

A Treatise of Algebra, both Historical and Practical. By JOHN WALLIS, D. D. Professor of Geometry in the University of Oxford; and a Member of the Royal Society, London.

THE Author, in his Preface to the Reader, has given us a full account of this learned Work; which shall be here Reprinted in his own words.

It contains an Account of the Original, Progress, and Advancement of (what we now call) *Algebra*, from time to time; shewing its true Antiquity (as far as I have been able to trace it;) and by what Steps it hath attained to the Height at which now it is.

That it was in use of old among the *Grecians*, we need not doubt; but studiously concealed (by them) as a great Secret.

Examples we have of it in *Euclid*, at least in *Theo*, upon him; who ascribes the invention of it (amongst them) to *Plato*.

Other Examples we have of it in *Pappus*, and the effects of it in *Archimedes*, *Apollonius*, and others, though obscurely covered and disguised.

But we have no professed Treatise of it (among them) ancients than that of *Diophantus*, first published (in *Latin*) by *Xylander*, and since (in *Greek* and *Latin*) by *Bachetus*, with divers Additions of his own; and Re-printed lately with some Additions of Monsieur *Fermat*.

That it was of ancient use also among the *Arabs*, we have reason to believe, (and perhaps sooner than amongst the *Greeks*;) which they are supposed to have received (not from the *Greeks*, but) from the *Persians*, and these from the *Indians*.

From the *Arabs* (by means of the *Saracens* and *Moors*) it was brought into *Spain*, and thence into *England* (to-

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gether with the use of the Numeral Figures, and other Parts of *Mathematical Learning*, and particularly the *Astronomical*,) before *Diophantus* seems to have been known amongst us: And from those we have the name of ALGEBRA.

And indeed most of the *Greek Learning* came to us the same way; the first Translations of *Euclid*, *Ptolemy*, and others, into *Latin*, being from the *Arabick Copies*, and not from the *Greek Originals*.

The use of the *Numeral Figures* (which we now have, but the *Greeks* had not) was a great advantage to the improvement of *Algebra*.

These Figures seem to have come in use, in these Parts, about the Eleventh Century (or rather in the Tenth Century, about the middle of it, if not sooner;) though some others think, not 'till about the middle of the Thirteenth; and it seems they did scarce come to be of common use 'till about that time.

Archimedes (in his *Arenarius*) had laid a good Foundation of such a way of Computation, (as he hath indeed, there and elsewhere, of most of those new Improvements, which later Ages have advanced;) Though he has not fitted a Notation thereunto.

The *Sexagesimal Fractions* (introduced, as it seems, by *Ptolemy*) did but imperfectly supply the want of such a Method of Numeral Figures.

The use of these Numeral Figures hath received two great Improvements. The one is that of *Decimal Parts*, which seems to have been introduced (silently and unobserved) by *Regiomontanus*, in his *Trigonometrical Canons*, about the Year 1450; but much advanced in the last and present Century, by *Simon Stevin*, and *Mr. Briggs*, &c.

And this is much to be preferred before *Ptolemy's Sexagesimal way*, as is shewed by the comparative use of both.

And therefore *Briggs*, *Gellibrand*, and others, have attempted the introducing of this, even in those cases
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where the Sexagesimal is yet in use: Which doth in good measure, now obtain; (and, daily more and more.) And would, no doubt, have obtained absolutely e're this time, did not the Old Tables heretofore Calculated, make it somewhat necessary to retain (in part) the Sexagesimal.

The other Improvement is that of *Logarithms*, which is of great use, especially in *Astronomical* and other *Trigonometrical* Calculations; introduced by the Lord *Neper*, and perfected by Mr. *Briggs* (about the beginning of this Century.) The ground and practice of which is here declared.

And these things, though they be not properly Parts of *Algebra*, are yet of great advantage in the practice of it.

The first Printed Author which Treats of *Algebra* is *Lucas Pacciolus*, or *Lucas de Burgo*, a Minorite Fryer, of whom we have a Treatise in *Italian*, Printed at *Venice* in the Year 1494, (soon after the first Invention of Printing,) and Re-printed there, a while after.

But he therein mentions *Leonardus Pisanus*, and divers others more ancient than himself, from whom he Learned it; but whose Works are not now extant.

This Fryer *Lucas*, in his *Summa Arithmetica & Geometrica*, (for he hath other Works extant) hath a very full Treatise of Arithmetick in all the parts of it; in *Integers*, *Fractions*, *Surds*, *Binomials*; Extraction of *Roots*, *Quadratick*, *Cubick*, &c. and the several Rules of *Proportion*, *Fellowship*, about *Accompts*, *Alligation*, and *False Position*, (so fully, that very little hath been thereunto added to this day:) And (after all this) of *Algebra*, with the Appurtenances thereunto, (as *Surd Roots*, *Negative Quantities*, *Binomials*, *Roots Universal*, the use of the Signs *Plus Minus*, or $+$, $-$, &c.) as far as *Quadratick Equations* reach, but no farther.

And this he tells us was derived from the *Arabs*, (to whom we are beholden for this kind of Learning,) without

taking notice of *Diophantus* (or any other *Greek* Author) who it seems was not known here in those days.

After him followed *Stiphelius* (a good Author,) and others by him cited, who also proceed no farther than *Quadratick* Equations.

Afterwards *Scipio Ferreus*, *Cardan*, *Tartalea*, and others, proceeded to the Solution of (some) *Cubick* Equations.

And *Bombelli* goes yet farther, and shews how to reduce a *Biquadratick* Equation (by the help of a *Cubick*) to two *Quadraticks*.

And *Nonnius* or *Nunnez* (in *Spanish*;) *Ramus*, *Schone-rus*, *Salignacus*, *Clavius*, and others, (in *Latine*,) *Record*, *Digs*, and some others of our own, (in *English*;) did (in the last Century) pursue the same Subject, in different ways; but (for the most part) proceeded no farther than *Quadratick* Equations.

In the mean time, *Diophantus*, first by *Xylander* (in *Latin*) and afterwards by *Bachetus* (in *Greek* and *Latin*) was made publick; whose method differs much from that of the *Arabs* (whom those others followed,) and particularly in the order of denominating the Powers; as taking no notice of *Surfolids*, but using only the names of *Square* and *Cube*, with the Compounds of these.

And hitherto no other than the unknown Quantities were wont to be denoted in *Algebra* by particular Notes or Symbols; but, the known Quantities, by the ordinary Numeral Figures.

The next great step, for the improvement of *Algebra*, was that of *Specious Arithmetick*, first introduced by *Vieta* about the Year 1590.

This *Specious Arithmetick*, which gives Notes or Symbols (which he calls *Species*) to Quantities both known and unknown, doth (without altering the manner of demonstration, as to the substance,) furnish us with a short and convenient way of Notation; whereby the
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whole process of many Operations is at once exposed to the Eye in a short Synopsis.

By help of this he makes many Discoveries, in the process of *Algebra*, not before taken notice of.

He introduceth also his Numeral *Exegesis*, of affected Equations, extracting the Roots of these in Numbers. Which had before been applied to single Equations, such as the extracting the Roots of *Squares*, *Cubes*, &c. singly proposed; but had not been applied (or but rarely) to Equations affected.

And in the Denomination of Powers, he follows the order of *Diophantus*; not that derived from the *Arabs*, which others had before used.

The method of *Vieta* is followed, and much improved, by Mr. *Oughtred* in his *Clavis* (first published in the Year 1631.) and other Treatises of his; and he doth therein, in a brief compendious method, declare in short, what had before been the Subject of large Volumes: and doth, in few small pieces of his, give us the Substance and marrow of all (or most of) the Ancient Geometry.

And for this reason, I have here inserted a pretty full account of his method, together with an Institution for the practice of *Algebra* according thereunto. And, though much of it had been before taught in the Authors above mentioned, yet this I judged the most proper place to insert such an Institution, because by him delivered in the most compendious form.

And in pursuance of his method, and as an Exemplification thereof, I have here added (beside some Examples of his own) a Discourse of *Angular Sections*, and several things thereon depending. But this (that it might not seem too great a Digression in the body of the Book) I have subjoined at the end as a Treatise by it self; as, for the like Reason, I have done some other things; to which the principal Treatise doth (in the proper places) refer.

Mr. *Harriot* was contemporary with Mr. *Oughtred*,

(but elder than he, and died before him,) and left many good things behind him in writing. Of which there is nothing hitherto made publick, but only his *Algebra* or *Analytice*, which was published by Mr. *Warner*, soon after that of Mr. *Oughtred*, in the same Year 1631.

He alters the way of Notation, used by *Vieta* and *Oughtred*, for another more convenient.

And he hath also made a strange improvement of *Algebra*, by discovering the true construction of *Compound Equations*, & how they may be raised by a Multiplication of *Simple Equations*, and may therefore be resolved into such.

By this means he shews the number of Roots (real or imaginary) in every Equation, and the Ingredients of all the Coefficients, in each degree of Affection.

He shews also how to increase or diminish the *Roots* (yet unknown) by any Excess, or in any Proportion assigned; to destroy some of the intermediate Terms; to turn Negative Roots into Affirmative, or these into those; with many other things very advantagious in the practice of *Algebra*.

And amongst other things, teacheth (thereby) to resolve, not only *Quadraticks*, but all *Cubick* Equations; even those whose Roots have, by others, been thought *Inexplicable*, and but *Imaginary*.

In sum, He hath taught (in a manner) all that which hath since passed for the *Cartesian* method of *Algebra*; there being scarce any thing of (pure) *Algebra* in *Des Cartes*, which was not before in *Harriot*; from whom *Des Cartes* seems to have taken what he hath (that is purely *Algebra*) but without naming him.

But the Application thereof to *Geometry*, or other particular Subjects, (which *Des Cartes* pursues,) is not the business of that Treatise of *Harriot*, (but what he hath handled in other Writings of his, which have not yet the good hap to be made publick;) the design of this being purely *Algebra*, abstract from particular Subjects.

Of

Of this treatise here is the fuller account inserted, because the Book it self hath been but little known abroad; that it may hence appear to what estate *Harriot* had brought *Algebra* before his death.

After this follows an account of *Dr. Pell's* method, who hath a particular way of Notation, by keeping a Register (in the Margin) of the severall Steps in his Demonstrations, with References from one to another.

Of this, some Examples are here inserted of his own, and others in imitation thereof; with intimation how that innumerable Solutions of Undetermined Cafes are by his method easily discoverable, where great Mathematicians have thought it a great work to find out some one.

On this occasion there is a farther Discourse of *Undetermined Questions*, and the Limitation of them, and particularly of the Rule of *Alligation*; and of (what they call) *Geometrical Places*; which are of a like nature, and but the *Geometrical* Construction of (some of) these *Undetermined Questions*.

After this is a Discourse of *Negative Squares*, and the Roots of them; on which depend (what they call) *Imaginary Roots* of Impossible Equations; shewing, what is the true Import thereof in nature, with divers *Geometrical* Constructions suiting thereunto.

And here also (though by way of Digression, as to the principal Subject) is account given of severall *Geometrical* Constructions, not only of *Quadratick*, but even of *Cubick* and *Biquadratick* Equations.

Then follows a Discourse of the method of *Exhaustions* (used by Ancients and Moderns,) with the foundation of it.

And in pursuance thereof, the *Geometria Indivisibilium* of *Cavalierius*; shewing the true import thereof, and its agreement with the Ancients method of *Exhaustions*; as being but a compendious Expression thereof,
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and grounded thereupon; not any way contrary or repugnant thereunto.

Consequent to this, is the *Arithmetica Infinitorum*, which depends also on the method of Exhaustions; taking that to be Equal, which is proved to differ by less than any assignable Quantity. And a Vindication of the method of Demonstration therein used.

And lastly, the method of infinite *Series*, (as of late they have been called) or continual *Approximations*, (grounded on the same Principles;) arising Principally from *Division*, and Extraction of *Roots* in *Species*, Infinitely continued.

With several Examples of the Application thereof, to the Squaring of Curve-lined Figures, Rectifying of Curve Lines, Planing of Curve Surfaces, and many other perplexed Inquiries.

Which is an *Arithmetick of Infinites upon Infinites*. For when as the *Quotient* of Division, or the *Root* extracted in *Species*, doth not Terminate, but run on Infinitely, (much after the manner of some ordinary Fractions, when reduced to Decimals;) an Infinite Series of these (continued as far as is thought necessary,) is Collected according to the method, in the Arithmetick of Infinites, for Terminated Magnitudes.

This was introduced by Mr. *Isaac Newton*, and hath been pursued by Mr. *Nicholas Mercator*, and others.

And it is of great use for Rectifying of *Curve Lines*, Squaring of *Curve-lined Figures*, and other abstruse Difficulties in *Geometry*; especially where the Enquiry doth not end in a determinate Proportion, explicable according to the commonly received ways of Notation.

And on this occasion, is inserted a Discourse of *Infinite Progressions Geometrical*; (which when decreasing, become Equivalent to Finite Magnitudes,) first used by *Archimedes*, and since pursued by *Toricellius*, *San-Vincent*, *Tacquet*, and others. With the Result of two or more such Progressions compounded. Several

Several other Discourses are, partly inserted in their proper places, and partly subjoyned at the end, that they might not seem too great Digressions.

And particularly a Treatise of the *Cono-Cuneus* (a Body Compounded of a *Cone*, and a *Wedge*.) with the Sections thereof; considered in the same manner as the Sections of a *Cone* use to be considered.

A Treatise of *Angular Sections*, (a Subject handled by *Vieta*, and others,) with other things thereon depending; together with a short (but full) account of *Trigonometry*.

A further Treatise of the *Angle of Contact*; in pursuance of a former Treatise on that Subject. Wherein is further discoursed what concerns the *Composition of Magnitudes*, *Inceptives of Magnitudes*, *Composition of Motions*, and other things hereunto relating.

A Treatise of *Combinations*, *Alternations*, and *Aliquot Parts*: a Subject discoursed of, by *Schooten*, *Pell*, *Kersey*, and others.

With many other things, which may be seen in the Table of Chapters; but, more fully, in the Treatise it self. Much of which are Additions of my own, where I apprehended a defect (in what I met with in others) which seemed needfull to be supplied.

But I do not pretend so to have gleaned all those Authors who have Written on this Subject, as to have left nothing worthy to be there sought, in the Authors themselves, (especially as to the Accommodation thereof to particular Subjects:) But have rather directed to those Authors where such things are to be found.

And I have been the less able so to do (if I would have done it,) because I did not designedly read them over to this purpose, nor (when I did read them) did make Collections (as I went along) in order to such a design. But have rather (out of my memory) inserted (in their proper places) such things as the Order and Method of

the Discourse seemed to call for ; and (on such occasions) had recourse to the respective Authors.

Those who desire a fuller account of such things as I have but briefly touched ; may, for that purpose, consult *Vieta*, *Oughtred*, *Harriot*, *Cartes*, *Slufus*, and others; and (in English) Mr. *Kerfy*; who hath published a Compleat Volume of *Algebra* (with the Appurtenances thereunto) in Two parts.

But my design being, to trace this of *Analyticks* (as the *Greeks* call'd it) or *Algebra* (as the *Arabs*) from its first Original (as near as I could) by the several Steps whereby it hath proceeded : Mine Ey was chiefly on the several Advances which from time to time it hath made; omitting, for the most part, the Accommodations thereof to particular Subjects.

And herein I have endeavoured, all along, to be just to every one : Ascribing, as near as I could, every Step of advance to its own Author; or at least to the most ancient of those in whom I found it.

If I have any where missed of this (ascribing to a latter what was due to some former Writer;) it is either because I had not read the more ancient, or did not there heed it when I so read, or at least did not remember it, when I was Writing. And I shall be willing to be rectified, in what I have any where mistaken.

There may yet perhaps (notwithstanding all my care) be some difficulty to satisfy all Readers, as to what I have, or what I have not taken notice of. Who may think there are divers things omitted (and doubtless there are so) which might deserve to be taken notice of; or but briefly touched, which might have deserved a fuller discourse; and some things inserted, which (in their opinion) might have been spared, or needed not to have been so fully handled.

But as to such things, I must be content to leave my self to the Readers Candor; or leave the Readers themselves

elves to fatisfie one another Amongft whom, fome may be found to Blame, what another Commends, and fome to Commend, what another Blames.

And I have endeavoured all along to represent the fentiments of others with Candor, and to the beft advantage: Not Studioufly feeking opportunities of Cavilling, or greedily catching at them if offered. (For there is no man can Write fo warily, but that he may fometime give opportunity of Cavilling, to thofe who feek it.) And have been carefull to put the beft Con- ftruction on their Words and Meaning; and, if need be (as fometimes there is) to help an incommodious expref- fion, by one (as at leaft appeared to me) more intelli- gible and better agreeing (or more fully) to their own meaning; (without reproaching them for the want of fuch :) For it many times happens, that a man lights on a good notion; which he hath not the happinefs to exprefs fo intelligibly, as perhaps another may do for him. And if here (fometimes) I have fo done (as I think I have;) I do not therein wrong, either the Author or the Reader.

*Apologia pro Circuitione Sanguinis; qua responde-
tur Æmylio Parisano, medico Veneto, Authore
GEORGIO ENTIO; Editio altera, auctior
& accuratior Lond. 1685. 8^{vo}.*

THE Learned Author having thought fit to give us a fecond Edition of this Book, it may not be amifs to take notice of fome particulars added in this Impref- fion.

In his Epiftle to Dr. *Harvey*, he fhews how little truth there is in *Father Paul's* being the Inventor of the *Circu- lation*; for as much as the papers Written by him on this Subject, and found in his Study after his Death, were no

more then notes taken at the reading of Dr. *Harvey's* Book, which was lent him by a Country-man of his lately returned to *Venice* from *England*, where he had been *Ambassador* from that *State*, and was presented by the *Author* with one of these Books; the Truth of which appears from a Letter written by *Father Fulgentio* to Dr. *Harvey*, expressing as much.

In the Book it self he takes occasion to give some *Comparative* account of the *Spleen* in several creatures.

He enquires into the reason why the Air is hotter in Summer then in Winter, which he thinks does not proceed, from that the rays of the *Sun* fall on the Earth, nearer to a perpendicular, in the former then in the latter season; the *Angulus Incidentiæ* made by the Rays not being to be considered so much as the *number* of them; he supposes this effect may more rationally be deduced partly from the Northern Winds blowing much in Winter, but chiefly from the *Density* of the *Air* which defends us from the *Sun*, and prevails more (he thinks) by 40 to one, in the Winter, then in the Summer.

To prove that the Blood does not nourish, he urges the disproportion between some parts, and the quantity of Blood conveyd to them; thus a great deal of Blood is carryed to the Intestines, and but little to the Head, in proportion to the bigness of each part: he urges the largeness of the Veins in respect to the Arterys: and the smalness of the recruit the Blood receives by the *Ductus Thoracicus*, not large enough, the Author thinks, to convey matter for the nourishment of the whole body; with several other particulars to the same effect.

He answers the arguments of Dr. *Needham*, and Dr. *Mayow*, against the *Biolychnium*.

Printed at the Theater in Oxford, for Sam. Smith at the Prince's Arms in St. Paul's Church-yard London; and Hen. Clements Book-seller in Oxford.