Background (1A)

-
-

Copyright (c) 2010, 2011 Young W. Lim.

Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice and Octave.

Complex Multiplication

Given Complex Value $C = I_c + j\,Q_c$

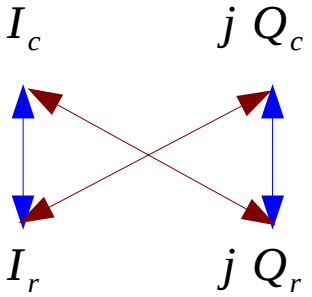
Rotated Complex Value $C' = I_c' + j\,Q_c'$

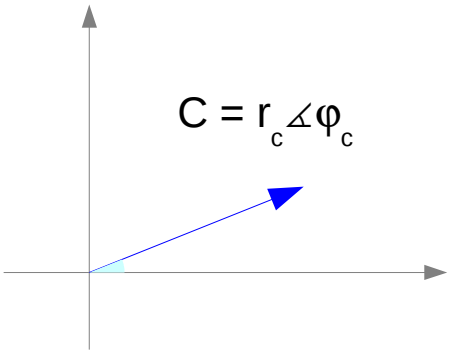
Rotation Value $R = I_r + j\,Q_r$

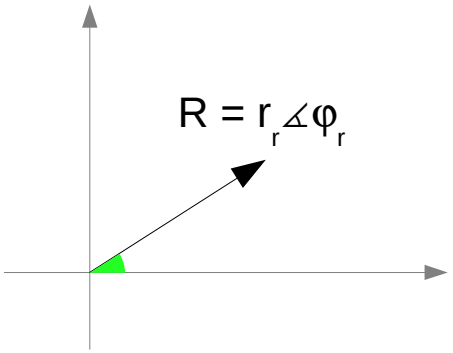
$$C' = C \cdot R$$

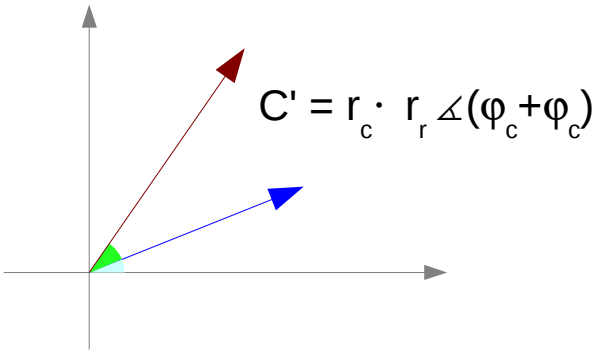
$$I_c' + j\,Q_c' = (I_c + j\,Q_c) \cdot (I_r + j\,Q_r)$$

$$= (I_c I_r - Q_c Q_r) + j\,(Q_c I_r + I_c Q_r)$$





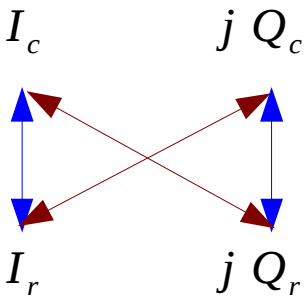




Adding / Subtracting Phase

To add R' phase to C

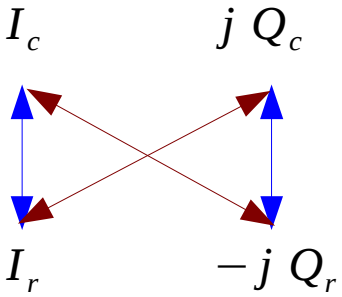
$$C' = C \cdot R$$



$$I_c' + j\,Q_c' = (I_c + j\,Q_c) \cdot (I_r + j\,Q_r)$$
$$= (I_c I_r - Q_c Q_r) + j\,(Q_c I_r + I_c Q_r)$$

To sub R' phase to C

$$C' = C \cdot R^*$$

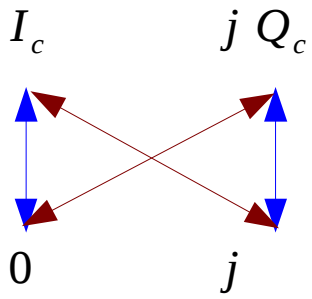


$$I_c' + j\,Q_c' = (I_c + j\,Q_c) \cdot (I_r - j\,Q_r)$$
$$= (I_c I_r + Q_c Q_r) + j\,(Q_c I_r - I_c Q_r)$$

Adding / Subtracting 90 Degrees

To add R' phase to C

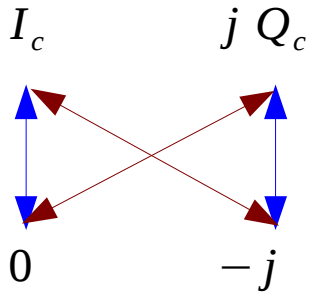
$$C' = C \cdot R$$



$$I_c' + j\,Q_c' = (I_c + j\,Q_c) \cdot (0 + j)$$
$$= (-Q_c) + j\,(I_c)$$

To sub R' phase to C

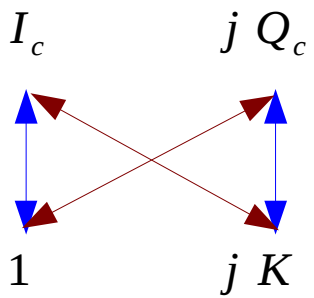
$$C' = C \cdot R^*$$



$$I_c' + j\,Q_c' = (I_c + j\,Q_c) \cdot (0 - j)$$
$$= (Q_c) + j\,(-I_c)$$

Adding / Subtracting atan(K)

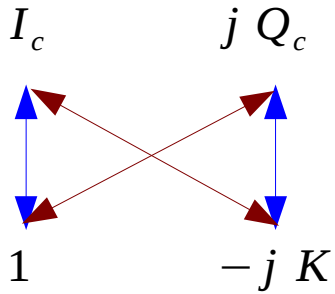
To add R' phase to C



$$C' = C \cdot R$$

$$\begin{aligned} I_c' + j\,Q_c' &= (I_c + j\,Q_c) \cdot (1 + j\,K) \\ &= (I_c - K\,Q_c) + j\,(Q_c + K\,I_c) \\ &= (I_c - 2^{-L} Q_c) + j\,(Q_c + 2^{-L} I_c) \end{aligned}$$

To sub R' phase to C

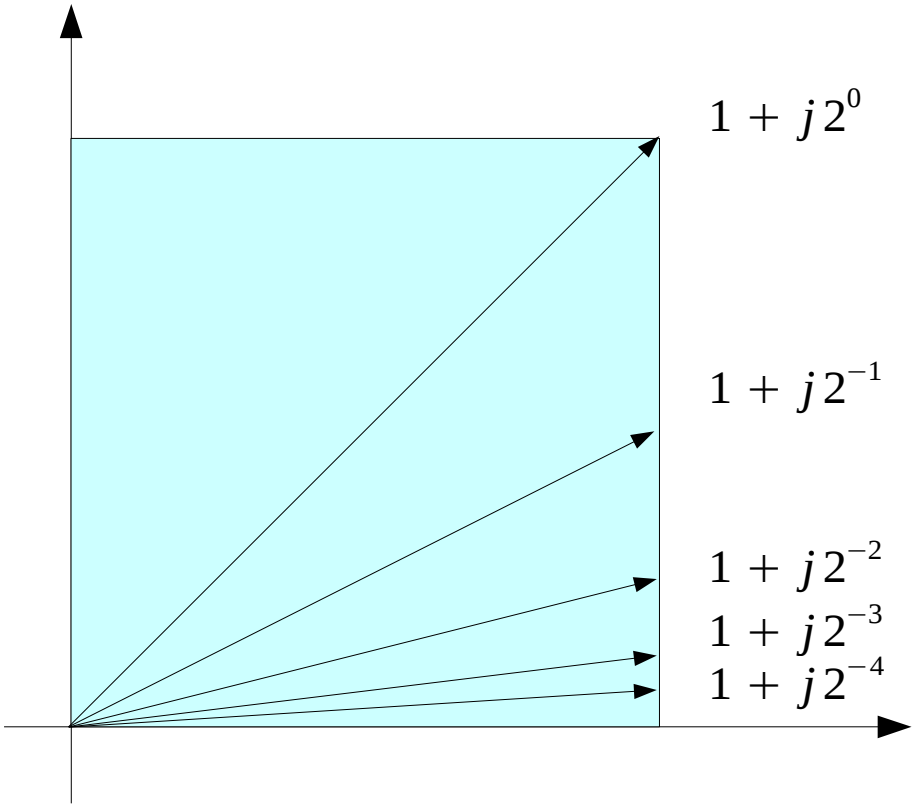


$$C' = C \cdot R^*$$

$$\begin{aligned} I_c' + j\,Q_c' &= (I_c + j\,Q_c) \cdot (1 - j\,K) \\ &= (I_c + K\,Q_c) + j\,(Q_c - K\,I_c) \\ &= (I_c + 2^{-L} Q_c) + j\,(Q_c - 2^{-L} I_c) \end{aligned}$$

$$K = \frac{1}{2^L}, \quad L = 0, 1, 2, \cdots$$

atan(K)



$1 + j2^{0}$

$1 + j2^{-1}$

$1 + j2^{-2}$

$1 + j2^{-3}$

$1 + j2^{-4}$

$$\theta_0 = \tan^{-1}(2^{0}) = 45.00000$$

$$\theta_1 = \tan^{-1}(2^{-1}) = 26.56505$$

$$\theta_2 = \tan^{-1}(2^{-2}) = 14.03624$$

$$\theta_3 = \tan^{-1}(2^{-3}) = 7.12502$$

$$\theta_4 = \tan^{-1}(2^{-4}) = 3.57633$$

$$K = \frac{1}{2^{L}}, \quad L = 0, 1, 2, \cdots$$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A “graphical interpretation” of the DFT and FFT, by Steve Mann
[4] CORDIC FAQ, www.dspguru.com