Pf: Lagr. interp. error

$G(0) = 0 = G(1)$

($t=0$, $t=1$)

Actually, $]0,1[$ $\equiv (0,1)$ open interv.

Rolle's thm $\Rightarrow \exists \xi_1 \in [0,1]$ st $G^{(1)}(\xi_1) = 0$

We have $G^{(1)}(0) = 0$ Why?

$G^{(1)}(t) = e^{(1)}(t) - 5t^4 e^{(1)}(1)$ (Const)

$e(t) := \alpha(t) - \alpha_2(t)$

$\alpha(t)$: $\int_{-t}^{+t} \cdots$; $\alpha_2(t)$: Simpson

$\alpha(t) = \int_{-t}^{k} \cdots + \int_{k}^{+t} \cdots \quad k \in [-t, t]$ (HW)

$\alpha^{(1)}(t) = F(-t) + F(t)$

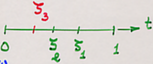
$\alpha_2^{(1)}(t) = \frac{1}{3}\left[F(-t) + 4F(0) + F(t)\right] + \frac{t}{3}\left[F^{(1)}(-t) + F^{(1)}(t)\right]$

$$e^{(1)}(t) = \alpha^{(1)}(t) - \alpha_2^{(1)}(t)$$

$$e^{(1)}(0) = 2F(0) - [2F(0) + 0] = 0$$

$$\Rightarrow G^{(1)}(0) = 0 .$$

Recall $G^{(1)}(\xi_1) = 0$ $]0, \xi_1[$



Rolle's thm $\Rightarrow$

$\exists\, \xi_2 \in [0, \xi_1]$

$G^{(1)}(0) = 0$ $G^{(1)}(\xi_1) = 0$ st $G^{(2)}(\xi_2) = 0$

Again, $G^{(2)}(0) = 0$ HW $\in\,]0, \xi_2[$

Rolle's thm $\Rightarrow \exists\, \xi_3 \in [0, \xi_2]$ st

$$G^{(3)}(t) = e^{(3)}(t) - 60 t^2 e(1) \qquad G^{(3)}(\xi_3) = 0$$

$$e^{(3)}(t) \equiv -\frac{t}{3}\left[F^{(3)}(t) - F^{(3)}(-t)\right]$$

↑ HW

use DMVT

$$G^{(3)}(\xi_3) = -\frac{\xi_3}{3}\left[F^{(3)}(\xi_3) - F^{(3)}(-\xi_3)\right] - 60(\xi_3)^2 e(1) = 0$$

$$G^{(3)}(\xi_3) = -\frac{\xi_3}{3}\left[2\xi_3 F^{(4)}(\xi_4)\right] - 60(\xi_3)^2 e(1)$$

$\xi_3 - (-\xi_3)$

$= 0$

Since $\xi_3 \neq 0$ why? since $\xi_3 \in]0, \xi_2[$

Solve for $e(1) = -\frac{1}{90} F^{(4)}(\xi_4)$

$$= -\frac{(b-a)^4}{1440} f^{(4)}(\xi)$$

HW: Also rel. betw. ξ and ξ_4.

Goal, p. 14-2.

///

E_2 : (2) p. 14-2

Thm: p. 14-1

(end proof)