WLS

WLS estimates regression model with different weights for different cases.

Notation

The following notation is used throughout this chapter unless otherwise stated:

n	The number of cases
p	The number of parameters for the model
$\mathbf{y}$	$n \times 1$ vector with element y_i, which represents the observed dependent variable for case i
$\mathbf{X}$	$n \times p$ matrix with element x_{ij}, which represents the observed value of the *i*th case of the *j*th independent variable
β	$p \times 1$ vector with element β_j, which represents the regression coefficient of the *j*th independent variable
$\mathbf{w}$	$n \times 1$ vector with element w_i, which represents the weight for case i

Model

The linear regression model has the form of

$$y_i = \mathbf{X}_i' \beta + \varepsilon_i , \quad i = 1, \ldots, n \qquad (\mathbf{1})$$

where $\mathbf{X}_i$ is the vector of covariates for the *i*th case, $\mathbf{E}(\varepsilon_i) = 0$, and $\operatorname{var}(\varepsilon_i) = w_i^{-1}\sigma^2$. Assuming that $\varepsilon_1, \ldots, \varepsilon_n$ follow a normal distribution, the log-likelihood function of model **(1)** is

$$L = 0.5\left\{ -n\ln 2\pi - n\ln\sigma^2 + \sum_{i=1}^{n}\ln w_i - \frac{\sum_{i=1}^{n} w_i\left(y_i - \mathbf{X}_i'\beta\right)^2}{\sigma^2} \right\} \qquad \textbf{(2)}$$

Computational Details

The algorithm used to obtain the weighted least-square estimates for the parameters in the model is the same as the REGRESSION procedure with regression weight. For details of the algorithm and statistics (the ANOVA table and the variables in the equation), see REGRESSION.

After the estimation is finished, the log-likelihood function shown in **(2)** is estimated by

$$\hat{L} = 0.5\left\{ -n\ln 2\pi - n\ln\hat{\sigma}^2 + \sum_{i=1}^{n}\ln w_i - (n-p) \right\}$$

where $\hat{\sigma}^2$ is the mean square error in the ANOVA table.