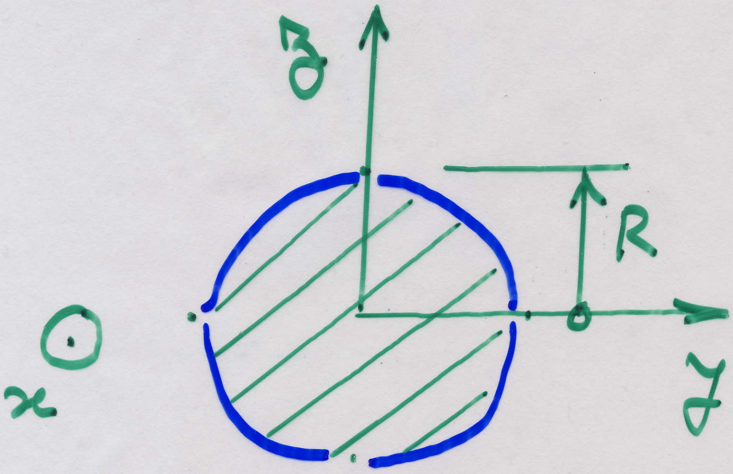
Mtg 10: Wed, 17 Sep 08. EAS 4200c (10-1

Axial members: Cont'd

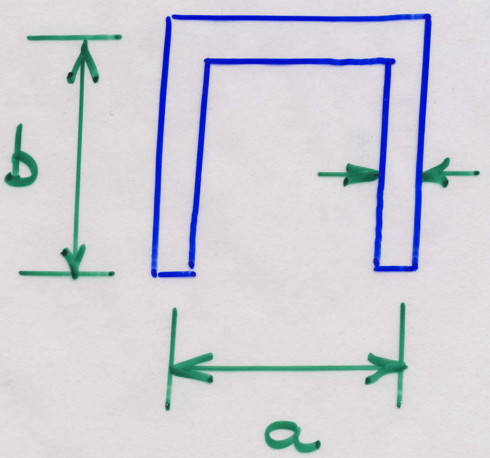
Stringers

Fig. 1.2 in book: Solid circular cross section.



Case 1

Case 2



HW: 1) Derive $I_y^{(1)}$ (mom. of inertia for case 1 wrt y axis) by integration using polar coord.

$$I_y = \iint_A z^2 \underbrace{dy\,dz}_{dA}$$

Also derive I_y expression for rectangular cross section.

2) Distribute material in case 1 into case 2, such that $a = b$

$$A^{(2)} = (3a)\, t = A^{(1)} = \pi R^2$$

Also assume $t = \frac{a}{10}$

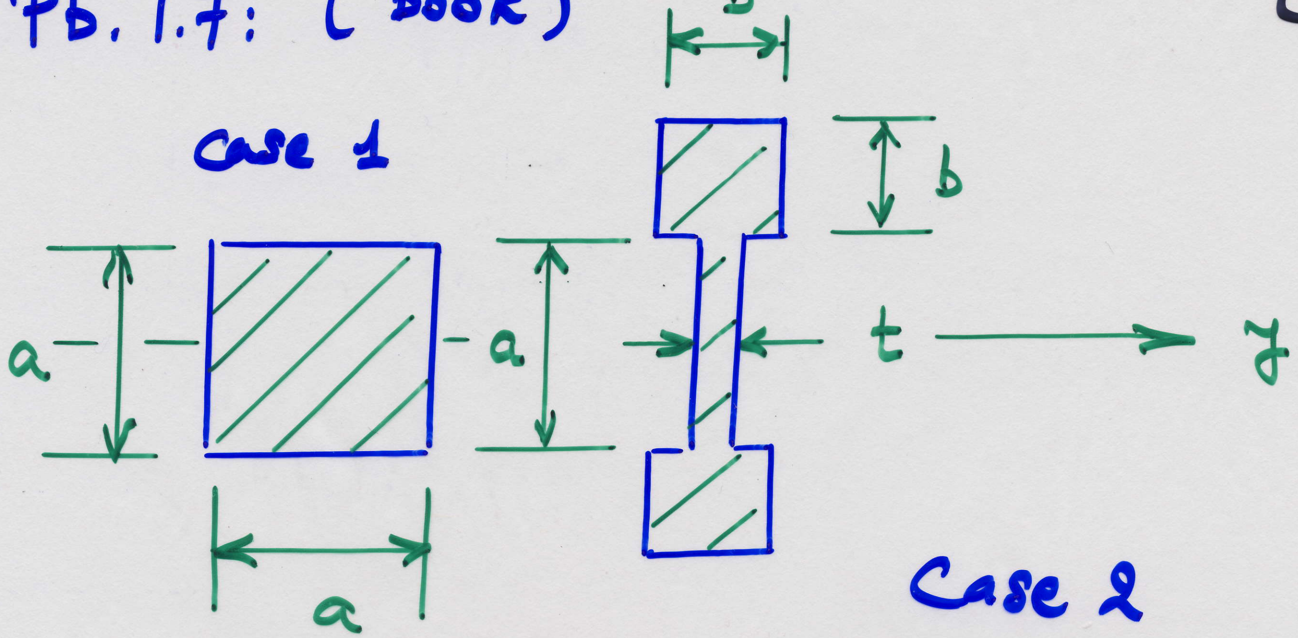
Find $I_y^{(2)}$ and compare to $I_y^{(1)}$.

For case 2, the y axis passes thru centroid of channel cross section (and justify – fair comparison)

(Jeff).

HW: Pb. 1.7, p. 18 w/ modification
use circles instead of $\underbrace{\text{rectangles}}_{\text{squares}}$

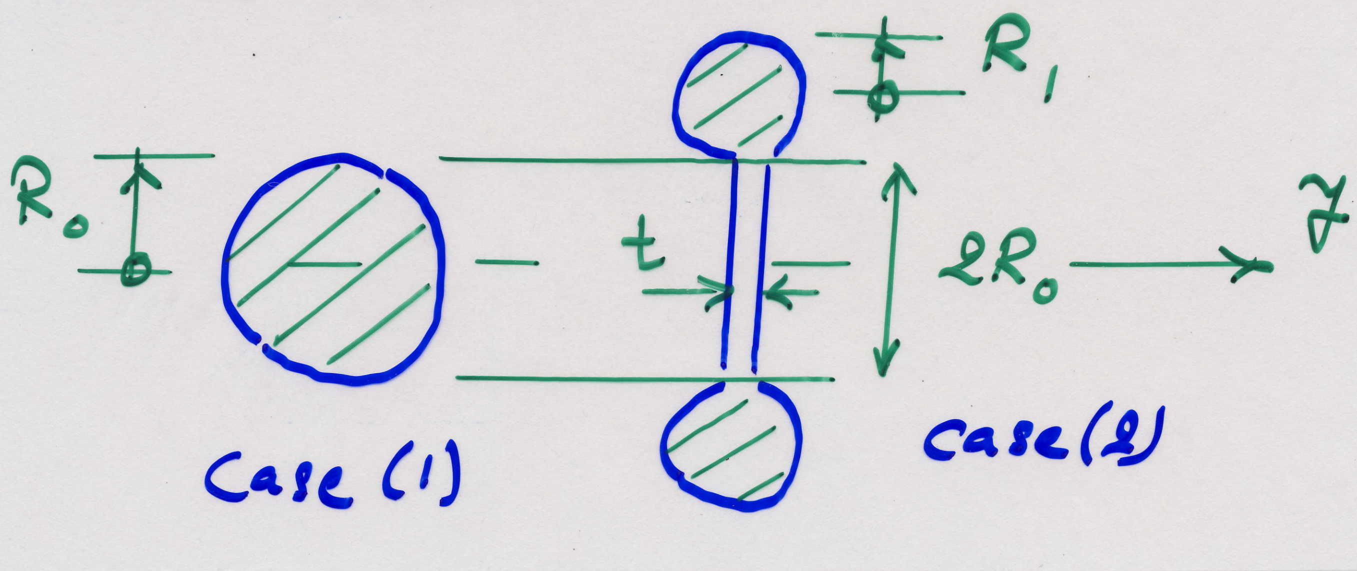
Pb. 1.7: (book)



Data: $a = 4$ cm, $t = 0.2$ cm $\Big\}$ b is such that $A^{(2)} = A^{(1)}$

Q: Find $I_y^{(1)}$ and $I_y^{(2)}$ and Compare

Modifications:



<u>Data:</u>

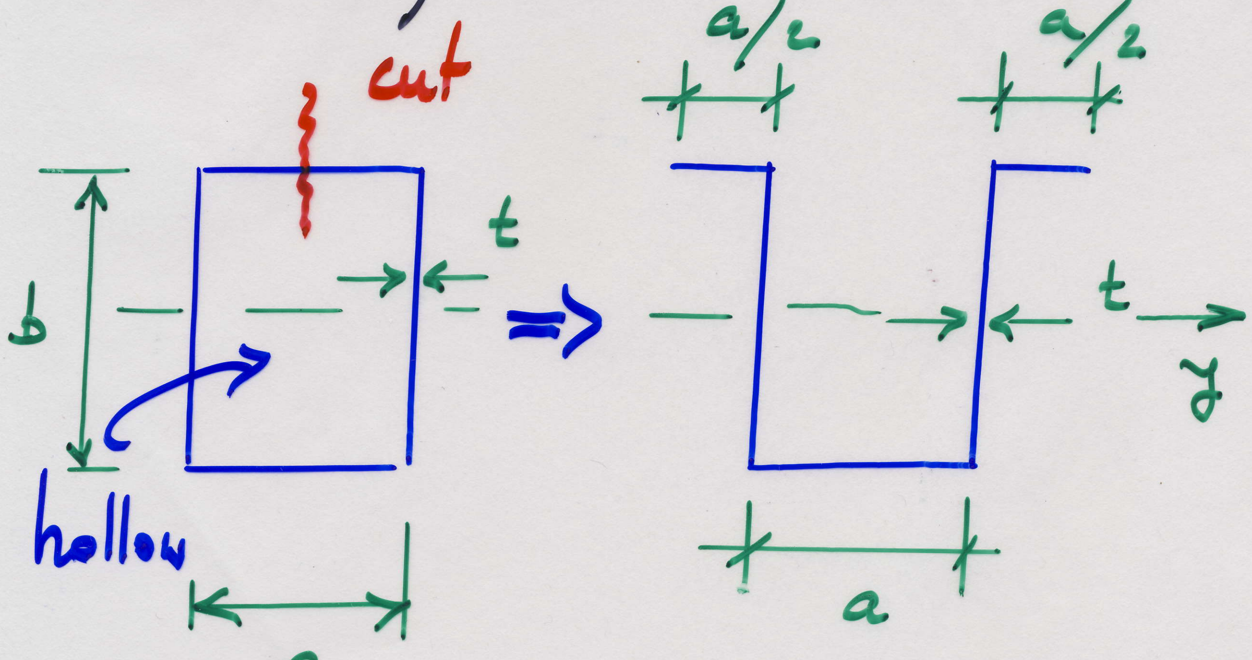
$$R_0 = 10 \text{ cm}$$

$$t = \frac{1}{10} R_0$$

$$R_1 \text{ st } A^{(1)} = A^{(2)}$$

Q: Find $I_y^{(1)}$ and $I_y^{(2)}$ and compare.

Q: why not hollow (closed, thin-walled) cross sections? (Pb. 1.1)



case 1

$I_y^{(1)}$

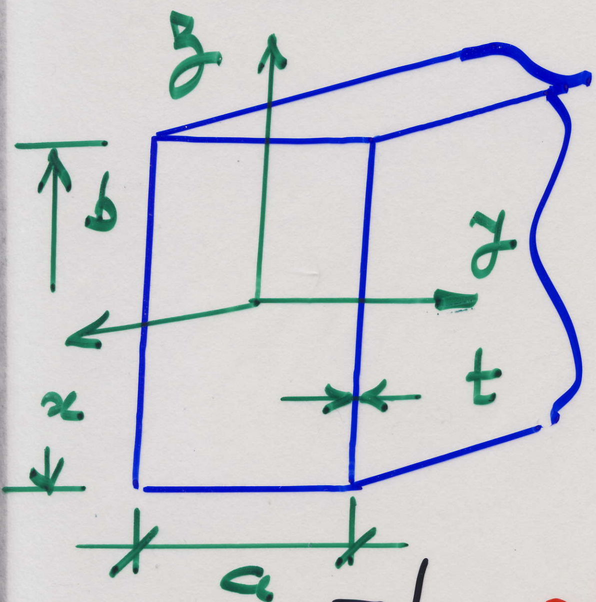
case 2

$I_y^{(2)} = I_y^{(1)}$

Flow "ahead" we have done:

Pb. 1.1:

$$\tau = \frac{T}{2abt}$$



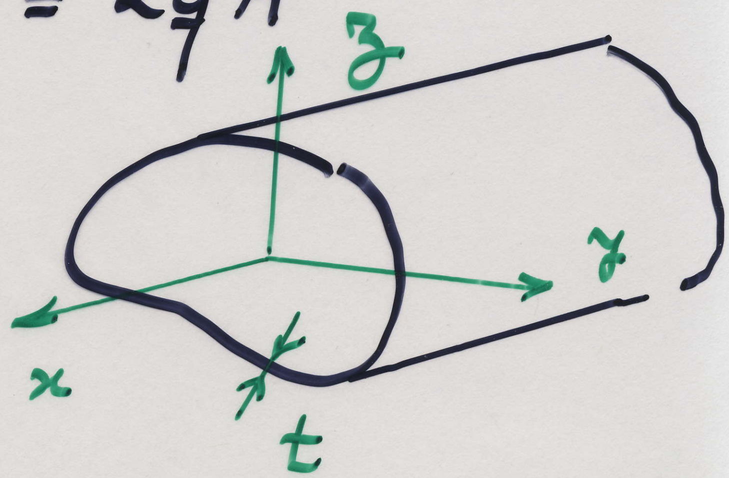
$\underline{q} = \tau t$ shear flow

$\bar{A} = ab$ "average" area of cross section

Sec 3.5, p. 85, (3.49)

$$T = \oint \rho \underline{q}\, ds = \iint_{\bar{A}} 2\underline{q}\, dA$$

$$= 2\underline{q}\bar{A}$$



Ad hoc method | Method based on elasticity theory

And there are more ...

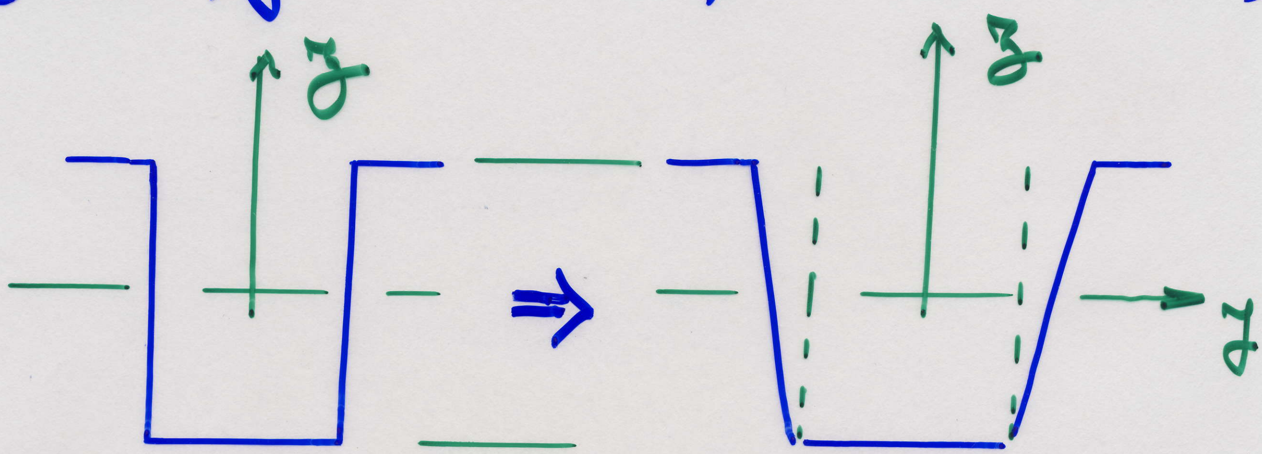
Stringers: cont'd from p. 10-4

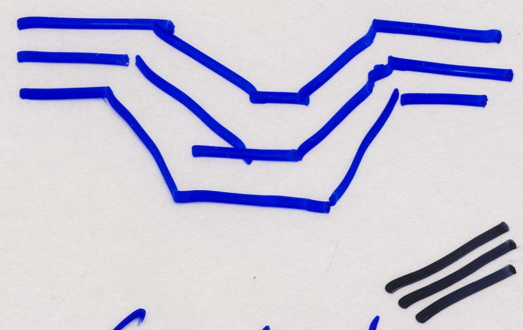
(Jared) Reasons to use open, thin walled cross section:

- manufacturing of stringers: stamping of thin flat sheets of metal
- construction of aircraft: rivetting

Q: why "vertical" walls of a stringer

(see fig. in HW1 of Team vadsam)





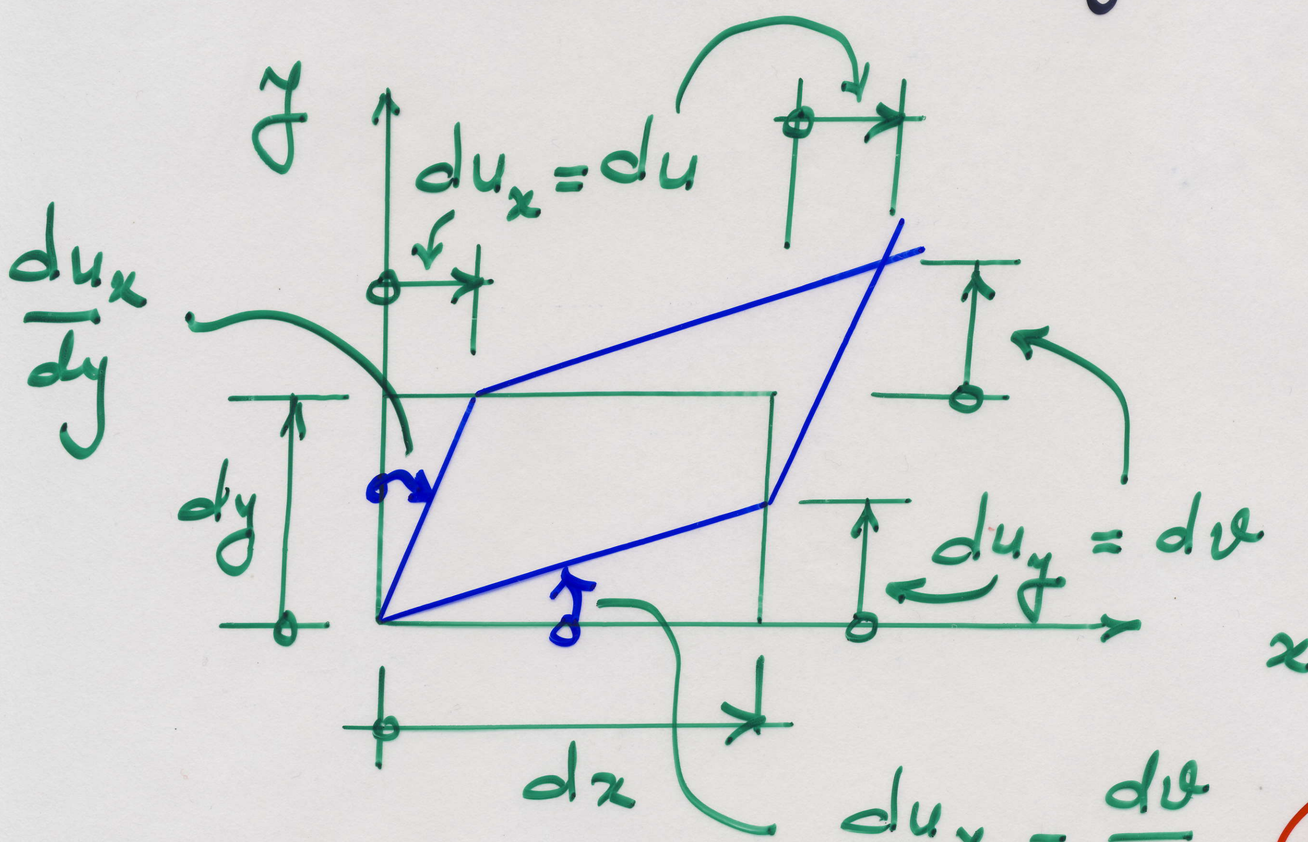
(and stringers)

Shear panels, shear stress/strain L11-3
also chap 2 (elasticity)

Engineering shear strain
$\underbrace{\text{Engineering}}$ (not tensorial)

$$\gamma = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}$$

$$u \equiv u_x \quad , \quad v \equiv u_y \qquad \text{(disp.)}$$



$$\frac{du_y}{dx} = \frac{dv}{dx}$$ (small angle)

$\alpha \cong \tan \alpha$

γ = change in the right angle ($90°$) due to shear deform.

ε_{xy} = tensorial shear strain

$$= \frac{1}{2}\gamma_{xy}$$

Shear panel: cont'd, pp. 4-5

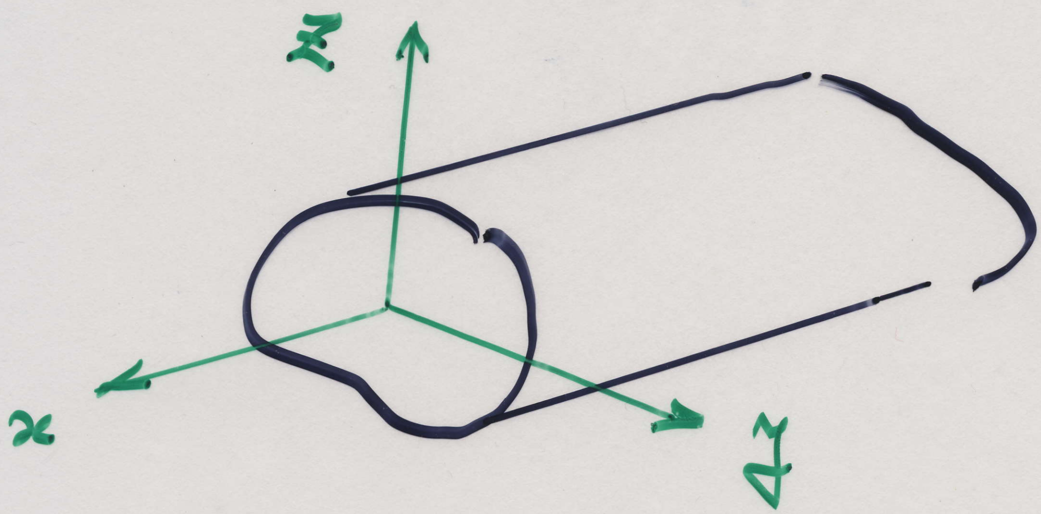
Shear flow: pp. 85-86, (3.49a-c) (3.50)

curved panels:

Notations: lowercase $z = \mathfrak{z}$

uppercase $\not{Z}$

Unified notation for EAS 4200C
EML 4500

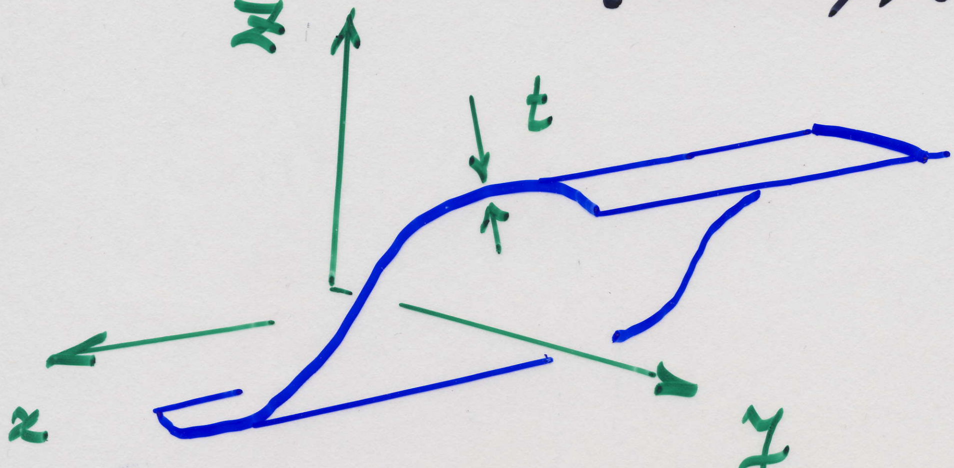


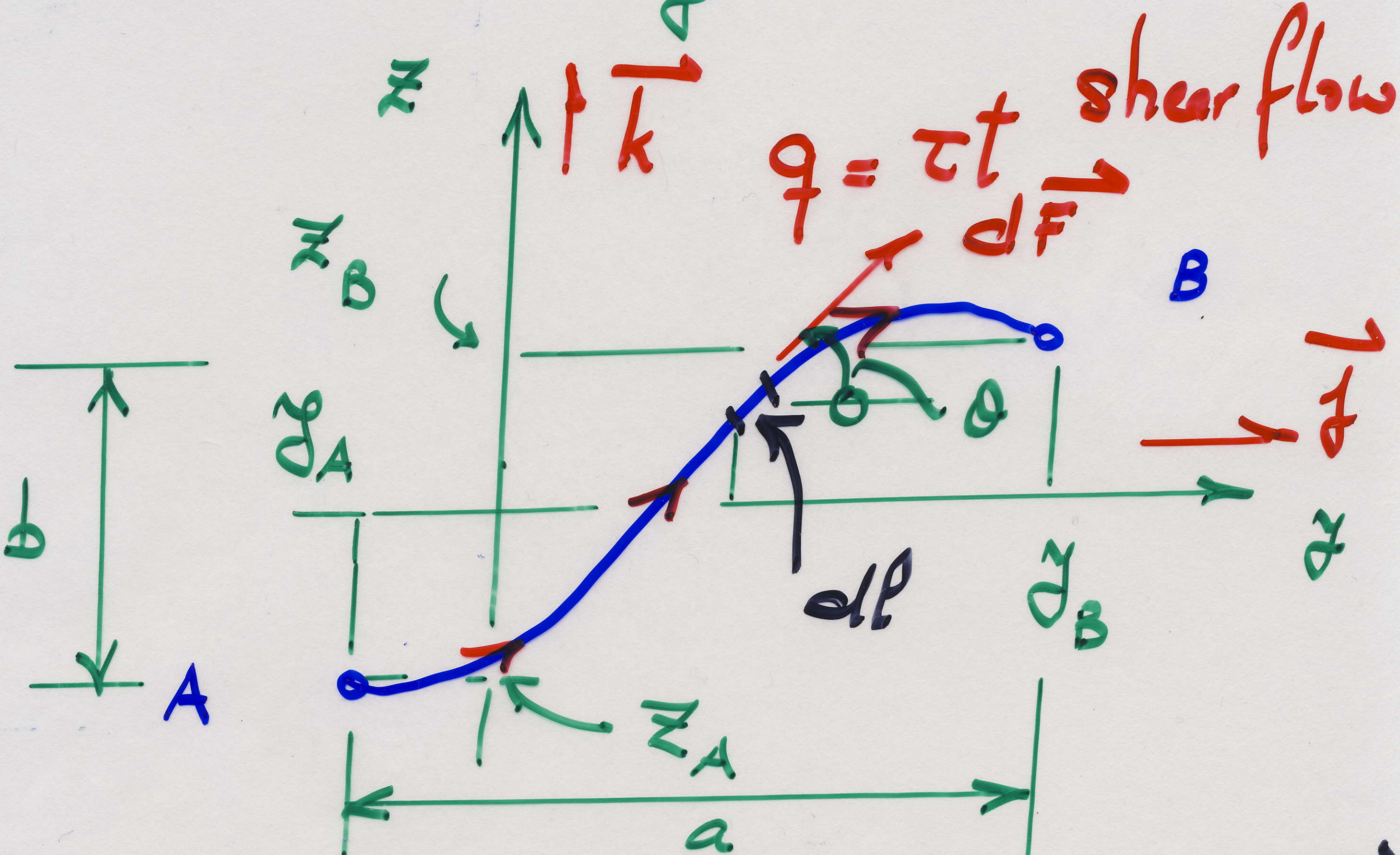
Book: sometimes (x, y) used for axes in plane of cross section

p. 20, Fig. 2.2 } bar elem
p. 64, Fig. 3.2 }
p. 115, Fig. 4.1 ← beam elem

Curved panels: $\underbrace{(1.4),\ (1.5)}_{\text{pb. 1.5, p. 17}}$ p. 5

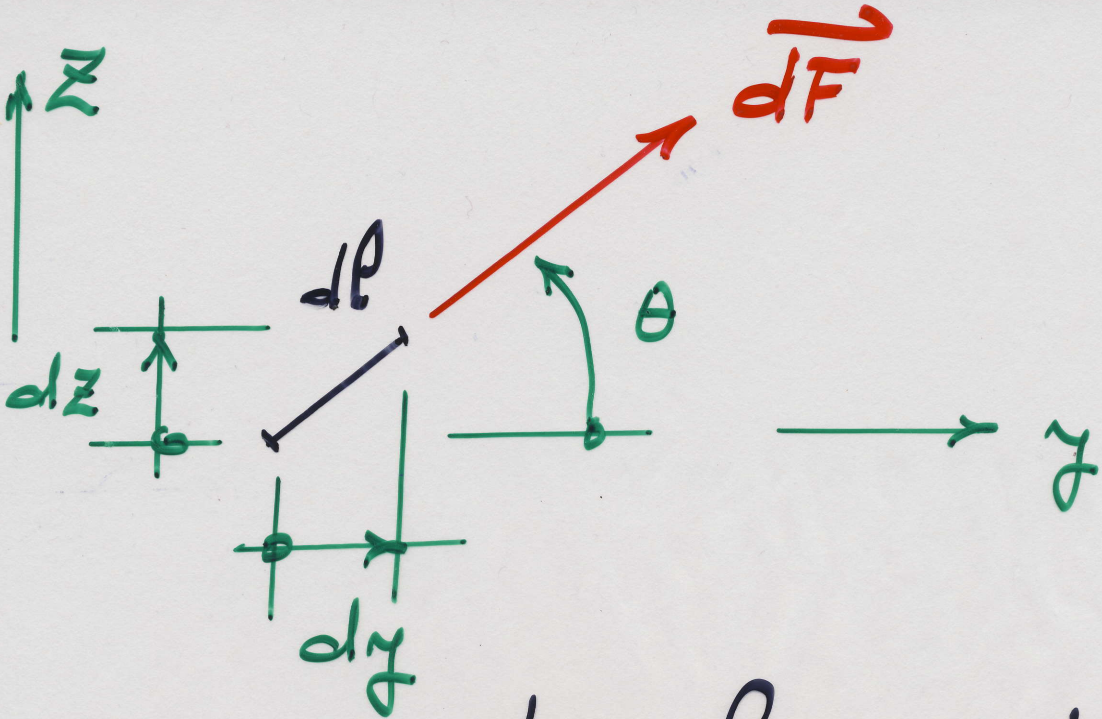
(3.49b), (3.49c), p. 86





$$\vec{dF} = q\,\vec{dl} = q\left(dl_{\cancel{x}\,y}\,\vec{j} + dl_z\,\vec{k}\right)$$

$$= q\left(\underbrace{dl\cos\theta}_{dy}\,\vec{j} + \underbrace{dl\sin\theta}_{dz}\,\vec{k}\right)$$



Resultant shear force vector

$$\vec{F} = \int_A^B \vec{dF} = \underbrace{q}_{\text{constant wrt }(y,z)}\left(\left(\int_A^B dy\right)\vec{j} + \left(\int_A^B dz\right)\vec{k}\right)$$

$$\boxed{\vec{F} = q\left(a\vec{j} + b\vec{k}\right)}$$

cf. (1.4) (1.5)

$$\vec{F} = F_y\vec{j} + F_z\vec{k} \Rightarrow \left.\begin{array}{l} F_y = qa \\ F_z = qb \end{array}\right\} \text{(eq 3.49)}$$

$$\Rightarrow \boxed{\frac{F_y}{F_z} = \frac{a}{b}}$$

Resultant mag. $\|\vec{F}\|$

$$\|\vec{F}\| = \left[(F_y)^2 + (F_z)^2 \right]^{1/2}$$

$$\boxed{\underbrace{\|\vec{F}\|}_{R} = q \underbrace{\left[a^2 + b^2 \right]^{1/2}}_{d}}$$

d = length of straight line AB.

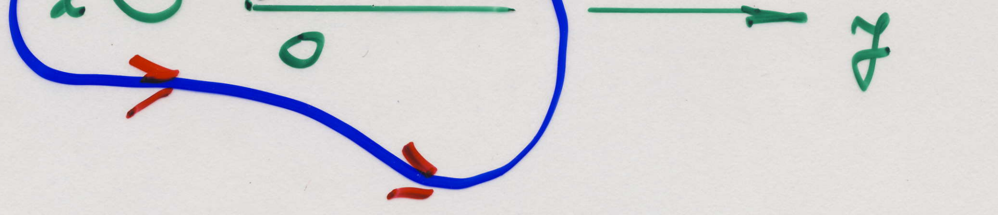
cf. (3.49a)

(Eddie)

Relate $R = \|\vec{F}\|$ to $\underbrace{T = 2q\bar{A}}_{(3.48)}$

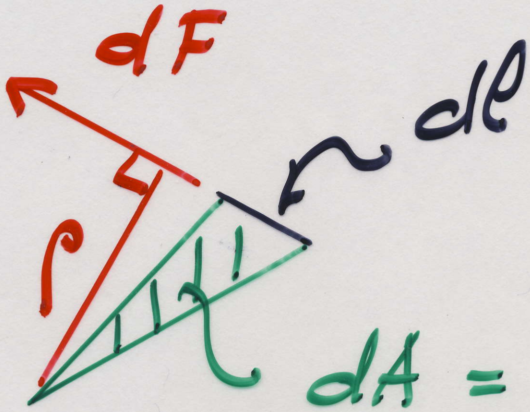
Closed thin-walled cross section:

$\vec{T} = T\vec{\imath}$



$$\vec{dT} = \vec{r} \times \vec{dF}$$

$$\Rightarrow dT = \rho \, dF = \rho \, (q \, dl)$$

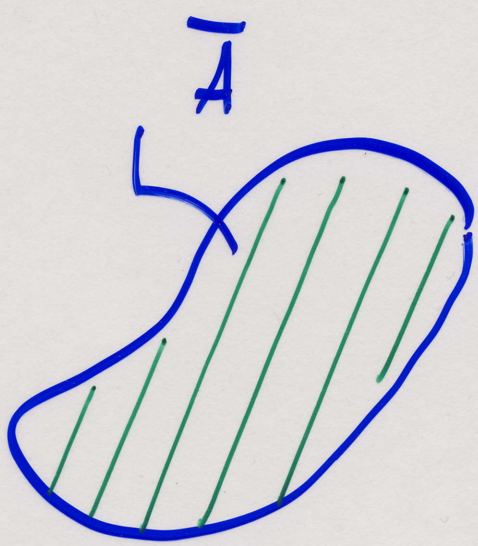


$$dA = \frac{1}{2} \rho \, dl$$

$$T = \oint dT = q \oint \underbrace{\rho \, dl}_{2dA}$$

$$= 2q \int_{\bar{A}} dA$$

$$T = 2q\bar{A}$$



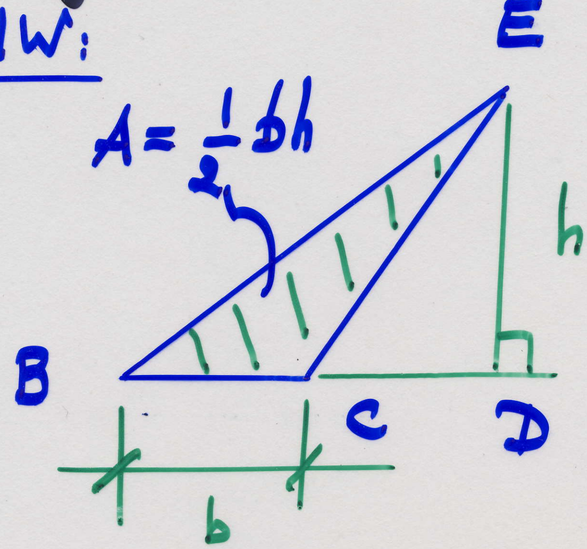
Mtg 13: Wed, 24 Sep08.

Exam 1

Mtg 14: Fri, 26 Sep 08. EAS4200C

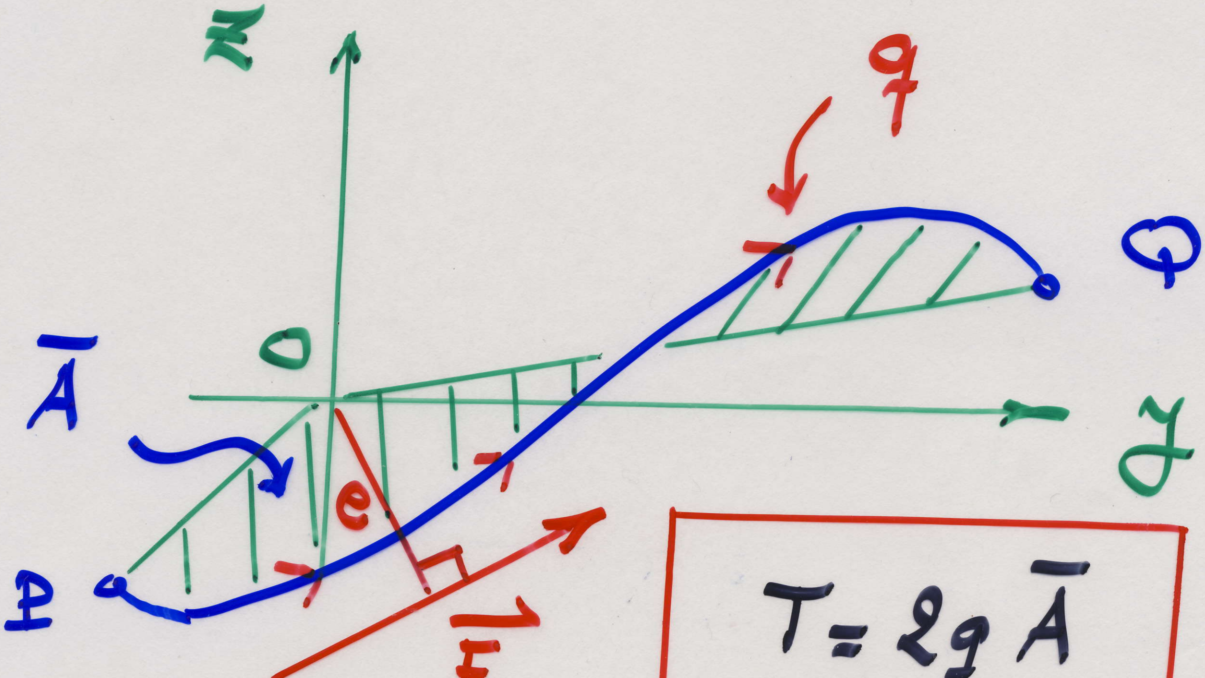
(Mtg 13 = Exam 1, Wed, 24 Sep)

HW:



Prove this eq.

Open thin-walled cross section:

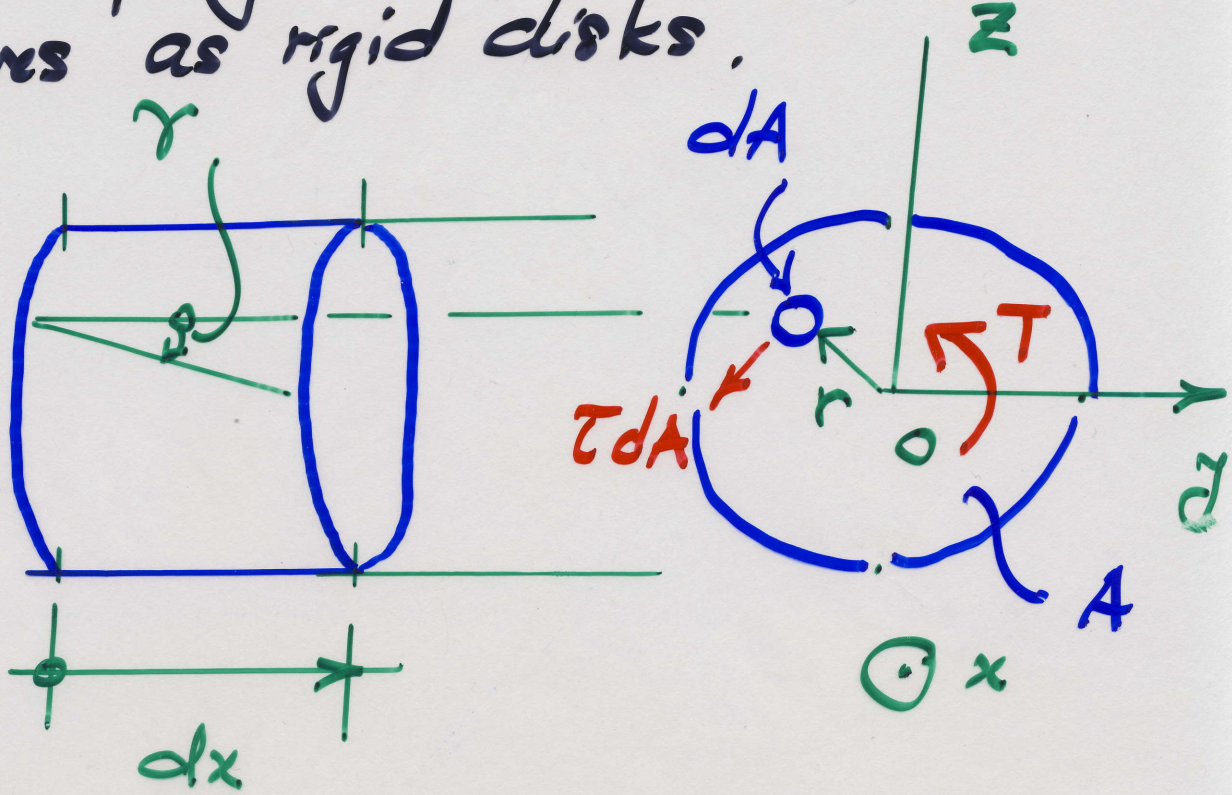


$$\boxed{T = 2q\bar{A}} \quad (3.49d)$$

$$\boxed{Re = T = 2q\bar{A}} \quad (3.50)$$

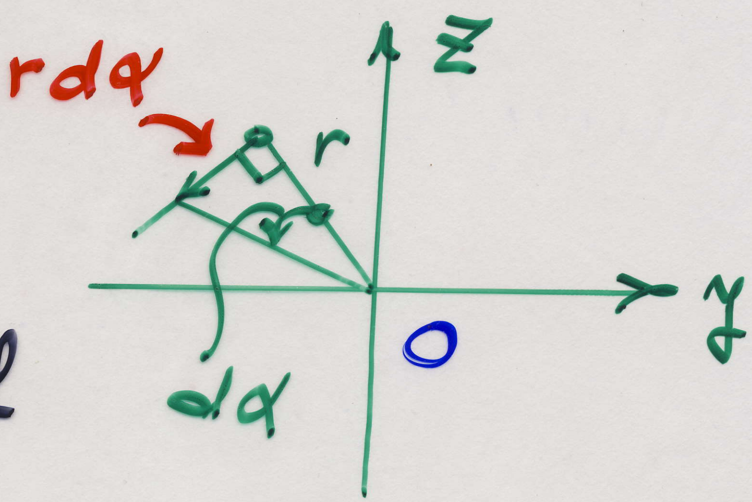
Uniform bar w/ circular cross section

non-warping case: cross section behaves as rigid disks.



$$T = \iint_A r \underbrace{\tau}_{= G\gamma} dA \quad \text{(Hooke's law)}$$

$$\gamma = \frac{r\,d\alpha}{dx}$$



$$\frac{d\alpha}{dx} =: \theta$$ rate of twist

$$T = \int_A r\, G\, (r\theta)\, \underbrace{dA}_{dy\,dz \,=\, r\,dr\,dw}$$

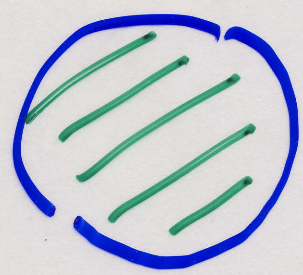
indep of (y, z)

$$= G\theta \left(\underbrace{\int_A r^2\, dA}_{J} \right)$$

J 2nd polar area moment of inertia

a = radius of circ. cross section

<u>HW</u>: $\boxed{J = \frac{1}{2}\pi a^4}$ solid circ. cross sect.



<u>Hollow</u> circ. cross sect.

<u>thin-walled</u> $(t \ll a)$

$r_i = a$ (inner radius)

$r_o = b$ (outer radius)

$$J = \frac{1}{2}\pi\left(\underbrace{b^4}_{(b^2)^2} - \underbrace{a^4}_{(a^2)^2}\right)$$

$$= \frac{1}{2}\pi \underbrace{(b-a)}_{t}\underbrace{(b+a)}_{\cong 2\bar{r}}\underbrace{(b^2+a^2)}_{\cong 2\bar{r}^2}$$

aver. radius

$$\bar{r} := \frac{a+b}{2}$$

$$b^2 \cong \bar{r}^2$$
$$a^2 \cong \bar{r}^2$$

<u>HW</u>: show this approx. rigorously

$$\Rightarrow \boxed{J = 2\pi t \bar{r}^3} \quad (1.12)$$

$$= \left(2\pi^{-1/2} t\right)\left(\underbrace{\pi \bar{r}^2}_{\bar{A}}\right)^{3/2}$$

J is prop. to $\bar{A}^{3/2}$ with $(2\pi^{-1/2}\, t)$ being prop. factor.

<u>HW</u>: Compare solid circ. cross sect. to hollow thin walled cross-sect

Fig. 1.8.: (a), (b)

(a) Solid circ. cross sect. $r_o^{(a)} = 1\,cm$

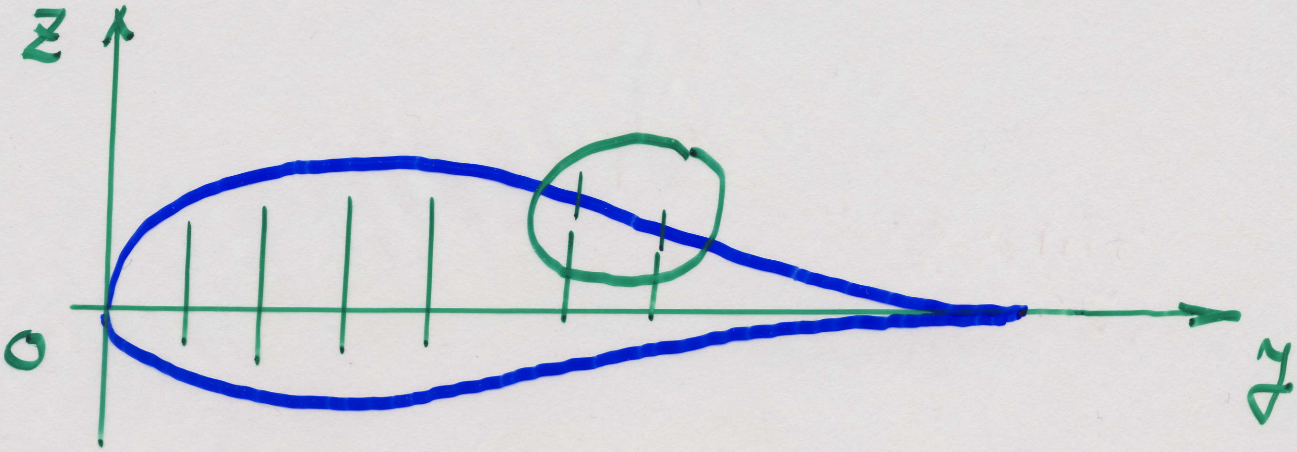
(b) Thin-walled (hollow) circ. cross sect.

$$r_i^{(b)} = 5\,cm, \quad t = 0.1\,cm.$$

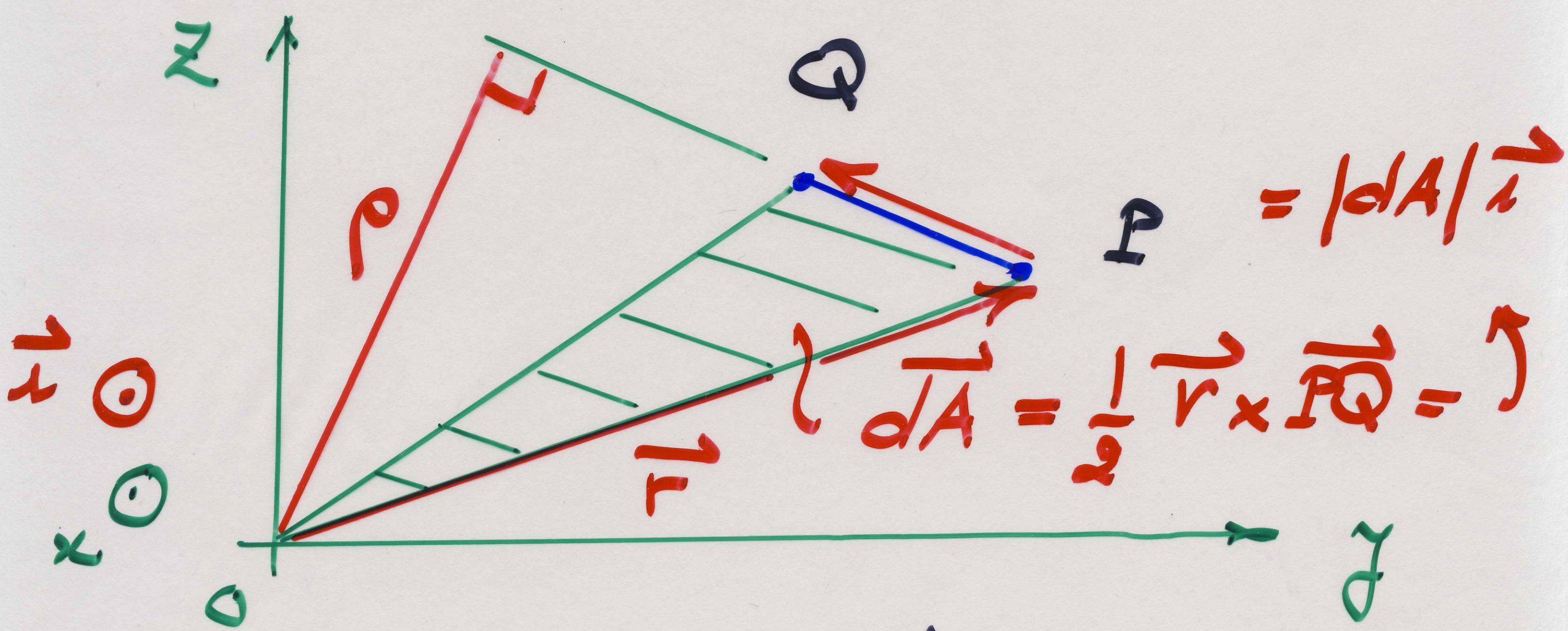
- Comp. areas $A_{(a)}$ and $A_{(b)}$
- Comp. $J_{(a)}$ and $J_{(b)}$
- Comp. $J_{(a)} / J_{(b)}$
- Find $r_i^{(c)}$ with $t = 0.02\, r_i^{(c)}$ st $J_{(c)} = J_{(a)}$. Comp. $A_{(a)}/A_{(c)}$

Mtg 15: Mon, 29 Sep 08. EAS 4200c 15-1

HW3: NACA 4-digit airfoil series.



ns = number of segments to discretize the y axis.



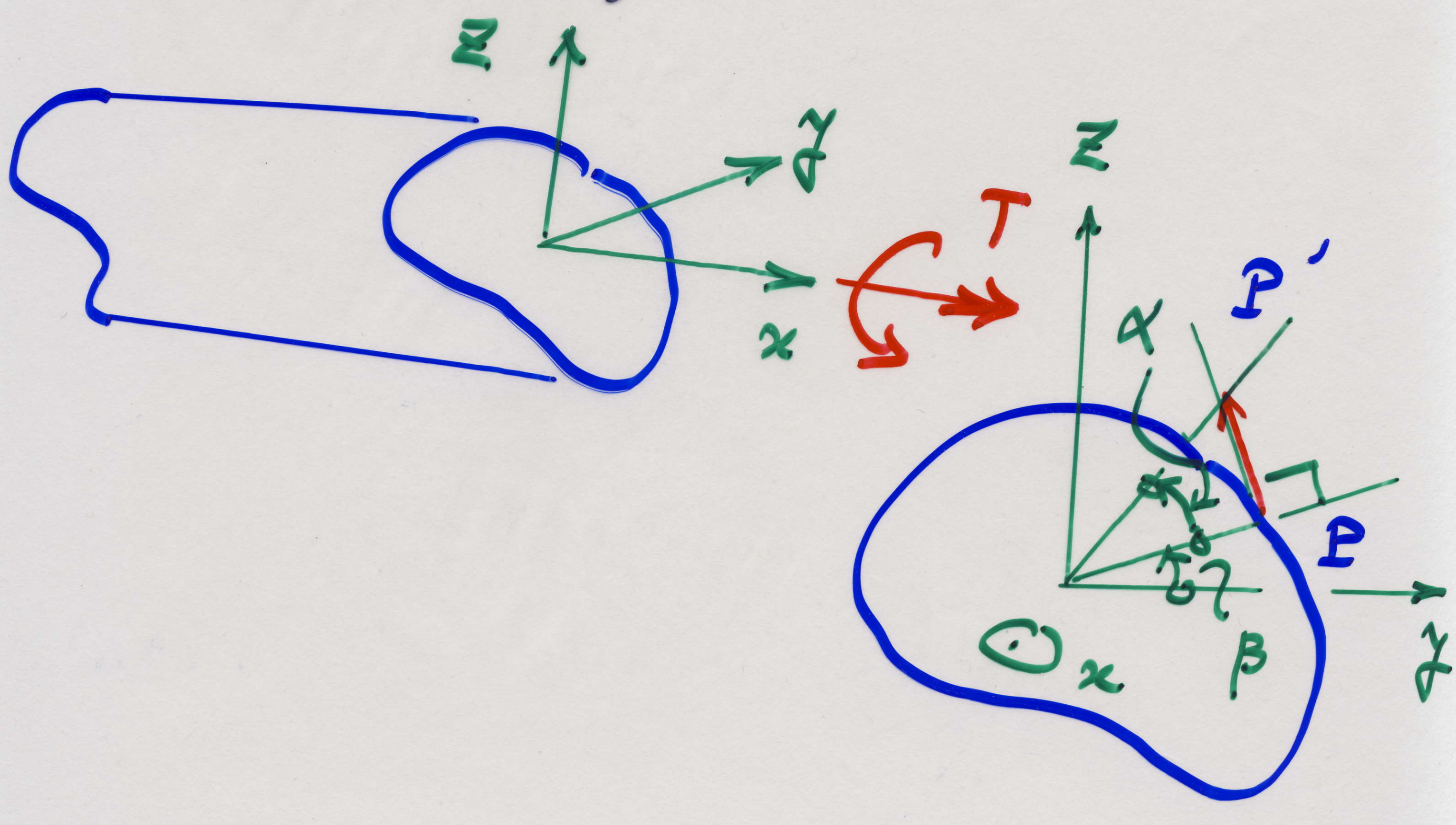
$$\vec{dT} = \vec{r} \times \underbrace{\vec{dF}}_{q\,\vec{PQ}} = q \underbrace{\vec{r} \times \vec{PQ}}_{(2\,dA)\,\vec{\imath}}$$

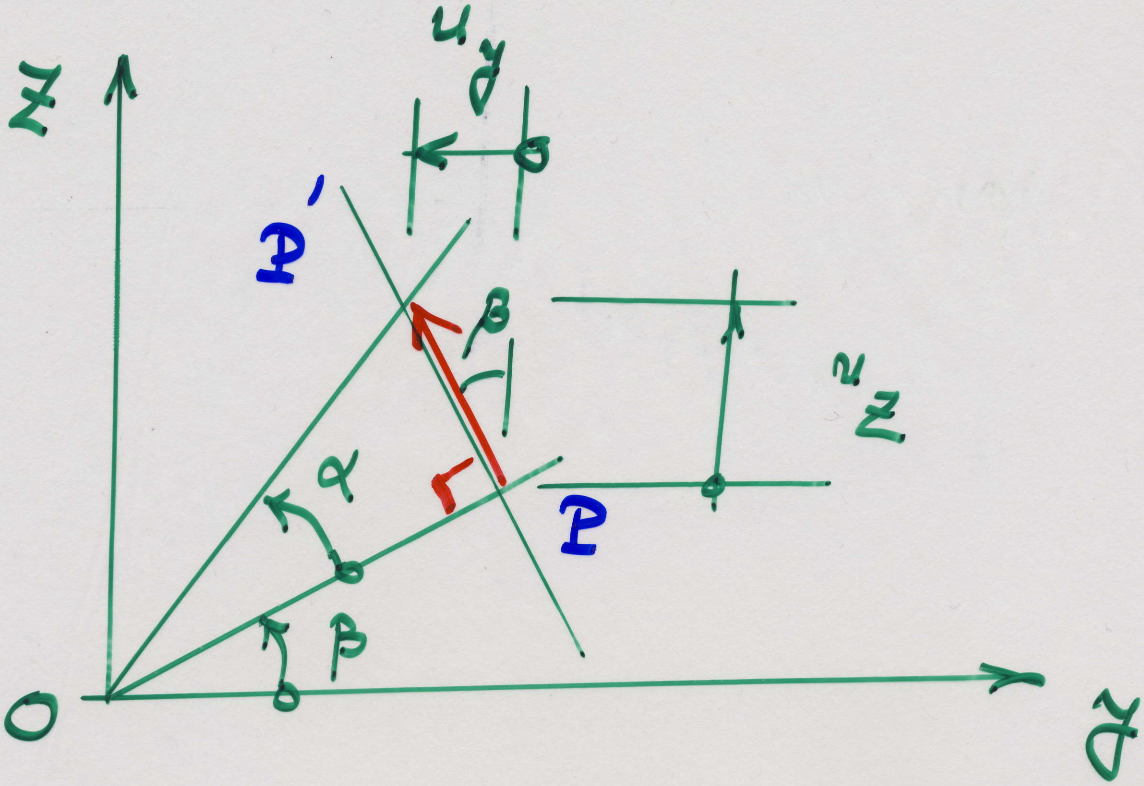
$$\|\overrightarrow{PQ}\| = d\ell$$

(end HW3 Pb)

Torsion of uniform, non circular bars (leads to warping of cross section)

warping = axial disp. along x axis (i.e., along bar length) of a pt on the deformed (rotated) cross section.





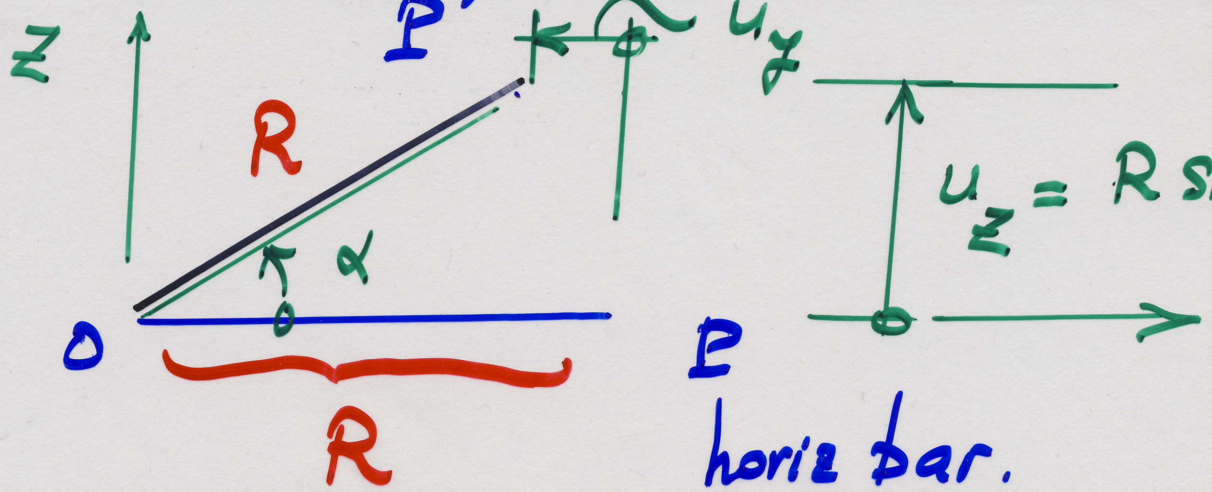
u_y = y-comp. of disp vector $\overrightarrow{PP'}$

u_z = z comp. of $\overrightarrow{PP'}$

Q: why $\overrightarrow{PP'} \perp OP$?
(why P' <u>not</u> on circle centered at O w/ radius OP ?)
Clearly $OP' > OP$.

Chris: Since α is very small so disp is $\perp$ to OP. Why?

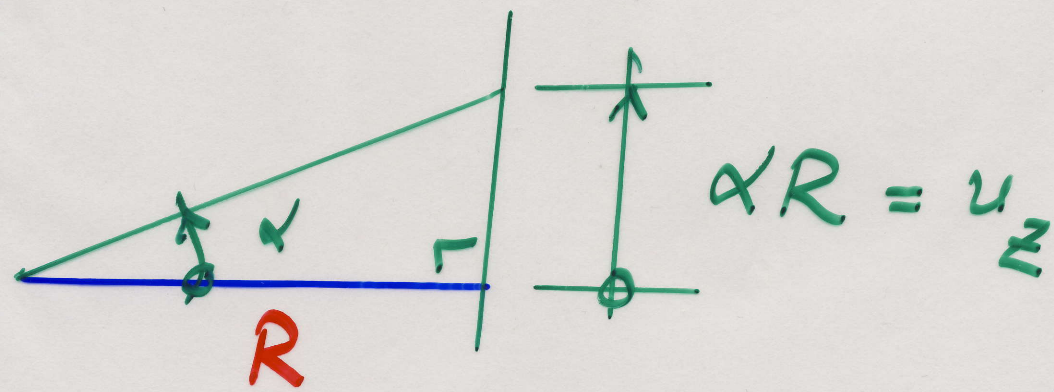
Note



horiz bar.

$$OP = OP' = R$$

$$\begin{cases} u_z = R\sin\alpha \cong R\alpha & (\alpha \text{ small}) \\ u_y = R(1-\underbrace{\cos\alpha}_{\cong 1}) \cong 0 & (\text{first order}) \end{cases}$$



$$u_y = ? \quad -\underbrace{(PP')}_{(OP)\,\alpha}\sin\beta = -\underbrace{\alpha}_{\theta_x}\underbrace{(OP)\sin\beta}_{z_P}$$

(end note) ///

$\theta = \dfrac{\alpha}{x}$ = rate of twist

$\underbrace{u_y}_{y\text{-disp (horiz)}}$

(1) $$\boxed{u_y = -\theta x z}$$ (3.11)

$$u_z = +\underbrace{(PP')}_{(OP)\,\alpha}\cos\beta = +\underbrace{\alpha}_{\theta x}\, y_P$$

(2) $$\boxed{u_z = +\theta x y}$$ (3.12)

u_z: $\underbrace{z\text{-disp}}_{\text{vertical}}$ of a pt on cross sect.

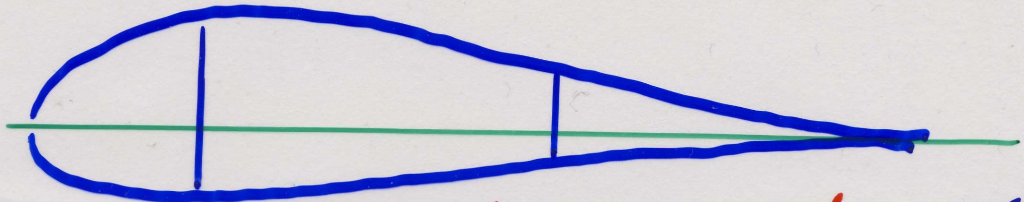
Warping disp. along x-axis

(3) $$\boxed{u_x = \theta\, \psi(y, z)}$$ (3.13)

Kinematic assump: Eqs. (1), (2), (3)

Deriv. cont'ed after road map.

Roadmap for torsional Analysis of aircraft wing:



multicell section (Sec. 3.6)

A. Kinematic assump. (Sec. 3.2)

B. Strain-disp rel. (")

C. Equil. eq (stresses) (Chap. 2, Sec. 3.2)

D. Prandtl stress func. ϕ (") (3.15)

E. Strain compatibility eq (3.17)

F. Eq. for ϕ (3.19)

G. Bound. cond. for ϕ (3.24)

H. $T = 2 \iint_A \phi \, dA$ (3.25)

$T = GJ\theta$, $J = \frac{-4}{\nabla^2 \phi} \iint_A \phi \, dA$

p. 74

I. Thin-walled cross section

p. 5-2 : Ad-hoc assump. on ~~shear fow.~~ flow

Formal deriv. Sec (3.5)

(we did the end of this sect., the beginning is more challenging.)

$$\boxed{T = 2q\bar{A}} \quad (3.48)$$

~~H.~~ J. Twist angle θ : Method 1

$$\boxed{\theta = \frac{1}{2G\bar{A}} \oint \frac{q}{t}\, ds} \quad (3.56)$$

Script "s" = curvilinear coord along thin wall.

K. Sec 3.6 on multicell thin walled cross section.

Roadmap: Cont'd

K. Multicell section:

cell $i = 1, \ldots, \underbrace{n_{cell}}_{n_c}$

p. 16-2: $n_c = 3$

K1. $$\boxed{T = 2 \sum_{i=1}^{n_c} q_i \bar{A}_i}$$ (3.62) and p. 93

q_i = shear flow in cell i

$\bar{A}_i$ = "average" area in cell i.

Define: $T_i = 2 q_i \bar{A}_i$ torque generated by one cell.

$$\Rightarrow T = \sum_{i=1}^{n_c} T_i$$

K2. Shape of airfoil is "rigid" in the plane (y, z) (but can warp out of plane)

$$\boxed{\theta = \theta_1 = \ldots = \theta_{n_c}}$$

$$\boxed{\theta_i = \frac{1}{2 G_i \bar{A}_i} \oint \frac{q_i}{t_i} \, ds}$$

p. 16-3 applied to cell i.

G_i = shear mod. of cell i

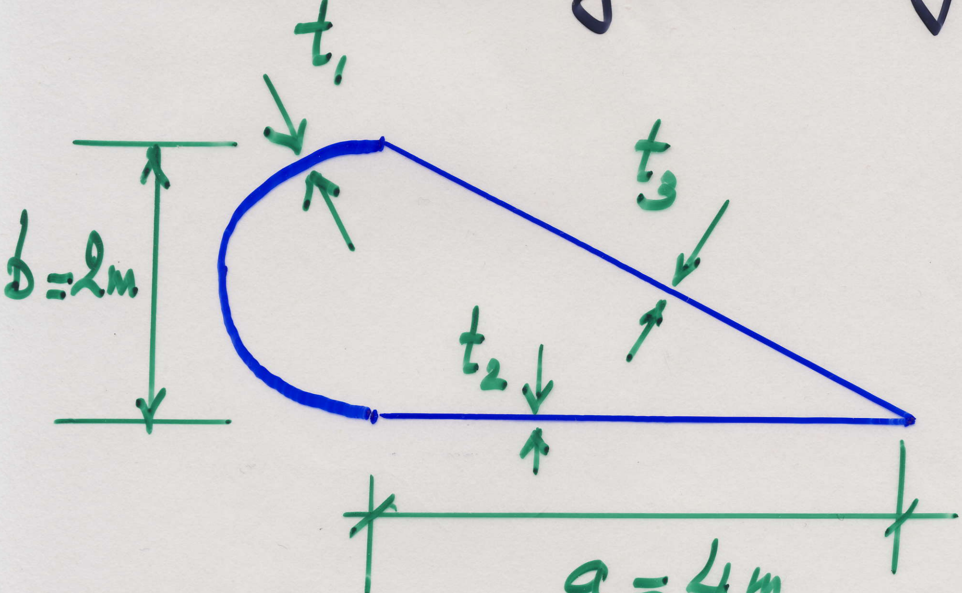
t_i = thickness in cell i

$\underbrace{t_i}_{t_i(s)}$

s curv. coord. along cell wall.

Pb. 1.1 : rect. single cell section.

Now a more general single cell section:



$t_1 = 0.008\, m$

$t_2 = t_3 = 0.01\, m$

(cf. p. 90)

$$\bar{A} = \frac{1}{2}\pi \left(\frac{b}{2}\right)^2 + \frac{1}{2}\, b\, a = HW$$

Shear flow: $T = 2q\bar{A}$

$\Rightarrow \quad q = \dfrac{T}{2\bar{A}}$ ← variable.

Twist angle:

Jared $\quad \theta = \dfrac{1}{2G\bar{A}} \displaystyle\sum_{j=1}^{3} \dfrac{q_j \, l_j}{t_j} = \dfrac{T}{GJ}$

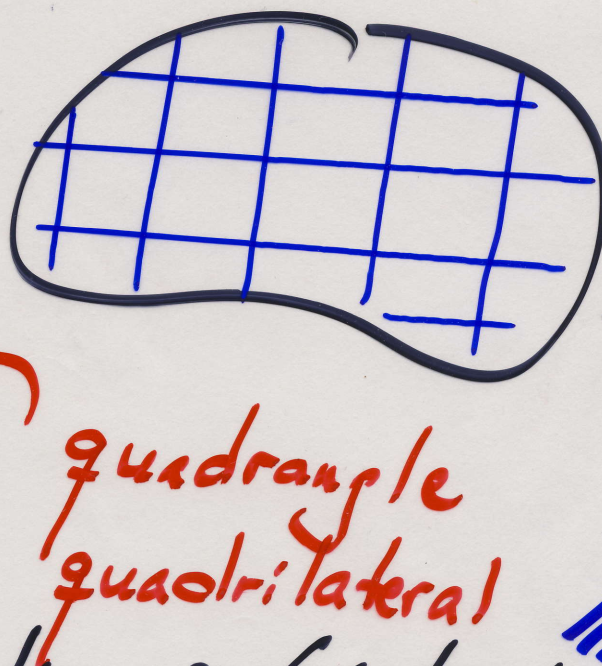
Deduce J.

$j = 1, 2, 3$ index for seg. number.

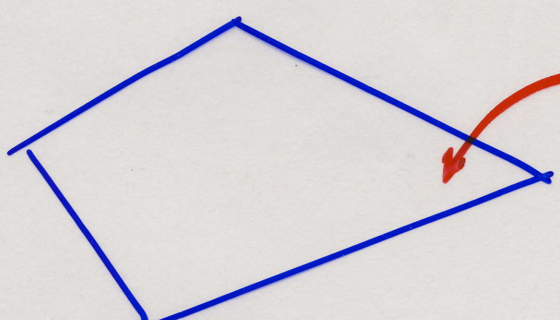
Note: $\int$: elongated S, standing for summation (continuous)

Σ : (discrete) sum

Kevin: Riemann sum
quadrature



Wikipedia

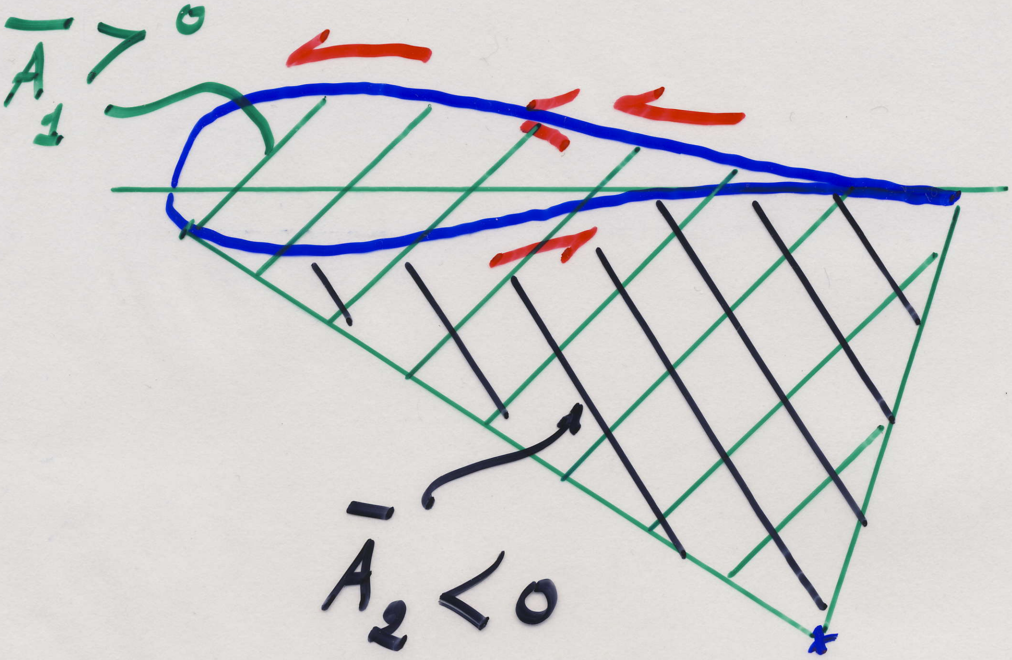


quadrangle
quadrilateral

cubature = int. volumes (cubes)
squaring (quadrature) of the circle

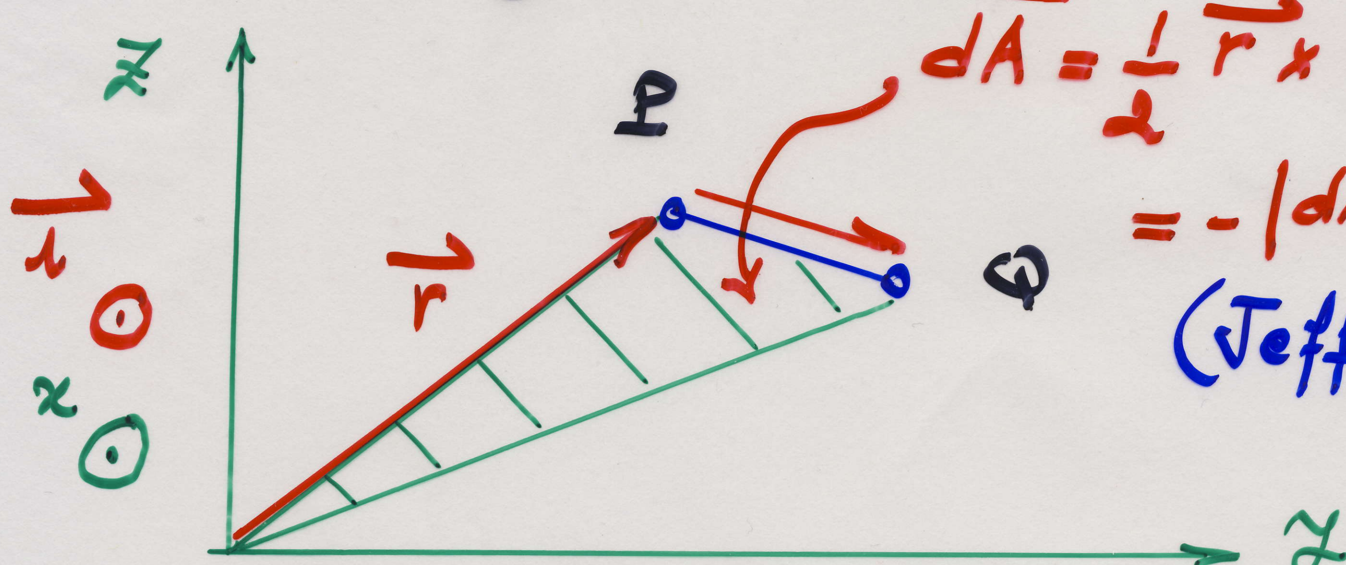
Mtg 18: Mon, 6 Oct 08. EAS 4200 C

Note: Quadrature of NACA airfoil.



$$\bar{A} = \underbrace{\bar{A}_1}_{>0} + \underbrace{\bar{A}_2}_{<0} = \bar{A}_1 - |\bar{A}_2|$$

Matt: Many if statements in code?

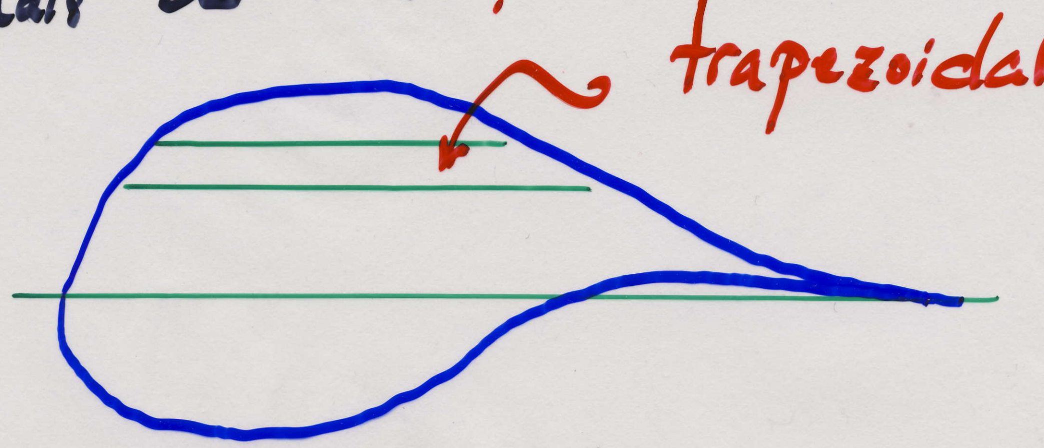


$$\overrightarrow{dA} = \frac{1}{2}\,\vec{r} \times \overrightarrow{PQ}$$

$$= -|dA|\,\vec{\imath}$$

(Jeff)

Matt, Ray: How about using trapezoidals to int. ?



Quad. by triangles follows deriv. of $T = 2q\bar{A}$.

Disadvantages: change in curvature

* not as elegant as the other quadrature (triangles)

Back to single-cell airfoil (p. 17-3)

Shear flow is constant:

$$q = q_1 = q_2 = q_3$$

$$\theta = \frac{1}{2G\bar{A}} q \sum_{j=1}^{3} \frac{l_j}{t_j}$$

rate of twist angle

$$= \frac{1}{2G\bar{A}} q \left[\frac{\cancel{2}\pi\left(\frac{b}{2}\right)}{t_1} + \frac{a}{t_2} + \frac{\sqrt{a^2+b^2}}{t_3} \right]$$

$$= (\text{HW})\, q$$

Max. shear stress τ_{max}

$$\tau_{max} = \frac{q}{\min\{t_1, t_2, t_3\}}$$

If $\tau_{max} = \tau_{allow}$ (given)

and since $q = \dfrac{T}{2\bar{A}}$, then

$$T_{allow} = 2\bar{A}\,\tau_{allow} \min\{t_1, t_2, t_3\}$$

Let $\tau_{allow} = 100$ GPa. Find T_{allow}

HW.