Sequence (3A)

- Partial Sum Sequence
- Difference Sequence
- Summation of k^2, k^3, k^4

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Partial Sum Sequence – Summary

Given Sequence a_n		$b_n = \sum_{k=1}^{n} a_k$		Partial Sum Sequence b_n
$a_n = 2$	$O(1)$	→	$O(n)$	$b_n = 2 \cdot n$
$a_n = n+1$	$O(n)$	→	$O(n^2)$	$b_n = \frac{1}{2}n^2 + \frac{3}{2}n$
$a_n = 3^{(n-1)}$	$O(3^n)$	→	$O(3^n)$	$b_n = \frac{1}{2} \cdot 3^n - \frac{1}{2}$

Partial Sum Sequence (1)

$a_n = 2$

$(a_n) = (2,\ 2,\ 2,\ 2,\ 2,\ 2,\ \cdots\ )$

$$b_n = \sum_{k=1}^{n} a_k$$

$b_1 = 2 \qquad\qquad\qquad\qquad = 2 \cdot 1$

$b_2 = 2+2 \qquad\qquad\qquad = 2 \cdot 2$

$b_3 = 2+2+2 \qquad\qquad = 2 \cdot 3$

$b_4 = 2+2+2+2 \qquad\quad = 2 \cdot 4$

$b_5 = 2+2+2+2+2 \qquad = 2 \cdot 5$

$b_6 = 2+2+2+2+2+2 = 2 \cdot 6$

$b_n = 2 \cdot n$

$(b_n) = (2,\ 4,\ 6,\ 8,\ 10,\ 12,\ \cdots\ )$

Partial Sum Sequence (2)

$$a_n = n + 1$$

$$(a_n) = (2,\ 3,\ 4,\ 5,\ 6,\ 7,\ \cdots\ )$$

$$b_n = \sum_{k=1}^{n} a_k$$

$$b_1 = 2 \qquad = 2$$
$$b_2 = 2+3 \qquad = 2\cdot(2+3)/2$$
$$b_3 = 2+3+4 \qquad = 3\cdot(2+4)/2$$
$$b_4 = 2+3+4+5 \qquad = 4\cdot(2+5)/2$$
$$b_5 = 2+3+4+5+6 \qquad = 5\cdot(2+6)/2$$
$$b_6 = 2+3+4+5+6+7 \qquad = 6\cdot(2+7)/2$$

$$b_n = \frac{1}{2}n^2 + \frac{3}{2}n$$

$$(b_n) = (2,\ 5,\ 9,\ 14,\ 20,\ 27,\ \cdots\ )$$

Partial Sum Sequence (3)

$$a_n = 3^{(n-1)}$$

$$(a_n) = (1, 3, 9, 27, 81, 243, \cdots)$$

$$b_n = \sum_{k=1}^{n} a_k$$

$$b_1 = 1 = (3^1 - 1)/2$$

$$b_2 = 1+3 = (3^2 - 1)/2$$

$$b_3 = 1+3+9 = (3^3 - 1)/2$$

$$b_4 = 1+3+9+27 = (3^4 - 1)/2$$

$$b_5 = 1+3+9+27+81 = (3^5 - 1)/2$$

$$b_6 = 1+3+9+27+81+243 = (3^6 - 1)/2$$

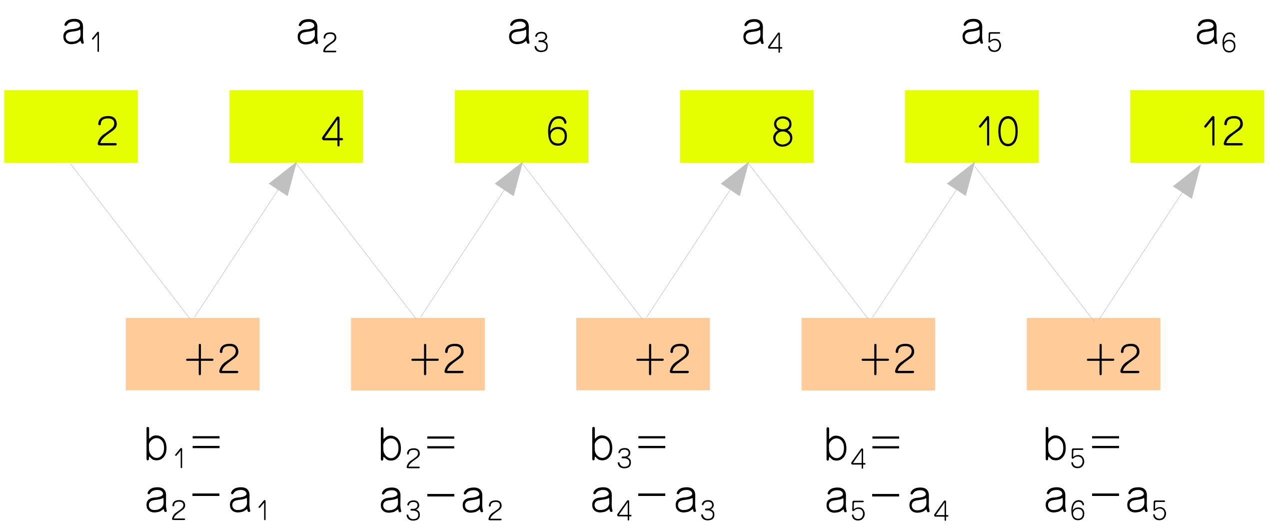
$$b_n = \frac{1}{2} \cdot 3^n - \frac{1}{2}$$

$$(b_n) = (1, 4, 13, 40, 121, 364, \cdots)$$

Difference Sequence – Summary

Given Sequence a_n		$b_n = a_{n+1} - a_n$		Difference Sequence b_n
$a_n = 2 \cdot n$	$O(n)$	→	$O(1)$	$b_n = 2$
$a_n = \frac{1}{2}n^2 + \frac{3}{2}n$	$O(n^2)$	→	$O(n)$	$b_n = n+1$
$a_n = \frac{1}{2} \cdot 3^n - \frac{1}{2}$	$O(3^n)$	→	$O(3^n)$	$b_n = 3^{(n-1)}$

Difference Sequence (1)



$$a_n = a_1 + \sum_{k=1}^{n-1} b_k$$

$$b_n = a_{n+1} - a_n$$

$$\left(\sum_{k=1}^{n-1} b_k = a_n - a_1 \right)$$

$$b_n = a_{n+1} - a_n = 2$$

$$a_n = a_1 + \sum_{k=1}^{n-1} b_k = 2 + \sum_{k=1}^{n-1} 2 = 2 \cdot n$$

Difference Sequence (2)

a_1	a_2	a_3	a_4	a_5	a_6
2	5	9	14	20	27

+3	+4	+5	+6	+7
$b_1 = a_2 - a_1$	$b_2 = a_3 - a_2$	$b_3 = a_4 - a_3$	$b_4 = a_5 - a_4$	$b_5 = a_6 - a_5$

$$a_n = a_1 + \sum_{k=1}^{n-1} b_k$$

$$b_n = a_{n+1} - a_n$$

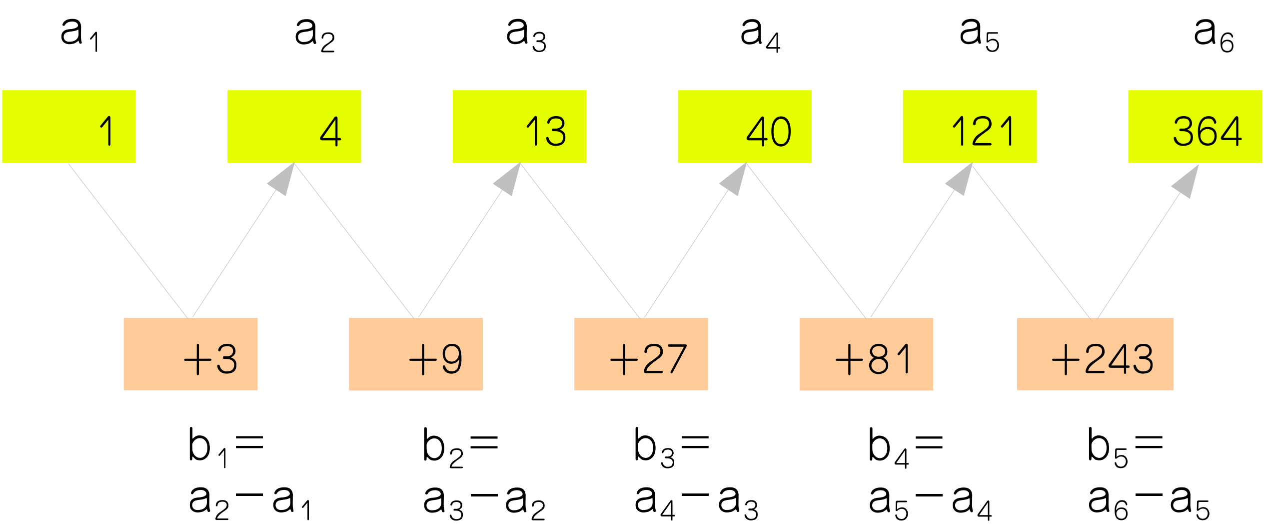
$$\left(\sum_{k=1}^{n-1} b_k = a_n - a_1 \right)$$

$$b_n = a_{n+1} - a_n = n+2$$

$$a_n = a_1 + \sum_{k=1}^{n-1} b_k = 2 + \sum_{k=1}^{n-1} (k+2) = 2 + \frac{(n-1)n}{2} + 2(n-1)$$

$$= \frac{1}{2}n^2 + \frac{3}{2}n$$

Difference Sequence (3)



$$a_n = a_1 + \sum_{k=1}^{n-1} b_k$$

$$b_n = a_{n+1} - a_n$$

$$\left(\sum_{k=1}^{n-1} b_k = a_n - a_1 \right)$$

$$b_n = a_{n+1} - a_n = 3^n$$

$$a_n = a_1 + \sum_{k=1}^{n-1} b_k = 1 + \sum_{k=1}^{n-1} 2^k = 1 + \frac{3(3^{n-1} - 1)}{(3-1)}$$

$$= \frac{1}{2} 3^n - \frac{1}{2}$$

Partial Sum Chain – Observation (1)

$a_n = 6$

$b_n = 6 \cdot n$

$c_n = 3 \cdot n(n+1)$

$d_n = n(n+1)(n+2)$

$O(1)$

$$b_n = \sum_{k=1}^{n} a_k$$

$O(n)$

$$c_n = \sum_{k=1}^{n} b_k$$

$O(n^2)$

$$d_n = \sum_{k=1}^{n} c_k$$

$O(n^3)$

Partial Sum Chain: Observation (2)

1	2	3	4	5	6	7	8	9	10	index
6	6	6	6	6	6	6	6	6	6	$a_n = 6$
6	12	18	24	30	36	42	48	54	60	$b_n = \sum_{k=1}^{n} a_k = 6 \cdot n$
6	18	36	60	90	126	168	216	270	330	$c_n = \sum_{k=1}^{n} b_k = 3 \cdot n \cdot (n+1)$
6	24	60	120	210	336	504	720	990	1320	$d_n = \sum_{k=1} c_k = n \cdot (n+1) \cdot (n+2)$

Summation of Consecutive Integer Product

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}$$

$$\sum_{k=1}^{n} k(k+1)(k+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

$$\sum_{k=1}^{n} k(k+1)(k+2)(k+3) = \frac{n(n+1)(n+2)(n+3)(n+4)}{5}$$

Summation of k, k^2, and k^3

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \left\{\frac{n(n+1)}{2}\right\}^2 = \left\{\sum_{k=1}^{n} k\right\}^2$$

Summation of k^2

$$\sum_{k=1}^{n} k^2 = \sum_{k=1}^{n} k(k+1) - \sum_{k=1}^{n} k$$

$$= \frac{n(n+1)(n+2)}{3} - \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)(2n+1)}{6}$$

Summation of k^3

$$\sum_{k=1}^{n} k^3 = \sum_{k=1}^{n} k(k+1)(k+2) - 2\cdot\sum_{k=1}^{n} k(k+1) - \sum_{k=1}^{n} k^2$$

$$= \frac{n(n+1)(n+2)(n+3)}{4} - \frac{n(n+1)(n+2)\cdot 2}{3} - \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)}{12}\{3(n^2+5n+6)-8(n+2)-2(2n+1)\}$$

$$= \frac{n(n+1)}{12}\{3n(n+1)\}$$

$$= \left\{\frac{n(n+1)}{2}\right\}^2$$

References

[1] http://en.wikipedia.org/
[2] http://planetmath.org/
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[4] Smith, R. T., Minton, R. B. "Calculus: Concepts & Connections," Mc Graw Hill
[5] 홍성대, "기본/실력 수학의 정석," 성지출판