FACTOR

Extraction of Initial Factors

Principal Components Extraction (PC)

The matrix of factor loadings based on factor m is

$$\Lambda_m = \Omega_m \Gamma_m^{1/2}$$

where

$$\Omega_m = (\omega_1, \omega_2, \ldots, \omega_m)$$
$$\Gamma_m = \mathrm{diag}(|\gamma_1|, |\gamma_2|, \ldots, |\gamma_m|)$$

The communality of variable i is given by

$$h_i = \sum_{j=1}^{m} |\gamma_j| \omega_{ij}^2$$

Analyzing a Correlation Matrix

$\gamma_1 \geq \gamma_2 \geq \ldots \geq \gamma_m$ are the eigenvalues and ω_i are the corresponding eigenvectors of **R**, where **R** is the correlation matrix.

Analyzing a Covariance Matrix

$\gamma_1 \geq \gamma_2 \geq \ldots \geq \gamma_m$ are the eigenvalues and ω_i are the corresponding eigenvectors of Σ, where $\Sigma = (\sigma_{ij})_{n \times n}$ is the covariance matrix.

The rescaled loadings matrix is $\Lambda_{mR} = [diag\Sigma]^{-1/2} \Lambda_m$.

The rescaled communality of variable i is $h_{iR} = \sigma_{ii}^{-1} h_i$.

Principal Axis Factoring

Analyzing a Correlation Matrix

An iterative solution for communalities and factor loadings is sought. At iteration *i*, the communalities from the preceding iteration are placed on the diagonal of **R**, and the resulting **R** is denoted by $\mathbf{R}_i$. The eigenanalysis is performed on $\mathbf{R}_i$ and the new communality of variable *j* is estimated by

$$h_{j(i)} = \sum_{j=1}^{m} \left|\gamma_{k(i)}\right| \omega_{jk(i)}^2$$

The factor loadings are obtained by

$$\Lambda_{m(i)} = \Omega_{m(i)} \Gamma_{m(i)}^{1/2}$$

Iterations continue until the maximum number (default 25) is reached or until the maximum change in the communality estimates is less than the convergence criterion (default 0.001).

Analyzing a Covariance Matrix

This analysis is the same as analyzing a correlation matrix, except Σ is used instead of the correlation matrix **R**. Convergence is dependent on the maximum change of *rescaled* communality estimates.

At iteration i, the rescaled loadings matrix is $\Lambda_{m(i)R} = [diag\Sigma]^{-1/2} \Lambda_{m(i)}$. The rescaled communality of variable *i* is $h_{j(i)R} = \sigma_{ii}^{-1} h_{j(i)}$.

Maximum Likelihood (ML)

The maximum likelihood solutions of Λ and ψ^2 are obtained by minimizing

$$F = tr\left[\left(\Lambda\Lambda' + \psi^2\right)^{-1}\mathbf{R}\right] - \log\left|\left(\Lambda\Lambda' + \psi^2\right)^{-1}\mathbf{R}\right| - p$$

with respect to Λ and ψ, where p is the number of variables, Λ is the factor loading matrix, and ψ^2 is the diagonal matrix of unique variances.

The minimization of F is performed by way of a two-step algorithm. First, the conditional minimum of F for a given y is found. This gives the function $f(\psi)$, which is minimized numerically using the Newton-Raphson procedure. Let $\mathbf{x}^{(s)}$ be the column vector containing the logarithm of the diagonal elements of y at the sth iteration; then

$$\mathbf{x}^{(s+1)} = \mathbf{x}^{(s)} - \mathbf{d}^{(s)}$$

where $\mathbf{d}^{(s)}$ is the solution to the system of linear equations

$$\mathbf{H}^{(s)}\mathbf{d}^{(s)} = \mathbf{h}^{(s)}$$

and where

$$\mathbf{H}^{(s)} = \left(\partial^2 f(\psi) \big/ \partial x_i \partial x_j\right)$$

and $\mathbf{h}^{(s)}$ is the column vector containing $\partial f(\psi) \big/ \partial x_i$. The starting point $\mathbf{x}^{(1)}$ is

$$\mathbf{x}_i^{(1)} = \begin{cases} \log\left[(1 - m/2p) \big/ r^{ii}\right] & \text{for ML and GLS} \\ \left[(1 - m/2p) \big/ r^{ii}\right]^{1/2} & \text{for ULS} \end{cases}$$

where m is the number of factors and r^{ii} is the ith diagonal element of $\mathbf{R}^{-1}$.

The values of $f(\psi)$, $\partial f / \partial x_i$, and $\partial^2 f \big/ \partial x_i \partial x_j$ can be expressed in terms of the eigenvalues

$$\gamma_1 \le \gamma_2 \le \ldots \le \gamma_p$$

and corresponding eigenvectors

$$\omega_1, \omega_2, \ldots, \omega_\pi$$

of matrix $\psi \mathbf{R}^{-1} \psi$. That is,

$$f(\psi) = \sum_{k=m+1}^{p} \left(\log \gamma_k + \gamma_k^{-1} - 1 \right)$$

$$\frac{\partial f}{\partial x_i} = \sum_{k=m+1}^{p} \left(1 - \gamma_k^{-1} \right) \omega_{ik}^2$$

$$\frac{\partial^2 f}{\partial x_i \partial x_j} = -\delta_{ij} \frac{\partial f}{\partial x_i} + \sum_{k=m+1}^{p} \omega_{ik} \omega_{jk} \left(\sum_{n=1}^{m} \frac{\gamma_k + \gamma_n - 2}{\gamma_k - \gamma_n} \omega_{in} \omega_{jn} + \delta_{ij} \right)$$

where

$$\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

The approximate second-order derivatives

$$\frac{\partial^2 f}{\partial x_i \partial x_j} \cong \left(\sum_{k=m+1}^{p} \omega_{ik} \omega_{jk} \right)^2$$

are used in the initial step and when the matrix of the exact second-order derivatives is not positive definite or when all elements of the vector $\mathbf{d}$ are greater than 0.1. If $\partial^2 f / \partial x_i^2 < 0.05$ (Heywood variables), the diagonal element is replaced by 1 and the rest of the elements of that column and row are set to 0. If the value of $f(\psi)$ is not decreased by step $\mathbf{d}$, the step is halved and halved again until the value of $f(\psi)$ decreases or 25 halvings fail to produce a decrease. (In this case, the computations are terminated.) Stepping continues until the largest absolute value of the elements of $\mathbf{d}$ is less than the criterion value (default 0.001) or until the maximum number of iterations (default 25) is reached. Using the converged value of ψ (denoted by $\hat{\psi}$), the eigenanalysis is performed on the matrix $\hat{\psi}\mathbf{R}^{-1}\hat{\psi}$. The factor loadings are computed as

$$\hat{\Lambda}_m = \hat{\psi} \Omega_m \left(\Gamma_m^{-1} - \mathbf{I}_m \right)^{1/2}$$

where

$$\Gamma_m = \text{diag}(\gamma_1, \gamma_2, \ldots, \gamma_m)$$
$$\Omega_m = (\omega_1, \omega_2, \ldots, \omega_m)$$

Unweighted and Generalized Least Squares (ULS, GLS)

The same basic algorithm is used in ULS and GLS methods as in maximum likelihood, except that

$$f(\psi) = \begin{cases} \sum_{k=m+1}^{p} \frac{\gamma_k^2}{2} & \text{for ULS} \\ \sum_{k=m+1}^{p} \frac{(\gamma_k - 1)^2}{2} & \text{for GLS} \end{cases}$$

for the ULS method, the eigenanalysis is performed on the matrix $\mathbf{R} - \psi^2$, where $\gamma_1 \geq \gamma_2 \geq \ldots \geq \gamma_p$ are the eigenvalues. In terms of the derivatives, for ULS

$$\frac{\partial f}{\partial x_i} = 2x_i \sum_{k=m+1}^{p} \gamma_k \omega_{ik}^2$$

$$\frac{\partial^2 f}{\partial x_i \partial x_j} = 4\left[x_i x_j \sum_{k=m+1}^{p} \omega_{ik} \omega_{jk} \sum_{n=1}^{m} \frac{\gamma_k + \gamma_n}{\gamma_k - \gamma_n} \omega_{ik} \omega_{jk} + \delta_{ij} \sum_{k=m+1}^{p} \left(x_i^2 - \frac{\gamma_k}{2} \right) \omega_{ik}^2 \right]$$

and

$$\frac{\partial^2 f}{\partial x_i \partial x_j} = 4 x_i x_j \left(\sum_{k=m+1}^{p} \omega_{ik} \omega_{jk} \right)^2$$

For GLS

$$\frac{\partial f}{\partial x_i} = \sum_{k=m+1}^{p} \left(\gamma_k^2 - \gamma_k\right)\omega_{ik}^2$$

$$\frac{\partial^2 f}{\partial x_i \partial x_j} = \delta_{ij}\frac{\partial f}{\partial x_i} + \sum_{k=m+1}^{p} \gamma_k \omega_{ik} \omega_{jk} \left(\sum_{n=1}^{m} \gamma_n \frac{\gamma_k + \gamma_n - 2}{\gamma_k - \gamma_n} \omega_{in}\omega_{jn} + r^{ii} \exp\left[\left(x_i + x_j\right)/2\right] \right)$$

and

$$\frac{\partial^2 f}{\partial x_i \partial x_j} = \left(\sum_{k=m+1}^{p} \omega_{ik}\omega_{jk} \right)^2$$

Also, the factor loadings of the ULS method are obtained by

$$\hat{\Lambda}_m = \Omega_m \Gamma_m^{1/2}$$

The chi-square statistics for *m* factors for the ML and GLS methods is given by

$$\chi_m^2 = \left(W - 1 - \frac{2p+5}{6} - \frac{2m}{3} \right) f(\hat{\psi})$$

with $\left((p-m)^2 - p - m\right)/2$

Alpha (Harman, 1976)

Iteration for Communalities

At each iteration i:

- The eigenvalues $\left(r_{(i)}\right)$ and eigenvectors $\left(\boldsymbol{\Omega}_{(i)}\right)$ of

  $$\mathbf{H}_{(i-1)}^{1/2}(\mathbf{R}-\mathbf{I})\mathbf{H}_{(i-1)}^{1/2}+\mathbf{I}$$ are computed.

- The new communalities are

  $$h_{k(i)}\left(\sum_{j=1}^{m}\left|\gamma_{j(i)}\right|\omega_{kj(i)}^{2}\right)h_{k(i-1)}$$

  The initial values of the communalities, $\mathbf{H}_0$, are

  $$h_{io}=\begin{cases}1-1/r^{ii} & |\mathbf{R}|\geq 10^{-8} \text{ and all } 0\leq h_{io}\leq 1\\ \max_j\left|r_{ij}\right| & \text{otherwise}\end{cases}$$

  where r^{ii} is the ith diagonal entry of $\mathbf{R}^{-1}$.

  If $|\mathbf{R}|\geq 10^{-8}$ and all r^{ii} are equal to one, the procedure is terminated. If for some i, $\max_j\left|r_{ij}\right|>1$, the procedure is terminated.

- Iteration stops if any of the following are true:

$$\max_k \left| h_{k(i)} - h_{k(i-1)} \right| < \text{EPS}$$

$$i = \text{MAX}$$

$$h_{k(i)} = 0 \text{ for any } k$$

Final Communalities and Factor Pattern Matrix

The communalities are the values when iteration stops, unless the last termination criterion is true, in which case the procedure terminates. The factor pattern matrix is

$$\mathbf{F}_m = \mathbf{H}_{(f)}^{1/2} \Omega_{m(f)} \Gamma_{m(f)}^{1/2}$$

where f is the final iteration.

Image (Kaiser, 1963)

Factor Analysis of a Correlation Matrix

- Eigenvalues and eigenvectors of $\mathbf{S}^{-1}\mathbf{R}\mathbf{S}^{-1}$ are found.

$$\mathbf{S}^2 = \text{diag}\left(1/r^{11}, \ldots, 1/r^{nn} \right)$$
$$r^{ii} = i\text{th diagonal element of } \mathbf{R}^{-1}$$

- The factor pattern matrix is

  $$\mathbf{F}_m = \mathbf{S}\Omega_m\left(\Lambda_m - I_m\right)\Lambda_m^{-1/2}$$

  where Ω_m and Λ_m correspond to the *m* eigenvalues greater than 1.

  If $m = 0$, the procedure is terminated.

- The communalities are

  $$h_i = \sum_{j=1}^{m}\left(\gamma_j - 1\right)^2 \omega_{ij}^2 \Big/ \left(\gamma_j r^{ii}\right)$$

- The image covariance matrix is

  $$\mathbf{R} + \mathbf{S}^2\mathbf{R}^{-1}\mathbf{S}^2 - 2\mathbf{S}^2$$

- The anti-image covariance matrix is

  $$\mathbf{S}^2\mathbf{R}^{-1}\mathbf{S}^2$$

Factor Analysis of a Covariance Matrix

We are using the covariance matrix Σ instead of the correlation matrix $\mathbf{R}$. The calculation is similar to the correlation matrix case.

The rescaled factor pattern matrix is $\mathbf{F}_{mR} = [diag\Sigma]^{-1/2}\mathbf{F}_m$. The rescaled communality of variable *i* is $h_{iR} = \sigma_{ii}^{-1}h_i$.

Factor Rotations

Orthogonal Rotations (Harman, 1976)

Rotations are done cyclically on pairs of factors until the maximum number of iterations is reached or the convergence criterion is met. The algorithm is the same for all orthogonal rotations, differing only in computations of the tangent values of the rotation angles.

- The factor pattern matrix is normalized by the square root of communalities:

  $$\Lambda_m^* = \mathbf{H}^{-1/2}\Lambda_m$$

  where

  $\Lambda_m = \left(\underline{\lambda}_1, \ldots, \underline{\lambda}_m\right)$ is the factor pattern matrix

  $\mathbf{H} = \operatorname{diag}\left(h_1, \ldots, h_n\right)$ is the diagonal matrix of communalities

- The transformation matrix $\mathbf{T}$ is initialized to $\mathbf{I}_m$

- At each iteration i

  (1) The convergence criterion is

  $$SV_{(i)} = \sum_{j=1}^{m}\left(n\sum_{k=1}^{n}\lambda_{kj(i)}^{*4} - \left(\sum_{k=1}^{n}\lambda_{kj(i)}^{*2}\right)^2\right)\Bigg/ n^2$$

  where the initial value of $\Lambda_{m(1)}^*$ is the original factor pattern matrix. For subsequent iterations, the initial value is the final value of $\Lambda_{m(i-1)}^*$ when all factor pairs have been rotated.

  (2) For all pairs of factors $\left(\lambda_j, \lambda_k\right)$ where $k > j$, the following are computed:

  (a) Angle of rotation

  $$P = 1/4\tan^{-1}\left(X/Y\right)$$

  where

$$X = \begin{cases} D - 2AB/n & \text{Varimax} \\ D - mAB/n & \text{Equimax} \\ D & \text{Quartimax} \end{cases}$$

$$Y = \begin{cases} C - \left(A^2 - B^2\right)/n & \text{Varimax} \\ C - m\left(A^2 - B^2\right)/2n & \text{Equimax} \\ C & \text{Quartimax} \end{cases}$$

$$u_{p(i)} = f^{*2}_{pj(i)} - f^{*2}_{pk(i)} \qquad v_{p(i)} = 2 f^{*}_{pj(i)} f^{*}_{pk(i)} \qquad p = 1, \ldots, n$$

$$A = \sum_{p=1}^{n} u_{p(i)} \qquad B = \sum_{p=1}^{n} v_{p(i)}$$

$$C = \sum_{p=1}^{n} \left[u^2_{p(i)} - v^2_{p(i)} \right] \qquad D = \sum_{p=1}^{n} 2 u_{p(i)} v_{p(i)}$$

If $\left|\sin(P)\right| \le 10^{-15}$, no rotation is done on the pair of factors.

(b) New rotated factors

$$\left(\underline{\tilde{\lambda}}_{j(i)}, \underline{\tilde{\lambda}}_{k(i)}\right) = \left(\underline{\lambda}^{*}_{j(i)}, \underline{\lambda}^{*}_{k(i)}\right) \begin{vmatrix} \cos(P) & -\sin(P) \\ \sin(P) & \cos(P) \end{vmatrix}$$

where $\underline{\lambda}^{*}_{j(i)}$ are the last values for factor *j* calculated in this iteration.

(c) Accrued rotation transformation matrix

$$\left(\tilde{t}_j, \tilde{t}_k\right) = \left(t_j, t_k\right) \begin{vmatrix} \cos(P) & -\sin(P) \\ \sin(P) & \cos(P) \end{vmatrix}$$

where t_j and t_k are the last calculated values of the *j*th and *k*th columns of *T*.

(d) Iteration is terminated when

$$\left| SV_{(i)} - SV_{(i-1)} \right| \le 10^{-5}$$

or the maximum number of iterations is reached.

(e) Final rotated factor pattern matrix

$$\tilde{\Lambda}_m = \mathbf{H}^{1/2} \Lambda^*_{m(f)}$$

where

$\Lambda^*_{m(f)}$ is the value of the last iteration.

(f) Reflect factors with negative sums

If

$$\sum_{i=1}^{n} \tilde{\lambda}_{ij(f)} < 0$$

then

$$\underline{\tilde{\lambda}}_j = -\underline{\tilde{\lambda}}_{j(f)}$$

(g) Rearrange the rotated factors such that

$$\sum_{j=1}^{n} \tilde{\lambda}^2_{j1} \ge \cdots \ge \sum_{j=1}^{n} \tilde{\lambda}^2_{jm}$$

(h) The communalities are

$$h_j = \sum_{i=1}^{m} \tilde{\lambda}^2_{ji}$$

Oblique Rotations

The direct oblimin method (Jennrich and Sampson, 1966) is used for oblique rotation. The user can choose the parameter δ. The default value is $\delta = 0$.

(a) The factor pattern matrix is normalized by the square root of the communalities

$$\Omega_m^* = \mathbf{H}^{-1/2}\Lambda_m$$

where

$$h_j = \sum_{k=1}^{m} \lambda_{jk}^2$$

If no Kaiser is specified, this normalization is not done.

(b) Initializations

The factor correlation matrix $\mathbf{C}$ is initialized to $\mathbf{I}_m$. The following are also computed:

$$s_k = \begin{cases} 1 & \text{if Kaiser} \\ h_k & \text{if no Kaiser} \end{cases} \qquad k = 1, \ldots, n$$

$$u_i = \sum_{j=1}^{n} \lambda_{ji}^{*2} \qquad i = 1, \ldots, m$$

$$v_i = \sum_{j=1}^{n} \lambda_{ji}^{*4}$$

$$x_i = v_i - (\delta/n)\, u_i^2$$

$$D = \sum_{i=1}^{m} u_i$$

$$G = \sum_{i=1}^{m} x_i$$

$$H = \sum_{k=1}^{n} s_i^2 - (\delta/n)\, D^2$$

$$FO = H - G$$

(c) At each iteration, all possible factor pairs are rotated. For a pair of factors $\underline{\lambda}_p^*$ and $\underline{\lambda}_q^*$ $(p \neq q)$, the following are computed:

$$D_{pq} = D - u_p - u_q$$

$$G_{pq} = G - x_p - x_q$$

$$s_{pq,i} = s_i - \lambda_{ip}^{*2} - \lambda_{iq}^{*2}$$

$$y_{pq} = \sum_{i=1}^{n} \lambda_{ip}^{*} \lambda_{iq}^{*}$$

$$z_{pq} = \sum_{i=1}^{n} \lambda_{ip}^{*2} \lambda_{iq}^{*2}$$

$$T = \sum_{i=1}^{n} s_{pq,i}\, \lambda_{ip}^{*2} - (\delta/n) u_p D_{pq}$$

$$Z = \sum_{i=1}^{n} s_{pq,i}\, \lambda_{ip}^{*} \lambda_{iq}^{*} - (\delta/n) y_{pq} D_{pq}$$

$$P = \sum_{i=1}^{n} \lambda_{ip}^{*3} \lambda_{iq}^{*} - (\delta/n) u_p y_{pq}$$

$$R = z_{pq} - (\delta/n) u_p u_q$$

$$P' = \tfrac{3}{2}\left(c_{pq} - P/x_p\right)$$

$$Q' = \tfrac{1}{2}\left(x_p - 4c_{pq}P + R + 2T\right)\Big/ x_p$$

$$R' = \tfrac{1}{2}\left(c_{pq}(T + R) - P - Z\right)\Big/ x_p$$

- A root, a, of the equation

  $b^3 + P'b^2 + Q'b + R = 0$ is computed, as well as:

  $$A = 1 + 2c_{pq}a + a^2$$
  $$t_1 = |A|^{1/2}$$
  $$t_2 = a/t_1$$

- The rotated pair of factors is

  $$\left(\tilde{\underline{\lambda}}_p^*, \tilde{\underline{\lambda}}_q^*\right) = \left(\underline{\lambda}_p^*, \underline{\lambda}_q^*\right) \begin{vmatrix} t_1 & -a \\ 0 & 1 \end{vmatrix}$$

  These replace the previous factor values.

- New values are computed for

  $$\tilde{u}_p = |A| u_p$$

  $$\tilde{x}_p = A^2 x_p$$

  $$\tilde{v}_q = \sum_{i=1}^{n} \tilde{\lambda}_{iq}^{*4}$$

  $$\tilde{u}_q = \sum_{i=1}^{n} \tilde{\lambda}_{iq}^{*2}$$

  $$\tilde{x}_q = \tilde{v}_q - (\delta/n)\tilde{u}_q^2$$

  $$\tilde{S}_k = S_{pq,k} + \tilde{\lambda}_{kp}^{*2} + \tilde{\lambda}_{kq}^{*2}$$

  $$\tilde{D} = D_{pq} + \tilde{u}_p + \tilde{u}_q$$

  $$\tilde{G} = G_{pq} + \tilde{x}_p + \tilde{x}_q$$

  All values designated as $\tilde{V}$ replaces V and are used in subsequent calculations.

- The new factor correlations with factor p are

$$\tilde{c}_{ip} = t_1^{-1} c_{ip} + t_2 c_{iq} \qquad (i \neq p)$$

$$\tilde{c}_{pi} = \tilde{c}_{ip}$$

$$\tilde{c}_{pp} = 1$$

- After all factor pairs have been rotated, iteration is terminated if

MAX iterations have been done

or

$$\left| F1_{(i)} - F1_{(i-1)} \right| < (FO)(EPS)$$

where

$$F1_{(i)} = \tilde{H} - \tilde{G}$$

$$\tilde{H} = \sum_{k=1}^{n} \tilde{s}_k^2 - (\delta/n)\tilde{D}^2$$

$$F1_{(0)} = FO$$

Otherwise, the factor pairs are rotated again.

- The final rotated factor pattern matrix is

$$\tilde{\lambda}_m = \mathbf{H}^{1/2} \tilde{\lambda}_m^*$$

where $\tilde{\lambda}_m$ is the value in the final iteration.

- The factor structure matrix is

  $$\mathbf{S} = \tilde{\Lambda}_m \tilde{\mathbf{C}}_m$$

  where $\tilde{\mathbf{C}}_m$ is the factor correlation matrix in the final iteration.

Promax Rotation

Hendrickson and White (1964) proposed a computationally fast rotation. The speed is achieved by first rotating to an orthogonal varimax solution and then relaxing the orthogonality of the factors to better fit simple structure.

- Varimax rotation is used to get an orthogonal rotated matrix $\Lambda_R = \{\lambda_{ij}\}$.

- The matrix $\mathbf{P} = (p_{ij})_{p \times m}$ is calculated, where

  $$p_{ij} = \left| \frac{\lambda_{ij}}{\left(\sum_{j=1}^{m} \lambda_{ij}^2 \right)^{1/2}} \right|^{k+1} \left(\sum_{j=1}^{m} \lambda_{ij}^2 \right)^{1/2} / \lambda_{ij}$$

  Here, k ($k > 1$) is the power of promax rotation.

- The matrix **L** is calculated.

  $$\mathbf{L} = (\Lambda'_R \Lambda_R)^{-1} \Lambda'_R \mathbf{P}$$

- The matrix **L** is normalized by column to a transformation matrix

  $$\mathbf{Q} = \mathbf{LD}$$

  where $\mathbf{D} = (diag(\mathbf{L}'\mathbf{L}))^{-1/2}$ is the diagonal matrix that normalizes the columns of **L**.

At this stage, the rotated factors are

$$f_{promax\_temp} = \mathbf{Q}^{-1} f_{varimax}.$$

Because

$$\mathrm{var}(f_{promax\_temp}) = (\mathbf{Q}'\mathbf{Q})^{-1},$$

and the diagonal elements do not equal 1, we must modify the rotated factor to

$$f_{promax} = \mathbf{C} f_{promax\_temp}$$

where

$$\mathbf{C} = \{diag((\mathbf{Q'Q})^{-1})\}^{-1/2}$$

The rotated factor pattern is

$$\Lambda_{promax} = \Lambda_{varimax} \mathbf{QC}^{-1}$$

The correlation matrix of the factors is

$$\mathbf{R}_{ff} = \mathbf{C}(\mathbf{Q'Q})^{-1}\mathbf{C'}$$

The factor structure matrix is

$$\Lambda_S = \Lambda_{promax} \mathbf{R}_{ff}$$

Factor Score Coefficients (Harman, 1976)

Regression[1]

$$\mathbf{W} = \begin{cases} \Lambda_m \Gamma_m^{-1} & \text{PC without rotation} \\ \Lambda_m \left(\Lambda'_m \Lambda_m\right)^{-1} & \text{PC with rotation} \\ \mathbf{R}^{-1}\mathbf{S}_m & \text{otherwise} \end{cases}$$

where

$\mathbf{S}_m$ = factor structure matrix

$\mathbf{S}_m = \Lambda_m$ for orthogonal rotations

For PC without rotation if any $|\gamma_i| \le 10^{-8}$, factor score coefficients are not computed. For PC with rotation, if the determinant of $\Lambda'_m \Lambda_m$ is less than 10^{-8}, the coefficients are not computed. Otherwise, if **R** is singular, factor score coefficients are not computed.

[1] This algorithm applies to SPSS 7.0 and later releases.

Bartlett

$$\mathbf{W} = \mathbf{J}^{-1}\Lambda'\mathbf{U}^{-2}$$

where

$$\mathbf{J} = \Lambda'\mathbf{U}^{-2}\Lambda$$
$$\mathbf{U}^2 = \mathbf{R} - \hat{\mathbf{R}}$$

Anderson Rubin

$$\mathbf{W} = \left(\Lambda'\mathbf{U}^{-2}\mathbf{R}\mathbf{U}^{-2}\Lambda\right)^{-1/2}\Lambda'\mathbf{U}^{-2}$$

where the symmetric square root of the parenthetical term is taken.

Optional Statistics (Dziubin and Shirkey, 1974)

- The anti-image covariance matrix $\mathbf{A} = \left(a_{ij}\right)$ is given by

$$a_{ij} = \frac{r^{ij}}{r^{ii} r^{jj}}$$

- The chi-square value for Bartlett's test of sphericity is

$$\chi^2 = -\left(W - 1 - \frac{2p+5}{6}\right)\log|\mathbf{R}|$$

with $p(p-1)/2$ degrees of freedom.

- The Kaiser-Mayer-Olkin measure of sample adequacy is

$$KMO_j = \frac{\sum_{i\neq j} r_{ij}^2}{\sum_{i\neq j} r_{ij}^2 + \sum_{i\neq j} a_{ij}^{2*}} \qquad KMO = \frac{\sum\sum_{i\neq j} r_{ij}^2}{\sum\sum_{i\neq j} r_{ij}^2 + \sum\sum_{i\neq j} a_{ij}^{2*}}$$

where a_{ij}^* is the anti-image correlation coefficient.

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