Appendix 2: Significance of a *t* Statistic

An exact formula is used for 30 or fewer degrees of freedom, the Cornish-Fisher approximation otherwise.

Notation

The following notation is used in this appendix:

X	Absolute value of the *t* variable
k	Integer degrees of freedom
Q	Two-sided significance level

Computation (Abramowitz and Stegun, 1965)

(1) For $k \le 30$,

$$Q = 1 - A(k)$$

where, for *k* odd

$$A(k) = \begin{cases} \dfrac{2}{\pi}\left\{\theta + \sin\theta\left[\cos\theta + \dfrac{2}{3}\cos^3\theta + \cdots + \dfrac{2\cdot 4\ldots(k-3)}{3\cdot 5\ldots(k-2)}\cos^{k-2}\theta\right]\right\} & k \ne 1 \\ \dfrac{2\theta}{\pi} & k = 1 \end{cases}$$

For *k* even,

$$A(k) = \sin\theta\left\{1 + \frac{1}{2}\cos^2\theta + \frac{1\cdot 3}{2\cdot 4}\cos^4\theta + \cdots + \frac{1\cdot 3\ldots(k-3)}{2\cdot 4\ldots(k-2)}\cos^{k-2}\theta\right\}$$

and

$$\theta = \arctan \frac{X}{\sqrt{k}}$$
$$\sin(\theta) = \frac{\tilde{\theta}}{\sqrt{1+\tilde{\theta}^2}}$$
$$\cos(\theta) = \frac{1}{\sqrt{1+\tilde{\theta}^2}}$$

where

$$\tilde{\theta} = \frac{X}{\sqrt{k}}$$

(2) If $k > 30$ and $X < 5\times10^{-5}$,

$$Q = 1$$

If $k > 30$ and $X \geq 5\times10^{-5}$,

$$Q = 2\left(Q_N(X) + \frac{XR\exp\left(-X^2/2\right)}{\sqrt{2\pi}} \right)$$

where $Q_N(X)$ is the normal one-tailed significance probability, and

$$R = \frac{\left(X^2+1\right)}{4k} + \frac{-3+X^2\left(-5+X^2\left(-7+3X^2\right)\right)}{96k^2}$$
$$+\frac{(-15)+X^2\left(-3+X^2\left(6+X^2\left(14+X^2\left(-11+X^2\right)\right)\right)\right)}{384k^3}$$
$$+\frac{945+X^2\left(-915+X^2\left(-213+X^2\left(-939+X^2\left(-2141+X^2\left(2225+X^2\left(-375+15X^2\right)\right)\right)\right)\right)\right)}{92160k^4}$$
$$+\frac{17955+X^2\left(5355+X^2\left(180+X^2\left(1140+X^2\left(2490+X^2\left(5994+X^2\left(-7516+X^2\left(1764+X^2\left(-133+3X^2\right)\right)\right)\right)\right)\right)\right)\right)\right)}{368640k^5}$$

References

Abramowitz, N., and Stegun, I., eds. 1965. *Handbook of mathematical functions*. National Bureau of Standards, Applied Mathematics Series 55. Washington, D.C.: U.S. Government Printing Office.