Prof. Dr. H. Brenner **Osnabrück WS 2011/2012**

Mathematics for natural sciences I

Exercise sheet 16

Warm-up-exercises

Exercise 16.1. Find a zero for the function

$$f : \mathbb{R} \longrightarrow \mathbb{R},\ x \longmapsto f(x) = x^2 + x - 1,$$

in the interval $[0,1]$ using the interval bisection method with a maximum error of $1/100$.

Exercise 16.2. Let

$$f : [0,1] \longrightarrow [0,1[$$

be a continuous function. Prove that f is not surjective.

Exercise 16.3. Give an example of a bounded interval $I \subseteq \mathbb{R}$ and a continuous function

$$f : I \longrightarrow \mathbb{R}$$

such that the image of f is bounded, but the function admits no maximum.

Exercise 16.4. Let

$$f : [a,b] \longrightarrow \mathbb{R}$$

be a continuous function. Show that there exists a continuous extension

$$\tilde{f} : \mathbb{R} \longrightarrow \mathbb{R}$$

of f.

Exercise 16.5. Let

$$f : I \longrightarrow \mathbb{R}$$

be a continuous function defined over a real interval. The function has at points $x_1, x_2 \in I$, $x_1 < x_2$, local maxima. Prove that the function has between x_1 and x_2 has at least one local minimum.

Exercise 16.6. Determine directly, for which $n \in \mathbb{N}$ the power function

$$\mathbb{R} \longrightarrow \mathbb{R},\ x \longmapsto x^n,$$

has an extremum at the point zero.

Exercise 16.7. Show that the Intermediate value theorem for continuous functions from $\mathbb{Q}$ to $\mathbb{Q}$ does not hold.

Exercise 16.8. Determine the limit of the sequence

$$x_n = \sqrt{\frac{7n^2 - 4}{3n^2 - 5n + 2}},\ n \in \mathbb{N}.$$

Hand-in-exercises

Exercise 16.9. (2 points)

Determine the minimum of the function

$$f : \mathbb{R} \longrightarrow \mathbb{R},\ x \longmapsto x^2 + 3x - 5.$$

Exercise 16.10. (4 points)

Find for the function

$$f : \mathbb{R} \longrightarrow \mathbb{R},\ x \longmapsto f(x) = x^3 - 3x + 1,$$

a zero in the interval $[0, 1]$ using the interval bisection method, with a maximum error of $1/200$.

Exercise 16.11. (2 points)

Determine the limit of the sequence

$$x_n = \sqrt[3]{\frac{27n^3 + 13n^2 + n}{8n^3 - 7n + 10}},\ n \in \mathbb{N}.$$

The next task uses the notion of an even and an odd function.

A function

$$f : \mathbb{R} \longrightarrow \mathbb{R}$$

is called *even*, if for all $x \in \mathbb{R}$ the equality

$$f(x) = f(-x)$$

holds.

A function

$$f : \mathbb{R} \longrightarrow \mathbb{R}$$

is called *odd*, if for all $x \in \mathbb{R}$ the equality

$$f(x) = -f(-x)$$

holds.

Exercise 16.12. (4 points)

Let

$$f : \mathbb{R} \longrightarrow \mathbb{R}$$

be a continuous function. Show that one can write

$$f = g + h$$

with a continuous even function g and a continuous odd function h.

The following task uses the notion of *fixed point*.

Let M be a set and let

$$f : M \longrightarrow M$$

be a function. An element $x \in M$ such that $f(x) = x$ is called a *fixed point* of the function.

Exercise 16.13. (4 points)

Let

$$f : [a, b] \longrightarrow [a, b]$$

be a continuous function from the interval $[a, b]$ into itself. Prove that f has a fixed point.