# Calculations

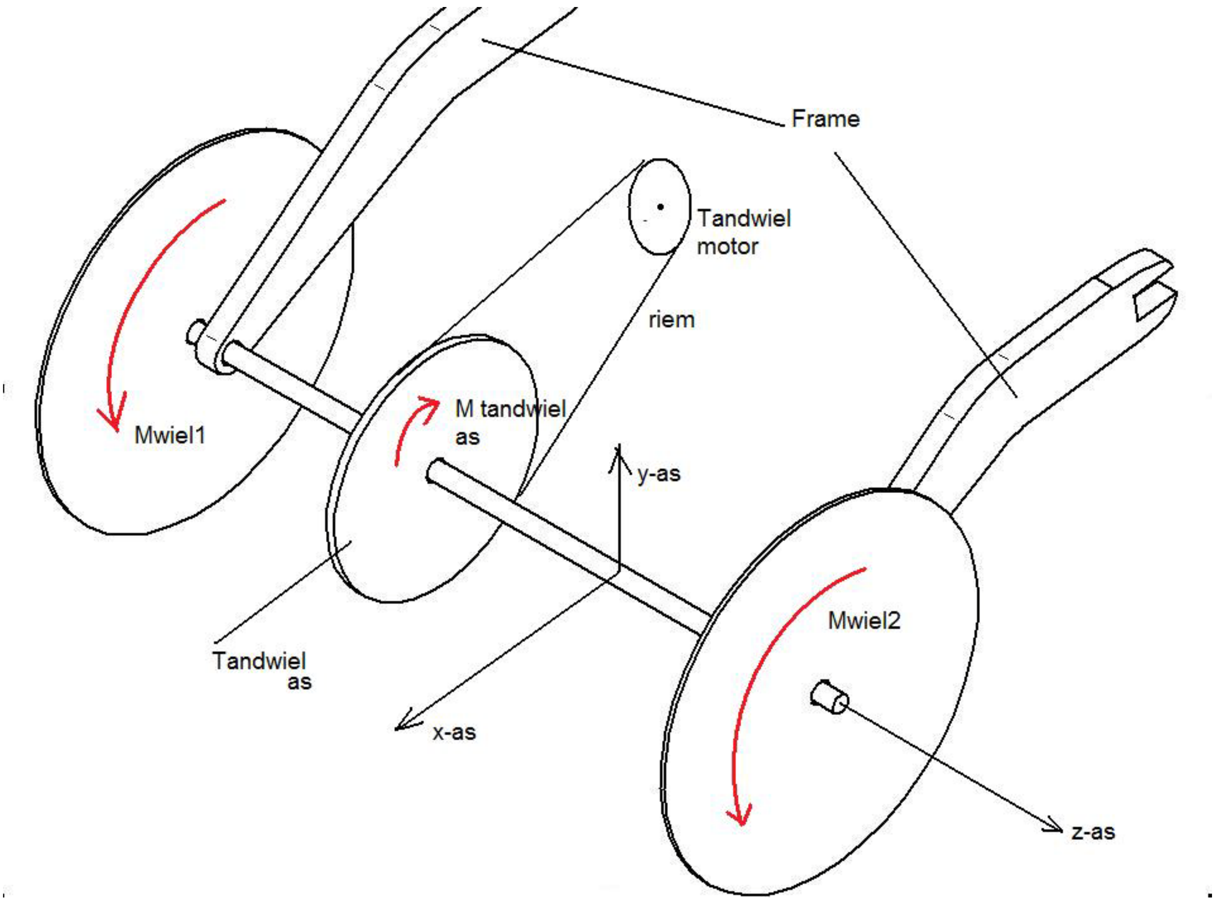
## 1. Influences on the mechanical stress of the shaft

The following elements are part of the mechanical stress on the shaft, assuming we hold the vehicle stil with our hand:

- Weight of the frame on the shaft
- Normal force on the shaft (reaction of the ground against the weight of the vehicle)
- Bending stresses
- Torsion stresses
- Shear stresses
- Reaction moment of the ground against the momentum in the gear
- Pushing forces (exterior force to keep the vehicle in place), moments on the wheels and transmission

## 2. Stressed parts

*A.Sketch of the moments around the z-axis*



*B.Moments in the motor-to-gear and shaft-to-wheel transmission.*

- The torsion constant of the motor is equal to $T_{motor}$ = 8,55mNm/A.
- The current created by the solar panel at maximum power equals:
  I = 0,76 A (measurement in week 2)

Hence the motor will transmit a moment of 8,55.0,76 = 6,498 mNm to the gear attached on it.

Next, we calculate the torque created by this moment:

$$F_{gear\ motor} = F_{gear\ shaft} = T_{motor}/r_{gear\ motor}$$

$$F_{gear\ motor} = 0,006498/0,0045 = 1,44N$$

From this torque we derive the moments around the wheel-shaft (z-as):

$$M_{gear\ shaft} = 1,44N.0,026m = 0,0375Nm$$

The sum of the moments around the z-as should equal 0:

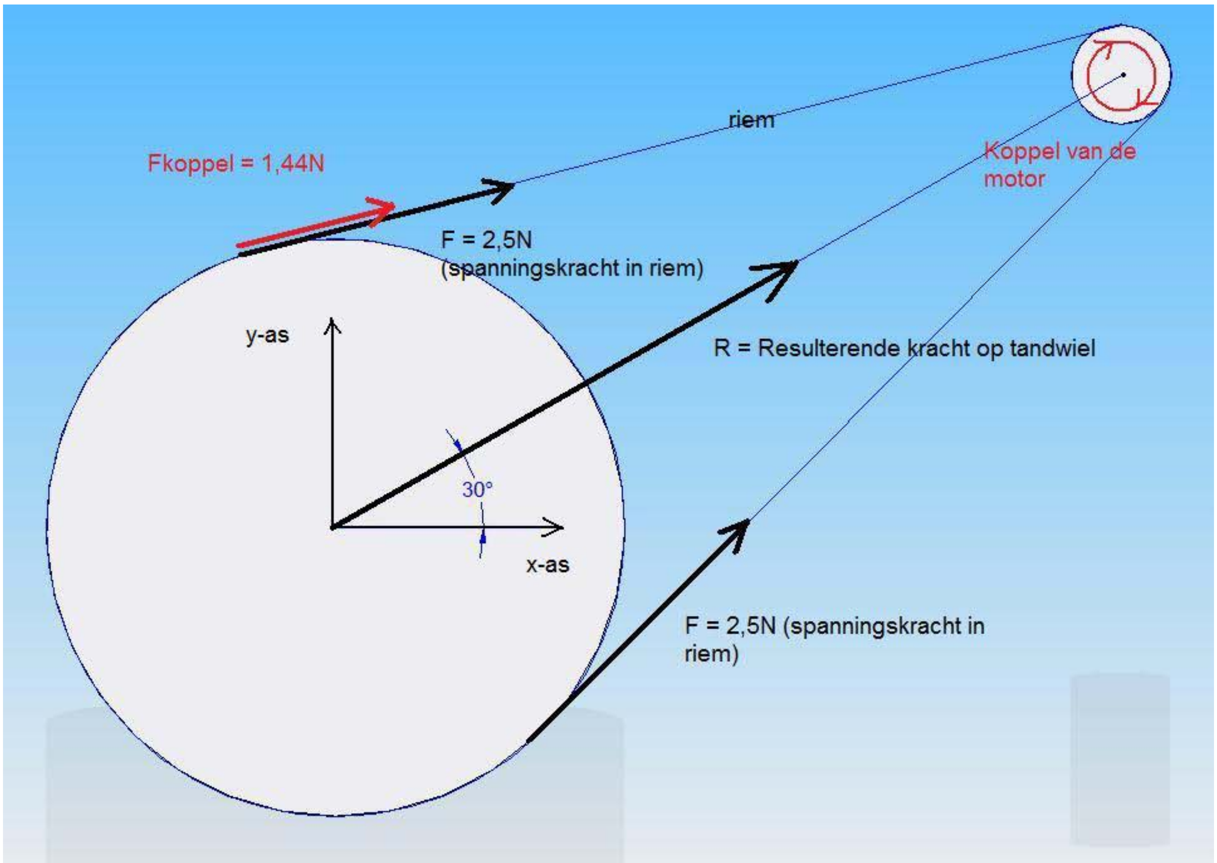
$$M_{gear\ shaft} - M_{wheel1} - M_{wheel2} = 0$$

$$M_{wheel\ 1} = M_{wheel\ 2}$$

$$M_{gear\ shaft} = 2.M_{wheel1}$$

$$M_{wheel1} = 0,0375Nm/2 = 0,01875Nm = M_{wheel2}$$

*C.Force R on the gear due to the belt tension and the motor torque*




This force R results from:

- The tension force in the belt (rubber)
- The torque transmitted by the motor

When the belt is tightened around the gears, the belt is being stretched about 10% compared to its normal state. From previous measurements we are able to state that this 10% stretch corresponds to 0,625 N/mm² tension in the belt, meaning the belt is being pulled with a force of 2,5 N on both sides ($A_{belt}$ = 4 mm² ). If we don't count the motor torque, the gear experiences a force R of:

$$2.\cos(15°).2{,}5N = 4{,}83N$$

Furthermore, the torque pulls the belt with an additional force of 1,44N (however, the belt can't set pushing forces, only pulling). Thus, a part of the belt will relatively experience more tension. We calculate the component according to the R-line:

$$\cos(15°).1{,}44N = 1{,}39N$$

Hereby we neglect the component perpendicular to the R-line since its negligibly small (sin(15°).1,44N = 0,37N).

The total R force should equal:

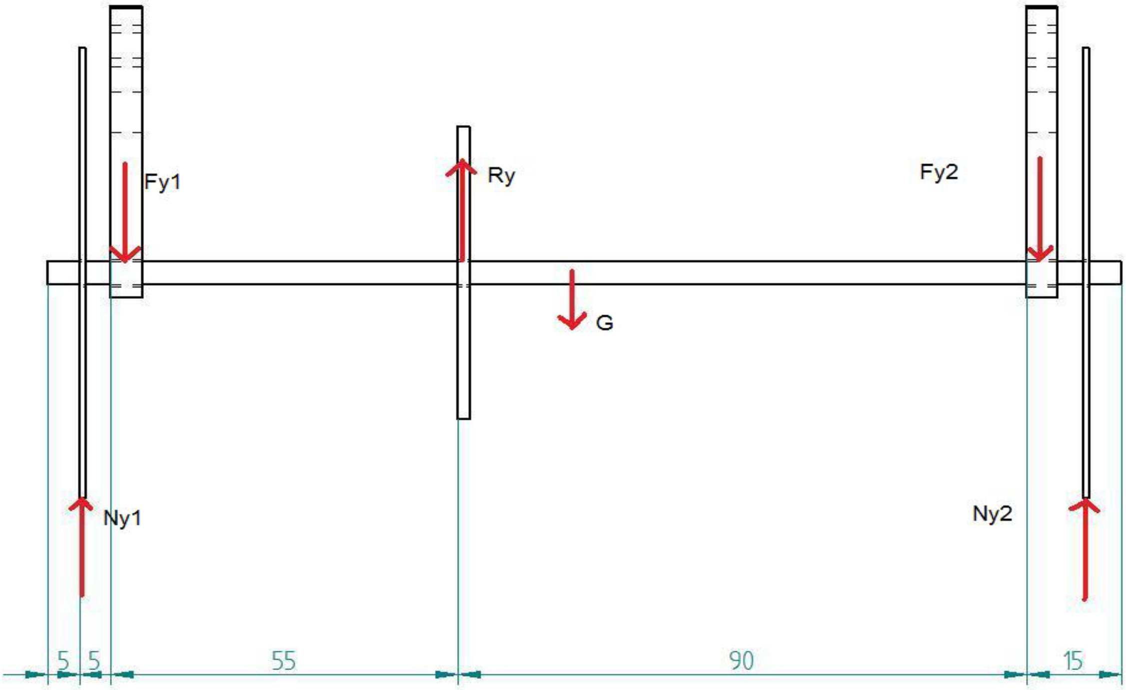
$$4{,}83N + 1{,}39N = 6{,}22N$$

And the components according to the x-axis and y-axis:

$$R_x = \cos(30°).R = 5{,}39N$$

$$R_y = \sin(30°).R = 3{,}11N$$

*D.Forces in the y-direction*




We free the shaft, wheels and gear from the rest of the vehicle and consider these as our working system. The working forces in the y-direction are:

- Weight of shaft, gear and wheels (G)
- Weight of the frame on the shaft ($F_{y1}$ en $F_{y2}$)
- Normal forces ($N_{y1}$ en $N_{y2}$)
- Forces created by the tension in the belt transmission ($R_y$)

We assume the vehicle shape is symmetric and approximately half the weight of the car stresses on the back-shaft. The mass of the vehicle is 870 g, the mass of the wheels, gear and shaft (G) is negligible. We calculate the normal forces $N_{y1}$ en $N_{y2}$ out of the measured weight of the vehicle:

$$N_{y1} = N_{y2} = 0{,}87kg.9{,}81\ m/s^2.\ 0{,}5.0{,}5 = 2{,}13\ N$$

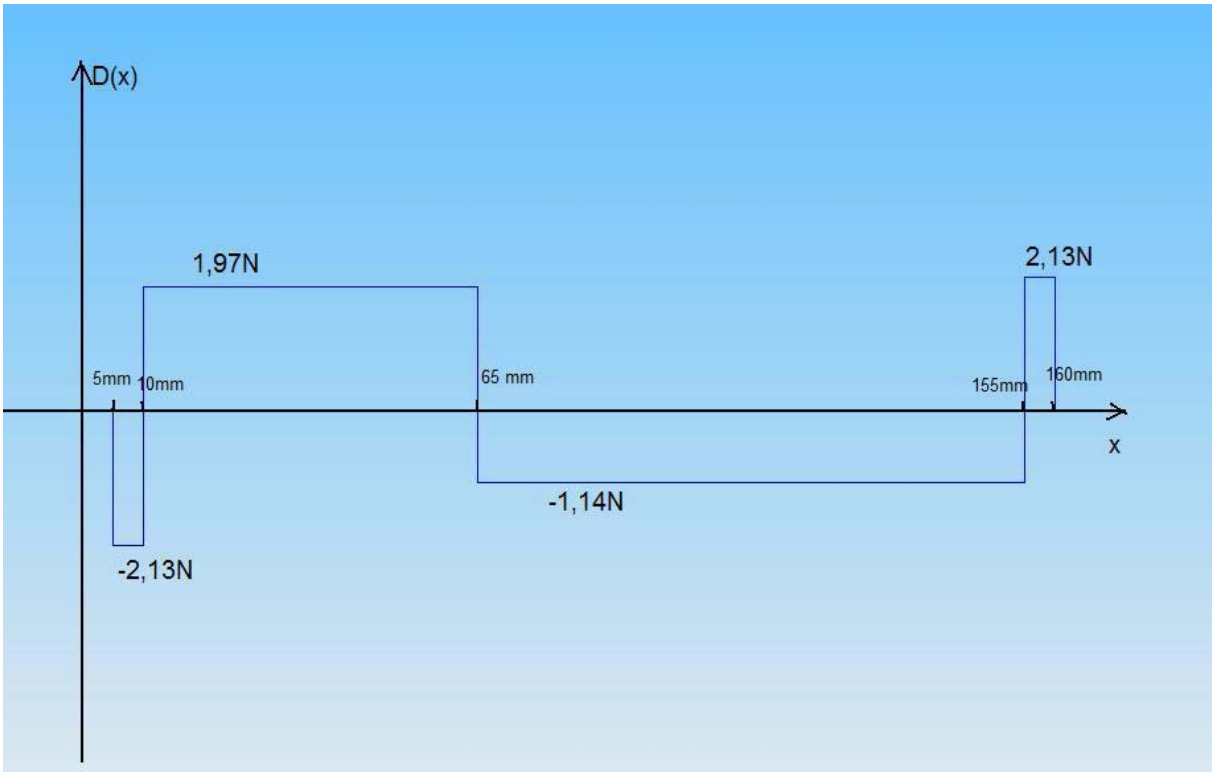
Next, we become a system of two equations by assuming the sum of all forces in the y-direction equals zero and the sum of the moments around the x-axis also equals zero. (take the $N_{y1}$-line as a reference).

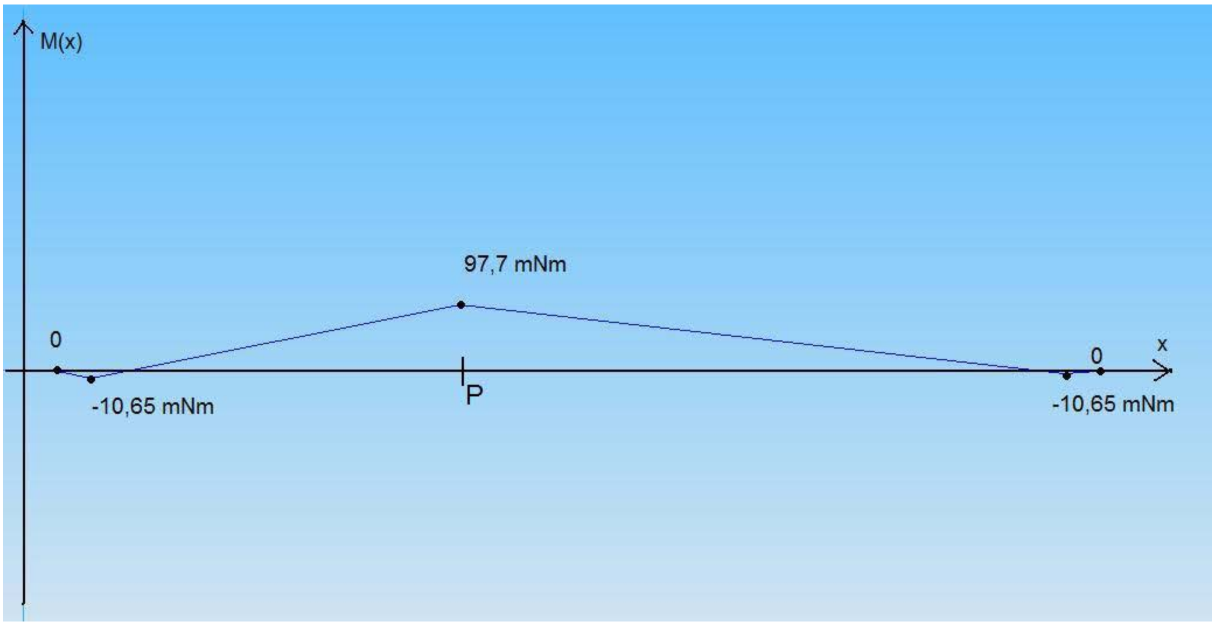
- $N_{y1} + N_{y2} - F_{y1} - F_{y2} + R_y = 0$
- $(-0,005).F_{y1} + 0,06.\ R_y - 0,155.F_{y2} + 0,16.N_{y2} = 0$

Resolving this sytem we become:

- $N_{y1} = N_{y2} = 2,13$ N
- $R_y = 3,11$ N
- $F_{y1} = 4,1$ N
- $F_{y2} = 3,27$ N

*Lateral force- & moment-lines according to the y-axis:*

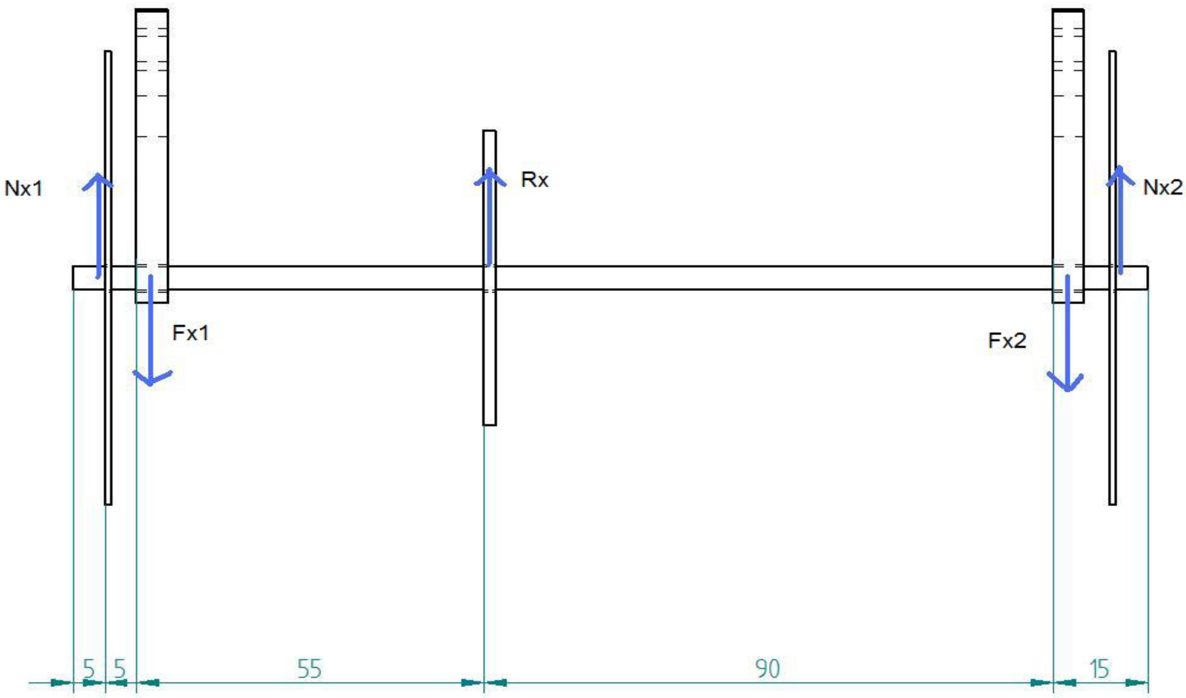





$$D_{y\text{-}max} = 1{,}97 \text{ N}$$

$$M_{y\text{-}max} = 97{,}7 \text{ mNm}$$

*E.Forces in the x-direction*




We free the shaft, wheels and gear from the rest of the vehicle and consider these as our working system.

The working forces in the x-direction are:

- Pushing forces of the frame against the shaft in order to hold stil the vehicle. ($F_{x1}$ en $F_{x2}$)
- Reaction forces of the ground due to the torque in the wheels ($N_{x1}$ en $N_{x2}$)
- Forces created by the tension in the belt transmission ($R_x$)

We calculate the force the wheel exercises on the ground this way: we know  the moments on the 2 wheels equal $M_{wiel1}$=$M_{wiel2}$= 0,01875Nm, we divide this amount by the radius of the wheels and what we obtain is the force the wheels exercise on the ground: F = 0,46875N. The reaction force of the ground against the wheels is hence equal in magnitude, but contrary in direction:

$$N_{x1} = N_{x2} = 0{,}46875 \text{ N}$$

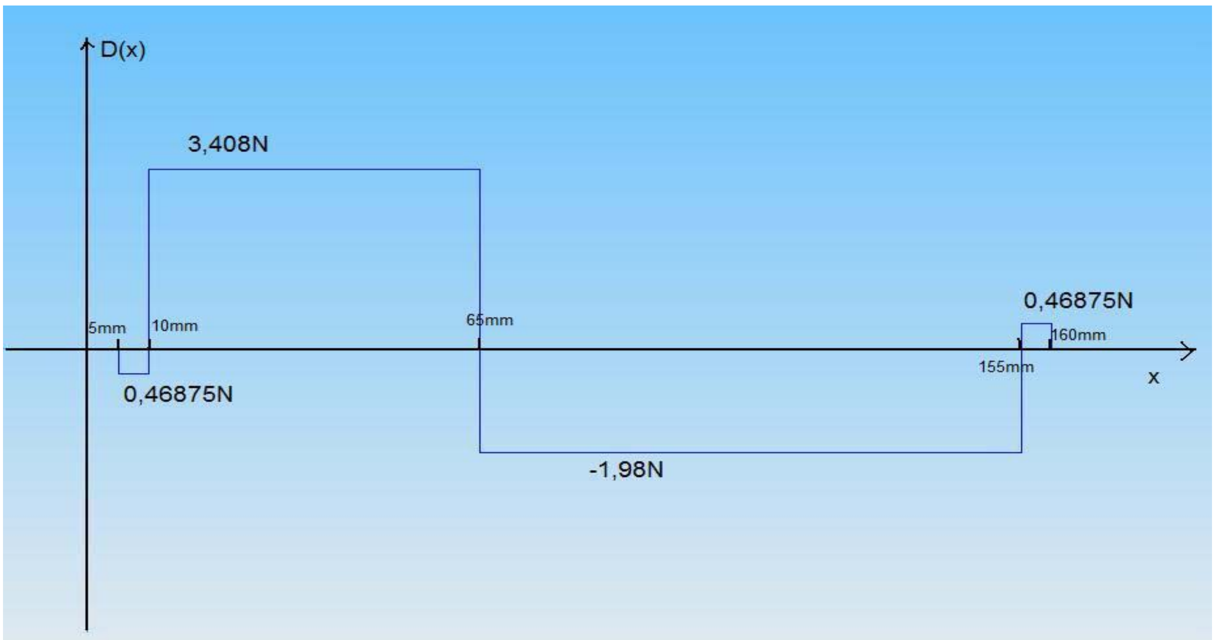
Next, we become a system of two equations by assuming the sum of all forces in the y-direction equals zero and the sum of the moments around the x-axis also equals zero. (take the $N_{x1}$-line as a reference).

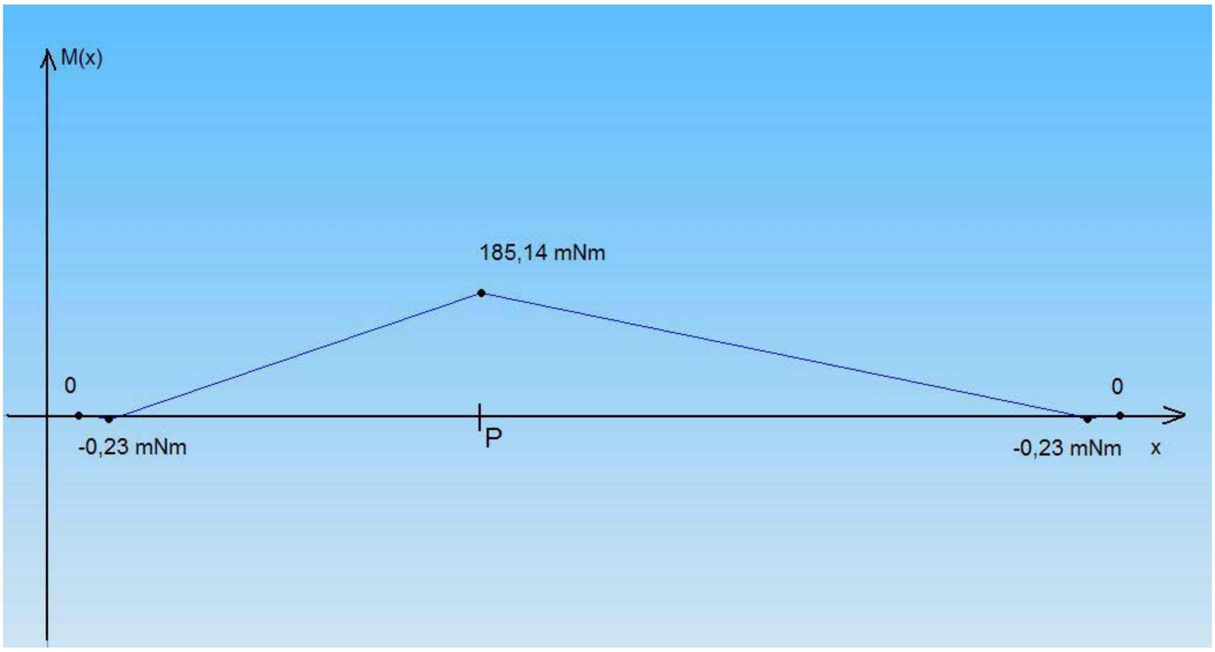
- $N_{x1} + N_{x2} - F_{x1} - F_{x2} + R_x = 0$
- $(-0{,}005).F_{x1} + 0{,}06.R_y - 0{,}155.F_{x2} + 0{,}16.N_{x2} = 0$

Resolving this sytem we become:

- $N_{x1} = N_{x2} =$ 0,46875 N
- $R_y$ = 5,39 N
- $F_{x1}$ = 3,88 N
- $F_{x2}$ = 2,45 N

*Lateral force- & moment-lines according to the x-axis:*






$$D_{x\text{-}max} = 3{,}408\ N$$

$$M_{x\text{-}max} = 185{,}14\ mNm$$

## 3. Maximal Shear Stresses

We find the biggest shear stress on the place where the biggest lateral forces are working, and since the radius of the shaft doesn't change, we use the next formula:

$$T_{max} = D_{max}/A$$

- The diameter of the shaft is 0,004 m
- The surface of a cross section of the shaft equals: $pi.0{,}002^2 = 1{,}256.10^{-5}m^2$
- We find the maximal lateral forces between the left bearing and the shaft-gear. It is the resulting force of $D_{x\text{-}max}$ en $D_{y\text{-}max}$.

$$D_{max} = \sqrt{Dxmax^2 + Dymax^2} = 3{,}93N$$

$$T_{max} = 3{,}93\ N\ /\ 1{,}256.10^{-5}m^2 = 313\ kPa$$

## 4. Maximal Bending Stress

The moment-lines are maximal amounting point P (ref. images). This point P matches the location of the gear on the shaft. We now calculate $M_{max}$ out of $M_{x\text{-}max}$ and $M_{y\text{-}max}$ with Pythagoras' law:

$$M_{max} = \sqrt{Mxmax^2 + Mymax^2} = 209{,}33 \text{ mNm}$$

The <u>maximal bending stresses</u> are to calculate with the next formula:

$$\sigma \max = \frac{\text{M max. y}}{\text{I}}$$

where:

- $M_{max}$ is the resulting moment
- y (0,002 m) is the distance between the point (where the maximal stresses rule) and the neutral line (the line where no tensions exercise)
- I is the surface moment of inertia ($1{,}26.1^{0\text{-}11}$)

When we fill in these values, we obtain for $\sigma_{max}$ :

$$\sigma_{max} = 33{,}33 \text{ MPa.}$$

## 5. Maximal Torsion Stress

We get maximal torsion stresses amounting the transmission-gear on the shaft. We already calculated the gear has a moment of $M_{tandwiel\ as}$ = 0,0375 Nm.

So the torsion stresses are to calculate with the next formula:

$$\text{T max} = \frac{\text{M (gear shaft). r}}{\text{Ip}}$$

where:

- $M_{gear\text{-}shaft}$ is the moment around the gear
- r is the radius of the shaft (0,002m)
- $I_p$ is the moment of inertia($2{,}51.10^{-11}$)

When we fill in these values, we obtain fot $T_{max}$ :

$$T_{max} = 2{,}988 \text{ MPa.}$$

## 6. Examination of the vehicle in dynamic state

When we remove our imaginary hand holding the vehicle stil, some elements will be different. We investigate them here below.

1. When we put our hand against the vehicle to stop it, we exercise a force -F on the frame which is contrary to the force F pushing the vehicle forward due to the gear torque. When we remove our hand, the reaction forces, frame against shaft, will become smaller in magnitude and <u>the sum of the forces in the x-direction won't equal zero anymore.</u> The vehicle would thus experience an acceleration a = F/m, the shaft is being translated in the x-direction in relation with the ground, also the drive-direction of the vehicle.
2. The sum of the moments around the z-axis won't equal zero anymore, the wheel-shaft experiences a resulting moment M, forcing it into rotation in clockwise direction
3. Air resistance, a force pointing against the driving-direction of the vehicle
4. Ground friction, a force pointing against the driving-direction of the vehicle