Exploring Sets and Logic

General Mathematics for School Learners

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The Source of Resources
Mathematical, Scientific, Technical, Vocational & Occupational

Luutu Suleiman

PREFACE

Exploring Sets and Logic is a supplementary as well as revision and practice textbook for school learners in the age range of 13 – 16 years. However, it can selectively be used by a large section of people from teachers, teacher trainers and trainees, adult learners, and so on for various purposes.

The book covers the subject area in eight (8) units, sequentially well developed from basic, intermediate, through to a higher level of learning.

The work therein constitutes:

- Introductory matter for each Unit.
- Learning/studying objectives for each unit
- Concepts explanatory notes, involving illustration as much as possible
- Worked out examples (various for a unit)
- Graded exercises (various for a unit)
- Revision paper
- Answers

The core aim of this work is to create a one stop centre for a particular subject area for the target group.

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CONTENTS

UNIT 1

The Concept of Sets

INTRODUCTION

When you look around our environment, you can see a lot of different and similar things. Such things may include plants, buildings, roads, and many more. The concept of sets is therefore concerned with identifying, sorting and grouping of things following a certain common identity.

Often we shall be directly or indirectly identifying, sorting or grouping while dealing with sets.

Objectives

Having gone through the explanations, worked out examples, activities and exercises, you should be able to:

1. *Identify the common characteristics of items, sort and group them according to their similarities.*
2. *Find out the possible ways by which a given group can be sub-grouped.*
3. *Well elaborate the meaning of a set.*
4. *Represent and describe sets in the various possible ways - naming, listing, word description and venn diagram representations.*
5. *Describe a set by its name and listing of its members*
6. *Understand the meaning of and be able to use set notations.*

1.1 Identifying, Sorting and Grouping of things

To know whether some things are similar or not, we must be able to understand what they are and what makes them similar or different from each other. By doing this, we are ***identifying*** them. When we know what each thing is, we can then select them according to what they are; and this means ***sorting*** them. Having known the similar things, we can then put them together according to their similarity. This is known as ***grouping***. Can you tell the common characteristic for a mango, orange, pawpaw and an apple?

Activity 1.1

1 Collect a number of things around you and name them.
2 Find the different ways you can group the collected items.
3 Clearly list the names of things in each group.

Group of plants

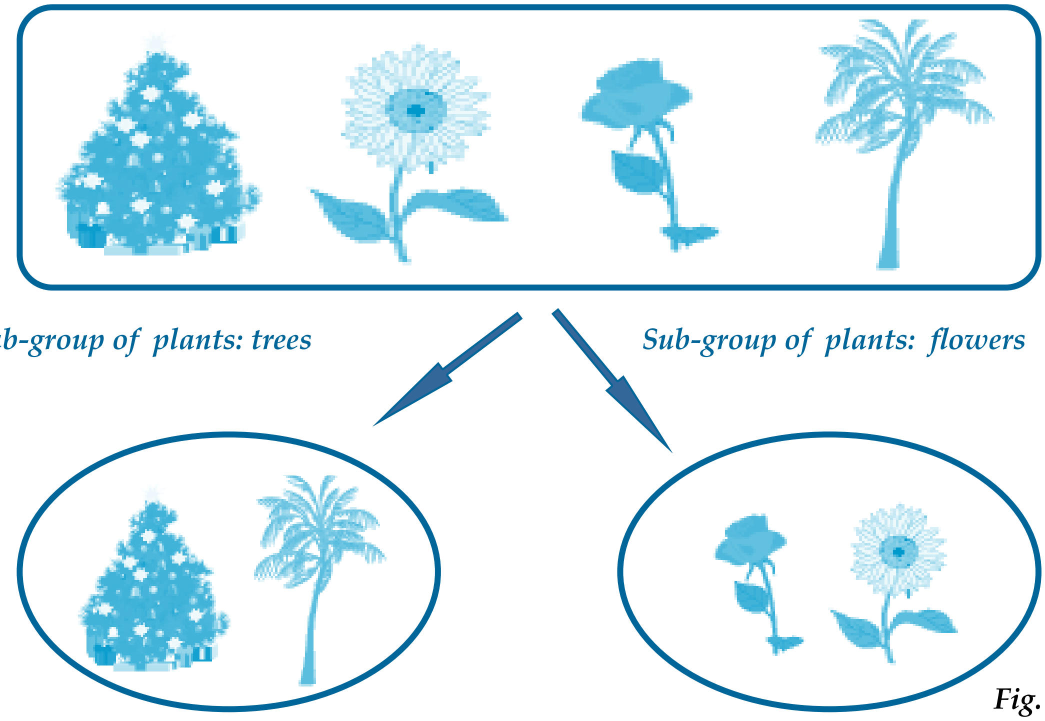


Fig. 1.1

A Christmas tree, sunflower, rose and a palm tree are all *plants*; but they can be sub-grouped according to type. The possible types of plants can be *trees* and *flowers*. Study above to understand this sub-grouping.

- The sub-group; *trees* constitutes a *Christmas tree* and a *palm tree*.
- And the sub-group; *flowers* constitutes a *sunflower* and a *rose*.

1.2 Meaning and Representation of a Set

When we talk of animals we can still come up with many different types. These may include mammals, reptiles, birds, amphibians and the fish. Mammals can further form a set of herbivorous, carnivorous and omnivorous animals.

To group things, we depend on their similarities or differences. A group of similar things will always form a set. We can therefore define a set as a collection of well defined things with something in common.

A set can be made of things such as objects, numerals, letters, words, symbols, plants, animals and so on. Special names are used to refer to a number of sets, say, a swarm for bees, a flock for birds, a herd for cattle, etc.

Sets can also be named as follows;

- a set of consonants of the English alphabet,
- a set of months of the year,
- a set of days of the week, etc.

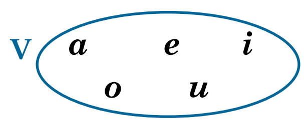
A set is usually denoted a capital letter, and items in it shown listed or it is defined with in a pair of curl brackets, { }. If a set A = {*a, b, c, d, e, f*}; then the letters *a*, *b*, *c*, *d*, *e* and *f* are members of the set A. Members of a set are also called its ***elements***.

Sets can be represented using Venn diagrams. Venn diagrams are a pictorial representation of sets with in closed loop boundaries. The loop of a particular set can take the shape of a circle, oval, rectangle, square or any other shape.

Set description: **V** = {vowels of the English alphabet}

Set elements: **V** = {*a, e, i, o, u*}

On a Venn diagram:



1.3 Description of Sets

Members of the same set should obey and follow a similar rule. A set is always labeled as a specific group and simply represented by a letter. If set A = {0, 2, 4, 6, 8, 10}, then, set A is a set of all even numbers up to and including 10.

Activity 1.2

State the common characteristic (s) of the elements of each of the following sets

P = {Chair, Cupboard, Bench, Book shelf, Bed} and

Q = {Rubber, Plastic, Wood, Cork, Paper, Cloth}

1.3.1 Naming of a set and listing of its members

Given a set Q, such that; Q = {A set of Even numbers}, then the members of set Q are listed as; Q = {0, 2, 4, 6, 8, 10, 12, 14, 16 …}.

You should be able to get the name of a set with the help of a common characteristic between or among the members of that set.

Example 1

List all the members of each of the sets;

A = {a set of letters that form the word "country"}
B = {a set of the seven colours of the rainbow}
C = {a set of all prime numbers between 4 and 25}
D = {a set of all Vowels of the English alphabet}

Solution

A = {*c, o, u, n, t, r, y*}
B = {Red, Orange, Yellow, Green, Blue, Indigo, Violet}
C = {5, 7, 11, 13, 17, 19, 23}
D = {*a, e, i, o, u*}

Example 2

List the members of a set of the first 10 composite numbers.

Solution

A composite number is one with other factors other than 1 and the number itself. If the set of the first 10 composite numbers is C,
then set C = {4, 6, 8, 9, 10, 12, 14, 15, 16, 18}

Example 3

Name the following sets:

P = {Lion, Leopard, Chitter, Tiger, Jaguar}
Q = {Corrolla, Carina, Corsa, Corona}
R = {Square, Rectangle, Parallelogram, Rhombus, Trapezium, Kite}
S = {Red, Blue, Green}

Solution

P = {A set of wild animals of the cat family}
Q = {A set of Toyota company cars with names starting with letter C}
R = {A set of four sided plane figures, quadrangles or quadrilaterals}
S = {A set of primary colours}

Exercise 1a

List down all the possible members of each of the following sets.

1 **P** = {Polygons with 4 sides}

2 **Q** = {Solid figures with six faces}

3 **R** = {East African countries with names starting with consonants}

4 **S** = {Multiples of 3 which are even}

5 **T** = {Multiples of 4 between 1 and 100 which are divisible by 6}

Given the members of the following sets state their names;

6 **U** = {Ream of papers, Clip board, Staples, Pen}

7 **V** = {Jan, Feb, Mar, Apr, May, Jun, Jul, Aug, Sept, Oct, Nov, Dec}

8 **W** = {B, C, D, F, G, H, J, K, L, M, N, P, Q, R, S, T, V, W, X, Y, Z}

9 **X** = {C, L, O, S, E, D}

10 **Y** = {2, 4, 6, 8, 10, 12 … }

If set S = {1, 2, 3, 4, 5, 6, 7, 8 ... }: state *true* or *false*.

11 S is a set of numbers grater than zero.

12 S is a set of counting numbers

13 124 is not a member of set S.

14 All prime numbers are members of set S.

15 S is a set of numbers exactly divisible by two.

16 S = { 1, 2, 3 . . . 9, 10, 11 . . . }

1.3.2 Set notations

Symbols are used as a language to represent set terminologies, relations or operations. This system is known as ***set notation***.

Member of; ∈

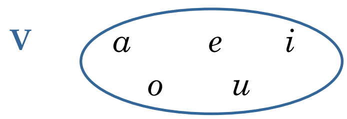
If set V = {*a, e, i, o, u*}, then, members of set V are *a, e, i, o* and *u*. "*a*" is a member of set V; represented as: $a \in V$. "2" is not a member of set V, and this can be represented as: 2 ◎ V.

Number of; n(A)

The notation, n(A) means, "number of elements in set A".

For set P = {2, 3, 5, 7}, then n(P) = 4

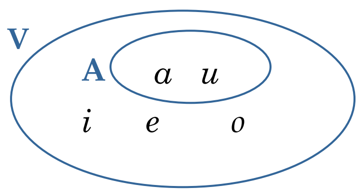
EXPLORING SETS AND LOGIC

Given a set V: 

, then n(V) = 5

Contained in; ⊃

If set V = {*a, e, i, o, u*} and another set A = {*a, u}* we realize that all members of set A are also in set V. So, set A is contained in set V and this is represented as A ⊃ V.



For a set B = {1, 2, 3}, we realize that it is not contained in set V and this is represented as B ⊅ V.

Set V is also known to be the superset for set A. A superset sometimes is also referred to as a universal or a mother set.

Subset of; ⊂

For the set V = {*a, e, i, o, u*} and A = {*a, u*}; set A is a subset of set V. This is represented as A ⊂ V. If a set B = {1, 2, 3}, we realized that B is not a sub set of set V, that is, B ⊄ V.

Exercise 1b

Given sets A = {*a, b, c, d, e, f*} and B = {0, 2, 4, 6, 8}, state *true* or *false* in the following;

1 b ∈ A

2 e ∈ B

3 c ∈ A

4 2 ∈ A

5 n(A) = n(B)

6 Ø ⊂ B

7 A ⊂ B

8 A ⊃ B

9 {2} ⊃ B

10 {a, b, c} ⊃ A

11 A ⊂ {a, b, c, d, e}

12 {2, 4, 8} ⊂ B

UNIT 2 Types of Sets

INTRODUCTION

Sets can be categorized according to their number of elements. A set can be with or without elements.

Objectives

Having gone through the explanations, worked out examples, activities and exercises, you should be able to:

1. *Identify the members of a given set.*
2. *Tell and describe finite and infinite sets.*
3. *Tell and describe an empty set.*

2.1 Finite and Infinite sets

Finite sets are sets with a countable number of elements.
A set of books in my bag, fingers on my palms and birds in the world are examples of finite sets. A set W, of days of the week is finite, because the number of its elements is specific.

W = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

Infinite sets are sets with an endless list of elements.
A set of all counting numbers, a set of all odd numbers or a set of all composite numbers are examples of infinite sets. A set C, of all counting numbers is infinite because its number of elements is endless.
Counting numbers can never be completed, therefore, for a set they can be represented as; **C** = {1, 2, 3, 4, 5, 6 … }.

While identifying sets, avoid confusing finite and infinite sets in some cases. For example, a set of all living creatures in the world may be confused to be infinite, but it is not. It may be very difficult to count the creatures, but they are countable.

Activity 1.3

1 Do you think a set of primary school children in China is infinite or finite? Why?
2 State whether the number of cells in a human body are infinite or finite.
Explain your argument.

Example 1

Given that; **A** = {All even numbers},
B = {All prime numbers which are even} and
C = {Square numbers which are also rectangular numbers}.
State whether each of the sets above is finite or infinite.

Solution

Set A is an infinite set, since it is, **A** = {0, 2, 4, 6 … }
Set B is a finite set, since it is, **B** = {2}
Set C is an infinite set since all square numbers are rectangular, and all numbers are rectangular except the prime numbers.

2.2 Empty sets " Ø " or " { } "

An ***empty set*** is a set which has no members.

Example 1

Think of and write down any five empty sets

Solution

Set **A** = {Elephants studying in our classroom},
Set **B** = {Pregnant men}
Set **C** = {Even numbers which are odd},
Set **D** = {Triangles with 4 sides} and
Set **E** = {Multiples of 10 with 2 as the last digit}

A set of rivers in our class is an empty set. This is because we don't have rivers in our class. A set of crying stones is an empty set, since there are no stones which cry. An empty set is also known as a ***null set***.

Exercise 2a

1 Think of and write down any five empty sets.

2 State *true* or *false*

(a) $\varnothing \subset Q$, where Q is any set.

(b) If A = {All even prime numbers}, then A = { }

(c) If M = {All prime multiples of 5}, then M = { }

(d) If T = {All square numbers that are cubic}, then T = { }

3 List any five finite sets

4 List any five infinite sets.

5 State *true* or *false*

(a) A set of dust particles in the atmosphere is an infinite set.

(b) A set of birds in the Wild Life Center, Entebbe is a finite set.

(c) A set of all presidents in Africa is an infinite set.

(d) A set of human cells in the whole world is a finite set.

(e) A set of all natural numbers is an infinite set.

UNIT 3 Relationships between Sets

INTRODUCTION

Sets may be categorized according to how they relate to each other. These set relations can also go with certain operation

Objectives

Having gone through the explanations, worked out examples, activities and exercises, you should be able to:

1. *Identify the relationship for equal and equivalent sets.*
2. *Identify the relationship for universal and subsets.*
3. *Tell the properties associated with related sets.*

3.1 Equal " = " and Equivalent " ≡ " sets

Equal sets are sets with similar members. Therefore, two sets are said to be equal if they contain exactly the same members. For example, if Set A = {1, 3, 8, 9}, and Set B = {3, 8, 9, 1}, we can say that set A equals to set B, represented as; Set A = Set B. If set P = {letters in the word queen} and set Q = {*q, u, e, n*}, we still conclude that Set P = Set Q. Note that letter "e" is not repeated in set Q.

Example 1

State whether the sets below are equal or not.

(a) **A** = {1, 3, 5, 7} and **B** = {1, 3, 5, 7}
(b) **C** = {0, 2, 4, 6} and **D** = {0, 2, 4}
(c) **E** = {1, 2, 3 ... } and **F** = {1, 2, 3, 4, 5 ... }
(d) **G** = {*a, e, i, o, u*} and **H** = {Vowels of the English Alphabet}

Solution

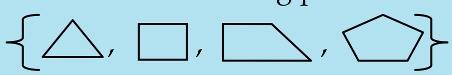
(a) Set A = Set B, since all members in A are also in B.
(b) Set C ≠ Set D, since 6 is only in C and not in D, (≠ means "not equal to").
(c) Set E = Set F. They both start from 1 and are continuous.
(d) Set G = Set H. Set H is the naming of Set G.

Equivalent sets are sets with an equal number of members or elements. The elements may not necessarily be the same. Therefore all equal sets are also equivalent, but not

all equivalent sets are equal.
If P = {1, 4, 8, 12} and Q = {3, 8, 4, 9}; then P is equivalent to Q, represented as P ≡ Q. The number of elements in P is equal to the number of elements in Q, that is, four elements in each set. If U = {1, 3} and V = {0, 2, 4}, then U ⊚ V.
(U is not equivalent to V).

Example 1

State whether each of the following pair of sets is equivalent or not.

(a) Set **A** =  and Set **B** = { L, M, N, P }

(b) Set **C** = {3, 6, 9, 12} and Set **D** = {9, 12, 15, 18}

(c) Set **E** = {0, 2, 4, 6, 8} and Set **F** = {0, 2, 4, 6, 8}

(d) Set **G** = {1, 2, 3} and Set **H** = {1, 2, 3, 4}

Solution

(a) Set A ≡ Set B, since they have the same number of elements.

(b) Set C ≡ Set D, since they have the same number of elements.

(c) Set E ≡ Set F, (Also E = F)

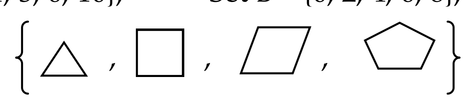
(d) Set G ⊚ H, since they have a different number of elements.

Activity 1.4

Given that, Set P = {the first 5 prime numbers}, and
Set Q = {the first 5 odd numbers}. State whether the sets P and Q are equal, equivalent or neither of these and give a reason (s) why?

Exercise 3a

Given that, Set A = {1, 3, 6, 10}, Set B = {0, 2, 4, 6, 8},

Set C =  ,

Set D = {1, 2, 3, 4 …} , Set E = { 1, 3, 6, 10, 15 …} ,

Set F = {1, 3, 6, 10, 15} and Set G = {L, M, N, O, P}

State *true* or *false* and give a reason (s) why?

1 Set C = Set A
5 Set F ≡ Set A
9 Set G = Set C
2 Set D ≡ Set G
6 Set B ≡ Set G
10 Set B ≠ Set F

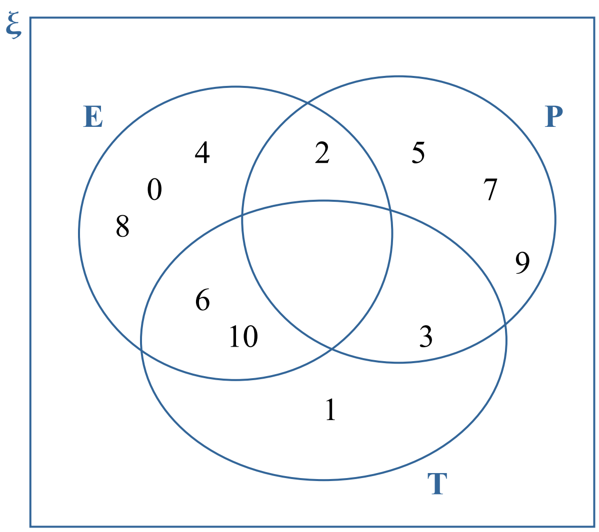
3 Set D = Set E	**7** Set F = Set A	**11** Set A ⊚ Set C
4 Set B = Set G	**8** Set D ⊚ Set E	**12** Set E ≠ Set F

3.2 Universal set, ξ and subsets, ⊂

A universal set is one which exhausts all the members of the same characteristics, hence being with all the elements for a particular description. Given a universal set ; ξ = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, of whole numbers, we may obtain the following subsets from it.

Subsets	*Members of the subsets*
E = {Even numbers}	**E** = {0, 2, 4, 6, 8, 10}
P = {Prime numbers}	**P** = {2, 3, 5, 7, 9}
T = {Triangular numbers}	**T** = {1, 3, 6, 10}

Conclusively, set E, P and T are sub sets of the universal set given before. Study fig. 3.1 on that follows for the Venn diagram representation.



Symbolically;

E ⊂ ξ , **P** ⊂ ξ and **T** ⊂ ξ

The universal set also contains sets E, P and T.

Fig. 3.1

Always note that;

(i) If X ⊂ Y, it means that all members of set X are also in set Y. For example, where Set X = {1, 3} and Set Y = {1, 2, 3, 4}.

(ii) If set R = set S, then R ⊂ S and S ⊂ R. For example, if set R = {1, 2, 3, 4, 5} and S = R, then set R is a sub set of set S, and set S is a sub set of set R. Therefore; for equal sets, one is a subset of the other.

(iii) Any set is a subset of itself.

(iv) An empty set is a subset of any set.

(v) A proper subset is one which does **NOT** contain **ALL** the members of its mother set and it is not empty.

Example 1

Given the sets; Q = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, R = {2, 5, 7}
S = {2, 3, 5, 7, 11} T = {1, 3, 5, 7} U = {2, 5, 7}

State *true* or *false*: **1** $R \subset T$ **2** $S \subset Q$ **3** $R \subset U$ **4** $Q \subset U$ **5** $T \subset Q$

Solution

1 False **2** False **3** True **4** False **5** True

Study the following table carefully.

No. of elements in universal set (n)	Universal set (ξ)	Subsets listed, (s)	No. of Subsets (S)
0	ξ_0 = { }	s_1 = { }	**1**
1	ξ_1 = {a}	s_1 = { }, s_2 = {a}	**2**
2	ξ_2 = {a, b}	s_1 = { }, s_2 = {a}, s_3 = {b}, s_4 = {a, b}	**4**
3	ξ_3 = {a, b, c}	s_1 = { }, s_2 = {a}, s_3 = {b}, s_4 = {c}, s_5 = {a, b}, s_6 = {a, c}, s_7 = {b, c}, s_8 = {a, b, c}	**8**
4	ξ_4 = {a, b, c, d}	s_1 = { }, s_2 = {a}, s_3 = {b}, s_4 = {c}, s_5 = {d}, s_6 = {a, b}, s_7 = {a, c}, s_8 = {a, d}, s_9 = {b, c}, s_{10} = {b, d}, s_{11} = {c, d}, s_{12} = {a, b, c}, s_{13} = {a, b, d}, s_{14} = {a, c, d}, s_{15} = {b, c, d}, s_{16} = {a, b, c, d}	**16**

Fig. 3.2

Activity 1.5

1 State *true* or *false* for $\varnothing \subset R$, where R is any set, and explain.
2 Give explanations for all the answers in the example 1 above.

Activity 1.6

List all the subsets of $\xi = \{a, b, c, d, e\}$. How many subsets are there?

Have you realized how difficult it is to list all the sub sets of sets with five elements and more? We therefore use a formula to find subsets of a given set if we know the number of elements it has.

No. of subsets, $S = 2^n$*, where* ***n*** *is the number of elements in the set.*

Example 2

Find the number of subsets for each of the following sets:

$\xi_1 = \{A, B, C\}$ $\quad\xi_2 = \{a, e, i, o, u\}$ and

$\xi_3 = \{$ △ , □ , ◇ , ▱ , ⏢ , ⬠ $\}$

Solution

Let the number of subsets be: S

(i) $S_1 = 2^n$, but n = 3
$S_1 = 2^3$
$S_1 = 8$ subsets

(ii) $S_2 = 2^n$, but n = 5
$S_2 = 2^5$
$S_2 = 32$ subsets

(iii) $S_3 = 2^n$, but n = 6
$S_3 = 2^6$
$S_3 = 64$ sbsets

Exercise 3b

Given the sets; **A** = {1, 3, 5, 7, 9, 11 … 21} **B** = {2, 3, 5, 7, 11}
C = {5, 7, 9, 11, 13, 15} **D** = {1, 2, 3, 4, 5, 6, 7, 8, 9 …}

State *true* or *false*

1 $B \subset C$ **2** $B \subset A$ **3** $C \subset A$ **4** $A \subset D$ **5** $D \subset C$

6 (a) Given the sets below, find the number of subsets for each of the following sets.
A = {1, 2, 3, 4, 5, 6}, **B** = {*a, e, i, o, u*}, and **C** = {U, V, W, X, Y, Z, P, Q, R, S, T}
(b) If a set has 128 subsets, how many elements does it have?

UNIT 4 Operations on Sets

INTRODUCTION

With the aid of set concepts, we can carry out various operations. These help us with handling problems associated with sets.

Objectives

Having gone through the explanations, worked out examples, activities and exercises, you should be able to:

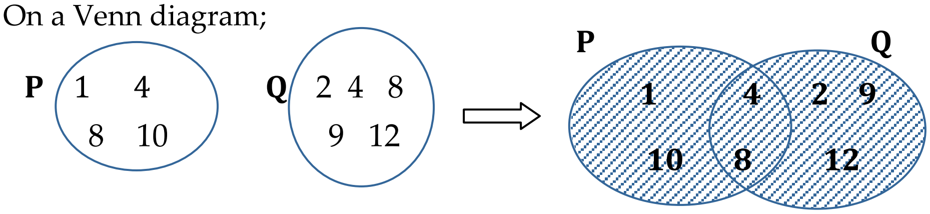
1. *Carry out the union of sets*
2. *Carry out the intersection of sets*
3. *Tell joint and disjoint sets (and also in terms of intersection)*
4. *Identify and tell the complement of a set.*
5. *Tell the set difference (and also in terms of set complement)*
6. *Make simple set operations illustration on venn diagrams*

4.1 Union of sets; ∪

Given the sets P = {1, 4, 8, 10} and Q = {2, 4, 8, 9, 12}, then the union of sets P and Q; P ∪ Q is given by a set of all members found in both sets.

Therefore, P ∪ Q = {1, 2, 4, 8, 9, 10, 12}.

On a Venn diagram;

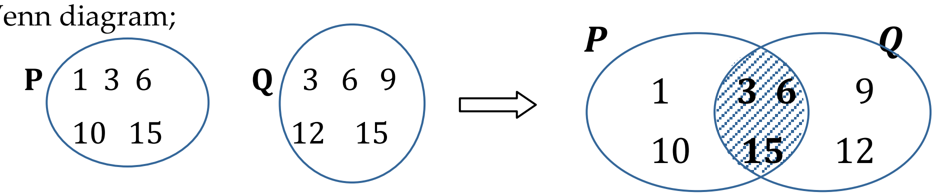


Note that members appearing in both sets are written once (are not repeated) in the union of sets. For example, 4 and 8 are common for both sets. The shaded area represents the union of sets.

4.2 Intersection of sets; ∩

For the sets P and Q, the intersection of such sets is the set containing all the elements which are common in both sets. Given a set P = {1, 3, 6, 10, 15} and Q = {3, 6, 9, 12, 15}, we see that the elements 3, 6 and 15 are common in both sets, so they represent the intersection of sets P and Q. Therefore, P ∩ Q = {3, 6, 15}

On a Venn diagram;



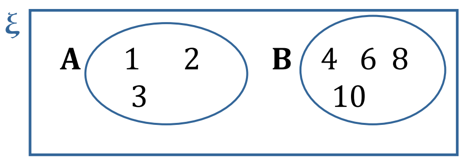
The shaded part represents the intersection of sets P and Q.

Joint and Disjoint sets

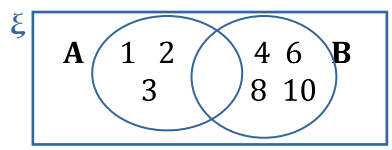
If any two sets have atleast a common element, then they are said to be ***joint***. If they have no any common elements, then they are said to be ***disjoint***. Therefore the intersection of any two disjoint sets is an empty set. A ∩ B = { } is a condition for which sets A and B are disjoint.

For sets **A** = {1, 2, 3} and **B** = {4, 6, 8, 10}; being disjoint.

On the Venn diagram;



or



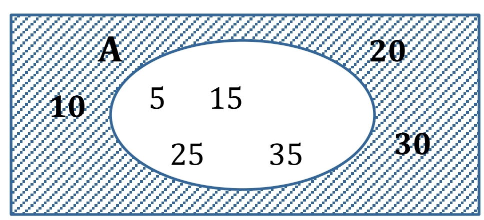
Note that;

The loop for set A does not cross that of set B.

4.3 Complement of a set, A'

This refers to the members existing in one set, but not existing in the other. Complement of a set A is denoted A' (A-complement). It means all the members out side set A, but in the universal set. Given the universal set;

ξ = {5, 10, 15, 20, 25, 30, 35} and set A = {5, 15, 25, 35}, elements which are in the universal set, but not in set A are 10, 20 and 30; therefore, A' = {10, 20, 30}.

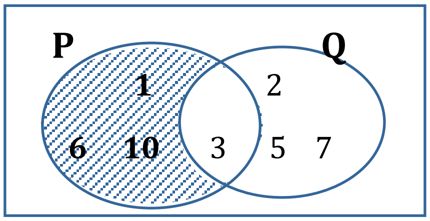


Set difference

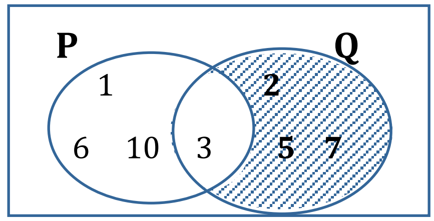
A set difference constitutes of elements of a set P, which are not elements of set Q. For set P = {1, 3, 6, 10} and Q = {2, 3, 5, 7}, then P difference Q; P – Q = {1, 6, 10}; (elements in set P and not in set Q).

On the venn diagram;

P – Q (Shaded)



Q – P (Shaded)



Exercise 4a

1 Given set P = {a, b, d, e, s} and set Q = {2, 4, 6, 8, 9}, state *true* or *false* in the following,

(i) b ∈ P **(ii)** e ∈ Q **(iii)** c ∈ P

(iv) 2 ∈ P **(v)** n(P) = n(Q) **(vi)** Ø ⊂ Q

(vii) P ⊂ Q **(viii)** P ⊃ Q **(ix)** {6} ⊃ Q

(x) {a, b} ⊃ P

2 Given sets P = { 1, 2, 3, 4 …}, Q = { 2, 3, 5, 7, 11} and R = { 0, 2, 4, 6, 8}

(a) Find **(i)** Q ∪ R **(ii)** Q ∩ R **(iii)** P ∪ R

(b) Find **(i)** P ∩ R **(ii)** n(Q ∩ R)

(c) State *true* or *false* **(i)** n(Q) = n(R) **(ii)** R ⊃ P

(iii) Q ⊂ P **(iv)** R ⊂ Q

3 Given sets, ξ = {0, 1, 2, 3 … 100} **P** = {2, 3, 5, 7, 11 … 97}

C = {4, 6, 8, 9, 10, 12, … 100} **E** = {0, 2, 4, 6, 8, 10, … 100}

D = {1, 3, 5, 7 … 99} **S** = {1, 4, 9, 16, … 100}

(a) Find **(i)** n(ξ) **(ii)** n(E ∪ D) **(iii)** D ∩ S

(iv) P ∩ C **(v)** E ∩ D **(vi)** P ∩ E

(vii) (P ∪ C)' **(viii)** n(P ∪ C)' **(ix)** n(E ∩ S)

(x) n(P)

(b) State *true* or *false*

(i) E ∪ D = ξ **(ii)** n(E) = n(D) **(iii)** E ∩ P = {2} **(iv)** C ∩ P = {0, 1}

(v) 1 ∈ P **(vi)** C ⊃ ξ **(vii)** E ∩ D = { } **(viii)** n(ξ) = 100

(ix) $E \equiv D$ (x) $n(P) + n(C) = n(\xi)$

4 Given the sets;

A = {All square numbers less than 1000}

B = {All multiples of 3 which are square numbers and less than 1000}

C = {All cubic numbers less than 1000}

D = {All square numbers which are cubic and are less than 1000}

(a) List the members of the sets A, B, C and D.

(b) Find (i) $A \cap B$ (ii) $A \cap B \cap C$ (iii) $B \cap D$

(iv) $(A \cap C)$ (v) $n(B \cap D)$ (vi) $n(A \cap C)$

(c) State *true* or *false*

(i) $(B \cap D) = C$ (ii) $A \supset B$ (iii) $n(B) = n(D)$

(iv) $n(B \cap D) = n(C)$ (v) $n(B) > n(A)$ (vi) $A \supset D$

UNIT 5 Sets Display on a Number line

INTRODUCTION

Sets with numerical members can be represented on a number line. This can be done by use of illustration or with the help of algebraic notation for inequalities.

Objectives

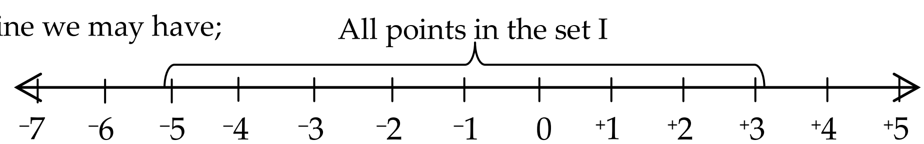
Having gone through the explanations, worked out examples, activities and exercises, you should be able to:

1. *Carry out representation of sets with numerical elements as ranges by illustration on a number line*
2. *Represent sets with numerical elements by the use of algebraic notation of inequalities.*
3. *Carry out graphical representations of set operations on a number line.*

5.1 Sets of integers

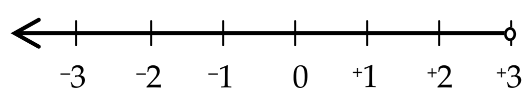
Points on a number line may have a similar description and such points form a set. A number line sharing a set of all integers from $^-5$ to $^+3$, includes integers;
I = { $^-5, ^-4, ^-3, ^-2, ^-1, 0, ^+1, ^+2, ^+3$ }

On a number line we may have;

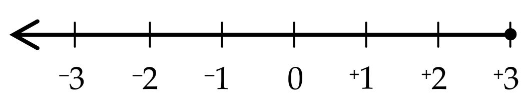


This set can also be written as:
(i) $^-6 < x < {}^+4$ or $^-5 \leq x \leq {}^+3$, where x is any one of the elements in the set I.

Given $x < 3$, we show it on a number line as:



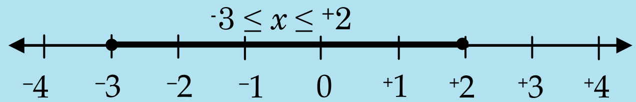
Whereas $x \leq 3$ is represented as:



Example 1

Given a set of integers I = {$^-3, ^-2, ^-1, 0, ^+1, ^+2$}. Show it on a number line solution

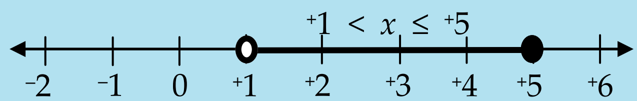
Solution



Example 2

On a number line show: ${}^{+}1 < x \le {}^{+}5$

Solution



Always remember that a colour filled cirle represents a point included in the set, and an unfilled circle is put at a point that is not included in the required set.

" ○ " at a point means **NOT INCLUDED** " ● " at a point means **INCLUDED**

Exercise 4a

Represent each of the following sets on a number line and name it as an inequality.

1 $\{{}^{+}2, {}^{+}3, {}^{+}4, {}^{+}5, {}^{+}6\}$

2 $\{{}^{-}7, {}^{-}6, {}^{-}5, {}^{-}4\}$

3 $\{{}^{-}2, {}^{-}1, 0, {}^{+}1, {}^{+}2, {}^{+}3, {}^{+}4\}$

4 $\{{}^{-}5, {}^{-}4, {}^{-}3, {}^{-}2, {}^{-}1\}$

5 $\{{}^{-}3, {}^{-}2, {}^{-}1, 0, {}^{+}1, {}^{+}2, {}^{+}3\}$

Show each of the following sets on a number line

6 ${}^{-}4 < x < {}^{-}1$

7 ${}^{-}5 < x < {}^{+}3$

8 ${}^{-}1 \le x < {}^{+}5$

9 ${}^{-}9 \le x \le {}^{+}1$

10 ${}^{+}2 > x > {}^{-}5$

11 ${}^{-}3 < x \le {}^{+}2$

12 ${}^{-}1 < x \le 0$

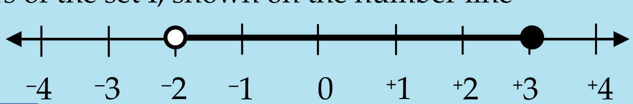
13 ${}^{-}7 < x < {}^{+}2$

14 ${}^{-}2 > x \ge {}^{-}5$

15 ${}^{+}3 \ge x > {}^{-}2$

Example 3

List the members of the set I, shown on the number line

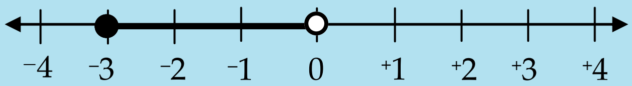


Solution

Set I = $\{{}^{-}2, {}^{-}1, 0, {}^{+}1, {}^{+}2\}$

Example 4

Give the set I shown on the number line



(a) List all the elements of set I.

(b) Write an inequality for Set I.

Solution

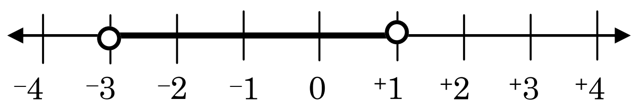
(a) I = {⁻3, ⁻2, ⁻1}

(b) $^-3 \le x < 0$ (or $0 > x \ge {}^-3$)

Exercise 5b

Given the representation of the sets shown on the number lines, list the members and show an inequality in each case.

1



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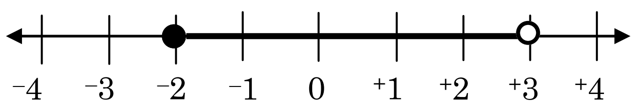

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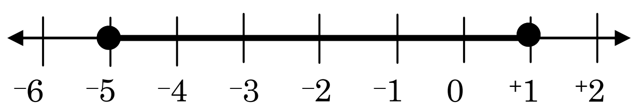

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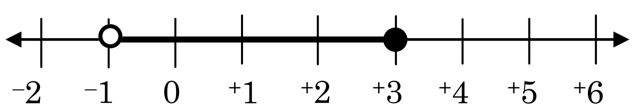

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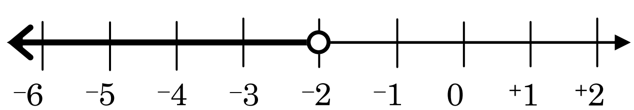

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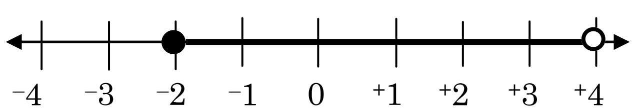


UNIT 6

Sets and Probability

INTRODUCTION

While watching a football match, a supporter always expects 3 possible results for his/her team, that is, a win, draw or a loss. All these three possibilities have equal chances of occurrence (or *are equally likely to occur*). If you sit for an examination, there are 2 possible results, that is, either you pass or fail. These two possibilities have equal chances of occurrence.

Carefully study the following terms as used in probability;

***Event* :** is an activity that takes place. For example, sitting for an examination.

***Outcome* :** is the actual result of an event. For example, passing the examination.

***Possibility* :** is an outcome that can happen

***Equally likely* :** a set of outcomes associated with a particular event are described as being equally likely when each occurs as readily as any other.

- The probability of an event is a measure of how likely that event is.
- The probability of an impossible event (e.g. the probability that a dead body talks) is equal to 0;
- The probability of an event that is certain (e.g. the probability that you will die) is equal to 1.
- Otherwise the probability must have a value which lies between 0 and 1.
- The probability (P) of any event to occur ranges from 0 to 1 inclusive, that is; $0 \leq P \leq 1$.
- Probability may be represented as a common fraction, a decimal fraction or as a percentage.

Objectives

Having gone through the explanations, worked out examples, activities and exercises, you should be able to:

1. *Elaborately use the concept of probability*
2. *To use the concept of sets with probability in problem solving*
3. *Exhaust all the possible subsets from a given set of elements*

6.1 The Probability Formula

The probability of an event A is represented as P(A);

$$P(\text{event}) = \frac{\textit{Number of ways that named outcome can happen}}{\textit{Number of all possible out comes which can be obtained from that event.}}$$

That is, $P(E) = \frac{n(E)}{n(S)}$ S can also mean sample space.

Example 1

What is the probability that a pregnant woman will produce a baby boy?

Solution

P (baby boy) = $\frac{n(E)}{n(S)}$, but, n(E) = 1 and n(S) = 2

Therefore, P (baby boy) = $^1/_2$

The probability that a pregnant woman will produce a baby boy is ½.

Example 2

What is the probability that Rangers FC will win the football match played against SC East Enders?

Solution

- A chance of winning is 1, [n(E) = 1].
- All possible outcomes are 3, that is making a win, a draw or a loss, [n(S) = 3].
  Therefore, P(a win) = $^1/_3$

So, the probability that Rangers FC wins the match is $^1/_3$.

Exercise 6a

1 What is the probability that you will never die?
2 What is the probability that petrol will catch fire if they are put together?
3 What is the probability that the word MAN has three letters?
4 What is the probability that the letter W is a number?
5 What is the probability that when you sit for an examination you will pass?

6 What is the probability that in a football match the Blues lose to SP?

7 What is the probability that today is a Thursday?

8 What is the probability that we are in the month of December?

9 What is the probability that a 3 shows up when a die is tossed?

10 What is the probability that a primary school pupil picked at random is in primary four? (The school has eight classes)

6.2 Sample space (Possibility Space)

All the possible ways in which an event is likely to occur are referred to as *sample space* usually denoted S. Other terms for sample space are possibility space or all possible outcomes.

All possible days in the week are; Sunday, Monday, Tuesday, Wednesday, Thursday, Friday and Saturday.
Therefore, n(S) = 7 for days of the week.

All possible months of the year are; January, February, March, April, May, June, July, August, September, October, November and December.
Therefore, n(S) = 12 for months of the year.

Example 1

List the possible outcomes of 3-lettered words you can write from letters A, F and R, where no letter is repeated. What is n(S) ?

Solution

From A, F and R, we can have all possible words as; AFR, ARF, FRA, RAF and RFA, leading to n(S) = 6.

Example 2

Given the digits 1, 3, 4 and 8, list down all the possible numbers that can be obtained without repeating any digit. What is n(S)?

Solution

From digits 1, 3, 4 and 8 we can have all possible numbers as;

1348	1384	1438	1483	1834	1843	
3148	3184	3418	3481	3814	3841	
4138	4183	4318	4381	4813	4831	
8134	8143	8314	8341	8413	8431	Therefore, n(S) = 24

Example 3

While fixing a day for a meeting, what is the probability of selecting a day which starts with a letter T?

Solution

Days starting with letter, T = { Tue, Thur }, then n(T) = 2
Sample space, S = {Sun, Mon, Tue, Wed, Thurs, Fri, Sat}, then n(S) = 7

$$P(T) = \frac{n(T)}{n(S)} \quad \Rightarrow \quad P(T) = {}^{2}/_{7}$$

Therefore the probability of selecting a day that starts with a letter T is ${}^{2}/_{7}$.

Exercise 6b

List down all the possible outcomes for the following events;

1 All 3-lettered words you can write from letters P, Q and R, where no letter is repeated

2 All even numbers less than 20

3 All prime numbers from 1 to 25

4 All 3-digit numbers you can write from digits 2, 3 and 5 without repeating any digit.

5 Given digits 1, 2 and 4 to be arranged to form 3 digit numbers without repeating any. What is the probability of selecting a 3-digit number from these which is a multiple of 4?

6 What is the probability of choosing a month of the year which starts with letter J?

7 An integer is chosen from the numbers 10, 11, 12, 13, 14, 15, 16, 17, 18, 19 and 20. What is the probability that the number chosen is;

(i) Prime (ii) a multiple of 4
(iii) Divisible by 5 (iv) Greater than 16

8 Given the following pairs of numbers (1, 3), (8, 3), (4, 0), (5, 4), (9, 6), (5, 7). What is the probability of selecting a pair with

(i) the sum of the 2 numbers greater than 10 ?

(ii) the difference between the 2 numbers even ?

(iii) one number exceeding the other by more than 2.

9 A storey building has five floors, each with five rooms. The floors are labeled with letters A, B, C, D and E, and the rooms are labeled with numbers 1, 2, 3, 4 and 5. The first room on floor A is labeled A1, the third room on floor B is labeled B3, etc.

(a) Show all the possible labeling of the rooms on the building.

(b) If one needs to rent any room apart from those on the floor B and any fifth room, show all the possible rooms he/she remains with for choice.

(c) If I choose to rent, what is the probability that am offered a room number 2 on any floor.

10 A basket contains 6 balls of the colours, Blue, Green, Orange, Pink, Red and Yellow.

(a) If I pick two balls at a time;

(i) Show all the possible colour combinations I can pick.

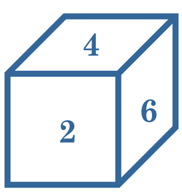
(ii) What is the probability of picking a yellow and any other ball ?

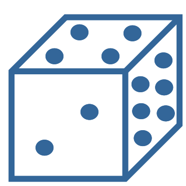
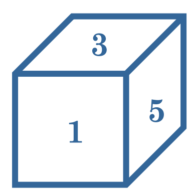
(b) If I pick three balls at a time;

(i) Show all the possible colour combinations I can pick.

(ii) What is the probability of picking a Green, Blue and Pink balls ?

6.3 The die and its tabulations



When a die is tossed all the digits 1, 2, 3, 4, 5 and 6 are equally likely to show up. Therefore the chance of having any of the faces showing up is $^1/_6$.

If two dice (dice is the plural of a die) are thrown, we can have all possible outcomes as shown in the table next page.

When two dice are tossed, all the possible outcomes are 36, that is, n(S) = 36 as shown in the table.

1	1, 1	2, 1	3, 1	4, 1	5, 1	6, 1
2	1, 2	2, 2	3, 2	4, 2	5, 2	6, 2
3	1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
4	1, 4	2, 4	3, 4	4, 4	5, 4	6,4
5	1, 5	2, 5	3, 5	4, 5	5, 5	6, 5
6	1, 6	2, 6	3, 6	4, 6	5, 6	6, 6

Therefore, when two dice are tossed, all the possible outcomes are 36, that is, n(S) = 36 as shown in the table.

Let us look at the problems involving tossing of dice.

Example 1

If two dice were tossed, what is the probability of obtaining the same numbers showing up?

Solution

N = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)};

So, n(N) = 6 and n(S) = 36.

$$P(\text{Same}) = \frac{n(N)}{n(S)} = {}^{6}/_{36} = {}^{1}/_{6}$$

∴ the probability of having the same numbers showing up on two die tossed is $^{1}/_{6}$.

Example 2

If two dice were tossed, what is the probability of obtaining two numbers making a sum greater than 10 ?

Solution

G = {(6, 5), (5, 6), (6, 6)}; So n(G) = 3 and n(S) = 36.

$$P(\text{Sum} > 10) = \frac{n(G)}{n(S)} = \frac{3}{36} = {}^{1}/_{12}$$

∴ the probability of having the sum of the two numbers showing up on two die tossed is greater than 10 is $^{1}/_{12}$.

Exercise 6c

1 If a coin is tossed, what is the probability that a tail shows up?

2 If two coins are tossed, what is the probability that heads are shown up?

3 If a die is tossed, what is the probability that an odd number is shown up?

4 If a die is tossed, what is the probability that a number divisible by 3 shows up?

5 If a die is tossed, what is the probability that a prime number shows up?

6 If two dice are tossed, what is the probability that the two numbers showing up have a sum equal to 6 ?

7 If two dice are tossed, what is the probability that the sum of the two numbers showing up is less than 5 ?

8 If two dice are tossed, what is the probability that the two numbers showing up make a difference of 2 ?

9 If two dice are tossed, what is the probability that the two numbers showing up are consecutive?

10 If two dice are tossed, what is the probability that the product of the two numbers showing up is less than or equal to 6 ?

11 Two dice were tossed, what is the probability that;

(a) the sum of the two numbers showing up is less than 2 ?

(b) the difference of the two numbers showing up is a factor of 12 ?

12 A tetrahedron has four faces with equal chances of any showing up. If it is marked 1, 2, 3 and 4.

(a) Tabulate all the possible outcomes if two tetrahedrons are tossed.

(b) What is the probability of tossing two tetrahedrons and the two numbers showing up have;

(i) their sum greater than 2 ?

(ii) their difference less than 3 ?

(iii) their product equals to their sum ?

UNIT 7 Further Venn diagrams

INTRODUCTION

Venn diagrams are pictorial representations of sets. It involves displaying the elements of the set or the number of these elements.

Members (or number of members) of given sets can be represented on a venn diagram for illustration of various purposes. These can extend to logical problems manipulation and probability as applications.

Objectives

Having gone through the explanations, worked out examples, activities and exercises, you should be able to:

1. *Understand the meaning of set regions on venn diagrams and be able to shade them according to their naming.*
2. *Represent elements of sets on venn diagrams*
3. *Represent the number of elements on venn diagrams*

7.1 Meaning of regions on Venn diagrams

Each region or part of the Venn diagram can be labeled according to its meaning. The figure below reveals the meaning of the different portions of a Venn diagram showing a universal set with two subsets.

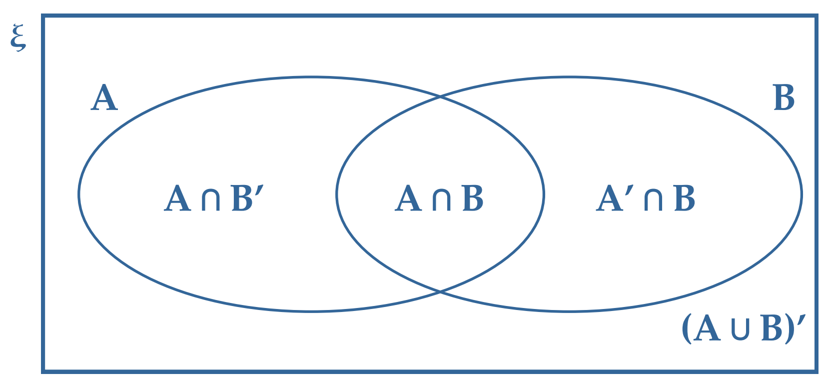


Fig. 7.1

- **$A \cap B$** means: members in both sets A and B
- **$(A \cup B)'$** means: members outside the union of sets A and B.

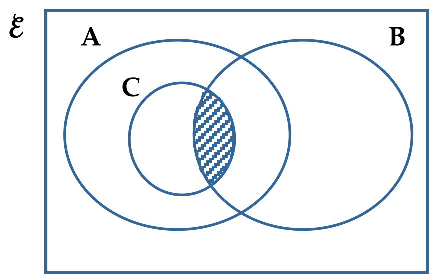
EXPLORING SETS AND LOGIC

- A ∩ B′ means: members in set A but not in set B (can also be A – B).
- A′ ∩ B means: members in set B but not in set A (can also be B – A).

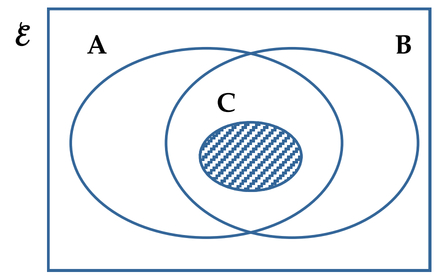
Shading Venn diagrams

We can use shading to represent the various regions of interest.

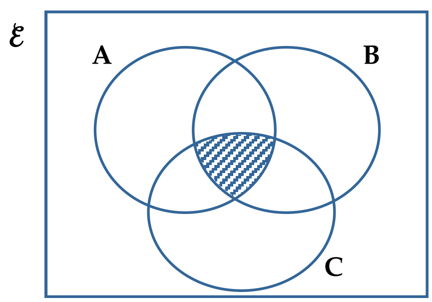
(a) **A ∩ B ∩ C (Also C ⚜ A)**



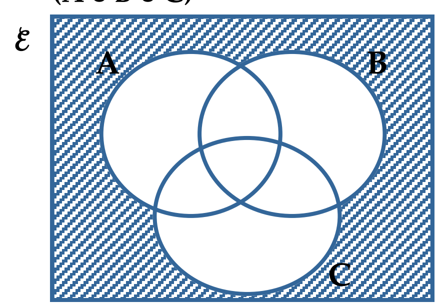
(b) **A ∩ B ∩ C (Also C ⚜ A, C ⚜ B)**



(c) **A ∩ B ∩ C**



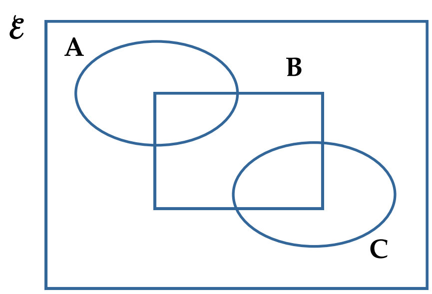
(d) **(A ∪ B ∪ C)'**



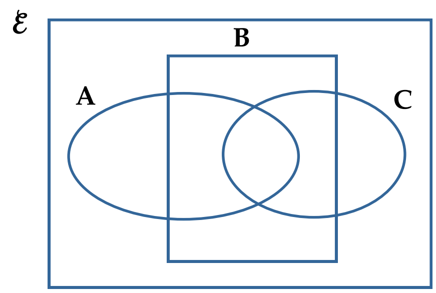
Exercise 7a

Given the following Venn diagrams, label all the regions given

1

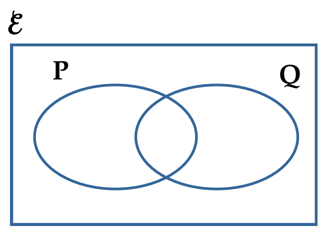


2

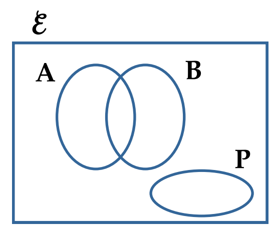


Show by shading the following regions

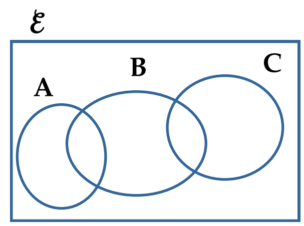
3 $(P \cup Q)'$



4 $(A \cap B) \cup P$

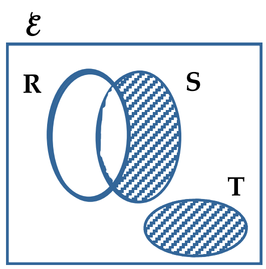


5 $(A \cap B) \cup (B \cap C)$

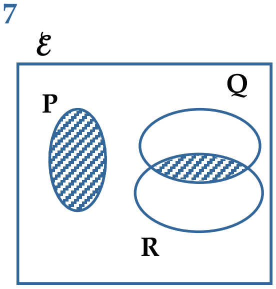


Name the shaded regions of the following Venn diagrams.

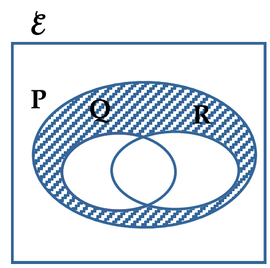
6



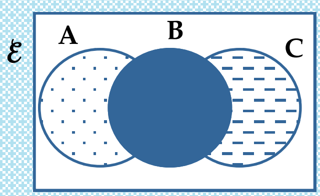
7



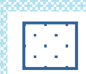
8

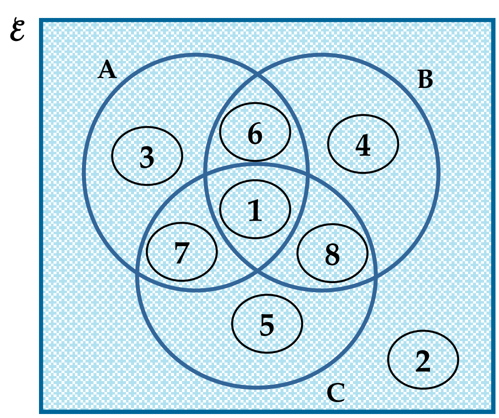


Activity 7.1



From the Venn diagram on the left, name the regions shaded as:

(a)  (b) 



1 ⟹ $A \cap B \cap C$

2 ⟹ $(A \cup B \cup C)'$

3 ⟹ $A \cap B' \cap C'$

4 ⟹ $A' \cap B \cap C'$

5 ⟹ $A' \cap B' \cap C$

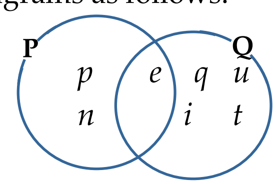
(6) ⟹ $(A \cap B) \cap C'$ (7) ⟹ $(A \cap C) \cap B'$ (8) ⟹ $(B \cap C) \cap A'$

These notations imply:

1. $A \cap B \cap C$ means: Members in all sets A, B and C.
2. $(A \cup B \cup C)'$ means: Members outside the union of sets A, B and C.
3. $A \cap B' \cap C'$ means: Members in set A, and not in sets B and C.
4. $A' \cap B \cap C'$ means: Members in set B, and not in sets A and C.
5. $A' \cap B' \cap C$ means: Members in set C, and not in sets A and B.
6. $(A \cap B) \cap C'$ means: Members in both sets A and B, but not in set C.
7. $(A \cap C) \cap B'$ means: Members in both sets A and C, but not in set B.
8. $(B \cap C) \cap A'$ means: Members in both sets B and C, but not in set A.

7.2 Representing elements of sets on Venn diagrams

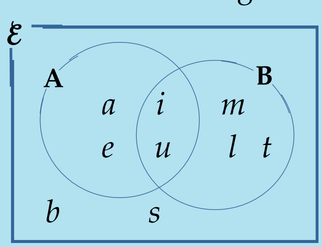
We can indicate specific elements of a set on Venn diagrams. Given that set P = {*p*, *e*, *n*} and set Q = {*q*, *u*, *i*, *t*, *e*}, then we can represent these two sets on Venn diagrams as follows:



Note that the commas are not shown on the Venn diagrams and that the element *e* is in both sets, so it is written once in a region shared by both the sets.

Example 1

Given the Venn diagram below;

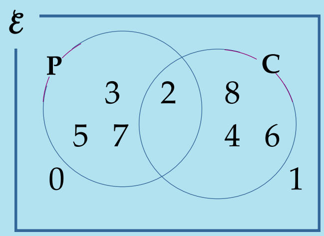


Find **(a)** A **(b)** B **(c)** ℰ
(d) $(A \cup B)'$ **(e)** $A' \cap B$

Solution

(a) A = {*a*, *e*, *i*, *u*} **(b)** B = {*i*, *l*, *m*, *t*, *u*} **(c)** ℰ = {*a*, *b*, *e*, *i*, *l*, *m*, *s*, *t*, *u*}
(d) $(A \cup B)'$ = {*b*, *s*} **(e)** $A' \cap B$ = {*l*, *m*, *t*}

Example 2



Given the Venn diagram on left;

Find (a) P
(b) C
(c) P ∩ C
(d) (P ∪ C)'

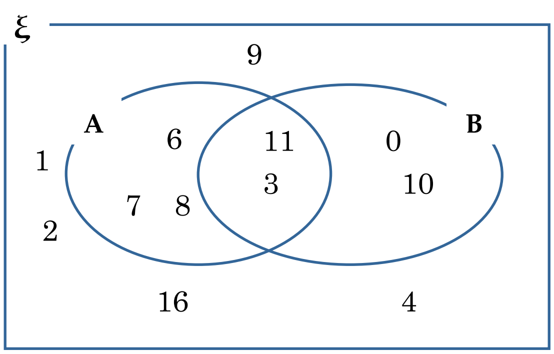
Solution

(a) P = {2, 3, 5, 7} (b) C = {2, 4, 6, 8} (c) P ∩ C = {2} (d) (P ∪ C)'= {0, 1}

Exercise 7b

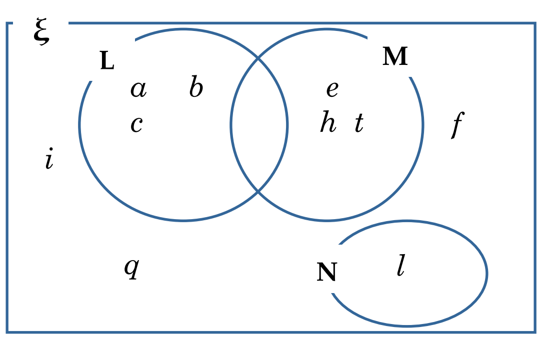
1 Given a universal set ℰ = {1, 2, 3, 4, 5, 6, 7, 8 . . . 15}, set **L** = {1, 3, 6, 10}, set **M** = {2, 3, 5, 7} and **N** = {3, 9, 12, 15}.
(a) Show the sets on a venn diagram.
(b) Find (i) L ∩ M ∩ N (ii) (L ∪ M ∪ N)'

2 Given the Venn diagram below;



Find the members of
(i) set A
(ii) set B only
(iii) A ∩ B
(iv) (A ∪ B)'
(v) n(ξ)

3 Given the Venn diagram below;

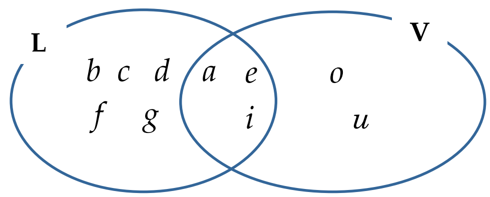


(a) Find, (i) L ∪ M
(ii) L ∩ M
(iii) (L ∪ M ∪ N)'

(b) Find, (i) n(L ∩ M)
(ii) n(M ∪ N)'
(iii) n(L)'

7.3 Representing the number of elements on Venn diagrams

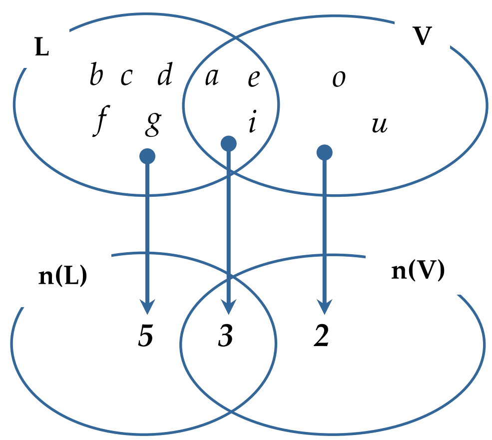
Given a set L = {*a, b, c, d, e, f, g, h, i*} and set V = {*a, e, i, o, u*}, their elements can be shown on Venn diagrams as follows:



We can however rewrite this as number of elements on Venn diagrams.

Elements of the sets L and V:

Number of elements for sets L and V:



We can now summarize that;

(i) n(L) = (5 + 3) = **8**

(ii) n(V) = (3 + 2) = **5**

(iii) n(L ∩ V) = **3**

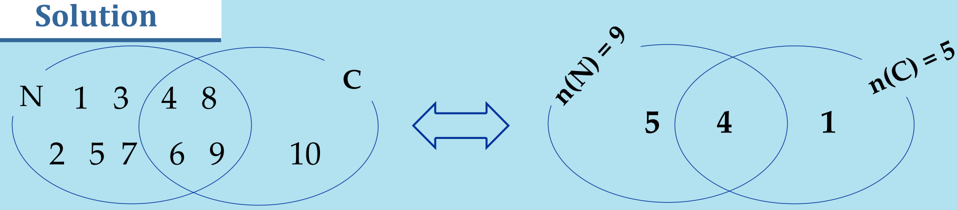
(iv) n(L ∪ V) = (5 + 3 + 2) = **10**

Example 2

Given that set N = {1, 2, 3, 4, 5, 6, 7, 8, 9} and set C = {4, 6, 8, 9, 10}. Represent their number of elements on a Venn diagram and find:

(i) n(N ∩ C) **(ii)** n(N ∪ C)

Solution



(i) N ∩ C = {4, 6, 8, 9}. So, n(N ∩ C) = 4

(ii) N ∪ C = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. So, n(N ∪ C) = 10

Exercise 7c

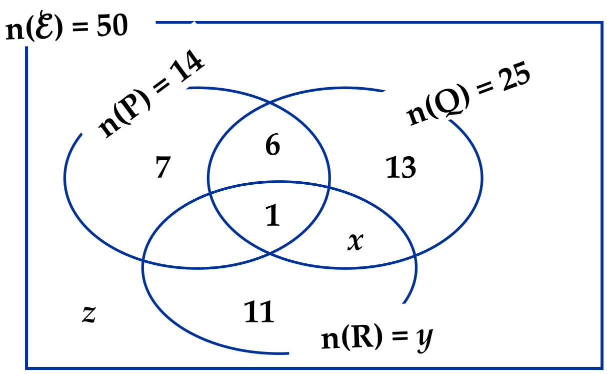
1 Given that set **P** = {2, 4, 6, 8} and set **Q** = {1, 2, 3, 4, 5, 6}, represent them on a Venn diagram and find; **(a)** n(P ∪ Q) **(b)** n(P ∩ Q)

2 Given that set R = {  }, and

set S = {  }

Show on Venn diagrams, the number of elements for the sets R and S, hence find
(i) n(R ∩ S) **(ii)** n(R ∪ S) **(iii)** n(R ∩ S)′ **(iv)** n[(R ∩ S′) ∪ (R′ ∩ S)]

3 Given the Venn diagram that follows,



(a) Find the values of letters x, y and z

(b) Find **(i)** n(P ∩ Q)
(ii) n(Q ∪ R)

UNIT 8 Venn diagrams and Logical problems

INTRODUCTION

We can use venn diagrams to solve various daily life problems which involve logic. By doing this we need to effectively remember all the concepts as earlier studied.

Objectives

Having gone through the explanations, worked out examples, activities and exercises, you should be able to:

1. *Interpret word problems involving sets theory and representing the information on venn diagrams.*
2. *Present and Interpret information on venn diagrams*
3. *Calculate the problems obtained from any given information*
4. *Further use venn diagrams to solve logical problems*

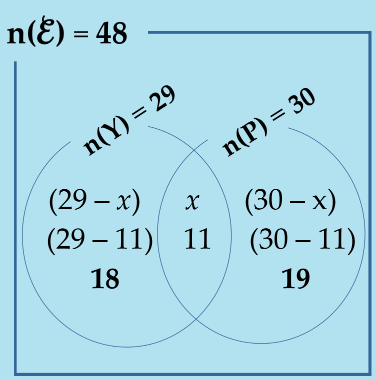
8.1 Problems involving Venn diagrams

The concept of representing sets on venn diagrams can also be extended to solving problems involving sets. These can be elementary or applied (that is, in daily life), tackled using word problems and illustrations.

Example 1

In a class of 48 students, 29 like Yams (Y) and 30 like Posho (P). Find the number of students who like both Yams and Posho using a venn diagram.

Solution



Let the number of those who like both Yams and Posho be **x**

If the number of all students who like Yams is 29, then those who like *Yams only will be (29 – x).*

Likewise for Posho only is (30 – x).

But; n(Y only) + n(M ∩ P) + n(P only) = n(ξ)

$$(29 - x) + x + (30 - x) = 48$$

$$29 - \cancel{x} + \cancel{x} + 30 - x = 48$$

$$59 - x = 48$$

$$^{-}x = 48 - 59$$

$$\frac{^{-}\cancel{1}x}{^{-}\cancel{1}} = \frac{^{-}11}{^{-}1}$$

$$\Rightarrow \mathbf{x = 11}$$

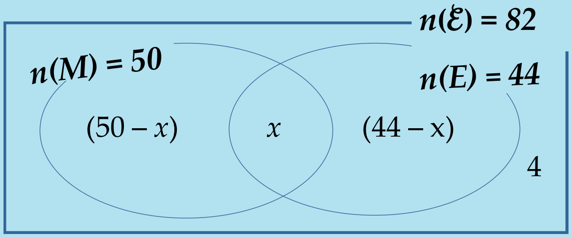
So, 11 students like both Matooke and Posho. *Check: Is 48 = 18 + 11 + 19 ? Yes.*

So, our calculations to find the value of x are accurate.

Example 2

In a class of 82 students, 50 like Mathematics (M) and 44 students like English (E). 4 students like neither of the two subjects. Find the number of students who like both Math and English.

Solution



n(ξ) = 82

n(E) = 44

n(M) = 50

n(M ∪ E)' = 4

$$(50 - x) + x + (44 - x) + 4 = 82$$

$$50 - x + x + 44 - x + 4 = 82$$

$$98 - x = 82$$

$$^{-}x = 82 - 98$$

$$^{-}x = {}^{-}16$$

$$\Rightarrow x = 16$$

Therefore, there are 16 students who like both Math and English in the class.

Exercise 8a

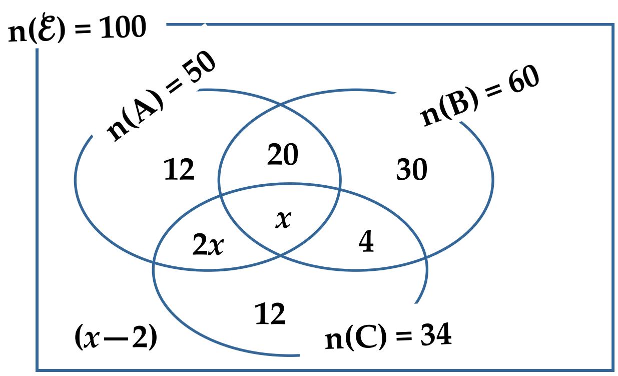
1 In a class of 48 students 20 like beans (B) and 30 like peas (P). How many students like both beans and peas?

2 In a class of 50 students 25 like meat (M) and 5 like both meat and chicken. How

many students like chicken only?

3 Of the 86 people who went for a film show 52 liked a Chinese (C) film only and 20 liked a Japanese (J) film only. How many people liked both the Japanese and Chinese films?

4 Given the Venn diagram below:
Find the value of x and hence find



(i) n(A ∩ B ∩ C)
(ii) n(A ∩ C)
(iii) n(A ∪ B ∪ C)'
(iv) n(A ∩ B)

5 In a class of 58 students, 20 like volleyball (V) only, 30 like Basket ball (B) and the rest like neither of the two games. If the number of students who like both the games is $^1/_3$ the number of those who like basket ball, find the number of students who like:

(i) Basket ball only?
(ii) Both of the games?
(iii) Volley ball?
(iv) Neither of the two games?

REVISION PAPER

1 **(a)** Name the following sets

(i) **S** = {pens, pencils, punching machine, clips, ruler, markers }

(ii) **E** = {calculator, mobile phone hand set, digital watch, computer}

(iii) **P** = {Democratic Party, Conservative Party}

(iv) **M** = { 8, 16, 24, 32, 36, 48 }

(v) **C** = {4, 6, 8, 9, 10, 12, 14, 15 …}

(vi) **B** = {Chicken, Duck, Turkey, Pigeon}

(vii) **S** = {Biology, Physics, Chemistry}

(viii) **R** = {Lizard, Snake, Chameleon}

(ix) **G** = {Closed plane figures with all sides and angles congruent}

(x) **L** = {Chord, Center, Diameter, Arc, Sector}

(b) List down the members of the following sets:

(i) **M** = {Even multiples of 7 that are divisible by 6}

(ii) **S** = {Square numbers that are cubic and even but less than 100}

(iii) **N** = {All letters forming your surname}

(iv) **T** = {Multiples of ten that are divisible by 6}

(v) **I** = {All instruments found in a geometry set}

(vi) **E** = {Multiples of 7 which are prime}

2 Given the sets below;

V = { All vowels of the English alphabet}

E = { All letters of the English alphabet}

N = { A, B, C, D, E, F}

P = { 2, 3, 5, 7, 11, 13 }

Q = { All even numbers which are square numbers}

R = {1, 3, 6, 10, 15 …}

C = { 4, 6, 8, 9, 10, 12, 14, 15 …}

(a) State ***true*** or ***false***

(i) $V \subset E$ **(ii)** $N \equiv P$ **(iii)** **V** = {*a*, *e*, *i*, *o*, *u*, *w*}

(iv) **R** = {Triangular numbers} **(v)** $n(E) = 27$

(vi) $n(Q) = 1$ **(vii)** $P \cap C = \varnothing$ **(viii)** $21 \in C$

(b) State which of the given sets are finite and infinite

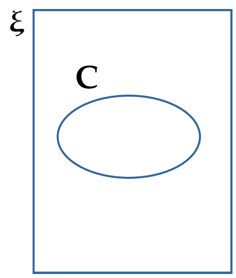
3 **(a)** Given the set D = {All consonants in the word "*diagrammatically*"}. Find the number of subsets which can be obtained from set D.

(b) Given the sets: **A** = { 1, 2, 3, 4 … 10} , **B** = { 0, 2, 4, 6 … 10}
C = { 1, 4, 9 … 100} , **D** = { 1, 8, 27 … 1000} , Find

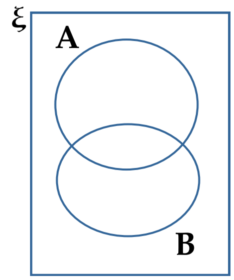
(i) n(D) **(ii)** $C \cap D$ **(iii)** $(A \cup B) \cap C$ **(iv)** $n(A \cap B)$ **(v)** $A \cap D$

4 **(a)** Show by shading the regions.

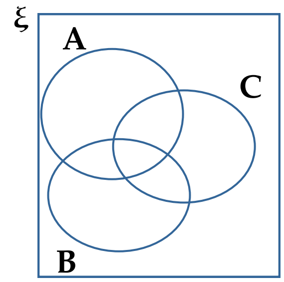
(i) C '



(ii) $A \cap B$

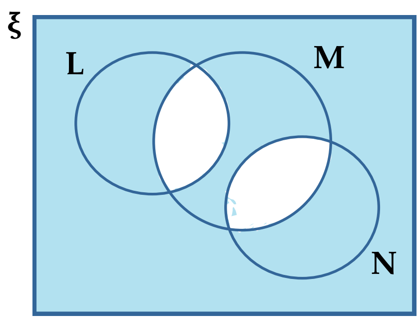


(iii) $(A \cup B) \cap C$

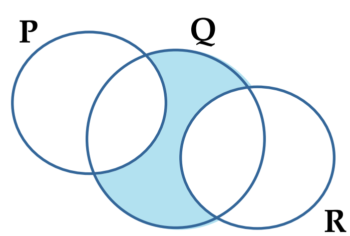


(b) Name the following shaded regions.

(i)

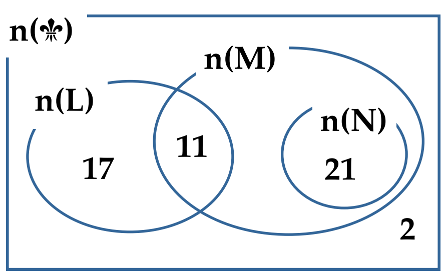


(ii)



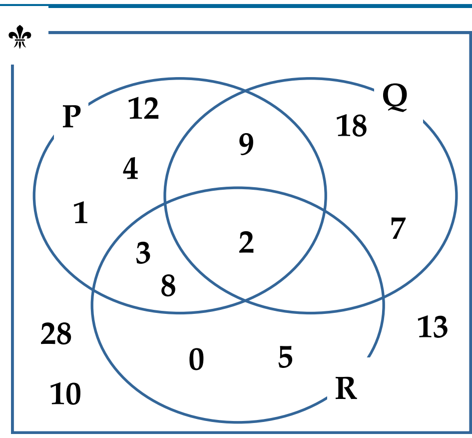
Given the Venn diagrams that follow, answer the questions as required:

5



Find **(i)** $n(L \cap N)$
(ii) $n(M \cap N)$
(iii) $n(N \cup L)'$
(iv) n(M)

6

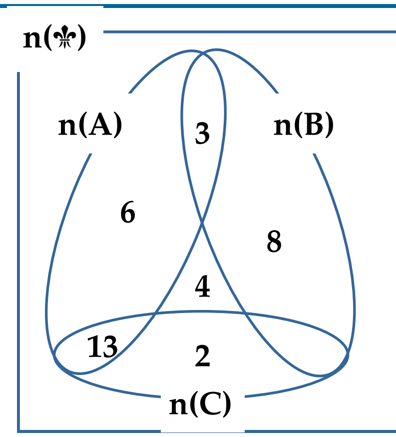


Find (i) (P ∩ R)

(ii) (P ∪ Q ∪ R)'

(iii) (P ∩ Q ∩ R)'

7



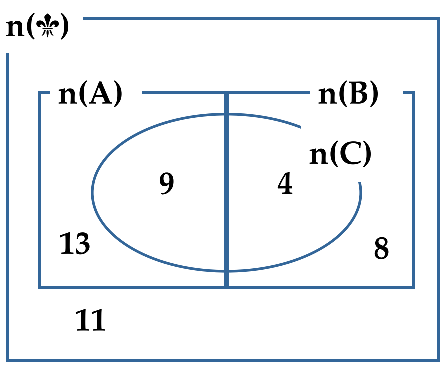
Find (i) n(A ∪ B ∪ C)'

(ii) n(B ∩ C)

(iii) n(A ∩ B ∩ C)

(iv) n(A)

8



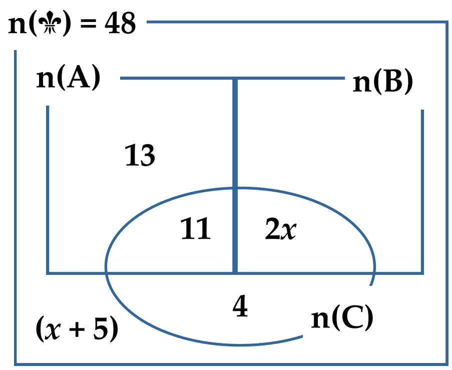
Find (i) n(A ∪ B)′

(ii) n(C)

(iii) n(B ∩ C)

(iv) n(A ∩ B)

9



Find (i) x

(ii) n(B ∩ C)

(iii) n(A ∪ B ∪ C)′

(iv) n(C)

10 In a group of 82 students, 47 like Volleyball, 32 like Basket ball and 15 like neither of the two games.

(a) Draw a venn-diagram to represent this information.

(b) How many students like both games?

(c) How many students like only one game?

11 In a class of 84 students, 30 like only Maths, 42 like only English. The number of

those who like neither of the two subjects is twice the number of those who like both of the subjects.

(a) Draw a Venn- diagram to represent this information.
(b) How many students like both the subjects?
(c) How many students like neither of the two subjects?

12 In a group of 52 students, 30 like Physics, 24 like Chemistry and twice the number of those who like both the subjects exceeds the number of students who like neither of the two subjects by 5.

(a) Draw a Venn-diagram to represent this information.
(b) How many students like both the subjects?
(c) How many students like neither of the subjects?

ANSWERS

EXERCISE 1A (Pages 4 - 5)

1. {square, rectangle, rhombus, parallelogram, trapezium, kite, chevron, oblong, arrowhead}

2. {cube, cuboid, frustrum, pentagon based pyramid}

3. {Kenya, Tanzania, Burundi}

4. {6, 12, 18, 24, 30, 36, 42, 48, . . . }

5. {12, 24, 36, 48, 60, 72, 84, 96}

6. stationary

7. Months of the year

8. Consonants of the English alphabet

9. Letters in the word "closed"

10. Even numbers from 2 onwords

11. true **12.** true **13.** false **14.** true **15.** false **16.** true

EXERCISE 1B (Page 6)

1. true **2.** false **3.** true **4.** true **5.** false **6.** true

7. false **8.** false **9.** false **10.** false **11.** false **12.** true

EXERCISE 2A (Pages 8 - 9)

1. {goats with eight legs}, {boys older than their mothers}, {egg laying cows}, {flying snakes}, {speaking stones}

2. **(a)** true **(b)** false **(c)** false **(d)** false

3. {Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune, Pluto}, {a, e, i, o, u}, {earth wire, live wire, neutral wire}, {monitor, keyboard, Central Processing Unit, Mouse}, {1, 2, 3, 4, 6, 12}, {arithmetic, algebra, geometry, statistics}, etc.

4. {1, 2, 3, 4, . . . }, {0, 2, 4, 6, . . . }, {1, 3, 5, 7, 9, . . . }, {1, 4, 9, 16, . . . }, { 1, 8, 27, 64, . . . }

5. **(a)** false **(b)** true **(c)** false **(d)** true **(e)** true

EXERCISE 3A (Pages 11 - 12)

1. false **2.** false **3.** false **4.** false **5.** true **6.** true

7. false **8.** true **9.** false **10.** true **11.** false **12.** true

EXERCISE 3B (Page14)

1. false **2.** false **3.** true **4.** true **5.** false

EXPLORING SETS AND LOGIC

6. (a) set A has 64 subsets, set B has 32 subsets and set C has 2048 subsets **(b)** 7 elements

EXERCISE 4A (Pages 17 - 18)

1. (i) true **(ii)** false **(iii)** false **(iv)** false
(v) true **(vi)** true **(vii)** false **(viii)** false
(ix) false **(x)** false

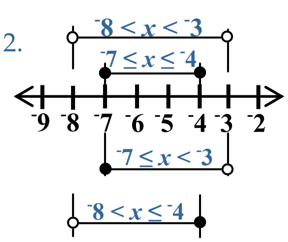
2. (a) (i) {0, 2, 3, 4, 5, 6, 7, 8, 11} **(ii)** {0, 1, 2, 3, 4, 5, 6, 7, 8, . . . }
(b) (i) {0, 2, 4, 6, 8} **(ii)** 1
(c) (i) true **(ii)** false **(iii)** true **(ii)** false

3. (a) (i) 101 **(ii)** 101 **(iii)** {1, 9, 25, 49, 81} **(iv)** { }
(v) { } **(vi)** { 2 } **(vii)** {0, 1} **(viii)** 2
(b) (i) true **(ii)** false **(iii)** true **(iv)** false
(v) false **(vi)** false **(vii)** true **(viii)** false
(ix) false **(x)** false

4. (a) A = {1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 225, 256, 289, 324, 361, 400, 441, 484, 529, 576, 625, 676, 729, 784, 841, 900, 961}
B = {9, 36, 81, 144, 225, 324, 441, 576, 729, 900}
C = {1, 8, 27, 64, 125, 216, 343, 512, 729}
D = {1, 64, 729}
(b) (i) {9, 36, 81, 144, 225, 324, 441, 576, 729, 900} **(ii)** {729} **(iii)** {729}
(iv) {1, 64, 729} (v) 1 (vi) 3
(c) (i) false **(ii)** true **(iii)** false **(iv)** false **(iv)** true

EXERCISE 5A (Page 20)

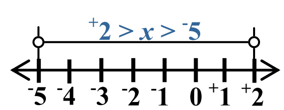
1. $^{+}3 < x < ^{+}7$, $^{+}2 \le x \le ^{+}6$, $^{+}3 < x \le ^{+}6$, $^{+}2 \le x < ^{+}7$

2.


3. $^{-}3 < x < ^{+}5$, $^{-}2 \le x \le ^{+}4$, $^{-}3 < x \le ^{+}4$, $^{-}2 \le x < ^{+}5$

4. $^{-}6 < x < 0$, $^{-}5 \le x \le ^{-}1$, $^{-}6 < x \le ^{-}1$, $^{-}5 \le x < \mathbf{0}$

5. $^{-}4 < x < ^{+}4$, $^{-}3 \le x \le ^{+}3$, $^{+}4 < x \le ^{+}3$, $^{-}3 \le x < ^{+}4$

10.


(Complete numbers 6— 9 and 11—15)

EXERCISE 5B (Page 21)

1.	{$^{-}2, {}^{-}1, 0$}	$^{-}3 < x < {}^{+}1$,	$^{-}2 \le x \le 0$,	$^{-}3 < x \le 0$,	$^{-}2 \le x < {}^{+}1$
2.	{$^{-}2, {}^{-}1, 0, {}^{+}1, {}^{+}2$}	$^{-}3 < x < {}^{+}3$,	$^{-}2 \le x \le {}^{+}2$,	$^{-}3 < x \le {}^{+}2$,	$^{-}2 \le x < {}^{+}3$
3.	{$^{-}5, {}^{-}4, {}^{-}3, {}^{-}2, {}^{-}1, 0, {}^{+}1$}	$^{-}6 < x < {}^{+}2$,	$^{-}5 \le x \le {}^{+}1$,	$^{-}6 < x \le {}^{+}1$,	$^{-}5 \le x < {}^{+}2$
4.	{$0, {}^{+}1, {}^{+}2, {}^{+}3$}	$^{-}1 < x < {}^{+}4$,	$0 \le x \le {}^{+}3$,	$^{-}1 < x \le {}^{+}3$,	$0 \le x < {}^{+}4$
5.	{ . . . $^{-}6, {}^{-}5, {}^{-}4, {}^{-}3$}	$^{-}2 > x$,	$x \le {}^{-}3$,	$x < {}^{-}3$,	$^{-}3 \ge x$ 6
6.	{$^{-}2, {}^{-}1, 0, {}^{+}1, {}^{+}2, {}^{+}3$}	$^{-}3 < x < {}^{+}4$,	$^{-}2 \le x \le {}^{+}3$,	$^{-}3 < x \le {}^{+}3$,	$^{-}2 \le x < {}^{+}4$

EXERCISE 6A (Pages 23 - 24)

1. 0	**2.** 1	**3.** 1	**4.** 0	**5.** $^{1}/_{2}$
6. $^{1}/_{3}$	**7.** $^{1}/_{7}$	**8.** $^{1}/_{12}$	**9.** $^{1}/_{6}$	**10.** $^{1}/_{8}$

EXERCISE 6B (Pages 25 - 26)

1. {PQR, PRQ, QPR, QRP, RPQ, RQP}

2. {0, 2, 4, 6, 8, 10, 12, 14, 16, 18}

3. {2, 3, 5, 7, 11, 13, 17, 19, 23}

4. {235, 253, 325, 352, 523, 532}

5. $^{1}/_{3}$

6. $^{1}/_{4}$

7. (i) $^{4}/_{11}$ **(ii)** $^{3}/_{11}$ **(iii)** $^{3}/_{11}$ **(iv)** $^{4}/_{11}$

8. (i) $^{1}/_{2}$ **(ii)** $^{1}/_{2}$ **(iii)** $^{1}/_{2}$

9. (a)

A1	**A2**	**A3**	**A4**	**A5**
B1	**B2**	**B3**	**B4**	**B5**
C1	**C2**	**C3**	**C4**	**C5**
D1	**D2**	**D3**	**D4**	**D5**
E1	**E2**	**E3**	**E4**	**E5**

(b)

A1	**A2**	**A3**	**A4**
C1	**C2**	**C3**	**C4**
D1	**D2**	**D3**	**D4**
E1	**E2**	**E3**	**E4**

(c) $^{1}/_{5}$

10. (a) (i)

Blue-Green	Blue-Orange	Blue-Pink	Blue-Red	Blue-Yellow
Green-Orange	Green-Pink	Green-Red	Green-Yellow	
Orange-Pink	Orange-Red	Orange-Yellow		
Pink-Red	Pink-Yellow			
Red-Yellow				

(ii) $^{1}/_{3}$

(b) (i) Blue-Green-Orange Blue-Green-Pink Blue-Green-Red Blue-Green-Yellow

Green-Orange-Pink   Green-Orange-Red   Green-Orange-Yellow
Orange-Pink-Red   Orange-Pink-Yellow
Pink-Red-Yellow

(ii) $\frac{1}{10}$

EXERCISE 6C (Page 28)

1. $\frac{1}{2}$ **2.** $\frac{1}{4}$ **3.** $\frac{1}{2}$ **4.** $\frac{1}{3}$ **5.** $\frac{1}{2}$

6. $\frac{5}{36}$ **7.** $\frac{1}{6}$ **8.** $\frac{2}{9}$ **9.** $\frac{5}{18}$ **10.** $\frac{7}{18}$

11. (a)

	1	2	3	4
1	1, 1	2, 1	3, 1	4, 1
2	1, 2	2, 2	3, 2	4, 2
3	1, 3	2, 3	3, 3	4, 3
4	1, 4	2, 4	3, 4	4, 4

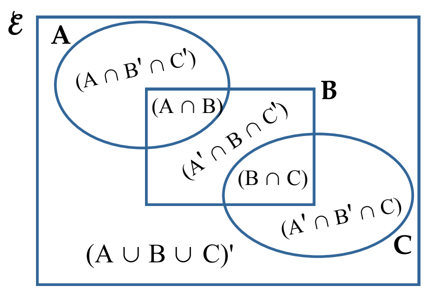
(b) (i) $\frac{15}{16}$

(ii) $\frac{7}{8}$

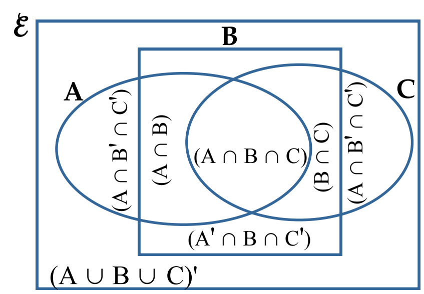
(iii) $\frac{1}{16}$

EXERCISE 7A (Pages 30 - 31)

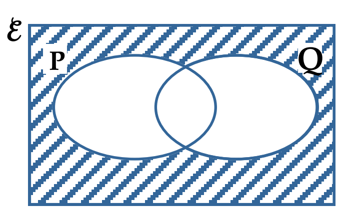
1.



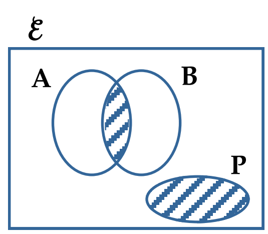
2.



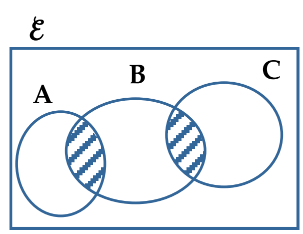
3.



4.



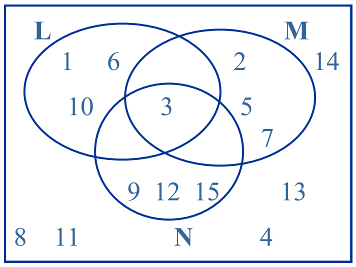
5.



6. (S ∩ R') ∪ T   **7.** P ∪ (R ∩ Q)   **8.** P— (Q ∪ R)

EXERCISE 7B (Page 33)

1. (a) $\mathcal{E}$



(b) (i) {3} **(ii)** {4, 8, 11, 13, 14}

2. (i) {3, 6, 7, 8, 11} **(ii)** {0, 10}

(iii) {3, 11} **(iv)** {1, 2, 4, 9, 16}

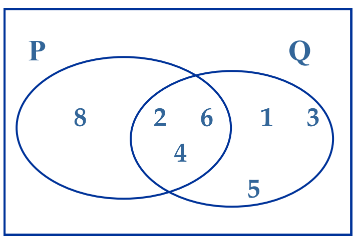
(v) {0, 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 16}

3. (a) (i) {*a*, *b*, *c*, *e*, *h*, *t*} **(ii)** { } **(iii)** {*f*, *i*, *q*}

(b) (i) 0 **(ii)** 3 **(iii)** 7

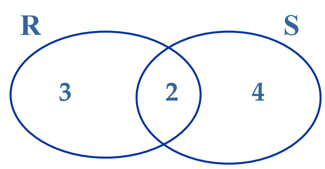
EXERCISE 7C (Page 35)

1. $\mathcal{E}$



(a) 7 **(b)** 3

2.



(i) 2 **(ii)** 9

(iii) 7 **(iv)** 7

3. (a) $x = 5$, $y = 7$ and $z = 7$ **(b) (i)** 7 **(ii)** 36

EXERCISE 8A (Pages 37 - 38)

1. 2 students **2.** 25 students **3.** 14 people

4. $x = 6$; **(i)** 6 **(ii)** 18 **(iii)** 4 **(iv)** 26

5. **(i)** 20 **(ii)** 10 **(iii)** 30 **(iv)** 8

REVISION PAPER (Pages 39 - 42)

1. (a) (i) stationary **(ii)** electronic equipment **(iii)** political parties

(iv) first 6 multiples of 8 **(v)** composite numbers **(vi)** domestic birds

(vii) science subjects **(viii)** reptiles **(ix)** regular polygons **(x)** circle parts

EXPLORING SETS AND LOGIC

(b) (i) {42, 84, 126, 168, 210} **(ii)** {64} **(iii)** "open answer" **(iv)** { }

(v) {compasses, dividers, set squares, protractors, sharpeners, rubbers, rulers} **(vi)** { 7 }

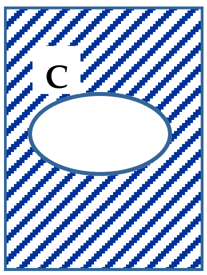
2. (a) (i) true **(ii)** true **(iii)** false **(iv)** true

(v) false **(vi)** false **(vii)** true **(viii)** true

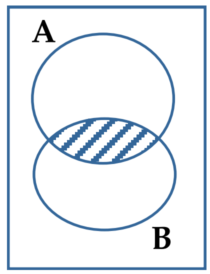
(b) sets V, E, N and P are finite sets Q, R and C are infinite

3. (a) 256 subsets **(b) (i)** 10 **(ii)** {64} **(iii)** {1, 4, 9} **(iv)** 5 **(v)** {1, 8}

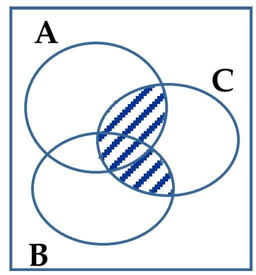
4. (a) (i)



(ii)



(iii)



(a) (i) $[(L \cap M) \cup (M \cap N)]'$ **(ii)** $Q \cap P' \cap R'$ [or $(Q - P) \cap (Q - R)$]

5. **(i)** 0 **(ii)** 21 **(iii)** 2 **(iv)** 32

6. **(i)** {2, 3, 8} **(ii)** {10, 13, 28} **(iii)** {0, 1, 3, 4, 5, 7, 8, 9, 10, 12, 13, 18, 28}

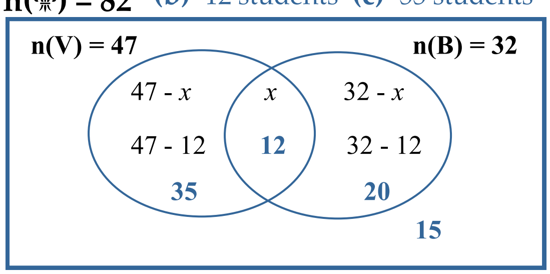
7. **(i)** 4 **(ii)** 0 **(iii)** 0 **(iv)** 22

8. **(i)** 11 **(ii)** 13 **(iii)** 4 **(iv)** 0

9. **(i)** 5 **(ii)** 10 **(iii)** 10 **(iv)** 25

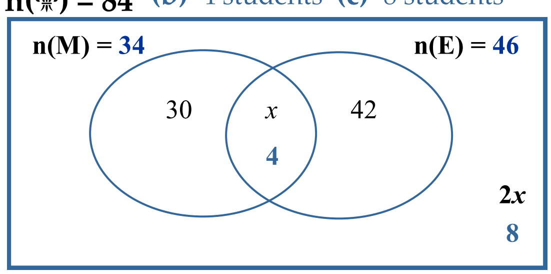
10. (a)

n(ξ) = 82 **(b)** 12 students **(c)** 55 students



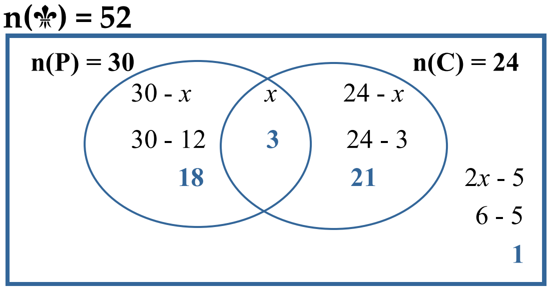
11. (a)

n(ξ) = 84 **(b)** 4 students **(c)** 8 students



12. (a)

n(ξ) = 52



(b) 3 students

(c) 1 student