RESOLVER EL SIGUIENTE SISTEMA DE ECUACIONES DIFERENCIALES

$$\frac{dx}{dt} - 4x + \frac{d^3y}{dt^3} = 6\,\text{sen}t \rightarrow (1) \qquad x(0)=0,\ y(0)=0$$

$$y'(0)=0,\ y''(0)=0$$

$$\frac{dx}{dt} + 2x - 2\frac{d^3y}{dt^3} = 0 \longrightarrow (2)$$

EL OBJETIVO ES HALLAR $x(t)$ e $y(t)$

DESPEJANDO $\frac{d^3y}{dt^3}$ EN (1).

$$\frac{d^3y}{dt^3} = 6\,\text{sen}t - \frac{dx}{dt} + 4x \rightarrow (3)$$

SUSTITUYENDO (3) EN (2).

$$\frac{dx}{dt} + 2x - 2\left[6\,\text{sen}t - \frac{dx}{dt} + 4x\right] = 0$$, SIMPLIFICANDO:

$$\Rightarrow \frac{dx}{dt} + 2x - 12\,\text{sen}t + 2\frac{dx}{dt} - 8x = 0$$

$$\Rightarrow 3\frac{dx}{dt} - 6x - 12\,\text{sen}t = 0$$, DIVIDIENDO ENTRE 3.

$$\Rightarrow \frac{dx}{dt} - 2x - 4\,\text{sen}t = 0 \Rightarrow \frac{dx}{dt} - 2x = 4\,\text{sen}t$$

PARA HALLAR $x(t)$, APLICAMOS LA TRANSFORMADA DE LAPLACE:

$$\mathcal{L}\{x'\} - 2\mathcal{L}\{x\} = 4\mathcal{L}\{\text{sen}t\}$$, CONSIDERANDO $x(0)=0$

$$\Rightarrow s\,X(s) - 2X(s) = 4\left[\frac{1}{s^2+1}\right]$$

$$\Rightarrow X(s)[s-2] = 4\left(\frac{1}{s^2+1}\right) \Rightarrow X(s) = 4\left[\frac{1}{(s^2+1)(s-2)}\right]$$

USANDO FRACCIONES PARCIALES:

$$\Rightarrow X(s) = 4\left[\frac{1}{5}\left(\frac{1}{s-2}\right) - \frac{1}{5}\left(\frac{s+2}{s^2+1}\right)\right] = \frac{4}{5}\left[\frac{1}{s-2}\right] - \frac{4}{5}\left(\frac{s+2}{s^2+1}\right)$$

LUEGO, CALCULANDO LA INVERSA:

$$\mathcal{L}^{-1}\{X(s)\} = x(t) = \frac{4}{5}\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} - \frac{4}{5}\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} - \frac{8}{5}\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}$$

$$\Rightarrow x(t) = \frac{4}{5}e^{2t} - \frac{4}{5}\cos t - \frac{8}{5}\,\text{sen}\,t \longrightarrow (4)$$

PARA CALCULAR $y(t)$, USAMOS (3), A SABER:

$$\frac{d^3y}{dt^3} = 6\,\mathrm{sen}\,t - \frac{dx}{dt} + 4x \rightarrow 3$$, OBSERVAMOS QUE

NECESITAMOS $x(t)$ y $\frac{dx}{dt}$, PERO DADO QUE:

$$x(t) = \frac{4}{5}e^{2t} - \frac{4}{5}\cos t - \frac{8}{5}\,\mathrm{sen}\,t$$ y DERIVANDO, TENEMOS $\frac{dx}{dt}$

$$\Rightarrow \frac{dx}{dt} = \frac{8}{5}e^{2t} + \frac{4}{5}\,\mathrm{sen}\,t - \frac{8}{5}\cos t \rightarrow (5)$$

LUEGO, SUSTITUYENDO (4) Y (5) EN (3):

$$\frac{d^3y}{dt^3} = 6\,\mathrm{sen}\,t - \left(\frac{8}{5}e^{2t} + \frac{4}{5}\,\mathrm{sen}\,t - \frac{8}{5}\cos t\right) + 4\left(\frac{4}{5}e^{2t} - \frac{4}{5}\cos t - \frac{8}{5}\,\mathrm{sen}\,t\right)$$

$$\Rightarrow \frac{d^3y}{dt^3} = 6\,\mathrm{sen}\,t - \frac{8}{5}e^{2t} - \frac{4}{5}\,\mathrm{sen}\,t + \frac{8}{5}\cos t + \frac{16}{5}e^{2t} - \frac{16}{5}\cos t - \frac{32}{5}\,\mathrm{sen}\,t$$

$$\Rightarrow \frac{d^3y}{dt^3} = -\frac{6}{5}\,\mathrm{sen}\,t + \frac{8}{5}e^{2t} - \frac{8}{5}\cos t \rightarrow (6)$$

LUEGO, APLICANDO LA TRANSFORMADA DE LAPLACE A (6)
CON $y(0)=0$, $y'(0)=0$ Y $y''(0)=0$, TENEMOS:

$$s^3 Y(s) = -\frac{6}{5}\left(\frac{1}{s^2+1}\right) + \frac{8}{5}\left(\frac{1}{s-2}\right) - \frac{8}{5}\left(\frac{s}{s^2+1}\right)$$

$$\Rightarrow Y(s) = -\frac{6}{5}\left[\frac{1}{s^3(s^2+1)}\right] + \frac{8}{5}\left[\frac{1}{s^3(s-2)}\right] - \frac{8}{5}\left[\frac{1}{s^2(s^2+1)}\right] \rightarrow (7)$$

FINALMENTE, DESARROLLANDO FRACCIONES PARCIALES Y
APLICANDO LA INVERSA DE LAPLACE A (7), TENEMOS:

$$y(t) = \left(-\frac{3}{5}t^2 - \frac{6}{5}\cos t + \frac{6}{5}\right) + \left(-\frac{2}{5}t^2 - \frac{2}{5}t + \frac{1}{5}e^{2t} - \frac{1}{5}\right) + \left(-\frac{8}{5}t + \frac{8}{5}\,\mathrm{sen}\,t\right)$$

$$\Rightarrow y(t) = -\frac{6}{5}\cos t + \frac{8}{5}\,\mathrm{sen}\,t - \frac{1}{5}e^{2t} - t^2 - 2t + 1 \rightarrow (8)$$

ASÍ, EL SISTEMA QUEDÓ RESUELTO CON (4) Y (8).

$$x(t) = \frac{4}{5}e^{2t} - \frac{4}{5}\cos t - \frac{8}{5}\,\mathrm{sen}\,t$$

$$y(t) = -\frac{6}{5}\cos t + \frac{8}{5}\,\mathrm{sen}\,t - \frac{1}{5}e^{2t} - t^2 - 2t + 1$$