Unitary Principle and Mechanical Non-solution Proving of Neomorphic Dirac Equation

Rui Chen

*Shenzhen Institute of Mathematics and Physics,Shenzhen, 518028,China**

It further introduces the method of using the unitary principle to test the logic of theoretical physics. As one of the important examples, the non-existence of solution of the system of recurrence relations from a neomorphic Dirac equation is proven in this paper. Also, we use the method of mechanical proving to show the reliability of this pivotal result. It clearly denotes that the corresponding neomorph of the Dirac equation has no real solution, and there is not any formal solution of the neomorphic Dirac equations given in the various related literatures to satisfy the original Dirac equation. Furthermore, it shows that the formal solutions of the neomorphic Dirac equations conceal a basic mathematical contradiction that one equals zero. Consequently, constructing any new form of the Dirac equation requires great consideration.

PACS numbers: 03.65.Ge, 03.65.Pm

I. INTRODUCTION

In theoretical physics, there is often some mathematical reasoning of incomprehension and the relative results expressed by the mathematical formulas are indistinguishable in experimental observation because they have the same magnitude. It needs for an effective logical criterion to make conclusions accurately and concisely on such issues. For 25 years, by researching some typical problems such as the real solutions and formal solutions of the wave equation with the condition for determining solution and so on, we find that the unitary principle[1] is just one of such logical criterions. The unitary principle is a general principle that can widely disclose the logic contradictions hidden in natural science and mathematical perjuries. The reliability of the principle consists in expatiating on a simple fact. **Describing the law of nature can choose different metrologies, there are definite transforms among different metrologies, but the law of nature does not change per se because of choosing different metrologies. As the different mathematical forms in the different metrologies for describing the**

*Electronic address: rdchen@australia.edu

same law of nature are transformed into the same metrologies, it must be the same as the form in the present metrologies, 1=1, the transformation is unitary.

Applying the unitary principle to quantum mechanics, we focus on the existence and uniqueness of solution of the various forms of the Dirac equations. It is well known that the relativistic Dirac equation[2–5] has been regarded as the accurate wave equation for describing the law of motion of microcosmic particles. Its wide application produced many important results, while also bringing some specious solutions and conclusions. In mathematical method, it usually transforms the Dirac equation into a system of the first order differential equations to solve. However, in order to obtain the exact solution in the different ways, many papers introduced some function transformations to translate the original Dirac equation into the neomorphic Dirac equation for new components of wave function. We found that the new form of the solution for the new components could not yield the standard solution. According to the unitary principle, introducing a correct function transformation to a differential equation, it does not change the intrinsic solution for the original function. Otherwise, the uniqueness of solution of differential equation would be destroyed. Consequently, the new form for the solution of the neomorphic Dirac equation should be an unreal solution of the Dirac equations. Here we use the method of mechanical proving to show the non-existence of solutions of the neomorphic Dirac equations which are quoted widely in various scientific literatures. The problem is rooted in the quantum mechanics, but the argumentation is purely mathematical.

In treating the wave equation for the quantum bound state, we often pay higher attention to obtaining the anticipant formula of quantum energy. Solving the Schrödinger equation or the Klein-Gordon equation for the bound state, the boundary condition requires that the power series in the formal solution must terminate with the discretionary finite term. It deduces the corresponding first order recurrence relations[6] for determining the coefficients of the series and gives the eigenvalue set as well as the formula for the quantum energy. Solving the Dirac equation for at least two components of the wave function, finding its solution should translate the original equation into a system of differential equations for multi-components of the wave function. According to the boundary condition, two power series in the formal solution also must terminate with the discretionary finite terms. This means that two systems of recurrence relations for determining the coefficients of the series must terminate with the discretionary finite terms. As the solution of the Dirac equation, the Dirac wave function is hence obtained. Usually, it should show the existence and uniqueness of eigen-solution set for the wave equation. By the theorems of the optimum differential equations[7], one can prove the existence and uniqueness of solution of the Schrödinger equation and Klein-Gordon equation. However, most of the so-called second-order Dirac equation and new first-order Dirac equation pay attention to find the Dirac formula for the energy levels. The given solutions actually violate the existence and uniqueness of solution for the Dirac equation. These second-order Dirac equations and first-order Dirac-like equations have the substantial difference from the original Dirac equation, being called the neomorphic Dirac equation.

The computer can help us to validate the invalidation for the various formal solutions of the neomorphic Dirac equations, because it is reliable enough for accurately solving algebraic equations by the calculating machine. Solving the neomorphic Dirac equation is uniformly translated to solve the corresponding system of recurrence relation and the latter is finally translated to solve the algebraic equation with the finite elements. By the Wolfram Mathematica, the system of recurrence relation from the neomorphic Dirac equation has no solution. If the calculating machine's negation to the neomorphic Dirac equations is still misdoubted in theoretical physics, one should find the reason why the calculating machine proves the neomorphic Dirac equations having not any solution. This will not only give the last word to the issue but also develop the mechanical method for checking the logic of theoretical physics.

II. NEOMORPHIC DIRAC EQUATION AND NEOMORPHIC DIRAC RECURRENCE RELATIONS

It is generally believed that the Dirac equation succeeds in describing the fine structure of the hydrogen and hydrogen-like atom[8–10]. In fact, solving the original Dirac equation is very easy. We cannot understand why the so-called second-order forms of the Dirac equation[11–18] and the imitated first-order Dirac equations are introduced more and more in many literatures. Are they all necessary or true? Now, we discuss a neomorphic Dirac system of recurrence relations from the formal solution to a neomorphic Dirac system of first-order differential equations obtained by introducing a function transformation to the original Dirac equation. In a modern quantum mechanics book[19], considering the Coulomb interaction energy of a point nucleus and a particle of charge $-e$ is $V = -Ze^2/r$, it wrotes the simplified radial equations for a Dirac particle as the following form

$$\begin{aligned}
\frac{dG}{dr} &= -\frac{\kappa}{r}G + \left(\frac{E+m_0c^2}{\hbar c} + \frac{Z\alpha}{r}\right)F \\
\frac{dF}{dr} &= \frac{\kappa}{r}F - \left(\frac{E-m_0c^2}{\hbar c} + \frac{Z\alpha}{r}\right)G
\end{aligned} \tag{1}$$

where $\kappa = \pm 1, \pm 2, \cdots$, $\alpha = e^2/\hbar c \approx 1/137$ is the fine-structure constant, m_0 is the rest mass of the electron, c is the velocity of light in the vacuum, $\hbar = h/2\pi$ and h is the Plank constant, E is the energy eigenvalue parameter. In quantum mechanics, the standard solution to the Dirac differential equations (1) involves the formula for the energy level and the Dirac functions

$$
\begin{aligned}
E &= \frac{mc^2}{\sqrt{1+\frac{\alpha^2}{\left(n_r+\sqrt{\kappa^2-Z^2\alpha^2}\right)^2}}} \\
F &= \exp\left(-\frac{\sqrt{m_0^2c^4-E^2}}{\hbar c}r\sum_{\nu=0}^{n} b_\nu r^{\sqrt{\kappa^2-Z^2\alpha^2}+\nu}\right) \\
G &= \exp\left(-\frac{\sqrt{m_0^2c^4-E^2}}{\hbar c}r\sum_{\nu=0}^{n} d_\nu r^{\sqrt{\kappa^2-Z^2\alpha^2}+\nu}\right)
\end{aligned}
\tag{2}
$$

where $n = 0, 1, 2, \cdots$, and the coefficient b_ν and d_ν satisfy the coupled recurrence relations

$$
\begin{aligned}
&\left(\sqrt{\kappa^2-Z^2\alpha^2}+\nu-\kappa\right)b_\nu + Z\alpha d_\nu - \frac{\sqrt{m_0^2c^4-E^2}}{\hbar c}b_{\nu-1} - \frac{m_0c^2-E}{\hbar c}d_{\nu-1} = 0 \\
&Z\alpha b_\nu - \left(\sqrt{\kappa^2-Z^2\alpha^2}+\nu+\kappa\right)d_\nu - \frac{m_0c^2+E}{\hbar c}b_{\nu-1} + \frac{\sqrt{m_0^2c^4-E^2}}{\hbar c}d_{\nu-1} = 0
\end{aligned}
\tag{3}
$$

Some literatures search the different method to treat the Dirac equation (1). It mainly introduces the function transformation to transform the Dirac equation for the original components of the wave function F and G into another form for the new components of the wave function. In fact, it is identical for introducing various function transformations to construct the different neomorphic Dirac equations. Those different function transformations can be written in the same general form. For example, the mentioned book[19] writes down the following transformation to the original Dirac equation

$$
\begin{aligned}
\rho &= \frac{2\sqrt{m_0^2c^4-E^2}}{\hbar c}r \\
G(\rho) &= \sqrt{m_0c^2+E}e^{-\rho/2}\left[\phi_1(\rho)+\phi_2(\rho)\right] \\
F(\rho) &= \sqrt{m_0c^2-E}e^{-\rho/2}\left[\phi_1(\rho)-\phi_2(\rho)\right]
\end{aligned}
\tag{4}
$$

substituting this into the equation (1) will make it

$$
\begin{aligned}
&\frac{d\phi_1}{d\rho} - \left(1-\frac{Z\alpha E}{\hbar c\lambda\rho}\right)\phi_1 + \left(\frac{\kappa}{\rho}+\frac{Z\alpha m_0c^2}{\hbar c\lambda\rho}\right)\phi_2 = 0 \\
&\frac{d\phi_2}{d\rho} + \left(\frac{\kappa}{\rho}-\frac{Z\alpha m_0c^2}{\hbar c\lambda\rho}\right)\phi_1 - \frac{Z\alpha E}{\hbar c\lambda\rho}\phi_2 = 0
\end{aligned}
\tag{5}
$$

where $\lambda = \sqrt{m_0^2c^4-E^2}\Big/\hbar c$. Taking notice of the new first-order differential equation (5) being similar to the original Dirac equation, but it has the substantial difference from the original Dirac equation (1). The equation (5) belongs to the neomorphic Dirac equation. In order to obtain the formula of energy level, the mentioned book makes the ansatz of a power series expansion, which will terminate with the arbitrary term to become two polynomials. It writes down $\varphi_1 = \rho^\gamma\sum\limits_{n=0}^{\infty}\alpha_n\rho^n$, $\varphi_2 = \rho^\gamma\sum\limits_{n=0}^{\infty}\beta_n\rho^n$. Inserting this into equation (5) and comparing the coefficients of variable quantity yields two recurrence relations

$$
\begin{aligned}
&\left(n+\gamma+\frac{Z\alpha E}{\hbar c\lambda}\right)\alpha_n + \left(\kappa+\frac{Z\alpha m_0c^2}{\hbar c\lambda}\right)\beta_n = \alpha_{n-1} \\
&\left(\kappa-\frac{Z\alpha m_0c^2}{\hbar c\lambda}\right)\alpha_n + \left(n+\gamma-\frac{Z\alpha E}{\hbar c\lambda}\right)\beta_n = 0
\end{aligned}
\tag{6}
$$

where $\gamma = \sqrt{\kappa^2 - (Z\alpha)^2}$, $n = 0, 1, 2, \cdots, n$. This is a system of recurrence relations, which is now called the neomorphic Dirac system of recurrence relations! The relative literatures consider that this system of recurrence relations has the eigen-solution just reading the Dirac formula (2). Does it have the eigenvalue really? Here, we use the mechanical proving and mathematical proof respectively to show the nonexistence of the solution of the neomorphic Dirac system of recurrence relations (6). So that the neomorphic Dirac equation (5) does not have any eigen-solution, and the given formal solution in the corresponding literature is only a pseudo solution of the original Dirac equation.

III. MECHANICAL NON-SOLUTION PROVING OF NEOMORPHIC DIRAC EQUATION

For convenience of mechanical proving, we insert $\lambda = \sqrt{m_0^2 c^4 - E^2} \big/ \hbar c$ and $\gamma = \sqrt{\kappa^2 - (Z\alpha)^2}$ into the system of recurrence relations (6) and write down the original form expressed by the parameters α_{n-1}, α_n, β_n and E

$$\begin{aligned} &\left(n + \sqrt{\kappa^2 - Z^2\alpha^2} + \frac{Z\alpha E}{\sqrt{m_0^2 c^4 - E^2}}\right)\alpha_n + \left(\kappa + \frac{Z\alpha m_0 c^2}{\sqrt{m_0^2 c^4 - E^2}}\right)\beta_n = \alpha_{n-1} \\ &\left(\kappa - \frac{Z\alpha m_0 c^2}{\sqrt{m_0^2 c^4 - E^2}}\right)\alpha_n + \left(n + \sqrt{\kappa^2 - Z^2\alpha^2} - \frac{Z\alpha E}{\sqrt{m_0^2 c^4 - E^2}}\right)\beta_n = 0 \end{aligned} \tag{7}$$

Noticing that the system of recurrence relations (6) or (7) is obtained from the neomorphic Dirac equation (5), as being not equivalent to the system of recurrence relations that is obtained from the original Dirac equation (1), the recurrence relation (6) and (7) are called the neomorphic Dirac recurrence relations.

Wolfram Mathematica can accurately solve algebraic equations. We use this program to prove the nonexistence of eigen-solution for the system of recurrence relations (7). By the reduction to absurdity, if the system of recurrence relations (7) has the eigen-solution, it would have the solution for the ground state, which corresponds to the system of recurrence relations (7) terminating with an only term $n = 0$. Let $\alpha_1 = \alpha_2 = \cdots = 0$, $\beta_1 = \beta_2 = \cdots = 0$, substituting into the equations (7) gives a system of linear equations with three unknown numbers, α_0, β_0 and E. In order that the program identifies the signs, we make the replacements, $Z\alpha \to a$, $\kappa \to k$, α_0, $\alpha_0 \to x$, $\beta_0 \to y$,$E_0 \to z$. The format of solving this system of linear equations by Wolfram Mathematica is as follows

$$\begin{aligned} &\text{Solve}[\{\left(\sqrt{k^2 - \alpha^2} + \frac{az}{\sqrt{m^2c^4 - z^2}}\right)x + \left(k + \frac{amc^2}{\sqrt{m^2c^4 - z^2}}\right)y == 0, \\ &\quad \left(k - \frac{amc^2}{\sqrt{m^2c^4 - z^2}}\right)x + \left(\sqrt{k^2 - a^2} - \frac{az}{\sqrt{m^2c^4 - z^2}}\right)y == 0, \\ &\quad x == 0\}, \{x, y, z\}] \end{aligned} \tag{8}$$

or

$$\begin{aligned}
&\text{Solve}[\{\left(k+\frac{amc^2}{\sqrt{m^2c^4-z^2}}\right)y==0,\\
&\left(\sqrt{k^2-a^2}-\frac{az}{\sqrt{m^2c^4-z^2}}\right)y==0,\\
&x==0\},\{x,y,z\}]
\end{aligned}\tag{9}$$

Copy one of them into Mathematica Kernel, the output is"Solve::svars: Equations may not give solutions for all "solve" variables. Out[1]= {{y -> 0, x -> 0}}". It cannot give the solution for z, making it known that the system of recurrence relations (7) namely (6) only has a trivial solution but has no energy eigenvalue for the S state. Consequently, the neomorphic Dirac equation (5) has no useful solution for the S state.

IV. HIDDEN MATHEMATICAL CONTRADICTION IN FORMAL SOLUTIONS

Mechanical non-solution proving to the neomorphic Dirac equations (5) for the ground and the first excitation state made it clear to us that the neomorphic Dirac equations have no solution to any other excitation states yet. The formal solutions of those neomorphic Dirac equations given in the interrelated literatures are not the real solution. A differential equation without solution being endued with the pseudo solution must conceal some fateful mathematical contradictions.

We open up one of the mathematical contradictions hidden in the formal eigenvales of the neomorphic Dirac recurrence relation for the ground sate (6). Let $n=0$, the formal series solution of the neomorphic Dirac equation (5) takes $\varphi_{01}=\alpha_0\rho^{\sqrt{\kappa^2-Z^2\alpha^2}}$ and $\varphi_{02}=\beta_0\rho^{\sqrt{\kappa^2-Z^2\alpha^2}}$, the undetermined parameters α_0, β_0 and E_0 satisfy the following system of algebraic equations

$$\begin{aligned}
&\left(\sqrt{\kappa^2-Z^2\alpha^2}+\frac{Z\alpha E_0}{\sqrt{m_0^2c^4-E_0^2}}\right)\alpha_0+\left(\kappa+\frac{Z\alpha m_0c^2}{\sqrt{m_0^2c^4-E_0^2}}\right)\beta_0=0\\
&\left(\kappa-\frac{Z\alpha m_0c^2}{\sqrt{m_0^2c^4-E_0^2}}\right)\alpha_0+\left(\sqrt{\kappa^2-Z^2\alpha^2}-\frac{Z\alpha E_0}{\sqrt{m_0^2c^4-E_0^2}}\right)\beta_0=0\\
&\alpha_0=0
\end{aligned}\tag{10}$$

These equations can be obtained directly by the recurrence relations (7). Clearly, the third formula $\alpha_0=0$ is just a negation to all formal solutions, because $\alpha_0=0$ must read $\beta_0=0$, indicating that the ground state of the neomorphic Dirac equation does not exist! However, the relevant literatures often evade the intrinsic solution $\alpha_0=0$ and $\beta_0=0$ but discuss the general formal solution, thereby structure a formal mathematical logic to coin the anticipant deduction. For the ground state, however, this formal mathematical logic is equivalent to insert the third formula into the first and second formula in the system of equations (10) respectively, in order to obtain a non-trivial solution, formally, it makes a feint $\beta_0 \neq 0$ and only selects one expression from two necessary expressions

$$\kappa + \frac{Z\alpha m_0 c^2}{\sqrt{m_0^2 c^4 - E_0^2}} = 0, \quad \sqrt{\kappa^2 - Z^2\alpha^2} - \frac{Z\alpha E_0}{\sqrt{m_0^2 c^4 - E_0^2}} = 0 \tag{11}$$

Does the first equation come into existence? No, it does not! In most of the literatures on the neomorphic Dirac equations, the inconsistent formulas are often deleted but only the specious result such as the last formula in expressions (11) is chosen, the formal energy eigenvalues are hence given as follows

$$E_0 = m_0 c^2 \sqrt{1 - \left(\frac{Z\alpha}{\kappa}\right)^2} \tag{12}$$

This formula is as the same as the Dirac formula and has had a lot of acceptance. From the equations (10) to the single equation (12), it seems to be a small, nevertheless, fatal mathematical error actually! By (12), one obtains $m_0^2 c^4 - E_0^2 = m_0^2 c^4 (Z\alpha/\kappa)^2$ or $\sqrt{m_0^2 c^4 - E_0^2} = m_0 c^2 Z\alpha/\kappa$, substituting this and (12) into the first equation of (11) gives at once

$$\kappa = -\frac{Z\alpha m_0 c^2}{\sqrt{m_0^2 c^4 - E_0^2}} = -\frac{Z\alpha m_0 c^2}{m_0 c^2 Z\alpha}\kappa = -\kappa \tag{13}$$

leading to a mathematical contradiction $1 = -1$, it actually gives a stern negative on the existence of the solution to the equation (10), showing that the neomorphic Dirac system of recurrence relations (6) has no solution for the ground state. This is why the machine calculation (8) cannot solve the unknown number z that expresses the energy eigenvalue parameter E_0. In many literatures on the neomorphic Dirac equations, the inconsistent formulas are often deleted, and only the formal solution such as the last formula in (12) is chosen. One often pays attention to that if the formula of energy agrees with the Dirac formula, but not attaches importance to that if the logic is correct. In fact, $\alpha_0 = 0$ in the equations (10) implies that the factor of the wave function (4) for the ground state is inadvertently written as the form

$$\begin{aligned} G_0(\rho) &= \sqrt{m_0 c^2 + E} e^{-\rho/2} \varphi_{02}(\rho) \\ F_0(\rho) &= -\sqrt{m_0 c^2 - E} e^{-\rho/2} \varphi_{02}(\rho) \end{aligned} \tag{14}$$

It is well known that the Dirac equation for the hydrogen atom has not such solution. The main aim of the quantum mechanics accurately solving the wave equation for the bound quantum system is to obtain the energy eigenvalues. However, many relevant literatures shied away from the essential questions that the neomorphic Dirac equation has no solution, those pseudo solutions have been considered as the real solution. The unitary principle request that the two equations in (11) come into the existence at the same time, there is the trivial solution: $\alpha_0 = 0$, $\beta_0 = 0$ but E_0 has not determinative values. Namely, the pseudo eigen-solution (12) conceals the most mathematical paradox $1 = 0$, which has been covered up by some writing skills in all alike theories on the various neomorphic Dirac equation.

A theory, if conceals the basic contradiction such as $1 = -1$ or $1 \neq 1$ it must be incorrect. However, an incorrect theory often gives some specious deduction that cannot be distinguished from the correct theory. Its mathematical errors are hence ignored by us. For the system of recurrence relations (6), we often pay attention to the general case and find the solution to the system of differential equations (5) for the n-excited state

$$\begin{aligned}
\varphi_1 &= \alpha_0\rho^{\gamma} + \alpha_1\rho^{\gamma+1} + \cdots + \alpha_{n-1}\rho^{\gamma+n-1} + \alpha_n\rho^{\gamma+n} \\
\varphi_2 &= \beta_0\rho^{\gamma} + \beta_1\rho^{\gamma+1} + \cdots + \beta_{n-1}\rho^{\gamma+n-1} + \beta_n\rho^{\gamma+n}
\end{aligned} \tag{15}$$

Inserting this into (5) and using $\gamma = \sqrt{\kappa^2 - Z^2\alpha^2}$ as well as $\lambda = \sqrt{m_0^2c^4 - E^2}\Big/\hbar c$, one obtains the system of linear algebraic equation for five undetermined parameters α_0, β_0, α_1, β_1 and E_1, it is

$$\begin{aligned}
&\left(\sqrt{\kappa^2 - Z^2\alpha^2} + \frac{Z\alpha E_2}{\sqrt{m_0^2c^4 - E_n^2}}\right)\alpha_0 + \left(\kappa + \frac{Z\alpha m_0c^2}{\sqrt{m_0^2c^4 - E_n^2}}\right)\beta_0 = 0 \\
&\left(\kappa - \frac{Z\alpha m_0c^2}{\sqrt{m_0^2c^4 - E_n^2}}\right)\alpha_0 + \left(\sqrt{\kappa^2 - Z^2\alpha^2} - \frac{Z\alpha E_2}{\sqrt{m_0^2c^4 - E_n^2}}\right)\beta_0 = 0 \\
&\left(1 + \sqrt{\kappa^2 - Z^2\alpha^2} + \frac{Z\alpha E_2}{\sqrt{m_0^2c^4 - E_n^2}}\right)\alpha_1 + \left(\kappa + \frac{Z\alpha m_0c^2}{\sqrt{m_0^2c^4 - E_n^2}}\right)\beta_1 = \alpha_0 \\
&\left(\kappa - \frac{Z\alpha m_0c^2}{\sqrt{m_0^2c^4 - E_n^2}}\right)\alpha_1 + \left(1 + \sqrt{\kappa^2 - Z^2\alpha^2} - \frac{Z\alpha E_2}{\sqrt{m_0^2c^4 - E_n^2}}\right)\beta_1 = 0 \\
&\vdots \\
&\left(n + \sqrt{\kappa^2 - Z^2\alpha^2} + \frac{Z\alpha E_2}{\sqrt{m_0^2c^4 - E_n^2}}\right)\alpha_n + \left(\kappa + \frac{Z\alpha m_0c^2}{\sqrt{m_0^2c^4 - E_n^2}}\right)\beta_n = \alpha_{n-1} \\
&\left(\kappa - \frac{Z\alpha m_0c^2}{\sqrt{m_0^2c^4 - E_n^2}}\right)\alpha_n + \left(n + \sqrt{\kappa^2 - Z^2\alpha^2} - \frac{Z\alpha E_2}{\sqrt{m_0^2c^4 - E_n^2}}\right)\beta_n = 0 \\
&\alpha_n = 0
\end{aligned} \tag{16}$$

Of course, this system of equations can also be obtained directly from the system of linear recurrence relations (6). The first and second equations require that the determinant of the coefficient equal zero, this has no problem. Formally, combining the last three formulas of (15) gives

$$\begin{aligned} &\left(\kappa + \frac{Z\alpha m_0 c^2}{\sqrt{m_0^2 c^4 - E_n^2}}\right)\beta_n = \alpha_{n-1} \\ &\left(n + \sqrt{\kappa^2 - Z^2\alpha^2} - \frac{Z\alpha E_2}{\sqrt{m_0^2 c^4 - E_n^2}}\right)\beta_n = 0 \end{aligned} \tag{17}$$

in order to obtain the formal non-trivial solution, let $\beta_1 \neq 0$, it must order that the coefficients before β_n equals zero,$n + \sqrt{\kappa^2 - Z^2\alpha^2} - Z\alpha E_2 \Big/ \sqrt{m_0^2 c^4 - E^2} = 0$, producing the eigenvalues of the energy levels for the n-excited state

$$E_n = \frac{m_0 c^2}{\sqrt{1 + \frac{Z^2\alpha^2}{\left(n + \sqrt{\kappa^2 - Z^2\alpha^2}\right)^2}}} \tag{18}$$

It is just the Dirac formula! One only notices the eigenvalues of the energy levels, he would not consider if the wave functions for all states satisfy the original Dirac equations.

It should be pointed that finding the eigenfunction for the wave equation by terminating the formal series solution must use the mathematical induction. Giving a different treatment to the Dirac equation, only when the solution for the ground state and the first excited state is demonstrated, the general solution for the n-excited state can be considered as the real solution. (17) producing (18) is only a formal deduction of the system of recurrence relations (16), and the corresponding new form of the Dirac equation for the hydrogen-like atom (5) conceals the mathematical errors, however, have been considered as the correct deduction for accurately describing the fine-structure of the hydrogen-like atom. This situation can be seen in many literatures on the Dirac equation.

V. GENERAL PROOF OF NON-SOLUTION TO NEOMORPHIC DIRAC EQUATION

We now discuss the general method of treatment to the neomorphic Dirac system of recurrence relations (6) and prove its non-existence of solution. Although the mentioned literatures give the perplexing procedures to solve this system of coupled recurrence relations, it can be transformed into two uncoupled recurrence relations for each formal series. Directly, eliminating β_m in equations (6) gives the uncoupled recurrence relation for the coefficient α_m in the power series ϕ_1

$$\alpha_m = \frac{m + \gamma - \frac{Z\alpha E}{\hbar c\lambda}}{(m+\gamma)^2 - \kappa^2 + Z^2\alpha^2}\alpha_{m-1} \tag{19}$$

One the other hand, eliminating α_m in equations (6) will give another uncoupled recurrence relation for the coefficient β_m in the power series ϕ_2. The second equation of (6) can be written down in the following form

$$\alpha_m = -\frac{\hbar c\,(m+\gamma)\,\lambda - Z\alpha E}{\hbar c\kappa\lambda - Z\alpha m_0 c^2}\beta_m \tag{20}$$

this means

$$\alpha_{m-1} = -\frac{\hbar c\,(m-1+\gamma)\,\lambda - Z\alpha E}{\hbar c\kappa\lambda - Z\alpha m_0 c^2}\beta_{m-1} \tag{21}$$

substituting (20) and (21) into the first equation of (6) reads

$$\beta_m = \frac{m-1+\gamma-\frac{Z\alpha E}{\hbar c\lambda}}{(m+\gamma)^2-\kappa^2+Z^2\alpha^2}\beta_{m-1} \tag{22}$$

Consequently, the system of recurrence relations (6) is equivalent to two uncoupled recurrence relations with the first order, (19) and (22). Because having the same energy eigenvalue parameters, they compose a system of equations for the same undetermined parameter E

$$\begin{aligned} \alpha_m &= \frac{m+\gamma-\frac{Z\alpha E}{\hbar c\lambda}}{(m+\gamma)^2-\kappa^2+Z^2\alpha^2}\alpha_{m-1} \\ \beta_m &= \frac{m-1+\gamma-\frac{Z\alpha E}{\hbar c\lambda}}{(m+\gamma)^2-\kappa^2+Z^2\alpha^2}\beta_{m-1} \end{aligned} \tag{23}$$

The above two recurrence relations each have a formal eigenvalue set. It is supposed that the linear recurrence relations terminate with the term $m = n$, namely, $\alpha_n \neq 0$, $\beta_n \neq 0$ and $\alpha_{n+1} = \alpha_{n+2} = \cdots = 0$, $\beta_{n+1} = \beta_{n+2} = \cdots = 0$. Substituting for equations (23) and using the sign $\lambda = \sqrt{m_0^2c^4-E^2}\Big/\hbar c$, it deduces that

$$\begin{aligned} E_n &= \frac{m_0c^2}{\sqrt{1+\frac{Z^2\alpha^2}{(n+1+\gamma)^2}}} \\ E_n &= \frac{m_0c^2}{\sqrt{1+\frac{Z^2\alpha^2}{(n+\gamma)^2}}} \end{aligned} \tag{24}$$

where $n = 0, 1, 2, \cdots$. These two formulas are similar but different actually. When looking from a mathematical point of view, two formulas have two different eigenvalue sets, the one and only eiegenvalues parameter for the same wave equation must satisfy two different eigenvalue sets, so it can only choose their intersection. However, when looking from a physical point of view, choosing the intersection would delete the energy for the ground state, falling short of the natural law. In fact, the energy eigenvalues for the ground state given by the first formula is just the energy eigenvalue for the first excited state given by the second formula. This strange result is inexplicable. For the same quantum system described by the same Dirac wave equation, the different solution methods produce two different energy eigenvalue sets, the formulas (24) ever caused the profound misconception. There are some antagonistic points of view considering that making substitution $n+1 \to n$ for the first formula it would be just the second formula. However, as $n = 0$, this substitution implies $0+1 \to 0$. As the subscript of the coefficient of series, the natural number n cannot be allowed to make such substantiation. The different results of the equations in (24) express the solution to the same energy eigenvalue parameter, it has to order

$m_0c^2 \Big/ \sqrt{1 + Z^2\alpha^2 \Big/ (n+1+\gamma)^2} = m_0c^2 \Big/ \sqrt{1 + Z^2\alpha^2 \Big/ (n+\gamma)^2}$, implying a most basic mathematics mistake $1 = 0$! Consequently, choosing anyone of formal solutions always yields the pseudo solution.

This procedure also verifies the veracity of the mechanical proving. The mechanical no-solution proving to the neomorphic Dirac equation reads that one of the variable quantities cannot be solved and the equations only have the triteness solution. There is not any reason to unceasingly argue, all neomorphic Dirac equations have no solution and deriving a neomorphic Dirac equation is useless in the Dirac theory.

VI. CONCLUSIONS

According to the unitary principle, whichever function transformation is introduced to transform the original Dirac equation into a neomorphic Dirac equation, substituting the obtained formal solution into the corresponding function transformation must read the standard solution of the original Dirac equation. Did one[20–25] ever note that it could not separate the standard exact solution (2) into the form (4) or its similar forms? This shows that the function transformation (4) is useless for the Dirac equation, and the corresponding formal solutions of the neomorphic Dirac equations are the false solutions! It is easy to treat the problem of the neomorphic Dirac equation in mathematics. However, because the formal expressions are described as the correct theory and were concealed by the results of the experimental observation, the firm conclusions about those pseudo solutions of the neomorphic Dirac equations have been difficult to accept by us. There exists the other congener problems concealed in the theoretical physics. From 1985 to now, we have disclosed and corrected some pivotal mathematical errors concealed in theoretical physics. In very few published Chinese papers[26] and English papers[27], it is very euphemistic to indicate those mathematical errors. Usually, disclosing an incorrectness of mathematical deduction in theoretical physics, we at least make the calculations and argumentation in various aspects, only their conclusions are identical can we express our points of view. It should be pointed out that those discoveries can be directly obtained by using the unitary principle. Generally verifying and revising most of the principled mathematical mistakes and specious conclusions concealed in theoretical physics still requires quite a long time. Using the mechanical proving to show the non-solution of the neomorphic Dirac equation only gives a precedent to check theoretical physics by machine. It establishes a credible foundation to correct some self-contradictory physical theories. Any logic of theoretical physics and its results must withstand the strict proof of mathematics[28].

Is there a general method that can widely check physics theory, which is similar to the method for mechanical geometry theorem proving[29, 30]? Now, we should pay attention to a fact: because of neglecting some simple mathematical logics, in recent centuries, we have possibly created those anticipant results in some incorrect logic, therefore missing the certain important deductions.

Acknowledgments

I wish to thank Professor Walter Greiner for helpful discussions, and Robert Montalbano for English help.

---

[1] Chen,R., Unitary principle and real solution of Dirac equation, Arxiv prepring arXiv: 0908.4320 (2009).

[2] Dirac, P. A. M., The Quantum Theory of the Electron, *Proc. R. Soc. London, Ser. A* **117**, 610-624 (1928), **118** (779), 351-361 (1928).

[3] Dirac, P. A. M., *The principles of quantum mechanics*, 4th ed. (Clarendon Press, Oxford, 1962, pp270).

[4] Schiff, L. I., *Quantum Mechanics*, (McGraw Hill Press, New York, 1949, pp326).

[5] Bjorken, J. D., Drell, S.D., *Relativistic Quantum Mechanics*, (McGraw–Hill, New York, 1964, pp52.-60)

[6] Brualdi, R. A., *Introductory Combinatories*, (Pearson Education, Inc., 5 Edition, 2004).

[7] Chen, R., The Uniqueness of the Eigenvalue Assemblage for Optimum Differential Equations, *Chin. J. Engin. Math* **20** (2002): 121-124(2003).

[8] Darwin, C. G., The Wave Equations of the Electron, *Proc. R. Soc. London, Ser. A* **118**, 654 (1928).

[9] Gordon, W., Über den Stoß zweier Punktladungen nach der Wellenmechanik, *Z. Phys.* **48**, 11 (1928).

[10] Dirac, P. A. M., *The principles of Quantum Mechanics*, Clarendon Press, Oxford, 1958, pp270.

[11] Biedenharn, L. C., Remarks on the relativistic Kepler problem, *Phys Rev.*, **126**, 845 (1962).

[12] Wong, M. K. F., Yeh, H. Y., Simplified solution of the Dirac equation with a Coulomb potential, *Phys. Rev. D* **25**, 3396 (1982).

[13] Su, J. Y., Simplified solution of the Dirac-Coulomb equation, *Phys. Rev. A* **32**, 3251 (1985).

[14] Ciftci, H., Hall, R, L., Saad, N., Iterative solutions to the Dirac equation, *Phys.Rev. A* **72**, 022101 (2005).

[15] Pudlak, M., Pincak, R., Osipov, V. A, Low-energy electronic states in spheroidal fullerenes, *Phys. Rev. B* **74,** 235435 (2006).

[16] Voitkiv, A. B., Najjari, B., Ullrich, J., Excitation of heavy hydrogenlike ions in relativistic collisions, *Phys. Rev. A* **75**, 062716 (2007).

[17] Ferrer, F., Mathur, H., Vachaspati, T., Starkman, G., Zero modes on cosmic strings in an external magnetic field, *Phys. Rev. D* **74**, 025012 (2006).

[18] Boulanger, N., Spindel, P., Buisseret, F., Bound states of Dirac particles in gravitational fields, *Phys. Rev. D* **74**, 125014 (2006).

[19] Greiner, W., *Relativistic Quantum Mechanics: Wave Equations*, Springer, 2000, 3rd Edition, pp225.

[20] Crater, H. W., Wong, C. Y., Alstine, P. V., Tests of two-body Dirac equation wave functioquarkonium and positronium into two photons, *Phys. Rev. D* **74**, 054028 (2006).ns in the decays of

[21] Alhaidari, A. D., Bahlouli, H., Al-Hasan, A., Abdelmonem, M. S., Relativistic scattering with a spatially dependent effective mass in the Dirac equation, *Phys. Rev. A* **75**, 062711 (2007).

[22] Korol, V. A., Kozlov, M. G., Relativistic corrections to the isotope shift in light ions, *Phys. Rev. A* **76**, 022103 (2007).

[23] Amaro, J. E., Barbaro, M. B., Caballero, J. A., Donnelly, T. W., Udias, J. M., Final-state interactions and superscaling in the semi-relativistic approach to quasielastic electron and neutrino scattering, *Phys. Rev. C* **75**, 034613 (2007).

[24] Alberto, P., Castro, A. S., Malheiro, M., Spin and pseudospin symmetries and the equivalent spectra of relativistic spin -1/2 and spin -0 particles, *Phys. Rev. C* **75**, 047303 (2007).

[25] Lamata, L., León, J., Schätz, T., Solano, E., Dirac Equation and Quantum Relativistic Effects in a Single Trapped Ion, *Phys. Rev. Lett.* **98**, 253005 (2007).

[26] Chen, R., The Problem of Initial Value for the Plane Transverse Electromagnetic Mode. *Acta Phys. Sin-CH ED* **49**, 2518 (2000).

[27] Chen, R., New Exact Solution of Dirac-Coulomb Equation with Exact Boundary Condition. *Int. J. Theor. Phys.* **47**, 881 (2008).

[28] Christiansen, T., Isophasal, isopolar, and isospectral Schrodinger operators and elementary complex analysis. *Am. J. Math.* **130** 49-58(2008).

[29] Wu, W-T,. Basic principles of mechanical theorem proving in elementary geometries. *J. Autom. Reasoning* **2**: 221-252(1986).

[30] Wu, W-T, *Mechanical theorem proving in geometries: Basic principles*, Springer (1994).