SEASON

Based on the multiplicative or additive model, the SEASON procedure decomposes the existing series into three components: trend-cycle, seasonal, and irregular.

Model

Multiplicative Model

$$X_t = TC_t S_t I_t, \quad t = 1, \ldots, n$$

Additive Model

$$X_t = TC_t + S_t + I_t, \quad t = 1, \ldots, n$$

where TC_t is the “trend-cycle” component, S_t is the “seasonal” component, and I_t is the “irregular” or “random” component.

The procedure for estimating the seasonal component is:

(1) Smooth the series by the moving average method; the moving average series reflects the trend-cycle component.

(2) Obtain the seasonal-irregular component by dividing the original series by the smoothed values if the model is multiplicative, or by subtracting the smoothed values from the original series if the model is additive.

(3) Isolate the seasonal component from the seasonal-irregular component by computing the medial average (average) of the specific seasonal relatives for each unit of periods if the model is multiplicative (additive).

Moving Average Series

Based on the specified method and period *p*, the moving average series Z_t for X_t is defined as follows:

p is even, weight all points equally

$$Z_t = \begin{cases} \sum_{j=t-\frac{p}{2}}^{t+\frac{p}{2}-1} X_j \big/ p & t = \frac{p}{2}+1, \ldots, n-\frac{p}{2}+1 \\ \text{SYSMIS} & \text{otherwise} \end{cases}$$

p is even, weights unequal

$$Z_t = \begin{cases} \left(\left(X_{t-\frac{p}{2}} + X_{t+\frac{p}{2}} \right) \Big/ 2p + \left(\sum_{j=t-\frac{p}{2}+1}^{t+\frac{p}{2}-1} X_j \right) \right) \Big/ p, & t = \frac{p}{2}+1, \ldots, n-\frac{p}{2} \\ \text{SYSMIS} & \text{otherwise} \end{cases}$$

p is odd

$$Z_t = \begin{cases} \left(\sum_{j=t-\left[\frac{p}{2}\right]}^{t+\left[\frac{p}{2}\right]} X_j \right) \Big/ p, & t = \left[\frac{p}{2}\right]+1, \ldots, n-\left[\frac{p}{2}\right] \\ \text{SYSMIS} & \text{otherwise} \end{cases}$$

Ratios or Differences (Seasonal-Irregular Component)

Multiplicative Model

$$SI_t = \begin{cases} \text{SYSMIS}, & \text{if } Z_t = \text{SYSMIS} \\ (X_t / Z_t) \times 100, & \text{otherwise} \end{cases}$$

Additive Model

$$SI_t = \begin{cases} \text{SYSMIS}, & \text{if } Z_t = \text{SYSMIS} \\ X_t - Z_t, & \text{otherwise} \end{cases}$$

Seasonal Factors (Seasonal Components)

Multiplicative Model

$$F_t = \begin{cases} \text{medial average } \left(SI_{t+p}, SI_{t+2p}, \ldots, SI_{t+qp}\right), & 1 \le t \le L - \left[\frac{L}{p}\right]p \\ \text{medial average } \left(SI_{t+p}, SI_{t+2p}, \ldots, SI_{t+(q-1)p}\right), & L - \left[\frac{L}{p}\right]p < t \le \left[\frac{p}{2}\right] \\ \text{medial average } \left(SI_t, SI_{t+p}, \ldots, SI_{t+(q-1)p}\right), & \left[\frac{p}{2}\right] < t \le p \end{cases}$$

where

$$\begin{array}{lll} L = n - \frac{p}{2} + 1, & q = \left[\frac{L}{p}\right], & \text{if } p \text{ is even and all points are weighted equally} \\ L = n - \left[\frac{p}{2}\right], & q = \left[(n - p/2)/p\right], & \text{otherwise} \end{array}$$

and the medial average of a series is the mean value of the series after the smallest and the largest values are excluded. The seasonal factor is defined as

$$SAF_t = F_t \frac{100p}{\sum_{t=1}^{p} F_t}, \quad t = 1, \ldots, p$$

Additive Model

F_t is defined as the arithmetic average of the series shown above. Then

$$SAF_t = F_t - \overline{F},$$

where

$$\overline{F} = \sum_{t=1}^{p} F_t / p$$

Seasonally Adjusted Series (SAS)

$$SAS_t = \begin{cases} (X_t / SAF_m)100, & \text{if model is multiplicative} \\ X_t - SAF_m, & \text{if model is additive} \end{cases}$$

where

$$m = t - [t/p]p$$

Smoothed Trend-Cycle Series

The smoothed trend-cycle series (STC) is obtained by applying a 3×3 moving average on seasonally adjusted series (SAS). Thus,

$$STC_t = \frac{1}{9}\left[(SAS)_{t-2} + 2(SAS)_{t-1} + 3(SAS)_t + 2(SAS)_{t+1} + (SAS)_{t+2}\right],$$
$$t = 2, \ldots, n-2$$

and for the two end points on the beginning and end of the series

$$(STC)_2 = \frac{1}{3}\left[(SAS)_1 + (SAS)_2 + (SAS)_3\right]$$
$$(STC)_{n-1} = \frac{1}{3}\left[(SAS)_{n-2} + (SAS)_{n-1} + (SAS)_n\right]$$

$$(STC)_1 = (STC)_2 + \frac{1}{2}\left[(STC)_2 - (STC)_3\right]$$
$$(STC)_n = (STC)_{n-1} + \frac{1}{2}\left[(STC)_{n-1} - (STC)_{n-2}\right]$$

Irregular Component

For $t = 1, \ldots, n$

$$I_t = \begin{cases} (SAS)_t / (STC)_t, & \text{if model is multiplicative} \\ (SAS)_t - (STC)_t, & \text{if model is additive} \end{cases}$$

References

Makridakis, S., Wheelwright, S. C., and McGee, V. E. 1983. *Forecasting*: *Methods and Applications*, 2nd ed. New York: John Wiley & Sons, Inc.