

Spectra (1A)

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Periodogram

Periodogram

A truncated sample function

$$\begin{aligned}x_T(t) &= x(t) \quad -T/2 < t < +T/2 \\ &= 0 \quad \textit{otherwise}\end{aligned}$$

Fourier Transform

$$\begin{aligned}X_T(\omega) &= \int_{-\infty}^{+\infty} x_T(t) e^{-j\omega t} dt & x_T(t) &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} X_T(\omega) e^{+j\omega t} d\omega \\ X_T(f) &= \int_{-\infty}^{+\infty} x_T(t) e^{-j2\pi f t} dt & x_T(t) &= \int_{-\infty}^{+\infty} X_T(f) e^{+j2\pi f t} df\end{aligned}$$

Parseval's Theorem

$$\int_{-\infty}^{+\infty} |x_T(t)|^2 dt = \int_{-\infty}^{+\infty} |X_T(f)|^2 df$$

Average Power

$$\frac{1}{T} \int_{-\infty}^{+\infty} x_T^2(t) dt = \int_{-\infty}^{+\infty} \frac{|X_T(f)|^2}{T} df$$

Average Power

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Average Power

$$\frac{1}{T} \int_{-\infty}^{+\infty} x_T^2(t) dt = \int_{-\infty}^{+\infty} \frac{|X_T(f)|^2}{T} df$$

Raw Power Spectral Density

Average Power

$$\frac{1}{T} \int_{-\infty}^{+\infty} x_T^2(t) dt = \int_{-\infty}^{+\infty} \frac{|X_T(f)|^2}{T} df \Rightarrow \int_{-\infty}^{+\infty} \hat{S}_{xx}(f) df$$

Raw Power Spectral Density

$$\frac{|X_T(f)|^2}{T} = \hat{S}_{xx}(f)$$

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\infty}^{+\infty} x_T^2(t) dt = \lim_{T \rightarrow \infty} \int_{-\infty}^{+\infty} \frac{|X_T(f)|^2}{T} df$$

$$E \left[\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\infty}^{+\infty} x_T^2(t) dt \right] = E \left[\lim_{T \rightarrow \infty} \int_{-\infty}^{+\infty} \frac{|X_T(f)|^2}{T} df \right]$$

Power Spectral Density

Average Power

$$\frac{1}{T} \int_{-\infty}^{+\infty} x_T^2(t) dt = \int_{-\infty}^{+\infty} \frac{|X_T(f)|^2}{T} df \Rightarrow \int_{-\infty}^{+\infty} \hat{S}_{xx}(f) df$$

Raw Power Spectral Density

$$\frac{|X_T(f)|^2}{T} = \hat{S}_{xx}(f)$$

$$\begin{aligned} E \left[\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\infty}^{+\infty} x_T^2(t) dt \right] &= E \left[\lim_{T \rightarrow \infty} \int_{-\infty}^{+\infty} \frac{|X_T(f)|^2}{T} df \right] \\ \text{Var}(x(t)) = \sigma_x^2 &= \lim_{T \rightarrow \infty} \int_{-\infty}^{+\infty} \frac{E[|X_T(f)|^2]}{T} df \Rightarrow \int_{-\infty}^{+\infty} \hat{S}_{xx}(f) df \end{aligned}$$

Power Spectral Density

$$\lim_{T \rightarrow \infty} \frac{E[|X_T(f)|^2]}{T} = S_{xx}(f)$$

References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] K.H. Shin, J.K. Hammond, Fundamentals of Signal Processing for Sound and Vibration Engineers, Wiley, 2008