

DLTI z-Transform

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Finding ZIR & ZSR Using Laplace Transform (1)

$$y[n] = 2x[n] - x[n-1] + 3x[n-2] + \frac{9}{20}y[n-1] - \frac{1}{20}y[n-2]$$

initial condition $y[1]=3, y[-2]=2$

input $x[n] = u[n]$

$y[n]$	\leftrightarrow	$Y[z]$
$y[n-1]$	\leftrightarrow	$y[-1] + z^{-1}Y[z]$ $= z^{-1}Y[z] + 3$
$y[n-2]$	\leftrightarrow	$y[-2] + z^{-1}y[-1] + z^{-2}Y[z]$ $= z^{-2}Y[z] + 3z^{-1} + 2$

$x[n]$	\leftrightarrow	$X[z] = \frac{z}{z-1}$
$x[n-1]$	\leftrightarrow	$\frac{1}{z-1}$
$x[n-2]$	\leftrightarrow	$\frac{1}{z(z-1)}$

$$Y[z] = 2\frac{z}{z-1} - \frac{1}{z-1} + \frac{3}{z(z-1)} + \frac{9}{20}(z^{-1}Y[z] + 3) - \frac{1}{20}(z^{-2}Y[z] + 3z^{-1} + 2)$$

initial condition terms

Finding ZIR & ZSR Using Laplace Transform (2)

$$y[n] = 2x[n] - x[n-1] + 3x[n-2] + \frac{9}{20}y[n-1] - \frac{1}{20}y[n-2]$$

initial condition $y[1]=3, y[-2]=2$

input $x[n] = u[n]$

$$Y[z] = 2\frac{z}{z-1} - \frac{1}{z-1} + \frac{3}{z(z-1)} + \frac{9}{20}(z^{-1}Y[z]+3) - \frac{1}{20}(z^{-2}Y[z]+3z^{-1}+2)$$

$$\frac{Y[z]}{z} = \frac{2z^2 - z + 3}{(z-1)(z-\frac{1}{3})(z-\frac{1}{4})} + \frac{(\frac{5}{4}z - \frac{3}{20})}{(z-\frac{1}{5})(z-\frac{1}{4})}$$

init cond terms

input terms

$$\frac{Y[z]}{z} = \frac{\frac{20}{3}}{(z-1)} + \frac{72}{(z-\frac{1}{5})} + \frac{\frac{230}{3}}{(z-\frac{1}{4})} - \frac{2}{(z-\frac{1}{5})} + \frac{\frac{13}{4}}{(z-\frac{1}{4})}$$

$$y[n] = \frac{30}{3} + 72\left(\frac{1}{5}\right)^n - \frac{230}{3}\left(\frac{1}{4}\right)^n - 2\left(\frac{1}{5}\right)^n + \frac{13}{4}\left(\frac{1}{4}\right)^n = \frac{20}{3} + 70\left(\frac{1}{5}\right)^n - \frac{881}{12}\left(\frac{1}{4}\right)^n$$

Zero State Resp

Zero Input Resp

$$X(z) = \frac{1}{1-0.5z^{-1}}$$

$$X(z) = \frac{(0.5)^{-1}z}{0.5z^{-1}-1}$$

$$X(z) = -\frac{(0.5)^{-1}z}{1-0.5z^{-1}}$$

$$\frac{a}{1-r}$$

Inifinite Geometric Series

$$\frac{a}{1-r}$$

$$a=1$$

Initial Term

$$a=-(0.5)^{-1}z$$

$$r=0.5z^{-1}$$

Common Ratio

$$r=(0.5)^{-1}z$$

$$|0.5z^{-1}| < 1$$

$$|(0.5)^{-1}z| < 1$$

$$1 + 0.5, 0.5^2, 0.5^3$$

$$, 0.5, 0.5^2, 0.5^3$$

References

- [1] <http://en.wikipedia.org/>
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- [3] B.P. Lathi, Linear Systems and Signals (2nd Ed)
- [4] D. Sundararajan, A Practical Approach to Signals and Systems