

Bandpass Sampling (2B)

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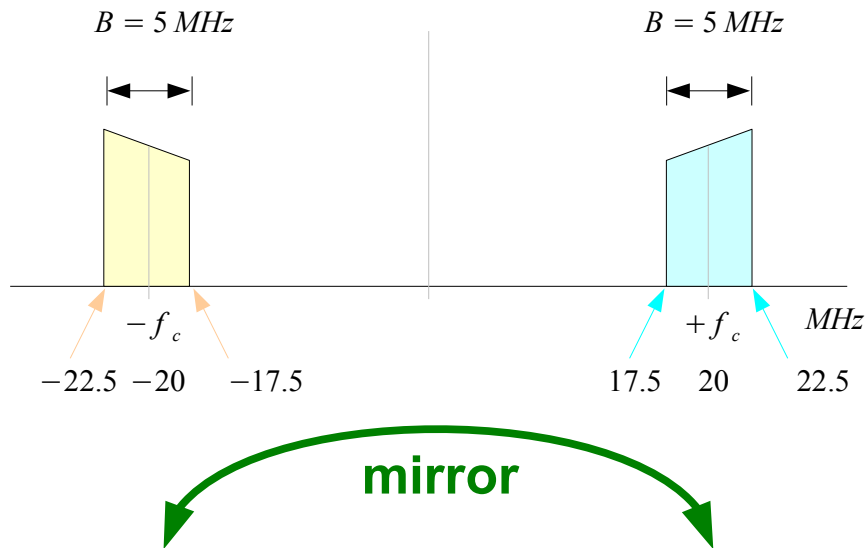
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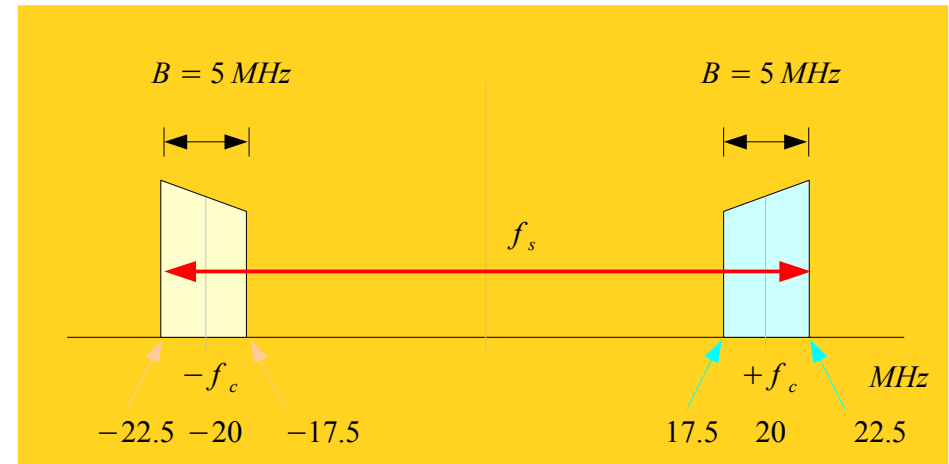
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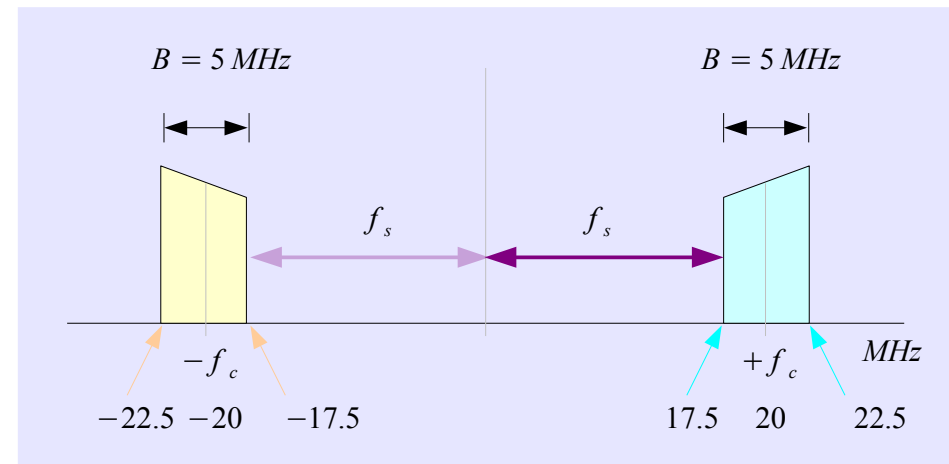
Band-limited Signal



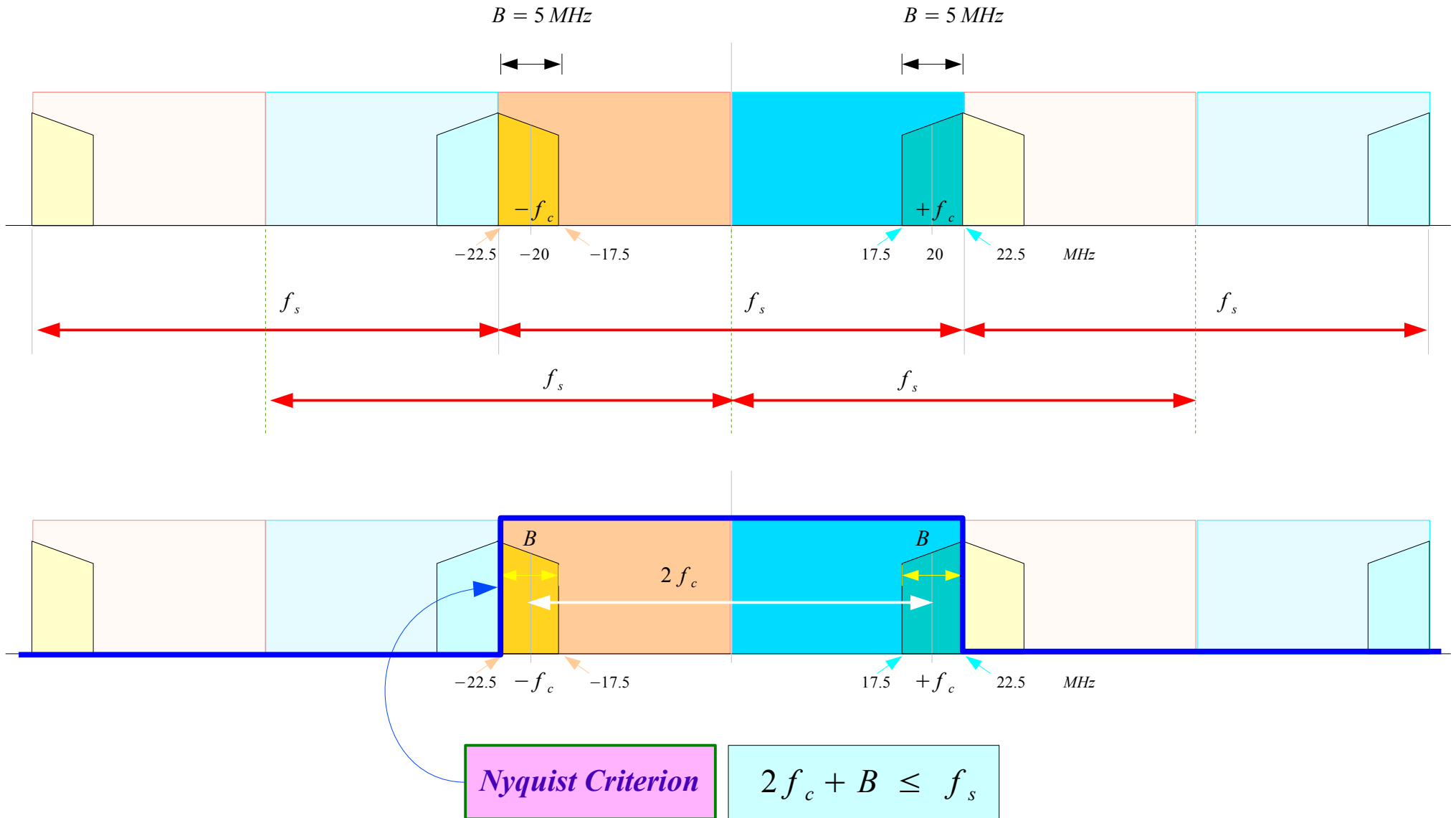
- Bandpass Sampling
- IF filtering
- Harmonic Sampling
- Sub-Nyquist Sampling



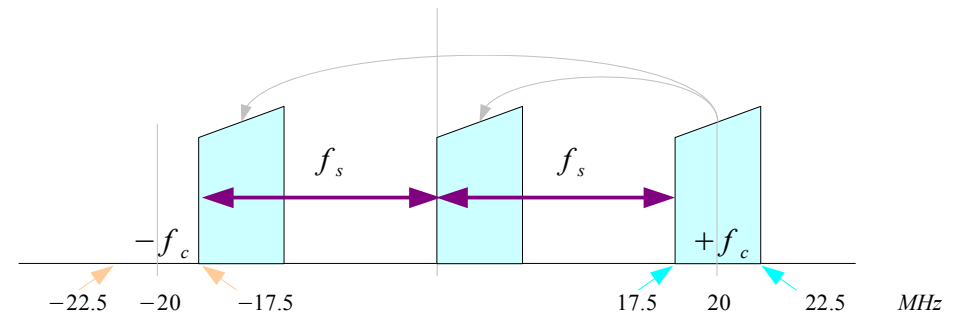
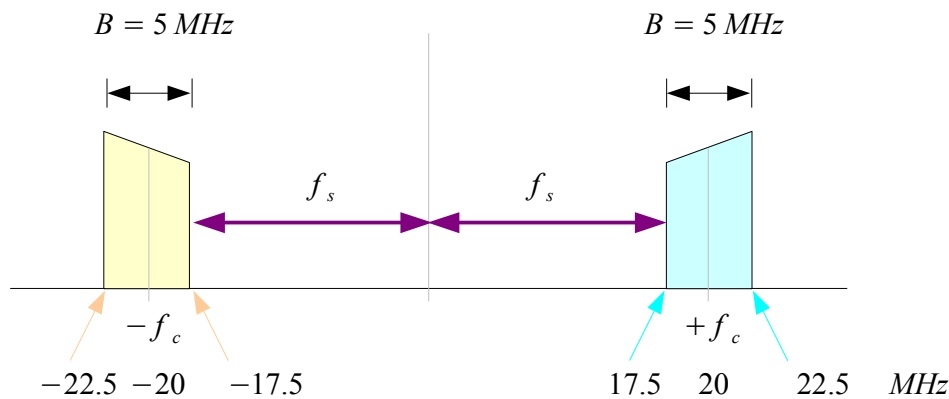
- Lowpass Sampling



Low-pass Signal Sampling

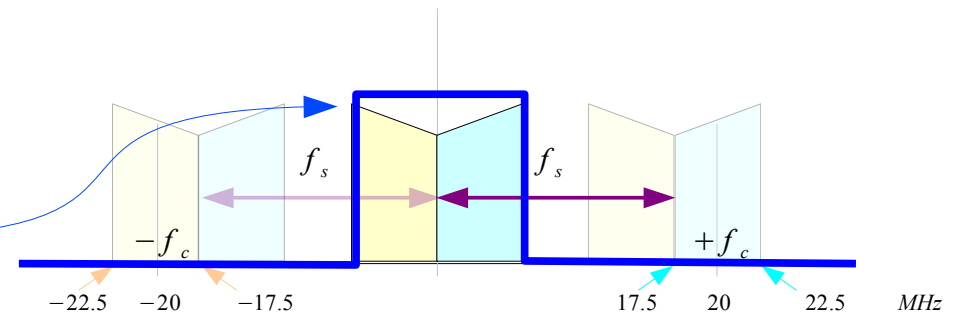
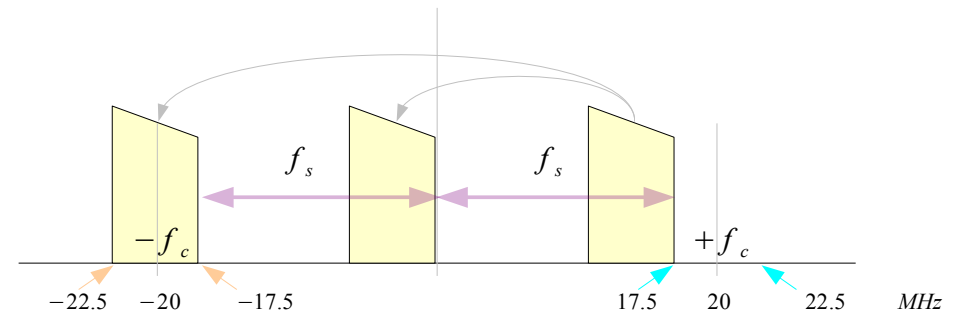


Band-pass Signal Sampling

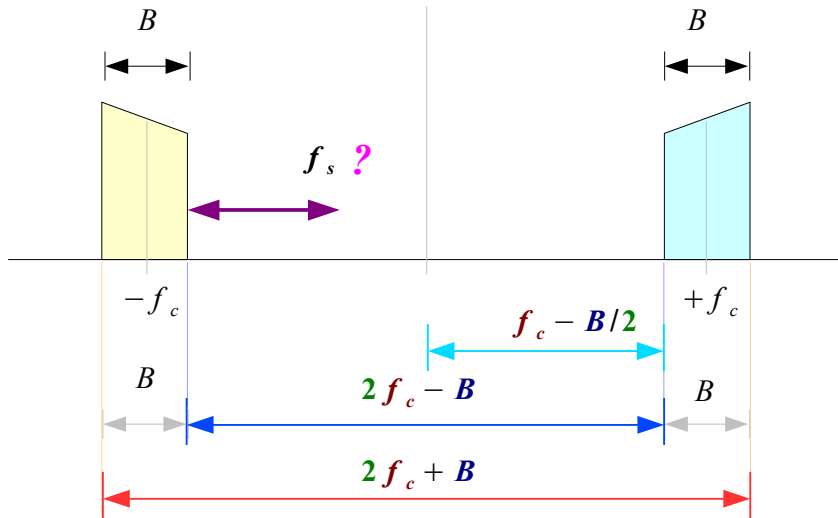


- **Bandpass Sampling**
- **IF filtering**
- **Harmonic Sampling**
- **Sub-Nyquist Sampling**

Nyquist Criterion $2B \leq f_s$



Sampling Frequency f_s (1)



- Bandpass Sampling
- IF filtering
- Harmonic Sampling
- Sub-Nyquist Sampling

Assume there are m multiples of f_s

$$2f_c - B = m \cdot f_s$$

Given an integer m

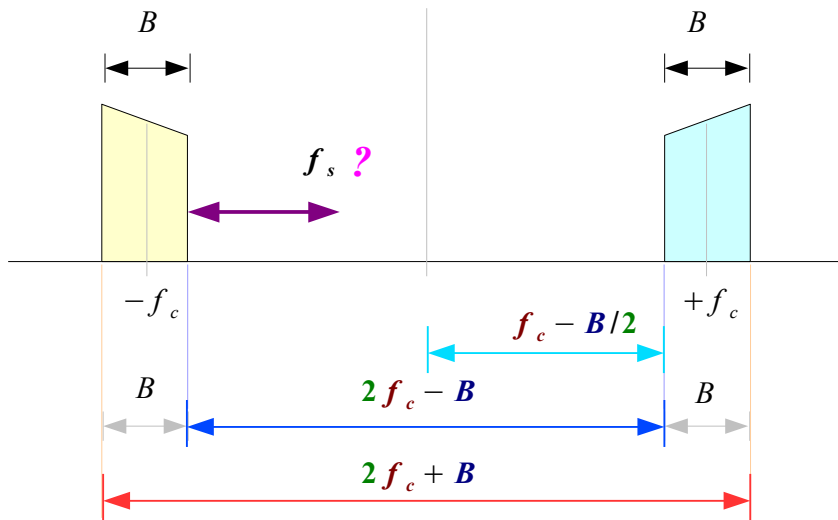
Max f_s condition

f_s can be decreased according to the following condition without introducing aliasing problems

$$2f_c + B = (m+1) \cdot f_s$$

Min f_s condition

Sampling Frequency f_s (2)



When $m = 6$

$$\begin{cases} \max f_s = \frac{2f_c - B}{6} \\ \min f_s = \frac{2f_c + B}{7} \end{cases}$$

$$\min f_s \frac{2f_c + B}{7} \leq f_s \leq \frac{2f_c - B}{6} \max f_s$$

Assume there are m multiples of f_s

$$2f_c - B = m \cdot f_s$$



Given an integer m

Max f_s condition

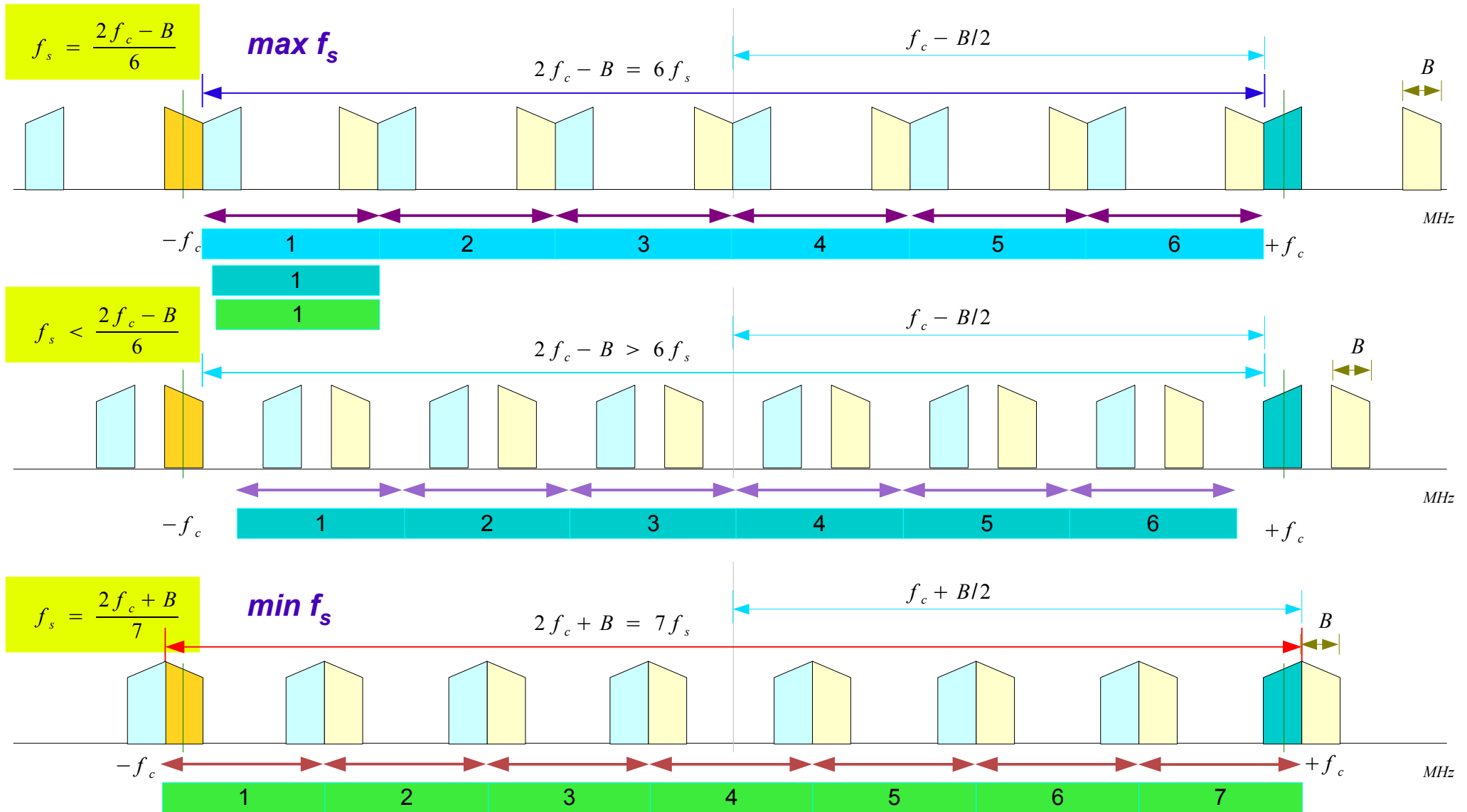
f_s can be decreased according to the following condition without introducing aliasing problems

$$2f_c + B = (m + 1) \cdot f_s$$

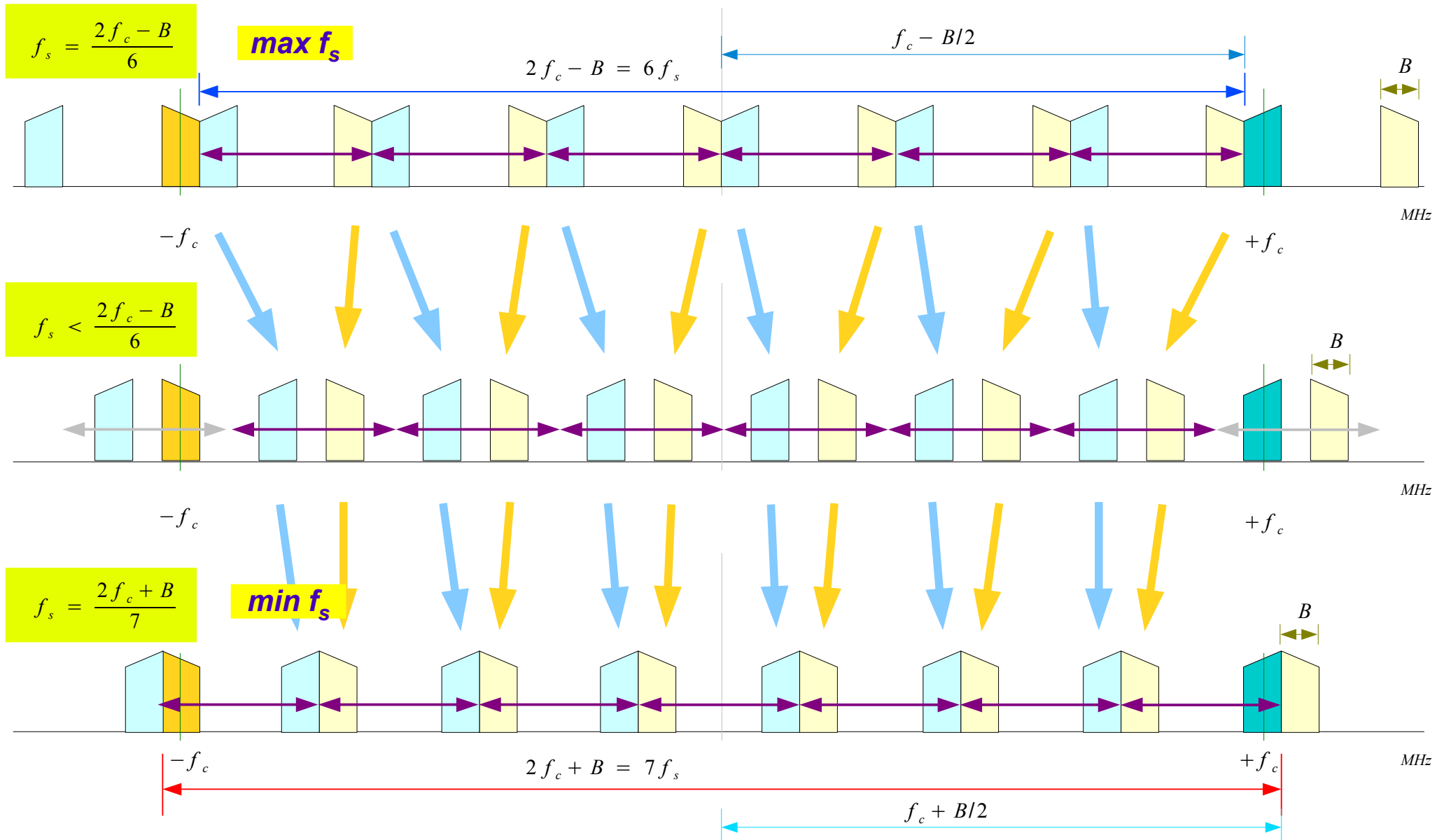


Min f_s condition

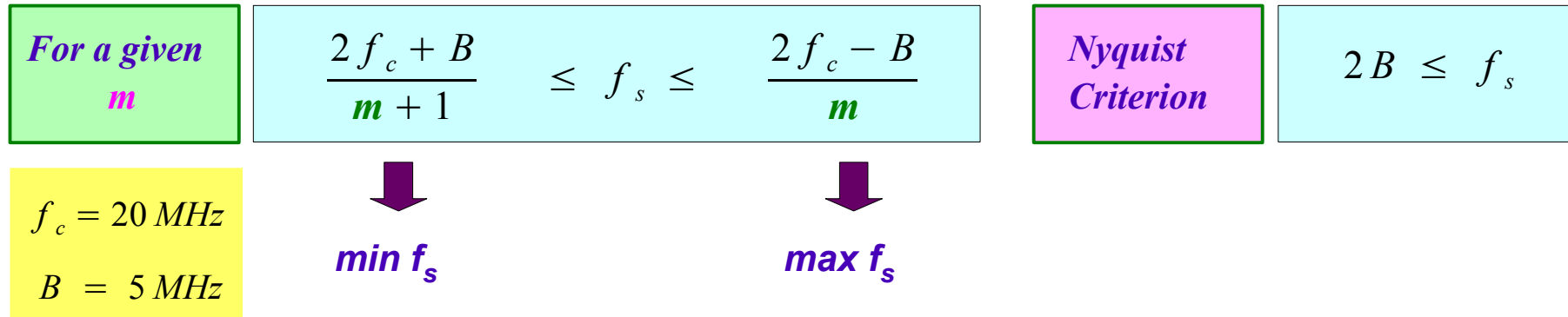
Sampling Frequency f_s (3)



Sampling Frequency f_s (4)



Range of f_s (1)



Optimum Sampling Frequency

$m = 1$	\rightarrow	$\frac{2 \cdot 20 + 5}{1 + 1} = 22.5 \leq f_s \leq \frac{2 \cdot 20 - 5}{1} = 35$	\rightarrow	$f_s = 22.5 \text{ MHz} \quad (10 \leq f_s)$
$m = 2$	\rightarrow	$\frac{2 \cdot 20 + 5}{2 + 1} = 15 \leq f_s \leq \frac{2 \cdot 20 - 5}{2} = 17.5$	\rightarrow	$f_s = 17.5 \text{ MHz} \quad (10 \leq f_s)$
$m = 3$	\rightarrow	$\frac{2 \cdot 20 + 5}{3 + 1} = 11.25 \leq f_s \leq \frac{2 \cdot 20 - 5}{3} = 11.67$	\rightarrow	$f_s = 11.25 \text{ MHz} \quad (10 \leq f_s)$
$m = 4$	\rightarrow	$\frac{2 \cdot 20 + 5}{4 + 1} = 9 \geq \frac{2 \cdot 20 - 5}{4} = 8.75$	\rightarrow	X
$m = 5$	\rightarrow	$\frac{2 \cdot 20 + 5}{5 + 1} = 7.5 \geq \frac{2 \cdot 20 - 5}{5} = 7.0$	\rightarrow	X

Range of f_s (2)

$$\frac{2f_c + B}{m+1} \leq f_s \leq \frac{2f_c - B}{m}$$

$$\frac{f_c + B/2}{B} = R$$

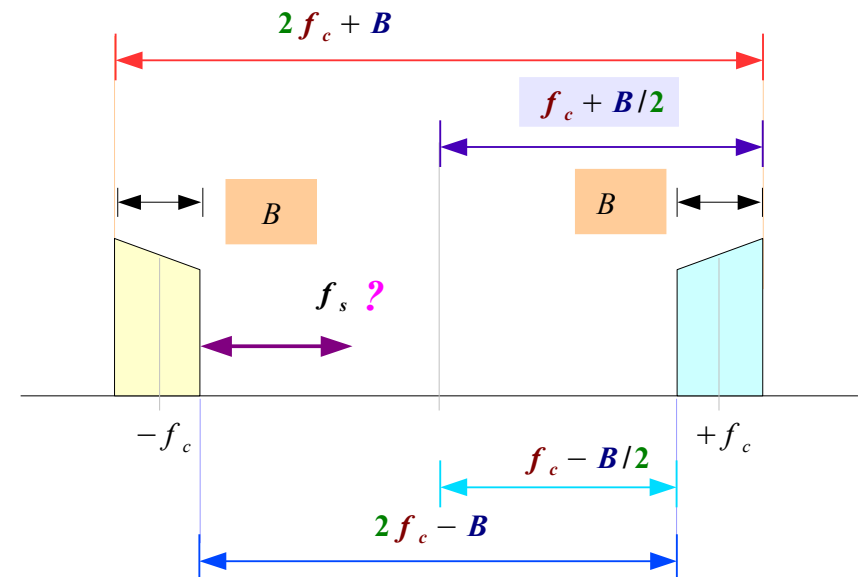
→ highest signal frequency
bandwidth B

$$\frac{2f_c + B}{(m+1)B} = \frac{f_{s,min}}{B} = g(m, R)$$

→ minimum sampling rate
bandwidth B

$$\frac{2(f_c + B/2)}{(m+1)B} = \frac{2R}{m+1} = g(m, R)$$

$m = 1$	$g(1, R) = R$	$m = 5$	$g(5, R) = \frac{1}{3}R$
$m = 2$	$g(2, R) = \frac{2}{3}R$	$m = 6$	$g(6, R) = \frac{2}{7}R$
$m = 3$	$g(3, R) = \frac{1}{2}R$	$m = 7$	$g(7, R) = \frac{1}{4}R$
$m = 4$	$g(4, R) = \frac{2}{5}R$	$m = 8$	$g(8, R) = \frac{2}{9}R$



Range of f_s (3)

$$\frac{2f_c + B}{m + 1} \leq f_s \leq \frac{2f_c - B}{m}$$

$$\frac{f_c + B/2}{B} = R \rightarrow \begin{array}{l} \text{highest signal frequency} \\ \text{bandwidth } B \end{array}$$

$$f_H = f_c + B/2$$

$$R = f_H / B$$

$$\frac{2f_c + B}{(m + 1)B} = g(m, R) \rightarrow \begin{array}{l} \text{minimum sampling rate} \\ \text{bandwidth } B \end{array}$$

$$f_{s, \min} = \frac{2f_c + B}{m + 1} = \frac{2f_H}{k}$$

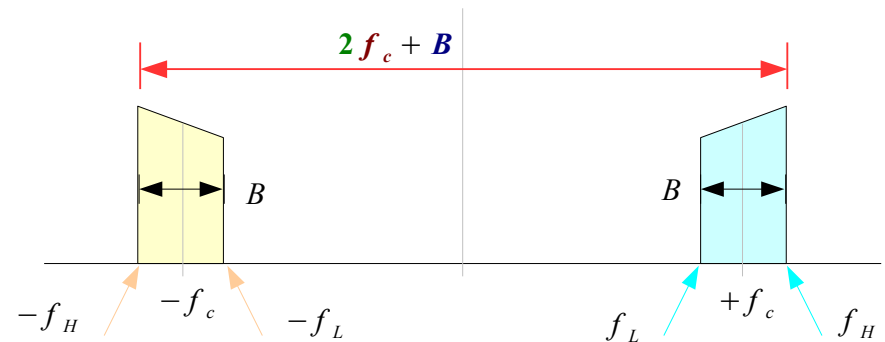
$$\frac{2f_c + B}{(m + 1)B} = g(m, R) = \frac{2R}{m + 1} = \frac{2R}{k} \quad \text{m + 1 = k}$$

$$g(m, R) = \frac{2f_H}{kB} = \frac{2R}{k}$$

k represents how many f_s are in $2f_c + B$ in

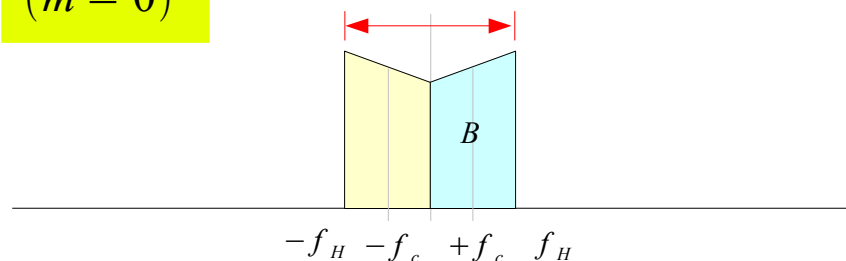
Min f_s condition

$$2f_c + B = (m + 1) \cdot f_s = k \cdot f_s$$



Example $k=1$

$k = 1$
 $(m = 0)$

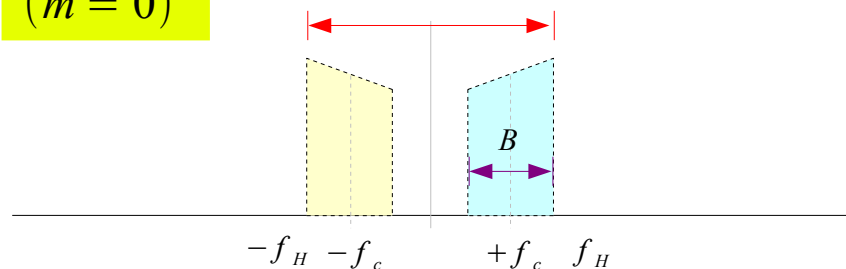


$$f_H = f_c + B/2 = 1B$$

$$R = f_H / B = 1$$

$$\frac{f_{s,min}}{B} = \frac{2f_H}{kB} = \frac{2f_H}{(m+1)B} = 2$$

$k = 1$
 $(m = 0)$

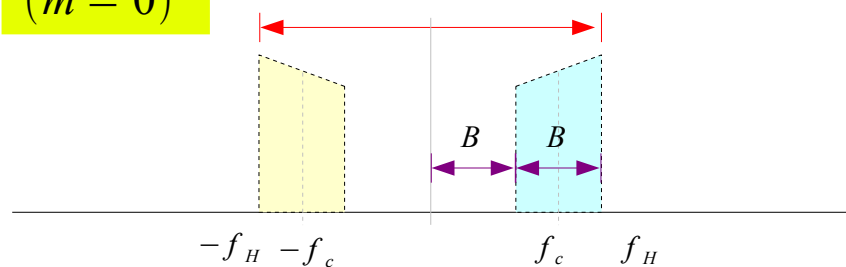


$$f_H = f_c + B/2 = 1.5B$$

$$R = f_H / B = 1.5$$

$$\frac{f_{s,min}}{B} = \frac{2f_H}{kB} = \frac{2f_H}{(m+1)B} = 3$$

$k = 1$
 $(m = 0)$



$$f_H = f_c + B/2 = 2B$$

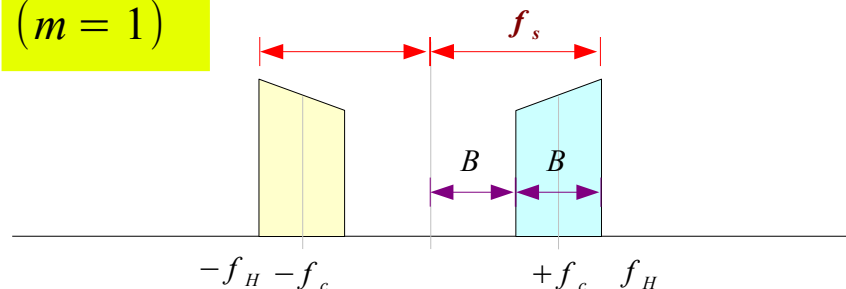
$$R = f_H / B = 2$$

$$\frac{f_{s,min}}{B} = \frac{2f_H}{kB} = \frac{2f_H}{(m+1)B} = 4$$



Example $k=2$

$k = 2$
 $(m = 1)$



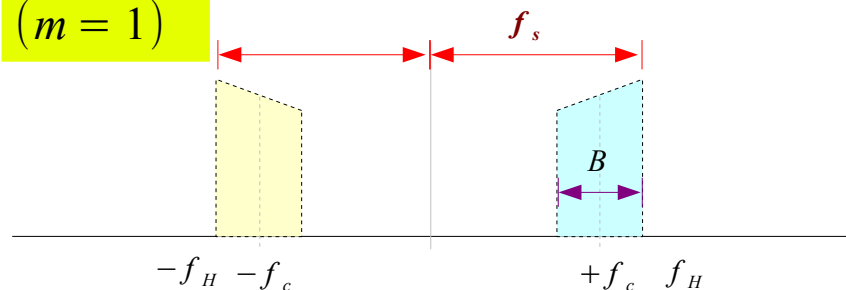
$$f_H = f_c + B/2 = 2B$$

$$R = f_H / B = 2$$

$$\frac{f_{s,min}}{B} = \frac{2f_H}{kB} = \frac{2f_H}{(m+1)B} = 2$$

$$\left(\frac{f_H}{B}, \frac{f_s}{B}\right) = (3, 3)$$

$k = 2$
 $(m = 1)$



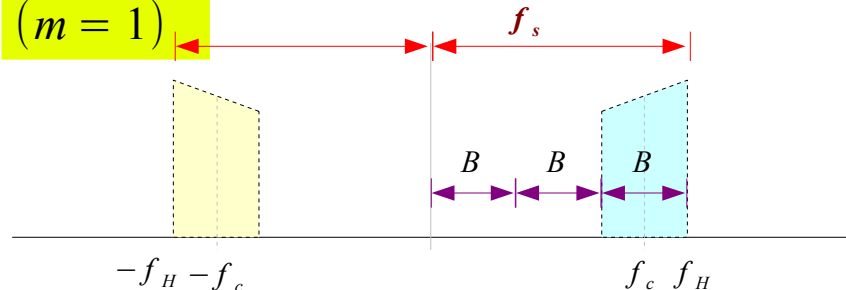
$$f_H = f_c + B/2 = 2.5B$$

$$R = f_H / B = 2.5$$

$$\frac{f_{s,min}}{B} = \frac{2f_H}{kB} = \frac{2f_H}{(m+1)B} = 2.5$$

$$\left(\frac{f_H}{B}, \frac{f_s}{B}\right) = (2, 2)$$

$k = 2$
 $(m = 1)$

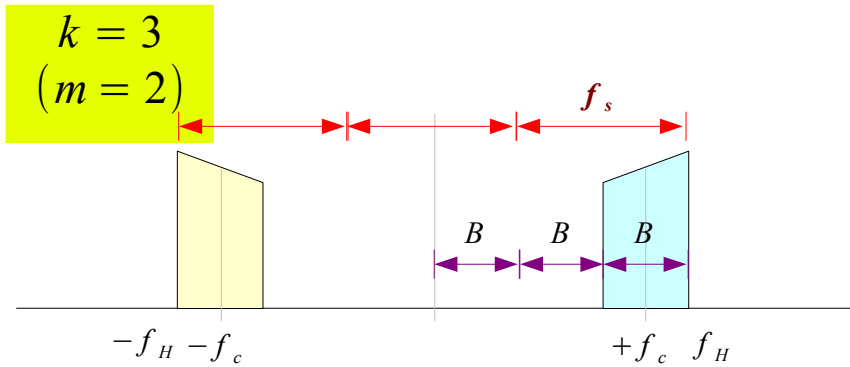


$$f_H = f_c + B/2 = 3B$$

$$R = f_H / B = 3$$

$$\frac{f_{s,min}}{B} = \frac{2f_H}{kB} = \frac{2f_H}{(m+1)B} = 3$$

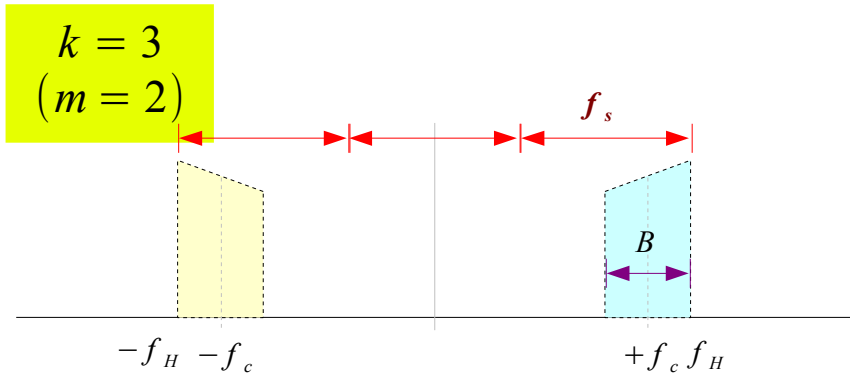
Example $k=3$



$$f_H = f_c + B/2 = 3B$$

$$R = f_H / B = 3$$

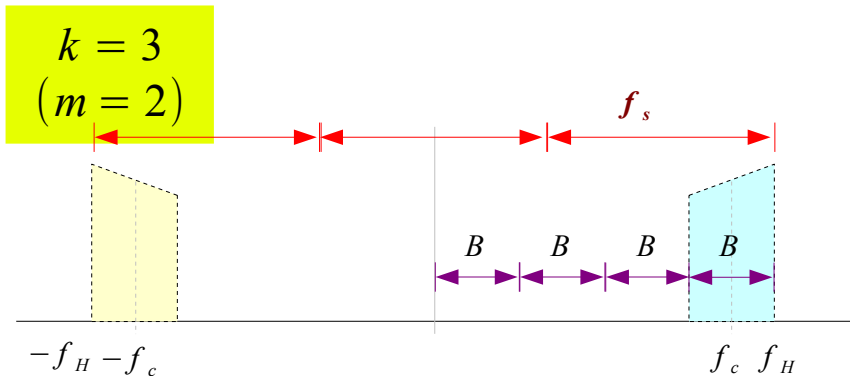
$$\frac{f_{s,min}}{B} = \frac{2f_H}{kB} = \frac{2f_H}{(m+1)B} = 2$$



$$f_H = f_c + B/2 = 3.5B$$

$$R = f_H / B = 3.5$$

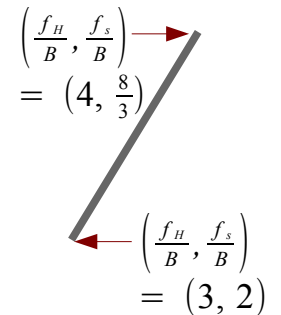
$$\frac{f_{s,min}}{B} = \frac{2f_H}{kB} = \frac{2f_H}{(m+1)B} = \frac{7}{3}$$

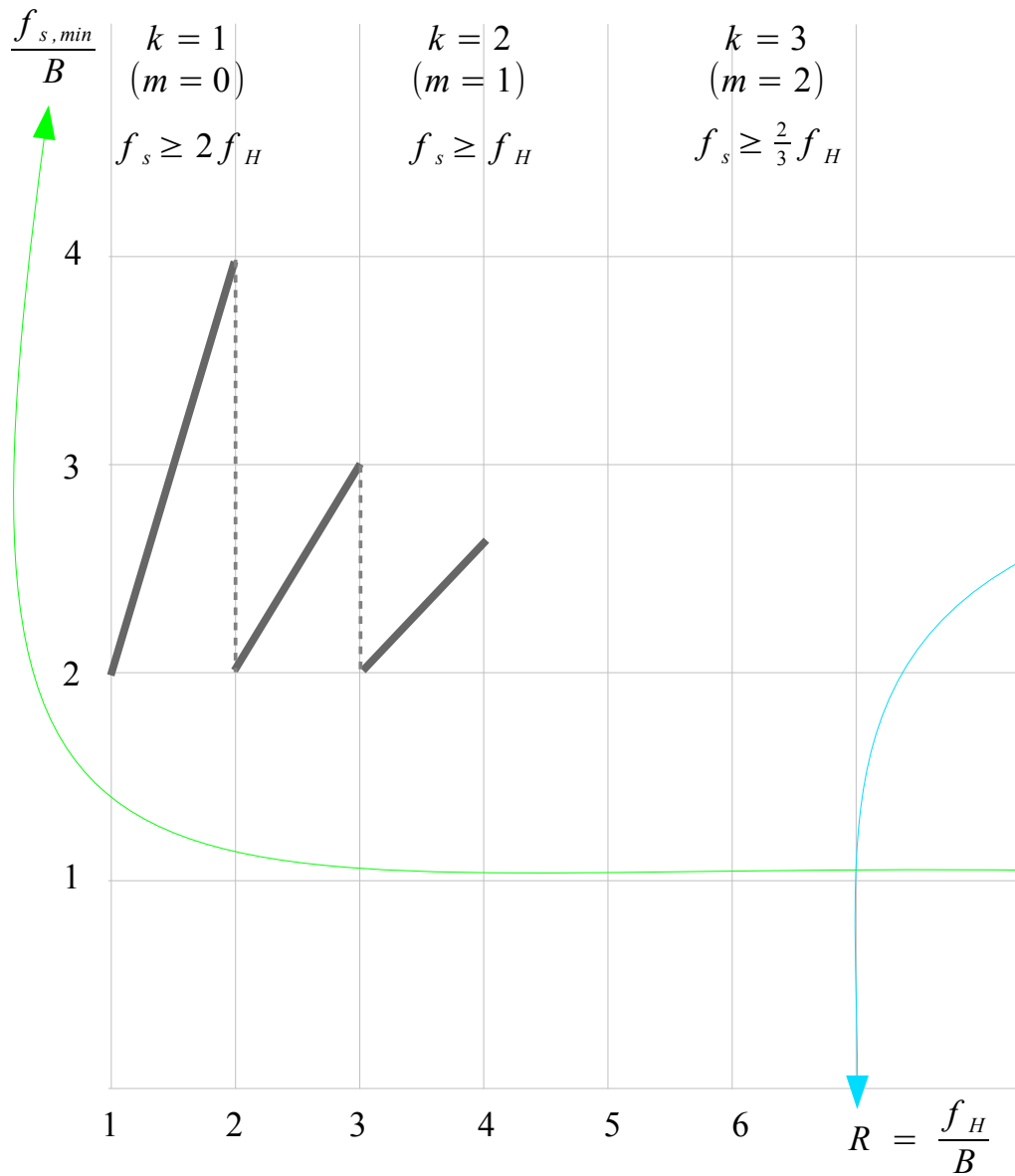


$$f_H = f_c + B/2 = 4B$$

$$R = f_H / B = 4$$

$$\frac{f_{s,min}}{B} = \frac{2f_H}{kB} = \frac{2f_H}{(m+1)B} = \frac{8}{3}$$





$$\frac{2f_c + B}{m + 1} \leq f_s \leq \frac{2f_c - B}{m}$$

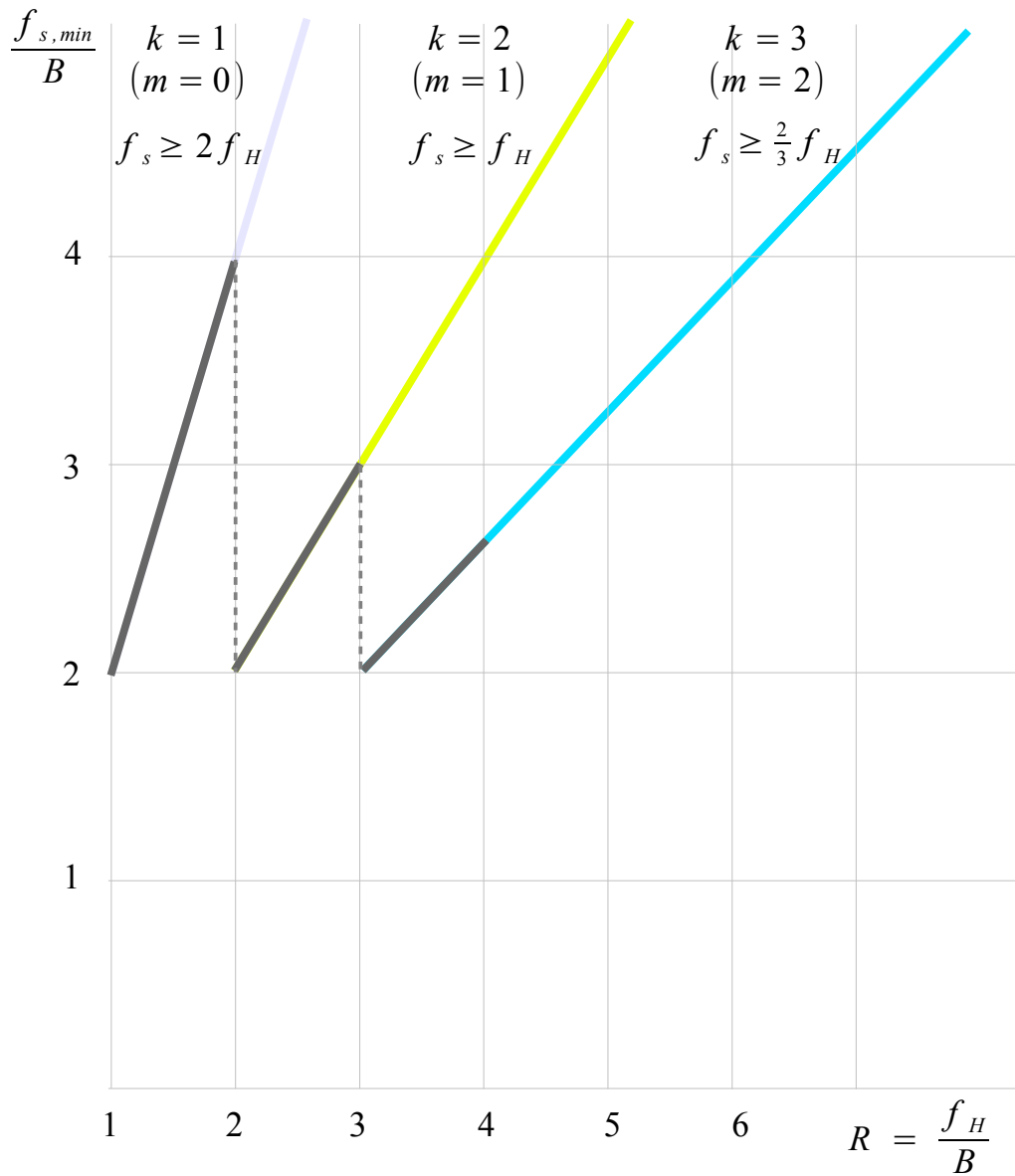
$$\frac{2f_c + B}{k} \leq f_s \leq \frac{2f_c - B}{k - 1}$$

$$\frac{f_c + B/2}{B} = R$$

highest signal frequency
bandwidth B

$$\frac{2f_c + B}{(m + 1)B} = \frac{f_{s,min}}{B} = g(m, R)$$

minimum sampling rate
bandwidth B



$$\frac{2f_c + B}{m + 1} \leq f_s \leq \frac{2f_c - B}{m}$$

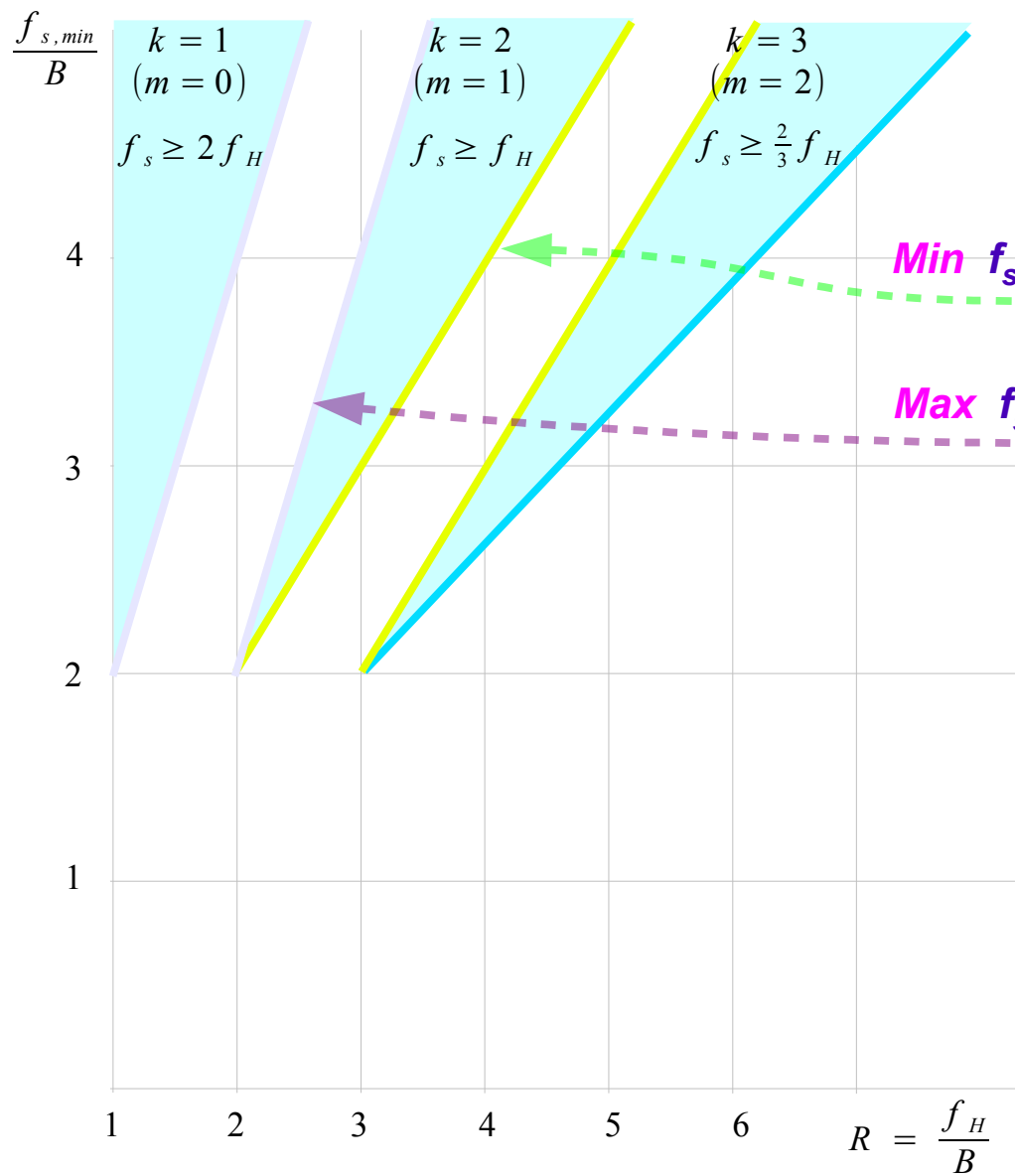
$$\frac{2f_c + B}{k} \leq f_s \leq \frac{2f_c - B}{k - 1}$$

$$\frac{f_c + B/2}{B} = R$$

highest signal frequency
bandwidth B

$$\frac{2f_c + B}{(m + 1)B} = \frac{f_{s,min}}{B} = g(m, R)$$

minimum sampling rate
bandwidth B



$\frac{2f_c + B}{m+1}$	$\leq f_s \leq$	$\frac{2f_c - B}{m}$
$\frac{2f_c + B}{k}$	$\leq f_s \leq$	$\frac{2f_c - B}{k-1}$

References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] A "graphical interpretation" of the DFT and FFT, by Steve Mann
- [4] R. G. Lyons, Understanding Digital Signal Processing, 1997