Appendix 3: Significance of a Chi-Square Statistic

For 30 or fewer degrees of freedom, an exact series expansion is used; otherwise the Peizer-Pratt approximation is used.

Notation

The following notation is used in this appendix:

X	Value of the chi-square statistic
k	Degrees of freedom
Q	Significance level (right-tail probability)

Computation

- If $X \le 0$ or $k < 1$,

  $$Q = 1$$

- If $k = 1$,

  $$Q = 2Q_N\left(\sqrt{X}\right)$$

  where $Q_N\left(\sqrt{X}\right)$ is the standard normal one-tailed significance probability.

- For $k \le 30$, an exact series expansion is used (Abramowitz and Stegun, 1965, eqs. 26.4.4 and 26.4.5)

$$Q = \begin{cases} 2Q_N\left(\sqrt{X}\right) + R\sqrt{\dfrac{2}{\pi}}\exp\left(\dfrac{-X}{2}\right) & k \text{ odd} \\ \exp\left(\dfrac{-X}{2}\right) \times (1+R) & k \text{ even} \end{cases}$$

where

$$R = \begin{cases} \displaystyle\sum_{r=1}^{(k-1)/2} \frac{X^{r-1/2}}{1 \cdot 3 \ldots (2r-1)} & k \text{ odd} \\ \displaystyle\sum_{r=1}^{(k-2)/2} \frac{X^r}{2 \cdot 4 \ldots 2r} & k \text{ even} \end{cases}$$

- If $k > 30$, the Peizer-Pratt approximation is used (Peizer and Pratt, 1968, eq 2.24a).
- If $X \geq 150$,

  $Q = 0$

  otherwise

  $Q = Q_N(Z)$

  where

$$Z = \begin{cases} \left(-\left(\frac{1}{3}+\frac{0.08}{k}\right)\right)\Big/\left(\sqrt{2k-2}\right) & \text{if } X = k-1 \\ \left(d\sqrt{(k-1)\log\left(\frac{k-1}{X}\right)+X-(k-1)}\right)\Big/\left|X-(k-1)\right| & \text{if } X \neq k-1 \end{cases}$$

where

$$d = X - k + 2/3 - 0.08/k$$

- If $Z < 0$,

$$Q = 1 - Q_N(Z)$$

References

Peizer, D. B., and Pratt, J. W. 1968. A normal approximation for binomial, F, beta and other common related tail probabilities. *Journal of the American Statistical Association*, 63: 1416–1456.