Spectra (1A)

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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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Periodogram

Periodogram

A truncated sample function

$$x_T(t) = x(t) \qquad -T/2 < t < +T/2$$
$$= 0 \qquad otherwise$$

Fourier Transform

$$X_T(\omega) = \int_{-\infty}^{+\infty} x_T(t)\, e^{-j\omega t}\, dt \qquad x_T(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X_T(\omega)\, e^{+j\omega t}\, d\omega$$

$$X_T(f) = \int_{-\infty}^{+\infty} x_T(t)\, e^{-j2\pi f t}\, dt \qquad x_T(t) = \int_{-\infty}^{+\infty} X_T(f)\, e^{+j2\pi ft}\, df$$

Parseval's Therorem

$$\int_{-\infty}^{+\infty} |x_T(t)|^2\, dt = \int_{-\infty}^{+\infty} |X_T(f)|^2\, df$$

Average Power

$$\frac{1}{T} \int_{-\infty}^{+\infty} x_T^2(t)\, dt = \int_{-\infty}^{+\infty} \frac{|X_T(f)|^2}{T}\, dt$$

Average Power

A truncated sample function

$$x_T(t) = x(t) \qquad -T/2 < t < +T/2$$

$$= 0 \qquad otherwise$$

Fourier Transform

$$X_T(\omega) = \int_{-\infty}^{+\infty} x_T(t)\, e^{-j\omega t}\, dt \qquad\qquad x_T(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X_T(\omega)\, e^{+j\omega t}\, d\omega$$

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Parseval's Therorem

$$\int_{-\infty}^{+\infty} |x_T(t)|^2\, dt = \int_{-\infty}^{+\infty} |X_T(f)|^2\, df$$

Average Power

$$\frac{1}{T} \int_{-\infty}^{+\infty} x_T^2(t)\, dt \qquad = \int_{-\infty}^{+\infty} \frac{|X_T(f)|^2}{T}\, dt$$

Raw Power Spectral Density

Average Power

$$\frac{1}{T}\int_{-\infty}^{+\infty} x_T^2(t)\,dt \quad = \int_{-\infty}^{+\infty} \frac{|X_T(f)|^2}{T}\,df \quad \Rightarrow \int_{-\infty}^{+\infty} \hat{S}_{xx}(f)\,df$$

Raw Power Spectral Density

$$\frac{|X_T(f)|^2}{T} = \hat{S}_{xx}(f)$$

$$\lim_{T\to\infty} \frac{1}{T}\int_{-\infty}^{+\infty} x_T^2(t)\,dt \quad = \lim_{T\to\infty} \int_{-\infty}^{+\infty} \frac{|X_T(f)|^2}{T}\,df$$

$$E\left[\lim_{T\to\infty} \frac{1}{T}\int_{-\infty}^{+\infty} x_T^2(t)\,dt\right] \quad = E\left[\lim_{T\to\infty} \int_{-\infty}^{+\infty} \frac{|X_T(f)|^2}{T}\,df\right]$$

Power Spectral Density

Average Power

$$\frac{1}{T}\int_{-\infty}^{+\infty} x_T^2(t)\,dt \quad = \int_{-\infty}^{+\infty} \frac{|X_T(f)|^2}{T}\,df \quad \Rightarrow \int_{-\infty}^{+\infty} \hat{S}_{xx}(f)\,df$$

Raw Power Spectral Density

$$\frac{|X_T(f)|^2}{T} = \hat{S}_{xx}(f)$$

$$E\left[\lim_{T\to\infty} \frac{1}{T}\int_{-\infty}^{+\infty} x_T^2(t)\,dt\right] \quad = E\left[\lim_{T\to\infty} \int_{-\infty}^{+\infty} \frac{|X_T(f)|^2}{T}\,df\right]$$

$$Var(x(t)) = \sigma_x^2 \quad = \lim_{T\to\infty} \int_{-\infty}^{+\infty} \frac{E\left[|X_T(f)|^2\right]}{T}\,df \quad \Rightarrow \int_{-\infty}^{+\infty} \hat{S}_{xx}(f)\,df$$

Power Spectral Density

$$\lim_{T\to\infty} \frac{E\left[|X_T(f)|^2\right]}{T} = S_{xx}(f)$$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] K.H. Shin, J.K. Hammond, Fundamentals of Signal Processing for Sound and Vibration Engineers, Wiley, 2008