

Linear Equations

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Linear Equations

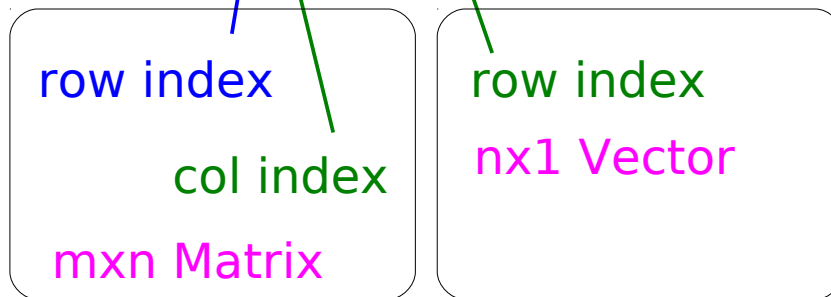
$$\begin{array}{ccccccc} a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n & = & b_1 \\ a_{21} x_1 + a_{22} x_2 + \cdots + a_{2n} x_n & = & b_2 \\ \vdots & & \vdots \\ a_{m1} x_1 + a_{m2} x_2 + \cdots + a_{mn} x_n & = & b_m \end{array}$$

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

Linear Equations

$$a_{11}x_1 + \quad + \quad \dots + a_{1n}x_n = b_1$$

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{bmatrix} b_1 \end{bmatrix}$$
$$\sum_{j=1}^n a_{1j} \cdot x_j = b_1$$

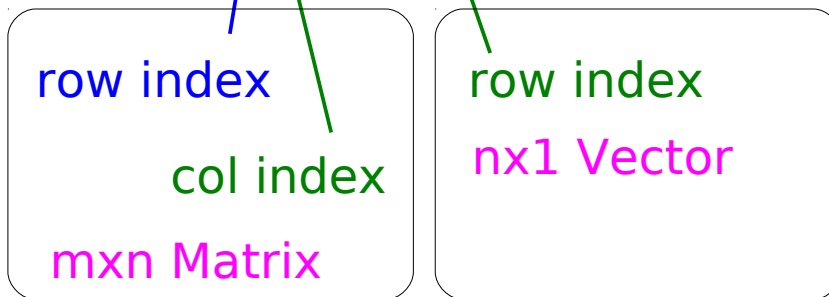


Linear Equations

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

$$\begin{bmatrix} a_{21} & a_{22} & \dots & a_{2n} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{bmatrix} b_2 \end{bmatrix}$$

$$\sum_{j=1}^n a_{2j} \cdot x_j = b_2$$

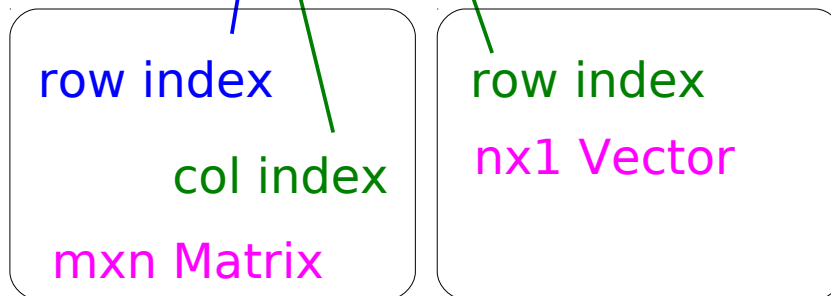


Linear Equations

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m$$

$$\begin{bmatrix} a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = b_m$$

$$\sum_{j=1}^n a_{mj} \cdot x_j = b_m$$



Echelon Forms (1)

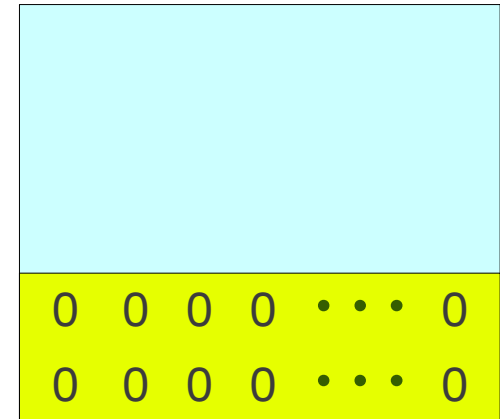
- zero rows → Should be grouped at the bottom
- non-zero row → A leading one
The 1st non-zero element should be one
- Any successive non-zero rows → The leading one of the lower row should be farther to the right than the leading one of the higher row

Echelon Forms (2)

zero rows



Should be grouped at the bottom



$$\begin{array}{ccccccc} 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{array}$$

Echelon Forms (3)

non-zero row



A leading one

The 1st non-zero element should be one

$$0 \quad \textcircled{1} \quad * \quad * \quad \dots \quad *$$

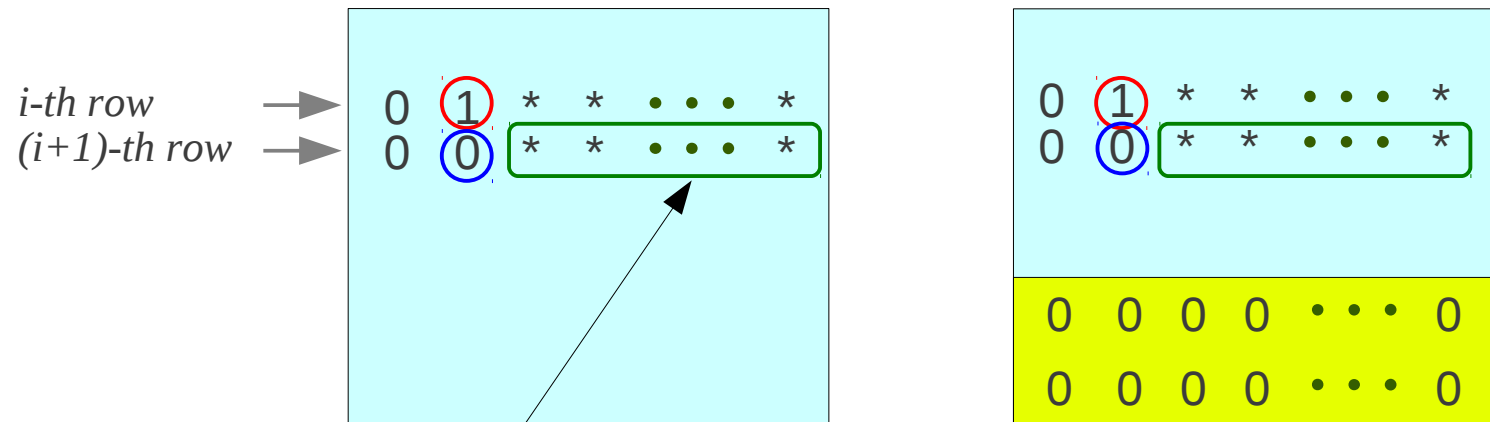
$$\begin{array}{cccccc} 0 & \textcircled{1} & * & * & \dots & * \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \end{array}$$

Echelon Forms (3)

Any successive
non-zero rows



The leading one of the lower row
should be farther to the right than
the leading one of the higher row



The possible location of the leading one

Could be like this $0 \quad 0 \quad 1 \quad * \quad \dots \quad *$


Or like this $0 \quad 0 \quad 0 \quad 1 \quad \dots \quad *$

Or like this $0 \quad 0 \quad 0 \quad \dots \quad 1$

Reduced Echelon Forms

- zero rows → Should be grouped at the bottom
- non-zero row → A leading one
The 1st non-zero element should be one
- Any successive non-zero rows → The leading one of the lower row should be farther to the right than the leading one of the higher row
- Any column that contains a leading one → All other elements except the leading one are all zeros

Reduced Echelon Forms

Any column that contains a leading one 

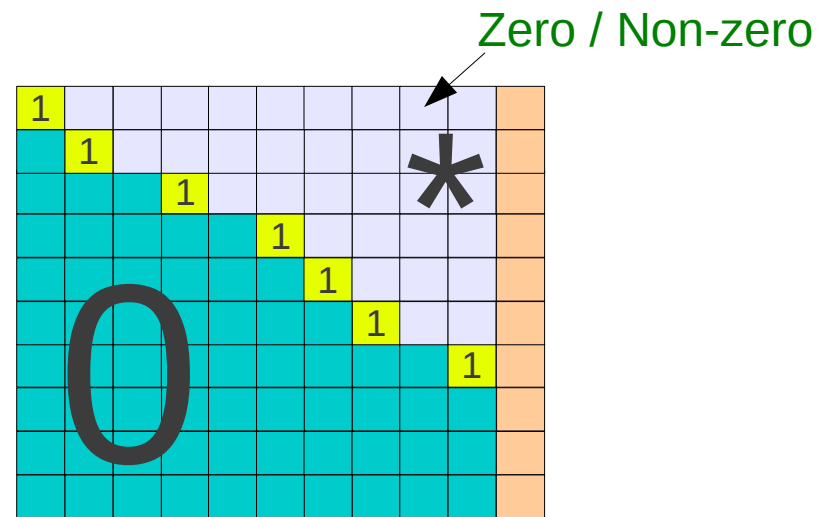
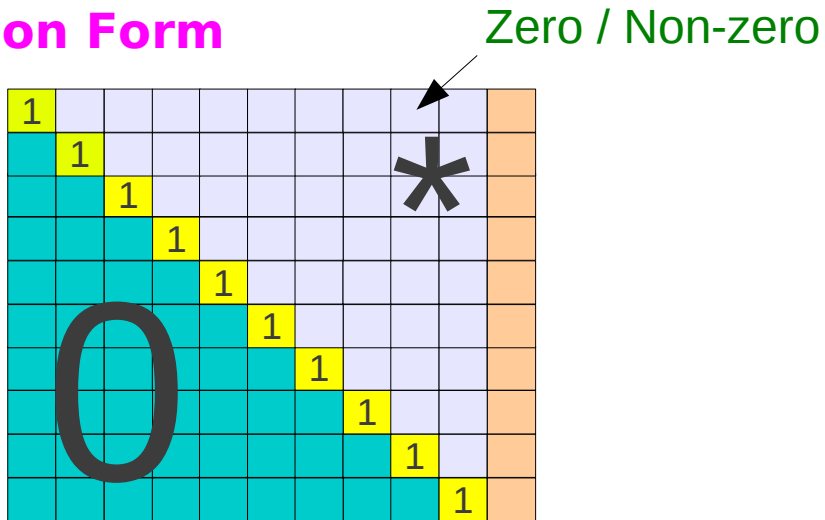
All other elements except the leading one are all zeros

0	0	*	*	...	*
	1				
	0				
	0				
	⋮				
	⋮				
	⋮				
	0				

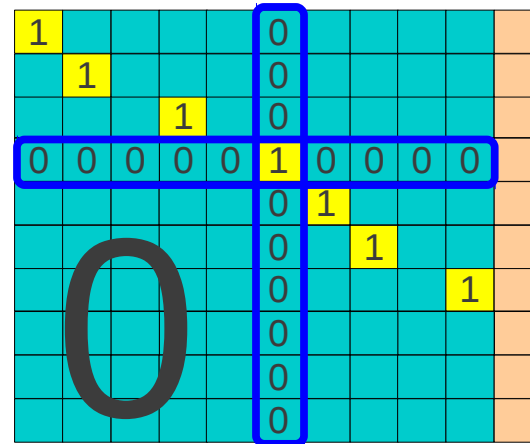
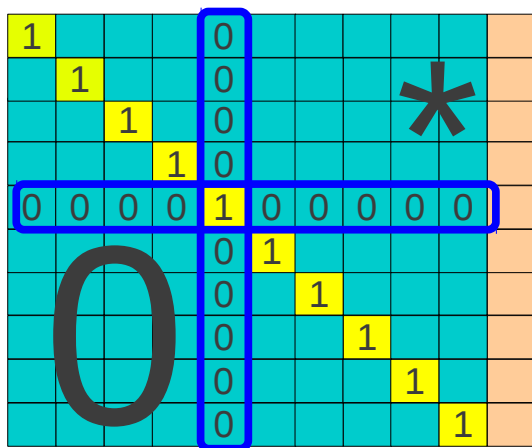
0	0	*	*	...	*
	1				
	0				
	0				
0	0	0	0	⋮	0
0	0	0	0	⋮	0

Examples

Echelon Form



Reduced Echelon Form



Common Emitter (1)

Common Emitter (2)

Common Emitter (2)

Maintain Magnetic Field

Storing Magnetic Energy

Dissipate Magnetic Energy

References

[1] <http://en.wikipedia.org/>

[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003