*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*) |
| **C and C++** | ssinf \| dsinf (*init*, *x*, *inc1x*, *inc2x*, *y*, *inc1y*, *inc2y*, *n*, *m*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*); |
| **PL/I** | CALL SSINF \| DSINF (*init*, *x*, *inc1x*, *inc2x*, *y*, *inc1y*, *inc2y*, *n*, *m*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*); |

*On Entry*

*init*

is a flag, where:

If *init* ≠ 0, trigonometric functions and other parameters, depending on arguments other than *x*, are computed and saved in *aux1*. The contents of *x* and *y* are not used or changed.

If *init* = 0, the discrete Fourier transforms of the given sequences are computed. The only arguments that may change after initialization are *x*, *y*, and *aux2*. All scalar arguments must be the same as when the subroutine was called for initialization with *init* ≠ 0.

Specified as: a fullword integer. It can have any value.

*x*

is the array X, consisting of *m* sequences of length *n*/2. Specified as: an array of (at least) length 1+(*n* / 2−1)*inc1x*+(*m*−1)*inc2x*, containing numbers of the data type indicated in Table 130. The first element in X must have a value of 0.0 (otherwise, incorrect results may occur).

*inc1x*

is the stride between the elements within each sequence in array X. Specified as: a fullword integer; *inc1x* > 0.

*inc2x*

is the stride between the first elements of the sequences in array X. (If *m* = 1, this argument is ignored.) Specified as: a fullword integer; *inc2x* > 0.

*y*

See “On Return” on page 782.

*inc1y*

is the stride between the elements within each sequence in array Y. Specified as: a fullword integer; *inc1y* > 0.

*inc2y*

is the stride between the first elements of the sequences in array `Y`. (If $m = 1$, this argument is ignored.) Specified as: a fullword integer; $inc2y > 0$.

*n*

is the transform length. However, due to symmetry, only the first $n/2$ values are given in the input and output. Specified as: a fullword integer; $n \le 37748736$ and must be one of the values listed in “Acceptable Lengths for the Transforms” on page 741. For all other values specified less than 37748736, you have the option of having the next larger acceptable value returned in this argument. For details, see “Providing a Correct Transform Length to ESSL” on page 38.

*m*

is the number of sequences to be transformed. Specified as: a fullword integer; $m > 0$.

*scale*

is the scaling constant *scale*. See “Function” on page 783 for its usage. Specified as: a number of the data type indicated in Table 130 on page 780, where $scale > 0.0$ or $scale < 0.0$.

*aux1*

is the working storage for this subroutine, where:

If $init \ne 0$, the working storage is computed.

If $init = 0$, the working storage is used in the computation of the Fourier transforms.

Specified as: an area of storage, containing *naux1* long-precision real numbers.

*naux1*

is the number of doublewords in the working storage specified in *aux1*. Specified as: a fullword integer; $naux1 \ge$ (minimum value required for successful processing). To determine a sufficient value, use the processor-independent formulas. For all other values specified less than the minimum value, you have the option of having the minimum value returned in this argument. For details, see “Using Auxiliary Storage in ESSL” on page 32.

*aux2*

has the following meaning:

If $naux2 = 0$ and error 2015 is unrecoverable, *aux2* is ignored.

Otherwise, it is the working storage used by this subroutine, which is available for use by the calling program between calls to this subroutine.

Specified as: an area of storage, containing *naux2* long-precision real numbers. On output, the contents are overwritten.

*naux2*

is the number of doublewords in the working storage specified in *aux2*. Specified as: a fullword integer, where:

If $naux2 = 0$ and error 2015 is unrecoverable, SSINF and DSINF dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, $naux2 \ge$ (minimum value required for successful processing). To determine a sufficient value, use the processor-independent formulas. For all other values specified less than the minimum value, you have the option of having the minimum value returned in this argument. For details, see “Using Auxiliary Storage in ESSL” on page 32.

*On Return*

*y*

has the following meaning, where:

If *init* ≠ 0, this argument is not used, and its contents remain unchanged.

If *init* = 0, this is array `Y`, consisting of the results of the *m* discrete Fourier transforms, where each Fourier transform is real and of length *n*. However, due to symmetry, only the first *n*/2 values are given in the output—that is, $y_{ki}$, *k* = 0, 1, ..., *n*/2−1 for each *i* = 1, 2, ..., *m*.

Returned as: an array of (at least) length 1+(*n* / 2−1)*inc1y*+(*m*−1)*inc2y*, containing numbers of the data type indicated in Table 130 on page 780.

*aux1*

is the working storage for this subroutine, where:

If *init* ≠ 0, it contains information ready to be passed in a subsequent invocation of this subroutine.

If *init* = 0, its contents are unchanged.

Returned as: the contents are not relevant.

### ***Notes***

1. *aux1* should **not** be used by the calling program between calls to this subroutine with *init* ≠ 0 and *init* = 0. However, it can be reused after intervening calls to this subroutine with different arguments.
2. When using the ESSL SMP library, for optimal performance, the number of threads specified should be the same for *init* ≠ 0 and *init* = 0.
3. For optimal performance, the preferred value for *inc1x* and *inc1y* is 1. This implies that the sequences are stored with stride 1. In addition, *inc2x* and *inc2y* should be close to *n*/2.

   It is possible to specify sequences in the transposed form—that is, as rows of a two-dimensional array. In this case, *inc2x* (or *inc2y*) = 1 and *inc1x* (or *inc1y*) is equal to the leading dimension of the array. One can specify either input, output, or both in the transposed form by specifying appropriate values for the stride parameters. For selecting optimal values of *inc1x* and *inc1y* for _SINF, you should use “STRIDE—Determine the Stride Value for Optimal Performance in Specified Fourier Transform Subroutines” on page 971. Example 3 in the STRIDE subroutine description explains how it is used for _SINF.

   If you specify the same array for `X` and `Y`, then *inc1x* and *inc1y* must be equal, and *inc2x* and *inc2y* must be equal. In this case, output overwrites input. If *m* = 1, the *inc2x* and *inc2y* values are not used by the subroutine. If you specify different arrays for `X` and `Y`, they must have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.

### ***Processor-Independent Formulas for SSINF for NAUX1 and NAUX2***

*NAUX1 Formulas:*

If *n* ≤ 16384, use *naux1* = 60000.
If *n* > 16384, use *naux1* = 20000+.30*n*.

*NAUX2 Formulas:*

If $n \leq 16384$, use *naux2* = 25000.
If $n > 16384$, use *naux2* = 20000+.32*n*.
For the transposed case, where *inc2x* = 1 or *inc2y* = 1, and where $n \geq 252$, add the following to the above storage requirements:
$(n/4+257)(\min(128, m))$.

***Processor-Independent Formulas for DSINF for NAUX1 and NAUX2***

*NAUX1 Formulas:*

If $n \leq 16384$, use *naux1* = 50000.
If $n > 16384$, use *naux1* = 20000+.60*n*.

*NAUX2 Formulas:*

If $n \leq 16384$, use *naux2* = 20000.
If $n > 16384$, use *naux2* = 20000+.64*n*.
For the transposed case, where *inc2x* = 1 or *inc2y* = 1, and where $n \geq 252$, add the following to the above storage requirements:
$(n/2+257)(\min(128, m))$.

***Function:*** The set of *m* real even discrete *n*-point Fourier transforms of the sine sequences of real data in array X, with results going into array Y, is expressed as follows:

$$y_{ki} = scale \sum_{j=0}^{n/2-1} x_{ji} \sin\left(jk\left(2\pi / n\right)\right)$$

for:

$k = 0, 1, ..., n/2-1$
$i = 1, 2, ..., m$

where:

$x_{0i} = 0.0$
$x_{ji}$ are elements of the sequences in array X, where each sequence contains the *n*/2 real nonredundant data $x_{ji}$, $j = 0, 1, ..., n/2-1$.
$y_{ki}$ are elements of the sequences in array Y, where each sequence contains the *n*/2 real nonredundant data $y_{ki}$, $k = 0, 1, ..., n/2-1$.
*scale* is a scalar value.

You can obtain the inverse sine transform by specifying *scale* = 4.0/*n*. Thus, if an X input is used with *scale* = 1.0, and its output is used as input on a subsequent call with *scale* = 4.0/*n*, the original X is obtained. See references [1 ], [4 ], [19 ], and [20 ].

Two invocations of this subroutine are necessary:

1. With *init* ≠ 0, the subroutine tests and initializes arguments of the program, setting up the *aux1* working storage.

2. With *init* = 0, the subroutine checks that the initialization arguments in the *aux1* working storage correspond to the present arguments, and if so, performs the calculation of the Fourier transforms.

These subroutines use a Fourier transform method with a mixed-radix capability. This provides maximum performance for your application.

***Error Conditions***

*Resource Errors:* Error 2015 is unrecoverable, *naux2* = 0, and unable to allocate work area.

*Computational Errors:* None

*Input-Argument Errors*

1. $n > 37748736$
2. *inc1x* or *inc1y* ≤ 0
3. *inc2x* or *inc2y* ≤ 0
4. $m \le 0$
5. *scale* = 0.0
6. The subroutine has not been initialized with the present arguments.
7. The length of the transform in *n* is not an allowable value. Return code 1 is returned if error 2030 is recoverable.
8. *naux1* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.
9. Error 2015 is recoverable or *naux2*≠0, and *naux2* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example 1:*** This example shows an input array `X` with a set of *m* sine sequences of length $n/2$, $\sin(jk(2\pi/n))$, $j = 0, 1, ..., n/2-1$, with the single frequencies $k = 1, 2, 3$. The Fourier transform of the sine sequence has $n/4$ in position *k* and zeros elsewhere. The arrays are declared as follows:

```
REAL*4     X(0:53),Y(0:53)
REAL*8     AUX1(414),AUX2(8960)
```

First, initialize `AUX1` using the calling sequence shown below with `INIT` ≠ 0. Then use the same calling sequence with `INIT` = 0 to do the calculation.

*Call Statement and Input*

```
            INIT  X INC1X INC2X Y INC1Y INC2Y  N   M   SCALE  AUX1  NAUX1  AUX2   NAUX2
             |    |  |     |    |  |     |     |   |     |     |      |     |      |
CALL SSINF(INIT, X , 1  , 18 , Y , 1  , 18 , 32 , 3 , SCALE, AUX1 , 414 , AUX2 , 8960)
```

```
INIT    =  1(for initialization)
INIT    =  0(for computation)
SCALE   =  1.0
```

`X` contains the following three sequences:

```
0.0000   0.0000   0.0000
0.1951   0.3827   0.5556
0.3827   0.7071   0.9239
0.5556   0.9239   0.9808
0.7071   1.0000   0.7071
0.8315   0.9239   0.1951
0.9239   0.7071  -0.3827
0.9808   0.3827  -0.8315
1.0000   0.0000  -1.0000
0.9808  -0.3827  -0.8315
0.9239  -0.7071  -0.3827
0.8315  -0.9239   0.1951
0.7071  -1.0000   0.7071
0.5556  -0.9239   0.9808
0.3827  -0.7071   0.9239
0.1951  -0.3827   0.5556
  .        .        .
  .        .        .
```

*Output:* Y contains the following three sequences:

```
0.0000  0.0000  0.0000
8.0000  0.0000  0.0000
0.0000  8.0000  0.0000
0.0000  0.0000  8.0000
0.0000  0.0000  0.0000
0.0000  0.0000  0.0000
0.0000  0.0000  0.0000
0.0000  0.0000  0.0000
0.0000  0.0000  0.0000
0.0000  0.0000  0.0000
0.0000  0.0000  0.0000
0.0000  0.0000  0.0000
0.0000  0.0000  0.0000
0.0000  0.0000  0.0000
0.0000  0.0000  0.0000
0.0000  0.0000  0.0000
  .       .       .
  .       .       .
```

***Example 2:*** This example shows an input array X with a set of three input spike sequences equal to the output of Example 1. This shows how you can compute the inverse of the transform in Example 1 by using *scale* = 4.0/*n*, giving as output the three sequences listed in the input for Example 1. First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation.

*Call Statement and Input*

```
            INIT  X INC1X INC2X Y INC1Y INC2Y  N   M   SCALE  AUX1  NAUX1  AUX2  NAUX2
             |    |  |     |    |  |     |     |   |     |     |      |     |      |
CALL SSINF(INIT, X , 1  , 18 , Y , 1  , 18 , 32 , 3 , SCALE, AUX1 , 414 , AUX2 , 8960)
```

INIT = 1(for initialization)
INIT = 0(for computation)
SCALE = 4.0/32
X =(same sequences as in output Y in Example 1)

*Output*

```
Y          =(same sequences as in output X in Example 1)
```

***Example 3:*** This example shows another computation using the same arguments initialized in Example 1 and using different input sequence data. The data for this example has frequencies $k$ = 14, 15, 17. Because only the sequence data has changed, initialization does not have to be done again.

*Call Statement and Input*

```
                INIT X INC1X INC2X Y INC1Y INC2Y  N   M   SCALE  AUX1  NAUX1  AUX2  NAUX2
                 |   |  |     |    |  |     |     |   |     |     |     |      |      |
CALL SSINF( 0 , X , 1  , 18 , Y , 1  , 18 , 32 , 3 , SCALE, AUX1 , 414 , AUX2 , 8960)
```

```
SCALE    =  1.0
```

X contains the following three sequences:

```
 0.0000   0.0000   0.0000
 0.3827   0.1951  -0.1951
-0.7071  -0.3827   0.3827
 0.9239   0.5556  -0.5556
-1.0000  -0.7071   0.7071
 0.9239   0.8315  -0.8315
-0.7071  -0.9239   0.9239
 0.3827   0.9808  -0.9808
 0.8573  -1.0000   1.0000
-0.3827   0.9808  -0.9808
 0.7071  -0.9239   0.9239
-0.9239   0.8315  -0.8315
 1.0000  -0.7071   0.7071
-0.9239   0.5556  -0.5556
 0.7071  -0.3827   0.3827
-0.3827   0.1951  -0.1951
   .        .        .
   .        .        .
```

*Output:* Y contains the following three sequences:

```
0.0000  0.0000   0.0000
0.0000  0.0000   0.0000
0.0000  0.0000   0.0000
0.0000  0.0000   0.0000
0.0000  0.0000   0.0000
0.0000  0.0000   0.0000
0.0000  0.0000   0.0000
0.0000  0.0000   0.0000
0.0000  0.0000   0.0000
0.0000  0.0000   0.0000
0.0000  0.0000   0.0000
0.0000  0.0000   0.0000
0.0000  0.0000   0.0000
8.0000  0.0000   0.0000
0.0000  8.0000  -8.0000
0.0000  0.0000   0.0000
  .       .        .
  .       .        .
```

# SCFT2 and DCFT2—Complex Fourier Transform in Two Dimensions

These subroutines compute the two-dimensional discrete Fourier transform of complex data.

*Table 131. Data Types*

| X, Y | *scale* | **Subroutine** |
|---|---|---|
| Short-precision complex | Short-precision real | SCFT2 |
| Long-precision complex | Long-precision real | DCFT2 |

**Note:** Two invocations of this subroutine are necessary: one to prepare the working storage for the subroutine, and the other to perform the computations.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SCFT2 \| DCFT2 (*init*, *x*, *inc1x*, *inc2x*, *y*, *inc1y*, *inc2y*, *n1*, *n2*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*) |
| **C and C++** | scft2 \| dcft2 (*init*, *x*, *inc1x*, *inc2x*, *y*, *inc1y*, *inc2y*, *n1*, *n2*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*); |
| **PL/I** | CALL SCFT2 \| DCFT2 (*init*, *x*, *inc1x*, *inc2x*, *y*, *inc1y*, *inc2y*, *n1*, *n2*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*); |

*On Entry*

*init*

is a flag, where:

If *init* ≠ 0, trigonometric functions and other parameters, depending on arguments other than *x*, are computed and saved in *aux1*. The contents of *x* and *y* are not used or changed.

If *init* = 0, the discrete Fourier transform of the given array is computed. The only arguments that may change after initialization are *x*, *y*, and *aux2*. All scalar arguments must be the same as when the subroutine was called for initialization with *init* ≠ 0.

Specified as: a fullword integer. It can have any value.

*x*

is the array X, containing the two-dimensional data to be transformed, where each element $x_{j1,j2}$, using zero-based indexing, is stored in X(*j1*(*inc1x*)+*j2*(*inc2x*)) for *j1* = 0, 1, ..., *n1*–1 and *j2* = 0, 1, ..., *n2*–1.

Specified as: an array of (at least) length 1+(*n1*–1)*inc1x*+(*n2*–1)*inc2x*, containing numbers of the data type indicated in Table 131. This array must be aligned on a doubleword boundary, and:

If *inc1x* = 1, the input array is stored in normal form, and *inc2x* ≥ *n1*.

If *inc2x* = 1, the input array is stored in transposed form, and *inc1x* ≥ *n2*.

See “Notes” on page 790 for more details.

*inc1x*

is the stride between the elements in array X for the first dimension.

If the array is stored in the normal form, *inc1x* = 1.

If the array is stored in the transposed form, *inc1x* is the leading dimension of the array and $inc1x \geq n2$.

Specified as: a fullword integer; $inc1x > 0$. If $inc2x = 1$, then $inc1x \geq n2$.

*inc2x*

is the stride between the elements in array X for the second dimension.

If the array is stored in the transposed form, $inc2x = 1$.

If the array is stored in the normal form, *inc2x* is the leading dimension of the array and $inc2x \geq n1$.

Specified as: a fullword integer; $inc2x > 0$. If $inc1x = 1$, then $inc2x \geq n1$.

*y*

See “On Return” on page 789.

*inc1y*

is the stride between the elements in array Y for the first dimension.

If the array is stored in the normal form, $inc1y = 1$.

If the array is stored in the transposed form, *inc1y* is the leading dimension of the array and $inc1y \geq n2$.

Specified as: a fullword integer; $inc1y > 0$. If $inc2y = 1$, then $inc1y \geq n2$.

*inc2y*

is the stride between the elements in array Y for the second dimension.

If the array is stored in the transposed form, $inc2y = 1$.

If the array is stored in the normal form, *inc2y* is the leading dimension of the array and $inc2y \geq n1$.

Specified as: a fullword integer; $inc2y > 0$. If $inc1y = 1$, then $inc2y \geq n1$.

*n1*

is the length of the first dimension of the two-dimensional data in the array to be transformed. Specified as: a fullword integer; $n1 \leq 37748736$ and must be one of the values listed in “Acceptable Lengths for the Transforms” on page 741. For all other values specified less than 37748736, you have the option of having the next larger acceptable value returned in this argument. For details, see “Providing a Correct Transform Length to ESSL” on page 38.

*n2*

is the length of the second dimension of the two-dimensional data in the array to be transformed. Specified as: a fullword integer; $n2 \leq 37748736$ and must be one of the values listed in “Acceptable Lengths for the Transforms” on page 741. For all other values specified less than 37748736, you have the option of having the next larger acceptable value returned in this argument. For details, see “Providing a Correct Transform Length to ESSL” on page 38.

*isign*

controls the direction of the transform, determining the sign *Isign* of the exponents of $W_{n1}$ and $W_{n2}$, where:

If *isign* = positive value, *Isign* = + (transforming time to frequency).

If *isign* = negative value, *Isign* = − (transforming frequency to time).

Specified as: a fullword integer; $isign > 0$ or $isign < 0$.

*scale*

is the scaling constant *scale*. See “Function” on page 791 for its usage. Specified as: a number of the data type indicated in Table 131 on page 787, where $scale > 0.0$ or $scale < 0.0$.

*aux1*

is the working storage for this subroutine, where:

If *init* ≠ 0, the working storage is computed.

If *init* = 0, the working storage is used in the computation of the Fourier transforms.

Specified as: an area of storage, containing *naux1* long-precision real numbers.

*naux1*

is the number of doublewords in the working storage specified in *aux1*. Specified as: a fullword integer; *naux1* ≥ (minimum value required for successful processing). To determine a sufficient value, use the processor-independent formulas. For all other values specified less than the minimum value, you have the option of having the minimum value returned in this argument. For details, see “Using Auxiliary Storage in ESSL” on page 32.

*aux2*

has the following meaning:

If *naux2* = 0 and error 2015 is unrecoverable, *aux2* is ignored.

Otherwise, it is the working storage used by this subroutine, which is available for use by the calling program between calls to this subroutine.

Specified as: an area of storage, containing *naux2* long-precision real numbers. On output, the contents are overwritten.

*naux2*

is the number of doublewords in the working storage specified in *aux2*. Specified as: a fullword integer, where:

If *naux2* = 0 and error 2015 is unrecoverable, SCFT2 and DCFT2 dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, *naux2* ≥ (minimum value required for successful processing). To determine a sufficient value, use the processor-independent formulas. For all other values specified less than the minimum value, you have the option of having the minimum value returned in this argument. For details, see “Using Auxiliary Storage in ESSL” on page 32.

*On Return*

*y*

has the following meaning, where:

If *init* ≠ 0, this argument is not used, and its contents remain unchanged.

If *init* = 0, this is array Y, containing the elements resulting from the two-dimensional discrete Fourier transform of the data in X. Each element $y_{k1,k2}$, using zero-based indexing, is stored in Y(*k1*(*inc1y*)+*k2*(*inc2y*)) for *k1* = 0, 1, ..., *n1*–1 and *k2* = 0, 1, ..., *n2*–1.

Returned as: an array of (at least) length 1+(*n1*–1)*inc1y*+(*n2*–1)*inc2y*, containing numbers of the data type indicated in Table 131 on page 787. This array must be aligned on a doubleword boundary, and:

If *inc1y* = 1, the output array is stored in normal form, and *inc2y* ≥ *n1*.

If *inc2y* = 1, the output array is stored in transposed form, and *inc1y* ≥ *n2*.

See “Notes” on page 790 for more details.

*aux1*

is the working storage for this subroutine, where:

If *init* ≠ 0, it contains information ready to be passed in a subsequent invocation of this subroutine.

If *init* = 0, its contents are unchanged.

Returned as: the contents are not relevant.

***Notes***

1. *aux1* should **not** be used by the calling program between program calls to this subroutine with *init* ≠ 0 and *init* = 0. However, it can be reused after intervening calls to this subroutine with different arguments.
2. When using the ESSL SMP library, for optimal performance, the number of threads specified should be the same for *init* ≠ 0 and *init* = 0.
3. If you specify the same array for X and Y, then *inc1x* must equal *inc1y*, and *inc2x* must equal *inc2y*. In this case, output overwrites input. If you specify different arrays X and Y, they must have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.
4. By appropriately specifying the *inc* arguments, this subroutine allows you to specify that it should use one of two forms of its arrays, the normal untransposed form or the transposed form. As a result, you **do not have to move any data**. Instead, the subroutine performs the adjustments for you. Also, either the input array or the output array can be in transposed form. The FFT computation is symmetrical with respect to *n1* and *n2*. They can be interchanged without the loss of generality. If they are interchanged, an array that is stored in the normal form appears as an array stored in the transposed form and vise versa. If, for performance reasons, the forms of the input and output arrays are different, then the input array should be specified in the normal form, and the output array should be specified in the transposed form. This can always be done by interchanging *n1* and *n2*.
5. Although the *inc* arguments for each array can be arbitrary, in most cases, one of the *inc* arguments is 1 for each array. If *inc1* = 1, the array is stored in normal form; that is, the first dimension of the array is along the columns. In this case, *inc2* is the leading dimension of the array and must be at least *n1*. Conversely, if *inc2* = 1, the array is stored in the transposed form; that is, the first dimension of the array is along the rows. In this case, *inc1* is the leading dimension of the array and must be at least *n2*. The rows of the arrays are accessed with a stride that equals the leading dimension of the array. To minimize cache interference in accessing a row, an optimal value should be used for the leading dimension of the array. You should use “STRIDE—Determine the Stride Value for Optimal Performance in Specified Fourier Transform Subroutines” on page 971 to determine this optimal value. Example 4 in the STRIDE subroutine description explains how it is used to find either *inc1* or *inc2*.

***Processor-Independent Formulas for SCFT2 for NAUX1 and NAUX2:*** The required values of *naux1* and *naux2* depend on *n1* and *n2*.

*NAUX1 Formulas*

If max(*n1*, *n2*) ≤ 8192, use *naux1* = 40000.
If max(*n1*, *n2*) > 8192, use *naux1* = 40000+1.14(*n1*+*n2*).

*NAUX2 Formulas*

If max(*n1*, *n2*) < 252, use *naux2* = 20000.
If max(*n1*, *n2*) ≥ 252, use *naux2* = 20000+(*r*+256)(*s*+1.14), where *r* = max(*n1*, *n2*) and *s* = min(64, *n1*, *n2*).

***Processor-Independent Formulas for DCFT2 for NAUX1 and NAUX2:*** The required values of *naux1* and *naux2* depend on *n1* and *n2*.

*NAUX1 Formulas*

If max(*n1*, *n2*) ≤ 2048, use *naux1* = 40000.
If max(*n1*, *n2*) > 2048, use *naux1* = 40000+2.28(*n1*+*n2*).

*NAUX2 Formulas*

If max(*n1*, *n2*) < 252, use *naux2* = 20000.
If max(*n1*, *n2*) ≥ 252, use *naux2* = 20000+(2*r*+256)(*s*+2.28), where
*r* = max(*n1*, *n2*) and *s* = min(64, *n1*, *n2*).

***Function:*** The two-dimensional discrete Fourier transform of complex data in array X, with results going into array Y, is expressed as follows:

$$y_{k1,k2} = scale \sum_{j1=0}^{n1-1} \sum_{j2=0}^{n2-1} x_{j1,j2}\ W_{n1}^{(Isign)j1k1}\ W_{n2}^{(Isign)j2k2}$$

for:

*k1* = 0, 1, ..., *n1*–1
*k2* = 0, 1, ..., *n2*–1

where:

$$W_{n1} = e^{-2\pi\left(\sqrt{-1}\right)/n1}$$
$$W_{n2} = e^{-2\pi\left(\sqrt{-1}\right)/n2}$$

and where:

$x_{j1,j2}$ are elements of array X.
$y_{k1,k2}$ are elements of array Y.
*Isign* is + or – (determined by argument *isign*).
*scale* is a scalar value.

For *scale* = 1.0 and *isign* being positive, you obtain the discrete Fourier transform, a function of frequency. The inverse Fourier transform is obtained with *scale* = 1.0/((*n1*)(*n2*)) and *isign* being negative. See references [1 ], [4 ], and [20 ].

Two invocations of this subroutine are necessary:

1. With *init* ≠ 0, the subroutine tests and initializes arguments of the program, setting up the *aux1* working storage.

2. With *init* = 0, the subroutine checks that the initialization arguments in the *aux1* working storage correspond to the present arguments, and if so, performs the calculation of the Fourier transform.

***Error Conditions***

*Resource Errors:* Error 2015 is unrecoverable, *naux2* = 0, and unable to allocate work area.

*Computational Errors:* None

*Input-Argument Errors*

1. *n1* > 37748736
2. *n2* > 37748736
3. *inc1x*|*inc2x*|*inc1y*|*inc2y* ≤ 0
4. *scale* = 0.0
5. *isign* = 0
6. The subroutine has not been initialized with the present arguments.
7. The length of one of the transforms in *n1* or *n2* is not an allowable value. Return code 1 is returned if error 2030 is recoverable.
8. *naux1* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.
9. Error 2015 is recoverable or *naux2*≠0, and *naux2* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example 1:*** This example shows how to compute a two-dimensional transform where both input and output are stored in normal form (*inc1x* = *inc1y* = 1). Also, *inc2x* = *inc2y* so the same array can be used for both input and output. The arrays are declared as follows:

```
COMPLEX*8  X(6,8),Y(6,8)
REAL*8     AUX1(20000), AUX2(10000)
```

Arrays X and Y are made equivalent by the following statement, making them occupy the same storage: EQUIVALENCE (X,Y). First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation.

*Call Statement and Input*

```
           INIT  X INC1X INC2X Y INC1Y INC2Y N1  N2 ISIGN SCALE  AUX1  NAUX1   AUX2  NAUX2
            |    |   |     |   |   |     |   |   |    |     |     |     |      |      |
CALL SCFT2(INIT, X , 1  ,  6 , Y , 1  ,  6 , 6 , 8 ,  1 , SCALE, AUX1, 20000 , AUX2, 10000)
```

INIT = 1(for initialization)
INIT = 0(for computation)
SCALE = 1.0
X is an array with 6 rows and 8 columns with (1.0, 0.0) in all locations.

*Output:* Y is an array with 6 rows and 8 columns having (48.0, 0.0) in location Y(1,1) and (0.0, 0.0) in all others.

***Example 2:*** This example shows how to compute a two-dimensional inverse Fourier transform. For this example, X is stored in normal untransposed form (*inc1x* = 1), and Y is stored in transposed form (*inc2y* = 1). The arrays are declared as follows:

```
COMPLEX*16  X(6,8),Y(8,6)
REAL*8      AUX1(20000), AUX2(10000)
```

First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation.

*Call Statement and Input*

```
            INIT  X INC1X INC2X Y INC1Y INC2Y N1  N2 ISIGN SCALE  AUX1   NAUX1   AUX2   NAUX2
             |    |  |     |    |  |     |    |   |    |     |     |      |       |      |
CALL DCFT2(INIT, X , 1  ,  6 , Y , 8  ,  1 , 6 , 8 , -1 , SCALE, AUX1 , 20000 , AUX2 , 10000)
```

```
INIT     =  1(for initialization)
INIT     =  0(for computation)
SCALE    =  1.0/48.0
X        =(same as output Y in Example 1)
```

*Output:* Y is an array with 8 rows and 6 columns with (1.0, 0.0) in all locations.

# SRCFT2 and DRCFT2—Real-to-Complex Fourier Transform in Two Dimensions

These subroutines compute the two-dimensional discrete Fourier transform of real data in a two-dimensional array.

*Table 132. Data Types*

| X, *scale* | Y | **Subroutine** |
|---|---|---|
| Short-precision real | Short-precision complex | SRCFT2 |
| Long-precision real | Long-precision complex | DRCFT2 |

**Note:** Two invocations of this subroutine are necessary: one to prepare the working storage for the subroutine, and the other to perform the computations.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SRCFT2 (*init*, *x*, *inc2x*, *y*, *inc2y*, *n1*, *n2*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*, *aux3*, *naux3*)<br>CALL DRCFT2 (*init*, *x*, *inc2x*, *y*, *inc2y*, *n1*, *n2*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*) |
| **C and C++** | srcft2 (*init*, *x*, *inc2x*, *y*, *inc2y*, *n1*, *n2*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*, *aux3*, *naux3*);<br>drcft2 (*init*, *x*, *inc2x*, *y*, *inc2y*, *n1*, *n2*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*); |
| **PL/I** | CALL SRCFT2 (*init*, *x*, *inc2x*, *y*, *inc2y*, *n1*, *n2*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*, *aux3*, *naux3*);<br>CALL DRCFT2 (*init*, *x*, *inc2x*, *y*, *inc2y*, *n1*, *n2*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*); |

*On Entry*

*init*

is a flag, where:

If *init* ≠ 0, trigonometric functions and other parameters, depending on arguments other than *x*, are computed and saved in *aux1*. The contents of *x* and *y* are not used or changed.

If *init* = 0, the discrete Fourier transform of the given array is computed. The only arguments that may change after initialization are *x*, *y*, and *aux2*. All scalar arguments must be the same as when the subroutine was called for initialization with *init* ≠ 0.

Specified as: a fullword integer. It can have any value.

*x*

is the array X, containing *n1* rows and *n2* columns of data to be transformed. The data in each column is stored with stride 1. Specified as: an *inc2x* by (at least) *n2* array, containing numbers of the data type indicated in Table 132. See “Notes” on page 796 for more details.

*inc2x*

is the leading dimension (stride between columns) of array X. Specified as: a fullword integer; *inc2x* ≥ *n1*.

*y*

See “On Return” on page 796.

*inc2y*

is the leading dimension (stride between columns) of array Y. Specified as: a fullword integer; *inc2y* ≥ ((*n1*)/2)+1.

*n1*

is the number of rows of data—that is, the length of the columns in array X involved in the computation. The length of the columns in array Y are (*n1*)/2+1. Specified as: a fullword integer; *n1* ≤ 37748736 and must be one of the values listed in "Acceptable Lengths for the Transforms" on page 741. For all other values specified less than 37748736, you have the option of having the next larger acceptable value returned in this argument. For details, see "Providing a Correct Transform Length to ESSL" on page 38.

*n2*

is the number of columns of data—that is, the length of the rows in arrays X and Y involved in the computation. Specified as: a fullword integer; *n2* ≤ 37748736 and must be one of the values listed in "Acceptable Lengths for the Transforms" on page 741. For all other values specified less than 37748736, you have the option of having the next larger acceptable value returned in this argument. For details, see "Providing a Correct Transform Length to ESSL" on page 38.

*isign*

controls the direction of the transform, determining the sign *Isign* of the exponents of $W_{n1}$ and $W_{n2}$, where:

If *isign* = positive value, *Isign* = + (transforming time to frequency).

If *isign* = negative value, *Isign* = − (transforming frequency to time).

Specified as: a fullword integer; *isign* > 0 or *isign* < 0.

*scale*

is the scaling constant *scale*. See "Function" on page 797 for its usage. Specified as: a number of the data type indicated in Table 132 on page 794, where *scale* > 0.0 or *scale* < 0.0.

*aux1*

is the working storage for this subroutine, where:

If *init* ≠ 0, the working storage is computed.

If *init* = 0, the working storage is used in the computation of the Fourier transforms.

Specified as: an area of storage, containing *naux1* long-precision real numbers.

*naux1*

is the number of doublewords in the working storage specified in *aux1*. Specified as: a fullword integer; *naux1* ≥ (minimum value required for successful processing). To determine a sufficient value, use the processor-independent formulas. For all other values specified less than the minimum value, you have the option of having the minimum value returned in this argument. For details, see "Using Auxiliary Storage in ESSL" on page 32.

*aux2*

has the following meaning:

If *naux2* = 0 and error 2015 is unrecoverable, *aux2* is ignored.

Otherwise, it is the working storage used by this subroutine, which is available for use by the calling program between calls to this subroutine.

Specified as: an area of storage, containing *naux2* long-precision real numbers. On output, the contents are overwritten.

*naux2*

is the number of doublewords in the working storage specified in *aux2*. Specified as: a fullword integer, where:

If *naux2* = 0 and error 2015 is unrecoverable, SRCFT2 and DRCFT2 dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, *naux2* ≥ (minimum value required for successful processing). To determine a sufficient value, use the processor-independent formulas. For all other values specified less than the minimum value, you have the option of having the minimum value returned in this argument. For details, see “Using Auxiliary Storage in ESSL” on page 32.

*aux3*

this argument is provided for migration purposes only and is ignored.

Specified as: an area of storage containing *naux3* long-precision real numbers.

*naux3*

this argument is provided for migration purposes only and is ignored.

Specified as: a fullword integer.

*On Return*

*y*

has the following meaning, where:

If *init* ≠ 0, this argument is not used, and its contents remain unchanged.

If *init* = 0, this is array Y, containing the results of the complex discrete Fourier transform of X. The output consists of *n2* columns of data. The data in each column is stored with stride 1. Due to complex conjugate symmetry, the output consists of only the first ((*n1*)/2)+1 rows of the array—that is, $y_{k1,k2}$, where *k1* = 0, 1, ..., (*n1*)/2 and *k2* = 0, 1, ..., *n2*–1.

Returned as: an *inc2y* by (at least) *n2* array, containing numbers of the data type indicated in Table 132 on page 794. This array must be aligned on a doubleword boundary.

*aux1*

is the working storage for this subroutine, where:

If *init* ≠ 0, it contains information ready to be passed in a subsequent invocation of this subroutine.

If *init* = 0, its contents are unchanged.

Returned as: the contents are not relevant.

***Notes***

1. *aux1* should **not** be used by the calling program between calls to this subroutine with *init* ≠ 0 and *init* = 0. However, it can be reused after intervening calls to this subroutine with different arguments.
2. When using the ESSL SMP library, for optimal performance, the number of threads specified should be the same for *init* ≠ 0 and *init* = 0.
3. If you specify the same array for X and Y, then *inc2x* must equal (2)(*inc2y*). In this case, output overwrites input. If you specify different arrays X and Y, they must have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.

4. For selecting optimal strides (or leading dimensions *inc2x* and *inc2y*) for your input and output arrays, you should use “STRIDE—Determine the Stride Value for Optimal Performance in Specified Fourier Transform Subroutines” on page 971. Example 5 in the STRIDE subroutine description explains how it is used for these subroutines.
5. Be sure to align array X on a doubleword boundary, and specify an even number for *inc2x*, if possible.

***Processor-Independent Formulas for SRCFT2 for NAUX1 and NAUX2:*** The required values of *naux1* and *naux2* depend on *n1* and *n2*.

*NAUX1 Formulas*

If max(*n1*/2, *n2*) $\le$ 8192, use *naux1* = 45000.
If max(*n1*/2, *n2*) $>$ 8192, use *naux1* = 40000+0.82*n1*+1.14*n2*.

*NAUX2 Formulas*

If *n1* $\le$ 16384 and *n2* $<$ 252, use *naux2* = 20000.
If *n1* $>$ 16384 and *n2* $<$ 252, use *naux2* = 20000+0.57*n1*.
If *n2* $\ge$ 252, add the following to the above storage requirements:
(*n2*+256)(1.14+*s*)
where *s* = min(64, 1+*n1*/2).

***Processor-Independent Formulas for DRCFT2 for NAUX1 and NAUX2:*** The required values of *naux1* and *naux2* depend on *n1* and *n2*.

*NAUX1 Formulas*

If *n* $\le$ 2048, use *naux1* = 42000.
If *n* $>$ 2048, use *naux1* = 40000+1.64*n1*+2.28*n2*,
where *n* = max(*n1*/2, *n2*).

*NAUX2 Formulas*

If *n1* $\le$ 4096 and *n2* $<$ 252, use *naux2* = 20000.
If *n1* $>$ 4096 and *n2* $<$ 252, use *naux2* = 20000+1.14*n1*.
If *n2* $\ge$ 252, add the following to the above storage requirements:
((2)*n2*+256) (2.28+*s*)
where *s* = min(64, 1+*n1*/2).

***Function:*** The two-dimensional complex conjugate even discrete Fourier transform of real data in array X, with results going into array Y, is expressed as follows:

$$y_{k1,k2} = scale \sum_{j1=0}^{n1-1} \sum_{j2=0}^{n2-1} x_{j1,j2} \; W_{n1}^{(Isign)j1k1} \; W_{n2}^{(Isign)j2k2}$$

for:

*k1* = 0, 1, ..., *n1*–1
*k2* = 0, 1, ..., *n2*–1

where:

$$W_{n1} = e^{-2\pi\left(\sqrt{-1}\right)/n1}$$
$$W_{n2} = e^{-2\pi\left(\sqrt{-1}\right)/n2}$$

and where:

$x_{j1,j2}$ are elements of array X.
$y_{k1,k2}$ are elements of array Y.
*Isign* is + or − (determined by argument *isign*).
*scale* is a scalar value.

The output in array Y is complex. For *scale* = 1.0 and *isign* being positive, you obtain the discrete Fourier transform, a function of frequency. The inverse Fourier transform is obtained with *scale* = 1.0/((*n1*)(*n2*)) and *isign* being negative. See references [1 ], [4 ], [19 ], and [20 ].

Two invocations of this subroutine are necessary:

1. With *init* ≠ 0, the subroutine tests and initializes arguments of the program, setting up the *aux1* working storage.
2. With *init* = 0, the subroutine checks that the initialization arguments in the *aux1* working storage correspond to the present arguments, and if so, performs the calculation of the Fourier transform.

### *Error Conditions*

*Resource Errors:* Error 2015 is unrecoverable, *naux2* = 0, and unable to allocate work area.

*Computational Errors:* None

*Input-Argument Errors*

1. *n1* > 37748736
2. *n2* > 37748736
3. *inc2x* < *n1*
4. *inc2y* < (*n1*)/2+1
5. *scale* = 0.0
6. *isign* = 0
7. The subroutine has not been initialized with the present arguments.
8. The length of one of the transforms in *n1* or *n2* is not an allowable value. Return code 1 is returned if error 2030 is recoverable.
9. *naux1* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.
10. Error 2015 is recoverable or *naux2*≠0, and *naux2* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example 1:*** This example shows how to compute a two-dimensional transform. The arrays are declared as follows:

```
COMPLEX*8  Y(0:6,0:7)
REAL*4     X(0:11,0:7)
REAL*8     AUX1(1000), AUX2(1000), AUX3(1)
```

First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation.

*Call Statement and Input*

```
            INIT  X  INC2X Y INC2Y N1  N2 ISIGN SCALE  AUX1   NAUX1  AUX2   NAUX2  AUX3  NAUX3
             |    |    |   |   |    |   |   |     |     |       |     |       |     |     |
CALL SRCFT2(INIT, X , 12 , Y , 7 , 12 , 8 , 1 , SCALE, AUX1 , 1000 , AUX2 , 1000 , AUX3 ,  0 )
```

INIT = 1(for initialization)
INIT = 0(for computation)
SCALE = 1.0

X is an array with 12 rows and 8 columns having 1.0 in location X(0,0) and 0.0 in all others.

*Output:* Y is an array with 7 rows and 8 columns with (1.0, 0.0) in all locations.

***Example 2:*** This example shows another transform computation with different data using the same initialized array AUX1 in Example 1.

*Call Statement and Input*

```
            INIT X  INC2X Y INC2Y N1   N2 ISIGN SCALE  AUX1  NAUX1  AUX2  NAUX2  AUX3  NAUX3
             |   |    |   |   |   |    |    |     |     |      |     |      |     |     |
CALL SRCFT2( 0 , X , 12 , Y , 7 , 12 , 8 ,  1 , SCALE, AUX1, 1000 , AUX2, 1000 , AUX3 ,  0 )
```

SCALE = 1.0
X is an array with 12 rows and 8 columns with 1.0 in all locations.

*Output:* Y is an array with 7 rows and 8 columns having (96.0, 0.0) in location Y(0,0) and (0.0, 0.0) in all others.

***Example 3:*** This example shows the same array being used for input and output, where *isign* = −1 and *scale* = 1/((N1)(N2)). The arrays are declared as follows:

```
COMPLEX*16  Y(0:8,0:7)
REAL*8     X(0:19,0:7)
REAL*8     AUX1(1000), AUX2(1000), AUX3(1)
```

Arrays X and Y are made equivalent by the following statement, making them occupy the same storage.

```
EQUIVALENCE (X,Y)
```

This requires *inc2x* ≥ 2(*inc2y*). First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation.

*Call Statement and Input*

```
            INIT  X  INC2X Y INC2Y N1   N2 ISIGN SCALE  AUX1   NAUX1  AUX2   NAUX2  AUX3  NAUX3
             |    |    |   |   |   |    |    |     |     |       |     |       |     |     |
CALL DRCFT2(INIT, X , 20 , Y , 9 , 16 , 8 , -1 , SCALE, AUX1 , 1000 , AUX2 , 1000 , AUX3 ,  0 )
```

```
INIT      =  1(for initialization)
INIT      =  0(for computation)
SCALE     =  1.0/128.0
```

```
                ┌                                                ┐
                │  2.0   2.0  -2.0  -2.0   2.0   2.0  -2.0  -2.0 │
                │  2.0  -2.0  -2.0   2.0   2.0  -2.0  -2.0   2.0 │
                │ -2.0  -2.0   2.0   2.0  -2.0  -2.0   2.0   2.0 │
                │ -2.0   2.0   2.0  -2.0  -2.0   2.0   2.0  -2.0 │
                │  2.0   2.0  -2.0  -2.0   2.0   2.0  -2.0  -2.0 │
                │  2.0  -2.0  -2.0   2.0   2.0  -2.0  -2.0   2.0 │
                │ -2.0  -2.0   2.0   2.0  -2.0  -2.0   2.0   2.0 │
                │ -2.0   2.0   2.0  -2.0  -2.0   2.0   2.0  -2.0 │
                │  2.0   2.0  -2.0  -2.0   2.0   2.0  -2.0  -2.0 │
X     =         │  2.0  -2.0  -2.0   2.0   2.0  -2.0  -2.0   2.0 │
                │ -2.0  -2.0   2.0   2.0  -2.0  -2.0   2.0   2.0 │
                │ -2.0   2.0   2.0  -2.0  -2.0   2.0   2.0  -2.0 │
                │  2.0   2.0  -2.0  -2.0   2.0   2.0  -2.0  -2.0 │
                │  2.0  -2.0  -2.0   2.0   2.0  -2.0  -2.0   2.0 │
                │ -2.0  -2.0   2.0   2.0  -2.0  -2.0   2.0   2.0 │
                │ -2.0   2.0   2.0  -2.0  -2.0   2.0   2.0  -2.0 │
                │   .     .     .     .     .     .     .     .  │
                │   .     .     .     .     .     .     .     .  │
                │   .     .     .     .     .     .     .     .  │
                │   .     .     .     .     .     .     .     .  │
                └                                                ┘
```

*Output:* Y is an array with 9 rows and 8 columns having (1.0, 1.0) in location Y(4,2) and (0.0, 0.0) in all others.

# SCRFT2 and DCRFT2—Complex-to-Real Fourier Transform in Two Dimensions

These subroutines compute the two-dimensional discrete Fourier transform of complex conjugate even data in a two-dimensional array.

*Table 133. Data Types*

| X | Y, *scale* | **Subroutine** |
|---|---|---|
| Short-precision complex | Short-precision real | SCRFT2 |
| Long-precision complex | Long-precision real | DCRFT2 |

**Note:** Two invocations of this subroutine are necessary: one to prepare the working storage for the subroutine, and the other to perform the computations.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SCRFT2 (*init*, *x*, *inc2x*, *y*, *inc2y*, *n1*, *n2*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*, *aux3*, *naux3*)<br>CALL DCRFT2 (*init*, *x*, *inc2x*, *y*, *inc2y*, *n1*, *n2*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*) |
| **C and C++** | scrft2 (*init*, *x*, *inc2x*, *y*, *inc2y*, *n1*, *n2*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*, *aux3*, *naux3*);<br>dcrft2 (*init*, *x*, *inc2x*, *y*, *inc2y*, *n1*, *n2*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*); |
| **PL/I** | CALL SCRFT2 (*init*, *x*, *inc2x*, *y*, *inc2y*, *n1*, *n2*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*, *aux3*, *naux3*);<br>CALL DCRFT2 (*init*, *x*, *inc2x*, *y*, *inc2y*, *n1*, *n2*, *isign*, *scale*, *aux1*, *naux1*, *aux2*, *naux2*); |

*On Entry*

*init*

is a flag, where:

If *init* ≠ 0, trigonometric functions and other parameters, depending on arguments other than *x*, are computed and saved in *aux1*. The contents of *x* and *y* are not used or changed.

If *init* = 0, the discrete Fourier transform of the given array is computed. The only arguments that may change after initialization are *x*, *y*, and *aux2*. All scalar arguments must be the same as when the subroutine was called for initialization with *init* ≠ 0.

Specified as: a fullword integer. It can have any value.

*x*

is the array X, containing *n2* columns of data to be transformed. Due to complex conjugate symmetry, the input consists of only the first ((*n1*)/2)+1 rows of the array—that is, $x_{j1,j2}$, *j1* = 0, 1, ..., (*n1*)/2, *j2* = 0, 1, ..., *n2*–1. The data in each column is stored with stride 1.

Specified as: an *inc2x* by (at least) *n2* array, containing numbers of the data type indicated in Table 133. This array must be aligned on a doubleword boundary.

*inc2x*

is the leading dimension (stride between columns) of array X. Specified as: a fullword integer; *inc2x* ≥ ((*n1*)/2)+1.

*y*

See “On Return” on page 803.

*inc2y*

is the leading dimension (stride between the columns) of array Y. Specified as: a fullword integer; *inc2y* ≥ *n1*+2.

*n1*

is the number of rows of data—that is, the length of the columns in array Y involved in the computation. The length of the columns in array X are (*n1*)/2+1. Specified as: a fullword integer; *n1* ≤ 37748736 and must be one of the values listed in “Acceptable Lengths for the Transforms” on page 741. For all other values specified less than 37748736, you have the option of having the next larger acceptable value returned in this argument. For details, see “Providing a Correct Transform Length to ESSL” on page 38.

*n2*

is the number of columns of data—that is, the length of the rows in arrays X and Y involved in the computation. Specified as: a fullword integer; *n2* ≤ 37748736 and must be one of the values listed in “Acceptable Lengths for the Transforms” on page 741. For all other values specified less than 37748736, you have the option of having the next larger acceptable value returned in this argument. For details, see “Providing a Correct Transform Length to ESSL” on page 38.

*isign*

controls the direction of the transform, determining the sign *Isign* of the exponents of $W_{n1}$ and $W_{n2}$, where:

If *isign* = positive value, *Isign* = + (transforming time to frequency).

If *isign* = negative value, *Isign* = − (transforming frequency to time).

Specified as: a fullword integer; *isign* > 0 or *isign* < 0.

*scale*

is the scaling constant *scale*. See “Function” on page 804 for its usage. Specified as: a number of the data type indicated in Table 133 on page 801, where *scale* > 0.0 or *scale* < 0.0.

*aux1*

is the working storage for this subroutine, where:

If *init* ≠ 0, the working storage is computed.

If *init* = 0, the working storage is used in the computation of the Fourier transforms.

Specified as: an area of storage, containing *naux1* long-precision real numbers.

*naux1*

is the number of doublewords in the working storage specified in *aux*. Specified as: a fullword integer; *naux1* ≥ (minimum value required for successful processing). To determine a sufficient value, use the processor-independent formulas. For all other values specified less than the minimum value, you have the option of having the minimum value returned in this argument. For details, see “Using Auxiliary Storage in ESSL” on page 32.

*aux2*

has the following meaning:

If *naux2* = 0 and error 2015 is unrecoverable, *aux2* is ignored.

Otherwise, it is the working storage used by this subroutine, which is available for use by the calling program between calls to this subroutine.

Specified as: an area of storage, containing *naux2* long-precision real numbers. On output, the contents are overwritten.

*naux2*

is the number of doublewords in the working storage specified in *aux2*. Specified as: a fullword integer, where:

If *naux2* = 0 and error 2015 is unrecoverable, SCRFT2 and DCRFT2 dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, *naux2* ≥ (minimum value required for successful processing). To determine a sufficient value, use the processor-independent formulas. For all other values specified less than the minimum value, you have the option of having the minimum value returned in this argument. For details, see “Using Auxiliary Storage in ESSL” on page 32.

*aux3*

this argument is provided for migration purposes only and is ignored.

Specified as: an area of storage, containing *naux3* long-precision real numbers.

*naux3*

this argument is provided for migration purposes only and is ignored.

Specified as: a fullword integer.

*On Return*

*y*

has the following meaning, where:

If *init* ≠ 0, this argument is not used, and its contents remain unchanged.

If *init* = 0, this is the array Y, containing *n1* rows and *n2* columns of results of the real discrete Fourier transform of X. The data in each column of Y is stored with stride 1.

Returned as: an *inc2y* by (at least) *n2* array, containing numbers of the data type indicated in Table 133 on page 801. See “Notes” for more details.

*aux1*

is the working storage for this subroutine, where:

If *init* ≠ 0, it contains information ready to be passed in a subsequent invocation of this subroutine.

If *init* = 0, its contents are unchanged.

Returned as: the contents are not relevant.

***Notes***

1. *aux1* should **not** be used by the calling program between program calls to this subroutine with *init* ≠ 0 and *init* = 0. However, it can be reused after intervening calls to this subroutine with different arguments.
2. When using the ESSL SMP library, for optimal performance, the number of threads specified should be the same for *init* ≠ 0 and *init* = 0.
3. If you specify the same array for X and Y, then (2)(*inc2x*) must equal *inc2y*. In this case, output overwrites input. If you specify different arrays X and Y, they must have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.

4. For selecting optimal strides (or leading dimensions *inc2x* and *inc2y*) for your input and output arrays, you should use “STRIDE—Determine the Stride Value for Optimal Performance in Specified Fourier Transform Subroutines” on page 971. Example 6 in the STRIDE subroutine description explains how it is used for these subroutines.
5. Be sure to align array Y on a doubleword boundary, and specify an even number for *inc2y*, if possible.

***Processor-Independent Formulas for SCRFT2 for NAUX1 and NAUX2:*** The required values of *naux1* and *naux2* depend on *n1* and *n2*.

*NAUX1 Formulas*

If max(*n1*/2, *n2*) ≤ 8192, use *naux1* = 45000.
If max(*n1*/2, *n2*) > 8192, use *naux1* = 40000+0.82*n1*+1.14*n2*.

*NAUX2 Formulas*

If *n1* ≤ 16384 and *n2* < 252, use *naux2* = 20000.
If *n1* > 16384 and *n2* < 252, use *naux2* = 20000+0.57*n1*.
If *n2* ≥ 252, add the following to the above storage requirements:
(*n2*+256)(1.14+*s*)
where *s* = min(64, 1+*n1*/2).

***Processor-Independent Formulas for DCRFT2 for NAUX1 and NAUX2:*** The required values of *naux1* and *naux2* depend on *n1* and *n2*.

*NAUX1 Formulas*

If *n* ≤ 2048, use *naux1* = 42000.
If *n* > 2048, use *naux1* = 40000+1.64*n1*+2.28*n2*,
where *n* = max(*n1*/2, *n2*).

*NAUX2 Formulas*

If *n1* ≤ 4096 and *n2* < 252, use *naux2* = 20000.
If *n1* > 4096 and *n2* < 252, use *naux2* = 20000+1.14*n1*.
If *n2* ≥ 252, add the following to the above storage requirements:
((2)*n2*+256) (2.28+*s*)
where *s* = min(64, 1+*n1*/2).

***Function:*** The two-dimensional discrete Fourier transform of complex conjugate even data in array X, with results going into array Y, is expressed as follows:

$$y_{k1,k2} = scale \sum_{j1=0}^{n1-1} \sum_{j2=0}^{n2-1} x_{j1,j2} \; W_{n1}^{(Isign)j1k1} \; W_{n2}^{(Isign)j2k2}$$

for:

*k1* = 0, 1, ..., *n1*–1
*k2* = 0, 1, ..., *n2*–1

where:

$$W_{n1} = e^{-2\pi\left(\sqrt{-1}\right)/n1}$$
$$W_{n2} = e^{-2\pi\left(\sqrt{-1}\right)/n2}$$

and where:

$x_{j1,j2}$ are elements of array X.
$y_{k1,k2}$ are elements of array Y.
*Isign* is + or − (determined by argument *isign*).
*scale* is a scalar value.

Because of the complex conjugate symmetry, the output in array Y is real. For *scale* = 1.0 and *isign* being positive, you obtain the discrete Fourier transform, a function of frequency. The inverse Fourier transform is obtained with *scale* = 1.0/((*n1*)(*n2*)) and *isign* being negative. See references [1 ], [4 ], and [20 ].

Two invocations of this subroutine are necessary:

1. With *init* ≠ 0, the subroutine tests and initializes arguments of the program, setting up the *aux1* working storage.
2. With *init* = 0, the subroutine checks that the initialization arguments in the *aux1* working storage correspond to the present arguments, and if so, performs the calculation of the Fourier transform.

### *Error Conditions*

*Resource Errors:* Error 2015 is unrecoverable, *naux2* = 0, and unable to allocate work area.

*Computational Errors:* None

*Input-Argument Errors*

1. *n1* > 37748736
2. *n2* > 37748736
3. *inc2x* < (*n1*)/2+1
4. *inc2y* < *n1*+2
5. *scale* = 0.0
6. *isign* = 0
7. The subroutine has not been initialized with the present arguments.
8. The length of one of the transforms in *n1* or *n2* is not an allowable value. Return code 1 is returned if error 2030 is recoverable.
9. *naux1* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.
10. Error 2015 is recoverable or *naux2*≠0, and *naux2* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example 1:*** This example shows how to compute a two-dimensional transform. The arrays are declared as follows:

```
REAL*4     Y(0:13,0:7)
COMPLEX*8  X(0:6,0:7)
REAL*8     AUX1(1000), AUX2(1000), AUX3(1)
```

First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation.

*Call Statement and Input*

```
            INIT  X INC2X Y  INC2Y N1   N2 ISIGN SCALE   AUX1  NAUX1   AUX2  NAUX2   AUX3  NAUX3
             |    |  |    |    |   |    |   |     |       |     |       |     |       |     |
CALL SCRFT2(INIT, X , 7 , Y , 14 , 12 , 8 , -1 , SCALE , AUX1 , 1000 , AUX2 , 1000 , AUX3 ,  0 )
```

```
INIT     =  1(for initialization)
INIT     =  0(for computation)
SCALE    =  1.0/96.0
```

X is an array with 7 rows and 8 columns with (1.0, 0.0) in all locations.

*Output*

```
         ┌                                         ┐
         | 1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |
         | 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |
         | 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |
         | 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |
         | 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |
         | 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |
Y    =   | 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |
         | 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |
         | 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |
         | 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |
         | 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |
         | 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |
         |  .    .    .    .    .    .    .    . |
         |  .    .    .    .    .    .    .    . |
         └                                         ┘
```

***Example 2:*** This example shows another transform computation with different data using the same initialized array AUX1 in Example 1.

*Call Statement and Input*

```
            INIT X INC2X Y  INC2Y N1   N2 ISIGN SCALE   AUX1   NAUX1  AUX2   NAUX2  AUX3  NAUX3
             |   |   |   |    |   |    |   |     |       |       |     |       |     |     |
CALL SCRFT2( 0 , X , 7 , Y , 14 , 12 , 8 , -1 , SCALE , AUX1 , 1000 , AUX2 , 1000 , AUX3 ,  0 )
```

```
SCALE    =  1.0/96.0
```

X is an array with 7 rows and 8 columns having (96.0, 0.0) in location X(0,0) and (0.0, 0.0) in all others.

*Output*

```
              ┌                                         ┐
              | 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0 |
              | 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0 |
              | 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0 |
              | 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0 |
              | 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0 |
              | 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0 |
Y     =       | 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0 |
              | 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0 |
              | 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0 |
              | 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0 |
              | 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0 |
              | 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0 |
              |  .    .    .    .    .    .    .    .  |
              |  .    .    .    .    .    .    .    .  |
              └                                         ┘
```

***Example 3:*** This example shows the same array being used for input and output. The arrays are declared as follows:

```
REAL*8      Y(0:17,0:7)
COMPLEX*16  X(0:8,0:7)
REAL*8      AUX1(1000), AUX2(1000), AUX3(1)
```

Arrays X and Y are made equivalent by the following statement, making them occupy the same storage.

```
EQUIVALENCE (X,Y)
```

This requires *inc2y* = 2(*inc2x*). First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation.

*Call Statement and Input*

```
             INIT  X INC2X Y  INC2Y N1  N2 ISIGN SCALE   AUX1   NAUX1  AUX2   NAUX2  AUX3  NAUX3
              |    |   |   |    |    |   |   |     |       |      |      |      |      |      |
CALL DCRFT2(INIT, X , 9 , Y , 18 , 16 , 8 , 1 , SCALE , AUX1 , 1000 , AUX2 , 1000 , AUX3 ,  0 )
```

```
INIT     =  1(for initialization)
INIT     =  0(for computation)
SCALE    =  1.0
```

X is an array with 9 rows and 8 columns having (1.0, 1.0) in location X(4,2) and (0.0, 0.0) in all others.

*Output*

$$Y = \begin{bmatrix}
2.0 & 2.0 & -2.0 & -2.0 & 2.0 & 2.0 & -2.0 & -2.0 \\
2.0 & -2.0 & -2.0 & 2.0 & 2.0 & -2.0 & -2.0 & 2.0 \\
-2.0 & -2.0 & 2.0 & 2.0 & -2.0 & -2.0 & 2.0 & 2.0 \\
-2.0 & 2.0 & 2.0 & -2.0 & -2.0 & 2.0 & 2.0 & -2.0 \\
2.0 & 2.0 & -2.0 & -2.0 & 2.0 & 2.0 & -2.0 & -2.0 \\
2.0 & -2.0 & -2.0 & 2.0 & 2.0 & -2.0 & -2.0 & 2.0 \\
-2.0 & -2.0 & 2.0 & 2.0 & -2.0 & -2.0 & 2.0 & 2.0 \\
-2.0 & 2.0 & 2.0 & -2.0 & -2.0 & 2.0 & 2.0 & -2.0 \\
2.0 & 2.0 & -2.0 & -2.0 & 2.0 & 2.0 & -2.0 & -2.0 \\
2.0 & -2.0 & -2.0 & 2.0 & 2.0 & -2.0 & -2.0 & 2.0 \\
-2.0 & -2.0 & 2.0 & 2.0 & -2.0 & -2.0 & 2.0 & 2.0 \\
-2.0 & 2.0 & 2.0 & -2.0 & -2.0 & 2.0 & 2.0 & -2.0 \\
2.0 & 2.0 & -2.0 & -2.0 & 2.0 & 2.0 & -2.0 & -2.0 \\
2.0 & -2.0 & -2.0 & 2.0 & 2.0 & -2.0 & -2.0 & 2.0 \\
-2.0 & -2.0 & 2.0 & 2.0 & -2.0 & -2.0 & 2.0 & 2.0 \\
-2.0 & 2.0 & 2.0 & -2.0 & -2.0 & 2.0 & 2.0 & -2.0 \\
. & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & .
\end{bmatrix}$$

# SCFT3 and DCFT3—Complex Fourier Transform in Three Dimensions

These subroutines compute the three-dimensional discrete Fourier transform of complex data.

*Table 134. Data Types*

| X, Y | *scale* | **Subroutine** |
|---|---|---|
| Short-precision complex | Short-precision real | SCFT3 |
| Long-precision complex | Long-precision real | DCFT3 |

**Note:** For each use, only one invocation of this subroutine is necessary. The initialization phase, preparing the working storage, is a relatively small part of the total computation, so it is performed on each invocation.

***Syntax***

| | |
|---|---|
| **Fortran** | CALL SCFT3 \| DCFT3 (*x*, *inc2x*, *inc3x*, *y*, *inc2y*, *inc3y*, *n1*, *n2*, *n3*, *isign*, *scale*, *aux*, *naux*) |
| **C and C++** | scft3 \| dcft3 (*x*, *inc2x*, *inc3x*, *y*, *inc2y*, *inc3y*, *n1*, *n2*, *n3*, *isign*, *scale*, *aux*, *naux*); |
| **PL/I** | CALL SCFT3 \| DCFT3 (*x*, *inc2x*, *inc3x*, *y*, *inc2y*, *inc3y*, *n1*, *n2*, *n3*, *isign*, *scale*, *aux*, *naux*); |

*On Entry*

*x*

is the array X, containing the three-dimensional data to be transformed, where each element $x_{j1,j2,j3}$, using zero-based indexing, is stored in X(*j1*+*j2*(*inc2x*)+*j3*(*inc3x*)) for *j1* = 0, 1, ..., *n1*–1, *j2* = 0, 1, ..., *n2*–1, and *j3* = 0, 1, ..., *n3*–1. The strides for the elements in the first, second, and third dimensions are assumed to be 1, *inc2x*( ≥ *n1*), and *inc3x*( ≥ (*n2*)(*inc2x*)), respectively.

Specified as: an array, containing numbers of the data type indicated in Table 134. This array must be aligned on a doubleword boundary. If the array is dimensioned X(LDA1,LDA2,LDA3), then LDA1 = *inc2x*, (LDA1)(LDA2) = *inc3x*, and LDA3 ≥ *n3*. For information on how to set up this array, see "Setting Up Your Data" on page 744. For more details, see "Notes" on page 811.

*inc2x*

is the stride between the elements in array X for the second dimension. Specified as: a fullword integer; *inc2x* ≥ *n1*.

*inc3x*

is the stride between the elements in array X for the third dimension. Specified as: a fullword integer; *inc3x* ≥ (*n2*)(*inc2x*).

*y*

See "On Return" on page 810.

*inc2y*

is the stride between the elements in array Y for the second dimension. Specified as: a fullword integer; *inc2y* ≥ *n1*.

*inc3y*

is the stride between the elements in array Y for the third dimension. Specified as: a fullword integer; *inc3y* ≥ (*n2*)(*inc2y*).

*n1*

is the length of the first dimension of the three-dimensional data in the array to be transformed. Specified as: a fullword integer; *n1* ≤ 37748736 and must be one of the values listed in "Acceptable Lengths for the Transforms" on page 741. For all other values specified less than 37748736, you have the option of having the next larger acceptable value returned in this argument. For details, see “Providing a Correct Transform Length to ESSL” on page 38.

*n2*

is the length of the second dimension of the three-dimensional data in the array to be transformed. Specified as: a fullword integer; *n2* ≤ 37748736 and must be one of the values listed in “Acceptable Lengths for the Transforms” on page 741. For all other values specified less than 37748736, you have the option of having the next larger acceptable value returned in this argument. For details, see “Providing a Correct Transform Length to ESSL” on page 38.

*n3*

is the length of the third dimension of the three-dimensional data in the array to be transformed. Specified as: a fullword integer; *n3* ≤ 37748736 and must be one of the values listed in “Acceptable Lengths for the Transforms” on page 741. For all other values specified less than 37748736, you have the option of having the next larger acceptable value returned in this argument. For details, see “Providing a Correct Transform Length to ESSL” on page 38.

*isign*

controls the direction of the transform, determining the sign *Isign* of the exponents of $W_{n1}$, $W_{n2}$, and $W_{n3}$, where:

If *isign* = positive value, *Isign* = + (transforming time to frequency).

If *isign* = negative value, *Isign* = − (transforming frequency to time).

Specified as: a fullword integer; *isign* > 0 or *isign* < 0.

*scale*

is the scaling constant *scale*. See “Function” on page 812 for its usage. Specified as: a number of the data type indicated in Table 134 on page 809, where *scale* > 0.0 or *scale* < 0.0.

*aux*

has the following meaning:

If *naux* = 0 and error 2015 is unrecoverable, *aux* is ignored.

Otherwise, it is a storage work area used by this subroutine.

Specified as: an area of storage, containing *naux* long-precision real numbers. On output, the contents are overwritten.

*naux*

is the number of doublewords in the working storage specified in *aux*. Specified as: a fullword integer, where:

If *naux* = 0 and error 2015 is unrecoverable, SCFT3 and DCFT3 dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, *naux* ≥ (minimum value required for successful processing). To determine a sufficient value, use the processor-independent formulas. For all other values specified less than the minimum value, you have the option of having the minimum value returned in this argument. For details, see “Using Auxiliary Storage in ESSL” on page 32.

*On Return*

*y*

is the array Y, containing the elements resulting from the three-dimensional discrete Fourier transform of the data in X. Each element $y_{k1,k2,k3}$, using zero-based indexing, is stored in Y(*k1*+*k2*(*inc2y*)+*k3*(*inc3y*)) for *k1* = 0, 1, ..., *n1*–1, *k2* = 0, 1, ..., *n2*–1, and *k3* = 0, 1, ..., *n3*–1. The strides for the elements in the first, second, and third dimensions are assumed to be 1, *inc2y*( ≥ *n1*), and *inc3y*( ≥ (*n2*)(*inc2y*)), respectively.

Returned as: an array, containing numbers of the data type indicated in Table 134 on page 809. This array must be aligned on a doubleword boundary. If the array is dimensioned `Y(LDA1,LDA2,LDA3)`, then `LDA1` = *inc2y*, (`LDA1`)(`LDA2`) = *inc3y*, and `LDA3` ≥ *n3*. For information on how to set up this array, see "Setting Up Your Data" on page 744. For more details, see "Notes."

***Notes***

1. If you specify the same array for `X` and `Y`, then *inc2x* must be greater than or equal to *inc2y*, and *inc3x* must be greater than or equal to *inc3y*. In this case, output overwrites input. When using the ESSL SMP library in a multithreaded environment, if *inc2x* > *inc2y* or *inc3x* > *inc3y*, these subroutines run on a single thread and issue an attention message.

   If you specify different arrays `X` and `Y`, they must have no common elements; otherwise, results are unpredictable. See "Concepts" on page 55.

2. You should use "STRIDE—Determine the Stride Value for Optimal Performance in Specified Fourier Transform Subroutines" on page 971 to determine the optimal values for the strides *inc2y* and *inc3y* for your output array. The strides for your input array do not affect performance. Example 7 in the STRIDE subroutine description explains how it is used for these subroutines. For additional information on how to set up your data, see "Setting Up Your Data" on page 744.

***Processor-Independent Formulas for SCFT3 for NAUX:*** Use the following formulas for calculating *naux*:

1. If max(*n2*, *n3*) < 252 and:

   If *n1* ≤ 8192, use *naux* = 60000.
   If *n1* > 8192, use *naux* = 60000+2.28*n1*.

2. If *n2* ≥ 252, *n3* < 252, and:

   If *n1* ≤ 8192, use *naux* = 60000+λ.
   If *n1* > 8192, use *naux* = 60000+2.28*n1*+λ,
     where λ = (*n2*+256)(*s*+2.28)
     and *s* = min(64, *n1*).

3. If *n2* < 252, *n3* ≥ 252, and:

   If *n1* ≤ 8192, use *naux* = 60000+ψ.
   If *n1* > 8192, use *naux* = 60000+2.28*n1*+ψ,
     where ψ = (*n3*+256)(*s*+2.28)
     and *s* = min(64, (*n1*)(*n2*)).

4. If *n2* ≥ 252 and *n3* ≥ 252, use the larger of the values calculated for cases 2 and 3 above.

***Processor-Independent Formulas for DCFT3 for NAUX:*** Use the following formulas for calculating *naux*:

1. If max(*n2*, *n3*) < 252 and:

   If *n1* ≤ 2048, use *naux* = 60000.
   If *n1* > 2048, use *naux* = 60000+4.56*n1*.

2. If *n2* ≥ 252, *n3* < 252, and:

   If *n1* ≤ 2048, use *naux* = 60000+λ.
   If *n1* > 2048, use *naux* = 60000+4.56*n1*+λ,
   where λ = ((2)*n2*+256)(*s*+4.56)
   and *s* = min(64, *n1*).

3. If *n2* < 252, *n3* ≥ 252, and:

   If *n1* ≤ 2048, use *naux* = 60000+ψ.
   If *n1* > 2048, use *naux* = 60000+4.56*n1*+ψ,
   where ψ = ((2)*n3*+256)(*s*+4.56)
   and *s* = min(64, (*n1*)(*n2*)).

4. If *n2* ≥ 252 and *n3* ≥ 252, use the larger of the values calculated for cases 2 and 3 above.

***Function:*** The three-dimensional discrete Fourier transform of complex data in array X, with results going into array Y, is expressed as follows:

$$y_{k1,k2,k3} = scale \sum_{j1=0}^{n1-1} \sum_{j2=0}^{n2-1} \sum_{j3=0}^{n3-1} x_{j1,j2,j3}\ W_{n1}^{(Isign)j1k1}\ W_{n2}^{(Isign)j2k2}\ W_{n3}^{(Isign)j3k3}$$

for:

*k1* = 0, 1, ..., *n1*–1
*k2* = 0, 1, ..., *n2*–1
*k3* = 0, 1, ..., *n3*–1

where:

$$W_{n1} = e^{-2\pi\left(\sqrt{-1}\right)/n1}$$
$$W_{n2} = e^{-2\pi\left(\sqrt{-1}\right)/n2}$$
$$W_{n3} = e^{-2\pi\left(\sqrt{-1}\right)/n3}$$

and where:

$x_{j1,j2,j3}$ are elements of array X.
$y_{k1,k2,k3}$ are elements of array Y.
*Isign* is + or – (determined by argument *isign*).
*scale* is a scalar value.

For *scale* = 1.0 and *isign* being positive, you obtain the discrete Fourier transform, a function of frequency. The inverse Fourier transform is obtained with *scale* = 1.0/((*n1*)(*n2*)(*n3*)) and *isign* being negative. See references [1 ], [4 ], [5 ], [19 ], and [20].

***Error Conditions***

*Resource Errors:* Error 2015 is unrecoverable, *naux* = 0, and unable to allocate work area.

*Computational Errors:* None

*Input-Argument Errors*

1. *n1* > 37748736
2. *n2* > 37748736
3. *n3* > 37748736
4. *inc2x* < *n1*
5. *inc3x* < (*n2*)(*inc2x*)
6. *inc2y* < *n1*
7. *inc3y* < (*n2*)(*inc2y*)
8. *scale* = 0.0
9. *isign* = 0
10. The length of one of the transforms in *n1*, *n2*, or *n3* is not an allowable value. Return code 1 is returned if error 2030 is recoverable.
11. Error 2015 is recoverable or *naux*≠0, and *naux* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example:*** This example shows how to compute a three-dimensional transform. In this example, INC2X ≥ INC2Y and INC3X ≥ INC3Y, so that the same array can be used for both input and output. The STRIDE subroutine is called to select good values for the INC2Y and INC3Y strides. (As explained below, STRIDE is not called for INC2X and INC3X.) Using the transform lengths (N1 = 32, N2 = 64, and N3 = 40) along with the output data type (short-precision complex: 'C'), STRIDE is called once for each stride needed. First, it is called for INC2Y:

```
CALL STRIDE (N2,N1,INC2Y,'C',0)
```

The output value returned for INC2Y is 32. Then STRIDE is called again for INC3Y:

```
CALL STRIDE (N3,N2*INC2Y,INC3Y,'C',0)
```

The output value returned for INC3Y is 2056. Because INC3Y is not a multiple of INC2Y, Y is not declared as a three-dimensional array. It is declared as a two-dimensional array, Y(INC3Y,N3).

To equivalence the X and Y arrays requires INC2X ≥ INC2Y and INC3X ≥ INC3Y. Therefore, INC2X is set equal to INC2Y( = 32). Also, to declare the X array as a three-dimensional array, INC3X must be a multiple of INC2X. Therefore, its value is set as INC3X = (65)(INC2X) = 2080.

The arrays are declared as follows:

```
COMPLEX*8  X(32,65,40),Y(2056,40)
REAL*8     AUX(30000)
```

Arrays X and Y are made equivalent by the following statement, making them occupy the same storage:

```
EQUIVALENCE (X,Y)
```

*Call Statement and Input*

```
            X INC2X INC3X   Y INC2Y INC3Y   N1   N2   N3 ISIGN SCALE   AUX    NAUX
            |   |     |     |   |     |     |    |    |    |     |      |      |
CALL SCFT3( X , 32 , 2080 , Y , 32 , 2056 , 32 , 64 , 40 , 1 , SCALE , AUX , 30000)
```

```
SCALE    =  1.0
```

X has (1.0,2.0) in location X(1,1,1) and (0.0,0.0) in all other locations.

*Output:* Y has (1.0,2.0) in locations Y(*ij*,*k*), where *ij* = 1, 2048 and *j* = 1, 40. It remains unchanged elsewhere.

# SRCFT3 and DRCFT3—Real-to-Complex Fourier Transform in Three Dimensions

These subroutines compute the three-dimensional discrete Fourier transform of real data in a three-dimensional array.

*Table 135. Data Types*

| X, *scale* | Y | **Subroutine** |
|---|---|---|
| Short-precision real | Short-precision complex | SRCFT3 |
| Long-precision real | Long-precision complex | DRCFT3 |

**Note:** For each use, only one invocation of this subroutine is necessary. The initialization phase, preparing the working storage, is a relatively small part of the total computation, so it is performed on each invocation.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SRCFT3 \| DRCFT3 (*x*, *inc2x*, *inc3x*, *y*, *inc2y*, *inc3y*, *n1*, *n2*, *n3*, *isign*, *scale*, *aux*, *naux*) |
| **C and C++** | srcft3 \| drcft3 (*x*, *inc2x*, *inc3x*, *y*, *inc2y*, *inc3y*, *n1*, *n2*, *n3*, *isign*, *scale*, *aux*, *naux*); |
| **PL/I** | CALL SRCFT3 \| DRCFT3 (*x*, *inc2x*, *inc3x*, *y*, *inc2y*, *inc3y*, *n1*, *n2*, *n3*, *isign*, *scale*, *aux*, *naux*); |

*On Entry*

*x*

is the array X, containing the three-dimensional data to be transformed, where each element $x_{j1,j2,j3}$, using zero-based indexing, is stored in X(*j1*+*j2*(*inc2x*)+*j3*(*inc3x*)) for *j1* = 0, 1, ..., *n1*–1, *j2* = 0, 1, ..., *n2*–1, and *j3* = 0, 1, ..., *n3*–1. The strides for the elements in the first, second, and third dimensions are assumed to be 1, *inc2x*( ≥ *n1*), and *inc3x*( ≥ (*n2*)(*inc2x*)), respectively.

Specified as: an array, containing numbers of the data type indicated in Table 135. If the array is dimensioned X(LDA1,LDA2,LDA3), then LDA1 = *inc2x*, (LDA1)(LDA2) = *inc3x*, and LDA3 ≥ *n3*. For information on how to set up this array, see “Setting Up Your Data” on page 744. For more details, see “Notes” on page 817.

*inc2x*

is the stride between the elements in array X for the second dimension. Specified as: a fullword integer; *inc2x* ≥ *n1*.

*inc3x*

is the stride between the elements in array X for the third dimension. Specified as: a fullword integer; *inc3x* ≥ (*n2*)(*inc2x*).

*y*

See “On Return” on page 816.

*inc2y*

is the stride between the elements in array Y for the second dimension. Specified as: a fullword integer; *inc2y* ≥ *n1*/2+1.

*inc3y*

is the stride between the elements in array Y for the third dimension. Specified as: a fullword integer; *inc3y* ≥ (*n2*)(*inc2y*).

*n1*

is the length of the first dimension of the three-dimensional data in the array to be transformed. Specified as: a fullword integer; *n1* ≤ 37748736 and must be one of the values listed in “Acceptable Lengths for the Transforms” on page 741. For all other values specified less than 37748736, you have the option of having the next larger acceptable value returned in this argument. For details, see “Providing a Correct Transform Length to ESSL” on page 38.

*n2*

is the length of the second dimension of the three-dimensional data in the array to be transformed. Specified as: a fullword integer; *n2* ≤ 37748736 and must be one of the values listed in “Acceptable Lengths for the Transforms” on page 741. For all other values specified less than 37748736, you have the option of having the next larger acceptable value returned in this argument. For details, see “Providing a Correct Transform Length to ESSL” on page 38.

*n3*

is the length of the third dimension of the three-dimensional data in the array to be transformed. Specified as: a fullword integer; *n3* ≤ 37748736 and must be one of the values listed in “Acceptable Lengths for the Transforms” on page 741. For all other values specified less than 37748736, you have the option of having the next larger acceptable value returned in this argument. For details, see “Providing a Correct Transform Length to ESSL” on page 38.

*isign*

controls the direction of the transform, determining the sign *Isign* of the exponents of $W_{n1}$, $W_{n2}$, and $W_{n3}$, where:

If *isign* = positive value, *Isign* = + (transforming time to frequency).

If *isign* = negative value, *Isign* = − (transforming frequency to time).

Specified as: a fullword integer; *isign* > 0 or *isign* < 0.

*scale*

is the scaling constant *scale*. See “Function” on page 818 for its usage. Specified as: a number of the data type indicated in Table 135 on page 815, where *scale* > 0.0 or *scale* < 0.0.

*aux*

has the following meaning:

If *naux* = 0 and error 2015 is unrecoverable, *aux* is ignored.

Otherwise, it is a storage work area used by this subroutine.

Specified as: an area of storage, containing *naux* long-precision real numbers. On output, the contents are overwritten.

*naux*

is the number of doublewords in the working storage specified in *aux*. Specified as: a fullword integer, where:

If *naux* = 0 and error 2015 is unrecoverable, SRCFT3 and DRCFT3 dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, *naux* ≥ (minimum value required for successful processing). To determine a sufficient value, use the processor-independent formulas. For all other values specified less than the minimum value, you have the option of having the minimum value returned in this argument. For details, see “Using Auxiliary Storage in ESSL” on page 32.

*On Return*

*y*

is the array Y, containing the elements resulting from the three-dimensional discrete Fourier transform of the data in X. Each element $y_{k1,k2,k3}$, using zero-based indexing, is stored in Y(*k1*+*k2*(*inc2y*)+*k3*(*inc3y*)) for *k1* = 0, 1, ..., *n1*/2, *k2* = 0, 1, ..., *n2*−1, and *k3* = 0, 1, ..., *n3*−1. Due to complex conjugate symmetry, the output consists of only the first *n1*/2+1 values along the first dimension of the array, for *k1* = 0, 1, ..., *n1*/2. The strides for the elements in the first, second, and third dimensions are assumed to be 1, *inc2y*( ≥ *n1*/2+1), and *inc3y*( ≥ (*n2*)(*inc2y*)), respectively.

Returned as: an array, containing numbers of the data type indicated in Table 135 on page 815. This array must be aligned on a doubleword boundary. If the array is dimensioned Y(LDA1,LDA2,LDA3), then LDA1 = *inc2y*, (LDA1)(LDA2) = *inc3y*, and LDA3 ≥ *n3*. For information on how to set up this array, see "Setting Up Your Data" on page 744. For more details, see "Notes."

### *Notes*

1. If you specify the same array for X and Y, then *inc2x* must be greater than or equal to (2)(*inc2y*), and *inc3x* must be greater than or equal to (2)(*inc3y*). In this case, output overwrites input. When using the ESSL SMP library in a multithreaded environment, if *inc2x* > (2)(*inc2y*) or *inc3x* > (2)(*inc3y*), these subroutines run on a single thread and issue an attention message.

   If you specify different arrays X and Y, they must have no common elements; otherwise, results are unpredictable. See "Concepts" on page 55.

2. To achieve the best performance, you should align array X on a doubleword boundary, and *inc2x* and *inc3x* should be even numbers. The strides for your input array do not affect performance as long as they are even numbers. In addition, you should use "STRIDE—Determine the Stride Value for Optimal Performance in Specified Fourier Transform Subroutines" on page 971 to determine the optimal values for the strides *inc2y* and *inc3y* for your output array. Example 8 in the STRIDE subroutine description explains how it is used for these subroutines. For additional information on how to set up your data, see "Setting Up Your Data" on page 744.

***Processor-Independent Formulas for SRCFT3 for NAUX:*** Use the following formulas for calculating *naux*:

1. If max(*n2*, *n3*) < 252 and:

   If *n1* ≤ 16384, use *naux* = 65000.
   If *n1* > 16384, use *naux* = 60000+1.39*n1*.

2. If *n2* ≥ 252, *n3* < 252, and:

   If *n1* ≤ 16384, use *naux* = 65000+λ.
   If *n1* > 16384, use *naux* = 60000+1.39*n1*+λ,
   where λ = (*n2*+256)(*s*+2.28) and *s* = min(64, 1+*n1*/2).

3. If *n2* < 252, *n3* ≥ 252, and:

   If *n1* ≤ 16384, use *naux* = 65000+ψ.
   If *n1* > 16384, use *naux* = 60000+1.39*n1*+ψ,
   where ψ = (*n3*+256)(*s*+2.28) and *s* = min(64, (*n2*)(1+*n1*/2)).

4. If *n2* ≥ 252 and *n3* ≥ 252, use the larger of the values calculated for cases 2 and 3 above.

If *inc2x* or *inc3x* is an odd number, or if array X is not aligned on a doubleword boundary, you should add the following amount to all the formulas given above:

*n2*(1+*n1*/2)

***Processor-Independent Formulas for DRCFT3 for NAUX:*** Use the following formulas for calculating *naux*:

1. If max(*n2*, *n3*) < 252 and:

   If *n1* ≤ 4096, use *naux* = 62000.
   If *n1* > 4096, use *naux* = 60000+2.78*n1*.

2. If *n2* ≥ 252, *n3* < 252, and:

   If *n1* ≤ 4096, use *naux* = 62000+λ.
   If *n1* > 4096, use *naux* = 60000+2.78*n1*+λ,
   where λ = ((2)*n2*+256)(*s*+4.56)
   and *s* = min(64, *n1*/2).

3. If *n2* < 252, *n3* ≥ 252, and:

   If *n1* ≤ 4096, use *naux* = 62000+ψ.
   If *n1* > 4096, use *naux* = 60000+2.78*n1*+ψ,
   where ψ = ((2)*n3*+256)(*s*+4.56)
   and *s* = min(64, *n2*(1+*n1*/2)).

4. If *n2* ≥ 252 and *n3* ≥ 252, use the larger of the values calculated for cases 2 and 3 above.

***Function:*** The three-dimensional complex conjugate even discrete Fourier transform of real data in array X, with results going into array Y, is expressed as follows:

$$y_{k1,k2,k3} = scale \sum_{j1=0}^{n1-1} \sum_{j2=0}^{n2-1} \sum_{j3=0}^{n3-1} x_{j1,j2,j3} \; W_{n1}^{(Isign)j1k1} \; W_{n2}^{(Isign)j2k2} \; W_{n3}^{(Isign)j3k3}$$

for:

$k1$ = 0, 1, ..., $n1$–1
$k2$ = 0, 1, ..., $n2$–1
$k3$ = 0, 1, ..., $n3$–1

where:

$$W_{n1} = e^{-2\pi\left(\sqrt{-1}\right)/n1}$$
$$W_{n2} = e^{-2\pi\left(\sqrt{-1}\right)/n2}$$
$$W_{n3} = e^{-2\pi\left(\sqrt{-1}\right)/n3}$$

and where:

$x_{j1,j2,j3}$ are elements of array X.
$y_{k1,k2,k3}$ are elements of array Y.
*Isign* is + or − (determined by argument *isign*).

*scale* is a scalar value.

The output in array Y is complex. For *scale* = 1.0 and *isign* being positive, you obtain the discrete Fourier transform, a function of frequency. The inverse Fourier transform is obtained with *scale* = 1.0/((*n1*)(*n2*)(*n3*)) and *isign* being negative. See references [1 ], [4 ], [5 ], [19 ], and [20].

### Error Conditions

*Resource Errors:* Error 2015 is unrecoverable, *naux* = 0, and unable to allocate work area.

*Computational Errors:* None

*Input-Argument Errors*

1. *n1* > 37748736
2. *n2* > 37748736
3. *n3* > 37748736
4. *inc2x* < *n1*
5. *inc3x* < (*n2*)(*inc2x*)
6. *inc2y* < *n1*/2+1
7. *inc3y* < (*n2*)(*inc2y*)
8. *scale* = 0.0
9. *isign* = 0
10. The length of one of the transforms in *n1*, *n2*, or *n3* is not an allowable value. Return code 1 is returned if error 2030 is recoverable.
11. Error 2015 is recoverable or *naux*≠0, and *naux* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example:*** This example shows how to compute a three-dimensional transform. In this example, INC2X ≥ (2)(INC2Y) and INC3X ≥ (2)(INC3Y), so that the same array can be used for both input and output. The STRIDE subroutine is called to select good values for the INC2Y and INC3Y strides. Using the transform lengths (N1 = 32, N2 = 64, and N2 = 40) along with the output data type (short-precision complex: 'C'), STRIDE is called once for each stride needed. First, it is called for INC2Y:

```
CALL STRIDE (N2,N1/2+1,INC2Y,'C',0)
```

The output value returned for INC2Y is 17. (This value is equal to N1/2+1.) Then STRIDE is called again for INC3Y:

```
CALL STRIDE (N3,N2*INC2Y,INC3Y,'C',0)
```

The output value returned for INC3Y is 1088. Because INC3Y is a multiple of INC2Y—that is, INC3Y = (N2)(INC2Y)—Y is declared as a three-dimensional array, Y(17,64,40). (In general, for larger arrays, these types of values for INC2Y and INC3Y are not returned by STRIDE, and you are probably not able to declare Y as a three-dimensional array.)

To equivalence the X and Y arrays requires INC2X ≥ (2)(INC2Y) and INC3X ≥ (2)(INC3Y). Therefore, the values INC2X = (2)(INC2Y) = 34 and INC3X = (2)(INC3Y) = 2176 are set, and X is declared as a three-dimensional array, X(34,64,40).

The arrays are declared as follows:

```
REAL*4     X(34,64,40)
COMPLEX*8  Y(17,64,40)
REAL*8     AUX(32000)
```

Arrays X and Y are made equivalent by the following statement, making them occupy the same storage:

```
EQUIVALENCE (X,Y)
```

*Call Statement and Input*

```
                X  INC2X INC3X  Y  INC2Y INC3Y  N1   N2   N3 ISIGN SCALE   AUX    NAUX
                |    |     |    |    |     |     |    |    |   |     |      |      |
CALL SRCFT3( X , 34 , 2176 , Y , 17 , 1088 , 32 , 64 , 40 , 1 , SCALE , AUX , 32000)
```

```
SCALE    =  1.0
```

X has 1.0 in location X(1,1,1) and 0.0 in all other locations.

*Output:* Y has (1.0,0.0) in all locations.

## SCRFT3 and DCRFT3—Complex-to-Real Fourier Transform in Three Dimensions

These subroutines compute the three-dimensional discrete Fourier transform of complex conjugate even data in a three-dimensional array.

*Table 136. Data Types*

| X | Y, *scale* | **Subroutine** |
|---|---|---|
| Short-precision complex | Short-precision real | SCRFT2 |
| Long-precision complex | Long-precision real | DCRFT2 |

**Note:** For each use, only one invocation of this subroutine is necessary. The initialization phase, preparing the working storage, is a relatively small part of the total computation, so it is performed on each invocation.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SCRFT3 \| DCRFT3 (*x*, *inc2x*, *inc3x*, *y*, *inc2y*, *inc3y*, *n1*, *n2*, *n3*, *isign*, *scale*, *aux*, *naux*) |
| **C and C++** | scrft3 \| dcrft3 (*x*, *inc2x*, *inc3x*, *y*, *inc2y*, *inc3y*, *n1*, *n2*, *n3*, *isign*, *scale*, *aux*, *naux*); |
| **PL/I** | CALL SCRFT3 \| DCRFT3 (*x*, *inc2x*, *inc3x*, *y*, *inc2y*, *inc3y*, *n1*, *n2*, *n3*, *isign*, *scale*, *aux*, *naux*); |

*On Entry*

*x*

is the array X, containing the three-dimensional data to be transformed, where each element $x_{j1,j2,j3}$, using zero-based indexing, is stored in X(*j1*+*j2*(*inc2x*)+*j3*(*inc3x*)) for *j1* = 0, 1, ..., *n1*/2, *j2* = 0, 1, ..., *n2*–1, and *j3* = 0, 1, ..., *n3*–1. Due to complex conjugate symmetry, the input consists of only the first *n1*/2+1 values along the first dimension of the array, for *j1* = 0, 1, ..., *n1*/2. The strides for the elements in the first, second, and third dimensions are assumed to be 1, *inc2x*( ≥ *n1*/2+1), and *inc3x*( ≥ (*n2*)(*inc2x*)), respectively.

Specified as: an array, containing numbers of the data type indicated in Table 136. This array must be aligned on a doubleword boundary. If the array is dimensioned X(LDA1,LDA2,LDA3), then LDA1 = *inc2x*, (LDA1)(LDA2) = *inc3x*, and LDA3 ≥ *n3*. For information on how to set up this array, see “Setting Up Your Data” on page 744. For more details, see “Notes” on page 823.

*inc2x*

is the stride between the elements in array X for the second dimension. Specified as: a fullword integer; *inc2x* ≥ *n1*/2+1.

*inc3x*

is the stride between the elements in array X for the third dimension. Specified as: a fullword integer; *inc3x* ≥ (*n2*)(*inc2x*).

*y*

See “On Return” on page 822.

*inc2y*

is the stride between the elements in array Y for the second dimension. Specified as: a fullword integer; *inc2y* ≥ *n1*+2.

*inc3y*

is the stride between the elements in array Y for the third dimension. Specified as: a fullword integer; *inc3y* ≥ (*n2*)(*inc2y*).

*n1*

is the length of the first dimension of the three-dimensional data in the array to be transformed. Specified as: a fullword integer; *n1* ≤ 37748736 and must be one of the values listed in “Acceptable Lengths for the Transforms” on page 741. For all other values specified less than 37748736, you have the option of having the next larger acceptable value returned in this argument. For details, see “Providing a Correct Transform Length to ESSL” on page 38.

*n2*

is the length of the second dimension of the three-dimensional data in the array to be transformed. Specified as: a fullword integer; *n2* ≤ 37748736 and must be one of the values listed in “Acceptable Lengths for the Transforms” on page 741. For all other values specified less than 37748736, you have the option of having the next larger acceptable value returned in this argument. For details, see “Providing a Correct Transform Length to ESSL” on page 38.

*n3*

is the length of the third dimension of the three-dimensional data in the array to be transformed. Specified as: a fullword integer; *n3* ≤ 37748736 and must be one of the values listed in “Acceptable Lengths for the Transforms” on page 741. For all other values specified less than 37748736, you have the option of having the next larger acceptable value returned in this argument. For details, see “Providing a Correct Transform Length to ESSL” on page 38.

*isign*

controls the direction of the transform, determining the sign *Isign* of the exponents of $W_{n1}$, $W_{n2}$, and $W_{n3}$, where:

If *isign* = positive value, *Isign* = + (transforming time to frequency).

If *isign* = negative value, *Isign* = − (transforming frequency to time).

Specified as: a fullword integer; *isign* > 0 or *isign* < 0.

*scale*

is the scaling constant *scale*. See “Function” on page 824 for its usage. Specified as: a number of the data type indicated in Table 136 on page 821, where *scale* > 0.0 or *scale* < 0.0.

*aux*

has the following meaning:

If *naux* = 0 and error 2015 is unrecoverable, *aux* is ignored.

Otherwise, it is a storage work area used by this subroutine. Specified as: an area of storage, containing *naux* long-precision real numbers. On output, the contents are overwritten.

*naux*

is the number of doublewords in the working storage specified in *aux*. Specified as: a fullword integer, where:

If *naux* = 0 and error 2015 is unrecoverable, SCRFT3 and DCRFT3 dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, *naux* ≥ (minimum value required for successful processing). To determine a sufficient value, use the processor-independent formulas. For all other values specified less than the minimum value, you have the option of having the minimum value returned in this argument. For details, see “Using Auxiliary Storage in ESSL” on page 32.

*On Return*

*y*

is the array Y, containing the elements resulting from the three-dimensional discrete Fourier transform of the data in X. Each element $y_{k1,k2,k3}$, using zero-based indexing, is stored in Y(*k1*+*k2*(*inc2y*)+*k3*(*inc3y*)) for *k1* = 0, 1, ..., *n1*−1, *k2* = 0, 1, ..., *n2*−1, and *k3* = 0, 1, ..., *n3*−1. The strides for the elements in the first, second, and third dimensions are assumed to be 1, *inc2y*( ≥ *n1*+2), and *inc3y*( ≥ (*n2*)(*inc2y*)), respectively.

Returned as: an array, containing numbers of the data type indicated in Table 136 on page 821. If the array is dimensioned Y(LDA1,LDA2,LDA3), then LDA1 = *inc2y*, (LDA1)(LDA2) = *inc3y*, and LDA3 ≥ *n3*. For information on how to set up this array, see "Setting Up Your Data" on page 744. For more details, see "Notes."

***Notes***

1. If you specify the same array for X and Y, then *inc2y* must equal (2)(*inc2x*) and *inc3y* must equal (2)(*inc3x*). In this case, output overwrites input. If you specify different arrays X and Y, they must have no common elements; otherwise, results are unpredictable. See "Concepts" on page 55.
2. To achieve the best performance, you should align array Y on a doubleword boundary, and *inc2y* and *inc3y* should be even numbers. In addition, you should use "STRIDE—Determine the Stride Value for Optimal Performance in Specified Fourier Transform Subroutines" on page 971 to determine the optimal values for the strides *inc2y* and *inc3y* for your output array. To obtain the best performance, you should use *inc2x* = *inc2y*/2 and *inc3x* = *inc3y*/2. Example 9 in the STRIDE subroutine description explains how it is used for these subroutines. For additional information on how to set up your data, see "Setting Up Your Data" on page 744.

***Processor-Independent Formulas for SCRFT3 for Calculating NAUX:*** Use the following formulas for calculating *naux*:

1. If max(*n2*, *n3*) < 252 and:

   If *n1* ≤ 16384, use *naux* = 65000.
   If *n1* > 16384, use *naux* = 60000+1.39*n1*.

2. If *n2* ≥ 252, *n3* < 252, and:

   If *n1* ≤ 16384, use *naux* = 65000+λ.
   If *n1* > 16384, use *naux* = 60000+1.39*n1*+λ,
   where λ = (*n2*+256)(*s*+2.28)
   and *s* = min(64, 1+*n1*/2).

3. If *n2* < 252, *n3* ≥ 252, and:

   If *n1* ≤ 16384, use *naux* = 65000+ψ.
   If *n1* > 16384, use *naux* = 60000+1.39*n1*+ψ,
   where ψ = (*n3*+256)(*s*+2.28)
   and *s* = min(64, (*n2*)(1+*n1*/2)).

4. If *n2* ≥ 252 and *n3* ≥ 252, use the larger of the values calculated for cases 2 and 3 above.

If *inc2y* or *inc3y* is an odd number, or if array Y is not aligned on a doubleword boundary, you should add the following amount to all the formulas given above:

(1+*n1*/2)(max(*n2*, *n3*))

***Processor-Independent Formulas for DCRFT3 for NAUX:*** Use the following formulas for calculating *naux*:

1. If max(*n2*, *n3*) < 252 and:

   If *n1* ≤ 4096, use *naux* = 62000.
   If *n1* > 4096, use *naux* = 60000+2.78*n1*.

2. If *n2* ≥ 252, *n3* < 252, and:

   If *n1* ≤ 4096, use *naux* = 62000+λ.
   If *n1* > 4096, use *naux* = 60000+2.78*n1*+λ,
   where λ = ((2)*n2*+256)(*s*+4.56)
   and *s* = min(64, *n1*/2).

3. If *n2* < 252, *n3* ≥ 252, and:

   If *n1* ≤ 4096, use *naux* = 62000+ψ.
   If *n1* > 4096, use *naux* = 60000+2.78*n1*+ψ,
   where ψ = ((2)*n3*+256)(*s*+4.56)
   and *s* = min(64, *n2*(1+*n1*/2)).

4. If *n2* ≥ 252 and *n3* ≥ 252, use the larger of the values calculated for cases 2 and 3 above.

***Function:*** The three-dimensional discrete Fourier transform of complex conjugate even data in array X, with results going into array Y, is expressed as follows:

$$y_{k1,k2,k3} = scale \sum_{j1=0}^{n1-1} \sum_{j2=0}^{n2-1} \sum_{j3=0}^{n3-1} x_{j1,j2,j3} \; W_{n1}^{(Isign)j1k1} \; W_{n2}^{(Isign)j2k2} \; W_{n3}^{(Isign)j3k3}$$

for:

*k1* = 0, 1, ..., *n1*–1
*k2* = 0, 1, ..., *n2*–1
*k3* = 0, 1, ..., *n3*–1

where:

$$W_{n1} = e^{-2\pi\left(\sqrt{-1}\right)/n1}$$
$$W_{n2} = e^{-2\pi\left(\sqrt{-1}\right)/n2}$$
$$W_{n3} = e^{-2\pi\left(\sqrt{-1}\right)/n3}$$

and where:

$x_{j1,j2,j3}$ are elements of array X.
$y_{k1,k2,k3}$ are elements of array Y.
*Isign* is + or – (determined by argument *isign*).
*scale* is a scalar value.

Because of the complex conjugate symmetry, the output in array Y is real. For *scale* = 1.0 and *isign* being positive, you obtain the discrete Fourier transform, a function of frequency. The inverse Fourier transform is obtained with *scale* = 1.0/((*n1*)(*n2*)(*n3*)) and *isign* being negative. See references [1 ], [4 ], [5 ], [19 ], and [20].

### *Error Conditions*

*Resource Errors:* Error 2015 is unrecoverable, *naux* = 0, and unable to allocate work area.

*Computational Errors:* None

*Input-Argument Errors*

1. *n1* > 37748736
2. *n2* > 37748736
3. *n3* > 37748736
4. *inc2x* < *n1*/2+1
5. *inc3x* < (*n2*)(*inc2x*)
6. *inc2y* < *n1*+2
7. *inc3y* < (*n2*)(*inc2y*)
8. *scale* = 0.0
9. *isign* = 0
10. The length of one of the transforms in *n1*, *n2*, or *n3* is not an allowable value. Return code 1 is returned if error 2030 is recoverable.
11. Error 2015 is recoverable or *naux*≠0, and *naux* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example:*** This example shows how to compute a three-dimensional transform. In this example, `INC2Y` = (2)(`INC2X`) and `INC3Y` = (2)(`INC3X`), so that the same array can be used for both input and output. The STRIDE subroutine is called to select good values for the `INC2Y` and `INC3Y` strides. (As explained below, STRIDE is not called for `INC2X` and `INC3X`.) Using the transform lengths (`N1` = 32, `N2` = 64, and `N3` = 40) along with the output data type (short-precision real: `'S'`), STRIDE is called once for each stride needed. First, it is called for `INC2Y`:

```
CALL STRIDE (N2,N1+2,INC2Y,'S',0)
```

The output value returned for `INC2Y` is 34. (This value is equal to `N1`+2.) Then STRIDE is called again for `INC3Y`:

```
CALL STRIDE (N3,N2*INC2Y,INC3Y,'S',0)
```

The output value returned for `INC3Y` is 2176. Because `INC3Y` is a multiple of `INC2Y`—that is, `INC3Y` = (`N2`)(`INC2Y`)—`Y` is declared as a three-dimensional array, `Y(34,64,40)`. (In general, for larger arrays, these types of values for `INC2Y` and `INC3Y` are not returned by STRIDE, and you are probably not able to declare `Y` as a three-dimensional array.)

A good stride value for `INC2X` is `INC2Y`/2, and a good stride value for `INC3X` is `INC3Y`/2. Also, to equivalence the `X` and `Y` arrays requires `INC2Y` = (2)(`INC2X`) and `INC3Y` = (2)(`INC3X`). Therefore, the values `INC2X` = `INC2Y`/2 = 17 and `INC3X` = `INC3Y`/2 = 1088 are set, and `X` is declared as a three-dimensional array, `X(17,64,40)`.

The arrays are declared as follows:

```
COMPLEX*8  X(17,64,40)
REAL*4     Y(34,64,40)
REAL*8     AUX(32000)
```

Arrays X and Y are made equivalent by the following statement, making them occupy the same storage:

```
EQUIVALENCE (X,Y)
```

*Call Statement and Input*

```
                X  INC2X INC3X  Y  INC2Y INC3Y  N1   N2   N3 ISIGN SCALE   AUX    NAUX
                |    |     |    |    |     |     |    |    |   |     |      |       |
CALL SCRFT3( X , 17 , 1088 , Y , 34 , 2176 , 32 , 64 , 40 , 1 , SCALE , AUX , 32000)
```

```
SCALE    =  1.0
```

X has (1.0,0.0) in location X(1,1,1) and (0.0,0.0) in all other locations.

*Output:* Y has 1.0 in all locations.

## Convolution and Correlation Subroutines

This section contains the convolution and correlation subroutine descriptions.

# SCON and SCOR—Convolution or Correlation of One Sequence with One or More Sequences

These subroutines compute the convolutions and correlations of a sequence with one or more sequences using a direct method. The input and output sequences contain short-precision real numbers.

**Note:** These subroutines are considered obsolete. They are provided in ESSL only for compatibility with earlier releases. You should use SCOND, SCORD, SDCON, SDCOR, SCONF, and SCORF instead, because they provide **better performance**. For further details, see "Convolution and Correlation Considerations" on page 745.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SCON \| SCOR (*init*, *h*, *inc1h*, *x*, *inc1x*, *inc2x*, *y*, *inc1y*, *inc2y*, *nh*, *nx*, *m*, *iy0*, *ny*, *aux1*, *naux1*, *aux2*, *naux2*) |
| **C and C++** | scon \| scor (*init*, *h*, *inc1h*, *x*, *inc1x*, *inc2x*, *y*, *inc1y*, *inc2y*, *nh*, *nx*, *m*, *iy0*, *ny*, *aux1*, *naux1*, *aux2*, *naux2*); |
| **PL/I** | CALL SCON \| SCOR (*init*, *h*, *inc1h*, *x*, *inc1x*, *inc2x*, *y*, *inc1y*, *inc2y*, *nh*, *nx*, *m*, *iy0*, *ny*, *aux1*, *naux1*, *aux2*, *naux2*); |

*On Entry*

*init*

is a flag, where:

If *init* ≠ 0, no computation is performed, error checking is performed, and the subroutine exits back to the calling program.

If *init* = 0, the convolutions or correlations of the sequence in *h* with the sequences in *x* are computed.

Specified as: a fullword integer. It can have any value.

*h*

is the array H, consisting of the sequence of length $N_h$ to be convolved or correlated with the sequences in array X. Specified as: an array of (at least) length $1+(N_h-1)|inc1h|$, containing short-precision real numbers.

*inc1h*

is the stride between the elements within the sequence in array H. Specified as: a fullword integer; *inc1h* > 0.

*x*

is the array X, consisting of *m* input sequences of length $N_x$, each to be convolved or correlated with the sequence in array H. Specified as: an array of (at least) length $1 + (m-1)inc2x + (N_x-1)inc1x$, containing short-precision real numbers.

*inc1x*

is the stride between the elements within each sequence in array X. Specified as: a fullword integer; *inc1x* > 0.

*inc2x*

is the stride between the first elements of the sequences in array X. Specified as: a fullword integer; *inc2x* > 0.

*y*

See "On Return" on page 829.

*inc1y*
: is the stride between the elements within each sequence in output array Y. Specified as: a fullword integer; *inc1y* > 0.

*inc2y*
: is the stride between the first elements of each sequence in output array Y. Specified as: a fullword integer; *inc2y* > 0.

*nh*
: is the number of elements, $N_h$, in the sequence in array H. Specified as: a fullword integer; $N_h > 0$.

*nx*
: is the number of elements, $N_x$, in each sequence in array X. Specified as: a fullword integer; $N_x > 0$.

*m*
: is the number of sequences in array X to be convolved or correlated. Specified as: a fullword integer; $m > 0$.

*iy0*
: is the convolution or correlation index of the element to be stored in the first position of each sequence in array Y. Specified as: a fullword integer. It can have any value.

*ny*
: is the number of elements, $N_y$, in each sequence in array Y. Specified as: a fullword integer; $N_y > 0$ for SCON and $N_y \geq -N_h+1$ for SCOR.

*aux1*
: is no longer used in the computation, but must still be specified as a dummy argument (for migration purposes from Version 1 of ESSL). It can have any value.

*naux1*
: is no longer used in the computation, but must still be specified as a dummy argument (for migration purposes from Version 1 of ESSL). It can have any value.

*aux2*
: is no longer used in the computation, but must still be specified as a dummy argument (for migration purposes from Version 1 of ESSL). It can have any value.

*naux2*
: is no longer used in the computation, but must still be specified as a dummy argument (for migration purposes from Version 1 of ESSL). It can have any value.

*On Return*

*y*
: is array Y, consisting of *m* output sequences of length $N_y$ that are the result of the convolutions or correlations of the sequence in array H with the sequences in array X. Returned as: an array of (at least) length $1 + (m-1)inc2y + (N_y-1)inc1y$, containing short-precision real numbers.

**Notes**

1. Output should not overwrite input; that is, input arrays X and H must have no common elements with output array Y. Otherwise, results are unpredictable. See “Concepts” on page 55.

2. When using the ESSL SMP library, for optimal performance, the number of threads specified should be the same for $init \neq 0$ and $init = 0$.

3. Auxiliary storage is not needed, but the arguments *aux1*, *naux1*, *aux2*, and *naux2* must still be specified. You can assign any values to these arguments.

***Function:*** The convolutions and correlations of a sequence in array `H` with one or more sequences in array `X` are expressed as follows:

Convolutions for SCON:

$$y_{ki} = \sum_{j=\max(0,k-N_x+1)}^{\min(N_h-1,k)} h_j x_{k-j,i}$$

Correlations for SCOR:

$$y_{ki} = \sum_{j=\max(0,-k\ )}^{\min(N_h-1,\ N_x-1-k)} h_j x_{k+j,i}$$

for:

$k$ = *iy0*, *iy0*+1, ..., *iy0*+$N_y$–1
$i$ = 1, 2, ..., $m$

where:

$y_{ki}$ are elements of the $m$ sequences of length $N_y$ in array `Y`.
$x_{ki}$ are elements of the $m$ sequences of length $N_x$ in array `X`.
$h_j$ are elements of the sequence of length $N_h$ in array `H`.
*iy0* is the convolution or correlation index of the element to be stored in the first position of each sequence in array `Y`.
min and max select the minimum and maximum values, respectively.

It is assumed that elements outside the range of definition are zero. See references [17] and [81 ].

Only one invocation of this subroutine is needed:

1. You do not need to invoke the subroutine with *init* ≠ 0. If you do, however, the subroutine performs error checking, exits back to the calling program, and no computation is performed.

2. With *init* = 0, the subroutine performs the calculation of the convolutions or correlations.

### *Error Conditions*

*Computational Errors:* None

*Input-Argument Errors*

1. *nh*, *nx*, *ny*, or *m* ≤ 0
2. *inc1h*, *inc1x*, *inc2x*, *inc1y*, or *inc2y* ≤ 0

***Example 1:*** This example shows how to compute a convolution of a sequence in H, which is a ramp function, and three sequences in X, a triangular function and its cyclic translates. It computes the full range of nonzero values of the convolution plus two extra points, which are set to 0. The arrays are declared as follows:

```
REAL*4   H(0:4999), X(0:49999), Y(0:49999)
REAL*8   AUX1, AUX2
```

*Call Statement and Input*

```
          INIT  H INC1H X INC1X INC2X Y INC1Y INC2Y NH NX  M IY0 NY  AUX1 NAUX1 AUX2 NAUX2
           |    |   |   |   |     |   |   |     |   |  |  |  |   |   |     |     |    |
CALL SCON(INIT, H , 1 , X , 1 ,  10 , Y , 1 ,  15 , 4, 10, 3, 0, 15, AUX1 , 0 , AUX2 , 0)
```

```
INIT    =  0(for computation)
H       =  (1.0, 2.0, 3.0, 4.0)
```

X contains the following three sequences:

```
1.0  2.0  3.0
2.0  1.0  2.0
3.0  2.0  1.0
4.0  3.0  2.0
5.0  4.0  3.0
6.0  5.0  4.0
5.0  6.0  5.0
4.0  5.0  6.0
3.0  4.0  5.0
2.0  3.0  4.0
```

*Output:* Y contains the following three sequences:

```
 1.0   2.0   3.0
 4.0   5.0   8.0
10.0  10.0  14.0
20.0  18.0  22.0
30.0  20.0  18.0
40.0  30.0  20.0
48.0  40.0  30.0
52.0  48.0  40.0
50.0  52.0  48.0
40.0  50.0  52.0
29.0  38.0  47.0
18.0  25.0  32.0
 8.0  12.0  16.0
 0.0   0.0   0.0
 0.0   0.0   0.0
```

***Example 2:*** This example shows how the output from Example 1 differs when the values for NY and *inc2y* are 10 rather than 15. The output is the same except that it consists of only the first 10 values produced in Example 1.

*Output:* Y contains the following three sequences:

```
 1.0   2.0   3.0
 4.0   5.0   8.0
10.0  10.0  14.0
20.0  18.0  22.0
30.0  20.0  18.0
40.0  30.0  20.0
48.0  40.0  30.0
52.0  48.0  40.0
50.0  52.0  48.0
40.0  50.0  52.0
```

***Example 3:*** This example shows how the output from Example 2 differs if the value for IY0 is 3 rather than 0. The output is the same except it starts at element 3 of the convolution sequences rather than element 0.

*Output:* Y contains the following three sequences:

```
20.0  18.0  22.0
30.0  20.0  18.0
40.0  30.0  20.0
48.0  40.0  30.0
52.0  48.0  40.0
50.0  52.0  48.0
40.0  50.0  52.0
29.0  38.0  47.0
18.0  25.0  32.0
 8.0  12.0  16.0
```

***Example 4:*** This example shows how to compute a correlation of a sequence in H, which is a ramp function, and three sequences in X, a triangular function and its cyclic translates. It computes the full range of nonzero values of the correlation plus two extra points, which are set to 0. The arrays are declared as follows:

```
     REAL*4   H(0:4999), X(0:49999), Y(0:49999)
     REAL*8   AUX1, AUX2
```

*Call Statement and Input*

```
          INIT  H INC1H X  INC1X INC2X Y  INC1Y INC2Y NH NX  M IY0  NY  AUX1  NAUX1 AUX2 NAUX2
           |    |   |   |   |      |   |   |      |   |  |   |  |   |   |      |     |     |
CALL SCOR(INIT, H , 1 , X , 1  , 10 , Y , 1  , 15 , 4, 10, 3, -3, 15, AUX1 ,  0 , AUX2 , 0)
```

INIT     =  0(for computation)
H        =  (1.0, 2.0, 3.0, 4.0)

X contains the following three sequences:

```
1.0  2.0  3.0
2.0  1.0  2.0
3.0  2.0  1.0
4.0  3.0  2.0
5.0  4.0  3.0
6.0  5.0  4.0
5.0  6.0  5.0
4.0  5.0  6.0
3.0  4.0  5.0
2.0  3.0  4.0
```

*Output:* Y contains the following three sequences:

```
 4.0   8.0  12.0
11.0  10.0  17.0
20.0  15.0  16.0
30.0  22.0  18.0
40.0  30.0  22.0
50.0  40.0  30.0
52.0  50.0  40.0
48.0  52.0  50.0
40.0  48.0  52.0
30.0  40.0  48.0
16.0  22.0  28.0
 7.0  10.0  13.0
 2.0   3.0   4.0
 0.0   0.0   0.0
 0.0   0.0   0.0
```

***Example 5:*** This example shows how the output from Example 4 differs when the values for NY and INC2Y are 10 rather than 15. The output is the same except that it consists of only the first 10 values produced in Example 4.

*Output:* Y contains the following three sequences:

```
 4.0   8.0  12.0
11.0  10.0  17.0
20.0  15.0  16.0
30.0  22.0  18.0
40.0  30.0  22.0
50.0  40.0  30.0
52.0  50.0  40.0
48.0  52.0  50.0
40.0  48.0  52.0
30.0  40.0  48.0
```

***Example 6:*** This example shows how the output from Example 5 differs if the value for IY0 is 0 rather than –3. The output is the same except it starts at element 0 of the correlation sequences rather than element –3.

*Output:* Y contains the following three sequences:

```
30.0  22.0  18.0
40.0  30.0  22.0
50.0  40.0  30.0
52.0  50.0  40.0
48.0  52.0  50.0
40.0  48.0  52.0
30.0  40.0  48.0
16.0  22.0  28.0
 7.0  10.0  13.0
 2.0   3.0   4.0
```

# SCOND and SCORD—Convolution or Correlation of One Sequence with Another Sequence Using a Direct Method

These subroutines compute the convolution and correlation of a sequence with another sequence using a direct method. The input and output sequences contain short-precision real numbers.

**Notes:**

1. These subroutines compute the convolution and correlation using direct methods. In most cases, these subroutines provide **better performance** than using SCON or SCOR, if you determine that SCON or SCOR would have used a direct method for its computation. For information on how to make this determination, see reference [4].
2. For long-precision data, you should use DDCON or DDCOR with the decimation rate, *id*, equal to 1.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SCOND \| SCORD (*h*, *inch*, *x*, *incx*, *y*, *incy*, *nh*, *nx*, *iy0*, *ny*) |
| **C and C++** | scond \| scord (*h*, *inch*, *x*, *incx*, *y*, *incy*, *nh*, *nx*, *iy0*, *ny*); |
| **PL/I** | CALL SCOND \| SCORD (*h*, *inch*, *x*, *incx*, *y*, *incy*, *nh*, *nx*, *iy0*, *ny*); |

*On Entry*

*h*
is the array H, consisting of the sequence of length $N_h$ to be convolved or correlated with the sequence in array X. Specified as: an array of (at least) length $1+(N_h-1)|inch|$, containing short-precision real numbers.

*inch*
is the stride between the elements within the sequence in array H. Specified as: a fullword integer; $inch > 0$ or $inch < 0$.

*x*
is the array X, consisting of the input sequence of length $N_x$, to be convolved or correlated with the sequence in array H. Specified as: an array of (at least) length $1+(N_x-1)|incx|$, containing short-precision real numbers.

*incx*
is the stride between the elements within the sequence in array X. Specified as: a fullword integer; $incx > 0$ or $incx < 0$.

*y*
See “On Return” on page 835.

*incy*
is the stride between the elements within the sequence in output array Y. Specified as: a fullword integer; $incy > 0$ or $incy < 0$.

*nh*
is the number of elements, $N_h$, in the sequence in array H. Specified as: a fullword integer; $N_h > 0$.

*nx*
is the number of elements, $N_x$, in the sequence in array X. Specified as: a fullword integer; $N_x > 0$.

*iy0*
is the convolution or correlation index of the element to be stored in the first position of the sequence in array `Y`. Specified as: a fullword integer. It can have any value.

*ny*
is the number of elements, $N_y$, in the sequence in array `Y`. Specified as: a fullword integer; $N_y > 0$.

*On Return*

*y*
is the array `Y` of length $N_y$, consisting of the output sequence that is the result of the convolution or correlation of the sequence in array `H` with the sequence in array `X`. Returned as: an array of (at least) length 1+($N_y$–1)|*incy*|, containing short-precision real numbers.

***Notes***

1. Output should not overwrite input—that is, input arrays `X` and `H` must have no common elements with output array `Y`. Otherwise, results are unpredictable. See “Concepts” on page 55.
2. If *iy0* and *ny* are such that output outside the basic range is needed, where the basic range is $0 \le k \le (nh+nx-2)$ for SCOND and $(-nh+1) \le k \le (nx-1)$ for SCORD, the subroutine stores zeros using scalar code. It is not efficient to store many zeros in this manner. It is more efficient to set *iy0* and *ny* so that the output is produced within the above range of *k* values.

***Function:*** The convolution and correlation of a sequence in array `H` with a sequence in array `X` are expressed as follows:

Convolution for SCOND:

$$y_k = \sum_{j=\max(0,k-N_x+1)}^{\min(N_h-1,k)} h_j x_{k-j}$$

Correlation for SCORD:

$$y_k = \sum_{j=\max(0,-k)}^{\min(N_h-1,\ N_x-1-k)} h_j x_{k+j}$$

for $k = iy0, iy0+1, ..., iy0+N_y-1$

where:

$y_k$ are elements of the sequence of length $N_y$ in array `Y`.
$x_k$ are elements of the sequence of length $N_x$ in array `X`.
$h_j$ are elements of the sequence of length $N_h$ in array `H`.
*iy0* is the convolution or correlation index of the element to be stored in the first position of each sequence in array `Y`.
min and max select the minimum and maximum values, respectively.

It is assumed that elements outside the range of definition are zero. See reference [4].

***Special Usage:*** SCORD can also perform the functions of SCON and SACOR; that is, it can compute convolutions and autocorrelations. To compute a convolution, you must specify a negative stride for H (see Example 9). To compute the autocorrelation, you must specify the two input sequences to be the same (see Example 10). In fact, you can also compute the autoconvolution by using both of these techniques together, letting the two input sequences be the same, and specifying a negative stride for the first input sequence.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. *nh*, *nx*, or *ny* ≤ 0
2. *inch*, *incx*, or *incy* = 0

***Example 1:*** This example shows how to compute a convolution of a sequence in H with a sequence in X, where both sequences are ramp functions.

*Call Statement and Input*

```
           H  INCH  X  INCX  Y  INCY  NH  NX IY0  NY
           |  |     |  |     |  |     |   |  |    |
CALL SCOND( H , 1  , X , 1  , Y , 1  , 4 , 8 , 0 , 11 )

H        =  (1.0, 2.0, 3.0, 4.0)
X        =  (11.0, 12.0, 13.0, 14.0, 15.0, 16.0, 17.0, 18.0)
```

*Output*

```
Y        =  (11.0, 34.0, 70.0, 120.0, 130.0, 140.0, 150.0, 160.0,
             151.0, 122.0, 72.0)
```

***Example 2:*** This example shows how the output from Example 1 differs when the value for IY0 is –2 rather than 0, and NY is 15 rather than 11. The output has two zeros at the beginning and end of the sequence, for points outside the range of nonzero output.

*Call Statement and Input*

```
           H  INCH  X  INCX  Y  INCY  NH  NX  IY0  NY
           |  |     |  |     |  |     |   |   |    |
CALL SCOND( H , 1  , X , 1  , Y , 1  , 4 , 8 , -2 , 15 )

H        =  (1.0, 2.0, 3.0, 4.0)
X        =  (11.0, 12.0, 13.0, 14.0, 15.0, 16.0, 17.0, 18.0)
```

*Output*

```
Y        =  (0.0, 0.0, 11.0, 34.0, 70.0, 120.0, 130.0, 140.0, 150.0,
             160.0, 151.0, 122.0, 72.0, 0.0, 0.0)
```

***Example 3:*** This example shows how the same output as Example 1 can be obtained when H and X are interchanged, because the convolution is symmetric in H and X. (The arguments are switched in the calling sequence.)

*Call Statement and Input*

```
            H  INCH  X  INCX  Y  INCY  NH  NX IY0  NY
            |  |     |   |    |   |    |   |   |   |
CALL SCOND( X , 1  , H , 1  , Y , 1  , 4 , 8 , 0 , 11 )

H        =  (1.0, 2.0, 3.0, 4.0)
X        =  (11.0, 12.0, 13.0, 14.0, 15.0, 16.0, 17.0, 18.0)
```

*Output*

```
Y        =  (11.0, 34.0, 70.0, 120.0, 130.0, 140.0, 150.0, 160.0,
             151.0, 122.0, 72.0)
```

***Example 4:*** This example shows how the output from Example 1 differs when a negative stride is specified for the sequence in H. By reversing the H sequence, the correlation is computed.

*Call Statement and Input*

```
            H  INCH  X  INCX  Y  INCY  NH  NX IY0  NY
            |    |   |   |    |   |    |   |   |   |
CALL SCOND( H , -1 , X , 1  , Y , 1  , 4 , 8 , 0 , 11 )

H        =  (1.0, 2.0, 3.0, 4.0)
X        =  (11.0, 12.0, 13.0, 14.0, 15.0, 16.0, 17.0, 18.0)
```

*Output*

```
Y        =  (44.0, 81.0, 110.0, 130.0, 140.0, 150.0, 160.0, 170.0,
             104.0, 53.0, 18.0)
```

***Example 5:*** This example shows how to compute the autoconvolution of a sequence by letting the two input sequences for H and X be the same. (X is specified for both arguments in the calling sequence.)

*Call Statement and Input*

```
            H  INCH  X  INCX  Y  INCY NH  NX IY0  NY
            |   |   |   |    |   |    |   |   |   |
CALL SCOND( X , 1 , X , 1  , Y , 1  , 4 , 4 , 0 , 7 )

X        =  (11.0, 12.0, 13.0, 14.0)
```

*Output*

```
Y        =  (121.0, 264.0, 430.0, 620.0, 505.0, 364.0, 196.0)
```

***Example 6:*** This example shows how to compute a correlation of a sequence in H with a sequence in X, where both sequences are ramp functions.

*Call Statement and Input*

```
            H  INCH  X  INCX  Y  INCY NH  NX   IY0  NY
            |   |    |   |    |   |    |   |    |   |
CALL SCORD( H , 1  , X , 1  , Y , 1  , 4 , 8 , -3 , 11 )

H        =  (1.0, 2.0, 3.0, 4.0)
X        =  (11.0, 12.0, 13.0, 14.0, 15.0, 16.0, 17.0, 18.0)
```

*Output*

```
Y        =  (44.0, 81.0, 110.0, 130.0, 140.0, 150.0, 160.0, 170.0,
             104.0, 53.0, 18.0)
```

***Example 7:*** This example shows how the output from Example 6 differs when the value for `IY0` is –5 rather than –3 and `NY` is 15 rather than 11. The output has two zeros at the beginning and end of the sequence, for points outside the range of nonzero output.

*Call Statement and Input*

```
            H  INCH  X  INCX  Y  INCY NH  NX   IY0  NY
            |  |     |  |     |  |    |   |    |    |
CALL SCORD( H , 1  , X , 1  , Y , 1  , 4 , 8 , -5 , 15 )

H        =  (1.0, 2.0, 3.0, 4.0)
X        =  (11.0, 12.0, 13.0, 14.0, 15.0, 16.0, 17.0, 18.0)
```

*Output*

```
Y        =  (0.0, 0.0, 44.0, 81.0, 110.0, 130.0, 140.0, 150.0, 160.0,
             170.0, 104.0, 53.0, 18.0, 0.0, 0.0)
```

***Example 8:*** This example shows how the output from Example 6 differs when `H` and `X` are interchanged (in the calling sequence). The output sequence is the reverse of that in Example 6. To get the full range of output, `IY0` is set to –`NX`+1.

*Call Statement and Input*

```
            H  INCH  X  INCX  Y  INCY NH  NX   IY0  NY
            |  |     |  |     |  |    |   |    |    |
CALL SCORD( X , 1  , H , 1  , Y , 1  , 4 , 8 , -7 , 11 )

H        =  (1.0, 2.0, 3.0, 4.0)
X        =  (11.0, 12.0, 13.0, 14.0, 15.0, 16.0, 17.0, 18.0)
```

*Output*

```
Y        =  (18.0, 53.0, 104.0, 170.0, 160.0, 150.0, 140.0, 130.0,
             110.0, 81.0, 44.0)
```

***Example 9:*** This example shows how the output from Example 6 differs when a negative stride is specified for the sequence in `H`. By reversing the `H` sequence, the convolution is computed.

*Call Statement and Input*

```
            H  INCH  X  INCX  Y  INCY  NH  NX  IY0  NY
            |    |   |  |     |  |     |   |   |    |
CALL SCORD( H , -1 , X , 1  , Y , 1  , 4 , 8 , -3 , 11 )

H        =  (1.0, 2.0, 3.0, 4.0)
X        =  (11.0, 12.0, 13.0, 14.0, 15.0, 16.0, 17.0, 18.0)
```

*Output*

```
Y        =  (11.0, 34.0, 70.0, 120.0, 130.0, 140.0, 150.0, 160.0,
             151.0, 122.0, 72.0)
```

***Example 10:*** This example shows how to compute the autocorrelation of a sequence by letting the two input sequences for H and X be the same. (X is specified for both arguments in the calling sequence.)

*Call Statement and Input*

```
            H  INCH  X  INCX  Y  INCY  NH  NX  IY0  NY
            |  |     |  |     |  |     |   |   |    |
CALL SCORD( X , 1  , X , 1  , Y , 1  , 4 , 4 , -3 , 7 )

X        =  (11.0, 12.0, 13.0, 14.0)
```

*Output*

```
Y        =  (154.0, 311.0, 470.0, 630.0, 470.0, 311.0, 154.0)
```

## SCONF and SCORF—Convolution or Correlation of One Sequence with One or More Sequences Using the Mixed-Radix Fourier Method

These subroutines compute the convolutions and correlations, respectively, of a sequence with one or more sequences using the mixed-radix Fourier method. The input and output sequences contain short-precision real numbers.

**Note:** Two invocations of these subroutines are necessary: one to prepare the working storage for the subroutine, and the other to perform the computations.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SCONF \| SCORF (*init*, *h*, *inc1h*, *x*, *inc1x*, *inc2x*, *y*, *inc1y*, *inc2y*, *nh*, *nx*, *m*, *iy0*, *ny*, *aux1*, *naux1*, *aux2*, *naux2*) |
| **C and C++** | sconf \| scorf (*init*, *h*, *inc1h*, *x*, *inc1x*, *inc2x*, *y*, *inc1y*, *inc2y*, *nh*, *nx*, *m*, *iy0*, *ny*, *aux1*, *naux1*, *aux2*, *naux2*); |
| **PL/I** | CALL SCONF \| SCORF (*init*, *h*, *inc1h*, *x*, *inc1x*, *inc2x*, *y*, *inc1y*, *inc2y*, *nh*, *nx*, *m*, *iy0*, *ny*, *aux1*, *naux1*, *aux2*, *naux2*); |

*On Entry*

*init*

is a flag, where:

If *init* ≠ 0, trigonometric functions, the transform of the sequence in *h*, and other parameters, depending on arguments other than *x*, are computed and saved in *aux1*. The contents of *x* and *y* are not used or changed.

If *init* = 0, the convolutions or correlations of the sequence that was in *h* at initialization with the sequences in *x* are computed. *h* is not used or changed. The only arguments that may change after initialization are *x*, *y*, and *aux2*. All scalar arguments must be the same as when the subroutine was called for initialization with *init* ≠ 0.

Specified as: a fullword integer. It can have any value.

*h*

is the array H, consisting of the sequence of length $N_h$ to be convolved or correlated with the sequences in array X. Specified as: an array of (at least) length $1+(N_h-1)|inc1h|$, containing short-precision real numbers.

*inc1h*

is the stride between the elements within the sequence in array H. Specified as: a fullword integer; *inc1h* > 0.

*x*

is the array X, consisting of *m* input sequences of length $N_x$, each to be convolved or correlated with the sequence in array H. Specified as: an array of (at least) length $1+(N_x-1)inc1x+(m-1)inc2x$, containing short-precision real numbers.

*inc1x*

is the stride between the elements within each sequence in array X. Specified as: a fullword integer; *inc1x* > 0.

*inc2x*

is the stride between the first elements of the sequences in array X. Specified as: a fullword integer; *inc2x* > 0.

*y*

See "On Return" on page 842.

*inc1y*
is the stride between the elements within each sequence in output array Y. Specified as: a fullword integer; *inc1y* > 0.

*inc2y*
is the stride between the first elements of each sequence in output array Y. Specified as: a fullword integer; *inc2y* > 0.

*nh*
is the number of elements, $N_h$, in the sequence in array H. Specified as: a fullword integer; $N_h > 0$.

*nx*
is the number of elements, $N_x$, in each sequence in array X. Specified as: a fullword integer; $N_x > 0$.

*m*
is the number of sequences in array X to be convolved or correlated. Specified as: a fullword integer; *m* > 0.

*iy0*
is the convolution or correlation index of the element to be stored in the first position of each sequence in array Y. Specified as: a fullword integer. It can have any value.

*ny*
is the number of elements, $N_y$, in each sequence in array Y. Specified as: a fullword integer; $N_y > 0$.

*aux1*
is the working storage for this subroutine, where:

If *init* ≠ 0, the working storage is computed.

If *init* = 0, the working storage is used in the computation of the convolutions.

Specified as: an area of storage, containing *naux1* long-precision real numbers.

*naux1*
is the number of doublewords in the working storage specified in *aux1*. Specified as: a fullword integer; *naux1* > 23 and *naux1* ≥ (minimum value required for successful processing). To determine a sufficient value, use the processor-independent formulas. For values between 23 and the minimum value, you have the option of having the minimum value returned in this argument. For details, see “Using Auxiliary Storage in ESSL” on page 32.

*aux2*
has the following meaning:

If *naux2* = 0 and error 2015 is unrecoverable, *aux2* is ignored.

Otherwise, it is the working storage used by this subroutine, which is available for use by the calling program between calls to this subroutine.

Specified as: an area of storage, containing *naux2* long-precision real numbers. On output, the contents are overwritten.

*naux2*
is the number of doublewords in the working storage specified in *aux2*. Specified as: a fullword integer, where:

If *naux2* = 0 and error 2015 is unrecoverable, SCONF and SCORF dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, *naux2* ≥ (minimum value required for successful processing). To determine a sufficient value, use the processor-independent formulas. For all other values specified less than the minimum value, you have the option of having the minimum value returned in this argument. For details, see “Using Auxiliary Storage in ESSL” on page 32.

*On Return*

*y*

has the following meaning, where:

If *init* ≠ 0, this argument is not used, and its contents remain unchanged.

If *init* = 0, this is array Y, consisting of *m* output sequences of length $N_y$ that are the result of the convolutions or correlations of the sequence in array H with the sequences in array X.

Returned as: an array of (at least) length $1+(N_y-1)inc1y+(m-1)inc2y$, containing short-precision real numbers.

*aux1*

is the working storage for this subroutine, where:

If *init* ≠ 0, it contains information ready to be passed in a subsequent invocation of this subroutine.

If *init* = 0, its contents are unchanged.

Returned as: the contents are not relevant.

***Notes***

1. *aux1* should **not** be used by the calling program between calls to this subroutine with *init* ≠ 0 and *init* = 0. However, it can be reused after intervening calls to this subroutine with different arguments.
2. When using the ESSL SMP library, for optimal performance, the number of threads specified should be the same for *init* ≠ 0 and *init* = 0.
3. If you specify the same array for X and Y, then *inc1x* and *inc1y* must be equal, and *inc2x* and *inc2y* must be equal. In this case, output overwrites input.
4. If you specify different arrays for X and Y, they must have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.
5. If *iy0* and *ny* are such that output outside the basic range is needed, the subroutine stores zeros. These ranges are: $0 \le k \le N_x+N_h-2$ for SCONF and $1-N_h \le k \le N_x-1$ for SCORF.

***Formulas for the Length of the Fourier Transform:*** Before calculating the necessary sizes of *naux1* and *naux2*, you must determine the length *n* of the Fourier transform. The value of *n* is based on *nf*. You can use one of two techniques to determine *nf*:

- Use the simple overestimate of *nf* = *nx*+*nh*−1. (If *iy0* = 0 and *ny* > *nh*+*nx*, this is the actual value, not an overestimate.)
- Use the values of the arguments *iy0*, *nh*, *nx*, and *ny* inserted into the following formulas to get a value for the variable *nf*:

  $iy0p = \max(iy0, 0)$
  $ix0 = \max((iy0p+1)-nh, 0)$
  $ih0 = \max((iy0p+1)-nx, 0)$
  $nd = ix0+ih0$
  $n1 = iy0+ny$
  $nxx = \min(n1, nx)-ix0$

*nhh* = min(*n1*, *nh*)−*ih0*
*ntt* = *nxx*+*nhh*−1
*nn1* = *n1*−*nd*
*iyy0* = *iy0p*−*nd*
*nzleft* = max(0, *nhh*−*iyy0*−1)
*nzrt* = min(*nn1*, *ntt*)−*nxx*
*nf* = max(12, *nxx*+max(*nzleft*, *nzrt*))

After calculating the value for *nf*, using one of these two techniques, refer to the formula or table of allowable values of *n* in "Acceptable Lengths for the Transforms" on page 741, selecting the value equal to or greater than *nf*.

***Processor-Independent Formulas for NAUX1 and NAUX2:*** The required values of *naux1* and *naux2* depend on the value determined for *n* in "Formulas for the Length of the Fourier Transform" on page 842.

*NAUX1 Formulas*

If $n \le 16384$, use *naux1* = 58000.
If $n > 16384$, use *naux1* = 40000+2.14*n*.

*NAUX2 Formulas*

If $n \le 16384$, use *naux2* = 30000.
If $n > 16384$, use *naux2* = 20000+1.07*n*.

***Function:*** The convolutions and correlations of a sequence in array H with one or more sequences in array X are expressed as follows.

Convolutions for SCONF:

$$y_{ki} = \sum_{j=\max(0,k-N_x+1)}^{\min(N_h-1,k)} h_j x_{k-j,i}$$

Correlations for SCORF:

$$y_{ki} = \sum_{j=\max(0,-k\ )}^{\min(N_h-1,\ N_x-1-k)} h_j x_{k+j,i}$$

for:

$k = iy0, iy0+1, ..., iy0+N_y-1$
$i = 1, 2, ..., m$

where:

$y_{ki}$ are elements of the *m* sequences of length $N_y$ in array Y.
$x_{ki}$ are elements of the *m* sequences of length $N_x$ in array X.
$h_j$ are elements of the sequence of length $N_h$ in array H.
*iy0* is the convolution or correlation index of the element to be stored in the first position of each sequence in array Y.
min and max select the minimum and maximum values, respectively.

These subroutines use a Fourier transform method with a mixed-radix capability. This provides maximum performance for your application. The length of the transform, *n*, that you must calculate to determine the correct sizes for *naux1* and *naux2* is the same length used by the Fourier transform subroutines called by this subroutine. It is assumed that elements outside the range of definition are zero. See references [17] and [81 ].

Two invocations of this subroutine are necessary:

1. With *init* ≠ 0, the subroutine tests and initializes arguments of the program, setting up the *aux1* working storage.
2. With *init* = 0, the subroutine checks that the initialization arguments in the *aux1* working storage correspond to the present arguments, and if so, performs the calculation of the convolutions.

***Error Conditions***

*Resource Errors:* Error 2015 is unrecoverable, *naux2* = 0, and unable to allocate work area.

*Computational Errors:* None

*Input-Argument Errors*

1. *nh*, *nx*, *ny*, or *m* ≤ 0
2. *inc1h*, *inc1x*, *inc2x*, *inc1y*, or *inc2y* ≤ 0
3. The resulting internal Fourier transform length *n*, is too large. See “Convolutions and Correlations by Fourier Methods” on page 747.
4. The subroutine has not been initialized with the present arguments.
5. *naux1* ≤ 23
6. *naux1* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.
7. Error 2015 is recoverable or *naux2*≠0, and *naux2* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example 1:*** This example shows how to compute a convolution of a sequence in H, where H and X are ramp functions. It calculates all nonzero values of the convolution of the sequences in H and X. The arrays are declared as follows:

```
REAL*4   H(8), X(10,1), Y(17)
```

Because this convolution is symmetric in H and X, you can interchange the H and X sequences, leaving all other arguments the same, and you get the same output shown below. First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation.

*Call Statement and Input*

```
           INIT  H INC1H X INC1X INC2X Y INC1Y INC2Y NH  NX  M IY0 NY  AUX1 NAUX1 AUX2 NAUX2
            |    |  |    |  |      |   |   |     |    |   |  | |   |   |     |     |    |
CALL SCONF(INIT, H , 1 , X , 1  ,  1 , Y,  1  ,  1  , 8, 10, 1, 0, 17, AUX1, 128, AUX2, 23)
```

INIT = 1(for initialization)
INIT = 0(for computation)

```
H        =  (1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0)
X        =  (11.0, 12.0, 13.0, 14.0, 15.0, 16.0, 17.0, 18.0, 19.0, 20.0)
```

*Output*

```
Y        =  (11.0, 34.0, 70.0, 120.0, 185.0, 266.0, 364.0, 480.0,
             516.0, 552.0, 567.0, 560.0, 530.0, 476.0, 397.0, 292.0,
             160.0)
```

***Example 2:*** This example shows how the output from Example 1 differs when the value for NY is 21 rather than 17, and the value for IY0 is −2 rather than 0. This yields two zeros on each end of the convolution.

*Output*

```
Y        =  (0.0, 0.0, 11.0, 34.0, 70.0, 120.0, 185.0, 266.0, 364.0,
             480.0, 516.0, 552.0, 567.0, 560.0, 530.0, 476.0, 397.0,
             292.0, 160.0, 0.0, 0.0)
```

***Example 3:*** This example shows how to compute the autoconvolution by letting the two input sequences be the same for Example 2. First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation.

*Call Statement and Input*

```
           INIT  H INC1H X INC1X INC2X Y INC1Y INC2Y NH   NX  M  IY0  NY  AUX1 NAUX1 AUX2 NAUX2
            |    |   |   |   |     |   |   |     |   |    |   |   |    |    |     |     |    |
CALL SCONF(INIT, H , 1 , H , 1  ,  1 , Y,  1  ,  1  , 8, 10, 1, -2, 21, AUX1, 128, AUX2, 23)
```

```
INIT     =  1(for initialization)
INIT     =  0(for computation)
```

*Output*

```
Y        =  (1.0, 4.0, 10.0, 20.0, 35.0, 56.0, 84.0, 120.0, 147.0,
             164.0, 170.0, 164.0, 145.0, 112.0, 64.0)
```

***Example 4:*** This example shows how to compute all nonzero values of the convolution of the sequence in H with the two sequences in X. First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation.

*Call Statement and Input*

```
           INIT  H INC1H X INC1X INC2X Y INC1Y INC2Y NH   NX  M IY0 NY  AUX1 NAUX1 AUX2 NAUX2
            |    |   |   |   |     |   |   |     |   |    |   | |   |   |     |     |    |
CALL SCONF(INIT, H , 1 , X,  1  , 10 , Y,  1  , 17  , 8, 10, 2, 0, 17, AUX1, 148, AUX2, 43)
```

```
INIT     =  1(for initialization)
INIT     =  0(for computation)
H        =  (1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0)
```

X contains the following two sequences:

```
11.0  12.0
12.0  13.0
13.0  14.0
14.0  15.0
15.0  16.0
16.0  17.0
17.0  18.0
18.0  19.0
19.0  20.0
20.0  11.0
```

*Output:* Y contains the following two sequences:

```
 11.0   12.0
 34.0   37.0
 70.0   76.0
120.0  130.0
185.0  200.0
266.0  287.0
364.0  392.0
480.0  516.0
516.0  552.0
552.0  578.0
567.0  582.0
560.0  563.0
530.0  520.0
476.0  452.0
397.0  358.0
292.0  237.0
160.0   88.0
```

***Example 5:*** This example shows how to compute a correlation of a sequence in H, where H and X are ramp functions. It calculates all nonzero values of the correlation of the sequences in H and X. The arrays are declared as follows:

```
REAL*4   H(8), X(10,1)
```

First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation.

*Call Statement and Input*

```
           INIT  H INC1H X INC1X INC2X Y INC1Y INC2Y NH  NX  M  IY0 NY  AUX1 NAUX1 AUX2 NAUX2
            |    |   |   |   |     |   |   |     |    |   |  |   |   |   |     |    |     |
CALL SCORF(INIT, H , 1 , X,  1  ,  1 , Y,  1  ,  1  , 8, 10, 1, -7, 17, AUX1, 128, AUX2, 23)
```

INIT = 1(for initialization)
INIT = 0(for computation)
H =(same as input H in Example 1)
X =(same as input X in Example 1)

*Output*

```
Y        =  (88.0, 173.0, 254.0, 330.0, 400.0, 463.0, 518.0, 564.0,
             600.0, 636.0, 504.0, 385.0, 280.0, 190.0, 116.0,
             59.0, 20.0)
```

***Example 6:*** This example shows how the output from Example 5 differs when the value for NY is 21 rather than 17, and the value for IY0 is −9 rather than 0. This yields two zeros on each end of the correlation.

*Output*

```
Y        =  (0.0, 0.0, 88.0, 173.0, 254.0, 330.0, 400.0, 463.0, 518.0,
             564.0, 600.0, 636.0, 504.0, 385.0, 280.0, 190.0, 116.0,
             59.0, 20.0, 0.0, 0.0)
```

***Example 7:*** This example shows the effect of interchanging H and X. It uses the same input as Example 5, with H and X switched in the calling sequence, and with IY0 with a value of −9. Unlike convolution, as noted in Example 1, the correlation is not symmetric in H and X. First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation.

*Call Statement and Input*

```
           INIT  H INC1H X INC1X INC2X Y INC1Y INC2Y NH  NX  M  IY0 NY  AUX1 NAUX1 AUX2 NAUX2
             |   |  |    |  |      |   |  |      |    |   |  |   |   |   |     |     |    |
CALL SCONF(INIT, X , 1 , H,  1  ,  1 , Y,  1  ,  1  , 8, 10, 1, -9, 17, AUX1, 128, AUX2, 23)
```

```
INIT     =  1(for initialization)
INIT     =  0(for computation)
```

*Output*

```
Y        =  (20.0, 59.0, 116.0, 190.0, 280.0, 385.0, 504.0, 636.0,
             600.0, 564.0, 518.0, 463.0, 400.0, 330.0, 254.0, 173.0,
             88.0)
```

***Example 8:*** This example shows how to compute the autocorrelation by letting the two input sequences be the same. First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation. Because there is only one H input sequence, only one autocorrelation can be computed. Furthermore, this usage does not take advantage of the fact that the output is symmetric. Therefore, you should use SACORF to compute autocorrelations, because it does not have either of these problems.

*Call Statement and Input*

```
           INIT  H INC1H X INC1X INC2X Y INC1Y INC2Y NH NX M  IY0 NY  AUX1 NAUX1 AUX2 NAUX2
             |   |  |    |  |      |   |  |      |   |  |  |   |   |   |     |     |    |
CALL SCONF(INIT, H , 1 , H,  1  ,  1 , Y,  1  ,  1 , 8, 8, 1, -7, 15, AUX1, 148, AUX2, 43)
```

```
INIT     =  1(for initialization)
INIT     =  0(for computation)
```

*Output*

```
Y        =  (8.0, 23.0, 44.0, 70.0, 100.0, 133.0, 168.0, 204.0, 168.0,
             133.0, 100.0 , 70.0, 44.0, 23.0, 8.0)
```

***Example 9:*** This example shows how to compute all nonzero values of the correlation of the sequence in H with the two sequences in X. First, initialize AUX1 using the calling sequence shown below with INIT ≠ 0. Then use the same calling sequence with INIT = 0 to do the calculation.

*Call Statement and Input*

```
              INIT  H INC1H X INC1X INC2X Y INC1Y INC2Y NH  NX  M  IY0 NY  AUX1 NAUX1 AUX2 NAUX2
               |    |   |   |   |     |   |   |     |    |   |  |   |   |   |     |    |     |
CALL SCONF(INIT, H , 1 , X,  1  , 10 , Y,  1  , 17  , 8, 10, 2, -7, 17, AUX1, 148, AUX2, 43)
```

```
INIT     =  1(for initialization)
INIT     =  0(for computation)
H        =(same as input H in Example 4)
X        =(same as input X in Example 4)
```

*Output:* Y contains the following two sequences:

```
 88.0   96.0
173.0  188.0
254.0  275.0
330.0  356.0
400.0  430.0
463.0  496.0
518.0  553.0
564.0  600.0
600.0  636.0
636.0  592.0
504.0  462.0
385.0  346.0
280.0  245.0
190.0  160.0
116.0   92.0
 59.0   42.0
 20.0   11.0
```

# SDCON, DDCON, SDCOR, and DDCOR—Convolution or Correlation with Decimated Output Using a Direct Method

These subroutines compute the convolution and correlation of a sequence with another sequence, with decimated output, using a direct method.

*Table 137. Data Types*

| ***h***, ***x***, ***y*** | **Subroutine** |
|---|---|
| Short-precision real | SDCON |
| Long-precision real | DDCON |
| Short-precision real | SDCOR |
| Long-precision real | DDCOR |

**Note:** These subroutines are the short- and long-precision equivalents of SCOND and SCORD when the decimation interval *id* is equal to 1. Because there is no long-precision version of SCOND and SCORD, you can use DDCON and DDCOR, respectively, with decimation interval *id* = 1 to perform the same function.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SDCON \| DDCON \| SDCOR \| DDCOR (*h*, *inch*, *x*, *incx*, *y*, *incy*, *nh*, *nx*, *iy0*, *ny*, *id*) |
| **C and C++** | sdcon \| ddcon \| sdcor \| ddcor (*h*, *inch*, *x*, *incx*, *y*, *incy*, *nh*, *nx*, *iy0*, *ny*, *id*); |
| **PL/I** | CALL SDCON \| DDCON \| SDCOR \| DDCOR (*h*, *inch*, *x*, *incx*, *y*, *incy*, *nh*, *nx*, *iy0*, *ny*, *id*); |

*On Entry*

*h*

is the array H, consisting of the sequence of length $N_h$ to be convolved or correlated with the sequence in array X. Specified as: an array of (at least) length $1+(N_h-1)|inch|$, containing numbers of the data type indicated in Table 137.

*inch*

is the stride between the elements within the sequence in array H. Specified as: a fullword integer; *inch* > 0 or *inch* < 0.

*x*

is the array X, consisting of the input sequence of length $N_x$, to be convolved or correlated with the sequence in array H. Specified as: an array of (at least) length $1+(N_x-1)|incx|$, containing numbers of the data type indicated in Table 137.

*incx*

is the stride between the elements within the sequence in array X. Specified as: a fullword integer; *incx* > 0 or *incx* < 0.

*y*

See “On Return” on page 850.

*incy*

is the stride between the elements within the sequence in output array Y. Specified as: a fullword integer; *incy* > 0 or *incy* < 0.

*nh*

is the number of elements, $N_h$, in the sequence in array H. Specified as: a fullword integer; $N_h > 0$.

*nx*

is the number of elements, $N_x$, in the sequence in array X. Specified as: a fullword integer; $N_x > 0$.

*iy0*

is the convolution or correlation index of the element to be stored in the first position of the sequence in array Y. Specified as: a fullword integer. It can have any value.

*ny*

is the number of elements, $N_y$, in the sequence in array Y. Specified as: a fullword integer; $N_y > 0$.

*id*

is the decimation interval *id* for the output sequence in array Y; that is, every *id*-th value of the convolution or correlation is produced. Specified as: a fullword integer; $id > 0$.

*On Return*

*y*

is the array Y of length $N_y$, consisting of the output sequence that is the result of the convolution or correlation of the sequence in array H with the sequence in array X, given for every *id*-th value in the convolution or correlation.

Returned as: an array of (at least) length $1+(N_y-1)|incy|$, containing numbers of the data type indicated in Table 137 on page 849.

***Notes***

1. If you specify the same array for X and Y, the following conditions must be true: $incx = incy$, $incx > 0$, $incy > 0$, $id = 1$, and $iy0 \geq N_h-1$ for _DCON and $iy0 \geq 0$ for _DCOR. In this case, output overwrites input. In all other cases, output should not overwrite input; that is, input arrays X and H must have no common elements with output array Y. Otherwise, results are unpredictable. See “Concepts” on page 55.
2. If *iy0* and *ny* are such that output outside the basic range is needed, where the basic range is $0 \leq k \leq (nh+nx-2)$ for SDCON and DDCON and is $(-nh+1) \leq k \leq (nx-1)$ for SDCOR and DDCOR, the subroutine stores zeros using scalar code. It is not efficient to store many zeros in this manner. If you anticipate that this will happen, you may want to adjust *iy0* and *ny*, so the subroutine computes only for *k* in the above range, or use the ESSL subroutine SSCAL or DSCAL to store the zeros, so you achieve better performance.

***Function:*** The convolution and correlation of a sequence in array H with a sequence in array X, with decimated output, are expressed as follows:

Convolution for SDCON and DDCON:

$$y_k = \sum_{j=\max(0,k-N_x+1)}^{\min(N_h-1,k)} h_j x_{k-j}$$

Correlation for SDCOR and DDCOR:

$$y_k = \sum_{j=\max(0,-k)}^{\min(N_h-1,\ N_x-1-k)} h_j x_{k+j}$$

for $k$ = *iy0*, *iy0*+*id*, *iy0*+(2)*id*, ..., *iy0*+($N_y$–1)*id*

where:

$y_k$ are elements of the sequence of length $N_y$ in array Y.
$x_k$ are elements of the sequence of length $N_x$ in array X.
$h_j$ are elements of the sequence of length $N_h$ in array H.
*iy0* is the convolution or correlation index of the element to be stored in the first position of the sequence in array Y.
min and max select the minimum and maximum values, respectively.

It is assumed that elements outside the range of definition are zero. See reference [4].

***Special Usage:*** SDCON and DDCON can also perform a correlation, autoconvolution, or autocorrelation. To compute a correlation, you must specify a negative stride for H. To compute the autoconvolution, you must specify the two input sequences to be the same. You can also compute the autocorrelation by using both of these techniques together, letting the two input sequences be the same, and specifying a negative stride for the first input sequence. For examples of this, see the examples for SCOND on page 836. Because SCOND and SDCON are functionally the same, their results are the same as long as the decimation interval *id* = 1 for SDCON.

SDCOR and DDCOR can also perform a convolution, autocorrelation, or autoconvolution. To compute a convolution, you must specify a negative stride for H. To compute the autocorrelation, you must specify the two input sequences to be the same. You can also compute the autoconvolution by using both of these techniques together, letting the two input sequences be the same and specifying a negative stride for the first input sequence. For examples of these, see the examples for SCORD on page 837. Because SCORD and SDCOR are functionally the same, their results are the same as long as the decimation interval *id* = 1 for SDCOR.

### *Error Conditions*

*Computational Errors:* None

*Input-Argument Errors*

1. *nh*, *nx*, or *ny* ≤ 0
2. *inch*, *incx*, or *incy* = 0
3. *id* ≤ 0

***Example 1:*** This example shows how to compute a convolution of a sequence in H with a sequence in X, where both sequences are ramp functions. It shows how a decimated output can be obtained, using the same input as “Example 1” on page 836 for SCOND and using a decimation interval ID = 2.

**Note:** For further examples of use, see the examples for SCOND on page 836. Because SCOND and SDCON are functionally the same, their results are the same as long as the decimation interval ID = 1 for SDCON.

*Call Statement and Input*

```
           H  INCH  X  INCX  Y  INCY  NH  NX IY0  NY  ID
           |  |     |  |     |  |     |   |  |    |   |
CALL SDCON( H , 1  , X , 1  , Y , 1  , 4 , 8 , 0 , 6 , 2 )

H        =  (1.0, 2.0, 3.0, 4.0)
X        =  (11.0, 12.0, 13.0, 14.0, 15.0, 16.0, 17.0, 18.0)
```

*Output*

```
Y        =  (11.0, 70.0, 130.0, 150.0, 151.0, 72.0)
```

***Example 2:*** This example shows how to compute a correlation of a sequence in H with a sequence in X, where both sequences are ramp functions. It shows how a decimated output can be obtained, using the same input as “Example 6” on page 837 for SCORD and using a decimation interval ID = 2.

**Note:** For further examples of use, see the examples for SCORD on page 837. Because SCORD and SDCOR are functionally the same, their results are the same as long as the decimation interval ID = 1 for SDCOR.

*Call Statement and Input*

```
           H  INCH  X  INCX  Y  INCY  NH  NX  IY0  NY  ID
           |  |     |  |     |  |     |   |   |    |   |
CALL SDCOR( H , 1  , X , 1  , Y , 1  , 4 , 8 , -3 , 6 , 2 )

H        =  (1.0, 2.0, 3.0, 4.0)
X        =  (11.0, 12.0, 13.0, 14.0, 15.0, 16.0, 17.0, 18.0)
```

*Output*

```
Y        =  (44.0, 110.0, 140.0, 160.0, 104.0, 18.0)
```

***Example 3:*** This example shows how to compute the same function as computed in “Example 1” on page 836 for SCOND. The input sequences and arguments are the same as that example, except a decimation interval ID = 1 is specified here for SDCON.

*Call Statement and Input*

```
           H  INCH  X  INCX  Y  INCY  NH  NX IY0  NY   ID
           |  |     |  |     |  |     |   |  |    |    |
CALL SDCON( H , 1  , X , 1  , Y , 1  , 4 , 8 , 0 , 11 , 1 )

H        =  (1.0, 2.0, 3.0, 4.0)
X        =  (11.0, 12.0, 13.0, 14.0, 15.0, 16.0, 17.0, 18.0)
```

*Output*

```
Y        =  (11.0, 34.0, 70.0, 120.0, 130.0, 140.0, 150.0, 160.0,
             151.0, 122.0, 72.0)
```

# SACOR—Autocorrelation of One or More Sequences

This subroutine computes the autocorrelations of one or more sequences using a direct method. The input and output sequences contain short-precision real numbers.

**Note:** This subroutine is considered obsolete. It is provided in ESSL only for compatibility with earlier releases. You should use SCORD, SDCOR, SCORF and SACORF instead, because they provide **better performance**. For further details, see reference [4].

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SACOR (*init*, *x*, *inc1x*, *inc2x*, *y*, *inc1y*, *inc2y*, *nx*, *m*, *ny*, *aux1*, *naux1*, *aux2*, *naux2*) |
| **C and C++** | sacor (*init*, *x*, *inc1x*, *inc2x*, *y*, *inc1y*, *inc2y*, *nx*, *m*, *ny*, *aux1*, *naux1*, *aux2*, *naux2*); |
| **PL/I** | CALL SACOR (*init*, *x*, *inc1x*, *inc2x*, *y*, *inc1y*, *inc2y*, *nx*, *m*, *ny*, *aux1*, *naux1*, *aux2*, *naux2*); |

*On Entry*

*init*

is a flag, where:

If $init \neq 0$, no computation is performed, error checking is performed, and the subroutine exits back to the calling program.

If $init = 0$, the autocorrelations of the sequence in *x* are computed.

Specified as: a fullword integer. It can have any value.

*x*

is the array X, consisting of *m* input sequences of length $N_x$, to be autocorrelated. Specified as: an array of (at least) length $1+(N_x-1)inc1x+(m-1)inc2x$, containing short-precision real numbers.

*inc1x*

is the stride between the elements within each sequence in array X. Specified as: a fullword integer; $inc1x > 0$.

*inc2x*

is the stride between the first elements of the sequences in array X. Specified as: a fullword integer; $inc2x > 0$.

*y*

See "On Return" on page 854.

*inc1y*

is the stride between the elements within each sequence in output array Y. Specified as: a fullword integer; $inc1y > 0$.

*inc2y*

is the stride between the first elements of each sequence in output array Y. Specified as: a fullword integer; $inc2y > 0$.

*nx*

is the number of elements, $N_x$, in each sequence in array X. Specified as: a fullword integer; $N_x > 0$.

*m*

is the number of sequences in array X to be correlated. Specified as: a fullword integer; $m > 0$.

*ny*

is the number of elements, $N_y$, in each sequence in array Y. Specified as: a fullword integer; $N_y > 0$.

*aux1*
is no longer used in the computation, but must still be specified as a dummy argument (for migration purposes from Version 1 of ESSL). It can have any value.

*naux1*
is no longer used in the computation, but must still be specified as a dummy argument (for migration purposes from Version 1 of ESSL). It can have any value.

*aux2*
is no longer used in the computation, but must still be specified as a dummy argument (for migration purposes from Version 1 of ESSL). It can have any value.

*naux2*
is no longer used in the computation, but must still be specified as a dummy argument (for migration purposes from Version 1 of ESSL). It can have any value.

*On Return*

*y*
is array Y, consisting of *m* output sequences of length $N_y$ that are the autocorrelation functions of the sequences in array X. Returned as: an array of (at least) length 1 + ($N_y$–1)*inc1y* + (*m*–1)*inc2y*, containing short-precision real numbers.

***Notes***

1. Output should not overwrite input; that is, input arrays X and H must have no common elements with output array Y. Otherwise, results are unpredictable. See “Concepts” on page 55.
2. When using the ESSL SMP library, for optimal performance, the number of threads specified should be the same for *init* ≠ 0 and *init* = 0.
3. Auxiliary storage is not needed, but the arguments *aux1*, *naux1*, *aux2*, and *naux2* must still be specified. You can assign any values to these arguments.

***Function:*** The autocorrelations of the sequences in array X are expressed as follows:

$$y_{ki} = \sum_{j=0}^{N_x-1-k} x_{ji} x_{j+k,i}$$

for:

$k = 0, 1, ..., N_y-1$
$i = 1, 2, ..., m$

where:

$y_{ki}$ are elements of the *m* sequences of length $N_y$ in array Y.
$x_{ji}$ and $x_{j+k,i}$ are elements of the *m* sequences of length $N_x$ in array X.

See references [17 ] and [81 ].

Only one invocation of this subroutine is needed:

1. You do not need to invoke the subroutine with *init* ≠ 0. If you do, however, the subroutine performs error checking, exits back to the calling program, and no computation is performed.
2. With *init* = 0, the subroutine performs the calculation of the convolutions or correlations.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. *nx*, *ny*, or *m* ≤ 0
2. *inc1x*, *inc2x*, *inc1y*, or *inc2y* ≤ 0 (or incompatible)

***Example 1:*** This example shows how to compute an autocorrelation for three short sequences in array X, where the input sequence length NX is equal to the output sequence length NY. This gives all nonzero autocorrelation values.

The arrays are declared as follows:

```
    REAL*4   X(0:49999), Y(0:49999)
    REAL*8   AUX1, AUX2
```

*Call Statement and Input*

```
          INIT  X INC1X INC2X Y INC1Y INC2Y NX  M   NY  AUX1 NAUX1 AUX2   NAUX2
           |    |  |     |    |  |     |    |   |   |    |     |    |       |
CALL SACOR(INIT, X , 1  ,  7 , Y , 1  ,  7 , 7 , 3 , 7 , AUX1 , 0 , AUX2 ,   0)
```

```
INIT     =  0(for computation)
```

X contains the following three sequences:

```
1.0  2.0  3.0
2.0  1.0  2.0
3.0  2.0  1.0
4.0  3.0  2.0
4.0  4.0  3.0
3.0  4.0  4.0
2.0  3.0  4.0
```

*Output:* Y contains the following three sequences:

```
59.0  59.0  59.0
54.0  50.0  44.0
43.0  39.0  30.0
29.0  27.0  24.0
16.0  18.0  21.0
 7.0  11.0  20.0
 2.0   6.0  12.0
```

***Example 2:*** This example shows how the output from Example 1 differs when the values for NY and INC2Y are 9 rather than 7. This shows that when NY is greater than NX, the output array is longer, and that part is filled with zeros.

*Output:* Y contains the following three sequences:

```
59.0  59.0  59.0
54.0  50.0  44.0
43.0  39.0  30.0
29.0  27.0  24.0
16.0  18.0  21.0
 7.0  11.0  20.0
 2.0   6.0  12.0
 0.0   0.0   0.0
 0.0   0.0   0.0
```

***Example 3:*** This example shows how the output from Example 1 differs when the value for NY is 5 rather than 7. Also, the values for INC1X and INC1Y are 3, and the values for INC2X and INC2Y are 1 rather than 7. This shows that when NY is less than NX, the output array is shortened.

*Output:* Y contains the following three sequences:

```
59.0  59.0  59.0
54.0  50.0  44.0
43.0  39.0  30.0
29.0  27.0  24.0
16.0  18.0  21.0
```

# SACORF—Autocorrelation of One or More Sequences Using the Mixed-Radix Fourier Method

This subroutine computes the autocorrelations of one or more sequences using the mixed-radix Fourier method. The input and output sequences contain short-precision real numbers.

**Note:** Two invocations of this subroutine are necessary: one to prepare the working storage for the subroutine, and the other to perform the computations.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SACORF (*init*, *x*, *inc1x*, *inc2x*, *y*, *inc1y*, *inc2y*, *nx*, *m*, *ny*, *aux1*, *naux1*, *aux2*, *naux2*) |
| **C and C++** | sacorf (*init*, *x*, *inc1x*, *inc2x*, *y*, *inc1y*, *inc2y*, *nx*, *m*, *ny*, *aux1*, *naux1*, *aux2*, *naux2*); |
| **PL/I** | CALL SACORF (*init*, *x*, *inc1x*, *inc2x*, *y*, *inc1y*, *inc2y*, *nx*, *m*, *ny*, *aux1*, *naux1*, *aux2*, *naux2*); |

*On Entry*

*init*

is a flag, where:

If *init* ≠ 0, trigonometric functions and other parameters, depending on arguments other than *x*, are computed and saved in *aux1*. The contents of *x* and *y* are not used or changed.

If *init* = 0, the autocorrelations of the sequence in *x* are computed. The only arguments that may change after initialization are *x*, *y*, and *aux2*. All scalar arguments must be the same as when the subroutine was called for initialization with *init* ≠ 0.

Specified as: a fullword integer. It can have any value.

*x*

is the array X, consisting of *m* input sequences of length $N_x$, to be autocorrelated. Specified as: an array of (at least) length $1+(N_x-1)inc1x+(m-1)inc2x$, containing short-precision real numbers.

*inc1x*

is the stride between the elements within each sequence in array X. Specified as: a fullword integer; *inc1x* > 0.

*inc2x*

is the stride between the first elements of the sequences in array X. Specified as: a fullword integer; *inc2x* > 0.

*y*

See “On Return” on page 858.

*inc1y*

is the stride between the elements within each sequence in output array Y. Specified as: a fullword integer; *inc1y* > 0.

*inc2y*

is the stride between the first elements of each sequence in output array Y. Specified as: a fullword integer; *inc2y* > 0.

*nx*

is the number of elements, $N_x$, in each sequence in array X. Specified as: a fullword integer; $N_x > 0$.

*m*

is the number of sequences in array X to be correlated. Specified as: a fullword integer; $m > 0$.

*ny*

is the number of elements, $N_y$, in each sequence in array Y. Specified as: a fullword integer; $N_y > 0$.

*aux1*

is the working storage for this subroutine, where:

If $init \neq 0$, the working storage is computed.

If $init = 0$, the working storage is used in the computation of the autocorrelations.

Specified as: an area of storage, containing *naux1* long-precision real numbers.

*naux1*

is the number of doublewords in the working storage specified in *aux1*. Specified as: a fullword integer; $naux1 > 21$ and $naux1 \geq$ (minimum value required for successful processing). To determine a sufficient value, use the processor-independent formulas. For values between 21 and the minimum value, you have the option of having the minimum value returned in this argument. For details, see “Using Auxiliary Storage in ESSL” on page 32.

*aux2*

has the following meaning:

If $naux2 = 0$ and error 2015 is unrecoverable, *aux2* is ignored.

Otherwise, it is the working storage used by this subroutine, which is available for use by the calling program between calls to this subroutine.

Specified as: an area of storage, containing *naux2* long-precision real numbers. On output, the contents are overwritten.

*naux2*

is the number of doublewords in the working storage specified in *aux2*. Specified as: a fullword integer, where:

If $naux2 = 0$ and error 2015 is unrecoverable, SACORF dynamically allocates the work area used by this subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, $naux2 \geq$ (minimum value required for successful processing). To determine a sufficient value, use the processor-independent formulas. For all other values specified less than the minimum value, you have the option of having the minimum value returned in this argument. For details, see “Using Auxiliary Storage in ESSL” on page 32.

*On Return*

*y*

has the following meaning, where:

If $init \neq 0$, this argument is not used, and its contents remain unchanged.

If $init = 0$, this is array Y, consisting of *m* output sequences of length $N_y$ that are the autocorrelation functions of the sequences in array X.

Returned as: an array of (at least) length $1+(N_y-1)inc1y+(m-1)inc2y$, containing short-precision real numbers.

*aux1*
is the working storage for this subroutine, where:

If *init* ≠ 0, it contains information ready to be passed in a subsequent invocation of this subroutine.

If *init* = 0, its contents are unchanged.

Returned as: the contents are not relevant.

***Notes***

1. *aux1* should **not** be used by the calling program between calls to this subroutine with *init* ≠ 0 and *init* = 0. However, it can be reused after intervening calls to this subroutine with different arguments.
2. When using the ESSL SMP library, for optimal performance, the number of threads specified should be the same for *init* ≠ 0 and *init* = 0.
3. If you specify the same array for X and Y, then *inc1x* and *inc1y* must be equal and *inc2x* and *inc2y* must be equal. In this case, output overwrites input.
4. If you specify different arrays for X and Y, they must have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.
5. If *ny* is such that output outside the basic range is needed, the subroutine stores zeros. This range is: 0 ≤ *k* ≤ *nx*–1.

***Formula for Calculating the Length of the Fourier Transform:*** Before calculating the necessary sizes of *naux1* and *naux2*, you must determine the length *n* of the Fourier transform. To do this, you use the values of the arguments *nx* and *ny*, inserted into the following formula, to get a value for the variable *nf*. After calculating *nf*, reference the formula or table of allowable values of *n* in “Acceptable Lengths for the Transforms” on page 741, selecting the value equal to or greater than *nf*. Following is the formula for determining *nf*:

*nf* = min(*ny*, *nx*)+*nx*+1

***Processor-Independent Formulas for NAUX1 and NAUX2:*** The required values of *naux1* and *naux2* depend on the value determined for *n* in “ Formula for Calculating the Length of the Fourier Transform” and the argument *m*.

*NAUX1 Formulas*

If *n* ≤ 16384, use *naux1* = 55000.
If *n* > 16384, use *naux1* = 40000+1.89*n*.

*NAUX2 Formulas*

If *n* ≤ 16384, use *naux2* = 50000.
If *n* > 16384, use *naux2* = 40000+1.64*n*.

***Function:*** The autocorrelations of the sequences in array X are expressed as follows:

$$y_{ki} = \sum_{j=0}^{N_x - 1 - k} x_{ji} x_{j+k,i}$$

for:

$k = 0, 1, ..., N_y - 1$
$i = 1, 2, ..., m$

where:

$y_{ki}$ are elements of the *m* sequences of length $N_y$ in array `Y`.
$x_{ji}$ and $x_{j+k,i}$ are elements of the *m* sequences of length $N_x$ in array `X`.

This subroutine uses a Fourier transform method with a mixed-radix capability. This provides maximum performance for your application. The length of the transform, *n*, that you must calculate to determine the correct sizes for *naux1* and *naux2* is the same length used by the Fourier transform subroutines called by this subroutine. See references [17] and [81 ].

Two invocations of this subroutine are necessary:

1. With *init* ≠ 0, the subroutine tests and initializes arguments of the program, setting up the *aux1* working storage.
2. With *init* = 0, the subroutine checks that the initialization arguments in the *aux1* working storage correspond to the present arguments, and if so, performs the calculation of the autocorrelations.

***Error Conditions***

*Resource Errors:* Error 2015 is unrecoverable, *naux2* = 0, and unable to allocate work area.

*Computational Errors:* None

*Input-Argument Errors*

1. *nx*, *ny*, or *m* ≤ 0
2. *inc1x*, *inc2x*, *inc1y*, or *inc2y* ≤ 0 (or incompatible)
3. The resulting correlation is too long.
4. The subroutine has not been initialized with the present arguments.
5. *naux1* ≤ 21
6. *naux1* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.
7. Error 2015 is recoverable or *naux2*≠0, and *naux2* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example 1:*** This example shows how to compute an autocorrelation for three short sequences in array `X`, where the input sequence length `NX` is equal to the output sequence length `NY`. This gives all nonzero autocorrelation values. The arrays are declared as follows:

```
REAL*4   X(0:49999), Y(0:49999)
REAL*8   AUX1(30788), AUX2(17105)
```

First, initialize `AUX1` using the calling sequence shown below with `INIT` ≠ 0. Then use the same calling sequence with `INIT` = 0 to do the calculation.

*Call Statement and Input*

```
                INIT  X INC1X INC2X Y INC1Y INC2Y NX  M   NY  AUX1  NAUX1 AUX2  NAUX2
                  |   |  |     |    |  |     |    |   |   |    |      |    |      |
CALL SACORF(INIT, X , 1  ,  7 , Y , 1  ,  7 , 7 , 3 , 7 , AUX1, 2959, AUX2, 4354)
```

```
INIT     =  1(for initialization)
INIT     =  0(for computation)
```

X contains the following three sequences:

```
1.0  2.0  3.0
2.0  1.0  2.0
3.0  2.0  1.0
4.0  3.0  2.0
4.0  4.0  3.0
3.0  4.0  4.0
2.0  3.0  4.0
```

*Output:* Y contains the following three sequences:

```
59.0  59.0  59.0
54.0  50.0  44.0
43.0  39.0  30.0
29.0  27.0  24.0
16.0  18.0  21.0
 7.0  11.0  20.0
 2.0   6.0  12.0
```

***Example 2:*** This example shows how the output from Example 1 differs when the value for NY and INC2Y are 9 rather than 7. This shows that when NY is greater than NX, the output array is longer and that part is filled with zeros.

*Output:* Y contains the following three sequences:

```
59.0  59.0  59.0
54.0  50.0  44.0
43.0  39.0  30.0
29.0  27.0  24.0
16.0  18.0  21.0
 7.0  11.0  20.0
 2.0   6.0  12.0
 0.0   0.0   0.0
 0.0   0.0   0.0
```

***Example 3:*** This example shows how the output from Example 1 differs when the value for NY is 5 rather than 7. Also, the values for INC1X and INC1Y are 3 rather than 1, and the values for INC2X and INC2Y are 1 rather than 7. This shows that when NY is less than NX, the output array is shortened.

*Output:* Y contains the following three sequences:

```
59.0  59.0  59.0
54.0  50.0  44.0
43.0  39.0  30.0
29.0  27.0  24.0
16.0  18.0  21.0
```

## Related-Computation Subroutines

This section contains the related-computation subroutine descriptions.

## SPOLY and DPOLY—Polynomial Evaluation

These subroutines evaluate a polynomial of degree *k*, using coefficient vector ***u***, input vector ***x***, and output vector ***y***:

$$y_i = u_0 + u_1 x_i + u_2 x_i^2 + \ldots + u_k x_i^k \qquad \text{for } i = 1,2,\ldots,n$$

where $u_k$, $x_i$, and $y_i$ are elements of ***u***, ***x***, and ***y***, respectively.

*Table 138. Data Types*

| ***u***, ***x***, ***y*** | **Subroutine** |
|---|---|
| Short-precision real | SPOLY |
| Long-precision real | DPOLY |

### *Syntax*

|  |  |
|---|---|
| **Fortran** | CALL SPOLY \| DPOLY (*u*, *incu*, *k*, *x*, *incx*, *y*, *incy*, *n*) |
| **C and C++** | spoly \| dpoly (*u*, *incu*, *k*, *x*, *incx*, *y*, *incy*, *n*); |
| **PL/I** | CALL SPOLY \| DPOLY (*u*, *incu*, *k*, *x*, *incx*, *y*, *incy*, *n*); |

*On Entry*

*u*
is the coefficient vector ***u*** of length *k*+1. It contains elements $u_0$, $u_1$, $u_0$, $u_1$, $u_2$, ..., $u_k$, which are stored in this order. Specified as: a one-dimensional array of (at least) length 1+*k*|*incu*|, containing numbers of the data type indicated in Table 138.

*incu*
is the stride for vector ***u***. Specified as: a fullword integer. It can have any value.

*k*
is the degree *k* of the polynomial. Specified as: a fullword integer; $k \geq 0$.

*x*
is the input vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length 1+(*n*–1)|*incx*|, containing numbers of the data type indicated in Table 138.

*incx*
is the stride for vector ***x***. Specified as: a fullword integer. It can have any value.

*y*
See “On Return.”

*incy*
is the stride for the output vector ***y***. Specified as: a fullword integer. It can have any value.

*n*
is the number of elements in input vector ***x*** and the number of resulting elements in output vector ***y***. Specified as: a fullword integer; $n \geq 0$.

*On Return*

*y*

is the output vector ***y*** of length *n*, containing the results of the polynomial evaluation. Returned as: a one-dimensional array of (at least) length 1+(*n*−1)|*incy*|, containing numbers of the data type indicated in Table 138.

***Note:*** Vectors ***u***, ***x***, and ***y*** must have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.

***Function:*** The evaluation of the polynomial:

$$y_i = u_0 + u_1 x_i + u_2 x_i^2 + \ldots + u_k x_i^k \qquad \text{for } i = 1,2,\ldots,n$$

is expressed as follows:

$$y_i = u_0 + x_i\,(u_1 + x_i\,(u_2 + \ldots + x_i\,(u_{k-1} + x_i u_k)\ \ldots) \qquad \text{for } i = 1, 2, \ldots, n$$

See reference [78 ] for Horner's Rule. If *n* is 0, no computation is performed. For SPOLY, intermediate results are accumulated in long precision.

SPOLY provides the same function as the IBM 3838 function POLY, with restrictions removed. DPOLY provides a long-precision computation that is not included in the IBM 3838 functions. See the *IBM 3838 Array Processor Functional Characteristics* manual.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. $k < 0$
2. $n < 0$

***Example 1:*** This example shows a polynomial evaluation with the degree, `K`, equal to 0.

*Call Statement and Input*

```
            U   INCU   K   X   INCX   Y  INCY  N
            |    |     |   |    |     |   |    |
CALL SPOLY( U , INCU , 0 , X , INCX , Y , 1  , 3 )
```

```
U        =  (4.0)
INCU     =(not relevant)
X        =(not relevant)
INCX     =(not relevant)
```

*Output*

```
Y        =  (4.0, 4.0, 4.0)
```

***Example 2:*** This example shows a polynomial evaluation, using a negative stride `INCU` for vector ***u***. For ***u***, processing begins at element `U(4)` which is 1.0.

*Call Statement and Input*

```
            U  INCU  K   X  INCX  Y  INCY  N
            |    |   |   |   |    |   |    |
CALL SPOLY( U , -1 , 3 , X , 1  , Y , 1  , 3 )

U        =  (4.0, 3.0, 2.0, 1.0)
X        =  (2.0, 1.0, -3.0)
```

*Output*

```
Y        =  (49.0, 10.0, -86.0)
```

***Example 3:*** This example shows a polynomial evaluation, using a stride `INCX` of `0` for input vector ***x***.

*Call Statement and Input*

```
            U  INCU  K   X  INCX  Y  INCY  N
            |   |    |   |   |    |   |    |
CALL SPOLY( U , 1  , 3 , X , 0  , Y , 1  , 3 )

U        =  (4.0, 3.0, 2.0, 1.0)
X        =  (2.0, . , . )
```

*Output*

```
Y        =  (26.0, 26.0, 26.0)
```

***Example 4:*** This example shows a polynomial evaluation, using a stride `INCX` greater than 1 for input vector ***x***, and a negative stride `INCY` for output vector ***y***. For ***y***, results are stored beginning at element `Y(5)`.

*Call Statement and Input*

```
            U  INCU  K   X  INCX  Y  INCY  N
            |   |    |   |   |    |    |   |
CALL SPOLY( U , 1  , 3 , X , 2  , Y , -2 , 3 )

U        =  (4.0, 3.0, 2.0, 1.0)
X        =  (2.0, . , -3.0, . , 1.0)
```

*Output*

```
Y        =  (10.0, . , -14.0, . , 26.0)
```

## SIZC and DIZC—I-th Zero Crossing

These subroutines find the position of the *i*-th zero crossing in vector ***x***. This is the *i*-th transition between positive and negative or negative and positive, where 0 is considered a positive value. It returns the position of the element in vector ***x*** where the *i*-th zero crossing is detected. The direction of the scan is either from the first element to the last or from the last element to the first, depending on the value you specify for the scan direction argument.

*Table 139. Data Types*

| ***x*** | **Subroutine** |
|---|---|
| Short-precision real | SIZC |
| Long-precision real | DIZC |

### *Syntax*

| **Fortran** | CALL SIZC \| DIZC (*x*, *idrx*, *n*, *i*, *ky*) |
|---|---|
| **C and C++** | sizc \| dizc (*x*, *idrx*, *n*, *i*, *ky*); |
| **PL/I** | CALL SIZC \| DIZC (*x*, *idrx*, *n*, *i*, *ky*); |

*On Entry*

*x*
is the target vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 139.

*idrx*
indicates the scan direction. If it is positive or 0, ***x*** is scanned from the first element to the last (1, *n*). If it is negative, ***x*** is scanned from the last element to the first (*n*, 1). Specified as: a fullword integer. It can have any value.

*n*
is the number of elements in vector ***x***. Specified as: a fullword integer; $n > 1$.

*i*
is the number of the zero crossing to be identified. Specified as: a fullword integer; $i > 0$.

*ky*
See “On Return.”

*On Return*

*ky*
is the integer vector ***ky*** of length 2, containing elements $ky_1$ and $ky_2$, where:

If the *i*-th zero crossing is found:

- $ky_1 = j$, where *j* is the position of the element $x_j$ at the point that the *i*-th zero crossing is found. The position is always relative to the beginning of the vector regardless of the scan direction.
- $ky_2 = i$

If the *i*-th zero crossing is not found:

- $ky_1 = 0$
- $ky_2 =$ the total number of zero crossings encountered in the scan.

Returned as: an array of (at least) length 2, containing fullword integers.

***Note:*** The *aux* and *naux* arguments, required in some earlier releases of ESSL, are no longer required by these subroutines. If your program still includes them, you do not have to change your program; it continues to run normally. It ignores these arguments. However, if you did any program checking for error code 2015, you may want to remove it, because this error no longer occurs. (You must not code these arguments in your C program.)

***Function:*** The *i*-th zero crossing in vector ***x*** is found by scanning vector ***x*** for *i* occurrences of TRUE for the following logical expressions. A zero crossing is defined here as a crossing either from a positive value to a negative value or from a negative value to a positive value, where 0 is considered a positive value. If the *i*-th zero crossing is found, the value of *j* at that point is returned in $ky_1$ as the position of the *i*-th zero crossing, and *i* is returned in $ky_2$.

If $idrx \geq 0$:

TRUE = $(x_{j-1} < 0$ and $x_j \geq 0)$ or $(x_{j-1} \geq 0$ and $x_j < 0)$ for $j = 2, n$

If $idrx < 0$:

TRUE = $(x_{j+1} < 0$ and $x_j \geq 0)$ or $(x_{j+1} \geq 0$ and $x_j < 0)$ for $j = n-1, 1$

If the position of the *i*-th zero crossing is not found, 0 is returned in $y_1$ and the number of zero crossings encountered in the scan is returned in $y_2$.

SIZC provides the same functions as the IBM 3838 functions NZCP and NZCN, with restrictions removed. It combines these functions into one ESSL subroutine. DIZC provides a long-precision computation that is not included in the IBM 3838 functions. See the *IBM 3838 Array Processor Functional Characteristics* manual.

### *Error Conditions*

*Computational Errors:* None

*Input-Argument Errors*

1. $n \leq 1$
2. $i \leq 0$

***Example 1:*** This example shows a scan of a vector ***x*** from the first element to the last. It is looking for the fifth zero crossing, which is encountered at position 9.

*Call Statement and Input*

```
           X  IDRX  N    I   KY
           |   |    |    |   |
CALL SIZC( X , 1  , 12 , 5 , KY )

X        =  (2.0, -1.0, -3.0, 3.0, 0.0, 8.0, -2.0, 0.0, -5.0, -3.0,
             2.0, -9.0)
```

*Output*

```
KY       =  (9, 5)
```

***Example 2:*** This example shows a scan of a vector ***x*** from the last element to the first. It is looking for the seventh zero crossing, which is encountered at position 3. Because `IDRX` is negative, `X` is scanned from the last element, `X(12)`, to the first element, `X(1)`.

*Call Statement and Input*

```
           X  IDRX  N    I   KY
           |    |   |    |   |
CALL SIZC( X , -1 , 12 , 7 , KY )

X        =  (2.0, -1.0, 3.0, -3.0, 0.0, -8.0, -2.0, 0.0, -5.0, -3.0,
             2.0, -9.0)
```

*Output*

```
KY       =  (3, 7)
```

***Example 3:*** This example shows a scan of a vector ***x*** when the *i*-th zero crossing is not found. It encounters seven zero crossings and returns this value in `KY(2)`.

*Call Statement and Input*

```
           X  IDRX  N    I    KY
           |   |    |    |    |
CALL SIZC( X , 1  , 12 , 10 , KY )

X        =  (2.0, -1.0, -3.0, 3.0, 0.0, 8.0, -2.0, 0.0, -5.0, -3.0,
             2.0, -9.0)
```

*Output*

```
KY       =  (0, 7)
```

## STREC and DTREC—Time-Varying Recursive Filter

These subroutines implement the first-order time-varying recursive equation, using initial value *s*, target vectors ***u*** and ***x***, and output vector ***y***.

*Table 140. Data Types*

| *s*, ***u***, ***x***, ***y*** | **Subroutine** |
|---|---|
| Short-precision real | STREC |
| Long-precision real | DTREC |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL STREC \| DTREC (*s*, *u*, *incu*, *x*, *incx*, *y*, *incy*, *n* , *iopt*) |
| **C and C++** | strec \| dtrec (*s*, *u*, *incu*, *x*, *incx*, *y*, *incy*, *n*, *iopt*); |
| **PL/I** | CALL STREC \| DTREC (*s*, *u*, *incu*, *x*, *incx*, *y*, *incy*, *n* , *iopt*); |

*On Entry*

*s*
is the scalar *s* used in the initial computation for $y_1$. Specified as: a number of the data type indicated in Table 140.

*u*
is the target vector ***u*** of length *n*. Specified as: a one-dimensional array of (at least) length 1+(*n*–1)|*incu*|, containing numbers of the data type indicated in Table 140.

*incu*
is the stride for target vector ***u***. Specified as: a fullword integer. It can have any value.

*x*
is the target vector ***x*** of length *n*. Specified as: a one-dimensional array of (at least) length 1+(*n*–1)|*incx*|, containing numbers of the data type indicated in Table 140.

*incx*
is the stride for target vector ***x.*** Specified as: a fullword integer. It can have any value.

*y*
See "On Return."

*incy*
is the stride for output vector ***y.*** Specified as: a fullword integer; $incy > 0$ or $incy < 0$.

*n*
is the number of elements in vectors ***u*** and ***x*** and the number of resulting elements in output vector ***y***. Specified as: a fullword integer; $n \geq 0$.

*iopt*
this argument has no effect on the performance of the computation, but still must be specified as: a fullword integer; $iopt = 0$ or 1.

*On Return*

*y*
is the vector ***y*** of length *n*, containing the results of the implementation of the first-order time-varying recursive equation. Returned as: a one-dimensional

array of (at least) length 1+($n$–1)|*incy*|, containing numbers of the data type indicated in Table 140.

***Note:*** Vectors ***u***, ***x***, and ***y*** must have no common elements; otherwise, results are unpredictable. See “Concepts” on page 55.

***Function:*** The first-order time-varying recursive equation is expressed as follows:

$y_1 = s+u_1x_1$
$y_2 = u_2y_1+u_1x_2$
.
.
.
$y_i = u_iy_{i-1}+u_1x_i$ for $i = 3, 4, ..., n$

STREC provides the same function as the IBM 3838 function REC, with restrictions removed. DTREC provides a long-precision computation that is not included in the IBM 3838 functions. See the *IBM 3838 Array Processor Functional Characteristics* manual.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. $incy = 0$
2. $n < 0$
3. $iopt \neq 0$ or 1

***Example 1:*** This example shows all strides INCU, INCX, and INCY equal to 1 for vectors ***u***, ***x***, and ***y***, respectively.

*Call Statement and Input*

```
            S    U  INCU  X  INCX  Y  INCY  N  IOPT
            |    |   |    |   |    |   |    |   |
CALL STREC( 1.0 , U , 1  , X , 1  , Y , 1  , 8 , 0 )

U        =  (1.0, 2.0, 3.0, 3.0, 2.0, 1.0, 1.0, 2.0)
X        =  (3.0, 2.0, 1.0, 1.0, 2.0, 3.0, 3.0, 2.0)
```

*Output*

```
Y        =  (4.0, 10.0, 31.0, 94.0, 190.0, 193.0, 196.0, 394.0)
```

***Example 2:*** This example shows a stride, INCU, that is greater than 1 for vector ***u***. The strides INCX and INCY for vectors ***x*** and ***y***, respectively, are 1.

*Call Statement and Input*

```
            S    U  INCU  X  INCX  Y  INCY  N  IOPT
            |    |   |    |   |    |   |    |   |
CALL STREC( 1.0 , U , 2  , X , 1  , Y , 1  , 4 , 0 )

U        =  (1.0, . , 3.0, . , 2.0, . , 1.0, . )
X        =  (3.0, 2.0, 1.0, 1.0, 2.0, 3.0, 3.0, 2.0)
```

*Output*

```
Y        =  (4.0, 14.0, 29.0, 30.0)
```

***Example 3:*** This example shows a stride, `INCU`, of 1 for vector ***u***, a stride, `INCX`, that is greater than 1 for vector ***x***, and a negative stride, `INCY`, for vector ***y***. For ***y***, results are stored beginning at element `Y(4)`.

*Call Statement and Input*

```
             S    U  INCU  X  INCX  Y  INCY  N  IOPT
             |    |  |     |  |     |    |   |  |
CALL STREC( 1.0 , U , 1  , X , 2  , Y , -1 , 4 , 1 )

U        =  (1.0, 2.0, 3.0, 3.0, 2.0, 1.0, 1.0, 2.0)
X        =  (3.0, . , 1.0, . , 2.0, . , 3.0)
```

*Output*

```
Y        =  (90.0, 29.0, 9.0, 4.0)
```

# SQINT and DQINT—Quadratic Interpolation

These subroutines perform a quadratic interpolation at specified points in the vector ***x***, using initial linear displacement in the samples *s*, sample interval *g*, output scaling parameter Ω, and sample reflection times in vector ***t***. The result is returned in vector ***y***.

Table 141. Data Types

| ***x***, *s*, *g*, Ω, ***t***, ***y*** | **Subroutine** |
|---|---|
| Short-precision real | SQINT |
| Long-precision real | DQINT |

### Syntax

| | |
|---|---|
| **Fortran** | CALL SQINT \| DQINT (*s*, *g*, *omega*, *x*, *incx*, *n*, *t*, *inct*, *y*, *incy*, *m*) |
| **C and C++** | sqint \| dqint (*s*, *g*, *omega*, *x*, *incx*, *n*, *t*, *inct*, *y*, *incy*, *m*); |
| **PL/I** | CALL SQINT \| DQINT (*s*, *g*, *omega*, *x*, *incx*, *n*, *t*, *inct*, *y*, *incy*, *m*); |

*On Entry*

*s*
is the scalar *s*, containing the initial linear displacement in samples. Specified as: a number of the data type indicated in Table 141.

*g*
is the scalar *g*, containing the sample interval. Specified as: a number of the data type indicated in Table 141; $g > 0.0$.

*omega*
is the output scaling parameter Ω. Specified as: a number of the data type indicated in Table 141.

*x*
is the vector ***x*** of length *n*, containing the trace data. Specified as: a one-dimensional array of (at least) length $1+(n-1)|incx|$, containing numbers of the data type indicated in Table 141.

*incx*
is the stride for vector ***x***. Specified as: a fullword integer; $incx > 0$ or $incx < 0$.

*n*
is the number of elements in vector ***x***. Specified as: a fullword integer; $n \geq 3$.

*t*
is the vector ***t*** of length *m*, containing the sample reflection times to be processed. Specified as: a one-dimensional array of (at least) length $1+(m-1)|inct|$, containing numbers of the data type indicated in Table 141.

*inct*
is the stride for vector ***t***. Specified as: a fullword integer; $inct > 0$ or $inct < 0$.

*y*
See “On Return” on page 873.

*incy*
is the stride for output vector ***y***. Specified as: a fullword integer; $incy > 0$ or $incy < 0$.

*m*
is the number of elements in vector ***t*** and the number of elements in output vector ***y***. Specified as: a fullword integer; $m \geq 0$.

*On Return*

*y*

is the vector ***y*** of length *m*, containing the results of the quadratic interpolation. Returned as: a one-dimensional array of (at least) length 1+(*m*–1)|*incy*|, containing numbers of the data type indicated in Table 141 on page 872.

***Function:*** The quadratic interpolation, which is expressed as follows:

$$y_i = \Omega \left( trace_{k_i} \left( f_i^2 - f_i \right) + 2\ trace_{k_i+1} \left( 1 - f_i^2 \right) + trace_{k_i+2} \left( f_i^2 + f_i \right) \right)$$

for $i = 1, 2, \ldots, m$

uses the following values:

***x*** is the vector containing the specified points.
*s* is the initial linear displacement in the samples.
*g* is a sample interval.
Ω is the output scaling parameter.
***t*** is the vector containing the sample reflection times.

and where ***trace***, ***k***, ***f***, and ***w*** are four working vectors, and *so* is a working scalar defined as:

$trace_1 = 3x_1 - 3x_2 + x_3$
$trace_{i+1} = x_i$ for $i = 1, 2, \ldots, n$
$so = s + 2.0$
$w_i = so + t_i / g$ for $i = 1, 2, \ldots, m$
$f_i$ = fraction part of $w_i$
$k_i+1$ = integer part of $w_i$

**Note:** Allowing $k_i+1$ to have a value of 2 results in performance degradation. If possible, avoid specifying a point at which this occurs.

If *n* or *m* is 0, no computation is performed.

SQINT provides the same function as the IBM 3838 function INT, with restrictions removed. DQINT provides a long-precision computation that is not included in the IBM 3838 functions. See the *IBM 3838 Array Processor Functional Characteristics* manual.

### Error Conditions

*Computational Errors:* The condition ($k_i+1 > n$) or ($k_i+1 \le 2$) has occurred, where *n* is the number of elements in vector ***x***. See “ Function” for how to calculate $k_i$.

- The lower range *l* and the upper range *j* of the vector are identified in the computational error message.
- The return code is set to 1.
- The ranges *l* and *j* of the vector can be determined at run time by using the ESSL error-handling facilities. To obtain this information, you must use ERRSET to change the number of allowable errors for error code 2100 in the ESSL error option table; otherwise, the default value causes your program to terminate when this error occurs. For details, see “What Can You Do about ESSL Computational Errors?” on page 48.

*Input-Argument Errors*

1. *n* < 3
2. *m* < 0
3. *g* ≤ 0
4. *incx* = 0
5. *inct* = 0
6. *incy* = 0

***Example 1:*** This example shows a quadratic interpolation, using vectors with strides of 1.

*Call Statement and Input*

```
               S     G   OMEGA  X  INCX  N   T  INCT  Y  INCY  M
               |     |     |    |   |    |   |   |    |   |    |
CALL SQINT( 2.0 , 1.0 , 1.0 , X , 1  , 8 , T , 1  , Y , 1  , 4 )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0)
T        =  (1.5, 2.5, 3.5, 4.5)
```

*Output*

```
Y        =  (9.0, 11.0, 13.0, 15.0)
```

***Example 2:*** This example shows a quadratic interpolation, using vectors with a positive stride of 1 and negative strides of −1.

*Call Statement and Input*

```
               S     G   OMEGA  X  INCX  N   T  INCT  Y  INCY  M
               |     |     |    |    |   |   |    |   |   |    |
CALL SQINT( 2.0 , 1.0 , 1.0 , X , -1 , 8 , T , -1 , Y , 1  , 4 )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0)
T        =  (1.5, 2.5, 3.5, 4.5)
```

*Output*

```
Y        =  (3.0, 5.0, 7.0, 9.0)
```

***Example 3:*** This example shows a quadratic interpolation, using vectors with a positive stride greater than 1 and negative strides less than −1.

*Call Statement and Input*

```
               S     G   OMEGA  X  INCX  N   T  INCT  Y  INCY  M
               |     |     |    |    |   |   |    |   |   |    |
CALL SQINT( 2.0 , 1.0 , 1.0 , X , -2 , 8 , T , -1 , Y , 2  , 4 )

X        =  (1.0, . , 3.0, . , 5.0, . , 7.0, . , 9.0, . , 11.0, . ,
             13.0, . , 15.0)
T        =  (1.36, 2.36, 3.36, 4.36)
```

*Output*

```
Y        =  (4.56, . , 8.56, . , 12.56, . , 16.56)
```

***Example 4:*** This example shows a quadratic interpolation, using vectors with positive strides and larger values for S and G than shown in the previous examples.

*Call Statement and Input*

```
             S      G   OMEGA  X  INCX  N   T  INCT  Y  INCY  M
             |      |     |    |   |    |   |   |    |   |    |
CALL SQINT( 3.0 , 10.0 , 1.0 , X , 1  , 8 , T , 2  , Y , 3  , 4 )

X        =  (1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0)
T        =  (1.5, . , 2.5, . , 3.5, . , 4.5)
```

*Output*

```
Y        =  (8.3, . , . , 8.5, . , . , 8.7, . , . , 8.9)
```

# SWLEV, DWLEV, CWLEV, and ZWLEV—Wiener-Levinson Filter Coefficients

These subroutines compute the coefficients of an n-point Wiener-Levinson filter, using vector ***x***, the trace for which the filter is to be designed, and vector ***u***, the right-hand side of the system, chosen to remove reverberations or sharpen the wavelet. The result is returned in vector ***y***.

*Table 142. Data Types*

| ***x***, ***u***, ***y*** | *aux* | **Subroutine** |
|---|---|---|
| Short-precision real | Long-precision real | SWLEV |
| Long-precision real | Long-precision real | DWLEV |
| Short-precision complex | Long-precision complex | CWLEV |
| Long-precision complex | Long-precision complex | ZWLEV |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SWLEV \| DWLEV \| CWLEV \| ZWLEV \| (*x*, *incx*, *u*, *incu*, *y*, *incy*, *n*, *aux*, *naux*) |
| **C and C++** | swlev \| dwlev \| cwlev \| zwlev (*x*, *incx*, *u*, *incu*, *y*, *incy*, *n*, *aux*, *naux*); |
| **PL/I** | CALL SWLEV \| DWLEV \| CWLEV \| ZWLEV (*x*, *incx*, *u*, *incu*, *y*, *incy*, *n*, *aux*, *naux*); |

*On Entry*

*x*

is the vector ***x*** of length *n*, containing the trace data for which the filter is to be designed.

For SWLEV and DWLEV, *x* represents the first row (or the first column) of a positive definite or negative definite symmetric Toeplitz matrix, which is the autocorrelation matrix for which the filter is designed.

For CWLEV and ZWLEV, *x* represents the first row of a positive definite or negative definite complex Hermitian Toeplitz matrix, which is the autocorrelation matrix for which the filter is designed.

Specified as: a one-dimensional array of (at least) length 1+(*n*–1)|*incx*|, containing numbers of the data type indicated in Table 142.

*incx*

is the stride for vector ***x***. Specified as: a fullword integer; *incx* > 0.

*u*

is the vector ***u*** of length *n*, containing the right-hand side of the system to be solved. Specified as: a one-dimensional array of (at least) length 1+(*n*–1)|*incu*|, containing numbers of the data type indicated in Table 142.

*incu*

is the stride for vector ***u***. Specified as: a fullword integer. It can have any value.

*y*

See “On Return” on page 877.

*incy*

is the stride for vector ***y***. Specified as: a fullword integer; *incy* > 0 or *incy* < 0.

*n*

is the number of elements in vectors ***x***, ***u***, and ***y***. Specified as: a fullword integer; *n* ≥ 0.

*aux*

has the following meaning:

If *naux* = 0 and error 2015 is unrecoverable, *aux* is ignored.

Otherwise, it is the storage work area used by these subroutines.

Specified as: an area of storage of length *naux*, containing numbers of the data type indicated in Table 142 on page 876.

*naux*

is the size of the work area specified by *aux*—that is, the number of elements in *aux*. Specified as: a fullword integer, where:

If *naux* = 0 and error 2015 is unrecoverable, SWLEV, DWLEV, CWLEV, and ZWLEV dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, *naux* ≥ 3*n*.

You cannot use dynamic allocation if you need the information returned in `AUX(1)`.

*On Return*

*y*

is the vector ***y*** of length *n*, containing the solution vector—that is, the coefficients of the n-point Wiener-Levinson filter. Returned as: a one-dimensional array of (at least) length 1+(*n*–1)|*incy*|, containing numbers of the data type indicated in Table 142 on page 876.

*aux*

is the storage work area used by these subroutines, where if *naux* ≠ 0:

If `AUX(1)` = 0.0, the input Toeplitz matrix is positive definite or negative definite.

If `AUX(1)` > 0.0, the input Toeplitz matrix is indefinite (that is, it is not positive definite and it is not negative definite). The value returned in `AUX(1)` is the order of the first submatrix of ***A*** that is indefinite. The subroutine continues processing. See reference [60] for information about under what circumstances your solution vector *y* would be valid.

All other values in *aux* are overwritten and are not significant.

Returned as: an area of storage of length *naux*, containing numbers of the data type indicated in Table 142 on page 876, where `AUX(1)`≥0.0.

**Notes**

1. For a description of a positive definite or negative definite symmetric Toeplitz matrix, see “Positive Definite or Negative Definite Symmetric Toeplitz Matrix” on page 71.
2. For a description of a positive definite or negative definite complex Hermitian Toeplitz matrix, see “Positive Definite or Negative Definite Complex Hermitian Toeplitz Matrix” on page 72.
3. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see “Using Auxiliary Storage in ESSL” on page 32.

***Function:*** The computation of the coefficients of an n-point Wiener-Levinson filter in vector ***y*** is expressed as solving the following system:

$$\boldsymbol{Ay} = \boldsymbol{u}$$

where:

- For SWLEV and DWLEV, matrix ***A*** is a real symmetric Toeplitz matrix whose first row (or first column) is represented by vector ***x***.

  For CWLEV and ZWLEV, matrix ***A*** is a complex Hermitian Toeplitz matrix whose first row is represented by vector ***x***.
- ***u*** is the vector specifying the right side of the system, chosen to remove reverberations or to sharpen the wavelet.
- ***y*** is the solution vector.

See reference [60 ], [27 ], and the *IBM 3838 Array Processor Functional Characteristics*.

If *n* is 0, no computation is performed. For SWLEV and CWLEV, intermediate results are accumulated in long precision.

SWLEV provides the same function as the IBM 3838 function WLEV, with restrictions removed. See the *IBM 3838 Array Processor Functional Characteristics* manual.

***Error Conditions***

*Resource Errors:* Error 2015 is unrecoverable, *naux* = 0, and unable to allocate work area.

*Computational Errors:* None

*Input-Argument Errors*

1. *n* < 0
2. *incx* ≤ 0
3. *incy* = 0
4. Error 2015 is recoverable or *naux*≠0, and *naux* is too small—that is, less than the minimum required value specified in the syntax for this argument. Return code 1 is returned if error 2015 is recoverable.

***Example 1:*** This example shows how to compute filter coefficients in vector ***y*** by solving the system ***Ay*** = ***u***. Matrix ***A*** is:

$$\begin{bmatrix} 50.0 & -8.0 & 7.0 & -5.0 \\ -8.0 & 50.0 & -8.0 & 7.0 \\ 7.0 & -8.0 & 50.0 & -8.0 \\ -5.0 & 7.0 & -8.0 & 50.0 \end{bmatrix}$$

This input Toeplitz matrix is positive definite, as indicated by the zero value in `AUX(1)` on output.

*Call Statement and Input*

```
           X  INCX  U  INCU  Y  INCY  N   AUX  NAUX
           |  |     |  |     |  |     |   |    |
CALL SWLEV( X , 1  , U , 1  , Y , 1  , 4 , AUX , 12 )
```

```
X        =  (50.0, -8.0, 7.0, -5.0)
U        =  (40.0, -10.0, 30.0, 20.0)
AUX      =(not relevant)
```

*Output*

```
Y        =  (0.7667, -0.0663, 0.5745, 0.5778)
AUX      =  (0.0, . , . , . , . , . , . , . , . , . , . , . )
```

***Example 2:*** This example shows how to compute filter coefficients in vector ***y*** by solving the system ***Ay*** = ***u***. Matrix ***A*** is:

$$\begin{bmatrix} 10.0 & -8.0 & 7.0 & -5.0 \\ -8.0 & 10.0 & -8.0 & 7.0 \\ 7.0 & -8.0 & 10.0 & -8.0 \\ -5.0 & 7.0 & -8.0 & 10.0 \end{bmatrix}$$

This input Toeplitz matrix is not positive definite, as indicated by the zero value in `AUX(1)` on output.

*Call Statement and Input*

```
           X  INCX  U  INCU  Y  INCY  N   AUX  NAUX
           |  |     |  |     |  |     |   |    |
CALL SWLEV( X , 1  , U , 1  , Y , 1  , 4 , AUX , 12 )
```

```
X        =  (10.0, -8.0, 7.0, -5.0)
U        =  (40.0, -10.0, 30.0, 20.0)
AUX      =(not relevant)
```

*Output*

```
Y        =  (5.1111, 5.5555, 12.2222, 10.4444)
AUX      =  (0.0, . , . , . , . , . , . , . , . , . , . , . )
```

***Example 3:*** This example shows a vector ***x*** with a stride greater than 1, a vector ***u*** with a negative stride, and a vector ***y*** with a stride of 1. It uses the same input Toeplitz matrix as in Example 2, which is not positive definite.

*Call Statement and Input*

```
           X  INCX  U  INCU  Y  INCY  N   AUX  NAUX
           |  |     |    |   |  |     |   |    |
CALL SWLEV( X , 2  , U , -2 , Y , 1  , 4 , AUX , 12 )
```

```
X        =  (10.0, . , -8.0, . , 7.0, . , -5.0)
U        =  (20.0, . , 30.0, . , -10.0, . , 40.0)
AUX      =(not relevant)
```

*Output*

```
Y        =  (5.1111, 5.5555, 12.2222, 10.4444)
AUX      =  (0.0, . , . , . , . , . , . , . , . , . , . , . )
```

***Example 4:*** This example shows how to compute filter coefficients in vector ***y*** by solving the system ***Ay*** = ***u***. Matrix ***A*** is:

$$\begin{bmatrix} (10.0, 0.0) & (2.0, -3.0) & (-3.0, 1.0) & (1.0, 1.0) \\ (2.0, 3.0) & (10.0, 0.0) & (2.0, -3.0) & (-3.0, 1.0) \\ (-3.0, -1.0) & (2.0, 3.0) & (10.0, 0.0) & (2.0, -3.0) \\ (1.0, -1.0) & (-3.0, -1.0) & (2.0, 3.0) & (10.0, 0.0) \end{bmatrix}$$

This input complex Hermitian Toeplitz matrix is positive definite, as indicated by the zero value in AUX(1) on output.

*Call Statement and Input*

```
            X  INCX  U  INCU  Y  INCY  N   AUX  NAUX
            |   |    |   |    |   |    |    |    |
CALL ZWLEV( X , 1  , U , 1  , Y , 1  , 4 , AUX , 12 )

X        =  ((10.0, 0.0), (2.0, -3.0), (-3.0, 1.0), (1.0, 1.0))
U        =  ((8.0, 3.0), (21.0, -5.0), (67.0, -13.0), (72.0, 11.0))
AUX      =(not relevant)
```

*Output*

```
Y        =  ((1.0, 0.0), (3.0, 0.0), (5.0, 0.0), (7.0, 0.0))
AUX      =  ((0.0, 0.0), . , . , . , . , . , . , . , . , . , . , . )
```

***Example 5:*** This example shows a vector ***x*** with a stride greater than 1, a vector ***u*** with a negative stride, and a vector ***y*** with a stride of 1. It uses the same input complex Hermitian Toeplitz matrix as in Example 4.

This input complex Hermitian Toeplitz matrix is positive definite, as indicated by the zero value in AUX(1) on output.

*Call Statement and Input*

```
            X  INCX  U  INCU   Y  INCY  N   AUX  NAUX
            |   |    |   |     |   |    |    |    |
CALL ZWLEV( X , 2  , U , -2  , Y , 1  , 4 , AUX , 12 )

X        =  ((10.0, 0.0), . , (2.0, -3.0), . , (-3.0, 1.0), . ,
             (1.0, 1.0))
U        =  ((72.0, 11.0), . , (67.0, -13.0), . , (21.0, -5.0), . ,
             (8.0, 3.0), . )
AUX      =(not relevant)
```

*Output*

```
Y        =  ((1.0, 0.0), (3.0, 0.0), (5.0, 0.0), (7.0, 0.0))
AUX      =  ((0.0, 0.0), . , . , . , . , . , . , . , . , . , . , . )
```

# Chapter 13. Sorting and Searching

The sorting and searching subroutines are described in this chapter.

## Overview of the Sorting and Searching Subroutines

The sorting and searching subroutines operate on three types of data: integer, short-precision real, and long-precision-real ( Table 143). The sorting subroutines perform sorts with or without index designations. The searching subroutines perform either a binary or sequential search.

*Table 143. List of Sorting and Searching Subroutines*

| **Descriptive Name** | **Integer Subroutine** | **Short-Precision Subroutine** | **Long-Precision Subroutine** | **Page** |
|---|---|---|---|---|
| Sort the Elements of a Sequence | ISORT | SSORT | DSORT | 884 |
| Sort the Elements of a Sequence and Note the Original Element Positions | ISORTX | SSORTX | DSORTX | 886 |
| Sort the Elements of a Sequence Using a Stable Sort and Note the Original Element Positions | ISORTS | SSORTS | DSORTS | 889 |
| Binary Search for Elements of a Sequence X in a Sorted Sequence Y | IBSRCH | SBSRCH | DBSRCH | 892 |
| Sequential Search for Elements of a Sequence X in the Sequence Y | ISSRCH | SSSRCH | DSSRCH | 896 |

## Use Considerations

It is important to understand the concept of stride for sequences when using these subroutines. For example, in the sort subroutines, a negative stride causes a sequence to be sorted into descending order in an array. In the search subroutines, a negative stride reverses the direction of the search. See “How Stride Is Used for Vectors” on page 58.

## Performance and Accuracy Considerations

1. The binary search subroutines provide better performance than the sequential search subroutines because of the nature of the searching algorithms. However, the binary search subroutines require that, before the subroutine is called, the sequence to be searched is sorted into ascending order. Therefore, if your data is already sorted, a binary search subroutine is faster. On the other hand, if your data is in random order and the number of elements being searched for is small, a sequential search subroutine is faster than doing a sort and binary search.

2. When doing multiple invocations of the binary search subroutines, you get better overall performance from the searching algorithms by doing fewer invocations and specifying larger search element arrays for argument *x*.

3. If you do not need the results provided in array RC by these subroutine, you get better performance if you do not request it. That is, specify 0 for the *iopt* argument.

## Sorting and Searching Subroutines

This section contains the sorting and searching subroutine descriptions.

## ISORT, SSORT, and DSORT—Sort the Elements of a Sequence

These subroutines sort the elements of sequence ***x***.

*Table 144. Data Types*

| ***x*** | **Subroutine** |
|---|---|
| Integer | ISORT |
| Short-precision real | SSORT |
| Long-precision real | DSORT |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL ISORT \| SSORT \| DSORT (*x*, *incx*, *n*) |
| **C and C++** | isort \| ssort \| dsort (*x*, *incx*, *n*); |
| **PL/I** | CALL ISORT \| SSORT \| DSORT (*x*, *incx*, *n*); |

*On Entry*

*x*
is the sequence ***x*** of length *n*, to be sorted. Specified as: a one-dimensional array of (at least) length 1+(*n*–1)|*incx*|, containing numbers of the data type indicated in Table 144.

*incx*
is the stride for both the input sequence ***x*** and the output sequence ***x***. If it is positive, elements are sorted into ascending order in the array, and if it is negative, elements are sorted into descending order in the array.

Specified as: a fullword integer. It can have any value.

*n*
is the number of elements in sequence ***x***. Specified as: a fullword integer; $n \geq 0$.

*On Return*

*x*
is the sequence ***x*** of length *n*, with its elements sorted into designated order in the array. Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 144.

***Function:*** The elements of input sequence ***x*** are sorted into ascending order, in place and using a partition sort. The elements of output sequence ***x*** can be expressed as follows:

$$x_1 \leq x_2 \leq x_3 \leq \ldots \leq x_n$$

By specifying a negative stride for sequence ***x***, the elements of sequence ***x*** are assumed to be reversed in the array, $(x_n, x_{n-1}, \ldots , x_1)$, thus producing a sort into descending order within the array. If *n* is 0 or 1 or if *incx* is 0, no sort is performed. See reference [72].

### *Error Conditions*

*Resource Errors:* Unable to allocate internal work area.

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Example 1:*** This example shows a sequence ***x*** with a positive stride.

*Call Statement and Input*

```
            X  INCX  N
            |   |    |
CALL ISORT( X , 2  , 5 )

X        =  (2, . , -1, . , 5, . , 4, . , -2)
```

*Output*

```
X        =  (-2, . , -1, . , 2, . , 4, . , 5)
```

***Example 2:*** This example shows a sequence ***x*** with a negative stride.

*Call Statement and Input*

```
            X   INCX  N
            |    |    |
CALL ISORT( X , -1  , 5 )

X        =  (2, -1, 5, 4, -2)
```

*Output*

```
X        =  (5, 4, 2, -1, -2)
```

# ISORTX, SSORTX, and DSORTX—Sort the Elements of a Sequence and Note the Original Element Positions

These subroutines sort the elements of sequence ***x***. The original positions of the elements in sequence ***x*** are returned in the indices array, `INDX`. Where equal elements occur in the input sequence, they do not necessarily remain in the same relative order in the output sequence.

**Note:** If you need a stable sort, you should use ISORTS, SSORTS, or DSORTS rather than these subroutines.

*Table 145. Data Types*

| ***x*** | **Subroutine** |
|---|---|
| Integer | ISORTX |
| Short-precision real | SSORTX |
| Long-precision real | DSORTX |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL ISORTX \| SSORTX \| DSORTX (*x*, *incx*, *n*, *indx*) |
| **C and C++** | isortx \| ssortx \| dsortx (*x*, *incx*, *n*, *indx*); |
| **PL/I** | CALL ISORTX \| SSORTX \| DSORTX (*x*, *incx*, *n*, *indx*); |

*On Entry*

*x*

is the sequence ***x*** of length *n*, to be sorted. Specified as: a one-dimensional array of (at least) length 1+(*n*−1)|*incx*| elements, containing numbers of the data type indicated in Table 145.

*incx*

is the stride for both the input sequence ***x*** and the output sequence ***x***. If it is positive, elements are sorted into ascending order in the array, and if it is negative, elements are sorted into descending order in the array.

Specified as: a fullword integer. It can have any value.

*n*

is the number of elements in sequence ***x***. Specified as: a fullword integer; $n \geq 0$.

*indx*

See “On Return.”

*On Return*

*x*

is the sequence ***x*** of length *n*, with its elements sorted into designated order in the array. Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 145.

*indx*

is the array, referred to as `INDX`, containing the *n* indices that indicate, for the elements in the sorted output sequence, the original positions of those elements in input sequence ***x***.

**Note:** It is important to remember that when you specify a negative stride, ESSL assumes that the order of the input and output sequence elements in the X array is reversed; however, the elements in INDX are not reversed. See "Function" on page 887.

Returned as: a one-dimensional array of length $n$, containing fullword integers; $1 \leq$ (INDX elements) $\leq n$.

***Function:*** The elements of input sequence ***x*** are sorted into ascending order, in place and using a partition sort. The elements of output sequence ***x*** can be expressed as follows:

$x_1 \leq x_2 \leq x_3 \leq ... \leq x_n$

Where equal elements occur in the input sequence, they do not necessarily remain in the same relative order in the output sequence.

By specifying a negative stride for ***x***, the elements of input sequence ***x*** are assumed to be reversed in the array, $(x_n, x_{n-1}, ... , x_1)$, thus producing a sort into descending order within the array.

In addition, the INDX array contains the $n$ indices that indicate, for the elements in the sorted output sequence, the original positions of those elements in input sequence ***x***. (These are not the positions in the array, but rather the positions in the sequence.) For each element $x_j$ in the input sequence, becoming element $xx_k$ in the output sequence, the elements in INDX are defined as follows:

INDX($k$) = $j$     for $j = 1, n$ and $k = 1, n$
where $xx_k = x_j$

To understand INDX when you specify a negative stride, you should remember that both the input and output sequences, ***x***, are assumed to be in reverse order in array X, but INDX is not affected by stride. The sequence elements of ***x*** are assumed to be stored in your input array as follows:

X = $(x_n, x_{n-1}, ... , x_1)$

The sequence elements of ***x*** are stored in your output array by ESSL as follows:

X = $(xx_n, xx_{n-1}, ... , xx_1)$

where the elements $xx_k$ are the elements $x_j$, sorted into descending order in X. As an example of how INDX is calculated, if $xx_1 = x_{n-1}$, then INDX(1) = $n$–1.

If $n$ is 0, no computation is performed. See reference [72 ].

***Error Conditions***

*Resource Errors:* Unable to allocate internal work area.

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Example 1:*** This example shows how to sort a sequence ***x*** into ascending order by specifying a positive stride.

*Call Statement and Input*

```
             X  INCX  N   INDX
             |   |    |    |
CALL ISORTX( X , 2  , 5 , INDX )

X        =  (2, . , -1, . , 5, . , 1, . , -2)
```

*Output*

```
X        =  (-2, . , -1, . , 1, . , 2, . , 5)
INDX     =  (5, 2, 4, 1, 3)
```

***Example 2:*** This example shows how to sort a sequence ***x*** into descending order by specifying a negative stride. Therefore, both the input and output sequences are assumed to be reversed in the array X. The input sequence is assumed to be stored as follows:

X = ($x_5$, $x_4$, $x_3$, $x_2$, $x_1$) = (2, -1, 5, 1, -2)

The output sequence is stored by ESSL as follows:

X = ($xx_5$, $xx_4$, $xx_3$, $xx_2$, $xx_1$) = (5, 2, 1, -1, -2)

As a result, INDX is defined as follows:

INDX = ($indx_1$, $indx_2$, $indx_3$, $indx_4$, $indx_5$) = (1, 4, 2, 5, 3)

For example, because output sequence element $xx_4$ = 2 is input sequence element $x_5$, then INDX(4) = 5.

*Call Statement and Input*

```
             X   INCX  N   INDX
             |    |    |    |
CALL ISORTX( X , -1  , 5 , INDX )

X        =  (2, -1, 5, 1, -2)
```

*Output*

```
X        =  (5, 2, 1, -1, -2)
INDX     =  (1, 4, 2, 5, 3)
```

## ISORTS, SSORTS, and DSORTS—Sort the Elements of a Sequence Using a Stable Sort and Note the Original Element Positions

These subroutines sort the elements of sequence ***x*** using a stable sort; that is, where equal elements occur in the input sequence, they remain in the same relative order in the output sequence. The original positions of the elements in sequence ***x*** are returned in the indices array `INDX`.

**Note:** If you need a stable sort, then you should use these subroutines rather than ISORTX, SSORTX, or DSORTX.

*Table 146. Data Types*

| ***x***, *work* | **Subroutine** |
|---|---|
| Integer | ISORTS |
| Short-precision real | SSORTS |
| Long-precision real | DSORTS |

***Syntax***

| | |
|---|---|
| **Fortran** | CALL ISORTS \| SSORTS \| DSORTS (*x*, *incx*, *n*, *indx*, *work*, *lwork*) |
| **C and C++** | isorts \| ssorts \| dsorts (*x*, *incx*, *n*, *indx*, *work*, *lwork*); |
| **PL/I** | CALL ISORTS \| SSORTS \| DSORTS (*x*, *incx*, *n*, *indx*, *work*, *lwork*); |

*On Entry*

*x*
: is the sequence ***x*** of length *n*, to be sorted. Specified as: a one-dimensional array of (at least) length 1+(*n*–1)|*incx*| elements, containing numbers of the data type indicated in Table 146.

*incx*
: is the stride for both the input sequence ***x*** and the output sequence ***x***. If it is positive, elements are sorted into ascending order in the array, and if it is negative, elements are sorted into descending order in the array.

Specified as: a fullword integer. It can have any value.

*n*
: is the number of elements in sequence ***x***. Specified as: a fullword integer; $n \geq 0$.

*indx*
: See "On Return" on page 890.

*work*
: is the storage work area used by this subroutine. Its size is specified by *lwork*. Specified as: an area of storage, containing numbers of the data type indicated in Table 146.

*lwork*
: is the size of the work area specified by *work*— that is, the number of elements in *work*. Specified as: a fullword integer; $lwork \geq n/2$.

**Note:** This is the value to achieve optimal performance. The sort is performed regardless of the value you specify for *lwork*, but you may receive an attention message.

*On Return*

*x*

is the sequence ***x*** of length $n$, with its elements sorted into designated order in the array. Returned as: a one-dimensional array, containing numbers of the data type indicated in Table 146 on page 889.

*indx*

is the array, referred to as `INDX`, containing the $n$ indices that indicate, for the elements in the sorted output sequence, the original positions of those elements in input sequence ***x***.

**Note:** It is important to remember that when you specify a negative stride, ESSL assumes that the order of the input and output sequence elements in the `X` array is reversed; however, the elements in `INDX` are not reversed. See “Function.”

Returned as: a one-dimensional array of length $n$, containing fullword integers; $1 \leq$ (`INDX` elements) $\leq n$.

***Function:*** The elements of input sequence ***x*** are sorted into ascending order using a partition sort. The sorting is stable; that is, where equal elements occur in the input sequence, they remain in the same relative order in the output sequence. The elements of output sequence ***x*** can be expressed as follows:

$$x_1 \leq x_2 \leq x_3 \leq \ldots \leq x_n$$

By specifying a negative stride for ***x***, the elements of input sequence ***x*** are assumed to be reversed in the array, $(x_n, x_{n-1}, \ldots , x_1)$, thus producing a sort into descending order within the array.

In addition, the `INDX` array contains the $n$ indices that indicate, for the elements in the sorted output sequence, the original positions of those elements in input sequence ***x***. (These are not the positions in the array, but rather the positions in the sequence.) For each element $x_j$ in the input sequence, becoming element $xx_k$ in the output sequence, the elements in `INDX` are defined as follows:

$\mathtt{INDX}(k) = j$ for $j = 1, n$ and $k = 1, n$
where $xx_k = x_j$

To understand `INDX` when you specify a negative stride, you should remember that both the input and output sequences, ***x***, are assumed to be in reverse order in array `X`, but `INDX` is not affected by stride. The sequence elements of ***x*** are assumed to be stored in your input array as follows:

$$\mathtt{X} = (x_n, x_{n-1}, \ldots , x_1)$$

The sequence elements of ***x*** are stored in your output array by ESSL as follows:

$$\mathtt{X} = (xx_n, xx_{n-1}, \ldots , xx_1)$$

where the elements $xx_k$ are the elements $x_j$, sorted into descending order in `X`. As an example of how `INDX` is calculated, if $xx_1 = x_{n-1}$, then $\mathtt{INDX}(1) = n-1$.

If $n$ is 0, no computation is performed. See references [28 ] and [72 ].

***Error Conditions***

*Resource Errors:* Unable to allocate internal work area.

*Computational Errors:* None

*Input-Argument Errors:* $n < 0$

***Example 1:*** This example shows how to sort a sequence ***x*** into ascending order by specifying a positive stride. Because this is a stable sort, the −1 elements remain in the same relative order in the output sequence, indicated by `INDX(2)` = 2 and `INDX(3)` = 4.

*Call Statement and Input*

```
             X  INCX  N   INDX   WORK  LWORK
             |   |    |    |      |      |
CALL ISORTS( X , 2  , 5 , INDX , WORK ,  5  )

X        =  (2, . , -1, . , 5, . , -1, . , -2)
```

*Output*

```
X        =  (-2, . , -1, . , -1, . , 2, . , 5)
INDX     =  (5, 2, 4, 1, 3)
```

***Example 2:*** This example shows how to sort a sequence ***x*** into descending order by specifying a negative stride. Therefore, both the input and output sequences are assumed to be reversed in the array `X`. The input sequence is assumed to be stored as follows:

> `X` = ($x_5$, $x_4$, $x_3$, $x_2$, $x_1$) = (2, -1, 5, -1, -2)

The output sequence is stored by ESSL as follows:

> `X` = ($xx_5$, $xx_4$, $xx_3$, $xx_2$, $xx_1$) = (5, 2, -1, -1, -2)

As a result, `INDX` is defined as follows:

> `INDX` = ($indx_1$, $indx_2$, $indx_3$, $indx_4$, $indx_5$) = (1, 2, 4, 5, 3)

For example, because output sequence element $xx_4 = 2$ is input sequence element $x_5$, then `INDX`(4) = 5. Also, because this is a stable sort, the −1 elements remain in the same relative order in the output sequence, indicated by `INDX(2)` = 2 and `INDX(3)` = 4.

*Call Statement and Input*

```
             X   INCX  N   INDX   WORK  LWORK
             |    |    |    |      |      |
CALL ISORTS( X , -1  , 5 , INDX , WORK ,  5  )

X        =  (2, -1, 5, -1, -2)
```

*Output*

```
X        =  (5, 2, -1, -1, -2)
INDX     =  (1, 2, 4, 5, 3)
```

## IBSRCH, SBSRCH, and DBSRCH—Binary Search for Elements of a Sequence X in a Sorted Sequence Y

These subroutines perform a binary search for the locations of the elements of sequence ***x*** in another sequence ***y***, where ***y*** has been sorted into ascending order. The first occurrence of each element is found. When an exact match is not found, the position of the next larger element in ***y*** is indicated. The locations are returned in the indices array `INDX`, and, optionally, return codes indicating whether the exact elements were found are returned in array `RC`.

*Table 147. Data Types*

| ***x***, ***y*** | **Subroutine** |
|---|---|
| Integer | IBSRCH |
| Short-precision real | SBSRCH |
| Long-precision real | DBSRCH |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL IBSRCH \| SBSRCH \| DBSRCH (*x*, *incx*, *n*, *y*, *incy*, *m*, *indx*, *rc*, *iopt*) |
| **C and C++** | ibsrch \| sbsrch \| dbsrch (*x*, *incx*, *n*, *y*, *incy*, *m*, *indx*, *rc*, *iopt*); |
| **PL/I** | CALL IBSRCH \| SBSRCH \| DBSRCH (*x*, *incx*, *n*, *y*, *incy*, *m*, *indx*, *rc*, *iopt*); |

*On Entry*

*x*
is the sequence ***x*** of length *n*, containing the elements for which sequence ***y*** is searched. Specified as: a one-dimensional array, containing numbers of the data type indicated in Table 147. It must have at least 1+(*n*–1)|*incx*| elements.

*incx*
is the stride for sequence ***x***. Specified as: a fullword integer. It can have any value.

*n*
is the number of elements in sequence ***x*** and arrays `INDX` and `RC`. Specified as: a fullword integer; $n \geq 0$.

*y*
is the sequence ***y*** of length *m*, to be searched, where ***y*** must be sorted into ascending order.

**Note:** Be careful in specifying the stride for sequence ***y***. A negative stride reverses the direction of the search, because the order of the sequence elements is reversed in the array.

Specified as: a one-dimensional array of (at least) length 1+(*m*–1)|*incy*|, containing numbers of the data type indicated in Table 147.

*incy*
is the stride for sequence ***y***. Specified as: a fullword integer. It can have any value.

*m*
is the number of elements in sequence ***y***. Specified as: a fullword integer; $m \geq 0$.

*indx*
See “On Return” on page 893.

*rc*
See “On Return” on page 893.

*iopt*
has the following meaning, where:

If *iopt* = 0, the *rc* argument is not used in the computation.

If *iopt* = 1, the *rc* argument is used in the computation.

Specified as: a fullword integer; *iopt* = 0 or 1.

*On Return*

*indx*
is the array, referred to as `INDX`, containing the *n* indices that indicate the positions of the elements of sequence ***x*** in sequence ***y***. The first occurrence of the element found in sequence ***y*** is indicated in array `INDX`. When an exact match between an element of sequence ***x*** and an element of sequence ***y*** is not found, the position of the next larger element in sequence ***y*** is indicated. When the element in sequence ***x*** is larger than all the elements in sequence ***y***, then *m*+1 is indicated in array `INDX`.

Returned as: a one-dimensional array of length *n*, containing fullword integers; 1 ≤ (`INDX` elements) ≤ *m*+1.

*rc*
has the following meaning, where:

If *iopt* = 0, then *rc* is not used, and its contents remain unchanged.

If *iopt* = 1, it is the array, referred to as `RC`, containing the *n* return codes that indicate whether the elements in sequence ***x*** were found in sequence ***y***. For *i* = 1, *n*, elements `RC`(*i*) = 0 if $x_i$ matches an element in sequence ***y***, and `RC`(*i*) = 1 if an exact match is not found in sequence ***y***.

Returned as: a one-dimensional array of length *n*, containing fullword integers; `RC`(*i*) = 0 or 1.

***Notes***

1. The elements of ***y*** must be sorted into ascending order; otherwise, results are unpredictable. For details on how to do this, see “ISORT, SSORT, and DSORT—Sort the Elements of a Sequence” on page 884.
2. If you do not need the results provided in array `RC` by these subroutines, you get better performance if you do not request it. That is, specify 0 for the *iopt* argument.

***Function:*** These subroutines perform a binary search for the first occurrence (or last occurrence, using negative stride) of the locations of the elements of sequence ***x*** in another sequence ***y***, where ***y*** must be sorted into ascending order before calling this subroutine. The first occurrence of each element is found. Two arrays are returned, containing the results of the binary searches:

- `INDX`, the indices array, contains the positions of the elements of sequence ***x*** in sequence ***y***. When an exact match between values of elements in sequences ***x*** and ***y*** is not found, the location of the next larger element in sequence ***y*** is indicated in array `INDX`.

- RC, the return codes array, indicates for each element in sequence ***x*** whether the exact element was found in sequence ***y***. If you do not need these results, you get better performance if you set *iopt* = 0.

The results returned for the INDX and RC arrays are expressed as follows:

```
For $i = 1, n$
    for all $y_j \geq x_i$, $j = 1, m$ , INDX($i$) = min($j$)
    if all $y_j < x_i$, $j = 1, m$ , INDX($i$) = $m$+1
And for $i = 1, n$
    if $x_i = y_{\mathrm{INDX}(i)}$, RC($i$) = 0
    if $x_i \neq y_{\mathrm{INDX}(i)}$, RC($i$) = 1
```

where:

***x*** is a sequence of length *n*, containing the search elements.
***y*** is a sequence of length *m* to be searched. It must be sorted into ascending order.
INDX is the array of length *n* of indices.
RC is the array of length *n* of return codes.

See reference [72 ]. If *n* is 0, no search is performed. If *m* is 0, then:

INDX($i$) = 1 and RC($i$) = 1     for $i = 1, n$

It is important to note that a negative stride for sequence ***y*** reverses the direction of the search, because the order of the sequence elements is reversed in the array. For more details on sorting sequences, see “Function” on page 884.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. $n < 0$
2. $m < 0$
3. *iopt* ≠ 0 or 1

***Example 1:*** This example shows a search where sequences ***x*** and ***y*** have positive strides, and where the optional return codes are returned as part of the output.

*Call Statement and Input*

```
              X  INCX  N   Y  INCY  M    INDX   RC  IOPT
              |   |    |   |   |    |     |     |    |
CALL IBSRCH( X , 2  , 5 , Y , 1  , 10 , INDX , RC , 1 )

X        =  (-3, . , 125, . , 30, . , 20, . , 70)
Y        =  (10, 20, 30, 30, 40, 50, 60, 80, 90, 100)
```

*Output*

```
INDX     =  (1, 11, 3, 2, 8)
RC       =  (1, 1, 0, 0, 1)
```

***Example 2:*** This example shows the same calling sequence as in Example 1, except that it includes the IOPT argument, specified as 1. This is equivalent to using the calling sequence in Example 1 and gives the same results.

*Call Statement and Input*

```
             X  INCX  N   Y  INCY  M    INDX   RC  IOPT
             |  |     |   |  |     |    |      |   |
CALL IBSRCH( X , 2  , 5 , Y , 1  , 10 , INDX , RC , 1  )
```

***Example 3:*** This example shows a search where sequence ***x*** has a negative stride, and sequence ***y*** has a positive stride. The optional return codes are not requested, because IOPT is specified as 0.

*Call Statement and Input*

```
             X   INCX  N   Y  INCY  M    INDX   RC  IOPT
             |   |     |   |  |     |    |      |   |
CALL IBSRCH( X , -2  , 5 , Y , 1  , 10 , INDX , RC , 0 )

X        =  (-3, . , 125, . , 30, . , 20, . , 70)
Y        =  (10, 20, 30, 30, 40, 50, 60, 80, 90, 100)
```

*Output*

```
INDX     =  (8, 2, 3, 11, 1)
RC       =(not relevant)
```

***Example 4:*** This example shows a search where sequence ***x*** has a positive stride, and sequence ***y*** has a negative stride. As shown below, elements of ***y*** are in descending order in array Y. The optional return codes are not requested, because IOPT is specified as 0.

*Call Statement and Input*

```
             X  INCX  N   Y   INCY  M    INDX   RC  IOPT
             |  |     |   |   |     |    |      |   |
CALL IBSRCH( X , 2  , 5 , Y , -1  , 10 , INDX , RC , 0  )

X        =  (-3, . , 125, . , 30, . , 20, . , 70)
Y        =  (100, 90, 80, 60, 50, 40, 30, 30, 20, 10)
RC       =(not relevant)
```

*Output*

```
INDX     =  (1, 11, 3, 2, 8)
```

# ISSRCH, SSSRCH, and DSSRCH—Sequential Search for Elements of a Sequence X in the Sequence Y

These subroutines perform a sequential search for the locations of the elements of sequence ***x*** in another sequence ***y***. Depending on the sign of the *idir* argument, the search direction indicator, the location of either the first or last occurrence of each element is indicated in the resulting indices array `INDX`. When an exact match between elements is not found, the position is indicated as 0.

*Table 148. Data Types*

| ***x***, ***y*** | **Subroutine** |
|---|---|
| Integer | ISSRCH |
| Short-precision real | SSSRCH |
| Long-precision real | DSSRCH |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL ISSRCH \| SSSRCH \| DSSRCH (*x*, *incx*, *n*, *y*, *incy*, *m*, *idir*, *indx*) |
| **C and C++** | issrch \| sssrch \| dssrch (*x*, *incx*, *n*, *y*, *incy*, *m*, *idir*, *indx*); |
| **PL/I** | CALL ISSRCH \| SSSRCH \| DSSRCH (*x*, *incx*, *n*, *y*, *incy*, *m*, *idir*, *indx*); |

*On Entry*

*x*

is the sequence ***x*** of length *n*, containing the elements for which sequence ***y*** is searched. Specified as: a one-dimensional array of (at least) length 1+(*n*–1)|*incx*|, containing numbers of the data type indicated in Table 148.

*incx*

is the stride for sequence ***x***. Specified as: a fullword integer. It can have any value.

*n*

is the number of elements in sequence ***x*** and array `INDX`. Specified as: a fullword integer; *n* ≥ 0.

*y*

is the sequence ***y*** of length *m* to be searched.

**Note:** Be careful in specifying the stride for sequence ***y***. A negative stride reverses the direction of the search, because the order of the sequence elements is reversed in the array.

Specified as: a one-dimensional array of (at least) length 1+(*m*–1)|*incy*|, containing numbers of the data type indicated in Table 148.

*incy*

is the stride for sequence ***y***. Specified as: a fullword integer. It can have any value.

*m*

is the number of elements in sequence ***y***. Specified as: a fullword integer; *m* ≥ 0.

*idir*

indicates the search direction, where:

If *idir* ≥ 0, sequence ***y*** is searched from the first element to the last (1, *n*), thus finding the first occurrence of the element in the sequence.

If *idir* < 0, sequence ***y*** is searched from the last element to the first (*n*, 1), thus finding the last occurrence of the element in the sequence.

Specified as: a fullword integer. It can have any value.

*indx*

See On Return.

*On Return*

*indx*

is the array, referred to as INDX, containing the *n* indices that indicate the positions of the elements of sequence ***x*** in sequence ***y***, where:

If *idir* ≥ 0, the first occurrence of the element found in sequence ***y*** is indicated in array INDX.

If *idir* < 0, the last occurrence of the element found in sequence ***y*** is indicated in array INDX.

In all cases, if no match is found, 0 is indicated in array INDX.

Returned as: a one-dimensional array of length *n*, containing fullword integers; 0 ≤ (INDX elements) ≤ *m*.

***Function:*** These subroutines perform a sequential search for the first occurrence (or last occurrence, using a negative *idir*) of the locations of the elements of sequence ***x*** in another sequence ***y***. The results of the sequential searches are returned in the indices array INDX, indicating the positions of the elements of sequence ***x*** in sequence ***y***. The positions indicated in array INDX are calculated relative to the first sequence element position—that is, the position of $y_1$. When an exact match between values of elements in sequences ***x*** and ***y*** is not found, 0 is indicated in array INDX for that position.

The results returned in array INDX are expressed as follows:

```
For $i = 1, n$
    for all $y_j = x_i, j = 1, m$
        INDX($i$) = min($j$), if $idir \geq 0$
        INDX($i$) = max($j$), if $idir < 0$
    if all $y_j \neq x_i, j = 1, m$
        INDX($i$) = 0
```

where:

***x*** is a sequence of length *n*, containing the search elements.
***y*** is a sequence of length *m* to be searched.
INDX is the array of length *n* of indices.

See reference [72 ]. If *n* is 0, no search is performed.

It is important to note that a negative stride for sequence ***y*** reverses the direction of the search, because the order of the sequence elements is reversed in the array.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. $n < 0$
2. $m < 0$

***Example 1:*** This example shows a search where sequences ***x*** and ***y*** have positive strides, and the search direction indicator, *idir*, is positive.

*Call Statement and Input*

```
             X  INCX  N   Y  INCY  M  IDIR  INDX
             |   |    |   |   |    |   |     |
CALL ISSRCH( X , 1  , 3 , Y , 2  , 8 , 1  , INDX )

X        =  (0, 12, 3)
Y        =  (0, . , 8, . , 12, . , 0, . , 1, . , 4, . , 0, . , 2)
```

*Output*

```
INDX     =  (1, 3, 0)
```

***Example 2:*** This example shows a search where sequences ***x*** and ***y*** have positive strides, and the search direction indicator, *idir*, is negative.

*Call Statement and Input*

```
             X  INCX  N   Y  INCY  M   IDIR  INDX
             |   |    |   |   |    |    |     |
CALL ISSRCH( X , 2  , 3 , Y , 2  , 8 , -1  , INDX )

X        =  (0, . , 12, . , 3)
Y        =  (0, . , 8, . , 12, . , 0, . , 1, . , 4, . , 0, . , 2)
```

*Output*

```
INDX     =  (7, 3, 0)
```

***Example 3:*** This example shows a search where sequences ***x*** and ***y*** have negative strides, and the search direction indicator, *idir*, is positive.

*Call Statement and Input*

```
             X   INCX  N   Y   INCY  M  IDIR  INDX
             |    |    |   |    |    |   |     |
CALL ISSRCH( X , -1  , 3 , Y , -2  , 8 , 1  , INDX )

X        =  (0, 12, 3)
Y        =  (0, . , 8, . , 12, . , 0, . , 1, . , 4, . , 0, . , 2)
```

*Output*

```
INDX     =  (0, 6, 2)
```

***Example 4:*** This example shows a search where sequences ***x*** and ***y*** have negative strides, and the search direction indicator, *idir*, is negative.

*Call Statement and Input*

```
              X   INCX  N   Y   INCY  M   IDIR  INDX
              |    |    |   |    |    |    |     |
CALL ISSRCH( X , -2  , 3 , Y , -1  , 8 , -1  , INDX )

X        =  (0, . , 12, . , 3)
Y        =  (0, 8, 12, 0, 1, 4, 0, 2)
```

*Output*

```
INDX     =  (0, 6, 8)
```

# Chapter 14. Interpolation

The interpolation subroutines are described in this chapter.

## Overview of the Interpolation Subroutines

The interpolation subroutines provide the capabilities of doing polynomial interpolation, local polynomial interpolation, and one- and two-dimensional cubic spline interpolation (Table 149).

*Table 149. List of Interpolation Subroutines*

| Descriptive Name | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|
| Polynomial Interpolation | SPINT | DPINT | 903 |
| Local Polynomial Interpolation | STPINT | DTPINT | 908 |
| Cubic Spline Interpolation | SCSINT | DCSINT | 911 |
| Two-Dimensional Cubic Spline Interpolation | SCSIN2 | DCSIN2 | 917 |

## Use Considerations

Polynomial interpolation (SPINT and DPINT) is a global scheme. As the number of data points increases, the degree of the interpolating polynomial is raised; therefore, the graph of the interpolating polynomial tends to be oscillatory.

Local polynomial interpolation (STPINT and DTPINT) is a local scheme. The data generated is affected only by locally grouped data points. The degree of the local interpolating polynomial is usually lower than a global interpolating polynomial.

## Performance and Accuracy Considerations

1. Doing extrapolation with SPINT and DPINT is not encouraged unless you know the consequences of doing polynomial extrapolation.
2. If performance is the overriding consideration, you should investigate using the general signal processing subroutines, DQINT and SQINT.
3. There are some ESSL-specific rules that apply to the results of computations on the workstation processors using the ANSI/IEEE standards. For details, see "What Data Type Standards Are Used by ESSL, and What Exceptions Should You Know About?" on page 45.

# Interpolation Subroutines

This section contains the interpolation subroutine descriptions.

# SPINT and DPINT—Polynomial Interpolation

These subroutines compute the Newton divided difference coefficients and perform a polynomial interpolation through a set of data points at specified abscissas.

*Table 150. Data Types*

| ***x***, ***y***, ***c***, ***t***, ***s*** | **Subroutine** |
|---|---|
| Short-precision real | SPINT |
| Long-precision real | DPINT |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SPINT \| DPINT (*x*, *y*, *n*, *c*, *ninit*, *t*, *s*, *m*) |
| **C and C++** | spint \| dpint (*x*, *y*, *n*, *c*, *ninit*, *t*, *s*, *m*); |
| **PL/I** | CALL SPINT \| DPINT (*x*, *y*, *n*, *c*, *ninit*, *t*, *s*, *m*); |

*On Entry*

*x*

is the vector ***x*** of length *n*, containing the abscissas of the data points used in the interpolations. The elements of ***x*** must be distinct. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 150.

*y*

is the vector ***y*** of length *n*, containing the ordinates of the data points used in the interpolations. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 150.

*n*

is the number of elements in vectors ***x, y,*** and ***c***—that is, the number of data points. Specified as: a fullword integer; $n \ge 0$.

*c*

is the vector ***c*** of length *n*, where:

If $ninit \le 0$, all elements of ***c*** are undefined on entry.

If $ninit > 0$, ***c*** contains the Newton divided difference coefficients, $c_j$ for $j = 1, ninit$, for the interpolating polynomial through the data points $(x_j,y_j)$ for $j = 1, ninit$. If $ninit < n$, the values of $c_j$ for $j = ninit+1, n$ are undefined.

Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 150.

*ninit*

indicates the following:

If $ninit \le 0$, this is the first call to this subroutine with the data in ***x*** and ***y***; therefore, none of the Newton divided difference coefficients in ***c*** have been initialized.

If $ninit > 0$, a previous call to this subroutine was made with the data points $(x_j, y_j)$ for $j = 1, ninit$, where:

- If $ninit = n$, all the Newton divided difference coefficients in ***c*** were computed for the data points. No additional coefficients are computed on this entry.

- If $ninit < n$, the first *ninit* Newton divided difference coefficients in ***c*** were computed for the data points $(x_j, y_j)$ for $j = 1, ninit$. The coefficients are updated for the additional data points $(x_j, y_j)$ for $j = ninit+1, n$ on this entry.

Specified as: a fullword integer; $ninit \leq n$.

*t*

is the vector ***t*** of length *m*, containing the abscissas at which interpolation is to be done. Specified as: a one-dimensional array of (at least) length *m*, containing numbers of the data type indicated in Table 150 on page 903.

*s*

See “On Return.”

*m*

is the number of elements in vectors ***t*** and ***s***—that is, the number of interpolations to be performed. Specified as: a fullword integer; $m \geq 0$.

*On Return*

*c*

is the vector ***c*** of length *n*, containing the coefficients of the Newton divided difference form of the interpolating polynomial through the data points $(x_j,y_j)$ for $j = 1, n$. Returned as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 150 on page 903.

*ninit*

is the number of coefficients, *n*, in output vector ***c***. (If you call this subroutine again with the same data, this value should be specified for *ninit*.) Returned as: a fullword integer; $ninit = n$.

*s*

is the vector ***s*** of length *m*, containing the resulting interpolated values; that is, each $s_i$ is the value of the interpolating polynomial evaluated at $t_i$. Returned as: a one-dimensional array of (at least) length *m*, containing numbers of the data type indicated in Table 150 on page 903.

***Notes***

1. In your C program, argument *ninit* must be passed by reference.
2. Vectors ***x, y,*** and ***t*** must have no common elements with vectors ***c*** and ***s***, and vector ***c*** must have no common element with vector ***s***; otherwise, results are unpredictable.
3. The elements of vector ***x*** must be distinct; that is, $x_i \neq x_j$ if $i \neq j$ for $i, j = 1, n$.

***Function:*** Polynomial interpolation is performed at specified abscissas, $t_i$ for $i = 1, m$, in vector ***t***, using the method of Newton divided differences through the data points:

$(x_j, y_j)$ for $j = 1, n$

where:

$x_j$ are elements of vector ***x***.
$y_j$ are elements of vector ***y***.

The interpolated value at each $t_i$ is returned in $s_i$ for $i = 1, m$. See references [15] and [52 ]. The interpolating values returned in ***s*** are computed using the Newton divided difference coefficients, as defined in the following section.

The divided difference coefficients, $c_j$ for $j = 1, n$, are returned in vector ***c***. These coefficients can then be reused on subsequent calls to this subroutine, using the same data points $(x_j, y_j)$, but with new values of $t_i$. If the number of data points is increased from one call this subroutine to the next, the new coefficients are computed, and the existing coefficients are updated (not recomputed). This feature can be used to test for the convergence of the interpolations through a sequence of an increasingly larger set of points.

The values specified for *ninit* and *m* indicate which combination of functions are performed by this subroutine: computing the coefficients, performing the interpolation, or both. If $m = 0$, only the divided difference coefficients are computed. No interpolation is performed. If $n = 0$, no computation or interpolation is performed.

For SPINT, the Newton divided differences and interpolating values are accumulated in long precision.

*Newton Divided Differences and Interpolating Values:* The Newton divided differences of the following data points:

$(x_j, y_j)$ for $j = 1, n$
where $x_j \neq x_l$ if $j \neq l$ for $j, l = 1, n$

are denoted by $\delta_k y_j$ for $k = 0, 1, 2, ..., n–1$ and $j = 1, 2, ..., n–k$, and are defined as follows:

For $k = 0$ and 1:
$\delta_0 y_j = y_j$ for $j = 1, 2, ..., n$
$\delta_1 y_j = (y_{j+1} - y_j) / (x_{j+1} - x_j)$ for $j = 1, 2, ..., n–1$
For $k = 2, 3, ..., n–1$:
$\delta_k y_j = (\delta_{k-1} y_{j+1} - \delta_{k-1} y_j) / (x_{j+k} - x_j)$ for $j = 1, 2, ..., n–k$

The value *s* of the Newton divided difference form of the interpolating polynomial evaluated at an abscissa *t* is given by:

$$s = y_n + (t–x_n)\ \delta_1 y_{n-1}$$
$$+ (t–x_{n-1})\ (t–x_n)\ \delta_2 y_{n-2}$$
$$+ ...+(t–x_2)\ (t–x_3)\ ...\ (t–x_n)\ \delta_{n-1} y_1$$

Therefore, on output, the coefficients in vector ***c*** are as follows:

$c_n = y_n$
$c_{n-1} = \delta_1 y_{n-1}$
$c_{n-2} = \delta_2 y_{n-2}$
.
.
.
$c_1 = \delta_{n-1} y_1$

Also, the interpolating values in ***s***, in terms of ***c***, are as follows for $i = 1, m$:

$$s_i = c_n + (t_i–x_n)\ c_{n-1}$$
$$+ (t_i–x_{n-1})\ (t_i–x_n)\ c_{n-2}$$
$$+ ...$$
$$+ (t_i–x_2)\ (t_i–x_3)\ ...\ (t_i–x_n)\ c_1$$

### *Error Conditions*

*Computational Errors:* None

*Input-Argument Errors*

1. *n* < 0
2. *ninit* > *n*
3. *m* < 0

***Example 1:*** This example shows a quadratic polynomial interpolation on the initial call with the specified data points; that is, NINIT = 0, and C contains all undefined values. On output, NINIT and C are updated with new values.

*Call Statement and Input*

```
           X   Y   N   C  NINIT  T   S   M
           |   |   |   |    |    |   |   |
CALL SPINT( X , Y , 3 , C ,  0  , T , S , 2 )
```

```
X        =  (-0.50, 0.00, 1.00)
Y        =  (0.25, 0.00, 1.00)
C        =  ( . , . , . )
T        =  (-0.2, 0.2)
```

*Output*

```
C        =  (1.00, 1.00, 1.00)
NINIT    =  3
S        =  (0.04, 0.04)
```

***Example 2:*** This example shows a quadratic polynomial interpolation on a subsequent call with the same data points specified in Example 1, but using a different set of abscissas in T. In this case, NINIT = N = 3, and C contains the values defined on output in Example 1. On output here, the values in NINIT and C are unchanged.

*Call Statement and Input*

```
           X   Y   N   C  NINIT  T   S   M
           |   |   |   |    |    |   |   |
CALL SPINT( X , Y , 3 , C ,  3  , T , S , 2 )
```

```
X        =  (-0.50, 0.00, 1.00)
Y        =  (0.25, 0.00, 1.00)
C        =  (1.00, 1.00, 1.00)
T        =  (-0.10, 0.10)
```

*Output*

```
C        =  (1.00, 1.00, 1.00)
NINIT    =  3
S        =  (0.01, 0.01)
```

***Example 3:*** This example is the same as Example 2 except that it specifies additional data points on the subsequent call to the subroutine. In this case, 0 < NINIT < N. On output here, the values in NINIT and C are updated. The interpolating polynomial is a degree of 4.

*Call Statement and Input*

```
             X   Y   N   C  NINIT  T   S   M
             |   |   |   |    |    |   |   |
CALL SPINT( X , Y , 5 , C ,  3  , T , S , 2 )

X        =  (-0.50, 0.00, 1.00, -1.00, 0.50)
Y        =  (0.25, 0.00, 1.00, 1.10, 0.26)
C        =  (1.00, 1.00, 1.00, . , . )
T        =  (-0.10, 0.10)
```

*Output*

```
C        =  (0.04, -0.06, 1.02, -0.56, 0.26)
NINIT    =  5
S        =  (0.0072, 0.0130)
```

## STPINT and DTPINT—Local Polynomial Interpolation

These subroutines perform a polynomial interpolation at specified abscissas, using data points selected from a table of data.

*Table 151. Data Types*

| ***x***, ***y***, ***t***, ***s***, *aux* | **Subroutine** |
|---|---|
| Short-precision real | STPINT |
| Long-precision real | DTPINT |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL STPINT \| DTPINT (*x*, *y*, *n*, *nint*, *t*, *s*, *m*, *aux*, *naux*) |
| **C and C++** | stpint \| dtpint (*x*, *y*, *n*, *nint*, *t*, *s*, *m*, *aux*, *naux*); |
| **PL/I** | CALL STPINT \| DTPINT (*x*, *y*, *n*, *nint*, *t*, *s*, *m*, *aux*, *naux*); |

*On Entry*

*x*

is the vector ***x*** of length *n*, containing the abscissas of the data points used in the interpolations. The elements of ***x*** must be distinct and sorted into ascending order. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 151.

*y*

is the vector ***y*** of length *n*, containing the ordinates of the data points used in the interpolations. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 151.

*n*

is the number of elements in vectors ***x*** and ***y***—that is, the number of data points. Specified as: a fullword integer; $n \geq 0$.

*nint*

is the number of data points to be used in the interpolation at any given point. Specified as: a fullword integer; $0 \leq nint \leq n$.

*t*

is the vector ***t*** of length *m*, containing the abscissas at which interpolation is to be done. For optimal performance, ***t*** should be sorted into ascending order. Specified as: a one-dimensional array of (at least) length *m*, containing numbers of the data type indicated in Table 151.

*s*

See "On Return" on page 909.

*m*

is the number of elements in vectors ***t*** and ***s***—that is, the number of interpolations to be performed. Specified as: a fullword integer; $m \geq 0$.

*aux*

has the following meaning:

If *naux* = 0 and error 2015 is unrecoverable, *aux* is ignored.

Otherwise, it is the storage work area used by this subroutine. Its size is specified by *naux*.

Specified as: an area of storage, containing numbers of the data type indicated in Table 151. On output, the contents are overwritten.

*naux*

is the size of the work area specified by *aux*—that is, the number of elements in *aux*. Specified as: a fullword integer, where:

If *naux* = 0 and error 2015 is unrecoverable, STPINT and DTPINT dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, $naux \geq nint+m$.

*On Return*

*s*

is the vector ***s*** of length *m*, containing the resulting interpolated values; that is, each $s_i$ is the value of the interpolating polynomial evaluated at $t_i$. Returned as: a one-dimensional array of (at least) length *m*, containing numbers of the data type indicated in Table 151 on page 908.

***Notes***

1. Vectors ***x, y,*** and ***t*** must have no common elements with vector ***s*** or work area *aux*; otherwise, results are unpredictable. See “Concepts” on page 55.
2. The elements of vector ***x*** must be distinct and must be sorted into ascending order; that is, $x_1 < x_2 < ... < x_n$. Otherwise, results are unpredictable. For details on how to do this, see “ISORT, SSORT, and DSORT—Sort the Elements of a Sequence” on page 884.
3. The elements of vector ***t*** should be sorted into ascending order; that is, $t_1 \leq t_2 \leq t_3 \leq ... \leq t_m$. Otherwise, performance is affected.
4. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see “Using Auxiliary Storage in ESSL” on page 32.

***Function:*** Polynomial interpolation is performed at specified abscissas, $t_i$ for $i = 1, m$, in vector ***t***, using *nint* points selected from the following data:

$(x_j, y_j)$ for $j = 1, n$

where:

$x_1 < x_2 < x_3 < ... < x_n$
$x_j$ are elements of vector ***x***.
$y_j$ are elements of vector ***y***.

The points $(x_j, y_j)$, used in the interpolation at a given abscissa $t_i$, are chosen as follows, where $k = nint/2$:

For $t_i \leq x_{k+1}$, the first *nint* points are used.
For $t_i > x_{n-nint+k}$, the last *nint* points are used.
Otherwise, points *h* through $h+nint-1$ are used, where:
$x_{h+k-1} < t_i \leq x_{h+k}$

The interpolated value at each $t_i$ is returned in $s_i$ for $i = 1, m$. See references [15] and [52 ]. If *n, nint*, or *m* is 0, no computation is performed. For a definition of the polynomial interpolation function performed through a set of data points, see “Function” on page 904.

For STPINT, the Newton divided differences and interpolating values are accumulated in long precision.

***Error Conditions***

*Resource Errors:* Error 2015 is unrecoverable, *naux* = 0, and unable to allocate work area.

*Computational Errors:* None

*Input-Argument Errors*

1. $n < 0$
2. $nint < 0$ or $nint > n$
3. $m < 0$
4. Error 2015 is recoverable or $naux \neq 0$, and *naux* is too small—that is, less than the minimum required value specified in the syntax for this argument. Return code 1 is returned if error 2015 is recoverable.

***Example 1:*** This example shows interpolation using two data points—that is, linear interpolation—at each $t_i$ value.

*Call Statement and Input*

```
            X   Y   N   NINT   T   S   M   AUX  NAUX
            |   |   |    |     |   |   |    |    |
CALL STPINT( X , Y , 10 ,  2  , T , S , 5 , AUX , 7  )

X        =  (0.0, 0.4, 1.0, 1.5, 2.1, 2.6, 3.0, 3.4, 3.9, 4.3)
Y        =  (1.0, 2.0, 3.0, 4.0, 5.0, 5.0, 4.0, 3.0, 2.0, 1.0)
T        =  (-1.0, 0.1, 1.1, 1.2, 3.9)
```

*Output*

```
S        =  (-1.5000, 1.2500, 3.2000, 3.4000, 2.0000)
```

***Example 2:*** This example shows interpolation using three data points—that is, quadratic interpolation—at each $t_i$ value.

*Call Statement and Input*

```
            X   Y   N   NINT   T   S   M   AUX  NAUX
            |   |   |    |     |   |   |    |    |
CALL STPINT( X , Y , 10 ,  3  , T , S , 5 , AUX , 8  )

X        =  (0.0, 0.4, 1.0, 1.5, 2.1, 2.6, 3.0, 3.4, 3.9, 4.3)
Y        =  (1.0, 2.0, 3.0, 4.0, 5.0, 5.0, 4.0, 3.0, 2.0, 1.0)
T        =  (-1.0, 0.1, 1.1, 1.2, 3.9)
```

*Output*

```
S        =  (-2.6667, 1.2750, 3.2121, 3.4182, 2.0000)
```

# SCSINT and DCSINT—Cubic Spline Interpolation

These subroutines compute the coefficients of the cubic spline through a set of data points and evaluate the spline at specified abscissas.

*Table 152. Data Types*

| ***x***, ***y***, ***C***, ***t***, ***s*** | **Subroutine** |
|---|---|
| Short-precision real | SCSINT |
| Long-precision real | DCSINT |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SCSINT \| DCSINT (*x*, *y*, *c*, *n*, *init*, *t*, *s*, *m*) |
| **C and C++** | scsint \| dcsint (*x*, *y*, *c*, *n*, *init*, *t*, *s*, *m*); |
| **PL/I** | CALL SCSINT \| DCSINT (*x*, *y*, *c*, *n*, *init*, *t*, *s*, *m*); |

*On Entry*

*x*

is the vector ***x*** of length *n*, containing the abscissas of the data points that define the spline. The elements of ***x*** must be distinct and sorted into ascending order. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 152.

*y*

is the vector ***y*** of length *n*, containing the ordinates of the data points that define the spline. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 152.

*c*

is the matrix ***C*** with elements $c_{jk}$ for $j = 1, n$ and $k = 1, 4$ that contain the following:

If *init* ≤ 0, all elements of ***c*** are undefined on entry.

If *init* = 1, $c_{11}$ contains the spline derivative at $x_1$.

If *init* = 2, $c_{21}$ contains the spline derivative at $x_n$.

If *init* = 3, $c_{11}$ contains the spline derivative at $x_1$, and $c_{21}$ contains the spline derivative at $x_n$.

If *init* > 3, ***c*** contains the coefficients of the spline computed for the data points $(x_j,y_j)$ for $j = 1, n$ on a previous call to this subroutine.

Specified as: an *n* by (at least) 4 array, containing numbers of the data type indicated in Table 152.

*n*

is the number of elements in vectors ***x*** and ***y*** and the number of rows in matrix ***C***—that is, the number of data points. Specified as: a fullword integer; *n* ≥ 0.

*init*

indicates the following, where in those cases for uninitialized coefficients, this is the first call to this subroutine with the data in ***x*** and ***y***:

If *init* ≤ 0, the coefficients are uninitialized. The second derivatives of the spline at $x_1$ and $x_n$ are set to zero. (These are free end conditions, also called natural boundary conditions.)

If *init* = 1, the coefficients are uninitialized. The value in $c_{11}$ is used as the spline derivative at $x_1$.

If *init* = 2, the coefficients are uninitialized. The value in $c_{21}$ is used as the spline derivative at $x_n$.

If *init* = 3, the coefficients are uninitialized. The value in $c_{11}$ is used as the spline derivative at $x_1$ and the value in $c_{21}$ is used as the spline derivative at $x_n$.

If *init* > 3, the coefficients in ***c*** were computed for data points ($x_j$, $y_j$) for $j = 1, n$ on a previous call to this subroutine.

Specified as: a fullword integer. It can have any value.

*t*

is the vector ***t*** of length *m*, containing the abscissas at which the spline is evaluated. Specified as: a one-dimensional array of (at least) length *m*, containing numbers of the data type indicated in Table 152 on page 911.

*s*

See "On Return."

*m*

is the number of elements in vectors ***t*** and ***s***—that is, the number of points at which the spline interpolation is evaluated. Specified as: a fullword integer; $m \geq 0$.

*On Return*

*c*

is the matrix ***C***, containing the coefficients of the spline through the data points ($x_j$,$y_j$) for $j = 1, n$. Returned as: an *n* by (at least) 4 array, containing numbers of the data type indicated in Table 152 on page 911.

*init*

is an indicator that is set to indicate that the coefficients have been initialized. (If you call this subroutine again with the same data, this value should be specified for *init*.) Returned as: a fullword integer; *init* = 4.

*s*

is the vector ***s*** of length *m*, containing the resulting values of the spline; that is, each $s_i$ is the value of the spline evaluated at $t_i$. Returned as: a one-dimensional array of (at least) length *m*, containing numbers of the data type indicated in Table 152 on page 911.

**Notes**

1. In your C program, argument *init* must be passed by reference.
2. Vectors ***x, y,*** and ***t*** must have no common elements with matrix ***C*** and vector ***s***, and matrix ***C*** must have no common elements with vector ***s***; otherwise, results are unpredictable.
3. The elements of vector ***x*** must be distinct and must be sorted into ascending order; that is, $x_1 < x_2 < \ldots < x_n$. Otherwise, results are unpredictable. For details on how to do this, see "ISORT, SSORT, and DSORT—Sort the Elements of a Sequence" on page 884.

***Function:*** Interpolation is performed at specified abscissas, $t_i$ for $i = 1, m$, in vector ***t***, using the cubic spline passing through the data points:

$(x_j, y_j)$ for $j = 1, n$

where:

$x_1 < x_2 < x_3 < ... < x_n$
$x_j$ are elements of vector ***x***.
$y_j$ are elements of vector ***y***.

The value of the cubic spline at each $t_i$ is returned in $s_i$ for $i = 1, m$. See references [15 ] and [52 ]. The coefficients of the spline, $c_{jk}$ for $j = 1, n$ and $k = 1, 4$, are returned in matrix ***C***. These coefficients can then be reused on subsequent calls to this subroutine, using the same data points ($x_j$, $y_j$), but with new values of $t_i$. The cubic spline values returned in ***s*** are computed using the coefficients as follows:

$$s_i = c_{j1} + c_{j2} (x_j–t_i) + c_{j3} (x_j–t_i)^2 + c_{j4} (x_j–t_i)^3 \quad \text{for } i = 1, m$$

where:

$j = 1$ for $t_i \leq x_1$
$j = k$ for $x_1 < t_i \leq x_n$, such that $x_{k–1} < t_i \leq x_k$
$j = n$ for $x_n < t_i$

The values specified for *m* and *init* indicate which combination of functions are performed by this subroutine:

- If $m = 0$ and $init > 3$, no computation is performed.
- If $m = 0$ and $init \leq 3$, only the coefficients are computed, and no interpolation is performed.
- If $m \neq 0$ and $init > 3$, the coefficients are not computed, and the interpolation is performed.
- If $m \neq 0$ and $init \leq 3$, the coefficients are computed, and the interpolation is performed.

In addition, if $n = 0$, no computation is performed.

The values specified for *n* and *init* determine the type of spline function:

- If $n = 1$, the constructed spline is a constant function.
- If $n = 2$ and $init = 0$, the constructed spline is a line through the points.
- If $n = 2$ and $init = 1$, the constructed spline is a cubic function through the points whose derivative at $x_1$ is $c_{11}$.
- If $n = 2$ and $init = 2$, the constructed spline is a cubic function through the points whose derivative at $x_n$ is $c_{21}$.
- If $n = 2$ and $init = 3$, the constructed spline is a cubic function through the points whose derivative at $x_1$ is $c_{11}$ and at $x_n$ is $c_{21}$.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. $n < 0$
2. $m < 0$

***Example 1:*** This example computes the spline coefficients through a set of data points with no derivative value specified. It also evaluates the spline at the abscissas specified in `T`. On output, `INIT` and `C` are updated with new values.

*Call Statement and Input*

```
             X   Y   C   N  INIT  T   S   M
             |   |   |   |   |    |   |   |
CALL SCSINT( X , Y , C , 6 , 0  , T , S , 4 )

X        =  (1.000, 2.000, 3.000, 4.000, 5.000, 6.000)
Y        =  (0.000, 1.000, 2.000, 1.100, 0.000, -1.000)
C        =(not relevant)
T        =  (-1.000, 2.500, 4.000, 7.000)
```

*Output*

```
        ┌                                ┐
        |  0.000  -0.868   0.000  -0.132 |
        |  1.000  -1.264   0.396  -0.132 |
C    =  |  2.000  -0.076  -1.585   0.660 |
        |  1.100   1.267   0.243  -0.609 |
        |  0.000   1.010   0.014   0.076 |
        | -1.000   0.995   0.000   0.005 |
        └                                ┘

INIT     =  4
S        =  (-2.792, 1.649, 1.100, -2.000)
```

***Example 2:*** This example computes the spline coefficients through a set of data points with a derivative value specified at the right endpoint. It also evaluates the spline at the abscissas specified in `T`. On output, `INIT` and `C` are updated with new values.

*Call Statement and Input*

```
             X   Y   C   N  INIT  T   S   M
             |   |   |   |   |    |   |   |
CALL SCSINT( X , Y , C , 6 , 2  , T , S , 4 )

X        =  (1.000, 2.000, 3.000, 4.000, 5.000, 6.000)
Y        =  (0.000, 1.000, 2.000, 1.100, 0.000, -1.000)

        ┌                ┐
        |  .    .   .   . |
        | 0.1   .   .   . |
C    =  |  .    .   .   . |
        |  .    .   .   . |
        |  .    .   .   . |
        |  .    .   .   . |
        └                ┘

T        =  (-1.000, 2.500, 4.000, 7.000)
```

*Output*

```
                ┌                               ┐
                |  0.000  -0.865   0.000  -0.135 |
                |  1.000  -1.270   0.405  -0.135 |
C     =         |  2.000  -0.054  -1.621   0.675 |
                |  1.100   1.188   0.379  -0.667 |
                |  0.000   1.303  -0.494   0.291 |
                | -1.000   0.100   1.897  -0.797 |
                └                               ┘

INIT     =  4
S        =  (-2.810, 1.652, 1.100, 1.794)
```

***Example 3:*** This example computes the spline coefficients through a set of data points with a derivative value specified at both endpoints. It does not evaluate the spline at any points. On output, `INIT` and `C` are updated with new values. Because arrays are not needed for arguments *t* and *s*, the value 0 is specified in their place.

*Call Statement and Input*

```
             X   Y   C   N  INIT  T   S   M
             |   |   |   |   |    |   |   |
CALL SCSINT( X , Y , C , 6 , 3  , 0 , 0 , 0 )

X        =  (1.000, 2.000, 3.000, 4.000, 5.000, 6.000)
Y        =  (0.000, 1.000, 2.000, 1.100, 0.000, -1.000)

                ┌                ┐
                | -1.0  .   .   . |
                |  0.1  .   .   . |
C     =         |   .   .   .   . |
                |   .   .   .   . |
                |   .   .   .   . |
                |   .   .   .   . |
                └                ┘
```

*Output*

```
                ┌                               ┐
                |  0.000   1.000   3.230   1.230 |
                |  1.000  -1.770  -0.460   1.230 |
C     =         |  2.000   0.079  -1.389   0.310 |
                |  1.100   1.152   0.316  -0.568 |
                |  0.000   1.312  -0.476   0.264 |
                | -1.000  -0.100   1.888  -0.788 |
                └                               ┘

INIT     =  4
```

***Example 4:*** This example evaluates the spline at a set of points, using the coefficients obtained in Example 3.

*Call Statement and Input*

```
             X   Y   C   N  INIT  T   S   M
             |   |   |   |   |    |   |   |
CALL SCSINT( X , Y , C , 6 , 4  , T , S , 4 )

X        =  (1.000, 2.000, 3.000, 4.000, 5.000, 6.000)
Y        =  (0.000, 1.000, 2.000, 1.100, 0.000, -1.000)
```

```
C       =(same as output C in Example 3 )
T       =  (-1.000, 2.500, 4.000, 7.000)
```

*Output*

```
C       =(same as output C in Example 3 )
S       =  (24.762, 1.731, 1.100, 1.776)
INIT    =  4
```

# SCSIN2 and DCSIN2—Two-Dimensional Cubic Spline Interpolation

These subroutines compute the interpolation values at a specified set of points, using data defined on a rectangular mesh in the x-y plane.

*Table 153. Data Types*

| ***x***, ***y***, ***Z***, ***t***, ***u***, *aux*, ***S*** | **Subroutine** |
|---|---|
| Short-precision real | SCSIN2 |
| Long-precision real | DCSIN2 |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SCSIN2 \| DCSIN2 (*x*, *y*, *z*, *n1*, *n2*, *ldz*, *t*, *u*, *m1*, *m2*, *s*, *lds*, *aux*, *naux*) |
| **C and C++** | scsin2 \| dcsin2 (*x*, *y*, *z*, *n1*, *n2*, *ldz*, *t*, *u*, *m1*, *m2*, *s*, *lds*, *aux*, *naux*); |
| **PL/I** | CALL SCSIN2 \| DCSIN2 (*x*, *y*, *z*, *n1*, *n2*, *ldz*, *t*, *u*, *m1*, *m2*, *s*, *lds*, *aux*, *naux*); |

*On Entry*

*x*

is the vector ***x*** of length *n1*, containing the x-coordinates of the data points that define the spline. The elements of ***x*** must be distinct and sorted into ascending order. Specified as: a one-dimensional array of (at least) length *n1*, containing numbers of the data type indicated in Table 153.

*y*

is the vector ***y*** of length *n2*, containing the y-coordinates of the data points that define the spline. The elements of ***y*** must be distinct and sorted into ascending order. Specified as: a one-dimensional array of (at least) length *n2*, containing numbers of the data type indicated in Table 153.

*z*

is the matrix ***Z***, containing the data at $(x_i, y_j)$ for $i = 1, n1$ and $j = 1, n2$ that defines the spline. Specified as: an *ldz* by (at least) *n2* array, containing numbers of the data type indicated in Table 153.

*n1*

is the number of elements in vector ***x*** and the number of rows in matrix ***Z***—that is, the number of x-coordinates at which the spline is defined. Specified as: a fullword integer; $n1 \geq 0$.

*n2*

is the number of elements in vector ***y*** and the number of columns in matrix ***Z***—that is, the number of y-coordinates at which the spline is defined. Specified as: a fullword integer; $n2 \geq 0$.

*ldz*

is the leading dimension of the array specified for *z*. Specified as: a fullword integer; $ldz > 0$ and $ldz \geq n1$.

*t*

is the vector ***t*** of length *m1*, containing the x-coordinates at which the spline is evaluated. Specified as: a one-dimensional array of (at least) length *m1*, containing numbers of the data type indicated in Table 153.

*u*

is the vector ***u*** of length *m2*, containing the y-coordinates at which the spline is evaluated. Specified as: a one-dimensional array of (at least) length *m2*, containing numbers of the data type indicated in Table 153.

*m1*
is the number of elements in vector ***t***—that is, the number of x-coordinates at which the spline interpolation is evaluated. Specified as: a fullword integer; $m1 \geq 0$.

*m2*
is the number of elements in vector ***u***—that is, the number of y-coordinates at which the spline interpolation is evaluated. Specified as: a fullword integer; $m2 \geq 0$.

*s*
See “On Return.”

*lds*
is the leading dimension of the array specified for *s*. Specified as: a fullword integer; $lds > 0$ and $lds \geq m1$.

*aux*
has the following meaning:

If $naux = 0$ and error 2015 is unrecoverable, *aux* is ignored.

Otherwise, it is the storage work area used by this subroutine. Its size is specified by *naux*.

Specified as: an area of storage, containing numbers of the data type indicated in Table 151 on page 908. On output, the contents are overwritten.

*naux*
is the size of the work area specified by *aux*—that is, the number of elements in *aux*. Specified as: a fullword integer, where:

If $naux = 0$ and error 2015 is unrecoverable, SCSIN2 and DCSIN2 dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, $naux \geq (10)(\max(n1, n2))+(n2+1)(m1)+2(m2)$.

*On Return*

*s*
is the matrix ***S*** with elements $s_{kh}$ that contain the interpolation values at $(t_k, u_h)$ for $k = 1, m1$ and $h = 1, m2$. Returned as: an *lds* by (at least) *m2* array, containing numbers of the data type indicated in Table 153 on page 917.

***Notes***

1. The cyclic reduction method used to solve the equations in this subroutine can generate underflows on well-scaled problems. This does not affect accuracy, but it may decrease performance. For this reason, you may want to disable underflow before calling this subroutine.

2. Vectors ***x***, ***y***, ***t***, and ***u***, matrix ***Z***, and the *aux* work area must have no common elements with matrix ***S***; otherwise, results are unpredictable.

3. The elements within vectors ***x*** and ***y*** must be distinct. In addition, the elements in the vectors must be sorted into ascending order; that is, $x_1 < x_2 < ... < x_{n1}$ and $y_1 < y_2 < ... < y_{n2}$. Otherwise, results are unpredictable. For details on how to do this, see “ISORT, SSORT, and DSORT—Sort the Elements of a Sequence” on page 884.

4. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see “Using Auxiliary Storage in ESSL” on page 32.

***Function:*** Interpolation is performed at a specified set of points:

$(t_k, u_h)$ for $k = 1, m1$ and $h = 1, m2$

by fitting bicubic spline functions with natural boundary conditions, using the following set of data, defined on a rectangular grid, $(x_i, y_j)$ for $i = 1, n1$ and $j = 1, n2$:

$z_{ij}$ for $i = 1, n1$ and $j = 1, n2$

where $t_k$, $u_h$, $x_i$, $y_j$, and $z_{ij}$ are elements of vectors ***t***, ***u***, ***x***, and ***y*** and matrix ***Z***, respectively. In vectors ***x*** and ***y***, elements are assumed to be sorted into ascending order.

The interpolation involves two steps:

1. For each *j* from 1 to *n2*, the single variable cubic spline:

   $$s_{y_j}(x)$$

   with natural boundary conditions, is constructed using the data points:

   $(x_i, z_{ij})$ for $i = 1, n1$

   The following interpolation values are then computed:

   $$s_{y_j}(t_k) \qquad \text{for } k = 1, m1$$

2. For each *k* from 1 to *m1*, the single variable cubic spline:

   $$s_{t_k}(y),$$

   with natural boundary conditions, is constructed using the data points:

   $$\left(y_j, s_{y_j}(t_k)\right) \qquad \text{for } j = 1, n2$$

   The following interpolation values are then computed:

   $$s_{kh} = s_{t_k}(u_h) \qquad \text{for } l = 1, m2$$

See references [52 ] and [58 ]. Because natural boundary conditions (zero second derivatives at the end of the ranges) are used for the splines, unless the underlying function has these properties, interpolated values near the boundaries may be less satisfactory than elsewhere. If *n1*, *n2*, *m1*, or *m2* is 0, no computation is performed.

***Error Conditions***

*Resource Errors:* Error 2015 is unrecoverable, *naux* = 0, and unable to allocate work area.

*Computational Errors:* None

*Input-Argument Errors*

1. *n1* < 0 or *n1* > *ldz*
2. *n2* < 0
3. *m1* < 0 or *m1* > *lds*
4. *m2* < 0
5. *ldz* < 0
6. *lds* < 0
7. Error 2015 is recoverable or *naux*≠0, and *naux* is too small—that is, less than the minimum required value specified in the syntax for this argument. Return code 1 is returned if error 2015 is recoverable.

***Example:*** This example computes the interpolated values at a specified set of points, given by T and U, from a set of data points defined on a rectangular mesh in the x-y plane, using X, Y, and Z.

*Call Statement and Input*

```
             X   Y   Z   N1  N2 LDZ  T   U   M1  M2  S  LDS  AUX  NAUX
             |   |   |   |   |   |   |   |   |   |   |   |    |    |
CALL SCSIN2( X , Y , Z , 6 , 5 , 6 , T , U , 4 , 3 , S , 4 , AUX , 90 )

X        =  (0.0, 0.2, 0.3, 0.4, 0.5, 0.7)
Y        =  (0.0, 0.2, 0.3, 0.4, 0.6)

        ┌                                  ┐
        │ 0.000  0.008  0.027  0.064  0.216 │
        │ 0.008  0.016  0.035  0.072  0.224 │
Z    =  │ 0.027  0.035  0.054  0.091  0.243 │
        │ 0.064  0.072  0.091  0.128  0.280 │
        │ 0.125  0.133  0.152  0.189  0.341 │
        │ 0.343  0.351  0.370  0.407  0.559 │
        └                                  ┘

T        =  (0.10, 0.15, 0.25, 0.35)
U        =  (0.05, 0.25, 0.45)
```

*Output*

```
        ┌                    ┐
        │ 0.001  0.017  0.095 │
S    =  │ 0.003  0.019  0.097 │
        │ 0.016  0.031  0.110 │
        │ 0.043  0.059  0.137 │
        └                    ┘
```

# Chapter 15. Numerical Quadrature

The numerical quadrature subroutines are described in this chapter.

## Overview of the Numerical Quadrature Subroutines

The numerical quadrature subroutines provide Gaussian quadrature methods for integrating a tabulated function and a user-supplied function over a finite, semi-infinite, or infinite region of integration (Table 154).

*Table 154. List of Numerical Quadrature Subroutines*

| Descriptive Name | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|
| Numerical Quadrature Performed on a Set of Points | SPTNQ | DPTNQ | 925 |
| Numerical Quadrature Performed on a Function Using Gauss-Legendre Quadrature | SGLNQ† | DGLNQ† | 928 |
| Numerical Quadrature Performed on a Function Over a Rectangle Using Two-Dimensional Gauss-Legendre Quadrature | SGLNQ2† | DGLNQ2† | 931 |
| Numerical Quadrature Performed on a Function Using Gauss-Laguerre Quadrature | SGLGQ† | DGLGQ† | 937 |
| Numerical Quadrature Performed on a Function Using Gauss-Rational Quadrature | SGRAQ† | DGRAQ† | 940 |
| Numerical Quadrature Performed on a Function Using Gauss-Hermite Quadrature | SGHMQ† | DGHMQ† | 944 |
| † This subprogram is invoked as a function in a Fortran program. | | | |

## Use Considerations

This section contains some key points about using the numerical quadrature subroutines.

## Choosing the Method

The theoretical aspects of choosing the method to use for integration can be found in the references [26], [59 ], and [89 ].

## Performance and Accuracy Considerations

1. There are *n* function evaluations for a method of order *n*. Because function evaluations are expensive in terms of computing time, you should weigh the considerations for computing time and accuracy in choosing a value for *n*.
2. To achieve optimal performance in the _GLNQ2 subroutines, specify the first variable integrated to be the variable having more points. This allows both the subroutine and the function evaluation to achieve optimal performance. Details on how to do this are given in “Notes” on page 932.

3. There are some ESSL-specific rules that apply to the results of computations on the workstation processors using the ANSI/IEEE standards. For details, see “What Data Type Standards Are Used by ESSL, and What Exceptions Should You Know About?” on page 45.

## Programming Considerations for the SUBF Subroutine

This section describes how to design and code the *subf* subroutine for use by the numerical quadrature subrutines.

## Designing SUBF

For the Gaussian quadrature subroutines, you must supply a separate subroutine that is callable by ESSL. You specify the name of the subroutine in the *subf* argument. This subroutine name is selected by you. You should design the *subf* subroutine so it receives, as input, a tabulated set of points at which the integrand is evaluated, and it returns, as output, the values of the integrand evaluated at these points.

Depending on the numerical quadrature subroutine that you use, the *subf* subroutine is defined in one of the two following ways:

- For _GLNQ, _GLGQ, _GRAQ, and _GHMQ, you define the *subf* subroutine with three arguments: *t*, *y*, and *n*, where:

  *t* is an input array, referred to as T, of tabulated Gaussian quadrature abscissas, containing *n* real numbers, $t_i$, where $t_i$ is automatically provided by the ESSL subroutine and is determined by *n* and the Gaussian quadrature method chosen.

  *y* is an output array, referred to as Y, containing *n* real numbers, where for the integrand, the following is true: $y_i = f(t_i)$ for $i = 1, n$.

  *n* is a positive integer indicating the number of elements in T and Y.

- For _GLNQ2, you define the *subf* subroutine with six arguments: *s*, *n1*, *t*, *n2*, *z*, and *ldz*, where:

  *s* is an input array, referred to as S, of tabulated Gaussian quadrature abscissas, containing *n1* real numbers, $s_i$, where $s_i$ is automatically provided by the ESSL subroutine and is determined by *n1* and the Gaussian quadrature method.

  *n1* is a positive integer indicating the number of elements in S and the number of rows to be used in array ***Z***.

  *t* is an input array, referred to as T, of tabulated Gaussian quadrature abscissas, containing *n2* real numbers, $t_i$, where $t_i$ is automatically provided by the ESSL subroutine and is determined by *n2* and the Gaussian quadrature method.

  *n2* is a positive integer indicating the number of elements in T and the number of columns to be used in array ***Z***.

  *z* is an *ldz* by (at least) *n2* output array, referred to as Z, of real numbers, where for the integrand, the following is true: $z_{ij} = f(s_i, t_j)$ for $i = 1, n1$ and $j = 1, n2$.

  *ldz* is a positive integer indicating the size of the leading dimension of the array ***Z***.

## Coding and Setting Up SUBF in Your Program

Examples of coding a *subf* subroutine in Fortran are provided for each subroutine in this chapter. Examples of coding a *subf* subroutine in C, C++, and PL/I are provided in “Example 1” on page 933.

Depending on the programming language you use for your program that calls the numerical quadrature subroutines, you have a choice of one or more languages that you can use for writing *subf*. These rules and other langauge-related coding rules for setting up *subf* in your program are described in the following sections:

- “Setting Up a User-Supplied Subroutine for ESSL in Fortran” on page 111
- “Setting Up a User-Supplied Subroutine for ESSL in C” on page 131
- “Setting Up a User-Supplied Subroutine for ESSL in C++” on page 147

# Numerical Quadrature Subroutines

This section contains the numerical quadrature subroutine descriptions.

# SPTNQ and DPTNQ—Numerical Quadrature Performed on a Set of Points

These subroutines approximate the integral of a real valued function specified in tabular form, $(x_i, y_i)$ for $i = 1, n$. For more than four points, an error estimate is returned along with the resulting value.

*Table 155. Data Types*

| ***x***, ***y***, *xyint*, *eest* | **Subroutine** |
|---|---|
| Short-precision real | SPTNQ |
| Long-precision real | DPTNQ |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SPTNQ \| DPTNQ (*x*, *y*, *n*, *xyint*, *eest*) |
| **C and C++** | sptnq \| dptnq (*x*, *y*, *n*, *xyint*, *eest*); |
| **PL/I** | CALL SPTNQ \| DPTNQ (*x*, *y*, *n*, *xyint*, *eest*); |

*On Entry*

*x*
is the vector ***x*** of length *n*, containing the abscissas of the data points to be integrated. The elements of ***x*** must be distinct and sorted into ascending or descending order. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 155.

*y*
is the vector ***y*** of length *n*, containing the ordinates of the data points to be integrated. Specified as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 155.

*n*
is the number of elements in vectors ***x*** and ***y***—that is, the number of data points. The value of *n* determines the algorithm used by this subroutine. For details, see “Function” on page 926. Specified as: a fullword integer; $n \geq 2$.

*xyint*
See “On Return.”

*eest*
See “On Return.”

*On Return*

*xyint*
is the approximation *xyint* of the integral. Returned as: a number of the data type indicated in Table 155.

*eest*
has the following meaning, where:

If $n < 5$, it is undefined and is set to 0.

If $n \geq 5$, it is an estimate, *eest*, of the error in the integral, where *xyint*+*eest* tends to give a better approximation to the integral than *xyint*. For details, see references [26] and [59 ].

Returned as: a number of the data type indicated in Table 155.

***Notes***

1. In your C program, arguments *xyint* and *eest* must be passed by reference.
2. The elements of vector ***x*** must be distinct—that is, $x_i \neq x_j$ for $i \neq j$,—and they must be sorted into ascending or descending order; otherwise, results are unpredictable. For how to do this, see “ISORT, SSORT, and DSORT—Sort the Elements of a Sequence” on page 884.

***Function:*** The integral is approximated for a real valued function specified in tabular form, ($x_i$, $y_i$) for $i = 1, n$, where $x_i$ are distinct and sorted into ascending or descending order, and $n \geq 2$. If $y_i = f(x_i)$ for $i = 1, n$, then on output, *xyint* is an approximation to the integral of the following form:

$$\int_{x_1}^{x_n} f(x)dx$$

The algorithm used by this subroutine is based on the number of data points used in the computation, where:

- If $n = 2$, the trapezoid rule is used to do the integration.
- If $n = 3$, the parabola through the three points is integrated.
- If $n \geq 4$, the method of Gill and Miller is used to do the integration.

For $n \geq 5$, an estimate of the error *eest* is returned. For the method of Gill and Miller, it is shown that adding the estimate of the error *eest* to the result *xyint* often gives a better approximation to the integral than the result *xyint* by itself. For $n < 5$, an estimate of the error is not returned. In this case, a value of 0 is returned for *eest*. See references [59 ] and [26 ].

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* $n < 2$

***Example 1:*** This example shows the result of an integration, where the abscissas in X are sorted into ascending order.

*Call Statement and Input*

```
            X   Y   N    XYINT   EEST
            |   |   |      |      |
CALL SPTNQ( X , Y , 10 , XYINT , EEST )

X        =  (0.0, 0.4, 1.0, 1.5, 2.1, 2.6, 3.0, 3.4, 3.9, 4.3)
Y        =  (1.0, 2.0, 3.0, 4.0, 5.0, 4.5, 4.0, 3.0, 3.5, 3.3)
```

*Output*

```
XYINT    =  15.137
EEST     =  -0.003
```

***Example 2:*** This example shows the result of an integration, where the abscissas in X are sorted into descending order.

*Call Statement and Input*

```
            X   Y   N    XYINT   EEST
            |   |   |      |       |
CALL SPTNQ( X , Y , 10 , XYINT , EEST )

X        =  (4.3, 3.9, 3.4, 3.0, 2.6, 2.1, 1.5, 1.0, 0.4, 0.0)
Y        =  (3.3, 3.5, 3.0, 4.0, 4.5, 5.0, 4.0, 3.0, 2.0, 1.0)
```

*Output*

```
XYINT    =  -15.137
EEST     =  0.003
```

## SGLNQ and DGLNQ—Numerical Quadrature Performed on a Function Using Gauss-Legendre Quadrature

These functions approximate the integral of a real valued function over a finite interval, using the Gauss-Legendre Quadrature method of specified order.

*Table 156. Data Types*

| *a*, *b*, **Result** | **Subroutine** |
|---|---|
| Short-precision real | SGLNQ |
| Long-precision real | DGLNQ |

### *Syntax*

| | |
|---|---|
| **Fortran** | SGLNQ \| DGLNQ (*subf*, *a*, *b*, *n*) |
| **C and C++** | sglnq \| dglnq (*subf*, *a*, *b*, *n*); |
| **PL/I** | SGLNQ \| DGLNQ (*subf*, *a*, *b*, *n*); |

*On Entry*

*subf*

is the user-supplied subroutine that evaluates the integrand function. The subroutine should be defined with three arguments: *t*, *y*, and *n*. For details, see “ Programming Considerations for the SUBF Subroutine” on page 922.

Specified as: *subf* must be declared as an external subroutine in you application program. It can be whatever name you choose.

*a*

is the lower limit of integration, *a*. Specified as: a number of the data type indicated in Table 156.

*b*

is the upper limit of integration, *b*. Specified as: a number of the data type indicated in Table 156.

*n*

is the order of the quadrature method to be used. Specified as: a fullword integer; *n* = 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 16, 20, 24, 32, 40, 48, 64, 96, 128, or 256.

*On Return*

*Function value*

is the approximation of the integral. Returned as: a number of the data type indicated in Table 156.

### *Notes*

1. Declare the DGLNQ function in your program as returning a long-precision real number. Declare the SGLNQ, if necessary, as returning a short-precision real number.
2. The subroutine specified for *subf* must be declared as external in your program. Also, data types used by *subf* must agree with the data types specified by this ESSL subroutine. The variable *x*, described under “Function” on page 929, and

the argument *n* correspond to the *subf* arguments *t* and *n*, respectively. For details on how to set up the subroutine, see “ Programming Considerations for the SUBF Subroutine” on page 922.

***Function:*** The integral is approximated for a real valued function over a finite interval, using the Gauss-Legendre Quadrature method of specified order. The region of integration is from *a* to *b*. The method of order *n* is theoretically exact for integrals of the following form, where *f* is a polynomial of degree less than 2*n*:

$$\int_a^b f(x)dx$$

The method of order *n* is a good approximation when your integrand is closely approximated by a function of the form *f*(*x*), where *f* is a polynomial of degree less than 2*n*. See references [26] and [89 ]. The result is returned as the function value.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors:* *n* is not an allowable value, as listed in the syntax for this argument.

***Example:*** This example shows how to compute the integral of the function *f* given by:

$f(x) = x^2+e^x$

over the interval (0.0, 2.0), using the Gauss-Legendre method with 10 points:

$$\int_{0.0}^{2.0}\left(x^2+e^x\right)dx$$

The user-supplied subroutine FUN1, which evaluates the integrand function, is coded in Fortran as follows:

```
   SUBROUTINE FUN1 (T,Y,N)
   INTEGER*4 N
   REAL*4 T(*),Y(*)
   DO 1 I=1,N
1     Y(I)=T(I)**2+EXP(T(I))
   RETURN
   END
```

*Program Statements and Input*

```
EXTERNAL FUN1
        .
        .
        .
              SUBF    A     B    N
               |      |     |    |
XINT = SGLNQ( FUN1 , 0.0 , 2.0 , 10 )
        .
        .
        .
```

FUN1 =(see above)

*Output*

```
XINT     =  9.056
```

## SGLNQ2 and DGLNQ2—Numerical Quadrature Performed on a Function Over a Rectangle Using Two-Dimensional Gauss-Legendre Quadrature

These functions approximate the integral of a real valued function of two variables over a rectangular region, using the Gauss-Legendre Quadrature method of specified order in each variable.

*Table 157. Data Types*

| *a*, *b*, *c*, *d*, ***Z***, **Result** | **Subroutine** |
|---|---|
| Short-precision real | SGLNQ2 |
| Long-precision real | DGLNQ2 |

### *Syntax*

| | |
|---|---|
| **Fortran** | SGLNQ2 \| DGLNQ2 (*subf*, *a*, *b*, *n1*, *c*, *d*, *n2*, *z*, *ldz*) |
| **C and C++** | sglnq2 \| dglnq2 (*subf*, *a*, *b*, *n1*, *c*, *d*, *n2*, *z*, *ldz*); |
| **PL/I** | SGLNQ2 \| DGLNQ2 (*subf*, *a*, *b*, *n1*, *c*, *d*, *n2*, *z*, *ldz*); |

*On Entry*

*subf*

is the user-supplied subroutine that evaluates the integrand function. The subroutine should be defined with six arguments: *s*, *n1*, *t*, *n2*, *z*, and *ldz*. For details, see “ Programming Considerations for the SUBF Subroutine” on page 922.

Specified as: *subf* must be declared as an external subroutine in your application program. It can be whatever name you choose.

*a*

is the lower limit of integration, *a*, for the first variable integrated. Specified as: a number of the data type indicated in Table 157.

*b*

is the upper limit of integration, *b*, for the first variable integrated. Specified as: a number of the data type indicated in Table 157.

*n1*

is the order of the quadrature method to be used for the first variable integrated. Specified as: a fullword integer; *n1* = 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 16, 20, 24, 32, 40, 48, 64, 96, 128, or 256.

*c*

is the lower limit of integration, *c*, for the second variable integrated. Specified as: a number of the data type indicated in Table 157.

*d*

is the upper limit of integration, *d*, for the second variable integrated. Specified as: a number of the data type indicated in Table 157.

*n2*

is the order of the quadrature method to be used for the second variable integrated. Specified as: a fullword integer; *n2* = 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 16, 20, 24, 32, 40, 48, 64, 96, 128, or 256.

*z*

is the matrix ***Z***, containing the *n1* rows and *n2* columns of data used to evaluate the integrand function. (The output values from the *subf* subroutine are placed in ***Z***.) Specified as: an *ldz* by (at least) *n2* array, containing numbers of the data type indicated in Table 157.

*ldz*

is the size of the leading dimension of the array specified for *z*. Specified as: a fullword integer; *ldz* > 0 and *ldz* ≥ *n1*.

*On Return*

*Function value*

is the approximation of the integral. Returned as: a number of the data type indicated in Table 157 on page 931.

***Notes***

1. Declare the DGLNQ2 function in your program as returning a long-precision real number. Declare the SGLNQ2 function, if necessary, as returning a short-precision real number.
2. The subroutine specified for *subf* must be declared as external in your program. Also, data types used by *subf* must agree with the data types specified by this ESSL subroutine. For details on how to set up the subroutine, see " Programming Considerations for the SUBF Subroutine" on page 922.

***Function:*** The integral:

$$\int_c^d \int_a^b f(s,t)\; ds\; dt$$

is approximated for a real valued function of two variables *s* and *t*, over a rectangular region, using the Gauss-Legendre Quadrature method of specified order in each variable. The region of integration is:

(*a*, *b*) for *s*
(*c*, *d*) for *t*

The method gives a good approximation when your integrand is closely approximated by a function of the form *f*(*s*, *t*), where *f* is a polynomial of degree less than 2(*n1*) for *s* and 2(*n2*) for *t*. See the function description for "SGLNQ and DGLNQ—Numerical Quadrature Performed on a Function Using Gauss-Legendre Quadrature" on page 928 and references [26 ] and [89 ]. The result is returned as the function value.

***Special Usage:*** To achieve optimal performance in this subroutine and in the functional evaluation, specify the first variable integrated in this subroutine as the variable having more points. The first variable integrated is the variable in the inner integral. For example, in the following integration, *x* is the first variable integrated:

$$\int_{u1}^{u2} \int_{r1}^{r2} f(x,y)\; dx\; dy$$

This is the suggested order of integration if the *x* variable has more points than the *y* variable. On the other hand, if the *y* variable has more points, you make *y* the first variable integrated.

Because the order of integration does not matter to the resulting approximation, you may be able to reverse the order that *x* and *y* are integrated and get better performance. This can be expressed as:

$$\int_{u1}^{u2}\int_{r1}^{r2} f(x,y)\; dx\; dy \quad = \quad \int_{r1}^{r2}\int_{u1}^{u2} f(x,y)\; dy\; dx$$

Results are mathematically equivalent. However, because the algorithm is computed in a different way, results may not be bitwise identical.

Table 158 shows how to assign your variables to the _GLNQ2 and *subf* arguments for the *x*-*y* integration shown on the left and for the *y*-*x* integration shown on the right. For examples of how to do each of these, see “Example 1” and “Example 2” on page 935.

*Table 158. How to Assign Your Variables for x-y Integration Versus y-x Integration*

| **_GLNQ2 and SUBF Arguments** | **Variables for *x-y* Integration** | **Variables for *y-x* Integration** |
|---|---|---|
| For _GLNQ2: | | |
| *a* | *r1* | *u1* |
| *b* | *r2* | *u2* |
| *n1* | (order for *x*) | (order for *y*) |
| *c* | *u1* | *r1* |
| *d* | *u2* | *r2* |
| *n2* | (order for *y*) | (order for *x*) |
| For *subf*: | | |
| *s* | *x* | *y* |
| *t* | *y* | *x* |
| *n1* | (order for *x*) | (order for *y*) |
| *n2* | (order for *y*) | (order for *x*) |

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. *ldz* ≤ 0
2. *n1* > *ldz*
3. *n1* or *n2* is not an allowable value, as listed in the syntax for this argument.

***Example 1:*** This example shows how to compute the integral of the function *f* given by:

$f(x, y) = e^{x} \sin y$

over the intervals (0.0, 2.0) for the first variable *x* and (−2.0, −1.0) for the second variable *y*, using the Gauss-Legendre method with 10 points in the *x* variable and 5 points in the *y* variable:

$$\int_{-2.0}^{-1.0}\int_{0.0}^{2.0}\left(e^{x}\ \sin\ y\right)dx\; dy$$

Because the variable *x* has more points, it is the first variable integrated. This allows the SGLNQ2 subroutine and the FUN1 evaluation to achieve optimal performance. Therefore, the *x* and *y* variables correspond to S and T in the FUN1 subroutine. Also, the *x* and *y* variables correspond to the A, B, N1 and C, D, N2 sets of arguments, respectively, for SGLNQ2.

*Using Fortran for SUBF:* The user-supplied subroutine FUN1, which evaluates the integrand function, is coded in Fortran as follows:

```
   SUBROUTINE FUN1 (S,N1,T,N2,Z,LDZ)
   INTEGER*4 N1,N2,LDZ
   REAL*4 S(*),T(*),Z(LDZ,*)
   DO 1 J=1,N2
   DO 2 I=1,N1
2     Z(I,J)=EXP(S(I))*SIN(T(J))
1     CONTINUE
   RETURN
   END
```

**Note:** The computation for this user-supplied subroutine FUN1 can also be performed by using the following statements in place of the above DO loops, using T1 and T2 as temporary storage areas:

```
   .
   .
   .
   DO 1 I=1,N1
1     T1(I)=EXP(S(I))
   DO 2 J=1,N2
2     T2(J)=SIN(T(J))
   DO 3 J=1,N2
   DO 4 I=1,N1
4     Z(I,J)=T1(I)*T2(J)
3     CONTINUE
   .
   .
   .
```

When coding your application, this is the preferred technique. It reduces the number of evaluations performed and, therefore, provides better performance.

*Using C for SUBF:* The user-supplied subroutine FUN1, which evaluates the integrand function, is coded in C as follows:

```
void fun1(s, n1, t, n2, z, ldz)
float *s, *t, *z;
int *n1, *n2, *ldz;
{
     int i, j;
     for(j = 0; j < *n2; ++j, z += *ldz)
         {
          for(i = 0; i < *n1; ++i)
          z[i] = exp(s[i]) * sin(t[j]);
         }
}
```

*Using C++ for SUBF:* The user-supplied subroutine FUN1, which evaluates the integrand function, is coded in C++ as follows:

```
void fun1(float *s, int *n1, float *t, int *n2, float *z, int *ldz)
{
     int i, j;
     for(j = 0; j < *n2; ++j, z += *ldz)
         {
         for(i = 0; i < *n1; ++i)
          z[i] = exp(s[i]) * sin(t[j]);
         }
}
```

*Using PL/I for SUBF:* The user-supplied subroutine FUN1, which evaluates the integrand function, is coded in PL/I as follows:

```
FUN1: PROCEDURE(S,N1,T,N2,Z,LDZ) OPTIONS(FORTRAN,NOMAP);
DCL (N1,N2,LDZ,I,J) REAL FIXED BINARY(31,0);
DCL (S(10),T(10),Z(5,10)) REAL FLOAT DEC(16) ALIGNED CONNECTED;
DO J=1 TO N1;
   DO I=1 TO N2;
      Z(I,J)=EXP(S(J))*SIN(T(I));
   END;
END;
RETURN;
END FUN1;
```

*Program Statements and Input*

```
EXTERNAL FUN1
        .
        .
        .
                SUBF    A     B    N1     C      D    N2  Z   LDZ
                 |      |     |    |      |      |    |   |    |
XYINT = SGLNQ2( FUN1 , 0.0 , 2.0 , 10 , -2.0 , -1.0 , 5 , Z , 10 )
        .
        .
        .
```

```
FUN1      =(see sections above)
Z         =(not relevant)
```

*Output*

```
XYINT    =  -6.1108
```

***Example 2:*** This example shows how to reverse the order of integration of the variables *x* and *y*. It computes the integral of the function *f* given by:

$f(x, y) = \cos x \sin y$

over the intervals (0.0, 1.0) for the variable *x* and (0.0, 20.0) for the variable *y*, using the Gauss-Legendre method with 5 points in the *x* variable and 48 points in the *y* variable. Because the order of integration does not matter to the approximation:

$$\int_{0.0}^{20.0}\int_{0.0}^{1.0}(\cos x \sin y)\,dx\,dy = \int_{0.0}^{1.0}\int_{0.0}^{20.0}(\cos x \sin y)\,dy\,dx$$

the variable *y*, having more points, is the first variable integrated (performing the integration shown on the right.) This allows the SGLNQ2 subroutine and the FUN1 evaluation to achieve optimal performance. Therefore, the *x* and *y* variables correspond to T and S in the FUN2 subroutine. Also, the *x* and *y* variables correspond to the C, D, N2 and A, B, N1 sets of arguments, respectively, for SGLNQ2.

The user-supplied subroutine FUN2, which evaluates the integrand function, is coded in Fortran as follows:

```
   SUBROUTINE FUN2 (S,N1,T,N2,Z,LDZ)
   INTEGER*4 N1,N2,LDZ
   REAL*4 S(*),T(*),Z(LDZ,*)
   DO 1 J=1,N2
   DO 2 I=1,N1
2     Z(I,J)=COS(T(J))*SIN(S(I))
1     CONTINUE
   RETURN
   END
```

**Note:** The same coding principles for achieving good performance that are noted in “Example 1” on page 933 also apply to this user-supplied subroutine FUN2.

*Program Statements and Input*

```
EXTERNAL FUN2.
        .
        .
        .
                SUBF    A      B    N1    C     D    N2   Z  LDZ
                 |      |      |    |     |     |    |    |   |
YXINT = SGLNQ2( FUN2 , 0.0 , 20.0 , 48 , 0.0 , 1.0 , 5  , Z , 48 )
        .
        .
        .

FUN2       =(see above)
Z          =(not relevant)
```

*Output*

```
YXINT    =  0.4981
```

# SGLGQ and DGLGQ—Numerical Quadrature Performed on a Function Using Gauss-Laguerre Quadrature

These functions approximate the integral of a real valued function over a semi-infinite interval, using the Gauss-Laguerre Quadrature method of specified order.

*Table 159. Data Types*

| *a*, *b*, **Result** | **Subroutine** |
|---|---|
| Short-precision real | SGLGQ |
| Long-precision real | DGLGQ |

### *Syntax*

| | |
|---|---|
| **Fortran** | SGLGQ \| DGLGQ (*subf*, *a*, *b*, *n*) |
| **C and C++** | sglgq \| dglgq (*subf*, *a*, *b*, *n*); |
| **PL/I** | SGLGQ \| DGLGQ (*subf*, *a*, *b*, *n*); |

*On Entry*

*subf*

is the user-supplied subroutine that evaluates the integrand function. The subroutine should be defined with three arguments: *t*, *y*, and *n*. For details, see " Programming Considerations for the SUBF Subroutine" on page 922.

Specified as: *subf* must be declared as an external subroutine in your application program. It can be whatever name you choose.

*a*

has the following meaning, where:

If $b > 0$, it is the lower limit of integration.

If $b < 0$, it is the upper limit of integration.

Specified as: a number of the data type indicated in Table 159.

*b*

is the scaling constant *b* for the exponential. Specified as: a number of the data type indicated in Table 159; $b > 0$ or $b < 0$.

*n*

is the order of the quadrature method to be used. Specified as: a fullword integer; $n$ = 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 16, 20, 24, 32, 40, 48, or 64.

*On Return*

*Function value*

is the approximation of the integral. Returned as: a number of the data type indicated in Table 159.

### *Notes*

1. Declare the DGLGQ function in your program as returning a long-precision real number. Declare the SGLGQ function, if necessary, as returning a short-precision real number.
2. The subroutine specified for *subf* must be declared as external in your program. Also, data types used by *subf* must agree with the data types specified by this

ESSL subroutine. The variable *x*, described under “Function” on page 938, and the argument *n* correspond to the *subf* arguments *t* and *n*, respectively. For details on how to set up the subroutine, see “ Programming Considerations for the SUBF Subroutine” on page 922.

***Function:*** The integral is approximated for a real valued function over a semi-infinite interval, using the Gauss-Laguerre Quadrature method of specified order. The region of integration is:

$(a, \infty)$ if $b > 0$
$(-\infty, a)$ if $b < 0$

The method of order *n* is theoretically exact for integrals of the following form, where *f* is a polynomial of degree less than 2*n*:

$$\int_{a}^{\infty} f(x)e^{-bx}dx \qquad \text{if } b > 0$$

$$\int_{-\infty}^{a} f(x)e^{-bx}dx \qquad \text{if } b < 0$$

The method of order *n* is a good approximation when your integrand is closely approximated by a function of the form $f(x)e^{-bx}$, where *f* is a polynomial of degree less than 2*n*. See references [26] and [89 ]. The result is returned as the function value.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. $b = 0$
2. *n* is not an allowable value, as listed in the syntax for this argument.

***Example 1:*** This example shows how to compute the integral of the function *f* given by:

$f(x) = \sin(3.0x)e^{-1.5x}$

over the interval (–2.0, ∞), using the Gauss-Laguerre method with 20 points:

$$\int_{-2.0}^{\infty} \left(\sin(3.0x)e^{-1.5x}\right)dx$$

The user-supplied subroutine FUN1, which evaluates the integrand function, is coded in Fortran as follows:

```
   SUBROUTINE FUN1 (T,Y,N)
   INTEGER*4 N
   REAL*4 T(*),Y(*)
   DO 1 I=1,N
1     Y(I)=SIN(3.0*T(I))*EXP(-1.5*T(I))
   RETURN
   END
```

*Program Statements and Input*

```
EXTERNAL FUN1
        .
        .
        .
              SUBF     A     B    N
               |       |     |    |
XINT = SGLGQ( FUN1 , -2.0 , 1.5 , 20 )
        .
        .
        .

FUN1     =(see above)
```

*Output*

```
XINT     =  5.891
```

***Example 2:*** This example shows how to compute the integral of the function *f* given by:

$f(x) = \sin(3.0x)e^{1.5x}$

over the interval (–∞, –2.0), using the Gauss-Laguerre method with 20 points:

$$\int_{-\infty}^{-2.0} \sin(3.0x)e^{1.5x}dx$$

The user-supplied subroutine `FUN2`, which evaluates the integrand function, is coded in Fortran as follows:

```
   SUBROUTINE FUN2 (T,Y,N)
   INTEGER*4 N
   REAL*4 T(*),Y(*),TEMP
   DO 1 I=1,N
1     Y(I)=SIN(3.0*T(I))*EXP(1.5*T(I))
   RETURN
   END
```

*Program Statements and Input*

```
EXTERNAL FUN2
        .
        .
        .
              SUBF     A      B    N
               |       |      |    |
XINT = SGLGQ( FUN2 , -2.0 , -1.5 , 20 )
        .
        .
        .
FUN2     =  (see above)
```

*Output*

```
XINT     =  -0.011
```

## SGRAQ and DGRAQ—Numerical Quadrature Performed on a Function Using Gauss-Rational Quadrature

These functions approximate the integral of a real valued function over a semi-infinite interval, using the Gaussian-Rational quadrature method of specified order.

*Table 160. Data Types*

| *a*, *b*, **Result** | **Subroutine** |
|---|---|
| Short-precision real | SGRAQ |
| Long-precision real | DGRAQ |

### *Syntax*

| | |
|---|---|
| **Fortran** | SGRAQ \| DGRAQ (*subf*, *a*, *b*, *n*) |
| **C and C++** | sgraq \| dgraq (*subf*, *a*, *b*, *n*); |
| **PL/I** | SGRAQ \| DGRAQ (*subf*, *a*, *b*, *n*); |

*On Entry*

*subf*

is the user-supplied subroutine that evaluates the integrand function. The subroutine should be defined with three arguments: *t*, *y*, and *n*. For details, see “ Programming Considerations for the SUBF Subroutine” on page 922.

Specified as: *subf* must be declared as an external subroutine in your application program. It can be whatever name you choose.

*a*

has the following meaning, where:

If $a+b > 0$, it is the lower limit of integration.

If $a+b < 0$, it is the upper limit of integration.

Specified as: a number of the data type indicated in Table 160.

*b*

is the centering constant *b* for the integrand. Specified as: a number of the data type indicated in Table 160.

*n*

is the order of the quadrature method to be used. Specified as: a fullword integer; *n* = 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 16, 20, 24, 32, 40, 48, 64, 96, 128, or 256.

*On Return*

*Function value*

is the approximation of the integral. Returned as: a number of the data type indicated in Table 160.

***Notes***

1. Declare the DGRAQ function in your program as returning a long-precision real number. Declare the SGRAQ function, if necessary, as returning a short-precision real number.
2. The subroutine specified for *subf* must be declared as external in your program. Also, data types used by *subf* must agree with the data types specified by this ESSL subroutine. The variable *x*, described under “Function,” and the argument *n* correspond to the *subf* arguments *t* and *n*, respectively. For details on how to set up the subroutine, see “ Programming Considerations for the SUBF Subroutine” on page 922.

***Function:*** The integral is approximated for a real valued function over a semi-infinite interval, using the Gauss-Rational quadrature method of specified order. The region of integration is:

$(a, \infty)$ if $a+b > 0$
$(-\infty, a)$ if $a+b < 0$

The method of order *n* is theoretically exact for integrals of the following form, where *f* is a polynomial of degree less than 2*n*:

$$\int_a^\infty f\left(\frac{1}{x+b}\right)\frac{1}{(x+b)^2}\,dx \qquad \text{if } a+b>0$$

$$\int_{-\infty}^a f\left(\frac{1}{x+b}\right)\frac{1}{(x+b)^2}\,dx \qquad \text{if } a+b<0$$

The method of order *n* is a good approximation when your integrand is closely approximated by a function of the following form, where *f* is a polynomial of degree less than 2*n*:

$$f\left(\frac{1}{x+b}\right)\frac{1}{(x+b)^2}$$

See references [26 ] and [89 ]. The result is returned as the function value to a Fortran, C, C++, or PL/I program.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. $a+b = 0$
2. *n* is not an allowable value, as listed in the syntax for this argument.

***Example 1:*** This example shows how to compute the integral of the function *f* given by:

$f(x) = (e^{1.0/x}) / x^2$

over the interval (−∞, −2.0), using the Gauss-Rational method with 10 points:

$$\int_{-\infty}^{-2.0} \left( \frac{e^{1.0/x}}{x^2} \right) dx$$

The user-supplied subroutine FUN1, which evaluates the integrand function, is coded in Fortran as follows:

```
   SUBROUTINE FUN1 (T,Y,N)
   INTEGER*4 N
   REAL*4 T(*),Y(*),TEMP
   DO 1 I=1,N
        TEMP=1.0/T(I)
1       Y(I)=EXP(TEMP)*TEMP**2
   RETURN
   END
```

*Program Statements and Input*

```
EXTERNAL FUN1
        .
        .
        .
              SUBF     A     B    N
               |       |     |    |
XINT = SGRAQ( FUN1 , -2.0 , 0.0 , 10 )
        .
        .
        .

FUN1     =(see above)
```

*Output*

```
XINT     =  0.393
```

***Example 2:*** This example shows how to compute the integral of the function *f* given by:

$f(x) = (x-3.0)^{-2} + 10(x-3.0)^{-11}$

over the interval (4.0, ∞), using the Gauss-Rational method with 6 points:

$$\int_{4.0}^{\infty} \left( (x-3.0)^{-2} + 10(x-3.0)^{-11} \right) dx$$

The user-supplied subroutine FUN2, which evaluates the integrand function, is coded in Fortran as follows:

```
   SUBROUTINE FUN2 (T,Y,N)
   INTEGER*4 N
   REAL*4 T(*),Y(*),TEMP
   DO 1 I=1,N
        TEMP=1.0/(T(I)-3.0)
1       Y(I)=TEMP**2+10.0*TEMP**11
   RETURN
   END
```

*Program Statements and Input*

```
EXTERNAL FUN2
        .
        .
        .
              SUBF    A      B    N
               |      |      |    |
XINT = SGRAQ( FUN2 , 4.0 , -3.0 , 6 )
        .
        .
        .
FUN2     =  (see above)
```

*Output*

```
XINT     =  2.00
```

# SGHMQ and DGHMQ—Numerical Quadrature Performed on a Function Using Gauss-Hermite Quadrature

These functions approximate the integral of a real valued function over the entire real line, using the Gauss-Hermite Quadrature method of specified order.

*Table 161. Data Types*

| *a*, *b*, **Result** | **Subroutine** |
|---|---|
| Short-precision real | SGHMQ |
| Long-precision real | DGHMQ |

### *Syntax*

| | |
|---|---|
| **Fortran** | SGHMQ \| DGHMQ (*subf*, *a*, *b*, *n*) |
| **C and C++** | sghmq \| dghmq (*subf*, *a*, *b*, *n*); |
| **PL/I** | SGHMQ \| DGHMQ (*subf*, *a*, *b*, *n*); |

*On Entry*

*subf*

is the user-supplied subroutine that evaluates the integrand function. The subroutine should be defined with three arguments: *t*, *y*, and *n*. For details, see " Programming Considerations for the SUBF Subroutine" on page 922.

Specified as: *subf* must be declared as an external subroutine in your application program. It can be whatever name you choose.

*a*

is the centering constant *a* for the exponential. Specified as: a number of the data type indicated in Table 161.

*b*

is the scaling constant *b* for the exponential. Specified as: a number of the data type indicated in Table 161; $b > 0$.

*n*

is the order of the quadrature method to be used. Specified as: a fullword integer; $n = 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 16, 20, 24, 32, 40, 48, 64$, or 96.

*On Return*

*Function value*

is the approximation of the integral. Returned as: a number of the data type indicated in Table 161.

### *Notes*

1. Declare the DGHMQ function in your program as returning a long-precision real number. Declare the SGHMQ function, if necessary, as returning a short-precision real number.
2. The subroutine specified for *subf* must be declared as external in your program. Also, data types used by *subf* must agree with the data types specified by this ESSL subroutine. The variable *x*, described under "Function" on page 945, and the argument *n* correspond to the *subf* arguments *t* and *n*, respectively. For

details on how to set up the subroutine, see “ Programming Considerations for the SUBF Subroutine” on page 922.

***Function:*** The integral is approximated for a real valued function over the entire real line, using the Gauss-Hermite Quadrature method of specified order. The region of integration is from $-\infty$ to $\infty$. The method of order *n* is theoretically exact for integrals of the following form, where *f* is a polynomial of degree less than 2*n*:

$$\int_{-\infty}^{\infty} f(x) e^{-b(x-a)^2} dx$$

The method of order *n* is a good approximation when your integrand is closely approximated by a function of the following form, where *f* is a polynomial of degree less than 2*n*:

$$f(x) e^{-b(x-a)^2}$$

See references [26 ] and [89 ]. The result is returned as the function value to a Fortran, C, C++, or PL/I program.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. $b \le 0$
2. *n* is not an allowable value, as listed in the syntax for this argument.

***Example:*** This example shows how to compute the integral of the function *f* given by:

$$f(x) = x^2 e^{-2(x+5.0)^2}$$

over the interval ($-\infty$, $\infty$), using the Gauss-Hermite method with 4 points:

$$\int_{-\infty}^{\infty} \left( x^2 e^{-2(x+5.0)^2} \right) dx$$

The user-supplied subroutine FUN1, which evaluates the integrand function, is coded in Fortran as follows:

```
   SUBROUTINE FUN1 (T,Y,N)
   INTEGER*4 N
   REAL*4 T(*),Y(*)
   DO 1 I=1,N
1     Y(I)=T(I)**2*EXP(-2.0*(T(I)+5.0)**2)
   RETURN
   END
```

*Program Statements and Input*

```
EXTERNAL FUN1
        .
        .
        .
              SUBF     A     B    N
               |       |     |    |
XINT = SGHMQ( FUN1 , -5.0 , 2.0 , 4 )
        .
        .
        .
```

```
FUN1     =(see above)
```

*Output*

```
XINT     =  31.646
```

# Chapter 16. Random Number Generation

The random number generation subroutines are described in this chapter.

## Overview of the Random Number Generation Subroutines

Random number generation subroutines generate uniformly distributed random numbers or normally distributed random numbers (Table 162).

*Table 162. List of Random Number Generation Subroutines*

| Descriptive Name | Short-Precision Subroutine | Long-Precision Subroutine | Page |
|---|---|---|---|
| Generate a Vector of Uniformly Distributed Random Numbers | SURAND | DURAND | 948 |
| Generate a Vector of Normally Distributed Random Numbers | SNRAND | DNRAND | 951 |
| Generate a Vector of Long Period Uniformly Distributed Random Numbers | SURXOR | DURXOR | 955 |

## Use Considerations

If you need a very long period random number generator, you should use SURXOR and DURXOR, rather than SURAND and DURAND, respectively. The very long period of the generator used by SURXOR and DURXOR, $2^{1279}-1$, makes it useful in modern statistical simulations in which the shorter period of other generators can be exhausted during a single run. As a result, if you need a large number of random numbers, you can use these subroutines, because with this generator, you are not be requesting more than a small percentage of the entire period of the generator.

## Random Number Generation Subroutines

This section contains the random number generation subroutine descriptions.

# SURAND and DURAND—Generate a Vector of Uniformly Distributed Random Numbers

These subroutines generate vector ***x*** of uniform (0,1) pseudo-random numbers, using the multiplicative congruential method with a user-specified seed.

*Table 163. Data Types*

| ***x*** | *seed* | **Subroutine** |
|---|---|---|
| Short-precision real | Long-precision real | SURAND |
| Long-precision real | Long-precision real | DURAND |

**Note:** If you need a very long period random number generator, use SURXOR and DURXOR instead of these subroutines.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SURAND \| DURAND (*seed*, *n*, *x*) |
| **C and C++** | surand \| durand (*seed*, *n*, *x*); |
| **PL/I** | CALL SURAND \| DURAND (*seed*, *n*, *x*); |

*On Entry*

*seed*

is the initial value used to generate the random numbers. Specified as: a number of the data type indicated in Table 163. It should be a whole number; that is, the fraction part should be 0. (If you specify a mixed number, it is truncated.) Its value must be $1.0 \le seed < (2147483647.0 = 2^{31}-1)$.

**Note:** *seed* is always a long-precision real number, even in SURAND.

*n*

is the number of random numbers to be generated. Specified as: a fullword integer; $n \ge 0$.

*x*

See “On Return.”

*On Return*

*seed*

is the new seed that is to be used to generate additional random numbers in subsequent invocations of SURAND or DURAND. Returned as: a number of the data type indicated in Table 163. It is a whole number whose value is $1.0 \le seed < (2147483647.0 = 2^{31}-1)$.

*x*

is a vector of length *n*, containing the uniform pseudo-random numbers with values between 0 and 1. Returned as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 163.

***Note:*** In your C program, argument *seed* must be passed by reference.

***Function:*** The uniform (0,1) pseudo-random numbers are generated as follows, using the multiplicative congruential method:

$$s_i = (a(s_{i-1})) \bmod(m) = (a^i s_0) \bmod(m)$$
$$x_i = s_i/m \quad \text{for } i = 1, 2, ..., n$$

where:

$s_i$ is a random sequence.
$x_i$ is a random number.
$s_0$ is the initial seed provided by the caller.
$a = 7^5 = 16807.0$
$m = 2^{31}-1 = 2147483647.0$
$n$ is the number of random numbers to be generated.

See references [73 ] and [77 ]. If $n$ is 0, no computation is performed, and the initial seed is unchanged.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. $n < 0$
2. *seed* $< 1.0$ or *seed* $\geq 2147483647.0$

***Example 1:*** This example shows a call to SURAND to generate 10 random numbers.

*Call Statement and Input*

```
             SEED    N   X
               |     |   |
CALL SURAND( SEED , 10 , X )

SEED     =  80629.0
```

**Note:** It is important to note that SEED is a long-precision number, even though X contains short-precision numbers.

*Output*

```
SEED     =  759150100.0

X        =  (0.6310323,
             0.7603202,
             0.7015232,
             0.5014868,
             0.4895853,
             0.4602344,
             0.1603608,
             0.1832564,
             0.9899062,
             0.3535068)
```

***Example 2:*** This example shows a call to DURAND to generate 10 random numbers.

*Call Statement and Input*

```
             SEED    N   X
               |     |   |
CALL DURAND( SEED , 10 , X )

SEED     =  80629.0
```

*Output*

```
SEED     =  759150100.0

X        =  (0.6310323270182275,
             0.7603201953509451,
             0.7015232633340746,
             0.5014868557925740,
             0.4895853057920864,
             0.4602344475967038,
             0.1603607578018497,
             0.1832563756887132,
             0.9899062002030695,
             0.3535068129904134)
```

# SNRAND and DNRAND—Generate a Vector of Normally Distributed Random Numbers

These subroutines generate vector ***x*** of normally distributed pseudo-random numbers, with a mean of 0 and a standard deviation of 1, using Polar methods with a user-specified seed.

*Table 164. Data Types*

| ***x***, *aux* | *seed* | **Subroutine** |
|---|---|---|
| Short-precision real | Long-precision real | SNRAND |
| Long-precision real | Long-precision real | DNRAND |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SNRAND \| DNRAND (*seed*, *n*, *x*, *aux*, *naux*) |
| **C and C++** | snrand \| dnrand (*seed*, *n*, *x*, *aux*, *naux*); |
| **PL/I** | CALL SNRAND \| DNRAND (*seed*, *n*, *x*, *aux*, *naux*); |

*On Entry*

*seed*

is the initial value used to generate the random numbers. Specified as: a number of the data type indicated in Table 164. It must be a whole number; that is, the fraction part must be 0. Its value must be $1.0 \le seed < (2147483647.0 = 2^{31}-1)$.

**Note:** *seed* is always a long-precision real number, even in SNRAND.

*n*

is the number of random numbers to be generated. Specified as: a fullword integer; *n* must be an even number and $n \ge 0$.

*x*

See “On Return.”

*aux*

has the following meaning:

If $naux = 0$ and error 2015 is unrecoverable, *aux* is ignored.

Otherwise, it is the storage work area used by this subroutine. Its size must be greater than or equal to $n/2$.

Specified as: an area of storage, containing numbers of the data type indicated in Table 164. They can have any value.

*naux*

is the size of the work area specified by *aux*. Specified as: a fullword integer, where:

If $naux = 0$ and error 2015 is unrecoverable, SNRAND and DNRAND dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, $naux \ge n/2$.

*On Return*

*seed*

is the new seed that is to be used to generate additional random numbers in subsequent invocations of SNRAND or DNRAND. Returned as: a number of the data type indicated in Table 164. It is a whole number whose value is $1.0 \le seed < (2147483647.0 = 2^{31}-1)$.

*x*

is a vector of length *n*, containing the normally distributed pseudo-random numbers. Returned as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 164 on page 951.

**Notes**

1. In your C program, argument *seed* must be passed by reference.
2. Vector ***x*** must have no common elements with the storage area specified for *aux*; otherwise, results are unpredictable.
3. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see “Using Auxiliary Storage in ESSL” on page 32.

***Function:*** The normally distributed pseudo-random numbers, with a mean of 0 and a standard deviation of 1, are generated as follows, using Polar methods with a user-specified seed. The Polar method, which this technique is based on, was developed by G. E. P. Box, M. E. Muller, and G. Marsaglia and is described in reference [73].

1. Using *seed*, a vector of uniform (0,1) pseudo-random numbers, $u_i$ for $i = 1, n$, is generated by calling SURAND or DURAND, respectively. These $u_i$ values are then used in the subsequent steps.
2. All $(y_j, z_j)$ for $j = 1, n/2$ are set as follows, where each $(y, z)$ is a point in the square −1 to 1:

   $y_j = 2u_{2j-1}-1$
   $z_j = 2u_{2j}-1$
3. All $p_j$ for $j = 1, n/2$ are set as follows, where each $p$ measures the square of the radius of $(y, z)$:

   $$p_j = y_j^2 + z_j^2$$

   If $p_j \ge 1$, then $p_j$ is discarded, and steps 1 through 3 are repeated until $p_j < 1$.
4. All $x_i$ for $i = 1, n$ are set as follows to produce the normally distributed random numbers:

   $x_{2j-1} = y_j\ ((-2 \ln p_j) / p_j)^{0.5}$
   $x_{2j} = z_j\ ((-2 \ln p_j) / p_j)^{0.5}$
   for $j = 1, n/2$

If *n* is 0, no computation is performed, and the initial seed is unchanged.

***Error Conditions***

*Resource Errors:* Error 2015 is unrecoverable, *naux* = 0, and unable to allocate work area.

*Computational Errors:* None

*Input-Argument Errors*

1. *n* < 0 or *n* is an odd number
2. *seed* < 1.0 or *seed* ≥ 2147483647.0
3. Error 2015 is recoverable or *naux*≠0, and *naux* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example 1:*** This example shows a call to SNRAND to generate 10 random numbers.

*Call Statement and Input*

```
             SEED   N    X   AUX  NAUX
               |    |    |    |    |
CALL SNRAND( SEED , 10 , X , AUX , 5  )

SEED     =  80629.0
```

**Note:** It is important to note that SEED is a long-precision number, even though X contains short-precision numbers.

*Output*

```
SEED     =  48669425.0

X        =  (0.660649538,
             1.312503695,
             1.906438112,
             0.014065863,
            -0.800935328,
            -3.058144093,
            -0.397426069,
            -0.370634943,
            -0.064151444,
            -0.275887042)
```

***Example 2:*** This example shows a call to DNRAND to generate 10 random numbers.

*Call Statement and Input*

```
             SEED   N    X   AUX  NAUX
               |    |    |    |    |
CALL DNRAND( SEED , 10 , X , AUX , 5  )

SEED     =  80629.0
```

*Output*

```
SEED     =  48669425.0

X        =  (0.6606495655963802,
              1.3125037758861060,
              1.9064381379483730,
              0.0140658628770495,
             -0.8009353314494653,
             -3.0581441239248530,
             -0.3974260845722100,
             -0.3706349643478605,
             -0.0641514443372939,
             -0.2758870630332470)
```

## SURXOR and DURXOR—Generate a Vector of Long Period Uniformly Distributed Random Numbers

These subroutines generate a vector ***x*** of uniform [0,1) pseudo-random numbers, using the Tausworthe exclusive-or algorithm.

*Table 165. Data Types*

| ***x***, ***vseed*** | ***iseed*** | **Subroutine** |
|---|---|---|
| Short-precision real | Integer | SURXOR |
| Long-precision real | Integer | DURXOR |

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL SURXOR \| DURXOR (*iseed*, *n*, *x*, *vseed*) |
| **C and C++** | surxor \| durxor (*iseed*, *n*, *x*, *vseed*); |
| **PL/I** | CALL SURXOR \| DURXOR (*iseed*, *n*, *x*, *vseed*); |

*On Entry*

*iseed*

has the following meaning, where:

If *iseed* ≠ 0, *iseed* is the initial value used to generate the random numbers. You specify *iseed* ≠ 0 when you call this subroutine for the first time or when you changed *vseed* between calls to this subroutine.

If *iseed* = 0, *vseed* is used to generate the random numbers, where *vseed* was initialized by an earlier call to this subroutine. ESSL assumes you have not changed *vseed* between calls to this subroutine, when you specify *iseed* = 0.

Specified as: a fullword integer, as indicated in Table 165.

*n*

is the number of random numbers to be generated. Specified as: a fullword integer; *n* ≥ 0.

*x*

See "On Return."

*vseed*

is the work area used by this subroutine and has the following meaning, where:

If *iseed* ≠ 0, *vseed* is not used for input. The work area can contain anything.

If *iseed* = 0, *vseed* contains the seed vector generated by a preceding call to this subroutine. *vseed* is used in this computation to generate the new random numbers. It should not be changed between calls to this subroutine.

Specified as: a one-dimensional array of (at least) length 10000, containing numbers of the data type indicated in Table 165.

*On Return*

*iseed*

is set to 0 for subsequent calls to SURXOR or DURXOR. Returned as: a fullword integer, as indicated in Table 165.

*x*

is a vector of length *n*, containing the uniform pseudo-random numbers with the following values: $0 \le x < 1$. Returned as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 165.

*vseed*

is the work area used by these subroutines, containing the new seed that is to be used in subsequent calls to this subroutine. Returned as: a one-dimensional array of (at least) length 10000, containing numbers of the data type indicated in Table 165 on page 955.

***Notes***

1. You can generate the same vector *x* of random numbers by starting over and specifying your original nonzero *iseed* value.
2. Multiple calls to these subroutines with mixed sizes generate the same sequence of numbers as a single call the total length, assuming you specify the same initial *iseed* in both cases. For example, you can generate the same vector *x* of random numbers by calling this subroutine twice and specifying $n = 10$ or by calling this subroutine once and specifying $n = 20$. You need to specify the same *iseed* in the initial call in both cases, and *iseed* = 0 in the second call with $n = 10$.
3. Vector ***x*** must have no common elements with the storage area specified for *vseed*; otherwise, results are unpredictable.
4. In your C program, argument *iseed* must be passed by reference.

***Function:*** The pseudo-random numbers uniformly distributed in the interval [0,1) are generated using the Tausworthe exclusive-or algorithm. This is based on a linear-feedback shift-register sequence. The very long period of the generator, $2^{1279}-1$, makes it useful in modern statistical simulations where the shorter period of other generators could be exhausted during a single run. If you need a large number of random numbers, you can use these subroutines, because with this generator you do not request more than a small percentage of the entire period of the generator.

This generator is based on two feedback positions to generate a new binary digit:

$$z_k = z_{(k-p)} \oplus z_{(k-q)}$$

where:

$p > q$
$k = 1, 2, \ldots$
*z* is a bit vector.
and where:

$$\oplus \text{ is the bitwise exclusive-or operation.}$$

For details, see references [51 ], [71 ], and [91 ]. The values of *p* and *q* are selected according to the criteria stated in reference [97].

The algorithm initializes a seed vector of length *p*, starting with *iseed*. The seed vector is stored in *vseed* for use in subsequent calls to this subroutine with *iseed* = 0.

If *n* is 0, no computation is performed, and the initial seed is unchanged.

***Special Usage:*** For some specialized applications, if you need multiple sources of random numbers, you can specify different *vseed* areas, which are initialized with different seeds on multiple calls to this subroutine. You then get multiple sequences of the random number sequence provided by the generator that are sufficiently far apart for most purposes.

***Error Conditions***

*Computational Errors:* None.

*Input-Argument Errors*

1. *n* < 0
2. *iseed* = 0 and *vseed* does not contain valid data.

***Example 1:*** This example shows a call to SURXOR to generate 10 random numbers.

*Call Statement and Input*

```
             ISEED    N   X   VSEED
               |      |   |     |
CALL SURXOR( ISEED , 10 , X , VSEED )

ISEED    =  137
```

*Output*

```
ISEED    =  0

X        =  (0.6440868,
             0.5105118,
             0.4878680,
             0.3209075,
             0.6624528,
             0.2499877,
             0.0056630,
             0.7329214,
             0.7486335,
             0.8050517)
```

***Example 2:*** This example shows a call to SURXOR to generate 10 random numbers. This example specifies *iseed* = 0 and uses the *vseed* output generated from Example 1.

*Call Statement and Input*

```
             ISEED    N   X   VSEED
               |      |   |     |
CALL SURXOR( ISEED , 10 , X , VSEED )

ISEED    =  0
```

*Output*

```
ISEED    =  0

X        =  (0.9930249,
             0.0441873,
             0.6891295,
             0.3101060,
             0.6324178,
             0.3299408,
             0.3553145,
             0.0100013,
             0.0214620,
             0.8059390)
```

***Example 3:*** This example shows a call to DURXOR to generate 20 random numbers. This sequence of numbers generated are like those generated in Examples 1 and 2.

*Call Statement and Input*

```
             ISEED    N   X   VSEED
               |      |   |     |
CALL DURXOR( ISEED , 20 , X , VSEED )

ISEED    =  137
```

*Output*

```
ISEED    =  0

X        =   (0.64408693438956721,
              0.51051182536460882,
              0.48786801310787142,
              0.32090755617007050,
              0.66245283144861666,
              0.24998782843358081,
              0.00566308101257373,
              0.73292147005172925,
              0.74863359794102236,
              0.80505169697755319,
              0.99302499462139138,
              0.04418740640269125,
              0.68912952155409579,
              0.31010611495627916,
              0.63241786342211936,
              0.32994081459690583,
              0.35531452631408911,
              0.01000134413132581,
              0.02146199494672940,
              0.80593898487597615)
```

# Chapter 17. Utilities

The utility subroutines are described in this chapter.

## Overview of the Utility Subroutines

The utility subroutines perform general service functions that support ESSL, rather than mathematical computations (Table 166).

*Table 166. List of Utility Subroutines*

| Descriptive Name | Subroutine | Page |
|---|---|---|
| ESSL Error Information-Handler Subroutine | EINFO | 962 |
| ESSL ERRSAV Subroutine for ESSL | ERRSAV | 965 |
| ESSL ERRSET Subroutine for ESSL | ERRSET | 966 |
| ESSL ERRSTR Subroutine for ESSL | ERRSTR | 968 |
| Set the Vector Section Size (VSS) for the ESSL/370 Scalar Library | IVSSET‡ | |
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| Determine the Level of ESSL Installed | IESSL | 969 |
| Determine the Stride Value for Optimal Performance in Specified Fourier Transform Subroutines | STRIDE | 971 |
| Convert a Sparse Matrix from Storage-by-Rows to Compressed-Matrix Storage Mode | DSRSM | 981 |
| For a General Sparse Matrix, Convert Between Diagonal-Out and Profile-In Skyline Storage Mode | DGKTRN | 985 |
| For a Symmetric Sparse Matrix, Convert Between Diagonal-Out and Profile-In Skyline Storage Mode | DSKTRN | 991 |
| ‡ This subroutine is provided for migration from earlier releases of ESSL and is not intended for use in new programs. Documentation for this subroutines is no longer provided. | | |

## Use Considerations

This section describes what you use the utility subroutines for.

### Determining the Level of ESSL Installed

IESSL gets the level of ESSL and returns it to your program. The level consists of the following: version number, release number, modification number, and number of the most recently installed ESSL PTF. You can use this function to verify that you are running on or using the capabilities of the desired level.

### Finding the Optimal Stride(s) for Your Fourier Transforms

STRIDE is used to determine optimal stride values for your Fourier transforms when using any of the Fourier transform subroutines, except _RCFT and _CRFT. You must invoke STRIDE for each optimal stride you want computed. Sometimes you need a separate stride for your input and output data. For the three-dimensional Fourier transforms, you need an optimal stride for both the second and third dimensions of the array. The examples provided for STRIDE explain how it is used for each of the subroutines listed above.

After obtaining the optimal strides from STRIDE, you should arrange your data using these stride values. After the data is set up, call the Fourier transform subroutine. For additional information on how to set up your data, see “Setting Up Your Data” on page 744.

## Converting Sparse Matrix Storage

DSRSM is used to migrate your existing program from sparse matrices stored by rows to sparse matrices stored in compressed-matrix storage mode. This converts the matrices into a storage format that is compatible with the input requirements for some ESSL sparse matrix subroutines, such as DSMMX.

DGKTRN and DSKTRN are used to convert your sparse matrix from one skyline storage mode to another, if necessary, before calling the subroutines DGKFS/DGKFSP or DSKFS/DSKFSP, respectively.

## Utility Subroutines

This section contains the utility subroutine descriptions.

# EINFO—ESSL Error Information-Handler Subroutine

This subroutine returns information to your program about the data involved in a computational error that occurred in an ESSL subroutine. This is the same information that is provided in the ESSL messages; however, it allows you to check the information in your program at run time and continue processing. You pass the computational error code of interest to this subroutine in *icode*, and it passes back one or more pieces of information in the output arguments *inf1* and, optionally, *inf2*, as defined in Table 167. **You should use this subroutine only for those computational errors listed in the table. It does not apply to computational errors that do not return information.**

For multithreaded application programs, if you want the error handling capabilities that this subroutine provides to be implemented on each thread created by your program, this subroutine must be called from each thread. If your application creates multiple threads, the action performed by a call to this subroutine applies to the thread that this subroutine was invoked from. For an example, see "Example of Handling Errors in a Multithreaded Application Program" on page 127.

*Table 167. Computational Error Information Returned by EINFO*

| Error Code | Receiver | Type of Information |
|---|---|---|
| 2100 | *inf1* | Lower range of a vector |
| | *inf2* | Upper range of a vector |
| 2101 | *inf1* | Index of the eigenvalue that failed to converge |
| | *inf2* | Number of iterations after which it failed to converge |
| 2102 | *inf1* | Index of the last eigenvector that failed to converge |
| | *inf2* | Number of iterations after which it failed to converge |
| 2103 | *inf1* | Index of the pivot with zero value |
| 2104 | *inf1* | Index of the last pivot with nonpositive value |
| 2105 | *inf1* | Index of the pivot element near zero causing factorization to fail |
| 2107 | *inf1* | Index of the singular value that failed to converge |
| | *inf2* | Number of iterations after which it failed to converge |
| 2109 | *inf1* | Iteration count when it was determined that the matrix was not definite |
| 2114 | *inf1* | Index of the last eigenvalue that failed to converge |
| | *inf2* | Number of iterations after which it failed to converge |
| 2115 | *inf1* | Order of the leading minor that was discovered to have a nonpositive determinant |
| 2117 | *inf1* | Column number for which pivot value was near zero |
| 2118 | *inf1* | Row number for which pivot value was near zero |
| 2120 | *inf1* | Row number of empty row where factorization failed |
| 2121 | *inf1* | Column number of empty column where factorization failed |
| 2126 | *inf1* | Row number for which pivot value was unacceptable |
| 2145 | *inf1* | First diagonal element with zero value |

### Syntax

| Fortran | CALL EINFO (*icode*[, *inf1*[, *inf2*]]) |
|---|---|
| **C and C++** | einfo (*icode*, *inf1*, *inf2*); |
| **PL/I** | CALL EINFO (*icode*[, *inf1*[, *inf2*]]); |

*On Entry*

*icode*

has the following meaning, where:

If *icode* = 0, this indicates that the ESSL error option table is to be initialized. (You specify this value once in the beginning of your program before calls to ERRSET.)

If *icode* has any of the allowable error code values listed in Table 167 on page 962, this is the computational error code of interest. (You specify one of these values whenever you want information returned about a computational error.)

Specified as: a fullword integer; *icode* = 0 or an error code value indicated in Table 167 on page 962.

*inf1*

See "On Return."

*inf2*

See "On Return."

*On Return*

*inf1*

has the following meaning, where:

If *icode* = 0, this argument is not used in the computation. In this case, *inf1* is an optional argument, except in C and C++ programs.

If *icode* ≠ 0, then *inf1* is the first information receiver, containing numerical information related to the computational error.

Returned as: a fullword integer.

*inf2*

has the following meaning, where:

If *icode* = 0, this argument is not used in the computation.

If *icode* ≠ 0, then *inf2* is the second information receiver, containing numerical information related to the computational error. It should be specified when the error code provides a second piece of information, and you want the information.

In both of these cases, *inf2* is an optional argument, except in C and C++ programs. For more details, see "Notes."

Returned as: a fullword integer.

### Notes

1. If *icode* is not 0 and is not one of the error codes specified in Table 167 on page 962, this subroutine returns to the caller, and no information is provided in *inf1* and *inf2*.

2. If there are two pieces of information for the error and you specify one output argument, the second piece of information is not returned to the caller.
3. If there is one piece of information for the error and you specify two output arguments, the second output argument is not set by this subroutine.
4. In C and C++ programs you must code the *inf1* and *inf2* arguments, because they are not optional arguments.
5. In Fortran programs, *inf1* and *inf2* are optional arguments. This is an exception to the rule, because other ESSL subroutines do not allow optional arguments.
6. Examples of how to use EINFO are provided in Chapter 4 on page 111.

# ERRSAV—ESSL ERRSAV Subroutine for ESSL

The ERRSAV subroutine copies an ESSL error option table entry into an 8-byte storage area that is accessible to your program.

For multithreaded application programs, if you want the error handling capabilities that this subroutine provides to be implemented on each thread created by your program, this subroutine must be called from each thread. If your application creates multiple threads, the action performed by a call to this subroutine applies to the thread that this subroutine was invoked from. For an example, see “Example of Handling Errors in a Multithreaded Application Program” on page 127.

***Syntax***

| | |
|---|---|
| **Fortran** | CALL ERRSAV (*ierno*, *tabent*) |
| **C and C++** | errsav (*ierno*, *tabent*); |
| **PL/I** | CALL ERRSAV (*ierno*, *tabent*); |

*On Entry*

*ierno*
is the error number in the option table. The entry for *ierno* in the ESSL error option table is stored in the 8-byte storage area *tabent*. Specified as: a fullword integer; *ierno* must be one of the error numbers in the option table. For a list of these numbers, see Table 26 on page 51.

*tabent*
is the storage area where the option table entry is stored. Specified as: an area of storage of length 8-bytes.

***Note:*** Examples of how to use ERRSAV are provided in Chapter 4 on page 111.

# ERRSET—ESSL ERRSET Subroutine for ESSL

The ERRSET subroutine allows you to control execution when error conditions occur. It modifies the information in the ESSL error option table for the error number indicated. For a range of error messages, you can specify the following:

- How many times a particular error is allowed to occur before the program is terminated
- How many times a particular error message is printed before printing is suppressed
- Whether the ESSL error exit routine is to be invoked

For multithreaded application programs, if you want the error handling capabilities that this subroutine provides to be implemented on each thread created by your program, this subroutine must be called from each thread. If your application creates multiple threads, the action performed by a call to this subroutine applies to the thread that this subroutine was invoked from. For an example, see "Example of Handling Errors in a Multithreaded Application Program" on page 127.

***Syntax***

| | |
|---|---|
| **Fortran** | CALL ERRSET (*ierno*, *inoal*, *inomes*, *itrace*, *iusadr*, *irange*) |
| **C and C++** | errset (*ierno*, *inoal*, *inomes*, *itrace*, *iusadr*, *irange*); |
| **PL/I** | CALL ERRSET (*ierno*, *inoal*, *inomes*, *itrace*, *iusadr*, *irange*); |

*On Entry*

*ierno*
is the error number in the option table. The entry for *ierno* in the ESSL error option table is updated as indicated by the other arguments. Specified as: a fullword integer; *ierno* must be one of the error numbers in the option table. For a list of these numbers, see Table 26 on page 51.

*inoal*
indicates the number of errors allowed before each execution is terminated, where:

If *inoal* ≤ 0, the specification is ignored, and the number-of-errors option is not changed.

If *inoal* = 1, execution is terminated after one error.

If 2 ≤ *inoal* ≤ 255, then *inoal* specifies the number of errors allowed before each execution is terminated.

If *inoal* > 255, an unlimited number of errors is allowed.

Specified as: a fullword integer, where:

If *iusadr* = ENOTRM, then 2 ≤ *inoal* ≤ 255.

*inomes*
indicates the number of messages to be printed, where:

If *inomes* < 0, all messages are suppressed.

If *inomes* = 0, the number-of-messages option is not changed.

If 0 < *inomes* ≤ 255, then *inomes* specifies the number of messages to be printed.

If *inomes* > 255, an unlimited number of error messages is allowed.

Specified as: a fullword integer.

*itrace*

this argument is ignored, but must be specified. Specified as: a fullword integer where, *itrace* = 0, 1, or 2 (for migration purposes).

*iusadr*

indicates whether or not the ESSL error exit routine is to be invoked, where:

If *iusadr* is zero, the option table is not altered.

If *iusadr* is one, the option table is set to show no exit routine. Therefore, standard corrective action is to be used when continuing execution.

If *iusadr* = ENOTRM, the option table entry is set to the ESSL error exit routine ENOTRM. Therefore, the ENOTRM subroutine is to be invoked after the occurrence of the indicated errors. (ENOTRM must appear in an EXTERNAL statement in your program.)

Specified as: a 32-bit integer in a 32-bit environment or the name of a subroutine; *iusadr* = 0, 1, or ENOTRM.

Specified as: a 64-bit integer in a 64-bit environment or the name of a subroutine; *iusadr* = 0_8, 1_8, or ENOTRM.

*irange*

indicates the range of errors to be updated in the ESSL error option table, where:

If *irange* < *ierno*, the parameter is ignored.

If *irange* ≥ *ierno*, the options specified for the other parameters are to be applied to the entire range of error conditions encompassed by *ierno* and *irange*.

Specified as: a fullword integer.

### Notes

1. Examples of how to use ERRSET are provided in Chapter 4 on page 111.
2. If you specify ENOTRM for *iusadr*, then *inoal* must be in the following range: 2 ≤ *inoal* ≤ 255.

# ERRSTR—ESSL ERRSTR Subroutine for ESSL

The ERRSTR subroutine stores an entry in the ESSL error option table.

For multithreaded application programs, if you want the error handling capabilities that this subroutine provides to be implemented on each thread created by your program, this subroutine must be called from each thread. If your application creates multiple threads, the action performed by a call to this subroutine applies to the thread that this subroutine was invoked from. For an example, see "Example of Handling Errors in a Multithreaded Application Program" on page 127.

***Syntax***

| | |
|---|---|
| **Fortran** | CALL ERRSTR (*ierno*, *tabent*) |
| **C and C++** | errstr (*ierno*, *tabent*); |
| **PL/I** | CALL ERRSTR (*ierno*, *tabent*); |

*On Entry*

*ierno*
is the error number in the option table. The information in the 8-byte storage area *tabent* is stored into the entry for *ierno* in the ESSL error option table. Specified as: a fullword integer; *ierno* must be one of the error numbers in the option table. For a list of these numbers, see Table 26 on page 51.

*tabent*
is the storage area containing the table entry data. Specified as: an area of storage of length 8-bytes.

***Note:*** Examples of how to use ERRSTR are provided in Chapter 4 on page 111.

## IESSL—Determine the Level of ESSL Installed

This function returns the level of ESSL installed on your system, where the level consists of a version number, release number, and modification number, plus the fix number of the most recent PTF installed.

***Syntax***

| | |
|---|---|
| **Fortran** | CALL IESSL () |
| **C and C++** | iessl (); |
| **PL/I** | CALL IESSL (); |

*On Return*

*Function value*

is the level of ESSL installed on your system. It is provided as a fullword integer in the form *vvrrmmff*, where each two digits represents a part of the level:

- *vv* is the version number.
- *rr* is the release number.
- *mm* is the modification number.
- *ff* is the fix number of the most recent PTF installed.

Returned as: a fullword integer; *vvrrmmff* > 0.

***Notes***

1. To use IESSL effectively, you must install your ESSL PTFs in their proper sequential order. As part of the result, IESSL returns the value *ff* of the **most recent** PTF installed, rather than the **highest number** PTF installed. Therefore, if you do not install your PTFs sequentially, the *ff* value returned by IESSL does not reflect the actual level of ESSL.
2. Declare the IESSL function in your program as returning a fullword integer value.

***Function:*** The IESSL function enables you to determine the current level of ESSL installed on your system. It is useful to you in those instances where your program is using a subroutine or feature that exists only in certain levels of ESSL. It is also useful when your program is dependent upon certain PTFs being applied to ESSL.

***Example:*** This example shows several ways to use the IESSL function. Most typically, you use IESSL for checking the version and release level of ESSL. Suppose you are dependent on a new capability in ESSL, such as a new subroutine or feature, provided for the first time in ESSL Version 3. You can add the following check in your program before using the new capability:

```
IF IESSL() ≥ 3010000
```

By specifying 0000 for *mmff*, the modification and fix level, you are independent of the order in which your modifications and PTFs are installed.

Less typically, you use IESSL for checking the PTF level of ESSL. Suppose you are dependent on PTF 2 being installed on your ESSL Version 3 system. You want to know whether to call a different user-callable subroutine to set up your array data. You can add the following check in your program before making the call:

```
IF IESSL() ≥ 3010002
```

If your system support group installed the ESSL PTFs in their proper sequential order, this test works properly; otherwise, it is unpredictable.

## STRIDE—Determine the Stride Value for Optimal Performance in Specified Fourier Transform Subroutines

This subroutine determines an optimal stride value for you to use for your input or output data when you are computing large row Fourier transforms in any of the Fourier transform subroutines, except _RCFT and _CRFT. The strides determined by this subroutine allow your arrays to fit comfortably in various levels of storage hierarchy on your particular processor, thus allowing you to improve your run-time performance.

**Note:** This subroutine returns a single stride value. Where you need multiple strides, you must invoke this subroutine multiple times; for example, in the multidimensional Fourier transforms and, also, when input and output data types differ. For more details, see “Function” on page 972.

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL STRIDE (*n*, *incd*, *incr*, *dt*, *iopt*) |
| **C and C++** | stride (*n*, *incd*, *incr*, *dt*, *iopt*); |
| **PL/I** | CALL STRIDE (*n*, *incd*, *incr*, *dt*, *iopt*); |

*On Entry*

*n*

is the length *n* of the Fourier transform for which the optimal stride is being determined. The transform corresponding to *n* is usually a row transform; that is, the data elements are stored using a stride value.

Specified as: a fullword integer; $n > 0$.

*incd*

is the minimum allowable stride for the Fourier transform for which the optimal stride is being determined. For each situation in each subroutine, there is a specific way to compute this minimum value. This is explained in the examples starting on page 973.

Specified as: a fullword integer; $incd > 0$ or $incd < 0$.

*incr*

See “On Return” on page 972.

*dt*

is the data type of the numbers for the Fourier transform for which the optimal stride is being determined, where:

If *dt* = 'S', the numbers are short-precision real.

If *dt* = 'D', the numbers are long-precision real.

If *dt* = 'C', the numbers are short-precision complex.

If *dt* = 'Z', the numbers are long-precision complex.

Specified as: a single character; *dt* = 'S', 'D', 'C', or 'Z'.

*iopt*

is provided only for migration purposes from ESSL Version 1 and is no longer used; however, you must still specify it as a dummy argument. Specified as: a fullword integer; *iopt* = 0, 1, or 2.

*On Return*

*incr*

is the stride that allows you to improve your run-time performance in your Fourier transform computation on your particular processor. In general, this value differs for each processor you are running on.

Returned as: a fullword integer; *incr* > 0 or *incr* < 0 and |*incr*| ≥ |*incd*|, where *incr* has the same sign (+ or −) as *incd*.

***Notes***

1. In your C program, argument *incr* must be passed by reference.
2. All subroutines accept lowercase letters for the *dt* argument.
3. For each situation in each of the Fourier transform subroutines, there is a specific way to compute the value you should specify for the *incd* argument. Details on how to compute each of these values is given in the examples starting on page 973. See the example corresponding to the Fourier transform subroutine you are using.
4. Where different data types are specified for the input and output data in your Fourier transform subroutine, you should be careful to indicate the correct data type in the *dt* argument in this subroutine.
5. For additional information on how to set up your data, see “Setting Up Your Data” on page 744.

***Function:*** This subroutine determines an optimal stride, *incr*, for you to use for your input or output data when computing large row Fourier transforms. The stride value returned by this subroutine is based on the size and structure of your transform data, using:

- The size of each data item (*dt*)
- The minimum allowable stride for this transform (*incd*)
- The length of the transform (*n*)

This information is used in determining the optimal stride for the processor you are currently running on. The stride determined by this subroutine allows your arrays to fit comfortably in various levels of storage hierarchy for that processor, thus giving you the ability to improve your run-time performance.

You get only one stride value returned by this subroutine on each invocation. Therefore, in many instances, you may need to invoke this subroutine multiple times to obtain several stride values to use in your Fourier transform computation:

- For multidimensional Fourier transforms using several strides, this subroutine must be called once for each optimal stride you want to obtain. Successive invocations should go from the lower (earlier) dimensions to the higher (later) dimensions, because the results from the lower dimensions are used to calculate the *incd* values for the higher dimensions.
- Where input and output data have different data types and you want to obtain optimal strides for each, this subroutine must be called once for each data type.

Where multiple invocations are necessary, they are explained in the examples starting on page 973. The examples also explain how to calculate the *incd* values for each invocation. There are nine examples to cover the Fourier transform subroutines that can use the STRIDE subroutine.

After calling this subroutine and obtaining the optimal stride value, you then set up your input or output array accordingly. This may involve movement of data for input arrays or increasing the sizes of input or output arrays. To accomplish this, you may want to set up a separate subroutine with the stride values passed into it as arguments. You can then dimension your arrays in that subroutine, depending on the values calculated by STRIDE. For additional information on how to set up your data, see “Setting Up Your Data” on page 744.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. $n \leq 0$
2. $incd = 0$
3. $iopt \neq$ 0, 1, or 2
4. $dt \neq$ S, D, C, or Z

***Example 1—SCFT:*** This example shows the use of the STRIDE subroutine in computing one-dimensional row transforms using the SCFT subroutine.

If *inc2x* = 1, the input sequences are stored in the transposed form as rows of a two-dimensional array X(INC1X,N). In this case, the STRIDE subroutine helps in determining a good value of *inc1x* for this array. The required minimum value of *inc1x* is *m*, the number of Fourier transforms being computed. To find a good value of *inc1x*, use STRIDE as follows:

```
            N  INCD  INCR    DT   IOPT
            |   |     |      |    |
CALL STRIDE( N , M  , INC1X , 'C' , 0  )
```

Here, the arguments refer to the SCFT subroutine. In the following table, values of *inc1x* are given (as obtained from the STRIDE subroutine) for some combinations of *n* and *m* and for POWER2-P2SC with 64KB level 1 cache using the POWER2 library for ESSL:

| N | M | INC1X |
|---|---|---|
| 128 | 64 | 64 |
| 240 | 32 | 32 |
| 240 | 64 | 65 |
| 256 | 256 | 264 |
| 512 | 60 | 60 |
| 1024 | 64 | 65 |

The above example also applies when the output sequences are stored in the transposed form (*inc2y* = 1). In that case, in the above example, *inc1x* is replaced by *inc1y*.

In computing column transforms (*inc1x* = *inc1y* = 1), the values of *inc2x* and *inc2y* are not very important. For these, any value over the required minimum of *n* can be used.

***Example 2—DCOSF:*** This example shows the use of the STRIDE subroutine in computing one-dimensional row transforms using the DCOSF subroutine.

If *inc2x* = 1, the input sequences are stored in the transposed form as rows of a two-dimensional array `X(INC1X,N/2+1)`. In this case, the STRIDE subroutine helps in determining a good value of *inc1x* for this array. The required minimum value of *inc1x* is *m*, the number of Fourier transforms being computed. To find a good value of *inc1x*, use STRIDE as follows:

```
             N      INCD  INCR    DT  IOPT
             |       |      |      |     |
CALL STRIDE( N/2+1 , M  , INC1X , 'D' , 0  )
```

Here, the arguments refer to the DCOSF subroutine. In the following table, values of *inc1x* are given (as obtained from the STRIDE subroutine) for some combinations of *n* and *m* and for POWER2-P2SC with 64KB level 1 cache using the POWER2 library for ESSL:

| N | M | INC1X |
|---|---|---|
| 128 | 64 | 64 |
| 240 | 32 | 32 |
| 240 | 64 | 64 |
| 256 | 256 | 264 |
| 512 | 60 | 60 |
| 1024 | 64 | 65 |

The above example also applies when the output sequences are stored in the transposed form (*inc2y* = 1). In that case, in the above example, *inc1x* is replaced by *inc1y*.

In computing column transforms (*inc1x* = *inc1y* = 1), the values of *inc2x* and *inc2y* are not very important. For these, any value over the required minimum of *n*/2+1 can be used.

***Example 3—DSINF:*** This example shows the use of the STRIDE subroutine in computing one-dimensional row transforms using the DSINF subroutine.

If *inc2x* = 1, the input sequences are stored in the transposed form as rows of a two-dimensional array `X(INC1X,N/2)`. In this case, the STRIDE subroutine helps in determining a good value of *inc1x* for this array. The required minimum value of *inc1x* is *m*, the number of Fourier transforms being computed. To find a good value of *inc1x*, use STRIDE as follows:

```
             N    INCD  INCR    DT   IOPT
             |     |     |       |    |
CALL STRIDE( N/2 , M  , INC1X , 'D' , 0  )
```

Here, the arguments refer to the DSINF subroutine. In the following table, values of *inc1x* are given (as obtained from the STRIDE subroutine) for some combinations of *n* and *m* and for POWER2-P2SC with 64KB level 1 cache using the POWER2 library for ESSL:

| N | M | INC1X |
|---|---|---|
| 128 | 64 | 64 |
| 240 | 32 | 32 |
| 240 | 64 | 64 |
| 256 | 256 | 264 |
| 512 | 60 | 60 |
| 1024 | 64 | 65 |

The above example also applies when the output sequences are stored in the transposed form (*inc2y* = 1). In that case, in the above example, *inc1x* is replaced by *inc1y*.

In computing column transforms (*inc1x* = *inc1y* = 1), the values of *inc2x* and *inc2y* are not very important. For these, any value over the required minimum of *n*/2 can be used.

***Example 4—SCFT2:*** This example shows the use of the STRIDE subroutine in computing two-dimensional transforms using the SCFT2 subroutine.

If *inc1y* = 1, the two-dimensional output array is stored in the normal form. In this case, the output array can be declared as `Y(INC2Y,N2)`, where the required minimum value of *inc2y* is *n1*. The STRIDE subroutine helps in picking a good value of *inc2y*. To find a good value of *inc2y*, use STRIDE as follows:

```
             N   INCD  INCR    DT   IOPT
             |    |     |       |    |
CALL STRIDE( N2 , N1 , INC2Y , 'C' , 0  )
```

Here, the arguments refer to the SCFT2 subroutine. In the following table, values of *inc2y* are given (as obtained from the STRIDE subroutine) for some two-dimensional arrays with *n1* = *n2* and for POWER2-P2SC with 64KB level 1 cache using the POWER2 library for ESSL:

| N1 | N2 | INC2Y |
|---|---|---|
| 64 | 64 | 64 |
| 128 | 128 | 136 |
| 240 | 240 | 240 |
| 512 | 512 | 520 |
| 840 | 840 | 848 |

If the input array is stored in the normal form (*inc1x* = 1), the value of *inc2x* is not important. However, if you want to use the same array for input and output, you should use *inc2x* = *inc2y*.

If *inc2y* = 1, the two-dimensional output array is stored in the transposed form. In this case, the output array can be declared as `Y(INC1Y,N1)`, where the required minimum value of *inc1y* is *n2*. The STRIDE subroutine helps in picking a good value of *inc1y*. To find a good value of *inc1y*, use STRIDE as follows:

```
             N   INCD  INCR    DT   IOPT
             |    |     |       |    |
CALL STRIDE( N1 , N2 , INC1Y , 'C' , 0  )
```

Here, the arguments refer to the SCFT2 subroutine. In the following table, values of *inc1y* are given (as obtained from the STRIDE subroutine) for some combinations of *n1* and *n2* and for POWER2-P2SC with 64K level 1 cache using the POWER2 library for ESSL:

| N1 | N2 | INC1Y |
|---|---|---|
| 60 | 64 | 64 |
| 120 | 128 | 136 |
| 256 | 240 | 240 |
| 512 | 512 | 520 |
| 840 | 840 | 848 |

If the input array is stored in the transposed form (*inc2x* = 1), the value of *inc1x* is also important. The above example can be used to find a good value of *inc1x*, by replacing *inc1y* with *inc1x*. If both arrays are stored in the transposed form, a good value for *inc1y* is also a good value for *inc1x*. In that situation, the two arrays can also be made equivalent.

***Example 5—SRCFT2:*** This example shows the use of the STRIDE subroutine in computing two-dimensional transforms using the SRCFT2 subroutine.

For this subroutine, the output array is declared as `Y(INC2Y,N2)`, where the required minimum value of *inc2y* is *n1*/2+1. The STRIDE subroutine helps in picking a good value of *inc2y*. To find a good value of *inc2y*, use STRIDE as follows:

```
             N      INCD     INCR    DT   IOPT
             |        |        |      |    |
CALL STRIDE( N2 , N1/2 + 1 , INC2Y , 'C' , 0  )
```

Here, the arguments refer to the SRCFT2 subroutine. In the following table, values of *inc2y* are given (as obtained from the STRIDE subroutine) for some two-dimensional arrays with *n1* = *n2* and for POWER2-P2SC with 64KB level 1 cache using the POWER2 library for ESSL:

| N1 | N2 | INC2Y |
|---|---|---|
| 240 | 240 | 121 |
| 420 | 420 | 211 |
| 512 | 512 | 257 |
| 840 | 840 | 421 |
| 1024 | 1024 | 513 |
| 2048 | 2048 | 1032 |

For this subroutine, the leading dimension of the input array (*inc2x*) is not important. If you want to use the same array for input and output, you should use *inc2x* ≥ 2(*inc2y*).

***Example 6—SCRFT2:*** This example shows the use of the STRIDE subroutine in computing two-dimensional transforms using the SCRFT2 subroutine.

For this subroutine, the output array is declared as `Y(INC2Y,N2)`, where the required minimum value of *inc2y* is *n1*+2. The STRIDE subroutine helps in picking a good value of *inc2y*. To find a good value of *inc2y*, use STRIDE as follows:

```
             N     INCD    INCR    DT   IOPT
             |       |       |      |    |
CALL STRIDE( N2 , N1 + 2 , INC2Y , 'S' , 0  )
```

Here, the arguments refer to the SCRFT2 subroutine. In the following table, values of *inc2y* are given (as obtained from the STRIDE subroutine) for some two-dimensional arrays with *n1* = *n2* and for POWER2-P2SC with 64KB level 1 cache using the POWER2 library for ESSL:

| N1 | N2 | INC2Y |
|---|---|---|
| 240 | 240 | 242 |
| 420 | 420 | 422 |
| 512 | 512 | 514 |
| 840 | 840 | 842 |
| 1024 | 1024 | 1026 |
| 2048 | 2048 | 2064 |

For this subroutine, the leading dimension of the input array (*inc2x*) is also important. In general, *inc2x* = *inc2y*/2 is a good choice. This is also the requirement if you want to use the same array for input and output.

***Example 7—SCFT3:*** This example shows the use of the STRIDE subroutine in computing three-dimensional transforms using the SCFT3 subroutine.

For this subroutine, the strides for the input array are not important. They are important for the output array. The STRIDE subroutine helps in picking good values of *inc2y* and *inc3y*. This requires two calls to the STRIDE subroutine as shown below. First, you should find a good value for *inc2y*. The minimum acceptable value for *inc2y* is *n1*.

```
             N   INCD  INCR    DT   IOPT
             |    |     |      |    |
CALL STRIDE( N2 , N1 , INC2Y , 'C' , 0  )
```

Here, the arguments refer to the SCFT3 subroutine. Next, you should find a good value for *inc3y*. The minimum acceptable value for *inc3y* is (*n2*)(*inc2y*).

```
             N      INCD    INCR    DT   IOPT
             |       |       |      |    |
CALL STRIDE( N3 , N2*INC2Y, INC3Y , 'C' , 0  )
```

If *inc3y* turns out to be a multiple of *inc2y*, then `Y` can be declared a three-dimensional array as `Y(INC2Y,INC3Y/INC2Y,N3)`. For large problems, this may not happen. In that case, you can declare the `Y` array as a two-dimensional array `Y(0:INC3Y-1,0:N3-1)` or a one-dimensional array `Y(0:INC3Y*N3-1)`. Using zero-based indexing, the element *y(k1,k2,k3)* is stored in the following location in these arrays:

- For the two-dimensional array, location *(k1+k2*inc2y,k3)*
- For the one-dimensional array, location *(k1+k2*inc2y+k3*inc3y)*

In the following table, values of *inc2y* and *inc3y* are given (as obtained from the STRIDE subroutine) for some three-dimensional arrays with *n1* = *n2* = *n3* and for POWER2-P2SC with 64KB level 1 cache using the POWER2 library for ESSL:

| N1,N2,N3 | INC2Y | INC3Y |
|---|---|---|
| 30 | 30 | 900 |
| 32 | 32 | 1032 |
| 64 | 64 | 4112 |
| 120 | 120 | 14400 |
| 128 | 136 | 17416 |
| 240 | 240 | 57608 |
| 256 | 264 | 67592 |
| 420 | 420 | 176400 |

As mentioned before, the strides of the input array are not important. The array can be declared as a three-dimensional array. If you want to use the same array for input and output, the requirements are *inc2x* ≥ *inc2y* and *inc3x* ≥ *inc3y*. A simple thing to do is to use *inc2x* = *inc2y* and make *inc3x* a multiple of *inc2x* not smaller than *inc3y*. Then `X` can be declared as a three-dimensional array `X(INC2X,INC3X/INC2X,N3)`.

***Example 8—SRCFT3:*** This example shows the use of the STRIDE subroutine in computing three-dimensional transforms using the SRCFT3 subroutine.

For this subroutine, the strides for the input array are not important. They are important for the output array. The STRIDE subroutine helps in picking good values of *inc2y* and *inc3y*. This requires two calls to the STRIDE subroutine as shown below. First, you should find a good value for *inc2y*. The minimum acceptable value for *inc2y* is *n1*/2+1.

```
             N       INCD    INCR    DT   IOPT
             |        |       |      |    |
CALL STRIDE( N2 , N1/2 + 1 , INC2Y , 'C' , 0  )
```

Here, the arguments refer to the SRCFT3 subroutine. Next, you should find a good value for *inc3y*. The minimum acceptable value for *inc3y* is (*n2*)(*inc2y*).

```
             N      INCD     INCR    DT   IOPT
             |       |        |      |    |
CALL STRIDE( N3 , N2*INC2Y , INC3Y , 'C' , 0  )
```

If *inc3y* turns out to be a multiple of *inc2y*, then `Y` can be declared a three-dimensional array as `Y(INC2Y,INC3Y/INC2Y,N3)`. For large problems, this may not happen. In that case, you can declare the `Y` array as a two-dimensional array `Y(0:INC3Y-1,0:N3-1)` or a one-dimensional array `Y(0:INC3Y*N3-1)`. Using zero-based indexing, the element *y(k1,k2,k3)* is stored in the following location in these arrays:

- For the two-dimensional array, location *(k1+k2*inc2y,k3)*
- For the one-dimensional array, location *(k1+k2*inc2y+k3*inc3y)*

In the following table, values of *inc2y* and *inc3y* are given (as obtained from the STRIDE subroutine) for some three-dimensional arrays with *n1* = *n2* = *n3* and for POWER2-P2SC with 64KB level 1 cache using the POWER2 library for ESSL:

| N1,N2,N3 | INC2Y | INC3Y |
|---|---|---|
| 32 | 17 | 544 |
| 64 | 33 | 2112 |
| 120 | 61 | 7320 |
| 128 | 65 | 8328 |
| 240 | 121 | 29064 |
| 256 | 129 | 33032 |

As mentioned before, the strides of the input array are not important. The array can be declared as a three-dimensional array. If you want to use the same array for input and output, the requirements are *inc2x* ≥ 2(*inc2y*) and *inc3x* ≥ 2(*inc3y*). A simple thing to do is to use *inc2x* = 2(*inc2y*) and make *inc3x* a multiple of *inc2x* not smaller than 2(*inc3y*). Then `X` can be declared as a three-dimensional array `X(INC2X,INC3X/INC2X,N3)`.

***Example 9—SCRFT3:*** This example shows the use of the STRIDE subroutine in computing three-dimensional transforms using the SCRFT3 subroutine.

The STRIDE subroutine helps in picking good values of *inc2y* and *inc3y*. This requires two calls to the STRIDE subroutine as shown below. First, you should find a good value for *inc2y*. The minimum acceptable value for *inc2y* is *n1*+2.

```
            N     INCD    INCR    DT   IOPT
            |       |       |      |    |
CALL STRIDE( N2 , N1 + 2 , INC2Y , 'S' , 0  )
```

Here, the arguments refer to the SCRFT3 subroutine. Next, you should find a good value for *inc3y*. The minimum acceptable value for *inc3y* is (*n2*)(*inc2y*).

```
            N      INCD     INCR    DT   IOPT
            |        |        |      |    |
CALL STRIDE( N3 , N2*INC2Y , INC3Y , 'S' , 0  )
```

If *inc3y* turns out to be a multiple of *inc2y*, then `Y` can be declared a three-dimensional array as `Y(INC2Y,INC3Y/INC2Y,N3)`. For large problems, this may not happen. In that case, you can declare the `Y` array as a two-dimensional array `Y(0:INC3Y-1,0:N3-1)` or a one-dimensional array `Y(0:INC3Y*N3-1)`. Using zero-based indexing, the element *y(k1,k2,k3)* is stored in the following location in these arrays:

- For the two-dimensional array, location *(k1+k2*inc2y,k3)*
- For the one-dimensional array, location *(k1+k2*inc2y+k3*inc3y)*

In the following table, values of *inc2y* and *inc3y* are given (as obtained from the STRIDE subroutine) for some three-dimensional arrays with *n1* = *n2* = *n3* and for POWER2-P2SC with 64KB level 1 cache using the POWER2 library for ESSL:

| N1,N2,N3 | INC2Y | INC3Y |
|---|---|---|
| 32 | 34 | 1088 |
| 64 | 66 | 4224 |
| 120 | 122 | 14640 |
| 128 | 130 | 16656 |
| 240 | 242 | 58128 |
| 256 | 258 | 66064 |

For this subroutine, the strides (*inc2x* and *inc3x*) of the input array are also important. In general, *inc2x* = *inc2y*/2 and *inc3x* = *inc3y*/2 are good choices. These are also the requirement if you want to use the same array for input and output.

# DSRSM—Convert a Sparse Matrix from Storage-by-Rows to Compressed-Matrix Storage Mode

This subroutine converts either $m$ by $n$ general sparse matrix ***A*** or symmetric sparse matrix ***A*** of order $n$ from storage-by-rows to compressed-matrix storage mode, where matrix ***A*** contains long-precision real numbers.

***Syntax***

| | |
|---|---|
| **Fortran** | CALL DSRSM (*iopt*, *ar*, *ja*, *ia*, *m*, *nz*, *ac*, *ka*, *lda*) |
| **C and C++** | dsrsm (*iopt*, *ar*, *ja*, *ia*, *m*, *nz*, *ac*, *ka*, *lda*); |
| **PL/I** | CALL DSRSM (*iopt*, *ar*, *ja*, *ia*, *m*, *nz*, *ac*, *ka*, *lda*); |

*On Entry*

*iopt*

indicates the storage variation used for sparse matrix ***A*** storage-by-rows:

If $iopt = 0$, matrix ***A*** is a general sparse matrix, where all the nonzero elements in matrix ***A*** are used to set up the storage arrays.

If $iopt = 1$, matrix ***A*** is a symmetric sparse matrix, where only the upper triangle and diagonal elements are used to set up the storage arrays.

Specified as: a fullword integer; $iopt = 0$ or 1.

*ar*

is the sparse matrix ***A***, stored by rows in an array, referred to as AR. The *iopt* argument indicates the storage variation used for storing matrix ***A***. Specified as: a one-dimensional array, containing long-precision real numbers. The number of elements, *ne*, in this array can be determined by subtracting 1 from the value in IA($m$+1).

*ja*

is the array, referred to as JA, containing the column numbers of each nonzero element in sparse matrix ***A***. Specified as: a one-dimensional array, containing fullword integers; $1 \leq$ (JA elements) $\leq n$. The number of elements, *ne*, in this array can be determined by subtracting 1 from the value in IA($m$+1).

*ia*

is the row pointer array, referred to as IA, containing the starting positions of each row of matrix ***A*** in array AR and one position past the end of array AR. Specified as: a one-dimensional array of (at least) length $m$+1, containing fullword integers; IA($i$+1) $\geq$ IA($i$) for $i = 1, m+1$.

*m*

is the number of rows in sparse matrix ***A***. Specified as: a fullword integer; $m \geq 0$.

*nz*

is the number of columns in output arrays AC and KA that are available for use. Specified as: a fullword integer; $nz > 0$.

*ac*

See “On Return” on page 982.

*ka*

See “On Return” on page 982.

*lda*

is the size of the leading dimension of the arrays specified for *ac* and *ka*. Specified as: a fullword integer; $0 < lda \leq m$.

*On Return*

*nz*
is the maximum number of nonzero elements, *nz*, in each row of matrix ***A***, which is stored in compressed-matrix storage mode. Returned as: a fullword integer; (input argument) *nz* ≤ (output argument) *nz*.

*ac*
is the *m* by *n* general sparse matrix ***A*** or symmetric matrix ***A*** of order *n* stored in compressed-matrix storage mode in an array, referred to as AC. Returned as: an *lda* by at least (input argument) *nz* array, containing long-precision real numbers, where only the first (output argument) *nz* columns are used to store the matrix.

*ka*
is the array, referred to as KA, containing the column numbers of the matrix ***A*** elements that are stored in the corresponding positions in array AC. Returned as: an *lda* by at least (input argument) *nz* array, containing fullword integers, where only the first (output argument) *nz* columns are used to store the column numbers.

***Notes***

1. In your C program, argument *nz* must be passed by reference.
2. The value specified for input argument *nz* should be greater than or equal to the number of nonzero elements you estimate to be in each row of sparse matrix ***A***. The value returned in output argument *nz* corresponds to the *nz* value defined for compressed-matrix storage mode. This value is less than or equal to the value specified for input argument *nz*.
3. For a description of the storage modes for sparse matrices, see "Compressed-Matrix Storage Mode" on page 93 and "Storage-by-Rows" on page 99.

***Function:*** A sparse matrix ***A*** is converted from storage-by-rows (using arrays AR, JA, and IA) to compressed-matrix storage mode (using arrays AC and KA). The argument *iopt* indicates whether the input matrix ***A*** is stored by rows using the storage variation for general sparse matrices or for symmetric sparse matrices. See reference [70].

This subroutine is meant for existing programs that need to convert their sparse matrices to a storage mode compatible with some of the ESSL sparse matrix subroutines, such as DSMMX.

***Error Conditions***

*Computational Errors:* None

*Input-Argument Errors*

1. *iopt* ≠ 0 or 1
2. $m < 0$
3. $lda < 1$
4. $lda < m$
5. $nz \leq 0$
6. IA($m$+1) < 1
7. IA($i$+1)−IA($i$) < 0, for any $i = 1, m$
8. *nz* is too small to store matrix ***A*** in array AC, where:

- If *iopt* = 0 , AC and KA are not modified.
- If *iopt* = 1 , AC and KA are modified.

***Example 1:*** This example shows a general sparse matrix ***A***, which is stored by rows and converted to compressed-matrix storage mode, where sparse matrix ***A*** is:

```
┌                         ┐
| 11.0   0.0   0.0  14.0 |
|  0.0  22.0   0.0  24.0 |
|  0.0   0.0  33.0  34.0 |
|  0.0   0.0   0.0  44.0 |
└                         ┘
```

Because there is a maximum of only two nonzero elements in each row of ***A***, and argument *nz* is specified as 5, columns 3 through 5 of arrays AC and KA are not used.

*Call Statement and Input*

```
           IOPT  AR   JA   IA   M   NZ  AC   KA  LDA
            |    |    |    |    |   |   |    |    |
CALL DSRSM( 0  , AR , JA , IA , 4 , 5 , AC , KA , 4 )

AR       =  (11.0, 14.0, 22.0, 24.0, 33.0, 34.0, 44.0)
JA       =  (1, 4, 2, 4, 3, 4, 4)
IA       =  (1, 3, 5, 7, 8)
```

*Output*

```
NZ       =  2


            ┌                     ┐
            | 11.0  14.0  .  .  . |
AC    =     | 22.0  24.0  .  .  . |
            | 33.0  34.0  .  .  . |
            | 44.0   0.0  .  .  . |
            └                     ┘


            ┌               ┐
            | 1  4  .  .  . |
KA    =     | 2  4  .  .  . |
            | 3  4  .  .  . |
            | 4  4  .  .  . |
            └               ┘
```

***Example 2:*** This example shows a symmetric sparse matrix ***A***, which is stored by rows and converted to compressed-matrix storage mode, where sparse matrix ***A*** is:

```
┌                         ┐
| 11.0   0.0   0.0  14.0 |
|  0.0  22.0   0.0  24.0 |
|  0.0   0.0  33.0  34.0 |
| 14.0  24.0  34.0  44.0 |
└                         ┘
```

Because there is a maximum of only four nonzero elements in each row of ***A***, and argument *nz* is specified as 6, columns 5 and 6 of arrays AC and KA are not used.

*Call Statement and Input*

```
                IOPT  AR   JA   IA   M   NZ  AC   KA  LDA
                 |    |    |    |    |   |   |    |    |
CALL DSRSM( 1  , AR , JA , IA , 4 , 6 , AC , KA , 4 )

AR       =  (11.0, 14.0, 22.0, 24.0, 33.0, 34.0, 44.0)
JA       =  (1, 4, 2, 4, 3, 4, 4)
IA       =  (1, 3, 5, 7, 8)
```

*Output*

```
NZ       =  4

             ┌                             ┐
             | 11.0  14.0   0.0   0.0  .  . |
AC    =      | 22.0  24.0   0.0   0.0  .  . |
             | 33.0  34.0   0.0   0.0  .  . |
             | 44.0  24.0  34.0  14.0  .  . |
             └                             ┘

             ┌                  ┐
             | 1  4  4  4  .  . |
KA    =      | 2  4  4  4  .  . |
             | 3  4  4  4  .  . |
             | 4  2  3  1  .  . |
             └                  ┘
```

# DGKTRN—For a General Sparse Matrix, Convert Between Diagonal-Out and Profile-In Skyline Storage Mode

This subroutine converts general sparse matrix ***A*** of order *n* from one skyline storage mode to another—that is, between the following:

- Diagonal-out skyline storage mode
- Profile-in skyline storage mode

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL DGKTRN (*n*, *au*, *nu*, *idu*, *al*, *nl*, *idl*, *itran*, *aux*, *naux*) |
| **C and C++** | dgktrn (*n*, *au*, *nu*, *idu*, *al*, *nl*, *idl*, *itran*, *aux*, *naux*); |
| **PL/I** | CALL DGKTRN (*n*, *au*, *nu*, *idu*, *al*, *nl*, *idl*, *itran*, *aux*, *naux*); |

*On Entry*

*n*

is the order of general sparse matrix ***A***. Specified as: a fullword integer; $n \geq 0$.

*au*

is the array, referred to as AU, containing the upper triangular part of general sparse matrix ***A***, stored as follows, where:

If ITRAN(1) = 0, ***A*** is stored in diagonal-out skyline storage mode.

If ITRAN(1) = 1, ***A*** is stored in profile-in skyline storage mode.

Specified as: a one-dimensional array of (at least) length *nu*, containing long-precision real numbers.

*nu*

is the length of array AU. Specified as: a fullword integer; $nu \geq 0$ and $nu \geq (\text{IDU}(n+1)-1)$.

*idu*

is the array, referred to as IDU, containing the relative positions of the diagonal elements of matrix ***A*** in input array AU. Specified as: a one-dimensional array of (at least) length *n*+1, containing fullword integers.

*al*

is the array, referred to as AL, containing the lower triangular part of general sparse matrix ***A***, stored as follows, where:

If ITRAN(1) = 0, ***A*** is stored in diagonal-out skyline storage mode.

If ITRAN(1) = 1, ***A*** is stored in profile-in skyline storage mode.

**Note:** Entries in AL for diagonal elements of ***A*** are assumed not to have meaningful values.

Specified as: a one-dimensional array of (at least) length *nl*, containing long-precision real numbers.

*nl*

is the length of array AL. Specified as: a fullword integer; $nl \geq 0$ and $nl \geq (\text{IDL}(n+1)-1)$.

*idl*

is the array, referred to as IDL, containing the relative positions of the diagonal elements of matrix ***A*** in input array AL. Specified as: a one-dimensional array of (at least) length *n*+1, containing fullword integers.

*itran*

is an array of parameters, ITRAN(*i*), where:

- ITRAN(1) indicates the input storage mode used for matrix ***A***. This determines the arrangement of data in arrays AU, IDU, AL, and IDL on input, where:

  If ITRAN(1) = 0, diagonal-out skyline storage mode is used.

  If ITRAN(1) = 1, profile-in skyline storage mode is used.

- ITRAN(2) indicates the output storage mode used for matrix ***A***. This determines the arrangement of data in arrays AU, IDU, AL, and IDL on output, where:

  If ITRAN(2) = 0, diagonal-out skyline storage mode is used.

  If ITRAN(2) = 1, profile-in skyline storage mode is used.

- ITRAN(3) indicates the direction of sweep that ESSL uses through the matrix ***A***, allowing you to optimize performance (see “Notes” on page 987), where:

  If ITRAN(3) = 1, matrix ***A*** is transformed in the positive direction, starting in row or column 1 and ending in row or column *n*.

  If ITRAN(3) = –1, matrix ***A*** is transformed in the negative direction, starting in row or column *n* and ending in row or column 1.

Specified as: a one-dimensional array of (at least) length 3, containing fullword integers, where:

ITRAN(1) = 0 or 1
ITRAN(2) = 0 or 1
ITRAN(3) = –1 or 1

*aux*

has the following meaning:

If *naux* = 0 and error 2015 is unrecoverable, *aux* is ignored.

Otherwise, it is the storage work area used by this subroutine. Its size is specified by *naux*.

Specified as: an area of storage, containing *naux* long-precision real numbers.

*naux*

is the size of the work area specified by *aux*—that is, the number of elements in *aux*. Specified as: a fullword integer, where:

If *naux* = 0 and error 2015 is unrecoverable, DGKTRN dynamically allocates the work area used by this subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, *naux* ≥ 2*n*.

*On Return*

*au*

is the array, referred to as AU, containing the upper triangular part of general sparse matrix ***A***, stored as follows, where:

If ITRAN(2) = 0, ***A*** is stored in diagonal-out skyline storage mode.

If ITRAN(2) = 1, ***A*** is stored in profile-in skyline storage mode.

Returned as: a one-dimensional array of (at least) length *nu*, containing long-precision real numbers.

*idu*

is the array, referred to as IDU, containing the relative positions of the diagonal elements of matrix ***A*** in output array AU. Returned as: a one-dimensional array of (at least) length *n*+1, containing fullword integers.

*al*

is the array, referred to as AL, containing the lower triangular part of general sparse matrix ***A***, stored as follows, where:

If ITRAN(2) = 0, ***A*** is stored in diagonal-out skyline storage mode.

If ITRAN(2) = 1, ***A*** is stored in profile-in skyline storage mode.

**Note:** You should assume that entries in AL for diagonal elements of ***A*** do not have meaningful values.

Returned as: a one-dimensional array of (at least) length *nl*, containing long-precision real numbers.

*idl*

is the array, referred to as IDL, containing the relative positions of the diagonal elements of matrix ***A*** in output array AL. Returned as: a one-dimensional array of (at least) length *n*+1, containing fullword integers.

***Notes***

1. Your various arrays must have no common elements; otherwise, results are unpredictable.
2. The ITRAN(3) argument allows you to specify the direction of travel through matrix ***A*** that ESSL takes during the transformation. By properly specifying ITRAN(3), you can optimize the performance of the transformation, which is especially beneficial when transforming large matrices.

   The direction specified by ITRAN(3) should be opposite the most recent direction of access through the matrix performed by the DGKFS or DGKFSP subroutine, as indicated in the following table:

| Most Recent Computation Performed by DGKFS/DGKFSP | Direction Used by DGKFS/DGKFSP | Direction to Specify in ITRAN(3) |
|---|---|---|
| **Factor and Solve** | Negative | Positive (ITRAN(3) = 1) |
| **Factor Only** | Positive | Negative (ITRAN(3) = –1) |
| **Solve Only** | Negative | Positive (ITRAN(3) = 1) |

3. For a description of how sparse matrices are stored in skyline storage mode, see “Profile-In Skyline Storage Mode” on page 103 and “Diagonal-Out Skyline Storage Mode” on page 101.
4. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see “Using Auxiliary Storage in ESSL” on page 32.

***Function:*** A general sparse matrix ***A***, stored in diagonal-out or profile-in skyline storage mode is converted to either of these same two storage modes. (Generally, you convert from one to the other, but the capability exists to specify the same storage mode for input and output.) The argument ITRAN(3) indicates the direction in which you want the transformation performed on matrix ***A***, allowing you to optimize your performance in this subroutine. This is especially beneficial for large matrices.

This subroutine is meant to be used in conjunction with DGKFS and DGKFSP, which process matrices stored in these skyline storage modes.

***Error Conditions***

*Resource Errors:* Error 2015 is unrecoverable, *naux* = 0, and unable to allocate work area.

*Computational Errors:* None

*Input-Argument Errors*

1. $n < 0$
2. $nu < 0$
3. IDU($n$+1) > $nu$+1
4. IDU($i$+1) ≤ IDU($i$) for $i$ = 1, $n$
5. IDU($i$+1) > IDU($i$)+$i$ and ITRAN(1) = 0 for $i$ = 1, $n$
6. IDU($i$) > IDU($i$–1)+$i$ and ITRAN(1) = 1 for $i$ = 2, $n$
7. $nl < 0$
8. IDL($n$+1) > $nl$+1
9. IDL($i$+1) ≤ IDL($i$) for $i$ = 1, $n$
10. IDL($i$+1) > IDL($i$)+$i$ and ITRAN(1) = 0 for $i$ = 1, $n$
11. IDL($i$) > IDL($i$–1)+$i$ and ITRAN(1) = 1 for $i$ = 2, $n$
12. ITRAN(1) ≠ 0 or 1
13. ITRAN(2) ≠ 0 or 1
14. ITRAN(3) ≠ –1 or 1
15. Error 2015 is recoverable or *naux*≠0, and *naux* is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example 1:*** This example shows how to convert a 9 by 9 general sparse matrix ***A*** from diagonal-out skyline storage mode to profile-in skyline storage mode. Matrix ***A*** is:

$$\begin{bmatrix}
11.0 & 12.0 & 13.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
21.0 & 22.0 & 23.0 & 24.0 & 25.0 & 0.0 & 0.0 & 0.0 & 29.0 \\
31.0 & 32.0 & 33.0 & 34.0 & 35.0 & 0.0 & 37.0 & 0.0 & 39.0 \\
41.0 & 42.0 & 43.0 & 44.0 & 45.0 & 46.0 & 47.0 & 0.0 & 49.0 \\
0.0 & 0.0 & 0.0 & 54.0 & 55.0 & 56.0 & 57.0 & 58.0 & 59.0 \\
0.0 & 62.0 & 63.0 & 64.0 & 65.0 & 66.0 & 67.0 & 68.0 & 69.0 \\
0.0 & 0.0 & 0.0 & 74.0 & 75.0 & 76.0 & 77.0 & 78.0 & 79.0 \\
0.0 & 0.0 & 0.0 & 84.0 & 85.0 & 86.0 & 87.0 & 88.0 & 89.0 \\
91.0 & 92.0 & 93.0 & 94.0 & 95.0 & 96.0 & 97.0 & 98.0 & 99.0
\end{bmatrix}$$

Assuming that DGKFS last performed a solve on matrix ***A***, the direction of the transformation is positive; that is, ITRAN(3) is 1. This provides the best performance here.

**Note:** On input and output, the diagonal elements in AL do not have meaningful values.

*Call Statement and Input*

```
                N   AU   NU   IDU   AL   NL   IDL   ITRAN   AUX   NAUX
                |   |    |    |     |    |    |     |       |     |
CALL DGKTRN( 9 , AU , 33 , IDU , AL , 35 , IDL , ITRAN , AUX ,  18


AU       =  (11.0, 22.0, 12.0, 33.0, 23.0, 13.0, 44.0, 34.0, 24.0,
             55.0, 45.0, 35.0, 25.0, 66.0, 56.0, 46.0, 77.0, 67.0,
             57.0, 47.0, 37.0, 88.0, 78.0, 68.0, 58.0, 99.0, 89.0,
             79.0, 69.0, 59.0, 49.0, 39.0, 29.0)
IDU      =  (1, 2, 4, 7, 10, 14, 17, 22, 26, 34)
AL       =  ( . , . , 21.0, . , 32.0, 31.0, . , 43.0, 42.0, 41.0, . ,
             54.0, . , 65.0, 64.0, 63.0, 62.0, . , 76.0, 75.0, 74.0,
             . , 87.0, 86.0, 85.0, 84.0, . , 98.0, 97.0, 96.0, 95.0,
             94.0, 93.0, 92.0, 91.0)
IDL      =  (1, 2, 4, 7, 11, 13, 18, 22, 27, 36)
ITRAN    =  (0, 1, 1)
```

*Output*

```
AU       =  (11.0, 12.0, 22.0, 13.0, 23.0, 33.0, 24.0, 34.0, 44.0,
             25.0, 35.0, 45.0, 55.0, 46.0, 56.0, 66.0, 37.0, 47.0,
             57.0, 67.0, 77.0, 58.0, 68.0, 78.0, 88.0, 29.0, 39.0,
             49.0, 59.0, 69.0, 79.0, 89.0, 99.0)
IDU      =  (1, 3, 6, 9, 13, 16, 21, 25, 33, 34)
AL       =  ( . , 21.0, . , 31.0, 32.0, . , 41.0, 42.0, 43.0, . , 54.0,
             . , 62.0, 63.0, 64.0, 65.0, . , 74.0, 75.0, 76.0, . ,
             84.0, 85.0, 86.0, 87.0, . , 91.0, 92.0, 93.0, 94.0, 95.0,
             96.0, 97.0, 98.0, . )
IDL      =  (1, 3, 6, 10, 12, 17, 21, 26, 35, 36)
```

***Example 2:*** This example shows how to convert the same 9 by 9 general sparse matrix ***A*** in Example 1 from profile-in skyline storage mode to diagonal-out skyline storage mode.

Assuming that DGKFS last performed a factorization on matrix ***A***, the direction of the transformation is negative; that is, ITRAN(3) is –1. This provides the best performance here.

**Note:** On input and output, the diagonal elements in AL do not have meaningful values.

*Call Statement and Input*

```
                N   AU   NU   IDU   AL   NL   IDL   ITRAN   AUX   NAUX
                |   |    |    |     |    |    |     |       |     |
CALL DGKTRN( 9 , AU , 33 , IDU , AL , 35 , IDL , ITRAN , AUX ,  18


AU       =(same as output AU in Example 1)
IDU      =(same as output IDU in Example 1)
AL       =(same as output AL in Example 1)
IDL      =(same as output IDL in Example 1)
ITRAN    =  (1, 0, -1)
```

*Output*

`AU` =(same as input `AU` in Example 1)
`IDU` =(same as input `IDU` in Example 1)
`AL` =(same as input `AL` in Example 1)
`IDL` =(same as input `IDL` in Example 1)

# DSKTRN—For a Symmetric Sparse Matrix, Convert Between Diagonal-Out and Profile-In Skyline Storage Mode

This subroutine converts symmetric sparse matrix ***A*** of order *n* from one skyline storage mode to another—that is, between the following:

- Diagonal-out skyline storage mode
- Profile-in skyline storage mode

### *Syntax*

| | |
|---|---|
| **Fortran** | CALL DSKTRN (*n*, *a*, *na*, *idiag*, *itran*, *aux*, *naux*) |
| **C and C++** | dsktrn (*n*, *a*, *na*, *idiag*, *itran*, *aux*, *naux*); |
| **PL/I** | CALL DSKTRN (*n*, *a*, *na*, *idiag*, *itran*, *aux*, *naux*); |

*On Entry*

*n*

is the order of symmetric sparse matrix ***A***. Specified as: a fullword integer; *n* ≥ 0.

*a*

is the array, referred to as `A`, containing the upper triangular part of symmetric sparse matrix ***A***, stored as follows, where:

If `ITRAN(1) = 0`, ***A*** is stored in diagonal-out skyline storage mode.

If `ITRAN(1) = 1`, ***A*** is stored in profile-in skyline storage mode.

Specified as: a one-dimensional array of (at least) length *na*, containing long-precision real numbers.

*na*

is the length of array `A`. Specified as: a fullword integer; *na* ≥ 0 and *na* ≥ (`IDIAG`(*n*+1)−1).

*idiag*

is the array, referred to as `IDIAG`, containing the relative positions of the diagonal elements of matrix ***A*** in input array `A`. Specified as: a one-dimensional array of (at least) length *n*+1, containing fullword integers.

*itran*

is an array of parameters, `ITRAN`(*i*), where:

- `ITRAN(1)` indicates the input storage mode used for matrix ***A***. This determines the arrangement of data in arrays `A` and `IDIAG` on input, where:

  If `ITRAN(1) = 0`, diagonal-out skyline storage mode is used.

  If `ITRAN(1) = 1`, profile-in skyline storage mode is used.

- `ITRAN(2)` indicates the output storage mode used for matrix ***A***. This determines the arrangement of data in arrays `A` and `IDAIG` on output, where:

  If `ITRAN(2) = 0`, diagonal-out skyline storage mode is used.

  If `ITRAN(2) = 1`, profile-in skyline storage mode is used.

- `ITRAN(3)` indicates the direction of sweep that ESSL uses through the matrix ***A***, allowing you to optimize performance (see “Notes” on page 992), where:

If ITRAN(3) = 1, matrix ***A*** is transformed in the positive direction, starting in row or column 1 and ending in row or column *n*.

If ITRAN(3) = –1, matrix ***A*** is transformed in the negative direction, starting in row or column *n* and ending in row or column 1.

Specified as: a one-dimensional array of (at least) length 3, containing fullword integers, where:
ITRAN(1) = 0 or 1
ITRAN(2) = 0 or 1
ITRAN(3) = –1 or 1

*aux*

has the following meaning:

If *naux* = 0 and error 2015 is unrecoverable, *aux* is ignored.

Otherwise, it is the storage work area used by this subroutine. Its size is specified by *naux*.

Specified as: an area of storage, containing *naux* long-precision real numbers.

*naux*

is the size of the work area specified by *aux*—that is, the number of elements in *aux*. Specified as: a fullword integer, where:

If *naux* = 0 and error 2015 is unrecoverable, DSKTRN dynamically allocates the work area used by this subroutine. The work area is deallocated before control is returned to the calling program.

Otherwise, *naux* ≥ *n*.

*On Return*

*a*

is the array, referred to as A, containing the upper triangular part of symmetric sparse matrix ***A***, stored as follows, where:

If ITRAN(2) = 0, ***A*** is stored in diagonal-out skyline storage mode.

If ITRAN(2) = 1, ***A*** is stored in profile-in skyline storage mode.

Returned as: a one-dimensional array of (at least) length *na*, containing long-precision real numbers.

*idiag*

is the array, referred to as IDIAG, containing the relative positions of the diagonal elements of matrix ***A*** in output array A. Returned as: a one-dimensional array of (at least) length *n*+1, containing fullword integers.

***Notes***

1. Your various arrays must have no common elements; otherwise, results are unpredictable.
2. The ITRAN(3) argument allows you to specify the direction of travel through matrix ***A*** that ESSL takes during the transformation. By properly specifying ITRAN(3), you can optimize the performance of the transformation, which is especially beneficial when transforming large matrices.

   The direction specified by ITRAN(3) should be opposite the most recent direction of access through the matrix performed by the DSKFS or DSKFSP subroutine, as indicated in the following table:

| Most Recent Computation Performed by DSKFS/DSKFSP | Direction Used by DSKFS/DSKFSP | Direction to Specify in ITRAN(3) |
|---|---|---|
| **Factor and Solve** | Negative | Positive (`ITRAN(3) = 1`) |
| **Factor Only** | Positive | Negative (`ITRAN(3) = –1`) |
| **Solve Only** | Negative | Positive (`ITRAN(3) = 1`) |

3. For a description of how sparse matrices are stored in skyline storage mode, see "Profile-In Skyline Storage Mode" on page 103 and "Diagonal-Out Skyline Storage Mode" on page 101.
4. You have the option of having the minimum required value for *naux* dynamically returned to your program. For details, see "Using Auxiliary Storage in ESSL" on page 32.

***Function:*** A symmetric sparse matrix ***A***, stored in diagonal-out or profile-in skyline storage mode is converted to either of these same two storage modes. (Generally, you convert from one to the other, but the capability exists to specify the same storage mode for input and output.) The argument ITRAN(3) indicates the direction in which you want the transformation performed on matrix ***A***, allowing you to optimize your performance in this subroutine. This is especially beneficial for large matrices.

This subroutine is meant to be used in conjunction with DSKFS and DSKFSP, which process matrices stored in these skyline storage modes.

***Error Conditions***

*Resource Errors:* Error 2015 is unrecoverable, *naux* = 0, and unable to allocate work area.

*Computational Errors:* None

*Input-Argument Errors*

1. $n < 0$
2. $na < 0$
3. IDIAG($n$+1) > $na$+1
4. IDIAG($i$+1) ≤ IDIAG($i$) for $i$ = 1, $n$
5. IDIAG($i$+1) > IDIAG($i$)+$i$ and ITRAN(1) = 0 for $i$ = 1, $n$
6. IDIAG($i$) > IDIAG($i$–1)+$i$ and ITRAN(1) = 1 for $i$ = 2, $n$
7. ITRAN(1) ≠ 0 or 1
8. ITRAN(2) ≠ 0 or 1
9. ITRAN(3) ≠ –1 or 1
10. *naux* Error 2015 is recoverable or *naux*≠0, and is too small—that is, less than the minimum required value. Return code 1 is returned if error 2015 is recoverable.

***Example 1:*** This example shows how to convert a 9 by 9 symmetric sparse matrix ***A*** from diagonal-out skyline storage mode to profile-in skyline storage mode. Matrix ***A*** is:

```
┌                                                      ┐
| 11.0  12.0  13.0  14.0   0.0   0.0   0.0   0.0   0.0 |
| 12.0  22.0  23.0  24.0  25.0  26.0   0.0  28.0   0.0 |
| 13.0  23.0  33.0  34.0  35.0  36.0   0.0  38.0   0.0 |
| 14.0  24.0  34.0  44.0  45.0  46.0   0.0  48.0   0.0 |
|  0.0  25.0  35.0  45.0  55.0  56.0  57.0  58.0   0.0 |
|  0.0  26.0  36.0  46.0  56.0  66.0  67.0  68.0  69.0 |
|  0.0   0.0   0.0   0.0  57.0  67.0  77.0  78.0  79.0 |
|  0.0  28.0  38.0  48.0  58.0  68.0  78.0  88.0  89.0 |
|  0.0   0.0   0.0   0.0   0.0  69.0  79.0  89.0  99.0 |
└                                                      ┘
```

Assuming that DSKFS last performed a factorization on matrix ***A***, the direction of the transformation is negative; that is, `ITRAN(3)` is –1. This provides the best performance here.

*Call Statement and Input*

```
             N   A   NA   IDIAG   ITRAN   AUX   NAUX
             |   |   |      |       |      |     |
CALL DSKTRN( 9 , A , 33 , IDIAG , ITRAN , AUX ,  9  )


A        =  (11.0, 22.0, 12.0, 33.0, 23.0, 13.0, 44.0, 34.0, 24.0,
             14.0, 55.0, 45.0, 35.0, 25.0, 66.0, 56.0, 46.0, 36.0,
             26.0, 77.0, 67.0, 57.0, 88.0, 78.0, 68.0, 58.0, 48.0,
             38.0, 28.0, 99.0, 89.0, 79.0, 69.0)
IDIAG    =  (1, 2, 4, 7, 11, 15, 20, 23, 30, 34)
ITRAN    =  (0, 1, -1)
```

*Output*

```
A        =  (11.0, 12.0, 22.0, 13.0, 23.0, 33.0, 14.0, 24.0, 34.0,
             44.0, 25.0, 35.0, 45.0, 55.0, 26.0, 36.0, 46.0, 56.0,
             66.0, 57.0, 67.0, 77.0, 28.0, 38.0, 48.0, 58.0, 68.0,
             78.0, 88.0, 69.0, 79.0, 89.0, 99.0)
IDIAG    =  (1, 3, 6, 10, 14, 19, 22, 29, 33, 34)
```

***Example 2:*** This example shows how to convert the same 9 by 9 symmetric sparse matrix ***A*** in Example 1 from profile-in skyline storage mode to diagonal-out skyline storage mode.

Assuming that DSKFS last performed a solve on matrix ***A***, the direction of the transformation is positive; that is, `ITRAN(3)` is 1. This provides the best performance here.

*Call Statement and Input*

```
             N   A   NA   IDIAG   ITRAN   AUX   NAUX
             |   |   |      |       |      |     |
CALL DSKTRN( 9 , A , 33 , IDIAG , ITRAN , AUX ,  9
```

`A        =`(same as output `A` in Example 1)
`IDIAG    =`(same as output `IDIAG` in Example 1)
`ITRAN    =  (1, 0, 1)`

*Output*

```
A        =(same as input A in Example 1)
IDIAG    =(same as input IDIAG in Example 1)
```

# Part 3. Appendixes

# Appendix A. Basic Linear Algebra Subprograms (BLAS)

This appendix lists the ESSL subprograms corresponding to a subprogram in the standard set of BLAS.

## Level 1 BLAS

*Table 168. Level 1 BLAS Included in ESSL*

| Descriptive Name | Short-Precision Subprogram | Long-Precision Subprogram |
|---|---|---|
| Position of the First or Last Occurrence of the Vector Element Having the Largest Magnitude | ISAMAX<br>ICAMAX | IDAMAX<br>IZAMAX |
| Sum of the Magnitudes of the Elements in a Vector | SASUM<br>SCASUM | DASUM<br>DZASUM |
| Multiply a Vector X by a Scalar, Add to a Vector Y, and Store in the Vector Y | SAXPY<br>CAXPY | DAXPY<br>ZAXPY |
| Copy a Vector | SCOPY<br>CCOPY | DCOPY<br>ZCOPY |
| Dot Product of Two Vectors | SDOT<br>CDOTU<br>CDOTC | DDOT<br>ZDOTU<br>ZDOTC |
| Euclidean Length of a Vector with Scaling of Input to Avoid Destructive Underflow and Overflow | SNRM2<br>SCNRM2 | DNRM2<br>DZNRM2 |
| Construct a Givens Plane Rotation | SROTG<br>CROTG | DROTG<br>ZROTG |
| Apply a Plane Rotation | SROT<br>CROT<br>CSROT | DROT<br>ZROT<br>ZDROT |
| Multiply a Vector X by a Scalar and Store in the Vector X | SSCAL<br>CSCAL<br>CSSCAL | DSCAL<br>ZSCAL<br>ZDSCAL |
| Interchange the Elements of Two Vectors | SSWAP<br>CSWAP | DSWAP<br>ZSWAP |

## Level 2 BLAS

*Table 169 (Page 1 of 2). Level 2 BLAS Included in ESSL*

| Descriptive Name | Short-Precision Subprogram | Long-Precision Subprogram |
|---|---|---|
| Matrix-Vector Product for a General Matrix, Its Transpose, or Its Conjugate Transpose | SGEMV<br>CGEMV | DGEMV<br>ZGEMV |

*Table 169 (Page 2 of 2). Level 2 BLAS Included in ESSL*

| Descriptive Name | Short-Precision Subprogram | Long-Precision Subprogram |
|---|---|---|
| Rank-One Update of a General Matrix | SGER<br>CGERU<br>CGERC | DGER<br>ZGERU<br>ZGERC |
| Matrix-Vector Product for a Real Symmetric or Complex Hermitian Matrix | SSPMV<br>CHPMV<br>SSYMV<br>CHEMV | DSPMV<br>ZHPMV<br>DSYMV<br>ZHEMV |
| Rank-One Update of a Real Symmetric or Complex Hermitian Matrix | SSPR<br>CHPR<br>SSYR<br>CHER | DSPR<br>ZHPR<br>DSYR<br>ZHER |
| Rank-Two Update of a Real Symmetric or Complex Hermitian Matrix | SSPR2<br>CHPR2<br>SSYR2<br>CHER2 | DSPR2<br>ZHPR2<br>DSYR2<br>ZHER2 |
| Matrix-Vector Product for a General Band Matrix, Its Transpose, or Its Conjugate Transpose | SGBMV<br>CGBMV | DGBMV<br>ZGBMV |
| Matrix-Vector Product for a Real Symmetric or Complex Hermitian Band Matrix | SSBMV<br>CHBMV | DSBMV<br>ZHBMV |
| Matrix-Vector Product for a Triangular Matrix, Its Transpose, or Its Conjugate Transpose | STPMV<br>CTPMV<br>STRMV<br>CTRMV | DTPMV<br>ZTPMV<br>DTRMV<br>ZTRMV |
| Solution of a Triangular System of Equations with a Single Right-Hand Side | STPSV<br>CTPSV<br>STRSV<br>CTRSV | DTPSV<br>ZTPSV<br>DTRSV<br>ZTRSV |
| Matrix-Vector Product for a Triangular Band Matrix, Its Transpose, or Its Conjugate Transpose | STBMV<br>CTBMV | DTBMV<br>ZTBMV |
| Triangular Band Equation Solve | STBSV<br>CTBSV | DTBSV<br>ZTBSV |

## Level 3 BLAS

*Table 170 (Page 1 of 2). Level 3 BLAS Included in ESSL*

| Descriptive Name | Short-Precision Subprogram | Long-Precision Subprogram |
|---|---|---|
| Combined Matrix Multiplication and Addition for General Matrices, Their Transposes, or Conjugate Transposes | SGEMM<br>CGEMM | DGEMM<br>ZGEMM |
| Matrix-Matrix Product Where One Matrix is Real or Complex Symmetric or Complex Hermitian | SSYMM<br>CSYMM<br>CHEMM | DSYMM<br>ZSYMM<br>ZHEMM |

*Table 170 (Page 2 of 2). Level 3 BLAS Included in ESSL*

| **Descriptive Name** | **Short-Precision Subprogram** | **Long-Precision Subprogram** |
|---|---|---|
| Triangular Matrix-Matrix Product | STRMM<br>CTRMM | DTRMM<br>ZTRMM |
| Rank-K Update of a Real or Complex Symmetric or a Complex Hermitian Matrix | SSYRK<br>CSYRK<br>CHERK | DSYRK<br>ZSYRK<br>ZHERK |
| Rank-2K Update of a Real or Complex Symmetric or a Complex Hermitian Matrix | SSYR2K<br>CSYR2K<br>CHER2K | DSYR2K<br>ZSYR2K<br>ZHER2K |
| Solution of Triangular Systems of Equations with Multiple Right-Hand Sides | STRSM<br>CTRSM | DTRSM<br>ZTRSM |

# Appendix B. LAPACK

This appendix lists the ESSL subroutines corresponding to subroutines in the standard set of LAPACK.

## LAPACK

*Table 171. LAPACK Included in ESSL*

| **Descriptive Name** | **Short-Precision Subprogram** | **Long-Precision Subprogram** |
|---|---|---|
| General Matrix Factorization | SGETRF<br>CGETRF | DGETRF<br>ZGETRF |
| General Matrix, Its Transpose, or Its Conjugate Transpose Multiple Right-Hand Side Solve | SGETRS<br>CGETRS | DGETRS<br>ZGETRS |

# Glossary

This glossary defines terms and abbreviations used in this publication. If you do not find the term you are looking for, refer to the index portion of this book. This glossary includes terms and definitions from:

- *IBM Dictionary of Computing,* New York: McGraw Hill (1-800-2MC-GRAW), 1994.
- *American National Standard Dictionary for Information Systems,* ANSI X3.172-1990, copyright 1990 by the American National Standards Institute (ANSI). Copies may be purchased from the American National Standards Institute, 11 West 42nd Street, New York, New York 10036. Definitions are identified by the symbol (A) after the definition.
- *Information Technology Vocabulary,* developed by Subcommittee 1, Joint Technical Committee 1, of the International Organization for Standardization and the International Electrotechnical Commission (ISO/IEC JTC1/SC1). Definitions from published sections of these vocabularies are identified by the symbol (I) after the definition. Definitions taken from draft international standards, committee drafts, and working papers being developed by ISO/IEC JTC1/SC1 are identified by the symbol (T) after the definition, indicating that final agreement has not yet been reached among the participating National Bodies of SC1.

**APAR**. Authorized Program Analysis Report. A report of a problem caused by a suspected defect in a current unaltered release of a program.

**argument**. A parameter passed between a calling program and a SUBROUTINE subprogram, a FUNCTION subprogram, or a statement function.

**array**. An ordered set of data items identified by a single name.

**array element**. A data item in an array, identified by the array name followed by a subscript indicating its position in the array.

**array name**. The name of an ordered set of data items that make up an array.

**assignment statement**. A statement that assigns a value to a variable or array element. It is made up of a variable or array element, followed by an equal sign (=), followed by an expression. The variable, array element, or expression can be character, logical, or arithmetic. When the assignment statement is processed, the expression to the right of the equal sign replaces the value of the variable or array element to the left.

**Basic Linear Algebra Subprograms (BLAS)**. A standard, public domain, set of mathematical subroutines that perform linear algebra operations.

**BLAS**. Basic Linear Algebra Subprograms.

**cache**. A special-purpose buffer storage, smaller and faster than main storage, used to hold a copy of instructions and data obtained from main storage and likely to be needed next by the processor. (T)

**character constant**. A string of one or more alphanumeric characters enclosed in apostrophes. The delimiting apostrophes are not part of the value of the constant.

**character expression**. An expression in the form of a single character constant, variable, array element, substring, function reference, or another expression enclosed in parentheses. A character expression is always of type character.

**character type**. The data type for representing strings of alphanumeric characters; in storage, one byte is used for each character.

**column-major order**. A sequencing method used for storing multidimensional arrays according to the subscripts of the array elements. In this method the leftmost subscript position varies most rapidly and completes a full cycle before the next subscript position to the right is incremented.

**complex conjugate even data**. Complex data that has its real part even and its imaginary part odd.

**complex constant**. An ordered pair of real or integer constants separated by a comma and enclosed in parentheses. The first real constant of the pair is the real part of the complex number; the second is the imaginary part.

**complex type**. The data type for representing an approximation of the value of a complex number. A data item of this type consists of an ordered pair of real data items separated by a comma and enclosed in parentheses. The first item represents the real part of the complex number; the second represents the imaginary part.

**constant**. An unvarying quantity. The four classes of constants specify numbers (arithmetic), truth values (logical), character data (character), and hexadecimal data.

**data type**. The structural characteristics, features and properties of data that may be directly specified by a programming language; for example, integers, real numbers in Fortran; arrays in APL; linked lists in LISP; character string in SNOBOL.

**decimation**. The formation of a sequence containing every n-th element of another sequence.

**dimension of an array**. One of the subscript expression positions in a subscript for an array. In Fortran, an array may have from one to seven dimensions. Graphically, the first dimension is represented by the rows, the second by the columns, and the third by the planes. Contrast with rank. See also extent of a dimension.

**direct access storage**. A storage device in which the access time is in effect independent of the location of the data. (A)

**divide-by-zero exception**. The condition recognized by a processor that results from running a program that attempts to divide by zero.

**double precision**. Synonym for long-precision.

**expression**. A notation that represents a value: a primary appearing alone, or combinations of primaries and operators. An expression can be arithmetic, character, logical, or relational.

**extent of a dimension**. The number of different integer values that may be represented by subscript expressions for a particular dimension in a subscript for an array.

**external function**. A function defined outside the program unit that refers to it. It may be referred to in a procedure subprogram or in the main program, but it must not refer to itself, either directly or indirectly. Contrast with statement function.

**function**. In Fortran, a procedure that is invoked by referring to it in an expression and that supplies a value to the expression. The value supplied is the value of the function. See also external function, intrinsic function, and statement function. Contrast with subroutine.

**function reference**. A Fortran source program reference to an intrinsic function, to an external function, or to a statement function.

**general matrix**. A matrix with no assumed special properties such as symmetry. Synonym for matrix.

**integer constant**. A string of decimal digits containing no decimal point and expressing a whole number.

**integer expression**. An arithmetic expression whose values are of integer type.

**integer type**. An arithmetic data type capable of expressing the value of an integer. It can have a positive, negative, or 0 value. It must not include a decimal point.

**intrinsic function**. A function, supplied by Fortran, that performs mathematical or character operations.

**leading dimension**. For a two-dimensional array, an increment used to find the starting point for the matrix elements in each successive column of the array.

**logical constant**. A constant that can have one of two values: true or false. The form of these values in Fortran is: .TRUE. and .FALSE. respectively.

**logical expression**. A logical primary alone or a combination of logical primaries and logical operators. A logical expression can have one of two values: true or false.

**logical type**. The data type for data items that can have the value true or false and upon which logical operations such as .NOT. and .OR. can be performed. See also “data type.”

**long-precision**. Real type of data of length 8. Contrast with single precision and short-precision.

**main program**. In Fortran, a program unit, required for running, that can call other program units but cannot be called by them.

**mask**. To use a pattern of characters to control the retention or elimination of portions of another pattern of characters. (I)

**matrix**. A rectangular array of elements, arranged in rows and columns, that may be manipulated according to the rules of matrix algebra. (A) (I)

**multithreaded**. There may be one or more threads in a process, and each thread is executed by the operating system concurrently. An application program is multithreaded if more than one thread is executed concurrently.

**overflow exception**. A condition caused by the result of an arithmetic operation having a magnitude that exceeds the largest possible number.

**platform**. A mainframe or a workstation.

**primary**. An irreducible unit of data; a single constant, variable, array element, function reference, or expression enclosed in parentheses.

**program exception**. The condition recognized by a processor that results from running a program that improperly specifies or uses instructions, operands, or control information.

**PTF**. Program Temporary Fix. A temporary solution or by-pass of a problem diagnosed by IBM as resulting from a defect in a current unaltered release of the program. A report of a problem caused by a suspected defect in a current unaltered release of a program.

**pthread**. A thread that conforms to the POSIX Threads Programming Model.

**real constant**. A string of decimal digits that expresses a real number. A real constant must contain either a decimal point or a decimal exponent and may contain both. For example, the real constant 0.36819E+2 has the value +36.819.

**real type**. An arithmetic data type, capable of approximating the value of a real number. It can have a positive, negative, or 0 value.

**row-major order**. A sequencing method used for storing multidimensional arrays according to the subscripts of the array elements. In this method the rightmost subscript position varies most rapidly and completes a full cycle before the next subscript position to the left is incremented.

**scalar**. (1) A quantity characterized by a single number. (A) (I) (2) Contrast with vector.

**shape of an array**. The extents of all the dimensions of an array listed in order. For example, the shape of a three-dimensional array that has four rows, five columns, and three planes is (4,5,3) or 4 by 5 by 3.

**short-precision**. Real type data of length 4. Contrast with double precision and long-precision.

**single precision**. Synonym for short-precision.

**size of an array**. The number of elements in an array. This is the product of the extents of its dimensions.

**SMP**. Symmetric Multi-Processing.

**statement**. The basic unit of a program, that specifies an action to be performed, or the nature and characteristics of the data to be processed, or information about the program itself. Statements fall into two broad classes: executable and nonexecutable.

**statement function**. A procedure specified by a single statement that is similar in form to an arithmetic, logical, or character assignment statement. The statement must appear after the specification statements and before the first executable statement. In the remainder of the program it can be referenced as a function. A statement function may be referred to only in the program unit in which it is defined. Contrast with external function.

**statement label**. A number of from one through five decimal digits that is used to identify a statement. Statement labels can be used to transfer control, to define the range of a DO, or to refer to a FORMAT statement.

**statement number**. See “statement label.”

**stride**. The increment used to step through array storage to select the vector or matrix elements from the array.

**subprogram**. A program unit that is invoked by another program unit in the same program. In Fortran, a subprogram has a FUNCTION, SUBROUTINE, or BLOCK DATA statement as its first statement.

**subscript**. (1) A symbol that is associated with the name of a set to identify a particular subset or element. (A) (2) A subscript expression or set of subscript expressions, enclosed in parentheses and used with an array name to identify a particular array element.

**subscript expression**. An integer expression in a subscript whose value and position in the subscript determine the index number for the corresponding dimension in the referenced array.

**thread**. A thread is the element that is scheduled, and to which resources such as execution time, locks, and queues may be assigned. There may be one or more threads in a process, and each thread is executed by the operating system concurrently.

**thread-safe**. A subroutine which may be called from multiple threads of the same process simultaneously.

**type declaration**. The explicit specification of the type of a constant, variable, array, or function by use of an explicit type specification statement.

**underflow exception**. A condition caused by the result of an arithmetic operation having a magnitude less than the smallest possible nonzero number.

**variable**. (1) A quantity that can assume any of a given set of values. (A) (2) A data item, identified by a name, that is not a named constant, array, or array element, and that can assume different values at different times during program processing.

**vector**. A one-dimensional ordered collection of numbers.

**working storage**. A storage area provided by the application program for the use of an ESSL subroutine.

**workstation**. A workstation is a single-user, high-performance microcomputer (or even a minicomputer) which has been specialized in some way, usually for graphics output. Such a machine has a screen and a keyboard, but is also capable of extensive processing of your input before it is passed to the host. Likewise, the host's responses may be extensively processed before being passed along to your screen. A workstation may be intelligent enough to do much or all the processing itself.

# Bibliography

This bibliography lists the publications that you may need to use with ESSL and describes how to obtain them.

## References

Text books and articles covering the mathematical aspects of ESSL are listed in this section, as well as several software libraries available from other companies. They are listed alphabetically as follows:

- Publications are listed by the author's name. IBM publications that include an order number, other than an *IBM Technical Report* can be ordered through the Subscription Library Services System (SLSS). The non-IBM publications listed here should be obtained through publishers, bookstores, or professional computing organizations.
- Software libraries are listed by their product name. Each reference includes the names, addresses, and phone numbers of the companies from which they can be obtained.

Each citation in the text of this book is shown as a number enclosed in square brackets. It indicates the number of the item listed in the bibliography. For example, reference [1] cites the first item listed below.

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## ESSL Publications

This section lists the ESSL publications for each major task that you perform.

You can order the full set of ESSL for AIX publications through the Subscription Library Services System (SLSS) by specifying:

- The subject code 82
- The program number: 5765-C42

You can also order individual publications by specifying the individual order numbers. For example, to order a copy of the *ESSL Version 3 Release 1.1 Guide and Reference*, specify the order number SA22-7272. Contact your IBM Marketing Representative or Systems Engineer to order manuals through SLSS.

### Evaluation and Planning

*ESSL Products General Information*, GC23-0529—provides detailed information helpful in evaluating and planning for ESSL: Parallel ESSL, ESSL for AIX, and ESSL/370.

### Installation

*ESSL Version 3 Installation Memo*, GI10-0604—describes how to install ESSL on AIX. It is a packing list for the ESSL product when it is shipped. (One copy is delivered with each ESSL product.)

### Application Programming

*ESSL Version 3 Guide and Reference*, SA22-7272—contains ESSL guidance information for designing, coding, and running programs using ESSL, and contains complete reference information for coding calls to the ESSL subroutines. This manual is available in HTML and PostScript format on the product medium.

*ESSL Version 2 Release 2 Guide and Reference*, SC23-0526—contains ESSL guidance information for designing, coding, and running programs using ESSL/370, and contains complete reference information for coding calls to the ESSL/370 subroutines.

*Parallel ESSL Version 2 Guide and Reference*, SA22-7273—contains Parallel ESSL guidance information for designing, coding, and running programs using Parallel ESSL, and contains complete reference information for coding calls to the Parallel ESSL subroutines. This manual is available in HTML and PostScript format on the Parallel ESSL product medium.

## Related Publications

The related publications listed below may be useful to you when using ESSL.

### AIX for the RS/6000

*IBM AIX Calls and Subroutines Reference for IBM RS/6000*,(all volumes) SC23-2198

*AIX Performance Monitoring and Tuning Guide*, SC23-2365

### AIX Version 4 Release 2 for the RS/6000

*IBM AIX Version 4.1 and 4.2 Commands and Reference*, (all volumes) SBOF-1851

*IBM Version 4.1 and 4.2 General Programming Concepts: Writing and Debugging Programs*, SC23-2533

*IBM AIX Version 4.1 and 4.2 System Management Guide: Operating System and Devices*, SC23-2525

### AIX Version 4 Release 3 for the RS/6000

For the latest updates, visit the RS/6000 web site at: **http://www.rs6000.ibm.com/resource/aix_resource/Pubs**

*AIX Version 4.3 Commands and Reference*, (all volumes) SBOF-1877

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### XL Fortran

On the XL Fortran Version 5 Release 1.1 CD ROM and the XL High Performance Fortran Version 1 Release 3.1 tape, you may reference the following document available in both PostScript and HTML formats, which describes the changes for those releases: *Changes to XL Fortran for AIX and XL High Performance Fortran for AIX*

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