

# LMS Overview (1A)

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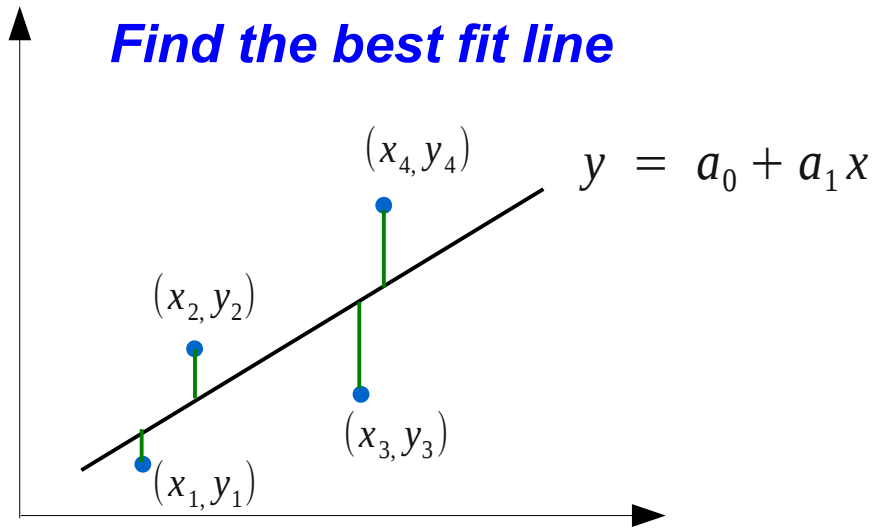
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# Linear Regression (1)



$a_0, a_1$  *unknowns*

$(x_i, y_i)$  *measured data*

*random*

$$S_r = \sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n (y_i - (a_0 + a_1 x_i))^2$$

# Linear Regression (2)

Sum of the square of the residuals

$$S_r = \sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n (y_i - (a_0 + a_1 x_i))^2$$

$a_0, a_1$  unknowns

$(x_i, y_i)$  measured data

random

Minimum Condition

$$\frac{\partial S_r}{\partial a_0} = 2 \sum_{i=1}^n (y_i - a_0 - a_1 x_i)(-1) = 0$$



$$\sum_{i=1}^n a_0 + \sum_{i=1}^n a_1 x_i = \sum_{i=1}^n y_i$$

$$\frac{\partial S_r}{\partial a_1} = 2 \sum_{i=1}^n (y_i - a_0 - a_1 x_i)(-x_i) = 0$$



$$\sum_{i=1}^n a_0 x_i + \sum_{i=1}^n a_1 x_i^2 = \sum_{i=1}^n y_i x_i$$

$$\begin{pmatrix} \sum_{i=1}^n 1 & \sum_{i=1}^n x_i \\ \sum_{i=1}^n x_i & \sum_{i=1}^n x_i^2 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n x_i y_i \end{pmatrix}$$

# Linear Regression (3)

$$\sum_{i=1}^n a_0 + \sum_{i=1}^n a_1 x_i = \sum_{i=1}^n y_i$$

$$n \cdot a_0 + \sum_{i=1}^n a_1 x_i = \sum_{i=1}^n y_i$$

$$a_0 = \frac{1}{n} \sum_{i=1}^n y_i - \frac{1}{n} \sum_{i=1}^n a_1 x_i$$

$$\sum_{i=1}^n a_0 x_i + \sum_{i=1}^n a_1 x_i^2 = \sum_{i=1}^n y_i x_i$$

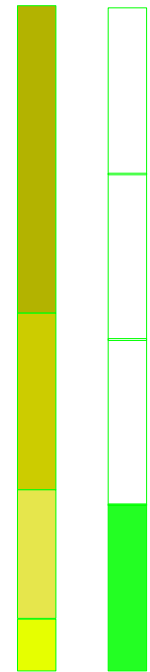
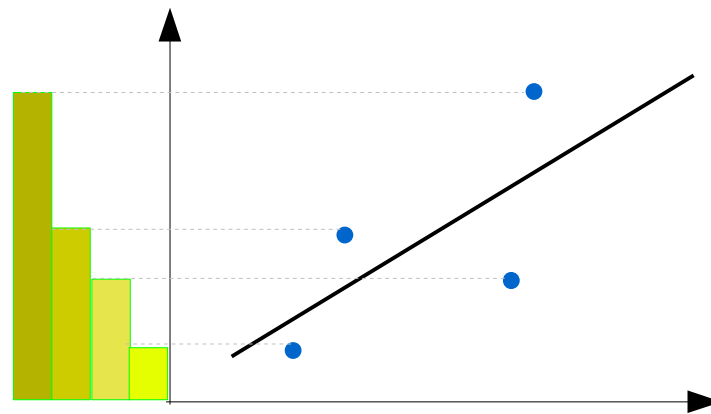
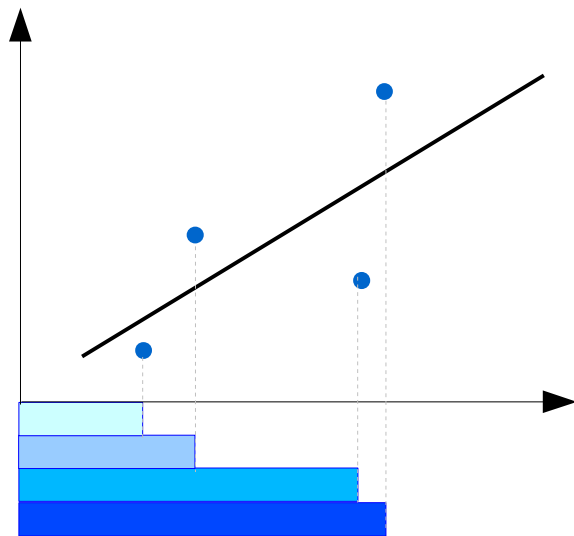
$$\left( \sum_{i=1}^n x_i \right) \left( \frac{1}{n} \sum_{i=1}^n y_i - \frac{1}{n} \sum_{i=1}^n a_1 x_i \right) + \sum_{i=1}^n a_1 x_i^2 = \sum_{i=1}^n y_i x_i$$

$$\frac{1}{n} \left( \sum_{i=1}^n x_i \right) \left( \sum_{i=1}^n y_i \right) - \frac{1}{n} \left( \sum_{i=1}^n x_i \right)^2 a_1 + \left( \sum_{i=1}^n x_i^2 \right) a_1 = \left( \sum_{i=1}^n y_i x_i \right)$$

$$n \left( \sum_{i=1}^n x_i^2 \right) a_1 - \left( \sum_{i=1}^n x_i \right)^2 a_1 = n \left( \sum_{i=1}^n y_i x_i \right) - \left( \sum_{i=1}^n x_i \right) \left( \sum_{i=1}^n y_i \right)$$

$$a_1 = \frac{n \left( \sum_{i=1}^n y_i x_i \right) - \left( \sum_{i=1}^n x_i \right) \left( \sum_{i=1}^n y_i \right)}{n \left( \sum_{i=1}^n x_i^2 \right) - \left( \sum_{i=1}^n x_i \right)^2}$$

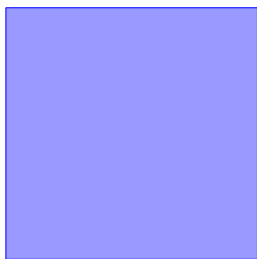
# Mean Values of $x_i, y_i$



$$\frac{1}{n} \sum_{i=1}^n x_i$$

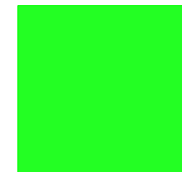


$$\frac{1}{n} \sum_{i=1}^n y_i$$

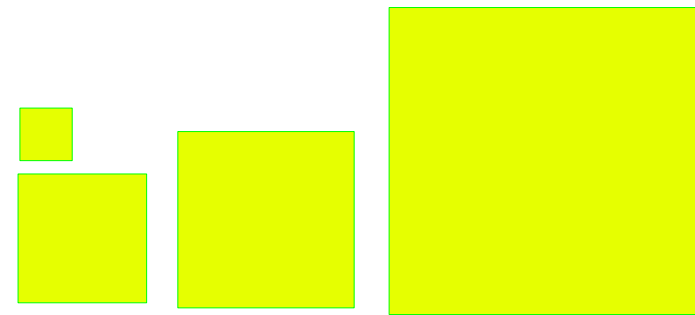
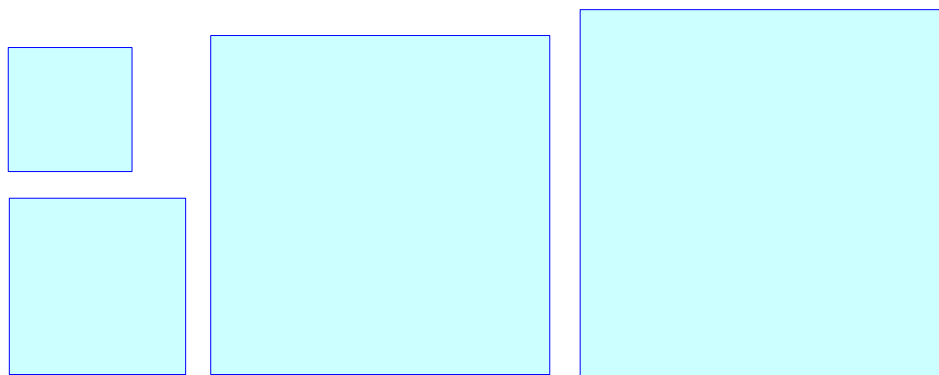
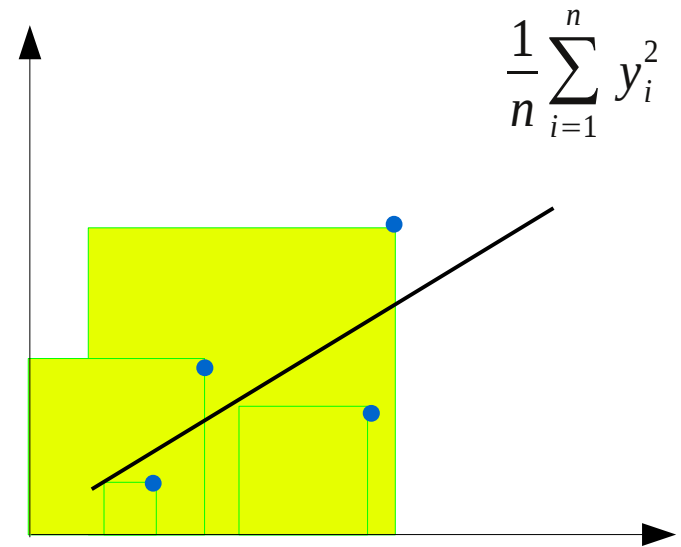
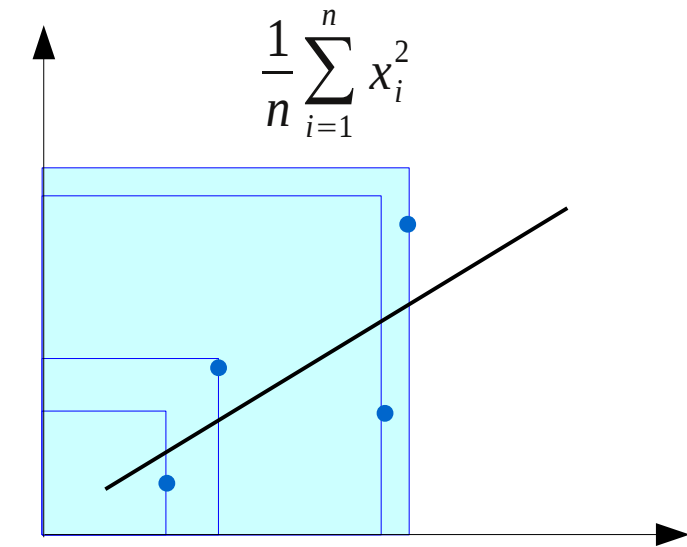


$$\left( \frac{1}{n} \sum_{i=1}^n x_i \right)^2$$

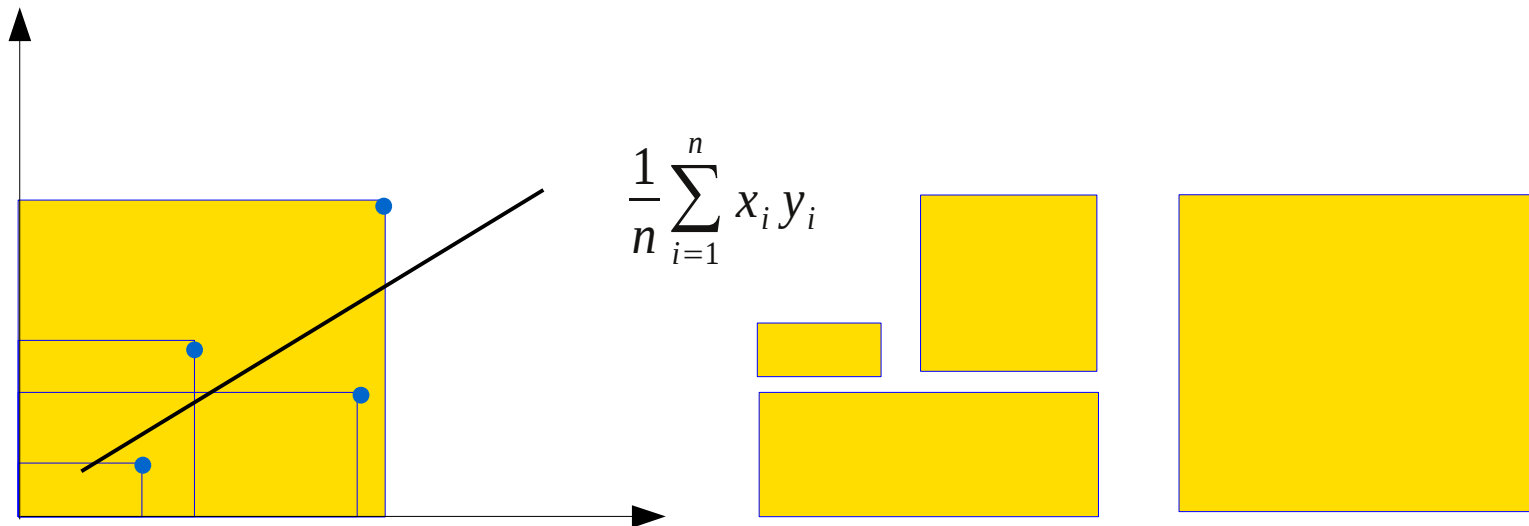
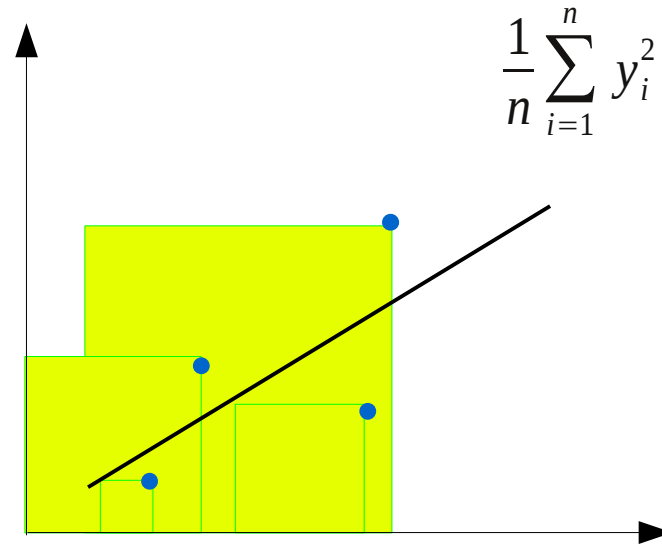
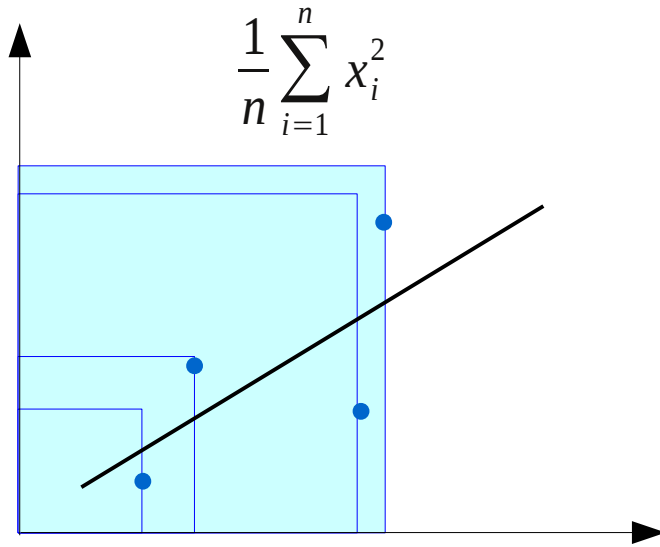
$$\left( \frac{1}{n} \sum_{i=1}^n y_i \right)^2$$



# Mean Values of $x_i^2$ , $y_i^2$ , $x_i y_i$ (1)

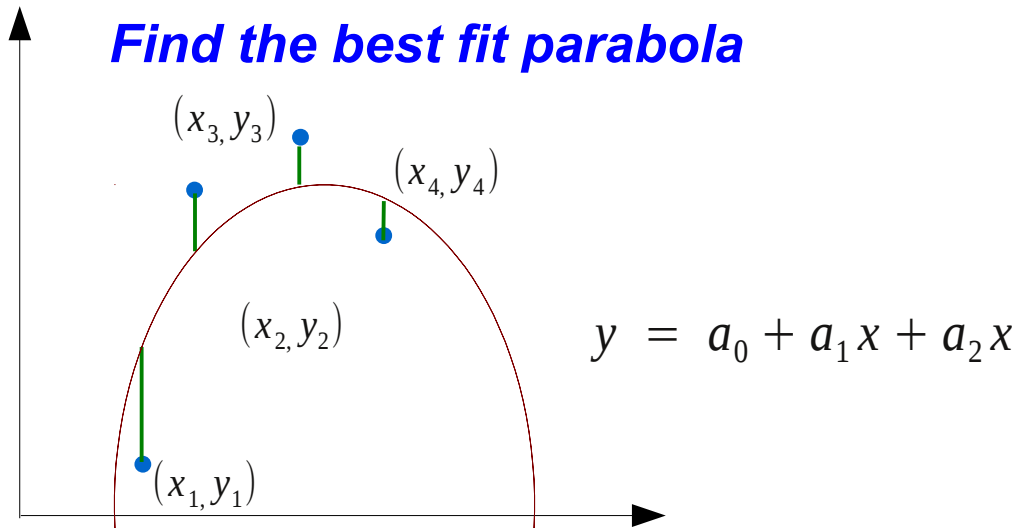


# Mean Values of $x_i^2$ , $y_i^2$ , $x_i y_i$ (2)





# Polynomial Regression (1)



$a_0, a_1, a_2$  *unknowns*

$(x_i, y_i)$  *measured data*

*random*

$$S_r = \sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n (y_i - (a_0 + a_1x_i + a_2x_i^2))^2$$

# Polynomial Regression (2)

*Sum of the square of the residuals*

$$S_r = \sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n (y_i - (a_0 + a_1 x_i + a_2 x_i^2))^2$$

*Minimum Condition*

$$\frac{\partial S_r}{\partial a_0} = 2 \sum_{i=1}^n (y_i - a_0 - a_1 x_i - a_2 x_i^2)(-1) = 0$$

$$\frac{\partial S_r}{\partial a_1} = 2 \sum_{i=1}^n (y_i - a_0 - a_1 x_i - a_2 x_i^2)(-x_i) = 0$$

$$\frac{\partial S_r}{\partial a_2} = 2 \sum_{i=1}^n (y_i - a_0 - a_1 x_i - a_2 x_i^2)(-x_i^2) = 0$$

$a_0, a_1, a_2$  *unknowns*

$(x_i, y_i)$  *measured data*

*random*

***Find the best fit parabola***

# Polynomial Regression (3)

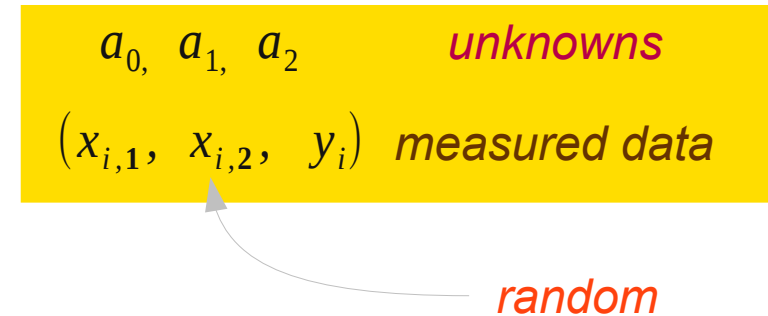
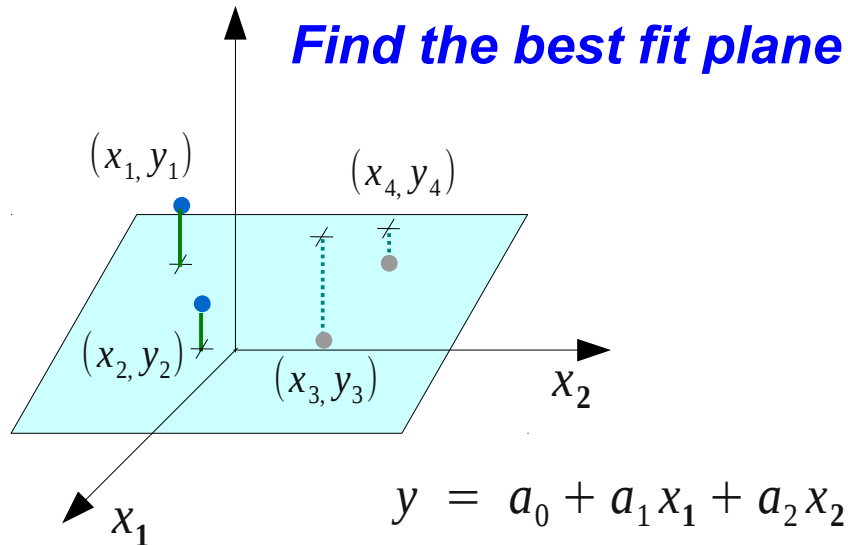
$$\left(\sum_{i=1}^n 1\right) \cdot a_0 + \left(\sum_{i=1}^n x_i\right) \cdot a_1 + \left(\sum_{i=1}^n x_i^2\right) \cdot a_2 = \left(\sum_{i=1}^n y_i\right)$$

$$\left(\sum_{i=1}^n x_i\right) \cdot a_0 + \left(\sum_{i=1}^n x_i^2\right) \cdot a_1 + \left(\sum_{i=1}^n x_i^3\right) \cdot a_2 = \left(\sum_{i=1}^n x_i y_i\right)$$

$$\left(\sum_{i=1}^n x_i^2\right) \cdot a_0 + \left(\sum_{i=1}^n x_i^3\right) \cdot a_1 + \left(\sum_{i=1}^n x_i^4\right) \cdot a_2 = \left(\sum_{i=1}^n x_i^2 y_i\right)$$

$$\begin{pmatrix} \left(\sum_{i=1}^n 1\right) & \left(\sum_{i=1}^n x_i\right) & \left(\sum_{i=1}^n x_i^2\right) \\ \left(\sum_{i=1}^n x_i\right) & \left(\sum_{i=1}^n x_i^2\right) & \left(\sum_{i=1}^n x_i^3\right) \\ \left(\sum_{i=1}^n x_i^2\right) & \left(\sum_{i=1}^n x_i^3\right) & \left(\sum_{i=1}^n x_i^4\right) \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} \left(\sum_{i=1}^n y_i\right) \\ \left(\sum_{i=1}^n x_i y_i\right) \\ \left(\sum_{i=1}^n x_i^2 y_i\right) \end{pmatrix}$$

# Multiple Linear Regression (1)

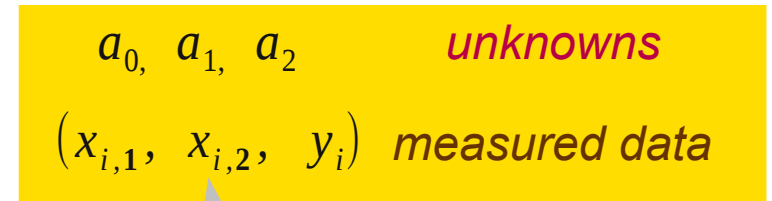


$$S_r = \sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n (y_i - (a_0 + a_1 x_{i,1} + a_2 x_{i,2}))^2$$

# Multiple Linear Regression (2)

*Sum of the square of the residuals*

$$S_r = \sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n (y_i - (a_0 + a_1 x_{i,1} + a_2 x_{i,2}))^2$$



*random*

*Minimum Condition*

$$\frac{\partial S_r}{\partial a_0} = 2 \sum_{i=1}^n (y_i - a_0 - a_1 x_{i,1} - a_2 x_{i,2})(-1) = 0$$

$$\frac{\partial S_r}{\partial a_1} = 2 \sum_{i=1}^n (y_i - a_0 - a_1 x_{i,1} - a_2 x_{i,2})(-x_{i,1}) = 0$$

$$\frac{\partial S_r}{\partial a_2} = 2 \sum_{i=1}^n (y_i - a_0 - a_1 x_{i,1} - a_2 x_{i,2})(-x_{i,2}) = 0$$

# Multiple Linear Regression (3)

$$\left( \sum_{i=1}^n 1 \right) \cdot a_0 + \left( \sum_{i=1}^n x_{i,1} \right) \cdot a_1 + \left( \sum_{i=1}^n x_{i,2} \right) \cdot a_2 = \left( \sum_{i=1}^n y_i \right)$$

$$\left( \sum_{i=1}^n x_{i,1} \right) \cdot a_0 + \left( \sum_{i=1}^n x_{i,1}^2 \right) \cdot a_1 + \left( \sum_{i=1}^n x_{i,1} x_{i,2} \right) \cdot a_2 = \left( \sum_{i=1}^n x_{i,1} y_i \right)$$

$$\left( \sum_{i=1}^n x_{i,2} \right) \cdot a_0 + \left( \sum_{i=1}^n x_{i,1} x_{i,2} \right) \cdot a_1 + \left( \sum_{i=1}^n x_{i,2}^2 \right) \cdot a_2 = \left( \sum_{i=1}^n x_{i,2} y_i \right)$$

$$\begin{pmatrix} \left( \sum_{i=1}^n 1 \right) & \left( \sum_{i=1}^n x_{i1} \right) & \left( \sum_{i=1}^n x_{i2} \right) \\ \left( \sum_{i=1}^n x_{i1} \right) & \left( \sum_{i=1}^n x_{i1}^2 \right) & \left( \sum_{i=1}^n x_{i1} x_{i2} \right) \\ \left( \sum_{i=1}^n x_{i2} \right) & \left( \sum_{i=1}^n x_{i1} x_{i2} \right) & \left( \sum_{i=1}^n x_{i2}^2 \right) \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} \left( \sum_{i=1}^n y_i \right) \\ \left( \sum_{i=1}^n x_{i1} y_i \right) \\ \left( \sum_{i=1}^n x_{i2} y_i \right) \end{pmatrix}$$

# Multiple Linear Regression – General (1)

Sum of the square of the residuals

$$S_r = \sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n \left( y_i - \beta_0 - \sum_{j=1}^m x_{ij} \beta_j \right)^2$$

$\beta_0, \beta_1, \dots, \beta_m$

*unknowns*

$(x_{i1}, x_{i2}, \dots, x_{im}, y_i)$  *measured data*

*random*

$$y = \beta_0 + \sum_{j=1}^m x_j \beta_j = \beta_0 + x_1 \beta_1 + x_2 \beta_2 + \dots + x_m \beta_m$$

Minimum Condition

$$\frac{\partial S_r}{\partial \beta_0} = 2 \sum_{i=1}^n (y_i - \beta_0 - \beta_1 x_{i1} - \beta_2 x_{i2} - \dots - \beta_m x_{im})(-1) = 0$$

$$\frac{\partial S_r}{\partial \beta_1} = 2 \sum_{i=1}^n (y_i - \beta_0 - \beta_1 x_{i1} - \beta_2 x_{i2} - \dots - \beta_m x_{im})(-x_{i1}) = 0$$

...

...

...

$$\frac{\partial S_r}{\partial \beta_m} = 2 \sum_{i=1}^n (y_i - \beta_0 - \beta_1 x_{i1} - \beta_2 x_{i2} - \dots - \beta_m x_{im})(-x_{im}) = 0$$

# Multiple Linear Regression – General (2)

$$\begin{pmatrix}
 \left( \sum_{i=1}^n 1 \right) & \left( \sum_{i=1}^n X_{i1} \right) & \left( \sum_{i=1}^n X_{i2} \right) & \cdots & \left( \sum_{i=1}^n X_{im} \right) \\
 \left( \sum_{i=1}^n X_{i1} \right) & \left( \sum_{i=1}^n X_{i1}^2 \right) & \left( \sum_{i=1}^n X_{i1} X_{i2} \right) & \cdots & \left( \sum_{i=1}^n X_{i1} X_{im} \right) \\
 \left( \sum_{i=1}^n X_{i2} \right) & \left( \sum_{i=1}^n X_{i2} X_{i1} \right) & \left( \sum_{i=1}^n X_{i2}^2 \right) & \cdots & \left( \sum_{i=1}^n X_{i2} X_{im} \right) \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
 \left( \sum_{i=1}^n X_{im} \right) & \left( \sum_{i=1}^n X_{im} X_{i1} \right) & \left( \sum_{i=1}^n X_{im} X_{i2} \right) & \cdots & \left( \sum_{i=1}^n X_{im}^2 \right)
 \end{pmatrix}
 \begin{pmatrix}
 \beta_0 \\
 \beta_1 \\
 \beta_2 \\
 \vdots \\
 \beta_m
 \end{pmatrix}
 =
 \begin{pmatrix}
 \left( \sum_{i=1}^n y_i \right) \\
 \left( \sum_{i=1}^n X_{i1} y_i \right) \\
 \left( \sum_{i=1}^n X_{i2} y_i \right) \\
 \vdots \\
 \left( \sum_{i=1}^n X_{im} y_i \right)
 \end{pmatrix}$$



# Multiple Linear Regression – General (3)

$m = 1$  *measured data*

1	$X_{11}$	$X_{12}$	...	$X_{1m}$
$X_{11}$	$X_{11}^2$	$X_{11}X_{12}$	...	$X_{11}X_{1m}$
$X_{12}$	$X_{12}X_{11}$	$X_{12}^2$	...	$X_{12}X_{1m}$
⋮	⋮	⋮		⋮
$X_{1m}$	$X_{1m}X_{11}$	$X_{1m}X_{12}$	...	$X_{1m}^2$

$m = 2$  *measured data*

1	$X_{21}$	$X_{22}$	...	$X_{2m}$
$X_{21}$	$X_{21}^2$	$X_{21}X_{22}$	...	$X_{21}X_{2m}$
$X_{22}$	$X_{22}X_{21}$	$X_{22}^2$	...	$X_{22}X_{2m}$
⋮	⋮	⋮		⋮
$X_{2m}$	$X_{2m}X_{21}$	$X_{2m}X_{22}$	...	$X_{2m}^2$

$m = 3$  *measured data*

1	$X_{31}$	$X_{32}$	...	$X_{3m}$
$X_{31}$	$X_{31}^2$	$X_{31}X_{32}$	...	$X_{31}X_{3m}$
$X_{32}$	$X_{32}X_{31}$	$X_{32}^2$	...	$X_{32}X_{3m}$
⋮	⋮	⋮		⋮
$X_{3m}$	$X_{3m}X_{31}$	$X_{3m}X_{32}$	...	$X_{3m}^2$

$m = 4$  *measured data*

1	$X_{41}$	$X_{42}$	...	$X_{4m}$
$X_{41}$	$X_{41}^2$	$X_{41}X_{42}$	...	$X_{41}X_{4m}$
$X_{42}$	$X_{42}X_{41}$	$X_{42}^2$	...	$X_{42}X_{4m}$
⋮	⋮	⋮		⋮
$X_{4m}$	$X_{4m}X_{41}$	$X_{4m}X_{42}$	...	$X_{4m}^2$

# Multiple Linear Regression – General (4)

$$\begin{array}{cccc}
 E\{1\} & E\{x_1\} & E\{x_2\} & E\{x_m\} \\
 1 & \bar{x}_1 & \bar{x}_2 & \bar{x}_m \\
 \frac{1}{n} \sum_{i=1}^n 1 & \frac{1}{n} \sum_{i=1}^n x_{i1} & \frac{1}{n} \sum_{i=1}^n x_{i2} & \frac{1}{n} \sum_{i=1}^n x_{im}
 \end{array}$$

1	$\bar{x}_1$	$\bar{x}_2$	...	$\bar{x}_m$
$\bar{x}_1$	$\bar{x}_1^2$	$\overline{x_1 x_2}$	...	$\overline{x_1 x_m}$
$\bar{x}_2$	$\overline{x_2 x_1}$	$\bar{x}_2^2$	...	$\overline{x_2 x_m}$
⋮	⋮	⋮	⋮	⋮
$\bar{x}_m$	$\overline{x_m x_1}$	$\overline{x_m x_2}$	...	$\bar{x}_m^2$

1	$x_{11}$	$x_{12}$	...	$x_{1m}$
$x_{11}$	$x_{11}^2$	$x_{11} x_{12}$	...	$x_{11} x_{1m}$
$x_{12}$	$x_{12} x_{11}$	$x_{12}^2$	...	$x_{12} x_{1m}$
⋮	⋮	⋮	⋮	⋮
$x_{1m}$	$x_{1m} x_{11}$	$x_{1m} x_{12}$	...	$x_{1m}^2$

$m = 4$  measured data

$m = 3$  measured data

$m = 2$  measured data

$m = 1$  measured data

# Least Square (1)

Sum of the square of the residuals

$$S_r = \sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n (y_i - f(x_i, \boldsymbol{\beta}))^2$$

$$\epsilon_i = (y_i - f(x_i, \boldsymbol{\beta}))$$

$\beta_1, \beta_2, \dots, \beta_m$  *unknowns*  
 $(x_{i1}, x_{i2}, \dots, x_{im}, y_i)$  *measured data*

*random*

Minimum Condition

$$\frac{\partial S_r}{\partial \beta_j} = 2 \sum_{i=1}^n \epsilon_i \frac{\partial \epsilon_i}{\partial \beta_j} = 0 \quad j = 1, \dots, m$$

$$\frac{\partial S_r}{\partial \beta_j} = -2 \sum_{i=1}^n \epsilon_i \frac{\partial f(x_i, \boldsymbol{\beta})}{\partial \beta_j} = 0 \quad j = 1, \dots, m \quad \leftarrow \quad \frac{\partial \epsilon_i}{\partial \beta_j} = -\frac{\partial f(x_i, \boldsymbol{\beta})}{\partial \beta_j}$$

$$y = \sum_{j=1}^m x_j \beta_j = x_1 \beta_1 + x_2 \beta_2 + \dots + x_m \beta_m$$

# Least Square (2)

*Sum of the square of the residuals*

$$S_r = \sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n (y_i - f(x_i, \boldsymbol{\beta}))^2$$

*Minimum Condition*

$$\frac{\partial S_r}{\partial \beta_j} = -2 \sum_{i=1}^n \epsilon_i \frac{\partial f(x_i, \boldsymbol{\beta})}{\partial \beta_j} = 0 \quad j = 1, \dots, m$$

$\beta_1, \beta_2, \dots, \beta_m$  *unknowns*  
 $(x_{i1}, x_{i2}, \dots, x_{im}, y_i)$  *measured data*

*random*

**Linear Least Square**

$$y = \sum_{j=1}^m x_j \beta_j = x_1 \beta_1 + x_2 \beta_2 + \dots + x_m \beta_m$$

$$f(x_i, \boldsymbol{\beta}) = \sum_{j=1}^m x_{ij} \beta_j = x_{i1} \beta_{i1} + x_{i2} \beta_{i2} + \dots + x_{im} \beta_{im}$$

# Linear Least Square (1)

Sum of the square of the residuals

$$S_r = \sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n \left( y_i - \sum_{j=1}^m x_{ij} \beta_j \right)^2$$

$$y_i = \sum_{j=1}^m x_{ij} \beta_j \quad \mathbf{X}\boldsymbol{\beta} = \mathbf{y}$$

$\beta_1, \dots, \beta_m$  *unknowns*  
 $(x_{i1}, x_{i2}, \dots, x_{im}, y_i)$  *measured data*

*random*

$$\mathbf{X} = \begin{pmatrix} x_{11} & x_{12} & x_{13} & \dots & x_{1m} \\ x_{21} & x_{22} & x_{23} & \dots & x_{2m} \\ x_{31} & x_{32} & x_{33} & \dots & x_{3m} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ x_{n1} & x_{n2} & x_{n3} & \dots & x_{nm} \end{pmatrix} \quad \boldsymbol{\beta} = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \vdots \\ \beta_m \end{pmatrix} \quad \mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_m \end{pmatrix}$$

# Linear Least Square (2)

Sum of the square of the residuals

$$S_r = \sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n \left( y_i - \sum_{j=1}^m x_{ij} \beta_j \right)^2$$

$$y = \sum_{j=1}^m x_j \beta_j = x_1 \beta_1 + x_2 \beta_2 + \cdots + x_m \beta_m$$

$\beta_1, \beta_2, \dots, \beta_m$  unknowns  
 $(x_{i1}, x_{i2}, \dots, x_{im}, y_i)$  measured data

random

Minimum Condition

$$\frac{\partial S_r}{\partial \beta_1} = 2 \sum_{i=1}^n (y_i - \beta_1 x_{i1} - \beta_2 x_{i2} - \cdots - \beta_m x_{im})(-x_{i1}) = 0$$

$$\frac{\partial S_r}{\partial \beta_2} = 2 \sum_{i=1}^n (y_i - \beta_1 x_{i1} - \beta_2 x_{i2} - \cdots - \beta_m x_{im})(-x_{i2}) = 0$$

...

...

...

$$\frac{\partial S_r}{\partial \beta_m} = 2 \sum_{i=1}^n (y_i - \beta_1 x_{i1} - \beta_2 x_{i2} - \cdots - \beta_m x_{im})(-x_{im}) = 0$$

# Linear Least Square (3)

$$\begin{pmatrix}
 \left( \sum_{i=1}^n X_{i1}^2 \right) & \left( \sum_{i=1}^n X_{i1} X_{i2} \right) & \left( \sum_{i=1}^n X_{i1} X_{i3} \right) & \cdots & \left( \sum_{i=1}^n X_{i1} X_{im} \right) \\
 \left( \sum_{i=1}^n X_{i2} X_{i1} \right) & \left( \sum_{i=1}^n X_{i2}^2 \right) & \left( \sum_{i=1}^n X_{i2} X_{i3} \right) & \cdots & \left( \sum_{i=1}^n X_{i2} X_{im} \right) \\
 \left( \sum_{i=1}^n X_{i3} X_{i1} \right) & \left( \sum_{i=1}^n X_{i3} X_{i2} \right) & \left( \sum_{i=1}^n X_{i3}^2 \right) & \cdots & \left( \sum_{i=1}^n X_{i3} X_{im} \right) \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
 \left( \sum_{i=1}^n X_{im} X_{i1} \right) & \left( \sum_{i=1}^n X_{im} X_{i2} \right) & \left( \sum_{i=1}^n X_{im} X_{i3} \right) & \cdots & \left( \sum_{i=1}^n X_{im}^2 \right)
 \end{pmatrix}
 \begin{pmatrix}
 \beta_1 \\
 \beta_2 \\
 \beta_3 \\
 \vdots \\
 \beta_m
 \end{pmatrix}
 =
 \begin{pmatrix}
 \left( \sum_{i=1}^n X_{i1} Y_i \right) \\
 \left( \sum_{i=1}^n X_{i2} Y_i \right) \\
 \left( \sum_{i=1}^n X_{i3} Y_i \right) \\
 \vdots \\
 \left( \sum_{i=1}^n X_{im} Y_i \right)
 \end{pmatrix}$$

# Linear Least Square (4)

## Normal Equations

$$\mathbf{X}\boldsymbol{\beta} = \mathbf{y}$$

$$\mathbf{X}^t \mathbf{X} \hat{\boldsymbol{\beta}} = \mathbf{X}^t \mathbf{y}$$

$$S_r = \sum_{i=1}^n \epsilon_i^2 = \sum_{i=1}^n \left( y_i - \sum_{j=1}^m x_{ij} \beta_j \right)^2$$

$$\frac{\partial S_r}{\partial \beta_j} = 2 \sum_{i=1}^n \epsilon_i \frac{\partial \epsilon_i}{\partial \beta_j} \quad (j = 1, 2, \dots, m) \quad \frac{\partial \epsilon_i}{\partial \beta_j} = -x_{ij}$$

$$\frac{\partial S_r}{\partial \beta_j} = 2 \sum_{i=1}^n \left( y_i - \sum_{j=1}^m x_{ij} \beta_j \right) (-x_{ij}) = 0 \quad (j = 1, 2, \dots, m)$$

$$\sum_{i=1}^n \sum_{k=1}^m x_{ij} x_{ik} \hat{\beta}_k = \sum_{i=1}^n \left( y_i - \sum_{j=1}^m x_{ij} \beta_j \right) (-x_{ij}) = 0 \quad (j = 1, 2, \dots, m)$$

$\beta_1, \dots, \beta_m$

*unknowns*

$(x_{i1}, x_{i2}, \dots, x_{im}, y_i)$  *measured data*

*random*







## References

- [1] <http://en.wikipedia.org/>
- [2] <http://numericalmethods.eng.usf.edu/>
- [3] S.C. Chapra, Applied Numerical Methods W/ml Engineering And Science