

An account of a Book. *Methodus Figurarum lineis rectis & curvis comprehensarum quadraturas determinandi* Authore J. Craige. 4to. Londini 1635.

THE great use of drawing the Tangents of Curve Lines, has made the most famous amongst the Modern Mathematicians endeavour to find out General Methods of finding the Tangents of Curve Lines, as may be seen from the several ways invented by *Des Cartes*, *Monsieur Fermat*, *Slusius*, *Dr. Barrow*, *Dr. Wallis*, *Tschurnehuy*, and *Leibnitzius*; But as yet none has attempted to invert this problem generally, that is, having the Tangent to find the Curve Line whose tangent it is. Therefore the Author of this Treatise perceiving that the doing of this would give a General Method of determining the Quadrature of any Curvilinear space, has laid down a rule for inverting *Slusius* his method mentioned in the *Philosophick Transactions* Num. 90. He has illustrated his Method of Quadratures by several Figures which have been already considered by Geometers. As for the Circle & Hyperbola, he asserts that their indefinite Quadratures are impossible, and therefore in these & such like cases, he expresses the Area by an infinite Series, which is easily done by his Method, except the Series consist of irrational termes, for in these he has recourse to *Leibnitzius* his method of finding Tangents, where the Calculation will be more tedious. By his resolving the Area of the Hyperbola into an infinite series, he comes to the same expression with that of *N. Mercator*: And in measuring the Zone of a Circle, his expression falls in with that invented by *Mr. Isaac Newton*, as *Mr. David Gregory* relates in his Treatise. He has subjoyned a Method of measuring the Curve Superficies made by the rotation of any Curve upon its *Ais*; with a small Animadversion on the Method of *Quadratrices*, published in the *Acta Lipsiensiæ Eruditorum* of October, 168.

Since the Publication of this Treatise, the Author is pleased to make the following Addition.

Z

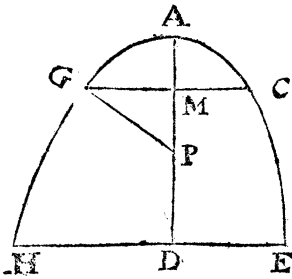
Addi

Additio ad Methodum Figurarum Quadraturas Determinandi. Autore Johanne Craige.

Quoniam omnium Figurarum Quadratura ex perfecta nostri primi problematis Solutione determinatur; propterea utile judicabam nonnulla addere, quae solutionem meam non modo plenius illustrant sed omnino perficiunt: non alio tamen sunt obscura, quin facile quisquam in istiusmodi rebus versatus, ex iis quae jam exposui, omnia supplere possit. Problema sic se habet. Data expressione Analytica linea inter ordinatam & Curvae perpendicularem designata, Invenire equationem naturam illius Curvae definientem. Hoc problema tres casus includit. 1. Cum expressio istius lineae talis est, qualis a vulgaribus tangentium Methodis exhibetur. 2. Cum ad simpliciores reducitur, facta divisione numeratoris & denominatoris per communem simplicem divisorem. 3. Cum expressio sit simplicissima, dividendo per divisorem compositum. Duos priores casus Regula, prout eam explicui, universim comprehendit; superest tantum ut ostendam quo pacto tertium pariter casum comprehendat.

Postquam expressio data per y multiplicatur, apponantur omnes termini qui sub maximo continentur (Terminorum magnitudinem e dimensionibus quantitatis y mensurans) & connectantur signo affirmativo vel negativo, prout libuerit; adhaerentur omnes illi termini (prius in coefficientis incognitis multiplicati) Quadrato quantitatis per x designata: eritque inde resultans aequatio quaesita, vel quaesitam includet; & determinationes coefficientium terminos aequationem constituentes a reliquis distinguent.

Sit in apposito schemate abscissa $AM=y$, ordinata $MC=z$, & Curva ACE proprietas $z^2 = \frac{a^2y^2 + y^4}{p^2}$, & invenienda sit



quadratura Area a lineis rectis & illa Curva comprehensa. Querenda est alia Curva A G H, in qua $PM =$

$$\sqrt{\frac{a^2y^2 + y^4}{p^2}} = z; \text{ ubi } PG \text{ Curva}$$

quaesita perpendicularem, & $MG=x$ illius ordinatam denotat. Cumq; hac expressio lineae PM in y multiplicata contineat sextam quantitatis y dimensionem, ideo appono omnes terminos sub

illa sexta dimensione contentos, unde resultans aequatio est.

$$\frac{na^6 + ma^5y + la^4y^2 + ha^3y^3 + ka^2y^4 + gay^5 + fy^6}{p^2} = x^4.$$

Ex hac aequatione inuenio valorem Lineae P M. quem comparo cum valore dato, unde

$$PM = \frac{ma^5 + 2la^4y + 3ha^3y^2 + 4ka^2y^3 + 5gay^4 + 6fy^5}{4p\sqrt{na^6 + ma^5y + la^4y^2 + ha^3y^3 + ka^2y^4 + gay^5 + fy^6}}$$

$$= \sqrt{\frac{a^2y^2 + y^4}{p^2}}. \text{ auferantur fractiones \& Signa radicalia, \& deter-}$$

minantur coefficientes n, m, l &c: (ut in prob: 2 tractatus nostri) & rejectis iis quarum determinationes absurdum involvunt: & caetera, in quibus nihil tale contingit, aequationem constituent. Sic

in exemplo proposito, erit $f = \frac{4}{9}$, $k = \frac{12}{9}$, $l = \frac{12}{9}$, $n = \frac{4}{9}$. sed dum g

determino, inuenio $240g = 144g$, quod absurdum involvit; & sic pro h comparatio erit $16h = 16h$ unde nullus illius valor, & pro m erit $4m = 4m$, quod iidem est absurdum: quapropter termini a quantitatibus g, h, m affecti ad aequationem non pertinebunt; unde reliqui a literis n, l, k, f affecti aequationem naturam

Curva definitam constituent. sc. $\frac{4a^6 + 12a^4y^2 + 12a^2y^4 + 4y^6}{9p^2}$

$$= x^4$$

$=x^4$, adeoq; $AMC = \sqrt{\frac{a^6 + 3a^4y^2 + 3a^2y^4 + y^6}{6p^2}} = \frac{x^2}{2}$. Exem. 2

Sit Curva Linea ACE talis proprietatis $z^2 = \frac{qy^2 + y^3}{p}$ & inveni-
nienda sit Quadratura spatii AMC. Querenda est Curva AGH

in qua sit $PM = \sqrt{\frac{qy^2 + y^3}{p}} = Z$ & quoniam hic valor in y mul-

tiplicatus continet quintam quantitatis y dimensionem, apponantur
omnes termini sub illa quinta dimensione, & aequentur quadrato
quantitatis per x designatae; unde aequatio resultans est.

$\frac{mq^2y^5 + mq^4y^4 + lq^3y^2 + kq^2y^3 + hqy^4 + fy^5}{p} = x^4$. atq; sola coeffi-

cientis (m) determinatio absurdam involvet, eruntq; reliqua,

$n = \frac{64}{225}, l = -\frac{16}{15}, k = -\frac{16}{45}, h = \frac{16}{15}, f = \frac{16}{25}$, unde aequatio curvam

quaesitam definiens est.

$\frac{64q^5}{225p} - \frac{16q^3y^2}{15p} - \frac{16q^2y^3}{45p} + \frac{16qy^4}{15p} + \frac{16y^5}{25p} = x^4$ adeoq;

$AMC = \sqrt{\frac{16q^5}{225p} - \frac{4q^3y^2}{15p} - \frac{4q^2y^3}{45p} + \frac{4qy^4}{15p} + \frac{4y^5}{25p}} = \frac{xx}{2}$

Exem. 3. Invenienda sit Quadratura spatii AMC, definita
natura Curvae ACE hac Aequatione $z^2 = \frac{2a}{4y+4a}$. Queratur

alia Curva AGH, in qua $PM = \sqrt{\frac{a^3}{4y+a}} = z$. Ex praemissis

constat Aequationem primam fore $\frac{na^2y^2 + ma^4y + 16a^5}{4a+4y} = 4x^2$

& determinationes Coefficientium $n = 16, m = 32, l = 16$.

Quibus substitutis, erit aequatio $\frac{16a^3y^2 + 32a^2y + 16a^5}{4y+4a} = x^4$

$= 4a^3y + 4a^4$: adeoq; $AMC = \sqrt{a^3y+a^4} = x^2$.

Notatu dignissimum est, has tres (sicut infinitas alias) Qua-
draturas abscissae AM (seu y) non convenire. Quoniam in istius-
modi

modi Figuris, simplicissima Area expressio huic portioni non respondet: attamen Quadratura abscissæ conveniens exinde parvo labore deducitur. Ut in Exem: 3. ubi Area est $\sqrt{a^2y + a^4}$; fiat $y = 0$, & erit Area $\sqrt{a^4} = a^2$, & subducatur hæc ex generali; proveniet Quadratura portionis abscissæ respondentis, sc. $\sqrt{a^2y + a^4} - a^2$. Quam observatiunculam mihi primus significavit Vir celeberrimus D. Isaacus Newton.

Tentetur jam idem processus in Circulo ACE, cujus diameter sit r , ac proinde $Z = \sqrt{ry - y^2}$, Quærenda est Curva AGH in qua $PM = \sqrt{ry - y^2} = Z$, sed ex dictis constat equationem primam fore $nr^4 + mr^3y + tr^2y^2 + hry^3 - ky^4 = x^4$: & singula coefficientium determinationes erunt impossibiles; adeoq; nulla datur Curva AGH in qua $PM = \sqrt{ry - y^2}$, ac proinde Circuli Quadratura indefinita est impossibilis. Fieri tamen potest ut sit aliqua hujusmodi Curva AGH, sed ex earum numero, quas post Cartesium Mechanicæ Geometria communiter appellant: sed quia harum usus non libenter admittunt Mathematici, præstat hujusmodi Quadraturas per series infinitas exhibere.

Benevole Lector

Ob inopiam Typorum Numeralium minusculorum, qui ad designandas quantitatum potestates supra Symbola dextrorsum apponi solent, festinante prælo, Typographus paulo majoribus usus est in eadem linea immediate sequentibus; ubicunq; itaq; offenderis a^3 , vel x^2 , &c. cubum vel quadratum, &c. e quantitate, cui suffigitur numerus, intelligas.

L O N D O N,

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