I. *Florum Geometricorum Manipulus Regiæ Societati exhibitus à* D. Guidone Grandi *Abbate* Camaldulenſi, Piſani *Lycæi Mathematico, R. S. S.*

SUOS Geometria hortos habet, in quibus æmula (an potius magiſtra?) naturæ ludere ſolet, ſua ipſius manu flores elegantiſſimos ſerens, irrigans, enutriens, quorum contemplatione cultores ſuos quandoque recreat, ac ſumma voluptate perfundit. Hos inter bonis avibus & ipſe quondam admiſſus, nonnullos decerpſi flores, vario frondium numero coronatos, quandoque & infinitis foliis ſibi per innumeros gyros circumpoſitis elegantiſſimè compactos, quorum exiguum hunc ſaltem manipulum vobis, Viri Clariſſimi, offere ſtatui, ut meum vobis obſequium aliquo argumento teſtatum facerem. An naturæ induſtria ſimili fortaſſe artificio florum, fruticum, arborum folia conſtruere ſatagat, tali proportione ſucci nutritii motum temperans, ac dirigens, ut eadem frondium figura dimanet, quales in variis ejuſmodi florum geometricorum foliis, juxta varias leges, quibus deſcribuntur. obſervare licet, Philoſophis diſcutiendum, ac decidendum relinquo, præcipuè verò ſolertiſſimis naturæ indagatoribus, qui magni *Neutoni* exemplo naturales leges ex profundioris Geometriæ principiis repetendas ſibi meritò perſuadent, quibus utique illuſtriſſimus cætus veſter, præ aliis maximè abundat. Valete.

DEFINITIONES.

I. *Flores Geometricos* generatim appello quaslibet figuras curvâ quadam per aliquot foliorum, sese ab uno centro expandentium, perimetrum recurrente circumscriptas, quales exhibent Figuræ 1, 2, 3, 4, 5; quos quidem flores, pro numero foliorum, bifolios, trifolios, tetrafolios, pentafolios, exafolios, &c. licebit nuncupare.

II. Cùm porrò innumeris modis ejusmodi flores generari possint, eam genesim hic speciatim consideramus, quæ per ramos a centro floris prodeuntes, æquales verò sinubus angulorum, iis angulis, quos cum data positione linea rami comprehendunt, in data aliqua ratione proportionalium, procedit: cujusmodi curvas *Rhodoneas* dudum appellavimus, eamque proportionem *Rhodoneæ* cuilibet propriam dicimus.

III. *Rhodoneam simplicem* appellamus, quæ una circulatione perficitur, *duplicem* quæ duplici, *triplicem* quæ triplici, & sic deinceps pro numero circulationum.

Itaque ad Rhodonearum descriptionem assumpto quolibet circulo, cujus centrum C (*Vid. Fig.* 6, *&* 7.) & ducto ubilibet radio CD ad radium positione datum CA utcunque inclinato, sit angulus ACD ad angulum ACG (sive arcus AD ad arcum AG) in data ratione *a* ad *b*, ductoque sinu GH, fiat CI æqualis GH; erit punctum I ad Rhodoneam supra definitam.

Ejusmodi Rhodonearum proprietates præcipuas enucleabimus, nec non spatia, & perimetros breviter dimetiemur sequentibus propositionibus.

PRO-

PROPOSITIO I.

Si fuerit arcus E A ad quadrantem A F (ſive angulus E C A ad rectum) ut *a* ad *b*, erit E C unus e maximis ramis Rhodoneæ, ſive erit E apex unius ex ejus foliis. (*Vid. Fig.* 6, *&* 7.)

Nam ex deſcriptione patet, ponendum eſſe ramum C E æqualem F C ſinui quadrantis A F, qui omnium ſinuum eſt maximus.

PROPOSITIO II.

Quodlibet folium Rhodoneæ circa axem C E hinc inde æquali, uniformi, & ſimili expanſione ſpargitur.

Factis enim hinc inde æqualibus angulis E C M, E C D, ob arcus æquales interceptos E M, E D, ſi fuerit arcus A M ad A N, ut A E ad A F, ut A D ad A G, nempe in data ratione *a* ad *b*, etiam reſidua E M, F N, itemque E D, F G in eadem ratione erunt, adeoque cùm antecedentia E M, E D æqualia ſint, etiam conſequentia F N, F G invicem æquabuntur, uti & reſidua ad quadrantes N K, G A, quorum ſinubus cùm æquari debeant rami Rhodoneæ C L, C I, & ipſi æquales erunt; quare ab axe C E hinc inde æquali, & uniformi expanſione ſpargitur quodlibet folium Rhodoneæ. Quod erat, &c.

COROLLARIA.

I. Ob æquales arcus E M, E D fit A E medius Arithmeticus inter A M, A D, qui intercipiunt æquales ramos Rhodoneæ; ideoque horum ſumma illius duplum adæquat, ſive æquatur toti A E P arcui ſectoris circumſcribentis unum Rhodoneæ folium.

II. Hinc etiam arcus M P æquatur A D.

III. Et eorundem arcuum A M, A D ſumma ad ſemiperipheriam A N K eſt in data ratione *a* ad *b*, quam habet A E ad quadrantem A F.

IV. Et ſector A P C Rhodoneæ circumſcriptus, eſt ad ſemicirculum in eadem data ratione *a* ad *b*, quam habet arcus A P, ſive ſumma duorum A M, A D ad ſemiperipheriam A N K.

PROPOSITIO III.

Numerus foliorum, quibus integra Rhodonea ſimplex compingitur, eſt ad unitatem, ut 2 *b* ad *a*.

Tot enim folia emergunt ex hac deſcriptione, quot ſectores unicuique folio circumſcripti, intra circulum diſponi poſſunt; ſed quilibet ſector eſt ad ſemicirculum, ex *Coroll.* 3. *præced.* ut *a* ad *b*, adeoque ad circulum ut *a* ad 2 *b*, quare numerus foliorum in una circulatione eſt ad unitatem ut 2 *b* ad *a*. Quod erat, &c.

COROLLARIA.

I. Hinc Rhodoneam ſimplicem deſcribere poſſumus, quæ datum foliorum numerum *m*, puta ſex, complectatur, ſi nempe pro ratione *a* ad *b* aſſumatur ratio 1 ad $\frac{m}{2}$ (in caſu propoſito 1 ad 3) quomodo erit 2 *b* ad *a*, ut *m* ad 1 (in propoſito ut 6 ad 1) adeoque prodibit datus foliorum numerus *m*.

II. Sed & Rhodoneam duplicem, triplicem, quadruplicem, &c. eadem arte componemus, dato foliorum numero in ſe recurrentem, ſi nimirum pro Rhodonea duplici ſumatur ratio 1 ad $\frac{m}{4}$, exiſtente dato numero m impari, alias prodiret Rhodonea ſimplex ſubduplo foliorum numero, quæ in ſecunda circulatione ſibimet ſuperponeretur, per eadem foliorum veſtigia recurrens. Pro Rhodonea triplici ratio 1 ad $\frac{m}{6}$, dummodo numerus m non ſit per 3 diviſibilis, alias iterum ſimplex Rhodonea prodiret ſubtriplo foliorum numero contenta. Similiter pro quadruplici Rhodonea ratio 1 ad $\frac{m}{8}$ inſerviet, dumodo numerus m ſit impar, alias Rhodonea ſimplex, aut duplex, ut antea oriretur; oportet enim in prima circulatione reſpectu Rhodoneæ duplicis haberi integrum aliquem foliorum numerum cum $\frac{1}{2}$ alterius folii, reſpectu triplicis cum $\frac{1}{3}$, vel $\frac{2}{3}$ folii, reſpectu quadruplicis cum $\frac{1}{4}$, vel $\frac{3}{4}$ alterius folii atque ita pariformiter in aliis.

PROPOSITIO IV.

Si ratio a ad b non ſit numeris effabilis, ſed arcus D A, G A ſint incommenſurabiles, innumera folia ſibimet per infinitas circulationes advoluta circumponentur.

Quælibet enim circulatio, præter certum foliorum integrorum numerum, partem folii ſuo toti incommenſurabilem comprehendet, nec unquam ad idem punctum deſcriptio revertetur, adeò ut æquatio ejuſmodi curvæ infinitorum ſit graduum. (*Vid. Fig.* 5.)

PROPOSITIO V.

At ſi ratio a ad b fuerit dupla, prodibit Rhodonea unifolia.

Nam

Nam ex *Prop.* 4. multitudo foliorum eſt ad unitatem ut 2 *b* ad *a*; ſed in hoc caſu *a* eſt 2, & *b* eſt 1, quare multitudo foliorum eſt ad unitatem ut 2 ad 2, ſive ut 1 ad 1; adeoque numerus foliorum eſt unitas. Et ſane arcus E A, qui ſit ad quadrantem A F ut *a* ad *b*, nempe in ratione dupla, eſt ſemiperipheria, adeoque ſemicirculus eſt ſector A F E circumſcriptus ſemifolio, cujus axis E C ex *Prop. prima*, ideoque integro folio circulus integer circumſcribitur.

COROLLARIA.

I. Facilis eſt hujuſmodi Rhodoneæ unifoliæ deſcriptio, ſi ſuper radio E C deſcribatur ſemicirculus, & ducta chorda E S D, in radio C D ponatur C I æqualis intervallo C S; nam cum C S ſit ſinus anguli C E S ad radium C E computatus, ejuſque anguli duplus ſit A C D, erit ramus C I ad Rhodoneam rationis duplæ, juxta geneſim præmiſſam.

II. Unde etiam, ſi centro C, quolibet intervallo C S, in dicto ſemicirculo arcus P S deſcribatur, & tantundem extendatur in I, ut ſint arcus P S, S I æquales, erit punctum I ad Rhodoneam; quippe C S perpendicularis chordæ E D bifariam ſecat in præcedenti deſcriptione angulum E C D; cumque ſit C M æqualis C S, punctum I eſt in arcu circulari, centro C per I, & S tranſeunte, qui continuatus in P remanet bifariam ſectus in S.

III. Et hinc patet, hanc Rhodoneam duplam eſſe circuli ſuper diametro E C deſcripti, ob quoſlibet arcus I S P duplos ipſorum S P, indeque dimidiam circumſcripti circuli, cujus diameter E A; id, quod conſonat infra generaliter demonſtrandis *Prop. octava.*

PROPOSITIO VI.

Ubi ratio *a* ad *b* eſt æqualitatis, efficitur Rhodonea bifolia, quæ nihil aliud eſt, quàm duplex circulus ſubduplæ diametri ad diametrum circuli, qui Rhodoneæ circumſcribitur. (*Vid. Fig.* 9.)

Nam ratio 2 *b* ad *a* erit ratio dupla, ergo ex *Prop. quarta* multitudo foliorum dupla erit unitatis: & ſane deſcripto circa radium F C, velut diametrum, ſemicirculo, quoniam ramus Rhodoneæ C I debet in hoc caſu æquari ſinui ipſiuſmet arcus A D, utique punctum I ad peripheriam dicti ſemircirculi pertinget, adeoque duplex circulus, circa radios F C, C V, velut diametros, deſcriptus, erit locus talium ramorum, id eſt, Rhodoneam ipſam bifoliam conſtituet.

COROLLARIUM.

Etiam hic conſtat Rhodoneam bifoliam dimidiam eſſe circuli circumſcripti, atque adeo æqualem unifoliæ Rhodoneæ præcedentis propoſitionis.

PROPOSITIO VII.

Quodlibet folium Rhodoneæ eſt ad quadrantem circularem ut *a* ad *b*.

Ductis enim radiis infinitè proximis C I D, C *i d*, & ductis ſinubus G H, *g h* correſpondentibus, nempe æquantibus (*Vid. Fig.* 10, & 11.) ramos interceptos C I, C *i*, deſcriptoque concentrico arcu I R, patet fore elementum C I *i* ſemifolii Rhodoneæ ad elementum G H *h g* quadrantis, ut $\frac{1}{2}$ arcus I R ad H *h*, eo quod baſes C *i*, *g h* trianguli elementaris C *i* I, & rectanguli elementaris *g h* H G æquentur; ergo duplum ipſius C I *i* ad G H

h g

hg eſt ut integra R I ad H h, nempe in ratione compoſita ex R I ad D d, & D d ad G g, & G g ad H h; ſed quia G g ad H h (ex theoria infinitè parvorum) eſt ut radius C g ad ſinum $g\,h$, nempe ut C D ad C I, vel D d ad R I, ratio G g ad H h elidit æqualem ſibi reciprocam R I ad D d; quare ſupereſt, ut ratio R I ad H h eadem ſit, quæ D d ad G g; ſed hæc eadem eſt quæ a ad b, cum in tali ratione ſint, tam A D ad A G, quam A d ad A g, adeoque & reſidua eandem rationem ſervent; ergo R I ad H h, ſive duplum elementare ſpatium C I i ad elementum quadrantis G H $h\,g$, eſt in dicta ratione a ad b, & hoc ſemper; igitur duplum ſemifolii C I E, nempe integrum folium Rhodoneæ, eſt ad quadrantem, ut a ad b; Quod erat, *&c.*

COROLLARIA.

I. Hinc ſemifolium C I E ad quadrantem eſt ut $\frac{1}{2}\,a$ ad b, (ſive ut a ad $2\,b$.

Item ſegmentum Rhodoneæ C I i ad ſemiſegmentum circuli A g h eſt in eadem ratione a ad $2\,b$.

PROPOSITIO VIII.

Quodlibet folium Rhodoneæ medietas eſt ſectoris circularis ſibi circumſcripti, & integra Rhodonea ſimplex medietas circuli, duplex duorum, triplex trium circulorum, &c.

Nam ex *præcedente* quodlibet folium eſt ad quadrantem ut a ad b, ideoque ad ſemicirculum ut a ad $2\,b$, ſed ex *Coroll.* 4. *Prop.* 2. ſemicirculus ad ſectorem folio circumſcriptum eſt ut b ad a; ergo ex æquo perturbatè quodlibet folium eſt ad circumſcriptum ſectorem, ut b ad $2\,b$, ſcilicet in ratione ſubdupla; quare & omnia

folia

folia Rhodoneæ ad omnes circumſcriptos ſectores, id eſt Rhodonea ſimplex ad circulum, duplex ad duos circulos, triplex ad tres, &c. in eadem ſubdupla ratione erit.

Aliter. Numerus foliorum ex *Prop.* 3, eſt ad unitatem, ideoque Rhodonea ipſa ad unum folium (ſi eſt ſimplex) ut 2 b ad a; ſed folium eſt ad quadrantem circuli, *ex præc.* ut a ad b, ergo Rhodonea ſimplex eſt ad quadrantem circuli ut 2 b ad b, ſcilicet in ratione dupla; quare ſimplex Rhodonea æquatur ſemicirculo. Similis diſcurſus Rhodoneis duplicibus, & triplicibus applicari poteſt; nam in illis numerus foliorum eſt ad unitatem ut 4 b ad a, in his verò ut 6 b ad a, &c.

COROLLARIA.

I. Quælibet Rhodonea ſimplex cuilibet ſimplici Rhodoneæ eidem circulo inſcriptæ æqualis eſt, quocunque foliorum numero conſtet, ſemper enim æqualis eſt ſpatio ejuſdem ſemicirculi.

II. Item quælibet Rhodonea duplex cuilibet duplici, & quælibet triplex cuivis triplici æqualis eſt, ob eandem rationem; quippe illa ſpecies eſt ſemper circulo æqualis, hæc ſeſquicirculo; & ſic de aliis. Oportet autem in duplici, aut triplici Rhodonea computare ſpatia foliorum, quæ ſibi ſuperponuntur, tanquam diſtincta eſſent.

PROPOSITIO IX.

Bifariam ſecto angulo E C A, quem axis folii Rhodoneæ cum tangente C A continet, per rectam C D, & ramo C I deſcripto arcu circulari I S T, erit lunula T E I quadrabilis, nempe ad quadratum radii, ut a ad 4 b. (*Vid. Fig.* 12.)

Cùm ſit enim quadrans F A ad A E, ut A G ad A D, qui eſt ipſius A E ſemiſſis, erit A G medietas quadrantis, ergo quadratum radii C G, vel C D, duplum eſt quadrati ſinus G H, ſive rami C I; ideoque ſector S C I ad ſectorem E C D ſimilem, ut 1 ad 2; ſector vero E C D ad F C G eſt ut *a* ad *b*; hæc enim eſt ratio arcuum E D, G F, ut eadem eſt integrorum E A, F A, & ablatorum A D, A G; ergo ex æquo ſector S C I ad ſectorem F C G erit ut *a* ad 2 *b*, nempe ut ſemifolium C I E ad quadrantem F G A C, vel ut ſegmentum Rhodoneæ C I ad ſegmentum A G H, vel ut reſiduum C E I C ad reſiduum F G H C, quare etiam reliquum ſemifolii S E I eſt ad reliquum triangulum C H G, aut tota lunula ad quadratum C H G P, in eadem ratione *a* ad 2 *b*, & ad quadratum radii C G, quod prædicti quadrati eſt duplum, erit ut *a* ad 4 *b*. Quod erat, &c.

COROLLARIA.

I. Cùm numerus foliorum Rhodoneæ ſimplicis ſit ad unitatem, adeoque etiam ſumma omnium lunularum, quas integra peripheria radio C T deſcripta abſcindit, ad unam lunulam T E I, ut 2 *b* ad *a*; ipſa vero lunula ad quadratum radii ut *a* ad 4 *b*, patet eſſe ſummam dictarum lunularum ad quadratum radii ut 2 *b* ad 4 *b*, nempe ſubduplam; hoc eſt ſummam talium lunularum æquare quadratum ipſum G H C P quadranti inſcriptum.

II. Unde ſumma lunularum, ex una Rhodonea per dictam peripheriam abſciſſarum, æquatur ſummæ lunularum ex qualibet alia Rhodonea, quotcunque foliorum fuerit, eidem circulo inſcripta ſimiliter determinatarum.

III. Cum

III. Cùm ejusdem sectoris E C A medietas sit tam semifolium E I C, quàm sector E C D, vel E D A, nec non sector C S V, fiunt segmentum C I æquale trilineo E I D, & semilunula E S I trilineo C I V æqualis, quod propterea erit pariter quadrabile, utpote ad triangulum C G H in data ratione *a* ad 2 *b*.

IV. Et summa horum trilineorum in qualibet Rhodonea pariter ejusdem erit quantitatis, utpote summæ lunularum ejusdem, vel cujuscunque alterius Rhodoneæ simplicis eidem circulo inscriptæ semper æqualis.

V. Adeoque si illa triangularia foliorum Rhodoneæ interstitia pro foliis computentur, flos inde totidem foliorum perfectè quadrabilis exurget, ut in *Fig.* 13.

PROPOSITIO X.

Ad quodlibet Rhodoneæ punctum I tangentem ducere.

Factum jam sit; ductaque ramo I C (*Fig.* 14, 15.) perpendicularis C M, conveniat cum tangente I M in M; & radio C I arcus I R infinitè parvus describatur usque ad alium ramum C *i* infinitè proximum, sintque ramis C I, C *i* æquales sinus G H, *g h*, & circuli tangens G L occurrat diametro in L. Erit ergo I C ad C M ut *i* R ad R I, nempe in ratione composita ex *i* R, seu *g* O, ad O G (hoc est *g h*, vel *i* C, ad *h* L) & O G, sive H *h*, ad R I (quæ ex *Prop.* 7. est eadem rationi *b* ad *a*) quare *i* C ad C M erit in ratione composita ex *i* C ad *h* L, & ex *b* ad *a*; sed eadem ratio *i* C ad C M componitur quoque ex *i* C ad *h* L, & *h* L ad C M; ergo oportet rationem *h* L, sive H L, ad C M esse datam, scilicet eam, quæ *b* ad *a*, ideoque si fiat, ut *b* ad *a*, ita subtangens circuli H L ad

CM ramo CI perpendicularem, juncta MI erit tangens Rhodoneæ in puncto I; Quod erat faciendum.

COROLLARIA.

I. Si fiat ut *a* ad *b*, ita CH ad CN ramo perpendicularem, juncta NI erit curvæ Rhodoneæ normalis; nam quia HL ad CM est ut *b* ad *a*, & CH ad CN ut *a* ad *b*, erit HL ad CM ut reciprocè CN ad CH; & ideo rectangulum MCN æquabitur rectangulo LHC, id est, quadrato GH, vel quadrato rami CI; ergo juncta NI est tangenti MI, seu curvæ Rhodoneæ in puncto I, perpendicularis.

II. Patet, tangentes angulorum CIM, & LGH, vel GCA semper esse in data ratione *a* ad *b*.

PROPOSITIO XI.

Si fiat ut *b* ad *a*, ita radius AC ad CQ, & semiaxibus FC, CQ describatur quadrans ellipsis FVQ, erit ejus perimetur æqualis perimetro semifolii Rhodoneæ ECI, & partes partibus correspondentibus. (*Vid. Fig.* 16, *&* 17.)

Erit enim ubique etiam GP ad VP, vel *gp* ad *up* in eadem ratione, quæ est AC ad CQ, id est, *b* ad *a*; quare & residua GO, VX in eadem ratione erunt. Quod si infinitè proximæ sint PG, *pg*, GH, *gh*, & correspondentes CI, C*i* cum arcu infinitè parvo IR, quoniam IR ad H*h*, vel GO ex *Prop.* 7. est ut *a* ad *b*, in qua etiam ratione erit VX ad eandem GO, patet ipsas IR, VX æquales fore; cum ergo & sint æquales R*i*, VX (ob æqualitatem quarumvis CI, GH, vel TV, nec non C*i*, *gh*, *tu*) patet subtensas quoque I*i*, V*u* æquales futuras. Singula igitur elementa, tum cur-

væ

væ Rhodoneæ E I C, tum ellipticæ F V Q invicem æquantur; quare & perimeter ſemifolii Rhodoneæ erit quadranti curvæ ellipticæ æqualis, & duo quælibet folia perimetrum habebunt integræ curvæ ellipſeos æqualem; Quod erat, &c.

COROLLARIA.

I. Patet, Rhodoneam eſſe ellipſim quandam contractam; nam ſi confluentibus in centrum C punctis T, *t*, ordinatæ elliptici quadrantis V T, *u t*, in ramos abeant a centro C diductos, quadrans ellipſis in ſemifolium Rhodoneæ contrahetur, eadem curvæ longitudine manente.

II. Hinc iterum patet, Rhodoneam eſſe medietatem ſectoris circularis circumſcripti; eſt enim ſemifolium E I C medietas quadrantis elliptici F V Q C, in quem expanderetur, ſi rami ab eorum centro diſſoluti fierent paralleli, & rectæ C Q perpendiculares; cumque quadrans ellipſis ſit ad quadrantem circularem, ut baſis Q C ad baſim C A, nempe ut *a* ad *b*, in qua etiam ratione eſt ſector E C A ad eundem quadrantem, ex *Prop. prima*, patet, ejuſmodi ſectorem æquari quadranti elliptico, ideoque duplum eſſe inſcripti folii Rhodoneæ.

III. Inſuper colligitur, æquales eſſe foliorum perimetros in Rhodoneis, quarum ratio ſit reciproca, & radii ſuorum circulorum in eadem reciproca ratione ſibi reſpondeant; nam ſi radius C F, vel E C *Figuræ* 17. æquaretur baſi ellipſis C Q *Figuræ* 16, & viciſſim radius C F iſtius æquaret baſim C Q ellipſis alterius *Figuræ*, patet, eandem ellipſim F V utrobique reſultare debere, quippe iiſdem ſemiaxibus deſcriptam, eamque fore utrivis folio iſoperimetram, exiſtente ibi ratione *a* ad *b*, hic

hic reciprocè *b* ad *a*. Exempli causa, si ratio *a* ad *b* sit subdupla, ut juxta *Prop.* 3. hinc proveniat Rhodonea tetrafolia, radio autem subduplo (adeoque æquali basi quadrantis ellipsis isoperimetræ) vicissim fiat Rhodonea juxta rationem duplam, quæ ex *Prop.* 5. unifolia evadet, erit hæc isoperimetra uni folio illius; nam basis quadrantis elliptici huic respondens basim habebit illius radio æqualem, adeoque eadem curva elliptica utrivis folio æqualis ostenditur.

IV. Si verò in eodem circulo duæ Rhodoneæ describantur, altera juxta rationem *a* ad *b*, altera juxta reciprocam *b* ad *a*, perimetros suarum foliorum habebunt ipsis rationum antecedentibus *a*, & *b* proportionales; nam si primæ Rhodoneæ tertia quædam Rhodonea similis describeretur in circulo, ad cujus radium prioris radius esset ut *a* ad *b*, esset perimetur primæ ad perimetrum tertiæ sibi similis in ipsa ratione radiorum *a* ad *b*. Verùm perimeter hujus tertiæ, ex *Coroll. præced.* æquaretur perimetro secundæ, utpotè reciproca ratione, & juxta reciprocos radios descriptæ, ergo perimeter primæ ad perimetrum secundæ est in eadem ratione *a* ad *b*.

PROPOSITIO XII.

Rhodoneam datæ rationis *a* ad *b* minoris inæqualitatis ex conica superficie secare.

Fiat ut *a* ad *b*, ita radius basis NB ad latus NC coni recti NCK, cujus basis diametro NK sit perpendicularis radius BF, (*Vid. Fig.* 18.) qui sit ad BR ut *b* ad *a*, & circa diametros BR, BF describantur semicirculi BLR, BSF, quos secet quilibet radius BG in punctis L, S, sitque GH diametro NK perpendicularis. Si super circulo BLR erecta superficies cylindrica intelligatur secare conicam in communi sectione

C I E, erit hæc (in planum explicata) ipſamet Rhodonea propoſitæ rationis. Nam communes ſectiones cylindricæ illius ſuperficiei cum planis triangulorum C B G, C B F per axem coni C B tranſeuntium, erunt rectæ L I, R E ipſi axi parallelæ, ideoque tam C I, ad B L, quàm C E ad B R erunt ut latus coni ad radium baſis, ſcilicet ut *b* ad *a* *ex conſtructione*, ſive ut F B ad B R, ſive S B ad B L; adeoque C E æquatur B F, & C I æquatur B S, ſive ſinui G H. Explicata autem ſuperficie conica in planum ſectorem circularem ipſi æqualem, radio C N deſcriptum, ejus angulus planus N C G ſubtendetur eodem arcu N G, ſubtendente in baſi coni angulum N B G; adeoque ut B N ad N C, ſive ut *a* ad *b*, ita erit angulus N C G ad ipſum N B G, cujus ſinui G H, ut vidimus, æquatur ramus C I folii C I E, cujus maximus ramus C E æquat radium B F circuli baſis; quare folium ipſum ad Rhodoneam pertinet in data ratione *a* ad *b* deſcriptam; Quod erat, &c.

COROLLARIA.

I. Cùm ſit etiam C E ad E O, ut C F ad F B, ut *b* ad *a*, ut F B ad B R ſintque C E, F B æquales, itidem æquales erunt B R, E O, & ſemicirculus B L R quarta pars erit ſemicirculi A E P duplum diametrum habentis, ſive erit medietas quadrantis A E O; eſt verò (ex noſtra Appendice de Fornicibus conicis, quam *Vivianeis* ſubjunximus jam inde ab anno 1698) ſuperficies conica A D E C ad ſuam baſim A D E O, ut ſuperficies ſemifolii C I E ad ſuam ichnographiam B L R, nempe in eadem ratione lateris coni ad radium baſis; ergo cum A D E O dupla ſit B L R, & ſuperficies A D E C ipſius ſemifolii C I E dupla erit, ut aliunde ſupra demonſtravimus ſectorem folio circumſcriptum illius duplum eſſe.

II. Cùm

II. Cùm oſtenſum ſit eſſe angulum A C I ad N B G, uti & A C E ad N B F, in data ratione *a* ad *b*, patet etiam in eadem ratione eſſe angulum reliquum I C E ad reliquum S B F, exiſtente (ut probavimus) ramo C I æquali ipſi B S; (*Vid. Fig.* 19.) unde ſi ſemicirculi C S E, in arcus concentricos, centro C deſcriptos, reſoluti, arcus quilibet P S, *p s* dividantur ad puncta I, *i*, ut ſit ſemper P I ad P S, *p i* ad *p s* in data illa ratione *a* ad *b*, erunt puncta I, *i* ſic inventa ad curvam Rhodoneam.

III. Imo etſi ratio *a* ad *b* majoris ſit inæqualitatis, adhuc Rhodoneas ope ſemicirculi deſcribere licebit generalius quam in *Coroll.* 2. *Prop.* 5. ſi arcus P S, *p s* producantur ad puncta I, *i*, ut ſint P I ad P S, *p i* ad *p s* in data ratione *a* ad *b*. Facto enim arcu E A R ad quadrantem E A in eadem ratione, ductoque radio C R, fiet angulus R C E ad A C E, ut angulus I C E ad angulum S C E. adeoque & reliquus *i* C R ad reliquum *s* C A, cujus ſinus æquatur C *s*, ſive C *i*, in eadem ratione erit *a* ad *b*; ideoque puncta I, *i* ſunt ad Rhodoneam datæ rationis.

IV. Et ſi arcus illi P S, *p s* in ſemicirculo deſcripti, tum dividantur in ratione *a* ad *b*, tum augeantur in reciproca ratione *b* ad *a*, curvæ interioris longitudo ad longitudinem exterioris erit ut *a* ad *b*, per *Coroll.* 4. *Prop. præcedentis.*

SCHOLION.

Verùm hæc, pro inſtituto noſtro, circa hujuſmodi curvas delibaſſe ſufficiat: quanquam alia etiam Rhodonearum ſymptomata enucleare in promptu eſſet, uti & alias florum ſpecies diverſâ geneſi efformatas exhibere facilè

 foret,

foret, quorum etiam folia (ut poſtremâ propoſitione folia Rhodonearum circa conicam ſuperficiem advoluta dedimus) circa aliquam conoidalem ſuperficiem convoluta deſcribere poſſemus, & quandam foliorum in calice floris latentium imaginem adumbrare, niſi jam tædio Lectorum parcendum eſſet. Unum hoc admonere non prætermittam, quod ex ultimò propoſita generali foliorum Rhodoneæ deſcriptione ſimpliciſſima ex circulo derivata, ſuſpicari quis non immeritò poſſet etiam prima naturalium foliorum ſtamina, quæ in floris, aut fruticis ſemine latent, non neceſſariò ſimilia eſſe foliis ipſis conſpicuis, & jam germinantibus, ſive adultis; ſicut enim ſi florum, & fruticum folia noſtras Rhodoneas reipſa imitarentur, poſſet quis concipere, illorum prima ſtamina ſeminibus cujuſlibet ſpeciei incluſa ſimpliciſſima circulari figurâ infinitè parvâ circumſcribi, ſed mox peculiari vi cujuſlibet ſingularis ſpeciei, dum germinant, ita determinari ſuccum nutritium, ut dum in longum eorum axis extenditur, per quaſdam undas, ſive gyros, ipſi origini ſui pedunculi, velut centro, circumpoſitos, expandatur, eoſque ſemper in determinata ratione, vel arctiores, vel ampliores, quàm ſi circularis primorum ſtaminum figura retinenda eſſet: quo poſito talis ſpecies foliorum Rhodoneæ, ac talis numerus, & forma exurgeret, qualem ratio illa determinaret. Ita etiamſi alia lege florum, & fruticum frondes natura moliatur, non neceſſe eſt earum figuram, uſque ad ipſa prima earundem ſtamina, ex quibus germinant, obſervari; ſed illa in quibuſlibet unius certæ, ac determinatæ figuræ eſſe poſſet, quæ tantum pro diverſa vi, determinante in ipſis expanſionem ſucci nutritii, in ſingulis ſpeciebus varianda foret, juxta diverſam rationem, quæ per ipſorum ſtaminum fibras dirigeretur. Sed ne extra chorum ſaltemus, hæc Philoſophis innuiſſe ſufficiat.

Fig: 1

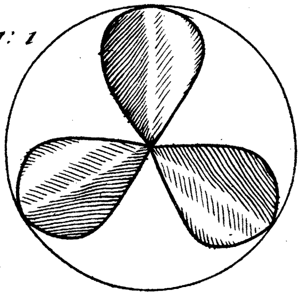


Fig: 2

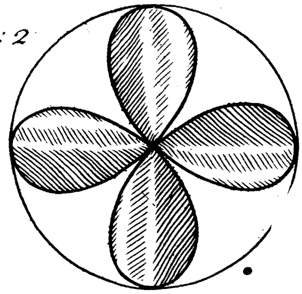


Fig: 3

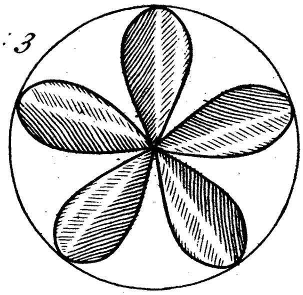


Fig: 4

Fig: 6

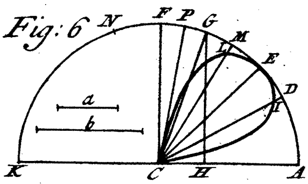


Fig: 7

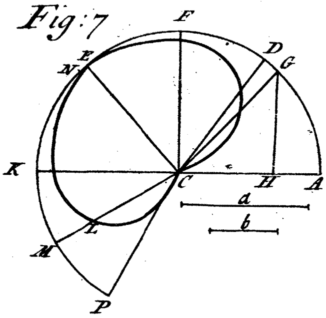


Fig: 8

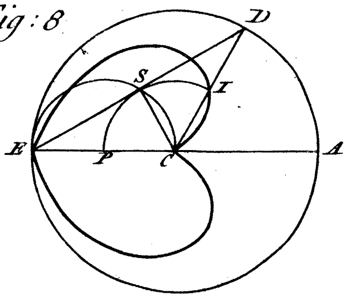


Fig: 9

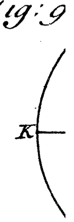


Fig: 11

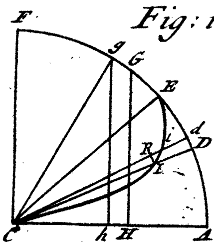


Fig: 12

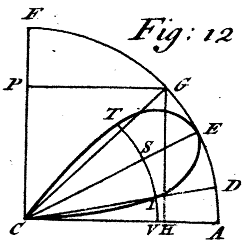


Fig: 13

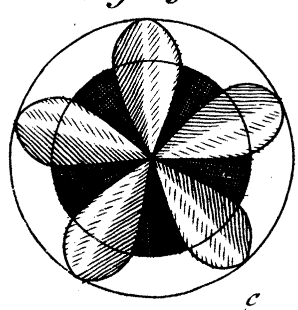


Fig:

Fig: 16

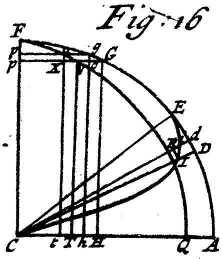


Fig: 17

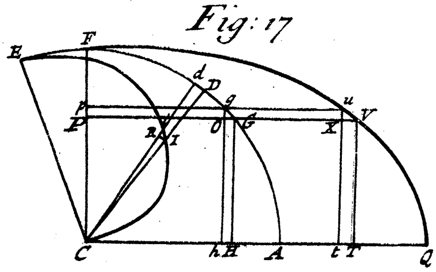
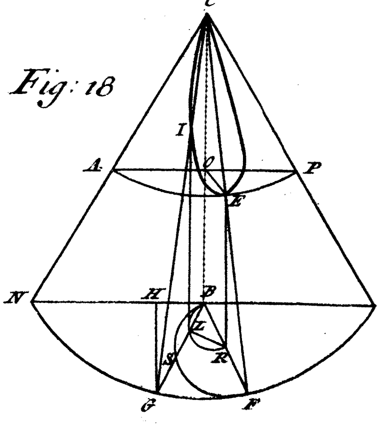


Fig: 18



Philosoph: Trans: No. 378.

ig: 4

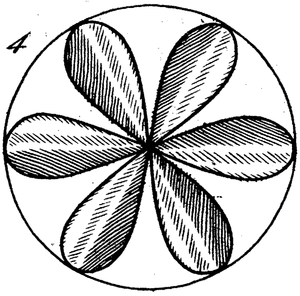


Fig: 5

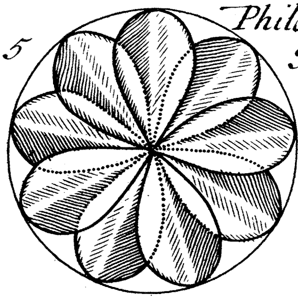


Fig: 9

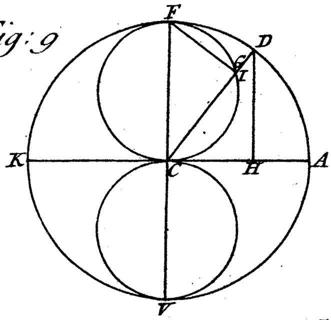


Fig: 10

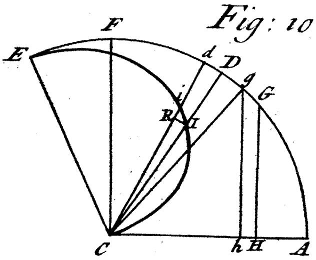


Fig: 14

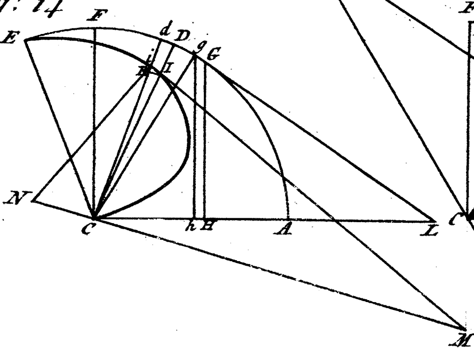
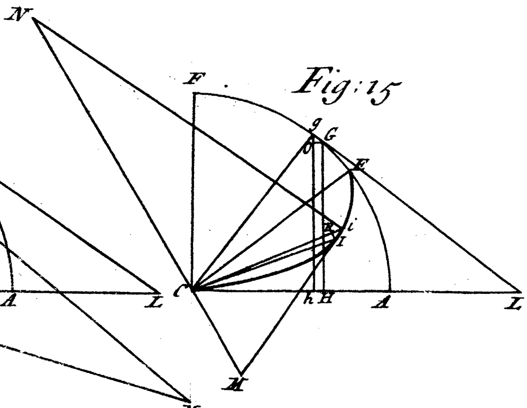


Fig: 15



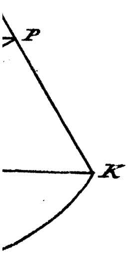


Fig: 19

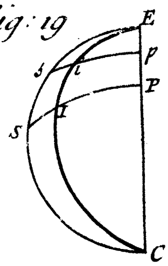


Fig: 20

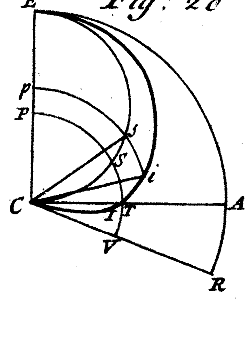


Fig: 1

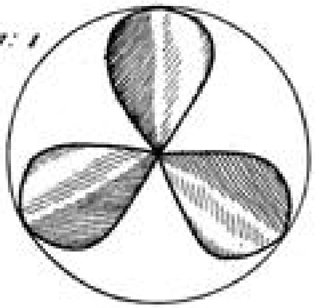


Fig: 2

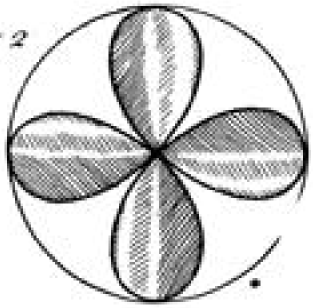


Fig: 3

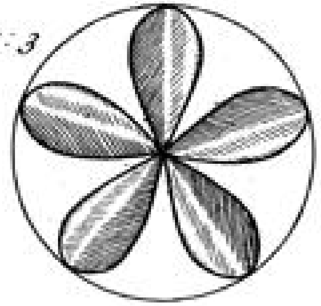


Fig: 4

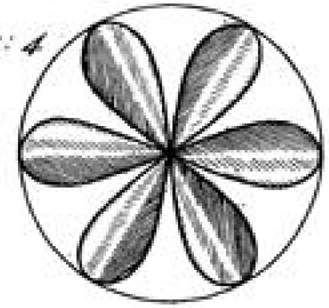


Fig: 5

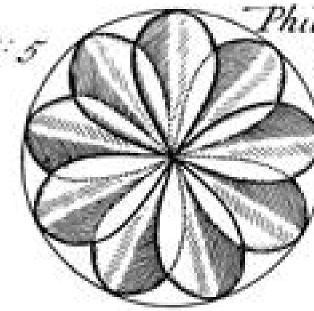


Fig: 6

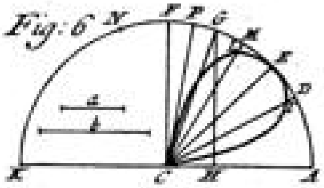


Fig: 7

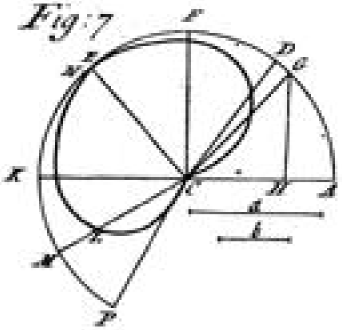


Fig: 8

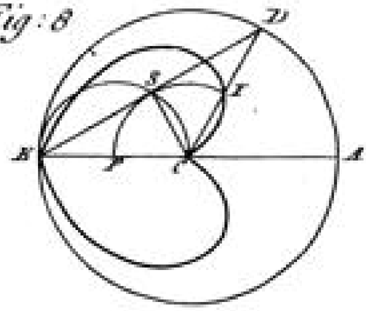


Fig: 9

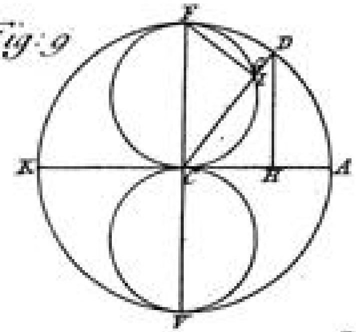


Fig: 10

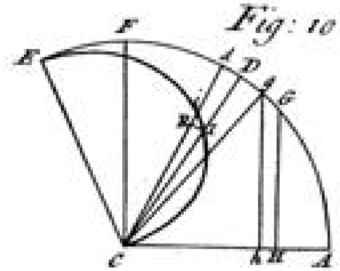


Fig: 11

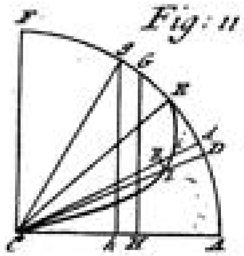


Fig: 12

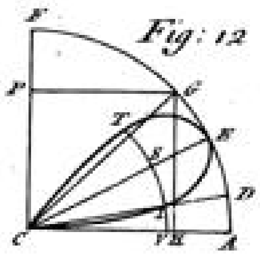


Fig: 13

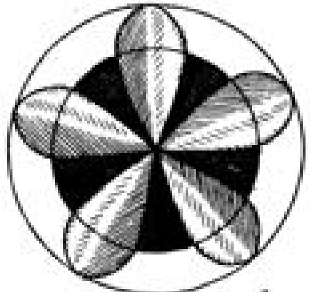


Fig: 14

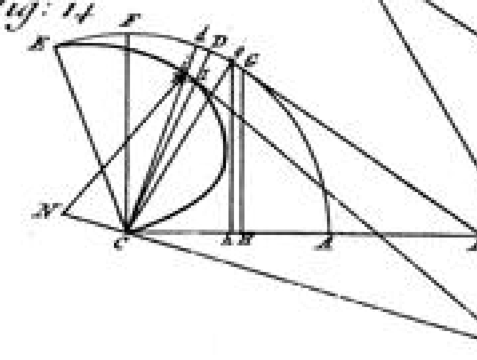


Fig: 15

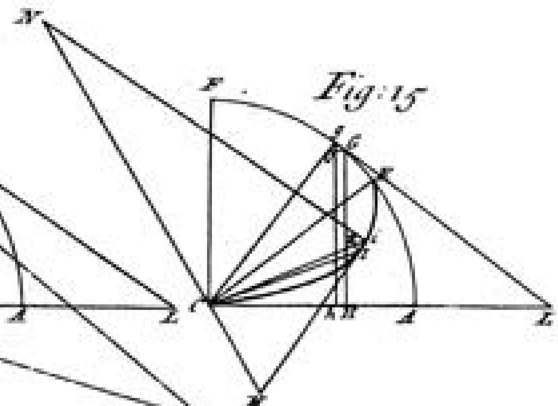


Fig: 16

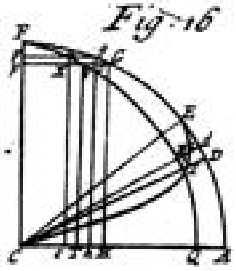


Fig: 17

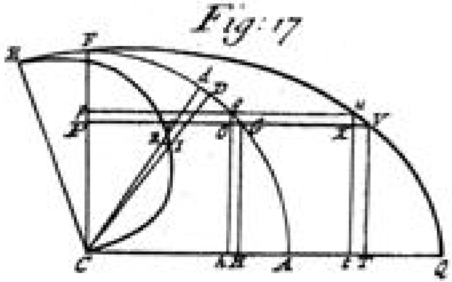


Fig: 18

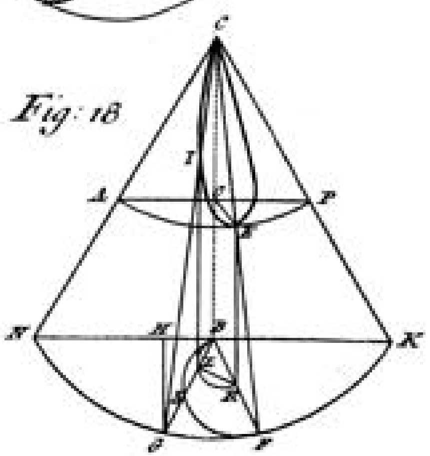


Fig: 19

Fig: 20