Appendix 4: Significance of an *F* Statistic

Either the Peizer-Pratt approximation or an exact algorithm suggested by O. G. Ludwig (1963) and improved by Bosten and Battiste is used.

Notation

The following notation is used in this appendix:

F	Value of the *F* statistic
k_1	Numerator degrees of freedom
k_2	Denominator degrees of freedom
Q_F	One-tailed significance level

Computation

- If $k_1 \geq 20$ and $k_2 \geq 20$ and if $F > 12, Q_F = C$
- Peizer-Pratt (1968, p.1420)

  The Peizer-Pratt approximation is used if $k_1 \geq 20$ and $k_2 \geq 20$ and $F \leq 12$, or $\min(k_1, k_2) \geq 6$ and $\max(k_1, k_2) > 2000$,

  $$Q_F = Q_N(Z)$$

  where

  $$Z = d \times \sqrt{\frac{1 + qg\left(\frac{S}{np}\right) + pg\left(\frac{T}{nq}\right)}{\left(n + \frac{1}{6}\right)pq}}$$

and

$$d = S + \frac{1}{6} - \left(n + \frac{1}{3}\right)p + 0.04\left(\frac{q}{k_2} - \frac{p}{k_1} + \frac{q - 0.5}{k_1 + k_2}\right)$$

$$S = \frac{k_2 - 1}{2} \quad T = \frac{k_1 - 1}{2}$$

$$n = \frac{k_1 + k_2 - 2}{2} \quad p = \frac{k_2}{k_1 F + k_2} \quad q = 1 - p$$

$$g(X) = \left(1 - X^2 + 2X\log(X)\right) \Big/ (1 - X)^2 \quad \text{for } X \neq 1,\ X > 0$$

$$g(0) = 1, \quad g(1) = 0$$

If $Z < 0$, $Q_F = 1 - Q_N(Z)$. If $Z > 10$, $Q_F = 0$.

- Exact Algorithm

  Determine the parameters for the incomplete beta functions

$$X = \frac{k_2}{k_2 + k_1 F}$$

$$a = \begin{cases} k_2/2 & X \le 0.5 \\ k_1/2 & X > 0.5 \end{cases}$$

$$b = \begin{cases} k_1/2 & X \le 0.5 \\ k_2/2 & X > 0.5 \end{cases}$$

$$Y = \begin{cases} X & X \le 0.5 \\ 1 - X & X > 0.5 \end{cases}$$

The F and incomplete beta functions are related as

$$Q_F(F, k_1, k_2) = \begin{cases} I_X(k_2/2, k_1/2) & X \le 0.5 \\ 1 - I_{1-X}(k_1/2, k_2/2) & X > 0.5 \end{cases}$$

The following is used for the computations:

$$I_Y(a,b) = INFSUM \times \frac{Y^a}{\Gamma(a+1)} \times \frac{\Gamma(PS+a)}{\Gamma(PS)} + FINSUM \times \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b+1)} \times Y^a (1-Y)^b$$

where

$$INFSUM = \sum_{i=0}^{\infty} \frac{(1-PS)_i\, a}{a+i} \frac{Y^i}{i!}$$

$$(1-PS)_i = \begin{cases} 1 & i = 0 \\ \dfrac{1+i-PS}{1-PS} & i > 0 \end{cases}$$

$$PS = \begin{cases} 1 & b \text{ is an integer} \\ b - [b] & \text{otherwise} \end{cases}$$

and

$[b] =$ largest integer less than or equal to b

$$FINSUM = \sum_{i=1}^{[b]} \frac{b(b-1)\ldots(b-i+1)}{(a+b-1)(a+b-2)\ldots(a+b-i)} \frac{1}{(1-Y)^i}$$

Summation of INFSUM terminates when a summand is less than 10^{-78} or less than $FINSUM \times 10^{-6}$.

References

Bosten, N. E., and Battiste, E. L. 1980. Remark on Algorithm 179. *Collected Algorithms from ACM*, 179–P 2–R1.

Ludwig, O. G. 1963. Algorithm 179: Incomplete beta ratio. *Communications of the ACM*, 6: 314.

Peizer, D. B., and Pratt, J. W. 1968. A normal approximation for binomial, F, beta and other common related tail probabilities. *Journal of the American Statistical Association*, 63: 1416–1456.