DFT Analysis (5B)

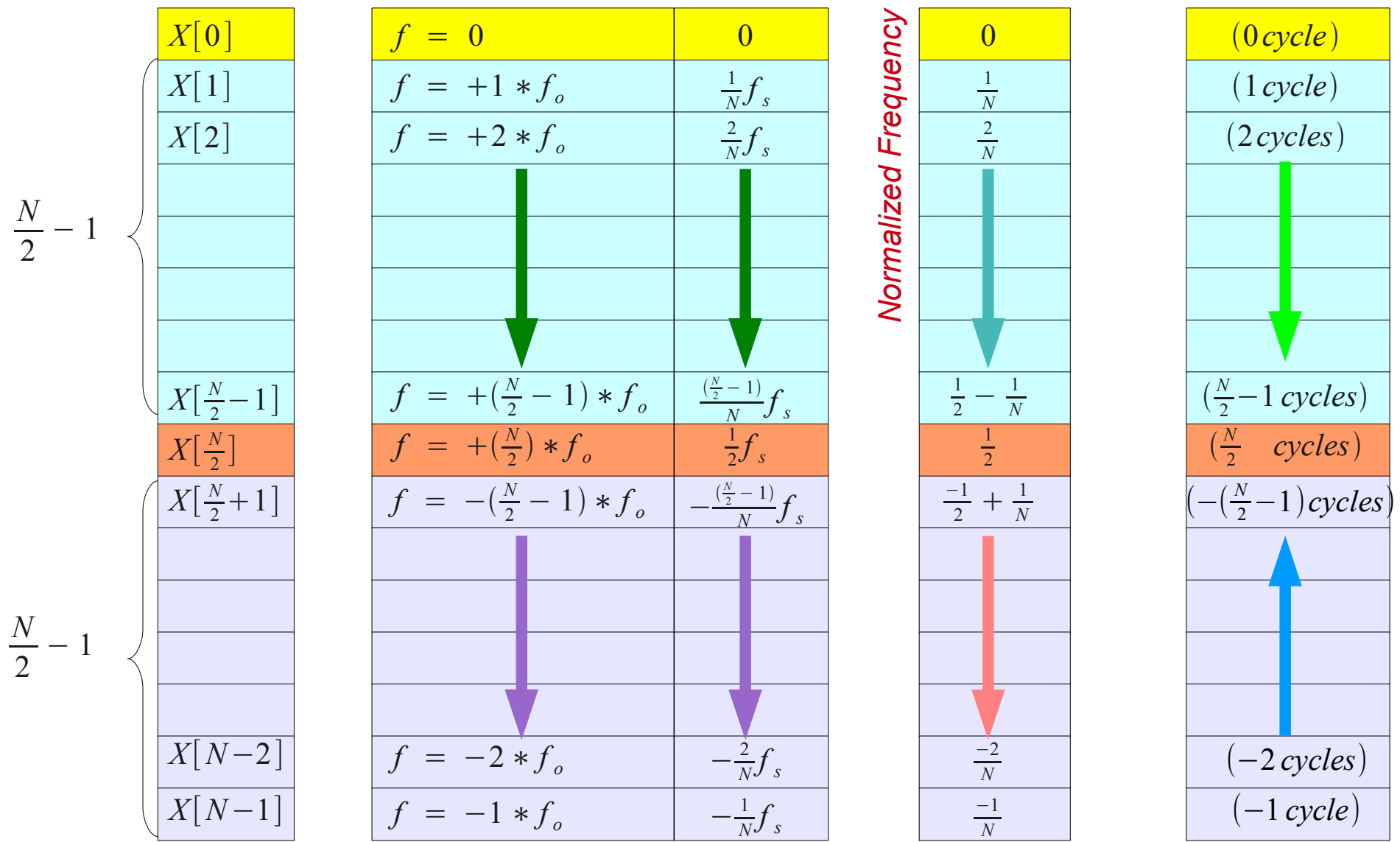Young Won Lim
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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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Frequency View of a DFT Matrix

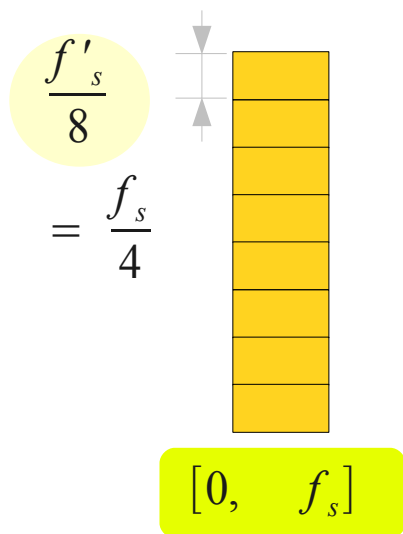|			
row 0	$f = 0$	$(0\,cycle)$	0
row 1	$f = -1 * f_o$	$(-1\,cycle)$	$\frac{-1}{N}$
row 2	$f = -2 * f_o$	$(-2\,cycles)$	$\frac{-2}{N}$
row $\left(\frac{N}{2}-1\right)$	$f = -\left(\frac{N}{2}-1\right) * f_o$	$\left(-\left(\frac{N}{2}-1\right)cycles\right)$	$\frac{-1}{2} + \frac{1}{N}$
row $\left(\frac{N}{2}\right)$	$f = -\left(\frac{N}{2}\right) * f_o$	$\left(\frac{N}{2}\ \ cycles\right)$	$\frac{-1}{2}$
row $\left(\frac{N}{2}+1\right)$	$f = +\left(\frac{N}{2}-1\right) * f_o$	$\left(\frac{N}{2}-1\ cycles\right)$	$\frac{1}{2} - \frac{1}{N}$
row $N-2$	$f = +2 * f_o$	$(2\,cycles)$	$\frac{2}{N}$
row $N-1$	$f = +1 * f_o$	$(1\,cycle)$	$\frac{1}{N}$

$$\frac{N}{2} - 1$$

$$\frac{N}{2} - 1$$

Normalized Frequency

$$f_o = \frac{f_s}{N}$$
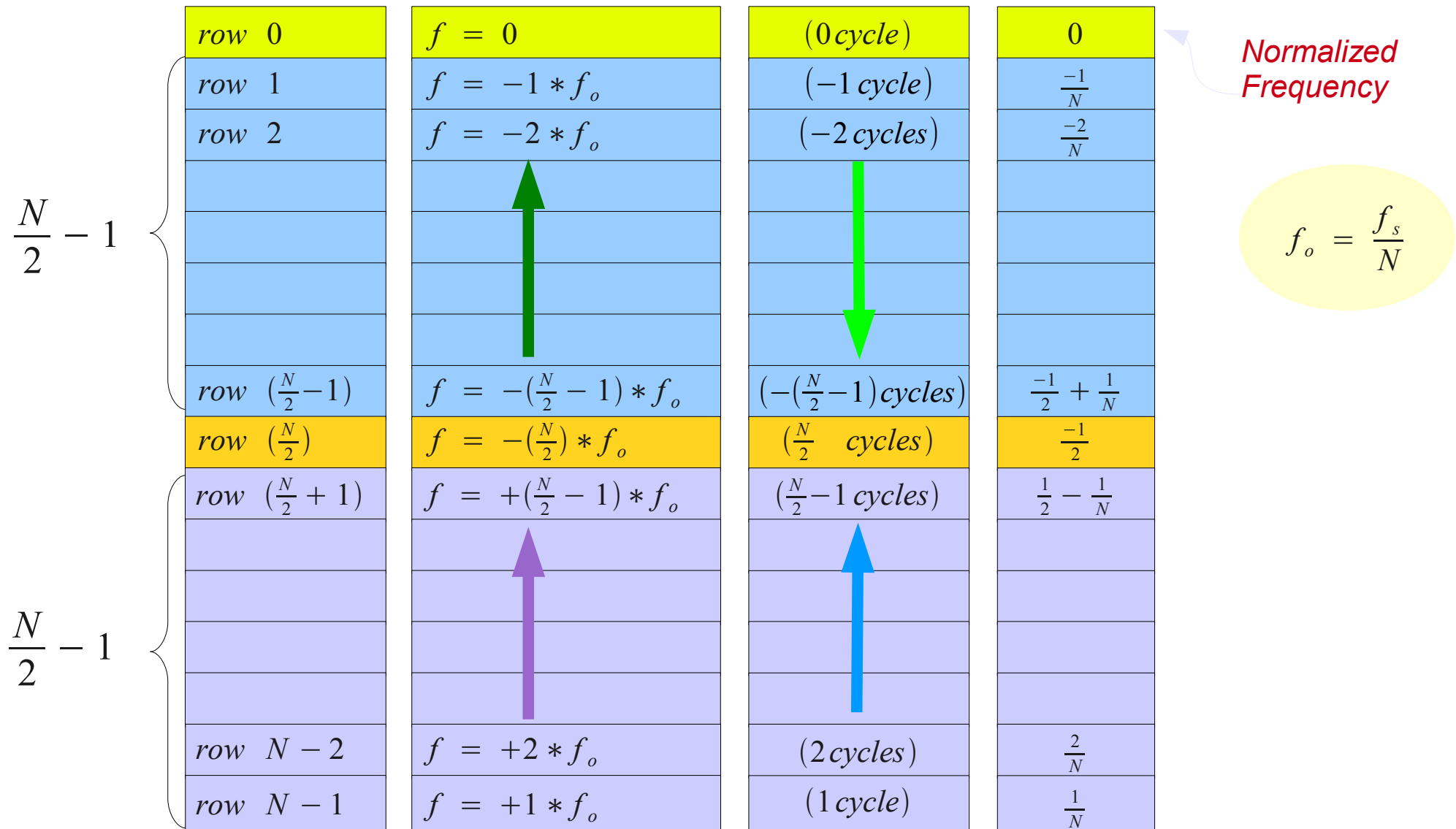
Frequency View of a X[i] Vector

$X[0]$	$f = 0$	0	0	$(0\,cycle)$
$X[1]$	$f = +1*f_o$	$\frac{1}{N}f_s$	$\frac{1}{N}$	$(1\,cycle)$
$X[2]$	$f = +2*f_o$	$\frac{2}{N}f_s$	$\frac{2}{N}$	$(2\,cycles)$
$X[\frac{N}{2}-1]$	$f = +(\frac{N}{2}-1)*f_o$	$\frac{(\frac{N}{2}-1)}{N}f_s$	$\frac{1}{2}-\frac{1}{N}$	$(\frac{N}{2}-1\,cycles)$
$X[\frac{N}{2}]$	$f = +(\frac{N}{2})*f_o$	$\frac{1}{2}f_s$	$\frac{1}{2}$	$(\frac{N}{2}\quad cycles)$
$X[\frac{N}{2}+1]$	$f = -(\frac{N}{2}-1)*f_o$	$-\frac{(\frac{N}{2}-1)}{N}f_s$	$\frac{-1}{2}+\frac{1}{N}$	$(-(\frac{N}{2}-1)\,cycles)$
$X[N-2]$	$f = -2*f_o$	$-\frac{2}{N}f_s$	$\frac{-2}{N}$	$(-2\,cycles)$
$X[N-1]$	$f = -1*f_o$	$-\frac{1}{N}f_s$	$\frac{-1}{N}$	$(-1\,cycle)$

$\frac{N}{2}-1$

$\frac{N}{2}-1$

Normalized Frequency

$\dfrac{f_s}{4}$

$\left[0, \quad \dfrac{f_s}{2} \right]$

4-pt DFT

$=$

τ

$T = 4\tau$

$f_s = \dfrac{1}{\tau}$

$\dfrac{f'_s}{8}$

$= \dfrac{f_s}{4}$

$\left[0, \quad f_s \right]$

$=$

8-pt DFT

$\tau/2$

$T = 8\tau$

$f'_s = \dfrac{2}{\tau}$

$= 2f_s$

$$\frac{f_s}{8}$$

$$=$$

8-pt DFT

$$\left[0, \quad \frac{f_s}{2}\right]$$

$$\tau$$

$$f_s \;=\; \frac{1}{\tau}$$

$$2T \;=\; 8\tau$$
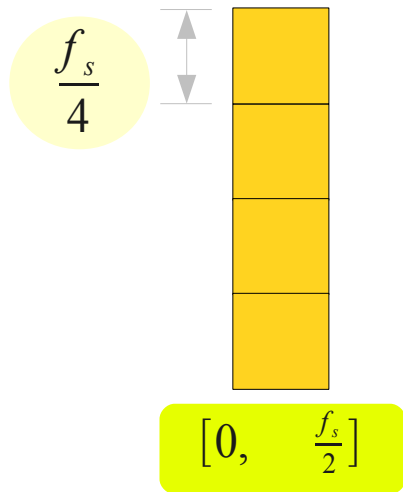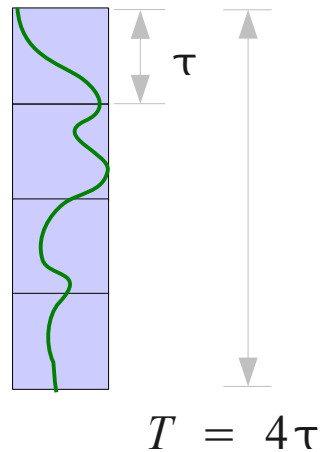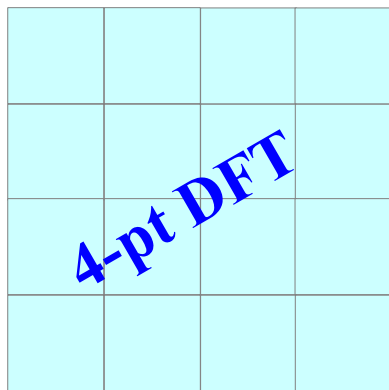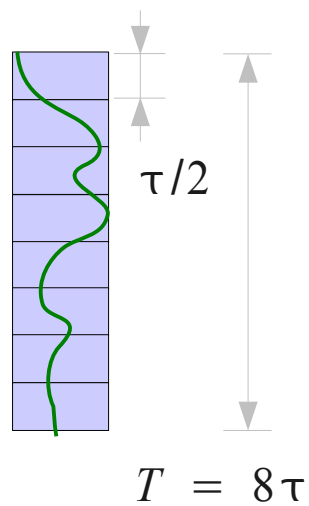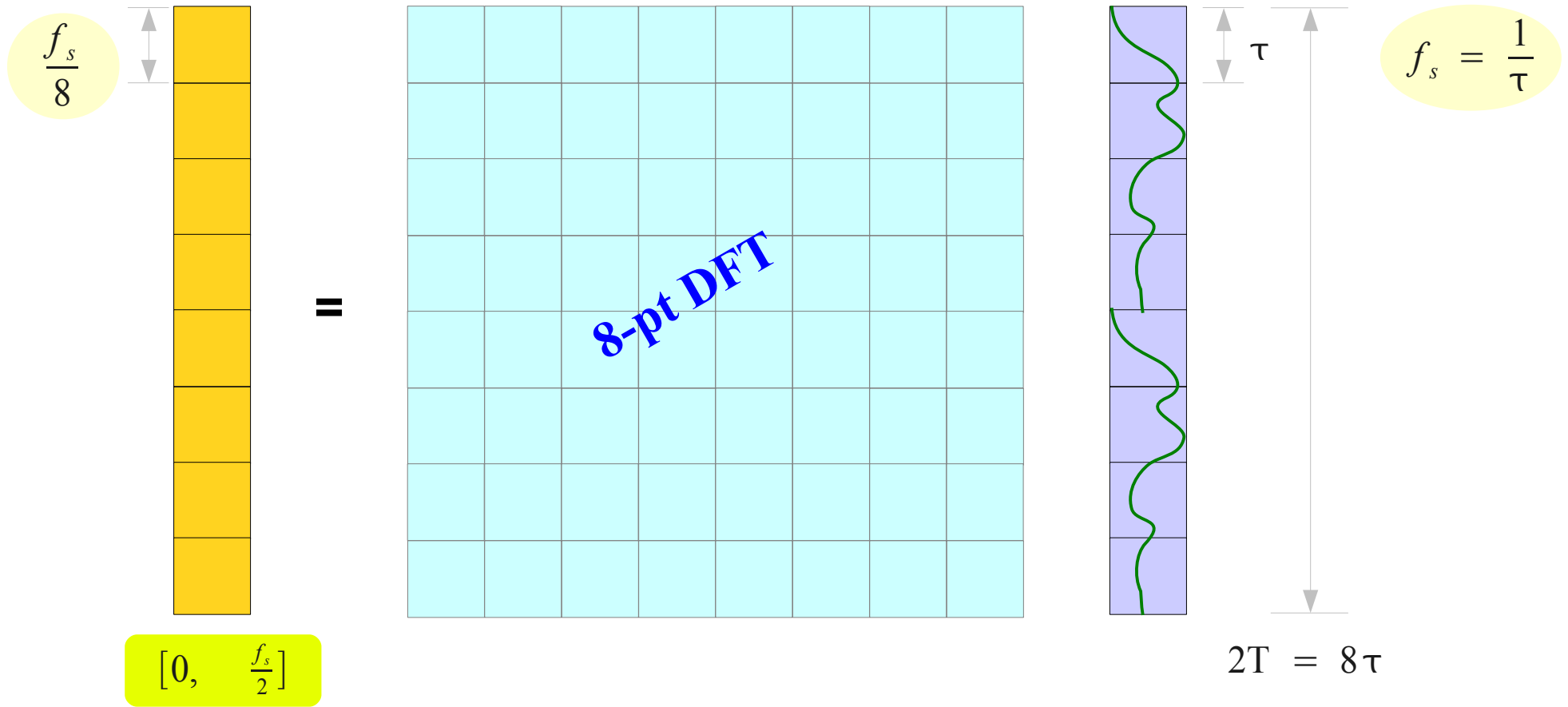
Data Truncation
Frequency Resolution
Zero Padding
Periodogram
Spectral Plot

Amplitude spectrum in quantity peak
Phase spectrum in radians
Amplitude spectrum in volts rms
Phase spectrum in degrees
Power spectrum

Signals without discontinuity
Signals with discontinuity

Sampling frequency is not an integer multiple
of the FFT length

Leakage

Periodic Signals

Frequency Spacing

$$\Delta f = \frac{1}{N \Delta t}$$

One Sided Fourier Series Coefficient

$$\frac{1}{N} X(k)$$

Two Sided Fourier Series Coefficient

$$\frac{1}{N} X(k) \qquad k = 0, \ \frac{N}{2}$$

$$\frac{2}{N} X(k) \qquad k = 1, \cdots, \frac{N}{2} - 1$$

Frequency Scale

$$k \Delta f$$

Aperiodic Signals

One Sided Fourier Series Coefficient

$$\Delta f = \frac{1}{N \Delta t}$$

One Sided Fourier Series Coefficient

$$\frac{\Delta t}{N} X(k)$$

Two Sided Fourier Series Coefficient

$$\frac{\Delta t}{N} X(k) \qquad k = 0, \ \frac{N}{2}$$

$$\frac{2 \Delta t}{N} X(k) \qquad k = 1, \cdots, \frac{N}{2} - 1$$

$$k \Delta f$$

Random Signals

One-sided Power Spectral Density

$$P = \sum_{k=0}^{N-1} S(k) \Delta f$$

One-sided Power Spectral Density

$$P = \sum_{k=0}^{N/2} S_1(k) \Delta f$$

$$S_1(k) = 2S(k) \qquad k = 1, ..., \frac{N}{2} - 1$$
$$S_1(k) = S(k) \qquad k = 0, \frac{N}{2}$$

Two Sided Fourier Series Coefficient

$$\frac{1}{N \Delta t} \sum x^2 \Delta t$$

$$\sum S \Delta f = \frac{1}{N \Delta t} \sum S$$

$$S(k) = \frac{\Delta t}{N} |X(k)|^2$$

$$k \Delta f$$

Amplitude Spectrum

$$A_k = \frac{1}{N}|X(k)| = \frac{1}{N}\sqrt{\Re^2(X(k)) + \Im^2(X(k))}$$

$$k = 0, 1, 2, \cdots, N-1$$

Power Spectrum

$$P_k = \frac{1}{N^2}|X(k)|^2 = \frac{1}{N^2}\{\Re^2(X(k)) + \Im^2(X(k))\}$$

$$k = 0, 1, 2, \cdots, N-1$$

One Sided Amplitude Spectrum

$$\bar{A}_k = \frac{1}{N}|X(0)| \quad k=0$$

$$\bar{A}_k = \frac{2}{N}|X(0)| \quad k=1, 2, \cdots, N/2$$

One Sided Power Spectrum

$$\bar{P}_k = \frac{1}{N^2}|X(0)|^2 \quad k=0$$

$$\bar{P}_k = \frac{2}{N^2}|X(0)|^2 \quad k=1, 2, \cdots, N/2$$

Frequency Bin

$$f = \frac{k f_s}{N}$$

Frequency Bin

$$f = \frac{k f_s}{N}$$

Phase Spectrum

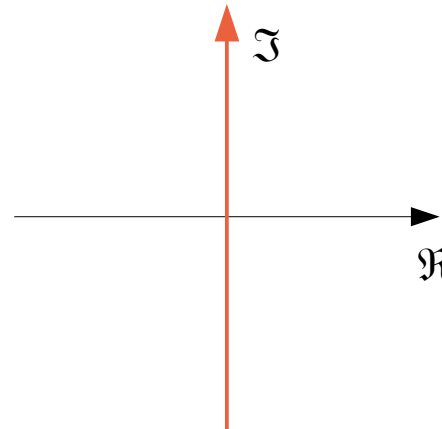$$\phi_k = \tan^{-1}\left(\frac{\Im(X(k))}{\Re(X(k))}\right) \quad k=0, 1, 2, \cdots, N-1$$

5B DFT Analysis

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$$\left[0, \quad \frac{f_s}{2} \right]$$

Fourier Transform

$f(t)$ A continuous sum of weighted exponential functions :

$$f(t)\, e^{-j\omega t}$$

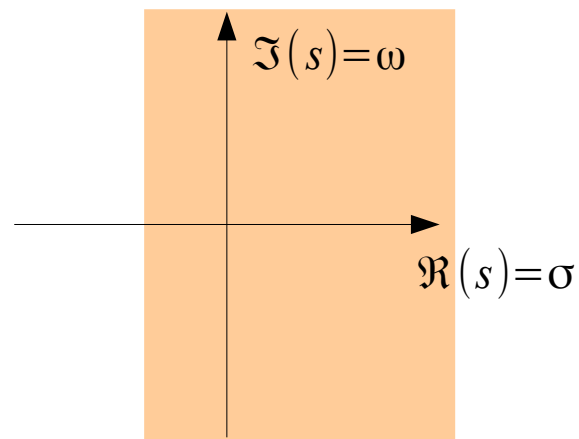$$-\infty \;<\; \omega \;<\; +\infty$$

Not so useful in transient analysis

Laplace Transform

$$f(t)\, e^{-st} \;=\; f(t)\, e^{-(\sigma + j\omega)t}$$

Linear Time Domain Analysis

Initial Condition

z Transform

$$f[n]\, z^{-n}$$

Discrete Time System

Difference Equation

$$z \;=\; e^{sT} \;=\; e^{\sigma T}\, e^{j\omega T}$$

$$\Im(s) = \omega$$

$$\Re(s) = \sigma$$

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References
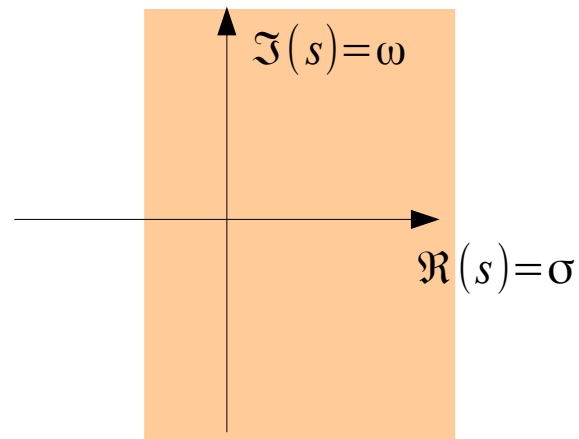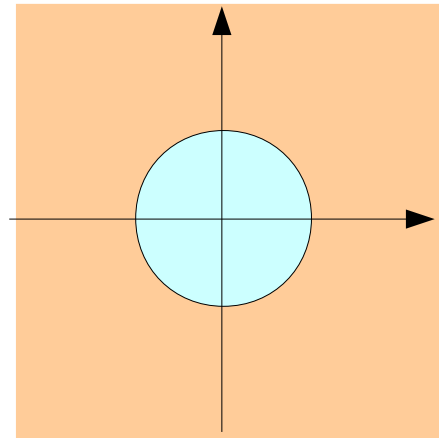
[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A "graphical interpretation" of the DFT and FFT, by Steve Mann