

Anti-aliasing Prefilter (6B)

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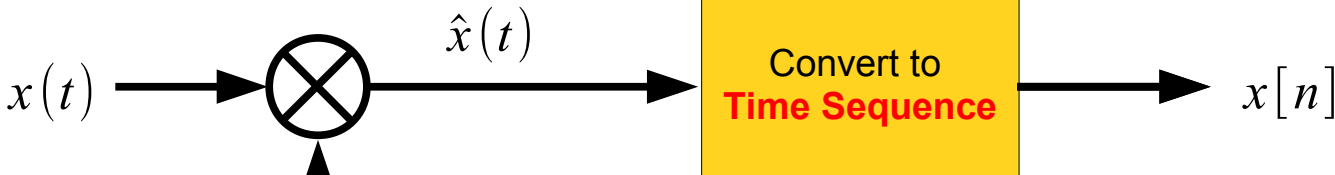
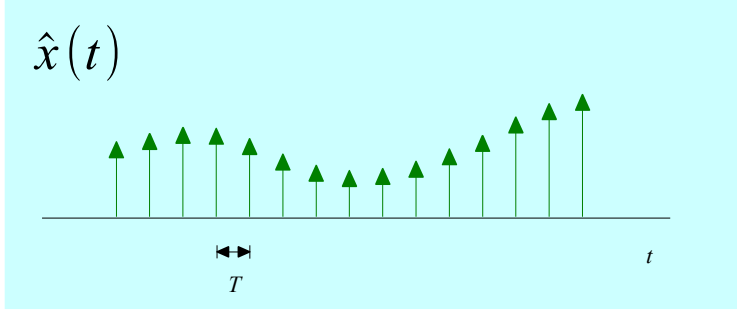
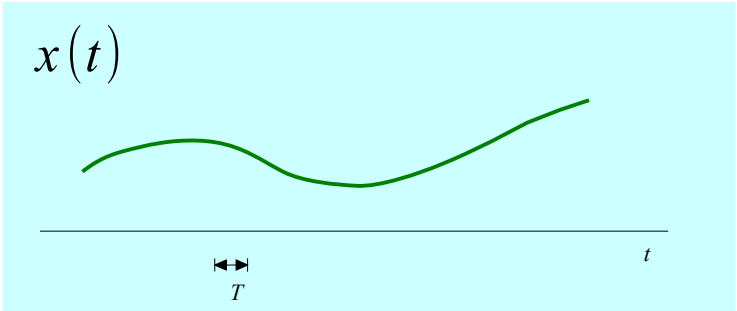
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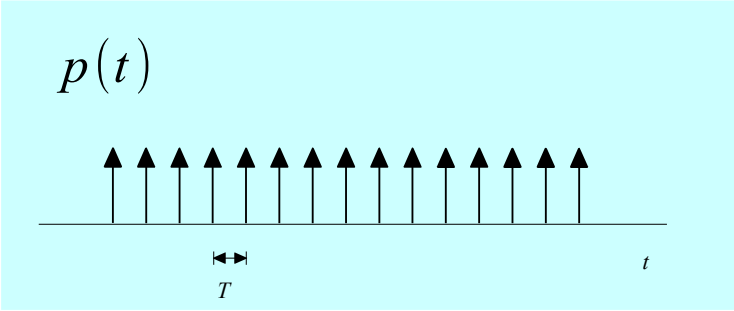
This document was produced by using OpenOffice and Octave.

Time Sequence

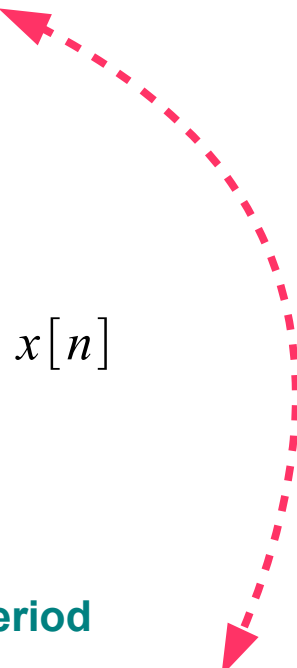
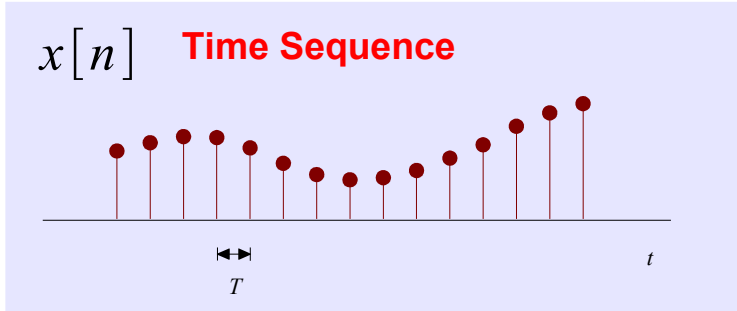


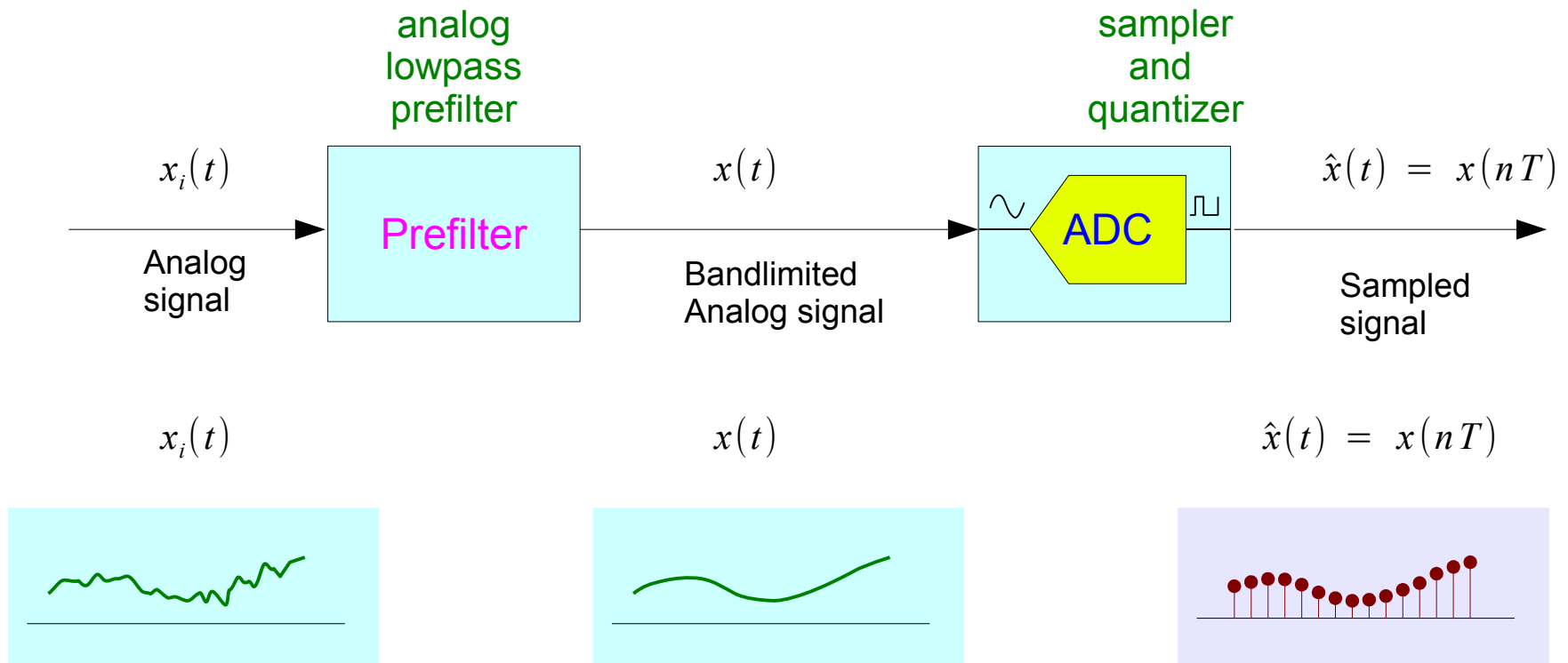
Ideal Sampling

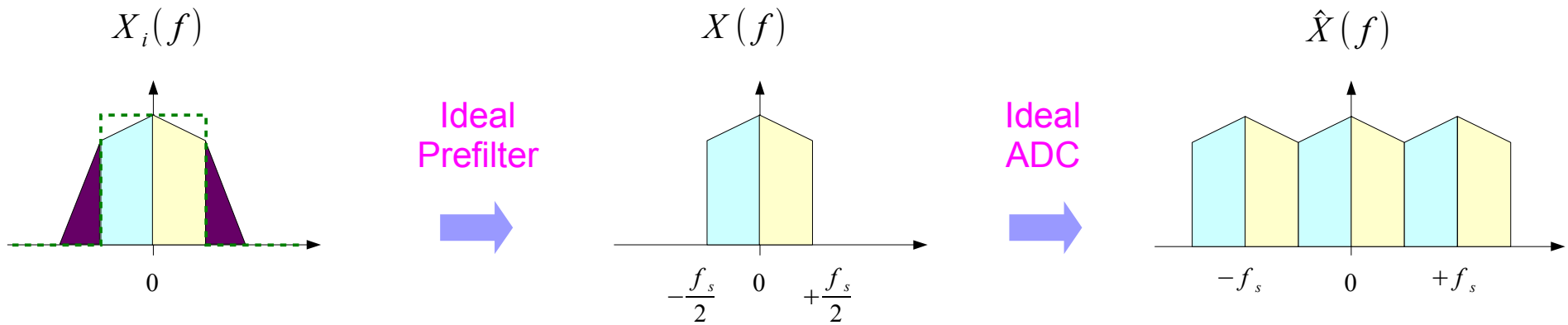
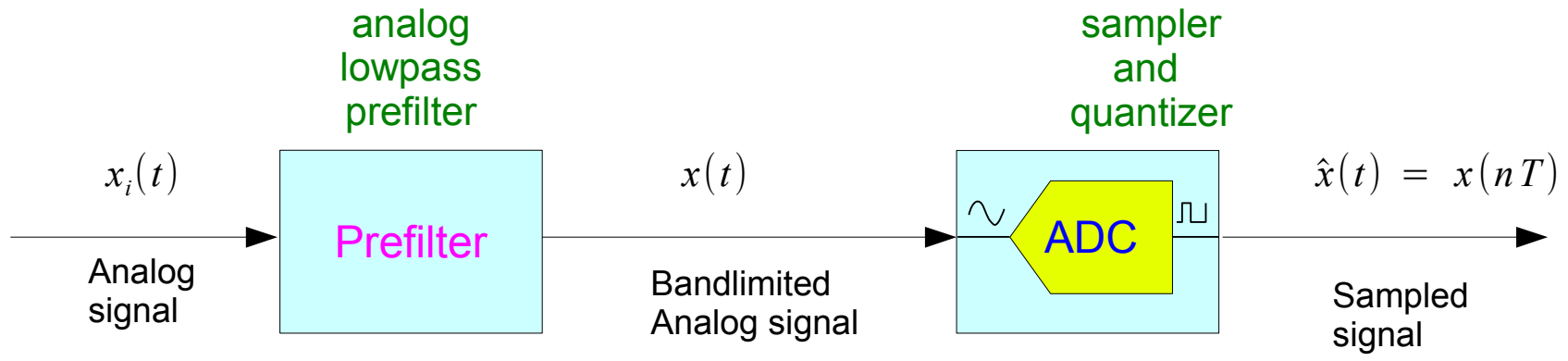
$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

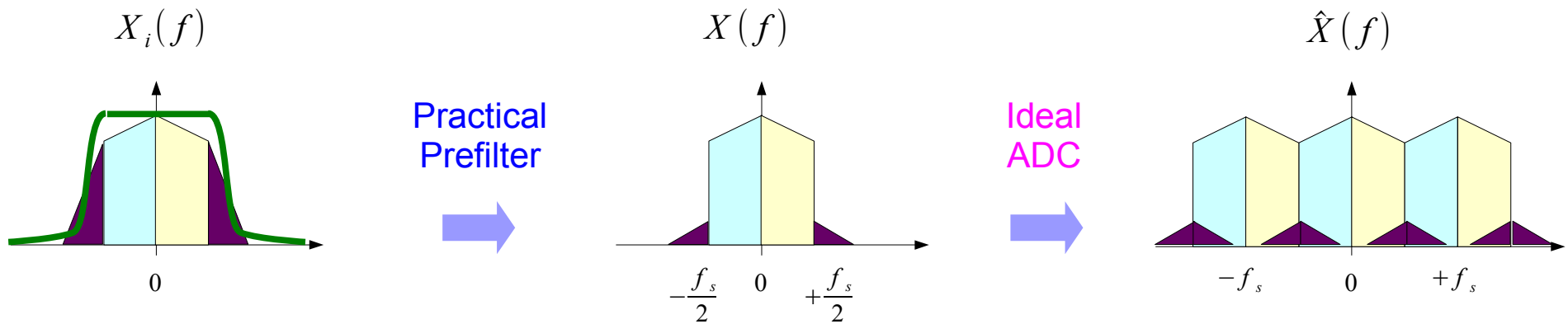
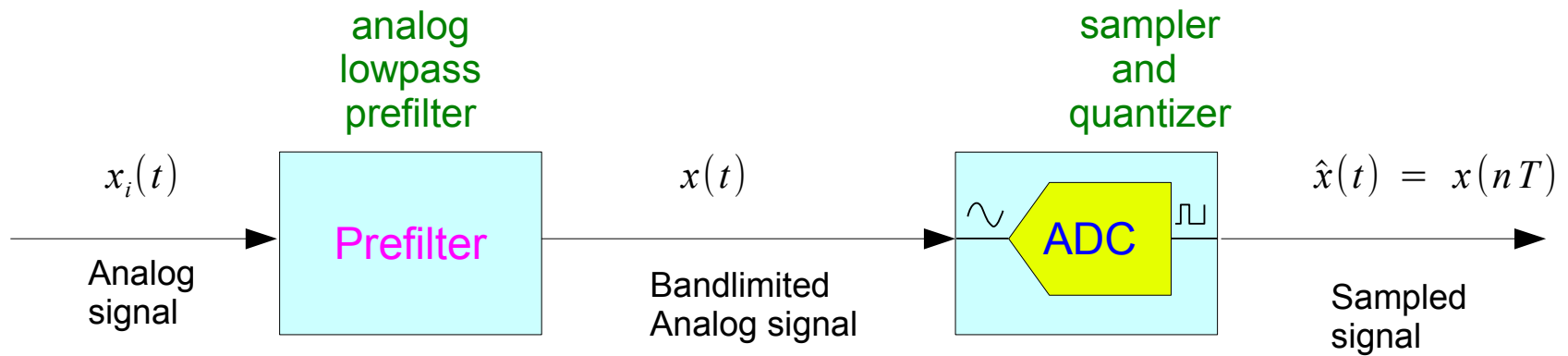


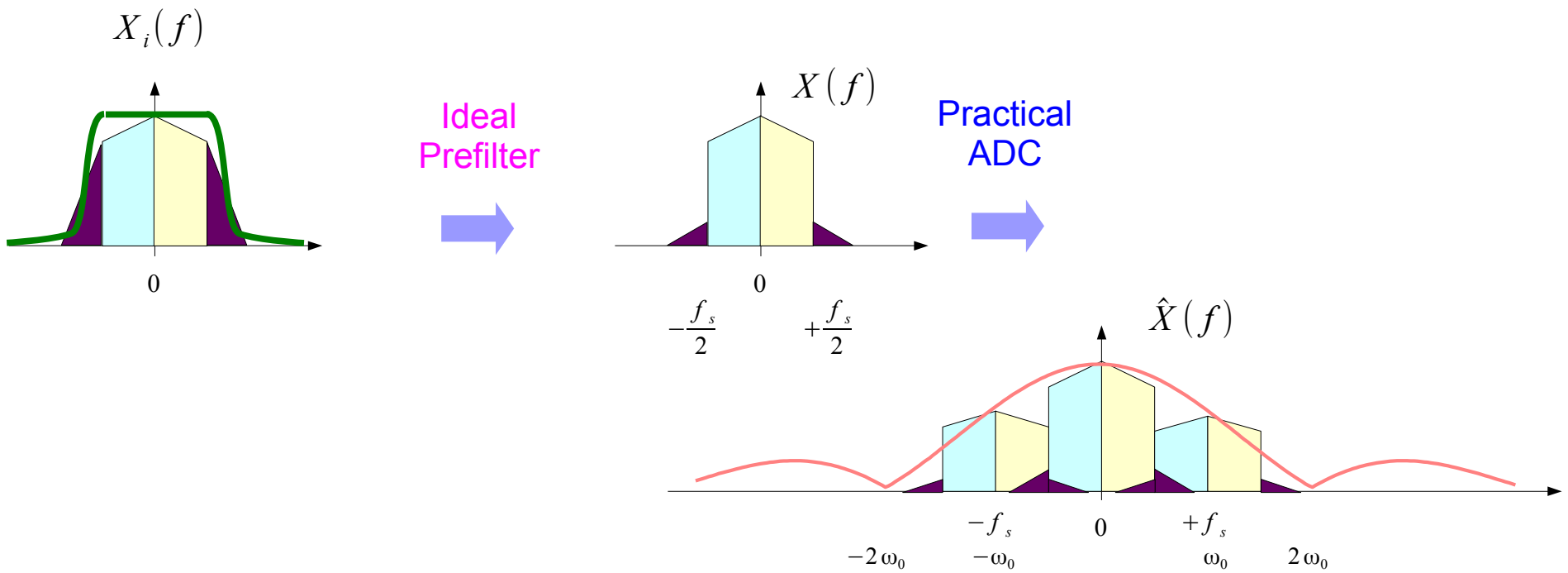
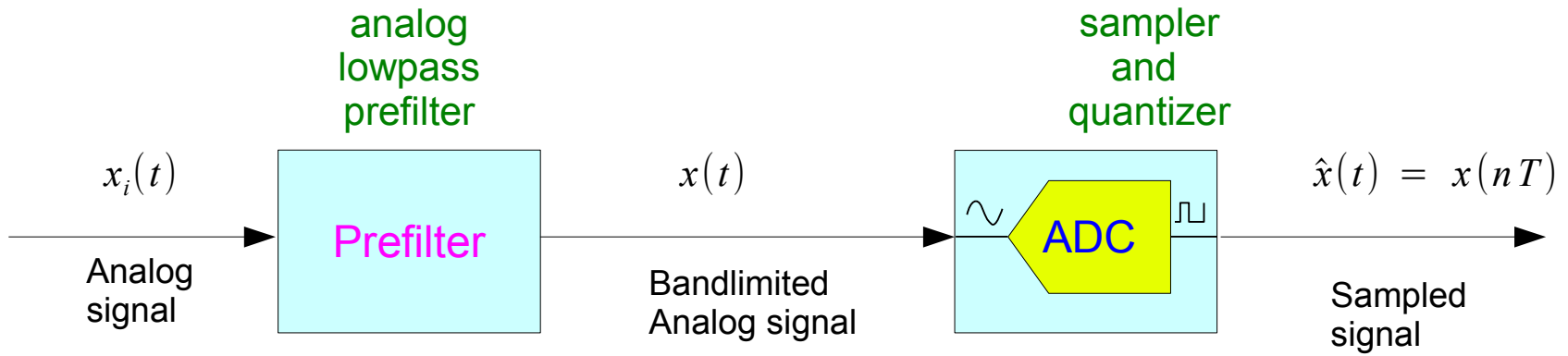
T Sampling Period

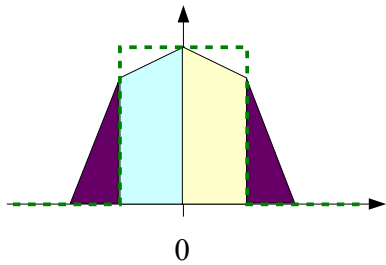








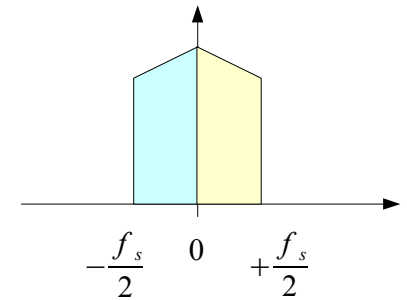




Not bandlimited signal

Lowpass Filter before sampling

Ideal Prefilter



Anti-aliasing Prefilter

Sampling Rate f_s

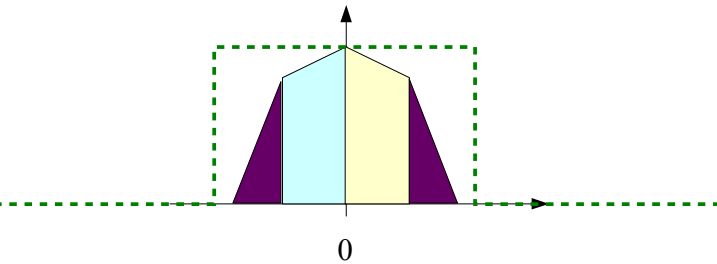
Cutoff Frequency

$$f_{max} \leq \frac{f_s}{2}$$

$$f_{max}$$

Spectrum replication

Bandlimited to max f

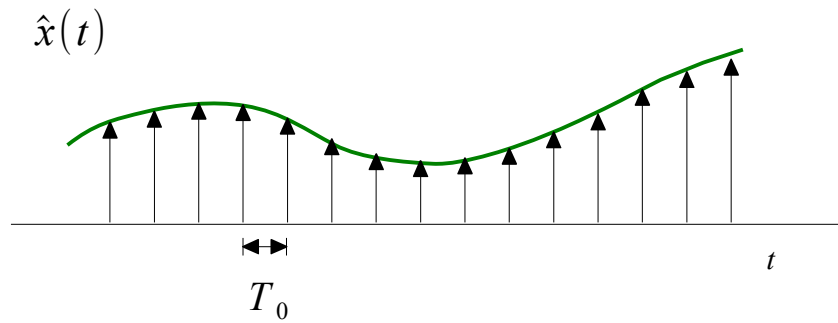


f_s must be chosen high enough so that after prefiltering operation the surviving signal spectrum in the Nyquist interval contain all the significant frequency component

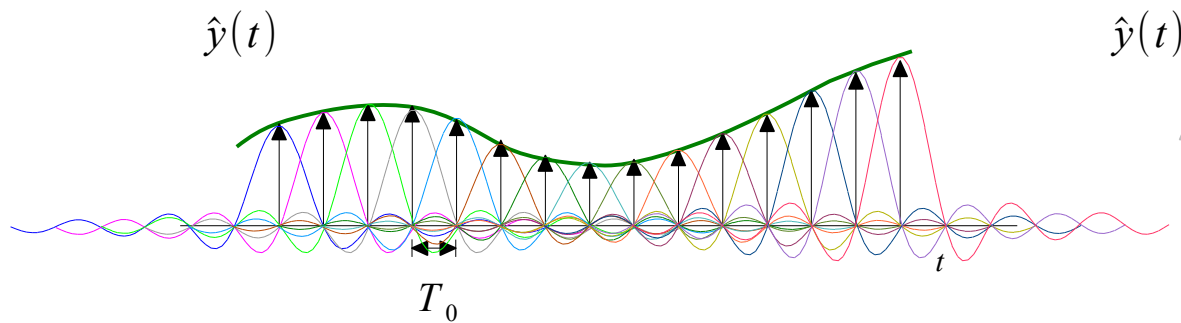
$$\left[-\frac{f_s}{2}, +\frac{f_s}{2} \right]$$

Sampler

Ideal Sampling

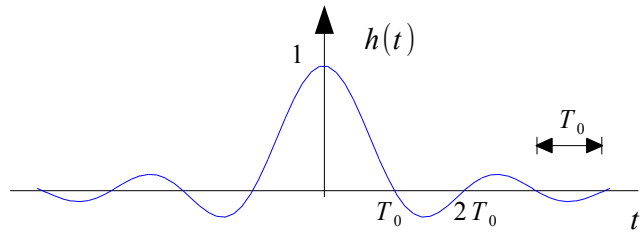


Ideal Reconstruction



CTFT of Reconstructors (1)

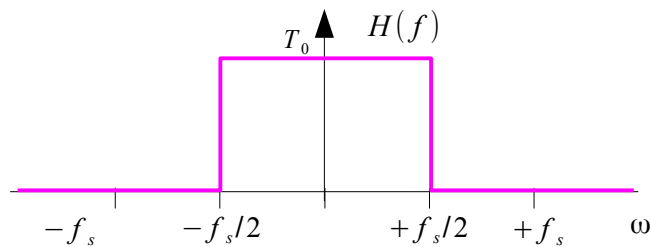
$t = \pm T_0, \pm 2T_0, \pm 3T_0, \dots \rightarrow h(t) = 0$



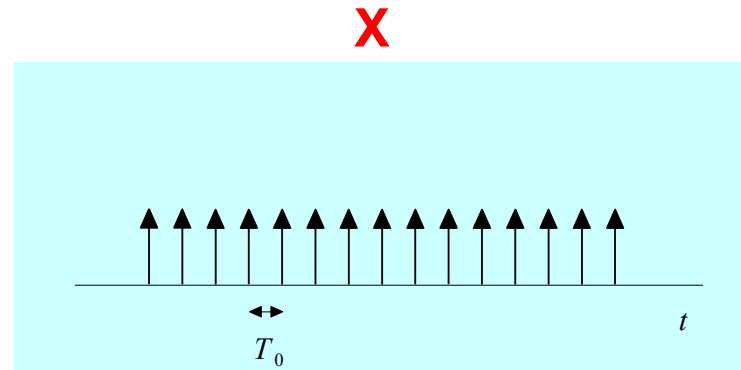
$$\frac{1}{T_0} \equiv f_s$$

$$h(t) = \frac{\sin(\pi t/T_0)}{\pi t/T_0} = \frac{\sin(\pi f_s t)}{\pi f_s t}$$

CTFT

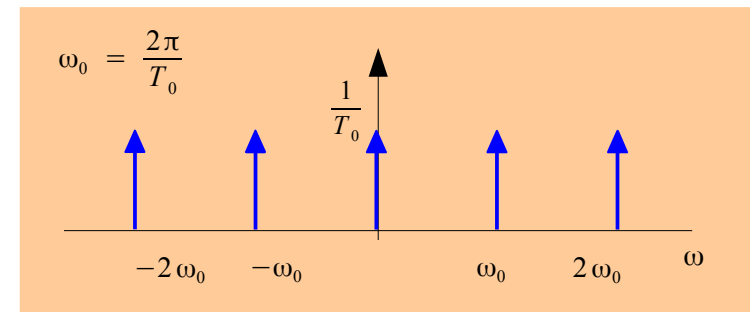


$$H(f) = \begin{cases} T_0, & |f| \leq f_s/2 \\ 0, & \text{otherwise} \end{cases}$$



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CTFT

Sampling (2)

Effect of sampling

$$f, \quad f \pm f_s, \quad f \pm 2f_s, \quad f \pm 3f_s, \quad \dots$$

Replace the original frequency f
With the replicated set of

Ideal reconstructor

Extracts from a sampled signal
All the frequency components
That lie within Nyquist interval

Removes all frequencies outside that interval

Lowpass filter

Cutoff frequency

$$\left[-\frac{f_s}{2}, +\frac{f_s}{2} \right]$$

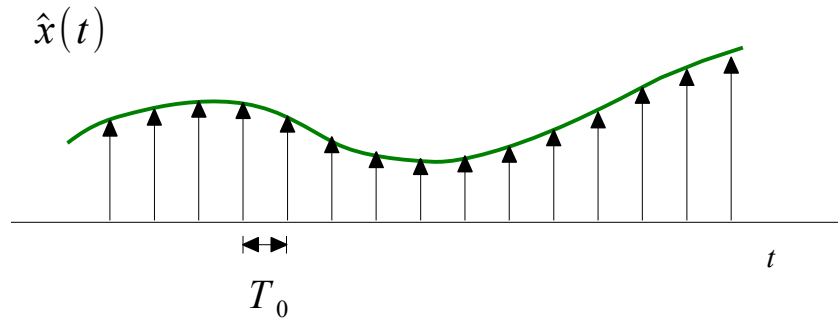
Sampling (2)

Guard Band

$$\delta = f_s - 2f_{max}$$

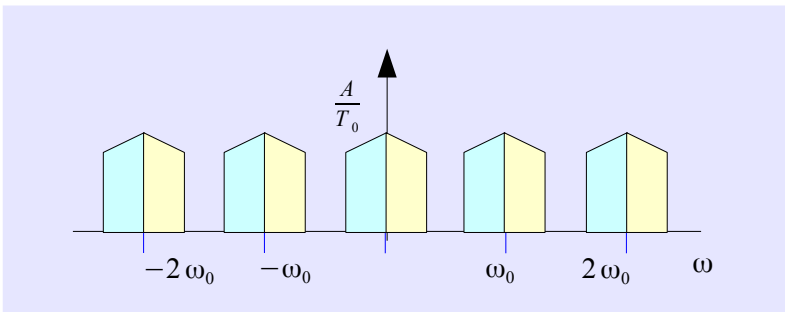
Sampling CTFT

Ideal Sampling

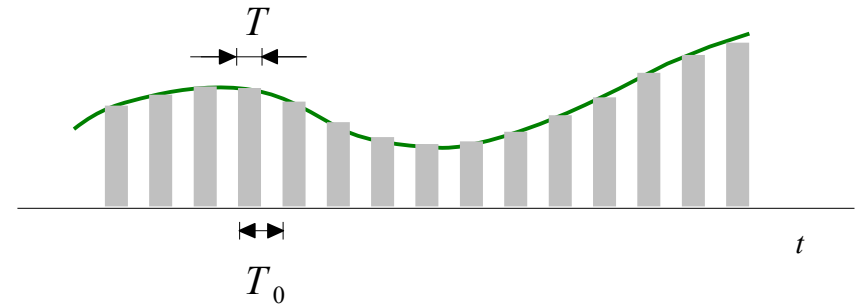


$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT_0) \delta(t-nT_0)$$

↓ CTFT

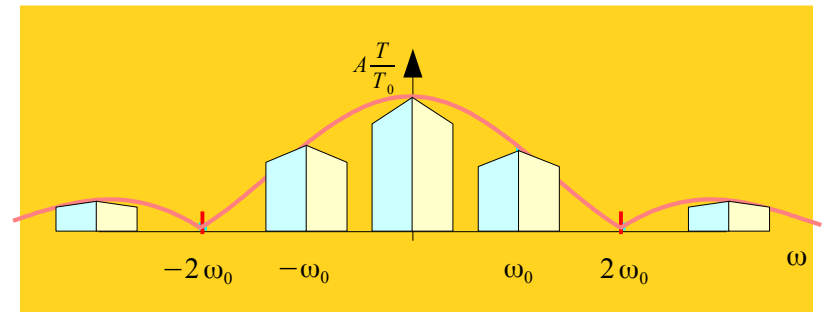


Practical Sampling



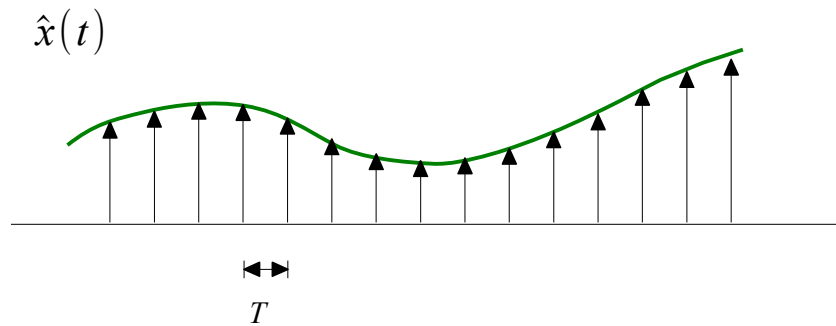
$$\hat{x}(t) \approx \sum_{n=-\infty}^{+\infty} x(nT_0) p(t-nT_0)$$

↓ CTFT



Sampler

Ideal Sampling

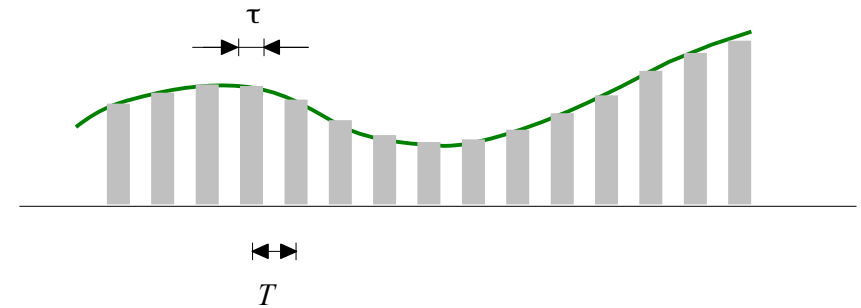


$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT) \delta(t-nT)$$

↓ CTFT

$$\hat{X}(f) = \int_{-\infty}^{+\infty} \hat{x}(t) e^{-j2\pi ft} dt$$

Practical Sampling

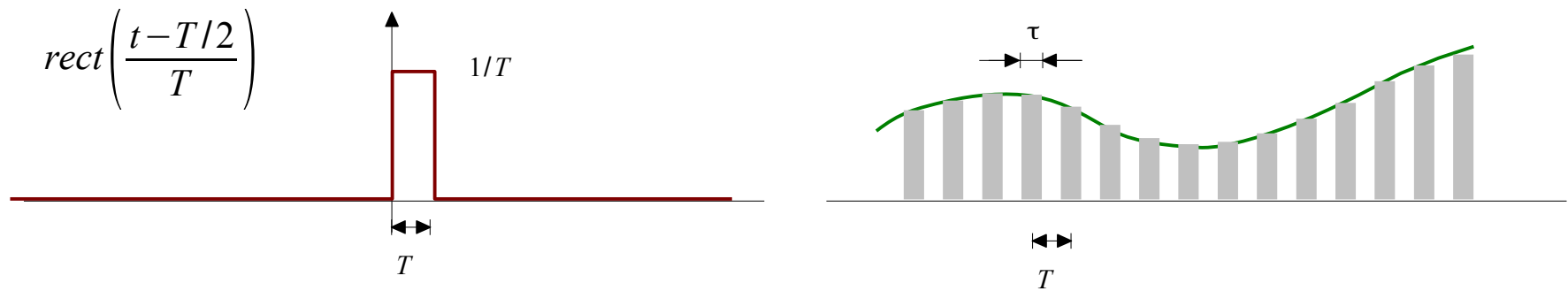


$$\hat{x}(t) \approx \sum_{n=-\infty}^{+\infty} x(nT) p(t-nT)$$

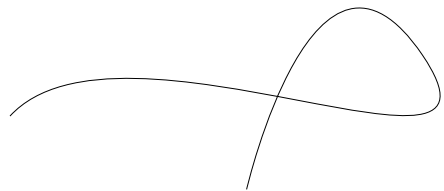
↓ CTFT

$$?$$

Zero Order Hold (ZOH)

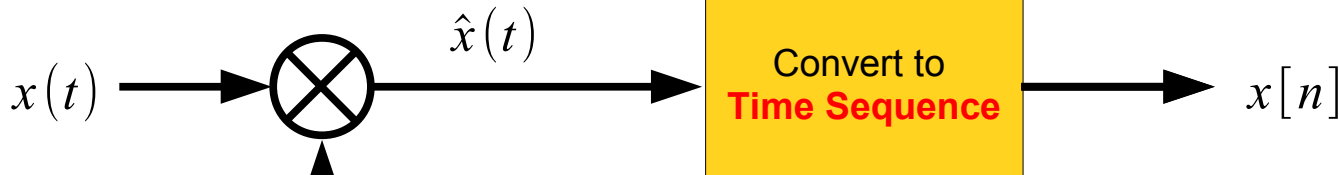


$$x_{ZOH}(t) = \sum_{n=-\infty}^{+\infty} x[n] \cdot \text{rect}\left(\frac{t - T/2 - nT}{T}\right)$$



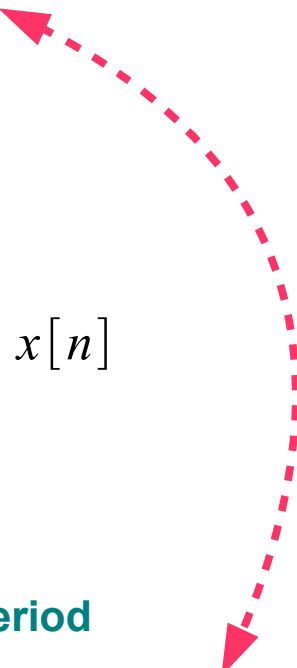
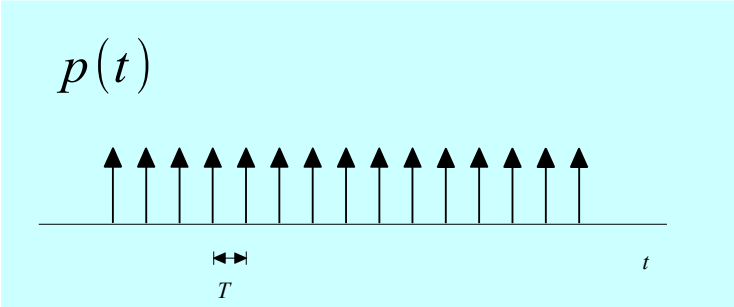
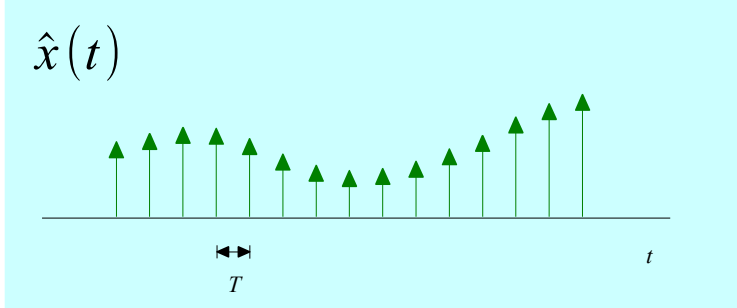
Time Sequence

Ideal Sampling



$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

T Sampling Period



References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] A "graphical interpretation" of the DFT and FFT, by Steve Mann
- [4] R. G. Lyons, Understanding Digital Signal Processing, 1997
- [5] AVR121: Enhancing ADC resolution by oversampling
- [6] S.J. Orfanidis, Introduction to Signal Processing
www.ece.rutgers.edu/~orfanidi/intro2sp