$$I = \int_0^1 \frac{e^x - 1}{x}\, dx$$ also Atk. 5.0.6 p 250

$$I_n = \int_0^1 \sum_{j=1}^{n} \frac{x^{j-1}}{j!}\, dx$$ class notes 2/2 eq.(1)

$$= \sum_{j=1}^{n} \int_0^1 \frac{x^{j-1}}{j!}\, dx = \sum_{j=1}^{n} \frac{x^j}{j!\, j}\Big|_0^1 = \sum_{j=1}^{n} \frac{1}{j!\cdot j}$$

as shown in 1.1.4 (Atk.)

$$f(x) - f_n(x) = \frac{x^n}{(n+1)!} e^{\xi}$$

$$I - I_n = \int_0^1 \frac{x^n}{(n+1)!} e^{\xi}\, dx \Rightarrow I - I_n \le \frac{e}{(n+1)!\,(n+1)}$$

Atk 5.0.8

For Trap. $x_0 = 0$ $x_n = 1$

$$I_n(f) = h\left(\tfrac{1}{2} f_0 + f_1 + f_2 + \dots f_{n-1} + \tfrac{1}{2} f_n\right)$$

$$E_n(f) = I(f) - I_n(f) = -\frac{h^3 n}{12} \frac{1}{n} \sum_{j=1}^{n} f''(\eta_j) \quad (5.1.6)$$

$$E_n(f) = -\frac{(b-a)h^2}{12} f''(\eta) \quad (5.1.7)$$

$\eta \in [a, b]$

Max $f''(\eta)$ whe $\eta = b = 1$

$$f'' = e^1 (1 - \cancel{2} + \cancel{2}) - 2 = e - 2$$

Simpson

$$I_n(f) = \frac{h}{3}\left(f_0 + 4f_1 + 2f_2 + 4f_3 + 2f_4 + \dots \right.$$
$$\left. 2f_{n-2} + 4f_{n-1} + f_n\right)$$

for $n =$ even 5.1.16

$$E_n(f) = I(f) - I_n(f) = \frac{h^5 (n/2)}{90} \cdot \frac{2}{n} \sum_{j=1}^{n/2} f''''(\eta_j)$$

5.1.17

or $E_n(f) = \frac{-h^4}{180}(b-a) f''''(\eta)$ $\eta \in [a, b]$

note that $f_0 = f(x=0) = 1$

$f(x) = \frac{e^x}{x} - \frac{1}{x}$ was shown in previous HW

$$f' = \frac{df}{dx} = -e^x x^{-2} + e^x \cdot x^{-1} + x^{-2} = e^x\left(\frac{1}{x} - \frac{1}{x^2}\right) + \frac{1}{x^2}$$

$$f'' = e^x\left(\frac{1}{x} - \frac{1}{x^2}\right) + e^x\left(-x^{-2} + 2x^{-3}\right) - \frac{2}{x^3}$$

$$f'' = e^x\left(\frac{1}{x} - \frac{2}{x^2} + \frac{2}{x^3}\right) - \frac{2}{x^3}$$

$$f''' = e^x\left(\frac{1}{x} - \frac{2}{x^2} + \frac{2}{x^3}\right) + e^x\left(-\frac{1}{x^2} + \frac{4}{x^3} - \frac{6}{x^4}\right) + \frac{6}{x^4}$$

$$= e^x\left(\frac{1}{x} - \frac{3}{x^2} + \frac{6}{x^3} - \frac{6}{x^4}\right) + \frac{6}{x^4}$$

$$f'''' = e^x\left(\frac{1}{x} - \frac{3}{x^2} + \frac{6}{x^3} - \frac{6}{x^4} - \frac{1}{x^2} + \frac{6}{x^3} - \frac{18}{x^4} + \frac{24}{x^5}\right) - \frac{24}{x^5}$$

$$= e^x\left(\frac{1}{x} - \frac{4}{x^2} + \frac{12}{x^3} - \frac{24}{x^4} + \frac{24}{x^5}\right) - \frac{24}{x^5}$$

$$\text{Max } f''''(\eta) = f''''(\eta = b) = e\left(1 - 4 + 12 - \cancel{24} + \cancel{24}\right) - 24$$

$$= 9e - 24 = 0.4645$$

1.8.3


```
% TITLE: Integration of for (exp(x)-1)/x about x = 0
%
% Initialize

Maxn=130;
ic=1;
n=2; % start n
cnt(1)=2; % vector that holds the n that needs to be locked at

while ((n*2)<Maxn)
    ic=ic+1;
    n=n*2;
    cnt(ic)=n;
 end

ice=ic; % The number of n locations to be analyzed up to and below Maxn

for i=1:1:Maxn
    I1(i)=0;
    I2(i)=0;
    I3(i)=0;
end
for i=1:1:Maxn
    EI1(i)=0;
    EI2(i)=0;
    EI3(i)=0;
end

% Interval
a = 0; b = 1;

% Produce the Integration using Taylor series

for ic=1:1:ice;
    n=cnt(ic);
    for j=1:1:n;
    I1(n)=I1(n)+1/factorial(j)/j;
    end;
    EI1(n)=exp(1)/factorial(n+1)/(n+1);
end;

% Produce the Integration using Trap Rule
for ic=1:1:ice;
    n=cnt(ic);
    h=(b-a)/n;
    for j=1:1:n-1;
        I2(n)=I2(n)+(exp(j*h)-1)/(j*h);
          end;
% f0 = 1 asymptoticly
        I2(n)=h*(I2(n)+.5*(1+(exp(n*h)-1)/(n*h)));
          EI2(n)=h^2*(b-a)/12*(exp(1)-2);
       end;
```

1.8.4


```
 % Produce the Integration using Simpson's Rule
if (Maxn > 3)
for ic=1:1:ice;
    n=cnt(ic);
    h=(b-a)/n;
    for j=1:2:n-1;
        I3(n)=I3(n)+(exp(j*h)-1)/(j*h);
          end;
    for j=2:2:n-2;
        I3(n-1)=I3(n-1)+(exp(j*h)-1)/(j*h);
          end;
% f0 = 1 asymptoticly
         I3(n)=h/3*(4*I3(n)+2*I3(n-1)+(1+(exp(n*h)-1)/(n*h)));
         EI3(n)=h^4*(b-a)/180*(9*exp(1)-24);
       end;
   end;
if (Maxn < 3)
 I3(n)=h/3*((1 + 4*(exp(h)-1)/(h))+ (exp(n*h)-1)/(n*h));
 EI3(n)=h^4*(b-a)/180*(9*exp(1)-24);
end;

% Print Integration Results and Errors
line=1;
for ic=1:1:ice;
n=cnt(ic);
Output(line,1)=n;
Output(line,2)=I1(n);
Output(line,3)=I2(n);
Output(line,4)=I3(n);
Output(line,5)=EI1(n);
Output(line,6)=EI2(n);
Output(line,7)=EI3(n);
line=line+1;
end;

xlswrite('1_8', Output);
```

1.8.5

	Taylor	Trap	Simp.	E_T	E_{Trap}	E_{simp}
n	I1	I2	I3	EI1	EI2	EI3
2	1.25	1.328292	1.318009	0.151016	0.014964	0.000161
4	1.315972	1.320505	1.317909	0.00453	0.003741	1.01E-05
8	1.317902	1.318553	1.317903	8.32E-07	0.000935	6.3E-07
16	1.317902	1.318065	1.317902	4.5E-16	0.000234	3.94E-08
32	1.317902	1.317943	1.317902	9.49E-39	5.85E-05	2.46E-09
64	1.317902	1.317912	1.317902	5.07E-93	1.46E-05	1.54E-10
128	1.317902	1.317905	1.317902	4.2E-220	3.65E-06	9.61E-12

both Taylor series & Simpson method
can get Accuracy of 1E-6 for this function
starting @ n=8
Trap. Method needs 128 sections (bins)
to get Accuracy of 1E-6

note it could be n=100 but we were
asked to look at n=2, 4, 8, 16 etc
in multiples of 2.