DLTI z-Transform

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Finding ZIR & ZSR Using Laplace Transform (1)

$$y[n] = 2x[n] - x[n-1] + 3x[n-2] + \frac{9}{20}y[n-1] - \frac{1}{20}y[n-2]$$

initial condition $y[1]=3, \quad y[-2]=2$

input $x[n] = u[n]$

$y[n]$	$\Leftrightarrow$	$Y[z]$
$y[n-1]$	$\Leftrightarrow$	$y[-1] + z^{-1}Y[z]$ $= z^{-1}Y[z]+3$
$y[n-2]$	$\Leftrightarrow$	$y[-2]+z^{-1}y[-1]+z^{-2}Y[z]$ $= z^{-2}Y[z]+3z^{-1}+2$

$x[n]$	$\Leftrightarrow$	$X[z] = \frac{z}{z-1}$
$x[n-1]$	$\Leftrightarrow$	$\frac{1}{z-1}$
$x[n-2]$	$\Leftrightarrow$	$\frac{1}{z(z-1)}$

$$Y[z] = 2\frac{z}{z-1} - \frac{1}{z-1} + \frac{3}{z(z-1)} + \frac{9}{20}(z^{-1}Y[z]+3) - \frac{1}{20}(z^{-2}Y[z]+3z^{-1}+2)$$

initial condition terms

Finding ZIR & ZSR Using Laplace Transform (2)

$$y[n] = 2x[n] - x[n-1] + 3x[n-2] + \frac{9}{20}y[n-1] - \frac{1}{20}y[n-2]$$

initial condition $y[1]=3, \quad y[-2]=2$

input $x[n] = u[n]$

$$Y[z] = 2\frac{z}{z-1} - \frac{1}{z-1} + \frac{3}{z(z-1)} + \frac{9}{20}\left(z^{-1}Y[z]+3\right) - \frac{1}{20}\left(z^{-2}Y[z]+3z^{-1}+2\right)$$

$$\frac{Y[z]}{z} = \frac{2z^2-z+3}{(z-1)(z-\frac{1}{3})(z-\frac{1}{4})} + \frac{\left(\frac{5}{4}z-\frac{3}{20}\right)}{(z-\frac{1}{5})(z-\frac{1}{4})}$$

init cond terms

input terms

$$\frac{Y[z]}{z} = \frac{\frac{20}{3}}{(z-1)} + \frac{72}{(z-\frac{1}{5})} + \frac{\frac{230}{3}}{(z-\frac{1}{4})} - \frac{2}{(z-\frac{1}{5})} + \frac{\frac{13}{4}}{(z-\frac{1}{4})}$$

$$y[n] = \frac{30}{3} + 72\left(\frac{1}{5}\right)^n - \frac{230}{3}\left(\frac{1}{4}\right)^n - 2\left(\frac{1}{5}\right)^n + \frac{13}{4}\left(\frac{1}{4}\right)^n \qquad = \frac{20}{3} + 70\left(\frac{1}{5}\right)^n - \frac{881}{12}\left(\frac{1}{4}\right)^n$$

Zero State Resp *Zero Input Resp*

ROC

$$X(z) = \frac{1}{1-0.5z^{-1}}$$

$$X(z) = \frac{(0.5)^{-1}z}{0.5z^{-1}-1}$$

$$X(z) = -\frac{(0.5)^{-1}z}{1-0.5z^{-1}}$$

Inifinite Geometric Series

$$\frac{a}{1-r}$$

$a=1$ *Initial Term*

$r=0.5z^{-1}$ *Common Ratio*

$|0.5z^{-1}|<1$

$1 + 0.5,\ 0.5^{2,}\ 0.5^{3}$

$$\frac{a}{1-r}$$

$a=-(0.5)^{-1}z$

$r=(0.5)^{-1}z$

$|(0.5)^{-1}z|<1$

$,\ 0.5,\ 0.5^{2,}\ 0.5^{3}$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] B.P. Lathi, Linear Systems and Signals (2nd Ed)
[4] D. Sundararajan, A Practical Approach to Signals and Systems