III. *An Essay on the Cohesion of Fluids.* *By* Thomas Young, *M. D. For. Sec. R. S.*

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I. *General Principles.*

It has already been asserted, by Mr. Monge and others, that the phenomena of capillary tubes are referable to the cohesive attraction of the superficial particles only of the fluids employed, and that the surfaces must consequently be formed into curves of the nature of linteariæ, which are supposed to be the results of a uniform tension of a surface, resisting the pressure of a fluid, either uniform, or varying according to a given law. Segner, who appears to have been the first that maintained a similar opinion, has shown in what manner the principle may be deduced from the doctrine of attraction, but his demonstration is complicated, and not perfectly satisfactory; and in applying the law to the forms of drops, he has neglected to consider the very material effects of the double curvature, which is evidently the cause of the want of a perfect coincidence of some of his experiments with his theory. Since the time of Segner, little has been done in investigating accurately and in detail the various consequences of the principle.

It will perhaps be most agreeable to the experimental philosopher, although less consistent with the strict course of logical argument, to proceed in the first place to the comparison of this theory with the phenomena, and to inquire afterwards for its foundation in the ultimate properties of matter. But it is necessary to premise one observation, which appears to be new, and which is equally consistent with theory and with experiment; that is, that for each combination of a solid and a fluid, there is an appropriate angle of contact between the surfaces of the fluid, exposed to the air, and to the solid. This angle, for glass and water, and in all cases where a solid is perfectly wetted by a fluid, is evanescent: for glass and mercury, it is about 140°, in common temperatures, and when the mercury is moderately clean.

II. *Form of the Surface of a Fluid.*

It is well known, and it results immediately from the composition of forces, that where a line is equably distended, the force that it exerts, in a direction perpendicular to its own, is directly as its curvature; and the same is true of a surface of simple curvature; but where the curvature is double, each curvature has its appropriate effect, and the joint force must be as the sum of the curvatures in any two perpendicular directions. For this sum is equal, whatever pair of perpendicular directions may be employed, as is easily shown by calculating the versed sines of two equal arcs taken at right angles in the surface. Now when the surface of a fluid is convex externally, its tension is produced by the pressure of the particles of the fluid within it, arising from their own weight, or from that of the surrounding fluid; but when the surface is concave, the tension is employed in counteracting the pressure of the atmosphere, or, where the atmosphere is excluded, the equivalent pressure arising from the weight of the particles suspended from it by means of their cohesion, in the same manner as, when water is supported by the atmospheric pressure in an inverted vessel, the outside of the vessel sustains a hydrostatic pressure proportionate to the height; and this pressure must remain unaltered, when the water, having been sufficiently boiled, is made to retain its situation for a certain time by its cohesion only, in an exhausted receiver. When, therefore, the surface of the fluid is terminated by two right lines, and has only a simple curvature, the curvature must be every where as the ordinate; and where it has a double curvature, the sum of the curvatures in the different directions must be as the ordinate. In the first case, the curve may be constructed by approximation, if we divide the height at which it is either horizontal or vertical into a number of small portions, and taking the radius of each portion proportional to the reciprocal of the height of its middle point above or below the general surface of the fluid, go on to add portions of circles joining each other, until they have completed as much of the curve as is required. In the second case, it is only necessary to consider the curve derived from a circular basis, which is a solid of revolution; and the centre of that circle of curvature, which is perpendicular to the section formed by a plane passing through the axis, is in the axis itself, consequently in the point where the normal of the curve intersects the axis: we must therefore here make the sum of this curvature, and that of the generating curve, always proportional to the ordinate. This may be done mechanically, by beginning at the vertex, where the two curvatures are equal, then, for each succeeding portion, finding the radius of curvature by deducting the proper reciprocal of the normal, at the beginning of the portion, from the ordinate, and taking the reciprocal of the remainder. In this case the analysis leads to fluxional equations of the second order, which appear to afford no solution by means hitherto discovered; but the cases of simple curvature may be more easily subjected to calculation.

III. *Analysis of the simplest Forms.*

Supposing the curve to be described with an equable angular velocity, its fluxion, being directly as the radius of curvature, will be inversely as the ordinate, and the rectangle contained by the ordinate and the fluxion of the curve will be a constant quantity; but this rectangle is to the fluxion of the area, as the radius to the cosine of the angle formed by the curve with the horizon; and the fluxion of the area varying as the cosine, *the area* itself will vary as the sine of this angle, and *will be equal to the rectangle contained by the initial ordinate, and the sine corresponding to each point of the curve in the initial circle of curvature.* Hence it follows, first, that *the whole area included by the ordinates where the curve is vertical and where it is horizontal, is equal to the rectangle contained by the ordinate and the radius of curvature;* and, secondly, that the area on the convex side of the curve, between the vertical tangent and the least ordinate, is equal to the whole area on the concave side of the curve between the same tangent and the greatest ordinate.

In order to find the ordinate corresponding to a given angular direction, we must consider that the fluxion of the ordinate at the vertical part, is equal to the fluxion of the circle of curvature there, that, in other places, it varies as the radius of curvature and the sine of the angle formed with the horizon conjointly, or as the ordinate inversely, and directly as the sine of elevation; therefore the fluxion of the ordinate multiplied by the ordinate is equal to the fluxion of any circle of curvature multiplied by its corresponding height, and by the sine, and divided by the radius: but the fluxion of the circle multiplied by the sine and divided by the radius, is equal to the fluxion of the versed sine; therefore the ordinate multiplied by its fluxion is equal to the initial height multiplied by the fluxion of the versed sine in the corresponding circle of curvature; and *the square of the ordinate is equal to the rectangle contained by the initial height and twice the versed sine, increased by a constant quantity.* Now at the highest point of the curve, the versed sine becomes equal to the diameter, and the square of the initial height to the rectangle contained by the initial height and twice the diameter, with the constant quantity: the constant quantity is therefore equal to the rectangle contained by the initial height and its difference from twice the diameter: *this constant quantity is the square of the least ordinate*, and *the ordinate is every where a mean proportional between the greatest height and the same height diminished by twice the versed sine of the angular depression in the corresponding circle of curvature.* Again, *at the vertical point, the square of the ordinate is equal to the square of the greatest height diminished by the rectangle contained by this height and the diameter of the corresponding circle of curvature*, a rectangle which is constant for every fluid, and which may be called *the appropriate rectangle: deducting this rectangle from the square of the ordinate at the vertical point, we have the least ordinate;* which consequently vanishes *when the square of the ordinate at the vertical point is equal to the appropriate rectangle; the horizontal surface becoming in this case an asymptote to the curve, and the* *square of the greatest ordinate being equal to twice the appropriate rectangle, and the greatest ordinate to twice the diameter of the corresponding circle of curvature:* so that, *if we suppose a circle to be described, having this ordinate for a diameter, the chord of the angular elevation in this circle will be always equal to the ordinate at each point*, and *the ordinate will vary as the sine of half the angle of elevation, whenever the curve has an asymptote.* Mr. FUSS has demonstrated, in the third volume of the *Acta Petropolitana*, some properties of the arch of equilibrium under the pressure of a fluid, which is the same as one species of the curves here considered. The series given by EULER in the second part of the same volume, for the elastic curve, may also be applied to these curves.

IV. *Application to the Elevation of particular Fluids.*

The simplest phenomena, which afford us data for determining the fundamental properties of the superficial cohesion of fluids, are their elevation and depression between plates and in capillary tubes, and their adhesion to the surfaces of solids which are raised in a horizontal situation to a certain height above the general surface of the fluids. When the distance of a pair of plates, or the diameter of a tube, is very minute, the curvature may be considered as uniform, and the appropriate rectangle may readily be deduced from the elevation, recollecting that the curvature in a capillary tube is double, and the height therefore twice as great as between two plates. In the case of the elevation of a fluid in contact with a horizontal surface, the ordinate may be determined from the weight required to produce a separation; and the appropriate rectangle may be found in this manner also, the angle of contact being properly considered, in this as well as in the former case. It will appear that these experiments by no means exhibit an immediate measure of the mutual attraction of the solid and fluid, as some authors have supposed.

Sir ISAAC NEWTON asserts, in his Queries, that water ascends between two plates of glass at the distance of one hundredth of an inch, to the height of about one inch; the product of the distance and the height being about .01; but this appears to be much too little. In the best experiment of MUSSCHENBROEK, with a tube, half of the product was .0196; in several of WEITBRECHT, apparently very accurate, .0214. In MONGE's experiments on plates, the product was 2.6 or 2.7 lines, about .0210. Mr. ATWOOD says that for tubes, the product is .0530, half of which is .0265. Until more accurate experiments shall have been made, we may be contented to assume .02 for the rectangle appropriate to water, and .04 for the product of the height in a tube by its bore. Hence, when the curve becomes infinite, is greatest ordinate is .2, and the height of the vertical portion, or the height of ascent against a single vertical plane .14, or nearly one-seventh of an inch.

Now when a horizontal surface is raised from a vessel of water, the surface of the water is formed into a lintearia to which the solid is a tangent at its highest point, and if the solid be still further raised, the water will separate: the surface of the water, being horizontal at the point of contact, cannot add to the weight tending to depress the solid, which is therefore simply the hydrostatic pressure of a column of water equal in height to the elevation, in this case one-fifth of an inch, and standing on the given surface. The weight of such a column will be $50\frac{1}{2}$ grains for each square inch; and in TAYLOR's well known experiment the weight required was 50 grains. But when the solid employed is small, the curvature of the horizontal section of the water, which is convex externally, will tend to counteract the vertical curvature, and to diminish the height of separation; thus if a disc of an inch in diameter were employed, the curvature in this direction would perhaps be equivalent to the pressure of about one-hundredth of an inch, and might reduce the height from .2 to about .19, and the weight in the same proportion. There is however as great a diversity in the results of different experiments on the force required to elevate a solid from the surface of a fluid, as in those of the experiments in capillary tubes: and indeed the sources of error appear to be here more numerous. Mr. ACHARD found that a disc of glass, $1\frac{1}{2}$ inch French in diameter, required, at 69° of FAHRENHEIT, a weight of 91 French grains to raise it from the surface of water; this is only 37 English grains for each square inch; at $44\frac{1}{2}$° the force was $\frac{1}{14}$ greater, or $39\frac{1}{2}$ grains; the difference being $\frac{1}{343}$ for each degree of FAHRENHEIT. It might be inferred from these experiments, that the height of ascent in a tube of a given bore, which varies in the duplicate ratio of the height of adhesion, is diminished about $\frac{1}{180}$ for every degree of FAHRENHEIT that the temperature is raised above 50°; there was however probably some considerable source of error in ACHARD's experiments, for I find that this diminution does not exceed $\frac{1}{1000}$. The experiments of Mr. DUTOUR make the quantity of water raised equal to 44.1 grains for each square inch. Mr. ACHARD found the force of adhesion of sulfuric acid to glass, at 69° of FAHRENHEIT, 1.26, that of water being 1, hence the height was as .69 to 1, and its square as .47 to 1, which is the corresponding proportion for the ascent of the acid in a capillary tube, and which does not very materially differ from the proportion of .395 to 1, assigned by BARRUEL for this ascent. MUSSCHENBROEK found it .8 to 1, but his acid was probably weak. For alcohol the adhesion was as .593, the height as .715, and its square as .510: the observed proportion in a tube, according to an experiment of MUSSCHENBROEK, was about .550, according to CARRE' from .400 to .440. The experiments on sulfuric ether do not agree quite so well, but its quality is liable to very considerable variations. DUTOUR found the adhesion of alcohol .58, that of water being 1.

With respect to mercury, it has been shown by Professor CASBOIS of Metz, and by others, that its depression in tubes of glass depends on the imperfection of the contact, and that when it has been boiled in the tube often enough to expel all foreign particles, the surface may even become concave instead of convex, and the depression be converted into an elevation. But in barometers, constructed according to the usual methods, the angle of the mercury will be found to differ little from 140°; and in other experiments, when proper precautions are taken, the inclination will be nearly the same. The determination of this angle is necessary for finding the appropriate rectangle for the curvature of the surface of mercury, together with the observations of the quantity of depression in tubes of a given diameter. The table published by Mr. CAVENDISH from the experiments of his father, Lord CHARLES CAVENDISH, appears to be best suited for this purpose. I have constructed a diagram, according to the principles already laid down, for each case, and I find that the rectangle which agrees best with the phenomena is .01. The mean depression is always .015, divided by the diameter of the tube: and in tubes less than half an inch in diameter, the curve is very nearly elliptic, and the central depression in the tube of a barometer may be found by deducting from the corresponding mean depression the square root of one-thousandth part of its diameter. There is reason to suspect a slight inaccuracy towards the middle of Lord CHARLES CAVENDISH's Table, from a comparison with the calculated mean depression, as well as from the results of the mechanical construction. The ellipsis approaching nearest to the curve may be determined by the solution of a biquadratic equation.

Diameter in inches.	Grains in an inch. C.	Mean depression by calculation. Y.	Central depression by observation. C.	Central depression by formula. Y.	Central depresson by diagram. Y.	Marginal depression by diagram. Y.
.6	972	.025	.005	(.001)	.005	.066
.5	675	.030	.007	.008	.007	.067
.4	432	.037	.015	.017	.012	.069
.35	331	.043	.025	.024	.017	.072
.30	243	.050	.036	.033	.027	.079
.25	169	.060	.050	.044	.038	.086
.20	108	.075	.067	.061	.056	.096
.15	61	.100	.092	.088	.085	.116
.10	27	.150	.140	.140	.140	.161

The square root of the rectangle .01, or .1, is the ordinate where the curve would become vertical if it were continued; but in order to find the height at which it adheres to a vertical surface, we must diminish this ordinate in the proportion of the sine of 25° to the sine of 45°, and it will become .06, for the actual depression in this case. The elevation of the mercury that adheres to the lower horizontal surface of a piece of glass, and the thickness at which a quantity of mercury will stand when spread out on glass, supposing the angle of contact still 140°, are found, by taking the proportion of the sines of 20° and of 70° to the sine of 45°, and are therefore .0484 and .1330 respectively. If, instead of glass, we employed any surface capable of being wetted by mercury, the height of elevation would be .141, and this is the limit of the thickness of a wide surface of mercury supported by a substance wholly incapable of attracting it. Now the hydrostatic pressure of a column of mercury .0484 in thickness on a disc of one inch diameter would be 131 grains; to this the surrounding elevation of the fluid will add about 11 grains for each inch of the circumference, with some deduction for the effect of the contrary curvature of the horizontal section, tending to diminish the height; and the apparent cohesion thus exhibited will be about 160 grains, which is a little more than four times as great as the apparent cohesion of glass and water. With a disc 11 lines in diameter Mr. DUTOUR found it 194 French grains, which is equivalent to 152 English grains, instead of 160, for an inch, a result which is sufficient to confirm the principles of the calculation. The depth of a quantity of mercury standing on glass I have found by actual observation, to agree precisely with this calculation. SEGNER says that the depth was .1358, both on glass and on paper: the difference is very trifling, but this measure is somewhat too great for glass, and too small for paper, since it appears from DUTOUR's experiments, that the attraction of paper to mercury is extremely weak.

If a disc of a substance capable of being wetted by mercury, an inch in diameter, were raised from its surface in a position perfectly horizontal, the apparent cohesion should be 381 grains, taking .141 as the height: and for a French circular inch, 433 grains, or 528 French grains. Now, in the experiments of MORVEAU, the cohesion of a circular inch of gold to the surface of mercury appeared to be 446 grains, of silver 429, of tin 418, of lead 397, of bismuth 372, of zinc 204, of copper 142, of metallic antimony 126, of iron 115, of cobalt 8: and this order is the same with that in which the metals are most easily amalgamated with mercury. It is probable that such an amalgamation actually took place in some of the experiments, and affected their results, for the process of amalgamation may often be observed to begin almost at the instant of contact of silver with mercury; and the want of perfect horizontality appears in a slight degree to have affected them all. A deviation of one-fiftieth of an inch would be sufficient to have produced the difference between 446 grains and 528; and it is not impossible that all the differences, as far down as bismuth, may have been accidental. But if we suppose the gold only to have been perfectly wetted by the mercury, and all the other numbers to be in due proportions, we may find the appropriate angle for each substance by deducting from 180°, twice the angle of which the sine is to the radius as the apparent cohesion of each to 446 grains; that is, for gold .1, for silver about .97, for tin .95, for lead .90, for bismuth .85, for zinc .46, for copper .32, for antimony .29, for iron .26, and for cobalt .02, neglecting the surrounding elevation, which has less effect in proportion as the surface employed is larger. GELLERT found the depression of melted lead in a tube of glass multiplied by the bore equal to about .0054.

It would perhaps be possible to pursue these principles so far as to determine in many cases the circumstances under which a drop of any fluid would detach itself from a given surface. But it is sufficient to infer, from the law of the superficial cohesion of fluids, that the linear dimensions of similar drops depending from a horizontal surface must vary precisely in the same ratio as the heights of ascent of the respective fluids against a vertical surface, or as the square root of the heights of ascent in a given tube: hence the magnitudes of similar drops of different fluids must vary as the cubes of the square roots of the heights of ascent in a tube. I have measured the heights of ascent of water and of diluted spirit of wine in the same tube, and I found them nearly as 100 to 64: a drop of water falling from a large sphere of glass weighed 1.8 grains, a drop of the spirit of wine about .85, instead of .82, which is nearly the weight that would be inferred from the consideration of the heights of ascent, combined with that of the specific gravities. We may form a conjecture respecting the probable magnitude of a drop by inquiring what must be the circumference of the fluid, that would support by its cohesion the weight of a hemisphere depending from it: this must be the same as that of a tube, in which the fluid would rise to the height of one-third of its diameter; and the square of the diameter must be three times as great as the appropriate product; or, for water .12; whence the diameter would be .35, or a little more than one-third of an inch, and the weight of the hemisphere would be 2.8 grains. If more water were added internally, the cohesion would be overcome, and the drop would no longer be suspended, but it is not easy to calculate what precise quantity of water would be separated with it. The form of a bubble of air rising in water is determined by the cohesion of the internal surface of the water exactly in the same manner as the form of a drop of water in the air. The delay of a bubble of air at the bottom of a vessel appears to be occasioned by a deficiency of the pressure of the water between the air and the vessel; it is nearly analogous to the experiment of making a piece of wood remain immersed in water, when perfectly in contact with the bottom of the vessel containing it. This experiment succeeds however far more readily with mercury, since the capillary cohesion of the mercury prevents its insinuating itself under the wood.

V. *Of apparent Attractions and Repulsions.*

The apparent attraction of two floating bodies, round both of which the fluid is raised by cohesive attraction, is produced by the excess of the atmospheric pressure on the remote sides of the solids above its pressure on their neighbouring sides: or, if the experiments are performed in a vacuum, by the equivalent hydrostatic pressure or suction derived from the weight and immediate cohesion of the intervening fluid. This force varies ultimately in the inverse ratio of the square of the distance; for, if two plates approach each other, the height of the fluid that rises between them is increased in the simple inverse ratio of the distance; and the mean action, or negative pressure, of the fluid on each particle of the surface is also increased in the same ratio. When the floating bodies are both surrounded by a depression, the same law prevails, and its demonstration is still more simple and obvious. The repulsion of a wet and a dry body does not appear to follow the same proportion: for it by no means approaches to infinity upon the supposition of perfect contact; its maximum is measured by half the sum of the elevation and depression on the remote sides of the substances, and as the distance increases, this maximum is only diminished by a quantity, which is initially as the square of the distance. The figures of the solids concerned modify also sometimes the law of attraction, so that, for bodies surrounded by a depression, there is sometimes a maximum, beyond which the force again diminishes: and it is hence that a light body floating on mercury, in a vessel little larger than itself, is held in a stable equilibrium without touching the sides. The reason of this will become apparent, when we examine the direction of the surface necessarily assumed by the mercury in order to preserve the appropriate angle of contact, the tension acting with less force when the surface attaches itself to the angular termination of the float in a direction less horizontal.

The apparent attraction produced between solids by the interposition of a fluid does not depend on their being partially immersed in it; on the contrary, its effects are still more powerfully exhibited in other situations; and, when the cohesion between two solids is increased and extended by the intervention of a drop of water or of oil, the superficial cohesion of these fluids is fully sufficient to explain the additional effect. When wholly immersed in water, the cohesion between two pieces of glass is little or not at all greater than when dry: but if a small portion only of a fluid be interposed, the curved surface, that it exposes to the air, will evidently be capable of resisting as great a force as it would support from the pressure of the column of fluid that it is capable of sustaining in a vertical situation; and in order to apply this force, we must employ in the separation of the plates, as great a force as is equivalent to the pressure of a column appropriate to their distance MORVEAU found that two discs of glass, 3 inches French in diameter, at the distance of one-tenth of a line, appeared to cohere with a force of 4719 grains, which is equivalent to the pressure of a column 23 lines in height: hence the product of the height and the distance of the plates is 2.3 lines, instead of 2.65, which was the result of MONGE'S experiments on the actual ascent of water. The difference is much smaller than the difference of the various experiments on the ascent of fluids; and it may easily have arisen from a want of perfect parallelism in the plates; for there is no force tending to preserve this parallelism. The error, in the extreme case of the plates coming into contact at one point, may reduce the apparent cohesion to one half.

The same theory is sufficient to explain the law of the force by which a drop is attracted towards the junction of two plates inclined to each other, and which is found to vary in the inverse ratio of the square of the distance; whence it was inferred by NEWTON that the primitive force of cohesion varies in the simple inverse ratio of the distance, while other experiments lead us to suppose that cohesive forces in general vary in the direct ratio of the distance. But the difficulty is removed by considering the state of the marginal surface of the drop. If the plates were parallel, the capillary action would be equal on both sides of the drop: but when they are inclined, the curvature of the surface at the thinnest part requires a force proportionate to the appropriate height to counteract it; and this force is greater than that which acts on the opposite side. But if the two plates are inclined to the horizon, the deficiency may be made up by the hydrostatic weight of the drop itself;

and the same inclination will serve for a larger or a smaller drop at the same place. Now when the drop approaches to the line of contact, the difference of the appropriate heights for a small drop of a given diameter will increase as the square of the distance decreases; for the fluxion of the reciprocal of any quantity varies inversely as the square of that quantity: and, in order to preserve the equilibrium, the sine of the angle of elevation of the two plates must be nearly in the inverse ratio of the square of the distance of the drop from the line of contact, as it actually appears to have been in HAUKSBEE's experiments.

VI. *Physical Foundation of the Law of superficial Cohesion.*

We have now examined the principal phenomena which are reducible to the simple theory of the action of the superficial particles of a fluid. We are next to investigate the natural foundations upon which that theory appears ultimately to rest. We may suppose the particles of liquids, and probably those of solids also, to possess that power of repulsion, which has been demonstratively shown by NEWTON to exist in aeriform fluids, and which varies in the simple inverse ratio of the distance of the particles from each other. In airs and vapours this force appears to act uncontrolled; but in liquids, it is overcome by cohesive force, while the particles still retain a power of moving freely in all directions; and in solids the same cohesion is accompanied by a stronger or weaker resistance to all lateral motion, which is perfectly independent of the cohesive force, and which must be cautiously distinguished from it. It is simplest to suppose the force of cohesion nearly or perfectly constant in its magnitude, throughout the minute distance to which it extends, and owing its apparent diversity to the contrary action of the repulsive force, which varies with the distance. Now in the internal parts of a liquid these forces hold each other in a perfect equilibrium, the particles being brought so near that the repulsion becomes precisely equal to the cohesive force that urges them together: but whenever there is a curved or angular surface, it may be found by collecting the actions of the different particles, that the cohesion must necessarily prevail over the repulsion, and must urge the superficial parts inwards with a force proportionate to the curvature, and thus produce the effect of a uniform tension of the surface. For, if we consider the effect of any two particles in a curved line on a third at an equal distance beyond them, we shall find that the result of their equal attractive forces bisects the angle formed by the lines of direction; but that the result of their repulsive forces, one of which is twice as great as the other, divides it in the ratio of one to two, forming with the former result an angle equal to one-sixth of the whole; so that the addition of a third force is necessary in order to retain these two results in equilibrium; and this force must be in a constant ratio to the evanescent angle which is the measure of the curvature, the distance of the particles being constant. The same reasoning may be applied to all the particles which are within the influence of the cohesive force: and the conclusions are equally true if the cohesion is not precisely constant, but varies less rapidly than the repulsion.

VII. *Cohesive Attraction of Solids and Fluids.*

When the attraction of the particles of a fluid for a solid is less than their attraction for each other, there will be an equilibrium of the superficial forces, if the surface of the fluid make with that of the solid a certain angle, the versed sine of which is to the diameter, as the mutual attraction of the fluid and solid particles is to the attraction of the particles of the fluid among each other. For, when the fluid is surrounded by a vacuum or by a gas, the cohesion of its superficial particles acts with full force in producing a pressure; but when it is any where in contact with a solid substance of the same attractive power with itself, the effects of this action must be as much destroyed as if it were an internal portion of the fluid. Thus, if we imagined a cube of water to have one of its halves congealed, without any other alteration of its properties, it is evident that its form and the equilibrium of the cohesive forces would remain undisturbed: the tendency of the new angular surface of the fluid water to contract would therefore be completely destroyed by the contact of a solid of equal attractive force. If the solid were of smaller attractive force, the tendency to contract would only be proportionate to the difference of the attractive forces or densities, the effect of as many of the attractive particles of the fluid being neutralised, as are equivalent to a solid of a like density or attractive power. For a similar reason, the tendency of a fluid to contract the sum of the surfaces of itself and a contiguous solid, will be simply as the density of the solid, or as the mutual attractive force of the solid and fluid. And it is indifferent whether we consider the pressure produced by these supposed superficial tensions, or the force acting in the direction of the surfaces to be compared. We may therefore inquire into the conditions of equilibrium of the three forces acting on the angular particles, one in the direction of the surface of the fluid only, a second in that of the common surface of the solid and fluid, and the third in that of the exposed surface of the solid. Now, supposing the angle of the fluid to be obtuse, the whole superficial cohesion of the fluid being represented by the radius, the part which acts in the direction of the surface of the solid will be proportional to the cosine of the inclination; and this force, added to the force of the solid, will be equal to the force of the common surface of the solid and fluid, or to the differences of their forces; consequently, the cosine added to twice the force of the solid, will be equal to the whole force of the fluid, or to the radius: hence the force of the solid is represented by half the difference between the cosine and the radius, or by half the versed sine; or, if the force of the fluid be represented by the diameter, the whole versed sine will indicate the force of the solid. And the same result follows when the angle of the fluid is acute. Hence we may infer, that if the solid have half the attractive force of the fluid, the surfaces will be perpendicular; and this seems in itself reasonable, since two rectangular edges of the solid are equally near to the angular particles with one of the fluid, and we may expect a fluid to rise and adhere to the surface of every solid more than half as attractive as itself; a conclusion which CLAIRAUT has already inferred, in a different manner, from principles which he has but cursorily investigated, in his treatise on the figure of the earth.

The versed sine varies as the square of the sine of half the angle: the force must therefore be as the square of the height to which the fluid may be elevated in contact with a horizontal surface, or nearly as the square of the number of grains expressing the apparent cohesion. Thus, according to the experiments of MORVEAU, on the suppositions already premised, we may infer that the mutual attraction of the particles of mercury being unity, that of mercury for gold will be .1 or more, that of silver about .94, of tin .90, of lead .81, of bismuth .72, of zinc .21, of copper .10, of antimony .08, of iron .07, and of cobalt .0004. The attraction of glass for mercury will be about one-sixth of the mutual attraction of the particles of mercury: but when the contact is perfect, it appears to be considerably greater.

Although the whole of this reasoning on the attraction of solids is to be considered rather as an approximation than as a strict demonstration, yet we are amply justified in concluding, that all the phenomena of capillary action may be accurately explained and mathematically demonstrated from the general law of the equable tension of the surface of a fluid, together with the consideration of the angle of contact appropriate to every combination of a fluid with a solid. Some anomalies, noticed by MUSSCHENBROEK and others, respecting in particular the effects of tubes of considerable lengths, have not been considered: but there is great reason to suppose that either the want of uniformity in the bore, or some similar inaccuracy, has been the cause of these irregularities, which have by no means been sufficiently confirmed to afford an objection to any theory. The principle, which has been laid down respecting the contractile powers of the common surface of a solid and a fluid, is confirmed by an observation which I have made on the small drops of oil which form themselves on water. There is no doubt but that this cohesion is in some measure independent of the chemical affinities of the substances concerned: tallow when solid has a very evident attraction for the water out of which it is raised; and the same attraction must operate upon an unctuous fluid to cause it to spread on water, the fluidity of the water allowing this powerful agent to exert itself with an unresisted velocity. An oil which has thus been spread is afterwards collected, by some irregularity of attraction, into thin drops, which the slightest agitation again dissipates: their surface forms a very regular curve, which terminates abruptly in a surface perfectly horizontal: now it follows from the laws of hydrostatics, that the lower surface of these drops must constitute a curve, of which the extreme inclination to the horizon is to the inclination of the upper surface as the specific gravity of the oil to the difference between its specific gravity and that of water: consequently since the contractile forces are held in equilibrium by a force which is perfectly horizontal, their magnitude must be in the ratio that has been already assigned; and it may be assumed as consonant both to theory and to observation, that the contractile force of the common surface of two substances, is proportional, other things being equal, to the difference of their densities. Hence, in order to explain the experiments of Boyle on the effects of a combination of fluids in capillary tubes, or any other experiments of a similar nature, we have only to apply the law of an equable tension, of which the magnitude is determined by the difference of the attractive powers of the fluids.

I shall reserve some further illustrations of this subject for a work which I have long been preparing for the press, and which I flatter myself will contain a clear and simple explanation of the most important parts of natural philosophy. I have only thought it right, in the present Paper, to lay before the Royal Society, in the shortest possible compass, the particulars of an original investigation, tending to explain some facts and establish some analogies, which have hitherto been obscure and unintelligible.