## FREQUENCIES

If the absolute value of any observation is greater than $10^{13}$, no calculations are done. For sorting of the observations, see Appendix 6. For information on percentiles for grouped data, see Appendix 8.

## Notation

The following notation is used throughout this chapter unless otherwise stated:

| $X_{k}$ | Value of the variable for case $k$ |
| :--- | :--- |
| $w_{k}$ | Weight for case $k$ |
| $N V$ | Number of distinct values the variable assumes |
| $N$ | Number of cases |
| $W$ | Sum of weights of the cases |

## Basic Statistics

The values are sorted into ascending order and the following calculated:

## Sum of Weights of Cases Having Each Value of $X$

$f_{j}=\sum_{i=1}^{N} w_{i} k_{i} \quad j=1,2, \ldots, N V$
where
$k_{i}= \begin{cases}1 & \text { if } X_{i}=X_{j} \\ 0 & \text { otherwise }\end{cases}$
where $X_{j}$ is the $j$ th largest distinct value of $X$.

## 2 FREQUENCIES

## Relative Frequency (Percentage) for each Value of $X$

$$
R f_{j}=\left(\frac{f_{j}}{W^{\prime}}\right) \times 100
$$

where

$$
W^{\prime}=\sum_{i=1}^{N V} f_{i} \text { (sum over all categories including those declared as missing values) }
$$

## Adjusted Frequency (Percentage)

$$
A f_{j}=\left(\frac{f_{j}}{W}\right) \times 100
$$

where

$$
W=\sum_{i=1}^{N V} f_{i} k_{i} \quad \text { (sum over nonmissing categories) }
$$

and

$$
k_{i}= \begin{cases}0 & \text { if } X_{i} \text { has been declared missing } \\ 1 & \text { otherwise }\end{cases}
$$

For all $X_{j}$ declared missing, an adjusted frequency is not printed.

## Cumulative Frequency (Percentage)

$$
C f_{j}=\sum_{i=1}^{j} f_{i}
$$

## Minimum

$$
\min _{k} X_{k}
$$

## Maximum

$$
\max _{k} X_{k}
$$

## Mode

Value of $X_{j}$ which has the largest observed frequency. If several are tied, the smallest value is selected.

## Range

Maximum - Minimum

## The $p$ th percentile

Find the first score interval ( $x 2$ ) containing more than $t p$ cases.
$p$ th percentile $= \begin{cases}x_{2} & \text { if } t p-c p_{1} \geq 100 / W \\ \left\{1-\left[(W+1) p / 100-c c_{1}\right]\right\} x_{1} & \text { if } t p-c p_{1}<100 / W \\ +\left[(W+1) p / 100-c c_{1}\right] x_{2} & \end{cases}$
where
$t p=(W+1) p / 100$
$c p_{1}<t p<c p_{2}$
$x_{1}$ and $x_{2}$ are the values corresponding to $c p_{1}$ and $c p_{2}$, respectively
$c c_{1}$ is the cumulative frequency up to $x_{1}$
$c p_{1}$ is the cumulative percent up to $x_{1}$

## 4 FREQUENCIES

## Mean

$$
\bar{X}=\frac{\sum_{j=1}^{N V} f_{j} X_{j}}{W}
$$

Moments about the mean are calculated as:

$$
M_{j}=\sum_{i=1}^{N V} f_{i}\left(X_{i}-\bar{X}\right)^{j} \quad j=2,3,4
$$

Variance

$$
S^{2}=\frac{M_{2}}{(W-1)}
$$

Standard Deviation

$$
S=\sqrt{S^{2}}
$$

Standard Error of the Mean

$$
S E M=\frac{S}{\sqrt{W}}
$$

Skewness (computed if $W \geq 3$ and $S^{2}>0$ ) (Bliss, 1967, p. 144)

$$
g_{1}=\frac{W M_{3}}{(W-1)(W-2) S^{3}} \quad \operatorname{se}\left(g_{1}\right)=\sqrt{\frac{6 W(W-1)}{(W-2)(W+1)(W+3)}}
$$

Kurtosis (computed if $W \geq 4$ and $S^{2}>0$ )

$$
g_{2}=\frac{W(W+1) M_{4}-3(W-1) M_{2}^{2}}{(W-1)(W-2)(W-3) S^{4}} \quad \operatorname{se}\left(g_{2}\right)=\sqrt{\frac{4\left(W^{2}-1\right) \operatorname{se}\left(g_{1}\right)^{2}}{(W-3)(W+5)}}
$$

## References

Blalock, H. M. 1972. Social statistics. New York: McGraw-Hill.

Bliss, C. I. 1967. Statistics in biology, Volume 1. New York: McGraw-Hill.

