Geometry Teacher's Edition - Teaching Tips

CK-12 Foundation

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Chapter 1

Geometry TE - Teaching Tips

1.1 Basics of Geometry

Points, Lines, and Planes

Pacing: This lesson should take approximately three class periods.

Goal: This lesson introduces students to the basic principles of geometry. Students will become familiar with three primary undefined geometric terms and how these terms are used to define other geometric vocabulary. Finally, students are introduced to the concept of dimensions.

Study Skills Tips! Start your students off on the correct foot – vocabulary is a necessity in geometry success! Devote five minutes of each class period to creating flash cards of the major terminology of this text. Use personal whiteboards to perform quick vocabulary checks. Or, better yet, visit Discovery School's puzzle maker and make your own word searches and crosswords (http://puzzlemaker.discoveryeducation.com/)!

Language Arts Connection! To give an example of why some words are undefined, use the concept of circularity. Students use a dictionary, either electronic or paper (yes, they are still printed!) to complete this activity. Ask students to look up the word *point* in their reference. Find a key word in that definition. Students should continue this process until the word *point* is found. Repeat this process for *line* and *plane.* The rationale behind this activity is for students to see there is no one way to define these geometric terms, thus allowing them to be undefined but recognizable.

Real World Connection! Have students identify real-life examples of points, lines, planes in the classroom, as well as sets of collinear and coplanar. For example, points could be chairs, lines could be the intersection of the ceiling and wall, and the floor is a great model of a plane. If your chairs are four-legged, this is a fantastic example of why 3 points determine a plane, not four. Four legged chairs tend to wobble, while $3-$legged stools remain stable.

To help students understand dimension, use the following table:

Table 1.1

Zero-dimensional	1-dimensional (length)	2-dimensional (length and width)	3-dimensional (length, width, and height)

Table 1.1: (continued)

Zero-dimensional	1-dimensional (length)	2-dimensional (length and width)	3-dimensional (length, width, and height)

Have students write abstract examples of each dimension (point, line, plane, prism, etc) in the first row. Then have students brainstorm real-life examples of each dimension. Complete the table by gathering the responses of various students.

Segments and Distances

Pacing: This lesson should take one class period

Goal: Students should be familiar with using rulers to measure distances. This lesson incorporates geometric postulates and properties to measurement, such as the Segment Addition Property.

Real World Connection! To review the concept of measurement, use a map of your community. Label several things on your map important to students – high school, grocery store, movie theatre, etc. Have students practice finding the distances between landmarks "as the crow flies."

Extension! Discuss with your students the rationale of using different units of distance – inch, foot, centimeter, mile, etc. Why are things measured in inches as opposed to fractional feet? This is also a great time to introduce the difference between the metric system and the U.S. measurement system. Have students perform research regarding why the United States continues to use its system while the majority of other countries use the metric system. Provide pros and cons to using each type of system.

Fun tip! Have students devise their own measurement device. Students can use their invention to measure a school hallway, parking lot, or football field. Engage in a whole-class discussion regarding the results.

Refresher! Students may need a refresher regarding multiplying units. Have the students write out the complete unit, as on page 18, and show students how units can be cross-canceled.

Look out! While the Segment Addition Property seems simple, students begin to struggle once proofs come into play. Remind students that the Segment Addition Property allows an individual to combine smaller measurements of a line segment into its whole.

Rays and Angles

Pacing: This lesson should take one class period

Goal: This lesson introduces students to rays and angles and how to use a protractor to measure angles. Several real world models are used to illustrate the concepts of angles.

Real World Connection! Have students Think-Pair-Share their answers to the opening question, "Can you think of other real-life examples of rays?" Choose several groups to share with the class.

Notation Tip! Beginning geometry students may get confused regarding the ray notation. Draw rays in different directions so students become comfortable with the concept that ray notation always points to the right, regardless of the drawn ray's orientation.

Teaching Strategy! Using a classroom sized protractor will allow students to check to make sure their calculations are the same as yours. Better yet, use an overhead projector or digital imager to demonstrate the proper way to use a protractor.

Teaching Strategy! A good habit for students is to name an angle using of all three letters. This becomes important when labeling vertices of triangles and labeling similar and congruent figures using the similarity statement. Furthermore, stress to students the use of double and triple arcs to denote angles of different measurements. Students can get caught up in the mass amounts of notation and forget this important concept, especially during triangle congruency.

Stress the parallelism between the Segment Addition Property and Angle Addition Property. Students will discover that many geometrical theorems and properties are quite similar, with perhaps one words changed. Yet, the meaning remains the same.

Arts and Crafts Time! Have students take a piece of paper and fold it at any angle of their choosing from the corner of the paper. Open the fold and refold the paper at a different angle, forming two "rays" and three angles. Show how the angle addition property can be used by asking students to measure their created angles and finding the sum – they should equal 90 degrees!

Physical Models! The angle formed at a person's elbow is a useful physical model of angles. Have the students put their arm straight out, illustrating a straight angle. Then have the student gradually turn their arm up (or down) gradually to demonstrate how the degree changes. Use several students as examples to show that the length of the forearm and bicep do not change the angle measurement.

Segments and Angles

Pacing: This lesson should take one to one and one-half class periods

Goal: The lesson introduces students to the concept of congruency and bisectors. Students will use algebra to write equivalence statements and solve for unknown variables.

Have Fun! Have students do a call-back, similar to what cheerleaders do. You call out "AB" and students would retort, "The distance between!" Continue this for several examples so students begin to see the difference between the distance notation and segment notation.

This is a great lesson for students to create a "dictionary" of all the notation and definitions learned thus far. In addition to the flashcards students are making, the dictionary provides an invaluable reference before assessments.

When teaching the Midpoint Postulate, reiterate to students that this really is the arithmetic average of the endpoints, incorporating algebra and statistics into the lesson.

Visualization! Students have not learned about a perpendicular bisector. Have students complete Example 3 without using their texts as guides. Have students show their bisectors. Hopefully your class will construct multiple bisectors, not simply those that are perpendicular. This helps students visualize that there are an infinite amount of bisectors, but only one that is perpendicular.

Fun Tip! To visualize the angle congruence theorem and provide a means of assessing the ability to use a protractor, give students entering your class an angle measure on a slip of paper (the measurements should repeat). Have the students construct the angle as a warm up. Then have the students find their "matching" partner and check their partner's angle using a protractor.

Physical Models! Once students have reviewed Example 5, have them copy the angle onto a sheet of notebook paper or patty paper and measure the degree of the bisector. Students will construct a fold at that particular angle measurement to see the angle bisector ray.

Real Life Application! Another method of illustrating angle bisectors is to show a compass rose, as shown below.

http://commons.wikimedia.org/wiki/File:Compass_rose_browns_00.svg

Students can see how directions such as SWS, NNW, etc bisect the traditional four-corner directions.

Angle Pairs

Pacing: This lesson should take one to one and one-half class periods

Goal: Angle pairs are imperative to geometry. This lesson introduces students to common angle pairs.

Inquiry Learning! Students should be encouraged to learn through self-discovery whenever possible. To illustrate the concept of the Linear Pair Postulate, offer several examples of linear pairs. Have students measure each angle and find the sum of the linear pair. Students should discover any linear pair of angles is supplementary.

To further illustrate the idea of vertical angles, extend the adjacent ray of the previous linear pairs to a line. Have students repeat the process of measuring the angles, notating the linear pairs. Students will come to the conclusion that the angles opposite in the "X" are equal.

Students tend to get confused with the term *vertical,* as in vertical angles. Vertical angles are named because the angles share a vertex, not necessarily because they are in a vertical manner.

Interdisciplinary Connection! NASA has developed many lesson plans that infuse science, technology, and mathematics. The following link will take you to a lesson plan incorporating the seasons and vertical angles. http://sunearthday.nasa.gov/2005/educators/AOTK_lessons.pdf

Classifying Triangles

Pacing: This lesson should take one class period

Goal: Students have previously experienced triangle terminology: scalene, equilateral, isosceles. This lesson incorporates these terms with other defining characteristics.

In Class Activity: Give pairs of students three raw pieces of spaghetti (you can also use non-bendy straws). Instruct one partner to recreate the below table while the second makes two breaks in the spaghetti. It is okay if some breaks away!

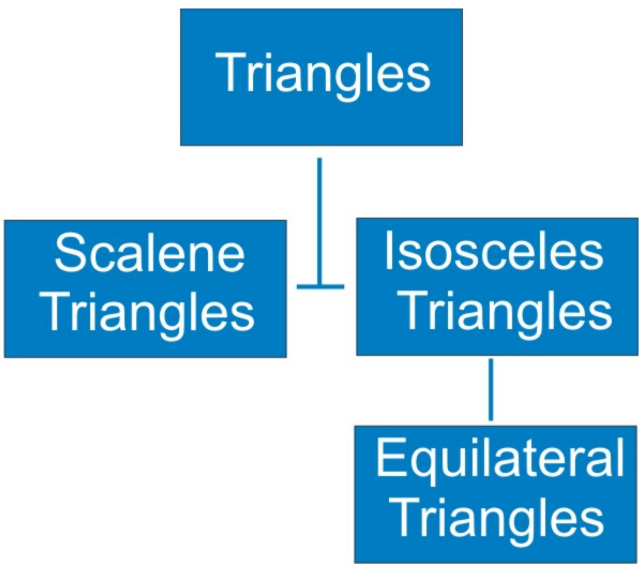
The students are to measure the three pieces formed by the two breaks and attempt to construct a triangle using these segments. Students will reach the conclusion that the sum of two segments must always be larger than the third if a triangle is to be formed. *The Triangle Inequality Theorem can be found in the lesson entitled Inequalities in Triangles*

Table 1.2

Segment 1 Length (in cm)	Segment 2 Length (in cm)	Segment 3 Length (in cm)	Can a triangle be formed (Yes/No)

Showing students the difference between line segments and curves, introduce cooked spaghetti. The flexibility of the spaghetti demonstrates to students that segments must be straight in order to provide rigidity and follow the definitions of polygons.

Students can express the concepts presented in this lesson using a Venn diagram or a hierarchy. If students are not familiar with a hierarchy, remind students a hierarchy is an ordering of related objects from the most general to the most specific. An example is shown below.



Classifying Polygons

Pacing: This lesson should take one class period

Goal: This lesson explains the characteristics of a polygon. Students should be able to classify polygons according to its number of sides and whether it's convex or concave.

Language Arts Connection! Have students find several high school textbooks and internet sites that provide a definition of polygon. Compare each definition for similarities and differences. Devise a workable classroom definition of a polygon, using the ones found as guides.

Study Skills Tip! Flashcards are imperative to geometry success! Students should construct flashcards of the important polygons. One side should be the drawing of the polygon with the reverse naming the polygon, listing the number of sides, and if applicable the sum of the interior angles in a polygon or how to separate the polygon into triangles (useful when determining polygonal area).

We love Pythagoras! When teaching the distance formula, relate this to Pythagorean's Theorem. The vertical distance (change in y) represents one leg of a right triangle and the horizontal distance (change in x) represents the other leg. Using $a^2 + b^2 = c^2$, students can derive the distance formula. Many students will use Pythagorean's Theorem as opposed to the distance formula when determining the length between two points.

What Did You Learn? Use this activity as a culminating activity or perhaps an alternative assessment. Devise a geometry scavenger hunt, listing the major concepts learned thus far. Equip students with a digital camera and their imagination. Ask students to find as many objects as possible, capture them with a photograph and incorporate the photos into a movie or slideshow presentation. Offer bonus points for such things as originality, nature made objects, etc.

Problem Solving in Geometry

Pacing: While this concept should permeate throughout this course, this particular lesson should take one class period.

Goal: Problem solving is necessary is daily life. Drawing diagrams, working backwards, checking multiple options, and answering reasoning questions are imperative for students to learn. This lesson introduces the key questions one should ask when problem solving and some strategies students can use.

Problem solving is essential in daily life. Encouraging students to reflect upon the strategies offered in this lesson and incorporating word problems into your daily routine will help students become successful problem solvers. Allow students to struggle through these problems, facilitating their knowledge rather than providing direct instruction.

Provide a chart for students to use as reference that ask the five essential questions:

- What is the problem asking for?
- What do I have that could be used to answer the question?
- What do I need to know to find the answer?
- Did I provide the information the problem requested?
- Does my answer make sense?

By having students begin their problem solving using a chart outlining these questions, students will begin to see when they have answered the problem completely. This helps students later in the textbook by providing a method to answer the age-old question, "When is my proof complete?"

Gather multiple story problems that require various forms of problem solving techniques. Use personal whiteboards to do an immediate check regarding students' progress.

1.2 Reasoning and Proof

Inductive Reasoning

Pacing: This lesson should take one class period

Goal: This lesson introduces students to inductive reasoning. Inductive reasoning applies easily to algebraic patterns, integrating algebra with geometry.

A great way to start this lesson is to further expand upon inductive reasoning. Inductive reasoning uses patterns to make generalizations. Simply put, inductive reasoning takes repeated specific examples and extends it to a general conjecture.

Begin by writing an arithmetic or a geometric sequence such as $1, 4, 7, 10 \ldots$ or $40, 20, 10, 5, \ldots$ Ask students to recognize the pattern and write the generalization in words (this also lends itself to exposure to sequences and series, a topic usually found in Advanced Algebra).

Challenge: Offer students this type of pattern: $14, 10, 15, 11, 16, 12, 17 \ldots$ The pattern here is to subtract four then add five.

Take the opportunity to further discuss the triangular numbers, as seen on page 76. The triangular numbers are formed such that the dots form a triangle and also a numerical pattern of $s(n) = \frac{1}{2}n^2 - n$.

In examples 1 and 2 on page 76, relate the even and odd numbers to a symbolic pattern. For example, even numbers can be represented by the expression $2n$, while all odd numbers can be represented by $2n + 1$.

Real Life Connection! Apply the idea of counterexample to real life situations. Begin by devising a statement, such as, "If the sun is shining, then you can wear shorts." While this is true for warm weather states such as Florida and California, for those living in the Midwest or Northern states, it is quite common to be sunny and 12 degrees! Have students create their own statements and encourage other students to find counterexamples.

Conditional Statements

Pacing: This lesson should take two class periods

Goal: This lesson introduces the all-important conditional statements. Students will gain an understanding of how converses, inverses, and contrapositives are formed from a conditional and further explore truth values of each of these statements.

The first portion of this lesson may be best taught using direct instruction and several visual aids. Design phrases you can laminate, such as "you are sixteen" and "you can drive." Adhere magnets to the back of the phrases (to stick to the white board), or you can use a SMART board. Begin by writing the words "IF" and "THEN," giving ample space to place your phrases. When discussing each type of conditional, show students how each is constructed by rearranging your phrases, yet leaving the words "IF" and "THEN" intact.

Have students create a chart listing the type of conditional, its symbolic form and an example. This allows students an easy reference sheet when trying to decipher between converse, conditional, contrapositive, and inverse.

Table 1.3

	Symbolic Form	Example
Conditional	$p \Rightarrow q$	

Table 1.3: (continued)

	Symbolic Form	Example
Inverse	$: p \Rightarrow: q$	
Converse	$q \Rightarrow p$	
Contrapositive	$: q \Rightarrow: p$	

Spend time reviewing example one on page 85 as a class. Stress the importance of counterexamples.

Interactive Lesson! Use the same setup as the opening activity when discussing biconditionals. Begin with a definition, such as example one on page 86. Set up your magnetic phrases in if and only if form, then illustrate to students how the biconditional can be separated into its conditional and converse.

Deductive Reasoning

Pacing: This lesson should take one class period

Goal: This lesson introduces deductive reasoning. Different than inductive reasoning, deductive reasoning begins with a generalized statement, and assuming the hypothesis is true, specific examples are deduced.

Differentiate between deductive and inductive reasoning to students by linking to the previous lessons. Deductive reasoning begins with a conjecture (hypothesis) and infers specific examples.

Stress example 5 with your students. Students can get confused with the inverse and contrapositive from the previous lesson that they make the mistake of using faulty reasoning.

When determining the truth value of $p^\wedge q$, students may be confused as to why the value is false if the hypothesis is false. Offer students a real life example. "If it is snowing, then it is cold." If the hypothesis is already false, stress that it doesn't matter the conclusion; the statement is not applicable.

Be sure the students understand the difference between $\wedge$ (exclusive) $\vee$ and (inclusive) before filling out the truth tables.

Real World Application! Show a portion of an episode of a courtroom drama scene. Ask students to apply the ideas of deductive and inductive reasoning to the lawyers. Determine which reasoning the prosecuting attorney is using. Is it different reasoning than what the defending attorney uses?

Algebraic Properties

Pacing: This lesson should take one class period

Goal: Students should have some familiarity with these properties. Here we can extend algebraic properties to geometric logic.

Fun Tip! Construct "I have, who has" cards for your class. Using the properties from this lesson (and other lessons if you have a large class), create as many cards as students in your class. The first card should read, "I have Reflexive Property of Equality. Who has the property that states if $a = b$, then $b = a$?" The next card should state, "I have the Symmetric Property of Equality. Who has...? Continue this process until the last card. The "Who has" of this card should state, "Who has the property that a equals a?" Shuffle the cards and give one to each student. Because the cards are all connected, it doesn't matter who starts. Time the class and then challenge the students to beat their previous time. Not only does this increase listening in the classroom, but it also reinforces the properties and encourages active participation.

Teaching Strategy: Use personal whiteboards or interactive "clickers" to do a spot check. Create two or three property questions each day and begin your class with these mini-quizzes. Encourage students to create flashcards for the properties and use them for two-column proofs.

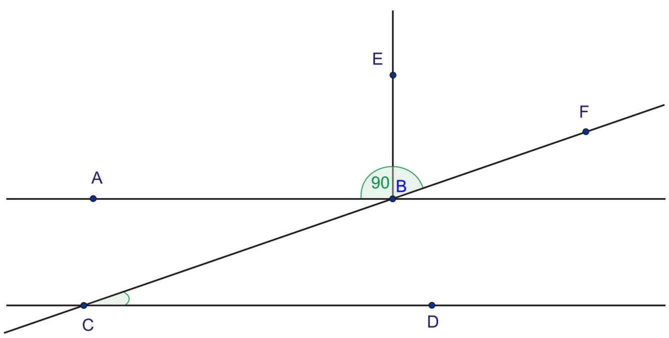
Stress to students that the properties of congruence can only be used when given congruence ($\cong$), not equality ($=$). This also hold true for the properties of equality; these properties are reserved for objects that are equivalent.

Have students list properties not mentioned in this lesson. Students may come up with the distributive property of the multiplying fractions property. Students may offer notions that are incorrect – take the time to have students learn from incorrect thoughts!

Diagrams

The best way to describe what you can and cannot assume is "Looks are deceiving." Reiterate to students that nothing can be assumed. The picture must literally say one thousand words using notation such as tic marks, angle arcs, arrows, etc.

Additional Example! Use the following diagram and ask your students to list everything they can assume from the drawing and those things that cannot be assumed. For the latter list, ask students to list additional information needed to clarify the drawing.



Two Column Proof

Pacing: While two-column proofs will be used for the remainder of the text, this lesson should take one to two class periods

Goal: Students are introduced to the format of a two-column proof in this lesson. The purpose of two-column proofs is not only to prove geometric theorems. Organizing one's thoughts in a logical manner allows students to become better writers and debaters.

Fun Tip! Use cut outs so students can begin to visualize two column proofs. Photocopy the following proof and cut it into sections. Shuffle the sections and place into an envelope. Give pairs of students the envelope and a sheet of paper with the given statement, the "to prove" statement, and the column separator. Have students sort through the rectangles and recreate the proof.

Given: $\overline{AB}$ bisects $\overline{DE}$; $\overline{DE}$ bisects $\overline{AB}$

Prove: $\triangle ABM \cong \triangle DCM$

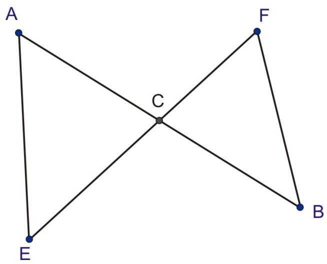


Table 1.4

Reason	Justification
Segment AD bisects segment BC Segment BC bisects segment AD	Given
Segment $AM \cong$ segment DM	Midpoint Postulate
Segment $BM \cong$ segment CM	Midpoint Postulate
$\angle AMB \cong \angle DMC$	Vertical Angles Theorem
$\triangle ABM \cong \triangle DCM$	Side-Angle-Side Congruence Theorem

Neat Idea! Whiteboard makers also make a 2-column personal whiteboard. You could purchase these to perform a spontaneous check or you could make your own. Purchase plain white paneling from a home improvement store. Cut out the desired lengths then use electrical tape to construct your proof T-chart.

1.3 Parallel and Perpendicular Lines

Lines and Angles

Pacing: This lesson should take one to one and one-half class periods

Goal: Students will be introduced to parallel, perpendicular, and skew lines in this lesson. Transversals and the angles formed by such are also introduced.

While example 1 shows students that it is possible for streets to be perpendicular or parallel, challenge students to find roads that begin as parallel then intersect (or begin perpendicular and then become parallel).

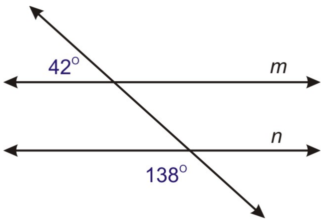
Visualization! Show a map that has zoned roads in the fashion of example 1, perhaps in rural Ohio or Kansas. Encourage students to compare this map with one of Atlanta, New York City, or Chicago.

Physical Model! Give each student a cube; it could be a die, box, etc. When discussing the definition of skew lines, have students point to the lines you are referencing. This provides students a physical model in addition to allowing you to do a quick assessment.

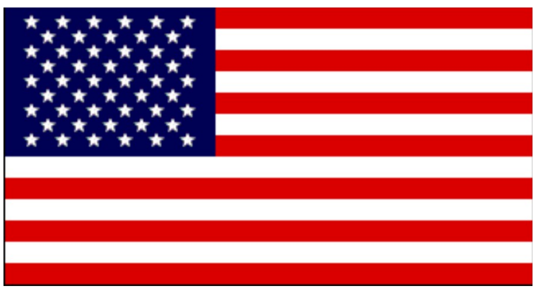
In Class Activity! Have students trace the top and bottom of a ruler to create pair of parallel lines. Then construct an oblique line crossing through both parallel lines. Instruct students to number all 8 angles and color code each of the terms found on page 131. Ask students to summarize each definition and how it relates to another angle in the diagram.

Vocabulary! There are five ways to determine parallel lines: showing congruent corresponding angles or congruent alternate interior angles, proving same side interior angles are supplementary, showing both lines are parallel to a third line or by showing both lines are perpendicular to the same line.

Have students prove the following lines are parallel using one of the above methods.



Challenge! There are 13 red and white alternating stripes on the United States Flag. Explain why the top red stripe must be parallel to the third white stripe. *Answer: Using the syllogism property and the idea of parallel lines, since each lines is parallel to the one before it, then the first red stripe must be parallel to the third white stripe. Also, the flag may look weird if the stripes were not parallel!*



http://www.steve4u.com/usflag/50star.gif

Parallel Lines and Transversals

Pacing: This lesson should take one class period

Goal: The textbook further extends the notion of transversals and parallel lines to illustrate the corresponding angles postulate and the alternate interior angles postulate. Additional theorems and postulates are proven in this lesson.

Use the in-class activity from the previous lesson as a refresher and guide to the lesson opener. This lesson provides several key theorems: corresponding angles postulate and alternate interior angles postulate. Students have experienced the definitions of these angles in previously lessons and have also been given brief introductory proofs. The goal of this lesson is to use these notions to prove alternate exterior angles are congruent and consecutive interior angles of parallel lines are supplementary.

In-Class Activity! Divide your class into six sections of pairs. Provide enough copies of the Corresponding Angles Postulate, its converse, the Alternate Interior Angles Postulate, its converse, and the Alternate Exterior Angles Postulate and its converse. Instruct each pair to prove their theorem, and then group homogenous sections in order to discuss the results. Taking a pair of each theorem and its respective converse to form a team of four, have the students discuss the proofs. As an assignment, have the groups create a visual poster of the proof of the theorem, the converse and its respective proof.

In-Class Activity! Demonstrate to students that these theorems do not apply to non-parallel lines. Each student should create two non-parallel lines and a transversal. Label the 8 angles formed, having students measure all angles. Students will see the alternate interior angles, corresponding angles, and vertical angles are not congruent, nor are the consecutive interior angles supplementary.

Proving Lines Parallel

Pacing: This lesson should take one class period

Goal: The converse of the previous lesson's theorems and postulates are provided in this lesson. Students are encouraged to read through this lesson and follow along with the proofs.

Vocabulary! The Parallel Lines Property can be stated, "If line l is parallel to line m, and line m is parallel to line n, then lines l and n are also parallel." Ask students to write the converse of this property and determine if it is true. If students determine the converse false, have them provide a counterexample.

Slopes of Lines

Pacing: This lesson should take one to two class periods

Goal: Students should feel comfortable with slopes and lines. Use this lesson as a review of key concepts needed to determine parallel and perpendicular lines in the coordinate plane.

Fun Fact! The word slope comes from the Middle English word *sloop*, meaning at an angle.

Most students have experienced slope in Algebra. However, students rarely have seen the delta symbol when determining slope. Use the following to stress the connection to pre-calculus:

$$\text{Slope} = \text{rate of change } \frac{\text{Rise}}{\text{run}} = \frac{\triangle y}{\triangle x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Vocabulary Connection! Ask students to brainstorm the many different interpretations of the word slope. Apply these to real world situations such as the slope of a mountain, or the part of a continent draining into a particular ocean (Alaska's North Slope), the slope of a wheelchair ramp, etc.

Include the synonym for slope – grade. Students should come up with more examples using this word.

Real Life Connection! Eldred Street in Los Angeles, California has a grade of 33%, Baldwin Street In Dunedin, New Zealand boasts a 35% incline, and Banton Avenue in Pittsburg, Pennsylvania officially measurers 37%! Have students reconstruct the incline of these streets using the rise over run notion of slope.

When discussing the rise over run triangles, begin making the right triangle connection to students, demonstrating that every rise/run triangle will form a 90 degree angle. When students are asked to find the distance between two oblique points, the distance formula is a derivation of Pythagorean's Theorem.

Fun Tip! To illustrate why vertical lines have an undefined slope, ask for volunteers for the following demonstration.

To illustrate a horizontal line, run a length of masking tape on your floor. Ask a student to walk over the line. Onlookers should see the student is walking at a zero incline (or slope).

To illustrate an oblique line, lay a 2" by 4" piece of wood on top of a chair, or something sturdy, creating a 2% – 3% incline. Ask a student to walk up the hill. Relate the percentage to a fraction, relating rise over run.

To illustrate a vertical, ask students to place their feet on a wall, lying parallel to the floor. Instruct the students to walk up the wall in this position, similar to what Spiderman can do. Students will tell you this is impossible! Dividing by zero is also impossible, thus illustrating why vertical lines have undefined (impossible) slopes.

To further demonstrate perpendicular slopes, use the formula to your advantage slope $= \frac{(y_2 - y_1)}{(x_2 - x_1)}$ so the slope of the line perpendicular must be $-\frac{(x_2 - x_1)}{(y_2 - y_1)}$

Students may find that making an xy T-chart is an easy way to construct a line. Whichever your preference, make sure students can see a variety of ways to begin to solve a problem.

Equations of Lines

Pacing: This lesson should take one to two class periods

Goal: This lesson reinforces key concepts learned during Algebra to prepare students for geometric connections. Students will review slope intercept form, standard form for a linear equation, and introduce equations for parallel and perpendicular lines.

Alternative Ways to Think! An alternative way to express slope-intercept form is $y = b + mx$. In some situations, this form will make much more sense to students that the "original" way. You could also try substituting m with a. Linear regressions found on graphing calculators often use this formula: $y = ax + b$. Students tend to feel frustrated with the constant replacement of variables. Determine which variable appear in later textbooks and feel free to use that variable from the beginning.

Inquiry based learning! Have students trace the top and bottom edges of a ruler onto a coordinate plane. Ask students to determine the equations for each line and compare the results. *Students should notice that, if done correctly, the slopes will be equal.* Follow this activity with the equations for parallel lines section.

Use the graph provided in example 3 for this activity. Once students have found the slope of the graphed equation, incorporate the previous lesson's concept to find the slope of the line perpendicular. Ask students to place a dot anywhere on the $y-$axis and use the newly found slope to construct a line. Using a projection device, ask several students to graph their equations. *Students should come to the conclusion there are infinitely many lines perpendicular in a coordinate plane.*

Algebra Review! Before discussing standard form for a linear equation, make sure students can clear fractions, something that is widely forgotten. During the warm up or opening set, ask students to clear the following fractions:

$$\frac{5}{6}x = 30 \qquad \frac{2}{3}x + 3 = 9 \qquad \frac{7}{6}x + \frac{1}{4} = \frac{1}{2}$$

This will allow you to determine the level of which you may have to re-teach before moving on to standard form.

Why do I need this? Many students ask why they need to know standard form. One reason is because many real life problems take form in a linear combination (standard form) approach. For example, one cheeseburger is \$1.69 and a small French fry is \$1.39. How many of each can you buy with \$15.25, excluding tax? The equation begins in standard form and many students will rewrite this into slope-intercept form.

Perpendicular Lines

Pacing: This lesson should take one class period

Goal: Students will extend their learning to include angle pairs formed with perpendicular lines. The properties presented in this lesson hold only for perpendicular lines pairs.

Extension: Connect the introduction of this lesson with circles. Draw a circle around the origin of the Cartesian plane found on page 180. Students should already know the sum of the degrees of a circle (360°). Demonstrate to students what angles are formed when the axes split the circle into four congruent segments. This will aid students when discussing circles in Chapter 9.

Example 3 can also be solved using the notion of vertical angles. To find $m\angle WHO = 90^\circ$, instruct students to visualize these two angles as being vertical angles.

Extension: Extend example 4 to review vertical angles. Turn ray L into a line and have students apply the Vertical Angle Theorem to the angles found in quadrant four. This will help keep Vertical Angles fresh in students' minds.

Take time to review how perpendicular angles are formed – *the product of slopes of perpendicular lines must equal* -1. Continuous review will help students prepare for the test.

Why Is This So? Students may question why the lesson is entitled "Perpendicular Lines" when most of the material presented is regarding angles. Explain to students that these properties only hold for lines intersecting at 90 degree angles. To further illustrate, have students construct non-perpendicular lines and attempt to draw in complementary adjacent angles.

Perpendicular Transversals

Pacing: This lesson should take one class period

Goal: The goal of this lesson is to introduce students to the concept that parallel lines are equidistant from each other and to prove lines parallel using the converse of the perpendicular to parallels theorem.

As students work through example 4, ask them to look at the slopes of the lines. Students should realize the slopes are the same, thus they will never intersect. Have students create their own property describing this concept.

Additional Example: Have students place a ruler in any direction on a coordinate plane. Then, by tracing the top and bottom of the ruler, the students will create parallel lines. Ask each student to find their equations for their personal lines. Check with a partner to see if the equations are correct.

Non-Euclidean Geometry

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to extend students' understanding of geometry beyond parallel and perpendicular lines, angle pairs, and abstract drawings. Most students will enjoy this lesson due to the real life application. However, even if students are unfamiliar with taxicabs, extend this lesson to rural areas with roads that intersect.

History Connection! Take time to discuss Euclid during the lesson. Show the following picture of his book, *Elements.* Go through the first five postulates. Use the following website to gather additional information. Or, have students write mini-reports of the impact Euclid had on present day Geometry. Offer "Euclid Day," a day of celebration on behalf of Euclid. The possibilities are endless!

Create a class discussion regarding Euclid's 5^{th} Postulate. "If two lines are cut by a transversal, and consecutive interior angles have a total measure of less than 180 degrees, then the lines will intersect on that side of the transversal." Mathematicians tried to prove this true, thus making it a theorem as opposed to a postulate for 2000 years. Since many mathematicians did not regard this as truth, non-Euclidean geometries were founded.

Other types of non-Euclidean geometry are: spherical geometry, hyperbolic geometry and elliptic geometry. In spherical geometry, straight lines are great spheres, so any two lines meet in two points. There are also no parallel lines (think longitude lines meeting at the poles). Hyperbolic geometry satisfied all Euclid's postulates except the parallel postulate, replacing it with "For any infinite straight line L and any point P not on it, there are many other infinitely extending straight lines that pass through P and which do not intersect L." Elliptic geometry replaces Euclid's parallel postulate with "through any point in the plane, there exist no lines parallel to a give line."

1.4 Congruent Triangles

Triangle Sums

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to familiarize students with the polygonal sum theorem and its specific application, the triangle sum theorem. Students will incorporate algebra to find unknown polygons given an interior angle sum and find an interior angle sum given a specific polygon.

Physical Models! Using the triangle from the introduction, have students measure the interior angles of the triangle. Then, by extending segments $\overline{AB}, \overline{BC}, \overline{AC}$, students see three new exterior angles and should measure these too. *Students should make the connection that the interior and exterior angles form a linear pair, and by the Linear Pair Theorem, are supplementary.*

Extension! The Triangle Sum Theorem is a special case of the Polygonal Sum Theorem, in which the sum of interior angles of an $n-$gon is found by the following formula:

$T = 180(n - 2)$, where $n \geq 3$

Ask students to brainstorm the reasoning behind $n \geq 3$. *Students should remember that a polygon cannot be formed with less than three segments.*

Physical Model! To demonstrate the explanation of the Triangle Sum Theorem found on page 209, students should draw a triangle and measure all three interior angles. Students can then rip or cut off any two angles and, like a puzzle, fit them with the third. The result is a straight line with a measurement of 180 degrees.

Technology Activity! Using a geometric software program, have students follow these steps:

1. Place 3 noncollinear points on the plane, labeled A, B, C. Connect these three points to form $\triangle ABC$.
2. Compute the measures of $\angle A, \angle B, \angle C$. *How can we classify this triangle? Is it scalene, equilateral, or isosceles? Is it acute, obtuse, or right?*
3. Find the sum of all three angles. *It should equal* 180 degrees.
4. Highlight points A, B (thus $\overline{AB}$), and point C. Using the appropriate menu, click on "construct a parallel line." There should a line parallel to $\overline{AB}$.
5. Locate points on the line parallel to $\overline{AB}$, calling them F and E.
6. Measure $\angle ABF$ and $\angle CBE$. Calculate the sum of these two angles and $\angle A$. *The sum should equal* 180 degrees.

Congruent Figures

Pacing: This lesson should take one class period

Goal: The goal of this lesson is to prepare students for the five triangle congruency theorems: Angle-side-angle, side-angle-side, side-side-side, angle-angle-side, and the special case of side-side-angle, the hypotenuse-leg theorem. This lesson provides a needed introduction by looking at congruent triangles in a non-formal manner.

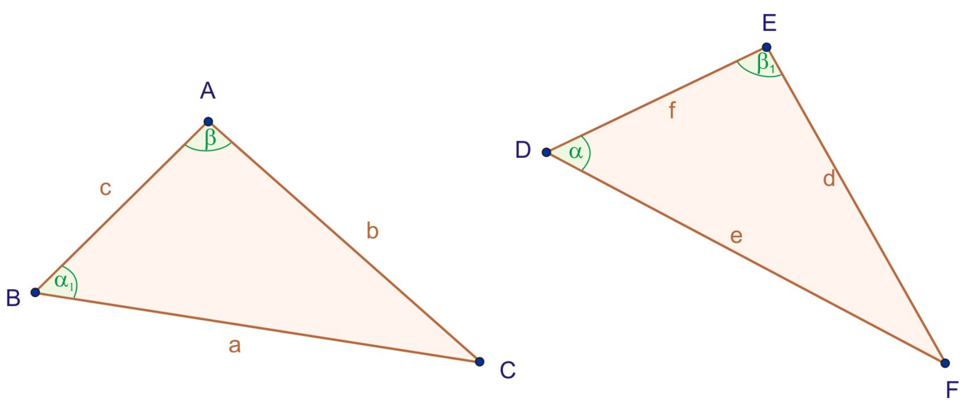
Notation, Notation, Notation! Revisit congruence notation from earlier lessons: $\cong$ Stress the importance of labeling each congruency statement such that the congruent vertices match. For example, $ABCD \cong LMPQ$ shows $\angle A \cong \angle L, \angle C \cong \angle P$, and so forth.

Stress the tic mark notation in relation to the congruency statement. Simply because the letters used are in alphabetical order does not necessarily mean they will line up this way in a congruency statement. Students must follow the tic marks around the figure when writing congruency statements.

Look Out! Students begin to become confused with notation at this point. Be consistent with notation. Have groups of students create classroom posters regarding symbols.

Use the following mantra, "Distances are equal and side lengths are congruent." While each lends to the other, students need to understand which value applies.

Ask students to determine if the below triangles are congruent and explain any reasoning. Use the following information: $\overline{DE} \cong \overline{AB}$ and $\overline{EF} \cong \overline{BC}$. *These are not congruent because the double tick marks do not match.*



Proof Using SSS

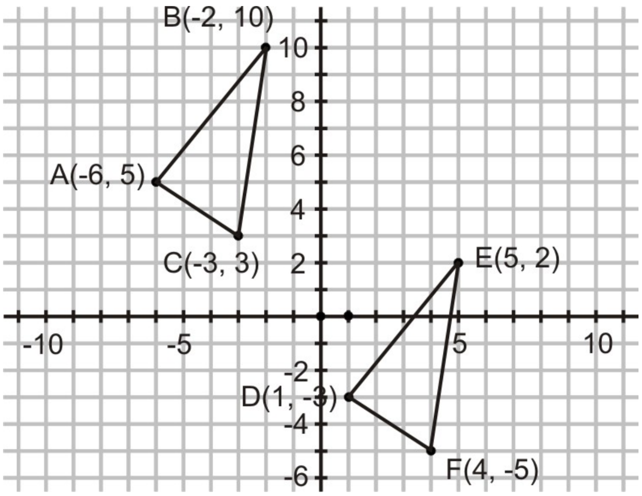
Pacing: This lesson should take one class period

Goal: This lesson introduces students to the formal concept of triangle congruency. The easiest for students to visualize is the side-side-side (SSS) Congruence Postulate.

Differentiation! For students struggling with the distance formula, encourage them to create a right triangle using the $\frac{\text{rise}}{\text{run}}$ of the line. Then students can use Pythagorean's Theorem $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$ to find the length of the segment.

Arts and Crafts Time! Students can visualize the SSS Congruence Postulate in the following way. Using three 8.5" $\times$ 11" sheets of paper, have students create three dowels by rolling tightly from corner to opposite corner. Cut the dowels to the following lengths: 4", 6", and 7." Using tape, glue, or staples, the students should create a triangle and compare their figure with the figures of several classmates. *Students should see that all triangles are congruent, helping to demonstrate the rationale behind the SSS Congruence Postulate.*

Background Information! The SSS Congruence Postulate can be proved using the idea of congruence. In theory, as mentioned in the lesson, these two triangles represent a slide of 7 units right and 8 units down. A slide, or *translation,* is an isometry, preserving distance and angle measure. Thus, since the distances are equal, the lengths are congruent.



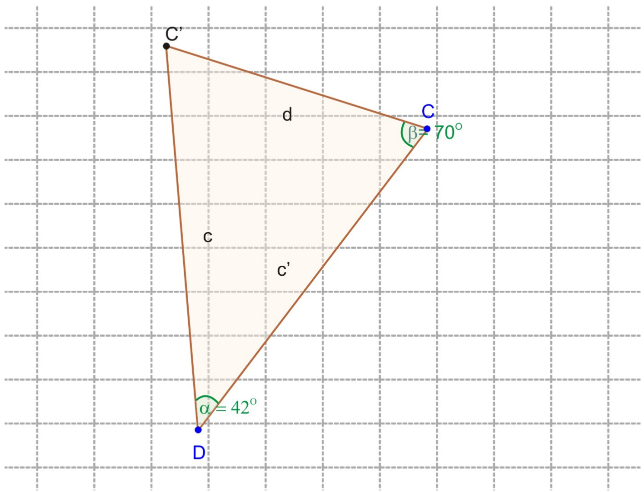
Proof Using ASA and AAS

Pacing: This lesson should take one class period

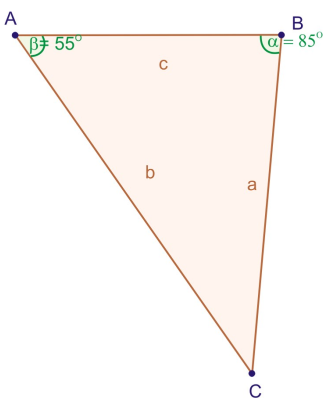
Goal: Students will learn how triangles can be determined congruent using Angle-Side-Angle and Angle-Angle-Side Theorems.

Look out! These two congruency postulates look identical to students. Use this method when explaining which to use. When **moving left to right of a triangle,** recite the information you have. If you have an angle-angle-side (or side-angle-angle), use the AAS Congruence Theorem. If you have an angle-side-angle, use the ASA Congruence Postulate. *In-class activity:* Have the students complete the following two activities:

Activity #1: Have students draw a 42 degree angle. Measure one ray 5 cm. Using the unmeasured ray, students will now draw a 70 degree angle. Connect both rays to form a triangle. Have students share their drawings. *All drawings should be congruent, according to AAS.*



Activity #2: Have students draw a 55 degree angle, measuring one ray 5 cm. Using this same ray, construct a second angle of measure of 85 degrees. Connect both rays to form a triangle. Have students share their drawings *(all drawings should be congruent, according to ASA).*



English Connection! There are three main types of proofs: two-column, flow charts, and paragraph form. Use all three methods when presenting this lesson (and subsequent lessons). Some students will be more comfortable with organizing information in a 2-column format and others will use a flow chart. Students with a strong language arts background will find that writing proofs using complete sentences in a paragraph will make the process of proving similar to drafting a persuasive essay.

Homework Check! Using the bonus question following #10 is a great review for students regarding the appropriate way to name angles. Be sure you review this question in class!

Proofs Using SAS and HL

Pacing: This lesson should take one class period

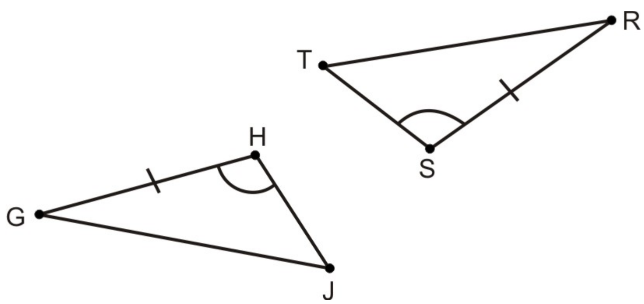
Goal: This lesson is to complete the triangle congruency postulates and theorems. There is a brief description of the anatomy of a right triangle and Pythagorean's Theorem. It also introduces the notion that AAA and SSA relationships will not produce congruent triangles.

Note Taking Time! Students need help organizing these congruency proofs. Use a table similar to the one shown below, projected on a digital imager or projector and take time to organize the material. Extend the organizer one more column and two more rows to accommodate the information gathered in the **Using Congruent Triangles** lesson.

Table 1.5

Congruence Type	Definition	Diagram
SSS		
SAS		
ASA		
AAS		
HL		

Extension! Use the following diagram from example 1 to further assess your students' understanding of Triangle Congruencies. Have students list the information needed for each of the four congruencies learned thus far: ASA, SSS, SAS, and AAS.



Triangle Anatomy! Understanding right triangle anatomy is crucial, especially once students move into trigonometry. Before discussing the Hypotenuse-leg Congruence Theorem, draw a blank right triangle. Have movable words of LEG, LEG, and HYPOTENUSE. Encourage students to correctly label the right triangle by moving the terms to the correct positions.

Pythagorean's Theorem! This is one of the most useful theorems in mathematics; it is used for distance, finding missing side lengths in a right triangle, and is the basis of the Law of Cosines. Use this silly memory device. You will use Pythagorean's Theorem enough times to cause *Post-traumatic Stress Disorder (PTSD).* Each time you say, "PTSD?" students should respond, "$a^2 + b^2 = c^2$!"

In-class Activity! Similar to the activities showing ASA and AAS congruencies, students will use the following two activities to show there are no such congruencies for AAA and SSA relationships.

Using Congruent Triangles

Pacing: This lesson should take one class period

Goal: The goal of this lesson to illustrate how congruent triangles can be used to determine congruent corresponding segments or vertices and find distance.

Extension! Using the graphic organizer from the **SAS and HL** lesson, include the information presented in the introduction.

Look out! This is approximately the lesson in which students will ask the question, "When will we ever need to know why triangles are congruent? How will this apply to my life?" Encourage students to research aviation, construction, manufacturing, and so forth to explore real world uses of triangles. Have students draft an essay with their findings and present it to the class. *Here are some real life examples: Architecture such as bridge construction and roof rafters; proving properties of other figures, such as parallelograms, squares, rhombuses; determining congruent sails on sailboats; ensuring stairs are the same (the risers have congruent triangles cut from them).*

Arts and Crafts Time! Students confuse "drawings" with "constructions." Stress to students that a true construction can only be made with the following tools: compass and a straightedge. Constructions cannot be measured using degrees from a protractor or units from a ruler. Once these two tools come into play, the construction is now considered a drawing.

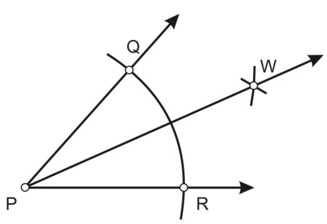
For a beginner warm up, encourage students to play with the compass. Many will make faces, animals, or flowers. Have students decorate their drawings and post them on a bulletin board.

Take time and go through the perpendicular bisector constructions as a class.

Extension! Have students continue to practice constructions by constructing an angle bisector using the following directions.

1. Have students draw an angle of their choice, labeling it $\angle B$.
2. Place the compass on B and draw an arc through both sides of the angle. Call the points of intersection A and C.
3. Place the compass on point B and draw an arc across the interior of the angle.
4. Without changing the radius of the compass, place the compass on point C and draw an arc across the interior of the angle.
5. Label the intersection of the two arc as D. Draw $\overrightarrow{BD}$.
6. $\overrightarrow{BD}$ is the bisector of $\angle ABC$.

An example is show below.



Isosceles and Equilateral Triangles

Pacing: This lesson should take one class period

Goal: There is a natural progression from triangle congruencies to isosceles and equilateral triangles. This lesson illustrates the special properties that arise from these two types of polygons.

It is helpful for students to reproduce the isosceles triangle drawing. They will benefit from using this diagram when completing the exercises.

Another useful theorem of isosceles triangles has an especially long name. The *Isosceles Triangle Coincidence Theorem* states, "If a triangle is isosceles, then the bisector of the vertex angle, the perpendicular bisector to the base, and the median to the base are the same line." Therefore, the perpendicular bisector to an isosceles triangle's base is the same line generated by the angle bisector of the vertex.

Because of this theorem, step 5 of the proof in example 1 can be alternatively justified using the HL Congruence Theorem. *$\overline{AD}$ creates two right angles, $\angle ADC$ and $\angle ADB$.*

An equilateral triangle is a special type of isosceles triangle. Some definitions of isosceles state, "An isosceles triangle has **at least two** sides of equal length." While this book does not use this exact definition, it is implied by using the Isosceles Triangle Base Angle Theorem with equilateral triangles.

Extension! Since the Isosceles Triangle Base Angle Theorem and its converse are true, have your students create a biconditional of this theorem. *The base angles of a triangle are congruent if and only if the triangle is isosceles.*

Extension! Students may fall into the trap of assuming figures must be both equiangular and equilateral. To show a counterexample to the belief, have students create equiangular polygons that are not equilateral. For example, students can draw a pentagon having five interior angle measurements of 108 degrees with varying side lengths.

Congruence Transformations

Pacing: This lesson should take one class period

Goal: This lesson introduces students to isometries, which can also be found in Chapter 12. The main focus of this lesson is on congruent triangle transformations.

Name That Transformation! Create slides or cards with images of transformations (preimage and image). Flash one at a time to the class. Have the students answer on a personal whiteboard or other monitoring system. Offer one point for each transformation the student correctly answers – offer double points if the student can explain why s/he chose that particular transformation.

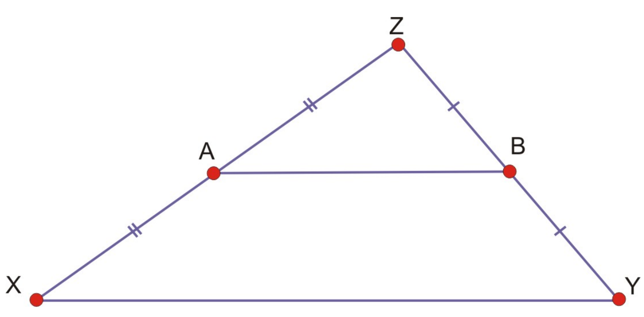
1.5 Relationships Within Triangles

Midsegments of a Triangle

Pacing: This lesson should take one class period

Goal: This lesson introduces students to the concept of midsegments and the properties they hold.

Extension: Have students find the other midsegments to the triangle of the introduction



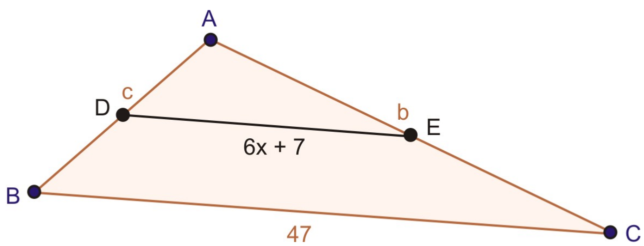
Students will need to measure XY to find its midpoint, call it C, and create two new line segments: $\overline{AC}$ and

$\overline{BC}$. Encourage students to analyze the three midsegments. What do the segments form? What relationships can be seen? *The midsegments form a second triangle.* $\overline{AB}$ *is parallel to* $\overline{XY}$, $\overline{AC}$ *is parallel to* $\overline{ZY}$, *and* $\overline{BC}$ *is parallel to* $\overline{XZ}$.

Discuss the proof of the first section of the Midsegment Theorem with your class. It may be helpful to begin with the figure above, and with each new justification, add it to the drawing.

Extension! Have your students take the paragraph proof of the Midsegment Theorem and rewrite it into 2-column form.

Additional Example: Find x, and the lengths of DE and BC.

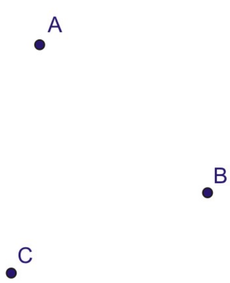


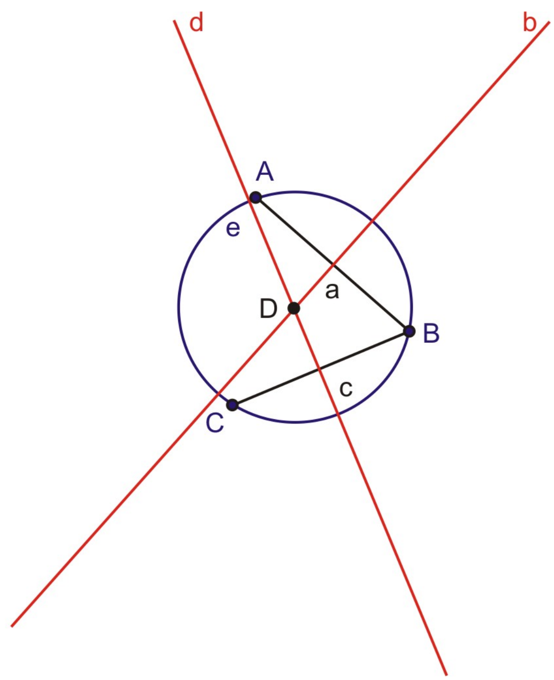
Perpendicular Bisectors in Triangles

Pacing: This lesson should take one class period

Goal: Students are introduced to the concept of the *circumcenter of a triangle.* The process of finding the circumcenter uses perpendicular bisectors, which students can either draw using a protractor or construct using the method found in the *Using Congruent Triangles* lesson.

Additional Example: Construct a circle passing through these three points:





Angle Bisectors in Triangles

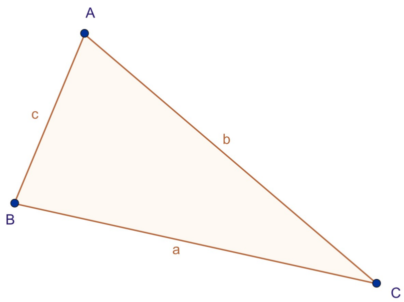
Pacing: This lesson should take one class period

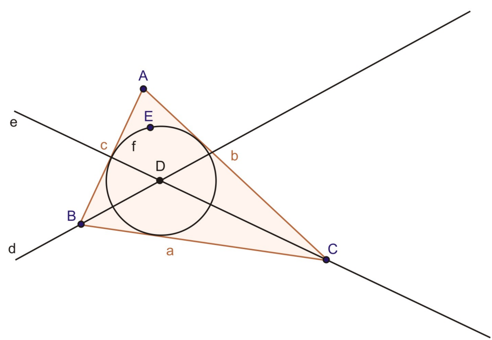
Goal: Students will learn how to inscribe circles within triangles using the concept of angle bisectors. Students should be familiar with angle bisectors, as there were presented in chapter one.

Summary: To inscribe circles within triangles, the center of the circle is the intersection of angle bisectors. To circumscribe circles about triangles, the center of the circle is the intersection of perpendicular bisectors.

To construct the angle bisectors, repeat the *In-class Activity* found in *Using Congruent Triangles* lesson.

Additional Example: Locate the circle inscribed within the following triangle.





Medians in Triangles

Pacing: This lesson should take one class period

Goal: This lesson introduces the centroid of a triangle. By now, students should be familiar with the three main intersection points regarding triangles: the circumcenter, the incenter, and the centroid.

History Connection! In addition to an infamous dictator, it appears Napoleon Bonaparte was an excellent mathematician. He was the top mathematics student in his school, taking algebra, trigonometry, and conics. His favorite class, however, was geometry. After graduation, Bonaparte interviewed for a position in the Paris Military School and was accepted due to his mathematical ability. Bonaparte completed the curriculum in one year (it took average students two or three years to complete) and was appointed to the mathematics section of the French National Institute.

During his reign, Bonaparte appointed such men as Gaspard Monge, Joseph Fourier, and Pierre Laplace to recruit teachers and reform the curriculum to emphasize mathematics. Napoleon's Theorem is named as such because, while Napoleon was not the first person to discover it, he supposedly found it independently.

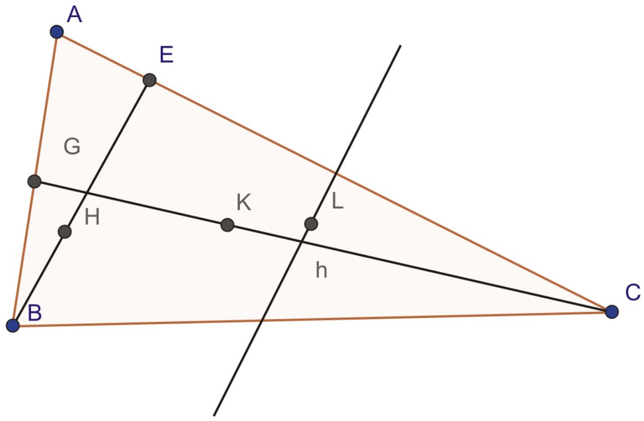
Altitudes in Triangles

Pacing: This lesson should take one class period

Goal: Students will learn how to construct an altitude and how this auxiliary line differs from the median. Altitudes are important in such geometrical concepts as area and volume.

Guided Discovery Questions! What is the difference between a median and an altitude? Is a median always an angle bisector? Can the perpendicular bisector be a median?

Review Question: Using the diagram below, ask students to label the following auxiliary items: median, circumcenter, incenter, orthocenter, altitude, perpendicular bisector



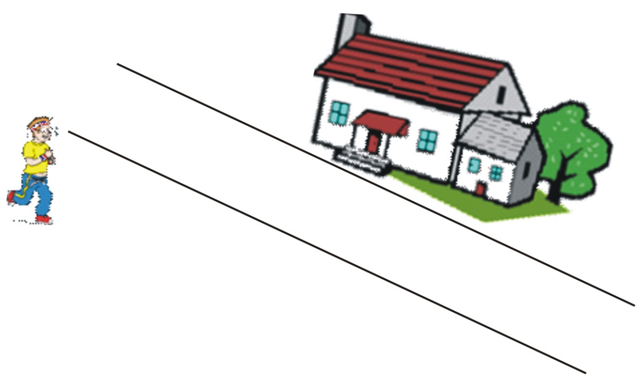
Inequalities in Triangles

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to familiarize students with the angle inequality theorems and the Triangle Inequality Theorem. The lesson further extends the concepts of perpendicular lines and triangles to deduce the shortest path between a point and a line is its perpendicular, thus leading to parallel lines.

If you have not used the in-class activity found in chapter 1, *Classifying Triangles* lesson, include it here. Otherwise, reintroduce the concept in this lesson.

Additional Example: Jerry is across the street in the following diagram. Draw the path she should travel to minimize the distance across the street.



Inequalities in Two Triangles

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to utilize the concept of inequality to determine corresponding angle measures and side lengths of a triangle.

Review! Be sure your students can solve inequalities. Try these as a warm-up or brief review.

1. $8x - 4 + x > -76 \quad x > -8$
2. $-3(4x - 1) \geq 15 \quad x \leq -1$
3. $8y - 33 > -1 \quad y > 4$

Additional Examples:

List the sides of each triangle in order from shortest to longest.

1. $\triangle ABC$ with $m\angle A = 90, m\angle B = 40, m\angle C = 50$. AC, AB, BC

2. $\triangle XYZ$ with $m\angle X = 51, m\angle Y = 59, m\angle Z = 70$. YZ, XZ, XY

List the angles of the triangle in order from largest to smallest.

3. $\triangle ABC$ where $AB = 10, BC = 3$, and $CA = 9$. $\angle C, \angle B, \angle A$,

Indirect Proof

Pacing: This lesson should take one class period

Goal: Students have seen several theorems proven using the indirect proof. Indirect proof is an invaluable resource to students attempting to prove theorems or postulates.

Indirect proofs typically have four sentences that can be summarized by the following acronym: ATBT.

A – Assume (the opposite of what you're trying to prove. Essentially, this is the negation of the conclusion of the conditional)

T – Then (by doing some mathematics or using reasoning, a conclusion can be made)

B – But (here lies the contradiction. The conclusion you made in the previous sentences defies a definition or previously proved theorem)

T – Therefore (your original conditional must be true)

Indirect proofs are usually used when the word *cannot* appears in the proof.

Example: Prove a triangle cannot have two right angles.

Assume a triangle can have two right angles. Then, using the Triangle Sum Theorem, $90 + 90 + x = 180$ degrees. Using the angle addition property and the addition property of equality, $x = 0$ (here lies your contradiction). But a triangle must have three angles greater than zero degrees. Therefore, a triangle cannot have two right angles.

Additional Example: Prove $\sqrt{15} \neq 4$.

Assume $\sqrt{15} = 4$. *Then, by squaring both sides,* $15 = 16$. But $15 \neq 16$. *Therefore,* $\sqrt{15} \neq 4$

1.6 Quadrilaterals

Interior Angles

Pacing: This lesson should take one class period

Goal: Students will use the Triangle Sum Theorem to derive the Polygonal Sum Theorem by dividing a convex polygon into triangles.

Inquiry Based Learning! Analyzing a pattern is another method to looking at the polygonal sum theorem. Begin by setting up the below chart. Ask students to fill in the second column, asking if the number of side lengths can form a polygon. Ask students to then complete the obvious interior angle sums such as triangle and quadrilateral. Encourage students to see a pattern and create its function. *Students should see that the "starting" sum is* 180 *and each subsequent polygonal sum is* 180 degrees *greater.*

Vocabulary! Reiterate to students that a diagonal is drawn from any vertex to a non-adjacent vertex.

Extension! Does this theorem work for non-convex polygons? Pose this question to students as you draw several non-convex polygons on the board. *Since the polygon is non-convex, the "indented" angle will always be obtuse, showing this theorem will only work for convex polygons.*

Additional Example: The sum of the interior angles of an $n-$gon is $3,960$ degrees. What is n? $n = 24$

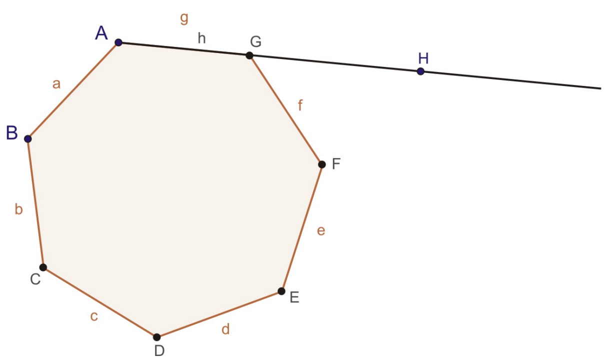
Exterior Angles

Pacing: This lesson should take one class period

Goal: This lesson introduces students to exterior angles of polygons. The Linear Pair Theorem is used to determine the measures of exterior angles.

Stress to students that there are two possibilities for exterior angles and it will be extremely important to label the angle correctly.

Additional Examples: Using what you have learned thus far, determine the measure of $\angle FGH$. *Students will use the Polygonal Sum Theorem to determine the sum of the interior angles in the heptagon is* 900°. *Dividing by* 7*, each interior angle has a measure of* 128.57°. $\angle FGH$ *forms a linear pair with* $\angle AGF$ *and are supplementary. Therefore, the measure of angle* $FGH = 51.43$ degrees.



Classifying Quadrilaterals

Pacing: This lesson should take one to two class periods

Goal: Students are introduced to the most common quadrilaterals and relationships they share. A Venn diagram is provided as a visual to allow students to visualize how a quadrilateral such as a square fits with a rectangle, parallelogram, and trapezoid.

Have students take the Venn diagram and transfer it to a hierarchy, showing the most general quadrilateral to the most specific quadrilateral.

Discussion! Begin a discussion with students regarding trapezoids and parallelograms. Some textbooks describe a trapezoid as, "a quadrilateral with at least one pair of parallel sides." Discuss this possible definition with your students. How would the Venn diagram change if this definition were accepted as true? Should it be accepted as true? Why is the definition of a trapezoid provided in this text stating "exactly one pair of parallel sides?"

Additional Examples: Ask students to answer the following questions, either on a personal whiteboard, journal entry, or in a Think-Pair-Share group.

1. *Always, sometimes, never.* All rhombi are squares. *Sometimes. A square is a special type of rhombus.*
2. *Always, sometimes, never.* All rhombi are parallelograms. *Always.*
3. *Always, sometimes, never.* Parallelograms are trapezoids. *This answer depends upon the discussion of your class.*

Pythagorean's Theorem AGAIN! Reiterate to your students the connection between Pythagorean's Theorem and the distance formula. This is especially helpful when determining the lengths of segments of a quadrilateral to determine its appropriate classification.

Trapezoids and Parallel Lines! Encourage your students to make the connection between consecutive interior angles of parallel lines and a trapezoid. A trapezoid is really a pair of parallel lines cut by two transversals. Therefore, consecutive interior angles are supplementary (according to Euclid's' 5^{th} Postulate).

Using Parallelograms

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to familiarize students with properties special to parallelograms. These properties are useful when proving a figure is a parallelogram. These properties also hold for rectangles, rhombi, and squares – those quadrilaterals that are "included" within parallelograms in the Venn diagram.

In-class Activity! Instead of using string, your students can also use raw spaghetti noodles. Be sure the segments are equal in length; otherwise, the models may not illustrate a parallelogram appropriately.

Making Connections! Make as many connections as possible. This will help your students see how geometrical concepts fit together. For example, students have learned parallel lines are equidistant from each other. Connect this to a parallelogram

Flash Fast Game! Have your students create flashcards with quadrilateral names on one side and important information or properties on the reverse. Have various types of quadrilaterals, both abstract and real world, ready to show students. Once students believe they have classified the quadrilateral, they are to hold up the appropriate name. You can keep score or use this as a summative assessment.

Proving Quadrilaterals are Parallelograms

Pacing: This lesson should take one class period

Goal: Students will use triangle congruence postulates and theorems to prove quadrilaterals are parallelograms. This lesson serves as an application of the concepts learned in the Triangle Congruence lesson.

Be sure to review each proof in the lesson with your class. Have the students perform a Think-Pair-Share by writing the conditional on the board and having students attempt to prove the statements on their own.

Another Way of Thinking! The proof of, "If a quadrilateral has two pairs of congruent sides, then it is a parallelogram," can be proven using the SSS Congruence Postulate. Instead of using same side interior angles, use the Reflexive Property to state $CE = CE$.

Rhombuses, Rectangles, and Squares

Pacing: This lesson should take one class period

Goal: This lesson demonstrates another application of triangle congruence. Students are shown important properties of rhombuses such as bisecting diagonals and opposite angles.

Refer students back to the Venn Diagram or the hierarchy of quadrilaterals. Make sure students understand that everything that falls within a rhombus possess the same characteristics and properties of a rhombus. Identifying this key relationship will help students understand this lesson.

Remind students that a diagonal is a segment drawn from one vertex to any non-adjacent vertex. *Question to think about:* Will there always be two diagonals for any quadrilateral? What is it is non-convex?

Arts and Crafts Time! Using patty paper and a pencil, have students trace a rectangle. Instruct students to fold the rectangle so the lower left angle fits on top of the upper right angle, thus forming a diagonal. Open the fold and repeat the process on the other diagonal. Overlay the patty paper onto a coordinate grid and have students work through the distance formula to determine the lengths of the diagonals.

Discuss the proof of this as a class, using the patty paper rectangle for further illustration, if necessary.

Be sure students can "take apart" a biconditional into its two separate statements. This may require more practice on behalf of your students before they can determine if the biconditional is true.

Additional examples: Separate these biconditionals into a conditional and its converse

1. The rain will fall if and only if it is cloudy. *If the rain will fall, then it is cloudy. If it is cloudy, then the rain will fall.*
2. An animal is a mammal if and only if it has whiskers. *If an animal is a mammal, then it has whiskers. If an animal has whiskers, then it is a mammal.*
3. An object is a circle if an only if it is the set of points equidistant from a single point. *If an object is a circle, then it is the set of points equidistant from a single point. If an object is the set of points equidistant from a single point, then it is a circle.*

Trapezoids

Pacing: This lesson should take one class period

Goal: This lesson introduces students to the special properties of trapezoids, especially those of the isosceles trapezoid.

Real Life Application! Show a photograph of the John Hancock Center, located in Chicago, Illinois. This structure was the first skyscraper to be built using exterior tube technology, instead of using internal beams as support. Each face of the John Hancock Center is comprised of six isosceles trapezoids.

Try This! Have each student draw three different isosceles trapezoids. Exchange papers with one student. Ask students to check one trapezoid to be sure it is isosceles by measuring non-base sides for congruence and equivalent diagonals. Have correcting students "sign off" on their opinions. Switch papers a second a third time, repeating the process.

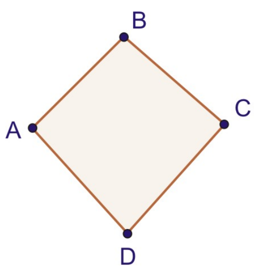
Kites

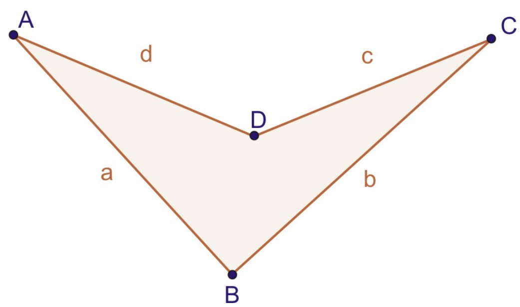
Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to present special properties of kites.

Visualization! Have students draw two isosceles triangles sharing the same base. By erasing the shared base, they have just drawn a kite! *The shared base represents one diagonal of the kite.*

Vocabulary! Using the phrase "ends of the kite" can be misleading for students. The ends do not always mean the endpoints of the longer diagonal. Be sure students can identify the ends of non-convex kites, and non-traditional kites (as shown below).





Beat the clock! Have students draw and cut out two copies of a scalene triangle. In one minute, have students form as many polygons as possible, drawing a sketch of each they form. *You can also use tangrams for this activity.*

1.7 Similarity

Ratios and Proportions

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to reinforce the algebraic concept of ratios and proportions. Proportions are necessary when discussing similarity of geometric objects.

What's the difference? Ratios and rates are both fractions. However, ratios compare same units, while rates compare different units. Ask students to brainstorm types of ratios and rates. *Rates are typically much easier for students to identify – miles per hour, cost per pound, etc.*

Look Out! Students easily get confused when we throw proportions into the mix. For some reason, students do not realize that the equal sign (=) in a proportion is different than a multiplication sign ($*$) when asking to find the product of two fractions. For example, students will attempt to solve these two statements the same way:

$$\frac{3}{4} * \frac{x}{7} \qquad\qquad \frac{3}{4} = \frac{x}{7}$$

Encourage your students to understand the difference between finding the product of two fractions and using the means-extremes method of cross-multiplication

Look Out! Another pitfall is cross-multiplication versus cross-reducing. You may have to take some time to discuss the difference and allow your students to practice doing both.

Additional Example: A model train is built $\frac{1}{64}$ scale. The stack of the model is 1.5". How tall is the real smokestack?

Food For Thought! "Why are these the means?" The best answer I have heard was, "The extremes are called such because they are on the far ends of the equation, meaning $ad = bc$." Before the fraction bar became commonplace, people would write fractions using the colon. $3 : 6 = 1 : 2$. Therefore, 6 and 1 represent the means (middle values), while 3 and 2 represent the extremes.

Properties of Proportions

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to demonstrate to students that the order in which you write the proportion is irrelevant, the answer comes out identical.

Try This! Prior to reading through the lesson, and using the following example, ask students to create their proportion. Look for several different proportions. Spread these students around the room. Have the remainder of the class match their proportion to the "totem pole." *This allows students to see that there is no one correct way to write a proportion.*

Question: A yardstick makes a shadow 6.5' long. Raul is $6'$ 3". How long is his shadow? *Be sure students convert a yardstick to* 3' *before continuing with the proportion.*

While there are many correct ways to write a proportion, encourage students to visualize what the proportion is stating. For example, the following are both correct (as are their reciprocals):

$$\frac{\text{length (stick)}}{\text{shadow (stick)}} = \frac{\text{length (person)}}{\text{shadow (person)}} \qquad\qquad \frac{\text{length (stick)}}{\text{length (person)}} = \frac{\text{shadow (stick)}}{\text{shadow (person)}}$$

Nonetheless, the units are still the same in each ratio. The first proportion uses length and shadow as units while the second uses stick and person.

Similar Polygons

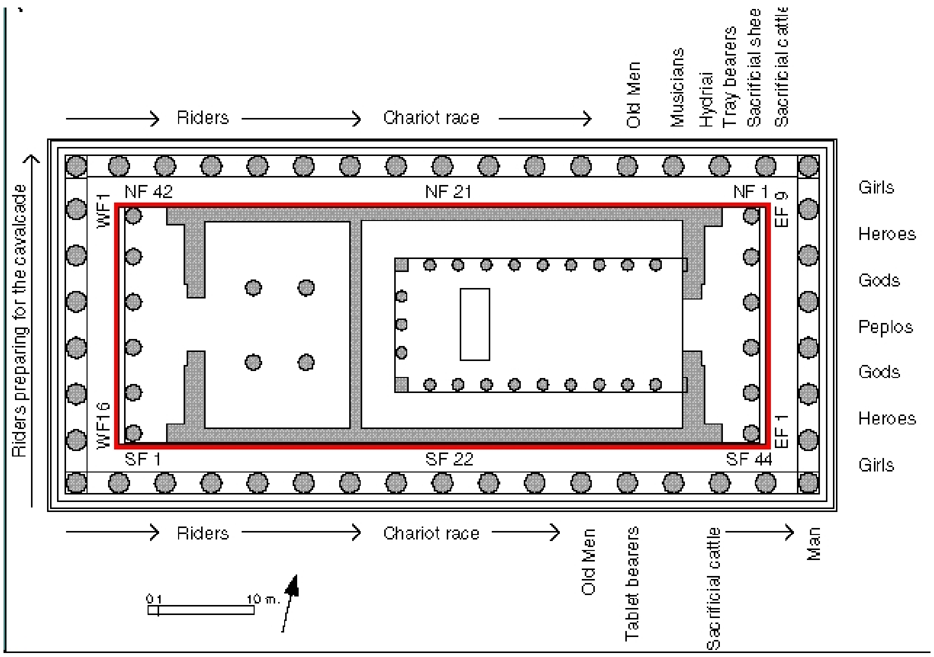
Pacing: This lesson should take one class period

Goal: This lesson connects the properties of proportions to similar polygons. An introduction to scale factors is also presented within this lesson.

An alternative way of determining a scale factor is by using the fraction $\frac{\text{image}}{\text{preimage.}}$ This relates to the notion that similar figures are formed by an applying an isometry, mapping an image onto its preimage. The first figure written in the similarity statement represents the preimage and the second figure represents its corresponding image. Therefore, the scale factor k, is a ratio of the length of the image to the preimage.

Extension! A golden rectangle is a rectangle in which the ratio of the length to the width is the Golden Ratio, approximately 1.6180339888. The Golden Ratio is denoted by the Greek letter Phi and can be found in nature, biology, art, and architecture. For example, golden rectangles can be found all over the Parthenon in Athens, Greece.

Using the diagram below, have your students measure distances and find how many golden rectangles can be found.



http://www.geom.uiuc.edu/~demo5337/s97b/art.htm

Similarity by AA

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to enable students to see the relationship between triangle similarity and proportions. While the angle-angle relationship does not necessarily lead to congruence, its properties are still imperative to similarity.

How Does it Work? Indirect measurement utilizes the Law of Reflection, stating that the angle at which a ray of light (ray of incidence) approaches a mirror will be the same angle in which the light bounces off (ray of reflection). This method is the basis of reflecting points in real world applications such as billiards and miniature golf.

Additional Example: Pere Noel is shopping for a Christmas tree. The tree can be no more than 4 meters tall. Mary finds a tree that casts a shadow of 2 m, whereas Mary (120 cm tall) casts a shadow of 0.8 m. Will the tree fit in Pere Noel's room? *Yes, the tree is* 3 meters *tall, therefore, it will fit in the room.*

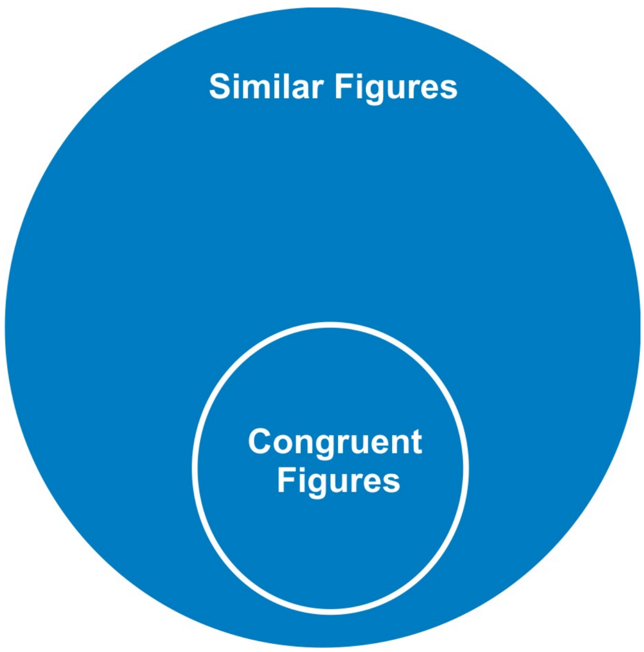
Similarity by SSS and SAS

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to extend the SSS and SAS Congruence Theorems to include similarity.

Visualization! Now may be a good time to discuss similarity and congruency by drawing a Venn diagram. Students may ask the question, "How can triangles be congruent and similar simultaneously?" The diagram below will help clear questions.

Similar figures are usually thought to be produced under dilations (size changes). However, congruent figures are a specific type of similarity transformation. Therefore, rotations, reflections, glide reflections, translations, and the identify transformation all yield similar figures.



Similarity Transformations

Pacing: This lesson should take one class period

Goal: Dilations produce similar figures. This lesson introduces the algorithm to produce similar figures using measurements and a scale factor, k

An easy way to remember expansions versus contractions is a rhyme. Have your students repeat the rhyme until it sticks! "A contraction is a proper fraction!" Improper fractions are mixed numbers, thus creating expansions.

Dilations can also be clarified using a photograph. School pictures are great examples of dilations. Suppose a typical photograph is 4"×6". An 8"×10" enlargement (expansion) does not alter the appearance. This is also true for shrinking photos for wallets. Using a base picture, bring in several enlargements and contractions to further illustrate this concept.

The above visualization can be used when discussing notation. The first dilation is denoted using the apostrophe (') symbol. Sub sequent transformations add an additional apostrophe (", '", and so on). Labeling each picture you've made with this notation will allow your students to visualize how to use image notation.

In-Class Activity! Reproduce the smiley face drawn below for each student. Using the algorithm presented in this lesson, instruct students to enlarge the smiley face by a scale factor of 3.

www.clker.com/clipart-4263.html

Self-Similarity (Fractals)

Pacing: This lesson should take one class period

Goal: Fractals, a term coined only approximately twenty years ago, are a newly discovered genre of mathematics. This lesson introduces students to popular fractals. Fractals possess self-similarity, thus maintaining properties of similarity.

History Connection! Mathematician Benoit Mandelbrot derived the term "fractal" from the Latin word *frangere*, meaning to fragment. A fractal is a geometric figure in which its branches are smaller versions of the "parent" figure. Most fractals are explained using higher level mathematics, however, students can create their own fractal patterns easily.

Additional Example: Make your own fractals! Follow these instructions to create a cauliflower fractal.

1. Hold your paper in landscape format.
2. Draw a horizontal segment $\overline{AB}$ in the center of the paper.
3. Find C, the midpoint of $\overline{AB}$.
4. Measure AC and take half of that distance. This is essentially $\frac{1}{4}AB$.
5. Draw $\overline{DC}$ (the length found in step 4) perpendicular to $\overline{AB}$.
6. Draw lines $\overline{AC}$ and $\overline{BC}$, forming $\triangle ADC$.
7. Repeat steps $3-6$ for $\overline{AC}$.
8. Repeat steps $3-6$ for $\overline{BC}$.

1.8 Right Triangle Trigonometry

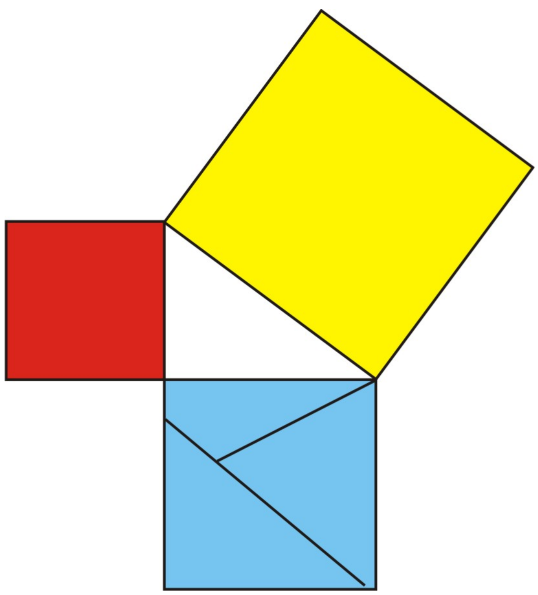
The Pythagorean Theorem

Pacing: This lesson should take one class period

Goal: This lesson introduces the Pythagorean Theorem. It is arguably one of the most important theorems in mathematics, allowing for a multitude of uses, including the Law of Cosines.

Visualization! Here is a second proof of Pythagorean's Theorem that students can do in class.

1. Reproduce the following diagram for each student.
2. Students cut the red square and the blue square from the triangle.
3. Cut along the lines within the blue square. Students should have four pieces.
4. Students will fit these four puzzle pieces onto the yellow triangle, proving that the combined area of the two smaller squares equal the area of the largest square. Hence, $a^2 + b^2 = c^2$



Illustration_to_Euclid's_proof_of_the_Pythagorean_theorem.svg

Additional Examples:

1. A rectangular park measures 500 m by 650 m. How much shorter is the path diagonally than walking around the outside edge?
2. Television sets are described according to its diagonal length. A 42" TV means the diagonal of the screen is 42" long. Suppose the TV below is 36" tall with a 47" diagonal. How wide is the TV?

Converse of the Pythagorean Theorem

Pacing: This lesson should take one class period

Goal: This lesson applies the converse of Pythagorean's Theorem to determine whether triangles are right, acute, or obtuse.

Additional Examples:

1. Can the following lengths form a right triangle? Explain your answer. $10, 15, 225$. *No,* $10^2 + 15^2 = 325$. *However, this will be much less than* 225^2.
2. Find an integer such that the three lengths represent an acute triangle: $9, 12,$ ______. *Sample:* $16, 17, 18, 22 \ldots$
3. Find an integer such that the three lengths represent an obtuse triangle: $8, 19,$ ______. *Sample:* $20, 10, 14, \ldots$

Using Similar Right Triangles

Pacing: This lesson may take two class periods, due to the difficulty of the material

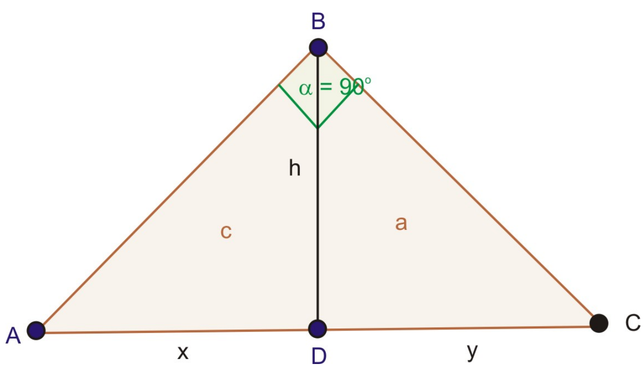
Goal: The concept of geometric mean is used in Advanced Algebra to determine the mean of a widespread data set. In geometry, the geometric mean is illustrated using a right triangle and its altitude.

Look Out! The concept of geometric mean is easy to comprehend, but difficult for student to apply. Spend time in class reviewing this lesson and using additional examples.

An alternative to the abstract formula for geometric mean is, "The altitude of the hypotenuse equals the geometric mean between the segments of the hypotenuse."

Additional Examples:

1. Consider the diagram below. Suppose $h = 12$ and $x = 8$. Find y. $y = 18$
2. Consider the diagram below. Suppose $a = ?, x = 9$ and $y = 11$. Find the value of a. $a = 6\sqrt{5}$
3. Consider the diagram below. Suppose $x = 5$ and $y = 15$. Find the value of the altitude, h. $h = 5\sqrt{3}$



Special Right Triangles

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to encourage the use of shortcuts to find values of special right triangles. These triangles are extremely useful when relating the trigonometric functions to the exact values found within the unit circle.

If students have trouble remembering these special shortcuts, encourage them to use Pythagorean's Theorem and simplify the answer. The resulting answer will equal the shortcut.

In-class Activity! Separate your class into six to ten groups. Write six to ten numbers on the board, one for each group. Instruct the groups to draw an isosceles right triangle with legs of the given length. Have the groups solve for the hypotenuse and share with the remaining groups.

What is the Connection? In any right isosceles triangle, if a leg has value of x, then by Pythagorean's Theorem, $x^2 + x^2 = h^2$. Adding like terms, you get $2x^2 = h^2$. To solve for the value of the hypotenuse, you must square root both sides, leaving the equation $x\sqrt{2} = h$. Therefore, the length of the hypotenuse in ANY right isosceles triangle is equal to leg$\sqrt{2}$

In any $30 - 60 - 90$ triangle, the relationship between the segments is as follows: Let the smallest leg have the value of d. The hypotenuse will always have length $2d$ and the other leg will always have length $d\sqrt{3}$. This again can be proved using Pythagorean's Theorem.

Tangent Ratios

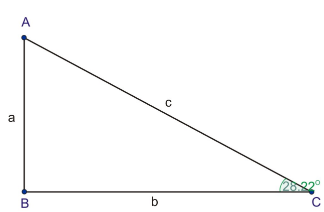
Pacing: This lesson should take one class period

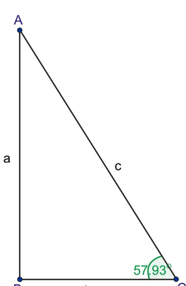
Goal: This lesson introduces the first trigonometric function, the tangent ratio. The tangent seems to be the most natural for students to understand, as opposed to the sine or cosine functions. Tangent ratios occur in many careers, from construction to machine operators.

Check Your Tech! If you are using calculators for the tangent (TAN), cosine (COS), and sine (SIN) functions, be sure to do a "Mode Check." Have each student check to ensure their calculator is set to degrees (DEG) instead of radians (RAD). Having a calculator in radians will provide incorrect answers and students at this level do not know what radians are to correct their answers.

Vocabulary Connection! Students must understand *adjacent* and *opposite* to be successful with trigonometric ratios. They have already had experience with adjacent in previous chapters. Begin by reviewing such vocabulary as adjacent angles and adjacent sides in regards to parallel lines and transversals.

Beginning Activity! Once the class has reviewed the term *adjacent,* offer students these triangles. Ask students to write the terms *adjacent* and *opposite* above the appropriate legs and label the hypotenuse. Always stress that the information stems from the given angle (not the 90 degree)!

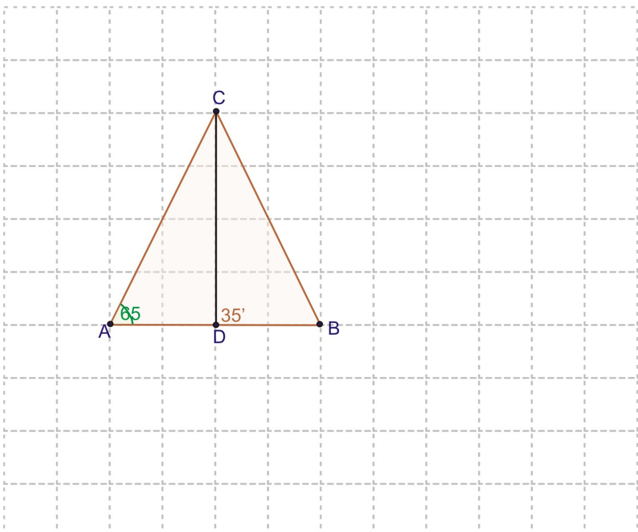




Extension! When discussing tangent values of common angles, such as $30°, 45°$, and $60°$, review how to rationalize the denominator with your students. This concept should have been presented in Algebra 1. While not as common with the increased use of technology, most standardized test questions will present

answers in completely simplified form. For example, the tangent $(30°) = \frac{1}{\sqrt{3}}$. The objective of rationalizing a denominator is to clear it of decimals, radicals, and complex values. To do so, multiply the fraction by a value of 1, in this case, $\frac{\sqrt{3}}{\sqrt{3}}$. The new expression becomes $\frac{\sqrt{3}}{\sqrt{3}}$, a simplified version of the tangent of 30 degrees.
Additional Examples:

1. Pradnya's kite is 50' away from her in the sky, forming a 27° angle with the ground. Pradnya is $4'$ 6" tall. How high is the kite from the ground? *Approximately* 30'
2. The roof pitch is always described in terms of rise/run. Suppose the roof makes a 65° with the horizontal truss and forms the triangle below. How tall is the peak of the roof? 37.53'



Arts and Crafts Time! Using paper, have each student create an astrolabe and use it to determine the height of a tall object, such as a skyscraper or tree.

1. Begin by folding an 8.5" × 11" sheet of notebook paper into a square and remove the excess.
2. Bisect on angle of the square − the segment represents a 45 − degree angle.
3. Bisect each 45−degree angle. There should be three creases – two 22.5 degree angles and one 45−degree angle.
4. Punch a hole in the opposite corner. Tie string through this hole and attach a pencil at the other end.
5. Go outside and line your astrolabe to the top of something, say a tree. Pretend you are hunting and plan to attack something with your astrolabe.
6. Gravity will show you the degree of your sight.
7. Use trigonometric functions to determine its height.

For more information, go to Berkley's website: http://cse.ssl.berkeley.edu/AtHomeAstronomy/activity_07.html

Sine and Cosine Ratios

Pacing: This lesson should take one to two class periods

Goal: The objective of this lesson is to complete the introduction of trigonometric functions by presenting the sine and cosine ratios.

Welcome to Camp $SOH-CAH-TOA$! To make trigonometry fun, invite students to Camp $SOH-CAH-TOA$. Wear a camp counselor outfit, arrange your students in a circle around a makeshift campfire, and begin with an old-fashioned tent (one you can use to point out right triangles).

Have students sketch your tent, splitting it into two right triangles at the altitude (good use of vocabulary!). State that the angle the tent makes to the ground is 55° (something you cannot use special triangles for). Ask students to label each triangle with the appropriate terms: adjacent leg, opposite leg, and hypotenuse. The question is, "How long is the outside edge of the tent?" Question why the tangent ratio cannot be used *(the question you want to answer is not the opposite nor adjacent leg).* Ask for additional ways to solve the problem. Present the sine and cosine ratios.

Additional Examples/Extensions:

1. Given the triangle below, find sine(A) and cos(B). What is special about these two answers?
2. Evaluate $(\cos(65))^2 + (\text{sine}(65))^2$. List as much as you can about this expression and the answer you received. Generalize this question. $(\cos(65))^2 + (\text{sine}(65))^2 = 1$, *the degrees of the sine and cosines are the same value, so the cosine of an angle square plus the sine of an angle squared should equal* 1.
3. Suppose a fireman's ladder is 39' long is placed against the side of a building at a 62 degree angle. How high will the ladder reach? 18.31'

Inverse Trigonometric Functions

Pacing: This lesson should take one to two class periods

Goal: In the previous two lessons, students used the special trigonometric values to determine approximate angle measurements. This lesson enables students to "cancel" a trigonometric function by applying its inverse to accurately find an angle measurement.

Using Previous Knowledge! Begin by listing several mathematical operations on the board in one column. In a second column, head it with "Inverse." Be sure students understand what an *inverse* means *(an inverse cancels an operation, leaving the original value undisturbed).*

For example,

Table 1.6

OPERATION	INVERSE
Addition	
Squaring	
Division	
Subtraction	
Tangent	
Sine	

The first four are typically easy for students *(Subtraction, square root, multiplication, and addition)*. You may have to lead students a little more on the last two *(inverse tangent and inverse sine)*. Students may say, "Un-tangent it." Use the correct terminology here, but also use their wording, if at all possible. Students will be able to cancel the trigonometric function using the inverse of that function, even though they may use incorrect terminology.

Outside at Camp $SOH - CAH - TOA$*!* Find the angle of inclination of the sun! Students love this activity, as it gets them outside and applying mathematics in real life. Explain how the Earth progresses around the sun, giving seasons. Also explain how the Earth's tilt lends to the number of hours of daylight. The combination of these principles describes the angle of elevation (inclination) of the sun in the sky. Create groups of three or four. One student is the statue, one student is the surveyor, and the third is the secretary. Once outside, the statue stands on a flat plane while the surveyor measures the statue's height and shadow length in the same units and relays this information to the secretary. The secretary draws a right-triangle sketch of the statue and shadow. The goal is to find the angle of elevation (the angle made between the horizon and sun).

Acute and Obtuse Triangles

Pacing: This lesson could take two to four class periods

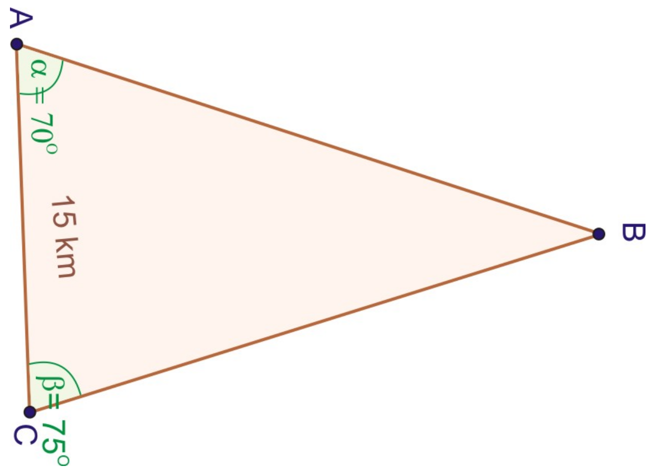
Goal: The purpose of this lesson is to extend trigonometric ratios to non-right triangles. This is done using to new laws: the Law of Sines and the Law of Cosines. The Law of Sines is much easier to present to students than the Law of Cosines. The Law of Sines uses proportions, while the Law of Cosines uses a general form of Pythagorean's Theorem.

Shortcut! Students confuse themselves regarding which law to use. A shortcut to use is as follows: If the triangle has more side information than angle information, use the Law of Cosines. If the triangle has more angle information than side information, use the Law of Sines. And of course, if you cannot solve it with the law you have chosen, "Choose the Other One!"

Relate to Triangle Similarity! Have students recall the five basic types of triangle similarity: SSS, SAS, AAS, AAA, ASA. Reading the given information from left to right, if the information ends in an "angle," use the Law of Sines. If the information ends in a "side," use the Law of Cosines.

Additional Examples:

1. A fire is spotted in Yellowstone National Park by two forest ranger stations. Fire Station A is 15 km from Fire Station B. The angle at which the fire is spotted by Fire Station B is 75 and the angle at which the fire is spotted by Fire Station A is 70 degrees. Which fire station should report to the fire? $AB = 25.26$ km *and* $BC = 24.57$. *Therefore, Fire Station A should report to the fire.*



2. Suppose $\triangle ABC$ has the following values: $m\angle C = 40, a = 8, b = 9$. Find c. $c = 5.89$

1.9 Circles

About Circles

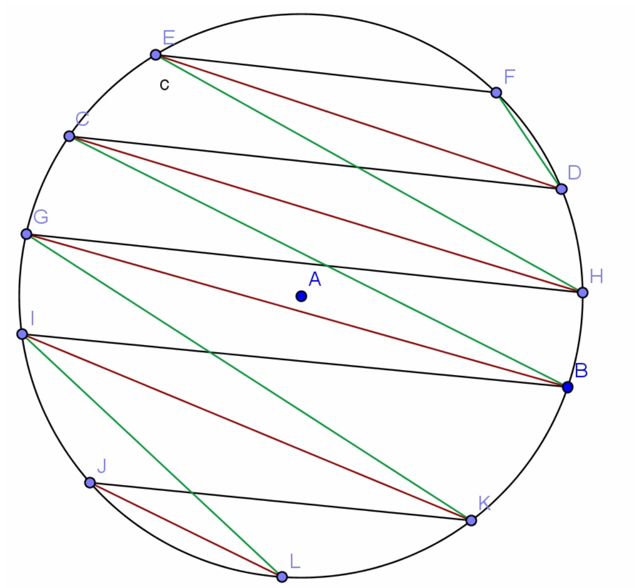
Pacing: This lesson could take two to four class periods

Goal: This lesson discusses numerous characteristics of circles. Inscribed polygons, equations of a circle, diameters, secants, tangents, and chords are all presented in this lesson.

Round Robin! Once students have read through this lesson, try this fun activity. Round robin tournaments are scheduled so each team players another exactly once. Circles and chords are used to schedule such a tournament.

Using a compass, have the students construct a circle that takes up most of an 8.5" $\times$ 11" sheet of copy paper. Decide upon a collegiate conference, such as the Big 10 and instruct students to evenly space these dots around the circle. *To do this, divide* 360° *by* 11. *This will give students the number of degrees before each new dot is placed.* Choosing 11 colored pencils, begin at one dot and draw a chord to a second dot (see diagram below). Draw parallel chords until all but one team has an opponent; the leftover team has a bye. This color represents week 1. Using a second color, start at another dot and connect it to a different team. Continue this process until all 11 weeks have been "scheduled."

If there is an even number of teams to be scheduled, place one dot in the center of the circle and set each remaining dot on the boundary. Start by drawing a radius from the center to any point, then draw chords to the remaining teams. There will be no byes with even numbered teams.



Visualization! Fill a clear bowl or container with water. Show how concentric circles are formed by dropping a rock into water, forming ripples. Concentric circles can also be formed when raindrops hit a body of water, such as a lake, puddle, etc.

Tangent Lines

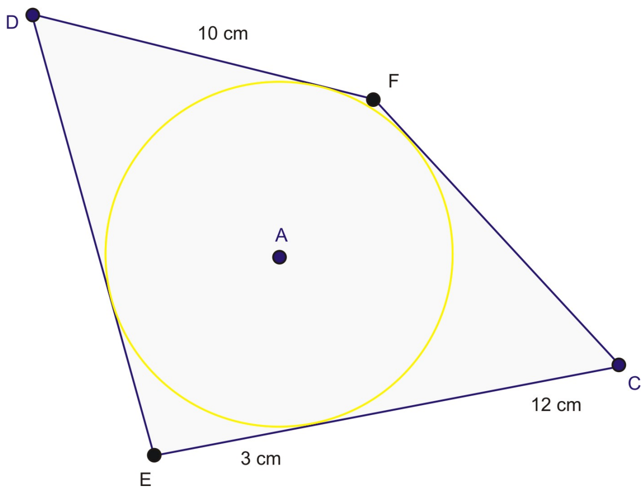
Pacing: This lesson should take one to two class periods

Goal: The purpose of this lesson is to connect the radius of a circle to its tangent. The real life applications of tangents are found in the additional examples below.

Connection! Tangents are used in Calculus; the slope of the tangent line represents the derivative. Students should make this connection visually once Calculus begins.

Additional Examples:

1. Find the perimeter of $ABCD$. 50 cm



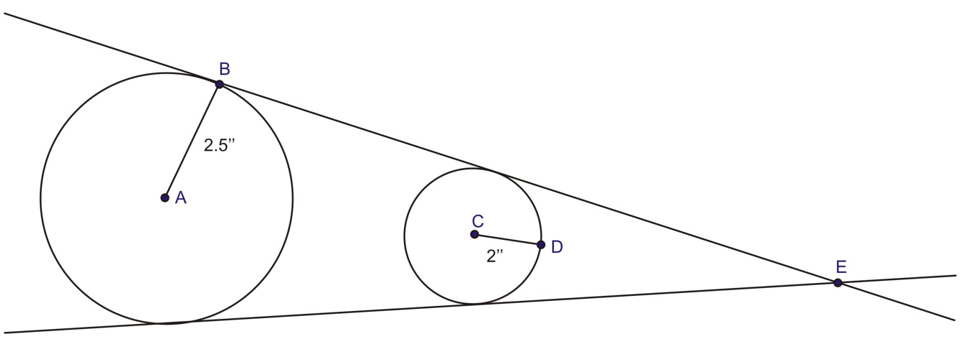
Common Tangent and Tangent Circles

Pacing: This lesson should take one class period

Goal: This lesson focuses on the properties shared with circles sharing common tangents.

Additional Example:

1. A dirt bike chain fits snugly around two gears, forming a diagram like the one below. Find the distance between the centers of the gears. Assume the distance from B to the top of the second circle is 30.25". 30.254"



Arc Measures

Pacing: This lesson should take one class period

Goal: This lesson introduces arc measurements. Central angle knowledge is important when creating and interpreting pie charts and when determining arc length and the area of a sector.

Students may need extra practice when it comes to finding the value of the arc. Set up several questions and use personal whiteboards to check students' understanding.

The Arc Addition Property should be familiar to students; it is quite similar to the Angle Addition Property.

Additional Example:

Identify the following in $\odot A$

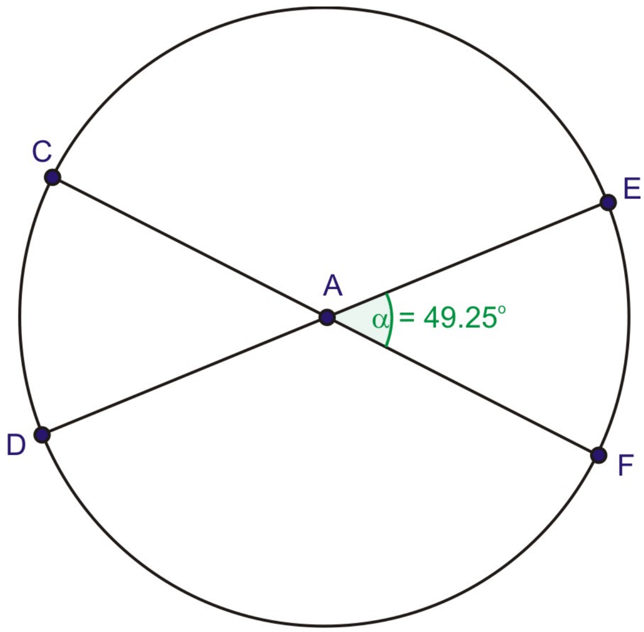
A. Minor arcs

B. Major arcs

C. Semicircles

D. Arcs with equal measurement

E. Identify the four major arcs that contain point F.



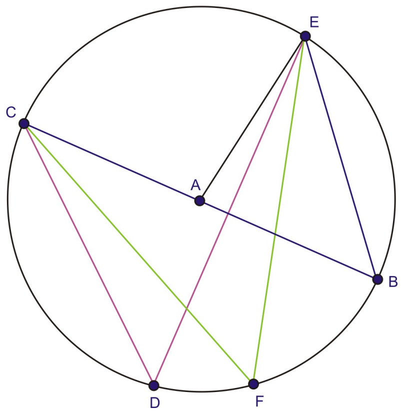
Inscribed Angles

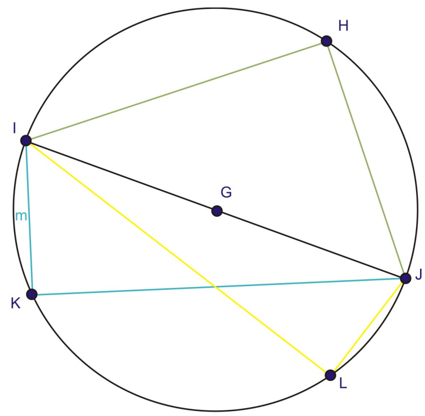
Pacing: This lesson should take one to one and one-half class periods

Goal: This lesson will demonstrate how to find measures of inscribed angles and how to find the measure of an angle formed by a tangent and a chord.

In-class activity! Have students copy the drawings below.

1. In $\odot A$, use a protractor to find the measures of $\angle CDE, \angle CFE$, and $\angle CBE$. Determine the measure of arc CE.
   (a) Write a hypothesis regarding measure arc CE and $\angle CDE$.
   (b) Write a hypothesis regarding the measures of $\angle CDE, \angle CFE$, and $\angle CBE$.
2. Use a protractor to measure $\angle IKJ, \angle ILJ$, and IHJ.
   (a) Write a hypothesis about an angle whose vertex is on the boundary of a circle and whose sides intersect the endpoints of the diameter.





Angles of Chords, Secants, and Tangents

Pacing: This lesson should take one class period

Goal: This goal of this lesson is to explain the formulas for determining angle measures formed by secants and tangents.

Vocabulary: According to www.encylcopedia.com, "A secant is a line that intersects a curved surface." A chord is a line segment connecting two points on the boundary of a circle. Thus, a chord is a segment of the secant.

Set up a chart similar to the one below to help students organize their formulas for this lesson and the next.

Table 1.7

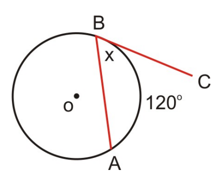
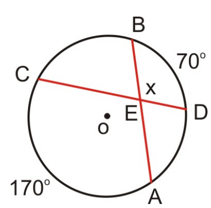
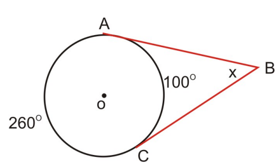
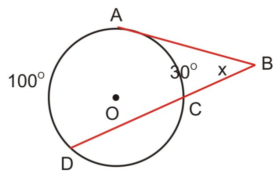
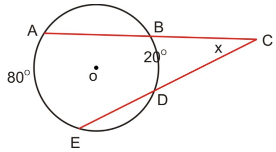
	Example	Angle Formula	Segment Length
Tangent Chord		$\frac{1}{2}$ (intercepted arc)	
Angles formed by two intersecting chords		Sum of intercepted arcs	
Angle formed by two intersecting tangents		Difference of intercepted arcs	

Table 1.7: (continued)

	Example	Angle Formula	Segment Length
Angle formed by two intersecting secants		Difference of intercepted arcs	
Angle formed by a tangent and secant intersection		Difference of intercepted arcs	

Segments of Chords, Secants, and Tangents

Pacing: This lesson should take one class period

Goal: This goal of this lesson is to explain the formulas for determining segment lengths formed by intersecting secants and tangents.

Organize! Finish the chart began in the previous lesson.

Table 1.8

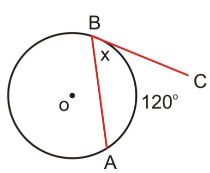
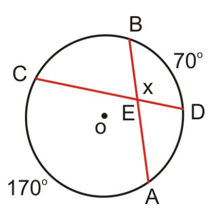
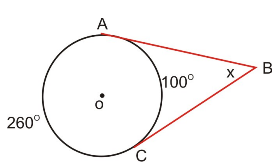
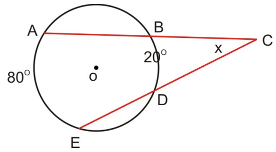
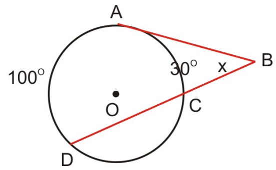
	Example	Angle Formula	Segment Length
Tangent Chord		$\frac{1}{2}$ (intercepted arc)	N/A
Angles formed by two intersecting chords		Sum of intercepted arcs	$a * b = c * d$ whole secant_1 $*$ external $\text{part}_1 =$ whole secant_2 $*$ external part_2
Angle formed by two intersecting tangents		Difference of intercepted arcs	N/A
Angle formed by two intersecting secants		Difference of intercepted arcs	$a * b = c * d$ whole secant_1 $*$ external $\text{part}_1 =$ whole secant_2 $*$ external part_2

Table 1.8: (continued)

	Example	Angle Formula	Segment Length
Angle formed by a tangent and secant intersection		Difference of intercepted arcs	$b * c = a^2$ whole secant $*$ external part $=$ tangent^2

1.10 Perimeter and Area

Triangles and Parallelograms

Pacing: This lesson should take one class period

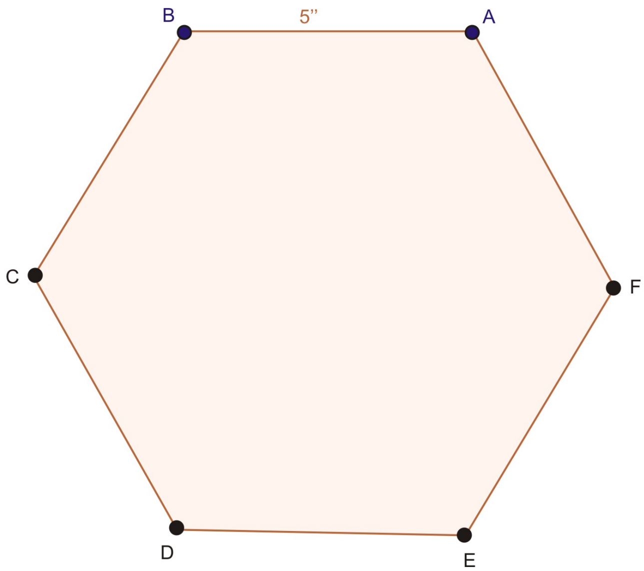
Goal: This lesson introduces students to the area formulas for parallelograms and rectangles. It also illustrates the relationships between these formulas.

Flashcards! Creating another set of flashcards will be second-nature to our students by now. These flashcards should also be double-sided. The blank side should be a sketch of the figure and its special name. The flip side should repeat its definition, the sum of its interior angles, the expression for its perimeter, the formula for its area, and the formula for its perimeter. Have students create flashcards as the chapter presents the figures; this lesson only covers triangles, rectangles, and parallelograms.

Visualization! Encourage students to see how a parallelogram can be transformed into a rectangle by performing the activity presented in the lesson.

Extra Credit? You may want to offer extra credit for students who can correctly determine the total area of the eight circles found in the introduction of this lesson. $16 - 8 * (\pi * (\frac{1}{2})^2) = 16 - 4\pi \text{ft}^2$.

Extension! Using the hexagon below, find its area. *Students use the concept of triangles and interior angles.* Note** This may be more appropriate for students to attempt once the trapezoid area formula has been presented.



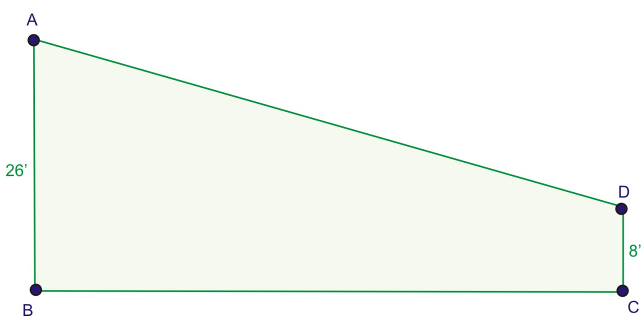
Trapezoids, Rhombi, and Kites

Pacing: This lesson should take one class period

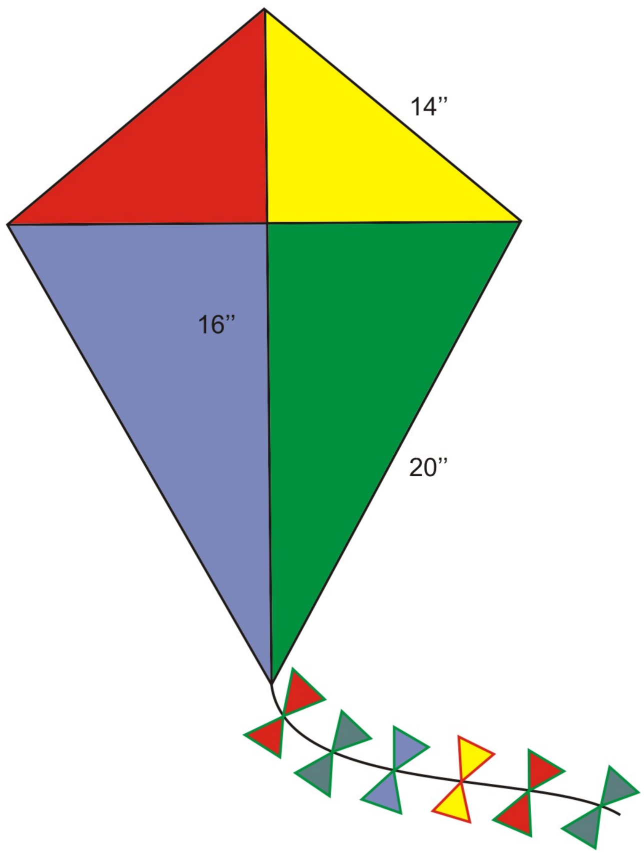
Goal: This lesson further expands upon area formulas to include trapezoids, rhombi, and kites.

Additional Examples:

1. Trina has a rectangular flower garden, as shown below. The area of the garden is $1,602 \text{ ft}^2$. How long is the bottom edge? *Students must realize the bottom edge represents the altitude of the trapezoid.*

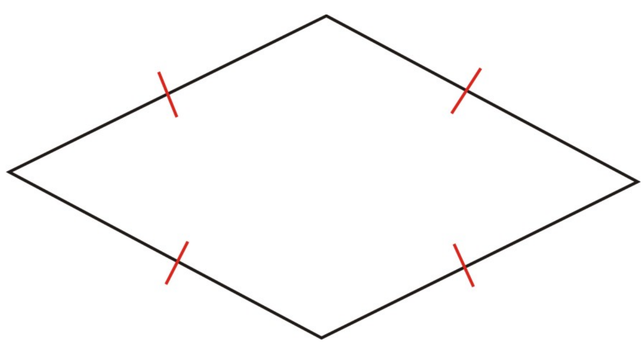


2. What is the area of the kite pictured at the right?



www.clker.com/clipart-kite.html

3. Assume the rhombus below has an area of 45 in^2. One diagonal measures 5 inches. What is the length of the second diagonal? *Extension:* How long is each segment of the rhombus?



Areas of Similar Figures

Pacing: This lesson should take one class period

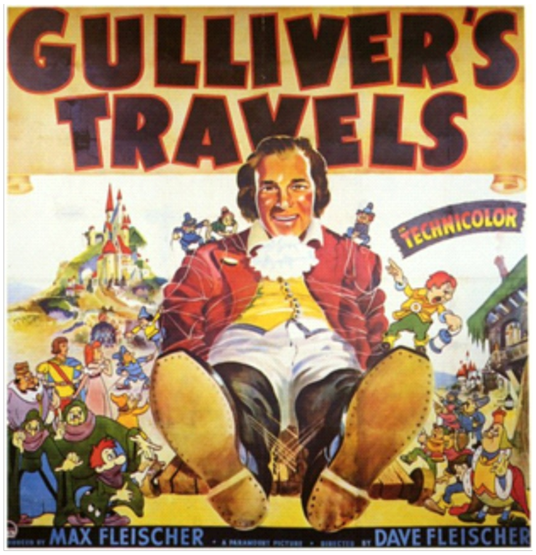
Goal: The purpose of this lesson is to connect similarity, areas, perimeters, and the scale factor k.

Look Out! Students may get confused when attempting to set up a proportion regarding similar area. Areas have a ratio of k^2, whereas perimeters have a ratio of k. This may be a good time to review the fraction

$\frac{\text{image area}}{\text{preimage area}} = \frac{k^2}{1}$. Encourage students to use this proportion when referring to area.

Could There Be Giants? Before reading the section "Why There Are No 12' Giants," discuss Robert Wadlow, the tallest man on record. The following website is the "official" Robert Wadlow information guide. Students are captivated at Wadlow's shoe size, his growth chart, and the photographs that can be found on this site. Encourage students to further research people of great height and do quick summaries of their findings.

Gulliver's Travels! Written by Jonathan Swift, Gulliver's Travels tells of a man who visits two worlds, one where people are 12 times his size (Brobdignagians), and another where people are $\frac{1}{12}$ his size (Lilliputians). Use these values for k to describe such things as the area of footprints.



i445.photobucket.com/.../mathewcmills/gull2.jpg

Circumference and Arc Length

Pacing: This lesson should take one to two class periods

Goal: The purpose of this lesson is to introduce the circumference formula and derive a formula for arc length (portions of the circumference)

Who Wants Pizza? Use pizza, pies, or cookies as a visual for this lesson's formulas. Give each student one of the aforementioned round objects and a piece of string. Ask students to measure how much string it takes to circle around the object. Explain to students that this is the circumference.

When discussing arc length, split the object into six, eight, ten, or twelve even sections. Revisit central angles and the fraction of the whole. This fraction $(\frac{1}{8}, \frac{1}{6}, \frac{1}{10}, \frac{1}{12})$ will be your multiplier to the entire circumference.

Look Out! Students can become confused regarding the Pi symbol (π). Students tend to view this as a variable instead of an approximate value.

Exact versus approximate. Students wonder why it is necessary to leave answer in exact value (π), instead of approximate (multiplying by 3.14). This is usually a teacher preference. By using the approximate value for Pi, the answer automatically has a rounding error. Rounding the decimal too short will cause a much larger error than using the decimal to the hundred-thousandths place. Whatever your preference, be sure to explain both methods to your students.

Circles and Sectors

Pacing: This lesson should take one to two class periods

Goal: The purpose of this lesson is to introduce the area of a circle formula and derive a formula for its fractional area, the sector.

Arts and Crafts Time! Use a compass to draw a large circle. Fold the circle horizontally and vertically along its diameters and cut into four 90° wedges. Fold each wedge into quarters and cut along lines. Students should have 16 wedges. Fit all 16 pieces together to form a parallelogram, where the width of the parallelogram is the radius of the circle and the length is some value b. *Students will see that the area of a sector must be a portion of the whole.*

Who Wants Pizza? Use pizza, pies, or cookies as a visual for this lesson's formulas. Give each student one of the aforementioned round objects. Illustrate area by discussing the amount of material needed to make the cookie, dough, etc. is an example of area. Have students discuss why the previously learned area formulas will not provide an accurate answer. Present the area of a circle formula and have students calculate the area of their individual object.

When discussing the area of a sector, divide the objects into $6, 8, 10$, or 12 even sections. Each section represents a fraction of the whole, thus can be modeled by determining the fraction and multiplying it by the entire area.

Additional Examples:

1. How much more pizza is in a 16" diameter pizza than a 12" diameter pizza? 87.96 in^2
2. Suppose a 14" pizza is cut into 10 slices. What is the area of two slices? 30.79 in^2

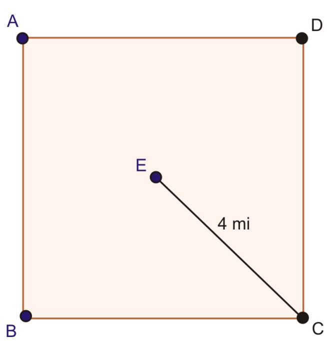
Regular Polygons

Pacing: This lesson should take one to two class periods

Goal: The purpose of this lesson is to introduce the formulas to determine the areas of regular polgyons by defining the *apothem.*

Additional Examples:

1. Find the area of a regular pentagon with 11.8 cm sides and a 9.2 cm apothem. 271.4 cm^2.
2. Find the area of a regular hexagon is its side length is 20". 1039.23 in^2.
3. Find the area of the regular quadrilateral below. 32 mi^2



Geometric Probability

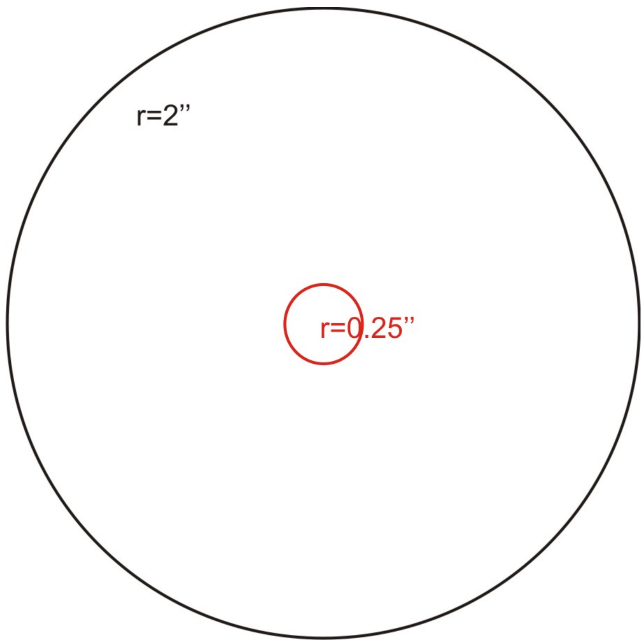
Pacing: This lesson should take one class period

Goal: Students will apply the formula for general probability to geometric objects.

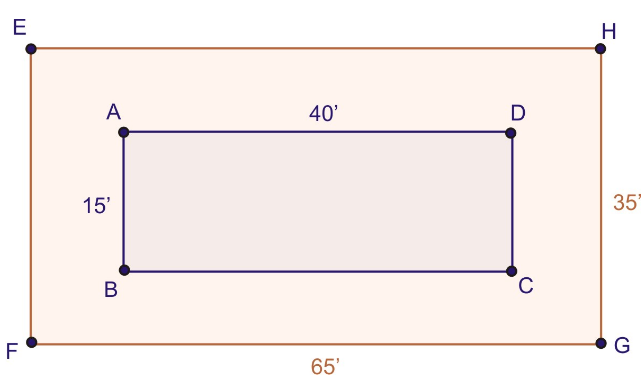
Extension! The probability formula can also be applied to areas. $P = \frac{\text{area of favorable outcome}}{\text{total area}}$. Use this formula for the following additional examples.

Additional Examples:

1. What is the probability of landing in the bulls-eye of the dartboard below? Probability = 1.56%



2. What is the probability that if you jump off the roof, you will land on the deck instead of the pool? Probability = 73.63%



1.11 Surface Area and Volume

The Polyhedron

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to introduce students to three-dimensional figures. Polyhedral figures are presented in this lesson and common terms such as edge, vertex, and face are explained.

Grocery Shopping! Begin gathering objects from home that you can use in subsequent lessons. Collect empty cereal, rice, or pasta boxes, empty or full cans of soup, Stackers containers (triangular prisms), etc. These objects will help students visualize nets, presented in the next lesson.

Arts and Crafts Time! Download the nets found on Mathforum's website. Have students color, cut, and adhere the edges together to form Platonic polyhedra. http://mathforum.org/alejandre/workshops/net.html

For More Information - Click on the following link to gather more information regarding polyhedral figures. http://mathforum.org/alejandre/workshops/unit14.html

Upcoming Vocabulary! Lateral face and lateral edge are two common vocabulary words students should learn. *Lateral face* is a non-base face (usually the sides). *Lateral edge* is the segment where two lateral faces meet.

Representing Solids

Pacing: This lesson should take one to two class periods

Goal: The purpose of this lesson is to introduce students to the various types of representations of solid figures. Most will come naturally to students and should be presented as a fun lesson.

Become an Architect! After discussing orthographic views, collect students into groups of 2 or 3. Offer each group a collection of wooden blocks. Their only rule is to build something – a building, house, the Parthenon, etc. Explain to students that architects will often visualize the completed 3-D building from two-dimensional drawings.

Once all the creations are complete, students will rotate to a different structure and sketch its top, front, back, and side views. *Students are drawing the orthographic views of a 3-D structure.* Students may complete additional drawings as an assignment, in-class activity, or extra credit.

Cross Section View, Using a Breadknife! When discussing cross-sections, bring in a loaf of bread and a breadknife. Illustrate the perpendicular cross by cutting through the bread vertically. You could also show non-perpendicular cross section by cutting through the bread at different angles.

Nets. Using the collected cereal boxes, have students turn these into nets of prisms and draw sketches. *Students will use real life objects to visualize nets.*

Prisms

Pacing: This lesson should take one to two class periods

Goal: This lesson introduces students to the surface area and volume formulas of prisms.

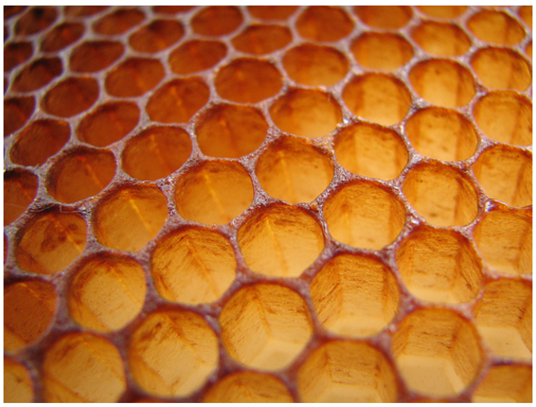
Flashcards! The focus of these flashcards is to organize $3-$dimensional formulas for area and volume. You could use the following chart to help students begin to organize their flashcards.

Table 1.9

	Surface Area Formula	Volume Formula
Cube		
Rectangular Prism		
Triangular Prism		
Hexagonal Prism		

Additional Examples:

1. How much honey can 65 honeycomb cells hold if each hexagonal cell is $\frac{1}{8}$" long by $\frac{1}{4}$" deep? 2.64 in^3



http://www.flickr.com/photos/justusthane/1252907196/

Cylinders

Pacing: This lesson should take one to two class periods

Goal: This lesson introduces students to the surface area and volume formulas of cylinders.

Organization! Add the following rows to your table began in the previous lesson.

Table 1.10

	Surface Area Formula	Volume Formula
Cylinder		

Visualization! Using soup cans or other cylindrical objects, show students the lateral face of a cylinder by peeling the label from the can. *Students will see the lateral face is a rectangle, not a circle.*

Additional Example:

1. A drinking straw has is 11" long with a 0.5" diameter. How much plastic is needed to form the straw? 17.28 in^2.

2. Using the same straw, how much liquid can it hold? 2.16 in^3

http://www.flickr.com/photos/cedsarlette/3002603565/

Pyramids

Pacing: This lesson should take one to two class periods

Goal: This lesson introduces students to the surface area and volume formulas of cylinders.

Lab Investigation! Before you read through the volume of a pyramid section, have your class complete this lab! This is a great way to demonstrate the relationship between the volume of a prism and the volume of a pyramid.

Fill a gallon jug with water colored with food coloring. Separate students into groups of three or four. Each group should receive a prism and its matching pyramid. **The bases and heights must be identical for this to work!** Instruct one student to measure the necessary values of the $3-$dimensional figures (altitude and lengths of base) while another student records the information. A third student will fill the **pyramidal figure** with colored water and pour it into the prism. The goal is to determine how many times it will take to fill the prism. *The answer should be approximately* 3.

Encourage students to write a hypothesis regarding the relationship between these two volumes. *Students should state that* $3 * \text{pyramid} = \text{prism}$.

Additional Examples:

1. Draw a net for a right pentagonal pyramid.

Cones

Pacing: This lesson should take one to two class periods

Goal: This lesson introduces students to the surface area and volume formulas of cones.

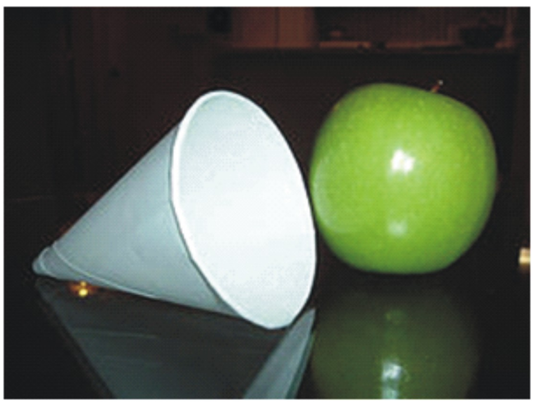
Additional Examples:

1. Draw the net for a right cone with diameter 3 cm and height 5 cm.

2. How much material is needed to make the waffle cone shown below with dimensions 12" tall with 5" diameter? 96.29 in^2

http://www.flickr.com/photos/goddess-arts/2886293009/

3. The conical water cup has dimensions 6" with a 3.5" diameter. How much can it hold? 57.73 in^3



http://www.flickr.com/photos/63965847@N00/27208445/

Cones

Pacing: This lesson should take one to two class periods

Goal: This lesson introduces students to the surface area and volume formulas of spheres.

Geography Connection! Using a globe, show students how a sphere is formed using rotation. The teardrop shaped pieces are called *gores.* Once placed together and folded so they meet at the top and bottom (poles), a sphere is formed.

The *great circle* is a cross section of a sphere cutting through the widest part of the sphere, the equator. Any other cross section is called a *small circle.* Using the globe, show students examples of each (i.e. The Artic Circle and the Equator).

Extra research! Have students research the different types of maps and list the pros and cons of each, relating to the true topography of Earth.

Additional Examples:

1. How much leather is needed to make a baseball with a 6.5" diameter? 132.72 in^2
2. A plant container is a hemisphere with a radius of 17." How much dirt can it hold? *Don't forget to*

divide your answer in half – 1286.22 in^3

Similar Solids

Pacing: This lesson should take one to two class periods

Goal: This lesson introduces students to the surface area and volume formulas of cones.

Gulliver's Travels Revisited! In a previous lesson, we connected Gulliver's Travels to areas of similar figures, such as the footprints of Gullivers versus the Lilliputians. Extend this topic to surface area and volume.

The surface area (amount of material needed to make clothing, etc.) has a ratio $\left(\frac{\text{image area}}{\text{preimage area}}\right)$ of k^2. Therefore, if a Lilliputian was $\frac{1}{12}$ the size of Gulliver and Gulliver was $\frac{1}{12}$ the size of a Brobdignagian, the amount of material needed to cloth a Brobdignagian would be 144^2 times the amount needed for a Lilliputian!

This also related to volume (weight). The ratio here would be k^3. Have fun and try lots of different ratios!

1.12 Transformations

Translations

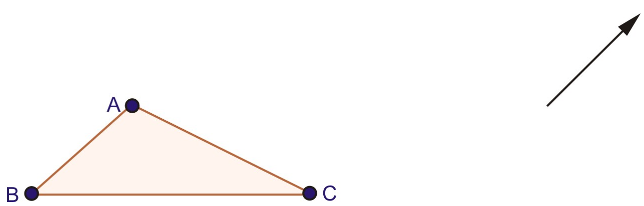
Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to introduce the concept of translations (slides) in the coordinate plane.

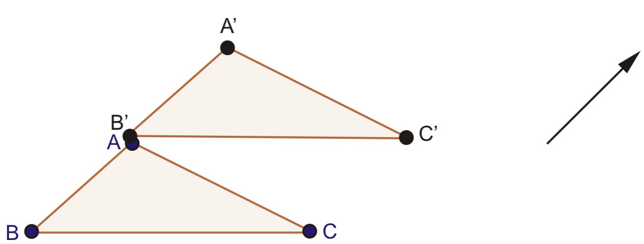
Vocabulary! The word *isometry* refers to any type of transformation...

Did You Know? Translations are formed by a composite of reflections over parallel lines. To illustrate this in further detail, see the **Reflections** lesson.

Vectors Outside the Coordinate Plane! Vectors can be used to translate an object not on a coordinate plane. A vector in this case tells the length and direction to translate a figure. Use the diagram below.



The black ray represents the vector. Therefore, $\triangle ABC$ should be translated NNE the length of the vector. The resulting translation is pictured below.



Additional Example:

Suppose $A'B'C'D'$ is the image of $ABCD$ under a translation by vector $c = (-8, 3)$. What are the vertices of the preimage is the image has vertices at the following locations: $A' = (0, 0), B' = (1, 5), C' = (-4, 4), D' = (-7, -10)? A = -8, -3), B = (9, 2), C = (4, 1), D = (-1, -13)$

Matrix Addition

Pacing: This lesson should take one class period

Goal: Matrices are useful in geometry, as well as algebra and business. This lesson introduces students to the basics of matrices, namely, matrix addition.

Basics of Matrices! Matrices are referred to according to its dimensions, rows by columns. To get students thinking about which is which, use this phrase. "You row ACROSS a lake and columns hold UP houses." By relating rowing across and columns up, students should correctly organize the information.

Matrices can only be added if the dimensions are equivalent. Because adding matrices requires adding the same cell, there must be equal numbers to combine.

Excel spreadsheets are excellent examples of matrices. If you have the ability to set up such spreadsheet matrices, students can see how businesses use these to organize and manipulative inventory.

Vocabulary! A 2×1 matrix organizing a point is called a *point matrix.* The $x-$values should go into row 1 and the $y-$values should go into row 2. The columns represent the points of the figure in the coordinate plane.

Additional Example: Target is processing its baby items inventory. Arrange the following into a matrix. Shirts: $24 - 2T, 0 - 3T, 9 - 4T$; shorts: $5 - 4T, 17 - 2T, 11 - 3T$; pants: $8 - 3T, 0 - 4T, 3 - 2T$.

Suppose another Target is shipping its excess inventory to this store. Write the sum of the two shipments into a single matrix.

Table 1.11

	$2T$	$3T$	$4T$
Shirts	25	6	7
Shorts	8	19	12
Pants	1	4	30

Reflections

Pacing: This lesson should take one class period

Goal: Reflections are an important concept in geometry. Many objects can be explained with reflections. This manipulation is also related to similarity and triangle congruence.

Matrix Multiplication! Matrix Multiplication can be quite difficult for students to compute by hand. Here is a way to use a graphing calculator to achieve the multiplication.

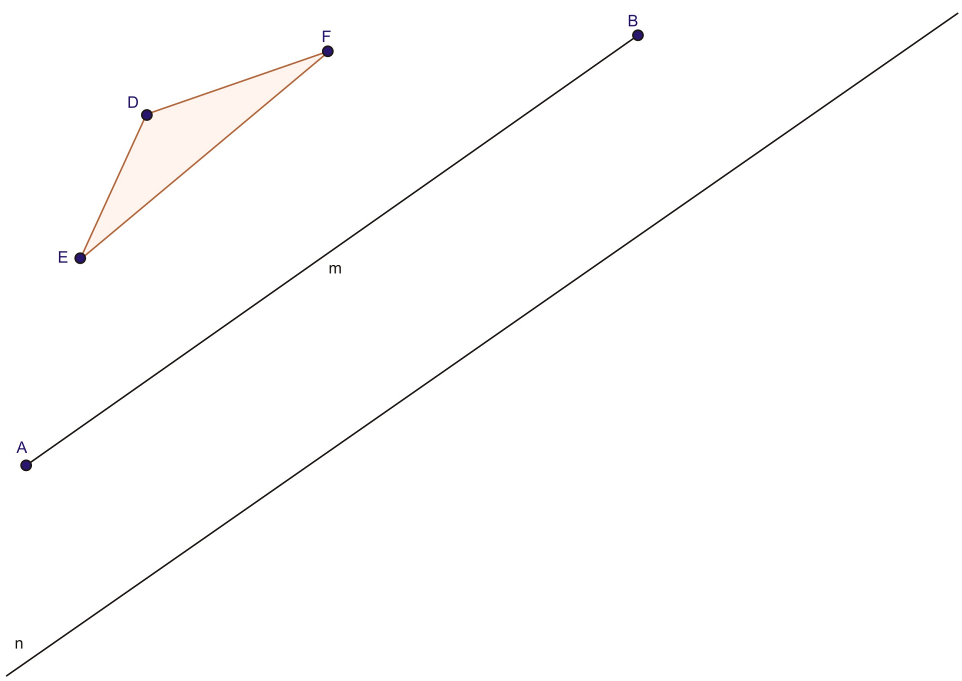
1. Find the Matrix menu. If your students are using a Texas Instrument product, it is located by typing the 2^{nd} and x^{-1} keys.
2. Edit matrix A by moving to the right to the edit menu and choosing $[A]$. Input the dimensions and data.

3. Choose 2^{nd} & MODE to quit the menu
4. Repeat steps 1 – 3 for the second matrix $[B]$.
5. Choose matrix $[A]$ by repeating step 1 and touching enter under the Name Menu
6. Choose the multiplication symbol
7. Repeat step 7 but choose $[B]$ instead.
8. Your working menu should look like this: $[A] * [B]$
9. Touch ENTER. The answer resulting is the product of the two matrices.

Extension In-Class Activity! Reflections can be performed without a coordinate plane, just as translations.

1. Using patty paper (or tracing paper), have students draw a small scalene triangle ($\triangle DEF$) on the right side of the paper.
2. Fold the paper so that $\triangle DEF$ is covered.
3. Trace $\triangle DEF$.
4. Unfold the patty paper and label the vertices as D', E', and F', the image points of the D, E, and F.
5. Darken in the fold – this is the reflecting line. Label a point on this line Q.
6. Use a ruler to draw $\overline{FF'}$. Mark the intersection of the reflecting line and $\overline{FF'}$ point M.
7. Measure the FM and F'M. What do you notice about these distances?
8. Measure $\angle FMQ$ and $\angle F$'MQ. What do you notice about these measurements?

Extension - Reflections and Translations! Use the diagram below. Reflect $\triangle CAT$ over line m, obtaining $\triangle C$'A'T'. Now reflect $\triangle C$'A'T' over line n, obtaining $\triangle C$"A"T". The resulting image is a translation of the preimage, double the distance between the parallel lines.



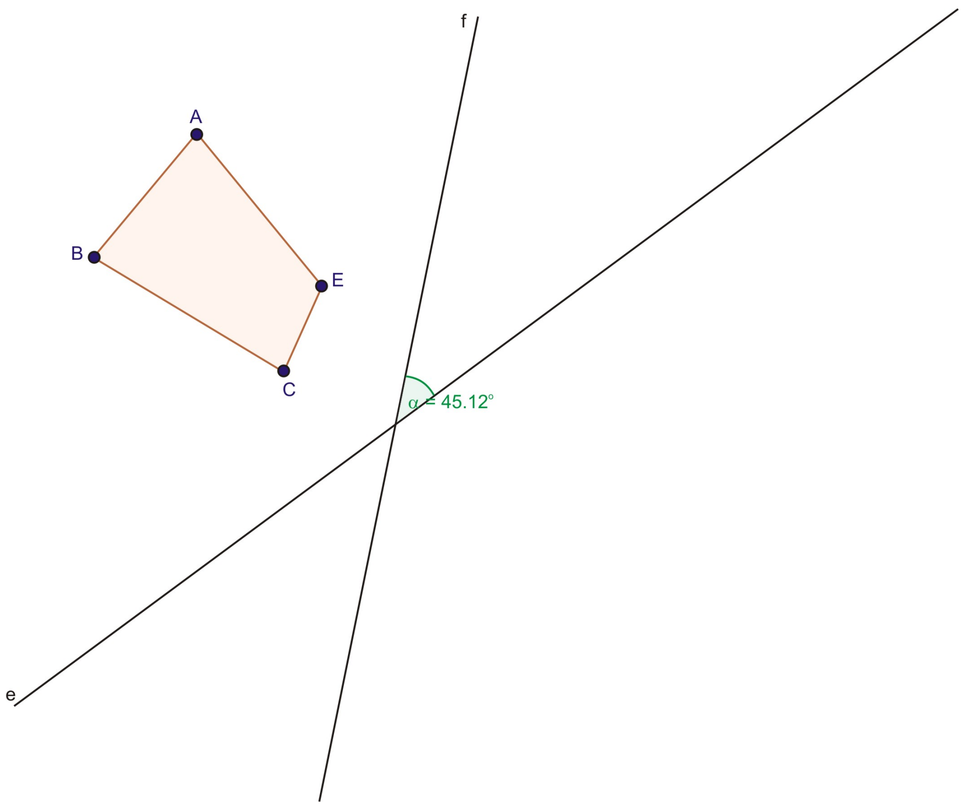
Rotations

Pacing: This lesson should take one class period

Goal: Rotations are also an important concept in geometry. Tires rotate in 360 degree increments, as do the hands on a clock. This lesson presents the concept of rotations and how matrix multiplication is used to compute the image points.

Extension – Reflections and Rotations! Just as translations are a composite of reflections over parallel lines, rotations are a composite of reflections. The only difference is that rotations occur when the reflecting lines intersect. Have your students complete the following:

Using the diagram below, reflect $ABCD$ over line f, resulting in $A'B'C'D$. Reflect $A'B'C'D'$ over line e, resulting in $A"B"C"D"$. The final image represents a rotation of the preimage double the acute angle formed by the intersecting reflecting lines.



Composition

Pacing: This lesson should take one class period

Goal: This lesson introduces students to the concept of composition. Composition is the process of applying two (or more) operations to an object. This concept is also in Advanced Algebra.

Look Out! Students can get easily confused when applying compositions. They may attempt to perform the composition from left to right, as in reading a sentence. Point out to the students they must begin with the object and, according to the order of operations, should perform the operation occurring within the parentheses first.

Notation! Composition notation can take two forms. To write the reflection of $ABCD$ over line m **following** a reflection over line n, you could:

A. Write $r_n \circ r_m(ABCD)$

B. Write $r_n(r_m(ABCD))$

The lowercase "r" stands for reflection and the subscript refers to the reflecting line.

Watch Your Feet! Your feet are a prime example of glide reflections. When you walk, one foot is translated above the other and are reflected about your body's center line.

Tessellations

Pacing: This lesson should take one class period

Goal: This lesson introduces students to how tessellations are formed and which type of polygons will tessellate the plane.

Research! Have your students research M.C. Escher, an artist who has designed numerous pieces of artwork using tessellations. Have each student choose a piece of artwork, outline its preimage figures and give a short presentation.

Create Your Own Escher Print! An activity many students love to do is designed an unique piece of art. Complete the following steps:

1. Cut out a 2" square from a sheet of copy paper.
2. Draw a curve between two consecutive vertices. Be careful to not cut off a vertex!
3. Cut out the curve and slide the cutout to the opposite side of the square and tape it in place.
4. Repeat this process with the remaining two sides of the square.
5. This is your template, or preimage. Begin with an 11" $\times$ 14" piece of copy paper. Trace your preiamge and continue the pattern by rotating and translating until you cover the entire sheet.
6. Color and post for a bulletin board.

Did You Know? The first person to discover how to tessellate with a pentagon was....

Symmetry

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to introduce the concept of symmetry. Students have experienced symmetry in previous lessons: isosceles triangles and various quadrilaterals. This lesson incorporates two-dimensional and three-dimensional figures and their lines of symmetry.

Maximum Points! Have your students write the alphabet in uppercase letters. Using one colored pencil, show which letters possess horizontal symmetry by drawing in the symmetry line. For example, B has a line of symmetry, as does E and K. Using a second color, draw in the vertical lines of symmetry. Have a contest to determine who can write the longest word possessing one type of symmetry. For example, MAXIMUM is a word where all the letters have vertical symmetry. KICKBOXED is another.

Project! Using a digital camera, have students (or groups thereof) take photographs of objects possessing symmetry, either rotational or reflective. Give points for the most original, the most nature made, etc. Create a slide show presentation in PowerPoint or Microsoft Movie Maker.

Vocabulary! When discussing rotational symmetry, some textbooks may refer to it as $n-$*fold rotational symmetry.* This simply means that the n is the number of times the figure can rotate onto itself. For example, a regular pentagon has $5-$fold rotation symmetry, because it can be rotated 5 times of 108 degrees before returning to its original position.

The Return of the Breadknife! The notion of cutting through a $3-$dimensional object with a breadknife was used in an earlier lesson to demonstrate cross sections. This concept can be used to illustrate planes of symmetry. The plane of symmetry essentially "cuts" through the $3-$dimensional solid so that each piece is identical.

Dilations

Pacing: This lesson should take one class period

Goal: The purpose of this lesson is to illustrate how scalar multiplication yield dilations. Figures under dilation are similar figures; all properties of the similarity chapter apply to these objects.

Vocabulary! Scale factors of the same value, such as S_2, are also called size changes. All properties of similar figures hold for size changes.

Extension! Scalar multiplication can be extended to multiplying the $x-$values and $y-$values by different values, yielding a non-similar figure. For example, you could multiply the points $(0, 2), (1, 7), (4, 5), (6, 2)$ by $S_{2,\ 3}$. The first value in the subscript is the multiplier for the $x-$values and the second value in the subscript is the multiplier for the $y-$values. The resulting ordered pairs are $(0, 6), (2, 21), (8, 15)$, and $(12, 6)$.

Technology! To find the image points of a size change, input a 2×2 matrix in matrix $[A]$, such as $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$. For a scale change, a matrix could look like this: $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$. Multiply the two matrices using the process found in **Reflections** lesson.

Additional Example:

1. Suppose the image JAR has the following matrix: $\begin{bmatrix} 2 & -4 & 8 \\ 6 & -3 & 5 \end{bmatrix}$, occurring under a size change $S_{\frac{1}{2}}$. What are the coordinates of the preimage TIP?