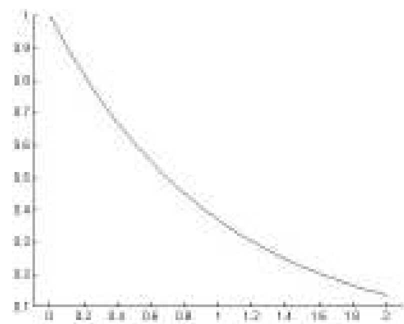
Resuelto por Ender Valdivieso Carnet 06-40411

Ejercicio 1

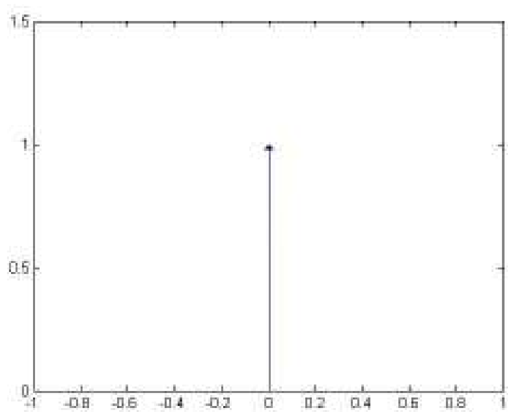
$$x_1(t) = e^{-t}u(t)u(t-2)$$

$$h_1(t) = \delta(t)$$

Gráfica de $x_1(t)$



Gráfica de $h(t)$



A priori conocemos que la función delta $\delta(t)$ es el elemento neutro en la convolución. Por ende, debemos obtener la misma señal como salida. Al realizar los cálculos tenemos:

$$x_1(t-\tau) = \begin{cases} e^{\tau - t}, & t-2 < \tau < t \\ 0, & resto \end{cases}$$

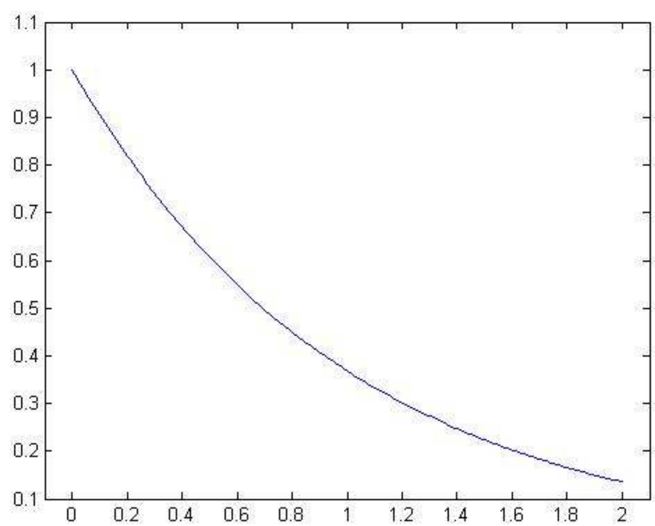
Para $0 < t < 2$

$$y_1(t) = \int_{-\infty}^{\infty} e^{\tau - t}\delta(\tau)d\tau$$

$$y_1(t) = e^{-t}\int_{-\infty}^{\infty} \delta(\tau)d\tau$$

$$y_1(t) = e^{-t}u(t)u(2-t)$$

Gráfica $y_1(t)$

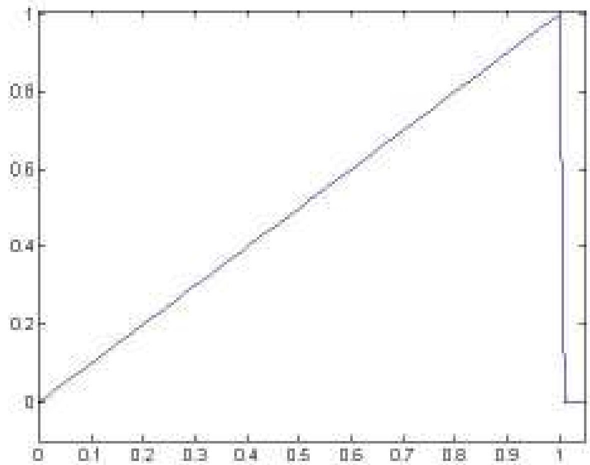


Ejercicio 2

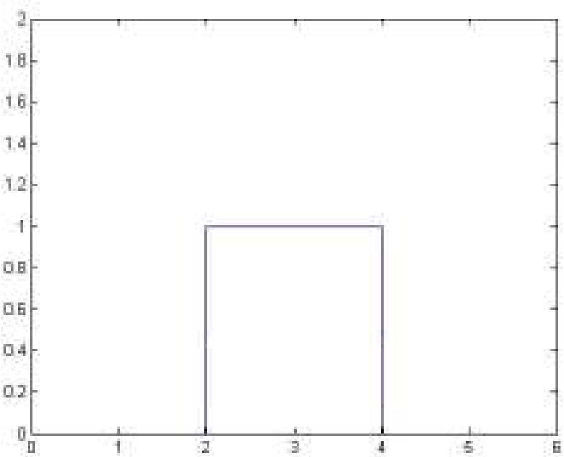
$$x_2(t) = r(t)u(1-t)$$

$$h_2(t) = tx|$$

Gráfica de $x_2(t)$



Gráfica de $h_2(t)$



$$h_2(t-\tau) = \begin{cases} 1, & t-4 < \tau < t-2 \\ 0, & resto \end{cases}$$

$$x_2(\tau) = \begin{cases} \tau, & 0 < \tau < 1 \\ 0, & resto \end{cases}$$

Para $2 < t < 3$

$$y_2(t) = \int_0^{t-2} \tau d\tau$$

$$y_2(t) = \frac{(t-2)^2}{2}$$

Para $3 < t < 4$

$$y_2(t) = \int_0^{1} \tau d\tau$$

$$y_2(t) = \frac{1}{2}$$

Para 4$< t < 5$

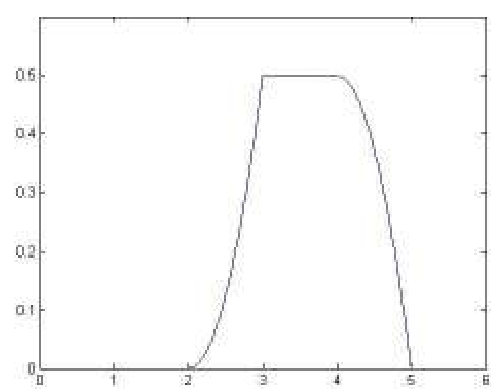
$$y_2(t) = \int_{t-4}^{1} \tau d\tau$$

$$y_2(t) = \frac{1}{2} - \frac{(t-4)^2}{2}$$

Entonces la función $y_2(t)$ quedaría definida de la forma

$$y_2(t) = \begin{cases} \frac{(t-2)^2}{2}, & 2 < \tau < 3 \\ \frac{1}{2}, & 3 < \tau < 4 \\ \frac{1}{2} - \frac{(t-4)^2}{2}, & 4 < \tau < 5 \end{cases}$$

Gráfica de $y_2(t)$

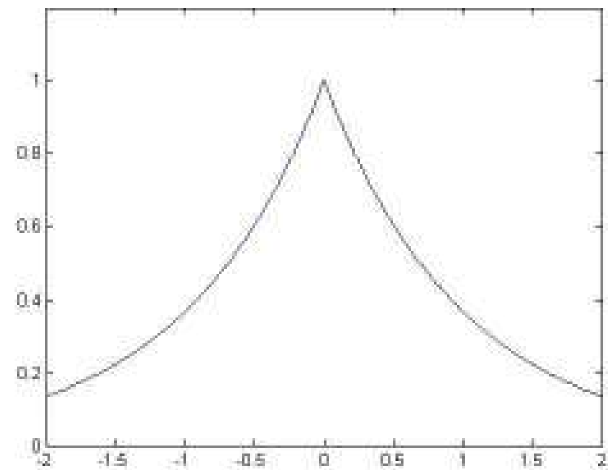


Ejercicio 3

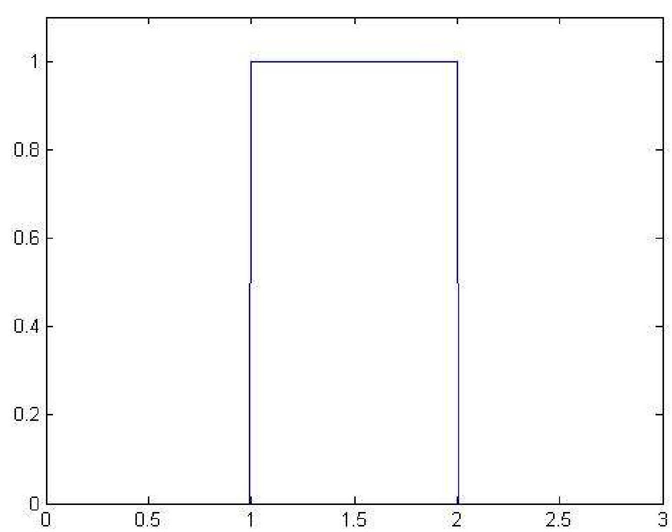
$$x_3(t) = e^{-|t|}u(t+2)u(2-t)$$

$$h_3(t) = u(t-1)u(2-t)$$

Gráfica $x_3(t)$



Gráfica de $h_3(t)$



$$x_3(\tau) = \begin{cases} e^{\tau}, & -2 < \tau < 0 \\ e^{-\tau}, & 0 < \tau < 2 \\ 0, & resto \end{cases}$$

$$h_3(t-\tau) = \begin{cases} 1, & t-2 < \tau < t-1 \\ 0, & resto \end{cases}$$

Para $-1 < t < 0$

$$y_3(t) = \int_{-2}^{t-1} e^{\tau} d\tau$$

$$y_3(t) = e^{t-1} - e^{-2}$$

Para $0 < t < 1$

$$y_3(t) = \int_{t-2}^{t-1} e^{\tau} d\tau$$

$$y_3(t) = e^{t-1} - e^{t-2}$$

Para $1 < t < 2$

$$y_3(t) = \int_{t-2}^{0} e^{\tau} d\tau + \int_{0}^{t-1} e^{-\tau} d\tau$$

$$y_3(t) = -e^{-t+1} - e^{t-2} + 2$$

Para $2 < t < 3$

$$y_3(t) = \int_{t-2}^{t-1} e^{-\tau} d\tau$$

$$y_3(t) = e^{-t+1} - e^{-t+2}$$

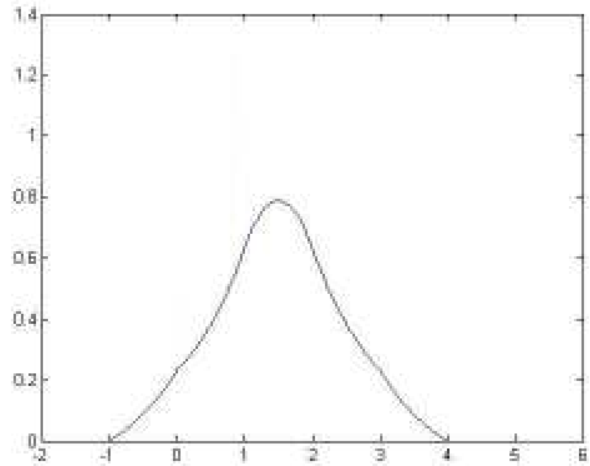
Para $3 < t < 4$

$$y_3(t) = \int_{t-2}^{2} e^{-\tau} d\tau$$

$$y_3(t) = e^{-t+2} - e^{-2}$$

La función sería $y_3(t) = 0$ para cualquier otro valor de t.

Gráfica de $y_3(t)$

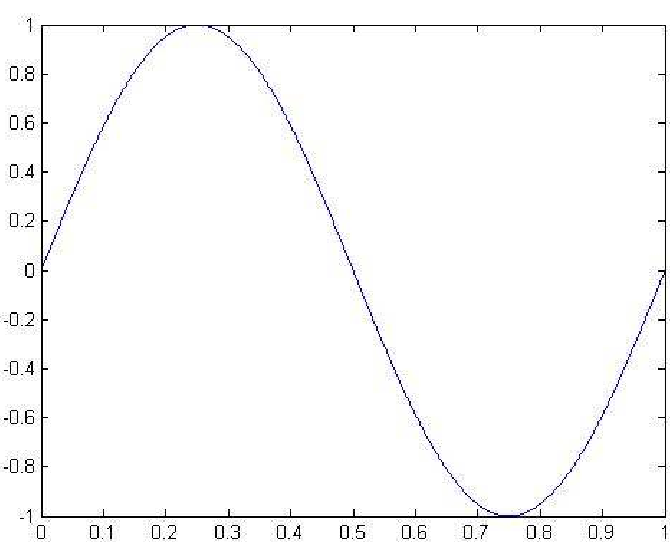


Ejercicio 4

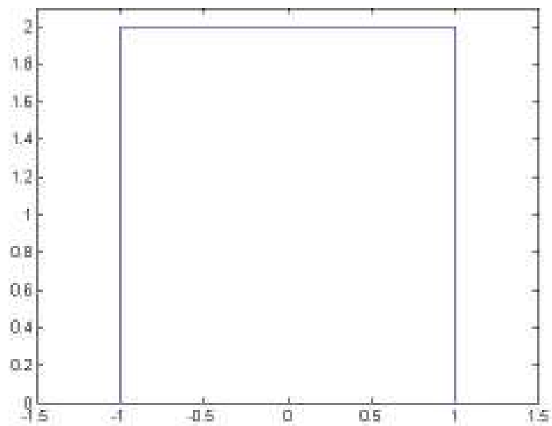
$$x_4(t) = \sin(2\pi t)\, u(t)u(1-t)$$

$$h_4(t) = 2u(t+1)u(1-t)$$

Gráfica de $x_4(t)$



Gráfica de $h_4(t)$



$$x_4(\tau) = \begin{cases} \sin(2\pi t), & 0 < \tau < 1 \\ 0, & resto \end{cases}$$

$$h_4(t-\tau) = \begin{cases} 2, & t-1 < \tau < t+1 \\ 0, & resto \end{cases}$$

Para $-1 < t < 0$

$$y_4(t) = \int_0^{t+1} 2\sin(2\pi t)\, d\tau$$

$$y_4(t) = 1 - \cos 2\pi t$$

Para $0 < t < 1$

$$y_4(t) = \int_0^{1} 2\sin(2\pi t)\, d\tau$$

$$y_4(t) = 0$$

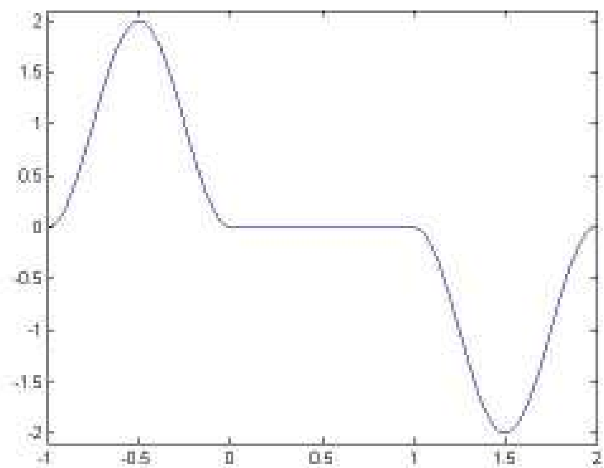
Para $1 < t < 2$

$$y_4(t) = \int_{t-1}^{1} 2\sin(2\pi t)\, d\tau$$

$$y_4(t) = \cos(2\pi t) - 1$$

$$y_4(t) = \begin{cases} 1 - \cos 2\pi t\,, & -1 < t < 0 \\ \cos(2\pi t) - 1\,, & 1 < t < 2 \\ 0, & resto \end{cases}$$

Gráfica de $y_4(\tau)$

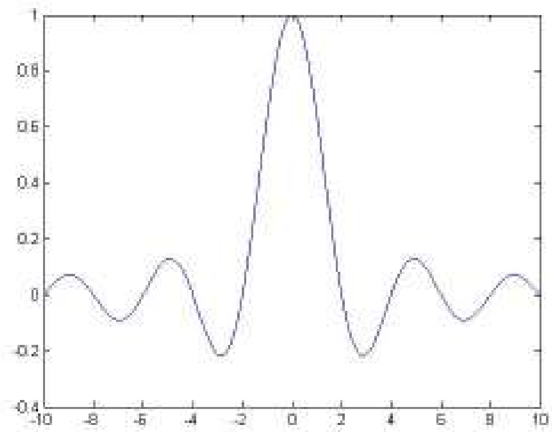


Ejercicio 5

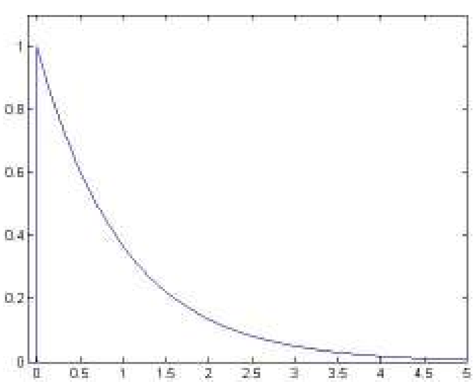
$$x_5(t) = \frac{\sin(\pi t)}{\pi t}$$

$$h_4(t) = e^{-t}u(t)$$

Gráfica de $x_5(t)$



Gráfica de $h_5(t)$



$$x_5(\tau) = \frac{\sin(\pi\tau)}{\pi\tau}$$

$$h_5(t-\tau) = \begin{cases} e^{\tau-t}, & \tau < t \\ 0, & resto \end{cases}$$

Para todo tiempo se cumple que

$$y_5(t) = \int_{-\infty}^{t} \frac{\sin(\pi\tau)}{\pi\tau} e^{\tau-t} d\tau$$