A DISCOURSE

Concerning GRAVITY, *and its Properties, wherein the Descent of* Heavy Bodies, *and the Motion of* Projects *is briefly, but fully handled: Together with the* Solution *of a* Problem *of great Use in* GUNNERY. *By* E. HALLEY.

NATURE amidst the great variety of *Problems* wherewith She exercises the Wits of Philosophical men, scarce affords any one wherein the Effect is more visible, and the Cause more concealed than in those of the *Phænomena* of *Gravity*. Before we can go alone, we must learn to defend our selves from the violence of its Impulse, by not trusting the *Center* of *Gravity* of our Bodies beyond our reach; and yet the Acutest Philosophers, and the subtilest Enquirers into the Original of this Motion, have been so far from satisfying their Readers, that they themselves seem little to have understood the Consequences of their own *Hypotheses*.

Des Cartes his Notion, I must needs confess to be to me Incomprehensible, while he will have the Particles of his *Celestial matter*, by being reflected on the Surface of the *Earth*, and so ascending therefrom, to drive down into their places those *Terrestrial Bodies* they find above them: This is as near as I can gather the scope of the 20, 21, 22, and 23 *Sections* of the last Book of his *Principia Philosophiæ*; yet neither he, nor any of his Followers can shew how a Body suspended *in libero æthere*, shall be carried downwards by a continual Impulse tending upwards, and acting upon all its parts equally: And besides the obscurity wherewith he expresses himself particularly, *Sect.* 23. does sufficiently argue according to his own Rules, the confused *Idea* he had of the thing he wrote.

Others, and among them Dr. *Vossius* asserts the Cause of the *Descent* of *heavy Bodies* to be the *Diurnal rotation* of the *Earth* upon its *Axis*, without considering, that according to the Doctrine of Motion fortified with Demonstration, all Bodies moved *in Circulo*, would recede from the Center of their Motion; whereby the contrary to *Gravity* would follow, and all loose Bodies would be cast into the Air in a *Tangent* to the *Parallel* of *Latitude*, without the intervention of some other Principle to keep them fast, such as is that of *Gravity*. Besides the effect of this Principle is throughout the whole Surface of the Glob found nearly equal, and *certain Experiment* seems to argue it rather less near the *Equinoctial*, than towards the *Poles*, which could not be by any means, if the *Diurnal rotation* of the *Earth* upon its *Axis* were the cause of *Gravity*, for where the Motion was swiftest, the Effect would be most considerable.

Others assign the *Pressure* of the *Atmosphere*, to be the Cause of this Tendency towards the *Center* of the *Earth*; but unhappily they have mistaken the *Cause* for the *Effect*, it being from undoubted Principles plain, that the *Atmosphere* has no other *Pressure* but what it derives from its *Gravity*, and that the Weight of the upper parts of the *Air*, pressing on the lower parts thereof, do so far bend the Springs of that *Elastick* Body, as to give it a force equal to the Weight that Compressed it, having of it self no force at all: And supposing it had, it will be very hard to explain the *Modus*, how that *Pressure* should occasion the *Descent* of a *Body* circumscribed by it, and pressed equally above and below, without some other force to draw, or thrust it downwards. But to demonstrate the contrary of this Opinion, an *Experiment* was long since shewn before the *Royal Society*, whereby it appeared

peared that the *Atmoſphere* was ſo far from being the Cauſe of *Gravity*, that the Effects thereof were much more Vigorous where the *preſſure* of the *Atmoſphere* was taken off; for a long *Glaſs-Receiver* having a light Down-feather included, being *evacuated* of *Air*, the Feather which in the *Air* would hardly ſink, did *in vacuo* deſcend with nearly the ſame *Velocity* as if it had been a Stone.

Some think to Illuſtrate this *Deſcent* of *Heavy Bodies*, by comparing it with the Vertue of the *Loadſtone*; but ſetting aſide the difference there is in the manner of their *Attractions*, the *Loadſtone* drawing only *in* and *about* its *Poles*, and the *Earth* near equally in all parts of its *Surface*, this Compariſon avails no more than to explain *ignotum per æque ignotum*.

Others aſſign a certain *Sympathetical attraction* between the *Earth* and its *Parts*, whereby they have, as it were, a deſire to be united, to be the Cauſe we enquire after: But this is ſo far from explaining the *Modus*, that it is little more, than to tell us in other terms, that *heavy bodies deſcend*, becauſe they *deſcend*.

This I ſay, not that I can pretend to ſubſtitute any *Solution* of this Important *Philoſophical Problem*, that ſhall more happily explicate the *Appearances* of *Gravity*, only it may be ſerviceable to thoſe with whom the Credit of great *Authors* ſways much, and who too-readily aſſent *in verba magiſtri*, to let them ſee that their Books are not always infallible: Beſides the detection of Errors is the firſt and ſureſt ſtep towards the diſcovery of Truth.

Tho' the Efficient Cauſe of *Gravity* be ſo obſcure, yet the final Cauſe thereof is clear enough; for it is by this ſingle *Principle* that the *Earth* and all the *Celeſtial Bodies* are kept from *diſſolution*: the leaſt of their *Particles* not being ſuffered to recede far from their *Surfaces*, without being immediately brought down again by vertue of this *Natural tendency*, which for their Preſervation, the Infinite Wiſdom of their *Creator* has Ordained to be towards each of their *Centers*;

nor can the *Globes* of the *Sun* and *Planets* otherwiſe be deſtroyed, but by taking from them this power of keeping their parts united.

The Affections or Properties of *Gravity*, and its manner of acting upon *Bodies falling*, have been in a great meaſure diſcovered, and moſt of them made out by *Mathematical demonſtration* in this our *Century*, by the accurate diligence of *Galilæus*, *Torricellius*, *Hugenius*, and others, and now lately by our worthy Country-man Mr. *Iſaac Newton*, (who has an incomparable *Treatiſe* of *Motion* almoſt ready for the *Preſs*) which Properties it may be very material here to enumerate, that they may ſerve for a Foundation to all thoſe that ſhall be willing to ſpend their Thoughts in ſearch of the true Cauſe of this *deſcent* of *Bodies*.

The firſt Property is, That by this principle of *Gravitation*, all Bodies do deſcend towards a Point, which either is, or elſe is very near to the Center of *Magnitude* of the *Earth* and *Sea*, about which the *Sea* forms it ſelf exactly into a *Spherical ſurface*, and the *Prominences* of the *Land*, conſidering the Bulk of the whole, differ but inſenſibly therefrom.

Secondly, That this Point or Center of *Gravitation*, is fixt within the *Earth*, or at leaſt has been ſo, ever ſince we have any *Authentick Hiſtory* : For a Conſequence of its Change, tho' never ſo little, would be the over-flowing of the low Lands on that ſide of the *Globe* towards which it approached, and the leaving new Iſlands bare on the oppoſite ſide, from which it receded ; but for this Two thouſand years it appears, that the low Iſlands of the *Mediterranean* *Sea* (near to which the ancienteſt Writers lived) have continued much at the ſame height above the Water, as they now are found ; and no *Inundations* or *Receſſes* of the *Sea* arguing any ſuch Change, are Recorded in Hiſtory ; excepting the *Univerſal Deluge*, which can no better way be accounted for, than by ſuppoſing this Center of *Gravitation* removed for a time, towards the middle of the then inhabited parts of the World ;

and

and a change of its place, but the two thoufandth part of the *Radius* of this *Globe*, were fufficient to bury the Tops of the higheft Hills under water.

Thirdly, That in all parts of the *Surface* of the *Earth*, or rather in all Points equidiftant from its *Center*, the force of *Gravity* is nearly equal; fo that the length of the *Pendulum* vibrating *feconds of time*, is found in all parts of the World to be very near the fame. 'Tis true at S. *Helena* in the *Latitude* of 16 Degrees *South*, I found that the *Pendulum* of my Clock which vibrated *feconds*, needed to be made fhorter than it had been in *England* by a very fenfible fpace, (but which at that time I neglected to obferve accurately) before it would keep time; and fince the like Obfervations has been made by the *French Obfervers* near the *Equinoctial*: Yet I dare not affirm that in mine it proceeded from any other Caufe, than the great height of my place of Obfervation above the *Surface* of the *Sea*, whereby the *Gravity* being diminifhed, the length of the *Pendulum* vibrating *feconds*, is proportionably fhortned.

Fourthly, That *Gravity* does equally affect all *Bodies*, without regard either to their *matter*, *bulk*, or *figure*; fo that the Impediment of the *Medium* being removed, the moft compact and moft loofe, the greateft and fmaleft *Bodies* would defcend the fame *fpaces* in equal times; the truth whereof will appear from the *Experiment* I beford cited. In thefe two laft particulars, is fhewn the great difference between *Gravity* and *Magnetifm*, the one affecting only *Iron*, and that towards its *Poles*, the other all *Bodies* alike in every part. As a *Corollary*; from hence it will follow, that there is no fuch thing as *pofitive levity*, thofe things that appear light, being only comparatively fo; and whereas feveral things rife and fwim in *fluids*, 'tis becaufe bulk for bulk, they are not fo heavy as thofe *fluids*; nor is there any reafon why *Cork*, for inftance, fhould be faid to be light becaufe it fwims on Water, any more than *Iron* becaufe it fwims on *Mercury*.

Fifthly, That this power encreafes as you defcend, and de-

decreaſes as you aſcend from the Center, and that in the proportion of the *Squares* of the *diſtances* therefrom *reciprocally*, ſo as at a double diſtance to have but a quarter of the force ; this property is the principle on which Mr. *Newton* has made out all the *Phænomena* of the *Cæleſtial Motions*, ſo eaſily and naturally that its truth is paſt diſpute. Beſides that, it is highly rational, that the *attractive* or *gravitating* power ſhould exert it ſelf more vigirouſly in a ſmall Sphere, and weaker in a greater, in proportion as it is contracted or expanded, and if ſo, ſeeing that the *ſurfaces* of *Spheres* are as the *Squares* of their *Radii*, this power at ſeveral diſtances will be as the *Squares* of thoſe *diſtances Reciprocally*, and then its whole action upon each *Spherical Surface*, be it great or ſmall will be alwaies equal. And this is evidently the rule of *Gravitation* towards the *Centers* of the *Sun*, *Jupiter*, *Saturn* and the *Earth*, and thence is reaſonably inferred, to be the general principle obſerved by *Nature*, in all the reſt of the *Celeſtial Bodies*.

Theſe are the principal affections of *Gravity*, from which the rules of the *fall* of *Bodies*, and the *motion* of *Projects* are *Mathamatically* deducible. Mr. *Iſaac Newton* has ſhewed how to define the ſpaces of the *deſcent* of a *Body*, let fall from any given highth, down to the *Center*. Suppoſing the *Gravitation* to increaſe, as in the fifth Property ; but conſidering the ſmalneſs of hight, to which any *Project* can be made aſcend, and over how little an *Arch* of the *Globe* it can be caſt by any of our *Engines*, we may well enough ſuppoſe the *Gravity* equal throughout, and the deſcents of *Projects* in parralel lines, which in truth are towards the *Center*, the difference being ſo ſmall as by no means to be diſcovered in *Practice*. The *Oppoſition* of the *Air*, 'tis true, is conſiderable againſt all light bodies moving through it, as likewiſe againſt ſmall ones (of which more hereafter) but in great and ponderous Shot, this Impediment is found by *Experience* but very ſmall, and may ſafely be neglected.

Pro-

Propositions concerning the Descent of heavy Bodies, and the Motion of Projects.

Prop. I. The *Velocities* of *falling Bodies*, are proportionate to the times from the beginning of their *falls*.

This follows, for that the action of *Gravity* being *continual*, in every space of time, the falling Body receives a new impulse, equal to what it had before, in the same space of time, received from the same power: For instance, in the first second of time, the falling Body has acquired a *Velocity*, which in that time would carry it to a certain distance, suppose 32 foot; and were there no new force, would descend at that rate with an *equable Motion*; but in the next second of time, the same power of *Gravity* continually acting thereon, superadds a new *Velosity* equal to the former; so that at the end of two seconds, the *Velecity* is double to what it was at the end of the first, and after the same manner may it be proved to be triple, at the end of the third second, and so on. Wherefore the *Velccities* of *falling Bodies*, are proportionate to the times of their *falls*, *Q. E. D.*

Prop. II. The *Spaces* described by the *fall* of a Body, are as the *Squares* of the times, from the beginning of the *Fall*.

Demonstration. Let A B (*Fig.*1. *Tab.*1.) represent the *time* of the *fall* of a *Body*, B C perpendicular to A B the *Velosity* acquired at the end of the *fall*, and draw the line A C, then divide the line A B representing the time into as many equal parts as you please, as b, b, b, b, &c. and through these points draw the lines bc, bc, bc, bc, &c. parallel to B C, 'tis manifest that the several lines, bc, represent the several *Velccities* of the falling Body, in such parts of the *time* as A b is of A B, by the former proposition. It is evident likewise that the *Area* A B C is the sum of all the lines bc being taken, according to the method of *Indivisibles*, infinitely many; so that the

the *Area* ABC repreſents the ſum of all the *Velocities*, between none and B C ſuppoſed infinitely many; which ſum is the ſpace deſended in the time repreſented by A B. And by the ſame reaſon the *Areas* A b c, will repreſent the ſpaces deſcended in the times A b; ſo then the ſpaces deſcended in the times AB, Ab, are as the *Areas* of the *Triangles* A B C, A b c, which by the 20 th of the 6 of *Euclid* are as the *Squares* of their *Homologous ſides* A B, A b, that is to ſay, of the *Times*: wherefore the deſcents of *falling Bodies*, are as the *Squares* of the times of their *fall*, *Q. E. D.*

Prop. III. The *Velocity* which a *falling Body* acquires in any ſpace of time, is double to that, wherewith it would have moved the ſpace, deſcended by an equable motion, in the ſame *time*.

Demonſtration, Draw the line E C parallel to A B and A E parallel to B C in the ſame *fig.* 1. and compleat the *Parallelogram* A B C E, it is evident that the *Area* thereof may repreſent the ſpace, a *Body* moved equably with the *Velocity* B C, would deſcribe in the time A B, and the *Triangle* A B C repreſents the *ſpace* deſcribed by the *fall* of a *Body*, in the ſame time A B, by the ſecond propoſition. Now the *Triangle* A B C is half of the *Parallelogram* A B C E, and conſequently the ſpace deſcribed by the *fall*, is half what would have been deſcribed by an *equable Motion* with the *Velocity* B C, in the ſame time; wherefore the *Velocity* B C at the end of the *fall*, is double to that *Velocity*, which in the time A B, would have deſcribed the *ſpace fallen*, repreſented by the *Triangle* A B C, with an *equable Motion*, *Q. E. D.*

Prop. IV. All *Bodies* on or near the ſurface of the *Earth*, in their *fall*, deſcend ſo, as at the end of the firſt ſecond of time, they have deſcribed 16 feet one inch *London Meaſure*, and acquired the *Velocity* of 32 feet two inches in a ſecond.

This is made out from the 25th propoſition of the ſecond part of that Excellent Treatice of Mr. *Hugenius de Horologio Oſcillatorio*; wherein he demonſtrates the time of the leaſt *Vibrations* of a *Pendulum*, to be to the time of the *fall* of a *Body*, from

from the height of half the length of the *Pendulum*, as the *Circumference* of a *Circle* to its *Diameter*; whence as a *Corollary* it follows, that as the *Square* of the *Diameter* to the *Square* of the *Circumference*, so half the length of the *Pendulum* vibrating *seconds*, to the *space* described by the *fall* of a *body* in a *second* of *time*: and the length of the *Pendulum* vibrating *seconds*, being found 39,125, or ⅛ Inches, the *descent* in a *second* will be found by the aforesaid *Analogy* 16 Foot and one Inch, and by the third Proposition, the *Velocity* will be double thereto; and near to this it hath been found by several *Experiments*, which by reason of the *swiftness* of the *fall*, cannot so exactly determine its *quantity*. The Demonstration of *Hugenius* being the Conclusion of a long train of *Consequences*. I shall for brevity sake omit; and refer you to his Book, where these things are more amply treated of.

From these four *Propositions*, all *Questions* concerning the *Perpendicular fall of bodies*, are easily *solved*, and either *Time*, *Height*, or *Velocity* being assigned, one may readily find the other two. From them likewise is the Doctrine of *Projects* deducible, assuming the two following *Axioms*; *viz*. That a *body* set a moving, will move on continually in a right *line* with an *equable motion*, unless some other force or impediment intervene, whereby it is accelerated, or retarded, or deflected.

Secondly, That a *Body* being agitated by two *motions* at a time, does by their *compounded forces* pass through the same *points*, as it would do, were the two *motions divided* and acted *successively*. As for instance, Suppose a *body* moved in the Line GF, (*Fig*.2.*Tab*.1.) from G to R, and there stopping, by another *impulse* suppose it moved in a *space* of *time* equal to the former, from R towards K, to V. I say, the *body* shall pass through the point V, tho' these two *several forces*, acted both in the *same time*.

Prop. V. The *Motion* of all *Projects* is in the *Curve* of a *Parabola*: Let the *line* GRF (in *fig*. 2.) be the *line* in which the *Project* is directed, and in which by the first *Axiom* it would move equal *ſpaces* in equal *times*, were it not deflected downwards by the force of *Gravity*. Let G B be the *Horizontal line*, and G C a *Perpendicular* thereto. Then the *line* G R F being divided into equal parts, anſwering to equal *ſpaces* of *time*, let the *deſcents* of the *Project* be laid down in *lines parallel* to GC, proportioned as the *ſquares* of the *lines* GS, GR, GL, GF, or as the *ſquares* of the *times*, from S to T, from R to V, from L to X, and from F to B, and draw the *lines* TH, VD, XY, BC parallel to GF; I ſay the Points T,V, X,B, are Points in the *Curve* deſcribed by the *Project*, and that that *Curve* is a *Parabola*. By the ſecond *Axiom* they are Points in the *Curve*; and the parts of the *deſcent* GH, GD, GY, GC,= to ST, RV, LX, FB, being as the *ſquares* of the *times* (by the *ſecond Prop.*) that is, as the *ſquares* of the *Ordinates*, HT, DU, YX, BC, equal to GS, GR, GL, GF, the *ſpaces* meaſured in thoſe times; and there being no other *Curve* but the *Parabola*, whoſe parts of the *Diameter* are as the *ſquares* of the *Ordinates*, it follows that the *Curve* deſcribed by a *Project*, can be no other than a *Parabola*: And ſaying, as RU the *deſcent* in any *time*, to GR or UD the *direct motion* in the ſame *time*, ſo is U D, to a *third* proportional; that *third* will be the *line* called by all Writers of *Conicks*, the *Parameter* of the *Parabola* to the *Diameter* GC, which is alwaies the ſame in *Projects* caſt with the ſame *Velocity*: And the *Velocity* being defined by the number of *feet* moved in a *ſecond* of *time*, the *Parameter* will be found by dividing the *ſquare* of the *Velocity*, by 16 *feet* 1 *inch*, the *fall* of a *body* in the ſame *time*.

Lemma.

The *Sine* of the double of any *Arch*, is equal to twice the *Sine* of that *Arch* into its *Co-ſine*, divided by *Radius*; and the *Verſed ſine* of the *double* of any *Arch* is equal to the *ſquare* of the *Sine* thereof divided by *Radius*.

Let the *Arch* BC (in *fig*.3.) be double the *Arch* BF, and A the *Center*; draw the *Radii* AB, AF, AC, and the *Chord* BDC,

BDC, and let fall BE perpendicular to AC, and the *Angle* EBC, will be equal to the *Angle* ABD, and the *Triangle* BCE, will be like to the *Triangle* BDA; wherefore it will be as AB to AD, ſo BC or twice BD; to BE, that is as *Radius* to *Co-ſine*, ſo twice *Sine*, to *Sine* of the double *Arch*. And as AB to BD, ſo twice BD or BC, to EC, that is as *Radius* to *Sine*, ſo twice that *Sine* to the *Verſed-ſine* of the double *Arch*; which two *Analogies* reſolved into *Equations*, are the *Propoſitions* contained in the *Lemma* to be proved.

Prop. VI. The *Horizontal* diſtances of *Projections* made with the ſame *Velocity*, at ſeveral *Elevations* of the *Line* of direction, are as the *Sines* of the doubled *Angles* of *Elevation*.

Let GB (*fig.*2.) the *Horizontal* diſtance be $=z$, the *ſine* of the *Angle* of *Elevation*, FGB, be $=s$, its *Co-ſine* $=c$, *Radius* $=r$, and the *Parameter* $=p$. It will be as c to s; ſo z to $\frac{sz}{c}$=FB=GC, and by reaſon of the *Parabola* $\frac{psz}{c}$=to the *ſquare* of CB, or GF,. Now as c to r, ſo is z to $\frac{zr}{c}$= GF, and its *ſquare* $\frac{zzrr}{cc}$ will be therefore=to $\frac{psz}{c}$: which *Equation* reduced, will be $\frac{psc}{rr}=z$. But by the former *Lemma* $\frac{2sc}{r}$ is equal to the *Sine* of the double *Angle*, whereof s is the *Sine*: wherefore 'twil be as Radius to *Sine* of double the *Angle* FGB, ſo is half the *Parameter*, to the *Horizontal rang* or *diſtance* ſought; and at the ſeveral *Elevations*, the *ranges* are as the *ſines* of the double *Angles* of *Elevation* *Q.E.D.*

Corollary.

Hence it follows, that half the *Parameter* is the greateſt R*andon*, and that that happens at the *Elevation* of 45 degrees, the *ſine* of whoſe double is R*adius*,. Likewiſe that the R*anges* equally diſtant above and below 45 are equal, as are the *sines* of all doubled *Arches*, to the *sines* of their doubled *Complements*.

Prop. VII. The *Altitudes* of *Projections* made with the same *Velocity*, at several *Elevations*, are as the *versed sines* of the doubled *Angles* of *Elevation*: As c is to s :: so is $\frac{psc}{rr}$ $=$GB to $\frac{pss}{rr}$$=$BF, and UK$=RU=$ $\frac{\text{BF}}{4}$, the *Altitude* of the *Projection* $=\frac{pss}{4rr}$. Now by the foregoing *Lemma* $\frac{2ss}{r}$$=$to the *versed sine* of the double *Angle*, and therefore it will be as *Radius*, to *versed sine* of double the *Angle* FGB, so an *8th* of *Parameters* to the height of the *Projection* VK; and so these heights at several *Elevations* are as the said *versed sines*, *Q. E. D.*

Corollary.

From hence it is plain, that the greatest *Altitude* of the perpendicular *Projection* is a *4th* of *Parameter*, or half the greatest *Horizontal Rang*; the *versed sine* of 180 degrees being $= 2r$.

Prop. VIII. The *Lines* GF, or times of the flight of a *Project* cast with the same degree of *velocity* at different *Elevations*, are as the *sines* of the *Elevations*.

As c is to r :: so is $\frac{psc}{rr}$$=$ GB by the *6 Prop.* to $\frac{ps}{r}$$=$GF, that is as *Radius* to *sine* of *Elevation*, so the *Parameter* to the *line* GF; so the *lines* GF are as the *sines* of *Elevation*, and the *Times* are proportional to those *Lines*; wherefore the *Times* are as the *Sines* of *Elevation*: *Ergo constat propositio.*

Prop. IX. Problem. A *Projection* being made as you please, having the Distance and Altitude, or Descent of an *Object*, through which the *Project* passes, together with the *Angle* of *Elevation* of the *line* of *Direction*; to find the *Parameter* and *Velocity*, that is (in *Fig.* 2.) having the *Angle* FGB, GM, and MX.

Solution. As *Radius* to *Secant* of FGB, so GM the *distance* given

given, to GL; and as Radius to *Tangent* of FGB, ſo GM to LM. Then LM−MX in *heights*, or + MX in *deſcents*; or elſe MX−ML, if the *direction* be below the *Horizontal-line*, is the *fall* in the *time* that the direct *impulſe* given in G would have carried the *Project* from G to L=LX=GY; then by reaſon of the *Parabola*; as LX or GY, is to GL or YX, :: ſo is GL to the *Parameter* ſought. To find the *Velocity* of the *Impulſe*, by *Prop.* 2. & 4, find the *time* in *ſeconds* that a *body* would fall the *ſpace* LX, and by that dividing the *line* GL, the *Quote* will be the *Velocity*, or *ſpace* moved in a *ſecond* ſought, which is alwaies a *mean proportional* between the *Parameter* and 16 *feet* 1 *inch*.

Prop. X. *Problem* 2. Having the *Parameter*, Horizontal diſtance, and height or deſcent of an *Object*, to find the *Elevations* of the *line* of *direction* neceſſary to hit the given *Object*; that is, having GM, MX, and the greateſt Randon equal to half the *Parameter*; to find the *Angles* FGB.

Let the *Tangent* of the *Angle* ſought be $=t$, the *Horizontal diſtance* GM$=b$, the Altitude of the *Object* MX$=h$, the *Parameter*$=p$, and Radius$=r$, and it will be,

As r to t, ſo b to $\frac{tb}{r}$=ML and $\frac{tb}{r} \mp h$ $\begin{matrix}\text{in aſcents}\\ \text{in deſcents}\end{matrix}$ =LX, and

$\frac{ptb}{r} \mp ph =$ GL *quad.* =XY *quad. ratione Parabolæ*; but

$bb \mp \frac{ttbb}{rr} =$ GL *quad.* 47. 1. *Euclid.* Wherefore

$\frac{ptb}{r} \mp ph = bb \mp \frac{ttbb}{rr}$ which Equation tranſpoſed, is

$\frac{ttbb}{rr} = \frac{ptb}{r} \mp ph - bb$, divided by bb is

$\frac{tt}{rr} = \frac{pt}{br} \mp \frac{ph}{bb} - 1$. this Equation ſhews the Queſtion to have two Anſwers, and the Roots thereof are $\frac{t}{r} = \frac{p}{2b} \mp \sqrt{\frac{pp \mp 4ph}{4bb} - 1}$ from which I derive the following Rule.

Divide half the *Parameter* by the Horizontal diſtance, and keep the Quote; *viz.* $\frac{p}{2b}$ then ſay, as *ſquare* of the *diſtance* given to the half *Parameter*, ſo half *Parameter* $\mp$ double height/deſcent to the *ſquare* of a *Secant* $= \frac{pp \mp 4ph}{4bb}$ the *Tangent* anſwering to that *Secant*, will be $\sqrt{\frac{pp \mp 4ph}{4bb} - 1}$ or rr: ſo then the ſum and difference of the afore-found *Quote*, and this *Tangent* will be the Roots of the *Equation*, and the *Tangents* of the *Elevations* ſought.

Note here, that in *Deſcents*, if the *Tangent* exceed the *Quote*, as it does when ph is more than bb, the *direction* of the lower *Elevation* will be below the *Horizon*, and if $ph=bb$, it muſt be directed *Horizontal*, and the *Tangent* of the upper *Elevation* will be $\frac{pr}{b}$: Note likewiſe, that if $4bb+4ph$ in *aſcents*, or $4bb-4ph$ in *deſcents*, be equal to pp, there is but one *Elevation* that can hit the *Object*, and its *Tangent* is $\frac{pr}{2b}$ and if $4bb+4ph$ in *aſcents*, or $4bb-4ph$ in *deſcents*, do exceed pp, the *Object* is without the reach of a *Project* caſt with that *Velocity*, and ſo the thing impoſſible.

From this *Equation* $4bb \mp 4ph = pp$ are determined the utmoſt limits of the reach of any *Project*, and the Figure aſſigned, wherein are all the *heights* upon each *Horizontal diſtance* beyond which it cannot paſs; for by reduction of that *Equation*, h will be found $= \frac{1}{4}p - \frac{bb}{p}$ in *heights*, and $\frac{bb}{p} - \frac{1}{4}p$ in *deſcents*; from whence it follows, that all the Points h are in the *Curve* of the *Parabola*, whoſe *Focus* is the Point from whence the *Project* is caſt, and whoſe *Latus rectum*, or *Parameter ad Axem* is $=p$. Likewiſe from the ſame *Equation* may the leaſt *Parameter* or *Velocity* be found capable to reach the

Object propoſed; for $bb = \frac{1}{4}pp \mp ph$ being reduced $\frac{1}{2}p$ will be $= \sqrt{bb + hh} \pm h$ in aſcents / in deſcents, which is the *Horizontal rang* at 45 degrees, that would juſt reach the *Object*, and the *Elevation* requiſite will be eaſily had; for dividing the ſo found *Semi-parameter* by the *Horizontal diſtance* given *b*, the *Quote* into *Radius* will be the *Tangent* of the *Elevation* ſought. This Rule may be of good uſe to all *Bombardiers* and *Gunners*, not only that they may uſe no more Powder than is neceſſary, to caſt their *Bombs* into the place aſſigned, but that they may ſhoot with much more certainty, for that a ſmall Error committed in the *Elevation* of the *Piece*, will produce no ſenſible difference in the fall of the Shot: For which Reaſons the *French* Engineers in their late Sieges have uſed Morter-pieces inclined conſtantly to the *Elevation* of 45, proportioning their Charge of Powder according to the diſtanceof the *Object* they intend to ſtrike on the *Horizon*.

And this is all that need to be ſaid concerning this *Problem*, of Shooting upon *hights* and *deſcents*. But if a *Geometrical* conſtruction thereof be required; I think I have one, that is as eaſy as any can be expected, which I deduce from the forgoing *Analytical Solution*, viz. $\frac{t}{r} = \frac{p}{2b} \pm \frac{\sqrt{\frac{1}{4}pp \pm ph - bb}}{bb}$, and tis this. Having made the right *Angle* L D A, *Tab.* 1. *fig.* 4. make D A, D F = *p*, or greateſt Rang, D G = *b* the *Horizontal* diſtance, and D B D C = *h*, the *Perpendicular hight* of the *Object*; and draw G B, and make DE = thereto. Then with the *Radius* A C and *center* E ſweep an Arch, which if the thing be poſſible, will Interſect the line A D in H; and the line D H being laid both waies from F will give the points K and L, to which draw the lines G L, G K; I ſay the *Angles* LGD, KGD are the Elevations required for hitting the *Object* B. But note that if B be below the *Horizon*, its *deſcent* D C = D B muſt be laid from A, ſo as to have A C = to A D + D C. *Note* likewiſe, that if in *deſcents* D H be greater than F D, and ſo K fall below D the

Angle

Angle KGD ſhall be the depreſſion below the *Horizon* : Now this Conſtruction ſo naturally follows from the *Equation*, that I ſhall need ſay no more about it.

Prop. XI. To determine the force or *Velocity* of a *Project*, in every point of the *Curve* it deſcribes.

To do this we need no other *præcognita*, but only the third Propoſition, Viz. that the *Velocity* of *falling Bodies*, is double to that which in the ſame time, would have deſcribed the ſpace *fallen* by an equable motion : For the *Velocity* of a Project, is compounded of the conſtant equal *Velocity* of the impreſſed motion, and the *Velocity* of the *fall*, under a given *Angle*, viz. the complement of the *Elevation* : For inſtance, in *Fig.* 2, in the time wherein a project would move from G to L, it deſcends from L to X, and by the third *Propoſition* has acquired a *Velocity*, which in that time would have carried it by an equable motion from L to Z or twice the deſcent L X; and drawing the line G Z, I ſay the *Velocity* in the point X, compounded of the *Velocities* G L and L Z under the *Angle* G L Z, is to the *Velocity* impreſt in the point G, as G Z is to G L ; this follows from our ſecond *Axiome* ; and by the 20 and 21.*Prop. lib.*1, *conic. Midorgii*, XO parallel and equal to G Z ſhall touch the *Parabola* in the point X. So that the *Velocities* in the ſeveral points, are as the lengths of the *Tangents* to the *Parabola* in thoſe points, intercepted between any two *Diameters* : And theſe again are as the *Secants* of the *Angles*, which thoſe *Tangents* continued make with the *Horizontal* line G B. From what is here laid down, may the comparative force of a *ſhott* in any two points of the *Curve*, be either *Geometrically* or *Arithmetically* diſcovered.

Corollary.

From hence it follows, that the force of a Shot is alwaies leaſt at U, or the *Vertex* of the *Parabola*, and that at equal diſtances therefrom, as at T and X, G and B its force is alwaies equal, and that the leaſt force in U is to that in G and

B, as

B, as *Radius* to the Secant of the *Angle* of *Elevation* F G B.

These *Propositions* considered, there is no question relating to *Projects*, which by the help of them may not easily be Solved; and tho' it be true that most of them are to be met withal, in *Galileus, Torricellius* and others, who have taken them from those Authors, yet their Books being Forreign, and not easy to come by, and their *Demonstrations* long and difficult, I thought it not amiss to give the whole *Doctrine* here in *English*, with such short *Analytical* Proof of my own, as might be sufficient to evince their Truth.

The Tenth *Proposition* containes a *Problem*, untouch't by *Torricellius*, which is of the greatest use in *Gunnery*, and for the sake of which this *Discourse* was principally intended; It was first Solved by Mr. *Anderson*, in his Book of the Genuine use and effects of the *Gunn*, Printed in the Year 1674; but his Solution required so much Calculation, that it put me upon search, whether it might not be done more easily, and thereupon in the Year 1678 I found out the rule I now publish, and from it the *Geometrical* Construction: Since which time there has a large *Treatise* of this Subject Entituled, *L'art de jetter les Bombes*, been Published in *France* by *Monsieur Blondel*, wherein he gives the Solutions of this *Problem* by *Messieurs Bout, Romer* and *de la Hire*; But none of them being the same with mine, or in my Opinion more easy, and most of them more Operose, and besides mine finding the *Tangent*, which generally determines the *Angle* better than its *Sine*, I thought my self obliged to Print it for the use of all such, as desire to be informed in the *Mathematical* part, of the Art of *Gunnery*.

Now these rules were rigidly true, were it not, as I said before, for the Opposition of the Medium, whereby not only the direct imprest Motion is continually retarded, by likewise the increase of the *Velocity* of the *fall*, so that the spaces described thereby, are not exactly as the squares of the times: But what this Opposition of the *Air* is, against several *Velocities, Bulks*, and *Weights*, is not so easy to determine. 'Tis certain that the weight of *Air*, to that of *Water*, is nearly as 1 to 800, whence the weight thereof, to that of any *Project* is given; tis very likely, that to the same *Velocity*, and *Magnitude*, but of different matter, the *Opposition* should be *reciprocally* as the weights of the shott; as likewise that to shott of the same *Velocity* and matter, but of different Sizes, it should be as the *Diameters reciprocally*: whence generally the *Opposition* to shott with the same *Velocity*, but of differing *Diameters*, and *Materials*, should be as their *Specifick Gravities* into their *Diameters reciprocally*; but whether the *Opposition*, to differing *Velocities* of the same shott, be as the *Squares* of those *Velocities*, or as the *Velocities* themselves, or otherwise, is yet a harder Question. However it be, tis certain, that in large Shott of *Mettal*, whose weight many Thousand times Surpasses that of the *Air*, and whose force is very great, in proportion to the *Surface* wherewith they press thereon; this *Opposition* is scarce discernable: For by several *Experiments* made with all Care and Circumspection with a *Morterpeice* Extraordinary well fixt to the *Earth* on purpose, which carried a Solid Brass Shott of $4\frac{1}{4}$ Inches *Diameter*, and of about 14 Pound weight, the *Ranges* above and below 45 *Degrees* were found nearly equal; if there were any difference, the under *Ranges* went rather the farthest, but those differences were usually less than the Errours committed in ordinary *Practice*, by the unequal Goodness and Dryness of the same sort of *Pouder*, by the Unfitness of the *Shott* to the *Bore*, and by the Looseness of the Carriage.

In a Smaller *Brass-Shott* of about an Inch and half *Diameter*, cast by a *Cross-Bow* which *ranged* it, at most about 400 *foot*, the Force being much more Equal than in the *Morterpeice*, this difference was found more Curiously, and Constantly and most Evidently, the under *Ranges* out went the upper. From which Trials I conclude, that altho' in small and light *Shott*, the *Opposition* of the *Air*, ought and must be accounted for; yet in *Shooting* of Great and Weighty *Bombs*, there need be very little or no allowance made; and so these *Rules* may

be

be put in *Practice* to all Intents and Purposes, as if this *Impediment* were absolutely Removed.

An Account of an Experiment *shewn before the* Royal Society, *of Shooting by the Rarefaction of the Air: By* Dr. D. Papin, R. S S.

WHereas ordinary Wind-Guns do their *Effect* by the Compression of the Air. *Ottho Ghericke* hath found a new Sort that shoots by Rarefaction; and he hath Publisht that device at large in his Book about P*neumatick Experiments*, but he doth not express how strong was the Effect. I have therefore had the Curiosity to try it my self by another Contrivance, which I take to be better than his: First, because I can make a Rarefaction much more perfect than he could do. Secondly, because his Device could not be used but for Guns of a small bore; but my way may be apply'd to the biggest bore that can be made by Workmen: So that one might by this means throw up vast Weights to a great distance.

A A is a Pipe very equal from one end to the other.

B B a small Pipe soder'd to a Hole near the end of the Pipe A A, and apply'd to the Plate of the *Pneumatick Engine*.

C C C C some kind of Stool to bear up the hinder part of the Pipe A A.

D. a peice of Lead fitted to the bore of the pipe A A.

The pipe A A is to be shut at both ends by *Valves* outwardly apply'd, and so the said pipe A A, though never so big, may be exhausted of Air by means of the *Pneumatick Engine*: Which done, the *Val e* towards D must be sudde..-ly open'd, so that the whole pressure of the *Atmosphere* acting upon the Lead D may drive it along the pipe A A with

 such

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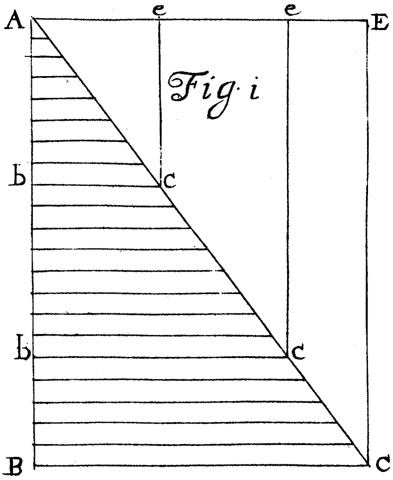
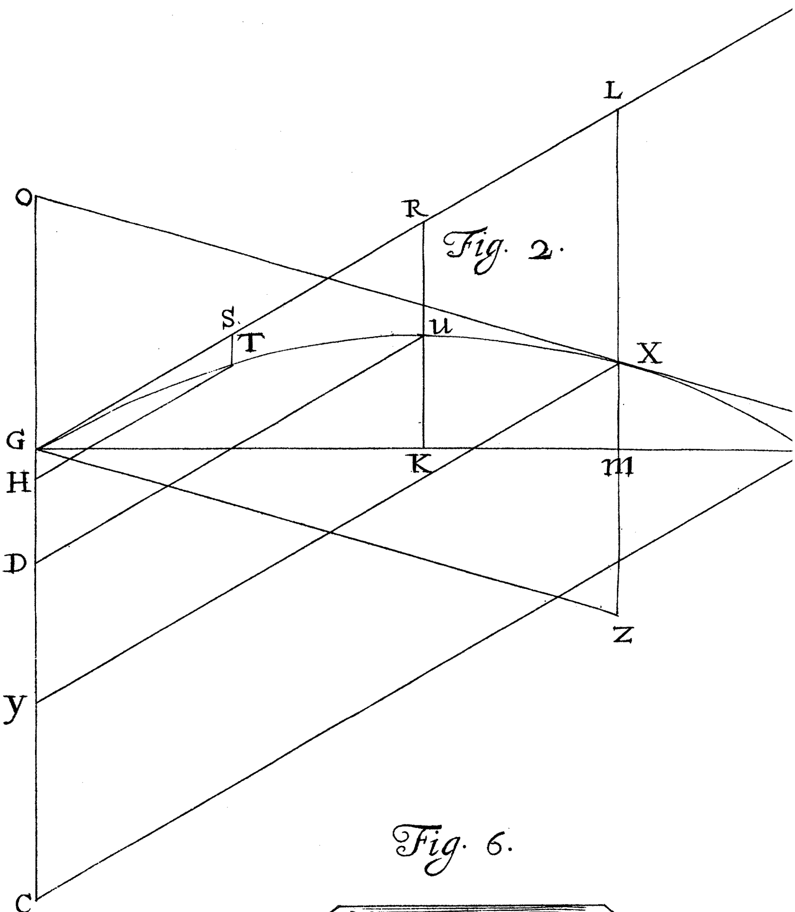


Fig. 1.



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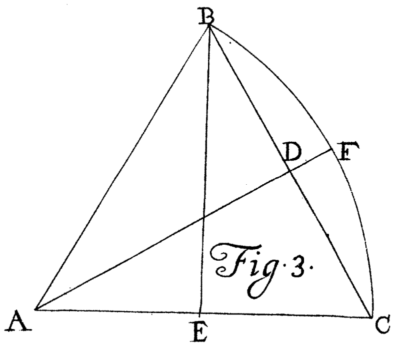

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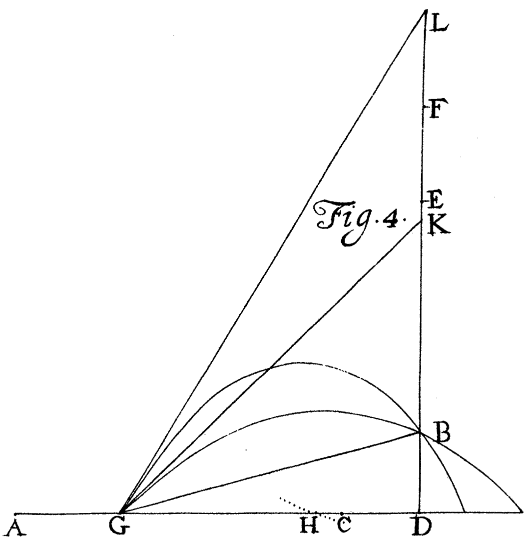
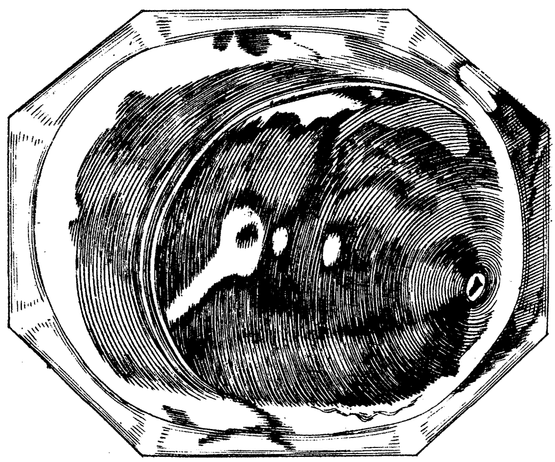


Fig. 4.

Fig. 6.



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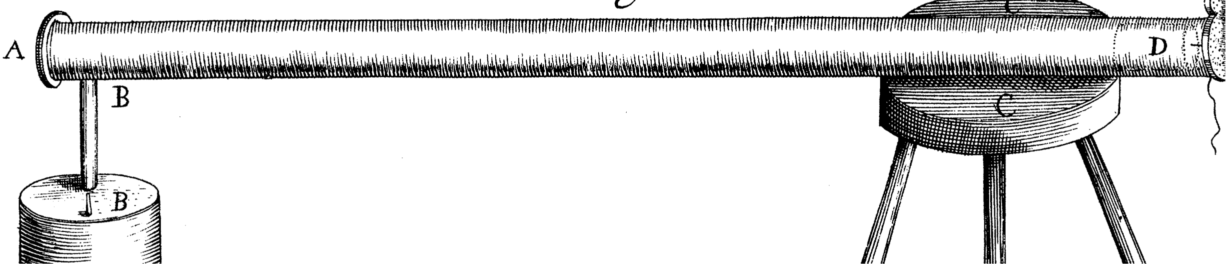

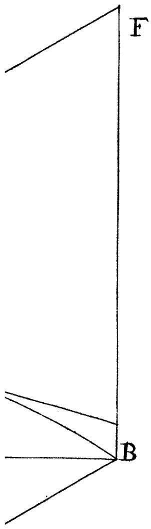
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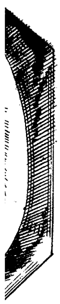

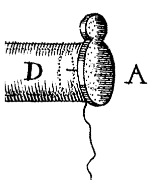
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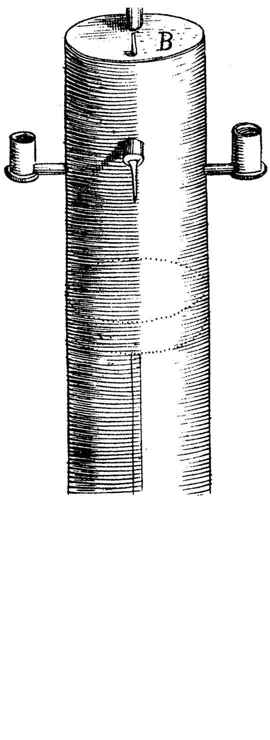
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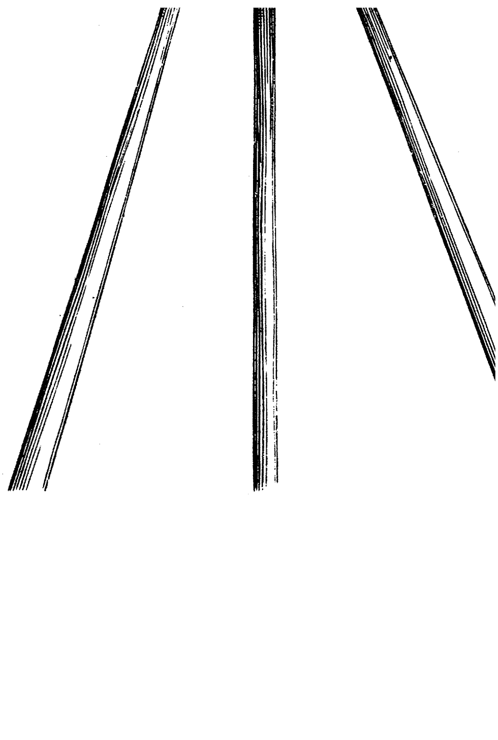

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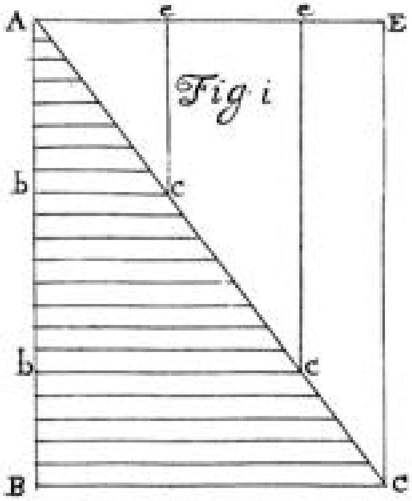

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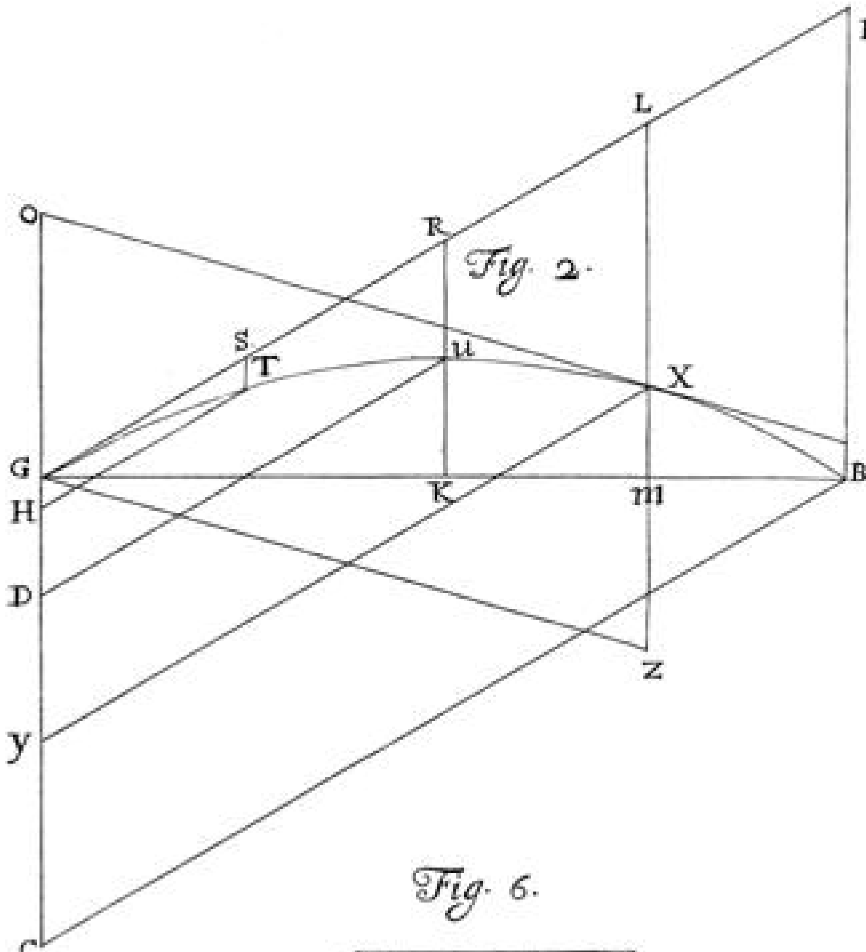

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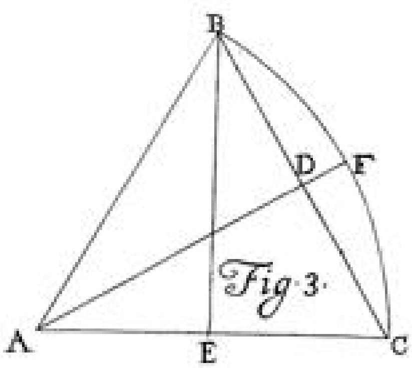

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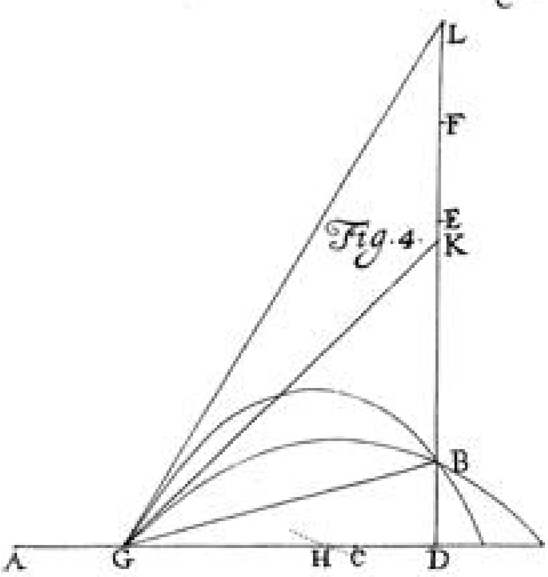

Fig. 4.

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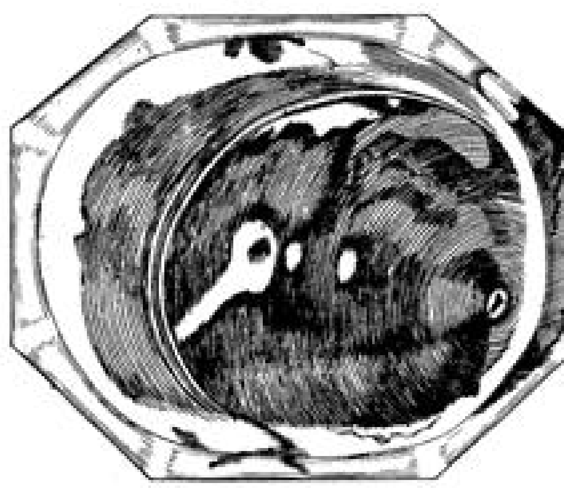

Fig. 5.