ORTHOPLAN

This procedure generates an orthogonal main-effects design. It will find the smallest orthogonal plan to fit the factors having at least as many combinations as requested.

Selecting the Plan

From a library of prepared plans, select the shortest plan that can be adapted to the design and that satisfies the minimum size requirement provided by the user. If no plan exists that satisfies the minimum size requirement, pick the largest plan that can be adapted.

Adapting the Prepared Plans

Generating Multiple Factors from One Column

A four-level factor can be transformed into three two-level factors using the rule in Table 1.

Table 1: Converting a four-level factor to three two-level factors

Original Code	A	B	C
0	0	0	0
1	0	1	1
2	1	0	1
3	1	1	0

An eight-level factor can be transformed into seven two-level factors using the rule in Table 2.

Table 2: Converting an eight-level factor to seven two-level factors

Original Code	A	B	C	D	E	F	G
0	0	0	0	0	0	0	0
1	1	0	0	1	1	0	1
2	0	1	0	1	0	1	1
3	1	1	0	0	1	1	0
4	0	0	1	0	1	1	1
5	1	0	1	1	0	1	0
6	0	1	1	1	1	0	0
7	1	1	1	0	0	0	1

A nine-level factor can be transformed into four three-level factors using the rule in Table 3.

Table 3: Converting a nine-level factor to four three-level factors

Original Code	A	B	C	D
0	0	0	0	0
1	0	1	1	2
2	0	2	2	1
3	1	0	1	1
4	1	1	2	0
5	1	2	0	2
6	2	0	2	2
7	2	1	0	1
8	2	2	1	0

Changing the Number of Levels in a Column

Any factor of m levels can be transformed into a factor of $n < m$ levels by many-to-one mappings without changing its orthogonality. Any mapping can be used; i mod n is used here.

Library of Prepared Plans

Plackett-Burman Plans

Plackett and Burman (1946) describe a series of plans that can be generated from a single column by rotation. The general algorithm for generating any of these plans is:

- Let L be the number of levels for which the plan is designed. No factor in the specific design can have more than L levels.
- Let N be the number of rows (combinations) finally to be generated. Note than $N = F + 1$ where F is defined below.
- Starting with a given column of $N - 1$ level codes, rotate one position to generate each new column.
- Finally, add a row of zeroes.

$\frac{(N-1)}{(L-1)}$ orthogonal columns can be generated in this fashion.

The Plackett-Burman plans used here are designated *PBL.F*, where L is the maximum number of levels and F is the number of factors:

Label	**Generating column**
PB 2.7	11101 00
PB 2.11	11011 10001 0
PB 2.15	11110 10110 01000
PB 2.19	11001 11101 01000 0110
PB 2.23	11111 01011 00110 01010 000
PB 2.31	00001 01011 10110 00111 11001 10100 1
PB 2.35	01011 10001 11110 11100 10000 10101 10010
PB 2.43	11001 01001 11011 11100 01011 10000 01000 11010 110
PB 2.47	11111 01111 00101 01110 01001 10110 00101 01100 00100 00
PB 2.59	11011 10101 00100 11101 11100 11111 00000 11000 01000 11011 01010 0010
PB 3.4	01220 211
PB 3.13	00101 21120 11100 20212 21022 2
PB 3.40	01111 20121 12120 20221 10201 10012 22021 00200 02222 10212 21210 10112 20102 20021 11012 00100
PB 5.6	04112 10322 42014 43402 3313
PB 7.8	01262 21605 32335 20413 11430 65155 61024 54425 03646 634

Addelman Plans

Addelman (1961) described general methods for generating orthogonal main effects plans. That paper included a number of such designs, and using those methods, the authors generated more.

Figure 1: 18 rows, 7 columns of 3 levels each

```
0000000  0021011
0112111  0100122
0221222  0212200
1011120  1002221
1120201  1111002
1202012  1220110
2022102  2010212
2101210  2122020
2210021  2201101
```

Figure 2: 8 rows, 1 column of 4 levels plus 4 columns of 2 levels

```
0  0000
0  1111
1  0011
1  1100
2  0101
2  1010
3  0110
3  1001
```

Figure 3: 16 rows, 5 columns of 4 levels each

```
00000  02231
10111  12320
20222  22013
30333  32102
01123  03312
11032  13203
21301  23130
31210  33021
```

Figure 4: 32 rows, 9 columns of 4 levels each

```
000000000  002130213
011231111  013301302
022312222  020222031
```

033123333 031013120
101111032 103021221
110320123 112210330
123203210 121333003
132032301 130102112
202223102 200313311
213012013 211122200
220131320 222001133
231300231 233230022
303332130 301202323
312103021 310033232
321020312 323110101
330211203 332321010

Figure 5: 64 rows, 21 columns of 4 levels each

000000000000000000000 000222233331111022220
111111111111111100000 111333322220000122220
222222222222222200000 222000011113333222220
333333333333333300000 333111100002222322220
123012301230123012301 123230132101032030121
032103210321032112301 032321023010123130121
301230123012301212301 301012310323210230121
210321032103210312301 210103201232301330121
231023102310231023102 231201331021320001322
320132013201320123102 320310220130231101322
013201320132013223102 013023113203102201322
102310231023102323102 102132002312013301322
312031203120312031203 312213030211203013023
203120312031203131203 203302121300312113023
130213021302130231203 130031212033021213023
021302130213021331203 021120303122130313023
000111122223333011110 000333311112222033330
111000033332222111110 111222200003333133330
222333300001111211110 222111133330000233330
333222211110000311110 333000022221111333330

123103223013210003211	123321010322301021031
032012332102301103211	032230101233210121031
301321001231032203211	301103232100123221031
210230110320123303211	210012323011032321031
231132020133102032012	231310213202013010232
320023131022013132012	320201302313102110232
013310202311320232012	013132031020231210232
102201313200231332012	102023120131320310232
312120321303021020313	312302112032130002133
203031230212130120313	203213003123021102133
130302103121203220313	130120330210312202133
021213012030312320313	021031221301203302133

Figure 6: 16 rows, 1 column of 8 levels plus 8 columns of 2 levels

0	00000000	0	11111111
1	01010101	1	10101010
2	00001111	2	11110000
3	01011010	3	10100101
4	00111100	4	11000011
5	01101001	5	10010110
6	00110011	6	11001100
7	01100110	7	10011001

Figure 7: 32 rows, 1 column of 8 levels plus 8 columns of 4 levels

0	00000000	0	22222222
1	01230123	1	23012301
2	02021313	2	20203131
3	03211230	3	21033012
4	00113322	4	22331100
5	01323201	5	23101023
6	02132031	6	20310213
7	03302112	7	21120330
0	11111111	0	33333333
1	10321032	1	32103210
2	13130202	2	31312020
3	12300321	3	30122103
4	11002233	4	33220011
5	10232310	5	32010132
6	13023120	6	31201302
7	12213003	7	30031221

Figure 8: 64 rows, 9 columns of 8 levels each

000000000	202222222	404444444	606666666
011234567	213016745	415670123	617452301
022456713	220647531	426021357	624203175
033651274	231473056	437215630	635037412
044517326	246735104	440153762	642371540
055723641	257501463	451367205	653145927
066172435	264350617	462536071	660714253
077346152	275164370	473702516	671520734
101111111	303333333	505555555	707777777
110325476	312107654	514761032	7165432210
123574602	321756420	527130246	725312064
132140365	330562147	536304721	734126503
145406237	347624015	541042673	743260451
154632750	356410572	550276314	752054136
167063524	356241706	563427160	761605342
176257043	374075261	572613407	770481625

Figure 9: 27 rows, 1 column of 9 levels plus 9 columns of 3 levels

0	000000000	3	011001111	6	022002222
0	112121212	3	120122020	6	101120101
0	221212121	3	202210202	6	210211010
1	000111122	4	011112200	7	022110011
1	112202001	4	120200112	7	101201220
1	221020210	4	202021021	7	210022102
2	000222211	5	011220022	8	022221100
2	112010120	5	120011201	8	101012012
2	221101002	5	202102110	8	210100221

Figure 10: 81 rows, 10 columns of 9 levels each

0000000000	0336258147	0663174285
1011111111	1347036258	1674285063
2022222222	2358147036	2685063174
3033333333	3360582471	3606417528
4044444444	4371360582	4617528306
5055555555	5382471360	5628306417
6066666666	6303825714	6630741852
7077777777	7314603825	7641852630
8088888888	8325714603	8652630741
0112345678	0448561723	0775426831
1120453786	1456372804	1783507642
2101534867	2437480615	2764318750
3145678012	3472804156	3718750264
4153786120	4480615237	4726831075
5134867201	5461723048	5707642183
6178012345	6415237480	6742183507
7186120453	7423048561	7750264318
8167201534	8404156372	8731075426
0221687354	0557813462	0884732516
1202768435	1538624570	1865840327
2210876543	2546705381	2873651408
3254021687	3581246705	3827165840
4235102768	4562057813	4808273651

5243210876	5570138624	5816084732
6287354021	6524570138	6851408273
7268435102	7505381246	7832516084
8276543210	8513462057	8840327165

Decision Rules

Each value of L (the maximum number of levels in the design) has a distinct decision rule. In their descriptions, the following notation is used:

- M The user-supplied minimum number of rows desired in the plan
- F The number of factors in the design

L = 2

If all factors have two levels, simply select the smallest two-level Plackett-Burman plan for which $N_{plan} \geq \max(M, F+1)$.

L = 3

Let P = the number of factors with more than two levels, and let $K = F + 2P$.

If $M < 9$ and $F < 6$ and $P < 2$, base the plan on Figure 2.

If $M < 10$ and $F < 5$, base the plan on PB 3.4.

Otherwise, if $M < 17$ and $K < 16$, base it on Figure 3.

Otherwise, if $M < 19$ and $K < 8$, base it on Figure 1.

Otherwise, if $M < 28$ and $K < 14$, base it on PB 3.13.

Otherwise, if $M < 65$ and $K < 22$, use the rules for $L = 4$.

Otherwise, if $F < 41$, base the plan on PB 3.40.

If $F > 40$, there are too many factors.

L = 4

Let P = the number of factors with more than two levels, and let $K = F + 2P$.

If $M < 9$ and $F < 6$ and $P < 2$, base the plan on Figure 2.

Otherwise, if $M < 17$ and $K \leq 15$, base it on Figure 3.

Otherwise, if $M < 26$ and $K < 19$, base it on PB 5.6.

Otherwise, if $M < 33$ and $K < 28$, base it on Figure 4.

Otherwise, if $M < 49$ and $K < 23$, use the rules for $L = 7$.

Otherwise, if $K < 64$, base the plan on Figure 5.

Otherwise, there are too many factors.

A four-level factor can be transformed into three two-level factors using the rule in Table 1.

L = 5

Create a plan based on the $L = 7$ rules.

If that plan has 26 or more rows and $M < 26$ and $F < 7$, base the plan on PB 5.6.

Otherwise, use the plan generated in step 1.

L = 6

Treat this case as $L = 7$.

L = 7

Generate the best plan based on $L = 8$.

If that plan has more than 49 rows and $M < 50$ and $F < 9$, base the plan on PB 7.8.

Otherwise, use the plan generated in step 1.

L = 8[1]

Let *P* be the number of factors with more than two levels, and *Q* be the number of factors with more than four levels.

If $M < 17$ and $F < 10$ and $P < 2$, then base the plan on Figure 6.

Otherwise, if $M < 28$ and $F < 11$ and only one factor has more than three levels, base the plan on the $L = 9$ rules.

Otherwise, if $M < 33$ and $Q < 2$ and $F + 2P + 4Q < 32$, base the plan on Figure 7.

Otherwise, if $M < 65$ and $F + 6P < 64$, base it on Figure 8.

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[1] This algorithm applies to SPSS 6.0 and later releases.

Otherwise, base the plan on the $L = 9$ rules.

An eight-level factor can be transformed into seven two-level factors using the rule in Table 2.

L = 9

Let *P* be the number of factors with more than three levels, and $K = F + 3P$.

If $M < 28$ and $F < 11$ and $P < 2$, then base the plan on Figure 9.

Otherwise, if $K < 41$, base it on Figure 10.

Otherwise, there are too many factors.

An nine-level factor can be transformed into four three-level factors using the rule in Table 3.

Randomization

After a basic plan has been selected, columns are selected at random (if possible) to fit the given design. If the basic plan is asymmetric; that is, one column has more levels than the others, then the factor in the plan with many levels must be assigned to the factor in the design with many levels, and the remaining plan factors must be assigned randomly to the remaining design factors.

If factors are to be transformed into multiple factors (for example, eight-level factors transformed into two-level factors), you can randomly assign columns from the plan to design factors with many levels first, then transform the remaining columns, and then select from the transformed columns at random the columns needed.

References

Plackett, R. L., and Burman, J. P. 1946. The design of optimum multifactorial experiments. *Biometrika*, 33: 305–325.

Addelman, S. 1962. Symmetrical and asymmetrical fractional factorial plans. *Technometrics*, 4: 47–58.