Introduction to Systems of Linear Equations

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Linear Equations

2-dim ➡ Line equation

$$a\,x \; + \; b\,y \; = \; c$$ (a, b not both 0)

3-dim ➡ Plane equation

$$a\,x \; + \; b\,y \; + \; c\,z \; = \; d$$ (a, b, c not both 0)

n-dim ➡ Hyper-Plane equation

$$a_1\,x_1 \; + \; a_2\,x_2 \; + \; \cdots \; + \; a_n\,x_n \; = \; b$$ (a_i not both 0)

(n=2) $a_1\,x_1 + a_2\,x_2 = b$ ➡ $a\,x + b\,y = c$

(n=3) $a_1\,x_1 + a_2\,x_2 + a_3\,x_3 = b$ ➡ $a\,x + b\,y + c\,z = d$

Homogeneous Linear Equations

2-dim ➡ Line equation

$$a\,x + b\,y = 0$$ (a, b not both 0)

3-dim ➡ Plane equation

$$a\,x + b\,y + c\,z = 0$$ (a, b, c not both 0)

n-dim ➡ Hyper-Plane equation

$$a_1\,x_1 + a_2\,x_2 + \cdots + a_n\,x_n = 0$$ (a_i not both 0)

(n=2) $a_1\,x_1 + a_2\,x_2 = 0$ ➡ $a\,x + b\,y = 0$

(n=3) $a_1\,x_1 + a_2\,x_2 + a_3\,x_3 = 0$ ➡ $a\,x + b\,y + c\,z = 0$

System of Linear Equation

$$
\begin{aligned}
\text{(Eq 1)} \Rightarrow \quad & a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n = b_1 \\
\text{(Eq 2)} \Rightarrow \quad & a_{21} x_1 + a_{22} x_2 + \cdots + a_{2n} x_n = b_2 \\
& \vdots \\
\text{(Eq m)} \Rightarrow \quad & a_{m1} x_1 + a_{m2} x_2 + \cdots + a_{mn} x_n = b_m
\end{aligned}
$$

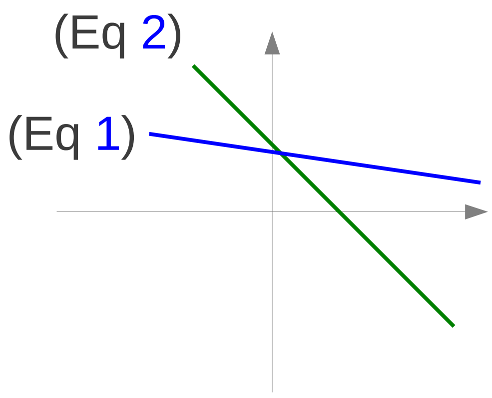
m equations

n unknowns

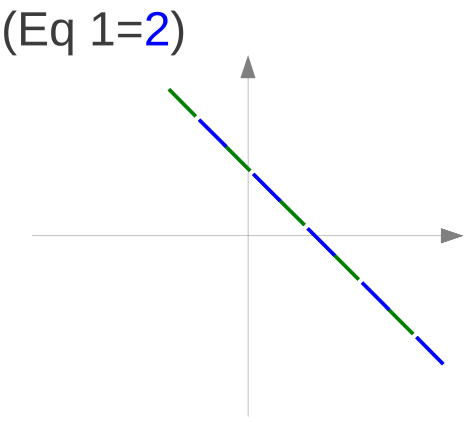
Linear Systems of 2 Unknowns

(Eq 1) $a_{11} x_1 + a_{12} x_2 = b_1$

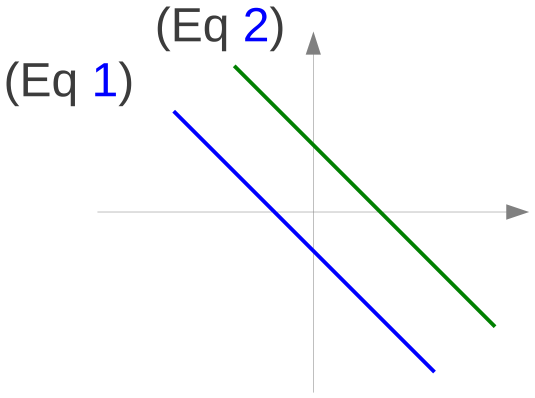
(Eq 2) $a_{21} x_1 + a_{22} x_2 = b_2$



One solution



Too many solutions



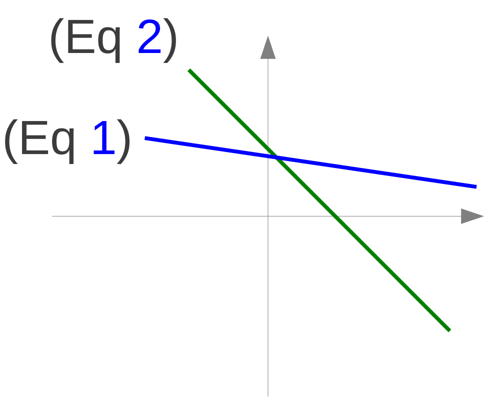
No solution

Linear Systems of 2 Unknowns

$$\begin{cases} x - y = 1 \\ 2x + y = 6 \end{cases}$$

$$2x - 2y = 2$$
$$2x + y = 6$$
$$-3y = -4$$
$$\left(\frac{7}{3}, \frac{4}{3}\right)$$

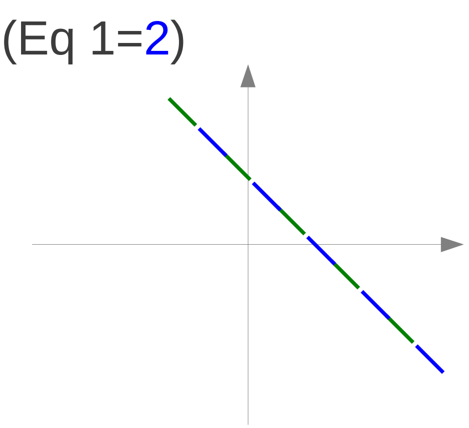
One solution



$$\begin{cases} 4x - 2y = 1 \\ 16x - 8y = 4 \end{cases}$$

$$4x - 2y = 1$$
$$4x - 2y = 1$$
$$0 = 0$$

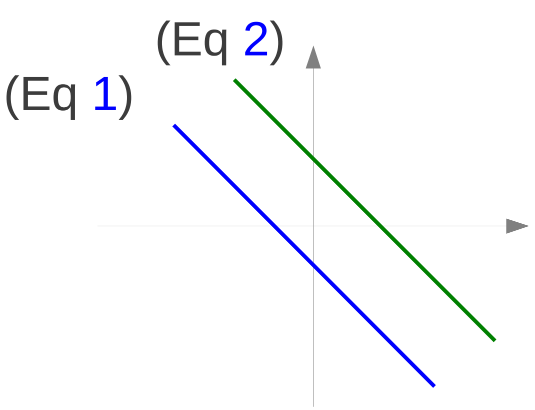
Too many solutions



$$\begin{cases} x + y = 4 \\ 3x + 3y = 6 \end{cases}$$

$$x + y = 4$$
$$x + y = 2$$
$$0 = 2$$

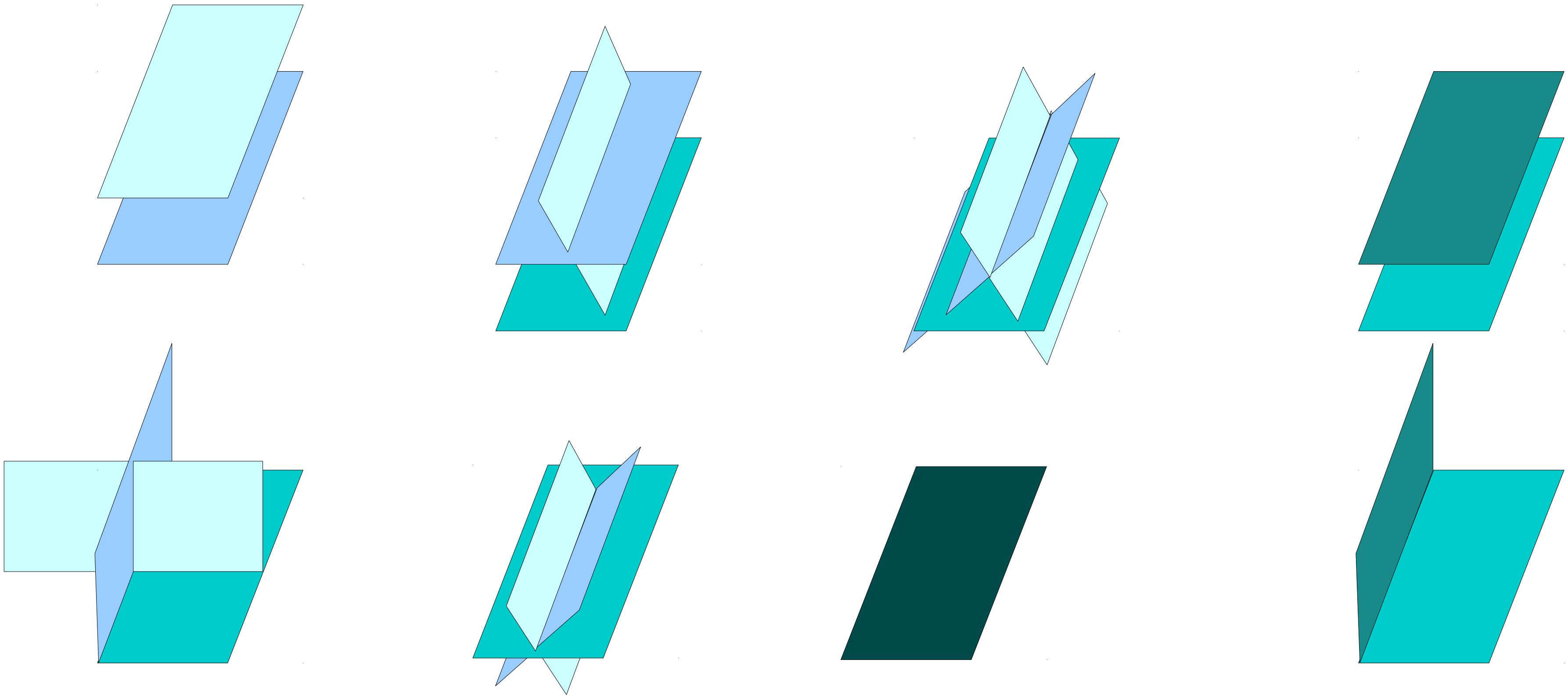
No solution



Linear Systems of 3 Unknowns

(Eq 1) $a_{11} x_1 + a_{12} x_2 + a_{13} x_3 = b_1$

(Eq 2) $a_{21} x_1 + a_{22} x_2 + a_{23} x_3 = b_2$



Linear Equations

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\
\vdots \qquad \vdots \qquad\qquad \vdots \quad &\quad \vdots \\
a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m
\end{aligned}$$

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \begin{matrix} = \\ = \\ \\ = \end{matrix} \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

Linear Equations

$$a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n = b_1$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{bmatrix} b_1 \end{bmatrix}$$

$$\sum_{j=1}^{n} a_{1j} \cdot x_j = b_1$$

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mxn Matrix

row index

nx1 Vector

Linear Equations

$$a_{21}\,x_1 + a_{22}\,x_2 + \cdots + a_{2n}\,x_n = b_2$$

$$\begin{bmatrix} a_{21} & a_{22} & \cdots & a_{2n} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{bmatrix} b_2 \end{bmatrix}$$

$$\sum_{j=1}^{n} a_{2j} \cdot x_j = b_2$$

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Linear Equations

$$a_{m1} x_1 + a_{m2} x_2 + \cdots + a_{mn} x_n = b_m$$

$$\begin{bmatrix} a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = b_m$$

$$\sum_{j=1}^{n} a_{mj} \cdot x_j = b_m$$

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nx1 Vector

Storing Magnetic Energy

References

[1] http://en.wikipedia.org/
[2] Anton, et al., Elementary Linear Algebra, 10th ed, Wiley, 2011
[3] Anton, et al., Contemporary Linear Algebra,