

**Einführung in die Algebra****Arbeitsblatt 1**

AUFGABE 1. (2 Punkte)

Bestimme die vier Bewegungen an einem Würfel mit den Eckpunkten  $(\pm 1, \pm 1, \pm 1)$  in Matrixschreibweise, die  $(1, 0, 0)$  auf  $(0, 0, -1)$  abbilden.

AUFGABE 2. (2 Punkte)

Wie viele (wesentlich verschiedene) Möglichkeiten gibt es, die Seiten eines Würfels von 1 bis 6 zu nummerieren, so dass die Summe gegenüberliegender Seiten stets 7 ergibt. Wie viele Möglichkeiten gibt es überhaupt?

AUFGABE 3. (3 Punkte)

Die Ecken  $(\pm 1, \pm 1, \pm 1)$  eines Würfels seien mit  $1, 2, 3, \dots, 8$  (oder ähnlich) bezeichnet (Skizze!). Beschreibe durch Wertetabellen, wie die folgenden (eigentlichen oder uneigentlichen) Würfelsymmetrien die Eckpunkte permutieren:

$$(1) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix},$$

$$(2) \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix},$$

$$(3) \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}.$$

Was passiert mit den Kantenmittelpunkten unter diesen Bewegungen?

AUFGABE 4. (2 Punkte)

Sei  $W$  der Würfel mit den Eckpunkten  $(\pm 1, \pm 1, \pm 1)$ . Fixiere eine Kantenmittelpunktachse (durch den Nullpunkt). Welche Bewegungen des Würfels lassen sich als Drehung um diese Achse beschreiben? Wie sehen diese Bewegungen in Matrixschreibweise aus, und was passiert dabei mit den Eckpunkten des Würfels?

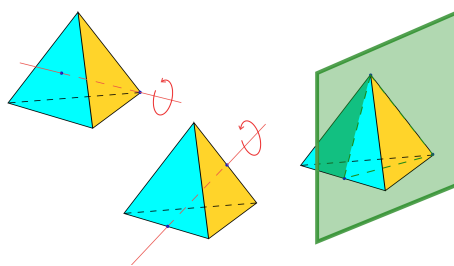
## AUFGABE 5. (2 Punkte)

Es sei  $W$  der Würfel mit den Eckpunkten  $(\pm 1, \pm 1, \pm 1)$ . Es sei  $\varphi$  eine Drittelrotation um die Raumdiagonale durch  $(1, 1, 1)$  und  $(-1, -1, -1)$ . Bestimme Ebenengleichungen für diejenigen Ebenen, auf denen je drei Eckpunkte liegen, die durch diese Drehung ineinander überführt werden.

## AUFGABE 6. (3 Punkte)

Bestimme die Koordinaten eines Tetraeders, bei dem der Nullpunkt der Mittelpunkt ist, die vier Eckpunkte des Tetraeders vom Nullpunkt den Abstand eins besitzen, der Punkt  $(0, 0, 1)$  ein Eckpunkt ist und ein weiterer Eckpunkt Koordinaten der Form  $(u, 0, v)$  besitzt.

## AUFGABE 7. (5 Punkte)



Man gebe für die in den obigen Skizzen angedeuteten Symmetrien des Tetraeders eine geeignete Matrixdarstellung.

## AUFGABE 8. (3 Punkte)

Betrachte ein Rechteck in der Ebene, das kein Quadrat sei, und dessen Mittelpunkt der Nullpunkt sei und dessen Seiten parallel zu den Koordinatenachsen liegen mögen. Bestimme die Matrizen, die die (eigentlichen und uneigentlichen) Symmetrien des Rechteckes beschreiben. Erstelle eine Verknüpfungstafel für diese Symmetriegruppe.

## Abbildungsverzeichnis

Quelle = Symmetries of the tetrahedron.svg, Autor = Benutzer  
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**Anhang 2: GFDL-Lizenz**

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