Prof. Dr. H. Brenner **Osnabrück WS 2011/2012**

Mathematics for natural sciences I

Exercise sheet 30

Warm-up-exercises

Exercise 30.1. Sketch the underlying vector fields of the differential equations

$$y' = \frac{1}{y},\ y' = ty^3 \text{ and } y' = -ty^3$$

as well as the solution curves given in Example 30.6, Example 30.7 and Example 30.8.

Exercise 30.2. Confirm by derivation that the curves we have found in Example 30.6, Example 30.7 and Example 30.8 are the solution curves of the differential equations

$$y' = \frac{1}{y},\ y' = ty^3 \text{ and } y' = -ty^3 .$$

Exercise 30.3. Interpret a location-independent differential equation as a differential equations with separable variables using the theorem for differential equations with separable variables.

Exercise 30.4. Determine all the solutions to the differential equation

$$y' = y ,$$

using the theorem for differential equations with separable variables.

Exercise 30.5. Determine all the solutions to the differential equation

$$y' = e^y ,$$

using the theorem for differential equations with separable variables.

Exercise 30.6. Determine all the solutions to the differential equation

$$y' = \frac{1}{\sin y} ,$$

using the theorem for differential equations with separable variables.

Exercise 30.7. Solve the differential equation

$$y' = ty$$

using the theorem for differential equations with separable variables.

Exercise 30.8. Consider the solutions

$$y(t) = \frac{g}{1 + \exp(-st)}$$

to the logistic differential equation we have found in Example 30.9.

a) Sketch up the graph of this function (for suitable s and g).

b) Determine the limits for $t \to \infty$ and $t \to -\infty$.

c) Study the monotony behavior of these functions.

d) For which t does the derivative of $y(t)$ have a maximum (For the function itself, this represents an *inflection point*).

e) Which symmetries have these functions?

Exercise 30.9. Find a solution for the ordinary differential equation

$$y' = \frac{t}{t^2 - 1} y^2$$

with $t > 1$ and $y < 0$.

Exercise 30.10. Determine the solutions for the differential equation ($y > 0$)

$$y' = t^2 y^3$$

using separation of variables. Where are the solutions defined?

Hand-in-exercises

Exercise 30.11. (3 points)

Prove that a differential equation of the shape

$$y' = g(t) \cdot y^2$$

with a continuous function

$$g : \mathbb{R} \longrightarrow \mathbb{R},\ t \longmapsto g(t),$$

on an interval I' has the solution

$$y(t) = -\frac{1}{G(t)},$$

where G is an antiderivative of g such that $G(I') \subseteq \mathbb{R}_+$.

Exercise 30.12. (3 points)

Determine all the solutions to the differential equation

$$y' = ty^2,\ y > 0\,,$$

using the theorem for differential equations with separable variables.

Exercise 30.13. (4 points)

Determine all the solutions to the differential equation

$$y' = t^3y^3,\ y > 0\,,$$

using the theorem for differential equations with separable variables.

Exercise 30.14. (5 points)

Determine the solutions to the differential equation

$$y' = ty + t$$

by using the approach for

a) inhomogeneous linear equations,

b) separated variables.